Lect-01

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Quantum Mechanics

1. Black body Radiation spectrum.
2. Photo electric effect
3. X-Ray spectrum
4. compton effect | Scattering
5. Hydrogen-Line spectrum
   OR. Atomic -

Domains where classical mechanics failed

a. where the velocity of object is very high ($10^8 ms^{-1}$) → (Theory of Relativity)
b. Atomic Realm (Quantum mechanics)

14 dec. 1900

→ Max. Planck produced a paper to explain Black body radiation.

$E = h\nu$

1905

→ Albert Einstein explain P.E.E by applyi Max planck's T.O.R.

1913 →

Neil's Bohr, gave Bohr's atomic model by introducing quantum condition on Angular momentum.

1923 A.H compton

1927

Davisson - Germer experiment

→ Particles have wave - character

Heisenberg (1925) | Erwin Schrodinger's (1927)

Matrix Mechanics | Wave mechanics

Postulates of quantum mechanics

Postulate 1:- The state of any physical system is given by space vector $\psi(x,t)$ in Hilbert space. This $\psi(x,t)$ contains all the information we need. Any superposition of state vectors is a state vector.

$$\psi'(x,t) = a\psi_1(x,t) + b\psi_2(x,t)$$

Postulate 2:- To every observables and/or dynamical variable, there corresponds a linear Harmition operator whose eigenfunction forms a complete basis.

Postulate 3:- The measurement of an observable $\hat{A}$ may be represented by the action of $\hat{A}$ on state vector $\psi(x,t)$ which results $a\psi$ where a is called eigen value of $\hat{A}$.

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Postulate 4:- The time evolution of the state vector $\psi(x,t)$ is given by the time dependent schrodinger's equation.

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\frac{-\hbar^2}{2m}\frac{-\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

Failure of classical (EM) Theory

→ (4) C.P could not explain the pheonomena related to atomic domain.
→ C.P could not explain the Hydrogen-spectra.
→ (1) This theory was not able to explain the stability of atom.
→ The enormous range of conductivity of metals compared to insulators also could not be explained.
→ The range of magnetic susceptibility is also very large, but it couldn't explained.
→ (3) Classical theory could not explained the emission of α-particle from a nucleus despite of the fact that it has insufficient energy.
→ The specific heat variation with temprature was not (5) explain by the classical theory.
→ (2) Black body radiation, PEE, Raman effect, compton effect was not successfully explained by classical Theory. Dual nature of particle etc.

Photoelectric Effect :-

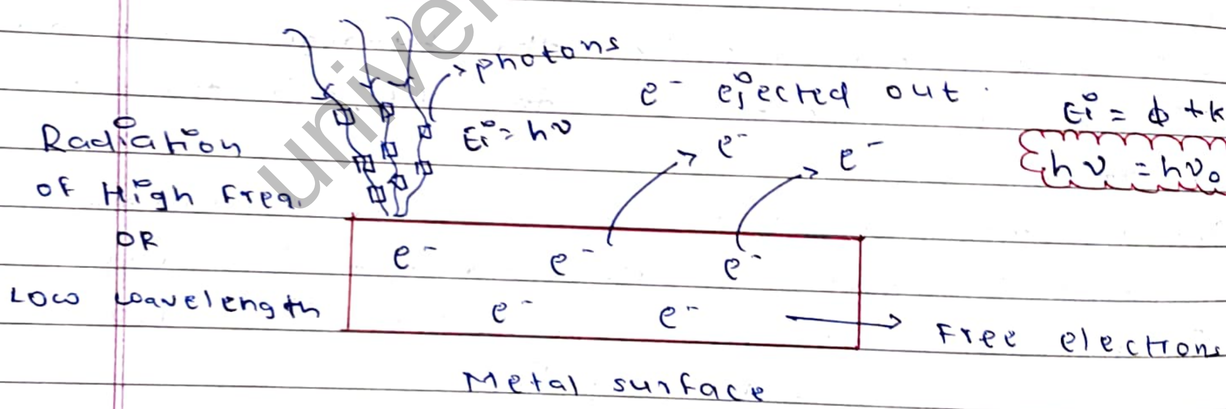
Random Light → Wave nature
→ Particle Nature

Particle Nature of Light.

Light → 1675 → Newton → Light → Particle corpuscle Theory.

1801 → YDSE Diffraction Polarisation } Light wave

1905 → Photoelectric } → Light → Particle



$E_i = \phi + KE$

$h\nu = h\nu_0 + KE$

① Light consists of particle called photons.
'wave packets'
'energ packets'
'quanta'

② Mass of photon is not defined in sense of Newtonian Mechanics. The Rest mass of photon Zero

3. All photons travel with 299,792,458 m/s in air vacum $\approx 3\times10^9$ m/s.

4. Photons are neutral in nature.

5. Each photon has a definite Energy & definite Momentum.

6. $E = \frac{hc}{\lambda}$ h ~ Planck's constant

$h = 6.62\times10^{-34}$ J/s

$E = \frac{1240\, eV}{\lambda\ (in\ nm)}$

$P = \frac{h}{\lambda}$.

$E = hF$

$v = f\lambda$.

$f = v/\lambda$.

$f = c/\lambda$.

6. All collisons b/w a photon & a matter particle are perfectly elastic.

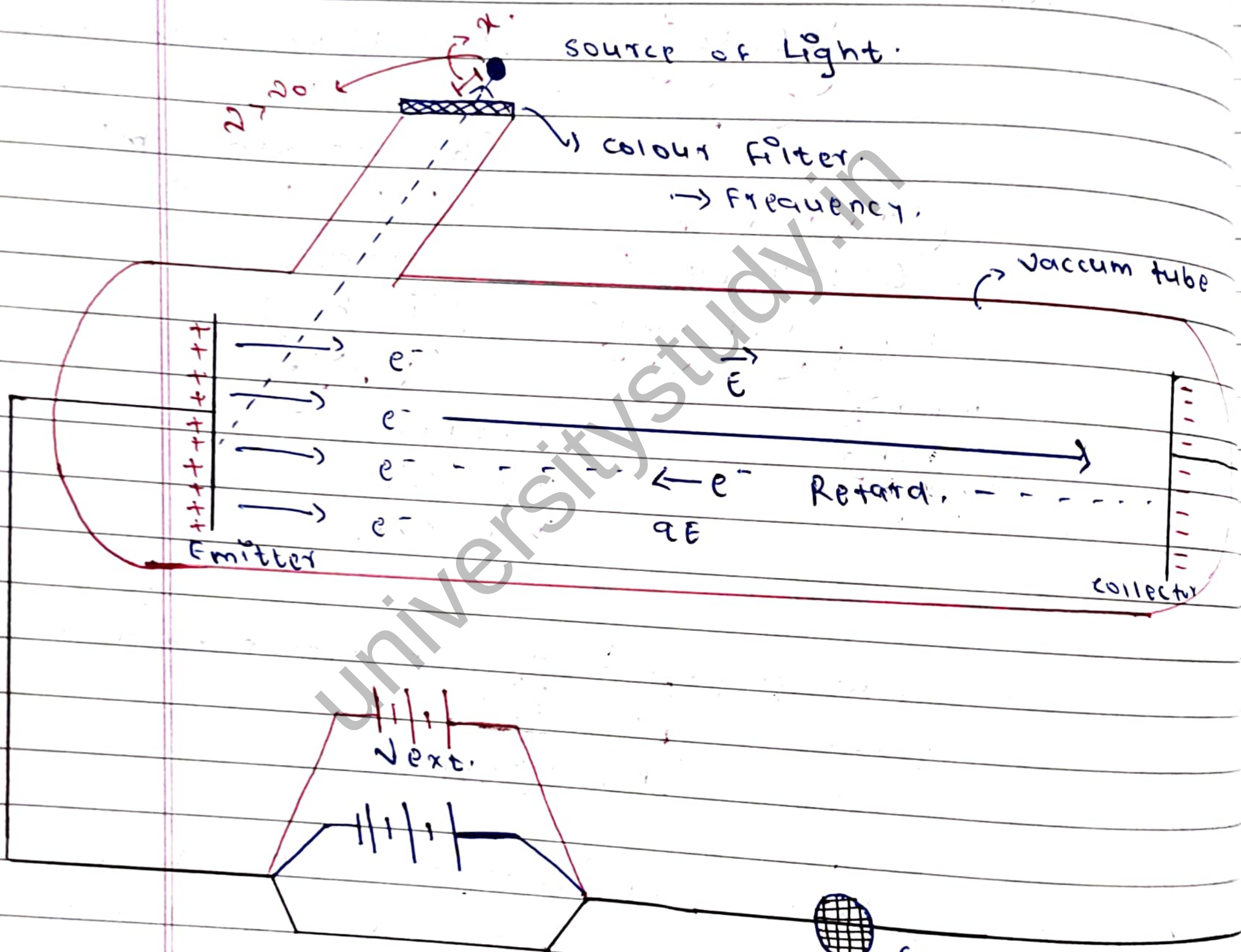
$\boxed{\vec{P_i} = \vec{P_f}}$

$\boxed{K.E_i = KE_f}$

7. On increasing intensity of Radiation (light), the no. of photons increased but the energy of each photon does not change.

