24-02-2022

Thursday, February 24, 2022 11:58 AM

Unit-3

Discrete

pmf ① Binomial

pmf ② Poisson

pdf ③ Normal distribution

$n \to \infty$

$p \to 0$

* Normal distribution: (Continuous distribution)

1733 De-Moivre

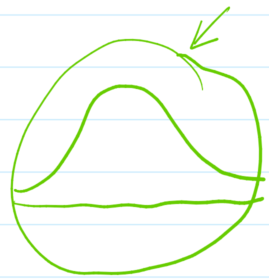
BD = limiting case

(1) n, the no. of trials is indefinitely large, $n \to \infty$ &

(2) neither p nor q is very small.

1809

Contin



d

pf

pmf

Σ

Conti

$\int$

Defn: A r.v X is said to follow normal distribution with parameters μ (called mean) and σ^2 (called variance) if its probability density function is given by –

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

or

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; -\infty < x < \infty, \ -\infty < \mu < \infty, \ \sigma > 0.$$

- When a r.v is normally distributed with mean μ and standard deviation σ, it is written as $X \sim N(\mu, \sigma^2)$.

§ Chief characteristics of Normal distribution:

The normal prob. curve with mean μ & S.D σ is given by –

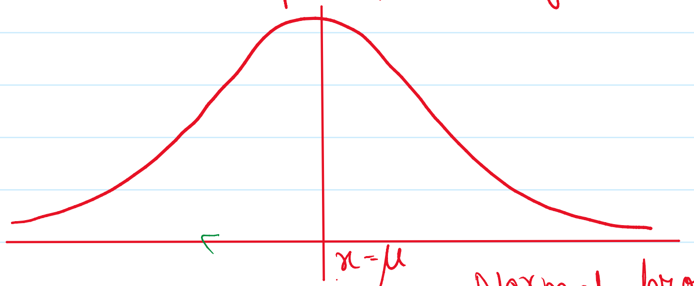
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty.$$

It has the foll^n properties:

$\sigma\sqrt{2\pi}$,

foll^n properties :

(i) The curve is bell-shaped & is symmetrical about the line $x=\mu$.



Normal prob. curve.

$x = -0.25$
$x = -1.25$

(ii) Mean, median, mode of the distribution coincide.

(iii) As x increases numerically, $f(x)$ decreases rapidly, the maximum probability occuring at the point $x=\mu$, and is given by –

$$f(x) = [p(x)]_{max} = \frac{1}{\sigma\sqrt{2\pi}} .$$

— x —

$$\boxed{Z = \frac{X - E(X)}{\sqrt{V(X)}}}$$

& If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X-\mu}{\sigma}$, is a standard normal variate with $E(Z) = 0$ and $Var(Z) = 1$ & we write $\underbrace{Z \sim N(0,1)}_{\text{st. normal variate.}}$

- The p.d.f of standard normal variate Z is given by –

$$\phi(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$\Rightarrow \boxed{\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} , \quad -\infty < z < \infty}$$

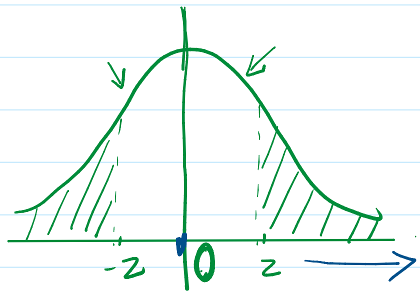
$$\Rightarrow \boxed{\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty < z < \infty}$$

and the corresponding distribution function $\Phi(z)$ is given by-

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \phi(u)\,du = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u^2/2}\,du.$$

Results :- (1) $\Phi(-z) = 1 - \Phi(z)$, $z > 0$

$$\Phi(-z) = P(Z \le -z) = P(Z \ge z) = 1 - P(Z \le z)$$
$$= 1 - \Phi(z).$$



$$\int_a^b f(x)\,dx = P(a \le X \le b) = P(a < X \le b) = P(a \le X < b) = P(a < X < b)$$

(2) The graph of $f(x)$ is famous bell-shaped curve. The top of the bell is directly above the mean μ. For large value of σ, the curve turns to flatten out & for small values of σ, it has a sharp peak.

- Normal distribution as a limiting case of B.D :

$$p(x) = {}^nC_x\, p^x q^{n-x} \quad ; x = 0, 1, 2, \dots, n.$$

Let us consider the standard binomial variate.

$$Z = \frac{X - E(X)}{\sqrt{V(X)}} = \frac{X - np}{\sqrt{npq}} \quad ; X = 0, 1, 2, \dots, n.$$

when $X = 0$, $Z = \dfrac{-np}{\sqrt{npq}} = -\sqrt{\dfrac{np}{q}}$ and

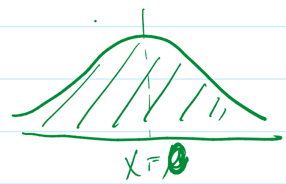
when $X = n$, $Z = \dfrac{n - np}{\ } = \dfrac{nq}{\ } = \sqrt{\dfrac{nq}{\ }}$

When $X = n$, $Z = \frac{n-np}{\sqrt{npq}} = \frac{nq}{\sqrt{npq}} = \sqrt{\frac{nq}{p}}$

Thus, in the limit as $n \to \infty$, $Z \to -\infty$ & $Z \to \infty$.

Hence, the distribution of X will be a continuous dist. over $-\infty$ to ∞.

- Mean = mode = median = μ = $E(x)$.



1

0.5

0.5