31-03-2022

Thursday, March 31, 2022 12:04 PM

Pb:- Fit a power curve of the form $Y = aX^b$ for the following data:

X	1	2	3	4
Y	2	4	3	1

Sol^n: $\log Y = \log a + b \log X \Rightarrow U = A + bV$

The normal equ's are –

✓ $\Sigma U = nA + b\Sigma V$

✓ $\Sigma UV = A\Sigma V + b\Sigma V^2$

X	Y	$U = \log Y$	$V = \log X$	UV	V^2
1	2	0.30103	0	0	0
2	4	0.60206	0.30103 ✓	0.18123	0.09061
3	3	0.47712	0.47712	0.22764	0.22764
4	1	0	0.60206 ✓	0	0.36247
		$\Sigma U = 1.38021$	$\Sigma V = 1.38021$	$\Sigma UV = 0.40887$	$\Sigma V^2 = 0.68072$

$\Sigma U = nA + b\Sigma V \Rightarrow 1.38021 = 4A + 1.38021 b \longrightarrow (1)$

$\Sigma UV = A\Sigma V + b\Sigma V^2 \Rightarrow 0.40887 = 1.38021A + 0.68072 b \longrightarrow (2)$

$Y = 2.8755\, X^{-0.329}$

$A = 0.474 = 0.4605$

$b = -0.329$

(2) Fitting of exponential curve: (i) $Y = ab^X$ (ii) $Y = ae^{bX}$.

(i) $Y = ab^X \Rightarrow \log Y = \log a + X \log b$

$\Rightarrow U = A + BX$

Normal equ's are –

$Y \to U$

Normal eqn's are –

$Y \to u$
$X \to X$

$$\left.\begin{aligned} \Sigma u &= nA + B\Sigma X \\ \Sigma uX &= A\Sigma X + B\Sigma X^2 \end{aligned}\right\}$$

(8) $Y = ab^X$.

X	1	2	3	4
Y	2	4	3	1

X	Y	$U = \log Y$	X^2	uX
1	2	0.30103	1	0.30103
2	4	0.60206	4	1.20412
3	3	0.47712	9	1.43136
4	1	0	16	0
$\Sigma X = 10$	$\Sigma Y = 10$	$\Sigma u = 1.38021$	$\Sigma X^2 = 30$	$\Sigma uX = 2.9365$

$\Sigma u = nA + B\Sigma X \Rightarrow 1.38021 = 4A + 10B \longrightarrow (1)$

$\Sigma uX = A\Sigma X + B\Sigma X^2 \Rightarrow 2.9365 = 10A + 30B \longrightarrow (2)$

$A = 0.602065$ $\qquad$ $B = -0.102805$

$\Rightarrow a = \text{antilog}(0.60205)$ $\qquad$ $\Rightarrow b = \text{antilog}(\bar{1}.897105)$ ✓

$= 3.9999 \simeq 4$ $\qquad$ $\Rightarrow b = 0.7446$

$\therefore \boxed{Y = 4(0.7446)^X}$ Ans//

-0.25
$= -1 + 0.75$
$= \bar{1}.75$

(ii) $Y = ae^{bX} \Rightarrow \log Y = \log a + bX \log e$

$\Rightarrow u = A + XB$,

$\left.\begin{aligned} A &= \log a \\ B &= b \log e \end{aligned}\right\}$

$$\Sigma u = nA + B\Sigma x$$
$$\Sigma ux = A\Sigma x + B\Sigma x^2$$

$B = b\log e$

$Y = ae^{bx}$ ✓

Eg:

X	1	2	3	4
Y	2	4	3	1

$a = 4$
$b = -2.367$

A B

$a = \text{antilog } A$

$b = \frac{B}{\log e}$

$\therefore Y = 4e^{-2.367x}$

$\log e = 0.4342$

Pb If the angle between two lines of regression is $0°$, then which of the foll^{n} is true for random variables X & Y?

(a) $r = 0$ ✗
(b) $r \in (-1, 1)$ ✗
(c) $r = +1$ or -1 ✓
(d) $r \in (-1, 0]$ ✗

$$\theta = \tan^{-1}\left\{\frac{1-r^2}{|r|}\,\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right\} = 0$$

$$\Rightarrow \frac{1-r^2}{|r|} = 0 \Rightarrow r^2 = 1$$
$$\Rightarrow r = +1 \text{ or } -1.$$

* $Y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(X - \bar{x})$

$X - \bar{x} = r\frac{\sigma_x}{\sigma_y}(Y - \bar{y})$

$\Rightarrow \sigma_x(Y - \bar{y}) = \sigma_y(X - \bar{x})$

$r = 1$
$r = -1$

$\sigma_x(Y - \bar{y}) = -\sigma_y(X - \bar{x})$

$= \sigma_Y (x - \bar{x})$

$r = -1$ $\quad \sigma_x (y - \bar{y}) = -\sigma_y (x - \bar{x})$

Pb:- If one of the regression co-efficients is $\frac{3}{2}$, then which of the foll" is a possible value for the other regression co-efficient?

$b_{xy} = \frac{3}{2}$ +ve > 1

b_{yx} +ve < 1

✗ (a) 3

✓ (b) $\frac{1}{2}$

✗ (c) 1

✗ (d) -1