Set is Unorder item of data item Which have at least One Opration in Set

$A = \{a, b, c\}$. $B = \{c, d\}$

→ (1) Union - $A \cup B = \{a, b, c, d\}$

→ (2) Intersection . $A \cap B = \{c\}$

→ (3) Diffrence $A - B = \{a, b\}$ $B - A = \{d\}$

→ (4) Cartesion Product $A \times B = \{(a,c), (a,d), (b,c), (b,d), (c,c), (c,d)\}$
(ordered pair)

→ (5) Power :- ~~Set~~ of all Sub Set of these Set Called Power Set. Dinoted by P( )

Ex
$B = \{c, d\} = \{\{\}, \{c\}, \{d\}, \{c,d\}\} = P(B)$

Formula $P(B) = 2^{B}$ → (Number of element in Set Called Cardinally) (~~1B1~~) (|1|)

Ex:- the set
$A = \{a, b, c, d\}$

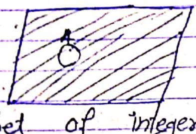
Cardinally of the Set is 4

$|P(A)| = 2^4 = 2 \times 2 \times 2 \times 2 \Rightarrow 16$

$P(A) = \{ \overset{1}{\{\}}, \overset{2}{\{a\}}, \overset{3}{\{b\}}, \overset{4}{\{c\}}, \overset{5}{\{d\}}, \overset{6}{\{a,b\}}, \overset{7}{\{a,c\}}, \underset{8}{\{a,d\}}, \underset{9}{\{b,c\}}, \underset{10}{\{b,d\}}, \underset{11}{\{c,d\}}, \underset{12}{\{d,a\}}, \underset{13}{\{d,b\}}, \underset{14}{\{d,c\}}, \underset{15}{\{a,b,c,d\}} \}$

$\subseteq$ - Subset — $\subset$: proper Set
$\in$ belong — $\Rightarrow$ Emplice that
$\exists$ there Exist — $\forall$: forall

Universal Set - U



$A = \{1, 5, 9, 10, 12, 15\}$

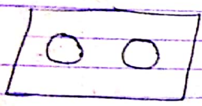
$\{1, 2, 3, 4, 6, 7, 8\}$ Universal Set of integer Value

Equality of Set

1 से 10,000 तक Set.

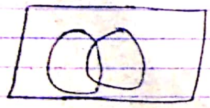
$\{x \in U \mid 1 < x < 10{,}000\}$

DisJoined Set :- Intersition is ϕ called Disjoined Set

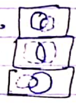


Compliment Set :- $\bar{A} = U - A$

for two set A & B



$A \cap \bar{B}$ =
$A \cap B$ =
$\bar{A} \cap B$ -



$|A \cup B|$
Resault : $|A| + |B| - |A \cap B|$

$|A \cup B| = |A \cap \bar{B}| + |A \cap B| + |\bar{A} \cap B|$ —— (1)

$|A| = |A \cap \bar{B}| + |A \cap B|$ —— (2)

$|B| = |A \cap B| + |\bar{A} \cap B|$ —— (3)

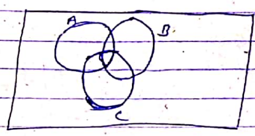
20/08/2019

from (1) (2) (3)

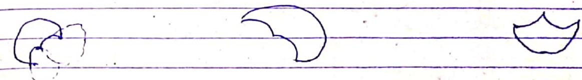
$|A \cup B| = |A| - \cancel{|A \cap B|} + \cancel{|A \cap B|} + |B| - |A \cap B|$

$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$ Prove

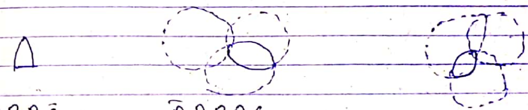
For three Set :-



$A \cup B \cup C$



$A \cap \bar{B} \cap \bar{C}$ — $\bar{A} \cap B \cap \bar{C}$ — $\bar{A} \cap \bar{B} \cap C$



$A \cap B \cap \bar{C}$ — $\bar{A} \cap B \cap C$ — $A \cap \bar{B} \cap C$

$A \cap B \cap C$

Application of principal of inclusion & Exclustion.

