Tuesday, January 11, 2022 9:59 AM

Unit - I Basics of Probability

Lecture 1

[redacted]

Unit 1: Basics of Probability

Eg: Tossing a coin
{H, T}

- **Random Experiment:** If in each trial of an experiment conducted under identical conditions, the outcome is not unique, but may be any one of the possible outcomes.

Eg2 :- Rolling a die {1, 2, 3, 4, 5, 6}

Eg3 Picking a card from a dack of 52 cards.

- **Outcome:** The result of an random experiment will be called an outcome.

Tossing a coin { Tail - outcome
Head - outcome.

- **Event**: The combination of outcomes are termed as events. → Subset of sample space.

set of all possible outcomes.

Eg: {Tail, Head} = S

E = Tail, Head, Tail and head

2^n

$2^2 = 4$ $\{\phi, \{H\}, \{T\}, \{H, T\}\}$.

Eg: Rolling a die $|S| = 6$. $|P(S)| = 2^6 = 64$

- **Trial**: Any particular performance of an random experiment is called trial.

Coin

- **Exhaustive Events:** The total number of possible outcomes of a random experiment is known as the exhaustive events or cases.

Eg: Rolling a die

- **Favourable Events:** The number of outcomes which entail the happening of the event.

Rolling a die E = getting even no.
= {2, 4, 6}

E' = {2, 4}

2, 4

1, 2

event.

$\varepsilon' = \{2,4\}$

$\varepsilon'' = \{1,2\}$

2,4

1,2

- **Equally likely Events:** When there is no reason to expect one event in preference of other, taking into consideration all relevant evidences.

H, T → equally likely events.

- **Mutually Exclusive Events:** If happening of any one of events precludes the happening of all others.

Coin Head

Tail

1, 2, 3, 4, 5, 6

1

- **Independent Events:** If occurrence or non-occurrence of an event is not affected by the supplementary knowledge concerning the occurrence of any other remaining events.

$A \longleftrightarrow B$

Eg: Tossing 2 coins.

H H
H T
T H
T T

A = getting H on 1st coin
B = " T on 2nd coin

Eg: Rolling a pair of die

A = getting 1 on 1st die
B = " 6 on 2nd die.

Rolling a pair of die $S=\{(1,1),(1,2),\dots\}$

ε' = getting the sum to be 5.

ε'' = getting . . to be 10.

Are they independent?

YES NO

$\underset{=}{5}$

$\underbrace{(1,4),(2,3),(3,2),(4,1)}$

- **Mathematical or Classical definition of Probability:** If a random experiment or a trial results in n exhaustive mutually exclusive and equally likely events outcomes, out of which m are favourable to the occurrence of an Event E, then

$$P(E)=\frac{Number\ of\ favourable\ cases}{Total\ number\ of\ exhaustive\ cases}=\frac{m}{n}$$

$$P(\bar{E})=\frac{n-m}{n}$$

$P(\varepsilon)$

$P(\bar{\varepsilon})=1-P(\varepsilon)$

$=1-\frac{m}{n}$.

$P(\bar{\varepsilon})$

A , B

$P(A)+P(B)=1$.

A, B $\underbrace{P(A)+P(B)=1}$

$\underbrace{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m}$ - mutually exclusive & exhaustive events.

Then,

$P(\varepsilon)\geq 0$

$0\leq P(\varepsilon)\leq 1$

$P(\varepsilon)\in[0,1]$.

$\varepsilon_1\cup\varepsilon_2\cup\dots\cup\varepsilon_m = S$

$\varepsilon_1\cap\varepsilon_2\cap\dots\cap\varepsilon_m=\phi$

$\varepsilon_i\cap\varepsilon_j=\phi$

$P(\varepsilon_1\cup\varepsilon_2\cup\dots\cup\varepsilon_m)=P(S)$

$\Rightarrow \underbrace{P(\varepsilon_1)+P(\varepsilon_2)+\dots+P(\varepsilon_m)}=1$

- **Statistical or Empirical definition of Probability:** If an experiment is performed repeatedly under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number becomes indefinitely large, is called the probability of happening of the event, it being assumed that the limit is unique.

