Propositional logic

Proposition:- A proposition is a declarative sentence that is either true or false.

Ex:
(I) New Delhi is the capital of India.
(II) $1+1=2$
(III) $2+2=3$

Proposition (I) and (II) are true, whereas (III) is false.

Propositional variables (or statement variables): that is the variable that represents propositions, are usually denoted by p, q, r, s .... The truth value of a true proposition is denoted by T and false proposition by F.

Compound Proposition:- Propositions constructed by combining one or more propositions are called compound proposition. Logical operators are used to combine propositions.

Definition:- Let p be a proposition. The negation of p, denoted by '$\neg p$' (also denoted by $\bar{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read as "not p".

Ex:- Find the negation of the proposition
"Today is Friday."

Sol:- It is not the case that today is Friday.
This negation can be more simply expressed as.
"Today is not Friday". or It is not Friday today

The truth table for '$\neg p$'

p	$\neg p$
T	F
F	T

Def: Let p and q be propositions. The conjuction of p and q denoted by $p \wedge q$, is the proposition "p and q". The conjuction $p \wedge q$ is true when both p and q are true and is false otherwise.

Truth table for conjuction of two proposition:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Def: Let p and q be propositions. The disjunction of p and q denoted by $p \vee q$, is the proposition "p or q". The disjunction $p \vee q$ is false when both p and q are false and is true otherwise. (See top for Truth table)

(This 'or' here is inclusive 'or')

Ex: "Students who have taken calculus or computer science can take this class"

Def: Let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement:-

Def:- Let p and q be propositions. The conditional statement $p \to q$ is the proposition, "if p, then q". The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In conditional statement $p \to q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence). Conditional statement is also called implication.

The truth table.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Other terminology used for $p \to q$

"If p then q" "p implies q"

"If p, q" "p only if q".

"p is sufficient for q"

Ex:- i) "If I am elected, then I will lower taxes."

ii) "If you get 100% on the final, then you will get an A."

These two statements will help us to understand '$p \to q$'

In case of (i) statement, suppose a politician who is elected and has not lowered the taxes then the population will feel cheated. Suppose he is not elected then still he may use his influence to lower taxes.

Similarly if we consider the statement (ii), suppose a student get 100% on the final exam, but still the 'A' grade is not awarded then he will feel cheated which is the case (T F F). Suppose he is not getting 100% on the final still he may manage to get A through other means.

Ex:- ① Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \to q$ as a statement in English.

Sol:- "If Maria learns discrete mathematics then she will find a good job"

or "For Maria to get a good job, it is sufficient for her to learn discrete Mathematics."

(ii) If it is sunny today, then we will go to the beach."

(iii) "If today is Friday, then $2+3=5$"
is true from the definition of a conditional statement, because its conclusion is true. (The truth value of the hypothesis does not matter.)

The conditional statement
"If today is Friday, then $2+3=6$"
is true everyday except Friday, even though $2+3=6$ is false.

So from above example it is clear that in logic the conditional statements are of more general kind than that we use in English language.

Converse, Contrapositive, and Inverse:—

Converse : The proposition $q \to p$ is called the converse of $p \to q$

Contrapositive:- The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$.

Inverse:- The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$.

Truth table

p	q	$\neg p$	$\neg q$	$p \to q$	Converse $q \to p$	Contrapositive $\neg q \to \neg p$	Inverse $\neg p \to \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

When two compound ~~statements~~ propositions always have the same truth value we call them equivalent.

So a conditional statement and its contrapositive are equivalent.

The converse and the inverse of a conditional statement are also equivalent but neither is equivalent to the original statement.

Ex:- What are the contrapositive, the converse, and the inverse of the conditional statement

"The home team wins whenever it is raining".

Sol: Since "q whenever p" is one of the ways to express the conditional statement $p \to q$, the original statement can be rewritten as.

"If it is raining, then the home team wins".

