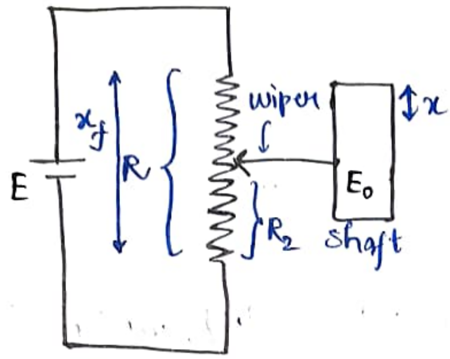
11-08-2025

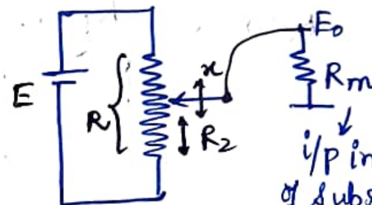
Potentiometric Displacement Sensing



$$R_2 = \frac{x}{x_F} R$$

$$E_0 = \frac{R_2}{R} E = \frac{x}{x_F} E$$

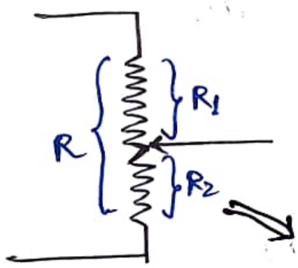
↳ Contact-based measurement :
- wear and tear (as wiper is in contact with fixed resistance winding)
- limited lifespan
- loading effect (due to R_m).
  [R_2 and R_m in parallel.]

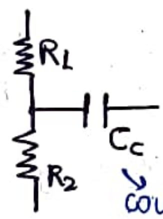


i/p impedance of subsequent electronics

Non-contact-based measurement :

→ wiper is separated by a small air-gap.





coupling capacitor to model the air-gap.

$$R_2 = \frac{x}{x_F} R$$

As $R_1 + R_2 = R$,

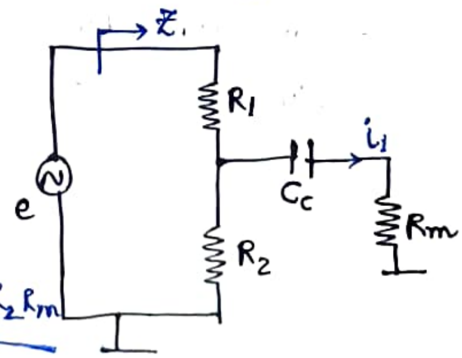
$$R_1 = \left[1 - \frac{x}{x_F}\right] R$$

$$Z = R_1 + R_2 \,||\, \left(R_m + \frac{1}{sC_c}\right)$$

$$= R_1 + \frac{R_2\left[R_m + \frac{1}{sC_c}\right]}{R_2 + R_m + \frac{1}{sC_c}}$$

$$= \frac{R_1R_2 + R_1R_m + (R_1+R_2)\frac{1}{sC_c} + R_2R_m}{R_2 + R_m + \frac{1}{sC_c}}$$

$$= \frac{R_1R_2 + R_m(R_1+R_2) + \frac{1}{sC_c}(R_1+R_2)}{R_2 + R_m + \frac{1}{sC_c}}$$



$$\Rightarrow Z = \frac{N}{R_2 + R_m + \frac{1}{sC_c}}$$

$$i_1 = \frac{e}{Z} \times \frac{R_2}{R_2 + R_m + \frac{1}{sC_c}}$$

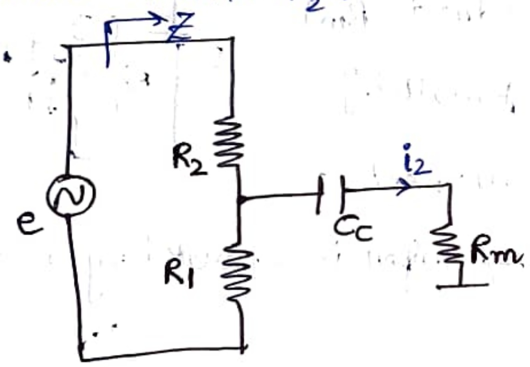
$$= \frac{eR_2}{N} \quad \text{(substitute } Z\text{)}$$

Let $e = E\sin(\omega t)$

(mag.) $I_1 = \frac{R_2}{|N|} \times E$ → dependent on $|N|$, which is dependent on R_m and C_c.

$$N = R_1R_2 + R_m(R_1+R_2) + \frac{1}{sC_c}(R_1+R_2)$$

Reverse R_1 and R_2:



$$i_2 = \frac{R_1 e}{N}$$

$$I_2 = \frac{R_1}{|N|} \times E$$

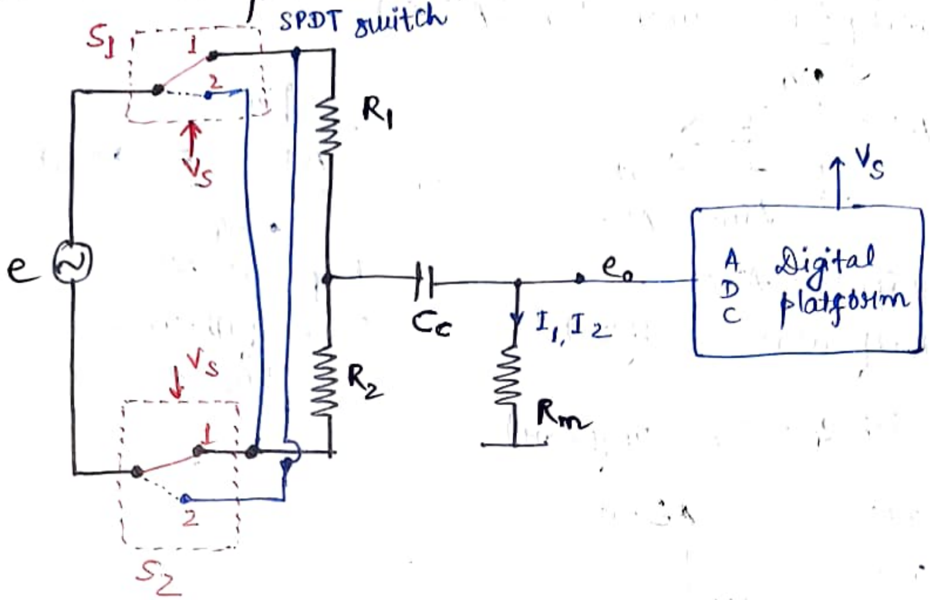
Ratiometric function,

$$F = \frac{I_1 - I_2}{I_1 + I_2} = \frac{R_2 - R_1}{R_2 + R_1}$$

$$F = 2\left(\frac{x}{x_F}\right) - 1$$

- Linear estimation of "x".
- No dependence on C_c, R_m, R.
- No wear & tear, loading.

Two-mode operation (Non-contact Potentiometer):



Mode-1:

V_s : logic-high

$S_1, S_2 \rightarrow 1$

Measure amplitude (E_{01}) of e_0:

$$E_{01} = I_1 R_m = \frac{R_2}{|N|} \times E R_m$$

Mode-2:

V_s : logic-low

$S_1, S_2 \rightarrow 2$

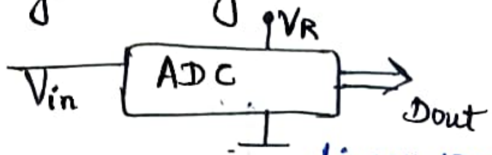
Measure amplitude (E_{02}) of e_0:

$$E_{02} = I_2 R_m = \frac{R_1}{|N|} \times E R_m$$

$$\therefore F = \frac{E_{01} - E_{02}}{E_{01} + E_{02}} = \frac{I_1 - I_2}{I_1 + I_2} = 2\left(\frac{x}{x_F}\right) - 1$$

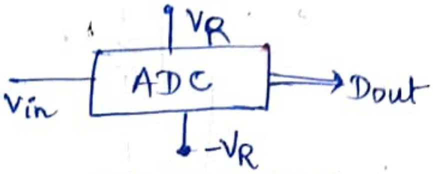
12-08-2025

Analog-to-Digital Converter (ADC)



Linear ADC

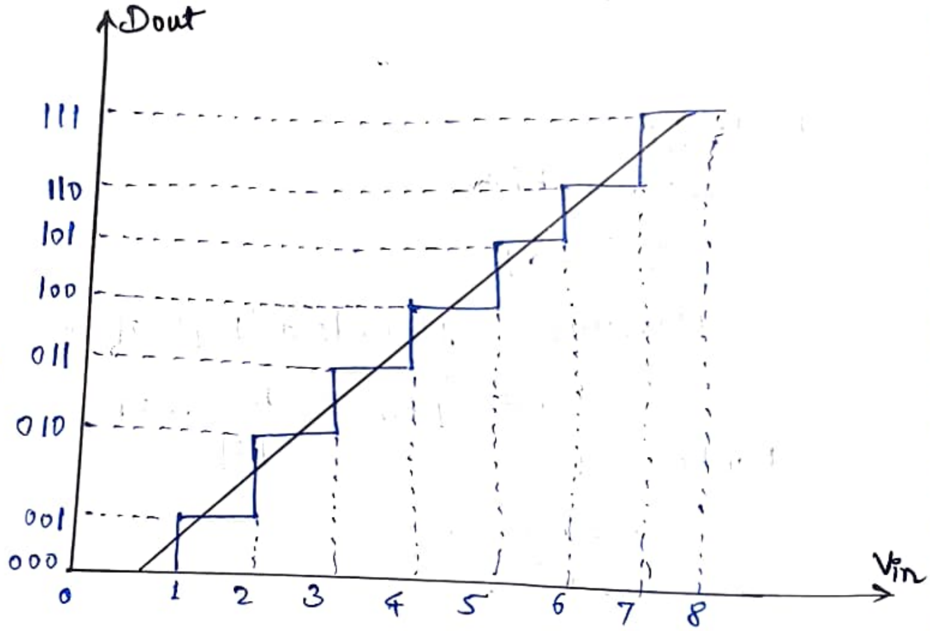
Unipolar ADC
(Four half wave rectified)



Bipolar ADC
(Four full-wave)
(sin, cos)

3-bit ADC with $V_R = 8V$ (unipolar):

Vin	Dout
[0,1]	000
[1,2]	001
[2,3]	010
[3,4]	~~100~~ 011
[4,5]	~~101~~ 100
[5,6]	101
[6,7]	110
[7,8]	111



Resolution (q): Min^m detectable voltage → depends on reference voltage, polarity, and no. of bits.

$$q = \frac{V_R}{2^n} \quad : \text{Unipolar ADC}$$

$$q = \frac{2V_R}{2^n} = \frac{V_R}{2^{(n-1)}} \quad : \text{Bipolar ADC}$$

In our case, input res^n, $q = \frac{8V}{2^3} = 1V$.

To reduce q, either reduce V_R or increase n.

$V_{in} \in [0, V_R]$.

$n \uparrow \Rightarrow$ size, cost, power $\uparrow$ | $V_R \downarrow \Rightarrow$ span of ADC decreases

For $V_R = 8V$, $n = 16$,

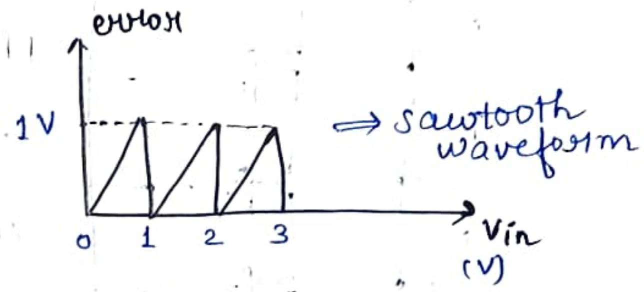
$$q = \frac{V_R}{2^n} = \frac{8}{2^{16}} = 120 \mu V.$$

In case of noisy sensor, if we decrease q, false detection may happen. so, we have to reduce noise as well.

Quantization Error = Vin − Analog Equivalent (Dout)

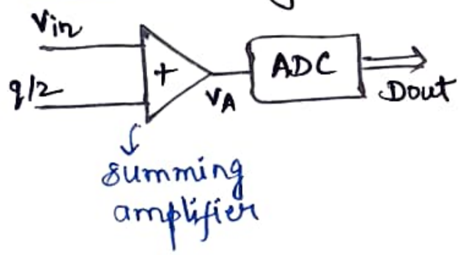
For $V_R = 8$, $n = 3$,

Vin	A(Dout)	error
0	0	0
0.5	0	0.5
1^- (just below)	0	1
1^+ (just above)	1	0

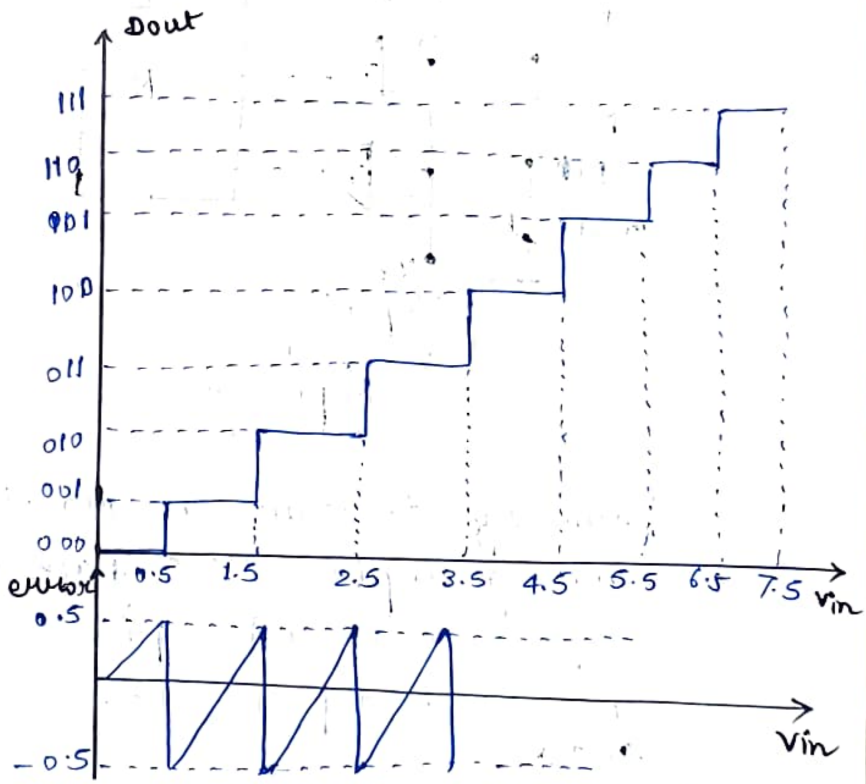


Maximum quantization error, q = 1.

How to mitigate this?



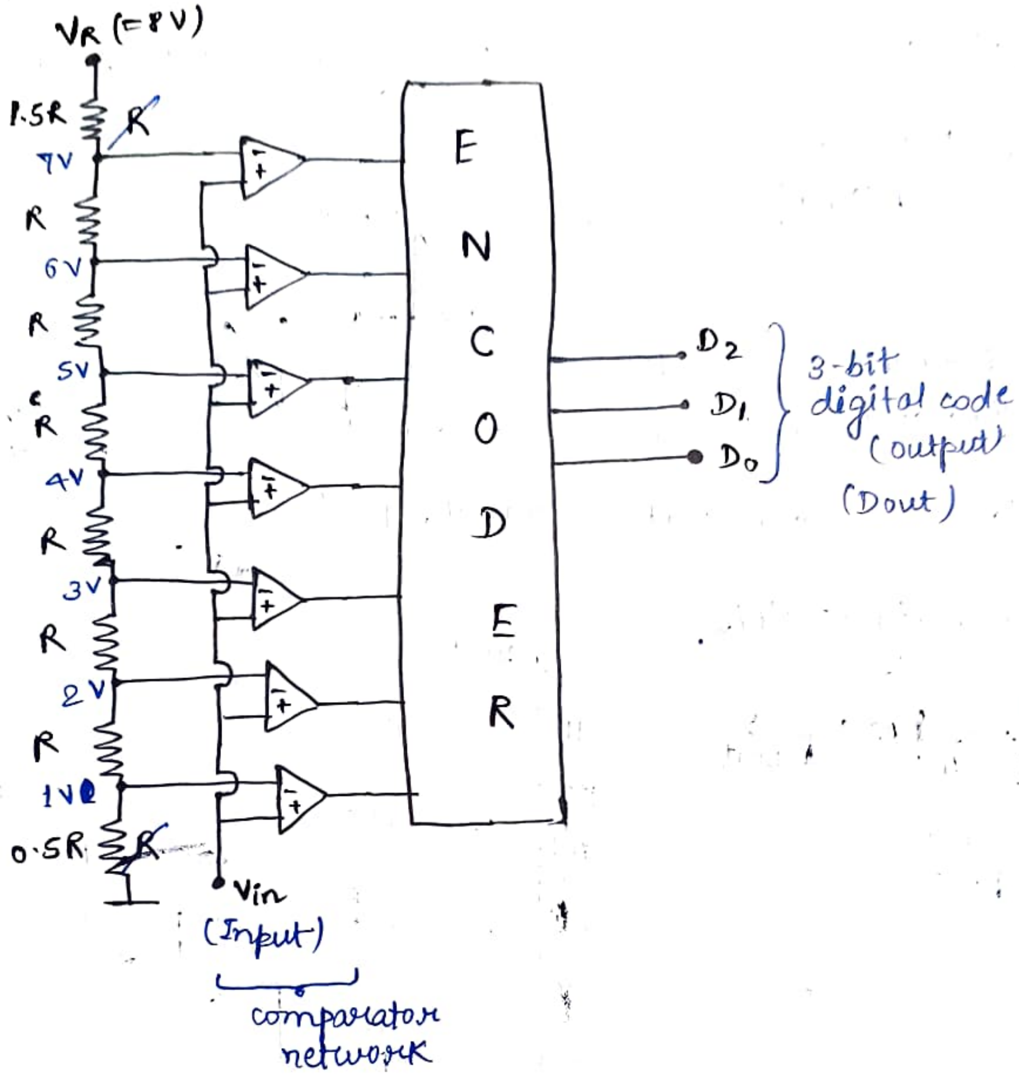
Max^m quantization error = q/2.



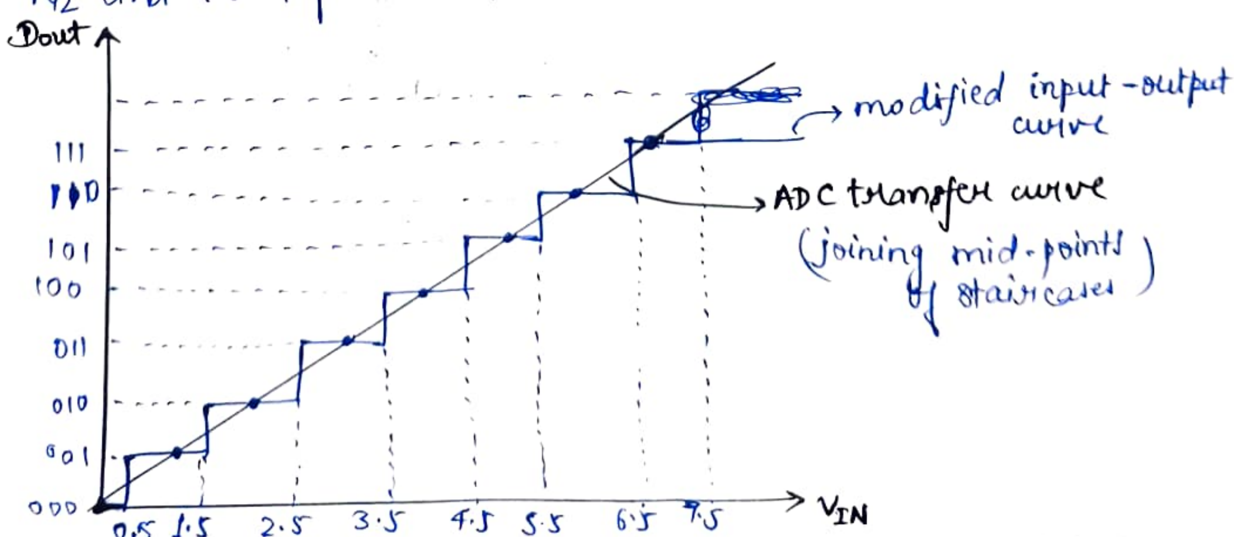
↳ Peak error value reduced to 0.5.

Flash ADC

3-bit Flash ADC:

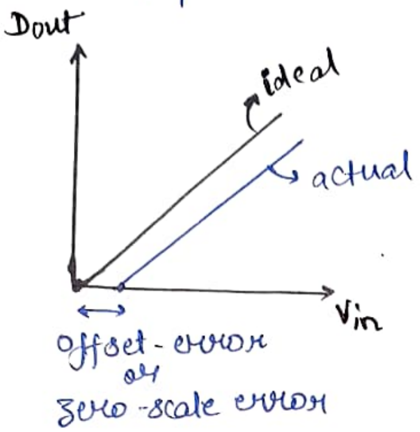


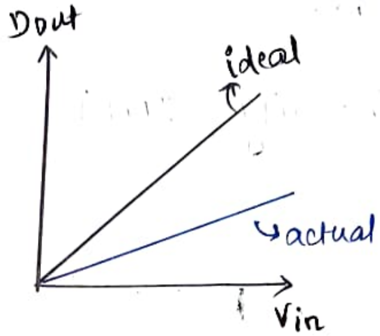
Previously, to reduce the quantization error, we used a summing amplifier. Here, flash ADC can be modified (instead of using summing amplifier) by making bottom resistance R/2 and the top one 3R/2.

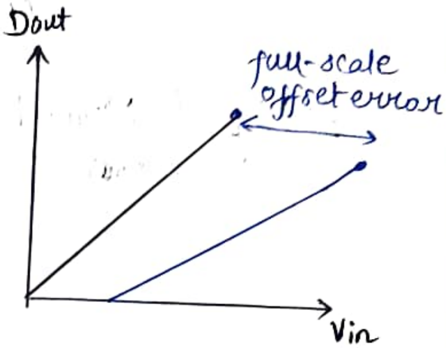


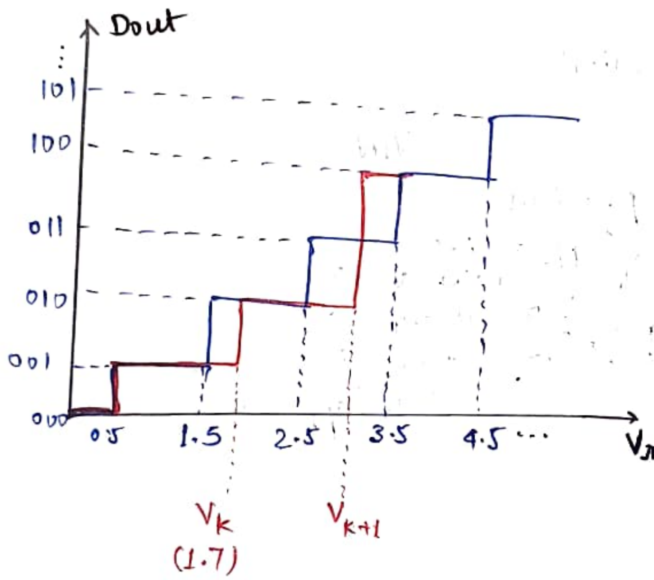
Offset and Gain Errors :

↳ Due to resistance tolerance and comparator tolerance (input offset), there will be some error in the actual response.









(Eg. all resistances have the same tolerance)

Ideal code width $= q$

Actual code width $\neq q$

$= V_{K+1} - V_K$

$= 1.2q$

(due to different R tol. & comp. i/p offset)

Difference $= 0.2q$

Differential Non-Linearity (DNL)

$$= \frac{(V_{K+1} - V_K) - q}{q}$$

$$= \frac{0.2q}{q} = 0.2$$

For $DNL_K = -1 \Rightarrow \frac{V_{K+1} - V_K - q}{q} = -1$

⇓

Missing code. $\Rightarrow V_{K+1} = V_K \Rightarrow 0$ code width

n-bit ADC → (n-1) bit ADC ⇝ one-level missing

For $DNL_K > 1 \Rightarrow V_{K+1} - V_K - q > q$

$\Rightarrow V_{K+1} - V_K > 2q$

may lead to missing code (may or may not) → will lead to non-linearity

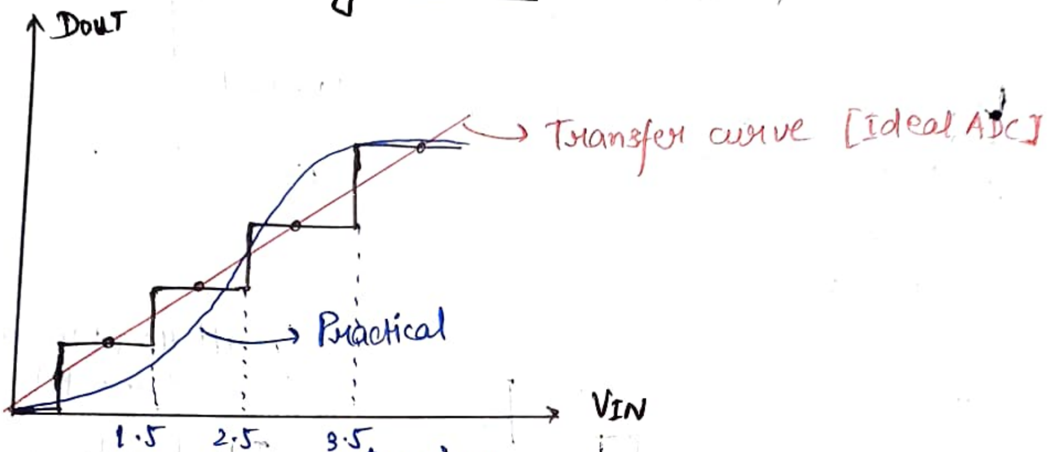
19-08-2025

DNL_{ADC} : worst-case (DNL_K)
↳ 8 for 3-bit ADC.

Eg. 0.2, −0.2, 0.3, −0.4 : DNL values

$DNL_{ADC} = \pm 0.4$.

Integral Non-Linearity (INL)



INL : Accumulation of previous DNLs.

$$INL_K = \frac{V_K - [Kq - q/2]}{q}$$

V_K : actual voltage at which transition happened

$Kq - q/2$: ~~actual~~ ideal voltage at which transition has happened

Eg. 0.5 1.7 3 3.5

1.5 2.5 → ideal transitions

$\underbrace{\quad}_{DNL = 0.2}$ $\underbrace{\quad}_{0.3}$

$INL = \frac{1.7 - 1.5}{1} = 0.2$ ($Kq - q/2$ = 1.5) $\frac{3 - 2.5}{1} = 0.5$ INL = 0

$$INL_{K+1} = \frac{V_{K+1} - [(K+1)q - q/2]}{q}$$

$$INL_{K+1} - INL_K = \frac{V_{K+1} - V_K - q}{q}$$

$$\therefore \boxed{INL_{K+1} - INL_K = DNL_K}.$$

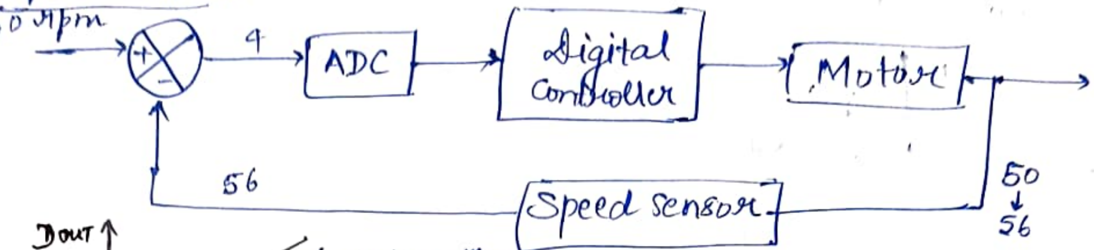
Datasheet gives:
- DNL
- INL

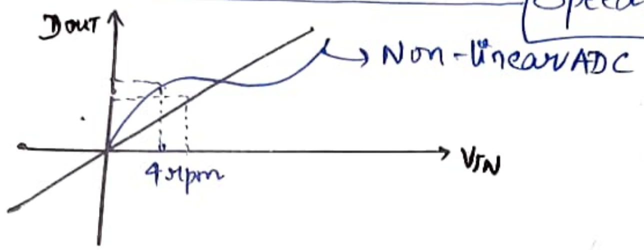
↳ which is more imp. depends on the end user

DNL → Define fine changes in ADC characteristics. (fine granularities) → useful for image processing

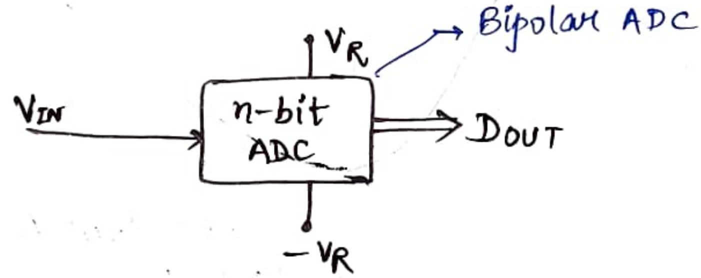
INL → global → Useful when ADC operates in a large span.

Eg. Motor-speed control :





Signal-to-Noise Ratio (SNR)

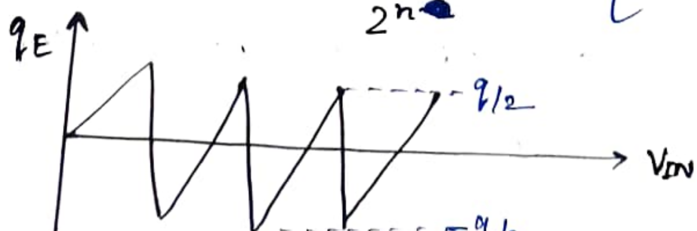


$$SNR = 10 \log \left(\frac{V_{sig}^2}{V_{noise}^2} \right).$$

Source of V_{noise}:

- Quantization error (q_E) ⇝ major source
- Noise from application.
- Random transients during switching.

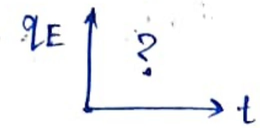
$$\max(q_E) = q/2$$
$$= \frac{V_R}{2^n} \qquad \left[\because q = \frac{V_R}{2^{n-1}} \right]$$

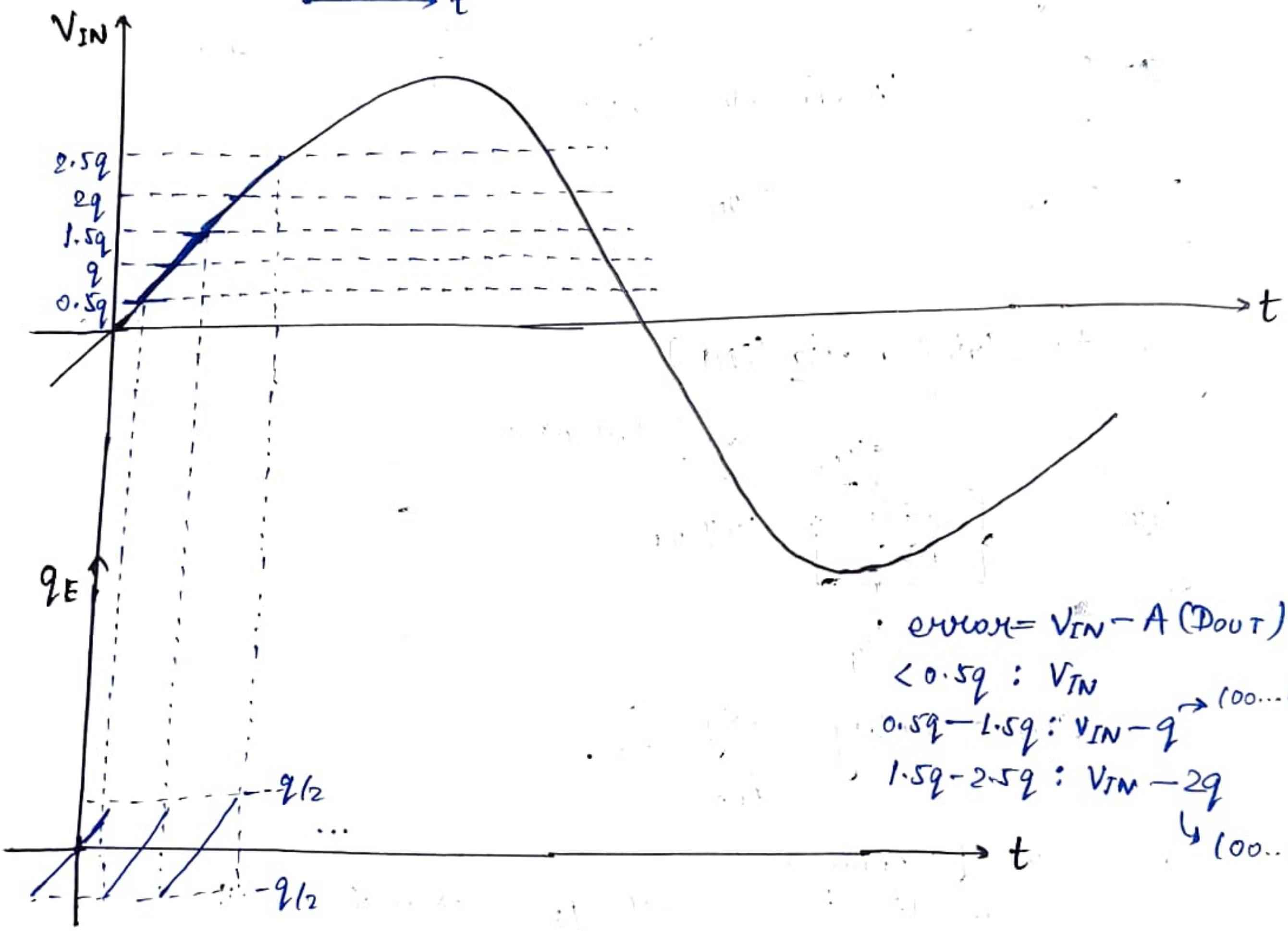


$V_{IN} = A \sin \omega t$

$V_{sig}^2 = \frac{A^2}{2}$ [= RMS value of V_{IN} ~ power of signal]

$V_{noise} \to$?





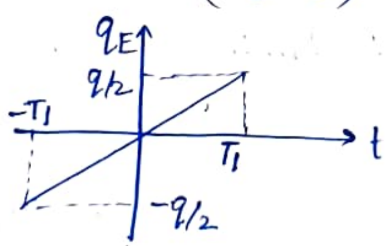
- error = $V_{IN} - A(D_{OUT})$

$< 0.5q$: V_{IN}

$0.5q - 1.5q$: $V_{IN} - q$ → (00...01)

$1.5q - 2.5q$: $V_{IN} - 2q$ ↳ (00..010)

↳ sawtooth $\left(-\frac{q}{2}, \frac{q}{2}\right)$



$q_E = s \cdot t$, s: slope → depends on V_{IN}

$s = \frac{q}{2T_1}$

$$V_{noise}^2 = \frac{1}{2T_1}\int_{-T_1}^{T_1} q_E^2\, dt$$

$$= \frac{1}{2T_1}\int_{-T_1}^{T_1}\left(\frac{q}{2T_1}\right)^2 t^2\, dt$$

$$= \frac{q^2}{8T_1^3}\left.\frac{t^3}{3}\right|_{-T_1}^{T_1}$$

$$= \frac{q^2}{24T_1^3}\,\cancel{\left(\frac{2T_1^3}{8}\right)}\; 2T_1^3$$

$$= \frac{q^2}{12} \quad \text{[Independent of } s\text{]}$$

22-08-2025

SNR

$$SNR = 10\log\left(\frac{V_{sig}^2}{V_{noise}^2}\right)$$

$$= 10\log\left(\frac{6A^2}{q^2}\right)$$

$$= 10\log 6 + 20\log(A/q)$$

$$= 10\log 6 + 20\log\left(\frac{A}{V_R}\cdot 2^{n-1}\right)$$

$$= 10\log 6 + 20\log\left(\frac{A}{V_R}\right) + 20(n-1)\log 2$$

$V_{sig}^2 = A^2/2$

$q = \frac{V_R}{2^{n-1}}$

$V_{noise}^2 = \frac{q^2}{12}$

$q_E = st$

$$\Rightarrow \boxed{SNR = 6.02n + 1.76 + 20\log\left(\frac{A}{V_R}\right)\ dB}$$

$A \in (0, V_R)$

$A = V_R \Rightarrow \boxed{SNR_{max} = 6.02n + 1.76\ dB}$

↳ Better to operate the ADC near the full-scale ⇒ May require pre-amplification of V_{IN}

For $V_R = 5\,V$

$A = 0.1\,V$

⇓

Poor SNR → Not wise choice

$n = 10 \Rightarrow SNR_{max} = 61.96\ dB \approx 62\ dB.$

$n = 12 \Rightarrow SNR_{max} = 74\ dB.$

$$SNR_{n+1} = SNR_n + 6.02$$

$A = V_R/2$, $n = 10$ $\Rightarrow$ $SNR = 6.02 \times 10 + 1.76 + 20\log(1/2)$

$= \cancel{58} \; 56$ dB.

