

# MatrixChainMultiplication-DP



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```
Codeium: Refactor | Explain
3
     public class BooleanParenthesis {
4
         //Given a boolean expression with following symbols.
 5
         // Symbols 'T' ---> true 'F' ---> false And
         // following operators filled between symbols // Operators & ---> boolean AND | ---> boolean OR ^ ---> boolean XOR
 6
 7
         // Count the number of ways we can parenthesize the expression so that the value of expression evaluates to true.
 8
         // For Example:
9
         // Expression: T|T&F^T
         // Ways: ((T|T)&(F^T)), (T|(T&(F^T))), (((T|T)&F)^T)
10
11
         // Expression: T^F|F
12
         // Ways: (T^(F|F)), ((T^F)|F)
13
         // Expression: T F^T&T
14
         // Ways: (T|(F^(T&T))), (((T|F)^T)&T), (T|((F^T)&T)), (T|((F^T)&T))
15
16
         //Approach: Recursive
         Codeium: Refactor | Explain | X
17
          public static void main(String[] args) {
             String s = "T|T&F^T";
18
              System.out.println(countWays(s, 0, s.length() - 1, true));
19
20
21
         Codeium: Refactor | Explain | Generate Javadoc | X
22
         public static int countWays(
23
                 String s, int i, int j, boolean isTrue) {
24
              if (i > j) {
25
                  return 0;
26
27
              if (i == j) {
28
29
                  if (isTrue) {
30
                      return s.charAt(i) == 'T' ? 1 : 0;
31
                  } else {
32
                      return s.charAt(i) == 'F' ? 1 : 0;
33
34
35
36
              int ans = 0;
37
              for (int k = i + 1; k < j; k += 2) {
                 int lT = countWays(s, i, k - 1, true);
38
                 int lF = countWays(s, i, k - 1, false);
39
```



```
if (s.charAt(k) == '&') {
43
44
                      if (isTrue) {
45
                          ans += 1T * rT;
46 ~
                        else {
                          ans += lT * rF + lF * rT + lF * rF;
47
48
49
                  } else if (s.charAt(k) == '|') {
                     if (isTrue) {
50
                          ans += lT * rT + lT * rF + lF * rT;
51
52 V
                      } else {
                          ans += 1F * rF;
53
54
                  } else if (s.charAt(k) == '^') {
55 V
56 V
                      if (isTrue) {
57
                          ans += lT * rF + lF * rT;
58
                      } else {
                          ans += lT * rT + lF * rF;
59
60
61
62
63
             return ans;
64
65
         //Approach: Memoization
66
         Codeium: Refactor | Explain | X
67
         public static int countWaysMemo(
                 String s, int i, int j, boolean isTrue) {
68
             int[][][] dp = new int[s.length() + 1][s.length() + 1][2];
69
70
             return countWaysMemo(s, i, j, isTrue, dp);
71
72
         Codeium: Refactor | Explain | Generate Javadoc | X
73 V
         public static int countWaysMemo(
                 String s, int i, int j, boolean isTrue, int[][][] dp) {
74
75 V
             if (i > j) {
76
                  return 0;
77
```

```
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```

```
78
 79
              if (i == j) {
 80
                  if (isTrue) {
 81
                       return s.charAt(i) == 'T' ? 1 : 0;
 82
                   } else {
 83
                       return s.charAt(i) == 'F' ? 1 : 0;
 84
 85
 86
 87
              if (dp[i][j][isTrue ? 1 : 0] != 0) {
                  return dp[i][j][isTrue ? 1 : 0];
 88
 89
 90
 91
              int ans = 0;
 92
              for (int k = i + 1; k < j; k += 2) {
                  int IT = countWaysMemo(s, i, k - 1, true, dp);
 93
                  int lF = countWaysMemo(s, i, k - 1, false, dp);
 94
                  int rT = countWaysMemo(s, k + 1, j, true, dp);
 95
                  int rF = countWaysMemo(s, k + 1, j, false, dp);
 96
 97
                  if (s.charAt(k) == '&') {
 98
 99
                       if (isTrue) {
                          ans += 1T * rT;
100
101
                       } else {
                           ans += 1T * rF + 1F * rT + 1F * rF;
102
103
                   } else if (s.charAt(k) == '|') {
104
                      if (isTrue) {
105
                           ans += lT * rT + lT * rF + lF * rT;
106
107
                       } else {
                          ans += lF * rF;
108
109
                    else if (s.charAt(k) == '^') {
110
111
                       if (isTrue) {
                          ans += 1T * rF + 1F * rT;
112
113
                       } else {
                          ans += 1T * rT + 1F * rF;
114
115
116
```

11/ }



```
121
          //Approach: Tabulation
          Codeium: Refactor | Explain | X
122
          public static int countWaysTab(String s, int i, int j, boolean isTrue) {
              int[][][] dp = new int[s.length() + 1][s.length() + 1][2];
123
124
              for (int gap = 0; gap < s.length(); gap++) {
125
                  for (i = 0, j = gap; j < s.length(); i++, j++) {
                      if (i == j) {
126
127
                          if (isTrue) {
128
                               dp[i][j][1] = s.charAt(i) == 'T' ? 1 : 0;
129
                           } else {
                               dp[i][j][0] = s.charAt(i) == 'F' ? 1 : 0;
130
131
                       } else {
132
133
                           for (int k = i + 1; k < j; k += 2) {
                              int 1T = dp[i][k - 1][1];
134
                              int 1F = dp[i][k - 1][0];
135
136
                               int rT = dp[k + 1][j][1];
                              int rF = dp[k + 1][j][0];
137
138
                              if (s.charAt(k) == '&') {
139
140
                                   if (isTrue) {
                                       dp[i][j][1] += lT * rT;
141
142
                                   } else {
143
                                       dp[i][j][0] += 1T * rF + 1F * rT + 1F * rF;
144
145
                               } else if (s.charAt(k) == '|') {
146
                                   if (isTrue) {
                                       dp[i][j][1] += 1T * rT + 1T * rF + 1F * rT;
147
148
                                   } else {
149
                                       dp[i][j][0] += 1F * rF;
150
151
                               } else if (s.charAt(k) == '^') {
152
                                   if (isTrue) {
153
                                       dp[i][j][1] += 1T * rF + 1F * rT;
154
                                   } else {
                                       dp[i][j][0] += 1T * rT + 1F * rF;
155
156
157
158
```





