CSE408
Minimum Spanning Tree(Prims,Kruskshal)

Lecture #26

Minimum Spanning Tree

- A minimum spanning tree connects all nodes in a given graph
- A MST must be a connected and undirected graph
- A MST can have weighted edges
- Multiple MSTs can exist within a given undirected graph

More about Multiple MSTs

- Multiple MSTs can be generated depending on which algorithm is used
- If you wish to have an MST start at a specific node
- However, if there are weighted edges and all weighted edges are unique, only one MST will exist

Real Life Application of a MST

A cable TV company is laying cable in a new neighborhood. If it is constrained to bury the cable only along certain paths, then there would be a graph representing which points are connected by those paths. Some of those paths might be more expensive, because they are longer, or require the cable to be buried deeper; these paths would be represented by edges with larger weights. A *minimum spanning tree* would be the network with the lowest total cost.

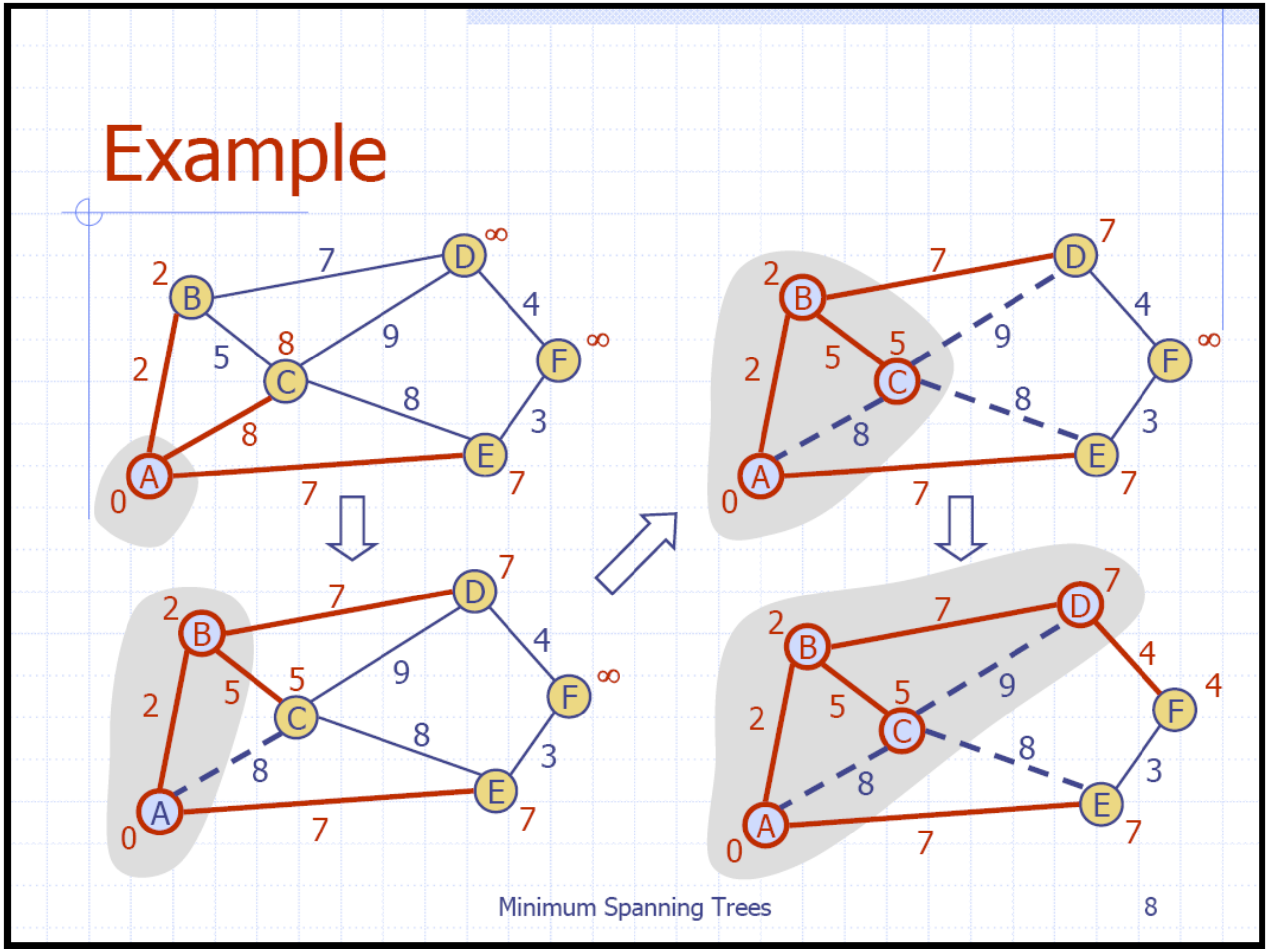
Prim’s Algorithm

- Initially discovered in 1930 by Vojtěch Jarník, then rediscovered in 1957 by Robert C. Prim
- Similar to Dijkstra’s Algorithm regarding a connected graph
- Starts off by picking any node within the graph and growing from there

Prim’s Algorithm Cont.

- Label the starting node, A, with a 0 and all others with infinite
- Starting from A, update all the connected nodes’ labels to A with their weighted edges if it less than the labeled value
- Find the next smallest label and update the corresponding connecting nodes
- Repeat until all the nodes have been visited

Prim’s Algorithm Example



Example

Minimum Spanning Trees

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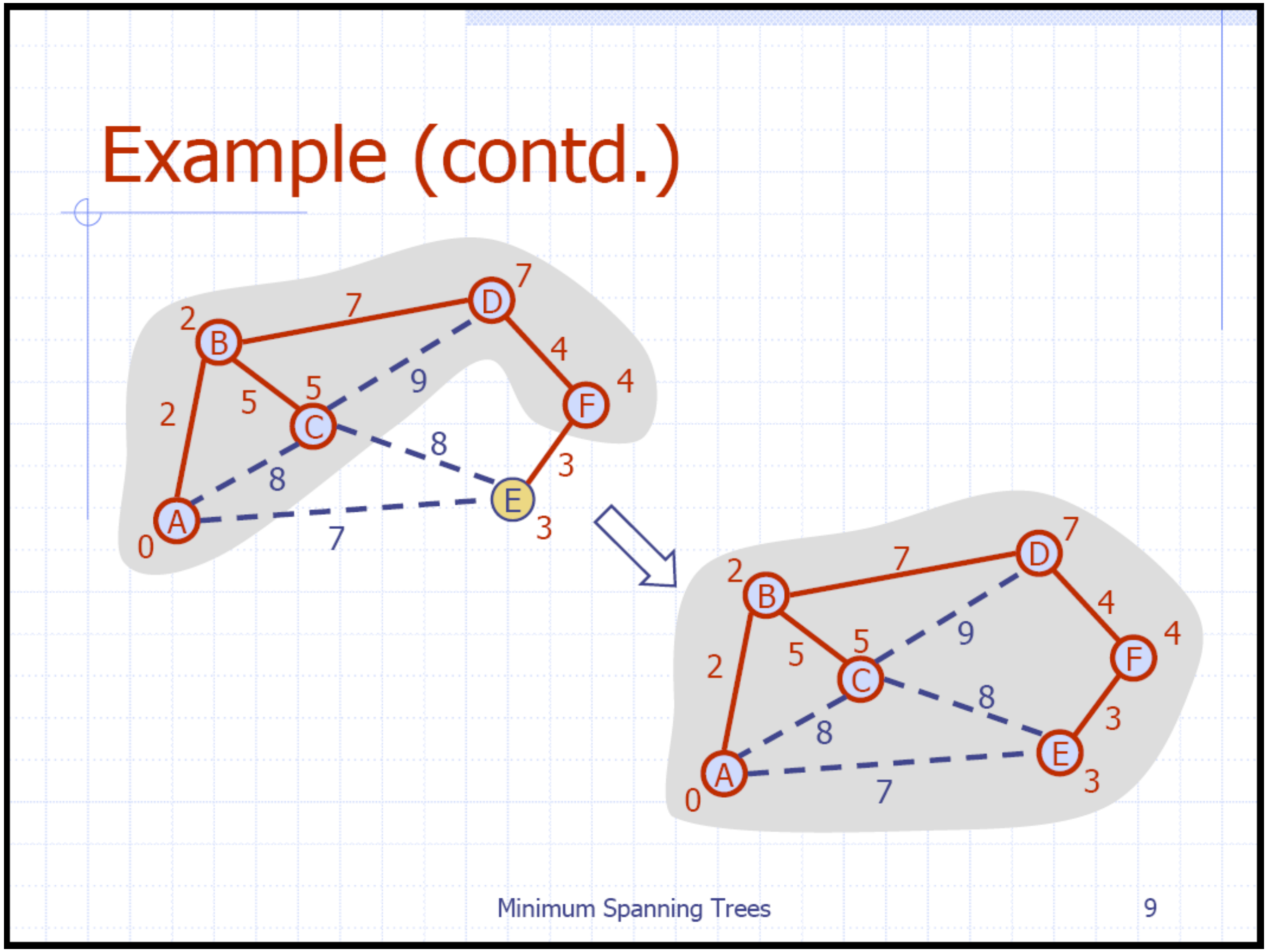
Prim’s Algorithm Example



