Indian Institute of Space Science and Technology

Quiz - II

B.Tech 2nd Semester

MA 121 - Differential Equations and Vector Calculus

Date : 13^{th} March, 2013 **Time: 9 am to 10 am** **Full Marks: 15**

Answer all questions

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1. (a) Find the general solution around $x = 0$ of the following differential equation

$$(x^2 - x)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$

[4]

(b) Does eigenvalues and eigenfunctions for the following differential equations exist? If yes, then find them

$$\frac{d^2y}{dx^2} + \lambda y = 0,$$
$$y(0) = 0, \;\; y(\pi) + y'(\pi) = 0.$$

[2]

(c) Show that between any two positive zeros of $J_0(x)$ there is a zero of $J_1(x)$. [2]

2. A point mass particle P in XYZ-space is spinning around Z axis with speed 3cm/sec in such a way that the projection of the trajectory of the particle on XY-plane is circular with radius $r(t)$ which is varying with respect to time t. Given that rate of change of the radius $r(t)$ is 2cm/sec and velocity of P along Z-axis is $\sqrt{\frac{2}{t}}$cm/sec. Suppose that at time $t = 0$ the particle was at the origine of XYZ-space. Find parametric representation the trajectory of P. Is the trajectory smooth? Find the arc length function of the trajectory with initial point $(0, 0, 0)$ and hence find the total length covered by the point mass when time $t = 30$sec. What is the position of the particle at time $t = 30$sec? [7]

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