Experiment 2:
Designing Low Pass filter by windowing

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Low pass filters actually have an infinite (duration) impulse response which is not practical to implement in real life, hence FIR filters provide a way to practically implement various filters. They are non-recursive digital filters as they do not have a feedback.

Window method is most commonly used method for designing FIR filters. The simplicity of design process makes this method very popular. A window is a finite array consisting of coefficients selected to satisfy the desirable requirements.

Digital FIR Filters are specified by:

- The Windowing function
- The filter order

These two requirements are interrelated and there exists a compromise between

- The sharpness of the filter and selectivity
- Stop band attenuation

The impulse response of a Low-Pass filter is given by:

$$W(n) = \frac{\sin\left(w_c(n-k)\right)}{\pi(n-k)} \; for\; n \neq k$$

$$\frac{w_c}{\pi} \; for\; n = k$$

N = Number of Samples

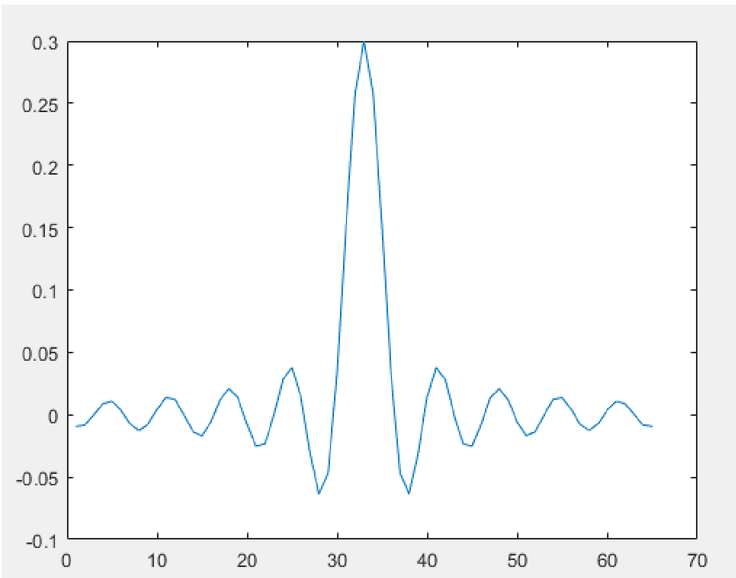
K = $\frac{N-1}{2}$

The various windowing functions in time domain are:

Rectangular window	$W(n) = 1; n = 0,1 \ldots N-1$ $0; otherwise$
Triangular window	$W(n) = 1 - 2\left(\frac{n-(N-1)/2}{N-1}\right); n = 0,1 \ldots N-1$ $0; otherwise$
Hanning window	$W(n) = 0.5 - 0.5cos\left(\frac{2\pi n}{N-1}\right); n = 0,1 \ldots N-1$ $0; otherwise$
Hamming window	$W(n) = 0.54 - 0.46cos\left(\frac{2\pi n}{N-1}\right); n = 0,1 \ldots N-1$ $0; otherwise$
Blackmann window	$W(n) = 0.42 - 0.5cos\left(\frac{2\pi n}{N-1}\right)$ $+0.08cos\left(\frac{4\pi n}{N-1}\right); n = 0,1..N-1$ $0; otherwise$

Now one of these windowing functions are multiplied to the impulse response of a low pass filter to make an FIR filter.

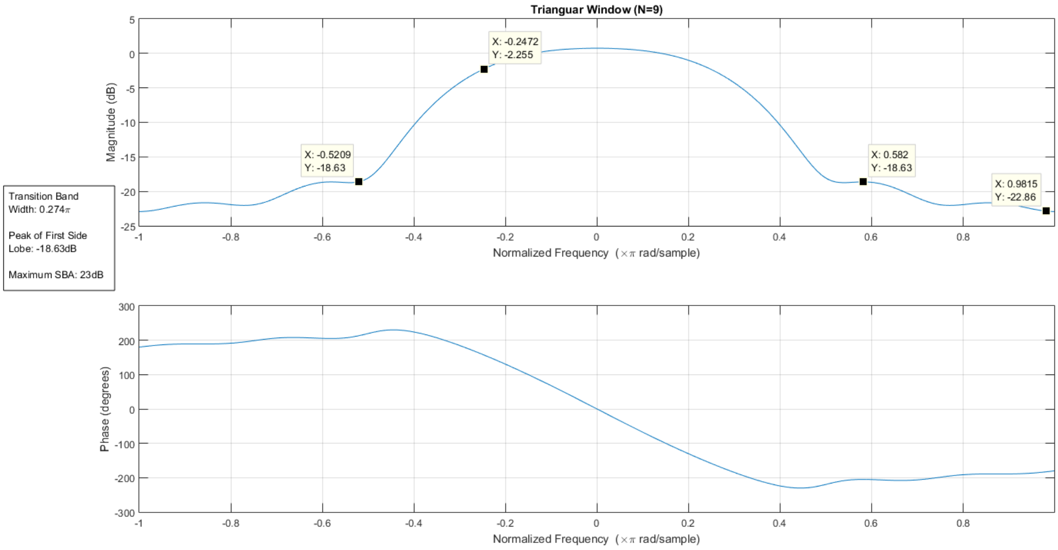
Once the desired time domain characteristics of the FIR Filter are established, the freqz function is used to find and plot the frequency response of the FIR filter.

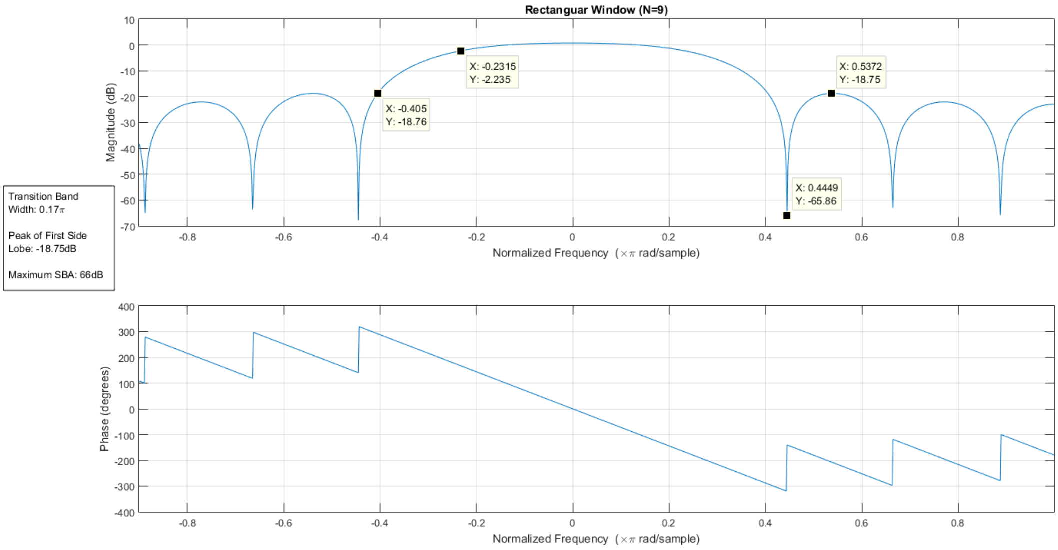


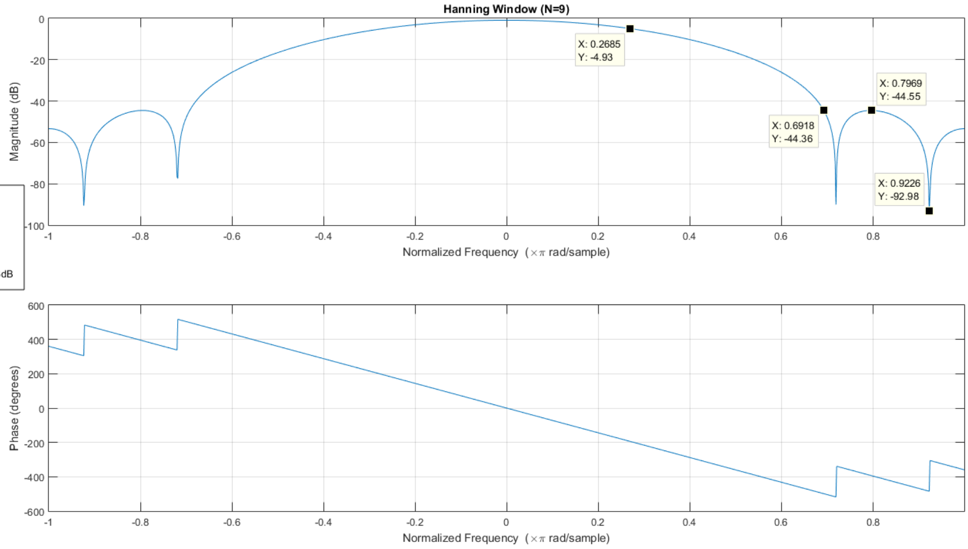
Impulse response of an ideal low-pass filter

Frequency Reponses:

