13-01-2022

Thursday, January 13, 2022 11:45 AM

Lecture-2

$A_i \cap A_j = \phi.$

$A_1, A_2, \ldots, A_n, \ldots$

Axiomatic Probability

$P(A)$ is the probability function defined on a σ −field $\boldsymbol{B}$ of events if the following properties or axioms hold.

1. For each $A \in \boldsymbol{B}$, $P(A)$ is defined, is real and $P(A) \geq 0$. (Axiom of non-negativity)
2. $P(S) = 1$. (Axiom of certainty)
3. If $\{A_n\}$ is any finite or infinite sequence of disjoint events in $\boldsymbol{B}$, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

(Axiom of additivity)

$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$

Theorems on Probability

T1. Probability of the impossible event is zero, i.e., $\boldsymbol{P(\phi) = 0}$

Pf: Impossible event has no sample point and the certain event S and the impossible event ϕ are mutually exclusive.

$\therefore S \cup \phi = S \Rightarrow P(S \cup \phi) = P(S).$

$\therefore$ Using Axiom of additivity, we get

$P(S) + P(\phi) = P(S) \Rightarrow \boxed{P(\phi) = 0.}$

$A_1 \cap A_2 = \phi$, then $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

T2. Probability of the complementary event $\bar{A}$ of A is $\boldsymbol{P(\bar{A}) = 1 - P(A)}$

Pf: A and $\bar{A}$ are mutually disjoint event, so that –

$A \cup \bar{A} = S \Rightarrow P(A \cup \bar{A}) = P(S)$

$\Rightarrow P(A \cup \bar{A}) = 1$ [Axiom of certainty].

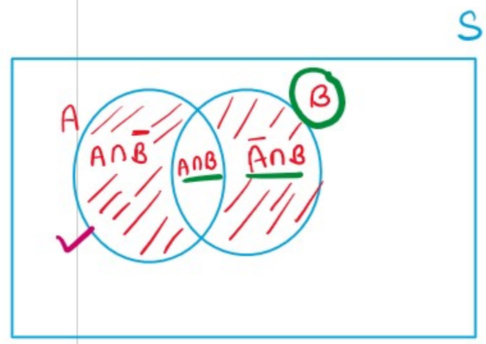
Using 3rd axiom, since $A \cap \bar{A} = \phi$, so

$P(A \cup \bar{A}) = P(A) + P(\bar{A})$

$\therefore P(A) + P(\bar{A}) = 1 \Rightarrow \boxed{P(\bar{A}) = 1 - P(A)}$

$A_1 \cap A_2 = \phi$
$\Rightarrow P(A_1 \cup A_2) = P(A_1) + P(A_2)$

T3. For any event $P(\bar{A} \cap B) = P(B) - P(A \cap B)$



$\Rightarrow$ $B = (A\cap B) \cup (\bar{A}\cap B)$,
where $\bar{A}\cap B$ & $A\cap B$ are disjoint events.
Hence, using Axiom 3 —

$P(B) = P(A\cap B) + P(\bar{A}\cap B)$

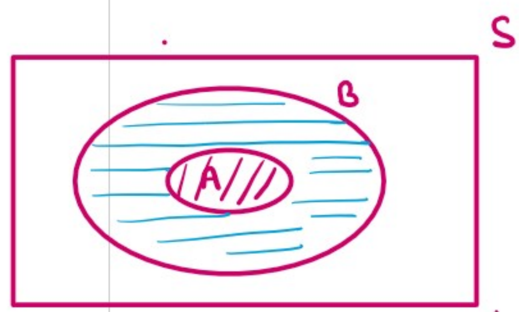
$\Rightarrow P(\bar{A}\cap B) = P(B) - P(A\cap B)$

① $P(A\cap\bar{B}) = P(A) - P(A\cap B)$

$A = (A\cap\bar{B}) \cup (A\cap B)$

$\Rightarrow P(A) = P(A\cap\bar{B}) + P(A\cap B) \Rightarrow P(A\cap\bar{B}) = P(A) - P(A\cap B).$

T4. If $A \subset B$**, then**
(i) $P(\bar{A} \cap B) = P(B) - P(A)$
(ii) $P(A) \leq P(B)$



Pf: (i) when $A\subset B$, then A and $\bar{A}\cap B$ are mutually exclusive, so that —

$B = A \cup (\bar{A}\cap B)$

$\Rightarrow P(B) = P(A) + P(\bar{A}\cap B)$ [Axiom 3]

$\Rightarrow P(\bar{A}\cap B) = P(B) - P(A).$

(ii) $P(\bar{A}\cap B) \geq 0 \Rightarrow P(B) - P(A) \geq 0 \Rightarrow P(B) \geq P(A)$

Hence, $A\subset B \Rightarrow P(A) \leq P(B)$. Proved//

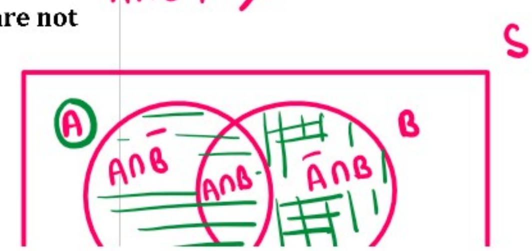
[redacted]

