07-04-2022

Thursday, April 7, 2022 12:00 PM

Unit-4 Point Estimation

* Sample mean is an unbiased estimation of population mean.

* sample variance is a biased estimator of population variance.

Pf: ✓ $E(s^2) = \left(\frac{n-1}{n}\right)\sigma^2 \Rightarrow E(s^2) \neq \sigma^2$.

✓ s^2 – sample variance

✓ σ^2 – pop. variance

n – sample size.

✓ $E(s^2) \neq \sigma^2$

$$E\left(\frac{n}{n-1}s^2\right) = \frac{n}{n-1}E(s^2)$$
$$= \frac{n}{n-1} \times \frac{n-1}{n}\sigma^2$$
$$= \sigma^2$$

$\therefore \left(\frac{n}{n-1}s^2\right)$ is an unbiased estimator of pop. variance. $\frac{n}{n-1}$

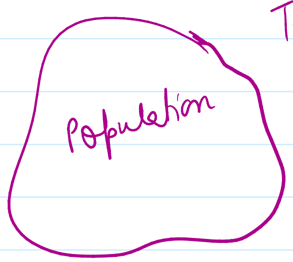
* Random sampling: (a) with replacement
(b) Without replacement.

(ii) Consistent estimators:

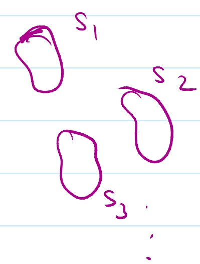
✓ $B(n,p)$

$P(\lambda)$

$N(\mu,\sigma^2)$



T_n



$n \to \infty$

$T_n \to \theta$

As $n \to \infty$, $T_n \to \theta$

μ

μ_i

As $n \to \infty$

$\mu_1, \mu_2, \ldots \mu_n \ldots$

$\mu_i \to \mu$

Defn: An estimator $T_n = T(x_1, x_2, \ldots, x_n)$, based on a random sample of

Def^n: An estimator $T_n = T(x_1, x_2, \ldots, x_n)$, based on a random sample of size n, is said to be a consistent estimator of θ, if T_n converges to θ

i.e as $n \to \infty$, $T_n \to \infty$.

* Invariance property of consistent estimators:

If T_n is a consistent estimator of θ and $\psi(\theta)$ is a continuous function of θ, then $\psi(T_n)$ is a consistent estimator of $\psi(\theta)$.

i.e as $n \to \infty$, $T_n \to \theta$. $\psi(\theta) \to \theta$

$\Rightarrow$ as $n \to \infty$, $\psi(T_n) \to \psi(\theta)$

§ Sufficient condition for consistency: Let $\{T_n\}$ be a sequence of estimators such that $\forall\ \theta \in \Theta$ (parameter space),

$$\begin{cases} \text{(i)} & E(T_n) \to \theta \text{ as } n \to \infty \\ \text{(ii)} & Var(T_n) \to 0 \text{ as } n \to \infty \end{cases}$$

Then, T_n is a consistent estimator of θ.

$\{n, p\}$
$\{\lambda\}$
$\{\mu, \sigma^2\}$

Q> Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of μ.

μ, σ^2

Sol^n: In sampling from $N(\mu, \sigma^2)$ population $\bar{x}$ the sample mean $\bar{x}$ is also normally distributed as $N\left(\mu, \frac{\sigma^2}{n}\right)$ i.e

$\frac{n \cdot \mu}{n}$

$\mu, \frac{\sigma^2}{n}$

$$E(\bar{x}) = \mu, \quad V(\bar{x}) = \frac{\sigma^2}{n}.$$

As $n \to \infty$, $E(\bar{x}) = \mu$ & $V(\bar{x}) = 0$

$\therefore$ Sample mean is a consistent estimator of μ.

$\therefore$ Sample mean is a consistent estimator of μ.

$$\boxed{\begin{aligned} &E(T_n) = \theta \\ &V(T_n) = 0 \\ &\text{as } n \to \infty \end{aligned}}$$

Q) which of the foll^n is an unbiased estimator of pop. variance if the sample size is 100.

(a) $\frac{100}{99} s^2$

(b) $\frac{99}{100} s^2$

(c) s^2

(d) $\frac{9}{10} s^2$

$E\left(\frac{n}{n-1} s^2\right) = \sigma^2$

§ If $E(T_n) > \theta$, then T_n is a positively biased estimator of θ.

If $E(T_n) < \theta$, then " " " -ve " " " θ.

$$\boxed{\text{Amount of biasedness} = E(T_n) - \theta.}$$

Q) X_1, X_2, X_3 is a random sample of size 3 from a population with mean value μ & variance σ^2. T_1, T_2, T_3 are the estimators used to estimate the mean value μ, where

$$T_1 = X_1 + X_2 - X_3, \quad T_2 = 2X_1 + 3X_3 - 4X_2,$$

$$T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$$

(a) Are T_1 & T_2 unbiased estimators?

(i) Yes

(ii) No

(iii) T_1 is unbiased & T_2 is biased

$$\begin{aligned} E(T_1) &= E(X_1 + X_2 - X_3) \\ &= E(X_1) + E(X_2) - E(X_3) \\ &= \mu + \mu - \mu \end{aligned}$$

(ii) No

(iii) T_1 is unbiased & T_2 is biased

(iv) T_1 is biased & T_2 is unbiased.

$-E(X_3)$

$= \mu + \mu - \mu$

$= \mu$

$E(T_2) = E(2X_1 + 3X_3 - 4X_2)$

$= 2\mu + 3\mu - 4\mu$

$= \mu$.

(b) For what value of λ, T_3 is an unbiased estimator of μ.

(i) 0

(ii) 1

(iii) 2

(iv) 3

$T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$

$E(T_3) = \mu$

$\Rightarrow \frac{1}{3}(\lambda\mu + \mu + \mu) = \mu$

$\Rightarrow \frac{1}{3}(\lambda + 2) = 1 \Rightarrow \lambda + 2 = 3$

$\Rightarrow \lambda = 1$

(iii) Efficient estimators: If T_1 is the most efficient estimator with variance V_1 and T_2 is any other estimator with variance V_2, then the efficiency of T_2 is defined as –

$$E = \frac{V_1}{V_2}.$$

E cannot exceed unity.

$V_1 \le V_2$

$\Rightarrow \frac{V_1}{V_2} \le 1$

✓

$V(T_1)$ $V(T_2)$

$= 3 \Leftrightarrow = 4$

(c) Which is the best estimator?

$T_1 = X_1 + X_2 - X_3$

$T_2 = 2X_1 + 3X_3 - 4X_2$

$T_3 = \frac{1}{3}(X_1 + X_2 + X_3)$

σ^2

$V(aX + b)$

$a^2 V(X) + 0$

(a) T_1

(b) T_2

$V(T_1) = V(X_1 + X_2 - X_3)$

(a) T_1

(b) T_2

(c) T_3

$$V(T_1) = V(X_1 + X_2 - X_3)$$

$$= Var(X_1) + Var(X_2) + Var(X_3)$$

$$= 3\sigma^2$$

$a^2 V(X) + 0$

$$V(T_2) = 4\sigma^2 + 9\sigma^2 + 16\sigma^2 = 29\sigma^2$$

$$V(T_3) = \frac{1}{9}(\sigma^2 + \sigma^2 + \sigma^2) = \frac{\sigma^2}{3} \checkmark$$