N = 9



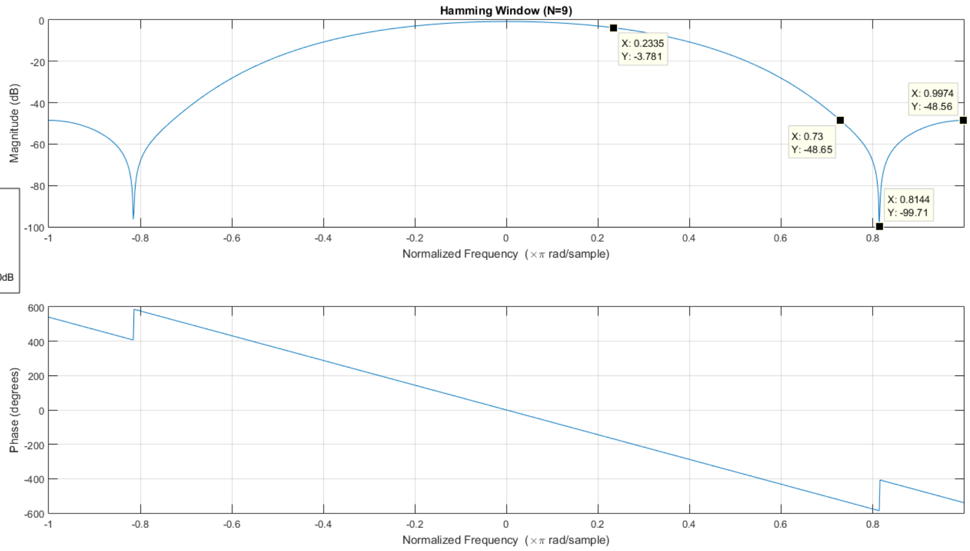




Transition Band Width: 0.42π

Peak of First Side Lobe: -44.55dB

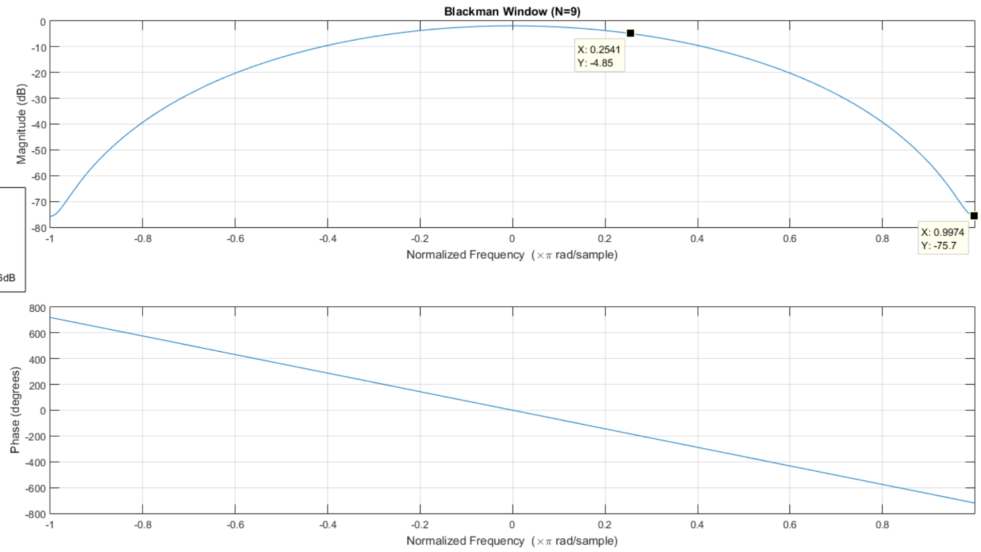
Maximum SBA: 93dB



Transition Band Width: 0.5π

Peak of First Side Lobe: -48.56dB

Maximum SBA: 100dB

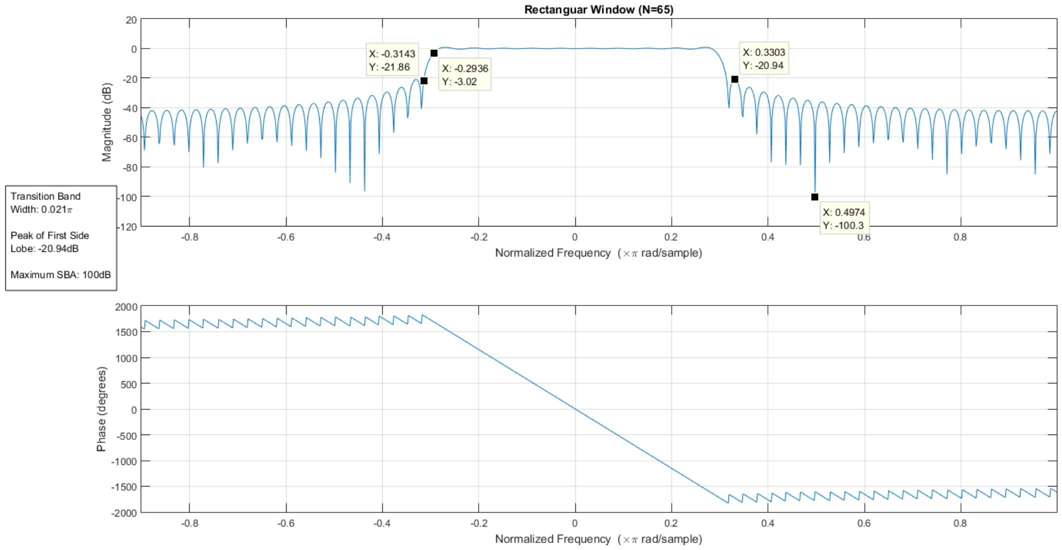


Transition Band Width: 0.65π

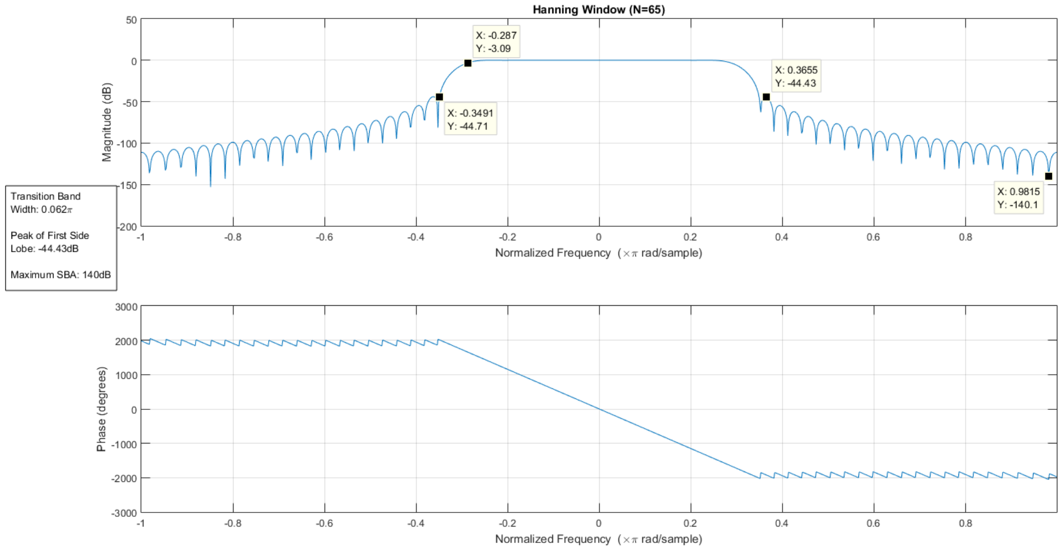
Peak of First Side Lobe: NA

Maximum SBA: 76dB

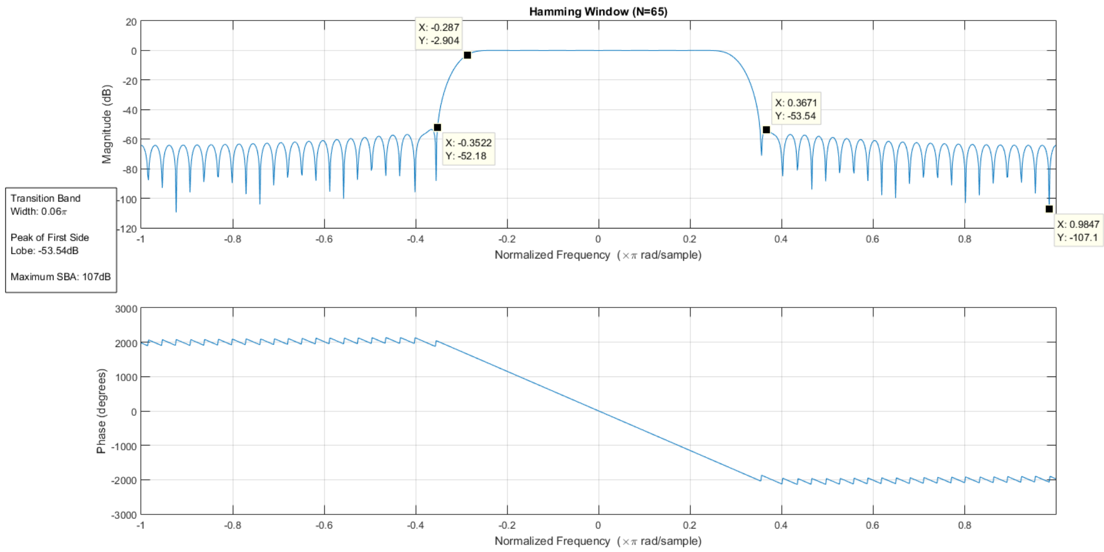
N = 65



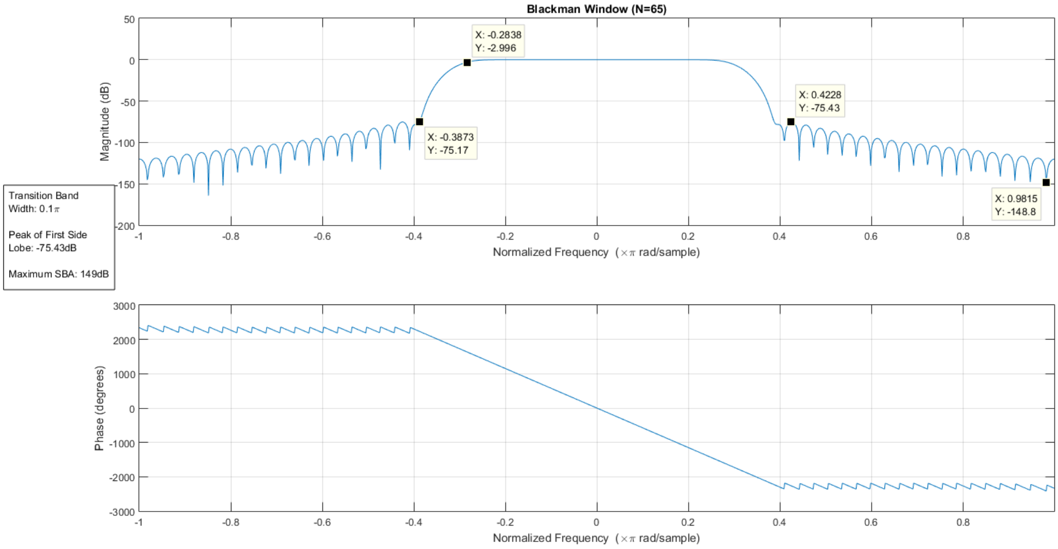
Rectanguar Window (N=65)
Magnitude (dB)
Phase (degrees)
Normalized Frequency (×π rad/sample)
X: -0.3143
Y: -21.86
X: -0.2936
Y: -3.02
X: 0.3303
Y: -20.94
X: 0.4974
Y: -100.3
Transition Band Width: 0.021π
Peak of First Side Lobe: -20.94dB
Maximum SBA: 100dB