The code in the images is focused on solving the "Boolean Parenthesization Problem." Let me break it down for you, write the code, explain its functionality with an example, and provide the output.

# **Problem Description:**

Given a boolean expression consisting of:

- Symbols: `'T'` (True) and `'F'` (False),
- Operators: `'&'` (AND), `'|'` (OR), and `'^'` (XOR),

Count the number of ways the expression can be parenthesized such that the result evaluates to `True` or `False`.

## **Approaches:**

- 1. **Recursive**: Count the number of ways recursively by considering every operator as a potential splitting point.
- 2. **Memoization**: Use a 3D array to store results of overlapping subproblems for optimization.
- 3. **Tabulation**: Use bottom-up dynamic programming.

# **Full Code Implementation:**

```
public class BooleanParenthesis {
    // Recursive Approach
    public static int countWays(String s, int i, int j, boolean isTrue) {
```



```
if (i > j) return 0;
   if (i == j) {
       if (isTrue) return s.charAt(i) == 'T' ? 1 : 0;
       else return s.charAt(i) == 'F' ? 1 : 0;
   }
   int ans = 0;
   for (int k = i + 1; k < j; k += 2) {
       int lT = countWays(s, i, k - 1, true);
       int lF = countWays(s, i, k - 1, false);
       int rT = countWays(s, k + 1, j, true);
       int rF = countWays(s, k + 1, j, false);
       if (s.charAt(k) == '&') {
            if (isTrue) ans += lT * rT;
            else ans += lF * rT + lT * rF + lF * rF;
       } else if (s.charAt(k) == '|') {
            if (isTrue) ans += lT * rT + lF * rT + lT * rF;
            else ans += lF * rF;
       } else if (s.charAt(k) == '^') {
            if (isTrue) ans += lT * rF + lF * rT;
            else ans += lT * rT + lF * rF;
       }
    return ans;
}
// Memoization Approach
public static int countWaysMemo(String s, int i, int j, boolean isTrue, int[][][] dp) {
   if (i > j) return 0;
   if (i == j) {
       if (isTrue) return s.charAt(i) == 'T' ? 1 : 0;
       else return s.charAt(i) == 'F' ? 1 : 0;
   if (dp[i][j][isTrue ? 1 : 0] != -1) return dp[i][j][isTrue ? 1 : 0];
   int ans = 0;
   for (int k = i + 1; k < j; k += 2) {
       int lT = countWaysMemo(s, i, k - 1, true, dp);
       int lF = countWaysMemo(s, i, k - 1, false, dp);
       int rT = countWaysMemo(s, k + 1, j, true, dp);
       int rF = countWaysMemo(s, k + 1, j, false, dp);
       if (s.charAt(k) == '&') {
            if (isTrue) ans += lT * rT;
            else ans += lF * rT + lT * rF + lF * rF;
       } else if (s.charAt(k) == '|') {
            if (isTrue) ans += lT * rT + lF * rT + lT * rF;
            else ans += lF * rF;
       } else if (s.charAt(k) == '^') {
            if (isTrue) ans += lT * rF + lF * rT;
```



```
else ans += lT * rT + lF * rF;
    return dp[i][j][isTrue ? 1 : 0] = ans;
// Tabulation Approach
public static int countWaysTab(String s) {
    int n = s.length();
    int[][][] dp = new int[n][n][2];
    for (int gap = 0; gap < n; gap++) {
        for (int i = 0, j = gap; j < n; i++, j++) {
            if (i == j) {
                dp[i][j][1] = s.charAt(i) == 'T' ? 1 : 0;
                dp[i][j][0] = s.charAt(i) == 'F' ? 1 : 0;
            } else {
                for (int k = i + 1; k < j; k += 2) {
                    int lT = dp[i][k - 1][1];
                    int lF = dp[i][k - 1][0];
                    int rT = dp[k + 1][j][1];
                    int rF = dp[k + 1][j][0];
                    if (s.charAt(k) == '&') {
                        dp[i][j][1] += lT * rT;
                        dp[i][j][0] += lF * rT + lT * rF + lF * rF;
                    } else if (s.charAt(k) == '|') {
                        dp[i][j][1] += lT * rT + lF * rT + lT * rF;
                        dp[i][j][0] += lF * rF;
                    } else if (s.charAt(k) == '^') {
                        dp[i][j][1] += lT * rF + lF * rT;
                        dp[i][j][0] += lT * rT + lF * rF;
            }
       }
    return dp[0][n - 1][1];
}
public static void main(String[] args) {
    String expression = "T|T&F^T";
    System.out.println("Recursive: " + countWays(expression, 0, expression.length() - 1, true));
    int[][][] dp = new int[expression.length()][expression.length()][2];
    for (int[][] row : dp)
        for (int[] col : row)
            java.util.Arrays.fill(col, -1);
    System.out.println("Memoization: " + countWaysMemo(expression, 0, expression.length() - 1, true, dp));
    System.out.println("Tabulation: " + countWaysTab(expression));
```



```
}
```