[Equivalent to one less bit]

$DNL = -1 \Rightarrow$ n-bit $\longrightarrow$ (n−1) bit

SNR $\rightarrow$ 6.02dB $\downarrow$

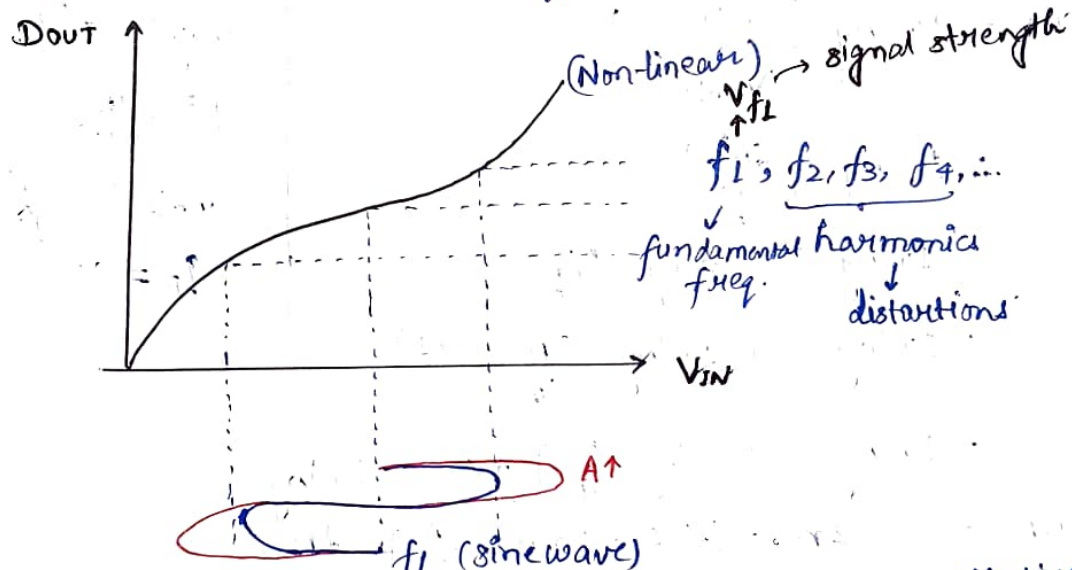
SNR $\xleftrightarrow[\text{relation}]{\text{some}}$ DNL

$V_{IN} = A \sin \omega t$

$\omega \uparrow$ $\rightarrow$ slew-rate effects $\Rightarrow$ SNR $\downarrow$

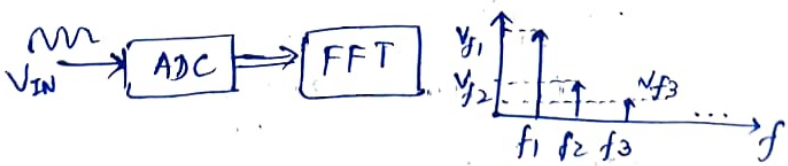
(reaches near BW of ADC opamp)

New parameter: THD $\xleftrightarrow[\text{to}]{\text{related}}$ INL



Total Harmonic Distortion (THD) $= 10 \log \left[\frac{D^2}{S^2} \right]$ (D: effective distortion, S: signal)

$$= 10 \log \left[\frac{V_{f2}^2 + V_{f3}^2 + V_{f4}^2 + \dots}{V_{f1}^2} \right]$$



- A $\uparrow \Rightarrow$ SNR $\uparrow$, $\Rightarrow V_{f1} \uparrow$ THD will become poorer (eg, −80dB → −60dB) (b/c we're using more portion of the non-linear curve)

$SNR = 10 \log \left[\frac{S^2}{N^2}\right]$; $THD = 10 \log \left[\frac{D^2}{S^2}\right]$

	Qs	harmonics
SNR:	✓	×
THD:	×	✓
SNDR:	✓	✓

SNDR ↓ Signal-to-Noise Distortion Ratio → should capture the effects of both Qs & harmonics.

$$SNDR = 10 \log \left[\frac{S^2}{N^2 + D^2}\right]$$

$$= 10 \log \left[\frac{1}{N^2/S^2 + D^2/S^2}\right]$$

$$= -10 \log \left[\frac{N^2}{S^2} + \frac{D^2}{S^2}\right]$$

$$= -10 \log \left[10^{-SNR/10} + 10^{THD/10}\right] \text{ dB}$$ → equal weightage to SNR & THD

Eg. $n = 10$

SNR = 62 dB

THD = −80 dB

$SNDR = -10 \log \left[10^{-6.2} + 10^{-8}\right]$

$= 61.93$ dB → SNR ~~detects~~ dictates SNDR.

Eg. $n = 12$

SNR = 74 dB

SNDR ≈ 68 dB → (6 dB ↓ compared to SNR)

↓

Effective no. of Bits = 11 (1-bit decrease)

Effective Number of Bits (ENoB):

$$ENoB = \frac{SNDR - 1.76}{6.02}$$ → Gives the overall performance of ADC.

Assignment: Look into the datasheet of any 2 commercially available ADC, and compare their parameters.

$$\boxed{\begin{aligned} SNR &= 6.02n + 1.76 \\ SNDR &= 6.02(ENoB) + 1.76 \end{aligned}}$$

26-08-2025

$SNR_{max} = 6.02n + 1.76$ dB
- ↳ Theoretical max value
- ↳ Only q_E considered
- ↳ Neglected application circuit noise and switching noise.
- ↳ Finite maxima, minima, discontinuities.

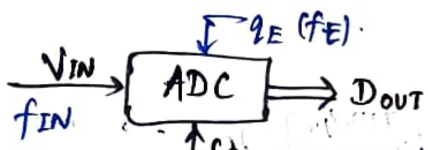
ADCs:

Flash ADC → Fastest.

Dual-slope ADC → Slow, good resolution

Sigma-Delta ADC → Faster, good resolution

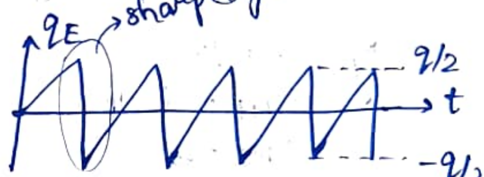
Sigma-Delta ADC:



$f_S = \frac{1}{T_S}$

$f_E \in [0, \infty)$

Gets undersampled
⇑
High-freq. component
⇑
sharp edge



$f_S \geq 2f_{IN}$: satisfy Nyquist criteria

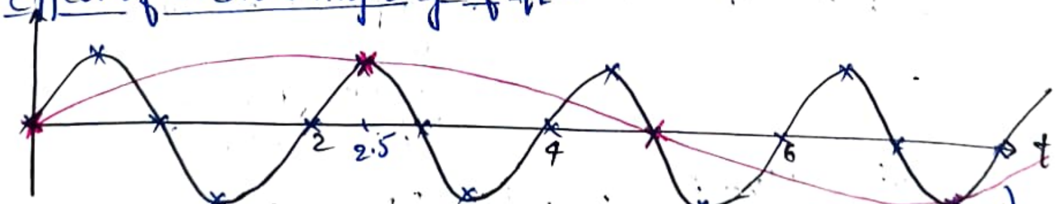
If $f_S = 2f_{IN} \Rightarrow SNR = 6.02n + 1.76$ (impulse)

Sub-Nyquist Sampling for q_E

With q_E,

$f_S < 2f_E$ as $0 < f_E < \infty$.

Effect of undersampling of q_E:

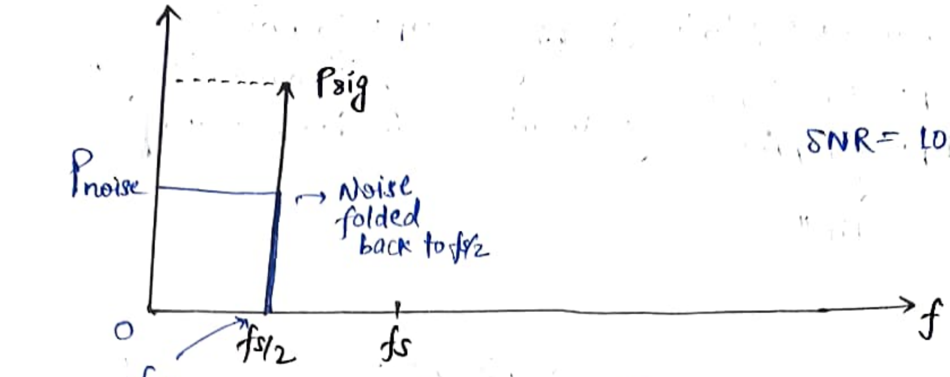


period of sampled signal = 10ms
⇓
100 Hz < $\boxed{f_S/2}$
= (500 − 400) Hz
Sub-nyquist sampling

$f_{IN} = 500$ HZ

$f_S = 500$ Hz
$T_S = 0.5$ ms
} same freq. as i/p wave

$f_S = 400$ HZ
$T_S = 2.5$ ms
}



$$SNR = 10 \log \left(\frac{P_{sig}}{P_{noise}} \right)$$

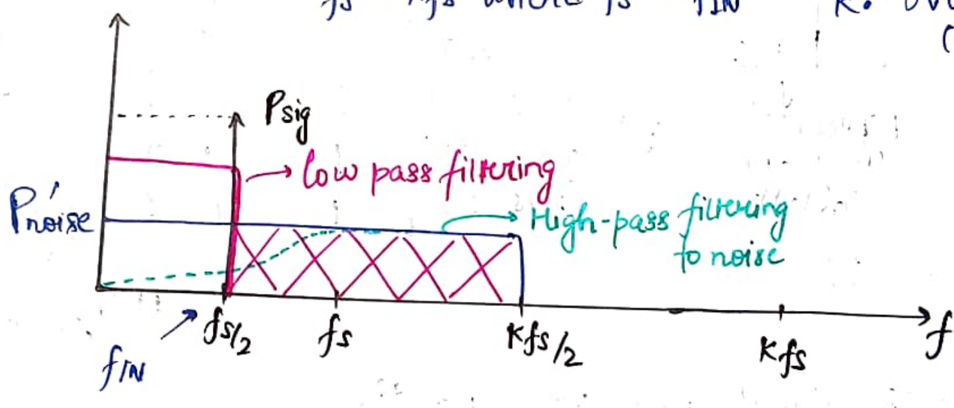
Oversampling: (For improvement)

$V_{IN} \longrightarrow$ ADC $\Longrightarrow D_{OUT}$

f_{IN}, f_E, q_E

Kf_s (sampling at a higher freq.)

$f_s \rightarrow Kf_s$ where $f_s = 2f_{IN}$ K: oversampling factor ($K > 1$)



$$P_{noise}' = \frac{P_{noise}}{K}$$

$$SNR' = 10 \log \left(\frac{P_{sig} \cdot K}{P_{noise}} \right)$$

$$= SNR + 10 \log K$$

Improvement in SNR (ΔSNR) $= 10 \log K$

$K = 4$, $\Delta SNR = 10 \log 4 = 6.02$ dB

1-bit-ADC $\xrightarrow{K=4}$ 2-bit ADC

1-bit ADC $\xrightarrow[m=15]{K=4^m}$ 16-bit ADC

$\boxed{K = 4^m}$, $\Delta SNR = 10 \log 4^m = 6.02\, m$ dB, for m-bit ↑

$\therefore K = 4^{15} = \underbrace{2^{30}}$

↓

~~exp~~ extremely large oversampling factor

↓

Not practical

Further improvement: Noise Shaping ⇒ HPF to noise
⇓
very low noise floor in desired region.

LPF to Signal, HPF to Noise } Circuit
↓
SNR''

29-08-2025

2-bit ADC

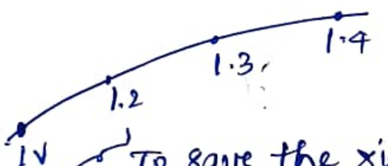
X	D1	D0	
(-1, -0.5)	0	0	1/4
(-0.5, 0)	0	1	2/4
(0, 0.5)	1	0	3/4
(0.5, 1)	1	1	4/4

↓ Represented using no. (coded)

3-bit ADC

X	D2	D1	D0	
(-1, -0.75)	0	0	0	1/8
(-0.75, -0.5)	0	0	1	2/8
(-0.5, -0.25)	0	1	0	3/8
(-0.25, 0)	0	1	1	4/8
(0, 0.25)	1	0	0	5/8 → ✓ → 1-bit error
3/8 ∈ (0.25, 0.5)	$\boxed{1}$	$\boxed{0}$	$\boxed{1}$	6/8 → ✓
(0.5, 0.75)	1	1	0	7/8
(0.75, 1)	1	1	1	8/8

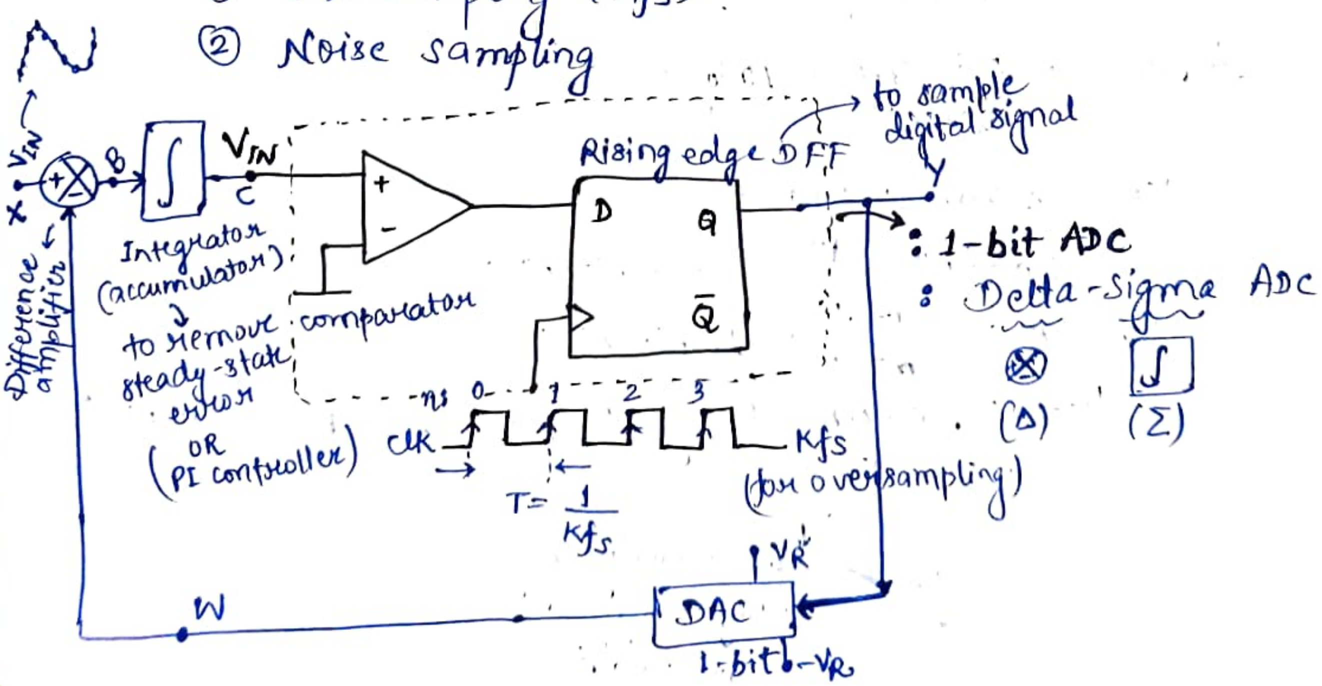
↓ mapping no./fraction



To save the signal space, we send the difference between values rather than individual samples.

Sigma-Delta ADC:

① Oversampling (Kfs)
② Noise sampling



$X \in (-V_R, V_R)$ → ref. voltages of ADC (also for DAC)

$V_R = 1$
$X \in (-1, 1)$

n: sampling instance

stored in capacitors in $\boxed{\int}$

$C_n = C_{n-1} + B_n$

Y_n = logic-high (1); $C_n \geq 0$ → $C > 0$ may not be the case

= logic-low (0); $C_n < 0$

$W_n = V_R$; $Y_n = 1$

$= -V_R$; $Y_n = 0$

$B_n = X_n - W_{n-1}$

$= X - W_{n-1}$; X should be constant for atleast few clock cycles.

Reset : $C_0 = 0$
$W_0 = 0$.

$B_1 = X - W_0 = X$

$C_1 = C_0 + B_1 = X$

clock cycles - $C_2 = C_1 + B_2$

n	X	B	C	Y	W(V)
1	3/8	3/8	3/8	1	1
2	3/8	-5/8	-2/8	0	-1
3	3/8	11/8	9/8	1	1
4	3/8	-5/8	4/8	1	1

(rows 1–4: 2-bit ADC; rows 1–8: 3-bit ADC)

i/p should be constant for 4 clk cycles (for 2-bit ADC) → can be assumed to be constant for a small interval of time

n	X	B	C	Y	W(V)
5	3/8	-5/8	-1/8	0	-1
6	3/8	11/8	10/8	1	1
7	3/8	-5/8	5/8	1	1
8	3/8	-5/8	0	~~1~~ 0	1

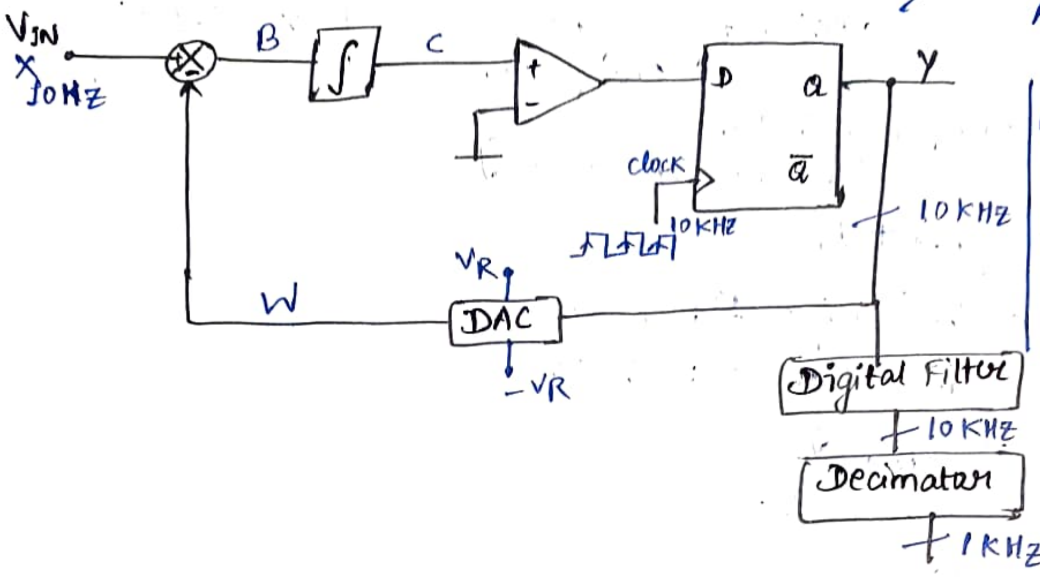
↓ for 3-bit ADC

0 → due to offset voltage of comparator

$\Sigma B = 0$ ⇒ s.s. error → 0 ⇒ $X = \overline{W}$

6/8 → Average

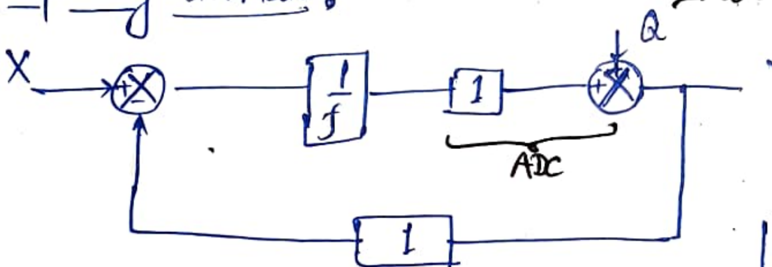
→ For n-bit Σ-Δ ADC, i/p should be constant for 2^n clk cycles.



→ 1st order Σ-Δ ADC

ADC
↳ Does not provide gain.
↳ only change the domain (from Analog to Digital)

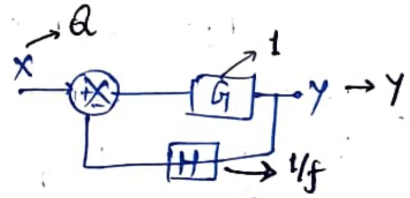
In frequency domain:



→ Quantization error (from comparator)

$$Y = \frac{X\,\frac{1}{f}}{1+\frac{1}{f}} + \frac{Q\cdot 1}{1+\frac{1}{f}}$$

$$\therefore Y = \underbrace{\frac{X}{f+1}}_{\text{LPF to X}} + \underbrace{\frac{Qf}{f+1}}_{\text{HPF to noise}} \quad : \text{Noise shaping}$$

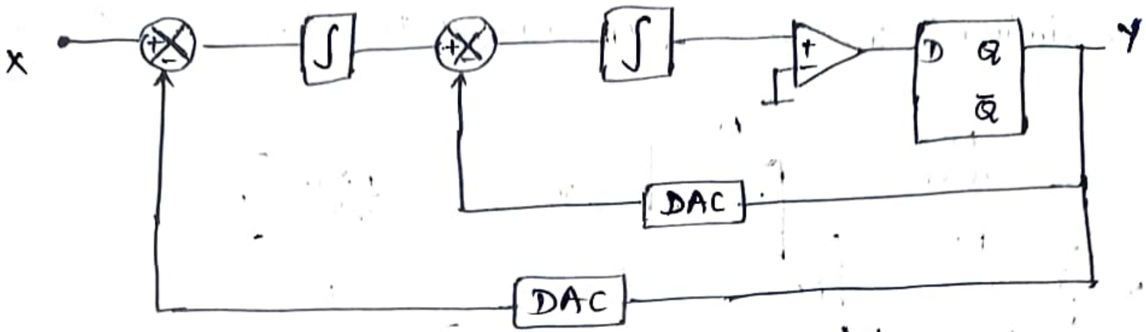


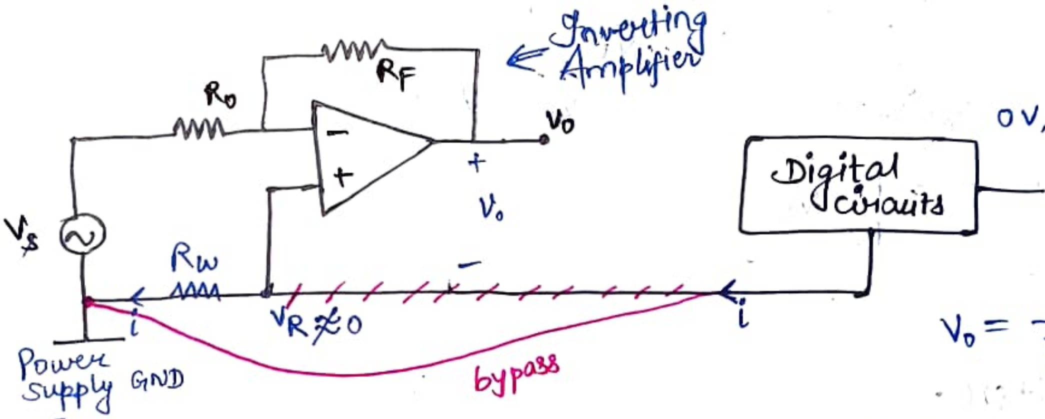
f	Y	
0	X	→ no noise
∞	Q	

: shaping noise towards high frequency

Oversampling factor, K	Improvement in SNR without noise shaping	Improvement in SNR with noise shaping
4	6 dB	
2		9 dB

2nd Order ADC (Σ-Δ):

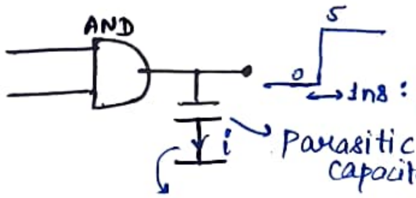




0V, 5V → logic 0,1

$$V_o = -\frac{R_F}{R_o} V_s$$

R_W: lead wire resistance



1ns : propagation delay

Parasitic capacitance : $C \simeq 50pF$

Rapid current

$$i = C\frac{\Delta V}{\Delta t}$$

$$= 50\times10^{-12} \times \frac{5}{10^{-9}}$$

$= 250mA$ → Return current from digital ckt. ⇒ Digital ckt. affecting analog o/p

$R_W = 0.5\Omega$

$V_R = iR_W = 125\,mV$ → Return voltage → Large due to large i.

↓

solution: Bypass the discharge current directly to ground.

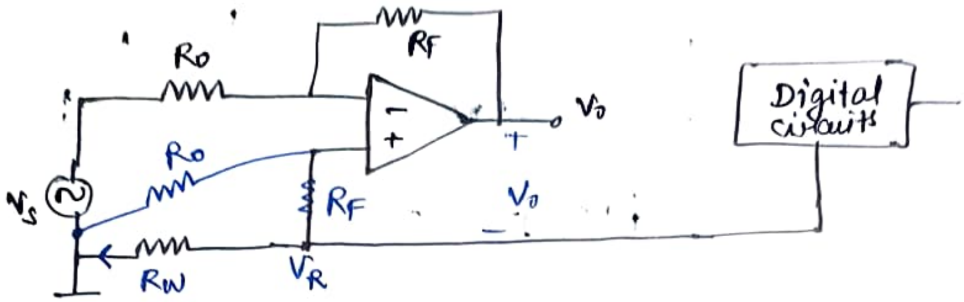
Suppose, if the bypass connection is not possible.
(no support of separate analog & digital GND)

$$V_o = \underbrace{-\frac{R_F}{R_o} V_s + V_R\left[1+\frac{R_F}{R_o}\right]}_{\text{superposition}} - V_R$$

[Convert to difference amplifier]

$$\Rightarrow V_o = -\frac{R_F}{R_o}\left[V_s - V_R\right] \rightsquigarrow \text{depends on } V_R$$

To eliminate the effect of V_R, add extra two resistors, R_o, R_F.



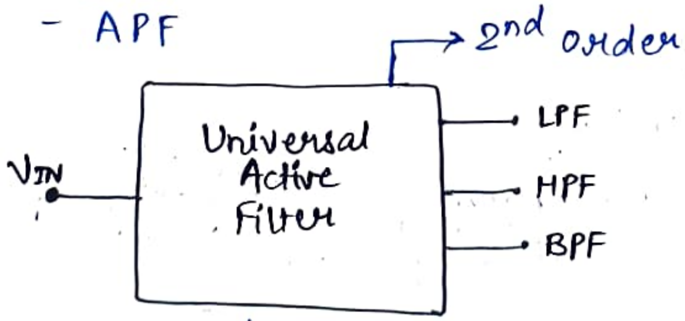
$R_W \ll R_F$

$$V_o = \underbrace{-\frac{R_F}{R_o}V_s}_{\text{Using } V_R = V_+ = 0} + \underbrace{\underbrace{\cancel{\frac{R_o}{R_o+R_F}V_R}}_{V_+}\cancel{\left(1+\frac{R_F}{R_o}\right)} - \cancel{V_R}}_{\text{using } V_s = 0}$$

$$\therefore \boxed{V_o = -\frac{R_F}{R_o}V_s}$$

Filters

- LPF
- HPF
- BPF
- BRF
- APF



↳ Ensure that we use min^m no. of opamps.

01-09-2025

Universal Active Filters (UAF)

LPF: $H(s) = \dfrac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

ω_n: natural frequency
ξ: damping ratio
K: ~~band~~ pass band gain
ω_0: center frequency

HPF: $H(s) = \dfrac{Ks^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

BPF: $H(s) = \dfrac{K(2\xi\omega_0 s)}{s^2 + 2\xi\omega_0 s + \omega_0^2}$ → Usually ω_0 is used instead of ω_n.

Quality Factor (Q): $Q = 1/2\xi$.

$\Rightarrow$ BPF: $H(s) = \dfrac{K\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$

BRF: $H(s) = \dfrac{K[s^2 + \omega_0^2]}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$

APF: $H(s) = \dfrac{K\left[s^2 - \frac{\omega_0}{Q}s + \omega_0^2\right]}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$

$|H(j\omega)| = K$

$\angle H(j\omega) \longrightarrow$ finite

LPF:

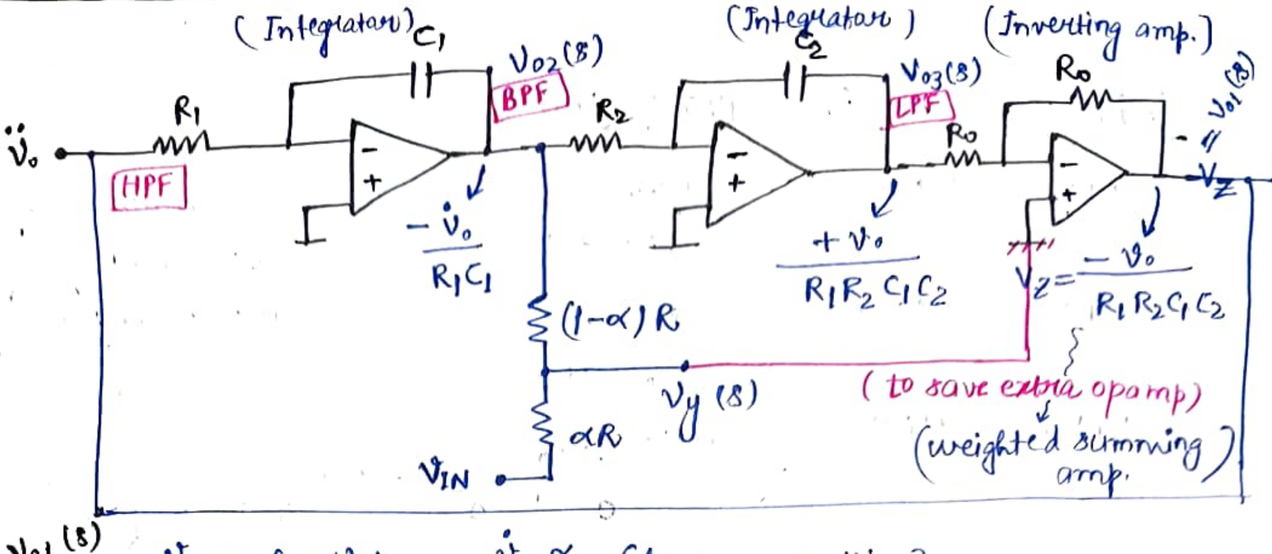
$H(s) = \dfrac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \dfrac{V_0(s)}{V_{IN}(s)}$

$\Rightarrow s^2 V_0(s) + 2\xi\omega_n s V_0(s) + \omega_n^2 V_0(s) = K\omega_n^2 V_{IN}(s)$

ILT $\Rightarrow$ $\ddot{v}_0 + 2\xi\omega_n \dot{v}_0 + \omega_n^2 v_0 = K\omega_n^2 v_{IN}$

$\ddot{v}_0 = -2\xi\omega_n \dot{v} + K\omega_n^2 v_{IN} - \omega_n^2 v_0$

↳ Build a circuit that obey this governing equation.



$$v_y = (1-\alpha)v_{IN} - \frac{\dot{v}_o \alpha}{R_1C_1}$$ (By superposition of voltage divider)

$$V_Z = -\frac{v_o}{R_1R_2C_1C_2} + 2v_y$$

$$\Rightarrow V_Z = -\frac{v_o}{R_1R_2C_1C_2} + 2(1-\alpha)v_{IN} - \frac{2\alpha}{R_1C_1}\dot{v}_o = \ddot{v}_o$$

[connect $\ddot{v}_o$ (i/p) to v_Z (o/p)]

$$\Rightarrow \ddot{v}_o + \frac{2\alpha}{R_1C_1}\dot{v}_o + \frac{v_o}{R_1C_1R_2C_2} = 2(1-\alpha)v_{IN}$$ [Mimics the differential eqn]

$$V_{o2}(s) = -\frac{1}{sR_1C_1}V_{o1}(s)$$

$$V_{o3}(s) = -\frac{1}{sR_2C_2}V_{o2}(s)$$

$$= \frac{1}{s^2R_1R_2C_1C_2}V_{o1}(s)$$

$$V_y(s) = (1-\alpha)V_{IN}(s) + \alpha V_{o2}(s)$$

$$V_{o1}(s)\,[= V_Z(s)] = -V_{o3}(s) + 2V_y(s)$$

$$\Rightarrow V_{o1}(s) = \frac{V_{o1}(s)}{s^2R_1R_2C_1C_2} + 2(1-\alpha)V_{IN}(s) - \frac{2\alpha V_{o1}(s)}{sR_1C_1}$$

$$\Rightarrow V_{o1}(s)\left[1 + \frac{2\alpha}{sR_1C_1} - \frac{1}{s^2R_1R_2C_1C_2}\right] = 2(1-\alpha)V_{IN}(s)$$

$$\Rightarrow V_{o1}(s)\left[\frac{s^2R_1R_2C_1C_2 + 2\alpha R_2C_2 s - 1}{s^2R_1R_2C_1C_2}\right] = 2(1-\alpha)V_{IN}(s)$$

$$\Rightarrow \frac{V_{o1}}{V_{IN}}(s) = \frac{2(1-\alpha)s^2}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1R_2C_1C_2}} \quad : \text{HPF}$$

Comparing with $\frac{Ks^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$,

$$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}},$$

$$2\xi\omega_n = \frac{2\alpha}{R_1C_1} \Rightarrow \xi = \alpha\sqrt{\frac{R_2C_2}{R_1C_1}},$$

$$K = 2(1-\alpha).$$

Also,

$$\frac{V_{o2}}{V_{IN}}(s) = \frac{-2(1-\alpha)\frac{s}{R_1C_1}}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1C_1R_2C_2}} \quad : \text{BPF}$$

$$\frac{V_{o3}}{V_{IN}}(s) = \frac{2(1-\alpha)\frac{1}{R_1R_2C_1C_2}}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1R_2C_1C_2}} \quad : \text{LPF}$$

BPF: comparing with $\frac{K\left[\frac{\omega_0}{Q}\right]s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$,

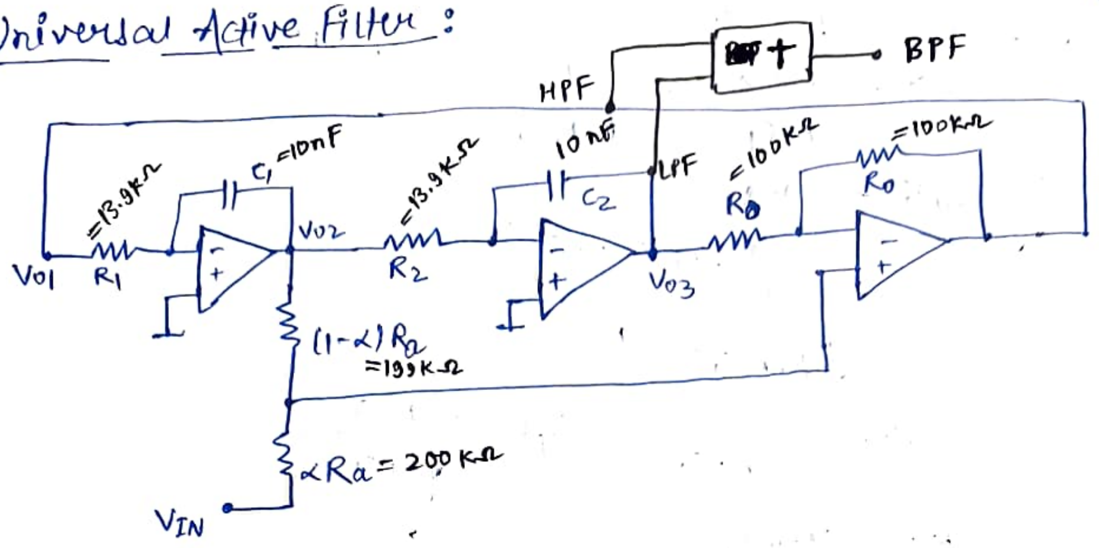
$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}},$$

$$\frac{\omega_0}{Q} = \frac{2\alpha}{R_1C_1} \Rightarrow Q = \frac{1}{2\alpha}\sqrt{\frac{R_1C_1}{R_2C_2}},$$

$$K\left(\frac{\omega_0}{Q}\right) = K\left(\frac{2\alpha}{R_1C_1}\right) = 2(1-\alpha)\frac{1}{R_1C_1}$$

$$\Rightarrow K = \frac{1-\alpha}{\alpha} = \frac{1}{\alpha} - 1$$

Universal Active Filter :



Eg. Design a BPF with center frequency of 1 KHz and Bandwidth of 10 Hz.

$$\frac{V_{o2}(s)}{V_{IN}} = \frac{K\, \omega_0/Q\; s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{2(1-\alpha)\, s/(R_1C_1)}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} \;;\quad Q = \frac{1}{2\alpha}\sqrt{\frac{R_1C_1}{R_2C_2}} = 100.$$

$f_0 = 1000$ Hz

$BW = 10\text{ Hz} = f_0/Q$

$Q = 100$

$$f_0 = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = 1000$$

$$= \frac{1}{2\pi RC} = 1000$$

Take $C = 10nF$;

$\Rightarrow R = 13.9\, k\Omega$

Take $R_1 = R_2 = R$

$C_1 = C_2 = C$

$\hookrightarrow \frac{1}{2\alpha} = 100 \Rightarrow \alpha = \frac{1}{200}$

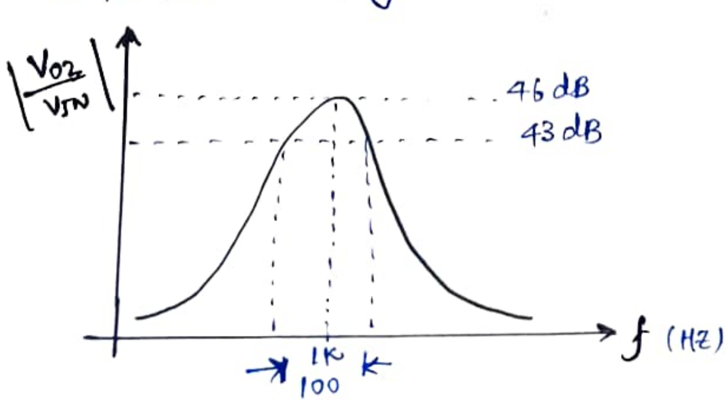
Take $R_a = 200\, k\Omega$

$\Rightarrow \alpha R_a = 1k\Omega$

$(1-\alpha) R_a = 199\, k\Omega$.

