26-2-2022

Saturday, February 26, 2022 12:57 PM

Unit-3

Normal Distribution

§ Moment generating function of Normal distribution :

If $X \sim N(\mu, \sigma^2)$, then -

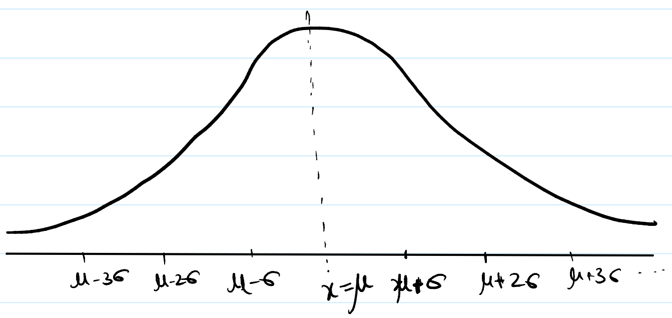
$$M_X(t) = e^{\mu t + \frac{t^2\sigma^2}{2}} = \exp\left(\mu t + \frac{t^2\sigma^2}{2}\right).$$

Pb Let $X \sim N(\mu, \sigma^2)$ with $\mu = 3, \sigma = 2$. Find the $M_X(t)$.

(a) e^{3t+3t^2} (c) e^{3t+2t^2}

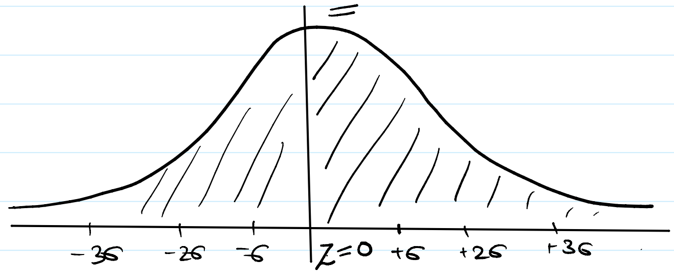
(b) e^{3t+4t^2} (d) e^{3t+t^2}

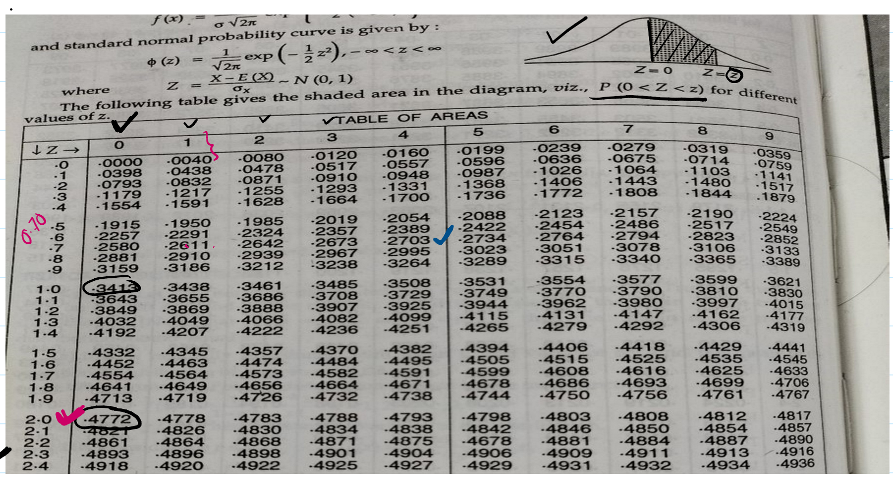
$$M_X(t) = e^{3t + \frac{t^2 2^2}{2}} = e^{3t+2t^2} \quad \text{Ans}$$



$X \to Z$

$\sigma = 0$





$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \ldots$

and standard normal probability curve is given by :

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right), -\infty < z < \infty$$

where $Z = \frac{X - E(X)}{\sigma_X} \sim N(0, 1)$

The following table gives the shaded area in the diagram, viz., $P(0 < Z < z)$ for different values of z.