# **Example:**

Input: `T|T&F^T`

Output:

Recursive: 4Memoization: 4Tabulation: 4

Explanation: There are **4 ways** to parenthesize the expression such that it evaluates to `**True**`:

1. `(T|((T&F)^T))`
2. `((T|T)&(F^T))`
3. `(((T|T)&F)^T)`
4. `(T|(T&(F^T)))`



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```
Codeium: Refactor | Explain
3 v public class EggDroppingProblemn {
 4
         //Given a certain number of floors and a certain number of eggs, find the minimum number of attempts needed to
 5
         // find the threshold floor from which the egg breaks.
 6
         //An egg that survives a fall can be used again.
 7
         //A broken egg must be discarded.
 8
         //For example, if the threshold is 16 and we have 2 eggs, then the minimum number of attempts is 4.
9
         //We can drop from floor 10, 16, 13, 14, 15
         //Approach: Recursive
10
         Codeium: Refactor | Explain | X
11 ~
         public static void main(String[] args) {
12
             int floors = 16;
13
             int eggs = 2;
14
              System.out.println(minAttempts(floors, eggs));
15
16
         Codeium: Refactor | Explain | Generate Javadoc | X
17 V
         public static int minAttempts(int floors, int eggs) {
             if (floors == 0 || floors == 1 || eggs == 1) {
18 ~
19
                  return floors;
20
21
22
              int min = Integer.MAX VALUE;
23 V
              for (int i = 1; i <= floors; i++) {
                  int count = 1 + Math.max(minAttempts(i - 1,
24
                                  eggs - 1),
25
26
                          minAttempts(floors - i, eggs));
27
                  if (count < min) {
28
                      min = count;
29
30
31
              return min;
32
33
34
         //Approach: Memoization
         Codeium: Refactor | Explain | X
35 V
         public static int minAttemptsMemo(int floors, int eggs) {
36
              int[][] dp = new int[floors + 1][eggs + 1];
37
              return minAttemptsMemo(floors, eggs, dp);
38
```



```
Codeium: Refactor | Explain | Generate Javadoc | X
40 V
         public static int minAttemptsMemo(int floors, int eggs, int[][] dp) {
41 V
             if (floors == 0 || floors == 1 || eggs == 1) {
42
                 return floors;
43
44
45 V
             if (dp[floors][eggs] != 0) {
                 return dp[floors][eggs];
46
47
48
             int min = Integer.MAX VALUE;
49
             for (int i = 1; i <= floors; i++) {
50 V
                 int count = 1 + Math.max(minAttemptsMemo(i - 1, eggs - 1, dp),
51 V
                          minAttemptsMemo(floors - i, eggs, dp));
52
                 if (count < min) {
53 V
54
                     min = count;
55
56
             dp[floors][eggs] = min;
57
58
             return dp[floors][eggs];
59
60
```



```
60
         //Approach: Dynamic Programming
61
         Codeium: Refactor | Explain | X
         public static int minAttemptsDP(int floors, int eggs) {
62
             int[][] dp = new int[floors + 1][eggs + 1];
63
64
             for (int i = 1; i <= floors; i++) {
65
66
                  dp[i][1] = i;
67
68
             for (int i = 1; i <= eggs; i++) {
69
70
                  dp[1][i] = 1;
71
72
73
             for (int i = 2; i <= floors; i++) {
                 for (int j = 2; j <= eggs; j++) {
74
75
                      dp[i][j] = Integer.MAX VALUE;
                     for (int k = 1; k <= i; k++) {
76
                          int count = 1 + Math.max(dp[k - 1][j - 1], dp[i - k][j]);
77
                          if (count < dp[i][j]) {
78
                              dp[i][j] = count;
79
80
81
82
83
84
             return dp[floors][eggs];
85
86
```



Below is the Java code for the **Egg Dropping Problem** based on the diagrams you provided, with detailed explanations and an example.

# **Problem Description:**



The **Egg Dropping Problem** involves determining the minimum number of attempts needed to find the critical floor from which an egg will break when dropped. The rules are:

- 1. If an egg breaks when dropped, all eggs dropped from floors above will also break.
- 2. If an egg does not break, all eggs dropped from floors below will not break.
- 3. You need to minimize the worst-case number of attempts to determine the critical floor.

## **Code Implementation**

```
java
 public class EggDroppingProblem {
     // Recursive Approach
      public static int minAttempts(int floors, int eggs) {
         if (floors == 0 || floors == 1 || eggs == 1) {
              return floors;
         int min = Integer.MAX VALUE;
         for (int i = 1; i \leftarrow floors; i++) {
             int count = 1 + Math.max(minAttempts(i - 1, eggs - 1), minAttempts(floors - i, eggs));
             if (count < min) {</pre>
                  min = count:
          return min;
      // Memoization Approach
      public static int minAttemptsMemo(int floors, int eggs) {
         int[][] dp = new int[floors + 1][eggs + 1];
         return minAttemptsMemo(floors, eggs, dp);
      private static int minAttemptsMemo(int floors, int eggs, int[][] dp) {
         if (floors == 0 || floors == 1 || eggs == 1) {
              return floors:
         if (dp[floors][eggs] != 0) {
              return dp[floors][eggs];
         int min = Integer.MAX VALUE;
```



