PRAYAS 2.0 2024

FOR JEE ASPIRANTS

(08 AUG 2023)

MATHEMATICS

CHAPTER-07

DETERMINANTS

Lecture - 01

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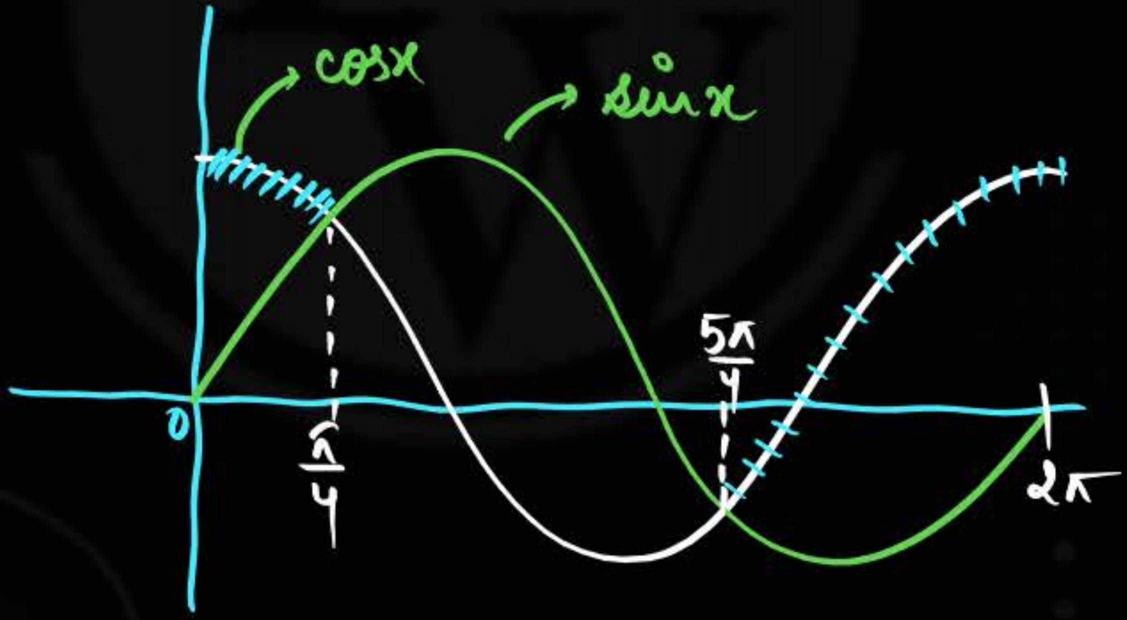
Today's Targets

1. Basics of Determinants
2. Properties of Determinants
3. Question Practice
4.

LAST CLASS

Solving Equations & Inequalities Graphically:

$\boxed{f(x) \geq g(x)}$



$\cos x \geq \sin x$

$\Downarrow$

$x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$

Generalise

(+ add $2n\pi$)

JEE-2006, 3M

The set of values of θ satisfying the inequation $2\sin^2\theta - 5\sin\theta + 2 > 0$, where $0 < \theta < 2\pi$, is

- [✓ A] $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},2\pi\right)$
- B $\left[0,\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6},2\pi\right]$
- C $\left[0,\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3},2\pi\right]$
- D None of the above

$$\underline{\underline{2\sin^2\theta}} - \underline{\underline{4\sin\theta}} - \sin\theta + 2 > 0$$

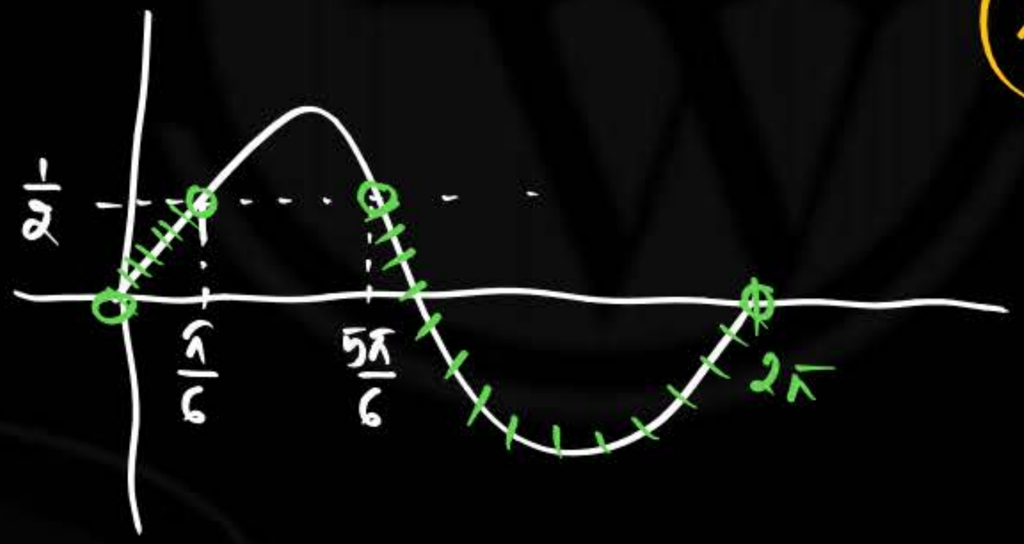
$$2\sin\theta(\sin\theta - 2) - (\sin\theta - 2) > 0$$

