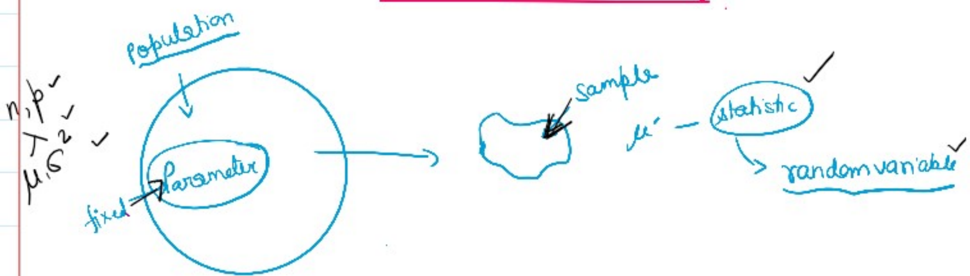
Unit-5 (Hypothesis testing)

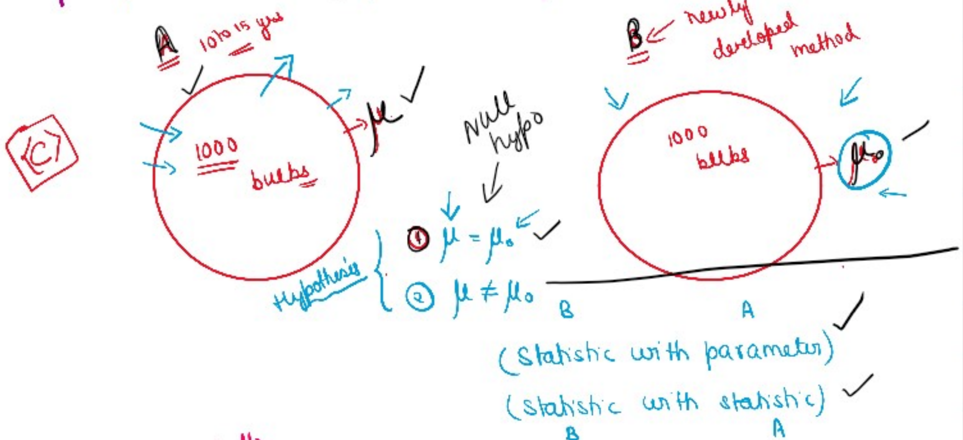
Random sampling

[redacted]

Unit-5 (Hypothesis testing)



Statistical hypothesis: It is some statement or assertion about a population or equivalently about the probability distribution characterising a population, which we want to verify on the basis of information available from a sample.



Alternative hypothesis

$\mu = \mu_0$

$\mu = \mu_0$
$\mu \neq \mu_0$

$\mu = \mu_0$ $\quad \mu ,\ \mu \neq \mu_0$

§ Test of a statistical hypothesis: It is a two-action decision problem after the experimental sample values have been obtained, the two actions being the acceptance or rejection of the hypothesis under consideration.

Two Types of Hypothesis:

(1) Null hypothesis (H_0)

(2) Alternative hypothesis (H_1 or H_A)

$H_0 : \mu = \mu_0 \longrightarrow$ Reject

$H_1 : \mu \neq \mu_0 \quad \mu > \mu_0 \quad \mu < \mu_0$

(1) Null hypothesis: — The neutral or non-committal of the statistician or decision-maker before the sample observations are taken is the keynote of null hypothesis.

For eg:, in case of light bulbs manufactured by method A & B, let μ & μ_0 denotes the resp. mean life.

Then, the null hypothesis is

$$H_0 : \mu = \mu_0 .$$

(2) Alternative hypothesis: It is denoted by H_1 or H_A & is true only

(2) Alternative hypothesis: It is denoted by H_1 or H_A & is true only when null hypothesis is rejected.

In the example of light bulbs, the alternative hypothesis is

H_1: $\mu \neq \mu_0$ or $\mu > \mu_0$ or $\mu < \mu_0$.

Eg: H_0: Husband is not guilty, innocent.

$\boxed{\text{Husband has murdered his wife.}}$

God

→ $\boxed{\text{tomorrow}}$ decision

A: Accepting H_0 → making a murder free.

B: Rejecting H_0 → $\boxed{\text{not favourable}}$

Type Error 2

§ Two types of errors:— H_0:

Type I error: Rejecting H_0; even if it is true.

Type II error: Accepting H_0; even if it is false.

The error of rejecting H_0 (accepting H_1) when H_0 is true is called Type I error and the error of accepting H_0 when H_0 is false (H_1 is true) is called type II error.

α = probability of type I error
= prob. of rejecting H_0 when it is true.

0

0

β = prob. of type II error

= prob. of accepting H_0 when it is false.

$\boxed{\alpha+\beta=1}$ $\boxed{\alpha \to 0}$. $\beta \to 1$.