Contrapositive: If home team does not win, then it is not raining.

Converse: "If the home team wins then it is raining".

Inverse: "If it is not raining, then the home team does not win".

Biconditional

Def:- Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

"p is necessary and sufficient for q"
"If p then q, and conversely."
"p iff q"

We note that $p \leftrightarrow q$ has exactly the same truth value as $(p \to q) \wedge (q \to p)$

Truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Ex:- p: "You can take the flight"
q: "You buy a ticket"
$p \leftrightarrow q$, "You can take the flight iff you buy a ticket"

Truth table of compound statement

1) $(p \vee \neg q) \to (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

2) How many rows appear in a truth table for each of these compound propositions.

(a) $p \to \neg p$ (b) $(p \vee \neg r) \wedge (q \vee \neg s)$

(c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$

(d) $(p \wedge \cancel{r} \wedge t) \leftrightarrow (q \wedge t)$

3) Construct a truth table for each of these compound statements

(a) $p \wedge \neg p$ (b) $p \vee \neg p$ (c) $(p \vee \neg q) \to q$ (d) $(p \vee q) \to (p \wedge q)$

(e) $(p \to q) \leftrightarrow (\neg q \to \neg p)$ (f) $(p \to q) \to (q \to p)$

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$\neg p \wedge q \quad (\neg p) \wedge q$

$p \wedge q \vee r \quad (p \wedge q) \vee r$

$p \vee q \rightarrow r \quad (p \vee q) \rightarrow r$

Translating English Sentences:

1) "You can access the Internet from campus <u>only if</u> you are a computer science major or you are not a freshman."

<u>Sol:</u> $p \rightarrow q$ mean "p only if q"

p: You can access the internet from Campus"
q: You are a computer science major.
r: You are a freshman

$$p \rightarrow (q \vee \neg r)$$

2) "You cannot ride the roller coster if you are under 4 feet tall unless you are older than 16 years old".

p: You can ride the roller coster.
q: You are under 4 feet.
r: You are older than 16 years old

$$\cancel{(q \wedge \neg 16) \rightarrow} \quad (q \wedge \neg r) \rightarrow \neg p.$$

<u>Def:</u> A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

Ex

```
   01 1011 0110
   11 0001 1101
  --------------
   11 1011 1111   bitwise OR.
   01 0001 0100   bitwise AND
   10 1010 1011   bitwise XOR
```

Def:- A compound proposition that is alway true, no matter what the truth value of the proposition that occur in it, is called tautology. A compound proposition that is always false is called a contradiction. A compound proposition that is neither a tautology nor a contradiction is called contingeny.

Ex:

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Logical Equivalence: Compound propositions that have the same truth values in all possible cases are called logically equivalent.

Def:- The compound propositions p and q are called logically equivalent if $p \longleftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes p and q are logically equivalent.

One way to determine the logical equivalence is truth table

Ex(i) Show that $\neg(p \vee q) \equiv \neg p \wedge \neg q$.

(ii) $p \rightarrow q \equiv \neg p \vee q$

Logical Equivalence:

Equivalence	Name
1) $p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
2) $p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws.
3) $p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
4) $\neg(\neg p) \equiv p$	Double negation law
5) $p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws.
6) $(p \vee q) \vee r \equiv p \vee (q \vee r)$. $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws

7) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ Distributive law

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

8) $\neg (p \wedge q) \equiv \neg p \vee \neg q$ De Morgan's law

$\neg (p \vee q) \equiv \neg p \wedge \neg q$

9) $p \vee (p \wedge q) \equiv p$ Absorption law

$p \wedge (p \vee q) \equiv p$

10) $p \vee \neg p \equiv T$ Negation laws.

$p \wedge \neg p \equiv F$

Logical Equivalences involving conditional statements.

1) $p \rightarrow q \equiv \neg p \vee q$

2) $p \rightarrow q \equiv \neg q \rightarrow \neg p$

3) $p \vee q \equiv \neg p \rightarrow q$.