Ejection of e^- from Metal surface → Work f...

When light of particular freq. is incident on a m... surface e^- are emmited from it, these e^- are called photoelectrons. The Effect is called photoelect... Effect.



* $\nu < \nu_0$ NO PEE will occur.

* E.F will set up in our tube
* e^- will be trapped in E.F & will accelerate...

→ Stopping potential → −ve voltage of collector at which photocurrent falls to zero.

→ −ve voltage at which e^- with highest KE also stops.

Analysis :

→ $\nu > \nu_0$ (Threshold frequency)

→ $\nu < \nu_0$ whatever may be time for which light is incident on metal There will be no PEE.

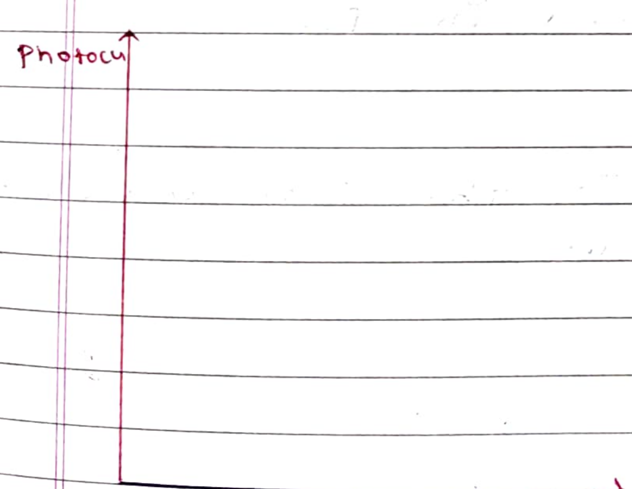
→ I of light ∝ no. of Photoelectrons ∝ current.

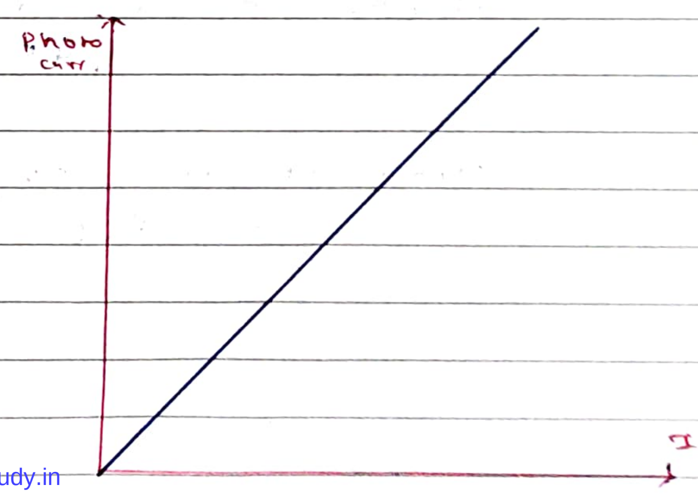
→ F of light ↑ ∝ maximum K.E of e^- ∝ stopping pot.

→ If $\nu = \nu_0$ (e^- will come out with zero KE) due to which metal will be +ve charged & absorbs back the e^-.

Observation :

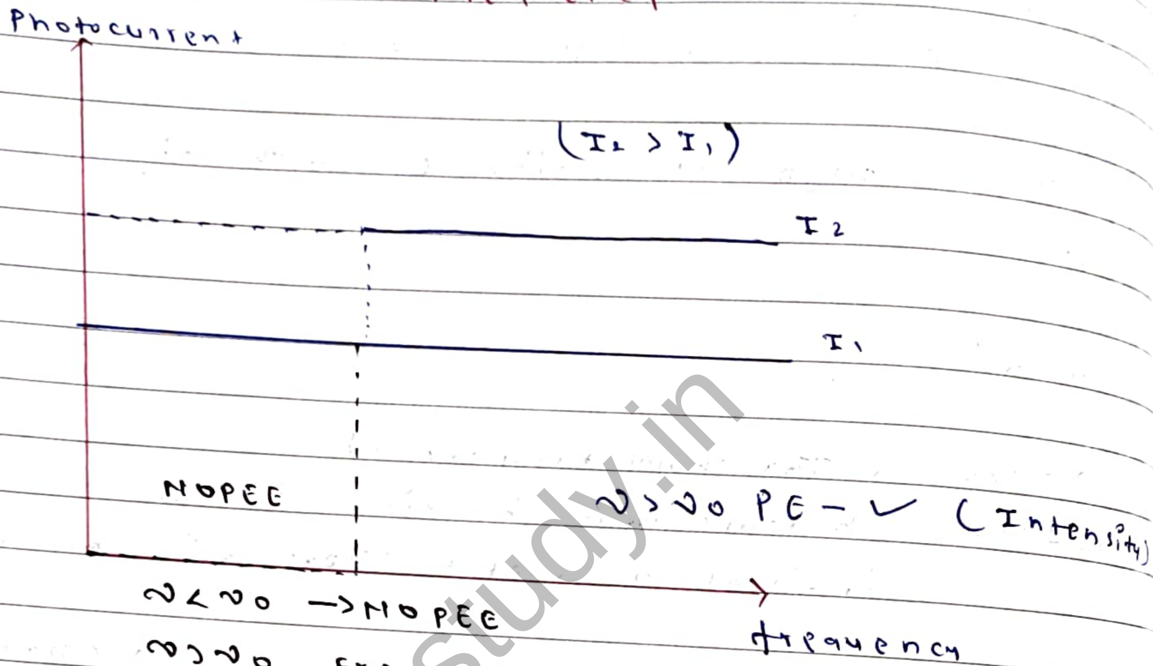
1. Intensity v/s Photocurrent.





$f < \nu_0$ $\nu < \nu_0$ $\nu > \nu_0$ >.

2. Photocurrent vs Frequency :



$\nu < \nu_0$ → NO PEE

$\nu > \nu_0$ Freq. will not decide photocurrent.

Photocurrent depends on intensity of light, & is independent of frequency.

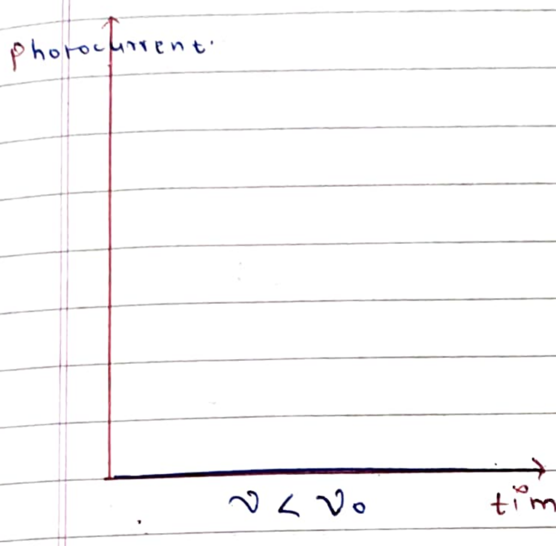
Threshold frequency :- Min. frequency at which PEE starts occuring (ν_0)