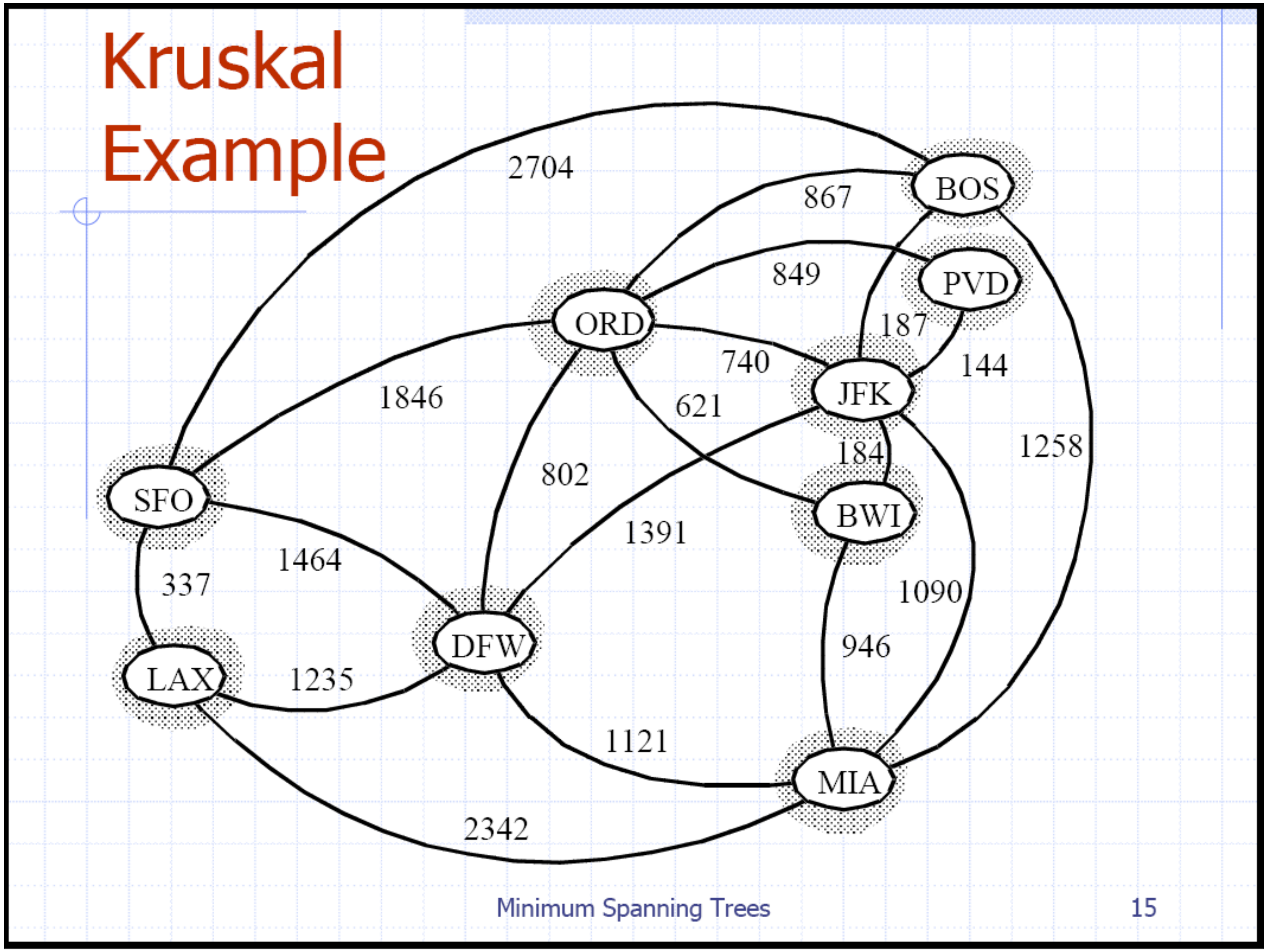
Example (contd.)

Minimum Spanning Trees

Kruskal’s Algorithm

- Created in 1957 by Joseph Kruskal
- Finds the MST by taking the smallest weight in the graph and connecting the two nodes and repeating until all nodes are connected to just one tree
- This is done by creating a priority queue using the weights as keys
- Each node starts off as it’s own tree
- While the queue is not empty, if the edge retrieved connects two trees, connect them, if not, discard it
- Once the queue is empty, you are left with the minimum spanning tree

Kruskal’s Algorithm Example



Kruskal Example

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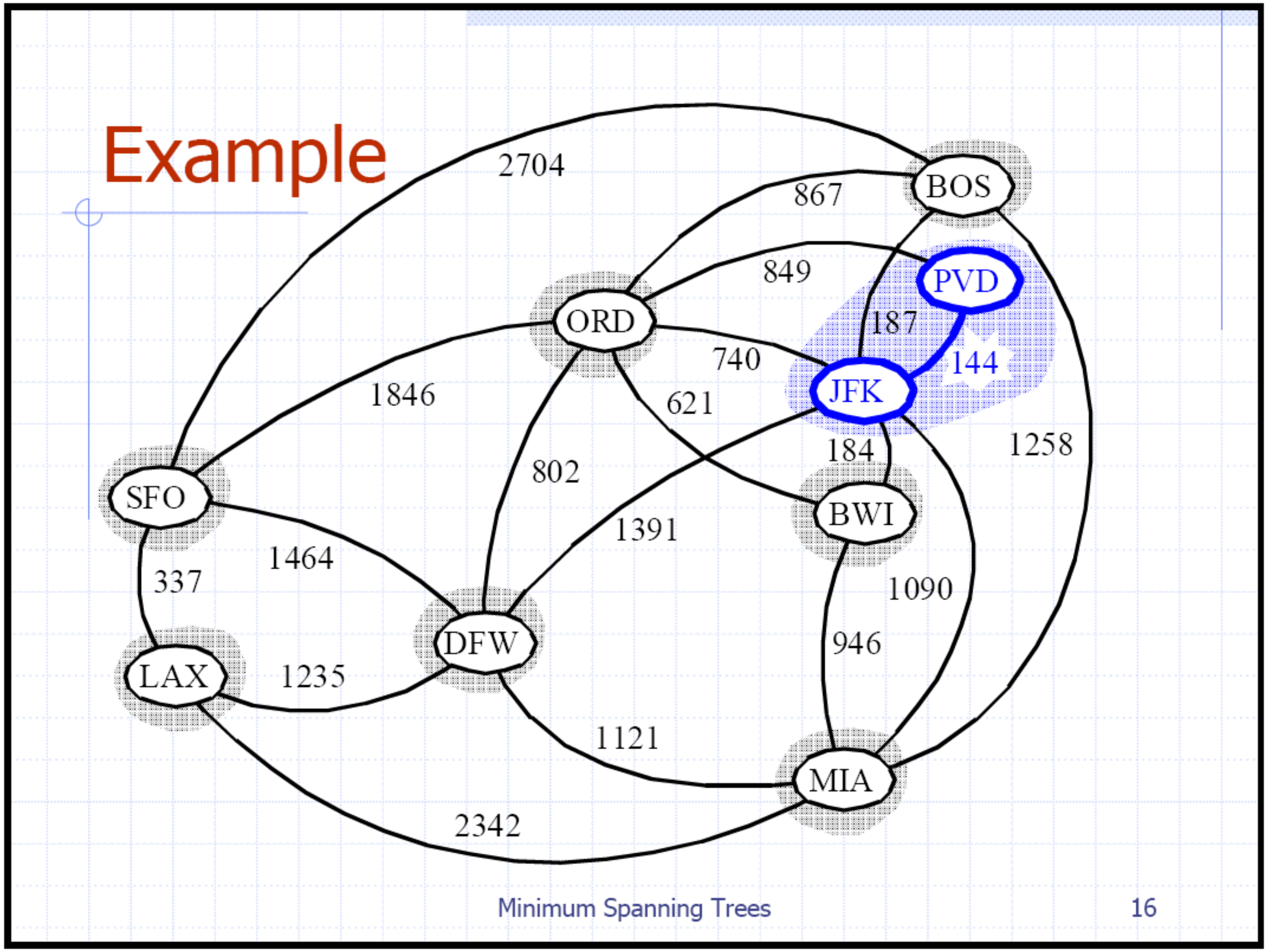
Kruskal’s Algorithm Example