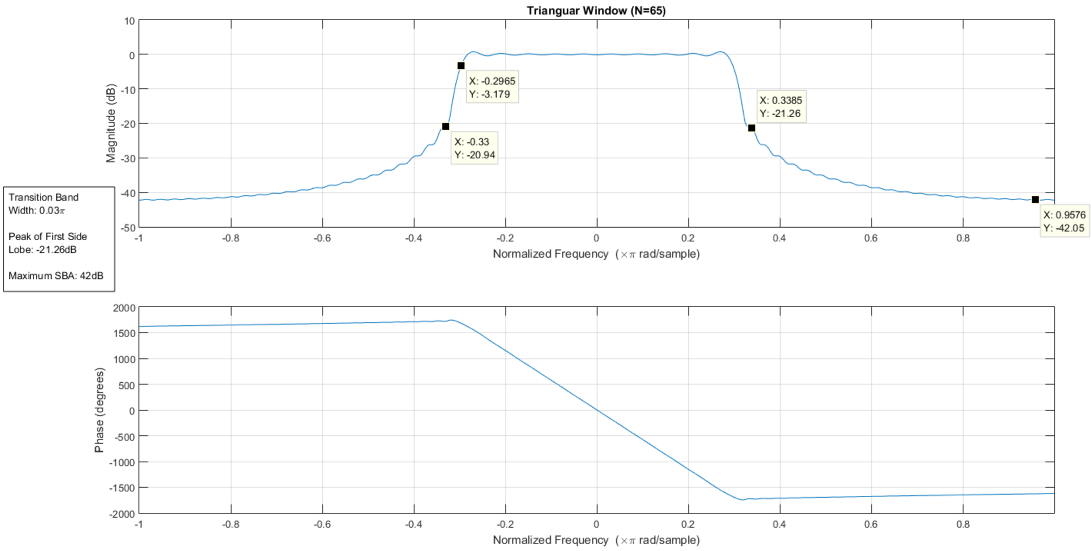


Hanning Window (N=65)
Magnitude (dB)
Phase (degrees)
Normalized Frequency (×π rad/sample)
X: -0.287
Y: -3.09
X: -0.3491
Y: -44.71
X: 0.3655
Y: -44.43
X: 0.9815
Y: -140.1
Transition Band Width: 0.062π
Peak of First Side Lobe: -44.43dB
Maximum SBA: 140dB

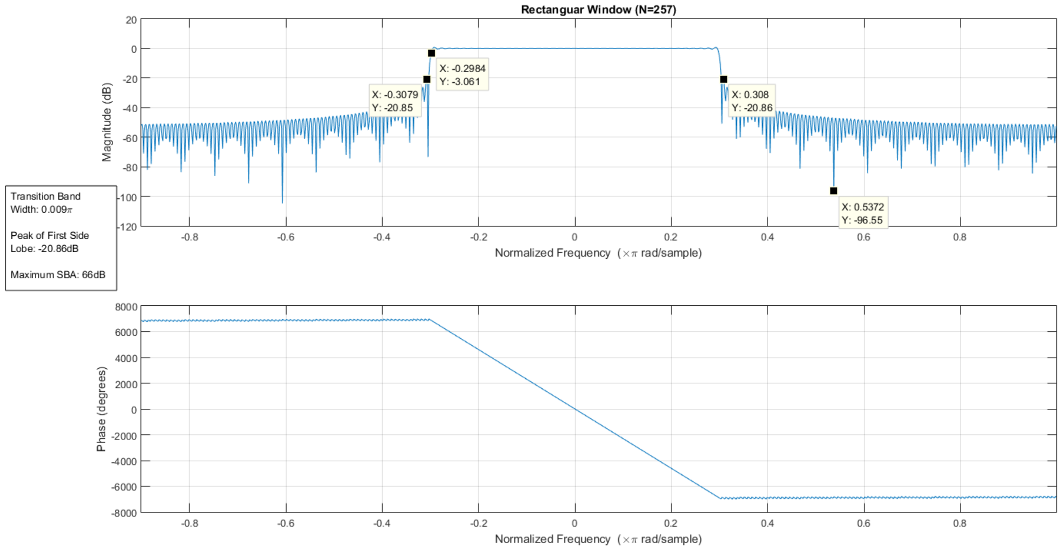


Hamming Window (N=65)
Magnitude (dB)
Phase (degrees)
Normalized Frequency (×π rad/sample)
X: -0.287
Y: -2.904
X: -0.3522
Y: -52.18
X: 0.3671
Y: -53.54
X: 0.9847
Y: -107.1
Transition Band Width: 0.06π
Peak of First Side Lobe: -53.54dB
Maximum SBA: 107dB

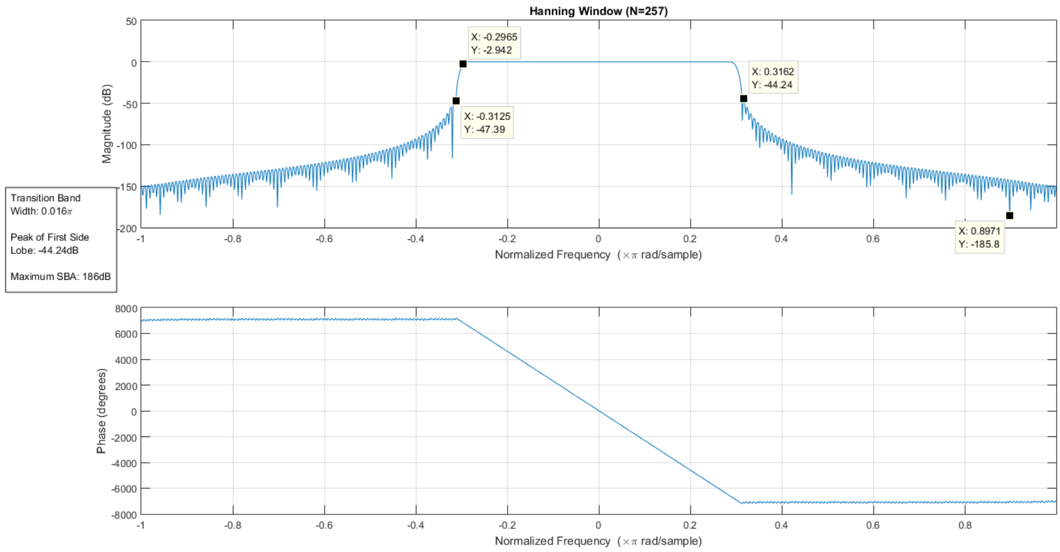

Blackman Window (N=65)
Magnitude (dB)
X: -0.2838
Y: -2.996
X: -0.3873
Y: -75.17
X: 0.4228
Y: -75.43
X: 0.9815
Y: -148.8
Normalized Frequency (×π rad/sample)
Transition Band Width: 0.1π
Peak of First Side Lobe: -75.43dB
Maximum SBA: 149dB
Phase (degrees)
Normalized Frequency (×π rad/sample)


Trianguar Window (N=65)
Magnitude (dB)
X: -0.2965
Y: -3.179
X: -0.33
Y: -20.94
X: 0.3385
Y: -21.26
X: 0.9576
Y: -42.05
Normalized Frequency (×π rad/sample)
Transition Band Width: 0.03π
Peak of First Side Lobe: -21.26dB
Maximum SBA: 42dB
Phase (degrees)
Normalized Frequency (×π rad/sample)