4) $p \wedge q \equiv \neg (q \rightarrow \neg p)$

5) $\neg (p \rightarrow q) \equiv p \wedge \neg q$

(6) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

(7) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$.

(8) $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

9) $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Logical Equivalences involving Biconditional

i) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

ii) $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

iii) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

iv) $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.

Ex: Show that 1) $\neg (p \rightarrow q) \equiv p \wedge \neg q$

Sol: $\neg (p \rightarrow q) \cancel{\equiv \neg (p \vee \neg q)} \equiv \neg (\neg p \vee q)$

$\equiv \neg (\neg p) \wedge \neg q \equiv p \wedge \neg q$

Ex: $\neg(p\vee(\neg p\wedge q) \equiv \neg p\wedge\neg q$

Sol: $\neg(p\vee(\neg p\wedge q)) \equiv \neg p\wedge \neg(\neg p\wedge q)$

$\equiv \neg p\wedge[\neg(\neg p)\vee\neg q]$

$\equiv \neg p\wedge[p\vee\neg q]$

$\equiv (\neg p\wedge p)\vee(\neg p\wedge\neg q)$

$\equiv F\vee(\neg p\wedge\neg q)$

$\equiv (\neg p\wedge\neg q)\vee F \equiv \neg p\wedge\neg q$

$\Rightarrow \neg(p\vee(\neg p\wedge q)) \equiv \neg p\wedge\neg q$

Ex: Show that $(p\wedge q)\rightarrow(p\vee q)$ is a tautology.

Sol: $(p\wedge q)\longrightarrow(p\vee q) \equiv \neg(p\wedge q)\vee(p\vee q)$

$\equiv (\neg p\vee\neg q)\vee(p\vee q)$

$\equiv (\neg p\vee p)\vee(\neg q\vee q)$

$\equiv T\vee T \equiv T$

Ex: Show that $(p\rightarrow\neg p)\wedge(\neg p\rightarrow p)$ is a contradiction.

Ex: - Show that $(p\vee q)\wedge(\neg p\vee r)\rightarrow(q\vee r)$ is a tautology.

Sol: The conclusion $(q\vee r)$ is true in every case except when q and r both are false.
But if q and r both are false, then one of $p\vee q$ or $\neg p\vee r$ is false, because one of p or $\neg p$ is false - Thus hypothesis $(p\vee q)\wedge(\neg p\vee r)$ is false. An implication in which the conclusion is true or the hypothesis is false is true. That completes the proof.

Q) What is the negation of the statement
"If you age is 18 years then you can vote" is

(a) Your age is not 18 years, and you cannot vote
(b) Your age is 18 years and you cannot vote
(c) Your age is not 18 years and you can vote
(d) Your age is not 18 years and you cannot vote.

Predicates and Quantifiers: -

Predicates:- Statements involving variables such as.

"$x>3$", "$x=y+3$", "$x+y=3$"

"Computer x is under attack by an intruder

and "computer x is functioning properly."

are often found in mathematical assertions, in computer program and in system specifications.

These statements are neither true nor false when the values of the variable are not specified.

The statement "x is greater than 3" has two parts. The first part, the variable x, is the subject of the statement. The second part - the predicate "is greater than 3" refers to a property that the subject of the statement can have.

We can denote the statement "x is greater than 3" by $P(x)$, where P denotes the predicate "is greater than 3" and x is the variable.

The $P(x)$ is also said to ~~have~~ be the value of the propositional function ~~at~~ P at x.

Once a value has been assigned to the variable x, the statement $P(x)$ becomes a proposition and has a truth value.

Ex 1) Let $P(x)$ denote the statement "$x>3$" what are the truth values of $P(4)$ and $P(2)$?