$$(\underbrace{\sin\theta - 2}_{\boxed{-ve}})(2\sin\theta - 1) > 0.$$

$$\xrightarrow{\div}$$

$$2\sin\theta - 1 < 0$$

$$\sin\theta < \frac{1}{2}$$



JEE (Adv.)-2020 (Paper-1)

Let $f : [0, 2] \to R$ be the function defined by

$$f(x) = (3 - \sin(2\pi x))\sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$$

HW

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2]\ ;\ f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is ____.

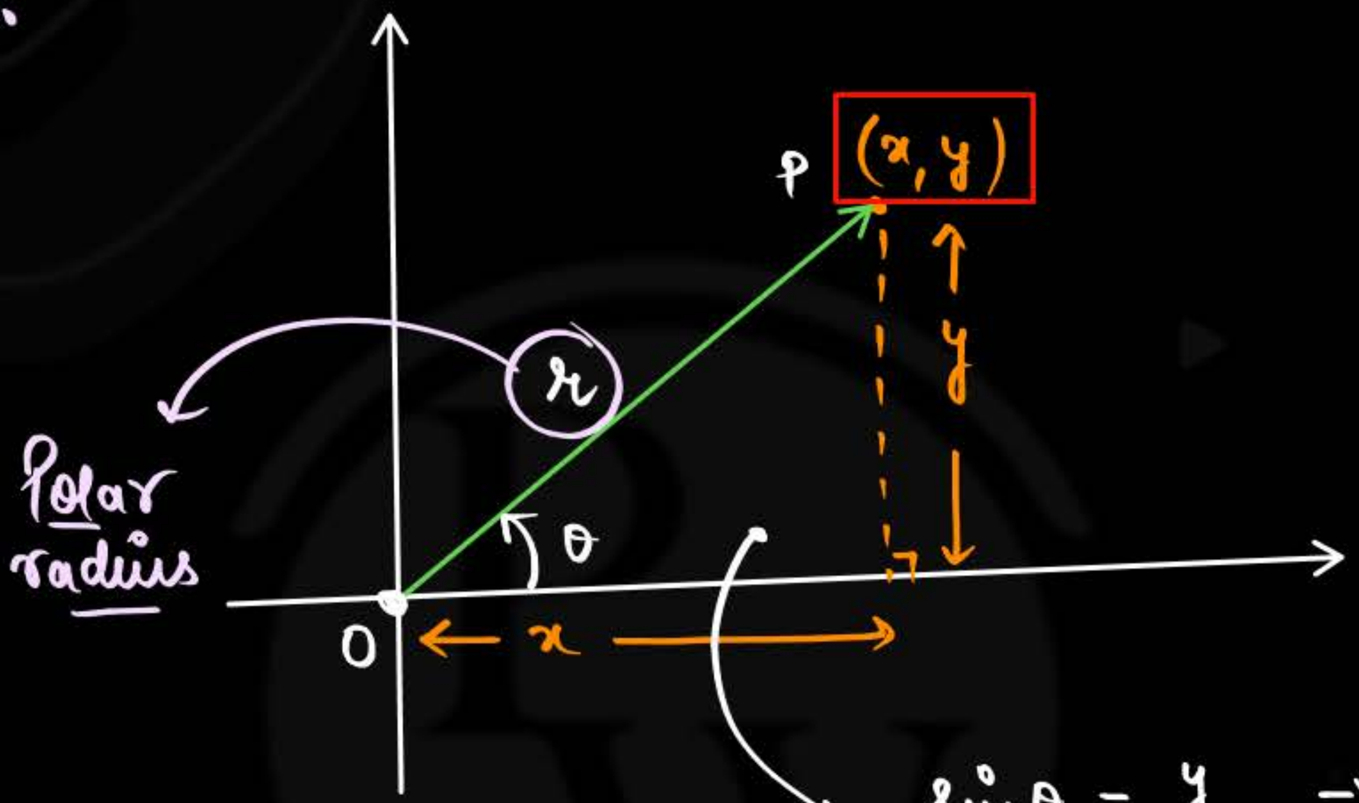
JEE (Adv.)-2015 (Paper-1)