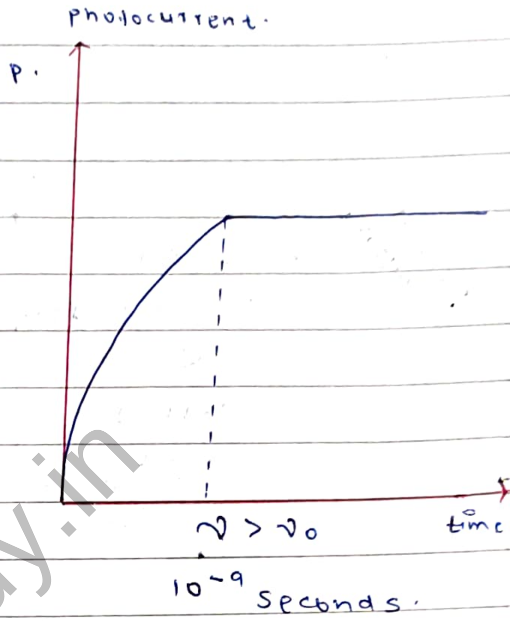
Work Function :- Min energy Required to remove an e^- from metal in PEE.

$$\boxed{\phi_0 = h\nu_0}$$

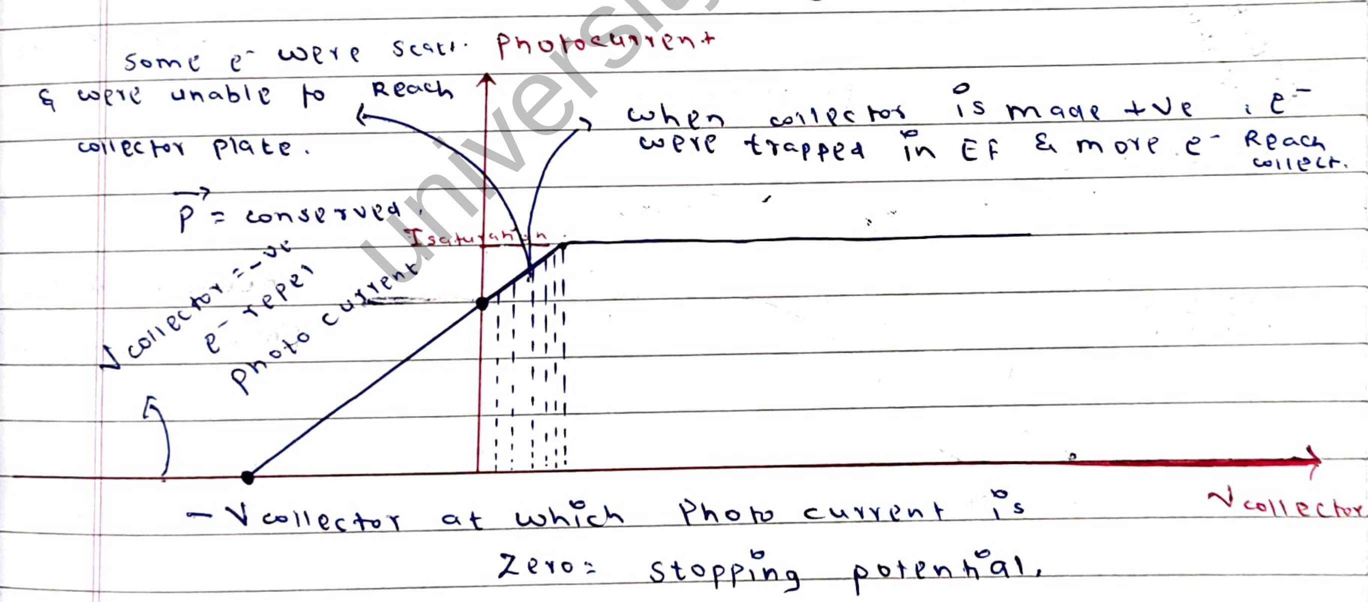
Cut-off-wavelength :- Is the Max wavelength upto which PEE will occur.

3. Photocurrent vs Time.





4. Photocurrent vs Voltage



Case 1 : Different Intensity

→ It will impact photocurrent

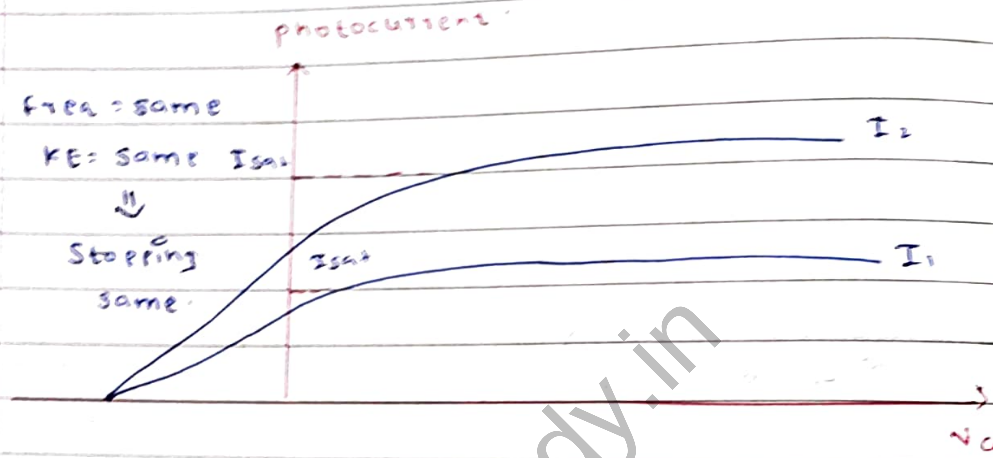
→ No change of Stopping Pot. (Freq)

freq = same

KE = same

⇓

Stopping same.



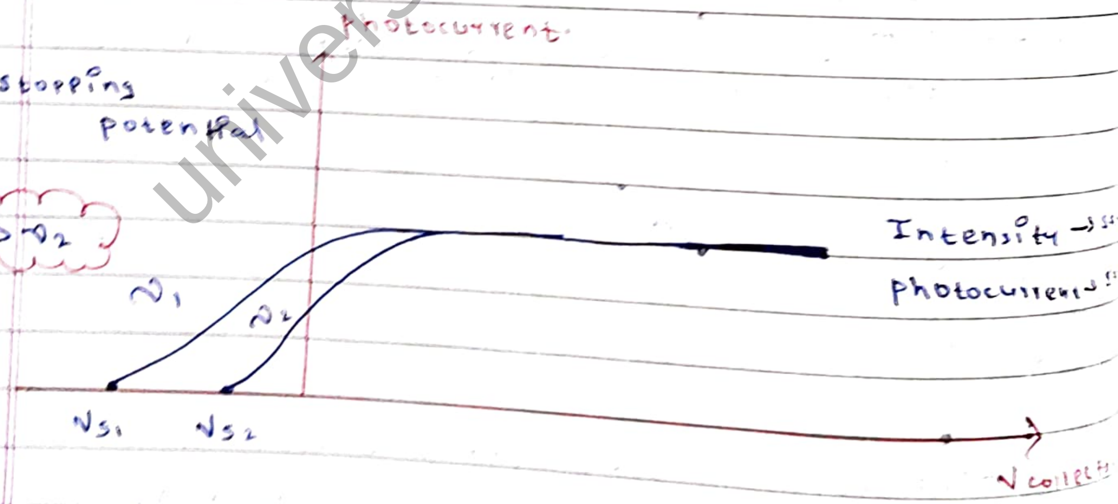
case 2 : Different frequency (Intensity constant

→ Saturated p.c

→ consta

V_s = stopping potential

$\nu_1 > \nu_2$

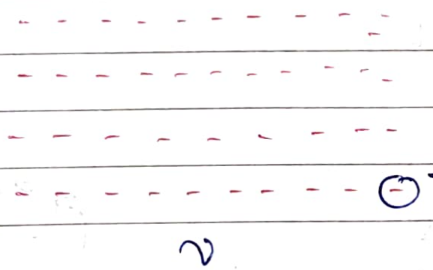


ALL GLORY COMES FROM DARING TO BEGIN.

Einstein Explanation of photoelectric effect.

quanta of lights → packet of energy.

- Radiation (observed by metal are in form of quanta (photon). Energy of each photons depends on frequency.