$K = \frac{1}{\alpha} - 1$

$\Rightarrow K = 199 \longrightarrow 20 \log 199 \approx 46\,dB$



Design for $K = 40\,dB \Rightarrow K = 100$.

$$K = \frac{1}{\alpha} - 1 = 100$$

$$\Rightarrow \alpha = \frac{1}{101}$$

$\downarrow$

Substitute in $Q = \frac{1}{2\alpha}\sqrt{\frac{R_1 C_1}{R_2 C_2}}$

↳ May not get $R_1 = R_2 = R_0$, $C_1 = C_2 = C$

IC: UAF42
↳ Realizes this topology

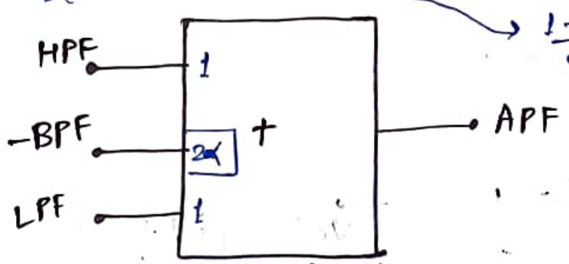
BRF: $$H(s) = \frac{K\left[s^2 + \omega_0^2\right]}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} = HPF + LPF$$

APF: $$H(s) = \frac{K\left[s^2 - \frac{\omega_0}{Q}s + \omega_0^2\right]}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$= HPF - BPF + LPF$$

$2(1-\alpha)$ $2(1-\alpha)$: gain

$\frac{1-\alpha}{\alpha}$ → should have same gain as HPF/LPF



Triple input summing amplifier

$$\frac{1-\alpha}{\alpha} \times 2\alpha = 2(1-\alpha)$$

$HPF + BPF + LPF = 1$

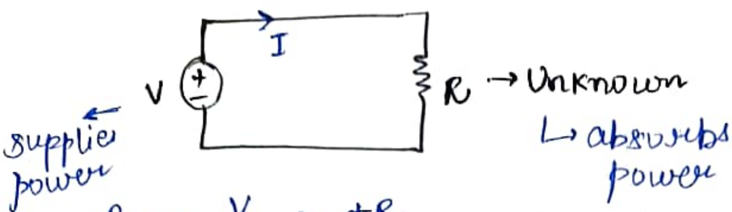
$\Rightarrow HPF + LPF = 1 - BPF$

$\therefore APF = 1 - 2BPF$

$$\Rightarrow \frac{V_{APF}}{V_{IN}} = 1 - 2\frac{V_{BPF}}{V_{IN}}$$

$\Rightarrow V_{APF} = V_{IN} - 2V_{BPF}$ → 2 i/p summing amp. required

Impedance Transformation Circuits

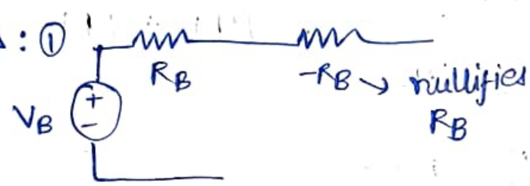


$$R_{eq} = \frac{V}{I} = +R$$

Negative Resistance : Delivers power ; source absorbs power.

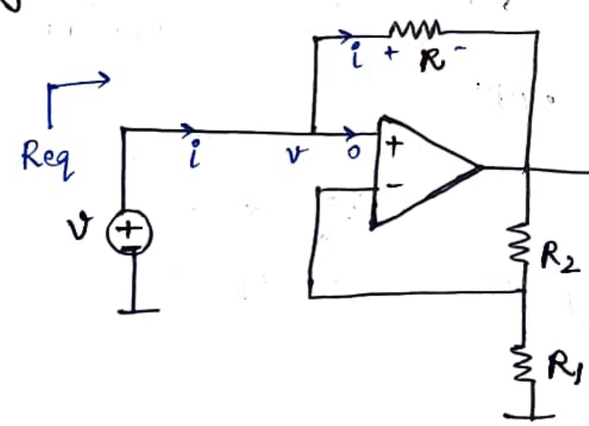
$$R_{eq} = -R$$

Applications : ①



② To change the damping ratio.

Negative Resistance Generator circuit :



$$v_o = v\left[1 + \frac{R_2}{R_1}\right]$$

$$iR = v - v_o$$

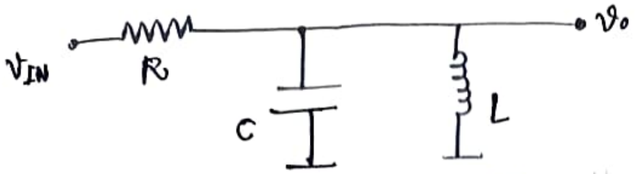
$$iR = -\frac{vR_2}{R_1} \Rightarrow \frac{v}{i} = -R\frac{R_1}{R_2}$$

Equivalent impedance from source,

$$\boxed{R_{eq} = \frac{v}{i} = -R\frac{R_1}{R_2}}$$ → Negative resistance
↳ Delivers power to the source.

↳ Negative Resistance circuit / Impedance Transformation Circuit.

$L \longrightarrow C$



At $\omega = \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow$ Parallel resonance
↳ very high resistance

↳ $V_0 = V_{IN}$

$\omega \neq \omega_0 \Rightarrow V_0 = 0$.

↳ cannot be ~~minitual~~ minitiatured.

↳ Mimic L using C, opamp, resistor.
(differential eqn of inductor)

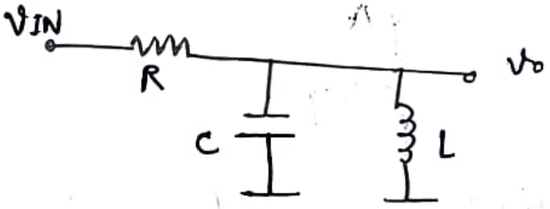
08-09-2025

~~Exdu~~

Impedance Transformation Circuits

→ Negative Resistance Generator
→ Inductance Simulator

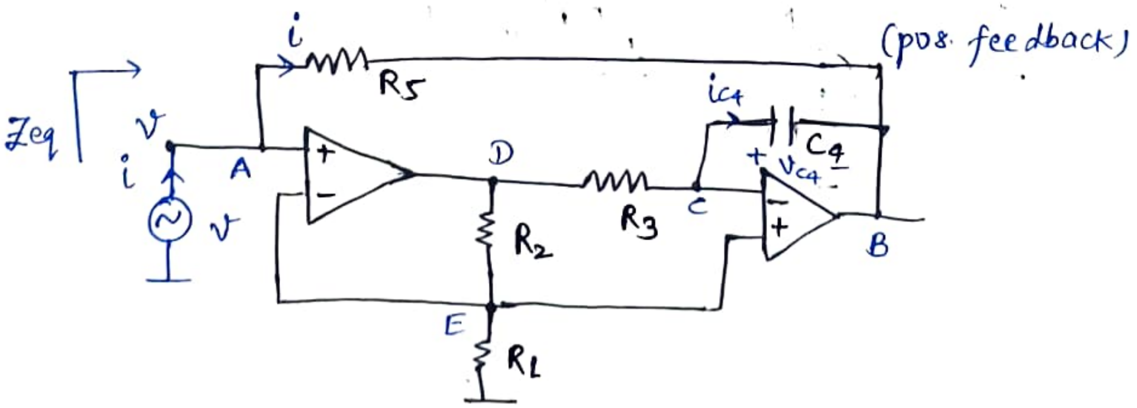
→ GIC



$$\frac{V_0}{V_{IN}} = \frac{s\omega_0/Q}{s^2 + \frac{s\omega_0}{Q} + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C}{L}}$$



$$v_A = v_E = v_C = v$$

$$v_D = v\left[1 + \frac{R_2}{R_1}\right]$$

$$v_{DC} = \frac{v R_2}{R_1} \quad (= v_{DE} = v_D - v_E)$$

$$i_{C4} = \frac{v R_2 / R_1}{R_3} = \frac{v R_2}{R_1 R_3}$$

$$v_{C4} = \frac{1}{C_4} \int i_{c4}\, dt$$

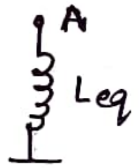
$$= \frac{1}{C_4} \frac{R_2}{R_1 R_3} \int v\, dt$$

$$i R_5 = v_C - v_B = v_{C4}$$

$$= \frac{R_2}{R_1 R_3 C_4} \int v\, dt$$

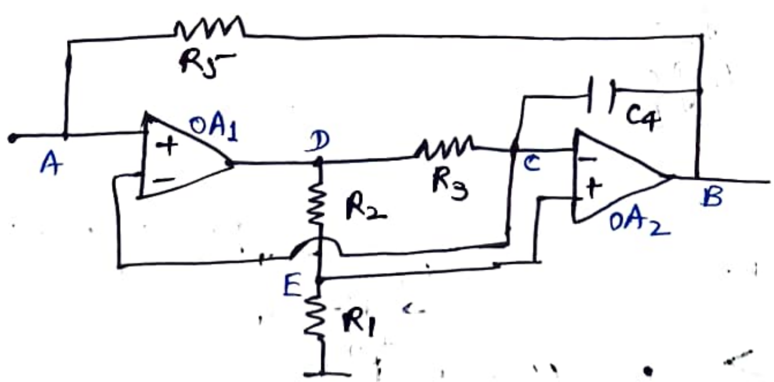
$$\Rightarrow v = \left(\frac{R_1 R_3 R_5 C_4}{R_2}\right) \frac{di}{dt}$$

$$\therefore v = L_{eq} \frac{di}{dt}$$

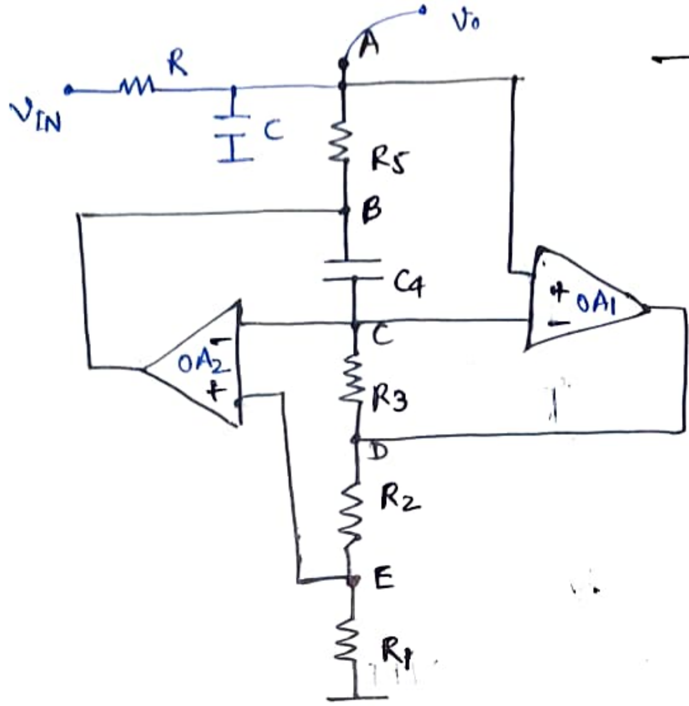


↳ can be minitiatured (opamp, capacitor, resistor).
↳ Mimics grounded inductance.

Modified circuit :



$$v_A = v_E = v_C = v$$



→ Easier way of Representation
↳ Found in books, research papers.

⇒ 

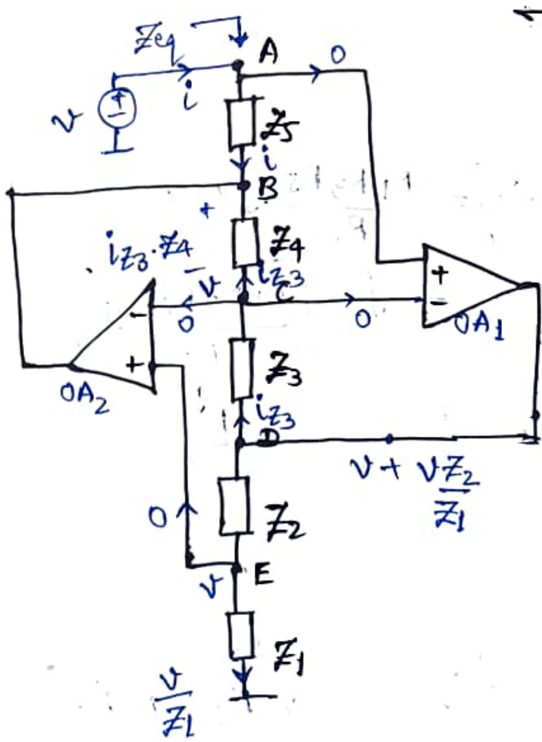
→ Not for discrete elements.

→ For VLSI minitiaturation

$$\omega_0 = \frac{1}{\sqrt{L_{eq} C}}$$

$$Q = R\sqrt{\frac{C}{L_{eq}}}$$

→ Generalized Impedance Converter (GIC)



$$i_{Z_3} = \frac{V_D - V_C}{Z_3} = \frac{v Z_2}{Z_1 Z_3}$$

$$V_B = v - i_{Z_3} \cdot Z_4$$

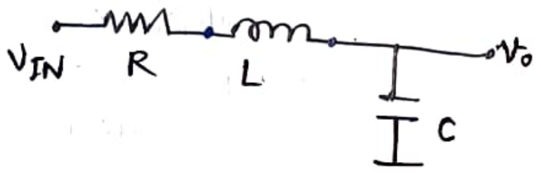
$$= v - v\frac{Z_2 Z_4}{Z_1 Z_3}$$

$$i = \frac{V_A - V_B}{Z_5}$$

$$\frac{V}{i} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

$$Z_{eq} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

→



→ LPF

Mimic the inductor
to create inductor free LPF, using GIC.

09-09-2025

Inductance Simulator:

GIC with

$Z_5 = R_5$

$Z_4 = 1/j\omega C_4$

$Z_3 = R_3$

$Z_2 = R_2$

$Z_1 = R_1$

$$\Rightarrow Z_{eq} = \frac{R_1 R_3 R_5}{R_2} j\omega C_4$$

$$= j\omega \underbrace{\left[\frac{R_1 R_3 R_5 C_4}{R_2}\right]}_{L_{eq}}$$

With

$Z_5 = 1/j\omega C_5$

$Z_4 = R_4$

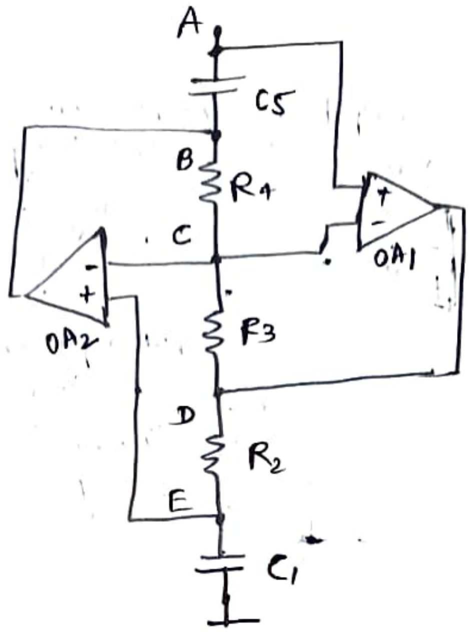
$Z_3 = R_3$

$Z_2 = R_2$

$Z_1 = 1/j\omega C_1$

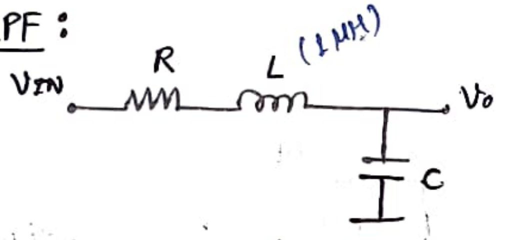
$$\Rightarrow Z_{eq} = -\frac{R_3}{\omega^2 C_1 C_5 R_2 R_4}$$

↳ Frequency Dependent Negative Resistance (FDNR)



Application: Double integrator $\left(\frac{1}{j\omega}\right)^2$

LPF:



$$\omega_n = \frac{1}{\sqrt{LC}}$$
$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

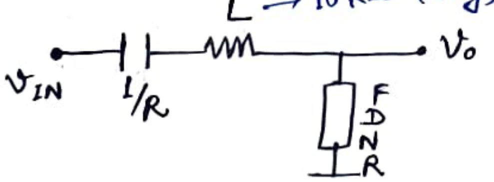
$$\frac{V_o}{V_{IN}} = \frac{\frac{1}{j\omega C}\frac{1}{F}}{\frac{j\omega L}{F} + \frac{R}{F} + \frac{1}{j\omega C}\frac{1}{F}}$$; choose $F = j\omega$.

$$= \frac{-\frac{1}{\omega^2 C}}{L + \frac{R}{j\omega\frac{1}{R}} - \frac{1}{\omega^2 C}}$$ → can be modelled using FDNR
↳ Replace capacitor by FDNR

Replace by a resistor

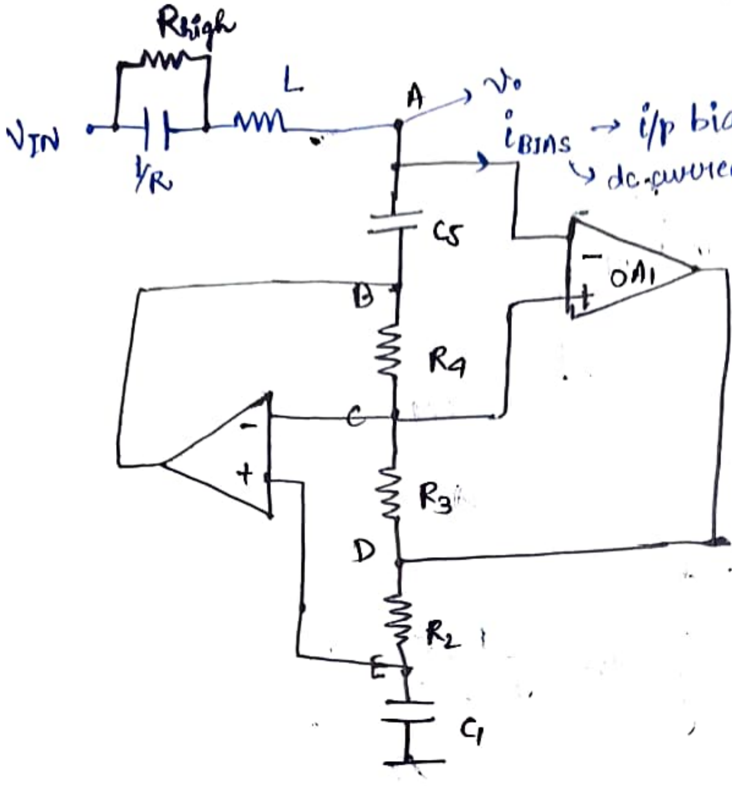
Replace by a capacitor

L → 10 kΩ (say) → choose a/c to ω_n, ξ requirement



[Previous values of R, L, C may not be satisfied.]

← Modelling floating inductor.

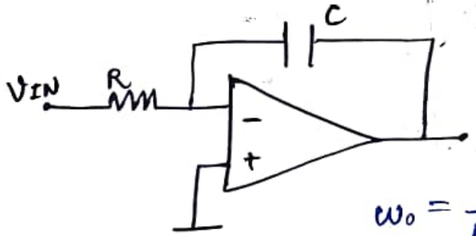


i_{BIAS} → i/p bias current should get a resistive path but is facing C_5 and 1/R capacitances
↳ dc current

⇓

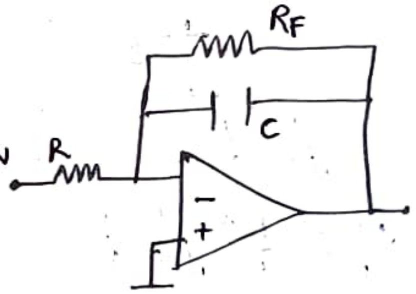
Put a parallel high value resistor across 1/R to provide a resistive path.

Miniaturize the circuits :



: Integrator/ HPF

$$\omega_0 = \frac{1}{RC}$$



: Active LPF

$$V_o = -\frac{1}{RC}\int V_{IN}\,dt$$

$$\frac{V_o}{V_{IN}} = -\frac{1}{j\omega RC} = -\frac{1}{j\omega/\omega_0}$$

$$\omega = \omega_0 \Rightarrow \left|\frac{V_o}{V_{IN}}\right| = 1$$

ω_0 : Unity gain frequency

$\omega_0 = 20\pi$ rad/s $= \frac{1}{RC}$

For $C = 1nF$,

$R = 15.9\ M\Omega$

↳ Cannot be implemented using MOS technology (can be max ~10kΩ)

$$\frac{V_o}{V_{IN}} = \frac{-R_F/R}{1 + j\omega R_F C}$$

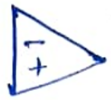
$G = -R_F/R$

$\omega_c = \frac{1}{R_F C}$: Corner frequency

$= 20\pi$ rad/sec [10 Hz]

For $C = 1nF$,

$R = 15.9\ M\Omega$

 $\Rightarrow$ MOS

 $\Rightarrow$ MOS capacitor

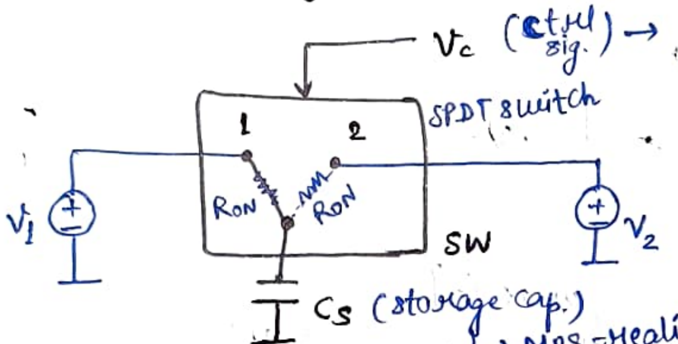
 $\Rightarrow$ MOS in triode ($< 10\,k\Omega$)

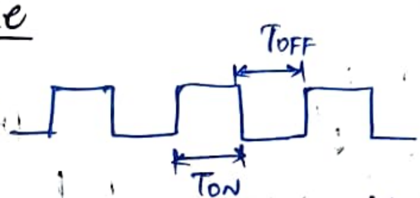
 $\Rightarrow$ MOS

Switched capacitor exhibits resistive behaviour, and can be used to implement high value resistor.

12-09-2025

Switched Capacitor Technique





Time period of ctrl sig.,

$$T_c = T_{ON} + T_{OFF}$$

$$f_c = \frac{1}{T_c}$$

$t \in T_{ON}$; $SW \to 1$, $Q = C_S V_1$

$t \in T_{OFF}$; $SW \to 2$, $Q = C_S V_2$

Differential charge :

$$\Delta Q = C_S(V_1 - V_2) \quad \text{every } T_c \text{ seconds.}$$

Average current flowing to the V_2 side,

$$i = \frac{\Delta Q}{T_c} = \frac{C_S(V_1 - V_2)}{T_c}$$

$$\frac{V_1 - V_2}{i} = \frac{T_c}{C_S} = \boxed{\frac{1}{f_c C_S} = R_{eq}}$$: Equ. resistance b/w pts. 1 & 2

V_1 and V_2 should be independent voltage sources (their voltages should not change due to capacitor charge).

V_1 and V_2 $\longrightarrow$ constant

Time constant associated with charging :

$$\tau = R_{ON} C_S$$

$T_{ON} \geq 5\tau$; $T_{OFF} \geq 5\tau$

$\therefore T_{c,min} = 10\tau = 10 R_{ON} C_S$

$R_{ON} = 100\,\Omega$
$C_S = 10\,pF$

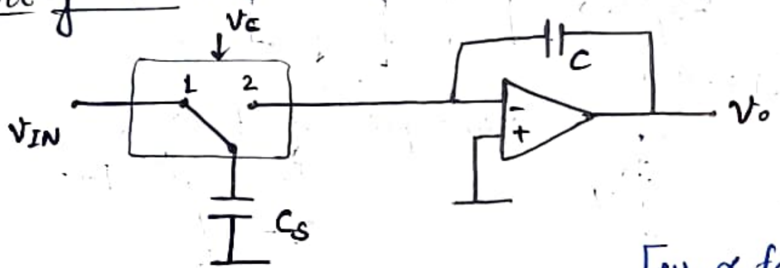
$\Rightarrow T_{c,min} = 10\,ns$

↳ I/p should remain constant for atleast 10 ns.

$$R_{eq} = \frac{T_c}{C_S} = \frac{10\,ns}{10\,pF} = 1\,k\Omega$$

If $T_c = 10\,\mu s$
$C_S = 10\,pF$
$\Rightarrow R_{eq} = 1\,M\Omega$. → A large resistance can be realised.

Integrator:



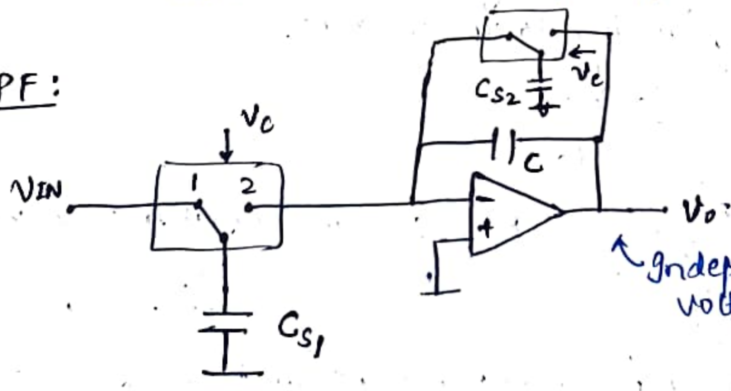
$[\omega_o \propto f_c]$

$$\omega_o = \frac{1}{R_{eq} C} = f_c\left(\frac{C_S}{C}\right)$$ → depends on clock freq. → stable & flexibly

C_s, C : MOS cap. → Have temperature coefficient.

$$\frac{C_S\,\cancel{(1+\alpha T)}}{C\,\cancel{(1+\alpha T)}}$$ → No effect of temp. coeff.

LPF:



Independent voltage source
→ doesn't depend on cap. current

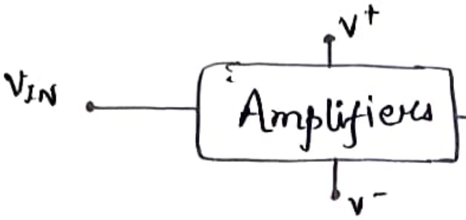
$$R_{eq1} = \frac{1}{f_c C_{S1}}$$

$$R_{eq2} = \frac{1}{f_c C_{S2}}$$

Gain : $G = -\dfrac{R_{eq2}}{R_{eq1}} = -\dfrac{C_{S1}}{C_{S2}}$ → Accurate & Stable

$$\omega_c = \frac{1}{R_{eq_2} C} = f_c \left(\frac{C_{S2}}{C} \right)$$

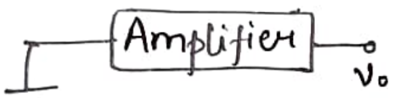
NOISE

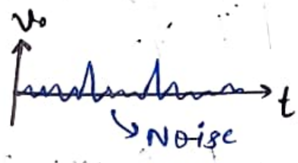


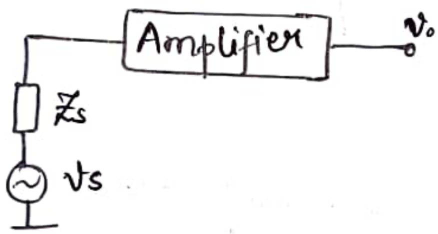
$V_o = A\, V_{IN} + \underset{\text{Suppose suppressed}}{\cancel{\text{External Disturbances}}} + \text{Internal Disturbances}$

External Disturbances ↓ Mitigated by: Shielding, guarding, PCB layouting

Internal Disturbances ↓ Due to internal parameters ⇓ Noise







→ Greater noise

Noise → affects the resolution of the device.

Sensors, Amplifiers, ADC } Each have their own resolution ⇒ Design amplifiers such that the overall resolution is dominated by sensors or ADC.

$\boxed{15\text{-}09\text{-}2025}$

Noise

- → Fundamentals
- → Types of electronic noise
- → Noise Bandwidth (B)
- → Noise analysis of passive circuits
- → Noise model of opamps
- → Noise analysis of opamp circuits
  - → Resolution & SNR.

2

Fundamentals

- Noise → Random
- Zero mean, finite variance
- Gaussian
- $x(t) \rightarrow$ noise
- $\bar{x} = 0$

Noise power:

$$(\Psi_x^2) = \lim_{T\to\infty} \left[\frac{1}{T} \int_{\langle T \rangle} x^2(t)\,dt \right]$$

Noise variance:

$$\sigma_x^2 = \lim_{T\to\infty} \left[\frac{1}{T} \int_{\langle T \rangle} [x(t) - \bar{x}]^2\,dt \right]$$

$$\bar{x} = 0 \Rightarrow \Psi_x^2 = \sigma_x^2$$

Frequency representation of noise:

$$PSD = \frac{\Psi^2(f, f+\Delta f)}{\Delta f}$$

$\Psi^2(f, f+\Delta f)$ → Noise power from f to $f+\Delta f$ (in Δf band)

↳ Power Spectral Density of noise

PSD = constant ⇒ White noise

PSD (f) ⇒ Frequency-dependent noise.

White noise:

$$\Psi_x^2(f_1, f_2) = PSD \times \Delta f = PSD\,(f_2 - f_1)$$

$\Delta f = 100$

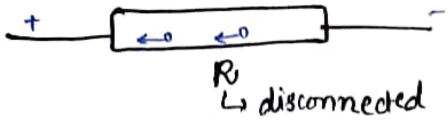
Noise power in $(100, 200) = (1000, 1100) = (10000, 10100)$

↳ Depends on freq. band and not its location.

$$\Psi_x^2(f_1, f_2) = \int_{f_1}^{f_2} PSD(f)\,df$$

Eg.

thermal voltage v → due to thermal energy → exist irrespective of whether the resistor is connected in a ckt.



Thermal Noise : Johnson Nyquist

noise power, $P_T = KTB$

K : Boltzmann's constant $= 1.38 \times 10^{-23}\ J/K$

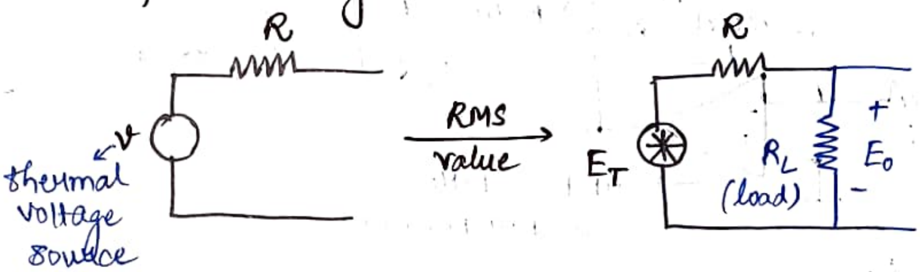
T : absolute temperature

B : Noise bandwidth

For $T = 298K$, $B = 1\ Hz$,

$$P_T = 4.1 \times 10^{-21}\ W$$

Model for a noisy resistor:



$$P_T = \left(\frac{E_0^2}{R_L}\right)_{max}$$

Max.^{m} power transfer occurs when

$R_L = R$

$\Rightarrow E_0 = E_T/2$

$$\therefore P_T = \frac{(E_T/2)^2}{R}$$

Noise power,

$$E_T^2 = 4P_T R = 4KTBR \quad \ldots (1)$$

$$\text{RMS value} = E_T = \sqrt{4KTBR}$$

$$PSD_T = \frac{E_T^2}{B} = 4KTR \quad \rightarrow \text{Unit}: \frac{V^2}{Hz}$$

Noise voltage density :

$$e_T = \sqrt{PSD_T}$$

$$= \sqrt{4KTR} \quad \longrightarrow \text{Unit}: \frac{V}{\sqrt{Hz}}$$

$\ldots (2)$

From ① and ②,

$$\boxed{E_T = e_T \sqrt{B}}$$

For $R = 1\,k\Omega$, $B = 1\,Hz$, $T = 298\,K$,

$E_T = \sqrt{4 \times 4.1\times10^{-21} \times 10^{3}} = 4\,nV$ → crucial for high precision applications

(4.1×10^{-21} ↑ P_T)

$E_T \downarrow \Rightarrow R\downarrow,\ T\downarrow,\ B\downarrow$

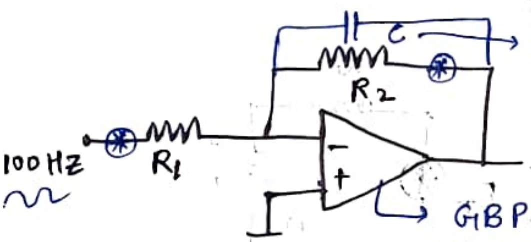
⇓

Liquid N_2 around electronics, $\Rightarrow -196°C = 77\,K$

⇓

$E_T = 2\,nV$

Liquid He $\Rightarrow 4\,K$



→ Reduces B ⇓ $E_T \downarrow$

$\left[\text{Corner freq.: } \frac{1}{2\pi R_2 C} \approx 10000\ Hz\right]$

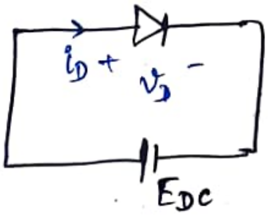
→ GBP = 5 MHz (Gain-B/W Product)

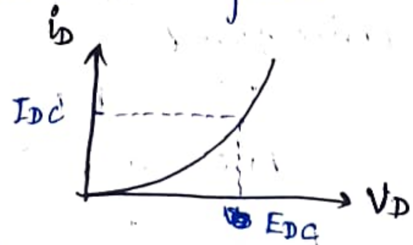
$\frac{R_2}{R_1} = 5$

Closed-loop bandwidth
$= \frac{5\,MHz}{5}$
$= 1\,MHz$.

↳ Reduce operating B/W by connecting a capacitor.

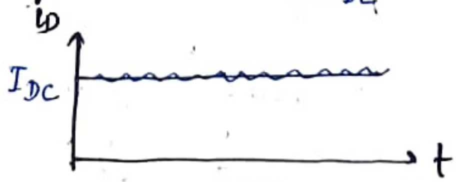
Shot Noise: observed in diodes when powered.





$i_D = I_{DC} + \hat{i}_{SH}(t)$

↳ shot/noise current



↳ Noise occurs only when the device is powered ON.
↳ Shot Noise

16-09-2025

Thermal Noise:

$V_T = \sqrt{4KTR}$

$e_T = \sqrt{PSD_T}$

$E_T^2 = e_T^2 B$ → Noise B/W

white : $PSD_T = 4KTR$

Shot Noise:

$i_D = I_{DC} + i_{SH}(t)$ → Instanteneous

$I_{SH}^2 = 2q I_{DC} B$: Square of RMS ... ① $[q = 1.6 \times 10^{-19} C]$

↳ Noise power

$PSD_{SH} = \frac{I_{SH}^2}{B} = 2q I_{DC}$ → constant for a given excitation

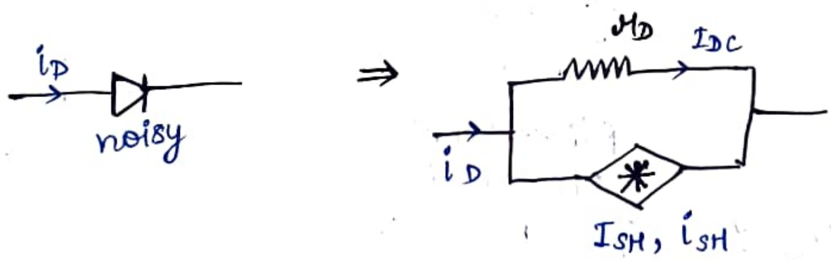
⇒ white noise

Current Noise Density:

$i_{SH} = \sqrt{PSD_{SH}} = \sqrt{2q I_{DC}}$ ... ②

$I_{SH}^2 = i_{SH}^2 \cdot B$

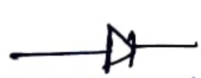
#



: Noise Model of a Diode

→ parallel, to provide a short path to i_{SH}.

Actual noise > Thermal noise



Actual noise > Shot noise

↳ Additional noise sources:

Excess noise → Source/reason is not well known as of today.

↳ only some observations.

- If current flows through non-homogeneous material, excess noise becomes large.

Types of resistors:

- Carbon composition $\xrightarrow{\text{made of}}$ Carbon + semiconductor
  ⇓
  Non-homogeneous
  ⇓
  Large excess ~~current~~ noise
  ⇓
  very ~~error~~ noise-prone
- Metal film
- Wire-wound → quiet

Transistors:

- MOSFET → Si & SiO_2 → excess noise → more noisy than BJT
- BJT → traps present in BE junction.