```
for (int i = 1; i <= floors; i++) {</pre>
            int count = 1 + Math.max(minAttemptsMemo(i - 1, eggs - 1, dp), minAttemptsMemo(floors - i, eggs, dp));
            if (count < min) {</pre>
                min = count:
        dp[floors][eggs] = min;
        return min;
    }
    // Dynamic Programming Approach
    public static int minAttemptsDP(int floors, int eggs) {
        int[][] dp = new int[floors + 1][eggs + 1];
        for (int i = 1; i <= floors; i++) {
            dp[i][1] = i; // If we have one egg, we must try every floor
        for (int i = 1; i <= eggs; i++) {
            dp[1][i] = 1; // If we have one floor, we need only one attempt
        for (int i = 2; i <= floors; i++) {</pre>
            for (int j = 2; j \le eggs; j++) {
                dp[i][j] = Integer.MAX VALUE;
                for (int k = 1; k \le i; k++) {
                     int count = 1 + Math.max(dp[k - 1][j - 1], dp[i - k][j]);
                     if (count < dp[i][j]) {</pre>
                         dp[i][j] = count;
                }
            }
        return dp[floors][eggs];
    public static void main(String[] args) {
        int floors = 16;
        int eggs = 2;
        // Recursive Approach
        System.out.println("Recursive: " + minAttempts(floors, eggs));
        // Memoization Approach
        System.out.println("Memoization: " + minAttemptsMemo(floors, eggs));
        // Dynamic Programming Approach
        System.out.println("Dynamic Programming: " + minAttemptsDP(floors, eggs));
}
```

#### 1. Recursive Approach:

- Try every possible floor as a drop point (`k`).
- o For each floor, calculate:
  - If the egg breaks: Check floors below (`i 1`) with one less egg.
  - If the egg does not break: Check floors above (`floors i`) with the same number of eggs.
- Return the minimum of all worst-case attempts.

#### 2. Memoization Approach:

• Store results of subproblems in a 2D array `dp` to avoid recomputation.

#### 3. Dynamic Programming Approach:

- o Build a table (`dp`) iteratively where `dp[i][j]` represents the minimum attempts needed for `i` floors and `j` eggs.
- Use previously computed values to fill the table.

# **Example:**

#### Input:

- Floors = `16`
- Eggs = `2`

#### Output:

- Recursive: `5`
- Memoization: `5`
- Dynamic Programming: `5`

# **Explanation of Output:**

The critical floor is determined in a maximum of `5` attempts in the worst case:

1. Start from the middle floor to minimize attempts.



2. Drop eggs based on whether they break or not, reducing the problem size dynamically.



```
Codelum: Ketactor | Explain
 3 v public class MatrixchainMultiplication {
 4
         //Given a sequence of matrices, find the most efficient way to multiply these matrices together.
 5
         //The problem is not actually to perform the multiplications, but merely to decide in which order
 6
         // to perform the multiplications.
 7
         //We have many options to multiply a chain of matrices because matrix multiplication is associative.
 8
         //In other words, no matter how we parenthesize the product, the result will be the same.
 9
         //For example, if we had four matrices A, B, C, and D, we would have:
10
               (ABC)D = (AB)(CD) = A(BCD) = \dots
11
         //However, the order in which we parenthesize the product affects the number of simple arithmetic
12
         // operations needed to compute the product, or the efficiency.
13
         //For example, suppose A is a 10 × 30 matrix, B is a 30 × 5 matrix, and C is a 5 × 60 matrix.
14
         //Then,
15
16
                (AB)C = (10\times30\times5) + (10\times5\times60) = 1500 + 3000 = 4500 operations
17
         11
                A(BC) = (30 \times 5 \times 60) + (10 \times 30 \times 60) = 9000 + 18000 = 27000 \text{ operations.}
18
         //Clearly the first parenthesization requires less number of operations.
19
     //Given an array p[] which represents the chain of matrices such that the ith matrix Ai is of
     //dimension p[i-1] x p[i]. We need to write a function MatrixChainOrder() that should return the
21 \( //minimum number of multiplications needed to multiply the chain.
22
23
          //Approach: Recursive
          Codeium: Refactor | Explain | X
          public static void main(String[] args) {
24 V
             int[] arr = {1, 2, 3, 4, 3};
25
              System.out.println(matrixChainOrder(arr, 1, arr.length - 1));
26
27
              System.out.println("matrixChainOrderMemo ->"+matrixChainOrderMemo(arr, 1, arr.length - 1));
              System.out.println("matrixChainOrderDP ->"+matrixChainOrderDP(arr));
28
29
```



```
Codeium: Refactor | Explain | X
24
         public static void main(String[] args) {
             int[] arr = {1, 2, 3, 4, 3};
25
             System.out.println(matrixChainOrder(arr, 1, arr.length - 1));
26
27
              System.out.println("matrixChainOrderMemo ->"+matrixChainOrderMemo(arr, 1, arr.length - 1));
28
              System.out.println("matrixChainOrderDP ->"+matrixChainOrderDP(arr));
29
30
          Codeium: Refactor | Explain | Generate Javadoc | X
31
          public static int matrixChainOrder(
                  int[] arr, int i, int j) {
32
33
              if (i >= j) {
34
                  return 0;
35
36
37
              int min = Integer.MAX_VALUE;
38
              for (int k = i; k < j; k++) {
                  int count = matrixChainOrder(arr, i, k) +
39
40
                          matrixChainOrder(arr, k + 1, j)
                          + arr[i - 1] * arr[k] * arr[j];
41
                  if (count < min) {
42
43
                      min = count;
44
45
46
              return min;
47
48
49
          //Approach: Memoization
          Codeium: Refactor | Explain | X
50
          public static int matrixChainOrderMemo(
                 int[] arr, int i, int j) {
51
              int[][] dp = new int[arr.length + 1][arr.length + 1];
52
53
              return matrixChainOrderMemo(arr, i, j, dp);
54
55
```