Energy of light = $h\nu$

→ Photon $E \propto \nu$

$\boxed{E = h\nu}$

ν

If n no. of Photons are incident $\boxed{E_T = nh\nu}$

- One photon interact with one electron only and interaction is instantenous process.
  (one-to-one interaction) (instaneous process.) $\sim 10^{-9}$ s.

- In the interaction the photon transfer complete energy to the electric.

- During the interaction the energy and momentum is conserved. → (collison process b/w 1 P & 1 e^-)
  (Momentum is conserved.)

- The I of light is directly proportional to no. of photons, hence if intensity increases the no. of photon incident increases which leads to more emission of electrons (rise of photocurrent).

$$I = \frac{E}{tA}$$

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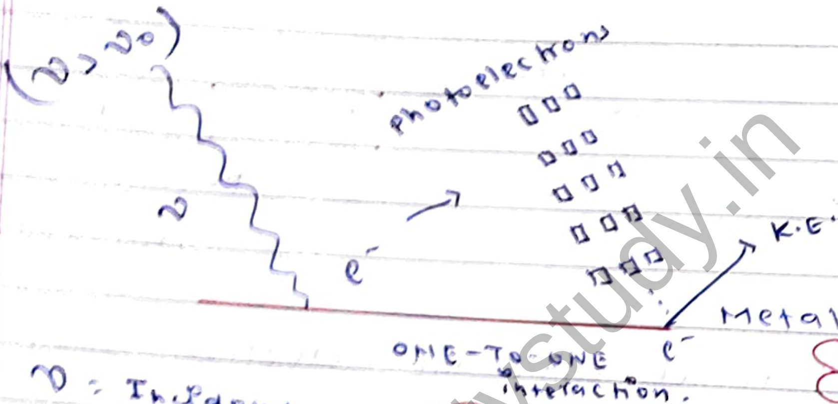
acc. to Einstein   Energy = $nh\nu$

n = Total no. of photons

$I \propto n$

($n\uparrow$, $I\uparrow$, no. of collisions $\uparrow$, no. of photo $e^-\uparrow$)

- Einstein's Equation of P.E.E



depends on Target

work function

$\boxed{\phi = h\nu_0}$

ν = Incident Frequency.

$\boxed{E_i = \phi_0 + KE_{e^-}}$

If $\nu < \nu_0$ $E_i < \phi_0$.

Forms

$\boxed{E_i = \phi_0 + KE}$

$\boxed{h\nu = h\nu_0 + \frac{1}{2} mv^2}$

$\boxed{\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \frac{1}{2} mv^2}$

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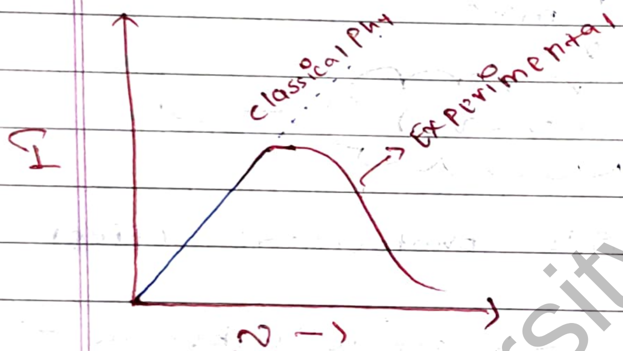
Black Body Radiation

$$I(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

h = Planck's constant

c = speed of light

ν = frequency.



Einstein Photoelectric Equation

→ has explained various law of PEE on the basis of Planck's law,

When a photon of energy $h\nu$ strikes on a metal surface a part of it's energy known as work function (ϕ) is used to liberate the electrons from the surface, whereas the remaining energy is spent for it's kinetic energy.

Thus, the energy of photon = the sum of work function and kinetic energy of the electron, that is

$$h\nu = w_0 + \frac{1}{2} m v^2 \quad \text{—— (1)}$$

Dual nature of a particle / Matter.

→ De broglie suggestion was based on the reasoning that nature is symmetrical & universe is composed of matter & Radiations, thus this two physical entities matter and Radiations should have symmetry, The wave associated with a moving particles is known as de-broglie wave, & the wave-length associated with this particle is known as de-broglie wave-length it is denoted by symbol (λ) =

De-broglie wave Equation.

Acc. to planck's quantum theory energy of photon with frequency ν is given as.

$$E = h\nu \longrightarrow (1).$$

The Einstein's Mass, Energy relation is given as

$$E = Mc^2 \longrightarrow (2).$$

From Eq (1) & (2).

$$mc^2 = h\nu$$

$$m = \frac{h\nu}{c^2} \longrightarrow (3).$$

→ particle is moving at speed of light

We know, $\vec{P}$ = Mass × velocity

$$P = mc. \longrightarrow (4).$$

when a min. threshold frequency is provided in that case, electron will just eject from the surface & velocity will become zero.

In that case $\nu = \nu_0$, $v = 0$

Eqn (1) will be modified as $h\nu_0 = \omega_0$ — (2).

Eq (1) & (2).

$$h\nu = h\nu_0 + \frac{1}{2} mv^2$$

$$\therefore \frac{1}{2} mv^2 = h(\nu - \nu_0) \quad \text{— (3)}$$

Eq - (3) is the Einstein's photoelectric effect.

$\nu < \nu_0$ → K.E will become -ve. which is not possible Hence, No PE emission is possible, if frequency of incident photon is less than threshold frequency.

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Explanation of de-broglie wave-Equation.

→ If mass or velocity is kept large, in that case, de-broglie wavelength will be small

→ De-broglie wavelength is independent on charge

→ If velocity = 0, in that case $\lambda \to \infty$.
such waves can't be visualized hence, the de-broglie wavelength is associated only with the moving particle.

De-broglie wavelength for an electron.

→ consider an electron with mass m, charge e & potential V so $E = eV$ — ①

→ ~~consider~~ The K.E of the electron is given as $\frac{1}{2}mv^2$ — ②.

on multiplying & dividing eq. ② (R.H.S) with mass.

$$E = \frac{1}{2} mv^2 \cdot \frac{m}{m}$$

$$= \frac{1}{2m}(mv)^2 \quad [\text{where } P = 2m].$$

$$E = \frac{P^2}{2m}$$

$$P = \sqrt{2mE} \quad — ③.$$

From Eq (3) & (4)

$$P = \frac{h\nu}{c^{\cancel{2}}} \times \cancel{c}$$

$$\boxed{P = \frac{h\nu}{c}} \quad \text{— (5)}$$

We know $c = \nu\lambda$ — (6)

From (5) & (6)

$$P = \frac{h\cancel{\nu}}{\cancel{\nu}\lambda}$$

$$P = \frac{h}{\lambda}$$

$$P\lambda = h$$

$$\boxed{\lambda = \frac{h}{P}} \quad \text{— (7)}$$

$$\lambda = \frac{h}{P} \quad \text{— (8)}$$

When a particle is moving with a velocity v then momentum $P = mv$.

From Eq. (8) & (7)

$$\lambda = \frac{h}{mv}$$

Above Equation is de-broglie wavelength Equation.

from Equation ① & ③.

$$p = \sqrt{2meV} \quad — ④.$$

We know de-b w — $\lambda = \frac{h}{p}$. — ⑤.

From ④ & ⑤,

$$\lambda = \frac{h}{\sqrt{2meV}}.$$

de-broglie wavelength for an electron,

Heisenberg Uncertainity Principle.

Acc. to U.P, it is impossible to measure simultaneously the exact position & momentum.

Mathematically, $\Delta x \, \Delta p \geq \frac{h}{4\pi}$

or

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}. \qquad \hbar = \frac{h}{2\pi}$$

In terms of Energy & time the uncertainity is given as

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

or

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

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Phase velocity (VP) & Group velocity (VG)

Phase velocity (VP)

→ It is the velocity with which planes of equal phase travel through a medium. the phase velocity is associated with a single wave.

The Equation of propogation of a wave is given as

$$y = a\sin(\omega t - kx) \quad -(1)$$

Where a represent amplitude.
ω is angular frequency
t is time
k is propagation constant
x is displacement.

Where $(\omega t - kx)$ is the phase of the wave
if the phase is constant then $\omega t - kx$ will be equal to constant.