Example

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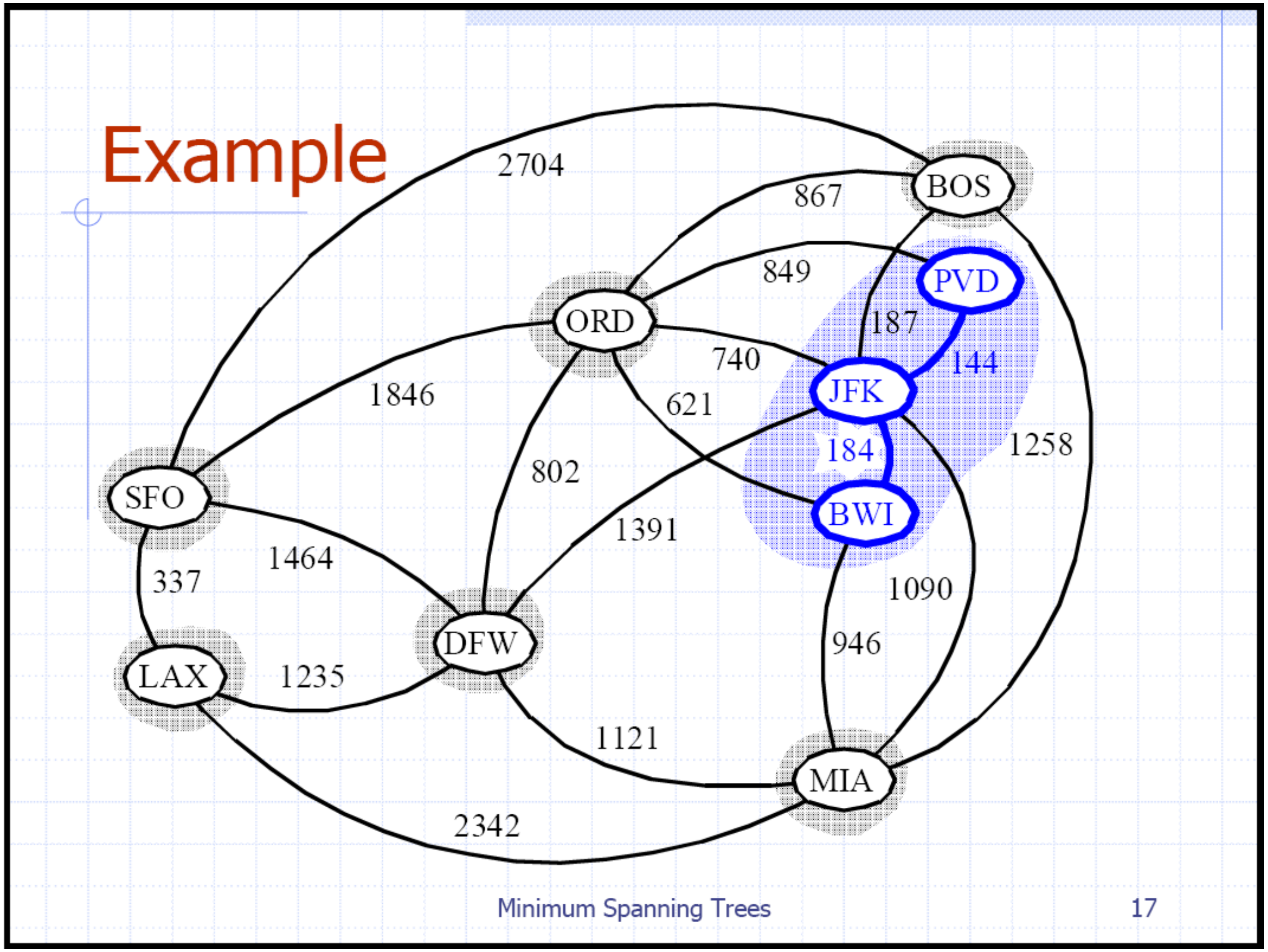
Kruskal’s Algorithm Example



Example

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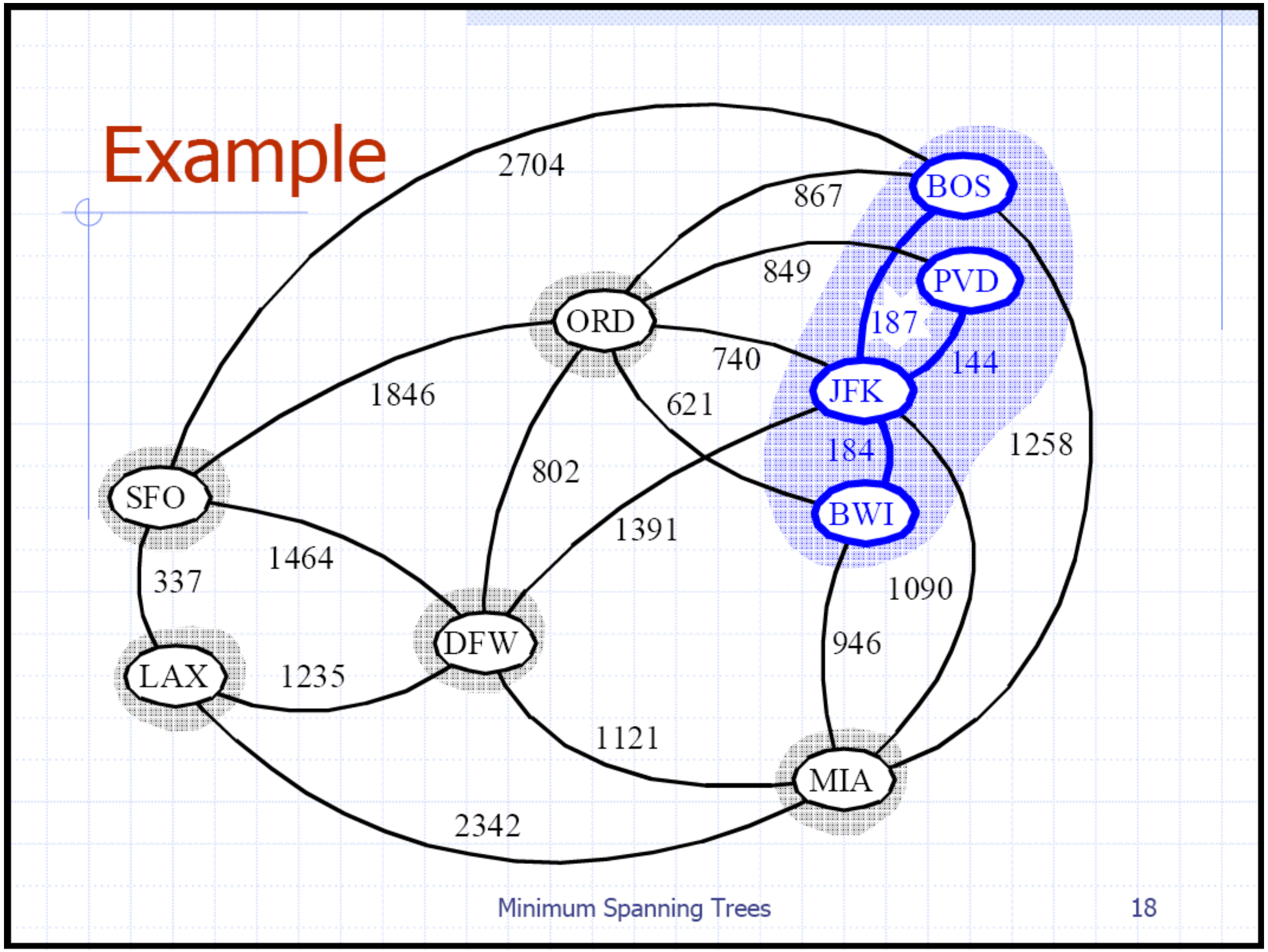
Kruskal’s Algorithm Example