The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$

HW

Polar Coordinates:

angle ↘ θ.



$r \equiv +ve/0$

$\theta \in [0, 2\pi)$

$$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta.$$

$$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta.$$

$$\boxed{(x, y)} \equiv \boxed{(r\cos\theta, r\sin\theta)}$$

Question

Two-variable, mila-jula kr (mixed)

(i) If $x^2 + 2xy - y^2 = 6$, then minimum value of $(x^2 + y^2)^2$ _____

(ii) If $x^2 + y^2 = 4$ & $a^2 + b^2 = 8$, then max. & min value of $(ax + by)$.

Put :-

(i) $\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\}$

$$r^2\cos^2\theta + 2(r\cos\theta \cdot r\sin\theta) - r^2\sin^2\theta = 6$$

Given:- $\boxed{r^2(\cos 2\theta) + r^2(\sin 2\theta) = 6.}$

$$r^2 = \frac{6}{\underbrace{\cos 2\theta + \sin 2\theta}_{\text{max.}}}\Bigg|_{\text{min.}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}.$$

$$(x^2+y^2)^2 = (r^2\cos^2\theta + r^2\sin^2\theta)^2 = r^4\Big|_{\text{min.}} = 18.$$

CSI

$$(ax+by)^2 \le (x^2+y^2)(a^2+b^2)$$

$$(ax+by)^2 \le 32$$

$$-4\sqrt{2} \le ax+by \le 4\sqrt{2}$$

(ii) $\boxed{x^2+y^2=4}$

$\boxed{a^2+b^2=8}$

$x = 2\cos\theta$

$y = 2\sin\theta.$

$a = 2\sqrt{2}\cos\alpha$

$b = 2\sqrt{2}\sin\alpha$

\# $ax+by = 2\sqrt{2}\cos\alpha \cdot 2\cos\theta + 2\sqrt{2}\sin\alpha \cdot 2\sin\theta.$

$= 4\sqrt{2}\left\{\cos(\alpha-\theta)\right\}$

max.

$\Downarrow$

$\cos(\alpha-\theta)=1$

$\Downarrow$

$\boxed{4\sqrt{2}}$

min.

$\Downarrow$

$\cos(\alpha-\theta)=-1$

$\Downarrow$

$\boxed{-4\sqrt{2}}$

DETERMINANTS

#Easy ✓

#Independent ✓

#Favourite of MAINS & ADV. ✓

BASICS OF DETERMINANT

\# $\boxed{\text{Number of Rows} = \text{Number of Column.}}$

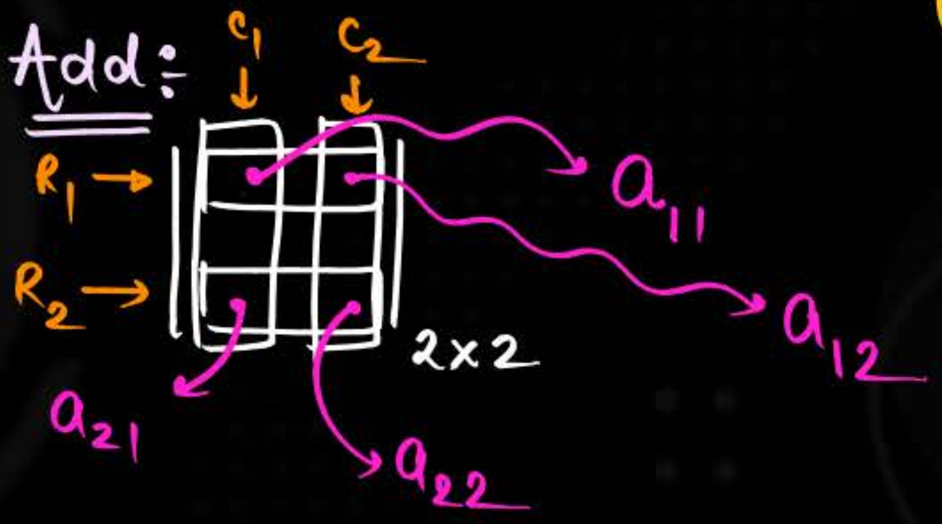
\# $\boxed{\text{Order} = \underset{\text{no. of Rows}}{n} \times \underset{\text{no. of column}}{n} \equiv n}$

