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Unit-2

Q A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$3k$	$2k$	k^2	$2k^2$	$7k^2+k$

(i) Find the value of k.

(a) 1 (b) -1 (c) $\frac{1}{10}$ (d) $-\frac{1}{10}$

Sol^n:

$0+k+2k+3k+2k+k^2+2k^2+7k^2+k=1$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0$

$\Rightarrow (10k-1)(k+1)=0$

$\Rightarrow k=\frac{1}{10}$ or $k=-1$ (prob. cannot be -ve)

$\therefore k = 1/10$

$\frac{7}{100}+\frac{1}{10}$

X	0	1	2	3	4	5	6	7
P(X)	0	1/10	2/10	3/10	2/10	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

$\boxed{\frac{8}{10}+\frac{20}{100} \quad \frac{4}{5}+\frac{1}{5}=1}$

Q Evalue $P(X<6)$ and $P(X\geq 6)$.

(a) $\frac{81}{100}, \frac{81}{100}$

(b) $\frac{19}{100}, \frac{81}{100}$

(c) $\frac{19}{100}, \frac{19}{100}$

(d) $\frac{81}{100}, \frac{19}{100}$

$P(X<6) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0+\frac{1}{10}+\frac{2}{10}+\frac{3}{10}+\frac{2}{10}+\frac{1}{100}$

$= \frac{8}{10}+\frac{1}{100}$

$= \frac{81}{100}$

$P(X\geq 6) = 1-P(X<6) = 1-\frac{81}{100} = \frac{19}{100}$ //

(iii) If $P(X\leq a) > \frac{1}{2}$, find the min. value of a.

0, 1, . . . , 7

X	0	1	2	3	4	5	6	7

X	0	1	2	3	4	5	6	7
$P(x)$	0	$1/10$	$2/10$	$3/10$	$2/10$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

(a) 3 (b) 4 (c) 5 (d) 6

$P(X \le 0) = 0 \not> \frac{1}{2}$

$P(X \le 1) = P(X=0) + P(X=1) = \frac{1}{10} \not> \frac{1}{2}$

$P(X \le 2) = 0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10} \not> \frac{1}{2}$

$P(X \le 3) = 0 + \frac{1}{10} + \frac{2}{10} + \frac{3}{10} = \frac{6}{10} = \frac{3}{5} = 0.6 > \frac{1}{2}$

$\therefore$ Min value of a s.t $P(X \le a) > \frac{1}{2}$ is $a = 3$.

(iv) Find the distribution function of X, $F(x)$.

$F(x) = P(X \le x)$ ✓

$F(0) = P(X \le 0) = 0$

$F(1) = P(X \le 1) = \frac{1}{10}$

$F(2) = P(X \le 2) = 3/10$

$F(3) = P(X \le 3) = 6/10 = 3/5$

$F(4) = P(X \le 4) = 8/10 = 4/5$

$F(5) = P(X \le 5) = \frac{4}{5} + \frac{1}{100} = 81/100$

$F(6) = P(X \le 6) = 81/100 + 2/100 = 83/100$

$F(7) = 1$

§ Probability density function of continuous r.v:

Let X be a continuous r.v taking values in an interval I_X. Then, the p.d.f, $f(x)$ satisfies the following conditions:

(i) $f(x) \geq 0 \ \forall x \in I_X$

(ii) $\int_{I_X} f(x)\,dx = 1$.

From the defn, we can write

$$\boxed{P(a \le X \le b) = \int^{b} f(x)\,dx}$$

$$\boxed{P(a \le X \le b) = \int_a^b f(x)\,dx}$$

* $P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^b f(x)\,dx + \int_b^{\infty} f(x)\,dx$

$$= 0 + \int_a^b f(x)\,dx + 0$$

$$= \int_a^b f(x)\,dx$$

* $\boxed{P(a \le X \le b) = P(a \le X < b) = P(a < X \le b) = P(a < X < b)}$

$[a,b]$ $\int_a^b f(x)\,dx$ $[a,b)$ $\int_a^b f(x)\,dx$ $(a,b]$ $\int_a^b f(x)\,dx$ (a,b) $\int_a^b f(x)\,dx$