$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3|$
$- |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$

~~$|A_1 \cap$~~

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = |A_1| + |A_2| + |A_3| + |A_4| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_1 \cap A_4| - |A_2 \cap A_3| - |A_2 \cap A_4| - |A_3 \cap A_4| + |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + |A_1 \cap A_3 \cap A_4| + |A_2 \cap A_3 \cap A_4| - |A_1 \cap A_2 \cap A_3 \cap A_4|$$

Q. Count the number of integer 1 to 1000 Which are not divigible by {2 3 5 7},

Sol:- A_1 = is set whose value is divigble by 2.

A_2 = — — 3

A_3 = — — 5

A_4 = — — 7

$A_1 \cup A_2 \cup A_3 \cup A_4$

$\overline{A_1} \cup \overline{A_2} \cup \overline{A_3} \cup \overline{A_4}$ — by De Morgans law

$|A_1| = \frac{1000}{2} = 500$ $|A_2| = \left\lfloor \frac{1000}{3} \right\rfloor = 333$

$|A_3| = \frac{1000}{5} = 200$ $|A_4| = \left\lfloor \frac{1000}{7} \right\rfloor = 142$

$|A_1 \cap A_2| = \left\lfloor \frac{1000}{6} \right\rfloor = 166.$

$|A_1 \cap A_3| = \left\lfloor \frac{1000}{10} \right\rfloor = 100$

$|A_1 \cap A_4| = \left\lfloor \frac{1000}{14} \right\rfloor = 71$

$|A_2 \cap A_3| = \frac{1000}{\cdot} = 66$ $|A_1 \cap A_3 \cap A_4| = 14$

$|A_1 \cap A_2 \cap A_4| = 23$

$|A_2 \cap A_4| = 47$

$|A_2 \cap A_3 \cap A_4| = 9$

$|A_3 \cap A_4| = 28$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 4.$

$|A_1 \cap A_2 \cap A_3| = 33$

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = 500 + 333 + 200 + 142 - 166 - 100 - 71 - 66 - 47 - 28 + 33 + 14 + 23 + 9 - 4$$

$$= 1175 - 478 + 79 - 4$$

$$= 1254 - 482$$

772

$$|\overline{A_1} \cup \overline{A_2} \cup \overline{A_3} \cup \overline{A_4}| = 1000 - 772 = 228$$

Principal of Mathmatical Induction :-

$$\left[n\frac{(n+1)}{2}\right]^2 = 1^3 + 2^3 + 3^3 \ldots n^3.$$

$f(n) = \frac{n(n+1)}{2}$

n term Satiesfiy these formula.

* n Should be positive

Proof : useing principal of mathmatical Induction has two part

(1) The Statement Holdes for positive integer I

(2) If Statment Hold

1 to 3 unit [7 th. addtion → Kennth Rosen]
4-5 [Compition Tech. by Ullman Hopcroft.]

then it must be hold for next large integer.

in mathmatical form

① $P(1)$ ② $\forall K \, (P(K) \to P(K+1))$

Step :-

① basis of Induction :- It shows that $P(n_0)$ true.

② Induction Hypothesis :- Assme that $P(K)$ is true.

③ Indicative Step :- It Shows that $P(K+1)$ is true on the basis of Induction Hypothesis.

Basis Of Induction :-

① $n = 1$ $S(1) = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$S(1)$ = true.

