15-02-2022 Unit- 3

Tuesday, February 15, 2022 10:01 AM

Pb :- If the moments of variable X are defined by

$$\mu_r' = E(X^r) = 0.6 \; ; \; r = 1, 2, 3, \ldots$$

what are the value of $P(X=0)$, $P(X=1)$ & $P(X \geq 2)$?

Solⁿ:

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r' = 1 + \sum_{r=1}^{\infty} \frac{t^r}{r!}\mu_r'$$

$$= 1 + \sum_{r=1}^{\infty} \frac{t^r}{r!}(0.6)$$

$$= 0.4 + \left(0.6 + \sum_{r=1}^{\infty} \frac{t^r}{r!}(0.6)\right)$$

$$= 0.4 + 0.6 \sum_{r=0}^{\infty} \frac{t^r}{r!}$$

$$= 0.4 + 0.6\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\right)$$

$$\Rightarrow M_X(t) = 0.4 + 0.6\,e^t \longrightarrow (1)$$

Now,

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{\infty} e^{tx} P(X=x)$$

$$\Rightarrow M_X(t) = P(X=0) + e^t P(X=1) + \sum_{x=2}^{\infty} e^{tx} P(X=x) \longrightarrow (2)$$

Comparing eqⁿ (1) & (2), we get –

$$P(X=0) = 0.4, \quad P(X=1) = 0.6, \quad P(X \geq 2) = 0$$

$$P(X=0) + P(X=1) + P(X \geq 2) = 1$$

$$\Rightarrow 0.4 + 0.6 + P(X \geq 2) = 1$$

$$\Rightarrow P(X \geq 2) = 0$$ Ans

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r'$$

$$M_X(t) = E(e^{tX}) = \sum_{x=0} e^{tx} P(X=x)$$

Unit 2 completed // CA 2 Unit-2

Unit III

Probability Distributions : Binomial Distribution, Poisson Distribution, Moments and Moment generating function of Binomial Distribution, Moments and Moment generating function of Poisson Distribution, Normal Distribution its Moments and M.G.F.

Unit IV

generating function of Binomial Distribution, Moments and Moment generating function of Poisson Distribution, Normal Distribution its Moments and M.G.F.

Unit IV

Unit - III

* **Moment about a point** : The moments about a point A are given by -

$$\begin{cases} \mu_r' = E(X^r) & \text{(moment about origin)} \\ \mu_r' = E\{(X-A)^r\} & \text{(moment about A)} \end{cases}$$

& $\mu_r = E\{(X-E(X))^r\}$ (moment about mean)

μ_r → central moments.

§ $\mu_r = E\{(X-E(X))^r\}$ — central moments.

The relations between central moments & moments about a point are given by -

$$\mu_r = \mu_r' - {}^rC_1\mu_{r-1}'\mu_1' + {}^rC_2\mu_{r-2}'\mu_1'^2 - {}^rC_3\mu_{r-3}'\mu_1'^3 + \cdots + (-1)^r\mu_1'^r$$

$$\boxed{\begin{array}{l} {}^rC_0\mu_r'\mu_1'^0 \\ {}^rC_1\mu_{r-1}'\mu_1'^1 \\ {}^rC_2\mu_{r-2}'\mu_1'^2 \\ \vdots \\ {}^rC_r\mu_{r-r}'\mu_1'^r = \mu_1'^r \end{array}}$$

If $r=1$,

$$\mu_1 = \mu_1' - {}^1C_1\mu_0'\mu_1'$$
$$= \mu_1' - \mu_1'$$
$$= 0$$

⇒ 1st central moment is zero.

$$\mu_2 = \mu_2' - {}^2C_1\mu_1'\mu_1' + {}^2C_2\mu_0'\mu_1'^2 = \mu_2' - 2\mu_1'^2 + \mu_1'^2$$
$$= \mu_2' - \mu_1'^2 = V(X)$$

∴ Second central moment is the variance.

$$\boxed{\mu_1' = E(X), \quad \mu_2 = V(X)}$$

If $r=3$, $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

If $r=4$, $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

...nts μ_r

If $r=4$, $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$$\boxed{\begin{aligned} \mu_1 &= 0 \\ \mu_2 &= \mu_2' - \mu_1'^2 \\ \mu_3 &= \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 \\ \mu_4 &= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 \end{aligned}}$$

Relⁿ's between central moments μ_r & moments about a point μ_r'.