Example

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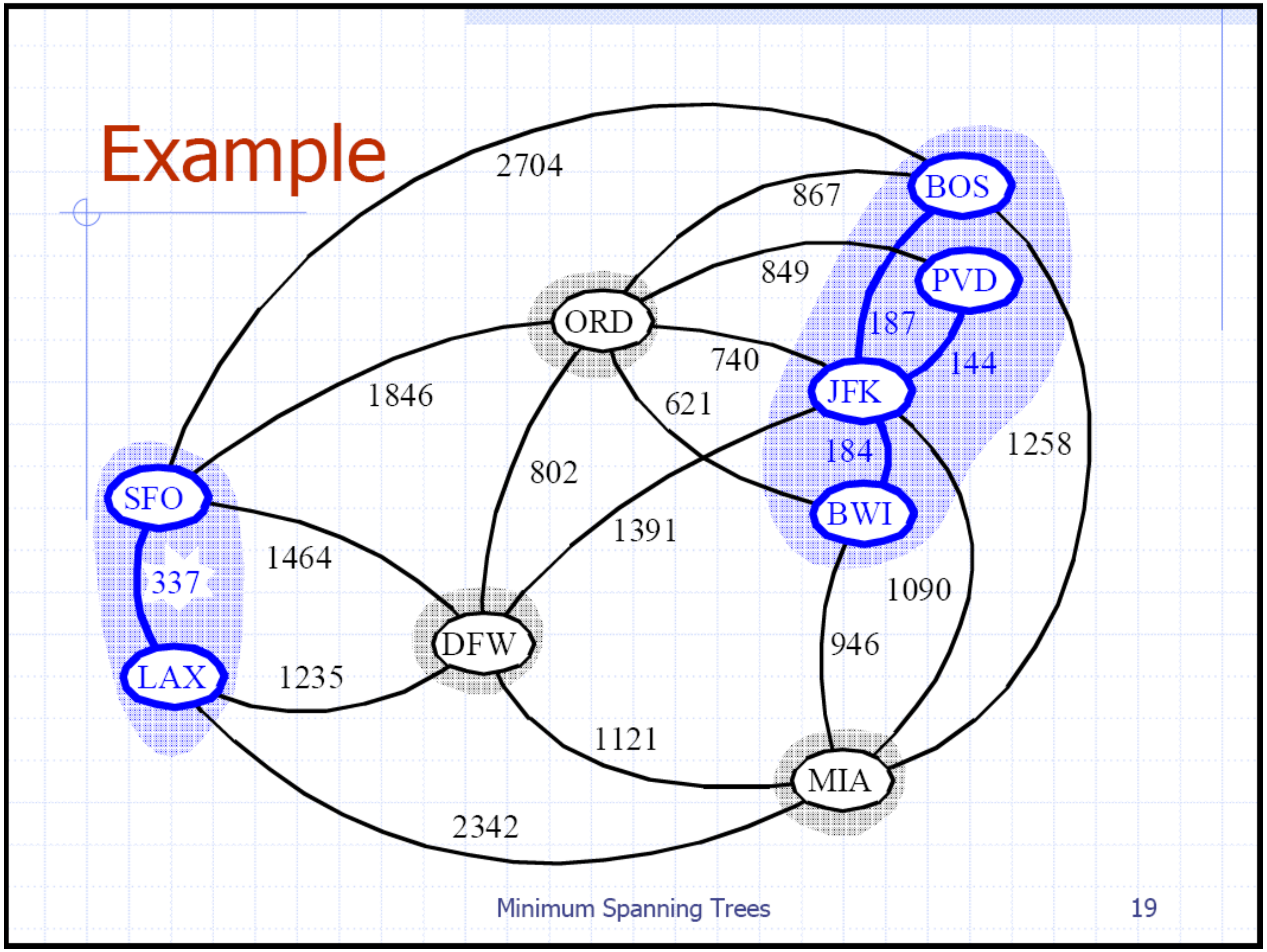
Kruskal’s Algorithm Example



