19 - 03 - 2022

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Unit-6 (Correlation & Regression)

§ **Repeated ranks (Continued):** If any two or more individuals are bracketed equal in any classification with respect to two characteristics A and B, or if there is more than one item with the same value in the series, the Spearman's formula breaks down.

To correct the errors in the formula, we give common ranks to the repeated items. This common rank is the average of the ranks which these items would have assumed if they were slightly diff^{n} from each other.

The new & corrected formula for repeated ranks is –

$$\rho = 1 - \frac{6\left(\Sigma d^2 + T_X + T_Y\right)}{n\left(n^2-1\right)}$$

here, $T_X = \Sigma \frac{m\left(m^2-1\right)}{12}$, where 'm' is the no. of times an item is repeating in X series

& $T_Y = \Sigma \frac{p\left(p^2-1\right)}{12}$, where 'p' is the no. of times an item is repeated in Y- series.

Q> Obtain the rank correlation co-efficient for the foll^{n} data:

X : 68 64 75 50 64 80 75 40 55 64

Y : 62 58 68 45 81 60 68 48 50 70

X	Y	Rank(x)	Rank(Y)	$d = x - y$	d^2
68	62	4	5	−1	1
64	58	6	7	−1	1
75	68	2.5	3.5	−1	1
50	45	9	10	−1	1
64	81	6	1	5	25
80	60	1	6	−5	25

64	81	6	1	5	25
80	60	7	6	-5	25
75	68	2.5	3.5	-1	1
90	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
				$\Sigma d = 0$	$\Sigma d^2 = 72$

$$T_X = \sum \frac{m(m^2-1)}{12} = \frac{2(2^2-1)}{12} + \frac{3(3^2-1)}{12} = \frac{1}{2} + 2 = \frac{5}{2}$$

$$T_Y = \sum \frac{p(p^2-1)}{12} = \frac{2(2^2-1)}{12} = \frac{1}{2}$$

$$\therefore \rho = 1 - \frac{6(\Sigma d^2 + T_X + T_Y)}{n(n^2-1)}$$

$$= 1 - \frac{6\left(72 + \frac{5}{2} + \frac{1}{2}\right)}{10(100-1)}$$

$$= 1 - \frac{6 \times 75}{10 \times 99} = 1 - \frac{5}{11}$$

$$= \frac{6}{11} = 0.545 \quad \text{Ans}$$

* The ranks (repeated) which are given to same items in a series are called tied ranks.

If the same item is at rank $m, m+1, m+2, \ldots, m+r$, then the common rank of all the same items is the average of $m, (m+1), \ldots, (m+r)$ i.e

$$\frac{m + (m+1) + (m+2) + \ldots + (m+r)}{(r+1)}.$$

Pb 5 players participated in a competition & their ranks in football & basketball are as follows:

A	B	d_i	Σd_i^2
1	2	-1	

Pb 5 players participated in a competition & their ranks in football & basketball are as follows:

(1,2), (2,3), (3,5) (4,1) and (5,4).

Calculate the [redacted] rank correlation co-effecient ?

1	2	-1	
2	3	-1	
3	5	-2	
4	1	3	
5	4	1	

$\Sigma d_i = 0$ $\Sigma d_i^2 = 16$

(a) 0.5
(b) 0.4
(c) -0.5
(d) 0.2 ✓

$$\rho = 1 - \frac{6\Sigma d_i^2}{5(25-1)}$$

$$= 1 - \frac{8 \times 16^4}{5 \times 24_4} = \frac{1}{5} = 0.2 //$$

Q)

	A	B	C
1st	1	3	6
	6	5	4
	5	8	9
	10	4	8
	3	7	1
	2	10	2
	4	2	3
	9	1	10
	7	6	5
	8	9	7

$\rho(A,B) = -0.5$

$\rho(B,C) = 0.2$

$\rho(A,C) = 0.7$

~~x~~

Regression

§ Regression - stepping back towards the average.

Sir Francis Galton