Sol: $P(4)$, T and $P(2)$, F

2) Let $Q(x,y)$ denote the statement "$x=y+3$". What are the truth values of the statement $Q(1,2)$ and $Q(3,0)$ Ans: $Q(1,2)$ F / $Q(3,0)$ T

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A statement involving n variables $x_1, x_2 \cdots x_n$ can be denoted by $p(x_1, x_2 \cdots x_n)$. A statement of the form $p(x_1, x_2 \cdots x_n)$ is the value of the propositional function p at the n-tuple $(x_1, x_2 \cdots x_n)$, and p is also called an n-place predicate or n-ary predicate.

Quantifiers:

Def: The universal quantification of $p(x)$ is the statement
"$p(x)$ for all values of x in the domain."

The notation $\forall x p(x)$ denotes the universal quantification of $p(x)$. Here $\forall$ is called universal quantifier. We read $\forall x p(x)$ as "For all x $p(x)$". or "For every x $p(x)$". Any ~~statement~~ element for which $p(x)$ is false is called a counterexample of $\forall x p(x)$.

Statement	When true	When false
$\forall x\, p(x)$	$p(x)$ is true for every x	There is an x for which $p(x)$ is false
$\exists x\, p(x)$	There is any x for which $p(x)$ is true	$p(x)$ is false for every x

Ex: 1) Let $p(x)$ be the statement "$x+1 > x$". What is the truth value of the quantification $\forall x p(x)$, where the domain consists of all real numbers?

Sol: $\forall x p(x)$ is true.

Remark:- Domain of discourse must be nonempty. If domain is empty $\forall x p(x)$ is true for any prop.

The Existential quantifier:-

Many mathematical statements assert that there is an element with a certain property. Such statements are expressed using existential quantification. With existential quantification, we form a proposition that is true if and only if $p(x)$ is true for atleast one value of x in the domain.

Def: The existential quantification of $p(x)$ is the proposition "There exists an element x in the domain such that $p(x)$"
We use the notation $\exists x\, p(x)$ for the existential quantification of $p(x)$. Here $\exists$ is called the existential quantifier.

The domain must always be specified when a statement $\exists x\, p(x)$ is used.

Ex: Let $p(x)$ denote the statement "$x > 3$". What is the truth value of the quantification $\exists x\, p(x)$, where the domain consists of all real numbers?

Sol: True.

§ Observe that the statement $\exists x\, p(x)$ is false if and only if there is no element x in the domain for which $p(x)$ is true. That is, $\exists x\, p(x)$ is false if and only if $p(x)$ is false for every element of the domain.

Ex:- Let $Q(x)$ denote the statement "$x = x+1$". What is the truth value of the quantification $\exists x\, Q(x)$, where the domain is the set of real numbers?

Sol: False.

Remark: Generally, an implicit assumption is made that all domain of discourse for quantifiers are non-empty. If the domain is empty, then $\exists x\, Q(x)$ is false whenever $Q(x)$ is a propositional function because when domain is empty there can be no element x in the domain for which $Q(x)$ is true.

When all elements in the domain can be listed - say $x_1, x_2 \cdots x_n$

$\forall x\, p(x)$ $\quad p(x_1) \wedge p(x_2) \wedge \cdots \wedge p(x_n)$ — because conjunction is true if all $p(x_1) \cdots p(x_n)$ are true

$\exists x\, p(x)$ $\quad p(x_1) \vee p(x_2) \vee \cdots \vee p(x_n)$

because disjunction is true iff at least one of $p(x_1) \cdots p(x_n)$ are true.

Ex: $\exists x\, P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and universe of discourse consists of positive integers not exceeding 4?

Sol: True.

Uniqueness quantifiers: $\exists!$ or $\exists_1$

The notation $\exists! P(x)$ or $\exists_1 x P(x)$ states that "there exists a unique x such that $P(x)$ is true".