Z=0 Z=z

TABLE OF AREAS

↓Z →	0	1	2	3	4	5	6	7	8	9
·0	·0000	·0040	·0080	·0120	·0160	·0199	·0239	·0279	·0319	·0359
·1	·0398	·0438	·0478	·0517	·0557	·0596	·0636	·0675	·0714	·0759
·2	·0793	·0832	·0871	·0910	·0948	·0987	·1026	·1064	·1103	·1141
·3	·1179	·1217	·1255	·1293	·1331	·1368	·1406	·1443	·1480	·1517
·4	·1554	·1591	·1628	·1664	·1700	·1736	·1772	·1808	·1844	·1879
·5	·1915	·1950	·1985	·2019	·2054	·2088	·2123	·2157	·2190	·2224
·6	·2257	·2291	·2324	·2357	·2389	·2422	·2454	·2486	·2517	·2549
·7	·2580	·2611	·2642	·2673	·2703	·2734	·2764	·2794	·2823	·2852
·8	·2881	·2910	·2939	·2967	·2995	·3023	·3051	·3078	·3106	·3133
·9	·3159	·3186	·3212	·3238	·3264	·3289	·3315	·3340	·3365	·3389
1·0	·3413	·3438	·3461	·3485	·3508	·3531	·3554	·3577	·3599	·3621
1·1	·3643	·3655	·3686	·3708	·3729	·3749	·3770	·3790	·3810	·3830
1·2	·3849	·3869	·3888	·3907	·3925	·3944	·3962	·3980	·3997	·4015
1·3	·4032	·4049	·4066	·4082	·4099	·4115	·4131	·4147	·4162	·4177
1·4	·4192	·4207	·4222	·4236	·4251	·4265	·4279	·4292	·4306	·4319
1·5	·4332	·4345	·4357	·4370	·4382	·4394	·4406	·4418	·4429	·4441
1·6	·4452	·4463	·4474	·4484	·4495	·4505	·4515	·4525	·4535	·4545
1·7	·4554	·4564	·4573	·4582	·4591	·4599	·4608	·4616	·4625	·4633
1·8	·4641	·4649	·4656	·4664	·4671	·4678	·4686	·4693	·4699	·4706
1·9	·4713	·4719	·4726	·4732	·4738	·4744	·4750	·4756	·4761	·4767
2·0	·4772	·4778	·4783	·4788	·4793	·4798	·4803	·4808	·4812	·4817
2·1	·4821	·4826	·4830	·4834	·4838	·4842	·4846	·4850	·4854	·4857
2·2	·4861	·4864	·4868	·4871	·4875	·4678	·4881	·4884	·4887	·4890
2·3	·4893	·4896	·4898	·4901	·4904	·4906	·4909	·4911	·4913	·4916
2·4	·4918	·4920	·4922	·4925	·4927	·4929	·4931	·4932	·4934	·4936

0.70

$z = 0 \quad z = 3$

$-\infty < Z < 0$

1 0.5

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$0 < Z < \infty$

0.5

$P(0 \leq Z \leq 3)$

$\boxed{.2703}$

$P(0 < Z < 0.74)$

1·8	·4641	·4649	·4656	·4664	·4671	·4678	·4686	·4693	·4699	·4706
1·9	·4713	·4719	·4726	·4732	·4738	·4744	·4750	·4756	·4761	·4767
2·0	·4772	·4778	·4783	·4788	·4793	·4798	·4803	·4808	·4812	·4817
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2·3	·4893	·4896	·4898	·4901	·4904	·4906	·4909	·4911	·4913	·4916
2·4	·4918	·4920	·4922	·4925	·4927	·4929	·4931	·4932	·4934	·4936
2·5	·4938	·4940	·4941	·4943	·4945	·4946	·4948	·4959	·4951	·4952
2·6	·4953	·4955	·4956	·4957	·4959	·4960	·4961	·4962	·4963	·4964
2·7	·4965	·4966	·4967	·4968	·4969	·4970	·4971	·4972	·4973	·4974
2·8	·4974	·4975	·4976	·4977	·4977	·4978	·4979	·4979	·4980	·4981
2·9	·4981	·4982	·4982	·4983	·4984	·4984	·4985	·4985	·4986	·4986
3·0	·4987	·4987	·4987	·4988	·4988	·4989	·4989	·4989	·4990	·4990
3·1	·4990	·4991	·4991	·4991	·4992	·4992	·4992	·4992	·4993	·4993
3·2	·4993	·4993	·4994	·4994	·4994	·4994	·4994	·4995	·4995	·4995
3·3	·4995	·4995	·4995	·4996	·4996	·4996	·4996	·4996	·4996	·4997
3·4	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4998
3·5	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998
3·6	·4998	·4998	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·7	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·9	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000