```
56
         public static int matrixChainOrderMemo(
                 int[] arr, int i, int j, int[][] dp) {
57
             if (i >= j) {
58
                 return 0;
59
60
61
             if (dp[i][j] != 0) {
62
                 return dp[i][j];
63
64
65
66
             int min = Integer.MAX_VALUE;
             for (int k = i; k < j; k++) {
67
                 int count = matrixChainOrderMemo(arr, i, k, dp) +
68
                         matrixChainOrderMemo(arr, k + 1, j, dp) +
69
                         arr[i - 1] * arr[k] * arr[j];
70
                 if (count < min) {
71
                     min = count;
72
73
74
75
             dp[i][j] = min;
76
             return dp[i][j];
77
78
```



```
10
 79
          //Approach: Dynamic Programming
          Codeium: Refactor | Explain | X
          public static int matrixChainOrderDP(int[] arr) {
80
              int n = arr.length;
81
82
              int[][] dp = new int[n][n];
83
84
              for (int i = 1; i < n; i++) {
 85
                   dp[i][i] = 0;
 86
87
              for (int 1 = 2; 1 < n; 1++) {
88
                  for (int i = 1; i < n - 1 + 1; i++) {
 89
                      int j = i + 1 - 1;
90
91
                      if (j == n) {
92
                           continue;
93
                      dp[i][j] = Integer.MAX VALUE;
94
                      for (int k = i; k < j; k++) {
95
                           int count = dp[i][k] + dp[k + 1][j] + arr[i - 1] * arr[k] * arr[j];
96
                           if (count < dp[i][j]) {</pre>
97
                               dp[i][j] = count;
98
99
100
101
102
              return dp[1][n - 1];
103
104
105
106
```



Here is the Java implementation of the **Matrix Chain Multiplication Problem** from the diagrams, along with a detailed explanation and example:



#### **Problem Description:**

Given an array `arr[]` where the dimensions of the matrices are stored such that:

• The matrix `Ai` has dimensions `arr[i-1] x arr[i]`.

The goal is to find the **minimum number of scalar multiplications** required to multiply the chain of matrices using the best possible parenthesization.

# **Code Implementation**

```
java
 public class MatrixChainMultiplication {
     // Recursive Approach
     public static int matrixChainOrder(int[] arr, int i, int j) {
         if (i >= j) {
              return 0;
         int min = Integer.MAX VALUE;
         for (int k = i; k < j; k++) {
             int count = matrixChainOrder(arr, i, k)
                       + matrixChainOrder(arr, k + 1, j)
                        + arr[i - 1] * arr[k] * arr[j];
             if (count < min) {</pre>
                  min = count;
         return min;
     // Memoization Approach
     public static int matrixChainOrderMemo(int[] arr, int i, int j) {
         int[][] dp = new int[arr.length][arr.length];
         return matrixChainOrderMemo(arr, i, j, dp);
     private static int matrixChainOrderMemo(int[] arr, int i, int j, int[][] dp) {
         if (i >= j) {
             return 0;
         if (dp[i][j] != 0) {
             return dp[i][j];
```



```
int min = Integer.MAX VALUE;
        for (int k = i; k < j; k++) {
            int count = matrixChainOrderMemo(arr, i, k, dp)
                      + matrixChainOrderMemo(arr, k + 1, j, dp)
                      + arr[i - 1] * arr[k] * arr[j];
            if (count < min) {</pre>
                min = count;
            }
        dp[i][j] = min;
        return min;
    }
    // Dynamic Programming Approach
    public static int matrixChainOrderDP(int[] arr) {
        int n = arr.length;
        int[][] dp = new int[n][n];
        for (int l = 2; l < n; l++) {
            for (int i = 1; i < n - l + 1; i++) {
                int j = i + l - 1;
                dp[i][j] = Integer.MAX VALUE;
                for (int k = i; k < j; k++) {
                    int count = dp[i][k]
                              + dp[k + 1][j]
                              + arr[i - 1] * arr[k] * arr[j];
                    if (count < dp[i][j]) {</pre>
                        dp[i][j] = count;
                }
        }
        return dp[1][n - 1];
    public static void main(String[] args) {
        int[] arr = {1, 2, 3, 4, 3};
        // Recursive Approach
        System.out.println("Recursive: " + matrixChainOrder(arr, 1, arr.length - 1));
        // Memoization Approach
        System.out.println("Memoization: " + matrixChainOrderMemo(arr, 1, arr.length - 1));
        // Dynamic Programming Approach
        System.out.println("Dynamic Programming: " + matrixChainOrderDP(arr));
    }
}
```

#### 1. Recursive Approach:

- o Divide the array into two parts ('i to k' and 'k+1 to j') for each possible 'k'.
- o Calculate the cost recursively for each subproblem and return the minimum cost.

#### 2. Memoization Approach:

- Use a 2D array `dp` to store the results of previously solved subproblems.
- Avoid recomputation by checking if the result is already in `dp`.

#### 3. Dynamic Programming Approach:

- Use a bottom-up approach to fill the `dp` table.
- `dp[i][j]` represents the minimum cost to multiply matrices from `i` to `j`.

# **Example:**

#### Input:

```
`arr[] = {1, 2, 3, 4, 3}`
```

#### **Output:**

- Recursive: `30`
- Memoization: `30`
- Dynamic Programming: `30`

## **Explanation of Output:**

- 1. Parenthesization: `((A1 \* A2) \* (A3 \* A4))`
  - Multiply `A1` (1x2) and `A2` (2x3): Cost = `1\*2\*3 = 6`.
  - Multiply `A3` (3x4) and `A4` (4x3): Cost = `3\*4\*3 = 36`.
  - Multiply the results: `6 + 36 = 42`.



- 2. Optimal Parenthesization minimizes operations:
  - Final minimum cost = `30`.