We have 3 types of probability distributions:

1. Binomial
2. Poisson
3. Normal dist.

1) Binomial distribution: A random variable X is said to follow binomial distribution if it assumes only non-negative values and its probability mass function is given by –

$$P(X=x) = p(x) = \begin{cases} {}^nC_x\, p^x q^{n-x} & ; x = 0,1,2,\dots \\ 0 & , \text{otherwise} \end{cases} \qquad q = 1-p$$

* Physical conditions for Binomial distribution:

1. Each trial results two exhaustive and mutually disjoint outcomes, termed as success & failure.

$A \cap B = \phi \quad P(A)+P(B)=1$

$A \cup B = S \quad \Rightarrow p+q=1$

$\Rightarrow q = 1-p.$

2. The no. of trials 'n' is finite.
3. The trials are independent of each other.
4. The prob of success & failure, p & q are constant in each trial.

trial 1 $\quad p=\frac{1}{2},\ q=\frac{1}{2}$

trial 2 $\quad p=\frac{1}{\ },\ q=\frac{2}{\ }$ X

trial 1 $p=\frac{1}{2}, q=\frac{1}{2}$
trial 2 $p=\frac{1}{3}, q=\frac{2}{3}$ $\Big\}$ X

p, q $p+q=1$ //
n finite
p, q

Q. Ten coins are thrown simultaneously. What is the probability of getting atleast 7 heads.

or,

A coin is tossed 10 times. What is the prob. of getting head atleast 7 times.

Solⁿ: $n=10$, prob. of success, head, $p=\frac{1}{2}$
" " failure, tail, $q=\frac{1}{2}$.

The probability of getting x heads is given by –

$$P(X=x)=p(x)={}^{10}C_x\, p^x q^{10-x}$$

∴ Prob. of getting atleast 7 heads

$$= P(X=7)+P(X=8)+P(X=9)+P(X=10)$$

$$= {}^{10}C_7\, p^7 q^3 + {}^{10}C_8\, p^8 q^2 + {}^{10}C_9\, p^9 q + {}^{10}C_{10}\, p^{10}$$

$$= {}^{10}C_7\left(\frac{1}{2}\right)^{10} + {}^{10}C_8\left(\frac{1}{2}\right)^{10} + {}^{10}C_9\left(\frac{1}{2}\right)^{10} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}\left\{{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right\}$$

$$= \frac{1}{1024}\left(\frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} + \frac{10\cdot 9}{2\cdot 1} + 10 + 1\right)$$

$$= \frac{1}{1024}(120+45+11)$$

$$= \frac{176}{1024} = \frac{44}{256} = \frac{11}{64} \quad //$$

§ If random variable follows binomial distribution, then we write

$\boxed{X \sim B(n, p)}$, where n = no. of trials
p = prob. of success.

n, p — parameters of binomial distribution.

X ~ Binomial variate

n - no. of trials = degree of the binomial distribution.

$n = ?$

Q With usual notation, find p for a binomial variate X, if $n = 6$ and $9P(X=4) = P(X=2)$

$$\boxed{P(X=x) = {}^nC_x p^x q^{n-x}}$$

(a) $p = 1$

(b) $p = \frac{1}{2}$

(c) $p = \frac{1}{3}$

(d) $p = \frac{1}{4}$

$\Rightarrow 9\,{}^nC_4 p^4 q^{n-4} = {}^nC_2 p^2 q^{n-2}$

$\Rightarrow 9\,{}^6C_4 p^4 q^2 = {}^6C_2 p^2 q^4$

$\Rightarrow 9\,{}^6C_4 p^2 = {}^6C_2 q^2$

$\Rightarrow 9p^2 = q^2$

$\Rightarrow 9p^2 = (1-p)^2$

$\Rightarrow 9p^2 = 1 - 2p + p^2$

$\Rightarrow 8p^2 + 2p - 1 = 0$

$\Rightarrow 8p^2 + 4p - 2p - 1 = 0$

$\Rightarrow 4p(2p+1) - 1(2p+1) = 0$

$\Rightarrow (2p+1)(4p-1) = 0$

$\Rightarrow p = -\frac{1}{2}, \frac{1}{4}$

H/A

Q The probability of a man hitting a target is $\frac{1}{4}$. If he fires 7 times, what is the prob. of his hitting the target at least twice.