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Unit-6 (Correlation & Regression)

§ $r(aX+b, cY+d) = \frac{ac}{|ac|} r(X,Y)$.

- If a, c are having same sign, then $r(aX+b, cY+d) = +r(X,Y)$
- If a, c are having opp. sign, then $r(aX+b, cY+d) = -r(X,Y)$.

- $\boxed{V(aX+bY) = a^2 V(X) + b^2 V(Y) + 2ab\, \mathrm{cov}(X,Y).}$

X, Y are ind. variables

If X & Y are ind. variables, $\mathrm{cov}(X,Y) = 0$

$\Rightarrow V(aX+bY) = a^2 V(X) + b^2 V(Y)$

Q) Let X & Y be two random variables such that $\sigma_X = 1$, $\sigma_Y = 2$ & $\mathrm{cov}(X,Y) = \frac{3}{2}$. Find $V(2X+3Y)$.

(a) 40

(b) 48

(c) 50

(d) 58

$$V(2X+3Y) = 2^2 V(X) + 3^2 V(Y) + 2\cdot2\cdot3\, \mathrm{cov}(X,Y)$$
$$= 2^2\cdot 1^2 + 3^2\cdot 2^2 + 12 \times \frac{3}{2}$$
$$= 4 + 36 + 18$$
$$= 58$$

Q) Calculate the correlation co-efficient for the foll^n heights (inches) of fathers (X) and their sons (Y)

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

y : 67 68 65 68 72 72 69 71

$$u = \frac{x-68}{1} \qquad v = \frac{y-69}{1}$$

x	y	u	v	u^2	v^2	uv
65	67	-3	-2	9	4	6
66	68	-2	-1	4	1	2
67	65	-1	-4	1	16	4
67	68	-1	-1	1	1	1
68	72	0	3	0	9	0
69	72	1	3	1	9	3
70	69	2	0	4	0	0
72	71	4	2	16	4	8
		$\Sigma u = 0$	$\Sigma v = 0$	$\Sigma u^2 = 36$	$\Sigma v^2 = 44$	$\Sigma uv = 24$

$$\bar{u} = \frac{1}{8}\Sigma u = \frac{1}{8}\times 0 = 0, \qquad \bar{v} = \frac{1}{8}\Sigma v = \frac{1}{8}\times 0 = 0$$

$$Cov(u,v) = \frac{1}{8}\Sigma uv - \bar{u}\bar{v} = \frac{1}{8}\times 24 - 0 = 3$$

$$\sigma_u^2 = \frac{1}{n}\Sigma u^2 - \bar{u}^2 = \frac{1}{8}\times 36 - 0 = 4.5$$

$$\sigma_v^2 = \frac{1}{n}\Sigma v^2 - \bar{v}^2 = \frac{1}{8}\times 44 - 0 = \frac{44}{8} = 5.5$$

$$\therefore r_{uv} = \frac{Cov(u,v)}{\sigma_u \sigma_v} = \frac{3}{\sqrt{4.5}\sqrt{5.5}} = \frac{3}{\sqrt{24.75}} = \frac{3}{4.974} = 0.603$$

Since the value r_{uv} is equal to r_{xy}, so $r(x,y) = 0.603$.

Thus, the corrⁿ is +ve but not perfect.

Q) A computer while calculating correlation co-effecient betⁿ x & y from 25 pairs of observations obtained the follⁿ results :-

Q) A computer while calculating correlation co-efficient bet x & y from 25 pairs of observations obtained the foll^n results :-

$n = 25$, $\Sigma X = 125$, $\Sigma X^2 = 650$, $\Sigma Y = 100$, $\Sigma Y^2 = 460$, $\Sigma XY = 508$.

It was, however, later discovered at the time of checking that he had copied down 2 pairs as

X	Y
✓ 6	14
✓ 8	6

while the correct values are

X	Y
✓ 8	12
✓ 6	8

obtain the correct value of r_{XY}.

Sol^n:

$$\Sigma X = 125 - 6 - 8 + 8 + 6 = 125$$
$$\Sigma Y = 100 - 14 - 6 + 12 + 8 = 100$$
$$\Sigma X^2 = 650 - 6^2 - 8^2 + 8^2 + 6^2 = 650$$
$$\Sigma Y^2 = 460 - 14^2 - 6^2 + 12^2 + 8^2 = 436$$
$$\Sigma XY = 508 - 84 - 48 + 96 + 48 = 520$$

$$\bar{X} = \frac{1}{n}\Sigma X = \frac{1}{25} \times 125 = 5$$
$$\sigma_X^2 = \frac{1}{25} 650 - 25 = 26 - 25 = 1$$
$$\bar{Y} = \frac{1}{n}\Sigma Y = \frac{1}{25} \times 100 = 4$$
$$\sigma_Y^2 = \frac{1}{25} \times 436 - 16 = \frac{36}{25}$$

$$\text{cov}(X,Y) = \frac{1}{25} \times 520 - 20$$
$$= \frac{20}{25} = \frac{4}{5}$$

$$\therefore r_{XY} = \frac{4/5}{\sqrt{1}\sqrt{36/25}} = \frac{4}{5} \times \frac{5}{6} = \frac{2}{3} = 0.67 \;//$$ Ans

H/W

Q) X_1 and X_2 are independent variables with means 5 and 10 resp. and standard deviations 2 and 3 resp.

Find $r(U,V)$, where

$$U = 3X_1 + 4X_2 \;\&\; V = 3X_1 - X_2$$