② Induction hypothesis :- Suppose $k \geq 1$

$S(K) = \frac{K(K+1)}{2}$: it is true (Suppose)

③ ~~Induction~~ Inductive Step :-

$S(K+1) = \{1, 2, 3, \ldots K, K+1\}$

Sum of n numbers ⇒ $\frac{n(n+1)}{2}$

$S(K+1) = \frac{K(K+1)}{2} + (K+1)$

$= \frac{K(K+1) + 2(K+1)}{2}$

$= \frac{(K+1)(K+2)}{2}$

$= \frac{(K+1)[(K+1)+1]}{2}$

Identity law ⇒ null sets $\cup$

Dominess law - $A \cap \emptyset = \emptyset$

Q.① Let A B & C are Sets then show that $(A \cup B) \subseteq (A \cup B \cup C)$

Q.② $(A \cap B \cap C) \subseteq (A \cap B)$

Q.③ $(A - B) - C \subseteq (A - C)$

Q.4 $(A - C) \cap (C - B) = \emptyset$

Q. ① $(A \cup B) \subseteq (A \cup B \cup C)$

Sol:- $A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 5, 7, 8\}$

$C = \{1, 5, 7, 10\}$

$(A \cup B) = \{1, 2, 3, 4, 5, 7, 8\}$

$A \cup B \cup C = \{1, 2, 3, 4, 5, 7, 8, 10\}$

$(A \cup B) \subseteq (A \cup B \cup C)$

Q ② $(A \cap B \cap C) \subseteq (A \cap B)$

Sol: $A = \{2, 4, 6, 8, 10\}$

$B = \{1, 3, 5, 7, 10\}$

$C = \{1, 2, 3, 4, 10\}$

$(A \cap B) = \{10\}$

$(A \cap B \cap C) = \{10\}$

Q. 3 $(A - B) - C \subseteq (A - C)$

$A = \{1, 2, 5, 7, 8\}$

$B = \{0, 3, 4, 6, 9\}$

$C = \{0, 1, 2, 3, 4, 5\}$

$A - B = \{1, 2, 5, 7, 8\}$

$(A - B) - C = \{0, 3, 4, 7, 8\}$

$A - C = \{0, 1, 2, 5, 7, 8\}$

$(A - B) - C \subseteq (A - C)$

Q. 4. $(A - C) \cap (C - B) = \emptyset$

Sol

$A = \{1, 2, 3, 4, 5\}$

$B = \{0, 1, 3, 5, 7\}$

$C = \{1, 2, 4, 6, 8\}$

$(A - C) = \{\cancel{1}, \cancel{2}, 3, 5\}$

$(C - B) = \{2, 4, 6, 8\}$

Ac $(A - C) \cap (C - B) = \{\ \} = \emptyset$

Principal of Mathmatical Induction

Q $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Sol:-

① Basis of Induction :- $P(1) = \left[\frac{1(1+1)}{2}\right]^2 = \left(\frac{2}{2}\right)^2 = 1$

② Induction hypothesis :- $P(K) = \left[\frac{K(K+1)}{2}\right]^2$ is true

③ Inductive Step :- $P(K+1) = 1^3 + 2^3 + 3^3 + \dots K^3 + (K+1)^3$

$$= \left[\frac{K(K+1)}{2}\right]^2 + (K+1)^3$$

$$= \frac{K^2(K+1)^2}{2^2} + (K+1)^3$$

$$= \frac{K^2(K+1)^2 + 2^2(K+1)^3}{2^2}$$

$$= \frac{(K+1)^2\left[K^2 + 2^2(K+1)\right]}{2^2}$$

$$\frac{(K+1)^2\left[K^2 + 4K + 4\right]}{4}$$

$$= \frac{(K+1)^2 (K+2)^2}{4}$$

$$= \frac{(K+1)^2 ((K+1)+1)^2}{2^2}$$

$$= \left[\frac{(K+1)((K+1)+1)}{2}\right]^2$$

$P(K) \Rightarrow P(K+1)$

D-Morgans law :-

Q Prove that If $n \geq 2$ then generalized D morgan law $\sim(P_1 \wedge P_2 \wedge \dots P_n) = (\sim P_1) \vee (\sim P_2) \dots (\sim P_n)$

① Basis of Induction $\sim(P_1 \wedge P_2) \equiv (\sim P_1) \vee (\sim P_2)$

② Indution hypothise - $[\neg(P) = \sim \rightarrow \text{Negation}]$

$\sim(P_1 \wedge P_2 \dots \wedge P_n) \equiv (\sim P_1) \vee (\sim P_2) \vee \dots (\sim P_n)$

③ Inductive Step :- $\sim(P_1 \wedge P_2 \wedge \dots \wedge P_n \wedge P_{n+1})$.