$\lim_{n\to\infty}\frac{N_\varepsilon}{N_T}$

Q1: Two unbiased dice are thrown. Find the probability that total of the number on the dice is greater than 8.

$S=\{(1,1),(1,2),\dots,(6,6)\}$.

$\varepsilon=\{(4,5),(3,6),(5,4),(6,3),(6,4),(4,6),(5,6),(6,5),(5,5),(6,6)\}$

$p=\frac{n(\varepsilon)}{n(S)}=\frac{10}{36}=\frac{5}{18}$

(A) $\frac{1}{9}$ **(B)** $\frac{5}{18}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{11}{18}$

$p=\frac{5}{18}$

Q2. In shuffling a pack of cards, four are accidentally dropped, find the chance that the missing cards should be one from each suit.

(A) $\frac{4}{52}$ **(B)** [redacted] **(C)** [redacted] **(D)** $\frac{4.\,{}^4P_1}{{}^{52}P_4}$

$4.\,{}^{13}C_1$

$({}^{13}C_1)^4$

$\underset{=}{4}$

13 13 13 13

13 13 13 13

(A) $\frac{4}{52}$ **(B)** [redacted] **(C)** [redacted] **(D)** $\frac{4.4_{P_1}}{52_{P_4}}$

$\frac{4\cdot{}^{13}C_1}{{}^{52}C_4}$ $\frac{({}^{13}C_1)^4}{{}^{52}C_4}$ $\frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4}$

4

13 13 13 13

1 1 1 1

39

Q3. A man is dealt 4 spade card from an ordinary pack of 52 cards. If he is given three more cards, find the probability that at least one of the additional cards is also a spade.

$P(\varepsilon) + P(\bar{\varepsilon}) = 1$

13
4
48 − 9 = 39

4 spade cards + 3 ← 1, 2, 3

52 − 4 = 48

ε = atleast one of additional 3 cards is spade.

$\bar{\varepsilon}$ = none of the 3 cards is spade.

$n(S) = {}^{48}C_3$

$n(\bar{\varepsilon}) = {}^{39}C_3$

$P(\bar{\varepsilon}) = \frac{{}^{39}C_3}{{}^{48}C_3}$

$\therefore P(\varepsilon) = 1 - P(\bar{\varepsilon}) = 1 - \frac{{}^{39}C_3}{{}^{48}C_3} = 0.4718.$

Q4. A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 officers of sales department and 1 chartered accountant. Find the probability that the chartered accountant must be in the committee.

3 + 4 + 2 + 1

10

$n(S) = {}^{10}C_4 = 210$

$n(\varepsilon) = {}^1C_1 \times {}^9C_3$

$= 1 \times \frac{9!}{3!\,6!}$

$= \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84.$

(A) $\frac{2}{5}$ **(B)** $\frac{13}{14}$ **(C)** $\frac{4}{35}$ **(D)** $\frac{1}{30}$

$\therefore P(\varepsilon) = \frac{84}{210} = \frac{2}{5}$

Q5. While naming of sections of CSE school of 2019 batch, find the following probabilities.

$P(Both\ vowels)$:

(A) $\frac{5}{676}$ (B) $\frac{25}{676}$ (C) $\frac{235}{676}$ (D) $\frac{210}{676}$

$P(Only\ one\ is\ vowel)$:

$P(Only\ one\ is\ vowel)$:

(A) $\frac{5}{676}$ (B) $\frac{25}{676}$ (C) $\frac{235}{676}$ (D) $\frac{210}{676}$

$P(None\ is\ consonant)$:

(A) $\frac{441}{676}$ (B) $\frac{348}{676}$ (C) $\frac{25}{676}$ (D) $\frac{210}{676}$

$P(Both\ are\ consonants)$:

Q6. In a random arrangement of the letters of the word "MATHEMATICS", find the probability that
(i) all the vowels come together.

[redacted]

I

Q7. 10 gifts are to be distributed among 4 children. Find the probability that (i) first child should get exactly 3 gifts.
(ii) One child should get exactly 3 gifts.