Excess Noise:

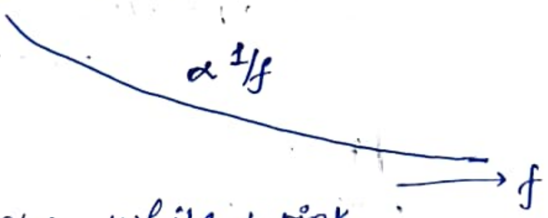
$$PSD_f = \frac{K^2}{f} \rightarrow \text{more dominant at low freq.}$$

$$\Psi_x^2 = \int_{f_1}^{f_2} PSD_f \, df = K^2 \ln\left(\frac{f_2}{f_1}\right)$$

↳ Noise power
↓
Same across different decades:
(10, 100) = (100, 1000) = (1000, 10000)

Different names:
Excess
Low-frequency
1/f - noise
Contact noise → occurs due to contact b/w two materials
Flicker noise
Pink noise

R O Y G B I V

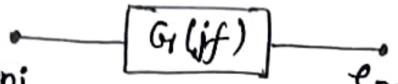


Noises: white + pink

$$PSD_{IC} = A_1 + \frac{A_2}{f}$$
(in ICs)

Noise Bandwidth (B)

↳ Applicable only for white noise.



e_{ni} — Input noise voltage density

e_{no} — Output noise voltage density

$$PSD_o(f) = e_{no}^2(f) = |G(jf)|^2 \, e_{ni}^2(f) \quad \ldots (1)$$

$$E_{NO}^2 = \int_0^\infty PSD_o(f)\,df \quad \ldots (2)$$

E_{NO} (Total) RMS noise

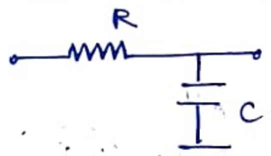
$$= \int_0^\infty |G(jf)|^2 \, e_{ni}^2(f)\,df$$

$$= e_{nw}^2 \underbrace{\int_0^\infty |G(jf)|^2\,df}_{B} \qquad [e_{ni} = e_{nw}]$$

$[\because E_T^2 = e_T^2 B]$

$$B = \int_0^\infty |G(jf)|^2\,df \quad \ldots (3)$$

Eg.



$$f_c = \frac{1}{2\pi RC}$$

$$G(jf) = \frac{1}{1 + jf/f_c} \Rightarrow |G(jf)|^2 = \frac{1}{1 + f^2/f_c^2}$$

$$B = \int_0^\infty \frac{1}{1 + f^2/f_c^2}\,df = f_c \tan^{-1}\left(\frac{f}{f_c}\right)\Big|_0^\infty = \frac{\pi}{2} f_c$$

$\therefore$ for $f_c = 100$ Hz,

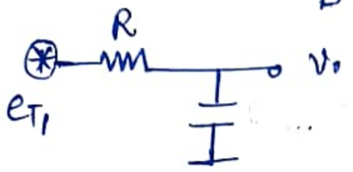
$B = 157$ Hz.

19-09-2025

Noise Bandwidth (B)

$$B = \int_0^\infty |G(jf)|^2 df$$

$$E_{NO}^2 = e_{nw}^2 \underbrace{\int_0^\infty |G(jf)|^2 df}_{B}$$

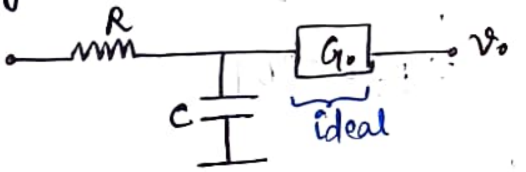


$$B = \frac{\pi}{2} f_c$$

$$E_{NO}^2 = 4KTRB$$

RMS o/p noise power

More general filter:



$$G(jf) = \frac{G_0}{1 + jf/f_c}$$

$$B = \int_0^\infty |G(jf)|^2 df = \int_0^\infty \frac{G_0^2}{1+(f/f_c)^2} \cdot df = G_0^2 \left(\frac{\pi}{2} f_c\right)$$

More generic expression: Same BW as previous.

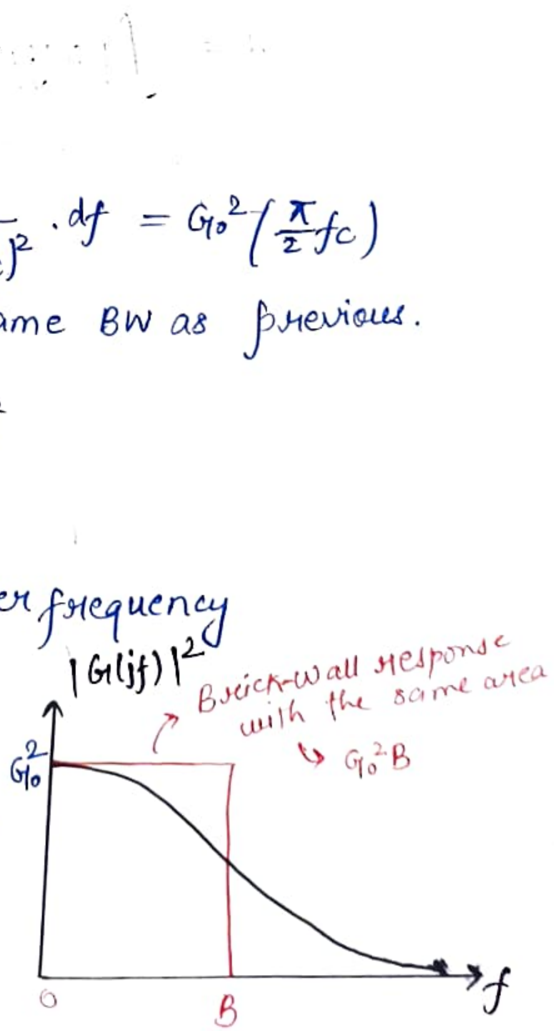
$$B = \frac{1}{G_0^2} \int_0^\infty |G(jf)|^2 df$$

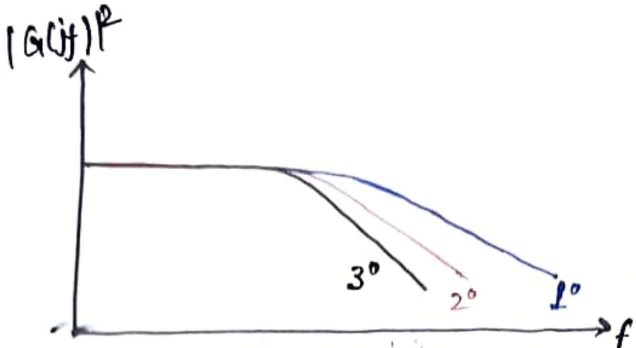
G_0 : pass-band gain

BPF: $G_0 \rightarrow$ gain @ center frequency

$$G_0^2 B = \int_0^\infty |G(jf)|^2 df$$

= area of power gain of the circuit



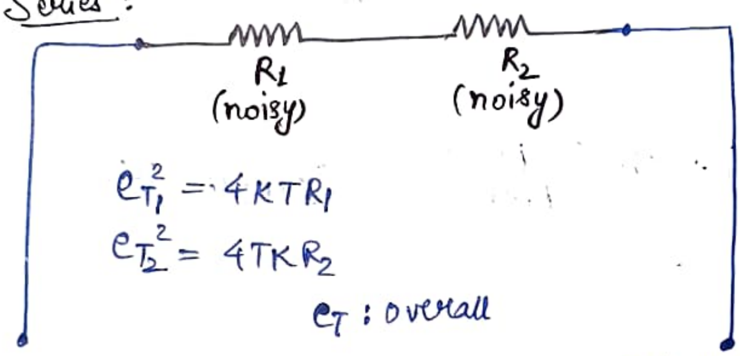


(all ideal)
↓
no noise

$B_3 < B_2 < B_1$

Noise Analysis of Passive Circuits

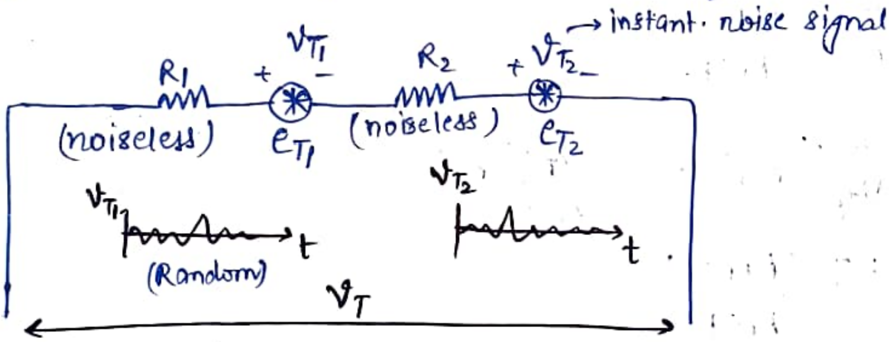
Series :



$$e_{T_1}^2 = 4KTR_1$$

$$e_{T_2}^2 = 4TKR_2$$

e_T : overall



$$v_T = v_{T_1} + v_{T_2}$$

$$X = f(\omega_1, \omega_2, \omega_3, \ldots)$$

$X \rightarrow \sigma_X$, $\omega_1 \rightarrow \sigma_{\omega_1}$, $\omega_2 \rightarrow \sigma_{\omega_2}$, $\omega_3 \rightarrow \sigma_{\omega_3}$

$$\sigma_X^2 = \left(\frac{\partial f}{\partial \omega_1}\right)^2 \sigma_{\omega_1}^2 + \left(\frac{\partial f}{\partial \omega_2}\right)^2 \sigma_{\omega_2}^2 + \left(\frac{\partial f}{\partial \omega_3}\right)^2 \sigma_{\omega_3}^2 + \ldots$$

[Propagation of variance in functions]

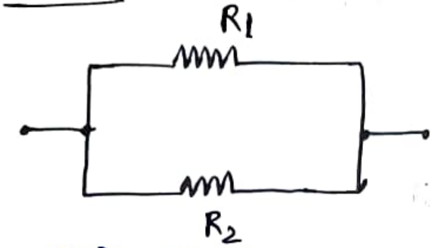
$$e_T^2 = \underbrace{\left(\frac{\partial v_T}{\partial v_{T_1}}\right)^2}_{1} \sigma_{T_1}^2 + \underbrace{\left(\frac{\partial v_T}{\partial v_{T_2}}\right)^2}_{1} \sigma_{T_2}^2$$

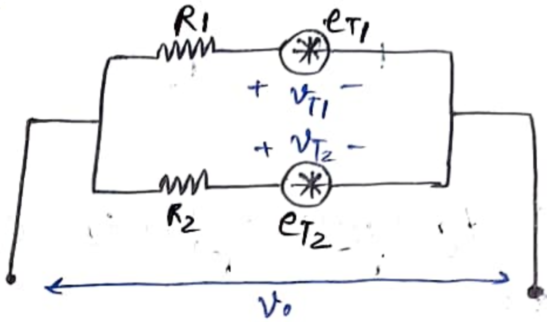
$$\Rightarrow e_T^2 = e_{T_1}^2 + e_{T_2}^2$$

$$\Rightarrow e_T^2 = 4KT\,(R_1 + R_2)$$

$$\Rightarrow e_T^2 = 4KT R_{eff}.$$

Parallel:





$$e_{T_1}^2 = 4KTR_1$$

$$e_{T_2}^2 = 4KTR_2$$

$$v_o = v_{T_1}\left(\frac{R_2}{R_1+R_2}\right) + v_{T_2}\left(\frac{R_1}{R_1+R_2}\right)$$ [Superposition]

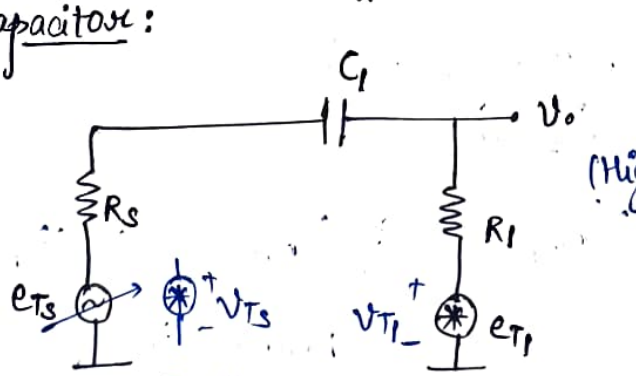
$$\frac{\partial v_o}{\partial v_{T_1}} = \left(\frac{R_2}{R_1+R_2}\right)$$

$$\frac{\partial v_o}{\partial v_{T_2}} = \left(\frac{R_1}{R_1+R_2}\right)$$

$$e_T^2 = \left(\frac{\partial v_T}{\partial v_{T_1}}\right)^2 \sigma_{T_1}^2 + \left(\frac{\partial v_T}{\partial v_{T_2}}\right)^2 \sigma_{T_2}^2$$

$$= 4KT \underbrace{\left(\frac{R_1 R_2}{R_1+R_2}\right)}_{R_{eff}}$$

With capacitor:



(High pass)

(Noise at the i/p are uncorrelated)

$$e_{T_1}^2 = 4KTR_1$$

$$e_{T_S}^2 = 4KTR_S$$

$$v_o = v_{T_1}\left[\frac{R_S + 1/j\omega C_1}{R_1 + R_S + 1/j\omega C_1}\right] + v_{T_S}\left[\frac{R_1}{R_1 + R_S + 1/j\omega C_1}\right]$$

$$= v_{T_1}\frac{(j\omega C_1 R_S + 1)}{D} + v_{T_S}\frac{j\omega C_1 R_1}{D},$$

$$D = 1 + j\omega C_1 (R_1 + R_S)$$

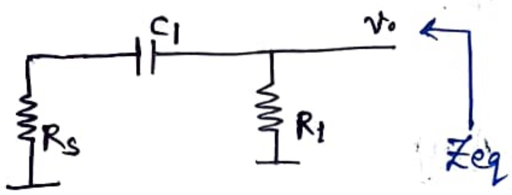
$$|D|^2 = 1 + \omega^2 C_1^2 (R_1 + R_S)^2$$

$$e_o^2 = \underset{\uparrow \atop 4KTR_1}{e_{T_1}^2}\,\frac{\omega^2 C_1 R_S^2 + 1}{|D|^2} + \underset{\uparrow \atop 4KTR_S}{e_{T_S}^2}\,\frac{\omega^2 C_1^2 R_1^2}{|D|^2}$$

$$= 4KT\left[\frac{1}{|D|^2}\left(\omega^2 C_1^2 R_S^2 R_1 + R_1 + \omega^2 C_1^2 R_1^2 R_S\right)\right]$$

$$= 4KT\underbrace{\left[\frac{1}{|D|^2}\left(R_1 + \omega^2 C_1^2 R_1 R_S (R_1 + R_S)\right)\right]}_{R'}$$

$R' \rightarrow$ complex $\rightarrow$ find without going through all this



$$Z_{eq} = R_1 \,||\, \left[R_S + \frac{1}{j\omega C_1}\right]$$

$$\text{Real}(Z_{eq}) = R'$$

$$\therefore e_o^2 = 4KT\,\text{Real}(Z_{eq}).$$

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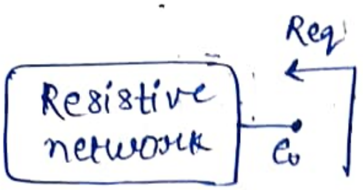
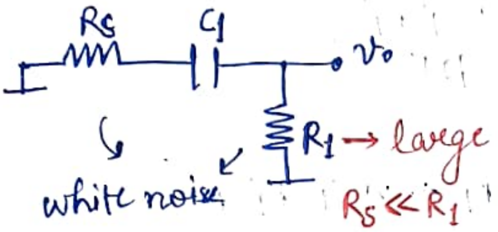


$e_0^2 = 4KTR_{eq}$

$R' =$ Real part (Z_{eq})

$e_0^2 = 4KTR'$



neglect, freq. dependent

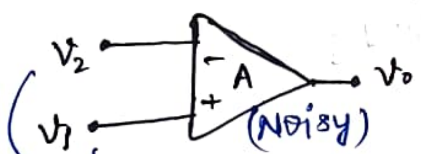
$$e_0^2 = 4KT\left[\frac{R_1\left(\cancel{1}+(\omega C_1)^2(R_1+R_S)R_S\right)}{\cancel{1}+(\omega C_1)^2(R_1+R_S)^2}\right]$$

o/p can be coloured

$$e_0^2 = 4KT\frac{R_1R_S}{R_1+R_S} \approx 4KTR_S$$

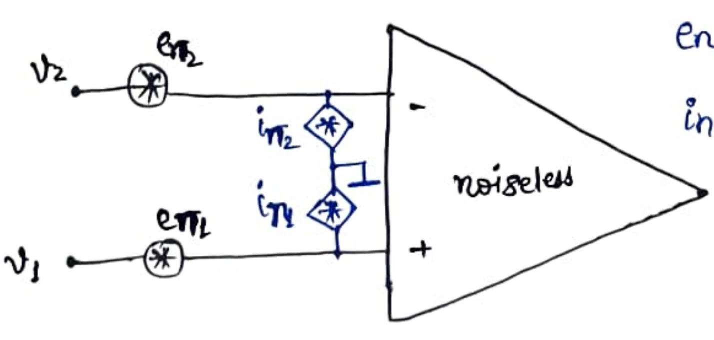
- Use of very large resistance need not lead to large o/p noise, it depends on the position of the resistance.

Noise Model of an OpAmp



$v_0 = A[v_1 - v_2]$

connected to base of different transistors
↳ having device resistances
↳ Noise : • Voltage noise
• Current noise



e_{n_1}, e_{n_2} : voltage noise spectral densities

i_{n_1}, i_{n_2} : current noise spectral densities

$e_n^2 = e_{n_1}^2 + e_{n_2}^2$: usually come in summation = white + pink

↳ equivalent voltage noise density

$$e_n^2 = A_1 + \frac{A_2}{f} = A_1 \left[1 + \frac{A_2/A_1}{f} \right]$$

A_2/A_1 → unit of freq.

A_1 → e_{nw}^2

↳ voltage noise which is white in nature

$$e_n^2 = e_{nw}^2 \left[1 + \frac{f_{CE}}{f} \right]$$

@ $f = f_{CE}$ (corner freq.), for voltage noise

$e_n^2 = 2 e_{nw}^2$

white noise = pink noise

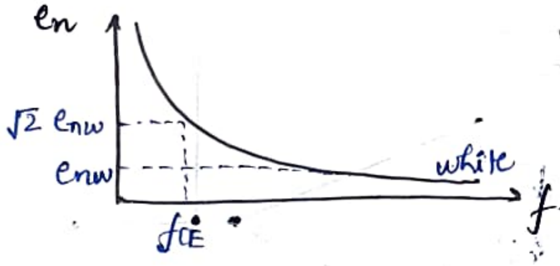
741 IC

$e_{nw} = 20\,nV/\sqrt{Hz}$

$f_{CE} = 200\,Hz$

$i_{nw} = 0.5\,pA/\sqrt{Hz}$

$f_{CI} = 2\,KHz$



[$f \to \infty$, $e_n = e_{nw}$]

$i_{n_1} \approx i_{n_2} = i_n$ [similar to i/p offset]

$$i_n^2 = i_{nw}^2 \left[1 + \frac{f_{CI}}{f} \right]$$, f_{CI} : corner frequency for current noise.

Eg. Estimate RMS input noise of 741 IC over audio range (20 Hz to 20000 Hz).

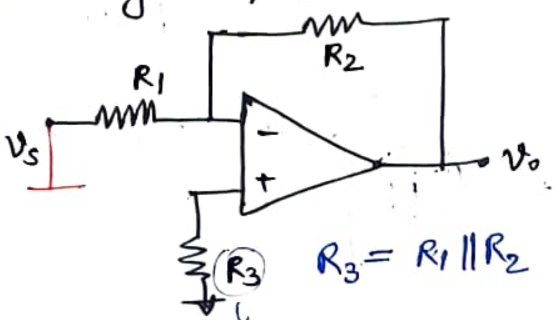
i_{n_1}, i_{n_2} : don't have resistive path to flow, so i/p spectral density will be dictated by e_{n_1}, e_{n_2}.

$$E_N^2 = \int_{f_L}^{f_H} e_n^2 \, df$$, $f_L = 20\,Hz$, $f_H = 20\,KHz$

$$= e_{nw}^2 \left[f_H - f_L + f_{CE} \ln\left(\frac{f_H}{f_L}\right) \right]$$

$\approx 2.9\,\mu V$

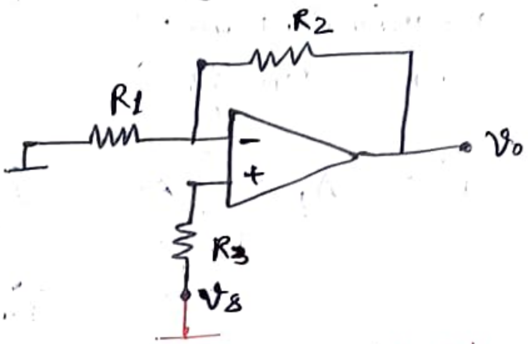
Inverting amplifier:



$R_3 = R_1 \| R_2$

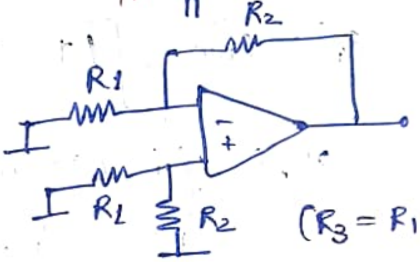
needed to compensate the error due to i/p offset current

Non-Inverting amplifier:



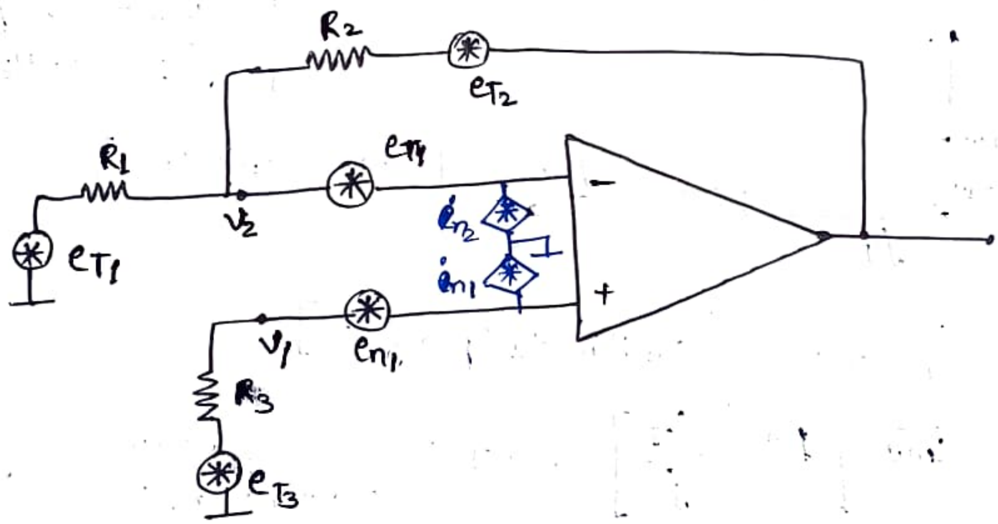
becomes same

Remove the source as i/p & noise are uncorrelated.



$(R_3 = R_1 \| R_2)$

- Noise analysis of a large class of configuration (inv., non-inv., summing) are the same. diff.



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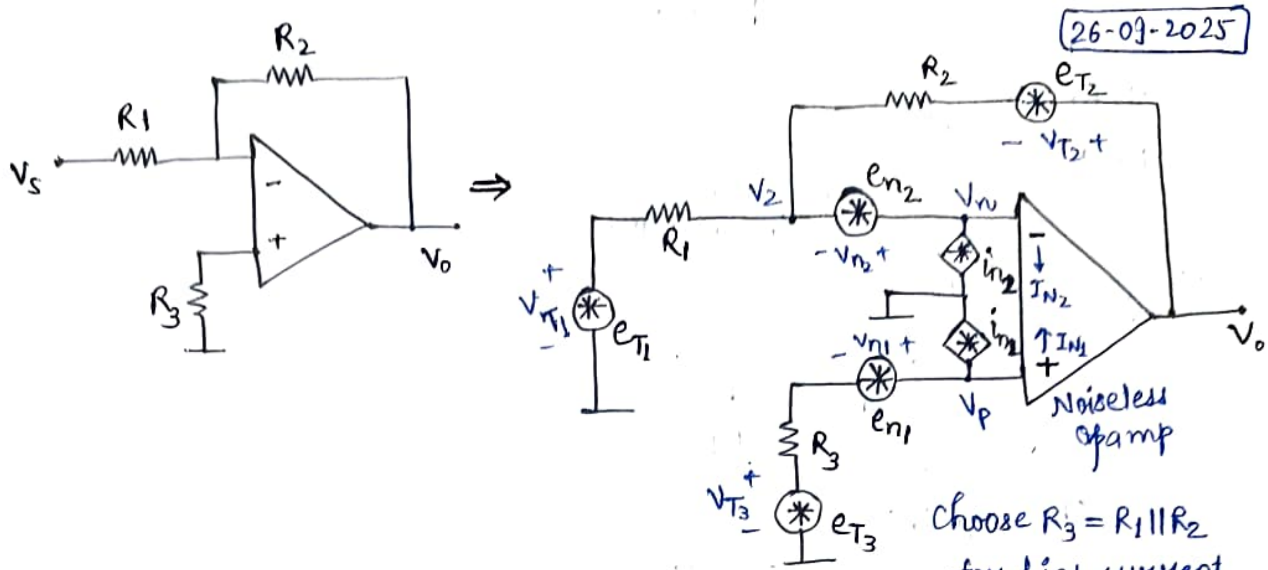


Choose $R_3 = R_1 \| R_2$ for bias current offset mitigation.

$v_p = v_{n_1} + v_{T_3} - I_{N_1} R_3 \quad \cdots ①$

$v_n = v_2 + v_{n_2} \Rightarrow v_2 = v_n - v_{n_2} \quad \cdots ②$

v_{n_i} : Instantaneous voltage
I_{N_i} : Instantaneous current due to i_{n_i}.

KCL @ v_2,

$$\frac{v_2 - v_{T_1}}{R_1} + \frac{v_2 - v_{T_2} - v_o}{R_2} + I_{N_2} = 0$$

$$\Rightarrow v_2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{v_{T_1}}{R_1} + \frac{v_{T_2}}{R_2} - \frac{v_o}{R_2} + I_{N_2} = 0$$

From eq^n ②, substitute v_2,

$$(v_n - v_{n_2})\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{v_{T_1}}{R_1} + \frac{v_{T_2}}{R_2} - \frac{v_o}{R_2} + I_{N_2} = 0$$

$$\Rightarrow v_n = v_{n_2} + v_{T_1}\frac{R_2}{R_1+R_2} - v_{T_1}\frac{R_1}{R_1+R_2} + \frac{v_o R_1}{R_1+R_2} - I_{N_2}(R_1 \| R_2) \quad \cdots ③$$

We know,

$$v_o = A(v_p - v_n)$$

$$\Rightarrow \frac{v_o}{A} = v_p - v_n = 0 \Rightarrow \boxed{v_p = v_n} \quad (\because A \to \infty)$$

Equate eq^n ① and ③,

$$v_{n_2} + v_{T_1}\frac{R_1}{R_1+R_2} - v_{T_2}\frac{R_1}{R_1+R_2} + \frac{v_o R_1}{R_1+R_2} - I_{N_2}(R_1 \| R_2)$$
$$= v_{n_1} + v_{T_3} - I_{N_1} R_3$$

$$\Rightarrow \frac{v_o R_1}{R_1+R_2} = v_{n_1} - v_{n_2} + v_{T_3} - v_{T_1}\frac{R_2}{R_1+R_2} + v_{T_2}\frac{R_1}{R_1+R_2} - I_{N_1}R_3 + I_{N_2}(R_1 \| R_2)$$

o/p due to noise (instantaneous)

$$\frac{v_o R_1}{R_1+R_2} = v_{ni}$$

i/p referred noise (instantaneous)

$$\Rightarrow \frac{v_o}{1+\frac{R_2}{R_1}} = v_{ni}$$

Noise gain, $\boxed{G_N = 1+\frac{R_2}{R_1}}$

$$v_o = G_N v_{ni}$$

$$\boxed{e_o^2 = G_N^2 e_{ni}^2}$$ ← spectral density

$$v_{ni} = v_{n_1} + v_{T_3} - v_{n_2} - v_{T_1}\frac{R_2}{R_1+R_2} + v_{T_2}\frac{R_1}{R_1+R_2} - I_{N_1}R_3 + I_{N_2}(R_1 \parallel R_2)$$

From v_{ni}, let's find e_{ni}^2,

$$e_{ni}^2 = \boxed{e_{n_1}^2 + e_{n_2}^2} + e_{T_3}^2 + \overset{4KTR_1}{e_{T_1}^2}\left(\frac{R_2}{R_1+R_2}\right)^2 + \overset{4KTR_2}{e_{T_2}^2}\left(\frac{R_1}{R_1+R_2}\right)^2$$

$$+ i_{n_1}^2 R_3^2 + i_{n_2}^2 (R_1 \parallel R_2)^2$$

Opamp manufacturer specify this, e_n^2 — white noise, $4KTR_3$

$$e_{ni}^2 = e_n^2 + 4KTR_3 + 4KTR_1\left(\frac{R_2}{R_1+R_2}\right)^2 + 4KTR_2\left(\frac{R_1}{R_1+R_2}\right)^2$$

$$+ i_{n_1}^2 R_3^2 + i_{n_2}^2 (R_1 \parallel R_2)$$

Approximation: $i_{n_1} \approx i_{n_2} \approx i_n$, $R_1 \parallel R_2 = R_3$

$$e_{ni}^2 = e_n^2 + 4KTR_3 + \overset{\text{after solving}}{4KTR_3} + i_n^2 R_3^2 + i_n^2 R_3^2$$

$$\Rightarrow e_{ni}^2 = e_n^2 + 8KTR_3 + 2 i_n^2 R_3^2$$

Now, substitute e_{ni} in $e_o^2 = G_N^2 e_{ni}^2$

$$e_o^2 = G_N^2 \left[e_n^2 + 8KTR_3 + 2 i_n^2 R_3^2\right]$$

$$\begin{cases} e_n^2 = e_{nw}^2 \left[1 + \frac{f_{CE}}{f}\right] & \rightarrow \text{white + pink} \\ 8KTR_3 & \rightarrow \text{white} \\ i_n^2 = i_{nw}^2 \left[1 + \frac{f_{CI}}{f}\right] & \rightarrow \text{white + pink} \end{cases}$$

Let $R_2/R_1 = 10$

<u>Case-A:</u>

$R_2 = 100\,\Omega$

$R_1 = 10\,\Omega$

$R_3 \rightarrow$ low

$e_{ni}^2 \approx e_n^2$

↑ short circuit noise

<u>Case-B:</u>

$R_2 = 1\,M\Omega$

$R_1 = 100k\,\Omega$

$R_3 \rightarrow$ high

$e_{ni}^2 \approx 2 i_n^2 R_3$

↑ open circuit noise

E_{NO}: RMS Output Noise $[f_L, f_H]$

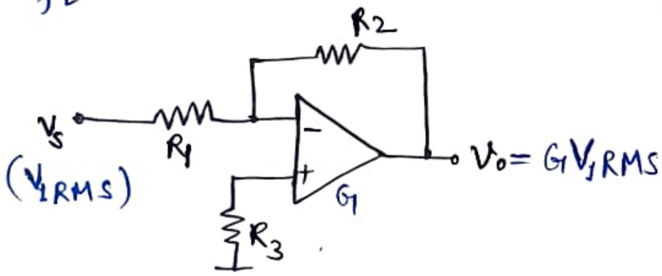
$$\boxed{E_{NO}^2 = \int_{f_L}^{f_H} e_o^2 \cdot df}$$

, E_{NO} dictates the output resolution of the amplifier.

$$\int_{f_L}^{f_H} e_n^2 \cdot df = [e_{nw}^2]\left[f_H - f_L + f_{cE} \ln\left(\frac{f_H}{f_L}\right)\right]$$

$$\int_{f_L}^{f_H} 8KTR_3\,df = 8\,KTR_3\,(f_H - f_L)$$

$$\int_{f_L}^{f_H} i_n^2\,df = i_{nw}^2\left[f_H - f_L + f_{cI} \ln\left(\frac{f_H}{f_L}\right)\right]$$



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$$SNR = 20\log\left(\frac{G_1 V_1 RMS}{E_{NO}}\right)$$

Inverting amplifier:

$$G_1 = R_2/R_1$$

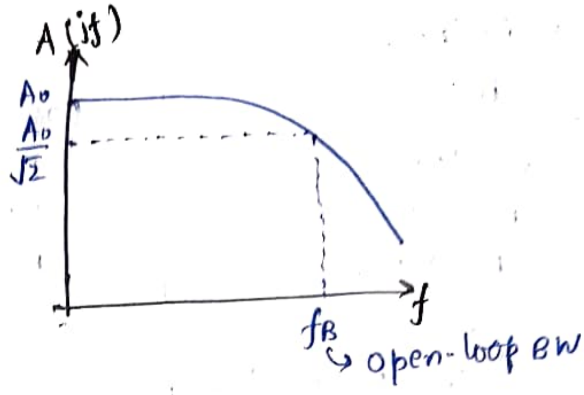
Non-inverting amplifier:

$$G_1 = 1 + R_2/R_1$$

→ We have assumed that $A \rightarrow \infty$, but it is ~~still~~ not pratical.

→ We also know that gain has frequency dependency.

$$\boxed{A(jf) = \frac{A_0}{1 + jf/f_B}}$$



Modified eqn ($A \neq \infty$):

$$\frac{v_o}{A(jf)} = v_p - v_n$$

$$v_{ni} = \frac{v_o}{G_N} \xrightarrow{f \text{ dep.}} \frac{v_o}{A(jf)} + \frac{v_o}{G_N} = v_{ni}$$

$$v_o = \frac{G_N A(jf)}{A(jf) + G_N} v_{ni}$$

$$= \frac{G_N}{1 + G_N/A(jf)} v_{ni}$$

$$= \frac{G_N v_{ni}}{1 + \frac{G_N}{A_0}\left(1 + \frac{jf}{f_B}\right)}$$

$$= \frac{G_N v_{ni}}{1 + \frac{G_N}{A_0} + \frac{jf}{\left(\frac{A_0 f_B}{G_N}\right)}}$$

As $G_N = 1 + \frac{R_2}{R_1} \ll A_0$,

$$v_o = \frac{G_N v_{ni}}{1 + jf/\left(\frac{A_0 f_B}{G_N}\right)}$$

$$\boxed{v_o = \frac{G_N v_{ni}}{1 + j(f/f_N)}}$$, where $f_N = \frac{A_0 f_B}{G_N}$ → Gain-BW product

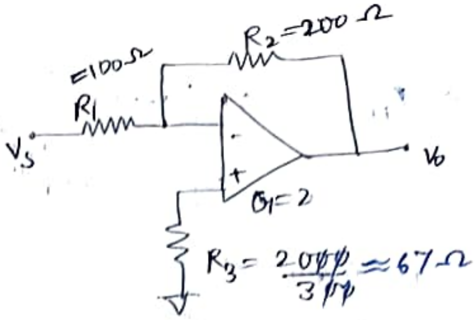
f_N ↳ Noise BW

$$\boxed{e_o^2 = \left(\frac{G_N^2}{1 + (f/f_N)^2}\right)^2 e_{ni}^2}$$

Now, $e_{ni}^2 = e_n^2 + 8kTR_3 + 2 i_n^2 R_3^2$

$E_{No}^2 = \int_{f_L}^{f_H} e_o^2 \, df$: RMS noise

Eg. Given;



$e_{nw} = 20\,nV/\sqrt{Hz}$
$f_{CE} = 200\,Hz$
$i_{nw} = 0.5\,pA/\sqrt{Hz}$
$f_{CI} = 2\,KHz$
$A_0 = 10^6$ (DC gain)

Unity-Gain-Bandwidth product $= 1\,MHz$.
RMS output noise above $0.1\,Hz$.
$f_L = 0.1\,Hz$, $f_H \to \infty$. Find RMS noise.

Soln: We know, $G_I = 2$, $G_N = 3$

$$f_N = \frac{A_0 f_B}{G_N} = \frac{1\,MHz}{3}$$

$$e_{ni}^2 \approx e_n^2 + \cancel{8KTR_3}^{\to 0} + \cancel{2 i_n^2 R_3^2}^{\to 0}$$

$$e_{ni}^2 \approx e_n^2 = e_{nw}^2 \left(1 + \frac{f_{CE}}{f}\right)$$

$$e_o^2 = \text{Gain}^2 \times e_{ni}^2$$
$$= \frac{G_N^2 e_{nw}^2}{1 + \frac{f^2}{f_N^2}} (1 + f_{CE}/f)$$

$$\Rightarrow e_o^2 = G_N^2 e_{nw}^2 \left[\frac{1}{1 + f^2/f_N^2} + \frac{f_{CE}/f}{1 + f^2/f_N^2}\right]$$

$\downarrow \int_{f_L}^{f_H} \cdot df$

$$E_{No}^2 = G_N^2 e_{nw}^2 \left[f_N \tan^{-1}\left(\frac{f}{f_N}\right) + \frac{f_{CE}}{2} \ln\left(\frac{f^2}{f^2 + f_N^2}\right)\right]_{0.1}^{\infty}$$

$(f \ll f_N)$

$$= G_N^2 e_{nw}^2 \left[\frac{\pi}{2} f_N + \frac{f_{CE}}{2} \ln(1) - f_N \frac{f_L}{f_N} - \frac{f_{CE}}{2} \ln\left(\frac{f_L^2}{f_L^2 + f_N^2}\right)\right]$$

$\approx f_N^2$

$$\Rightarrow E_{No}^2 = G_N^2 e_{nw}^2 \left[\frac{\pi}{2} f_N - f_L - \frac{f_{CE}}{\cancel{2}} \cancel{2} \ln\left(\frac{f_L}{f_N}\right)\right]$$

$$\Rightarrow \boxed{E_{No}^2 = G_N^2 e_{nw}^2 \left[\frac{\pi}{2} f_N - f_L + f_{CE} \ln\left(\frac{f_N}{f_L}\right)\right]}$$

$$\Rightarrow E_{No}^2 = (3)^2 (4\times10^{-16}) \left[\frac{\pi}{2}\left(\frac{10^6}{3}\right) - 0.1 + 200\ln\left(\frac{10^6}{3\times0.1}\right)\right]$$

$$= 5665.68\times10^{-12}\ V^2$$

$E_{No} \rightarrow$ Resolution of opamp.