Quantifiers with restricted domains:

Ex: What do the statements $\forall x<0 (x^2>0)$, $\forall y \neq 0 (y^3 \neq 0)$ and $\exists z>0 (z^2=2)$ mean, where the domain in each case consists of real numbers?

Sol: The statement $\forall x<0 (x^2>0)$ states that for every real number x with $x<0$, $x^2>0$.

That is, it states "~~The square of a negative real number is neg nonzero~~"

"The square of a negative real number is positive".

This statement is same as $\forall x (x<0 \rightarrow x^2>0)$

The statement $\forall y \neq 0 (y^3 \neq 0)$ states that for every real number y with $y \neq 0$, we have $y^3 \neq 0$. That is, it states that "The cube of every non-zero real number is nonzero." Note that this statement is equivalent to $\forall y (y \neq 0 \rightarrow y^3 \neq 0)$

Finally the statement $\exists z>0 (z^2=2)$ states that there exists a real number ~~zAa~~ z with $z>0$ such that $z^2=2$.

That is ~~the positive~~ "There is a positive square root of 2".

This statement is equivalent to $\exists z (z>0 \wedge z^2=2)$

Note that restriction of universal quantification is same as the universal quantification of a conditional statement.

For instance $\forall x(x<0 \rightarrow x^2>0)$ is another ways of expressing $\forall x<0\,(x^2>0)$

On the other hand, the restriction of existential quantification is same as the existential quantification of a conjuction.

For instance, $\exists z>0\,(z^2=2)$ is another way of expressing $\exists z\,(z>0 \wedge z^2=2)$

Bounding Variables:

Ex: $\exists x\,(x+y=1)$, the variable x is bound by the existential quantification $\exists x$, but the variable y is free because it is not bound by a quantifier and no value is assigned to this variable.

Logical Equivalency:

1) $\forall x\,(P(x) \wedge Q(x)) \equiv \forall x\,P(x) \wedge \forall x\,Q(x)$

2) $\neg \forall x\,P(x) \equiv \exists x\,\neg P(x)$

3) $\neg \exists x\,Q(x) \equiv \forall x\,\neg Q(x)$

Rules for negations of Quantifiers are called De Morgan's laws for quantifiers.

De morgan's law for quantifiers

Negation	eq statement
$\neg \exists x\,P(x)$	$\forall x\,\neg P(x)$
$\neg \forall x\,P(x)$	$\exists x\,\neg P(x)$

$\neg(P(x_1) \wedge P(x_2) \cdots P(x_n))$
$\equiv \neg P(x_1) \vee \neg P(x_2) \cdots \vee \neg P(x_n)$

$\neg(P(x_1) \vee P(x_2) - \vee P(x_n))$
$\equiv \neg P(x_1) \wedge \neg P(x_2) \cdots \wedge \neg P(x_n)$

Ex: What are the negation of the statements. "There is an honest politician" and "All Americans eat cheesbergers".

Sol. $\exists x\,H(x)$ $\quad \neg x\,H(x) \equiv \forall x\,\neg H(x)$

$\forall x\,C(x)$ $\quad \neg \forall x\,C(x) \equiv \exists x\,\neg C(x)$

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ex Write negation of $\forall x(x^2 > x)$ and $\exists x(x^2=2)$? (16)

Sol $\neg\forall x(x^2>x) \equiv \exists x \neg(x^2>x)$, which can be rewritten $\exists x(x^2 \le x)$

$\neg\exists x(x^2=2) \equiv \forall x \neg(x^2=2)$, which is eq to $\forall x(x^2 \neq 2)$

Ex : $\cancel{\neg(\forall}$ $\neg\forall x(P(x)\rightarrow Q(x)) \equiv \exists x(P(x)\wedge\neg Q(x))$

$\neg\forall x(P(x)\rightarrow Q(x)) \equiv \exists x\neg(P(x)\rightarrow Q(x))$

$\equiv \exists x \neg(\neg P(x) \vee Q(x))$

$\equiv \exists x (P(x)\wedge\neg Q(x))$

Ex:- "Every student in this class has studied Calculus." Express this statement by using predicate and quantifiers

Sol ~~$\forall x C(x)$~~ $\forall x C(x)$, Domain is students of the class.

or $\forall x(S(x)\rightarrow C(x))$, Domain is the set of all people

note that $\forall x(S(x)\wedge C(x))$ says that all people are students in this class and has studied calculus.