$\omega t - kx$ is phase
$\omega t - kx =$ constant $-(2)$.

on diff Eq. 2

$$\omega - k\frac{dx}{dt} = 0.$$

$$k\frac{dx}{dt} = \omega \quad \text{or} \quad \frac{dx}{dt} = \frac{\omega}{k}.$$

$$V_p = \frac{\omega}{k}.$$

Above expression represent the phase velocity of the wave.

where $\omega = 2\pi\nu$ & $k = \frac{2\pi}{\lambda}$.

Relationship b/w phase velocity & particle vel.

Since, phase velocity $v_p = \frac{\omega}{k}$ — (1)

Also $\omega = 2\pi\nu$ & $k = \frac{2\pi}{\lambda}$ — (3)
↳ (2)

$$v_p = \frac{\cancel{2\pi}\nu}{\frac{\cancel{2\pi}}{\lambda}}$$

$$v_p = \lambda\nu$$ — (4).

The energy of photon is given as $E = h\nu$
or $\nu = \frac{E}{h}$ — (5).

De-Broglie wave-equation is given as $\lambda = \frac{h}{mv}$ — (6).

$$v_p = \frac{\cancel{h}}{mv} \cdot \frac{E}{\cancel{h}}$$, $$v_p = \frac{E}{mv}$$ — (7)

From Einstein's Equation $E = mc^2$ — (8)

From (7) & (8)

$$v_p = \frac{\cancel{m}c^2}{\cancel{m}v}, \quad v_p = \frac{c^2}{v}$$

$\therefore$ A phase velocity is not associated with a particle in a motion.

Group Velocity (V_G).

→ The wave packet / bundle / Group, moves on it's own velocity known as group velocity.

consider a group of wave consisting two sine waves with same amplitude "a" and slightly different angular frequency "ω_1 & ω_2" & prop. constant "k_1 & k_2". This waves are represent as.

$$y_1 = a\sin(\omega_1 t - k_1 x) \quad — (1)$$

$$y_2 = a\sin(\omega_2 t - k_2 x) \quad — (2)$$

The super-imposition of this two waves will give

$$Y = y_1 + y_2. \quad — (3)$$

From Eq. (1) + (2) + (3)

$$Y = a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

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$$y = 2a\sin\left(\frac{\omega_1 t - k_1 x + \omega_2 t - k_2 x}{2}\right)\cos\left(\frac{\omega_1 t - k_1 x}{2} + \frac{\omega_2 t - k_2 x}{2}\right)$$

$$y = 2a\sin\left(\frac{\omega_1 t - k_1 x + \omega_2 t - k_2 x}{2}\right)\cos\left(\frac{\omega_1 t - k_1 x + \omega_2 t - k_2 x}{2}\right)$$

$$y = 2a\sin\left[\frac{(\omega_1+\omega_2)t}{2} - \left(\frac{k_1+k_2}{2}\right)x\right]\cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(k_1-k_2)x}{2}\right] \quad -(4)$$

$$\frac{\omega_1+\omega_2}{2} = \omega, \quad \frac{k_1+k_2}{2} = k.$$

$$y = 2a\sin\left[\omega t - kx\right].\cos\left[\frac{t\Delta\omega}{2} - \frac{x\Delta k}{2}\right] \quad -(5)$$

The general wave Equation is written as

$$y = a\sin(\omega t - kx) \quad -(6)$$

On comparing Eq. (5) & (6)

$$\text{Amplitude} = 2a\cos\left[\frac{t\Delta\omega}{2} - \frac{x\Delta k}{2}\right]$$

$$2a\cos\left[\frac{\Delta K}{2}\left[\frac{\Delta\omega}{\Delta k}\cdot t - x\right]\right]$$

where $\boxed{v_g = \frac{\Delta\omega}{\Delta k}}$.

The above expression represent the group velocity associated with the particle.

R/s b/w v_p & v_G.

since, $v_p = \frac{\omega}{k}$.

or $\omega = kv_p$ — ①.

On diff. both side w.r.t to k in Eq ①

ω ~~dk~~

$$\frac{d\omega}{dk} = v_p + k\frac{dv_p}{dk}$$

$$v_G = v_p + k\frac{dv_p}{dk} \quad \left[\because v_g = \frac{d\omega}{k}\right] \text{ — ②.}$$

$$k = \frac{2\pi}{\lambda}.$$

We know, $k = \frac{2\pi}{\lambda}$ — ③.

③.

On. diff. both side w.r.t 'λ'

$$\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}\cdot d\lambda \text{ — ④}$$

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on substituting ③ & ④. in Eq ②.

$$V_g = V_p + \frac{2\pi}{\lambda} \cdot \frac{dV_p}{\frac{-2\pi}{\lambda^2}}$$

$$V_g = V_p + \frac{\cancel{2\pi} \cdot \lambda \cdot dV_p}{-\cancel{2\pi}}$$

$$V_g = V_p - \lambda \frac{dV_p}{d\lambda}.$$

The above Equation Represent RL b/w Group Velocity & phase velocity.

where, λ is the dispersion in the medium

case ① :- when there is no dispersion, then $\lambda = 0$.

in that case $V_g = V_p$.

case ② :- For Normal dispersive medium
$V_p > V_G$.

Case ③ :- For anomolous dispersive medium
$V_p < V_G$.

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R/s b/w V_G & Particle velocity (v).

consider, a particle moving with velocity (v). The K.E is given as $E = \frac{1}{2} mv^2$

In terms of Momentum $E = \frac{p^2}{2m}$ $\left[\because p = mv \right]$ $\rightarrow$ (1)

since $E = \hbar \omega$ — (2)

$p = \hbar k$ — (3)

$E = h\nu$

$E = \frac{h}{2\pi} \cdot 2\pi\nu$

$E = \hbar \omega$

$\lambda = \frac{h}{p}$

$p = \frac{h}{\lambda}$

$p = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda}$

on substituting, Eq. (2) & (3) in Eq. (1).

$\hbar \omega = \frac{\hbar^2 k^2}{2m}$

$\omega = \frac{\hbar k^2}{2m}$ — (4).

on diff. Eq. (4) w.r.t k.

$\frac{d\omega}{dk} = \frac{2\hbar k}{2m}$

$\frac{d\omega}{dk} = \frac{\hbar k}{m}$

$V_g = \frac{\hbar k}{m}$ → (5)

$\left[\because V_g = \frac{d\omega}{dk} \right]$

From Eq. (3) & (5).

$V_g = \frac{p}{m}$

Since $p = mv$ so

$$v_g = \frac{\cancel{p} v}{\cancel{m}} \qquad \boxed{v_g = v}$$

Wave function (ψ)

The quantity whose variation make-up the matter waves is known as wave function. Matter wave are associated with the moving particle. Thus the wave function is the function which describes the matter wave associated with a moving particle. The wave function is a function of both space & time.

The wave function $\boxed{\psi = A + iB}$

A & B $\rightarrow$ constant.

The ψ can be expressed as

$$\boxed{\psi = A e^{i(kx - \omega t)}}$$

$\psi = 2+3i$
$\psi^* = 2-3i$

$$|\psi|^2 = (2-3i)(2+3i)$$
$$= (2)^2 - (3i)^2$$
$$= 4+9 = 13$$

Physical significance of ψ.

→ $|\psi|^2 = \psi^* \psi$

The wave function ψ has no direct physical significance, the probability of ψ gives the physical significance and the probability is written as $|\psi|^2 = \psi^* \psi$, where the probability should have real & +ve values

Normalization of ψ

→ As the particle must be found somewhere, so one can say that the probability of finding the particle in whole space is unity that is

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 = 1$$

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Schrodinger's Time dependent & Independent wave Equation.