$= 43.5\mu V$

For input v_s,

$V_s = \sin\omega t$

$V_{IRMS} = 1/\sqrt{2} \Rightarrow G\,V_{IRMS} = \sqrt{2}$

$SNR = 20\log\left(\frac{\sqrt{2}}{43.5\mu}\right)$

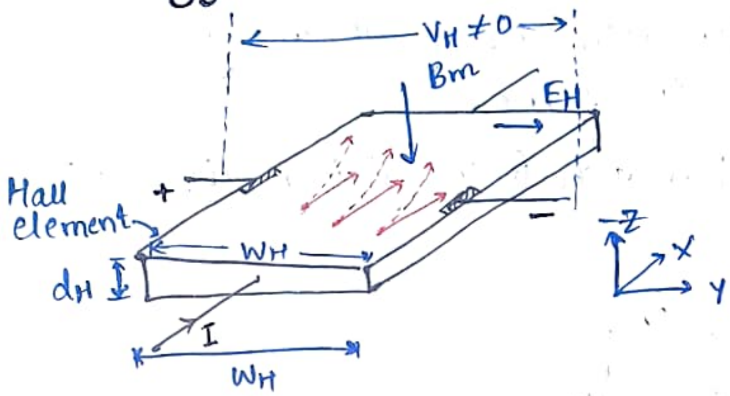
$\approx 90.2\ dB$

→ We may have to put a capacitor across R_2 if $f_H \neq \infty$, to increase SNR.

Magnetic Sensors:

Hall Effect Sensors

13-10-2025



When the magnetic field is applied, the force deflects the charges in y-dirⁿ.

E_H: Induced electric field
B_m: Measured magnetic field

~~Loren~~ Magnetic force,

$$F_B = \underset{\text{charge}}{q}\,(\underset{\substack{\text{drift velocity}\\ \text{of charges}}}{V_D} \times B_m) = q\,V_D B_m$$

Force due to Electric field,

$$F_E = qE_H = q\,V_D B_m$$

At equilibrium, $F_B = F_E$

Induced electric field, $E_H = V_D B_m$

Induced voltage: $\underset{\neq 0}{V_H}$ (Hall voltage)

$$E_H = V_H / W_H$$

$$\Rightarrow \frac{V_H}{W_H} = V_D B_m$$

$$\Rightarrow V_H = V_D B_m W_H$$

Drift velocity,

$$V_D = \frac{I}{n q W_H d_H}, \quad n: \text{no. of } e^{-}\text{s/volume}$$

$$\therefore V_H = B_m \left(\frac{I}{n q d_H}\right)$$

$$\Rightarrow \boxed{V_H = B_m \cdot K}, \quad K = \frac{I}{n q d_H} : \text{sensitivity}$$

: Linear Magnetic Field Sensor

For metal, $n \rightarrow$ high, $K \rightarrow$ low

Generally doped semiconductor is used.
↳ $n \rightarrow$ low
$K \rightarrow$ flexible

→ For pure Hall element, charge carriers will move in a straight line.

<u>Assumption so far</u>: Drift velocities of all charge carriers are the same.

There may be minor mismatch in v_D, giving different magnetic force.

$$v_{D_1},\ v_{D_2},\ v_{D_3}$$
$$\downarrow \qquad \downarrow \qquad \downarrow$$
$$q v_{D_1} B_m \quad q v_{D_2} B_m \quad q v_{D_3} B_m$$

Induced electric field will balance the average magnetic fields.

$$q\, v_{D_{AVG}} B_m = q E_H \Rightarrow \boxed{V_H = B_m\, v_{D_{AVG}}\, w_H}$$

↳ Path length of charge carriers increase
⇒ Resistance ↑ ⇒ Magneto-Resistance

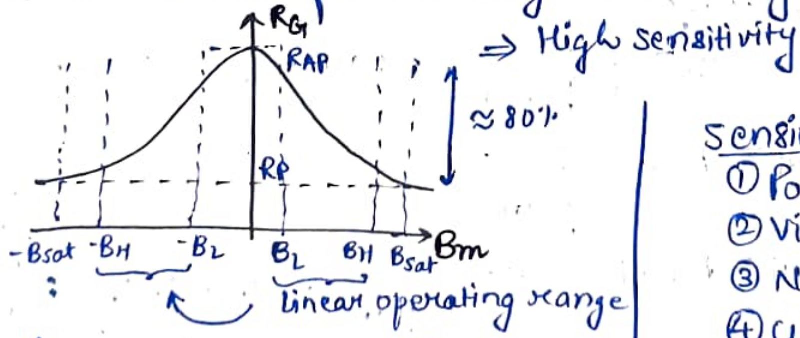
<u>Magneto-Resistance Technology</u>:

↳ Invented by Lord Kalvin
↳ Material used: Iron
↳ Change in resistance with magnetic field.
↳ Low sensitivity (≈ 2%)

New technology: GMR

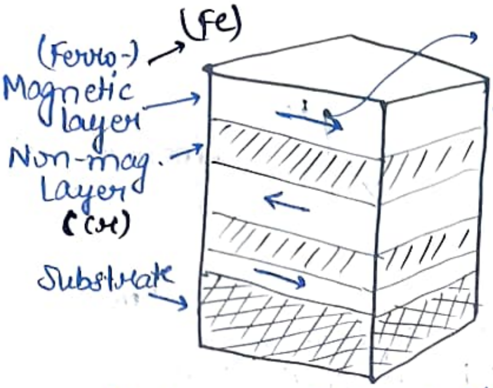
<u>Giant Magneto-Resistance (GMR)</u>:

↳ 'Giant' stands for the huge sensitivity of the material.



<u>Sensing Applications</u>:
① Position sensing
② Vital-sign monitor
③ Non-Destructive Evaluation
④ Current sensing

↳ Low cost, compact size
↳ Rugged Nature
↳ Non-contact sensing



→ Dirn of mag. moment

- Ferromagnetic layers sandwiched around a non-magnetic layer.
- Magnetic moments of Fe layers are in anti-parallel configuration (mag. field = 0)

Electron Spin: Electrons spin around their axis (apart from revolving around nucleus)

anticlkwise spin upward

⇓

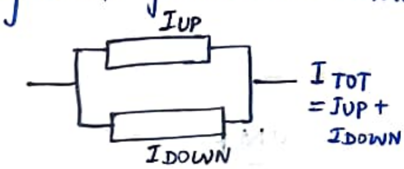
Upspin ($m_s = +\frac{1}{2}$)

clkwise spin downward

⇓

Downspin ($m_s = -\frac{1}{2}$)

- Electrons ~~spin~~ motion occurs in 2 separate parallel channels.



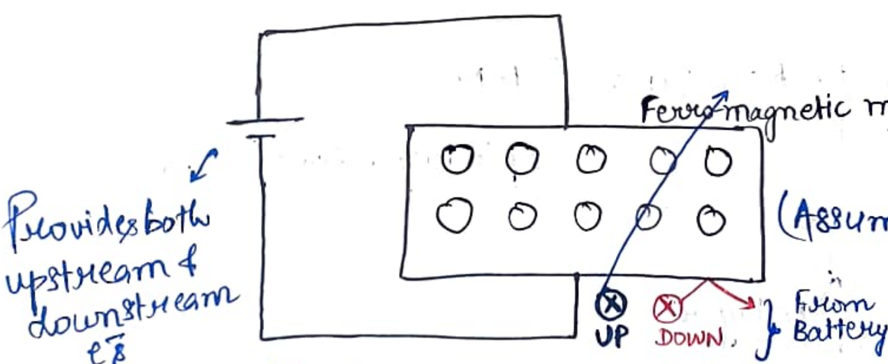
- For normal metals, (no. of e⁻s)

$N_{UPSPIN} = N_{DOWNSPIN}$

- For ferromagnetic materials,

$N_{UPSPIN} \neq N_{DOWNSPIN}$

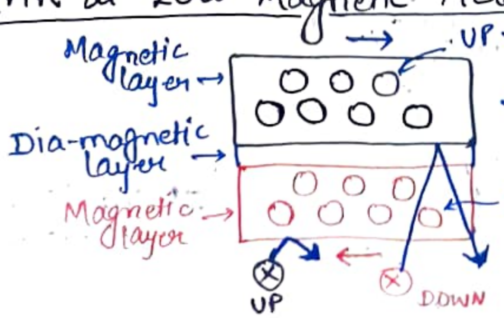
Spin-Dependant Conduction:



(Assume upstream states)

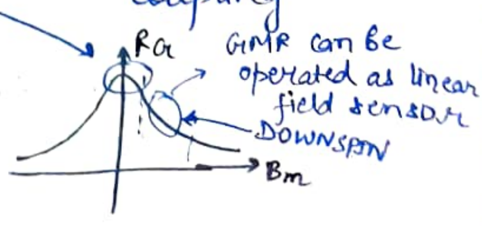
- Upstream e⁻s pass (dominate the conduction).
- Downstream e⁻s get scattered (blocked).

GMR at Zero Magnetic Field:

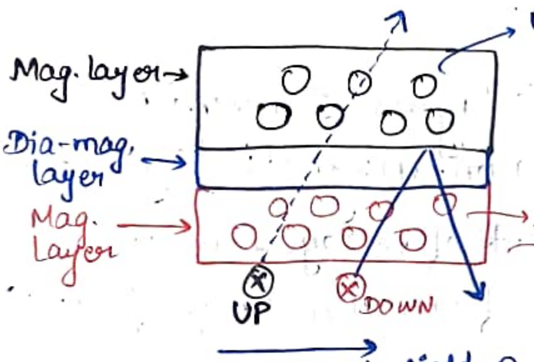


High Resistance

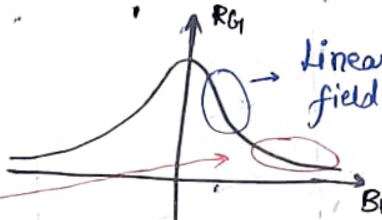
Anti-Ferromagnetic Coupling



→ Dirn of Magnetic moment



Magnetic field, B_M



Linear magnetic field sensor

Very low Resistance if $B_M \to B_{sat}$ (Saturation)

- Resistance decreases with applied magnetic field. (decreases linearly)

Axis of sensitivity → x-axis

In Hall-effect sensor, sensitivity was $\perp^r$ to plane of axis.

- sensitivity will depend on the thickness of non-magnetic material.
  Thickness ↓ ⇒ sensitivity ↑

21-10-2025

GMR

$$R_G = R_0 \left[1 - kB \right] \quad : \text{Linear}$$

R_G: GMR resistance, R_0: Base resistance, k: sensitivity

Tunneling Magneto-Resistance (TMR)

TMR structure:

- Spin dependent Tunneling

Low resistance — High resistance

⇓

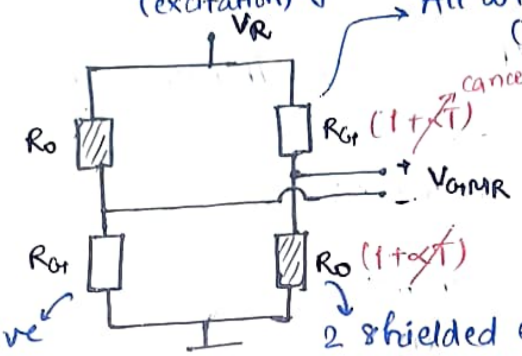
Large sensitivity to mag. field

- Carrier tunnel through insulator.
- Higher sensitivity than GMR.

CMR (Colossal Magneto-Resistance)

GMR IC :

Using wheatstone bridge configuration :



→ All are active GMR elements (of same materials, dimension)

V_{GMR} } Zero o/p regardless of i/p → Finite bridge o/p

2 active GMRs (Sensing MR)

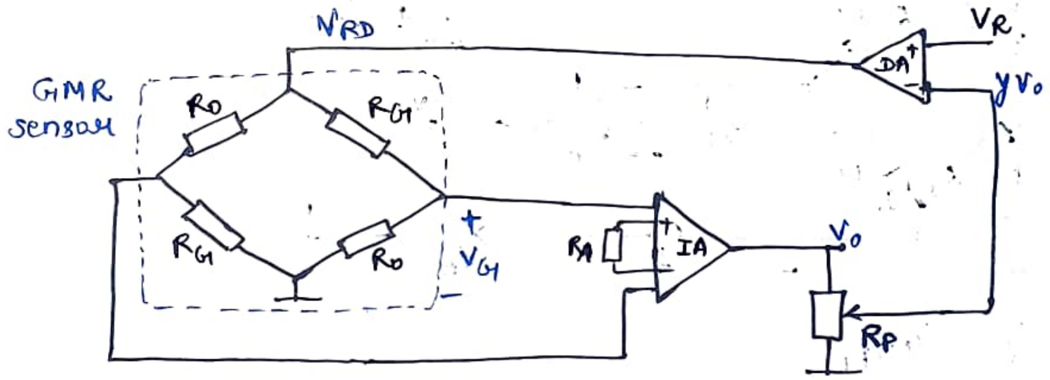
2 shielded elements (flux concentrators)
↳ Don't vary with field [Reference MR]

- Fields concentrate towards 2 active GMRs ⇒ Improved sensitivity
- $R_G = R_o(1 - KB)$
  - R_o cancels ⇒ Independent of R_o → vary by manufacturers

$$V_{GMR} = \left(\frac{R_o - R_G}{R_o + R_G}\right) V_R$$

⇝ Non-linearity (V_{GMR} vs. R_G)

Linearizer Circuit :



$$V_o = G V_G = G\left(\frac{R_o - R_G}{R_o + R_G}\right) V_{RD} \quad \ldots (1)$$

$V_o \downarrow yV_o$

$$V_{RD} = V_R - yV_o \quad \ldots (2)$$

$$\therefore V_o = G\left(\frac{R_o - R_G}{R_o + R_G}\right)(V_R - yV_o)$$

$$\Rightarrow V_o = \frac{G\left(\frac{R_o - R_G}{R_o + R_G}\right)V_R}{1 + yG\left(\frac{R_o - R_G}{R_o + R_G}\right)}$$

$$= \frac{G(R_o - R_G)V_R}{R_o + R_G + yG(R_o - R_G)} \quad \ldots (3)$$

- Get linear V_o vs. R_G.

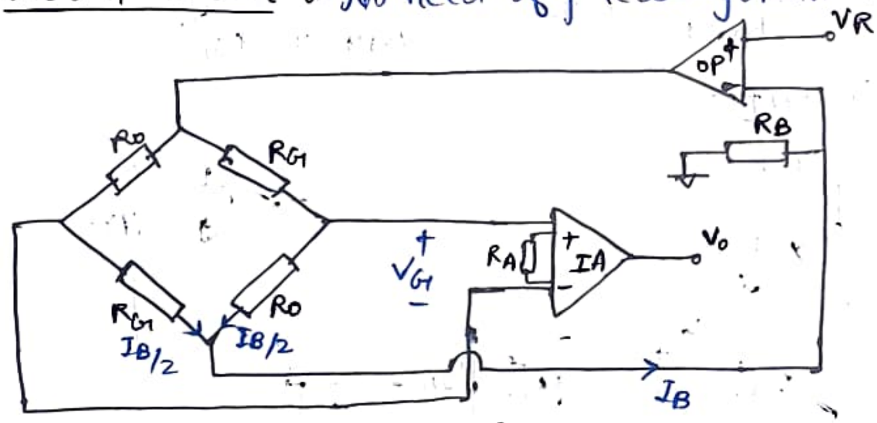
Take $\gamma G = 1$.

$$\Rightarrow \boxed{V_o = \frac{G(R_o - R_G)V_R}{2R_o}}$$: Feedback compensation

<u>Merits:</u>
- Improved NL (<0.7%)
- Simple
- sensitivity : 0.7V/mT
- Resolution : 0.2 μT
- Approx. cost : \$7

<u>Demerits:</u>
- Requires 2 I/As (DA using IA)
- Requires precise potentiometer to ensure $G\gamma = 1$.

<u>Improved circuit</u> : No need of precise potentiometer.

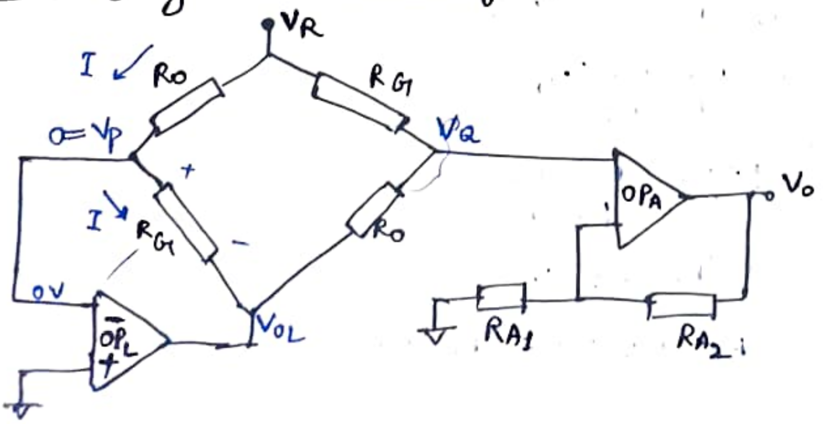


$$I_B = V_R/R_B$$

$$V_G = \frac{I_B}{2}(R_o - R_G)$$

$$V_o = G V_G = \frac{G I_B}{2}(R_o - R_G)$$

<u>Linearizing Front-End for GMR-sensors</u> (LFG)



$V_P = 0$ (virtual ground)

$I = V_R / R_0$

$$\therefore V_{OL} = -IR_{G1} = -\frac{V_R}{R_0} R_{G1}$$

$$V_Q = V_R \frac{R_0}{R_0 + R_{G1}} + \frac{V_{OL} R_{G1}}{R_0 + R_{G1}} \quad \text{(By superposition)}$$

$$= V_R \frac{R_0}{R_0 + R_{G1}} - \frac{\frac{V_R}{R_0} R_{G1}^2}{R_0 + R_{G1}}$$

$$= \frac{V_R}{R_0 + R_{G1}} \left(R_0 - \frac{R_{G1}^2}{R_0} \right)$$

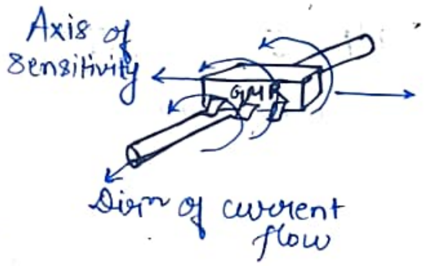
$$\therefore V_Q = \frac{V_R (R_0 - R_{G1})}{R_0}$$

- 2nd opamp for further amplification.

$$V_0 = \left(1 + \frac{R_{A2}}{R_{A1}} \right) V_Q$$

Current Sensing using GMR

- Non-intrusive measurement (no loading).
- current probes.
- Industrial process control
- PCB mounted current detection
- well-suited for High-current applications.

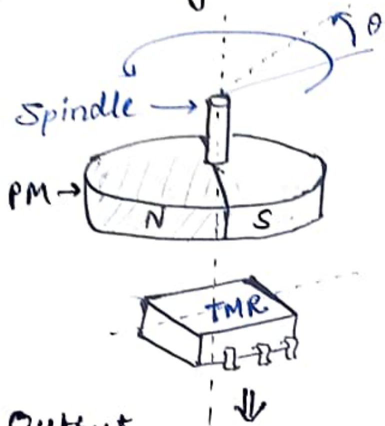


Angle sensors

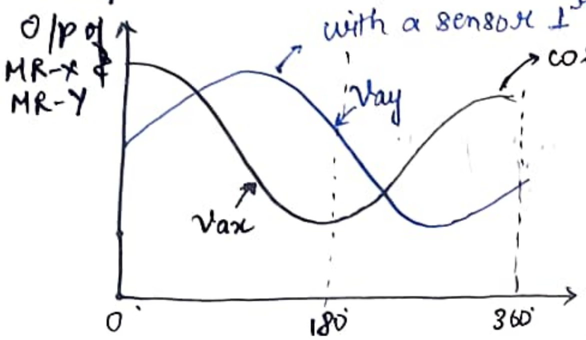
- Steering wheel
- Robotic arm
- Motor-positioning

24-10-2025

GMR/TMR Angle Sensors



Output



} used to detect angle

- Linear Transfer characteristics is desired.

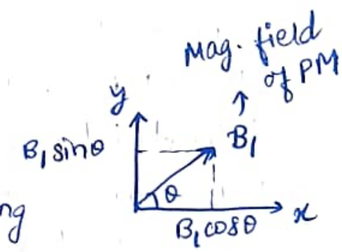
ATMR Angle Sensor with Full-Circle Range:

TMR IC → AAT001

Two half bridges: X Y

R_{x_1}, R_{x_2} aligned along x-dirn

R_{y_1}, R_{y_2} aligned along y-dirn



aligned along +ve x

$R_{x_2} \longrightarrow B_1 \cos\theta$

$R_{x_2} = R_0 [1 + K_T \cos\theta]$, R_0 : Nominal resistance

K_T : Transformation constant / sensitivity of half-bridges

aligned along -ve x

$R_{x_1} \longrightarrow -B_1 \cos\theta$

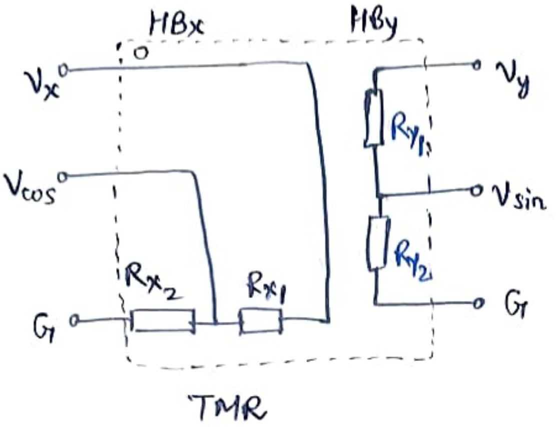
$R_{x_1} = R_0 [1 - K_T \cos\theta]$

offset can be removed (using DA)

$$V_{cos} = \frac{R_{x_2} V_x}{R_{x_1} + R_{x_2}} = \left(\frac{\cancel{1} + K_T \cos\theta}{2}\right) V_x = 0.5 K_T V_x \cos\theta$$

$$V_{SIN} = \frac{R_{y_1} V_y}{R_{y_1} + R_{y_2}} = \left(\frac{\cancel{1} + K_T \sin\theta}{2}\right) V_y = 0.5 K_T V_y \sin\theta$$

- Needs only one IC.
- Output should depend only on the angle.



We can choose V_x, V_y.

Choose $V_x = V_m \cos\omega t$, $V_y = V_m \sin\omega t$.

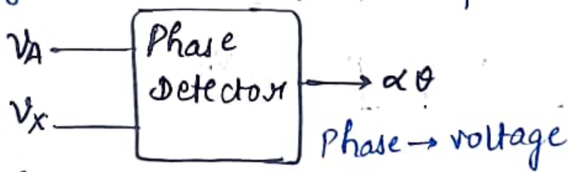
$\therefore$ $V_{cos} = 0.5\, K_T V_m \cos\omega t \cdot \cos\theta$

$V_{SIN} = 0.5\, K_T V_m \sin\omega t \cdot \sin\theta$.

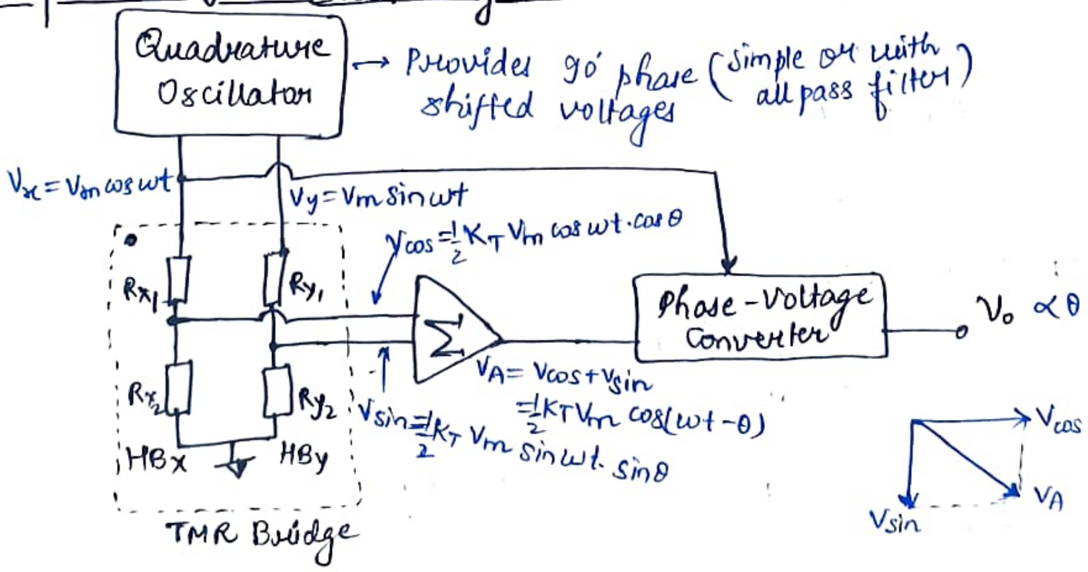
Using summing amplifier,

$V_A = V_{cos} + V_{SIN} = 0.5\, KT \cos(\omega t - \theta)$

Angle is converted to the phase angle b/w V_A and V_x.



Proposed Signal Conditioning Scheme:

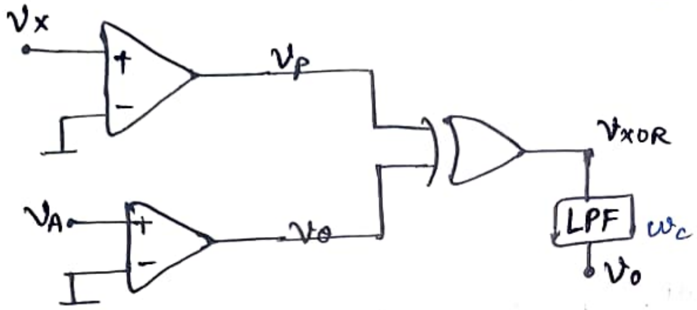


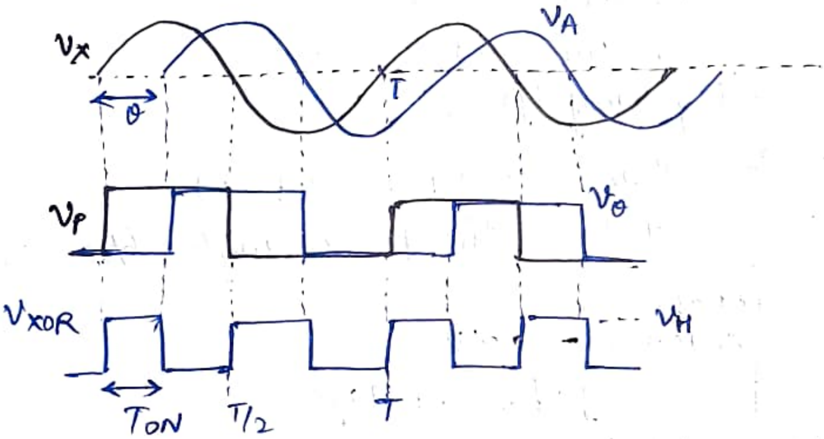
Phase Detector – Basic Circuit

$V_X = V_m \cos \omega t$

$V_A = 0.5 K_T V_m \cos(\omega t - \theta)$

We would want to detect 'θ' regardless of the amplitudes.





$T = 2\pi/\omega$

$T_{ON} \propto \theta$

$T_{ON} = \theta/\omega$

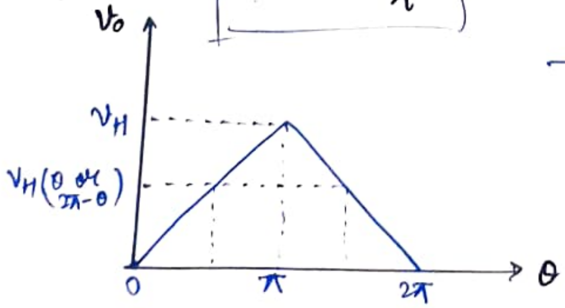
$\frac{T_{ON}}{T} = \frac{\theta}{2\pi}$

LPF: $\omega_c \ll \omega$,

$V_o = avg(V_{XOR})$

$= V_H \frac{T_{ON}}{T/2}$

$\therefore \boxed{V_o = V_H \frac{\theta}{\pi}}$ → V_H-dependent.



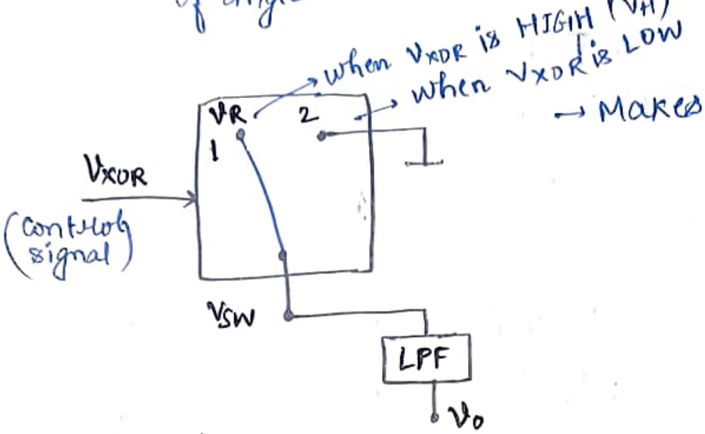
→ 180° phase-detector
[90° & 270° give the same obs.]

Eg. $\omega = 2000\pi$

$\omega_c = 20\pi$

Output should not contain any component of 2000π.

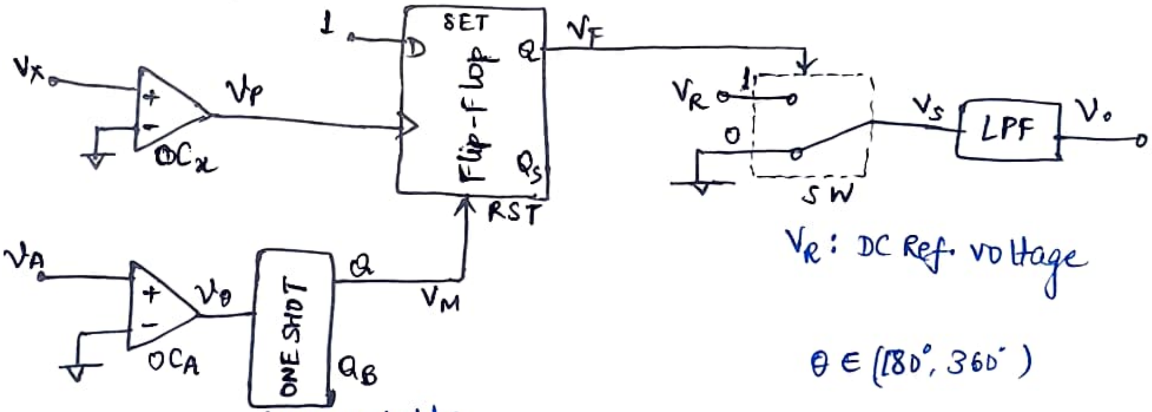
Rate of change of angle, $\frac{d\theta}{dt} \ll \omega$.



→ Makes V_o independent of V_H.

$\therefore \boxed{V_o = V_R \frac{\theta}{\pi}}$ → Improves precision, accuracy.

Phase-to-Voltage converter (360° phase detector)

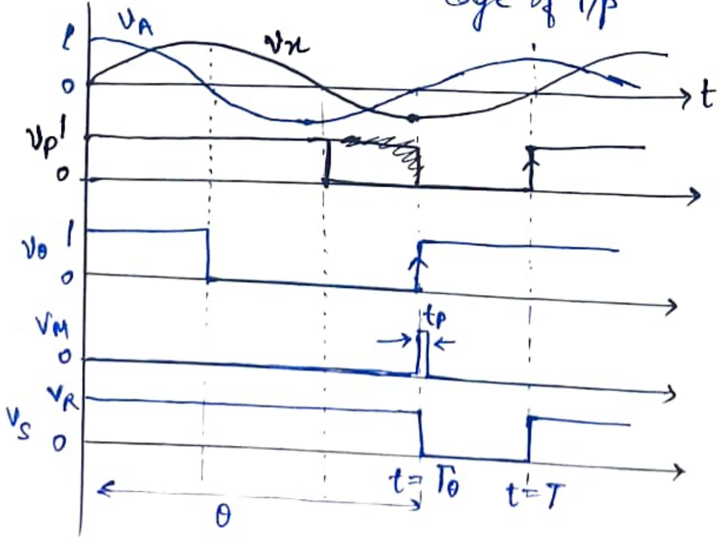


V_R: DC Ref. voltage

$\theta \in [180°, 360°)$

↳ Monostable multivibrator

↳ create a very short pulse at the rising edge of i/p



$V_x = V_m \cos \omega t$

$V_A = K_T V_m \cos(\omega t - \theta)$

$T_{ON} \propto \theta$

$\Rightarrow T_{ON} = \theta / \omega$

Switch o/p, $V_S = \begin{cases} V_R, & t \in T_{ON} \\ 0, & t \in T_{OFF} \end{cases}$

Avg. value, (o/p of LPF) $V_o = \frac{V_R T_{ON}}{T} = \frac{V_R \theta}{\cancel{\omega} \frac{2\pi}{\cancel{\omega}}} = \frac{V_R \theta}{2\pi}$

We get different T_{ON} for 90° and 270° phase diff. [for 1st-2nd and 3rd-4th quadrant], thus can be used for full 360° range phase-detection.

Dominant
Error Sources:

① we assumed to get 90° phase-shifted sinusoidal waves. But there may be some phase error (phasor) propagating to the output.