\# **Determinant of Order :** $\boxed{n \times n} \equiv$ order 'n'.

$$\text{Two} \equiv 2 \times 2 \Rightarrow \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\text{Three} \equiv 3 \times 3 \Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

\# Add :



\# A Determinant of order 1 is the number itself.

\# $\boxed{|3|_{1\times1} = 3.}$

$|-2|_{1\times1} = -2$

\# $a_{ij} \rightarrow \boxed{i^{th} \text{ row } \& \ j^{th} \text{ column.}}$

Value of 2×2 :-

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}_{2\times 2} = ad - bc$$

MINORS & CO-FACTORS

Minors :

Minors of an element is defined as the minor determinant obtained by deleting a particular row or column in which that element lies.

Cofactor :

It has no separate identity and is related to the minors as

$C_{ij} = (-1)^{i+j} M_{ij}$, where 'i' denotes the row and 'j' denotes the column.

$$(-1)^{1+1} = (-1)^2$$

\# $\overset{a_{11}}{\begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix}}$ 2×2

M_{11} = minor of $a_{11} = 4 \Rightarrow C_{11} = +4$

M_{21} = minor of $a_{21} = 3 \Rightarrow C_{21} = (-1)^{2+1} 3 = -3$

M_{12} = " " $a_{12} = -1 \Rightarrow C_{12} = (-1)^{1+2}(-1) = 1$

M_{22} = " " $a_{22} = 2 \Rightarrow C_{22} = (-1)^{2+2} 2 = 2$

$$\boxed{C_{ij} = \text{Cofactor of } a_{ij} = (-1)^{i+j} M_{ij}}$$

$$\begin{vmatrix} \overset{4}{2} & \overset{1}{3} \\ \underset{-3}{-1} & \underset{2}{4} \end{vmatrix}$$

$= 3\times1 + 4\times2 = \boxed{11}$

$= 2\times4 + 3\times1 = \boxed{11}$

$= (-1)(-3) + 4\times2 = \boxed{11}$

$= 2\times4 + (-1)(-3) = \boxed{11}$

\# $$\begin{vmatrix} \overset{+}{2} & \overset{-}{3} & \overset{+}{1} \\ \overset{-}{-1} & \overset{+}{5} & \overset{-}{1} \\ \overset{+}{4} & \overset{-}{2} & \overset{+}{0} \end{vmatrix}_{3\times3}$$

M_{11} = minor of $a_{11} = \begin{vmatrix} 5 & 1 \\ 2 & 0 \end{vmatrix} = 5\times0 - 1\times2 = -2$

M_{32} = minor of $a_{32} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 2 - (-1) = 3.$

$\vdots$

$C_{11} = (-1)^{1+1}(-2) = -2$

$C_{32} = (-1)^{3+2}(3) = -3.$

$$-14 = 12-20-6 = 3\times 4 + 5(-4)+2(-3) = \begin{vmatrix} 2 & \overset{4}{3} & 1 \\ -1 & 5^{-4} & 1 \\ 4 & 2 & 0 \end{vmatrix}$$

$$-14 = 4(-2) + 2(-3)+0(\;) = \begin{vmatrix} 2 & 3 & 1 \\ -1 & 5 & 1 \\ \underset{-2}{4} & \underset{-3}{2} & 0 \end{vmatrix}$$

$$\# \begin{vmatrix} \overset{\oplus}{2} & \overset{\ominus}{3} & \overset{\oplus}{1} \\ \overset{\ominus}{-1} & \overset{\oplus}{5} & \overset{\ominus}{1} \\ \overset{\oplus}{4} & \overset{\ominus}{2} & \overset{\oplus}{0} \end{vmatrix}_{3\times 3}$$

$$M_{11} = \text{minor of } a_{11} = \begin{vmatrix} 5 & 1 \\ 2 & 0 \end{vmatrix} = 5\times 0 - 1\times 2 = -2$$

$$M_{32} = \text{minor of } a_{32} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 2-(-1) = 3.$$

