02-02-2022 ## Lecture-8

Wednesday, February 2, 2022 2:58 PM

§ Types of Random Variable:

(a) Discrete random variable
(b) Continuous random variable.

(a) Discrete random variable :- If a r.v X takes a finite number or countably infinite number of values, then X is a discrete r.v.

The possible values taken by X are denoted by $x_1, x_2, \ldots, x_n, \ldots$ which terminate in finite case.

A real valued function X defined on a discrete sample space S is called a discrete r.v.

Eg: Let us toss two coins and note the no. of heads appeared.

X 0 1 2

Discrete

(b) Continuous random variable: A r.v X is said to be a continuous random variable if it takes all possible values in a given interval I_X. For eg: age, height, weight etc. are continuous random variable.

X — [4, 10]
continuous r.v

Example :- A pair of die is thrown & the sum is noted.

X 2 3 4 5 6 7 8 9 10 11 12

Discrete r.v

$S = \{(1,1), (1,2), (1,3), \ldots, (6,5), (6,6)\}$ Discrete

$X : S \rightarrow \{2, 3, \ldots, 12\}$
↳ Discrete r.v.

§ Probability mass function of discrete r.v :-

Let X be a discrete R.V which takes values $x_1, x_2, x_3, \ldots$ and let $P(X = x_i) = p_i$. Then, p_i is called the probability (mass) function if it satisfies the following conditions :-

$\boxed{p.f \text{ or } p.m.f}$

(i) $p_i \geq 0 \ \forall i$, and

(ii) $\sum_i p_i = 1$.

§ Probability distribution of the discrete R.V :

The collection of pairs (x_i, p_i), $i = 1, 2, 3, \ldots$

X	x_1	x_2	x_3	$\ldots$
$P(X = x_i)$	p_1	p_2	p_3	$\ldots$

is called the probability distribution or the discrete prob. dist. of r.v X.

p.d d.p.d

Eg: 3 coins are tossed simultaneously & X = no. of tails appeared. Write the probability distribution of X.

Solⁿ: $S = \{\underline{HHH}, \underbrace{HHT, HTH, THH}, \underbrace{HTT, THT, TTH}, \underline{TTT}\}$

X	0	1	2	3
$P(X = x_i)$	$1/8$	$3/8$	$3/8$	1

$p_1 = 1/8$, $p_2 = 3/8$, $p_3 = 3/8$, $p_4 = 1/8$

$p_i \geq 0$

& $\sum p_i = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

Q. From a lot of 12 items containing 3 defective items, a sample of 4 items are drawn at random without replacement. Let a random var X denote the no. of defective items in the sample. Find the prob. distribution of X.

denote the no. of defective items in the sample. Find the prob. distribution of X.

Soln: 12 – items, 3 – defective $\longrightarrow$ 4 items, 0 1 2 3

For $x = 0$, $p(x) = \dfrac{^9C_4}{^{12}C_4} = \dfrac{\frac{9!}{4!\,5!}}{\frac{12!}{4!\,8!}} = \dfrac{9!}{5!} \times \dfrac{8!}{12!} = \dfrac{\cancel{8}\cdot 7\cdot \cancel{6}}{\cancel{12}\cdot 11\cdot \cancel{10}} = \dfrac{14}{55}$.

For $x = 1$, $p(x) = \dfrac{^9C_3 \times {}^3C_1}{^{12}C_4} = \dfrac{28}{55}$

For $x = 2$, $p(x) = \dfrac{^9C_2 \times {}^3C_2}{^{12}C_4} = 12/55$

For $x = 3$, $p(x) = \dfrac{^9C_1 \times {}^3C_3}{^{12}C_4} = \dfrac{9\times 1}{^{12}C_4} = \dfrac{1}{55}$

$\therefore$ The prob. distribution is –

X, x	0	1	2	3
$p(X=x), p(x)$	$14/55$	$28/55$	$12/55$	$1/55$

$F(2.5) = P(X \le 2.5) = P(X=0) + P(X=1) + P(X=2)$
$= 14/55 + 28/55 + 12/55 = 54/55$

§ **Distribution function**: Let X be a random variable. Then, the function $F(x)$ defined by –

$F(x) = P(X \le x)$ is called the distribution func of X.

It has the folln properties: –

(i) $0 \le F(x) \le 1$

(ii) If $x_1 < x_2$, then $F(x_1) \le F(x_2)$

(iii) $P(a \le X \le b) = F(b) - F(a)$.

- $p(x_i) = P(X = x_i) = F(x_i) - F(x_{i-1})$.

$F(x)$ is also called the cumulative distribution func of X.

• $p(x_i) = P(X = x_i) = F(x_i) - F(x_{i-1})$.

$F(x)$ is also called the cumulative distribution fun^c of X.

Q A random variable X has the following prob. distribution:

X	0	1	2	3	4
$P(X=x_i)$	c	$2c$	$2c$	c^2	$5c^2$

Find the value of c?

(A) $-\frac{1}{6}$ (B) 1 (C) -1 (D) $\frac{1}{6}$

Sol^n: $c + 2c + 2c + c^2 + 5c^2 = 1$

$\Rightarrow 6c^2 + 5c - 1 = 0$

$\Rightarrow 6c^2 + 6c - c - 1 = 0$

$\Rightarrow 6c(c+1) - 1(c+1) = 0$

$\Rightarrow (6c-1)(c+1) = 0$

$\Rightarrow c = \frac{1}{6}, \; c = -1$ Not possible

Since p_i cannot be -ve, so $c = \frac{1}{6}$

X	0	1	2	3	4
$P(X=x_i)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{36}$	$\frac{5}{36}$

Find $P(X<3)$ and $P(1<X<4)$.

(a) $\frac{5}{6}, \frac{13}{36}$

(b) $\frac{5}{36}, \frac{13}{36}$

(c) $\frac{13}{36}, \frac{15}{36}$

(d) $\frac{5}{6}, \frac{5}{36}$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{1}{6} + \frac{1}{3} + \frac{1}{3}$

$= \frac{1}{6} + \frac{2}{3}$

$= \frac{5}{6}$

$P(1<X<4) = P(X=2) + P(X=3)$

$= \frac{1}{3} + \frac{1}{36} = \frac{13}{36}$