Oscillator signals: $V_x = V_m \cos\omega t$

$V_y = V_m \sin(\omega t + \psi)$, ψ: phase error of QO (°)

TMR sensor outputs:

$$V_{cos} = V_x K_s \cos\theta = V_m K_s \cos\omega t \cdot \cos\theta$$

$$V_{sin} = V_y K_s \sin\theta = V_m \cdot K_s \cdot \sin(\omega t + \psi) \cdot \sin\theta$$

Summation: $V_{S_1} = V_m K_s \cos\omega t \cos\theta + V_m K_s \sin(\omega t + \psi) \cdot \sin\theta$

$$V_{S_1} = V_{m_1} \cdot K_s \cdot \cos(\omega t - \phi_1)$$

$$\phi_1 = \tan^{-1}\left[\frac{\sin\theta \cos\psi}{\cos\theta + \sin\theta \sin\psi}\right] \rightarrow \text{phase-shift w.r.t. } V_x$$

$$= \tan^{-1}\left[\frac{\tan\theta \cos\psi}{1 + \tan\theta \sin\psi}\right]$$

$$\% \text{ error} = \frac{\phi_1 - \theta}{360°} \times 100\% \quad [\text{w.r.t. full scale}]$$

Maxm value of error occurs at

$$\theta = 90°, 270°$$

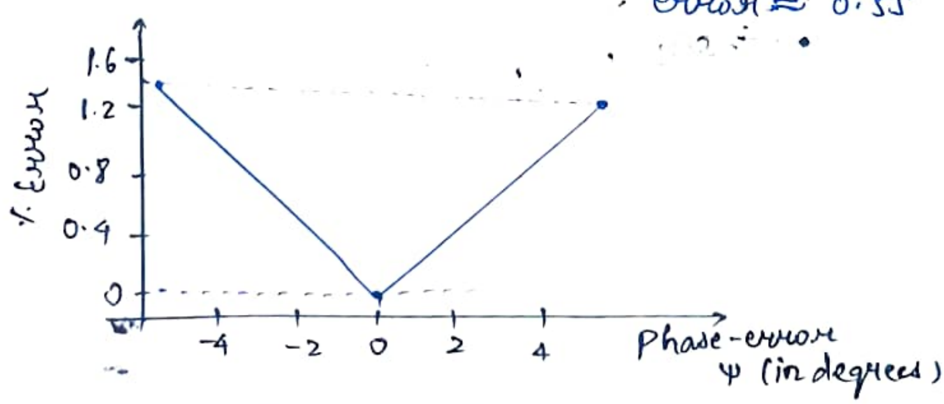
$$\theta = 90° \Rightarrow \phi_1 = \tan^{-1}[\cot\psi] = 90° - \psi$$

$$\% \text{ error} = \frac{\pm\psi}{360°} \times 100\% \quad (+ \text{ for } \theta = 270°)$$

$$\text{Maxm error (worst-case)} = \frac{|\psi|}{360°} \times 100\%$$

For phase-mismatch of $\psi = 2°$;

error ≈ 0.55



Enhanced Methodology:

Difference: $V_{S_2} = V_m K_S \cos\omega t \cdot \cos\theta - V_m K_S \sin(\omega t + \psi)\sin\theta$

$$= V_m K_S \cos(\omega t + \phi_2)$$

$$\phi_2 = \tan^{-1}\left[\frac{\tan\theta\cos\psi}{1-\tan\theta\sin\psi}\right] \rightarrow \text{phase-shift w.r.t. } V_x.$$

Define $D_\theta = \dfrac{\phi_1+\phi_2}{2}$

$$= \frac{1}{2}\left[\tan^{-1}\left(\frac{\tan\theta\cos\psi}{1+\tan\theta\sin\psi}\right) + \tan^{-1}\left(\frac{\tan\theta\cos\psi}{1-\tan\theta\sin\psi}\right)\right]$$

$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\dfrac{A+B}{1-AB}\right)$

$$= \frac{1}{2}\tan^{-1}(\tan 2\theta\cos\psi)$$

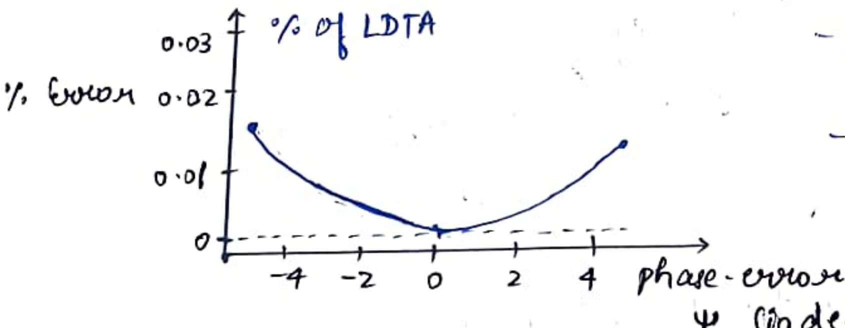
ψ: small $\Rightarrow \cos\psi \rightarrow 1$

$\Rightarrow D_\theta = \frac{1}{2}\tan^{-1}(\tan 2\theta) \approx \theta$ for $\theta \in (0^\circ, 360^\circ)$

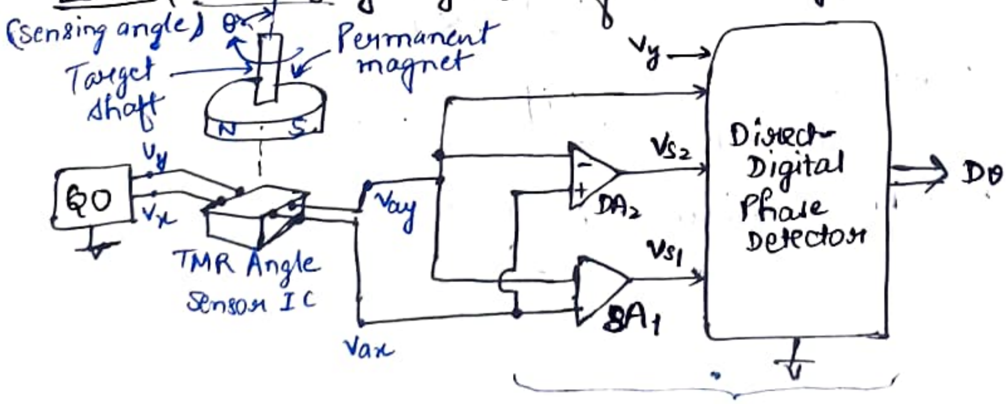
$$\%\text{ error} = \frac{D_\theta - \theta}{360^\circ} \times 100\%$$

→ Drastically reduced from the prev. approach

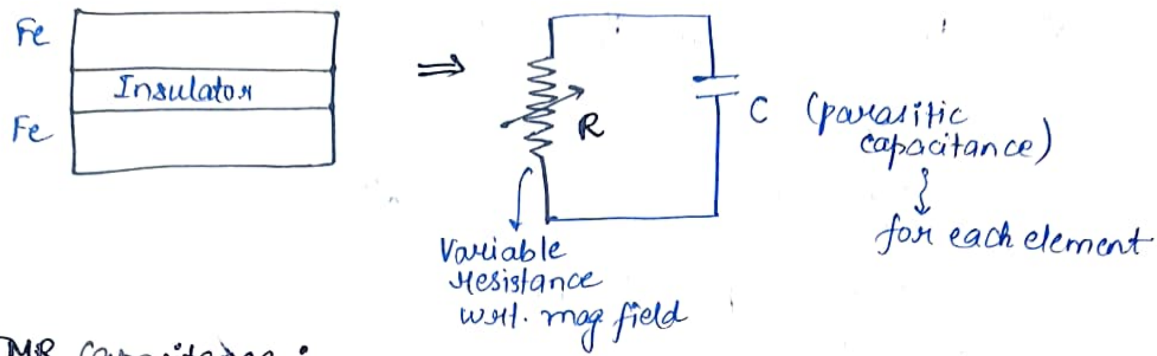
- Compensates the adverse effect of QO phase-error (ψ).
- Indep. of K_T and excitation voltage.



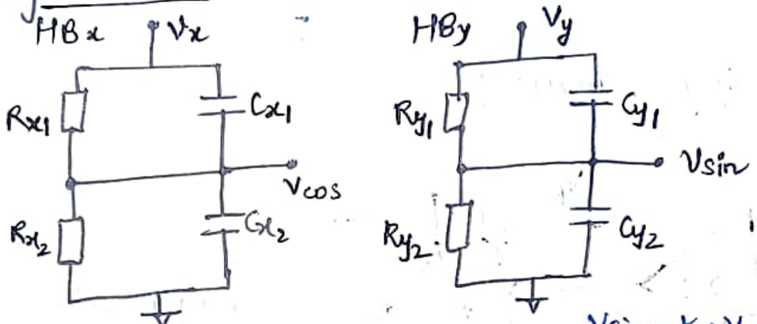
LDTA (Linearizing Digitizer for TMR Angle Sensor)



TMR element:



TMR Capacitance:



$V_{cos} = K_T V_M \cos(\omega t - \delta_{cos}) \cos\theta$

$V_{sin} = K_T V_M \sin(\omega t - \delta_{sin}) \sin\theta$

→ TMR capacitances do not affect the o/p.

→ Phase error of QO (Quadrature Oscillator)

→ Parasitic capacitances of TMR elements

↳ **Mitigation:**

Excitation using DC (open-circuited cap.)

$\cos\theta \rightarrow \theta$

Mitigation:
If QO is avoided by using a single sine-wave.

27-10-2025

Linear MR Angle Transducers using DC Excitation

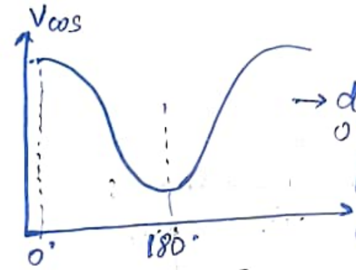
$V_{cos} = K_T V_x \cos\theta$, $V_{SIN} = K_T V_y \cos\theta$

↳ constant/dc for a given θ

$V_x = V_y = V_{DC}$

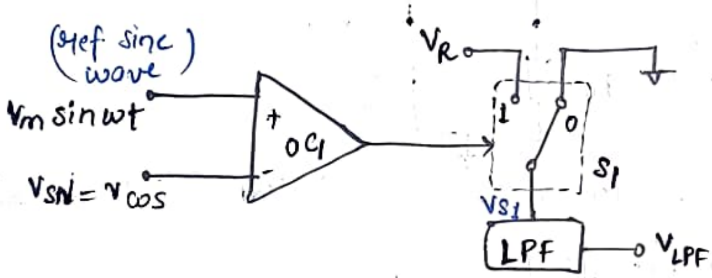
$V_{cos} = V_{SN}$ → o/p for sensor

$V_{SN} = V_{SN_0} \cos\theta$

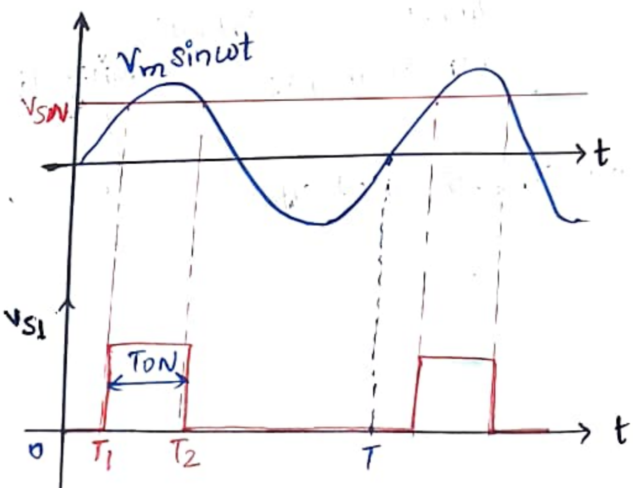


→ detects 0°–180° or 180–360°

Single conditioning Scheme :



→ Precision output with Half-Circle Range



$V_m \sin\omega t = V_{SN}$
$= V_m \cos\theta$

$\Rightarrow t = \frac{1}{\omega}\left(\frac{\pi}{2} \pm \theta\right)$

$T_1 = \frac{1}{\omega}\left(\frac{\pi}{2} - \theta\right)$, $T_2 = \frac{1}{\omega}\left(\frac{\pi}{2} + \theta\right)$

$T_{ON} = T_2 - T_1$
$= \frac{2\theta}{2\pi} \cdot T$

$\Rightarrow \frac{T_{ON}}{T} = \frac{\theta}{\pi}$

$V_{SN} = V_{SN_0} \cos\theta$

Choose $V_m = V_{SN_0}$

↳ may not match exactly
⇓
error will be significant near 0° & 180°.
(low sensitivity regions of sensor o/p)
(where rate of change is small)

$V_{LPF} = \text{average}\,(V_{st})$

$= V_R \dfrac{T_{ON}}{T}$

$$\boxed{V_{LPF} = V_R \frac{\theta}{\pi}}$$

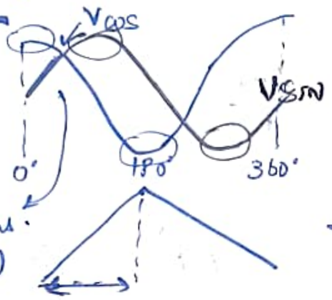
V_R : DC Ref. voltage $\left[\dfrac{d\theta}{dt} \ll \omega\right]$

$(0 < \theta < \pi)$

We have assumed that V_{SN} is constant for atleast few cycles.

So, change in $V_{SN} \ll \omega$.

(small rate of change) ⇓ (low sensitivity) ⇓ (High error) ⇓ Avoid it



High sens. (use this)

→ No difference b/w 90° & 270°.

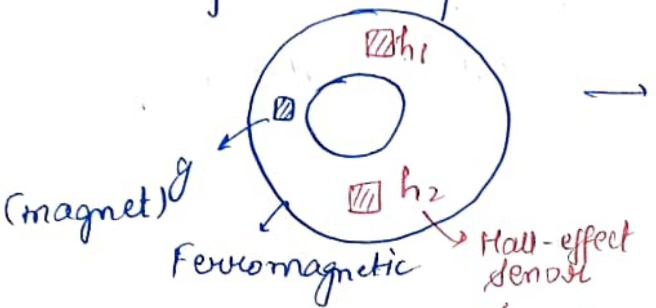
Avoid low sensitivity regions and use only high sensitivity regions → by switching between sin and cos.

↳ Difficult to use in through shaft

↳ ~~Hall~~ Use hall-effect sensor.

A linear Reluctance - Hall Effect Based Angle Sensor

When the shaft is inaccessible, attach the sensor as a ring at a particular position.



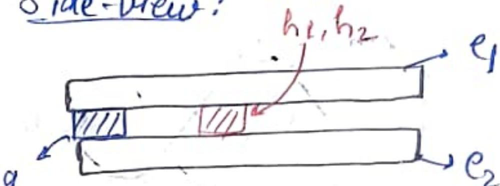
Mounted on some fixed mounting part in the airgap

g attached to rings in the airgap

↳ rotates with the rings

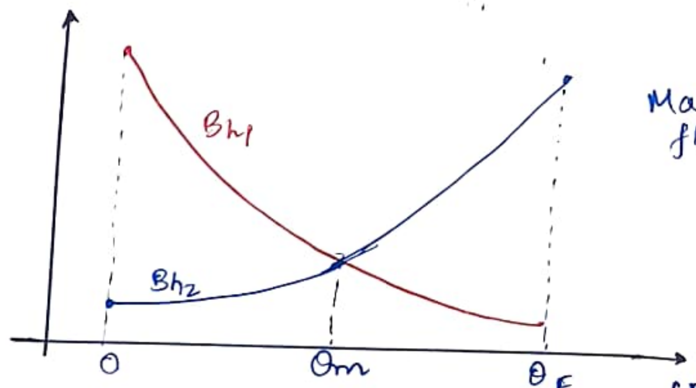
Use two such rings (e_1, e_2) in proximity with a small gap:

Side-view:



$e_1, e_2, g \to$ rotating parts

$h_1, h_2 \to$ fixed



Mag. flux $\begin{cases} B_{h_1} : \text{seen by } h_1 \\ B_{h_2} : \text{seen by } h_2 \end{cases}$

θ_F: full ~~angle~~-scale angle

θ_m: mid-scale angle

Ideal angle range $= 180°$

$\theta = 0 \Rightarrow g$ is closest to h_1.

Characteristic of the sensor is cubic in nature.

$$B_{h_1} \approx a_1\theta^3 + a_2\theta^2 + a_3\theta + a_4$$

$$B_{h_2} \approx b_1\theta^3 + b_2\theta^2 + b_3\theta + b_4.$$

Hall-effect sensor is linear.

$$\left.\begin{aligned} V_{H_1} &= V_B\,[1 + KB_{H_1}] \\ V_{H_2} &= V_B\,[1 + KB_{H_2}] \end{aligned}\right\} \text{Hall IC o/ps}$$

$$\Rightarrow V_{H_1}(\theta) = c_1\theta^3 + c_2\theta^2 + c_3\theta + c_4$$

$$V_{H_2}(\theta) = d_1\theta^3 + d_2\theta^2 + d_3\theta + d_4$$

29-10-2025

Linearization Methodology

Hall IC outputs, $V_{H_1}(\theta) = V_B(1 + K B_{H_1}(\theta))$
$V_{H_2}(\theta) = V_B(1 + K B_{H_2}(\theta))$

$$\Rightarrow V_{H_1}(\theta) = c_1\theta^3 + c_2\theta^2 + c_3\theta + c_4$$
$$V_{H_2}(\theta) = d_1\theta^3 + d_2\theta^2 + d_3\theta + d_4.$$

B_{H_2} can be represented as mirrored and shifted version of B_{H_1}.

$$B_{H_2}(\theta) = B_{H_1}(-\theta + \theta_F)$$
$$V_{H_2}(\theta) = V_{H_1}(-\theta + \theta_F)$$
$$\Rightarrow V_{H_2}(\theta) = c_1[-\theta + \theta_F]^3 + c_2[-\theta + \theta_F]^2 + c_3(-\theta + \theta_F) + c_4$$

coeff. of $\theta^3 = -c_1 = d_1$

$$\therefore V_{H_2}(\theta) = -c_1\theta^3 + d_2\theta^2 + d_3\theta + d_4.$$

$V_D = V_{H_1} - V_{H_2}$
↓
cubic
↓
crosses zero at θ_m; also symmetric about θ_m : real root : θ_m
Roots $\equiv \theta = \theta_m, \theta_m \pm \alpha j$

$V_A = V_{H_1} + V_{H_2}$
↓
quadratic
↓
never crosses zero : complex roots
↓
symmetric about θ_m : Roots $\equiv \theta_m \pm \beta j$
(real part of roots $= \theta_m$)

$$V_D = K_N(\theta - \theta_m)(\theta - (\theta_m - \alpha j))(\theta - (\theta_m + \alpha j))$$
$$= K(\theta - \theta_m)[\theta^2 - 2\theta_m\theta + \theta_m^2 + \alpha^2],$$

$K_N = 2c_1$
$K_D = c_1 + d_1$

$$V_A = K_D[\theta^2 - 2\theta\theta_m + \theta_m^2 + \beta^2]$$

Define $V_o(\theta) = \dfrac{V_{H_1}(\theta) - V_{H_2}(\theta)}{V_{H_1}(\theta) + V_{H_2}(\theta) + V_K} = \dfrac{V_D(\theta)}{V_A(\theta) + V_K}$, V_K : constant for a sensor unit.

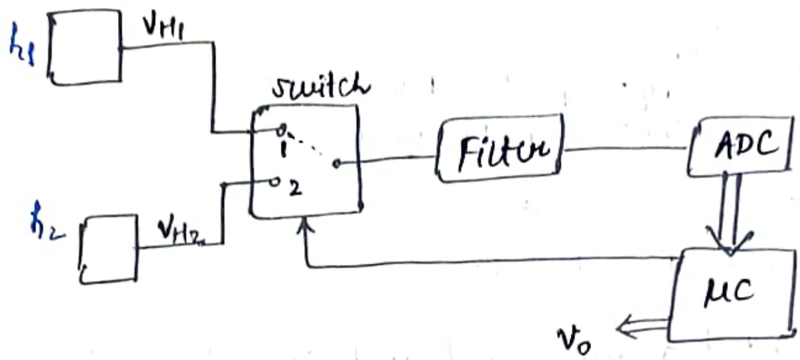
$$= \frac{K_N(\theta - \theta_m)[\theta^2 - 2\theta_m\theta + \theta_m^2 + \alpha^2]}{K_D[\theta^2 - 2\theta\theta_m + \theta_m^2 + \beta^2] + V_K}$$

$$= \frac{K_N}{K_D}\frac{(\theta - \theta_m)[\theta^2 - 2\theta_m\theta + \theta_m^2 + \alpha^2]}{[\theta^2 - 2\theta\theta_m + \theta_m^2 + \beta^2] + V_K/K_D}$$

Choose $V_K = K_D(\alpha^2 - \beta^2)$.

[Pole-zero cancellation idea]

$$\Rightarrow V_o(\theta) = \frac{K_N}{K_D}(\theta - \theta_m).$$



: Conventional Approach

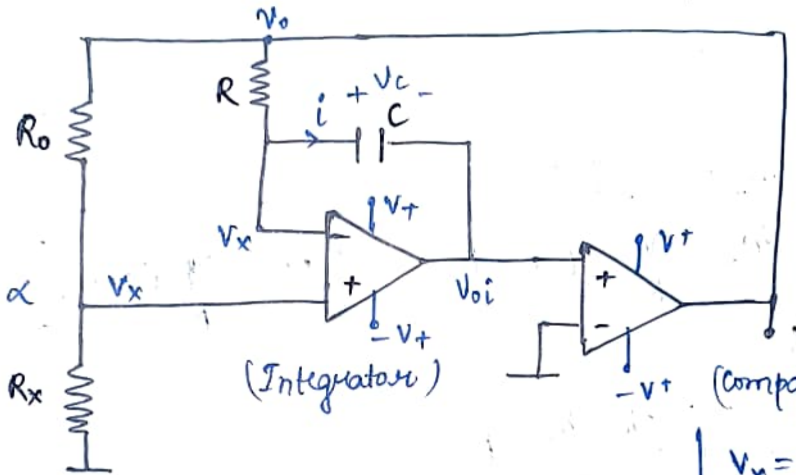
· First keep the switch (multiplexer) to pos. 1, then to 2. Give the control signal back to the switch from a microcontroller which configured to ~~to~~ calculate V_0, then gives V_0.

- Better dedicated circuit in the research paper.
- Cannot have a dedicated ADC for each sensor.

Resistive Sensors

Eg. GMR, LDR

- Digitizers to digitize resistive sensor without ADC.
- Relaxation - Oscillator → 2 o/ps ↗ Triangular ↘ Rectangular.



Convert triangular wave to rectangular

$V_x = \alpha V_o$; $\alpha = \dfrac{R_x}{R_x + R_o}$

[we need R_o, ~~as~~ otherwise f/b will be 1]

V_o :

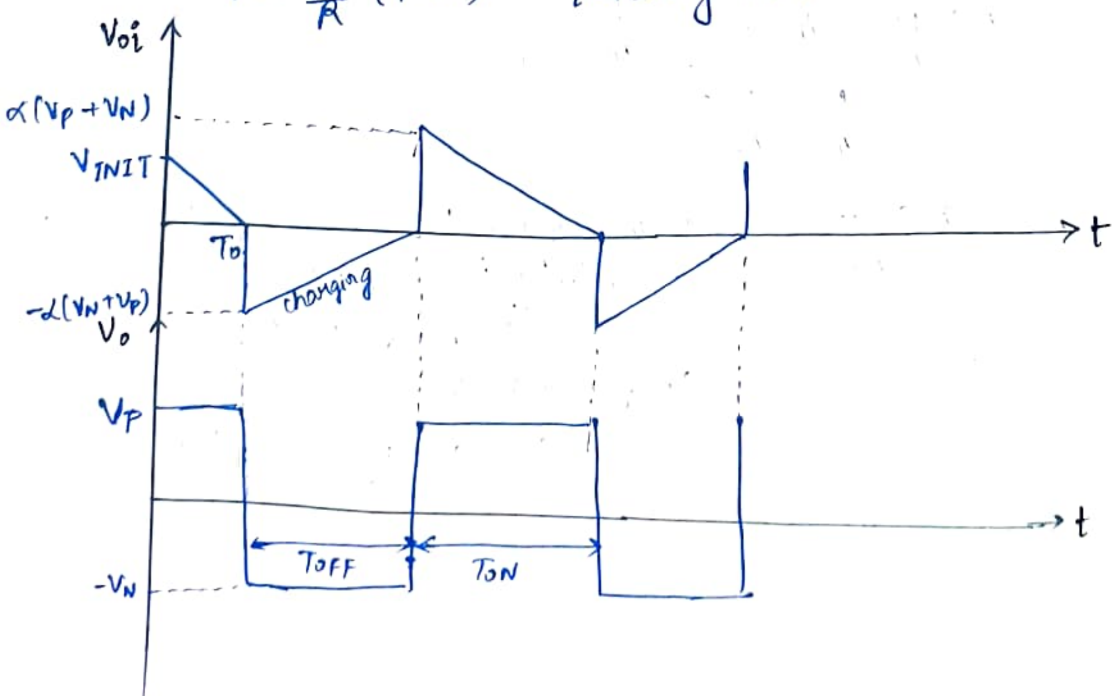
$V_p \approx V_+$ (drop due to C & E drop on o/p side transistor of comparator)

$- V_N \approx -V_+$

$$i = \frac{V_o - V_x}{R}$$

$$i = \frac{V_o}{R}(1-\alpha)$$

$$i = \frac{V_p}{R}(1-\alpha) \quad [\text{During } T_{ON}]$$



At T_0^-,

$V_{oi} \approx 0$, $V_x = \alpha V_P$

$V_{oi} + V_C = V_x$

$\Rightarrow V_C = V_x - V_{oi}$

$\Rightarrow V_C = \alpha V_P$ → stores voltage

$i = \frac{V_0}{R}(1-\alpha)$ → Negative after T_0.

At T_0^+,

$V_C = \alpha V_P$

$V_x = -\alpha V_N$

$V_{oi} = V_x - V_C$

$V_{oi} = -\alpha(V_N + V_P)$

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<u>T_{ON}</u>:

$$V_{oi} = \alpha[V_P + V_N] - \frac{1}{C}\int i\,dt$$

$$= \alpha[V_P + V_N] - \frac{V_P}{RC}(1-\alpha)t \qquad \left[\because i = \frac{V_P[1-\alpha]}{R}\right]$$

$$V_{oi}(T_{ON}) = 0$$

$$\Rightarrow T_{ON} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{V_P + V_N}{V_P}\right)RC$$

$$= \frac{R_x}{R_0}\left(\frac{V_P + V_N}{V_P}\right)RC$$

↳ R_x can be estimated from T_{ON} (timer)

↳ T_{ON} also depends on V_P, V_N.

$$T_{OFF} = \frac{R_x}{R_0}\left(\frac{V_N + V_P}{V_N}\right)RC \qquad [\text{Interchange } V_N \text{ \& } V_P \text{ in } T_{ON}]$$

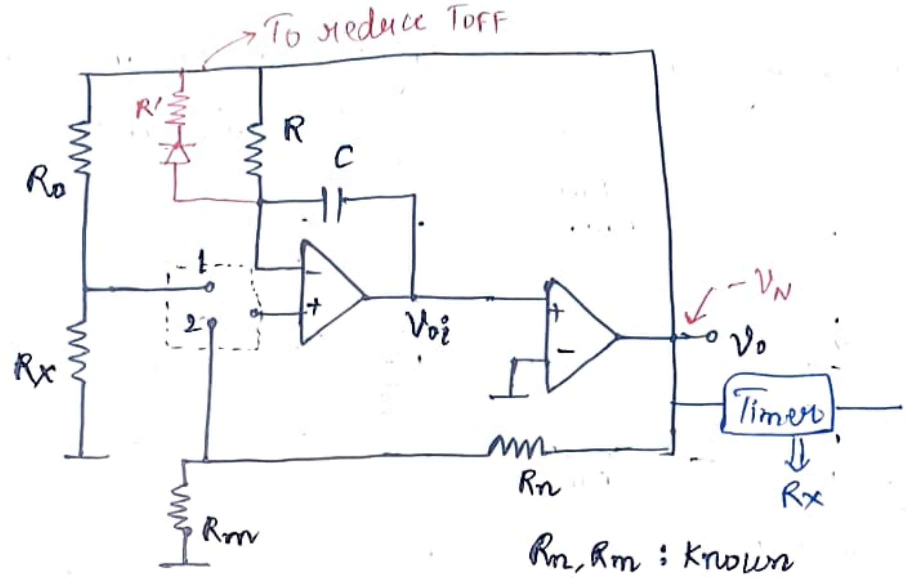
Define $F = \dfrac{T_{ON}\,T_{OFF}}{T_{ON} + T_{OFF}}$

$$= \frac{\left[RC\,\frac{R_x}{R_0}(V_P + V_N)\right]\left[\frac{1}{V_P}\cdot\frac{1}{V_N}\right]}{\left[\frac{1}{V_P} + \frac{1}{V_N}\right]}$$

$$= RC\frac{R_x}{R_0}$$ → If we find F, we can get correct estimation of R_x (indep. of V_P, V_N)

↳ Depends on C (having high drifts).

↳ Replace by 2 resistors.



R_n, R_m : known

Mode-1 : T_{ON_1}, T_{OFF_1}

$$T_{ON_1} = \frac{R_x}{R_0}\left(\frac{V_P+V_N}{V_P}\right)RC$$

Mode-2 : T_{ON_2}, T_{OFF_2}

$$T_{ON_2} = \left(\frac{R_m}{R_n}\right)\left(\frac{V_P+V_n}{V_P}\right)RC$$

$$F = \frac{T_{ON_1}}{T_{ON_2}} = \frac{R_x}{R_0}\cdot\frac{R_n}{R_m}$$ ... (Digital quantity)

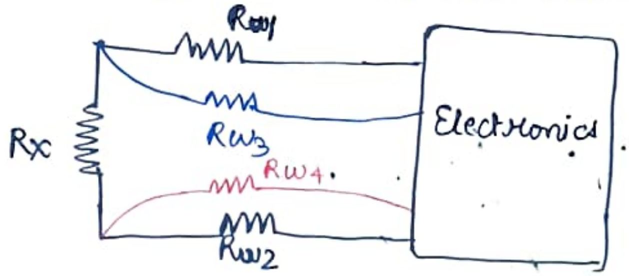
Conversion time $= T_{ON_1} + \underbrace{T_{OFF_1}} + T_{ON_2} + \underbrace{T_{OFF_2}}$

$$= \cancel{\left(\frac{R_x}{R_0}\right)\left(\frac{V_P+V_N}{V_P}\right)RC}$$

We want only T_{ON_1}, T_{ON_2} $\Rightarrow$ Reduce T_{OFF_1}, T_{OFF_2}.

$$T_{OFF} = \frac{R_x}{R_0}\left(\frac{V_N+V_P}{V_N}\right)(R' \parallel R)C$$

Usually, ~~etc~~ sensors are at remote locations connected by long connecting wires (with lead wire resistances).

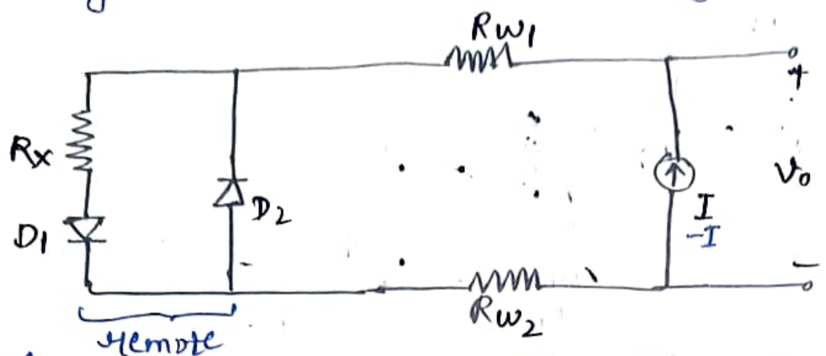


→ 3-wire measurement to mitigate [if $R_{w1} = R_{w2}$]

→ 4-wire measurement

$$R_x' = R_x + R_{w1} + R_{w2}.$$

We may have many sensors. So, try to reduce the wires connecting sensors and electronics by two-wire measurement.



Excite using a current source and measure the voltage.

When $I \to +ve$: current flows through R_{w1}, R_x, D_1, R_{w2}.

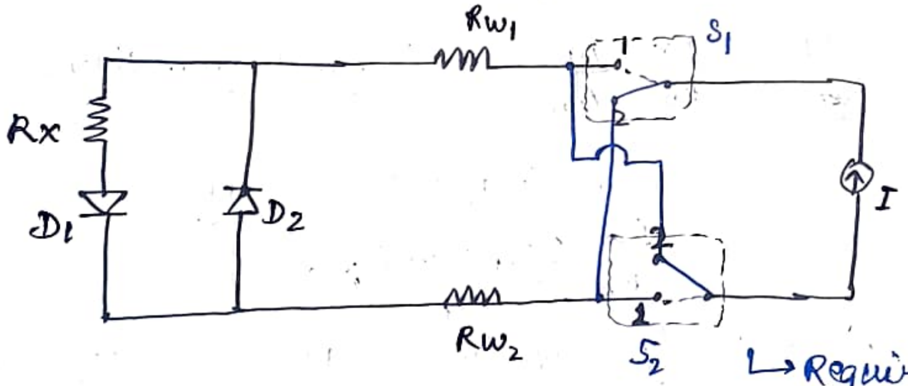
$$V_{o1} = I(R_{ON} + R_x + R_D + R_{w2})$$ [Across current source]

When $I \to -ve$:

$$V_{o2} = -I(R_{w1} + R_{D'} + R_{w2})$$

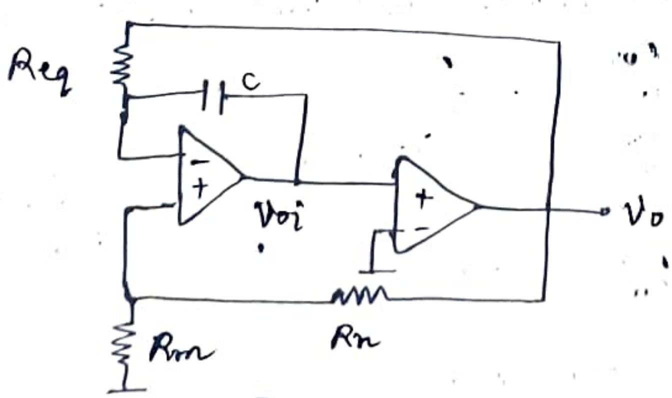
$$\Rightarrow V_{o1} + V_{o2} = IR_x$$ [Non-idealities cancel]

<u>Demerits</u>: Require ~~the~~ two matched diodes with same R_D.

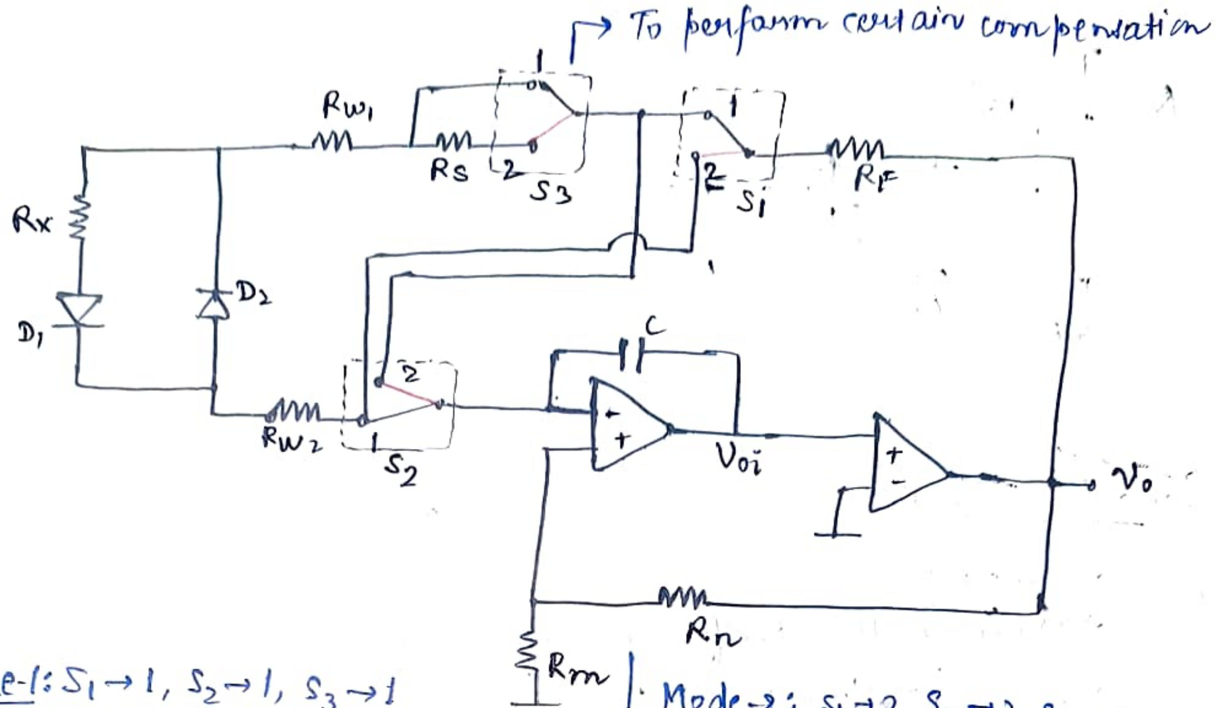


↳ Requires only one current source.