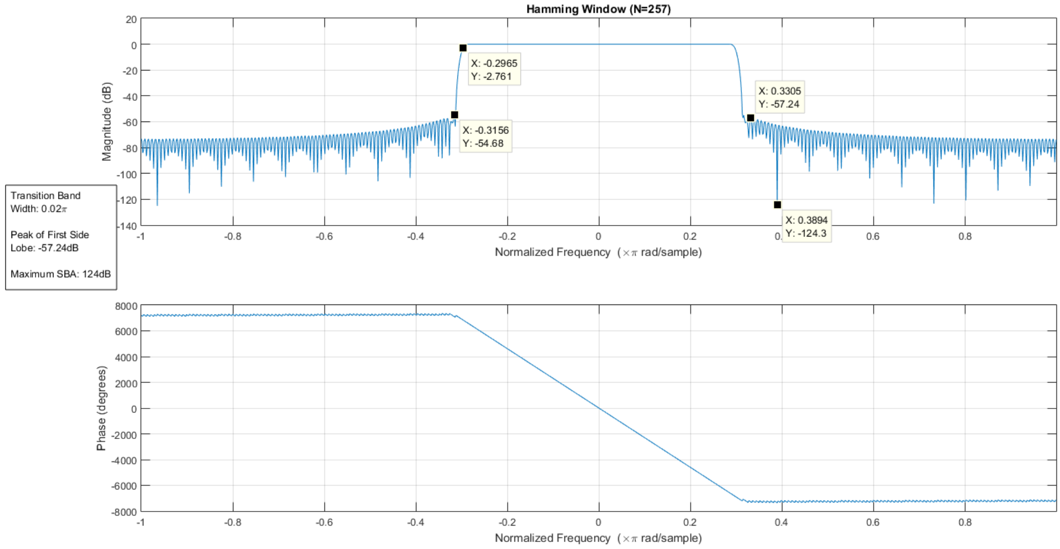
N = 257



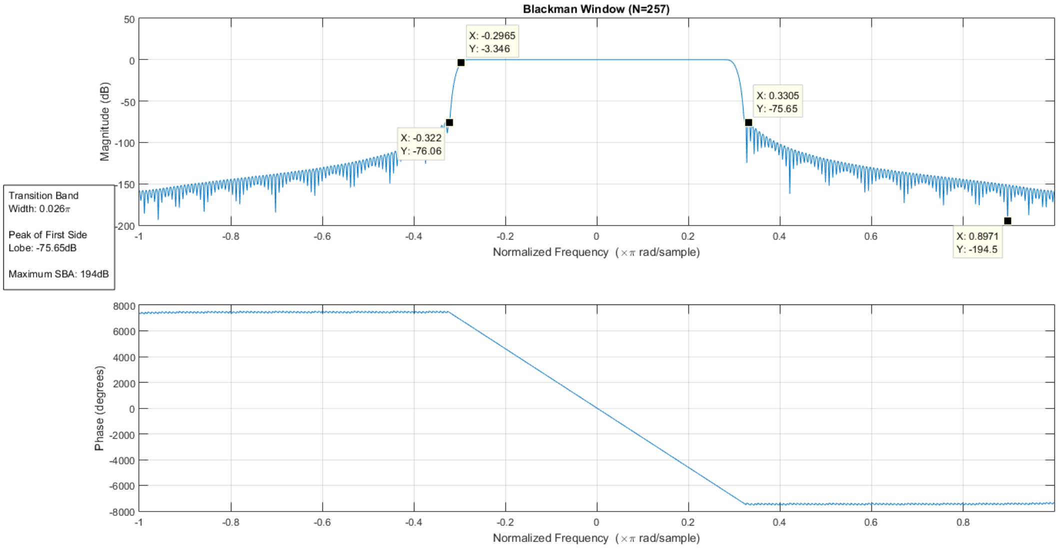
Rectanguar Window (N=257)
Magnitude (dB)
Normalized Frequency (×π rad/sample)
X: -0.2984
Y: -3.061
X: -0.3079
Y: -20.85
X: 0.308
Y: -20.86
X: 0.5372
Y: -96.55
Transition Band Width: 0.009π
Peak of First Side Lobe: -20.86dB
Maximum SBA: 66dB
Phase (degrees)
Normalized Frequency (×π rad/sample)



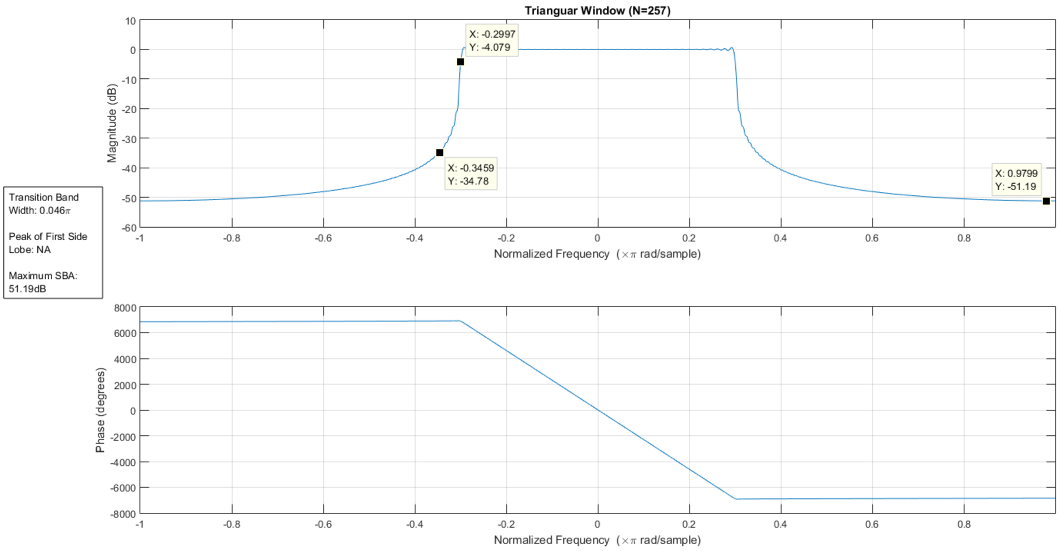
Hanning Window (N=257)
Magnitude (dB)
Normalized Frequency (×π rad/sample)
X: -0.2965
Y: -2.942
X: -0.3125
Y: -47.39
X: 0.3162
Y: -44.24
X: 0.8971
Y: -185.8
Transition Band Width: 0.016π
Peak of First Side Lobe: -44.24dB
Maximum SBA: 186dB
Phase (degrees)
Normalized Frequency (×π rad/sample)



Hamming Window (N=257)
Magnitude (dB)
Normalized Frequency (×π rad/sample)
X: -0.2965
Y: -2.761
X: -0.3156
Y: -54.68
X: 0.3305
Y: -57.24
X: 0.3894
Y: -124.3
Transition Band Width: 0.02π
Peak of First Side Lobe: -57.24dB
Maximum SBA: 124dB
Phase (degrees)
Normalized Frequency (×π rad/sample)



Blackman Window (N=257)
Magnitude (dB)
Normalized Frequency (×π rad/sample)
X: -0.2965
Y: -3.346
X: -0.322
Y: -76.06
X: 0.3305
Y: -75.65
X: 0.8971
Y: -194.5
Transition Band Width: 0.026π
Peak of First Side Lobe: -75.65dB
Maximum SBA: 194dB
Phase (degrees)
Normalized Frequency (×π rad/sample)



Trianguar Window (N=257)
Magnitude (dB)
Normalized Frequency (×π rad/sample)
X: -0.2997
Y: -4.079
X: -0.3459
Y: -34.78
X: 0.9799
Y: -51.19
Transition Band Width: 0.046π
Peak of First Side Lobe: NA
Maximum SBA: 51.19dB
Phase (degrees)
Normalized Frequency (×π rad/sample)

N = 9

Window	Transition Width(π)	Peak of First Lobe (dB)	Max Stop Band attenuation (dB)
Rectangle	0.17	-18.75	66
Triangle	0.274	-18.63	23
Hamming	0.5	-48.56	100
Hanning	0.42	-44.55	93
Blackmann	0.65	NA	76

N = 65

Window	Transition Width(π)	Peak of First Lobe (dB)	Max Stop Band attenuation (dB)
Rectangle	0.021	-20.94	100
Triangle	0.03	-21.26	42
Hamming	0.06	-53.54	107
Hanning	0.062	-44.43	140
Blackmann	0.1	-75.43	149

N = 257

Window	Transition Width(π)	Peak of First Lobe (dB)	Max Stop Band attenuation (dB)
Rectangle	0.009	-20.86	66
Triangle	0.046	NA	51.19
Hamming	0.02	-57.24	124
Hanning	0.016	-44.24	186
Blackmann	0.02	-57.24	194

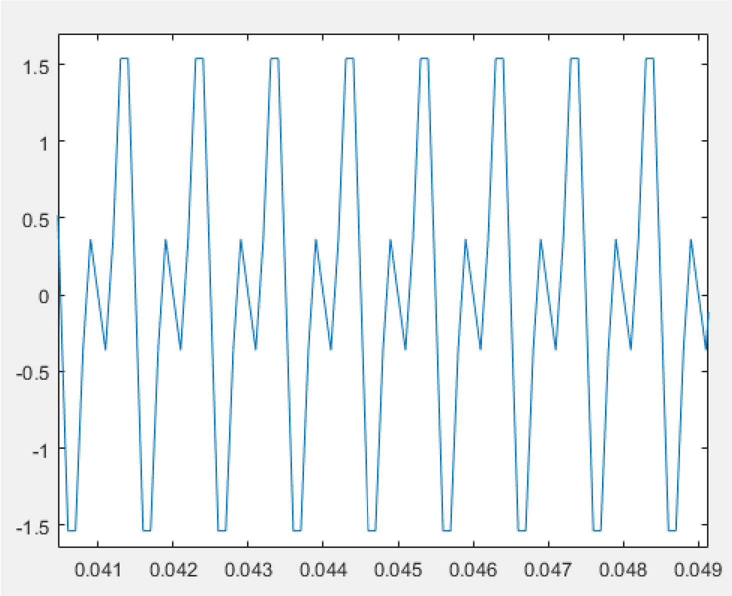
Code:

```
wc = 0.3*pi;
%%
hd = LPFilt(wc,N);
% plot(hd);
%%
w = rectWindow(N);
B1 = hd .* w;
figure;
freqz(B1,1,-0.9*pi:0.005:pi);
title(['Rectanguar Window (N=' num2str(N) ')']);
%%
w = triWindow(N);
B2 = hd .* w;
figure;
freqz(B2,1,-pi:0.005:pi);
title(['Trianguar Window (N=' num2str(N) ')']);
%%
w = hannWindow(N);
B3 = hd .* w;
figure;
freqz(B3,1,-pi:0.005:pi);
title(['Hanning Window (N=' num2str(N) ')']);
%%
w = hammWindow(N);
B4 = hd .* w;
figure;
freqz(B4,1,-pi:0.005:pi);
title(['Hamming Window (N=' num2str(N) ')']);
%%
w = blackWindow(N);
B5 = hd .* w;
figure;
freqz(B5,1,-pi:0.005:pi);
title(['Blackman Window (N=' num2str(N) ')']);
```

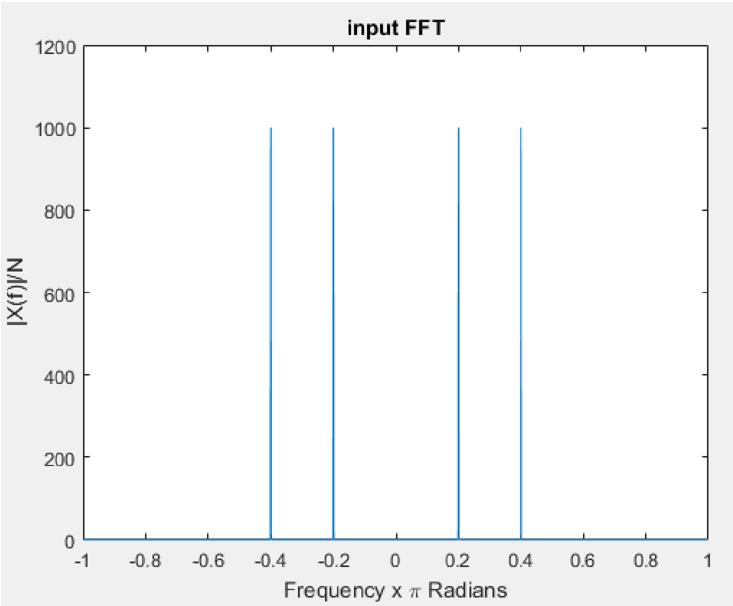
Part 2:

