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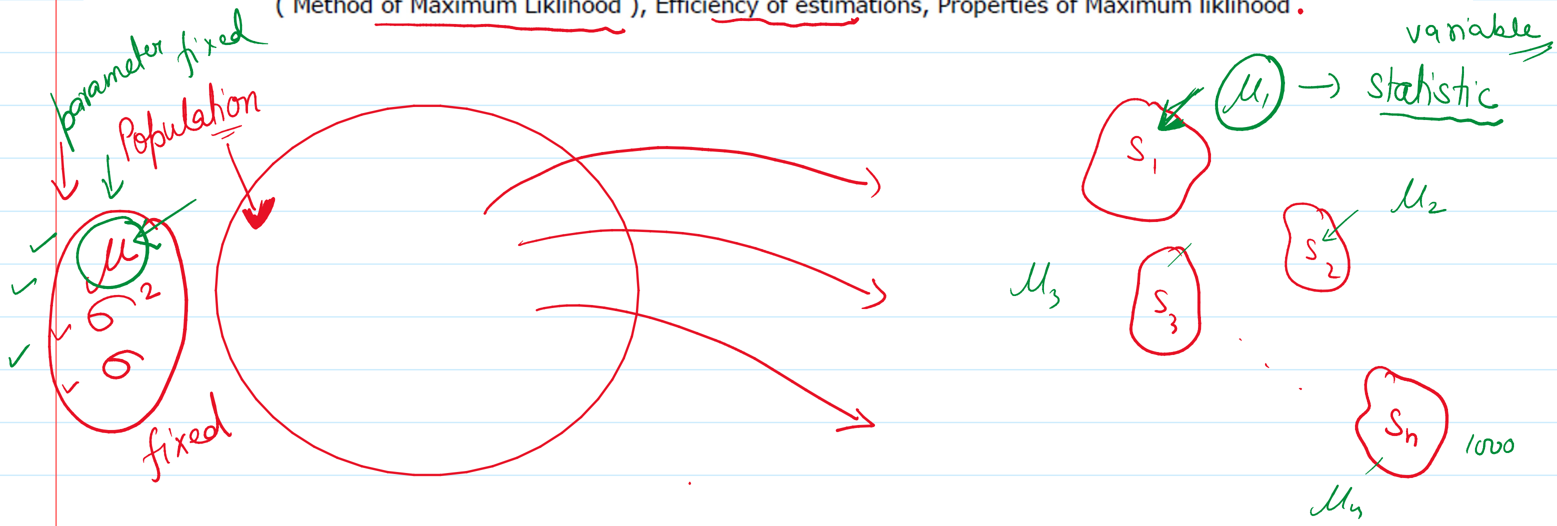
Unit-4

Unit IV

Point Estimation : Definition ,Unbiased Estimators, Consistent Estimators, Sufficient Estimator, MLE (Method of Maximum Liklihood), Efficiency of estimations, Properties of Maximum liklihood.



Definition :- Any function of the random sample $x_1, x_2, \ldots, x_n$ that are being observed, say $T_n(x_1, x_2, \ldots, x_n)$ is called a statistic. clearly, a statistic is a random variable.

If it is used to estimate unknown parameter θ of the distribution, it is called an estimator.

A particular value of the estimator, say $T_n(x_1, x_2, \ldots, x_n)$ is called an estimate of θ.

§ Characteristic of estimators:

(i) Unbiasedness or unbiased estimator

(ii) Consistency or consistent estimator

(iii) Efficiency or efficient estimator

(iv) Sufficiency or sufficient estimator

(i) Unbiased estimator: An estimator $T_n(x_1, x_2, \ldots, x_n)$ is said to be unbiased estimator of θ if $\boxed{E(T_n) = \theta}$.

(i) Unbiased estimator: An estimator $T_n(x_1, x_2, \dots, x_n)$ is said to be unbiased estimator of θ if $\boxed{E(T_n) = \theta}$.

Eg: $x_1, x_2, \dots, x_n$ is a random sample from a population $N(\mu, 1)$.
Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$, is an unbiased estimator of $\mu^2 + 1$.

$$\boxed{E(t) = \mu^2 + 1}$$

Sol^n:

$E(x_i) = \mu$, $\quad V(x_i) = 1 \quad \forall\ i = 1, 2, 3, \dots, n.$

$$\therefore E(t) = E\left\{\frac{1}{n}\sum_{i=1}^{n} x_i^2\right\} = \frac{1}{n}\left\{E\left(\sum_{i=1}^{n} x_i^2\right)\right\}$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(x_i^2)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{V(x_i) + \left(E(x_i)\right)^2\right\}$$

$$= \frac{1}{n}\sum_{i=1}^{n}(1 + \mu^2)$$

$$= \frac{1}{n}(1 + \mu^2)\sum_{i=1}^{n} 1$$

$$= \frac{1}{n} \times n(1 + \mu^2)$$

$\Rightarrow E(t) = \mu^2 + 1 \Rightarrow t$ is unbiased estimator of $\mu^2 + 1$.

$$\underbrace{E(t) = \theta}$$

Q) Let T be an unbiased estimator for θ. Then, which of the foll^n is correct?

(a) T^2 is also an unbiased estimator for θ^2. $\qquad \boxed{E(T_n) \neq \theta}$

(a) T^2 is also an unbiased estimator for θ^2.

(b) T^2 is an unbiased estimator of θ.

(c) T^2 is a biased estimator of θ^2

(d) None of these.

$\boxed{E(T_n) \neq \theta}$

T_n - biased estimator of θ

$$E(T) = \theta$$

$$E(T^2) = \theta^2 \quad \theta \quad \neq \theta^2$$

$$E(T^2) - \{E(T)\}^2 = V(T)$$

$$\Rightarrow E(T^2) = \{E(T)\}^2 + V(T)$$

$$\Rightarrow E(T^2) = \theta^2 + Var(T)$$

Since $Var(T) \geq 0$, $E(T^2) > \theta^2 \neq \theta^2$

✓ Sample mean is an unbiased estimator of population mean.

True False

$$E(\mu_i) = \mu$$

$x_1, x_2, \ldots, x_n$

✓ Sample variance is a biased estimator of population variance.

True False

$$E(\sigma_1^2) \neq \sigma^2$$

Consistent estimators