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Unit-2

§ Variance of a r.v X:

For a discrete random variable:

$$V(X) = E(X^2) - \{E(X)\}^2 \qquad p(x) - \text{p.m.f}$$

$$= \sum_x x^2 p(x) - \left(\sum_x x\,p(x)\right)^2$$

For a continuous random variable:

$$V(X) = E(X^2) - \{E(X)\}^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left\{\int_{-\infty}^{\infty} x f(x)\,dx\right\}^2 \qquad f(x) - \text{p.d.f}$$

Q Let X = no. of heads appeared on the toss of 2 coins. Find var(X).

Sol^n:

X	0	1	2
$p(X=x)$	$1/4$	$1/2$	$1/4$

$$E(X) = \sum_x x\,p(x) = 0\times\frac{1}{4} + 1\times\frac{1}{2} + 2\times\frac{1}{4} = 1.$$

$$E(X^2) = \sum_x x^2 p(x) = 0^2\times\frac{1}{4} + 1^2\times\frac{1}{2} \times 2^2\times\frac{1}{4} = 0 + \frac{1}{2} + 1 = \frac{3}{2}$$

$$\therefore \quad Var(X) = E(X^2) - \{E(X)\}^2$$

$$= \frac{3}{2} - 1^2$$

$$= \frac{1}{2} \,//$$

Q X = no. of tails appeared on the toss of 3 coins. Find var(X).

X	0	1	2	3
$p(X=x)$	$1/8$	$3/8$	$3/8$	$1/8$

$$E(X) = 0 + \frac{3}{8} + \frac{3}{4} + \frac{3}{8} = \frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2}$$

$E(X) = 0 + \frac{3}{8} + \frac{3}{4} + \frac{3}{8} = \frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2}$

$E(X^2) = \frac{3}{8} + \frac{3}{2} + \frac{9}{8} = \frac{3+12+9}{8} = \frac{24}{8} = 3$

$V(X) = E(X^2) - (E(X))^2 = 3 - \frac{9}{4} = \frac{3}{4}$

Properties of Variance:

(1) If X is a random variable, then −

$$V(aX+b) = a^2 V(X)$$

Pf:

$$\begin{aligned}
V(aX+b) &= E\{(aX+b)^2\} - \{E(aX+b)\}^2 \\
&= E(a^2X^2 + 2abX + b^2) - (aE(X) + b)^2 \\
&= a^2E(X^2) + \cancel{2ab}E(X) + \cancel{b^2} - a^2\{E(X)\}^2 - \cancel{2ab}E(x) - \cancel{b^2} \\
&= a^2\{E(X^2) - \{E(X)\}^2\} \\
&= a^2 V(X)
\end{aligned}$$

(2) If $b = 0$, then $V(aX+b) = V(aX) = a^2 V(X)$

(3) If $a = 0$, then $V(aX+b) = V(b) = 0 \Rightarrow$ Var of a cons. is zero.

(4) If $a = 1$, then $V(aX+b) = V(X+b) = V(X)$.

(5) $V(X_1 \pm X_2) = V(X_1) \pm V(X_2)$, if X_1, X_2 are independent r.v.s.

(6) $V(X_1 + X_2 + \dots + X_n) = V(X_1) + V(X_2) + \dots + V(X_n)$, X_i's are independent r.v.s

Q Let X be a r.v with the foll^n prob. dist.:

X	-3	6	9
$P(X=x)$	$1/6$	$1/2$	$1/3$

Find $V(X)$.

(a) $5/4$

(b) $25/$

$E(X) = -\frac{1}{2} + 3 + 3 = \frac{11}{2}$

$E(X^2) = 9 \times \frac{1}{6} + 36 \times \frac{1}{2} + 81 \times \frac{1}{3}$

$= \frac{3}{2} + 18 + 27$

(a) $^{5}/_{4}$

(b) $^{25}/_{4}$

(c) $^{45}/_{4}$

(d) $^{65}/_{4}$

$$E(x^2) = 9 \times \frac{1}{6} + 36 \times \frac{1}{2} + 81 \times \frac{1}{3}$$
$$= \frac{3}{2} + 18 + 27$$
$$= \frac{93}{2}$$

$$V(x) = \frac{93}{2} - \frac{121}{4} = \frac{186 - 121}{4} = \frac{65}{4}$$

$$V(2x) = 4V(x) = 4 \times {}^{65}/_{4} = 65.$$

$$V(2x+5) = 4V(x) = 65$$

H/A

Pb Two unbiased dice are thrown and the sum of the no.s is noted. Find the expectation & variance.

X = sum