22-01- 2022

Saturday, January 22, 2022 12:37 PM

* A, B are 2 independent events &

$P(A\cap\bar{B}) = \frac{3}{25}$

$P(\bar{A}\cap B) = \frac{8}{25}$

$\boxed{P(A) = ?}$

$P(A) = x$, $P(B) = y$.

[A, B are independent]

$P(A\cap\bar{B}) = P(A)P(\bar{B})$

$\Rightarrow x(1-y) = \frac{3}{25}$

$\Rightarrow x\cdot\left(1-\frac{8}{25-25x}\right) = \frac{3}{25}$

$P(\bar{A}\cap B) = \frac{8}{25}$

$\Rightarrow (1-x)y = \frac{8}{25}$.

$\Rightarrow y = \frac{8}{25(1-x)}$

(A) $1/25$ (B) $2/25$ (C) $3/25$ (D) $4/25$

$$x \times \frac{25-25x-8}{\cancel{25}(1-x)} = \frac{3}{\cancel{25}}$$

$$\Rightarrow x = \frac{1}{5}, \frac{3}{5}$$

$$\Rightarrow \frac{x(17-25x)}{1-x} = 3$$

$$\Rightarrow 17x - 25x^2 = 3-3x$$

$$\Rightarrow 25x^2 - 20x + 3 = 0$$

$$\Rightarrow 25x^2 - 15x - 5x + 3 = 0$$

$$\Rightarrow 5x(5x-3) - 1(5x-3) = 0$$

$x - xy = \frac{3}{25}$

$y - xy = \frac{8}{25}$

$y - x = \frac{1}{5}$

$P(A) < P(B)$

$x < y$

§ If A, B are independent events, then -

$P(A\cap B) = P(A)P(B)$

(a) $A, \bar{B}$ are independent

(b) $\bar{A}, B$ are independent

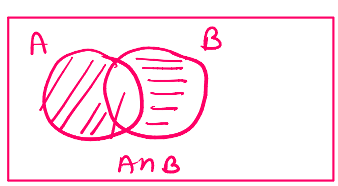
(c) $\bar{A}, \bar{B}$ are independent.

(a) $P(A\cap\bar{B}) = P(A) - P(A\cap B)$

$= P(A) - P(A)P(B)$

$= P(A)\{1-P(B)\}$

$= P(A)P(\bar{B})$



(b) $P(\bar{A}\cap B) = P(B) - P(A\cap B)$

(b) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
$= P(B) - P(A)P(B)$
$= (1 - P(A))\, P(B)$
$= P(\bar{A})\, P(B)$

(c) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$
$= 1 - P(A \cup B)$
$= 1 - \{P(A) + P(B) - P(A \cap B)\}$
$= 1 - P(A) - P(B) + P(A)P(B)$
$= 1\{1 - P(A)\} - P(B)\{1 - P(A)\}$
$= (1 - P(A))(1 - P(B)) = P(\bar{A})P(\bar{B})$ //

(1) Pairwise independent events:

$A_1, A_2, \ldots, A_n$

$P(A_i \cap A_j) = P(A_i)P(A_j) \quad \forall\ 1 \le i, j \le n,\ i \ne j.$

A_1, A_2, A_3 $\quad P(A_1 \cap A_2) = P(A_1)P(A_2)$
$P(A_2 \cap A_3) = P(A_2)P(A_3)$
$P(A_1 \cap A_3) = P(A_1)P(A_3)$

(2) Mutually independent events: $A_1, A_2, \ldots, A_n.$

$P(A_i \cap A_j) = P(A_i)P(A_j) \quad \forall\ 1 \le i, j \le n,\ i \ne j$
$P(A_i \cap A_j \cap A_k) = P(A_i)P(A_j)P(A_k) \ \forall\ 1 \le i, j, k \le n$
$i \ne j \ne k$
$\vdots$
$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1)P(A_2) \cdots P(A_n).$

Example 4·16. *A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn here is at least one ball of each colour.* **(Nagpur Univ. B.Sc., 1992)**

(A) 0·5275 (B) 0·6275 (C) 0·7275

(A) 0.5275 (B) 0.6275 (C) 0.7275 (D) 0.2275

	1 red	1 white	1 black
(1)	2	1	1
(2)	1	2	1
(3)	1	1	2

$${}^6C_2 \times {}^4C_1 \times {}^5C_1 = \frac{6!}{2! \times 4!} \times 4 \times 5 = 300$$

$${}^6C_1 \times {}^4C_2 \times {}^5C_1 = 6 \times 6 \times 5 = 180$$

$${}^6C_1 \times {}^4C_1 \times {}^5C_2 = 6 \times 4 \times 10 = 240$$

$$p = \frac{720}{{}^{15}C_4} = \frac{720}{\frac{15 \cdot \cancel{14} \cdot 13 \cdot \cancel{12}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}} = \frac{720}{7 \times 15 \times 13} = \frac{720}{1365}$$

$$= 0.5275$$

Example 4·18. *A problem in Statistics is given to the three students A,B and C whose chances of solving it are* 1/2, 3/4, *and* 1/4 *respectively.*

What is the probability that the problem will be solved if all of them try independently? **[Madurai Kamraj Univ. B.Sc.,1986; Delhi Univ. B.A.,1991]**

(A) $\frac{23}{32}$ (B) $\frac{25}{32}$ (C) $\frac{27}{32}$ (D) $\frac{29}{32}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) - P(A)P(B) + P(A)P(B)P(C)$$

$$= \frac{1}{2} + \frac{3}{4} + \frac{1}{4} - \frac{3}{8} - \frac{3}{16} - \frac{1}{8} + \frac{3}{32}$$

$$= \frac{16 + 24 + 8 - 12 - 6 - 4 + 3}{32} = \frac{48 - 22 + 3}{32}$$

$$= \frac{29}{32}$$

32

32

$= \frac{29}{32}$

(2)

$$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C})$$
$$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$$
$$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right)$$
$$= 1 - \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4} = 1 - \frac{3}{32} = \frac{29}{32}$$

Example 4·26. *The odds that person X speaks the truth are 3:2 and the odds that person Y speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point.*

Solⁿ: A : X speaks the truth $\bar{A}$: X lies
B : Y " " truth $\bar{B}$: Y lies

$P(A) = \frac{3}{3+2} = \frac{3}{5}$ $P(B) = \frac{5}{5+3} = \frac{5}{8}$

A, B

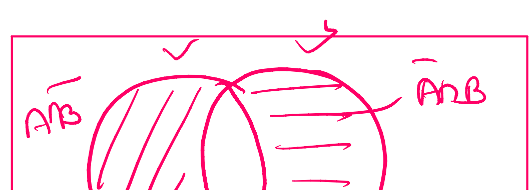
$P(A \cup B)$

$\Rightarrow P(\bar{A}) = \frac{2}{5}$ $P(\bar{B}) = \frac{3}{8}$.

Let E = event that X & Y will contradict each other.

i.e., E consists of 2 mutually exclusive events $\bar{A} \cap B$ & $A \cap \bar{B}$.

$$P(E) = P(\bar{A} \cap B) + P(A \cap \bar{B})$$
$$= P(\bar{A})\, P(B) + P(A)\, P(\bar{B})$$



$$= P(\bar{A})\,P(B) + P(A)\,P(\bar{B})$$

$$= \frac{2}{5} \times \frac{5}{8} + \frac{3}{5} \times \frac{3}{8}$$

$$= \frac{10}{40} + \frac{9}{40}$$

$$= \frac{19}{40} \quad = 0.475 \; //$$