Example

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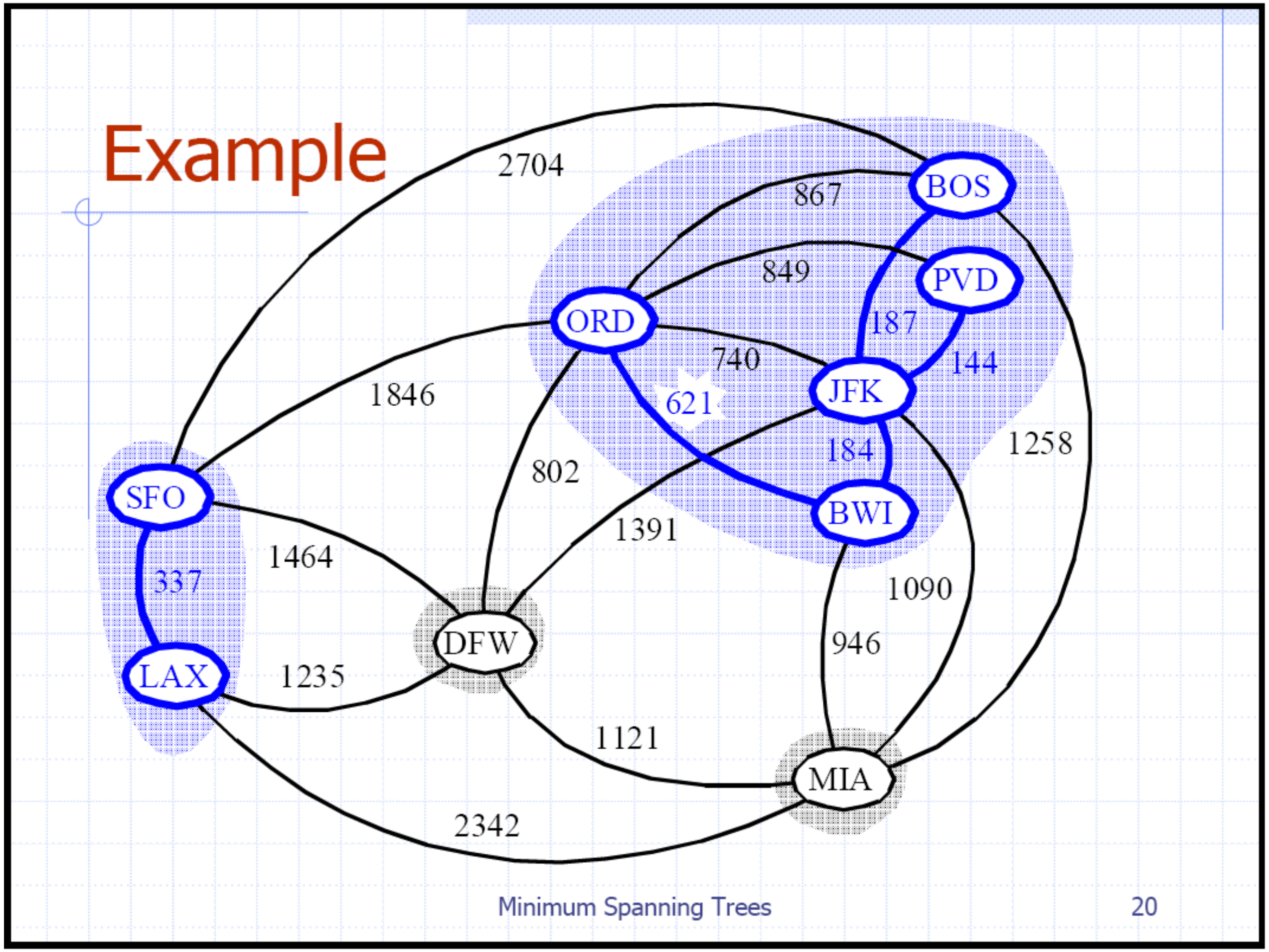
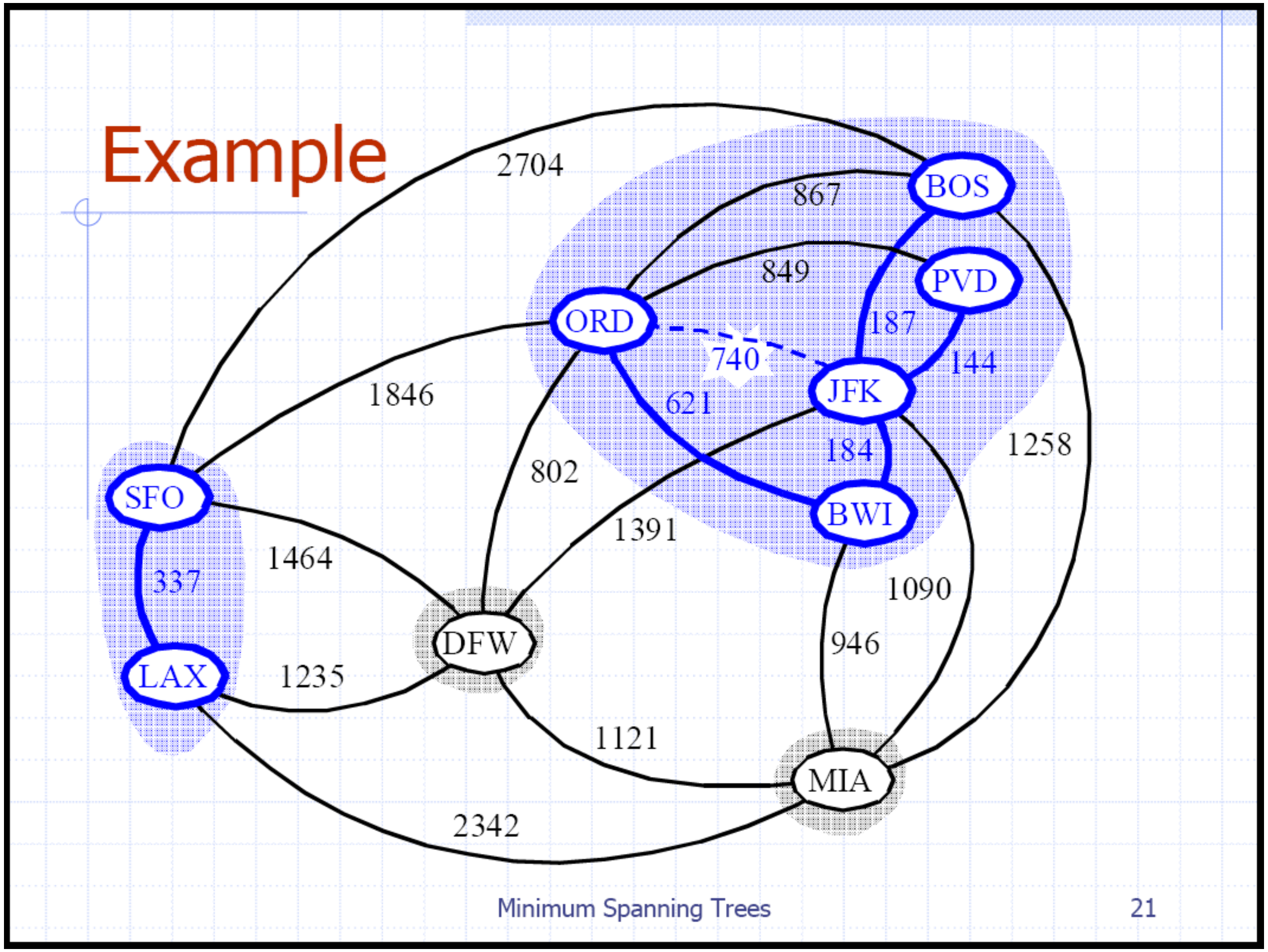
Kruskal’s Algorithm Example