A sinusoid signal is generated such that one component is in the passband and another in the stop band.

```
x = sin(2*pi*0.1*Fs*t)+sin(2*pi*0.8*Fs*t)
```



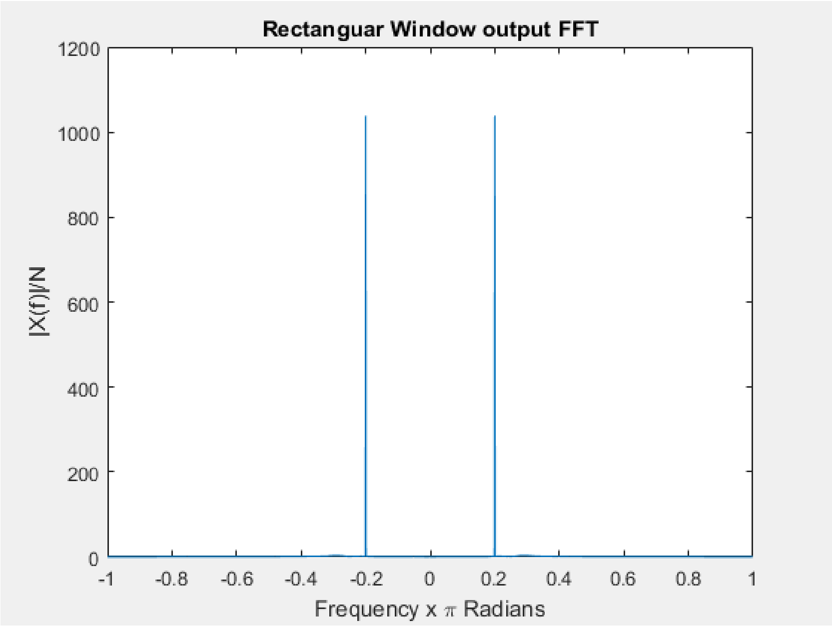
Generated Signal

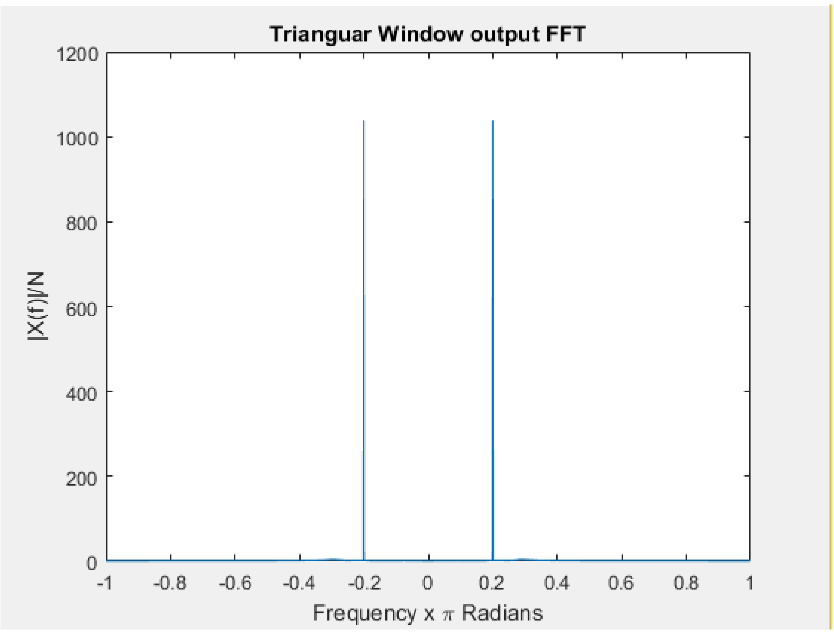


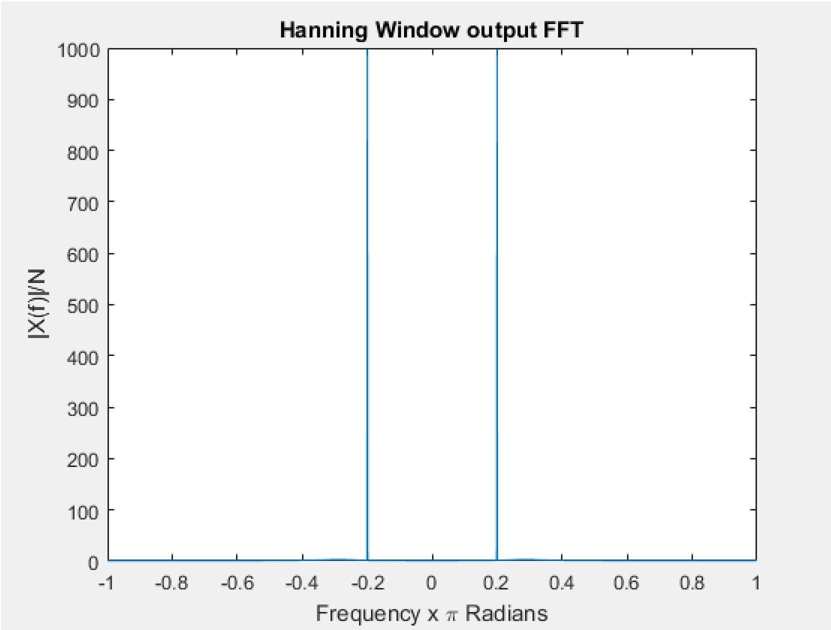
Input Signal DFT

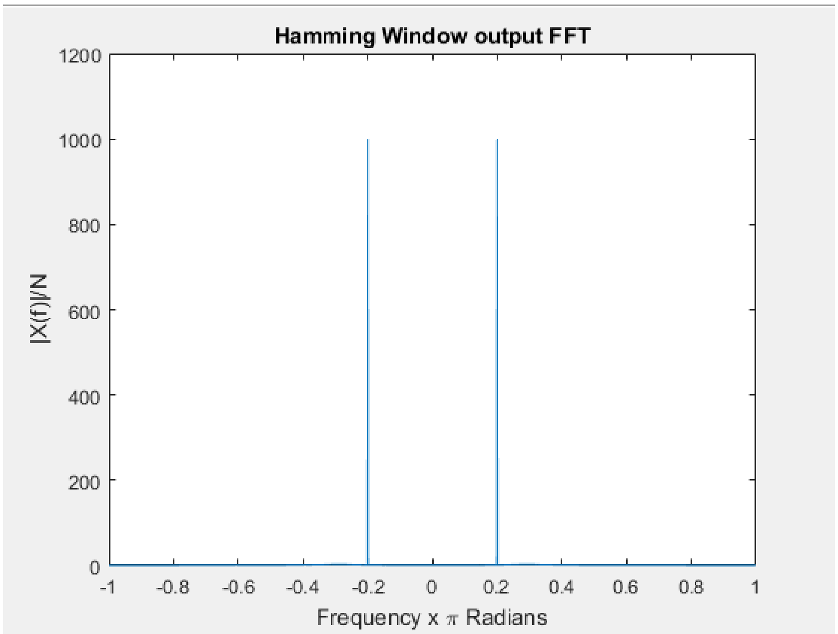
One of the components has a digital frequency of 0.2 π while the other is at 0.4 π. Since the cut-off is at 0.3 π. One is in the passband while the other is in the stopband.

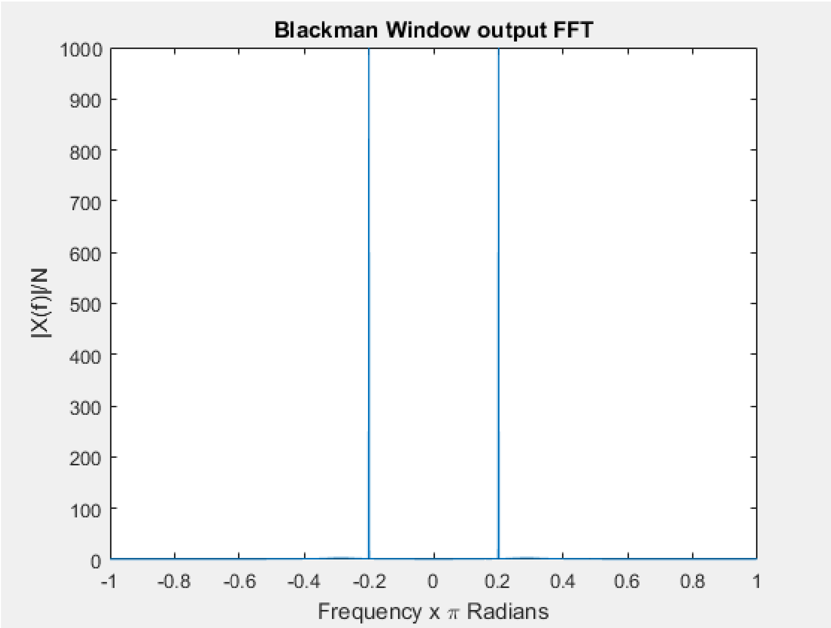
Filtered Output:





