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Unit-3

§ Binomial distribution:

$$M_X(t) = (q+pe^t)^n$$

$$M_{X-np}(t) = (qe^{-pt} + pe^{qt})^n.$$

Mode $\underbrace{(n+1)p}_{\in \mathbb{Z}} = m$, then mode $= m$ & $m-1$.

if $(n+1)p = m + f$, then mode $= m$.

m → integral, f → fraction

Q. If $X \sim B(n,p)$ with $n=11$ & $p=\frac{1}{2}$. Mode = ?

(a) 5
(b) 6
(c) 5 & 6
(d) 4 & 5 //

$(n+1)p = 12 \times \frac{1}{2} = 6 \in \mathbb{Z}.$

6 and 6-1

6 & 5

$n=12, p=\frac{1}{2}$

$13 \times \frac{1}{2} = 6.5 = 6 + \frac{1}{2}$

$X \sim B(n,p)$
Discrete

$P(X=x) = {}^nC_x p^x q^{n-x}$

2) Poisson distribution: This distribution was discovered by Simeon Dennis Poisson (1781-1840). He published it in 1837.

P.D is a limiting case of Binomial distribution under the foll^n conditions:

(a) n, the no. of trials, is indefinitely large, $n \to \infty$.

(b) p, the prob. of success in each trial is indefinitely small i.e, $p \to 0$.

(c) $\underline{np = \lambda}$ (say), is finite.

(c) $np = \lambda$ (say), if finite.

$$p = \frac{\lambda}{n}, \quad q = 1 - \frac{\lambda}{n}.$$

As $n \to \infty$, $p \to 0$, $q \to 1$

Here, λ is the parameter of Poisson distribution.

We used to denote $\boxed{X \sim P(\lambda)}$ when X is a Poisson Variate

Def^n: A r.v X is said to follow Poisson distribution, if it assumes non-negative values and its probability mass function is given by —

$$P(X=x) = p(x,\lambda) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x = 0,1,2,\dots, \lambda > 0 \\ 0, & \text{otherwise}. \end{cases}$$

Eg: ① No. of suicides reported in a particular city

$n \to \infty$, $p \to 0$, $q \to 1$.

② No. of printing mistakes at each page of a particular book.

$n \to \infty$, $p \to 0$, $q \to 1$.

③ No. of air accidents reported in a particular unit of time.

$n \to \infty$, $p \to 0$, $q \to 1$

Q Six coins are tossing 6400 times. Using the Poisson distribution, find the approx. prob of getting 6 heads 3 times.

Sol^n: The prob. of 6 heads in one toss,

$$p = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2^6}$$

p, $n = 6400$

Now, $\lambda = np = 6400 \cdot \frac{1}{2^6} = 100$

λ

$\therefore$ The prob. of getting 6 heads r times

000000

$e^{-\lambda}\lambda^{r}$

$\therefore$ The prob. of getting 6 heads r times 000000

$\boxed{\lambda > 0}$

$$\Rightarrow P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

$$\Rightarrow P(X=3) = \frac{e^{-100}\,100^3}{3!} = \frac{e^{-100}\cdot 1000000}{6} = \frac{10^6}{6\times(2.71)^{100}}$$

Ans

Q Let $X \sim P(\lambda)$ and $P(X=2) = 9(X=4) + 90(X=6)$. Find λ.

(a) $\lambda = 3$

(b) $\lambda = -3$

(c) $\lambda = 1$

(d) $\lambda = -1$.

$$\boxed{P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}} \qquad \lambda > 0$$

$$\Rightarrow \frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$$\Rightarrow \frac{\lambda^2}{2} = 9\frac{\lambda^4}{24} + \frac{\cancel{90}^{15}\lambda^6}{\cancel{6}\cdot\cancel{120}_{8}}$$

$$\Rightarrow \frac{\lambda^2}{2} = \frac{9\lambda^4}{24} + \frac{\lambda^6}{8}$$

$$\Rightarrow \frac{1}{2} = \frac{9\lambda^2}{24} + \frac{\lambda^4}{8}$$

$$\Rightarrow 1 = \frac{9\lambda^2}{12} + \frac{\lambda^4}{4}$$

$$\Rightarrow 3\lambda^4 + 9\lambda^2 - 12 = 0$$

$$\Rightarrow \lambda^4 + 3\lambda^2 - 4 = 0$$

$$\Rightarrow (\lambda^2+4)(\lambda^2-1) = 0$$

$\Rightarrow \lambda = \pm 2i$ ✗ $\qquad \lambda^2 = 1 \Rightarrow \lambda = \pm 1 \qquad \lambda > 0$

$\therefore \lambda = 1$ //

§ Moments about origin of Poisson distribution:

$$\mu_1' = E(X) = \sum_{x=0}^{\infty} x\,P(X=x) = \sum_{x=0}^{\infty} x\,p(x,\lambda)$$

$$= \sum^{\infty} \frac{e^{-\lambda}\lambda^x}{x!}$$

$x=0$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda}\left(1+\lambda+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots\right)$$

$$= \lambda e^{-\lambda} \cdot e^{\lambda}$$

$$= \lambda e^{-\lambda+\lambda} = \lambda e^{0} = \lambda$$

$$\boxed{\therefore \mu_1' = E(X) = \lambda}$$ 1st moment about origin.

λ – mean

$$\mu_2' = E(X^2) = \sum_{x=0}^{\infty} x^2 p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} \{x(x-1)+x\}\, p(x,\lambda)$$

$$\sum_{x=0}^{\infty} x(x-1)\, p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} \{x(x-1)+x\} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$0+0+\sum_{x=2}^{\infty}$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{(x-2)!} + \sum_{x=0}^{\infty} x\, p(x,\lambda)$$ $E(x)$

$$= e^{-\lambda}\lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= e^{-\lambda}\lambda^2\left(1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots\right)+\lambda$$

$$= e^{-\lambda}\cdot\lambda^2\cdot e^{\lambda}+\lambda$$

$$\Rightarrow \boxed{\mu_2' = \lambda^2+\lambda}$$ 2nd moment about origin.

H/A

$$\mu_3' = E(X^3) = \sum_{x=0}^{\infty} x^3 p(x,\lambda) = \lambda^3+3\lambda^2+\lambda$$

$$\mu_4' = E(X^4) = \sum_{x=0}^{\infty} x^4 p(x,\lambda) = \lambda^4+6\lambda^3+7\lambda^2+\lambda$$

$$E(X) = \mu_1' = \lambda$$

$$V(X) = \mu_2' - \mu_1'^2 = \lambda^2 + \lambda - \lambda^2$$
$$= \lambda$$

$\therefore$ Expection of a Poisson variate = Variance of the Poisson variate.

Exp > Vari
np > npq