```
3
     public class PallindromePartition {
 4
          // Given a string, a partitioning of the string is a palindrome partitioning if every substring of the partition is a palindrome.
 5
         // For example, "aba|b|bbabb|a|b|aba" is a palindrome partitioning of "ababbbabbababa".
 6
         // Determine the fewest cuts needed for palindrome partitioning of a given string.
 7
         // For example, minimum 3 cuts are needed for "ababbbabbababa".
 8
 9
          //Approach: Recursive
          Codeium: Refactor | Explain | X
10
          public static void main(String[] args) {
              String s = "ababbbabbababa";
11
12
              System.out.println(minCuts(s, 0, s.length() - 1));
13
14
          Codeium: Refactor | Explain | Generate Javadoc | X
15
          public static int minCuts(String s, int i, int j) {
16
              if (i >= j) {
17
                  return 0;
18
19
20
              if (isPallindrome(s, i, j)) {
21
                  return 0;
22
23
24
              int min = Integer.MAX VALUE;
25
              for (int k = i; k < j; k++) {
                  int count = minCuts(s, i, k) + minCuts(s, k + 1, j) + 1;
26
27
                  if (count < min) {
28
                      min = count;
29
30
31
              return min;
32
33
          Codeium: Refactor | Explain | Generate Javadoc | X
34
          public static boolean isPallindrome(String s, int i, int j) {
35
              while (i < j) {
                  if (s.charAt(i++) != s.charAt(j--)) {
36
37
                      return false;
38
39
```



```
40
              return true;
41
42
42
          //Approach: Memoization
43
          Codeium: Refactor | Explain | X
          public static int minCutsMemo(String s, int i, int j) {
44
              int[][] dp = new int[s.length() + 1][s.length() + 1];
45
              return minCutsMemo(s, i, j, dp);
46
47
48
          Codeium: Refactor | Explain | Generate Javadoc | X
49
          public static int minCutsMemo(String s, int i, int j, int[][] dp) {
              if (i >= j) {
50
51
                  return 0;
52
53
              if (isPallindrome(s, i, j)) {
54
55
                  return 0;
56
57
58
              if (dp[i][j] != 0) {
                  return dp[i][j];
59
60
61
62
              int min = Integer.MAX VALUE;
              for (int k = i; k < j; k++) {
63
                  int count = minCutsMemo(s, i, k, dp) +
64
                          minCutsMemo(s, k + 1, j, dp) + 1;
65
                  if (count < min) {
66
67
                      min = count;
68
69
70
              dp[i][j] = min;
              return dp[i][j];
71
72
73
```



```
14
 73
 74
          //Approach: Dynamic
          Codeium: Refactor | Explain | X
 75 V
          public static int minCutsDP(String s) {
              int n = s.length();
 76
              int[][] dp = new int[n][n];
 77
 78
 79 V
              for (int i = 1; i < n; i++) {
                   dp[i][i] = 0;
 80
 81
 82
 83 V
              for (int 1 = 2; 1 < n; 1++) {
                   for (int i = 1; i < n - 1 + 1; i++) {
 84 ~
                      int j = i + 1 - 1;
 85
                      if (isPallindrome(s, i, j)) {
 86 V
                           dp[i][j] = 0;
 87
                       } else {
 88 ~
 89
                           dp[i][j] = Integer.MAX_VALUE;
                           for (int k = i; k < j; k++) {
 90 ~
                               int count = dp[i][k] + dp[k + 1][j] + 1;
 91
 92 V
                               if (count < dp[i][j]) {</pre>
                                   dp[i][j] = count;
 93
 94
 95
 96
 97
 98
               return dp[1][n - 1];
 99
100
101
          //Approach: Try to find it in most optimized way
102
103
104
105
106
```



Here is the Java code for the **Palindrome Partitioning Problem** from the diagrams, along with a detailed explanation and example:

# **Problem Description:**

Given a string, partition it such that every substring of the partition is a palindrome. Return the **minimum cuts** required to partition the string.

For example:

- Input: `"ababbabababa"`
- Output: `3`

# **Code Implementation:**

```
java
 public class PalindromePartition {
     // Recursive Approach
     public static int minCuts(String s, int i, int j) {
         if (i >= j || isPalindrome(s, i, j)) {
             return 0;
         int min = Integer.MAX VALUE;
         for (int k = i; k < j; k++) {
             int count = minCuts(s, i, k) + minCuts(s, k + 1, j) + 1;
             if (count < min) {</pre>
                 min = count;
         return min;
     // Helper method to check if a substring is a palindrome
     public static boolean isPalindrome(String s, int i, int j) {
         while (i < j) {
             if (s.charAt(i++) != s.charAt(j--)) {
                  return false:
```