ADDITION THEOREM OF PROBABILITY

T5. If A **and** B **are any two events (subsets of sample space) and are not disjoint, then**

$A\cap B \neq \phi$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



Pf: We have —

$A\cup B = A\cup(\bar{A}\cap B)$, where A and

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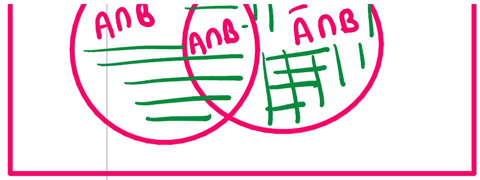

$$A\cup B = A\cup(\bar{A}\cap B)$$, where A and $\bar{A}\cap B$ are mutually disjoint.

$\therefore\ P(A\cup B) = P(A) + P(\bar{A}\cap B)$ [Using 3]

$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$

2nd method :-

$$P(A\cup B) = P(A) + P(\bar{A}\cap B)$$

$$= P(A) + P(\bar{A}\cap B) + P(A\cap B) - P(A\cap B)$$

$$= P(A) + \{P(\bar{A}\cap B) + P(A\cap B)\} - P(A\cap B)$$

$$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

3rd

$$P(A\cup B) = \frac{n(A\cup B)}{n(S)} = \frac{n(A)+n(B)-n(A\cap B)}{n(S)}$$

$$= n(A)/n(S) + n(B)/n(S) - n(A\cap B)/n(S)$$

$$\Rightarrow \boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

* $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$.

Q10. Two dice are tossed. Find the probability of getting 'an odd number on first die' or 'a total of 7'.

A = getting an odd no. on Ist die $= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

B = getting a total of 7.

$= \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$A\cap B = \{(1,6),(3,4),(5,2)\}$

$n(A) = 18$

$n(B) = 6$

$n(A\cap B) = 3$.

$$P(A\cup B) = \frac{n(A\cup B)}{n(S)} = \frac{18+6-3}{36} = \frac{21}{36} = \frac{7}{12}$$

$n(S)$ 36 36 12

$\frac{1}{1+3}$

Q11. An investment consultant predicts that the odds against the price of a certain stock will go up during the next week are 2:1 and the odds in favour of the price remaining same are 1:3. What is the probability that the price of the stock will go down during next week?

$= \frac{1}{2+1}$

→ A = stock price will go up

B = " " will remain same.

$P(A) = \frac{1}{2+1} = \frac{1}{3}$ $P(B) = \frac{1}{1+3} = \frac{1}{4}$.

$P(\bar{A} \cap \bar{B})$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{4} - 0$

$= \frac{7}{12}$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - \frac{7}{12}$

$= 5/12$ //

$m:n$ odd against

$P(A) = \frac{n}{m+n}$

odd in favour

$P(A) = \frac{m}{m+n}$

Q12. An MBA applies for a job in two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected at Y is 0.5. The probability of atleast one of his applications being rejected is 0.6. What is probability that

A = event that he is selected in firm X

B = event " " " in firm Y

$P(A) = 0.7 \Rightarrow P(\bar{A}) = 0.3$

$P(\bar{B}) = 0.5 \Rightarrow P(B) = 0.5.$

$P(\bar{A} \cup \bar{B}) = 0.6$

$\Rightarrow P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B}) = 0.6$

(i) he will be selected in both the firms?

$\Rightarrow 0.3 + 0.5 - P(\overline{A \cup B}) = 0.6$

$\Rightarrow P(\overline{A \cup B}) = 0.8 - 0.6 = 0.2$

$\Rightarrow 1 - P(A \cup B) = 0.2 \Rightarrow P(A \cup B) = 0.8$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.7 + 0.5 - 0.8$

$= 0.4$

(ii) he will be selected in one of the firms?

→ $0.8 = P(A \cup B)$

$0.8 = P(A \cup B)$

(iii) he will be selected in firm X only?

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.5.

$P(A \text{ only}) = P(A) - P(A \cap B) = 0.7 - 0.4 = 0.3$

Q. A card is drawn from a pack of 52 cards. What is the prob. of getting a king or a heart or a red card?

→ (a) $\frac{7}{13}$ (b) $\frac{5}{13}$ (c) $\frac{9}{13}$ (d) $\frac{1}{13}$

A = getting a king, B = getting a heart, C = getting a red card.

$P(A) = \frac{4}{52}$ $P(A \cap B) = \frac{1}{52}$ $P(A \cap B \cap C) = \frac{1}{52}$

$P(B) = \frac{13}{52}$ $P(B \cap C) = \frac{13}{52}$

$P(C) = \frac{26}{52}$ $P(A \cap C) = \frac{2}{52}$

$$\therefore P(A \cup B \cup C) = \frac{4}{52} + \frac{\cancel{13}}{52} + \frac{26}{52} - \frac{\cancel{1}}{52} - \frac{\cancel{13}}{52} - \frac{2}{52} + \frac{\cancel{1}}{52}$$

$$= \frac{28}{52} = \frac{7}{13} \;//$$

—x—