→ Time dependent wave Equation

$$-\frac{i\hbar\partial\psi}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}$$

→ Time independent wave Equation

$$\nabla^2\psi + \frac{2m}{\hbar}(E-V)\psi = 0.$$

Operators

1. **Energy operators $[\hat{E}]$**

$$\psi = Ae^{i(kx-\omega t)} \quad — (1)$$

$$P = \hbar k \Rightarrow k = \frac{P}{\hbar} \quad — (2)$$

$$E = \hbar\omega \quad \omega = \frac{E}{\hbar} \quad — (3)$$

From (1) (2) & (3)

$$\psi = Ae^{i(Rx-Et)} \quad — (4)$$

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}Ae^{i/\hbar(Px-Et)} \quad — (5).$$

From (4) & (5)

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi$$

$$E\psi = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t}$$

$$E\psi = \frac{-i\hbar}{i \cdot i} \frac{\partial \psi}{\partial t}$$

$$E\psi = \frac{i\hbar}{i \cdot i} \frac{\partial \psi}{\partial t}$$

$$E = i\hbar \frac{\partial}{\partial t}$$

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2. Momentum operator $[\hat{P}]$

$$\psi = A e^{i(kx - \omega t)} \quad \text{--- (1)}$$

(2)

(3)

(4)

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} P A e^{i/\hbar (Px - Et)}$$

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} P \psi$$

$$P\psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial x}$$

$$P\psi = \frac{i \hbar}{i \cdot i} \frac{\partial \psi}{\partial x}$$

$$P\psi = i\hbar \frac{\partial \psi}{\partial x}$$

$$P = i\hbar \frac{\partial}{\partial x}$$

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Kinetic Energy operator $(\hat{T})$

$$K.E = T = \frac{1}{2} m v^2$$

or

$$T = \frac{p^2}{2m}$$

$$[\hat{T}] = \left[\frac{p^2}{2m}\right]$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{T} = \frac{-h^2}{2m} \frac{\partial^2}{\partial x^2}$$

Hamiltonium operator $[\hat{H}]$

$$\hat{H} = \frac{-h^2}{2m}(\nabla^2) + V$$

Position operator $(\hat{x})$

$$\hat{x} = x$$

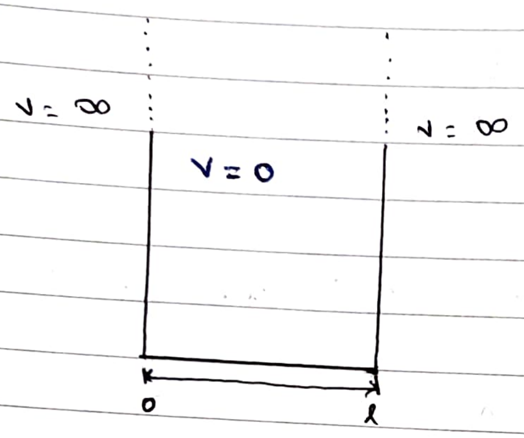
Potential operator $(\hat{V})$

$$\hat{V} = V$$

Q. Light of wavelength 5000 Å falls on a metal surface of work function (ϕ) = 1.9 eV. Find

(i) The energy of photons in eV
(ii) The K.E of photo-electrons
(iii) Stopping potential.

Particle inside 1-D box :-



→ Consider a particle of mass M, that is confined to move along x-axis, box of length l. The potential energy V = 0, inside the box & is infinite at the boundaries of the box, that is

$V = 0 \quad 0 < x < l$

$V = \infty \quad x = 0, x = l.$

Schrondinger 1-D wave equation (time independent) is given as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0.$$

Inside the box, the potential energy $V = 0$.

∴ S' time independent wave equation is given as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE\psi}{\hbar^2} = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0 \quad \text{--- (1)}$$

$E = \frac{1}{2} mv^2$

$E = p^2 / 2m$

$p = \hbar k$

$E = \frac{\hbar^2 k^2}{2m}$

$E =$

$k^2 = \frac{2mE}{\hbar^2}$

Where $$k^2 = \frac{2mE}{\hbar^2} \quad \text{--- (2)}$$

The general solution of Eq. (1) is given as

$$\psi = A \sin kx + B \cos kx \quad \text{--- (3)}$$

A & B are constant.

To find the value of A & B we have to apply the boundary condition:

B.C I : When $x = 0$, $\psi = 0$.

On applying the B.C - I on Equation - (3)

$x = 0$, $\psi = 0$.

$$0 = A \sin k(0) + B \cos(0) k$$

$$B = 0 \quad \text{--- (4)}$$

From Equation (3) & (4)

$$\psi = A \sin kx \quad \text{--- (5)}$$

B.C - II.

When $x = l$, $\psi = 0$.

on applying B.C - II, on, Equation (5)

$$A \sin kl = 0.$$
$$\sin kl = 0.$$

$$\sin kl = \sin n\pi$$

$$kl = n\pi, \quad n = 1, 2, 3, \ldots \infty.$$

$$k = \frac{n\pi}{l} \quad \text{— (6)}$$

On substituting Eq. 6, k, to Eq. (5)

$$\psi = A \sin \frac{n\pi}{l} \cdot x \quad \text{— (7)}$$

Since, there is only one particle & there is no chance of it's being located outside the box, ∵ the probability of finding the particle inside the box is 1. Thus, the normalization condition is given as

$$\int_0^l |\psi|^2 dx = 1$$

From Eq. (7)

$$A^2 \int_0^l \left[\sin^2\left(\frac{n\pi x}{l}\right)\right] dx = 1$$

$$\frac{A^2}{2} \cdot \int_0^l \left[2\sin^2\left(\frac{n\pi x}{l}\right)\right] dx = 1$$

$$\frac{A^2}{2} \int_0^l \left[1 - \cos\frac{2n\pi x}{l}\right] dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{\sin\left(\frac{2n\pi x}{l}\right)}{\frac{2n\pi}{l}}\right]_0^l = 1$$

$$\frac{A^2}{2}\left[x - \frac{l}{2n\pi}\sin\left(\frac{2n\pi x}{l}\right)\right]_0^l = 1$$

$$\frac{A^2}{2}\left[l - \frac{l}{2n\pi}\sin(2n\pi)\right] = 1$$

$$\frac{A^2}{2}\left[\frac{2n\pi l - l \quad 0}{2n\pi}\right]$$

$$\frac{A^2 l}{2} = 1$$

$$A = \sqrt{\frac{2}{l}} \quad \text{--- (8)}$$

On substituting a value from Eq. 8 to Eq. 7

** $$\boxed{\psi = \sqrt{\frac{2}{l}} \sin\left(\frac{n\pi x}{l}\right)}$$: Remember.

From Equation 2 & 6

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{l^2}$$

** $$\boxed{E = \frac{n^2\pi^2\hbar^2}{2ml^2}}$$

The above expression represent the energy of a particle confined in one-dimension box, it signifies that the energy is always discrete.

If $n = 1$

$$E = \frac{1\pi^2\hbar^2}{2ml^2}$$

$n = 2$

$$E = \frac{4\pi^2\hbar^2}{2ml^2}$$

$n = 3$

$$E = \frac{9\pi^2\hbar^2}{2ml^2}$$

$n = 4$

$$E = \frac{16\pi^2\hbar^2}{2ml^2}$$

UNIT-V - SOLID STATE PHYSICS

Free Electron Theory :- (classical approach)

Drude & Lorentz has proposed free electron theory on the basis of following assumption:

- In conductors (Metals), the free electrons or valence electrons are free to move inside a conductor just like a gas molecule.
- Since electrons behave like gas molecule, therefore the kinetic theory of gas will be applicable on electrons.
- In the absence of E.F, the electrons move randomly, whereas the collisons will be elastic (No loss of energy).
- When the E.F is applied, the free electrons will start accelerating opposite to the applied field. This is known as Drift current.
- There is no electrostatic force of attraction b/w + ions & electrons.

Electrical conductivity & Drift current

→ The electrical conductivity denoted by (σ) & is the measure of the ability of a material to conduct & electric current from it. It's units are S/m.

→ consider an electron, which is in motion in conductor under the influence of applied electric field. If e is the charge, m is the mass of electron, E is the E.F, Then the force on electron is given as

$$F = eE \quad \text{— (1)}.$$

Also,

$$F = ma \quad \text{—— (2)}$$

a = acceleration

from (1) & (2)

$$eE = ma$$

$$a = \frac{eE}{m}$$

Rate of change of velocity w.r.t time is acceleration,

$$\therefore \frac{dv}{dt} = \frac{eE}{m}$$

$$dv = \frac{eE}{m} dt$$

on integrating both side

$$\int dv = \frac{eE}{m} \int dt$$

$$v = \frac{eEt}{m} + c \quad \text{— (3)}$$

When, $t = 0$, $v = 0$.