*Types of tests: $\left\{\begin{array}{l}\text{1-tailed test}\\ \text{2-tailed test}\end{array}\right\}$ decided by alternative hypothesis.

$\mu \neq \mu_0 \} \to$ 2-tailed test

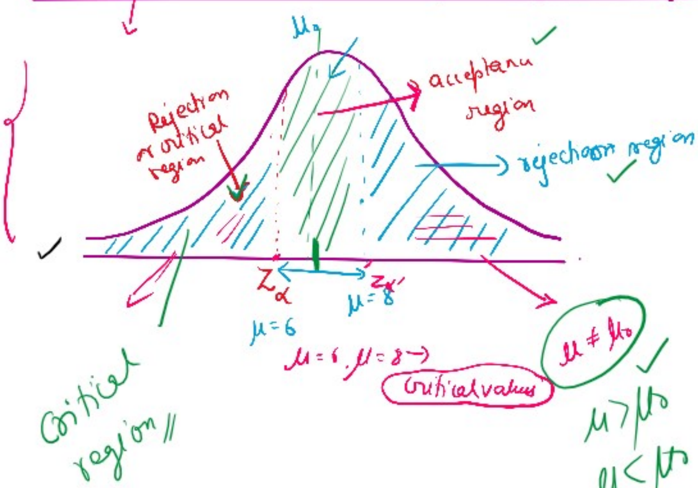
$\left.\begin{array}{l}\mu > \mu_0\\ \mu < \mu_0\end{array}\right\} \to$ 1-tailed test

$\mu \neq \mu_0$ Two tailed tests

$\mu > \mu_0$ One tailed

$\mu < \mu_0$ One tailed

§ Critical values and critical region (Rejection region):



$6 < \mu < 8$

$H_0 : \mu = \mu_0$

$H_A : \mu \neq \mu_0$

$\mu > \mu_0$ $\mu < \mu_0$

Lecture 35

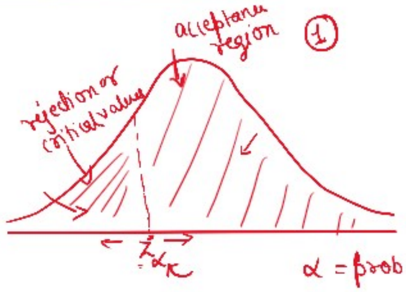
[redacted] Unit-5 (Hypothesis testing)

§ Critical values & critical region:

acceptance region (1) (2)

$\mu_0 \neq \mu$

$\mu_0 \neq \mu$



① H_0 : accepted & H_1 : rejected

② H_0 : rejected & H_1 : accepted

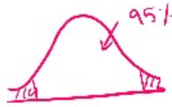
α = prob. of type 1 error. $\alpha \to 0$.

* If the observed value of statistic falls in the acceptance region, then H_0 is accepted & H_1 is rejected. On the other hand, if the observed value of statistic falls in the rejection region, then H_0 is rejected & H_1 is accepted.

* Level of significance : α, the prob. of type 1 error, is known as the level of significance of the test. Also known as the size of the critical region.

5% → H_0 is rejected

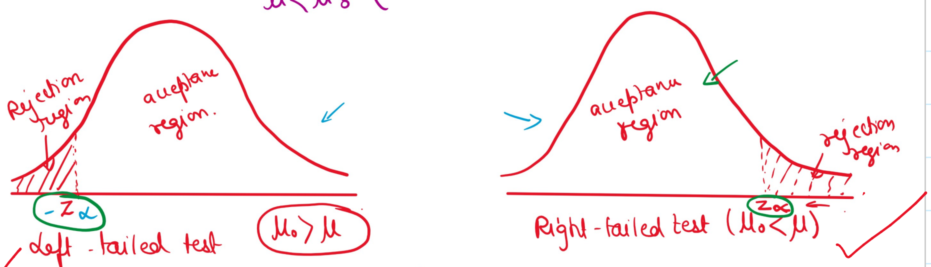
1% → H_0 is rejected.



$0 \leq \alpha \leq 1$

* Type of test: $\mu \neq \mu_0$ (Two-tailed test)

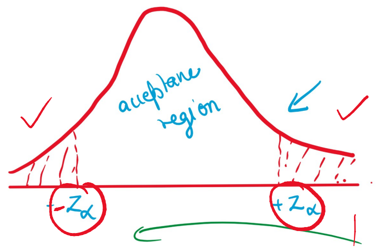
$\mu > \mu_0$, $\mu < \mu_0$ (One-tailed test)