$\sim[(P_1 \wedge P_2 \wedge P_3 \dots P_K) \wedge P_{K+1}]$

$\sim[(P_1) \wedge (P_2) \wedge (P_3) \wedge \dots (P_K)] \vee \sim(P_{K+1})$

$(\sim P_1) \vee (\sim P_2) \vee (\sim P_3) \dots \vee (\sim P_K) \vee P_{K+1}$

$P(K) \longrightarrow P(K+1)$

Principall of Identy :-

Commtive law $A \cup B = B \cup A$
$A \cap B = B \cap A$

Associative law $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

Destribute law $\cancel{(A \cap B) \cup C} = A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Scanned by CamScanner

De morgon law : $\overline{A \cup B} = \bar{A} \cap \bar{B}$
$\overline{A \cap B} = \bar{A} \cup \bar{B}$

Ques: Prove ~~that~~ if ~~$n \geq 2$~~

principal Strong Mathmatical Inducation :-

let P(n) be a statment which for each Integer n ~~Atom~~ may be true and false. then P(n) is True for all +ve integer. if thier intege $a \geq 1$ Such that

1) P(1), P(2), P(3), ... P(q) are all true.
2) When $K \geq a$ the assumtion is that P(i) is true for all integer. $1 < i < k$

The Step of Prove by Strong Mathmatical Induction,

① Basis of Induction :- P(1), P(2), ... P(q) are all true

② Strong Inditive Hypothesis :- Assum that P(i) is true for all intege i Such that $1 < i < k$, Where $K \geq q$

③ Inducutive hypothesis / Step :- Show that P(k+1) is true base of Inductive hypothesis.

$\underbrace{P(1), P(2), P(3) \ldots\ldots P(q)}_{①} \ldots P(K) \ldots P(k+1) \ldots\ldots P(n)$ ②

Permutation & Combination :~
↳ किसी चीज़ को arrange करना Permutation होता है

Principle of Counting :~

If one event Can Occur in n diff. ways following which ~~ante~~ another event Can occour n diff. ways. then the total number of occuren of the event in the given order. i.e. $n \times n$.

Q. Find the number the four latter word with or without meaning which Can be form out of the latter of the Word ROSE. Where the repetion of the word is not allowed.

Ans :- $4 \times 3 \times 2 \times 1 = 24$.

Permutation :- is an arrangement in Difinate Order of a number of object taken Some for all at a the time

$^{n}P_r = \frac{\lfloor n}{\lfloor n-r}$ $\quad {}^{n}P_1 = n$

Q Find the number of Permetion of the latter of the Word ALLAHABAD

Sol:— ALLAHBAD

A = 4
L = 2
H = 1
B = 1
D = 1

$$\frac{9!}{4!\,2!} \Rightarrow \frac{9\times8\times7\times6\times5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}\cdot2\times1}$$

= = — Ans

Q. Find the number of arrangement of the letter of the Word. INDEPENDANCE

Sol INDEPANDANCE = 12

N = 3
D = 2
E = 2
A = 2

$$\Rightarrow \frac{12!}{3!\,2!\,2!\,2!}$$

$$= \frac{12\times11\times10\times9\times8\times7\times\cancel{6}\times5\times\cancel{4}\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}\cdot2\times1\cdot2\times1\cdot2\times1}$$

= ~~9,979~~

$= 12\times11\times10\times9\times8\times7\times5\times3$

P is fix : $\frac{11!}{3!}$

Permution & Combination :-

Q Find the number of Permitions of the letter of the word ALLAHABAD.

$$\frac{9!}{4!\,2!} = \frac{9\times8\times7\times6\times5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}\cdot2\times1}$$

= Ans

Q. Find the number of diffrent letter arrangement that can be made from the letter of the word DAUGHTER

(1) all Vowel Comes to gather
(2) — do not —

3 Vowel.

(1) $8!$ DGHTR AUE

$6!\times3! = 6\times5\times4\times3\times2\times1\cdot3\times2\times1$

= Ans

(2) $8! = 8\times7\times6\times5\times4\times3\times2\times1$ — Ans

= (1) − (2)

How many words with and without meaning can be made form the letter of the Word MONDAY Assuming.

