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Unit-I (Basics of Probability)

1. (a) Which function defines a probability space on $S = (e_1, e_2, e_3)$

(i) $P(e_1) = \frac{1}{4}$, $P(e_2) = \frac{1}{3}$, $P(e_3) = \frac{1}{2}$ ×

(ii) $P(e_1) = \frac{2}{3}$, $P(e_2) = -\frac{1}{3}$, $P(e_3) = \frac{2}{3}$ ×

× (iii) $P(e_1) = -\frac{1}{4}$, $P(e_2) = \frac{1}{3}$, $P(e_3) = \frac{1}{2}$, and

(iv) $P(e_1) = 0$, $P(e_2) = \frac{1}{3}$, $P(e_3) = \frac{2}{3}$

Baye's theorem

(b) $P(S) = 1$

$\Rightarrow P(e_1) + P(e_2) + P(e_3) = 1.$

(a) $P(e) \geq 0$, ≤ 1.

(i) $\frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{3+4+6}{12} = \frac{13}{12} > 1$

$0 + \frac{1}{3} + \frac{2}{3} = 1,$ $\qquad P(e_i) \geq 0$

Q For two events A and B, P(A)=0.5, P(B)=0.6 and P(A or B)=0.8. Find the conditional probability P(A/B) and P(B/A).

$P(A \cup B) = 0.8$

$P(A\backslash B) = ?$ $\qquad P(B\backslash A) = ?$

$$\boxed{P(A\backslash B) = \frac{P(A\cap B)}{P(B)} \qquad P(B\backslash A) = \frac{P(A\cap B)}{P(A)}}$$

(a) 0.5, 0.6

(b) 0.5, 0.3

(c) 0.6, 0.6

(d) 0.5, 0.2

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow P(A\cap B) = P(A) + P(B) - P(A\cup B) = 0.5 + 0.6 - 0.8 = 0.3$

$\therefore P(A\backslash B) = \frac{0.3}{0.6} = 0.5$, $\quad P(B\backslash A) = \frac{P(B\cap A)}{P(A)} = \frac{0.3}{0.5} = 0.6$

- Let A and B be two events such that P(A)=0.6, P(B)=0.4 and P(A and B)=0.25. Find P(A/B) and P(B/A).

- Let A and B be two events such that P(A)=0.6, P(B)=0.4 and P(A and B)=0.25. Find P(A/B) and P(B/A).

(A) 5/8, 5/9 (B) 5/8, 5/12
(C) 5/4, 5/8 (D) 5/4, 8/9

$$P(A \backslash B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.4} = \frac{25}{40} = \frac{5}{8}$$

$$P(B \backslash A) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.6} = \frac{25}{60} = \frac{5}{12}$$

Q. A dice is thrown twice and the sum of the numbers appearing is noted to be 8. What is the conditional probability that the number 5 has appeared at least once?

Sol: A = event that sum of the no.s is 8
B = no. 5 has appeared atleast once.

$P(B \backslash A) = ?$

$S = \{(1,1), (1,2), (1,3), \ldots (6,6)\}$, $n(S) = 36$.

$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$P(A) = \frac{5}{36}$, $P(B) = \frac{11}{36}$

$B = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (5,6), (5,4), (5,3), (5,2), (5,1)\}$

$A \cap B = \{(3,5), (5,3)\}$ $\Rightarrow n(A \cap B) = 2 \Rightarrow P(A \cap B) = \frac{2}{36}$

$$\therefore P(B \backslash A) = \frac{P(A \cap B)}{P(A)} = \frac{2/36}{5/36} = \frac{2}{5} = 0.4$$

——— x ———

Q. Two cards are drawn one after the other from a well-shuffled deck of 52 cards. Find the probability that both are spade cards, if the first card is (i) replaced, (ii) not replaced.

$\frac{13}{52}$

Sol^n: (i)

Sol^n: (i) (a) $\frac{1}{12}$ (b) $\frac{1}{26}$ (c) $\frac{13}{208}$ (d) $\frac{1}{16}$

$\frac{13}{52}$

Probability that both are spades after replacing 1st card

$= \frac{13}{52} \times \frac{13}{52}$

$= \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16}$

(ii) Prob. that both are spades if the first card is not replaced

$= \frac{\cancel{13}}{\cancel{52}_{4}} \times \frac{\cancel{12}^{3}}{\cancel{51}_{17}} = \frac{1}{17}$

Q. Events E and F are given to be independent. Find P(F), if P(E) = 0.4 and P(E or F) = 0.55.

Solution: $P(E) = 0.4$

$P(E \cup F) = 0.55$

$P(F) = ?$

$P(E \cap F) = P(E)P(F)$

$= 0.4\, P(F).$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow 0.55 = 0.4 + x - 0.4x$

$\Rightarrow 0.6x = 0.15$

$\Rightarrow x = \frac{15}{60} = \frac{1}{4} = 0.25$

$\boxed{P(F) = 0.25}$

- A can solve 90% of the problems given in a book and B can solve 75%. What is the probability that at least one of them will solve a problem selected at random from the book ?

A, B

A 0.875 B 0.975 C 0.7775 D 0.675

will solve a problem selected at random from the book ?

A. 0.875 B. 0.975 C. 0.7775 D. 0.675

$X =$ A can solve the problem

$Y =$ B " " " "

$$\begin{array}{r} 1.65 \\ -0.0675 \\ \hline 1.5825 \end{array}$$

$P(X) = 0.9$ $P(Y) = 0.75$

$P(X \cap Y) = P(X)\,P(Y)$

$= 0.9 \times 0.75$

$= 0.675$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = 0.9 + 0.75 - 0.675 = 1.65 - 0.675$

$= 0.975$