# Numerical on Multiple Linear Regression solved using matrix concept

Q1. The data in the following table relate plant dry weight in grams($y$) to percent soil organic matter($x_1$) and kilograms of supplemental soil nitrogen added per 1000 square metres ($x_2$). Obtain the multiple regression equation.

Sol: The regression equation for the given data will be of the form:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

We want to estimate the coefficients $\beta_0$, $\beta_1$ and $\beta_2$ using matrices.

First, we have to write the data in matrix form

| $y$ | $x_1$ | $x_2$ |
|---|---|---|
| 78.5 | 7 | 2.6 |
| 74.3 | 1 | 2.9 |
| 104.3 | 11 | 5.6 |
| 87.6 | 11 | 3.1 |
| 95.9 | 7 | 5.2 |
| 109.2 | 11 | 5.5 |
| 102.7 | 3 | 7.1 |

$$Y = \begin{bmatrix} 78.5 \\ 74.3 \\ 104.3 \\ 87.6 \\ 95.9 \\ 109.2 \\ 102.7 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 7 & 2.6 \\ 1 & 1 & 2.9 \\ 1 & 11 & 5.6 \\ 1 & 11 & 3.1 \\ 1 & 7 & 5.2 \\ 1 & 11 & 5.5 \\ 1 & 3 & 7.1 \end{bmatrix}$$

Now we find $X^TX$ matrix

$$X^T \cdot X = \begin{bmatrix} 7 & 51 & 32 \\ 51 & 471 & 235 \\ 32 & 235 & 163.84 \end{bmatrix}$$

Find the inverse of the matrix $(X^TX)^{-1}$

$$(X^T \cdot X)^{-1} = \begin{bmatrix} 1.7996 & -0.0685 & -0.2532 \\ -0.0685 & 0.0101 & -0.0011 \\ -0.2532 & -0.0011 & 0.0571 \end{bmatrix}$$

The estimated regression coefficients is given by:

$$\hat{\beta} = (X^TX)^{-1}X^TY$$

$$= \begin{bmatrix} 51.6 \\ 1.5 \\ 6.72 \end{bmatrix}$$

$\therefore$ We get the multiple regression equation

$$\hat{Y} = 51.6 + \underline{1.5X_1} + 6.7X_2$$