$P(0 < Z < 2\cdot3) = 0\cdot4893$

0·5

$P(0 \leq Z \leq 3)$
$= 0\cdot4987$

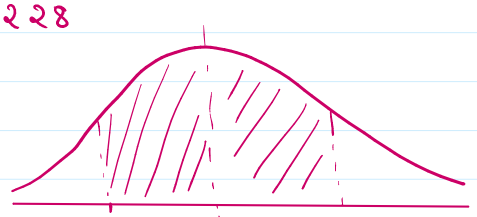
$P(1 \leq Z \leq 2)$
$= P(0 \leq Z \leq 2) - P(0 \leq Z \leq 1)$
$= 0\cdot4772 - 0\cdot3413$
$= \underline{0\cdot1359}$

Q. Let $X \sim N(\mu, \sigma^2)$, $\mu = 12$, $\sigma = 4$ $\sigma^2 = \text{var}(x)$.
Find $P(X \geq 20)$.

Sol^n: when $X = 20$, then $Z = \dfrac{X - E(X)}{\sqrt{v(x)}} = \dfrac{X - 12}{4} = \dfrac{20-12}{4} = 2.$

$\therefore P(X \geq 20) = P(Z \geq 2) = 0\cdot5 - P(0 < Z < 2)$
$= 0\cdot5 - 0\cdot4772$
$= 0\cdot0228$

② $P(0 \leq X \leq 12)$



when $X = 0$, $Z = \dfrac{0 - 12}{4} = -3$

when $X = 12$, $Z = \dfrac{12 - 12}{4} = 0$

$\therefore P(0 \leq X \leq 12) = P(-3 < Z < 0)$
$= P(0 < Z < 3)$
$= 0\cdot4987$

$Z = \dfrac{X - 12}{4}$
$\Rightarrow 4Z = X - 12$
$\Rightarrow X = 4Z + 12$

③ Find x such that $P(X > x) = 0\cdot24$.

$P(X > x) = 0\cdot24$
$\Rightarrow P(Z > z) = 0\cdot24$
$\Rightarrow 0\cdot5 - P(0 < Z < z) = 0\cdot24$
$\Rightarrow -P(Z < z) = -0\cdot26$
$P(0 < Z < z) = 0\cdot26$
$\Rightarrow$
$\Rightarrow z = 0\cdot72$
$\Rightarrow \dfrac{x - 12}{4} = 0\cdot72$
$\Rightarrow x - 12 = 2\cdot88$
$\Rightarrow x = 14\cdot88$ Ans

$X \quad \dfrac{X - 12}{4}$

$x \quad \dfrac{x - 12}{4}$

$\Rightarrow x - 12 = 2.88$

$\Rightarrow x = 14.88$ Ans

- Area under the normal curve is always 1.

§ Recurrence relation for the central moments:

$$\mu_{2n} = \sigma^2 (2n-1) \mu_{2n-2}$$

Eg: Let $\mu_2 = 2$, $\sigma^2 = 3$, find μ_6.

(a) 270 ✓
(b) 18
(c) 90
(d) 360

$\mu_4 = 3(2 \cdot 2 - 1)\mu_2$
$= 3 \times 3 \times 2$
$= 18$

$\mu_6 = 3(2 \cdot 3 - 1) 18$
$= 3 \times 5 \times 18$
$= 270$

Q $X \sim N(\mu, \sigma^2)$ with $\mu = 30$ & $\sigma = 5$. Find $P(26 \le X \le 40)$.

(a) 0.7653 ✓
(b) 0.7563
(c) 0.5375
(d) 0.6537

$X = 26$, $Z = \frac{26-30}{5} = \frac{-4}{5} = -0.8$

$X = 40$, $Z = \frac{10}{5} = 2$.

$\therefore P(26 \le X \le 40) = P(-0.8 < Z < 2)$
$= P(-0.8 < Z < 0) + P(0 < Z < 2)$
$= P(0 < Z < 0.8) + P(0 < Z < 2)$
$= 0.2881 + 0.4772$
$= 0.7653$

———x———

Unit-3

- Normal distribution is a limiting case of Poisson distribution as $\lambda \to \infty$.

—x—