(1) 4 letters are use at the time.
(2) all letter are — " —

Sol. ~~MONDAY~~ MONDAY

(1) $\frac{6!}{(6-4)!}$

(2) $6!$

Q. How many of disting letter of the word PERMUTIONS can we arrange it.
(i) word start with P & end with S
(ii) vowel all together.

Sol: PERMUTIONS

(1) $\lfloor 8 / \lfloor 2$

(2) $\frac{\lfloor 8}{\lfloor 2} \times \lfloor 5$ — 5 vowel, 7 letter disting, 2 T

Q. MISSISSIPPI

$\frac{\lfloor 11}{\lfloor 4 \lfloor 4 \lfloor 2}$: Number of permutation

M SS SS PP (IIII)

$\frac{\lfloor 8 \lfloor 4}{\lfloor 4 \lfloor 2}$ all vowel comes together

$${}^{n}P_{r} = \frac{\lfloor n}{\lfloor n-r} \qquad {}^{n}C_{r} = \frac{\lfloor n}{\lfloor n-r \lfloor r}$$

Q. A commite of 3 person is two constitute from a group of 2 men & 3 woman. In how many way this can ~~vido~~ ~~done~~ beddan.
(i) How many of these commite ~~with~~ would consiste of 1 man & 2 woman

${}^{2}C_{1} \times {}^{3}C_{2}$

PERMUTIONS (9)

PEUIO (EUIO) RMTNS = 564...

Q. What is the numbers of ~~boxes~~ of choosing 4 cards from peack of 52 playing card

Sol:-
(i) 4 cards are the same suit.

$${}^{52}C_{4} = {}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4}$$

(2) 4 cards belong to 4 diffrent suit.

$${}^{13}C_{1} \times {}^{13}C_{1} \times {}^{13}C_{1} \times {}^{13}C_{1}$$

(3) 4 cards of the same colour

$${}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4} + {}^{13}C_{4}$$

$${}^{26}C_{4} + {}^{26}C_{4}$$

Q. Find the numbers of the ways of select 9 balls from 6 red ball 5 white ball 5 blue colour

$${}^{9}C_{6} \times {}^{9}C_{5} \times {}^{9}C_{5}$$

(i) if each selection consite of 3 ball each

$${}^{16}C_{3} \quad {}^{16}C_{3} \quad {}^{16}C_{3}$$

$${}^{6}C_{3} \times {}^{5}C_{3} \times {}^{5}C_{3}$$

4 g 7 b $^4C_0$ $^7C_{11}$ $^4C_1$ $^7C_1$

5

Q A group consist of 4 girl 7 boys. In how many ways a team of 5 member be selected if the team has no girl.

(1) at least 1 boy 1 girl.

(1) $^4C_0 \times {}^7C_5$

(2) $(^4C_1 \times {}^7C_4) +$
$(^4C_2 \times {}^7C_3) +$
$(^4C_3 \times {}^7C_2) +$
$(^4C_4 \times {}^7C_1)$

Q. How many words with and without meaning can we found using all letter of the word EQUATION. So that vowel & consonant come together.

EQUATION → L8

QTN EUAIO → L2 L3 L5

also

Logic theory { Preposition logic, Prediate logic.

(1). Preposition logic :- It is normaly a single charector & may be a word used to represent one single statment.

also) that is

The oprater use preposition logic

oprater :- Disjunction ~~(∨)~~ OR (∨)

Conjuction AND (∧)

Negation NOT ¬

Implication →

Equivalence ↔

Note - Any prepasition logic expression while have any two possible value it is eather True & false.

Ex. (1) It is raining = R
(2) Car is Red = W
(3) Car is in Open.