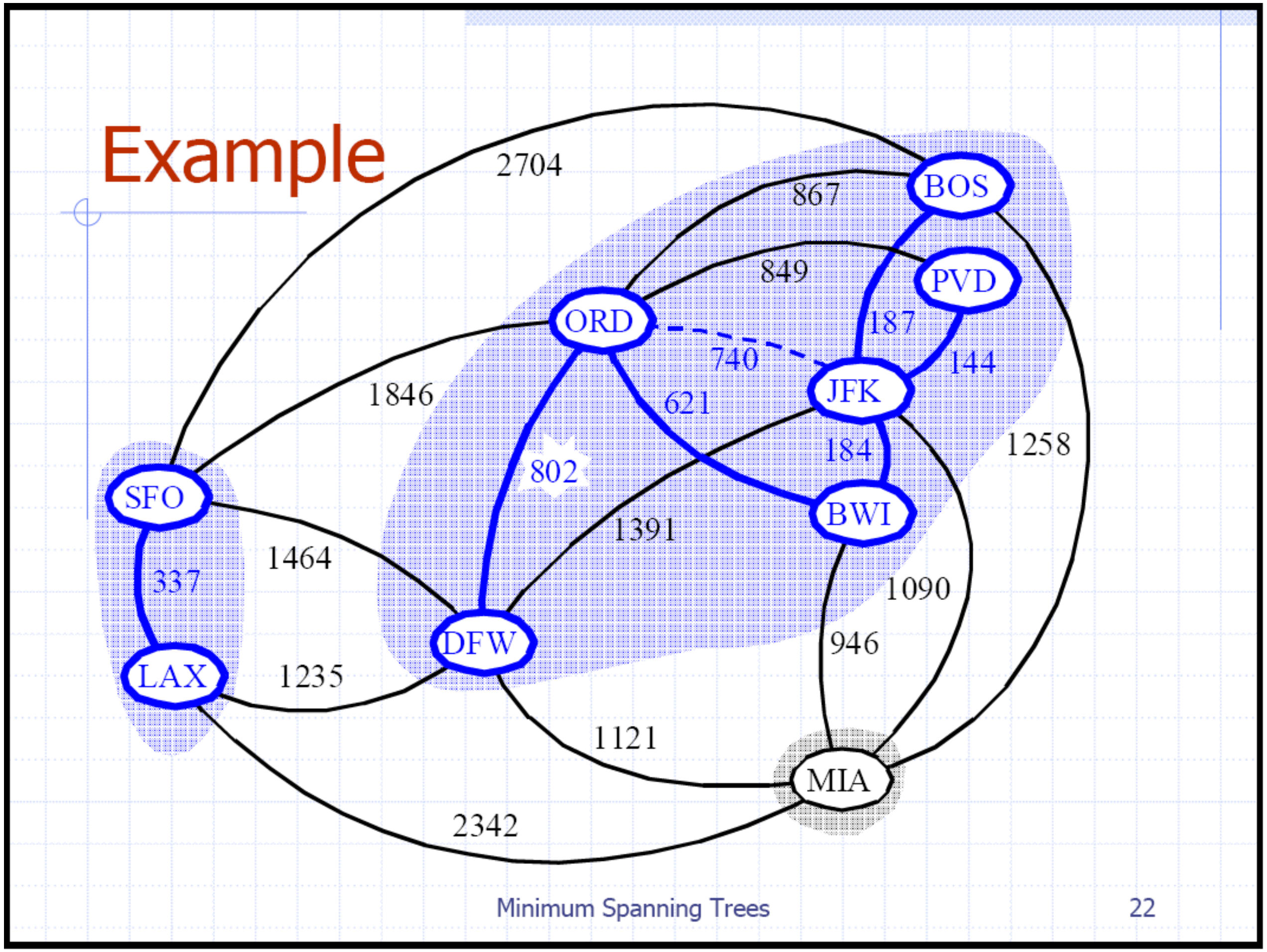
Kruskal’s Algorithm Example



Example

Minimum Spanning Trees

Kruskal’s Algorithm Example



Example

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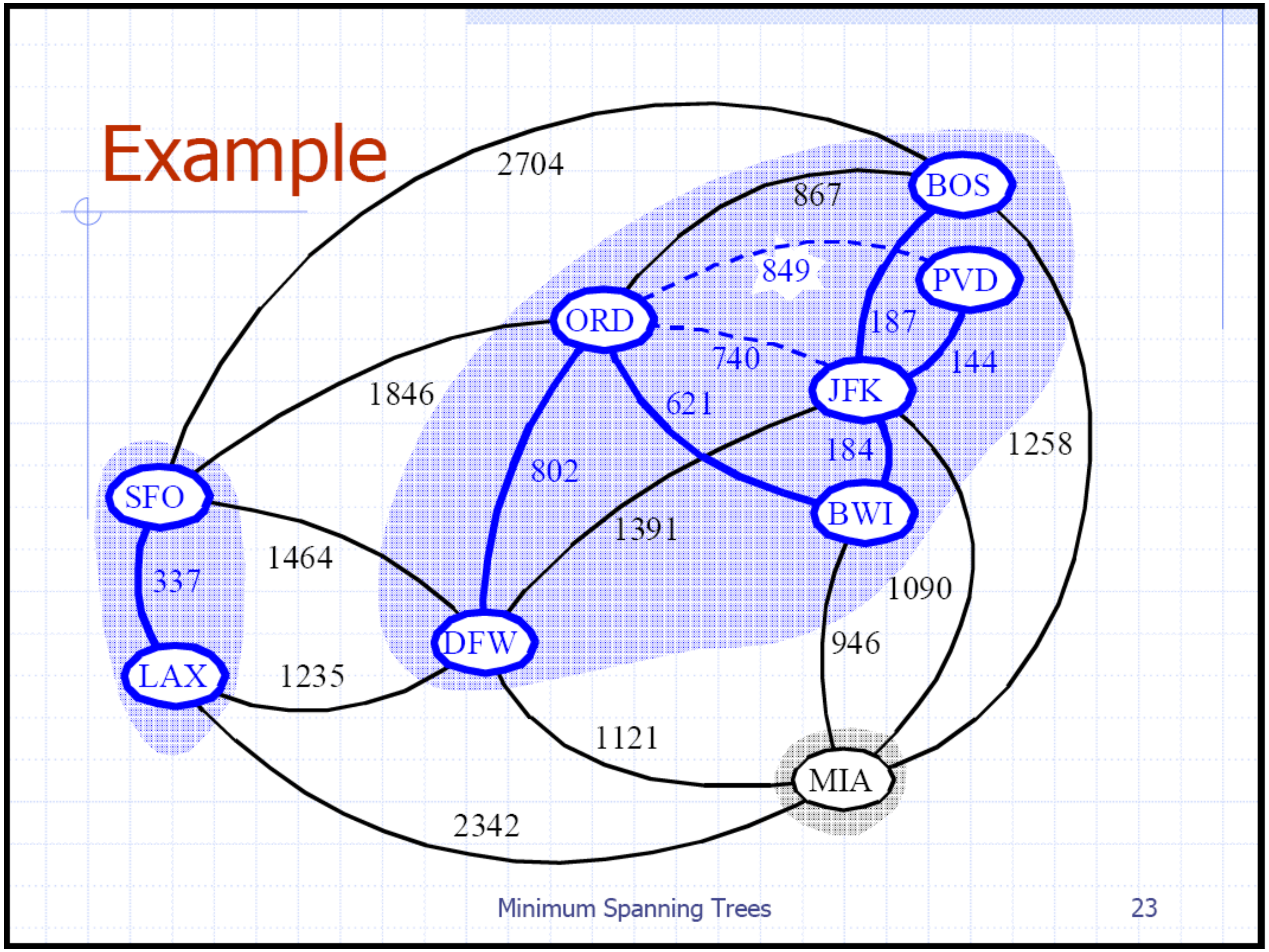
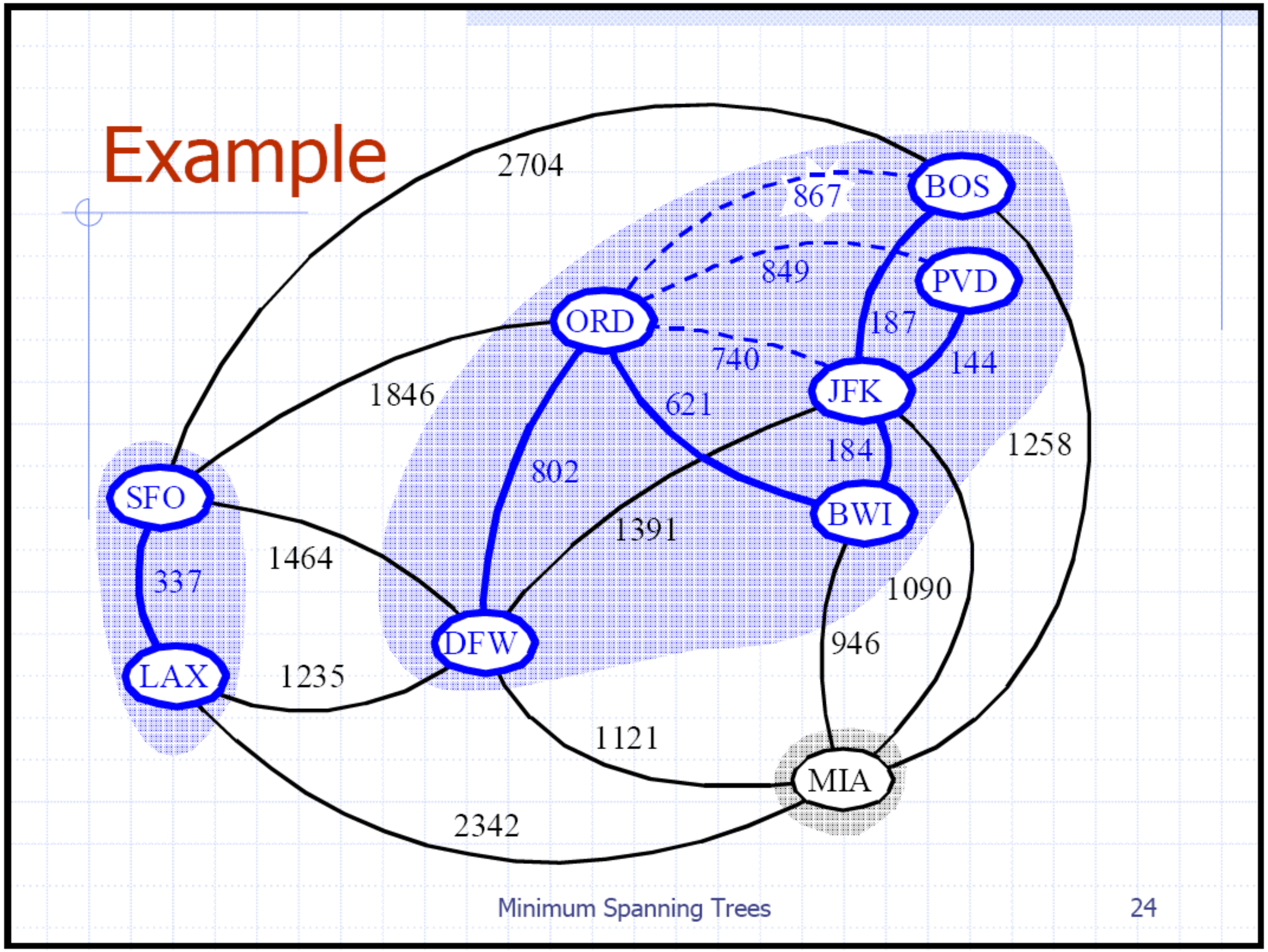
Kruskal’s Algorithm Example