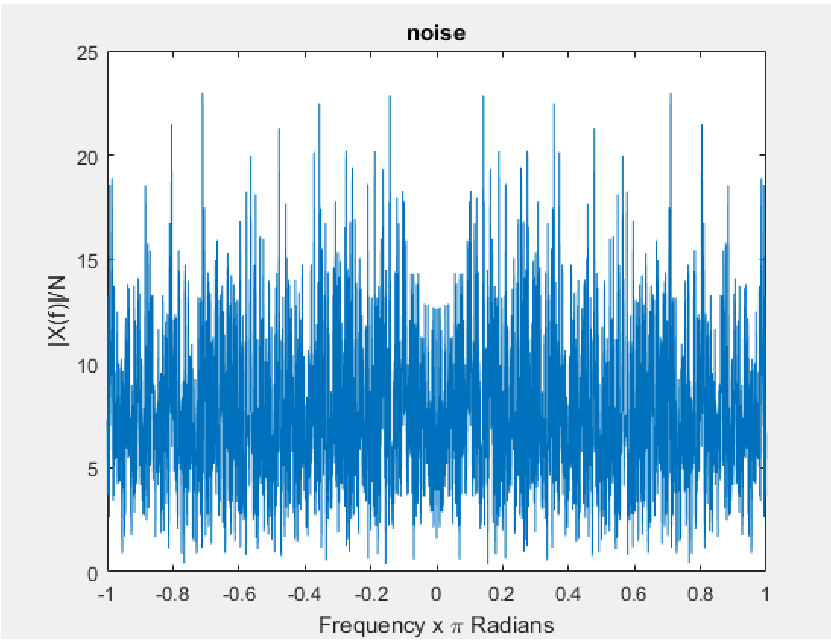

Hanning Window output FFT
|X(f)|/N
1000
900
800
700
600
500
400
300
200
100
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Frequency x π Radians


Hamming Window output FFT
|X(f)|/N
1200
1000
800
600
400
200
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Frequency x π Radians

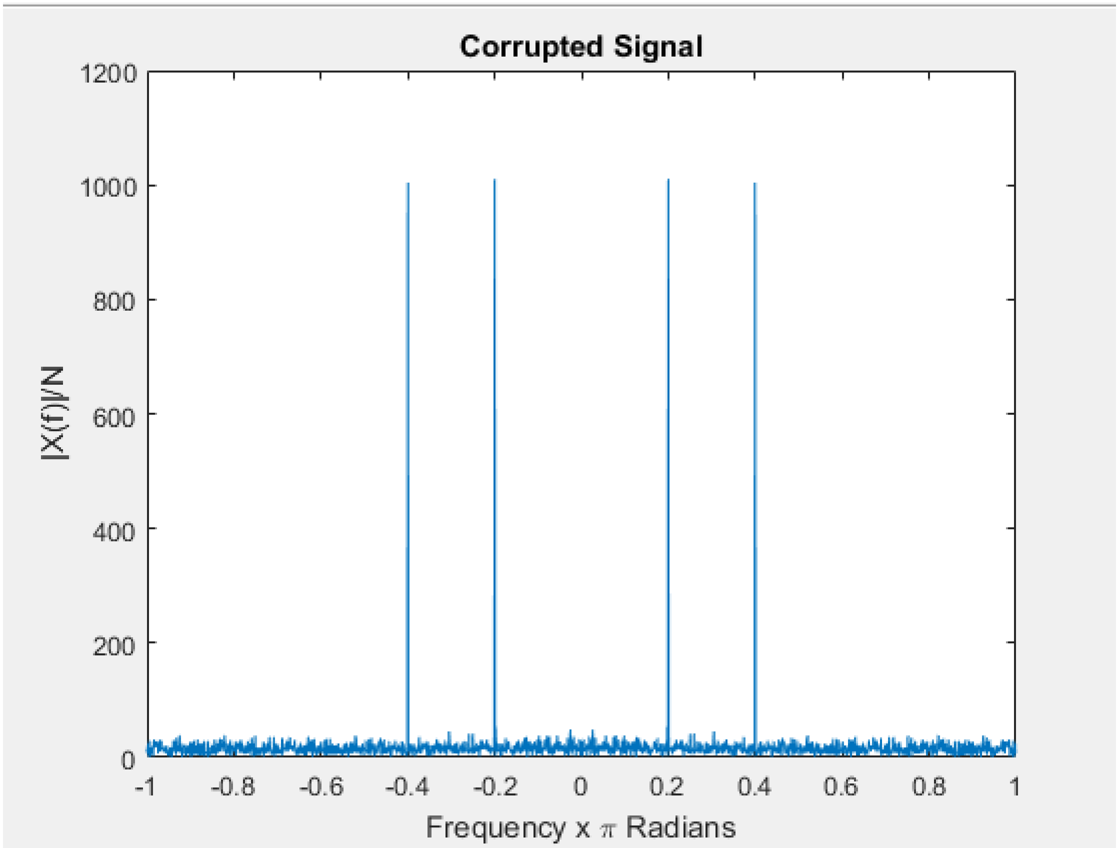


Addition of Noise:

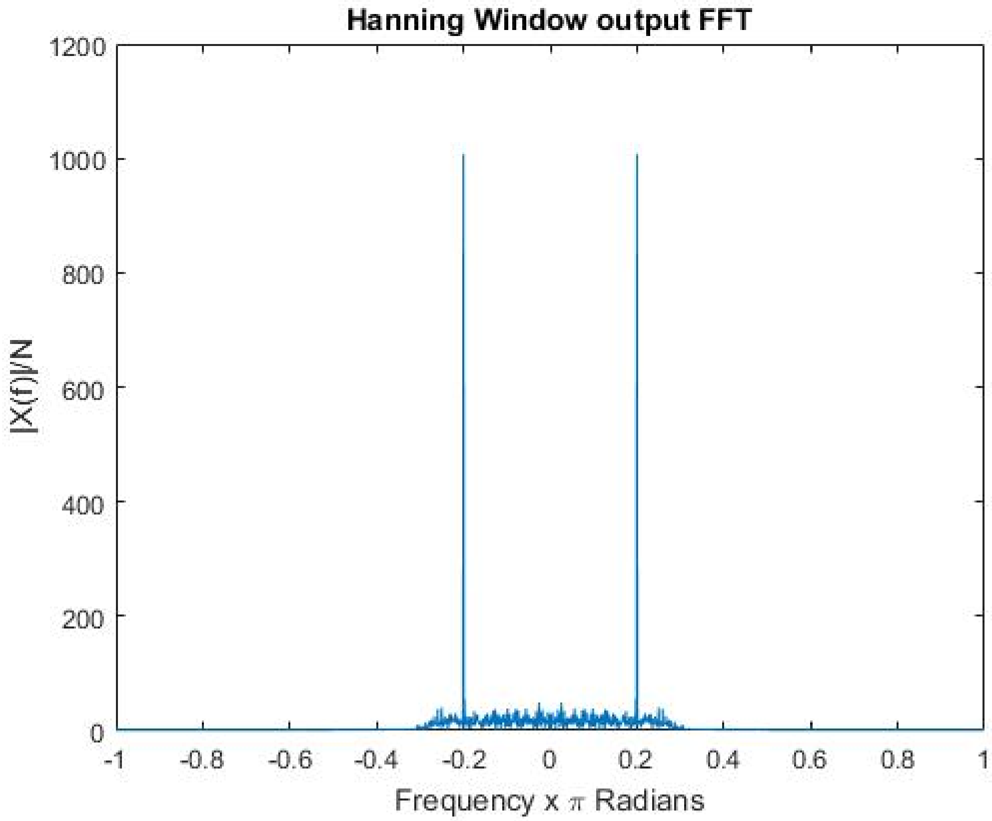
AWGN noise of a specific variance is now added to this signal.