Bring the sensor in the resistive feedback part of the electronics.



$$T_{ON} = \frac{R_m}{R_n}\left(\frac{V_P + V_N}{V_P}\right) R_{eq} C = K R_{eq}.$$



Mode-1: $S_1 \to 1,\ S_2 \to 1,\ S_3 \to 1$ | Mode-2: $S_1 \to 2,\ S_2 \to 2,\ S_3 \to 2$

$\hookrightarrow R_{eq_1} = R_F + R_{S_1} + R_{S_3} + R_{W_1} + R_x + \underset{\text{diode resistance}}{R_D} + R_{W_2} + \underset{\text{switch resistance}}{R_{S_2}}$

$\Rightarrow R_{eq_1} = R_x + \underbrace{R_{com.}}_{\text{all other common resistance}}$

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$$T_{ON_1} = R_{eq_1} \underbrace{C \left(\frac{V_p + V_N}{V_p} \right) \frac{R_m}{R_n}}_{K} = K R_{eq_1}$$

Mode-2:

$S_1 \to 2,\ S_2 \to 2,\ S_3 \to 2$

$R_{eq_2} = R_F + R_{S_1} + R_{W_2} + R_D + R_{W_1} + R_S + R_{S_3} + R_{S_2}$

$\Rightarrow R_{eq_2} = R_S + R_{com}$

$T_{ON_2} = K R_{eq_2}$

Mode-3:

$S_1, S_2 \to 2,\ S_3 \to 1$

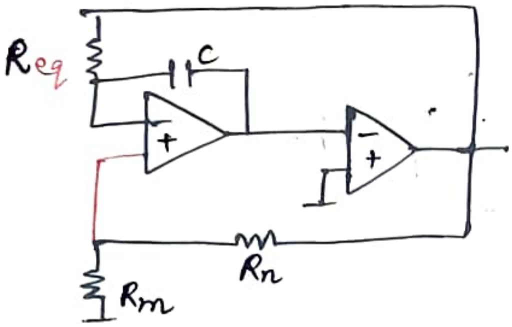
$R_{eq_3} = R_{com}$ (same as mode-2 but R_s is bypassed)

$T_{ON_3} = K R_{eq_3}$

$$F_2 = \frac{T_{ON_1} - T_{ON_3}}{T_{ON_2} - T_{ON_3}}$$

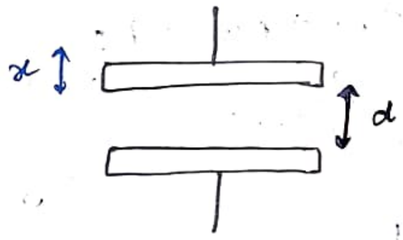
$$= \frac{R_{eq_1} - R_{eq_3}}{R_{eq_2} - R_{eq_3}} = \frac{R_x + \cancel{R_{com}} - \cancel{R_{com}}}{R_s + \cancel{R_{com}} - \cancel{R_{com}}} = \boxed{\frac{R_x}{R_s}}$$

$\to$ indep. of lead wires (R_{W}s)



Capacitive Sensors

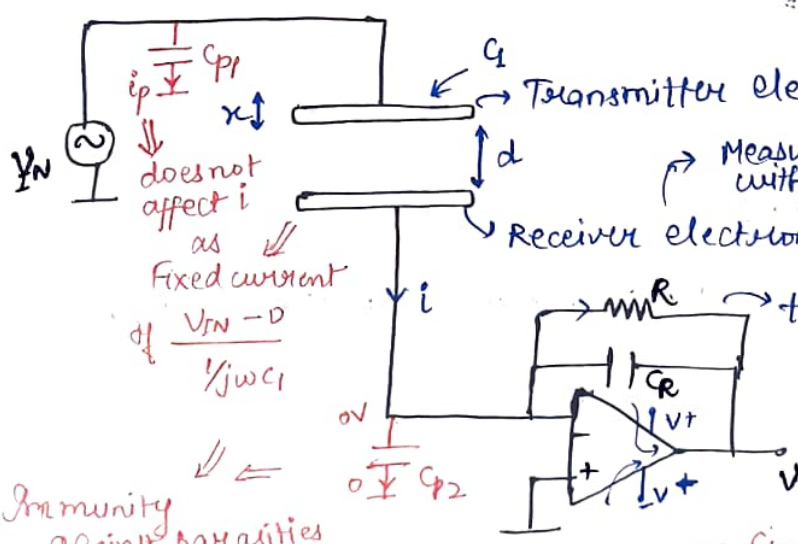
- compact
- No magnetic ~~shielf~~ shielding, only electrical shielding needed
- Low cost, low power



Nominal capacitance, $C_0 = \frac{\epsilon A}{d}$

When displacement b/w the plates change,

$$C_1 = \frac{\epsilon A}{d+x} = \frac{C_0}{1 + x/d}$$



→ Transmitter electrode

→ Receiver electrode

→ Measures without loading

(-ve feedback in amplifier, filter)

→ to permit the flow of bias current

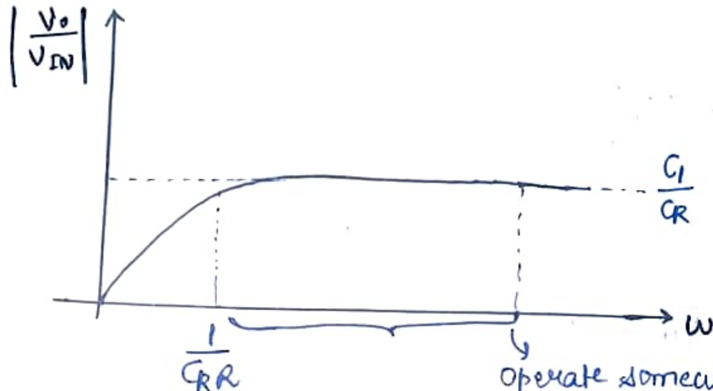
C_R: Known

→ C_{p1}, C_{p2} : parasities associated with tx & rx

C_1 → Practical cap.; made of actual plates

C_R → package.

$$\frac{V_o}{V_{IN}} = \frac{-j\omega C_1 R}{j\omega C_R R + 1} \Rightarrow \left|\frac{V_o}{V_{IN}}\right| = \frac{\omega C_1 R}{\sqrt{\omega^2 \cdot C_R^2 R^2 + 1}}$$



Operate somewhere here (flat region)

Choose $\omega \gg \frac{1}{C_R R}$

ω: freq. of excitation (V_{IN})

$\frac{1}{C_R R}$: corner freq.

$$\frac{V_o}{V_{IN}} = -\frac{C_1}{C_R}$$

[at high freq., $\frac{1}{R} \to 0 \Rightarrow \frac{V_o}{V_{IN}} = \frac{-j\omega C_1}{j\omega C_R}$]

$$V_o = -\frac{C_1 V_{IN}}{C_R}$$

[$C_1 V_{IN} = q_{IN}$]

$C_R \ll 1F$

$$V_o = -\frac{q_{IN}}{C_R}$$

→ Charge amplifier; charge is always conserved; draws ~~to~~ charge from power supply.

$$C_1 = \frac{C_0}{1 + x/d}$$

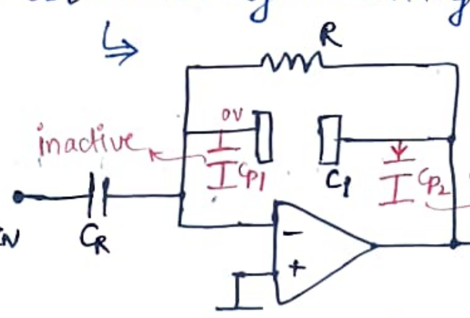
$$\frac{V_o}{V_{IN}} = -\frac{C_0}{C_R(1 + x/d)}$$

$$\boxed{\frac{V_o}{V_{IN}} = -\frac{C_R}{C_1} = -\frac{C_R}{C_0}(1 + x/d)}$$

[To get $V_o \propto x$]

↳ obtain this by interchanging capacitances.

↳



acting like load to opamp
↳ does not affect o/p

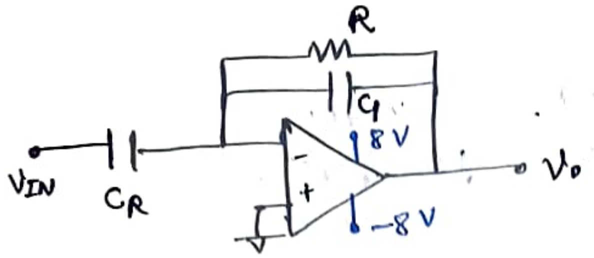
Again, immunity against C_{P1}, C_{P2}.

$$\omega \gg \frac{1}{C_1 R}$$

$x = 0 \Rightarrow$ we have an offset $\left(-\frac{C_R}{C_0}, -\frac{C_0}{C_R}\right) \Rightarrow$ limits sensor operating range

Change in x is very minimal $\Rightarrow$ Larger change in voltage.

$\Rightarrow$ Limits the ~~space~~ span of x (limited by power supply)



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$$\frac{V_o}{V_{IN}} = -\frac{C_R}{C_1} = -\frac{C_R}{C_0}\left[1 + x/d\right]$$

For $V_{IN} = V_m \sin\omega t$,

$$V_o = \underbrace{-\frac{C_R}{C_0}\left[1 + \frac{x}{d}\right] V_m}_{V_{OUT}} \sin\omega t$$

$$V_{OUT} = \frac{C_R}{C_0}\left[1 + x/d\right] V_m < 8\,V$$

$x = 0 \Rightarrow V_{OUT} = \frac{C_R}{C_0} V_m$

$= 7.5\,V$

for $V_m = 5\,V$

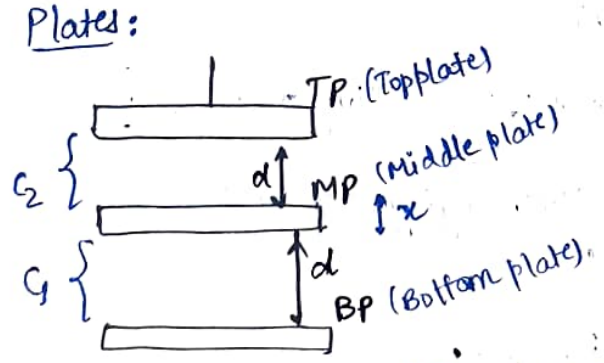
$C_0 = 10\,nF$

$C_R = 15\,nF$

$$7.5\left[1 + \frac{x}{d}\right] < 8\,V$$

$\Rightarrow x < 0.067d$ → Limited by ~~power supply~~ offset voltage

Plates:



→ Push-Pull capacitive sensor

OR

Differential sensor

$x = 0 \Rightarrow$ MP equidistant from TP & BP.

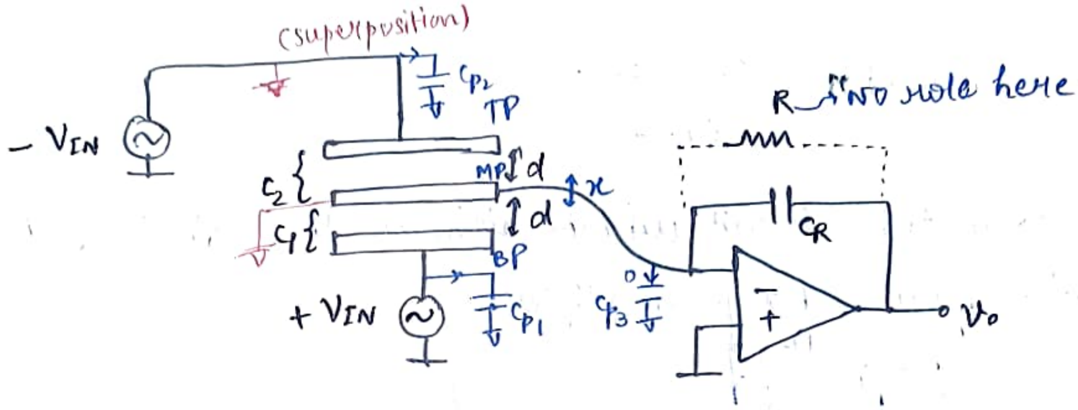
At $x = 0$, $C_1 = C_2 = C_0$

$$C_1 = \frac{C_0}{1 + x/d}$$

$$C_2 = \frac{C_0}{1 - x/d}$$

$$C_2 - C_1 = \frac{2\boxed{C_0}\, x/d}{1 - x^2/d^2}$$

$$x = 0 \Rightarrow C_2 - C_1 = 0$$



By superposition,

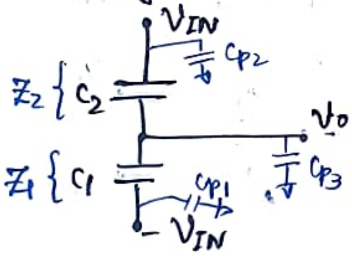
$$V_o = -\frac{C_1}{C_R} V_{IN} + \frac{C_2}{C_R} V_{IN}$$

$$\Rightarrow V_o = \frac{(C_2 - C_1)}{C_R} V_{IN}$$

→ No offset.
→ Non-linear w.r.t. x
→ Immunity against parasitics

↳ depends on $(C_2 - C_1)$ → depends on $C_0 \left(= \frac{\epsilon_0 A}{d}\right)$
↓
depends on capacitor dimension

Mitigation:



$$V_o = \frac{Z_1 V_{IN}}{Z_1 + Z_2} - \frac{Z_2 V_{IN}}{Z_1 + Z_2} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right) V_{IN} = \left(\frac{x}{d}\right) V_{IN}$$

$$Z_1 = \frac{1}{j\omega C_1} = \underbrace{\frac{1}{j\omega C_0}}_{\text{Nominal impedance}} [1 + x/d] = Z_0 [1 + x/d]$$

$$Z_2 = Z_0 [1 - x/d]$$

$$\Rightarrow V_o = \left(\frac{x}{d}\right) V_{IN}$$

→ Linear
→ No offset
→ Indep. from C_0.

Parasitic effect (C_p):

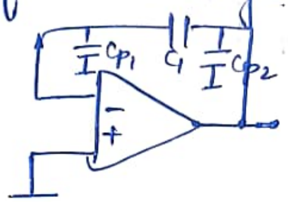
$C_{p1}, C_{p2} \rightarrow$ only draws some current $\Rightarrow$ No effect on o/p voltage

$C_{p3} \rightarrow$ affects V_o

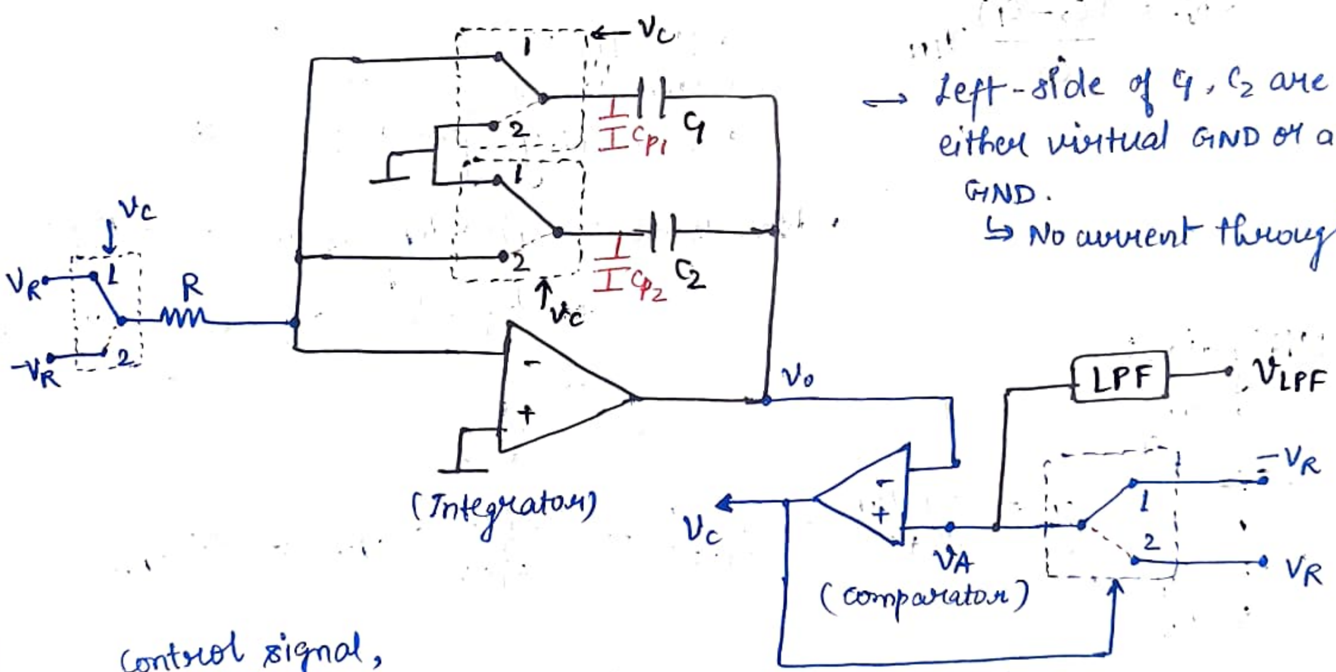
Ratiometric operation : $\left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)$

Make a ratiometric circuit which without the effect of C_p.

[Inspiration from charge amplifier:



]



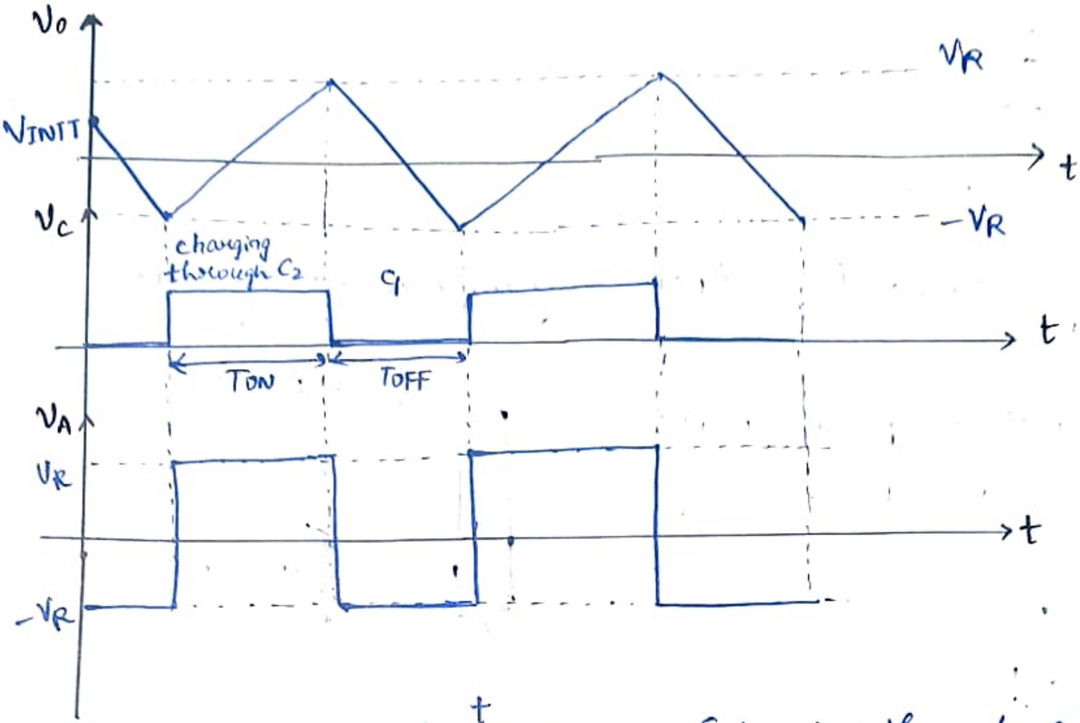
→ Left-side of C_1, C_2 are at either virtual GND or actual GND.

↳ No current through C_p's.

control signal,

V_C = Logic-low $\Rightarrow$ sw → 1

Logic-high $\Rightarrow$ sw → 2



$$V_o = -V_R + \frac{1}{C_2}\int_{T_0}^{t} i\,dt \quad \text{[Charging through } C_2\text{]}$$

$$= -V_R + \frac{1}{C_2}\int_{T_0}^{t} \frac{V_o}{R}\,dt$$

$$\Rightarrow V_o = -V_R + \frac{V_R}{RC_2}(t - T_0) \quad T_0 + T_{ON}$$

and, $V_o(T_0 + T_{ON}) = V_R$

$$\Rightarrow \begin{cases} T_{ON} = 2RC_2 \\ T_{OFF} = 2RC_1 \end{cases}$$

$$\left(\frac{T_{ON} - T_{OFF}}{T_{ON} + T_{OFF}}\right) = \frac{C_2 - C_1}{C_2 + C_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{x}{d}$$

$$avg(V_A) = \frac{V_R T_{ON} - V_R T_{OFF}}{T_{ON} + T_{OFF}}$$ → to get T_{ON}, T_{OFF} ratiometric

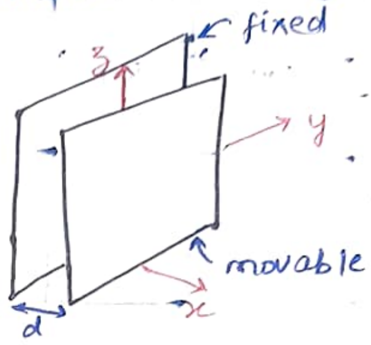
$$\Rightarrow avg(V_A) = V_R \frac{T_{ON} - T_{OFF}}{T_{ON} + T_{OFF}} = V_R \frac{x}{d}$$

$$= V_{LPF}$$

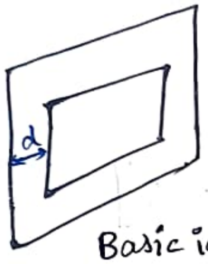
→ constant o/p for any given x, along with all other benefits.

$C = \frac{\epsilon A}{d}$, $V_o = F(C_o)$

- If our axis of interest is x, then any movement in z is undesired.

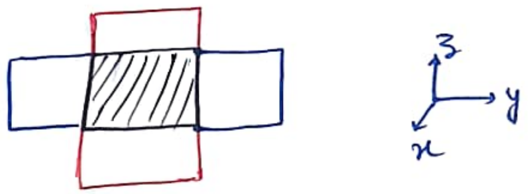


- To mitigate that, keep one plate smaller than the other.



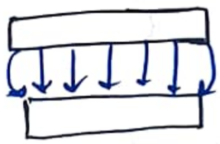
Basic idea

↓



- Any movement in x and y axis won't change effective overlap area.

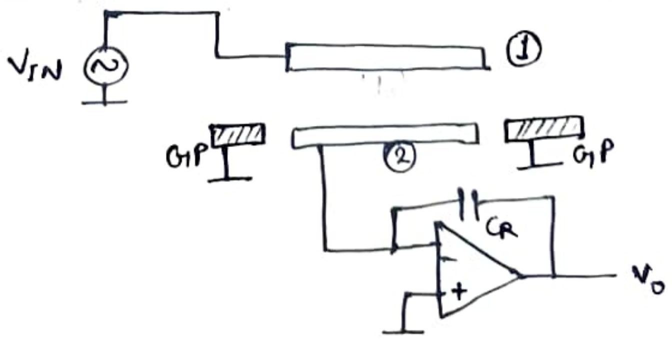
- Fringing effect:



$C_o = \frac{A\epsilon}{d}$

$C_{actual} \neq \frac{A\epsilon}{d}$

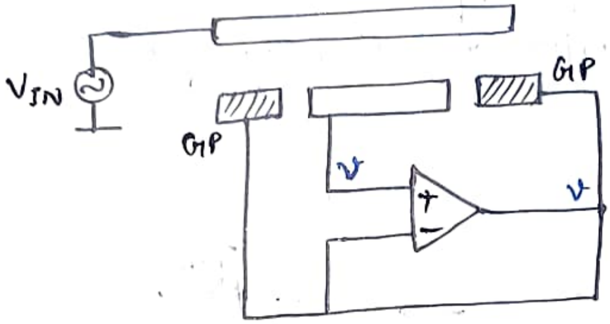
- To mitigate, use guard plate (GP):



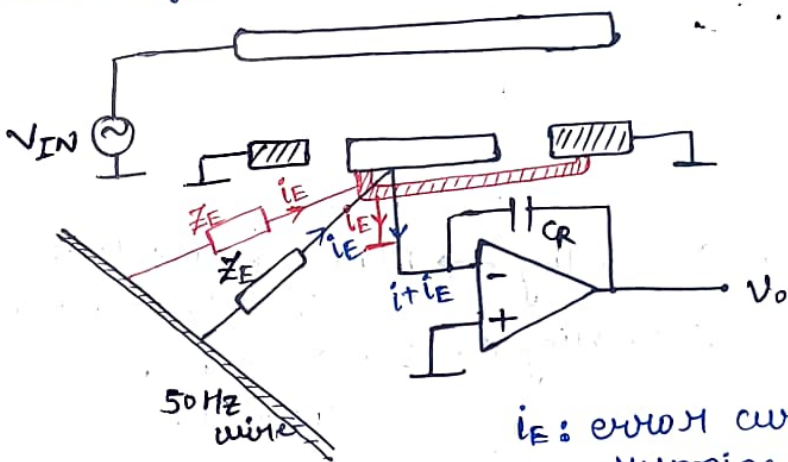
- Now, plate ② and guard plate should be at the same potential.
- Current due to fringing grounded to GP, hence won't contribute to the actual current.

How to use the concept of guarding for voltage measurement or results ?

↪ connect GP to the output of the opamp.



- Power-line interference :

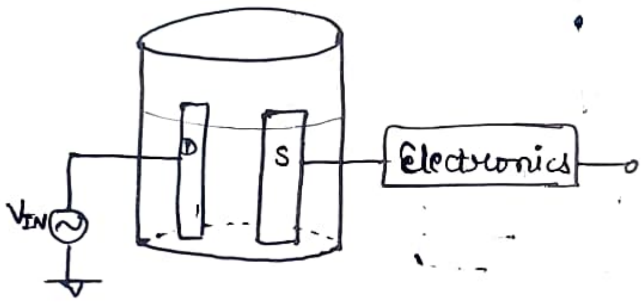


i_E : error current to 50 Hz running power supply.

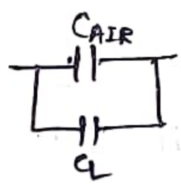
- To mitigate this, guard is suitably extended.
- Now, interference current is passed to gnd through guard plate shield.

Liquid Level Monitoring sensor

- Fuel level, coolant level.
- Capacitive system ⟷ Volumetric property
- Optical system ⟶ Surface property.
- Capacitive system have no sloshing system.
- Interference between two liquids.
- Independent of conductivity and viscosity of liquids.
- Dirty liquids.



⟶



Real Issues:

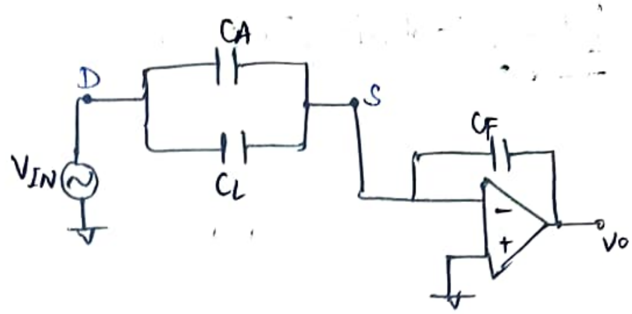
Tank can be Conducting → Place D & S inside the tank
(e-field cannot penetrate)

Insulating → D & S can be placed inside or outside the tank.

Liquid can be conducting → Electrodes should be insulated in case they are placed inside the tank.

Dielectric → works fine in either case.

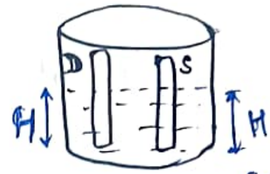
Dielectric Liquid



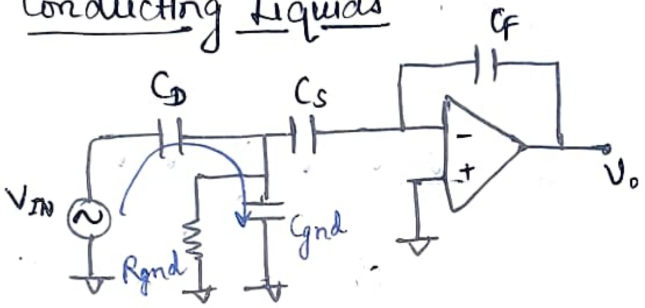
$C_L \gg C_A$

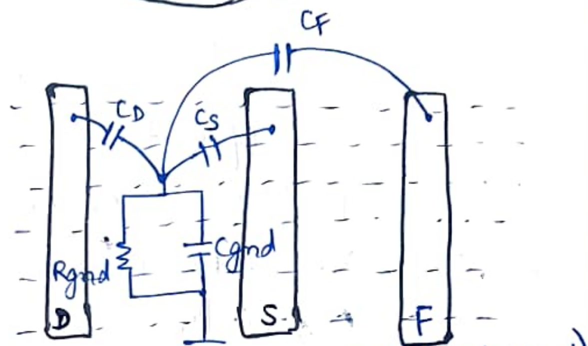
$$\frac{V_o}{V_{IN}} = -\frac{C_L}{C_F}$$

$H \uparrow,\ C_L \uparrow,\ V_o \uparrow$



Conducting Liquids





(extra electrode for feedback)

$C_{gnd} \gg C_D / C_S$

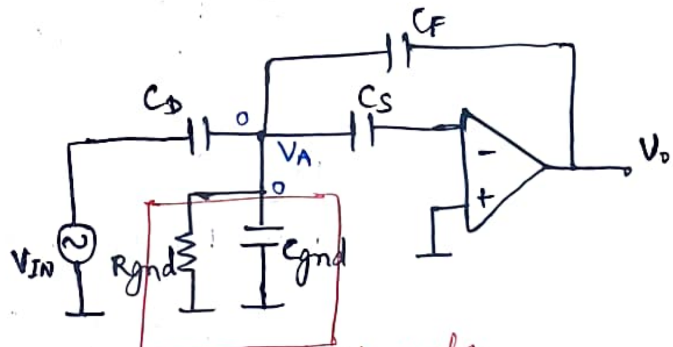
$\frac{1}{sC_{gnd}}$ will be very small, hence provide a gnd to V_{IN}.

$V_o \to 0$

- To mitigate this, bring one more electrode.
- C_F is now a feedback capacitor.

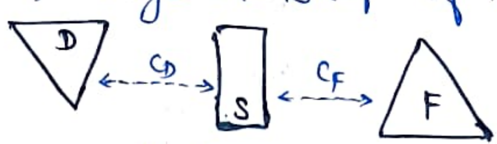
$V_A = 0$ (virtual gnd)

$$\frac{V_o}{V_{IN}} = -\frac{C_D}{C_F}$$



× No role

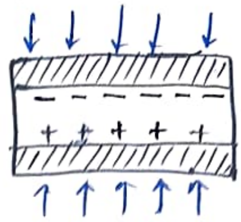
- But now $C_D = C_F$, it will be a constant output. [cannot get $V_o \propto$ height]
- To mitigate, change the shape of the electrode.



- Push-pull mechanism
- A good sensitivity.
  ↳ $\frac{C_D}{C_F} \uparrow$ with height

Book: Larey K. Baxter – Capacitive Sensors (For more detail about capacitive sensors)

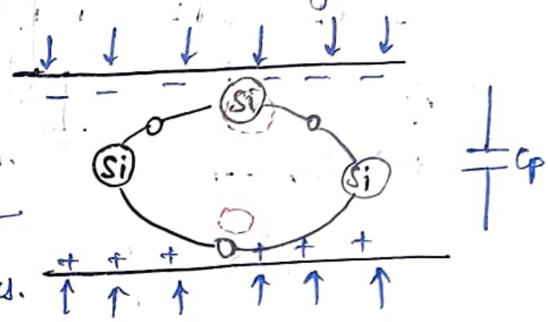
Piezoelectric Sensors



Material stress $\xrightarrow{\text{Piezoelectric}}$ Electric potential
(sensitivity of force, acceleration, pressure, ultrasonic receiver)

Electric potential $\xrightarrow{\text{inverse-piezo-electric}}$ Material stress
(actuators or development of ultrasonic transmitter)

- Piezo or inverse piezo-effect can be observed in asymmetrical crystal lattice (SiO_2).



If we apply stress, Si-atom will suffer less movement than O-atom.
⇒ Leads to relative displacement of +ve and -ve charge carriers.
⇒ Leads to potential difference.

$$\underset{\text{charge}}{q} \propto \underset{\text{force}}{F}$$

$\Rightarrow q = df$,
where d: charge sensitivity of piezo-material.

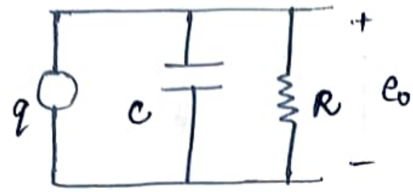
For SiO_2, $d = 2.3\ pC/N$

For $BaTi_2O_3$, $d = 140\ pC/N$

$C_p \rightarrow$ capacitance of the crystal.

$$v = \frac{q}{C_p} = \underbrace{\left(\frac{d}{C_p}\right)}_{\text{crystal voltage sensitivity}} f$$

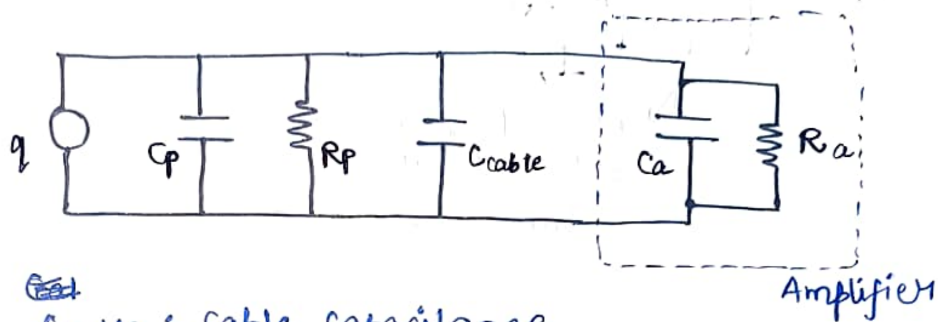
Equivalent circuit :



$R = R_p \parallel R_a$

$C = C_p + C_{cable} + C_a$

Actual circuit:



C_{cable} : cable capacitance

C_a : amplifier capacitance

R_a : amplifier resistance

Apply KCL,

$$\frac{dq}{dt} = C\frac{de_o}{dt} + \frac{e_o}{R}$$

$$\Rightarrow R\frac{dq}{dt} = RC\frac{de_o}{dt} + e_o$$

Apply Laplace,

~~Rsq~~

$$RsQ(s) = RC\,sE_o(s) + E_o(s)$$

$$\Rightarrow E_o(s) = \frac{RsQ}{1+sRC}$$

$$\Rightarrow \frac{E_o(s)}{Q} = \frac{Rs}{1+sRC}$$

Also, $Q(s) = dF(s)$.

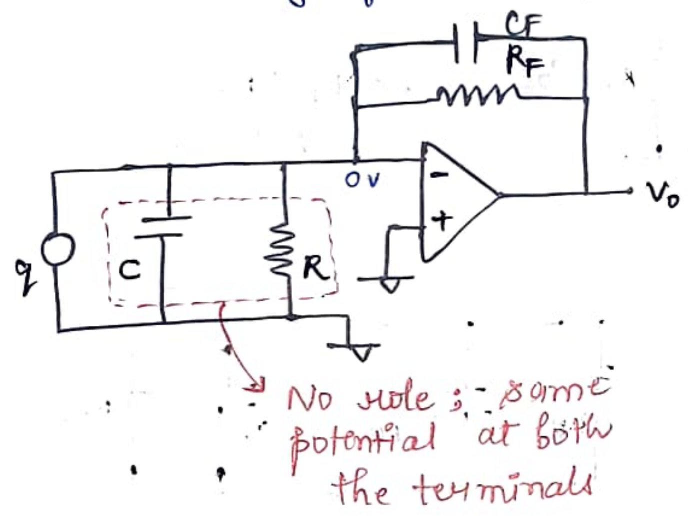
$$\Rightarrow \frac{E_o(s)}{F(s)} = \frac{Rds}{1+sRC}$$

$\because \tau = RC$

$$\therefore \frac{E_o(s)}{F(s)} = \frac{\tau \frac{d}{C} s}{\tau s + 1}$$

→ High-pass response

→ Piezo-electric can only sense varying (dynamic) forces but not steady forces.



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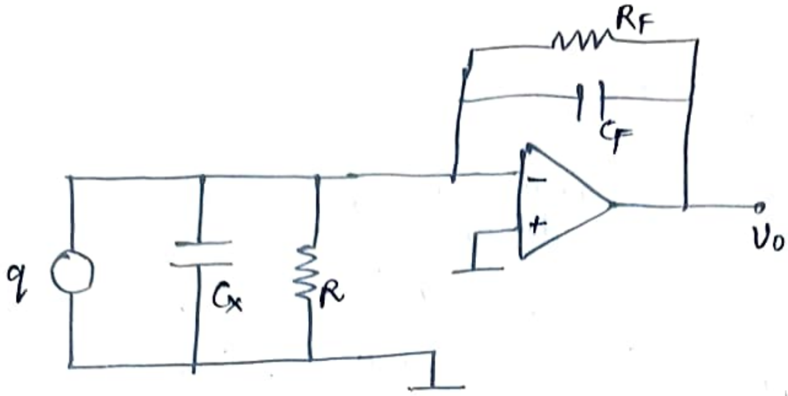


$$\frac{V_o}{F}(s) = \frac{-\tau\left(\frac{d}{C_F}\right)s}{\tau s + 1},$$

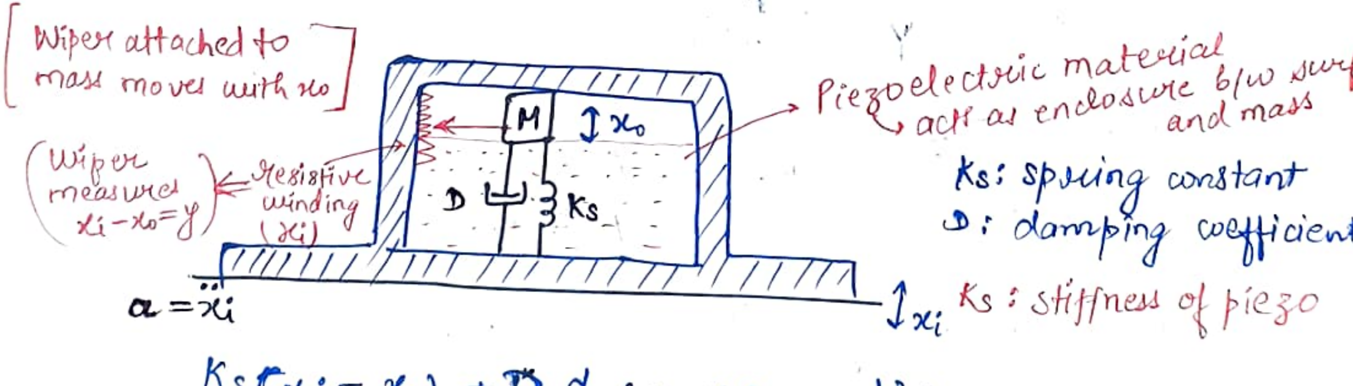
$$\tau = R_F C_F$$

$$\frac{dq}{dt} + \frac{V_o}{R_F} + C_F\frac{dV_o}{dt} = 0$$

$$R_F C_F \dot{V_o} + V_o = -R\dot{q}$$

Piezoelectric ~~Accelerati~~ Accelerometer :

↳ No reference frame (cannot fix any part) x_i : I/p displacement
$a = \ddot{x}_i \rightarrow$ to measure



Ks: spring constant
D: damping coefficient
Ks : stiffness of piezo

$$K_s(x_i - x_o) + D\frac{d}{dt}(x_i - x_o) = M\frac{d^2x_o}{dt^2}$$

Define $y = x_i - x_o \rightarrow$ relative displacement of x_i wrt. x_o.

$$\Rightarrow K_s y + D\frac{dy}{dt} = M\frac{d^2}{dt^2}(x_i - y)$$

$$\Rightarrow M\frac{d^2y}{dt^2} + D\frac{dy}{dt} + K_s y = M\frac{d^2x_i}{dt^2}$$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{D}{M}\frac{dy}{dt} + \frac{K_s}{M}y = a$$

$$\Rightarrow \ddot{y} + \frac{D}{M}\dot{y} + \frac{K_s}{M}y = a$$ → In inertial accelⁿ, 'a' can be found by determining 'y'.

$$\Rightarrow \frac{Y(s)}{A(s)} = \frac{1}{s^2 + \frac{D}{M}s + \frac{K_s}{M}}$$ → 2nd-order behaviour
↳ use ξ (damping ratio), ω_n (natural freq.).