§ Distribution function of a continuous random variable:

If X is a continuous random variable, then

$F(x) = P(X \le x)$ is called the cumulative d.f of the r.v X.

$$F(x) = P(X \le x) = P(-\infty < X \le x) = \int_{-\infty}^{x} f(x)\,dx.$$

$$\boxed{F(x) = \int_{-\infty}^{x} f(x)\,dx.}$$

* The prob. distribution of a continuous r.v is known if either its p.d.f or c.d.f is known.

Properties of c.d.f of a cont. r.v:

(i) $0 \le F(x) \le 1$, $-\infty < x < \infty$

(ii) $F(x)$ is a non-decreasing fun^c i.e $x_1 < x_2 \Rightarrow F(x_1) \le F(x_2)$

(iii) $F(\infty) = 1$ & $F(-\infty) = 0$

(iv) $F(x)$ is a cont. function of x on the right & may have countable no. of discontinuous.

(v) $f(x) = F'(x)$ at all points where $F(x)$ is differential.

$P(-\infty < X \leq a)$ $\qquad$ $(0, 3)$ $\qquad$ $\int_0^1 + \int_1^2 + \int_2^3$

0 $\qquad$ $1 \quad 2$

$(0,1)\ (1\ 2)\ (2\ 3)$

Q If the p.d.f of a continuous random variable is given by –

$$f(x) = \begin{cases} 0, & x < 0 \\ ax, & 0 \leq x \leq 2 \\ (4-x)a, & 2 \leq x \leq 4 \\ = 0, & x > 4 \end{cases}$$

$(-\infty, 0)$ $\quad$ $[0,2]$ $\quad$ $[2,4]$ $\quad$ $(4, \infty)$

0 $\qquad \int_0^4 = \int_0^2 + \int_2^4$ $\qquad 0$

(i) find the value of a.

Solⁿ:

$$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{-\infty}^{0} f(x)dx + \int_0^2 f(x)dx + \int_2^4 f(x)dx + \int_4^{\infty} f(x)dx = 1$$

$$\Rightarrow 0 + \int_0^2 ax\,dx + \int_2^4 (4-x)a\,dx + 0 = 1$$

$$\Rightarrow a\left[\frac{x^2}{2}\right]_0^2 + a\left[4x - \frac{x^2}{2}\right]_2^4 = 1$$

$$\Rightarrow \frac{a}{2}(2^2 - 0^2) + a\{(16-8) - (8-2)\} = 1$$

$$\Rightarrow 2a + a(8-6) = 1$$

$$\Rightarrow 4a = 1$$

$$\Rightarrow a = 1/4.$$

(ii) Find $P(X > 2.5)$

$$f(x) = \begin{cases} 0, & x < 0 \\ x/4, & 0 \leq x \leq 2 \\ (4-x)\frac{1}{4}, & 2 \leq x \leq 4 \\ 0, & x > 4 \end{cases}$$

$$\therefore P(X > 2.5) = P(2.5 < X < \infty) = \int_{2.5}^{\infty} f(x)dx$$

$$= \int^{4} f(x)dx$$

$$= \int_{2.5}^{4} f(x)\,dx$$

$$= \int_{2.5}^{4} (4-x)\frac{1}{4}\,dx$$

$$= \int_{2.5}^{4} \left(1-\frac{x}{4}\right)dx$$

$$= \left[x - \frac{x^2}{8}\right]_{2.5}^{4}$$

$$= (4-2) - \left(2.5 - \frac{6.25}{8}\right)$$

$$= 2 - \left(\frac{20-6.25}{8}\right) = 2 - \frac{13.75}{8}$$

$$= \frac{2.25}{8}$$

$$= \frac{225}{800} = \frac{45}{160}$$

$$= \frac{9}{32}$$ //

(iii) Find the c.d.f of X.

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