$x^2 > 9$

$\Rightarrow x < -3$

$x > 3$

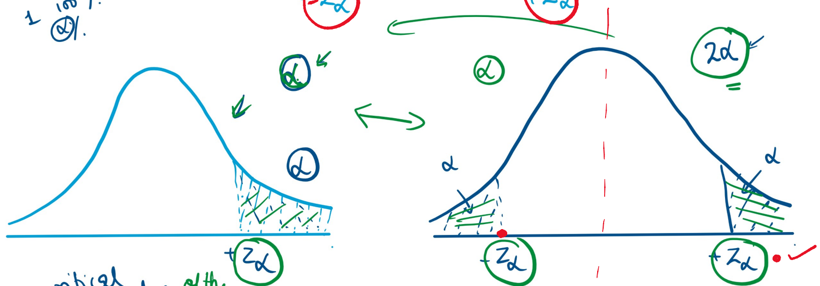


$\mu = \mu_0$

$\mu \neq \mu_0$

1 100%

α%



Z_α

critical value of the

The rejection region in a one test with level of significance α = the critical value in a two-tailed test with level of significance 2α.

Q) what does the error β stands for :-

a) Accepting H_0 when true

(b) Accepting H_0 when false

(c) Rejecting H_0 when true

(d) Rejecting H_1 when false

α = prob. of type 1 error = prob. of rejecting H_0 when it is true.

α = prob. of type 1 error = prob. of rejecting H_0 when it is true.

$\alpha \rightarrow$ level of significance.

(5%) $\alpha = 0.05$.

1

* Steps in performing a test:

(i) Set up null hypothesis.

(ii) Set up alternative hypothesis

(iii) Level of significance (α)

(iv) Calculation of observing value. (test-statistic)

(v) Conclusion.

(iv) Calculation of observing value. (test-statistic)

(v) Conclusion.

Z-test:-

Conditions for Z-test

(i) Sample size should be large (i.e $n>30$)

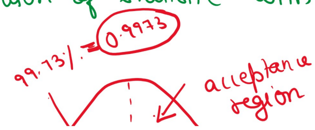
(ii) Parent population from which sample is drawn should follow Normal distributions.

Applications

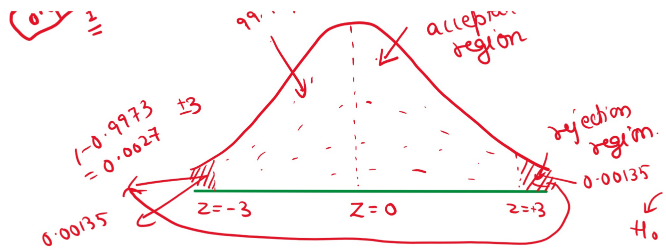
(1) comparison of statistic with parameter.

(2) comparison of statistic with another another.

0.9973
1



$z = \pm 3$



$z = \pm 3$

$Z_\alpha = ?$

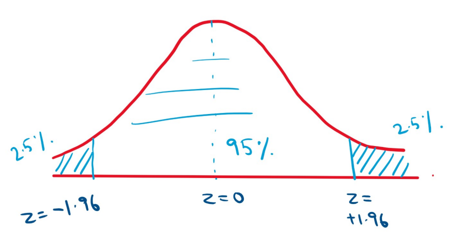
$Z_\alpha < -3$ or $Z_\alpha > 3$

H_0 : rejected

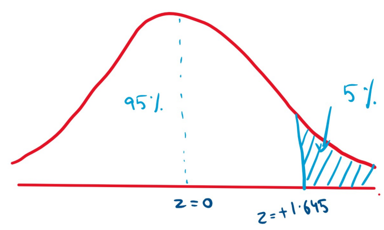
$\alpha = 0.0027$

* Level of significance, $\alpha = 0.05$ or 5%.

Two tailed
$\alpha = 5\%$



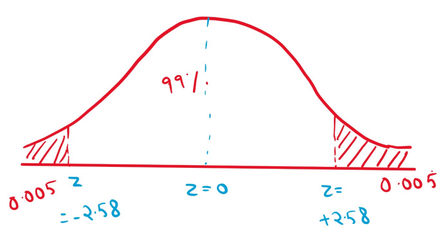
One-tailed
$\alpha = 5\%$



* Level of significance, $\alpha = 1\%$ or 0.01

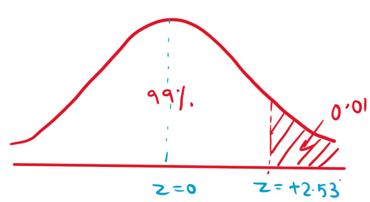
Two-tailed

$\alpha = 1\%$



One-tailed

$\alpha = 1\%$



* **Procedure:**
  (i) set up Null hypothesis
  (ii) set up alternative hypothesis
  (iii) level of significance
  (iv) Test statistic (Z)
  (v) **Conclusion** :- comparison of Z & Z_α.

actual parameter

§ **Proportion and probability:**

500 mangoes

50 bad

$\frac{50}{500} = \frac{1}{10}$

$\frac{50}{500} = \frac{1}{10}$

$= 0.1$

$\frac{1}{10}$th proportion is bad.

10 1%