If it rain car will be wet

(1) law of prepositional logic

(1) Commutive law :- $A \vee B = B \vee A$
$A \wedge B = B \wedge A$
$A \leftrightarrow B = B \leftrightarrow A$

(2) Associative law :-
$A \vee (B \vee C) = (A \vee B) \vee C$
$A \wedge (B \wedge C) = (A \wedge B) \wedge C$

$x + x' = 1$

Distribute law :-

~~At~~ $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$

De-morgan law :-

$\neg(A \vee B) = \neg A \wedge \neg B$

$\neg(A \wedge B) - \neg A \vee \neg B$

law Negation :- $\neg(\neg A) - A$

law of excluded middle :- $A \vee \neg A$ = True.

law of Contradiction ~ $A \wedge \neg A$ = Fales.

— " — implication ~ $A \rightarrow B = \neg A \vee B$

— " — equality :- $A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A)$

— " — (OR) Simplefication $A \vee A = A$

$A \wedge T = T$

$A \wedge F = A$

$A \vee (A \wedge B) = A$

law of AND Simplefial.

$A \wedge A = A$

$A \wedge T = A$

$A \wedge F = F$

Q $\neg(b \wedge \neg b)$:- $\neg(0) = 1$

~~$\neg b \vee$~~ b 1

0

A	B	A V B	A Λ B	(A)
			0	0
1	0	1	1	0
1	1	1	0	1
0	0	0	0	1
0	1	1		

Λ And V or

$\forall$ $\exists$ predicate

$(b \wedge (b \rightarrow c)) \rightarrow c$

Sol:- $(b \wedge (\neg b \vee c)) \rightarrow c$ by law of impligation.

~~$(b \wedge \neg b) \vee c \rightarrow c$~~

~~$0 \vee c \rightarrow c$~~

$(b \wedge \neg b) \vee (b \wedge c) \rightarrow c$ by Distribute law

$F \vee (b \wedge c) \rightarrow c$ by and Simplefication

$(b \wedge c) \rightarrow c$ by laws or Simplefication,

$\neg(b \wedge c) \vee c$ by implification.

$\neg b \vee \neg c \vee c$ by Demorgone law.

$\neg b \vee T$ exclude middle

T

propeties of preposinal logic gate.

(a) Satisfaiable

an expression is Satisfaible if there is Some interpretion for Which it's true.

(1) $\neg A \vee \neg B$ (S)
(2) $A \wedge B$ (S) (1)
(3) $\neg A \wedge B \wedge A$ {US}
(4) $\neg A$

(b) Contradiction :- an Expression is set to be Contradicty ~~tree~~ if no interpretion for Which is it true.
it also Called unSatisfiable :-

8(1) $\neg A \wedge B \wedge A$

(c) ~~Vaild~~ Valid :- an expression is set to be Valid if it is true for all possible interpretion. if it is also Called tautology.

(1) $\neg A \vee B \vee C$ / $(A \to B) \vee C$

(d) Equivalence :- two expression are equivelence if they have Same truth Value for every interpride.

$(A \to B) \wedge C$ $(\neg A \vee B) \wedge C$ $(\equiv)$

reasoning & interphracing Method.

(1) Modus Pones :- if $A \to B$ and A is given then B we Conclude B

(2) Chain rule :- $A \to B$ & $B \to C$

$A \to C$

(3) Principle of resolution :- this teknik Can only be apply if the knowledge is represented in Clause form

Claus :- it is a prepositinal logic expression Where only two oprater are allow these are negation ($\neg$) Disjunction ($\vee$) if $\neg$ is present then it must be with ~~individual~~ individual oprend.

(a) $(A \vee B) \to C$
$\neg(A \vee B) \vee C$ and opreter ~~breakd~~
$\neg A \wedge \neg B \vee C$

$\neg A$ $\neg B \vee C$

(1) $\neg A \vee B \vee C$ —(1)
(2) A —(2)

apply principle of resoulti . (1) & (2)

$B \vee C$

any term that appear in the Clause Can be Conceled by principle of resolution if Contradict is given.