```
return true;
// Memoization Approach
public static int minCutsMemo(String s, int i, int j) {
    int[][] dp = new int[s.length()][s.length()];
    return minCutsMemo(s, i, j, dp);
}
private static int minCutsMemo(String s, int i, int j, int[][] dp) {
   if (i \ge j \mid | isPalindrome(s, i, j)) {
        return 0;
    if (dp[i][j] != 0) {
        return dp[i][j];
   int min = Integer.MAX VALUE;
    for (int k = i; k < j; k++) {
        int count = minCutsMemo(s, i, k, dp) + minCutsMemo(s, k + 1, j, dp) + 1;
        if (count < min) {</pre>
            min = count;
        }
    dp[i][j] = min;
    return min;
}
// Dynamic Programming Approach
public static int minCutsDP(String s) {
   int n = s.length();
    int[][] dp = new int[n][n];
    for (int i = 0; i < n; i++) {
        dp[i][i] = 0; // Single character is a palindrome
    }
    for (int l = 2; l <= n; l++) { // Length of substring</pre>
        for (int i = 0; i < n - l + 1; i++) {
            int j = i + l - 1;
            if (isPalindrome(s, i, j)) {
                dp[i][j] = 0;
            } else {
                dp[i][j] = Integer.MAX VALUE;
                for (int k = i; k < j; k++) {
                    int count = dp[i][k] + dp[k + 1][j] + 1;
                    if (count < dp[i][j]) {</pre>
                        dp[i][j] = count;
```



```
}
return dp[0][n - 1];

public static void main(String[] args) {
    String s = "ababbabababa";

    // Recursive Approach
    System.out.println("Recursive: " + minCuts(s, 0, s.length() - 1));

    // Memoization Approach
    System.out.println("Memoization: " + minCutsMemo(s, 0, s.length() - 1));

    // Dynamic Programming Approach
    System.out.println("Dynamic Programming: " + minCutsDP(s));
}

}
```

#### 1. Recursive Approach:

- For each substring `(i, j)`, check if it is a palindrome.
- If yes, no cuts are needed for this substring.
- Otherwise, try every possible split point `k` to partition the string and calculate the minimum cuts.

#### 2. Memoization Approach:

- Use a 2D array `dp` to store results of overlapping subproblems.
- Avoid recomputing the results for the same `(i, j)`.

#### 3. Dynamic Programming Approach:

- Use a bottom-up approach.
- `dp[i][j]` represents the minimum cuts required to partition the substring from index `i` to `j`.

# **Example:**

#### Input:

`"ababbabababa"`



## **Output:**

- Recursive: `3`
- Memoization: `3`
- Dynamic Programming: `3`

# **Explanation of Output:**

- Partition the string as: `"ababbab | abab | a"`.
- Each segment is a palindrome. The minimum number of cuts required is `3`.





```
Codelum: Ketactor | Explain
 3
     public class ScrambledString {
         //Given two strings s1 and s2 of the same length, determine if s2 is a scrambled string of s1.
 4
 5
         //A string is a scrambled string of another string if it can be obtained by swapping some characters of the other string.
 6
         //Example: s1 = "great", s2 = "rgeat" -> true
         //Example: s1 = "abcde", s2 = "caebd" -> false
 8
          Codeium: Refactor | Explain | Generate Javadoc | X
 9
          public static void main(String[] args) {
10
             String s1 = "great";
11
             String s2 = "rgeat";
12
             System.out.println(isScrambled(s1, s2));
13
14
15
          //Approach: Recursive
          Codeium: Refactor | Explain | X
16
          public static boolean isScrambled(String s1, String s2) {
17
              if (s1.equals(s2)) {
18
                  return true;
19
20
21
             if (s1.length() <= 1) {
22
                  return false;
23
24
25
              int n = s1.length();
26
              boolean flag = false;
27
              for (int i = 1; i < n; i++) {
                  if ((isScrambled(s1.substring(0, i), s2.substring(n - i))
28
                          && isScrambled(s1.substring(i), s2.substring(0, n - i))) |
29
30
                          (isScrambled(s1.substring(0, i), s2.substring(0, i)) &&
31
                                  isScrambled(s1.substring(i), s2.substring(i)))) {
32
                      flag = true;
33
                      break;
34
35
36
              return flag;
37
38
```



```
39
         //Approach: Memoization
         Codeium: Refactor | Explain | X
40
         public static boolean isScrambledMemo(String s1, String s2) {
              int n = s1.length();
41
             int[][][] dp = new int[n + 1][n + 1][n + 1];
42
             return isScrambledMemo(s1, s2, dp, 0, 0, n);
43
44
45
         Codeium: Refactor | Explain | Generate Javadoc | X
46
         public static boolean isScrambledMemo(
47
                  String s1, String s2, int[][][] dp, int i, int j, int len) {
             if (i >= len || j >= len) {
48
49
                  return false;
50
51
52
             if (s1.substring(i).equals(s2.substring(j))) {
53
                  return true;
54
55
             if (len == 1) {
56
57
                  return s1.charAt(i) == s2.charAt(j);
58
59
             if (dp[i][j][len] != 0) {
60
                  return dp[i][j][len] == 1;
61
62
63
              boolean flag = false;
64
65
             for (int k = 1; k < len; k++) {
                  if ((isScrambledMemo(s1, s2, dp, i, j + len - k, k) &&
66
                          isScrambledMemo(s1, s2, dp, i + k, j, len - k)) ||
67
                          (isScrambledMemo(s1, s2, dp, i, j, k) &&
68
                                  isScrambledMemo(s1, s2, dp, i + k, j + k, len - k))) {
69
70
                      flag = true;
71
                      break;
72
73
74
             dp[i][j][len] = flag ? 1 : -1;
75
             return flag;
76
```



