01-02-2022

Tuesday, February 1, 2022 9:57 AM

H/A

Example 4·33. *In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C?*

Solⁿ: Let us define the events:

E_1 : event that a bolt selected at random is manufactured by machine A,

E_2 : " " " " " " " " " B,

E_3 : " " " " " " " " " C,

& E : event that the selected bolt is defected.

$P(E_1) = 0.25$, $P(E_2) = 0.35$, $P(E_3) = 0.40$

$P(E|E_1) = 0.05$, $P(E|E_2) = 0.04$, $P(E|E_3) = 0.02$

$$P(E_1|E) = \frac{P(E_1)P(E/E_1)}{\sum_{i=1}^{3} P(E_i)P(E/E_i)} = \frac{P(E_1)P(E/E_1)}{P(E)}, \quad P(E) = 0.25\times 0.05 + 0.35\times 0.04 + 0.40\times 0.02$$

$$= 0.0125 + 0.0140 + 0.0080 = 0.0345$$

$$= \frac{0.025\times 0.05}{0.0345} = \frac{125}{345} = \frac{25}{69}$$

$$P(E_2|E) = \frac{P(E_2)P(E|E_2)}{P(E)} = \frac{0.0140}{0.0345} = \frac{140}{345} = \frac{28}{69}$$

$$P(E_3|E) = \frac{P(E_3)P(E|E_3)}{P(E)} = \frac{0.0080}{0.0345} = \frac{16}{69}$$

—— x ——

CA I   8th Feb   10-11 AM   Unit-I

20 questions   10 of 1 mark (−0.25)   25% −ve marking.

10 of 2 mark (−0.50)

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Unit II

Random variables and its Characterization : Discrete and continuous random variables(in one dimension) and their distribution functions, Moments and Moment generating function of a random

Unit II

Random variables and its Characterization : Discrete and continuous random variables(in one dimension) and their distribution functions, Moments and Moment generating function of a random variable, Expectation and Variance of a random variable

mean

Unit-II

§ Random variables : A function whose domain is the sample space of the random experiment & range is the subset of set of real numbers is called a random variable.

Eg: Consider the exp. of tossing a coin.
Let X be a random variable defined as :-
X = no. of heads appeared.

$X : \{H, T\} \to \{0, 1\} \subset \mathbb{R}$.

	H		T
X	1		0
P(X)	$\frac{1}{2}$	+	$\frac{1}{2}$

Tossing two coins.
X = no. of head appears.

$X : \{HH, HT, TH, TT\} \to \{0, 1, 2\}$

X :	0		1		2
	TT		HT, TH		HH
P(X) :	$\frac{1}{4}$	+	$\frac{2}{4} = \frac{1}{2}$	+	$\frac{1}{4}$

③ Rolling two die together & noting the sum of the no. appeared.
X = sum of the no.s.

X :	2	3	4	5	6	7	8	9	10	11	12
P(X) :	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

= 1.

(1,1) (1,3)
(1,2) (2,2)
(2,1) (3,1)

$$F(5) = P(X \leq 5) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$$
$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} = \frac{10}{36} = \frac{5}{18} \text{ //}$$

Q Let a pair of die is thrown & the nos. are noted.
X = sum is noted.

Then, $P(X = 3 \text{ or } 5 \text{ or } 7 \text{ or } 9) = P(X=3) + P(X=5) + P(X=7) + P(X=9)$
$$= \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{4}{36}$$
$$= \frac{16}{36} = \frac{4}{9}$$

$$= \frac{16}{36} = \frac{4}{9}$$

Theorem :- Let X_1, X_2 are random variables and c is a constant, then -

① cX_1 is a random variable.

② $X_1 + X_2$ is a random variable.

③ $X_1 X_2$ is a random variable.

- If c_1, c_2 are cons. and X_1, X_2 are random variables. then $c_1X_1 + c_2X_2$ is also a random variable.

In fact $-X_1$ is also a random variable.

§ **Distribution function of a random variable**: Let X be a random variable. The function F defined for all real x by -

$F(x) = P(X \leq x)$, $-\infty < x < \infty$, is called the distribution function of the r.v X.

Also, it is known as cumulative d.f.

Clearly, the domain of the d.f is $(-\infty, \infty)$ & range is $[0,1]$.

Q Let X gives the no. of tails on tossing 3 coins together. Find the distribution function $F(2)$.

HHH
HHT, HTH, THH
HTT, THT, TTH
TTT

Sol^n:

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$

$= 7$

$8 \quad 8 \cdot 8$

$= \frac{7}{8}$ //

$F(3) = P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 1$ //

§ Properties of Distribution function:-

1) If F is the d.f of the r.v X and if $a<b$, then

$$P(a<X\le b) = F(b) - F(a).$$

Pf: The events $a<X\le b$ & $X \le a$ are disjoint and their union is $X \le b$. Hence, using addition theorem of prob -

$$P(a<X\le b) + P(X\le a) = P(X\le b)$$

$$\Rightarrow P(a<X\le b) = P(X\le b) - P(X\le a)$$

$$\Rightarrow P(a<X\le b) = F(b) - F(a) \quad //$$

2) If F is the d.f of one dimensional r.v X, then -

(a) $0 \le F(x) \le 1$,

(b) If $x<y$, then $F(x) \le F(y)$.

$x<y \Rightarrow F(x) < F(y)$

$F(3)=1$

$y=3$

X : no. of tail appeared

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

$x=2$, $y=2.5$ $\quad x<y$

$F(2) = P(X\le 2) = \frac{7}{8}$

$F(2.5) = P(X \le 2.5) = \frac{7}{8}$.

$x<y \Rightarrow F(x) = F(y)$

$x=2$, $y=2.5$, $z=3$.

$x<y$

$F(x) = F(y)$

$F(2) = \frac{7}{8}$

$F(2.5) = \frac{7}{8}$

$x<z$

$F(x) < F(z)$

$F(2) = \frac{7}{8}$

$F(3) = 1$

* c.d.f is monotonically non-decreasing & its value lies between 0 & 1.

3. If F is d.f. of one-dimensional r.v X, then –

$$F(-\infty) = \lim_{x \to -\infty} F(x) = 0 \quad \& \quad F(\infty) = \lim_{x \to \infty} F(x) = 1.$$

$$F(-\infty) = P(X \leq -\infty) = 0$$

$$F(\infty) = P(X \leq \infty) = 1$$

Random variables
prob. distribution
distribution func & its probabilities

———— x ————