15-01-2022

Saturday, January 15, 2022 12:52 PM

Lecture-3

Conditional Probability (Independent events)

CONDITIONAL PROBABILITY

A, B are any two events in a particular R.E.

$P(A|B)$ → conditional prob. of event A given event B.

$P(B|A)$ → " " " " B " " A.

Ex: A - getting a king ⟷ B - getting a face card.

$P(A) = \frac{4}{52} = \frac{1}{13}$

$P(A|B) = \frac{4}{12} = \frac{1}{3}$

MULTIPLICATION THEOREM OF PROBABILITY

T6. For two events A and B,

$$P(A \cap B) = P(A).P(B|A), P(A) > 0$$

$$= P(B).P(A|B), P(B) > 0$$

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Q13. From a city population, the probability of selecting (i) a male or a smoker is 7/10, (ii) a male smoker is 2/5, and (iii) a male, if a smoker is already selected is 2/3. Find the probability of selecting

A : a male is selected , B : a smoker is selected.

$P(A \cup B) = \frac{7}{10}$, $P(A \cap B) = \frac{2}{5}$ $P(A|B) = \frac{2}{3}$.

(a) a non-smoker

$P(\bar{B}) = ?$

we have — $P(A \cap B)$ ... $P(A \cap B)$ $2/5$ = 3

$P(\bar{B}) = ?$

we have — $P(A \setminus B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B) = \frac{P(A \cap B)}{P(A \setminus B)} = \frac{2/5}{2/3} = \frac{3}{5}$.

$\therefore P(\bar{B}) = 1 - \frac{3}{5} = \frac{2}{5}$ ✓

(b) a male

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

$P(A) = ?$

$P(A) = P(A \cup B) + P(A \cap B) - P(B) = \frac{7}{10} + \frac{2}{5} - \frac{3}{5} = \frac{5}{10} = \frac{1}{2}$.

(c) a smoker, if a male is first selected.

(A) $\frac{1}{5}$ **(B)** $\frac{1}{20}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{3}{10}$

$P(A \cup B) = \frac{7}{10}$, $P(A \cap B) = \frac{2}{5}$,

$P(A \setminus B) = \frac{2}{3}$

$P(B \setminus A) = ?$

$P(B \setminus A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{2/5}{1/2}$

$= \frac{4}{5}$

[redacted]

Q14. Sixty percent of the employees of the XYZ corporation are college

Q14. Sixty percent of the employees of the XYZ corporation are college graduates. Of these, ten percent are in sales. Of the employees who did not graduate from college, eighty percent are in sales. What is the probability that

A : An employee is a college graduate

B : An employee is in sales.

(i) an employee selected at random is in sales but not graduated from college.

(A) 0.60 (B) 0.48 (C) 0.32 (D) 0.80

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B|\bar{A}) = ?$

0.80

$P(B|\bar{A}) = \frac{P(\bar{A} \cap B)}{P(\bar{A})}$

$P(A) = 0.60$

$P(B|A) = 0.10$

$P(B|\bar{A}) = 0.80$

(ii) an employee selected at random is in sales.

$P(B) = ?$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$= P(A) \times P(B|A) + P(\bar{A}) \times P(B|\bar{A}) = 0.60 \times 0.10 + 0.40 \times 0.80$

$= 0.06 + 0.32 = 0.38.$

(iii) an employee selected is neither in sales nor a college graduate.

$P(\bar{A} \cap \bar{B}) = ?$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - 0.60 - 0.38 + 0.06$

$= 0.08$

INDEPENDENT EVENTS

A & B.

$P(A|B) = P(A)$ & $P(B|A) = P(B)$

A, B are independent events

$\frac{P(A \cap B)}{P(B)} = P(A)$

$\Rightarrow \boxed{P(A \cap B) = P(A)P(B)}$

Q15. One shot is fired from each of the three guns. A, B, C denote the events that the target is hit by the first, second and third guns resp. If

Q15. One shot is fired from each of the three guns. A, B, C denote the events that the target is hit by the first, second and third guns resp. If $P(A) = 0.5, P(B) = 0.6, P(C) = 0.8$. Assuming the events A, B, C independent, find the probability that

$\bar{A}, \bar{B}, \bar{C}$

$$P(A \cap B \cap \bar{C}) = P(A)\,P(B)\,P(\bar{C}).$$

(i) exactly two hits are registered.

(A) 0.70 (B) 0.40 (C) 0.26 (D) 0.46

$$P(A \cap B \cap \bar{C}) + P(\bar{A} \cap B \cap C) + P(A \cap \bar{B} \cap C)$$

$$= P(A)P(B)P(\bar{C}) + P(\bar{A})P(B)P(C) + P(A)P(\bar{B})P(C)$$

$$= 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.6 \times 0.8 + 0.5 \times 0.4 \times 0.8$$

$$= 0.06 + 0.24 + 0.16 = 0.46$$

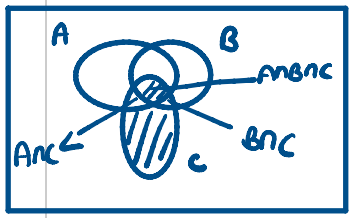
$P(\bar{A} \cap \bar{B} \cap C)$

$= 0.5 \times 0.4 \times 0.8$

$= 0.16$

(ii) target is hit by third gun only.

(A) 0.16 (B) 0.4 (C) 0.26 (D) 0.64



$$P(C \text{ only}) = P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$= 0.8 - 0.5 \times 0.8 - 0.6 \times 0.8 + 0.5 \times 0.6 \times 0.8$$

$$= 0.8 - 0.4 - 0.48 + 0.24$$

$$= 1.04 - 0.88 = 0.16$$

Q16. It is 8:5 against the wife who is 40 years old living till she is 70 and 4:3 against her husband now 50 living till he is 80. Find the probability that only wife will be alive.

H/A

Q17. A manager has two assistants that he bases his decision on information supplied independently by each one of them. The probability that he makes a mistake in his thinking is 0.005. The probability that an assistant gives wrong information is 0.3. Assuming that the mistakes made by the manager are independent of the information given by the assistants, find the probability that he reaches a wrong decision.

Q18. A **and** B **are two independent events such that** $P(A \cap \bar{B}) = \frac{3}{25}$ **and** $P(\bar{A} \cap B) = \frac{8}{25}$. **If** $P(A) < P(B)$, **then** $P(A) =$

(A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{5}$

Q19. A **and** B **throw alternatively with a pair of balanced dice.** A **wins if he throws a sum of six points before** B **throws a sum of seven points, while** B **wins if he throws a sum of seven points before** A **throws a sum of six points. If** A **begins the game, then the probability of winning of** B **is**