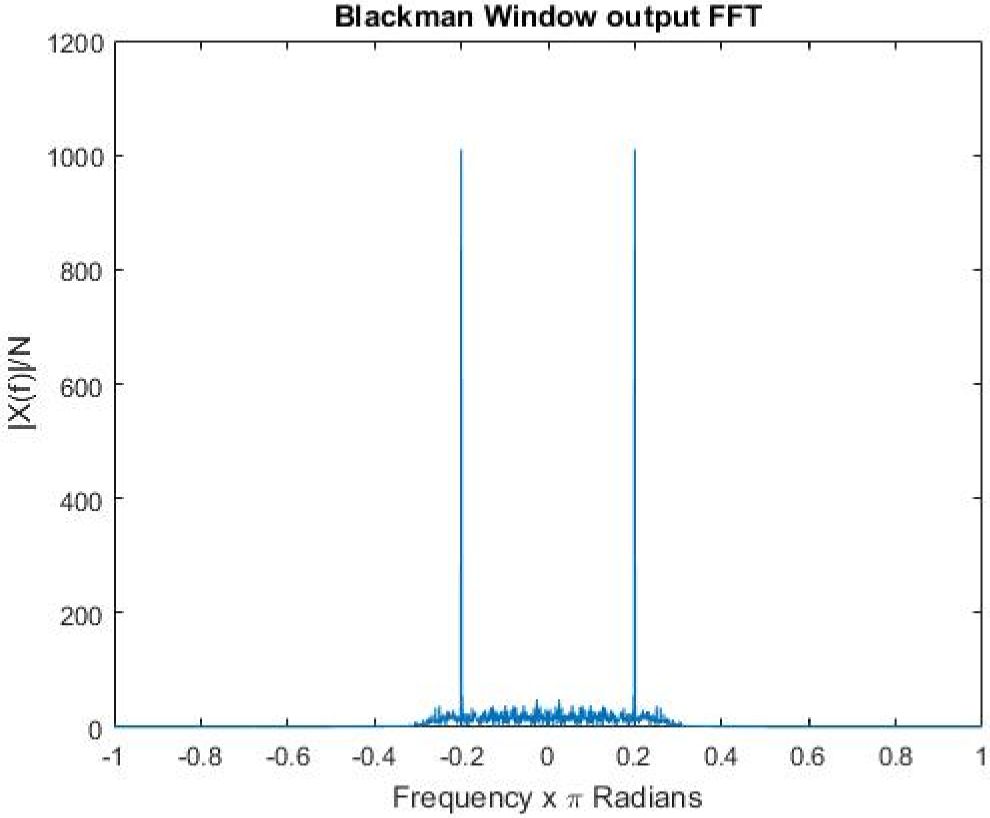


Power Spectral Density of Noise

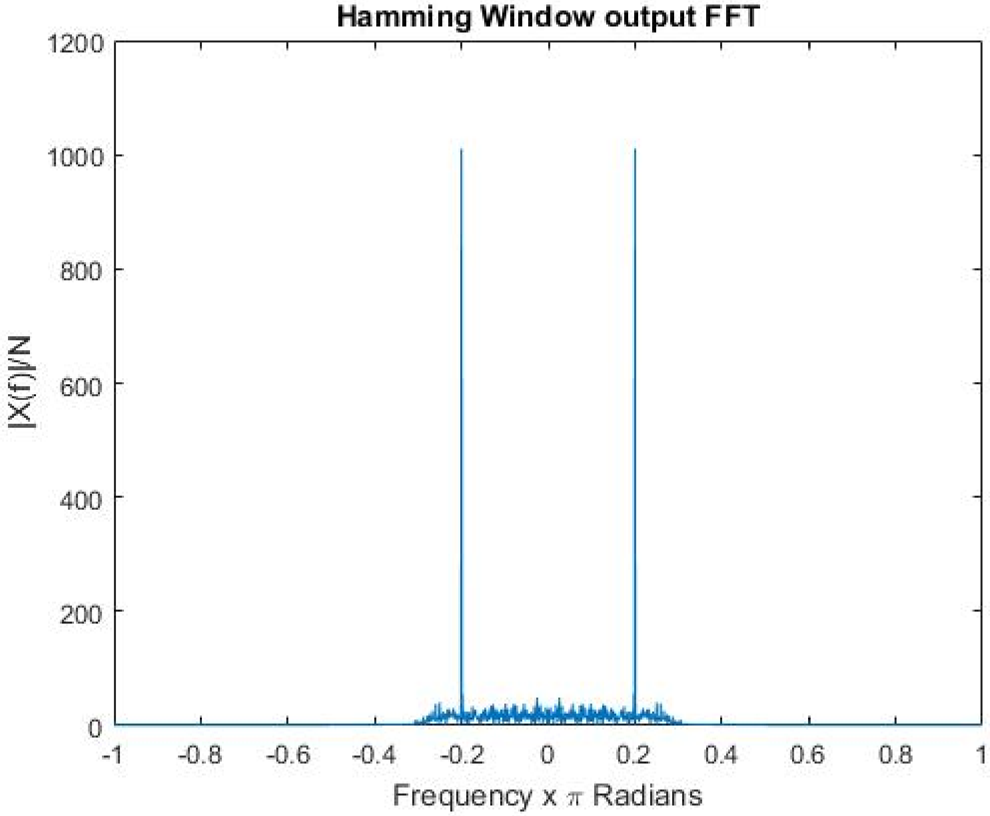


Filtered Output:

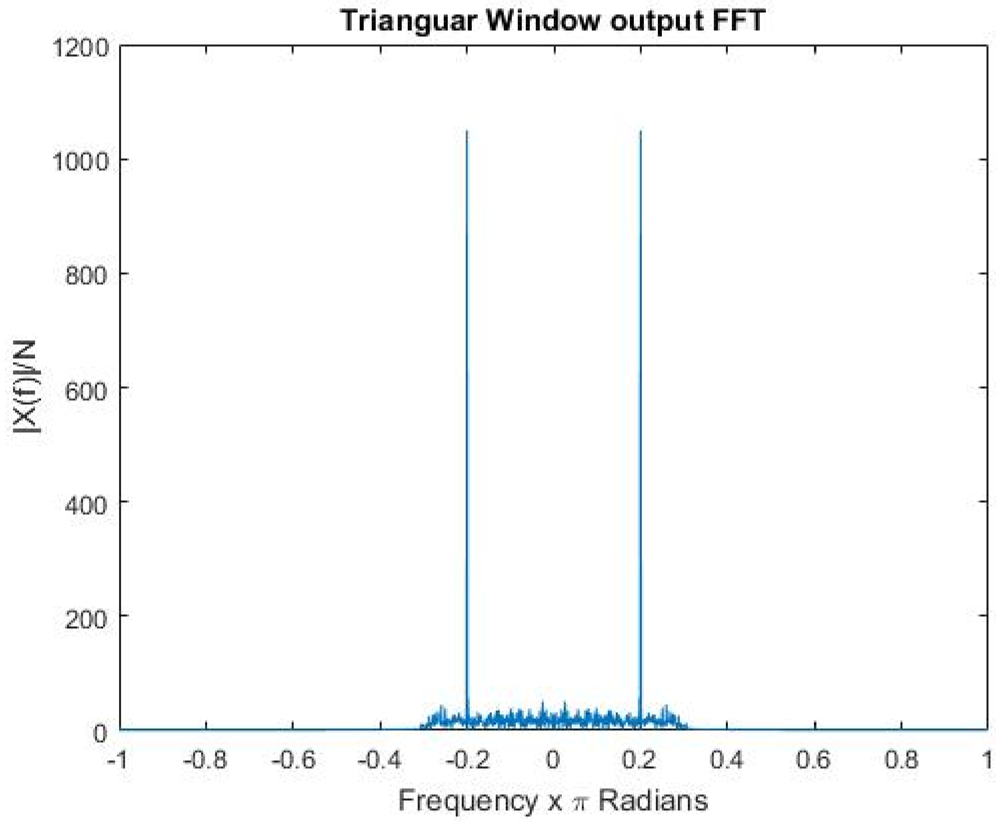


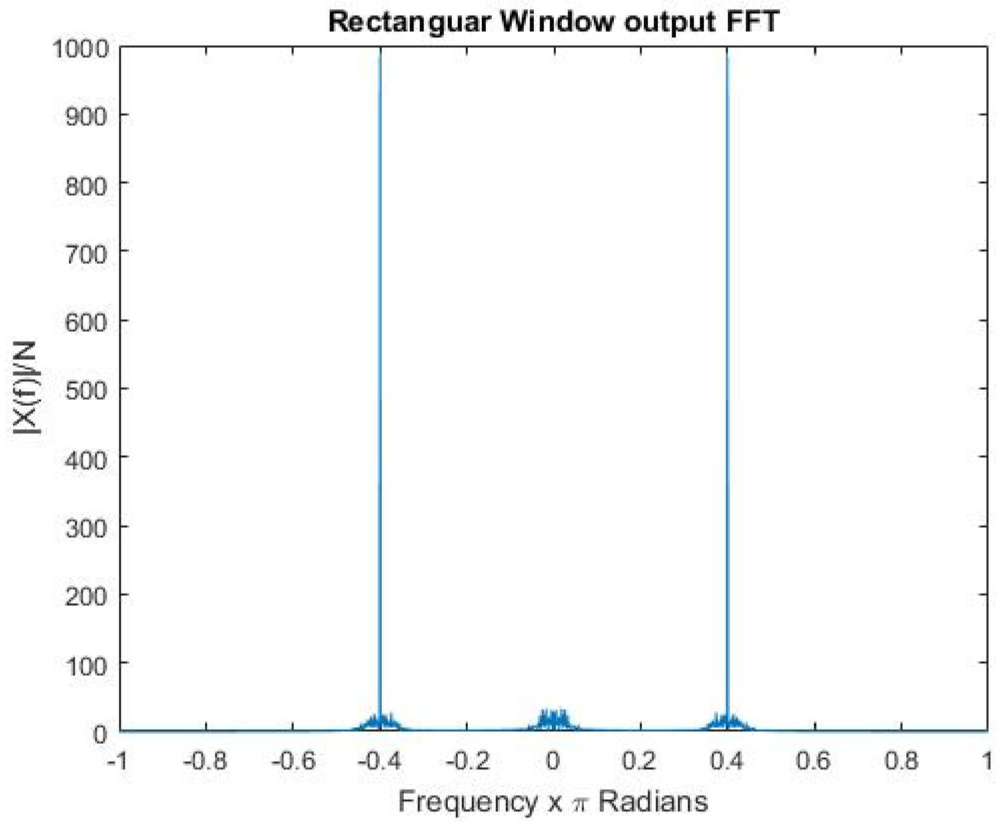


Blackman Window output FFT
1200
1000
800
600
400
200
0
|X(f)|/N
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Frequency x π Radians



Hamming Window output FFT
1200
1000
800
600
400
200
0
|X(f)|/N
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Frequency x π Radians


Trianguar Window output FFT
|X(f)|/N
Frequency x π Radians
1200
1000
800
600
400
200
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Rectanguar Window output FFT
|X(f)|/N
Frequency x π Radians
1000
900
800
700
600
500
400
300
200
100
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

SNR Noisy input signal = 7.95dB

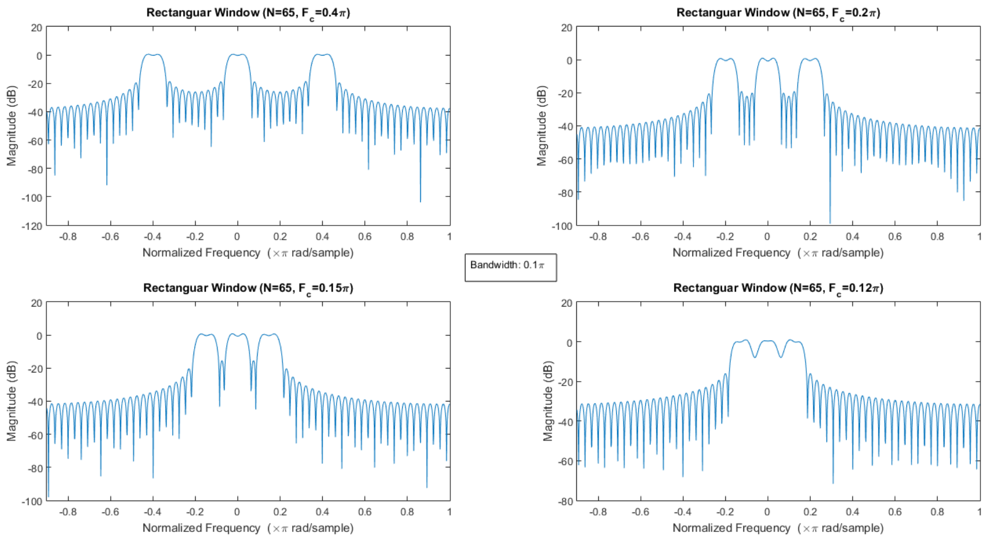
Window	**SNR (dB)**
Rectangular	17.69
Triangular	10.51
Hamming	10.47
Hanning	10.48
Blackmann	10.57