From Equation (3)

$$c = 0 \quad \text{— (4)}$$

From Equation (3) & (4)

$$v = \frac{eEt}{m}$$

When the transfers time is the collision time that is Relaxation time (τ)

Therefore $v = \frac{eE\tau}{m}$

When the velocity of electron is drift velocity this drift velocity is the velocity of electrons, when e^- starts moving opposite to the applied field. The V_d is the avg. velocity.

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So, the drift velocity is given as

$$V_d = \frac{eE\tau}{2m} \quad — (5)$$

A.C.T Ohm's law, the current density

** $$J = \sigma E \quad — (6)$$

Since, the current density is the current

$$J = \frac{I}{A} \quad — (7)$$

From Equation (6) & (7)

$$\frac{I}{A} = \sigma E$$

$$\sigma = \frac{I}{AE} \quad — (8)$$

The drift current is given as

$$I = eV_d n A. \quad — (9)$$

Where n = is no. of free electrons.

on substituting value of (9) in (8)

$$\sigma = \frac{eV_d n \cancel{A}}{E\cancel{A}}$$

$$\sigma = \frac{V_d e n}{E} \quad — (10)$$

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On substituting the value of v_d from equation (10) to Eq. (5).

$$\sigma = \frac{ne^2 \tau}{2m} \quad — (11)$$

Now, the mean free path (λ) – [The avg. dist. travelled by the electrons b/w successive collisions is known as mean free path]

$$\lambda = \tau v$$

$$\tau = \frac{\lambda}{v} \quad — (12)$$

On substituting the value of τ from (12) to (11).

$$\sigma = \frac{ne^2 \lambda}{2m} \quad — (13)$$

The above expression represent electrical conductivity of free electrons.

Temperature dependency of electric conductivity.

→ Lorentz has applied the K.T.G for free electrons to explain the temp dependency of sigma.

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$$\frac{1}{2} mv^2 = \frac{3}{2} KT$$

$$mv^2 = 3KT$$

$$mv = \frac{3KT}{v} \quad \text{— (14)}$$

From Eq. (13) (E) (14)

$$\sigma = \frac{ne^2 \lambda v}{6KT},$$

The above expression shows that the electrical conductivity, with the in temperature σ will decrease.

With the help of above relation, we can measure conductivity & Resistivity of the solids.

Wiedemann - Franz Law.

→ This Law gives the ration of Thermal conductivity & Electrical conductivity

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{K_B}{e}\right)^2 T$$

K → Thermal conductivity
σ → Electrical conductivity.
K_B → Boltzmann's constant.
T → Temperature.
e → Charge.

Diffusion current :-

→ The diffusion current is the current in which charge moves from higher concn to lower conn. under the effect of Electric field.

→ The diffusion current is caused due to unequal distribution inside solids, b/w electrons & holes.

Fermi - Dirac - Distribution.

• Fermi energy & Fermi levels

→ The energy of the highest occupied level at 0°c absolute energy is known as Fermi-energy & the energy level is represented as Fermi level.

$$f(E) = \begin{cases} 1 & E < E_f \\ 0 & E > E_f \end{cases}$$

- The Fermi-Dirac function is given as

$$\boxed{f(E) = \frac{1}{e^{(E-E_f)/KT} + 1}}$$

$\because$ E_f = Fermi Energy,

k = Boltzmann's constant.

Case 1 :- At absolute Temperature $T = 0$

$\frac{E-E_f}{KT}$, where $T = 0$.

$$\frac{E-E_f}{KT} = \begin{cases} +\infty & E > E_f \\ -\infty & E < E_f. \end{cases}$$

$$f(E) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1 \quad , \quad [e^{-\infty} = 0]$$

$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty+1} = 0.$$

Case 2 :- At some temperature T, $E = E_f$

$$f(E) = \frac{1}{e^{(E-E_f)KT} + 1}$$

$$f(E) = \frac{1}{2} \quad (e^0 = 1)$$

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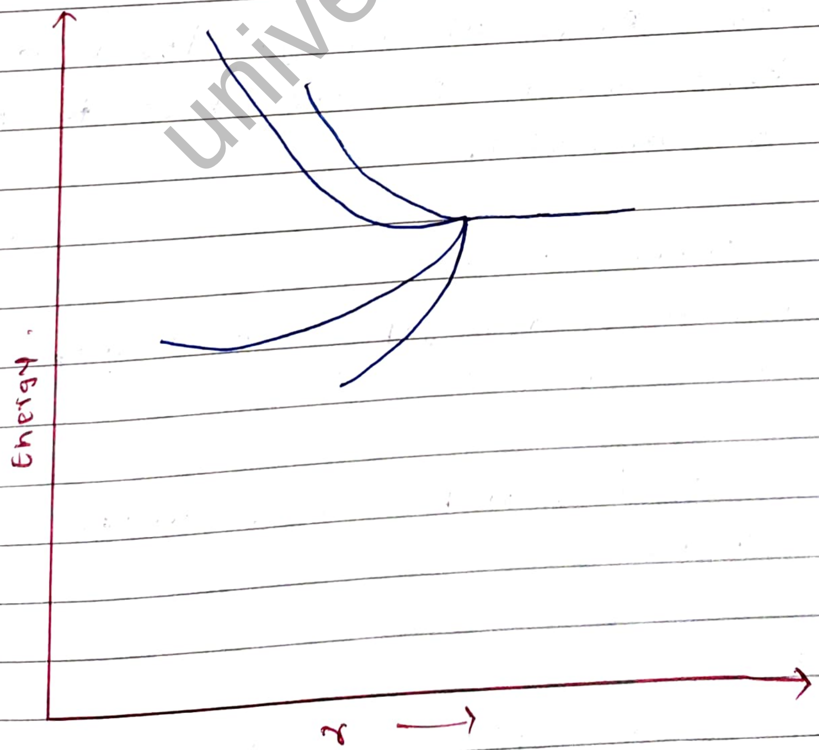
So, the Fermi level is the energy level at which the half probability of finding electron is $\frac{1}{2}$. Half.

Band Theory of solids, Formation of Allowed & Forbidden Energy bands.

Electrons are asumed to be tightly bound to the individual atoms, when atoms are bring together to form solids by interaction between the neighbouring atoms causes the electrons energy level of individual atoms to be split, this is how the bands are forming.

The band theory follows Pauli's exclusion principle.



(Interatomic distance) .

As the r decreases the energy level get split into energy band

Allowed Energy band.

→ Allowed energy bands are range of energy level within a material where electrons are allowed to exist. It is further categories into two types that is valence band & conduction band.

• Valence band.

→ Range of energy level where electrons are most likely to be found it is highest occupied energy level that is fully occupied at 0 absolute temp.

• Conduction band

→ It is the Range of energy level above valence band where e⁻ are free to move within the material.

Forbidden Energy bands

→ Also known as Energy gap & band gap where e⁻ are not allowed to exist.

Insulators, semiconductors & conductors in terms of Band Gap.

• conductors.

→ In such materials there is no forbidden gap b/w V.B & C.B

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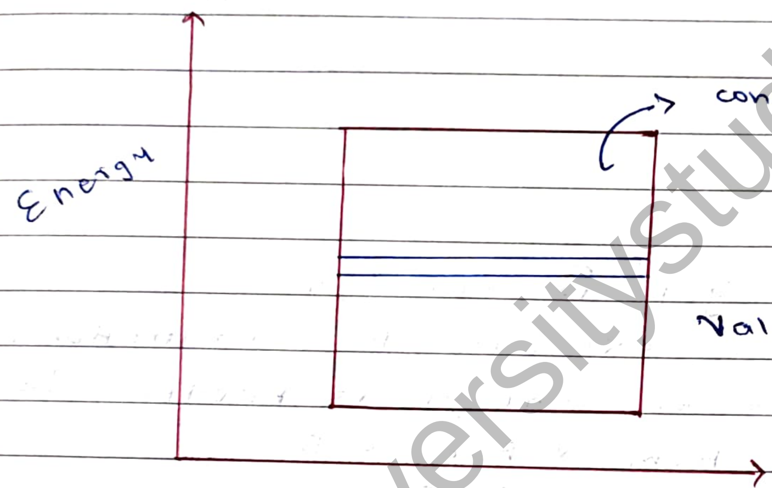
→ The e^- from valence band to conduction band can freely enter due to overlapping of bands.