$$\boxed{C_{ij} = \text{Cofactor of } a_{ij} = (-1)^{i+j} M_{ij}}$$

$$C_{11} = (-1)^{1+1}(-2) = -2$$

$$C_{32} = (-1)^{3+2}(3) = -3.$$

$$2\begin{vmatrix} 5 & 1 \\ 2 & 0 \end{vmatrix} - 3\begin{vmatrix} -1 & 1 \\ 4 & 0 \end{vmatrix} + 1\begin{vmatrix} -1 & 5 \\ 4 & 2 \end{vmatrix} = \begin{vmatrix} \overset{+}{2} & \overset{-}{3} & \overset{+}{1} \\ -1 & 5 & 1^{-} \\ 4 & 2 & 0^{+} \end{vmatrix}$$

$$\Downarrow$$

$$2(0-2)-3(0-4)+1(-2-20)$$

$$-14 = -4+12-22$$

$$= 1\begin{vmatrix} -1 & 5 \\ 4 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} + 0\begin{vmatrix} \sim \end{vmatrix}$$

$$= 1(-2-20)-1(4-12) = -22+8 = -14.$$

Question

$$\begin{vmatrix} \overset{+}{x} & \overset{-}{\sin\theta} & \overset{+}{\cos\theta} \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$ is independent of :

A x

[✓ **B**] θ

C both x and θ

D None of these

$$= +x\begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta\begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$$

$$= x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$$

$$= -x^3 - x + \underbrace{x\sin^2\theta} + \cancel{\sin\theta\cos\theta} - \cancel{\sin\theta\cos\theta} + \underbrace{x\cos^2\theta}$$

$$= -x^3 - \cancel{x} + \cancel{x}(\underbrace{\sin^2\theta + \cos^2\theta}_{1})$$

$$= -x^3.$$

Question

Find $\Delta = \begin{vmatrix} 1 & \overset{\ominus}{-2} & 3 \\ -1 & \overset{\oplus}{0} & -2 \\ -3 & \overset{\ominus}{4} & 1 \end{vmatrix}$ also show that $\Delta_C = \begin{vmatrix} 8 & 7 & -4 \\ 14 & 10 & 2 \\ 4 & -1 & -2 \end{vmatrix} = 324$.

$$\Delta = 2\begin{vmatrix} -1 & -2 \\ -3 & 1 \end{vmatrix} + 0\,\Big| \quad \Big| - 4\begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix}$$

$$= 2(-1-6) + 0 - 4(\underbrace{-2-(-3)}))$$

$$= -14 - 4$$

$$= -18.$$

$$\Delta = \begin{vmatrix} \overset{8}{1} & \overset{7}{-2} & \overset{-4}{3} \\ \overset{14}{-1} & \overset{10}{0} & \overset{2}{-2} \\ \underset{4}{-3} & \underset{-1}{4} & \underset{-2}{1} \end{vmatrix}$$

sabhi elements ki jagah, unke cofactors likh kr, ek naya det. bnao

cofactor det. of Δ $= \begin{vmatrix} 8 & 7 & -4 \\ 14 & 10 & 2 \\ 4 & -1 & -2 \end{vmatrix} = \Delta_C = (-18)^2.$

(#) Most Important Note :

$$\boxed{\Delta_c = (\Delta)^{n-1}}$$, $n \equiv$ order of det.

cofactor determinant

for 3×3 ⇒ $\boxed{\Delta_c = \Delta^2}$

for 2×2 ⇒ $\Delta_c = \Delta$

Show that:

$$\begin{vmatrix} cb-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

Sol.

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

cofactor det

$$\Delta_c = \begin{vmatrix} cb-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}_{3\times3} = \Delta^2.$$


TUNE ITNI JALDI KAISE SOLVE KIYA RE BABA
PROPERTIES KA CHAKKR BABU BAHIYA!!