Ex:- Consider the statement. The first two are called premises and third is called the conclusion.

"All lions are fierce"

"Some lions do not drink coffee"

"some fierce creatures do not drink coffee"

Assume that domain consists of all creatures, express the statement in argument using quantifiers

Sol: $P(x)$: "x is a lion"

$Q(x)$: "x is fierce"

$R(x)$: "x drinks coffee"

$\forall x(P(x)\rightarrow Q(x))$

$\exists x(P(x)\wedge\neg R(x))$

$\exists x(Q(x)\wedge\neg R(x))$

Note that the second statement cannot be written as $\exists x(P(x)\rightarrow\neg R(x))$. The reason is that $P(x)\rightarrow\neg R(x)$ is true whenever x is not a lion, so $\exists x(P(x)\rightarrow\neg R(x))$ is true as long as there is ~~one~~ atleast one creature that is not a lion, even if every lion drinks coffee.

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Ex: "All humming birds are richly colored".
"No large birds live on honey".
"Birds that do not live on honey are dull in color".
"Hummingbirds are small"

The three statements are premises and fourth is a valid conclusion.

$P(x)$: x is a hummingbird
$Q(x)$: x is large
$R(x)$: x lives on honey.
$S(x)$: x is richly colored

Domain consists of all birds. Express the statements in argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$ and $S(x)$

Sol: $\forall x\,(P(x) \rightarrow S(x))$
$\neg \exists x\,(Q(x) \wedge R(x))$
$\forall x\,(\neg R(x) \rightarrow \neg S(x))$
$\forall x\,(P(x) \rightarrow \neg Q(x))$

Q) Translate these specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy" $L(j)$ is "Print job j is lost" and $Q(j)$ is "Print job j is queued".

a) $\exists p\,(F(p) \wedge B(p)) \rightarrow \exists j\, L(j)$: If there is a printer that is both out of service and busy, then some job has been lost.

(b) $\forall p\, B(p) \rightarrow \exists j\, Q(j)$: If every printer is busy, then there is a job in the queue.

(c) $\exists j\,(Q(j) \wedge L(j)) \rightarrow \exists p\, F(p)$ If there is job that is both queued and lost, then some printer is out of service.

(d) $(\forall p\, B(p) \wedge \forall j\, Q(j)) \rightarrow \exists j\, L(j)$
If every printer is busy and every job is queued, then some job is lost.

8) Let $P(x)$, $Q(x)$, $R(x)$ and $S(x)$ be the statements "x is a baby", "x is logical", "x is able to manage a crocodile" and "x is despised", respectively. Suppose that universe of discourse consists of all people. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$ and $S(x)$.

(a) Babies are illogical

(b) Nobody is despised who can mange a crocodile.

(c) Illogical persons are despised

(d) Babies cannot manage a crocodile.

(e) Does (d) follows from (a), (b) and (c)? If not is there a correct conclusion?

Sol: (a) $\forall x\,(P(x) \rightarrow \neg Q(x))$

(b) $\forall x\,(R(x) \longrightarrow \neg S(x))$

(c) $\forall x\,(\neg Q(x) \longrightarrow S(x))$

(d) $\forall x\,(P(x) \longrightarrow \neg R(x))$

(e) Suppose x is a baby. Then by first statement (premise) x is illogical. So by third premise x is despised. The second premise says that if x could mange the crocodile, then x would not be despised. Therefore x cannot manage a crocodile.

Yes (d) follows from (a), (b), (C)