→ Because of overlapping very low potential diff. can cause continous flow of current.

→ The band gap for conductor is $E_g = 0\,eV$



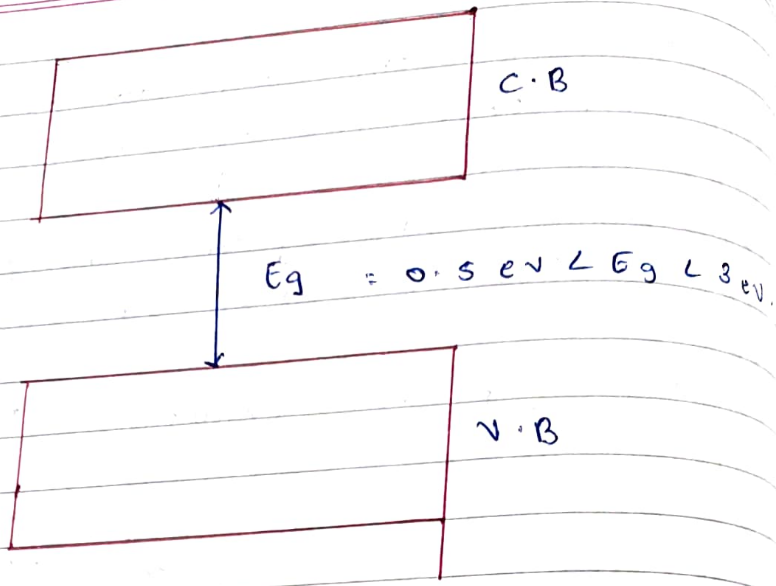
Semiconductors

→ In case of S.C. the forbidden gap is very small.

→ At 0K temp, the conduction band is empty & the valence band is completely filled.

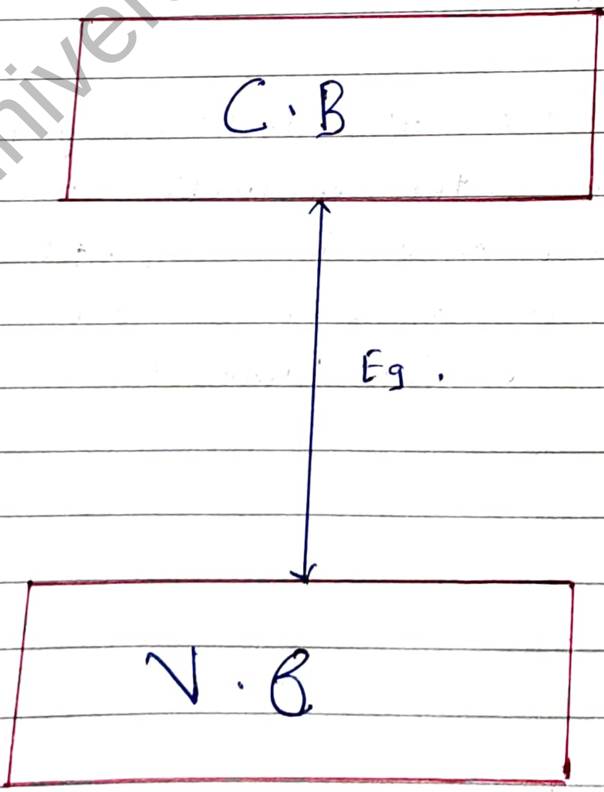
→ When a small amount of energy is supplied the electrons can easily jump from V.B to C.B.

→ The energy gap in case of S.C is $0.5\,eV < E_g < 3\,eV$.



Insulators

- → The forbidden gap is very large.
- → No electrons are available for conduction.
- → Large amount of energy is needed to move the electron from V.B to C.B.
- → The Energy gap is > 3 ev



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Semiconductors & it's Types.

Semiconductors are the materials which have conductivity in between conductors & insulators

$$10^{-8}\ S/cm < \sigma < 10^{3}\ S/cm.$$

Insulators

Conductors.

Types of semiconductor

The semi-conductors are categories as follow

Semiconductors
- Intrinsic
- Extrensic.
  - P-Type (acceptor ions)
  - N-Type. (donor ions)

Intrinsic semiconductors.

→ such types of materials are pure semiconductors for e.g. Si & Ge that are not doped with external impurity.

Extrinsic s.c.

→ In such type of materials some specific in are doped to modify their electrical properties.

P-Type

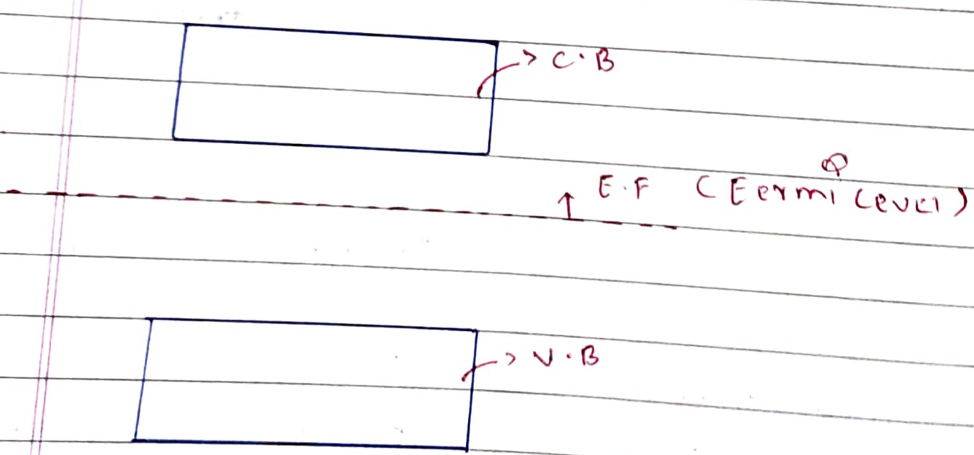
→ In such materials trivalent elements are doped, the doping will introduce excess holes, and the overall conductivity will de
For e.g. Group 13 elements.

N-Type

→ In such type of s.c the pentavalent impurity is doped. The doping will introdu excess e^-, which will enhance the electri conductivity.

Intrinsic & Extrinsic s.c on the basis of Fermi Level.

Intrinsic



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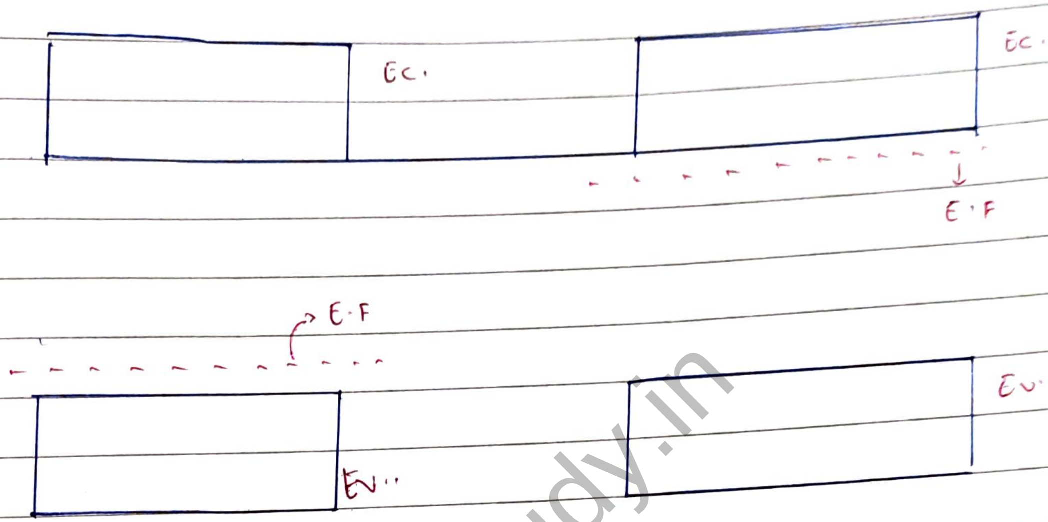
Extrinsic.

P- Type   N- Type.



→ The fermi level in terms of I.S.C will lies b/w V.B & C.B

→ In P-type the fermi level lies to near to the V.B

→ N - Type " " " "
" C.B.