INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

Class Test

B.Tech - IV Semester

MA221 - Integral Transforms, PDE and Calculus of Variations

Date: 28/04/2023 **Time: 1 Hour** **Max. Marks: 10**

Answer all questions.

1. Let $\bar{y}$ be the curve extremizing the functional
$$I(y) = \int_{x_0}^{x_1} F(x, y, y')dx,$$
with $y(x_0) = y_0, \quad y(x_1) = y_1$, satisfies the Euler-lagrage $(E-L)$ equation $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$. Then show that $\bar{y}$ satisfies the extended form of E-L equation,
$$\frac{\partial F}{\partial y} - \frac{\partial^2 F}{\partial x \partial y'} - \frac{\partial^2 F}{\partial y \partial y'} y' - \frac{\partial^2 F}{\partial y'^2} y'' = 0.$$

2. Reduce the Beltrami identity from the E-L equation for the extremum of a functional $I(y) = \int_{x_0}^{x_1} F(y, y')dx$. Using that, find the extremum of the functional $\int_{x_0}^{x_1} (y^2 - yy' + (y')^2)dx$.

3. Find the extremals of the functional $I = \int_0^1 (y'^2 + x^2 + Ry)\ dx$ under the conditions $(0,0), (1,0)$ and subject to the integral constraint $\int_0^1 y\ dx = \frac{1}{6}$.

4. Prove that the shortest distance between two points in a plane is a straightline.

5. Find the extremum of the functional $I(y) = \int_0^{\pi/2} \left[(y'')^2 - y^2 + x^2\right]\ dx$ subject to $y(0) = 1, \quad y'(0) = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = -1$

END