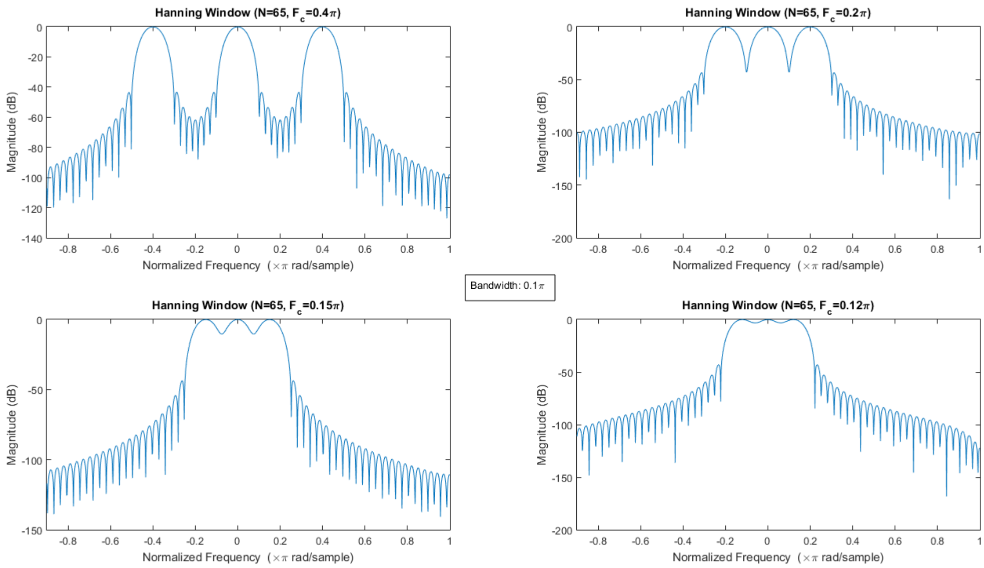
Code:

```
%% Generating Input Signal
Fs = 10 * 1e3;              % Sampling frequency
T = 1/Fs;               % Sampling period
L = 2000;               % Length of signal
t = (0:L-1)*T;          % Time vector
x = sin(2*pi*0.1*Fs*t)+sin(2*pi*0.8*Fs*t);
%plot(t,x);
r = rms(x);
plotdft(x,Fs,'input FFT')
%% Filtering using FIR Fiters
y = filtfilt(B1,1,x);
plotdft(y,Fs,'Rectanguar Window output FFT')

y = filtfilt(B2,1,x);
plotdft(y,Fs,'Trianguar Window output FFT')

y = filtfilt(B3,1,x);
plotdft(y,Fs,'Hanning Window output FFT')

y = filtfilt(B4,1,x);
plotdft(y,Fs,'Hamming Window output FFT')

y = filtfilt(B5,1,x);
plotdft(y,Fs,'Blackman Window output FFT')
%% Generation of noise
n = 0.4*r*randn(1,L);
plotdft(n,Fs,'noise')
%%
x1 = x+n;
plotdft(x1,Fs,'Corrupted Signal')
%% Filtering using FIR Fiters
y1 = filtfilt(B1,1,x1);
plotdft(y1,Fs,'Rectanguar Window output FFT')
snr(y1)
y1 = filtfilt(B2,1,x1);
plotdft(y1,Fs,'Trianguar Window output FFT')
snr(y1)
y1 = filtfilt(B3,1,x1);
plotdft(y1,Fs,'Hanning Window output FFT')
snr(y1)
y1 = filtfilt(B4,1,x1);
plotdft(y1,Fs,'Hamming Window output FFT')
snr(y1)
y1 = filtfilt(B5,1,x1);
plotdft(y1,Fs,'Blackman Window output FFT')
snr(y1)
```

Limitations of Windowing Method:

The windowing method for designing FIR filters is very effective, but is usually used to design filters with only a single passband. For multiple passbands, we have constructive interference in stop bands and degradation of filter sharpness as well as stop band attenuation. This can be seen in the following two screenshots, for the rectangular and hanning windows:





As we can see, in rectangular windowing technique, as the passbands are brought closer, we have an added passband ripple (>1.5dB, which is the accepted standard for a good filter)

The selectivity is also hampered. This is even more evident in the Hanning windowing method. Although any passband ripple has been avoided, due to the slow roll-off of the Hanning FIR filter, we have lesser sharpness and selectivity and even at F_c=0.15π, we have sufficient interference, keeping us from differentiating the bands clearly.

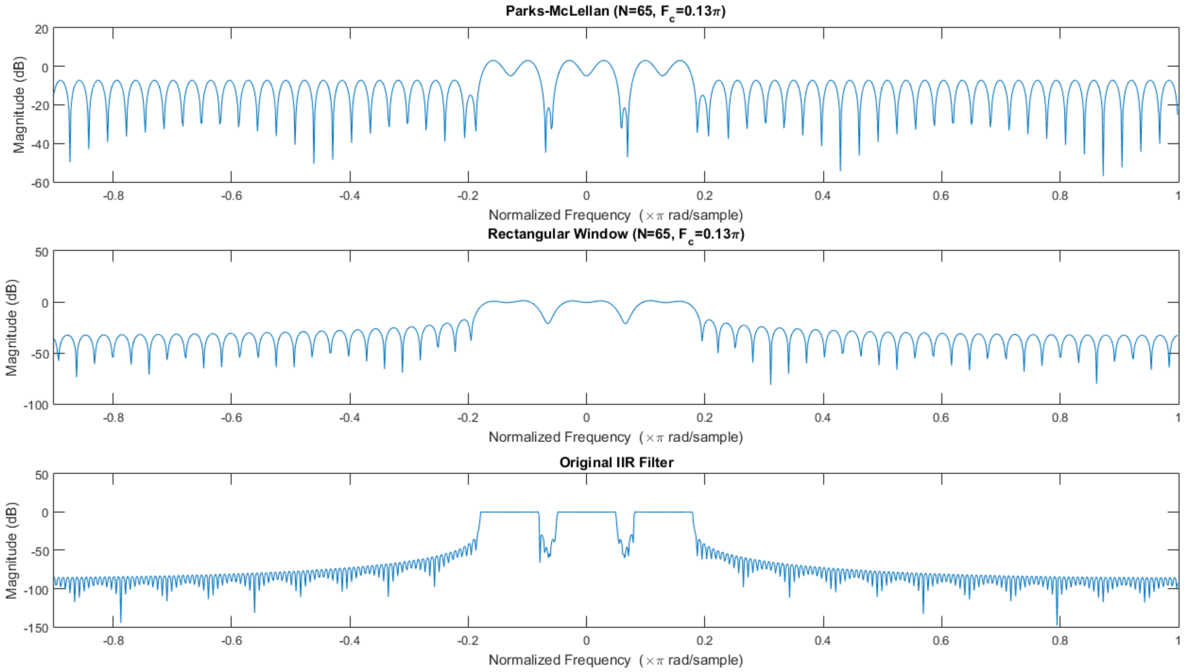
Code:

```
wc = 0.05*pi;                         %cutoff of band on either side
Fc = 0.13;                            %center frequency of pass band
N=65;
h1 = 2*BPFilt(wc,N,0.5*Fc)+LPFilt(wc,N); %compensate for division of power
in sidebands
w = rectWindow(N);
B1 = h1 .* w;
[A,~]=freqz(B1,1,-0.9*pi:0.005:pi);
%subplot(2,2,1)
plot(-0.9:1.9/size(A,2):1-1.9/size(A,2),20*log10(abs(A)));
xlim([-0.9 1]);
xlabel('Normalized Frequency  (\times\pi rad/sample)');
ylabel('Magnitude (dB)');
title(['Rectangular Window (N=' num2str(N) ', F_c=' num2str(Fc) '\pi)']);
```

Design of FIR filters using Parks-McClellan algorithm:

Parks-McClellan algorithm uses the Remez exchange algorithm and Chebyshev approximation theory to design optimal FIR filter corresponding to a given frequency response. This is optimal in the sense that the maximum error between the desired and actual response is minimized.

Here are the results, showing a comparison between the filter response for Parks-McClellan method, Windowing method and IIR filter (desired frequency response):



We do see some extra ripple, which is a characteristic of the Chebyshev approximation.

Code:

```
h2=2*BPFilt(wc,1001,0.5*Fc)+LPFilt(wc,1001);
[a,~]=freqz(h2,1,-0.9*pi:0.005:pi);
a(1:566)=[];
f=0:1/size(a,2):1-1/size(a,2);
b=firpm(N,f,abs(a));

subplot(3,1,1)
[A,~]=freqz(b,1,-0.9*pi:0.005:pi);
plot(-0.9:1.9/size(A,2):1-1.9/size(A,2),20*log10(abs(A)));
xlim([-0.9 1]);
xlabel('Normalized Frequency  (\times\pi rad/sample)');
ylabel('Magnitude (dB)');
title(['Parks-McLellan (N=' num2str(N) ', F_c=' num2str(Fc) '\pi)']);

subplot(3,1,2)
[A,~]=freqz(B1,1,-0.9*pi:0.005:pi);
plot(-0.9:1.9/size(A,2):1-1.9/size(A,2),20*log10(abs(A)));
xlim([-0.9 1]);
xlabel('Normalized Frequency  (\times\pi rad/sample)');
ylabel('Magnitude (dB)');
title(['Rectangular Window (N=' num2str(N) ', F_c=' num2str(Fc) '\pi)']);

subplot(3,1,3)
[A,~]=freqz(h2,1,-0.9*pi:0.005:pi);
plot(-0.9:1.9/size(A,2):1-1.9/size(A,2),20*log10(abs(A)));
xlim([-0.9 1]);
xlabel('Normalized Frequency  (\times\pi rad/sample)');
ylabel('Magnitude (dB)');
title('Original IIR Filter');
```