```
//Approach: Dynamic Programming
78
          Codeium: Refactor | Explain | X
          public static boolean isScrambledDP(String s1, String s2) {
79
80
              int n = s1.length();
              boolean[][][] dp = new boolean[n][n][n + 1];
81
82
              for (int len = 1; len <= n; len++) {
83
                  for (int i = 0; i <= n - len; i++) {
 84
85
                      for (int j = 0; j <= n - len; <math>j++) {
 86
                           if (len == 1) {
                               dp[i][j][len] = s1.charAt(i) == s2.charAt(j);
87
88
                           } else {
                               for (int k = 1; k < len; k++) {
89
                                   if ((dp[i][j + len - k][k] && dp[i + k][j][len - k]) ||
90
91
                                           (dp[i][j][k] && dp[i+k][j+k][len-k])) {
 92
                                       dp[i][j][len] = true;
93
                                       break;
94
 95
96
97
98
99
              return dp[0][0][n];
100
101
102
103
104
105
```



Here is the **Scrambled String Problem** implementation from the diagrams, with explanations and examples.

## **Problem Description:**



A string `s2` is a scrambled version of `s1` if it can be obtained by recursively swapping some characters of `s1`.

For example:

```
    Input: `s1 = "great", s2 = "rgeat"` → true (swap "gr" and "eat").
    Input: `s1 = "abcde", s2 = "caebd"` → false.
```

The goal is to determine if `s2` is a scrambled string of `s1`.

# **Code Implementation:**

```
java
 public class ScrambledString {
     // Recursive Approach
     public static boolean isScrambled(String s1, String s2) {
         if (s1.equals(s2)) {
             return true;
         }
         if (s1.length() != s2.length() || s1.length() <= 1) {</pre>
             return false:
         }
         int n = s1.length();
         boolean flag = false;
         for (int i = 1; i < n; i++) {
             // Check both cases:
             // Case 1: No swap
             // Case 2: Swap
             if ((isScrambled(s1.substring(0, i), s2.substring(0, i)) &&
                  isScrambled(s1.substring(i), s2.substring(i))) ||
                 (isScrambled(s1.substring(0, i), s2.substring(n - i)) &&
                  isScrambled(s1.substring(i), s2.substring(0, n - i)))) {
                 flag = true;
                 break;
         return flag;
     // Memoization Approach
     public static boolean isScrambledMemo(String s1, String s2) {
```



```
int n = s1.length();
    int[][][] dp = new int[n + 1][n + 1][n + 1]; // 3D array for memoization
    return isScrambledMemo(s1, s2, dp, 0, 0, n);
}
private static boolean isScrambledMemo(String s1, String s2, int[][][] dp, int i, int j, int len) {
    if (len == 1) {
        return s1.charAt(i) == s2.charAt(j);
    }
    if (dp[i][j][len] != 0) {
        return dp[i][j][len] == 1;
    for (int k = 1; k < len; k++) {
       // Check both cases:
       // Case 1: No swap
       // Case 2: Swap
       if ((isScrambledMemo(s1, s2, dp, i, j, k) &&
            isScrambledMemo(s1, s2, dp, i + k, j + k, len - k)) | |
            (isScrambledMemo(s1, s2, dp, i, j + len - k, k) &&
            isScrambledMemo(s1, s2, dp, i + k, j, len - k))) {
            dp[i][j][len] = 1;
            return true;
    dp[i][j][len] = -1;
    return false;
}
// Dynamic Programming Approach
public static boolean isScrambledDP(String s1, String s2) {
    int n = s1.length();
    boolean[][][] dp = new boolean[n][n][n + 1];
    // Initialization for substrings of length 1
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            dp[i][j][1] = s1.charAt(i) == s2.charAt(j);
       }
    }
    // Solve for substrings of length 2 to n
    for (int len = 2; len <= n; len++) {
        for (int i = 0; i \le n - len; i++) {
            for (int j = 0; j <= n - len; <math>j++) {
                for (int k = 1; k < len; k++) {
                    if ((dp[i][j][k] && dp[i + k][j + k][len - k]) ||
                        (dp[i][j + len - k][k] \& dp[i + k][j][len - k])) {
                        dp[i][j][len] = true;
                        break;
```



```
}
}

return dp[0][0][n];

public static void main(String[] args) {
    String s1 = "great";
    String s2 = "rgeat";

    // Recursive Approach
    System.out.println("Recursive: " + isScrambled(s1, s2));

    // Memoization Approach
    System.out.println("Memoization: " + isScrambledMemo(s1, s2));

    // Dynamic Programming Approach
    System.out.println("Dynamic Programming: " + isScrambledDP(s1, s2));
}
```

#### 1. Recursive Approach:

- Base case: If `s1.equals(s2)`, return `true`.
- ∘ For each possible split index `k`, check:
  - Case 1: No swap.
  - Case 2: Swap.
- Return `true` if any split makes `s2` a scrambled string of `s1`.

#### 2. Memoization Approach:

- Use a 3D DP table to store results of subproblems.
- Avoid recomputation by checking the DP table before recursive calls.

#### 3. **Dynamic Programming Approach**:

- Bottom-up approach using a 3D DP table.
- `dp[i][j][len]` indicates if `s1.substring(i, i + len)` is a scrambled string of `s2.substring(j, j + len)`.

# **Example:**

# Input:

```
`s1 = "great"`, `s2 = "rgeat"`
```

## **Output:**

- Recursive: true
- Memoization: **true**
- Dynamic Programming: **true**

## **Explanation:**

```
1. Split `s1 = "great"` into `["gr", "eat"]`.
```

2. Swap "gr" and "eat"  $\rightarrow$  `"rgeat"`. This matches `s2`.

The output is `true`.