Example

Minimum Spanning Trees

Kruskal’s Algorithm Example



Example

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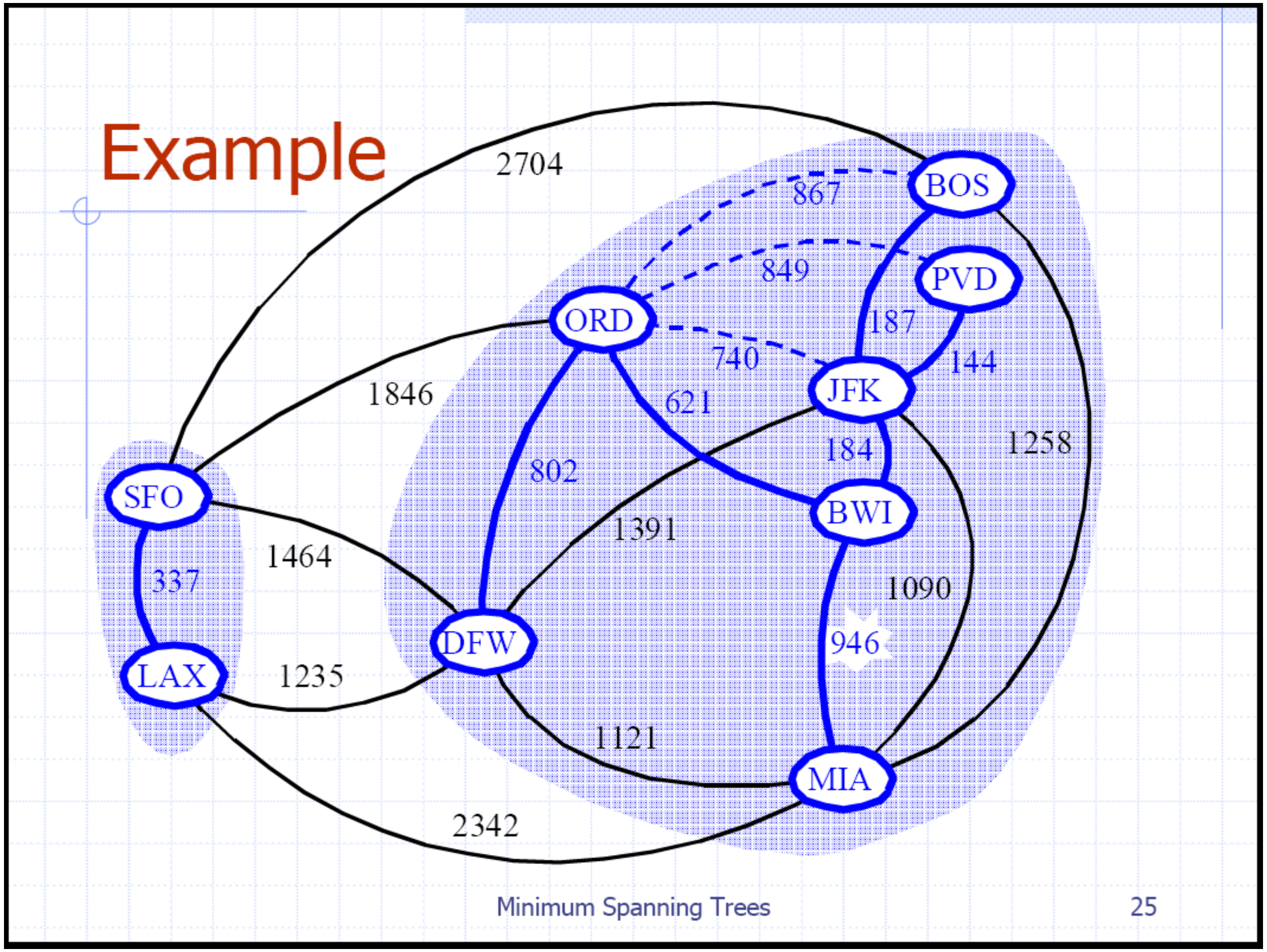
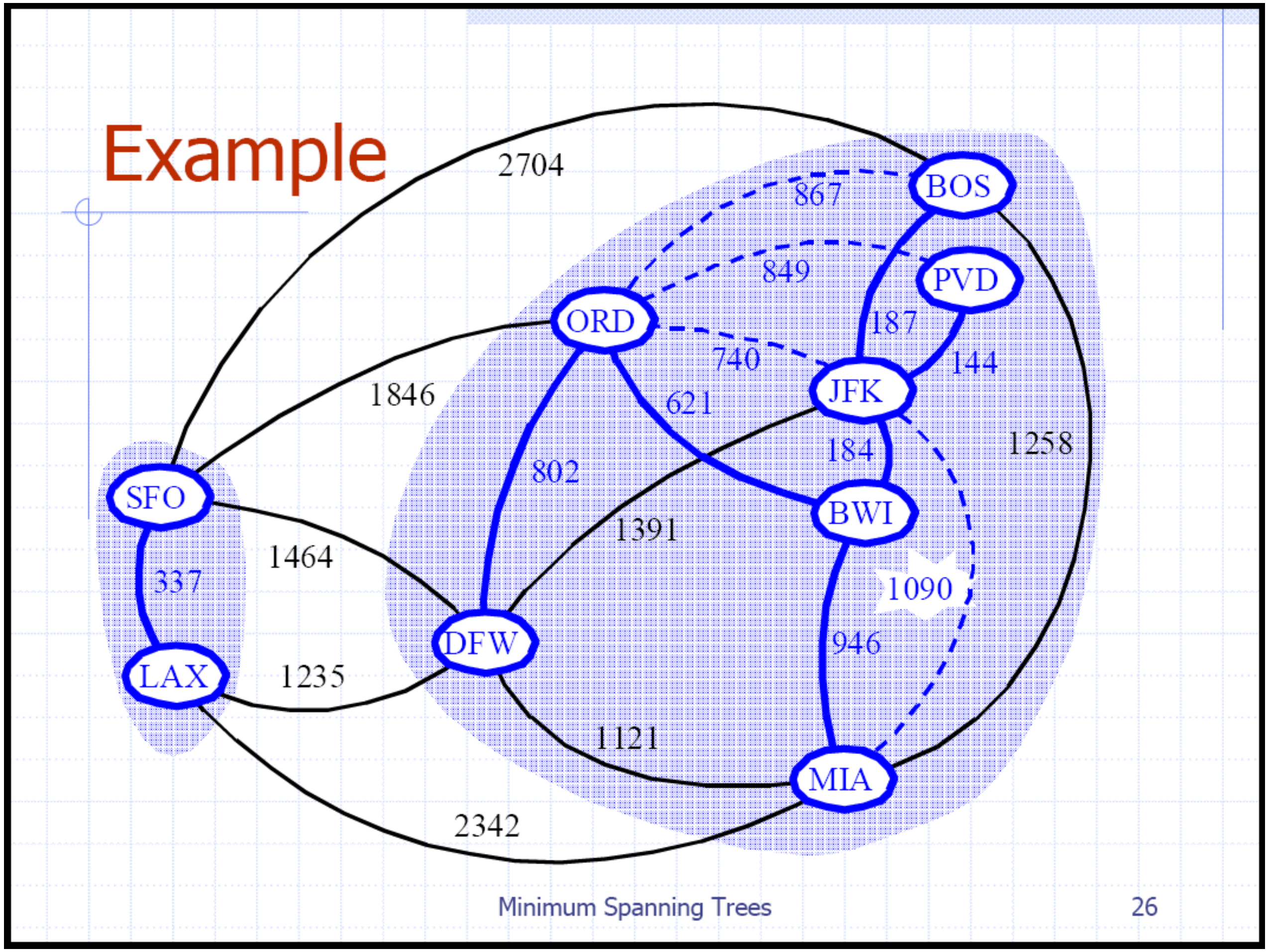
Kruskal’s Algorithm Example



