17-3-2022

Thursday, March 17, 2022 12:07 PM

Unit-6

§ Calculation of correlation co-effecient for a bivariate frequency distribution:

Q) The joint prob. distribution of X and Y is given below:

$Y \downarrow$ \ X	-1	$+1$
0	$\frac{1}{8}$	$\frac{3}{8}$
1	$\frac{2}{8}$	$\frac{2}{8}$

$P(X=-1, Y=0) = \frac{1}{8}$

$P(X=1, Y=0) = \frac{3}{8}$

$P(X=-1, Y=1) = \frac{2}{8}$

$P(X=1, Y=1) = \frac{2}{8}$

1

Find the correlation co-effecient of X and Y.

Soln: Computation of Marginal probabilities:-

Y \ X	-1	1	$g(y)$
0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{4}{8}$
1	$\frac{2}{8}$	$\frac{2}{8}$	$\frac{4}{8}$
$p(x)$	$\frac{3}{8}$	$\frac{5}{8}$	

$Cov(XY)$

$= E(XY) - E(X)E(Y)$

σ_X^2

σ_Y^2

we have, $E(X) = \sum x p(x)$

$= (-1) \times \frac{3}{8} + 1 \times \frac{5}{8} = \frac{2}{8} = \frac{1}{4}$

$E(X^2) = \sum x^2 p(x)$

$= (-1)^2 \times \frac{3}{8} + 1^2 \times \frac{5}{8}$

$= 1$

$E(Y) = \sum y g(y) = 0 \times \frac{4}{8} + 1 \times \frac{4}{8} = \frac{1}{2}$

$E(Y^2) = \sum y^2 g(y)$

$= 0^2 \times \frac{4}{8} + 1^2 \times \frac{4}{8}$

$= \frac{1}{2}$

$Var(X) = E(X^2) - (E(X))^2 = 1 - \frac{1}{16} = \frac{15}{16}$ $\Rightarrow \sigma_X = \frac{\sqrt{15}}{4}$

$Var(Y) = E(Y^2) - (E(Y))^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ $\Rightarrow \sigma_Y = \frac{1}{2}$.

$$Var(Y) = E(Y^2) - (E(Y))^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \Rightarrow \sigma_Y = \frac{1}{2}.$$

$$E(XY) = 0 \times (-1) \times \frac{1}{8} + 0 \times 1 \times \frac{3}{8} + 1 \times (-1) \times \frac{2}{8} + 1 \times 1 \times \frac{2}{8}$$

$$= -\frac{2}{8} + \frac{2}{8} = 0$$

$$\therefore Cov(X,Y) = E(XY) - E(X)E(Y) = 0 - \frac{1}{4} \cdot \frac{1}{2} = -\frac{1}{8}.$$

$$\therefore r_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{-1/8}{\sqrt{15}/8} = -\frac{1}{\sqrt{15}} = -0.2582.$$

---x---

§ Rank correlation: Let us suppose that a group of n individuals is arranged in order of merit or proficiency in possession of two characteristics A and B. These ranks in the 2 characteristics will, in general, be different.

For eg: if we consider the relation betn beauty & intelligence.

Let (x_i, y_i), $i = 1$ to n be the ranks of n individuals in two char. A & B respectively.

Pearsonian co-efficient of correlation between the ranks x_i's & y_i's is called the rank correlation co-effecient betn A & B for that group of individual.

§ Spearman's rank correlation co-effecient: Assuming that no two indivi-duals are bracketed equal in either classification, each of the variables X and Y takes the values $1, 2, \ldots, n$.

Hence, $\bar{x} = \bar{y} = \frac{1}{n}(1+2+3+\cdots+n) = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2}$.

$$\sigma_X^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2 = \frac{1}{n}(1^2+2^2+\cdots+n^2) - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n} \cdot \frac{n(n-1)(2n-1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{1}{n} \cdot \frac{n(n-1)(2n-1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{n^2-1}{12} = \sigma_Y^2$$

In general, $x_i \neq y_i$. Let $d_i = x_i - y_i$ $\quad \therefore \boxed{d_i = (x_i-\bar{x})-(y_i-\bar{y})}$

Squaring & summation over i from 1 to n, we get —

$$\sum_i d_i^2 = \sum_i \{(x_i-\bar{x})-(y_i-\bar{y})\}^2$$

$$\Rightarrow \sum_i d_i^2 = \sum(x_i-\bar{x})^2 + \sum(y_i-\bar{y})^2 - 2\sum(x_i-\bar{x})(y_i-\bar{y})$$

Dividing both sides by n, we get —

$$\frac{1}{n}\sum_i d_i^2 = \frac{1}{n}\sum(x_i-\bar{x})^2 + \frac{1}{n}\sum(y_i-\bar{y})^2 - 2 \cdot \frac{1}{n}\sum(x_i-\bar{x})(y_i-\bar{y})$$

$$\Rightarrow \frac{1}{n}\sum_i d_i^2 = \sigma_X^2 + \sigma_Y^2 - 2\,\mathrm{cov}(X,Y)$$

ρ — rank correl^n co-efficient

$$\Rightarrow \frac{1}{n}\sum d_i^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y$$

$$\Rightarrow \frac{1}{n}\sum d_i^2 = 2\sigma_X^2 - 2\rho\sigma_X^2$$

$$\Rightarrow (1-\rho) = \frac{\sum d_i^2}{2n\sigma_X^2}$$

$$\Rightarrow \rho = 1 - \frac{\sum d_i^2}{2n\sigma_X^2} = 1 - \frac{\sum d_i^2}{2n \cdot \frac{n^2-1}{12}}$$

$$\Rightarrow \rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)} \quad //$$

$$\boxed{\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)}}$$

Q) The ranks of same 16 students in Maths & Physics are as follows —

Ranks in (X) maths	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Ranks in (X) maths	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Ranks in physics (Y)	1	10	3	4	5	7	2	6	8	11	15	9	14	12	16	13

Calculate the rank correl^n co-effecient for proficiencies of this group in Maths & Physics.

d_i	0	-8	0	0	0	-1	5	2	1	-1	-4	3	-1	2	-1	3
d_i^2	0	64	0	0	0	1	25	4	1	1	16	9	1	4	1	9

$\Sigma d_i^2 = 136$

$$\therefore \rho = 1 - \frac{6 \Sigma d_i^2}{n(n^2-1)} = 1 - \frac{\cancel{6}^{3} \times \cancel{136}^{\cancel{34}\ 17}}{\cancel{16}_{\cancel{8}} \times 255}$$

$$= 1 - \frac{51}{255} = 1 - \frac{1}{5} = \frac{4}{5} = 0.8$$

Ex-10.18