PROPERTIES OF DETERMINANT

 # P-01 : TRANSPOSE LENE SE DETERMINANT VALUE NHI BADALTI.

converting

(all rows into column)

OR

(all column into Rows)

same

ex:- # $\Delta = \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} \xrightarrow{\text{Trans.}} \Delta^T = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$

$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 0 & -2 \\ -3 & 4 & 1 \end{vmatrix} \xrightarrow{\text{Transp.}} \Delta^T = \begin{vmatrix} 1 & -1 & -3 \\ -2 & 0 & 4 \\ 3 & -2 & 1 \end{vmatrix}$

P-02 : MULTIPLICATION OF DETERMINANT BY A NUMBER

ex:-

$$k\begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix} = \begin{vmatrix} ka & kb & kc \\ p & q & r \\ l & m & n \end{vmatrix}$$

$$= \begin{vmatrix} a & kb & c \\ p & kq & r \\ l & km & n \end{vmatrix}$$

- 6 different ways to multiply

Question

Show that : $\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = \begin{vmatrix} xyz & x & x^2 \\ xyz & y & y^2 \\ xyz & z & z^2 \end{vmatrix} = \begin{vmatrix} x & xy & x^2z \\ x & y^2 & y^2z \\ x & yz & z^3 \end{vmatrix}$

$R_1 \rightarrow$ 'x' common

$R_2 \rightarrow$ 'y' "

$R_3 \rightarrow$ 'z' "

$$xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \begin{vmatrix} xyz & x & x^2 \\ xyz & y & y^2 \\ xyz & z & z^2 \end{vmatrix}$$

$$= \begin{vmatrix} x & xy & x^2z \\ x & y^2 & y^2z \\ x & zy & z^3 \end{vmatrix}$$

Question

$f(x) =$

Let $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$, then $f(100)$ is

A ✓ 0

B 1

C 100

D -100

$$f(x) = (x+1)x \begin{vmatrix} 1 & x & 1 \\ 2 & (x-1) & 1 \\ 3x(x-1) & x(x-1)(x-2) & x(x-1) \end{vmatrix}$$

$$= (x-1)(x+1)\,x^2 \begin{vmatrix} \overset{+}{1} & \overset{-}{x} & \overset{+}{1} \\ 2 & x-1 & 1 \\ 3 & (x-2) & 1 \end{vmatrix}$$

$= 0$.

0

1

$1 + x - x - 1 = 0$.

$1(x-1-(x-2)) - x(2-3) + 1(2x-4-3x+3)$

DO IT BY YOURSELF (DIBY)

TRIGONOMETRY

JEE Mains-2021

Ans: C

The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to:

A 2

B 3

C 4

D 8

DIBY-19

JEE (Adv.)-2022

Ans: 01

Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha + \beta) = \frac{1}{3}$ and $\cos(\alpha - \beta) = \frac{2}{3}$, then the greatest integer less than or equal to $\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$ is

DIBY-20

Question

Ans: D

If $f(\theta) = \sin^3\theta + \sin^3\left(\theta + \frac{2\pi}{3}\right) + \sin^3\left(\theta + \frac{4\pi}{3}\right)$, then the value of $f\left(\frac{\pi}{18}\right) + f\left(\frac{7\pi}{18}\right)$ is equal to

A 3/4

B 1/4

C 1/8

D 0

DIBY-21

Challenger

Ans: 35

Let $f(x) = \frac{15}{x+1} + \frac{16}{x^2+1} - \frac{17}{x^3+1}$. Find the value of $f(\tan 15°) + f(\tan 30°) + f(\tan 45°) + f(\tan 60°) + f(\tan 75°)$.

DIBY-22

$\operatorname{cosec} x - \operatorname{cosec} 2x = \operatorname{cosec} 4x$. Find general values of x?

Ans: $(2m-1)\frac{\pi}{7}, m \neq 7k-3, k \in I$

DIBY-23

Solve the equation : $\sin^{10}x + \cos^{10}x = \frac{29}{16}\cos^4 2x$.

Ans: $x = \frac{n\pi}{4} + \frac{\pi}{4}, n \in$ Integer

DIBY-24

Solve for x and y : $1 - 2x - x^2 = \tan^2(x+y) + \cot^2(x+y)$

Ans: $x = -1, y = n\pi \pm \frac{\pi}{4} + 1$

DIBY-25

$$\tan^2 x - \left(\sqrt{3}+1\right)\tan x + \sqrt{3} < 0$$

Ans: $\bigcup_{n\in I}\left(n\pi+\frac{\pi}{4}, n\pi+\frac{\pi}{3}\right)$

DIBY-26

$$\sqrt{5-2\sin x} \geq 6\sin x - 1$$

Ans: $\bigcup_{n\in I}\left(2n\pi-\frac{7\pi}{6}, 2n\pi+\frac{\pi}{6}\right)$

squaring

ans

Check!

DAILY HOME WORK

*# All **DPP's** till date & Re-attempt all Classroom examples.*

 # future IITians

THANK

Y☺U