Example

Kruskal’s Algorithm Example



Example

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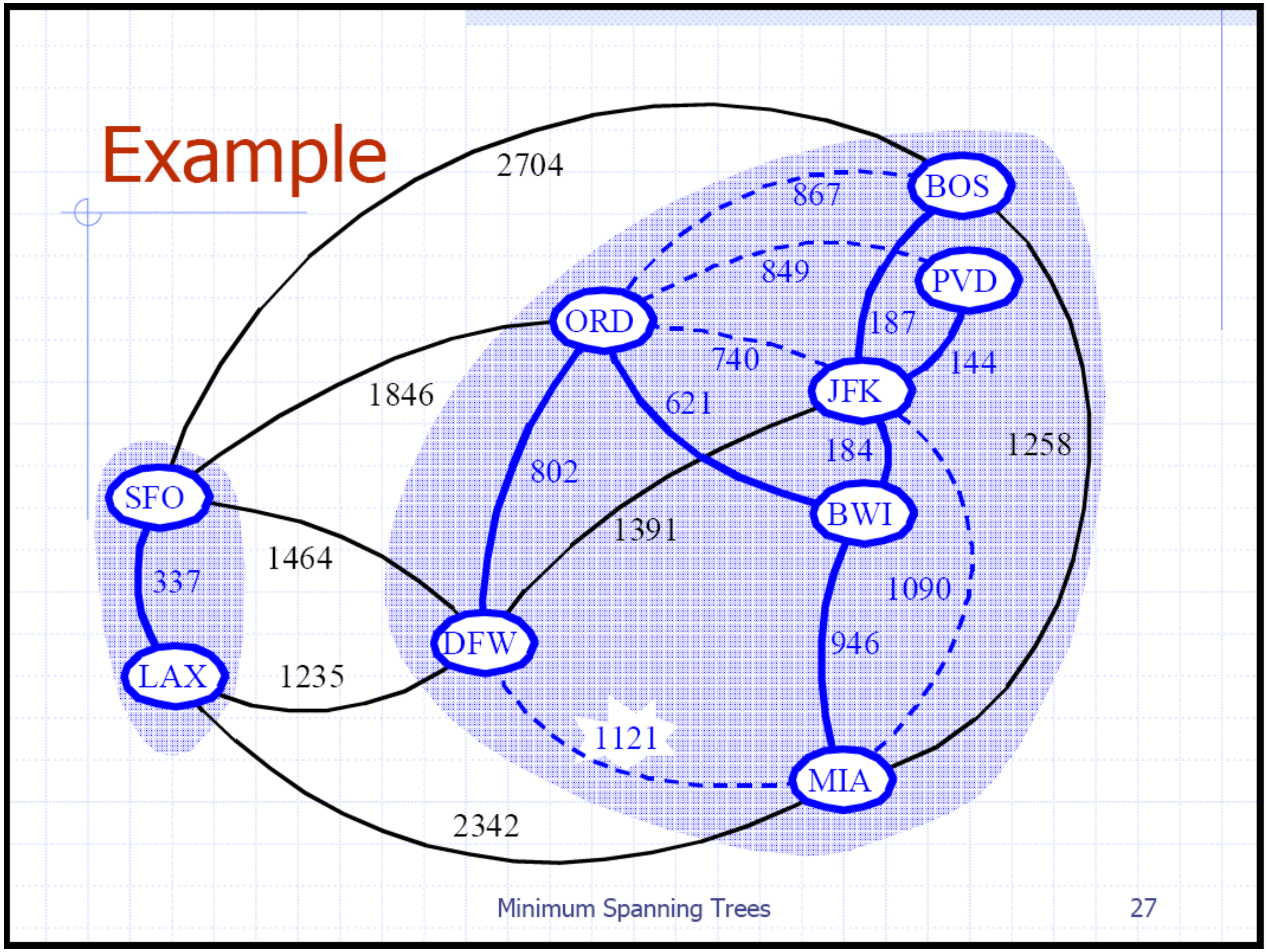
Kruskal’s Algorithm Example



Example

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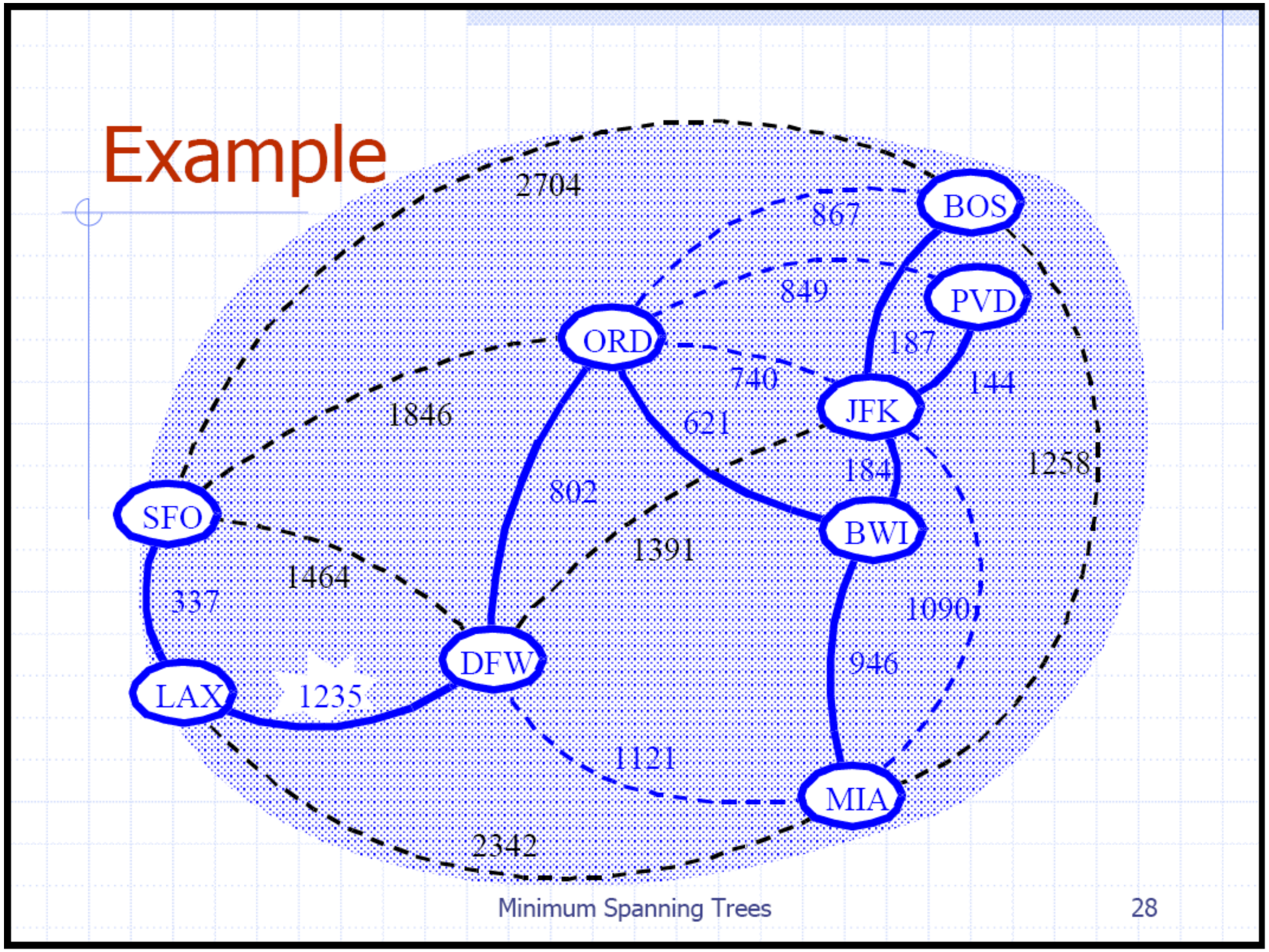
Kruskal’s Algorithm Example



Example

Thank You !!!