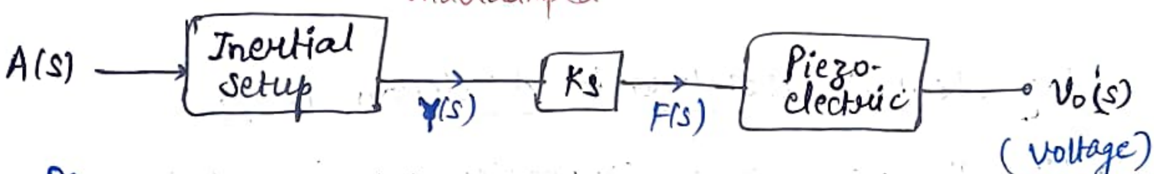
$$= \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where,

$$\omega_n = \sqrt{\frac{K_s}{M}}$$

$$\downarrow \xi = \frac{D}{2\sqrt{K_s M}}$$ → contributed due to friction (↓)

↳ Underdamped



Piezoelectric material experiences 'y' & thus a force

$$\left\{ \frac{F(s)}{Y(s)} = K_s \right.$$

$$\frac{V_o}{A}(s) = \frac{V_o}{F}(s) \cdot \frac{F}{Y}(s) \cdot \frac{Y(s)}{A(s)}$$

$$\frac{V_o}{A}(s) = \frac{\tau \frac{d}{c} s}{\tau s + 1} \cdot K_s \cdot \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2}$$ → 3rd-order instrument

Total dynamic force in the system,

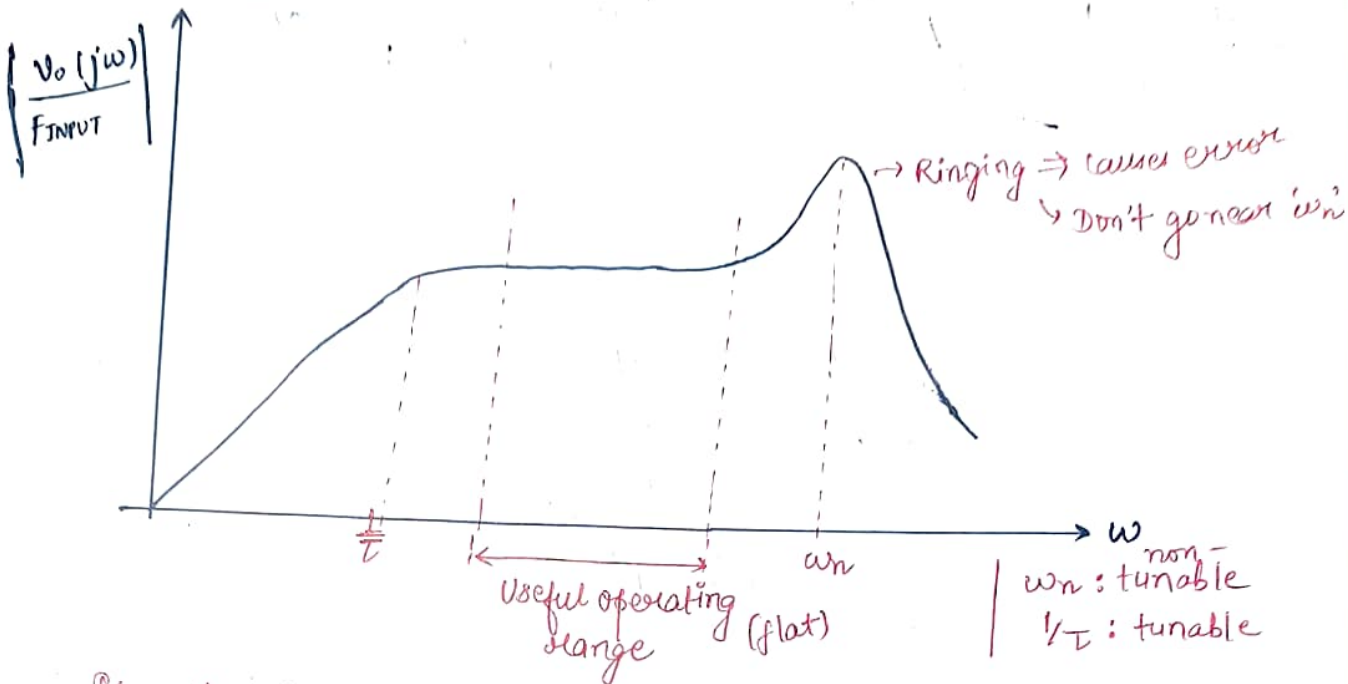
$F_{INPUT}(s) \approx M\,A(s)$ [Total mass $\approx$ A(s)]

$$\therefore \frac{V_o}{F_{INPUT}}(s) = \frac{\tau \frac{d}{c} s}{\tau s + 1} \cdot \frac{K_s/M}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

O/p w.r.t. I/p

$$= \frac{\tau\, d/c\; s}{\tau s + 1} \cdot \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

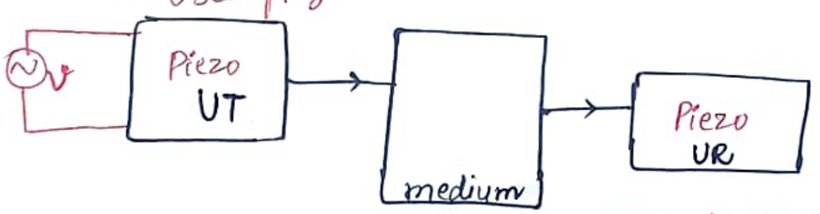
Take $s = j\omega$.



Piezoelectric accelerometer: Gives direct voltage output without requiring any batteries.

Ultrasonic Measurement System

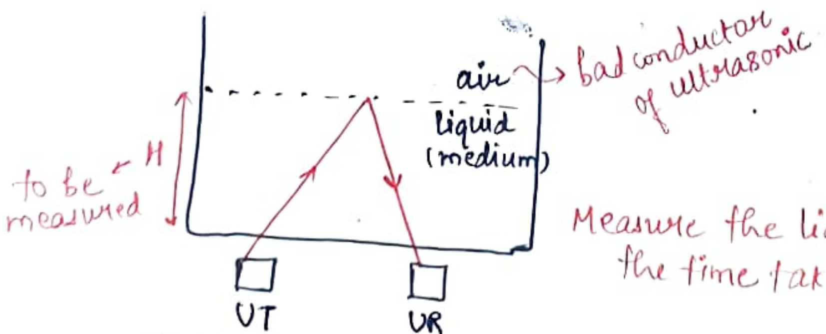
↳ Uses piezo



↳ contains info. about the physical quantity to measure

UT: Ultrasonic Transmitter

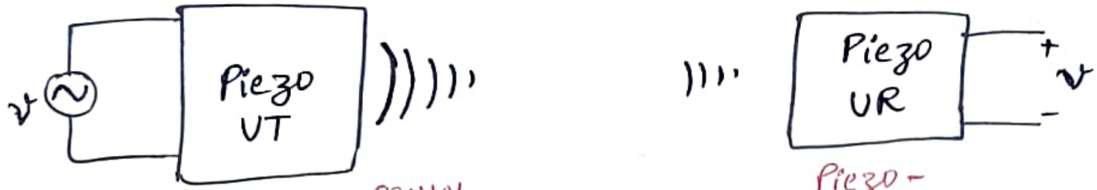
UR: Ultrasonic Receiver



Measure the liquid level by measuring the time taken by wave to reach UR.

Applications:
- level
- Flow rate
- Crack detection
- Blood flow rate

} measurement



Inverse Piezo-electric effect

causes vibration coupled through the medium (mechanical deformation)

Piezo-electric

Charge developed in piezo, $q = df$, d: charge sensitivity [UR]

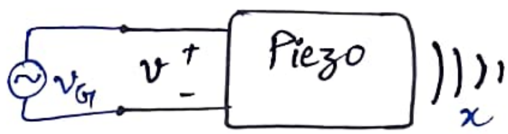
UT: $xq = dfx$

$x = d\left(\frac{fx}{q}\right)$ → work done per unit charge = voltage

$x = dv$

14-11-2025

Ultrasonic Transmitter

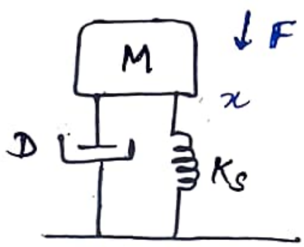


$x = dv$

K_s → Stiffness

$F = K_s x = (dK_s) v$

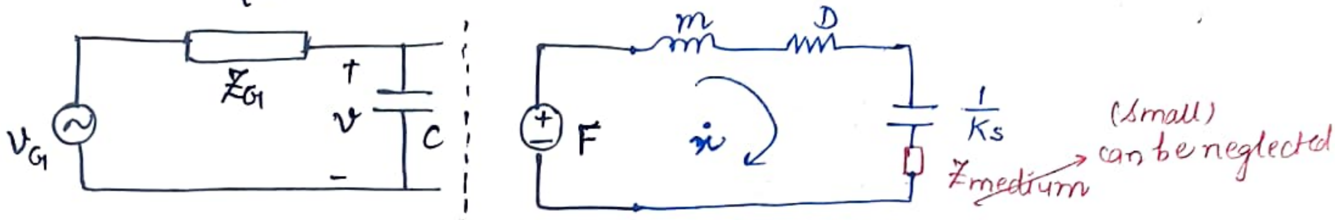
Equivalent Circuit:



$Z_{medium} \approx 0$

$F = m\ddot{x} + D\dot{x} + K_s x$

Electrical Equivalent:



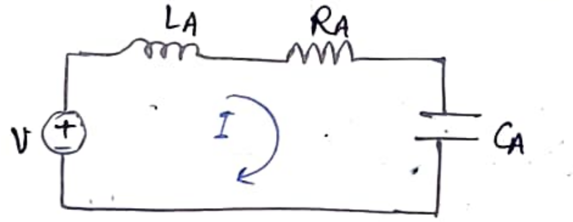
electromechanical
transduction

$$F = (d K_s) v$$

$$\text{Turn's ratio} = d K_s$$

Electrical model:

$$F = m\ddot{x} + D\dot{x} + K_s x$$



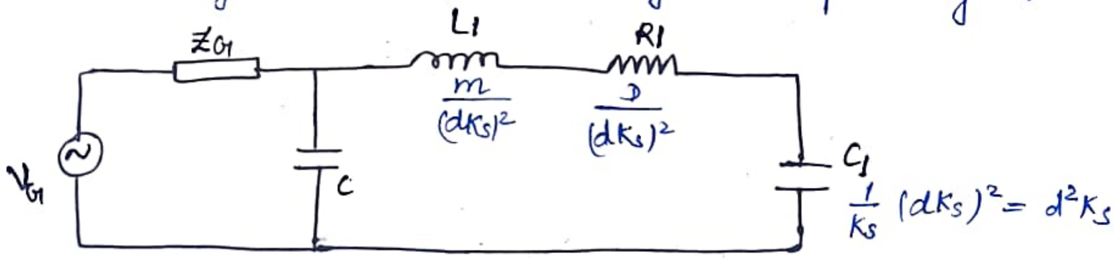
$$V = L_A \dot{I} + R_A I + \frac{1}{C_A}\int I\,dt$$

$F \longleftrightarrow V$ $\qquad$ $m \longleftrightarrow L_A$

$\dot{x} \longleftrightarrow I$ $\qquad$ $D \longleftrightarrow R_A$

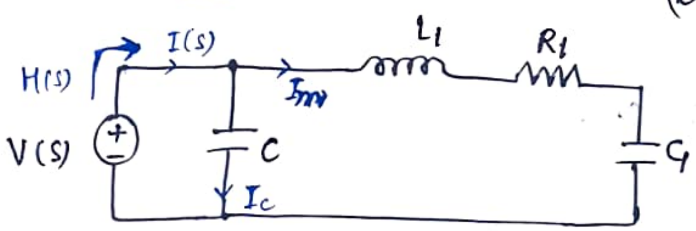
$K_s \longleftrightarrow \frac{1}{C_A}$

For easier analysis, move the secondary to the primary side.



↳ Butterworth Van Dyke model
(BVD model)

OR



$$H(s) = \frac{V(s)}{I(s)}$$

$$I(s) = I_c(s) + I_m(s)$$

$$I(s) = sC\,V(s) + \frac{V(s)}{sL_1 + R_1 + \frac{1}{sC_1}}$$

Put $s = j\omega$.

$$H(j\omega) = \frac{j(\omega^2 L_1 C_1 - 1) + \omega C_1 R_1}{j\omega^2 C C_1 R_1 + \omega(C + C_1) - \omega^3 L_1 C C_1}$$

Series resonance @ ω_s [due to L_1, C_1]

Numerator $\rightarrow$ minimum [Impedance, $H(j\omega) \rightarrow$ min]

$$\Rightarrow \omega_s^2 L_1 C_1 - 1 = 0$$

$$\Rightarrow \omega_s = \frac{1}{\sqrt{L_1 C_1}} = \sqrt{\frac{K_s}{M}} = \omega_n$$

[Series resonance appears at natural frequency]

$$L_1 \rightarrow \frac{m}{(dK_s)^2} \qquad C_1 \rightarrow d^2 K_s$$

Parallel resonance @ ω_p

Denominator $\rightarrow$ minimum [$\omega(C+C_1) - \omega^3 L_1 C C_1 = 0$]

[Imp., $H(j\omega) \rightarrow$ max]

$$\omega_p(C + C_1) = \omega_p^3 L_1 C C_1$$

$$\Rightarrow \omega_p^2 = \frac{C + C_1}{L_1 C C_1} = \frac{1}{L_1 C_1}\left[1 + \frac{C_1}{C}\right]$$

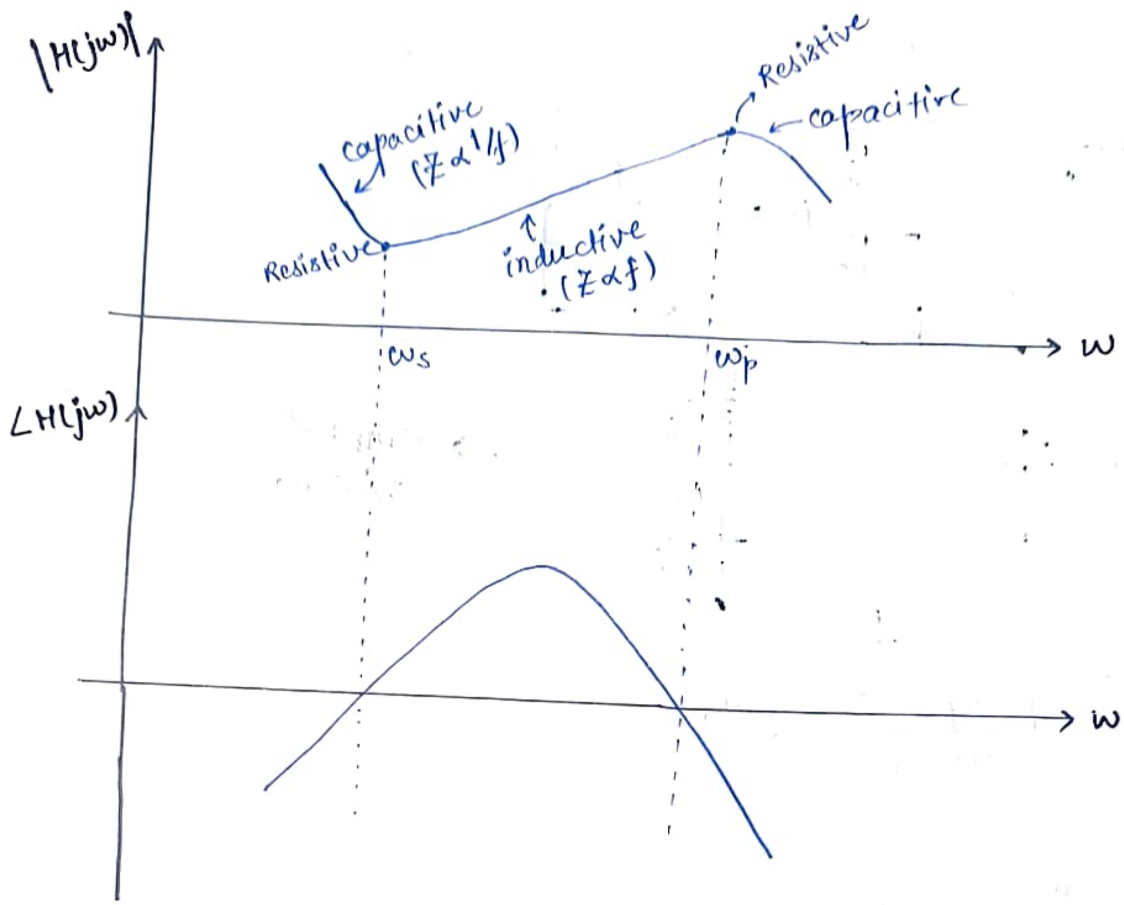
$$\Rightarrow \omega_p^2 = \omega_s^2\left[1 + \frac{C_1}{C}\right]$$

$$\therefore \omega_p = \omega_s\sqrt{1 + \frac{C_1}{C}}$$

$$C_1 \ll C$$

C_1: mechanical ~~electrical~~ side cap. C: electrical ~~mechanical~~ side cap.

$$\Rightarrow \omega_p \approx \omega_s.$$



@ $\underline{\omega_s}$:

- $|H(j\omega)|$ is minimum.
- $\dot{x}$ is high
- Mechanical power $[F\dot{x}]$ is high.

$\begin{bmatrix} \dot{x} \leftrightarrow I \\ F \leftrightarrow V \end{bmatrix}$

$\Rightarrow$ Operate at $\omega = \omega_s$ for maximum power output.
"Ultrasonic cleaning" $\rightarrow$ Application

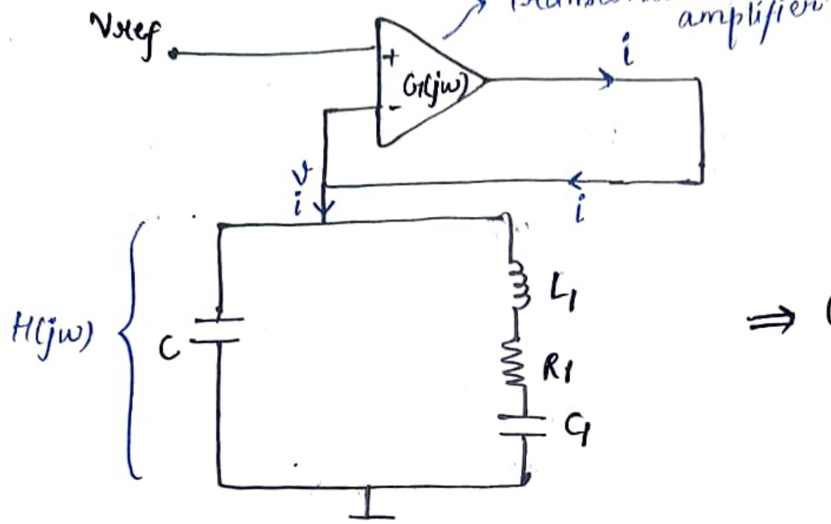
@ $\underline{\omega_p}$:

- $|H(j\omega)|$ is maximum.
- $\dot{x}$ is low.
- $F\dot{x}$ is low.

$\Rightarrow$ Z_{medium} will be very low
$\Rightarrow$ Precise low power.

- Bio-medical application.

Operate the crystal at one of the resonance frequencies:



⇒ Crystal Oscillator

$i = [V_{ref} - v]\, G(j\omega),$

$v = i\, H(j\omega)$

$$\Rightarrow \frac{i}{V_{ref}} = \frac{G(j\omega)}{1 + GH(j\omega)}$$

For resonance, $GH(j\omega) = -1$

$$\Rightarrow |G(j\omega_s)| = \frac{1}{|H(j\omega_s)|},$$

$$\angle G(j\omega) + \angle H(j\omega) = 180^\circ$$

$$\Rightarrow \angle G(j\omega_s) = 180^\circ - \angle H(j\omega_s).$$

If @ ω_s — $|H(j\omega)| = 1/5 \Rightarrow |G(j\omega)| = 5$

$\angle H(j\omega) = 20^\circ \Rightarrow \angle G(j\omega) = 160^\circ$

17-11-2025

Eg. $m = 0.05\ kg$

$K_s = 2 \times 10^9\ N/m$

$D = 200\ Ns/m$

$d = 2 \times 10^{-10}\ C/N$

$C = 1.6\ nF.$

(a) Series resonant frequency (ω_s)

$$\omega_s = \sqrt{\frac{K_s}{M}} = \sqrt{\frac{2 \times 10^9}{0.05}} = 2 \times 10^5\ rad/s.$$

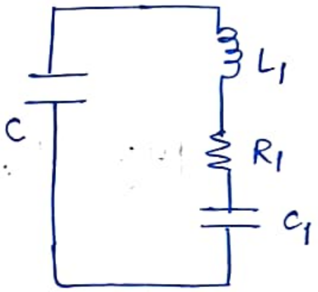
$$f_s = \frac{\omega_s}{2\pi} = 31.83\ KHz.$$

(b) Estimate parameters of BVD model.

$$L_1 = \frac{m}{(dK_s)^2} = \frac{0.05}{0.16} = 312.5\ mH$$

$$R_1 = \frac{D}{(dK_s)^2} = \frac{200}{0.16} = 1250\ \Omega$$

$$C_1 = d \cdot dK_s = 2 \times 10^{-10} \times 0.4 = 80\ pF.$$



$dK_s = 0.4$

$\therefore C_1 \ll C$, so $\omega_p \approx \omega_s$.

(c) Parallel resonant frequency.

$$\omega_p = \omega_s \sqrt{1 + \frac{C_1}{C}}$$

$$= 2 \times 10^5 \sqrt{1 + \frac{80}{1600}}$$

$$f_p = \frac{\omega_p}{2\pi} = 32.6\ KHz$$

$\therefore f_s \approx f_p.$

(d) Design an amplifier such that crystal operates at series resonance.

$$H(j\omega_n) = \frac{j[\omega_n^2 L_1C_1 - 1] + \omega_n^2 R_1 C_1}{j\omega_n^2 R_1 C C_1 + \omega_n (C + C_1) - \omega_n^3 L_1 C C_1}$$

$$\left[\omega_n = \omega_s = \frac{1}{\sqrt{L_1C_1}}\right]$$

$$\Rightarrow H(j\omega_n) = \frac{R_1 C_1}{j\omega_n R_1 C C_1 + C_1} = \frac{R_1}{j\omega R_1 C + 1}$$

$$= \frac{1250}{j2\times10^5 \times 1250 \times 1.6\times10^{-9} + 1}$$

$$= \frac{1250}{0.4j + 1}$$

$$|H(j\omega_n)| = \frac{1250}{\sqrt{0.16+1}} = 1160.1$$

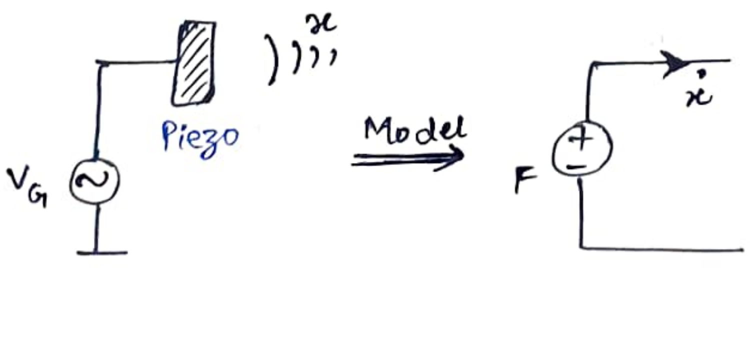
$$\angle H(j\omega_n) = -\tan^{-1}(0.4) = -22^\circ$$

$$|G(j\omega_n)| = \frac{1}{|H(j\omega_n)|} = \frac{1}{1161}$$

$$\angle G(j\omega_n) = 180^\circ - \angle H(j\omega_n) = 202^\circ \Downarrow -158^\circ$$

Specifications of amplifier.

Applications of Ultrasonic Transmission :

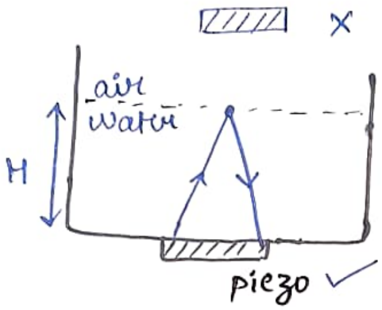


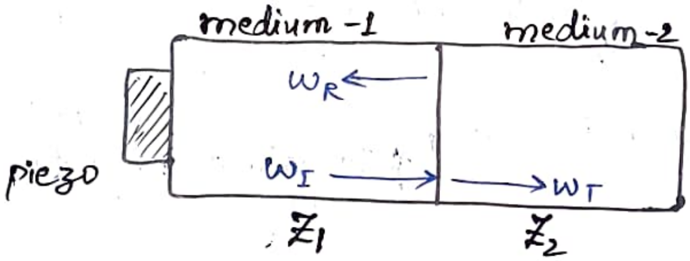
$$\left.\begin{array}{l}\text{Mechanical power} = F\dot{x} \\ \text{Mechanical impedance} = \dfrac{F}{\dot{x}}\end{array}\right\}\ \text{From equivalent circuit}$$

Mechanical terms:

Mechanical power intensity (W) = stress × velocity

Mechanical impedance $(Z) = \dfrac{\text{stress}}{\text{velocity}}$





Z-values:
- air : 430
- quartz : 1.5×10^7
- water : 0.15×10^7
- steel : 4.7×10^7
- AL : 1.7×10^7

W_I : mechanical power intensity incident on the medium at the interface.

Reflection coefficient $(\alpha_R) = \dfrac{W_R}{W_I} = \dfrac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$

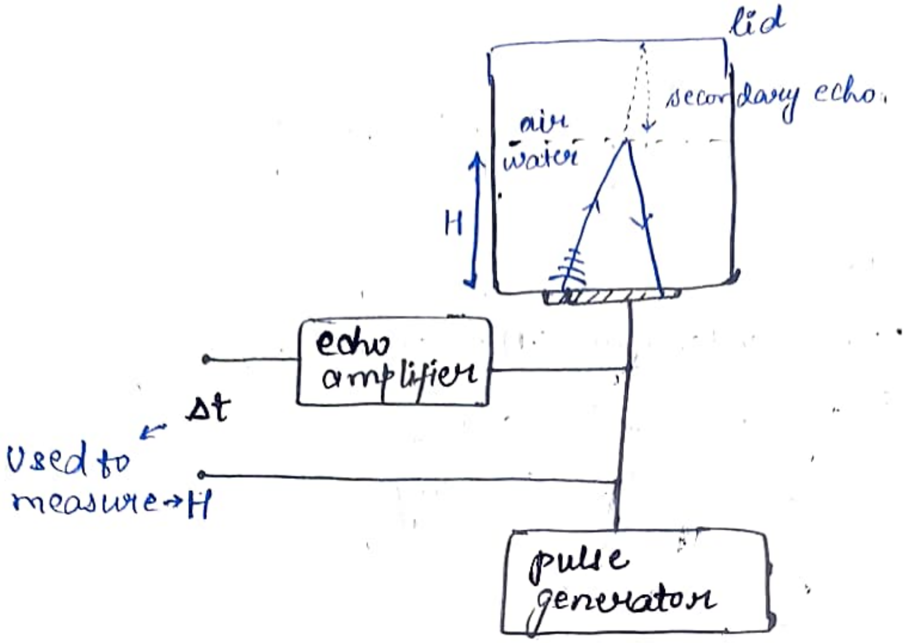
Transmission Coefficient $(\alpha_T) = \dfrac{W_T}{W_I} = \dfrac{4 Z_2 Z_1}{(Z_2 + Z_1)^2}$

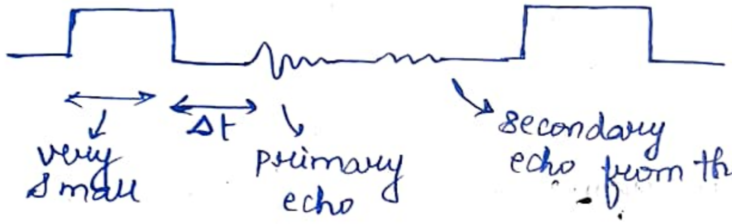
α_R, α_T : Known.

$$\boxed{\alpha_R + \alpha_T = 1}$$

$Z_2 \ll Z_1 \Rightarrow \alpha_R \to 1,\ \alpha_T \to 0 \Rightarrow$ No effective transmission

- Z_{air} is less than many Z_{medium}, so most of the power gets reflected.





↳ may be needed in some applications

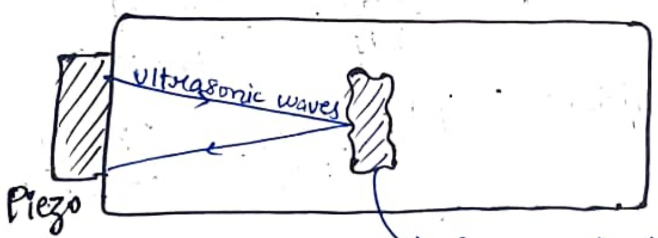
To estimate blood flow in artery in neck.

c: speed of ultrasound in water

$$\Delta t = \frac{2H}{c}$$

Non-Destructive Evaluation / Testing

↳ Access the quality of material.



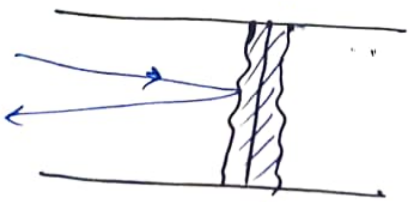
crack inside the material

↳ air

↳ Detect the presence and dimension of crack from received echo

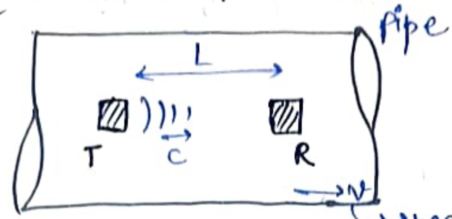
→ can also detect sub-surface cracks [Principle: low Z of air]

→ Access quality of welding:



↳ welding ⇒ Access its quality by quantity of air

- In a pipe: [Ultrasonic Flow Meters]



speed of fluid in pipe : v
speed of ultrasound : c
transit times : t_1, t_2

Transmit time, $t_1 = \dfrac{L}{c+v}$

Static fluid ($v=0$) : $t_2 = \dfrac{L}{c}$ [Calibration phase]

$v = ?$

$$t_2 - t_1 = \frac{Lv}{c(c+v)} \approx \frac{Lv}{c^2} \rightarrow \text{estimate } v \text{ from } (t_2 - t_1).$$

$(\because c \gg v)$

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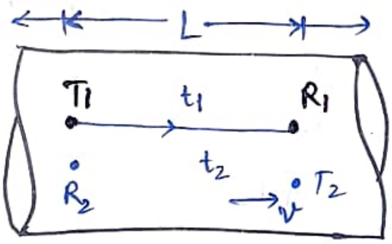


t_2-measurement in static fluid is difficult, so add another R-T pair, to avoid calibration phase.

$$t_1 = \frac{L}{c+v}$$

$$t_2 = \frac{L}{c-v}$$

$$\Delta t = t_2 - t_1 = \frac{L2v}{c^2 - v^2} \approx \frac{2Lv}{c^2}, \quad c^2 \gg v^2$$

$$\therefore \boxed{\Delta t \propto v}$$

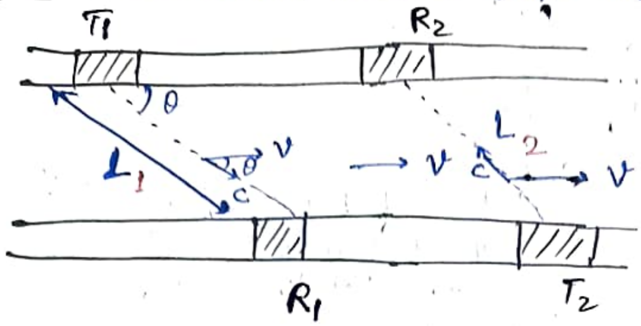
To get independence from c,

$$\frac{1}{t_1} - \frac{1}{t_2} = \frac{2v}{L}$$

Ultrasonic Flowmeter

- Transmit time (discussed)
  - Wetted
  - Clamp on → US probe embedded on wall/thickness of pipe
  - (Wetted, Clamp on): Clean fluids
- Doppler effect
  - Dirty fluids

Clamp-on: No direct contact with the liquid.



$$t_1 = \frac{L}{c + v\cos\theta}$$

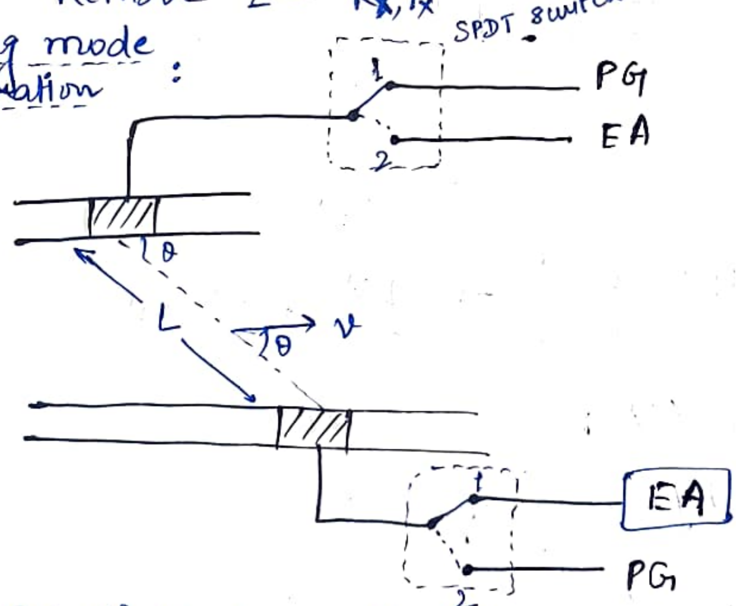
$$t_2 = \frac{L}{c - v\cos\theta}$$

$$\frac{1}{t_1} - \frac{1}{t_2} = \frac{2v\cos\theta}{L}$$

In reality, L_1 may not be same as L_2 (slight mismatch), making it dependent on 'c' $\Rightarrow \left(\frac{1}{t_1} - \frac{1}{t_2}\right)$ is no longer linear.

Solution: Remove 2nd R_x, T_x

Switching mode configuration:



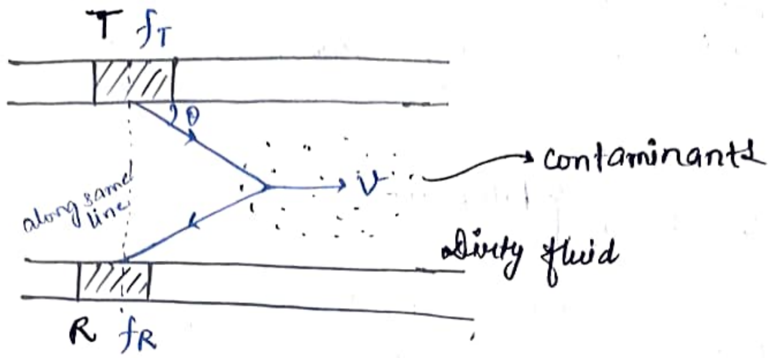
PG : Pulse generator (T_x)

EA : Echo amplifier (R_x)

Mode-1 : t_1

Mode-2 : t_2

Doppler Effect:



f_T : transmitted ultrasonic frequency

$$f' = f_T \frac{c - v\cos\theta}{c}$$

$$f_R = f' \frac{c}{c + v\cos\theta}$$

$$\Rightarrow f_R = f_T \left(\frac{c - v\cos\theta}{c + v\cos\theta}\right)$$

$$= f_T \frac{\left[1 - \frac{v}{c}\cos\theta\right]}{\left[1 + \frac{v}{c}\cos\theta\right]}, \quad v \ll c$$

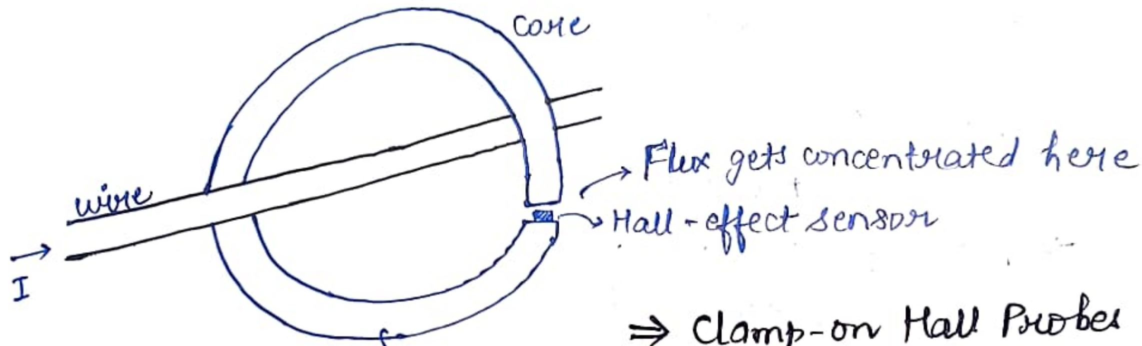
$$f_R \approx f_T \left[1 - \frac{2v}{c}\cos\theta\right]$$

$$\therefore \; f_T - f_R = \frac{2v}{c}\cos\theta \cdot f_T \quad \propto v$$

- There may be many f_R's $\Rightarrow$ Take the dominant one (from the fourier analysis)
- Calculating ~~proportionit~~ proportionality constant by exposing to known few 'v' values.

I and V Measurement

- ↳ In → smart grids
  - ↳ condition monitoring equipment.
  - ↳ power systems
- ↳ Heavy wires
- ↳ congested locations } Problems
- ↳ Option: Current transformer / Potential transformer } bulky.

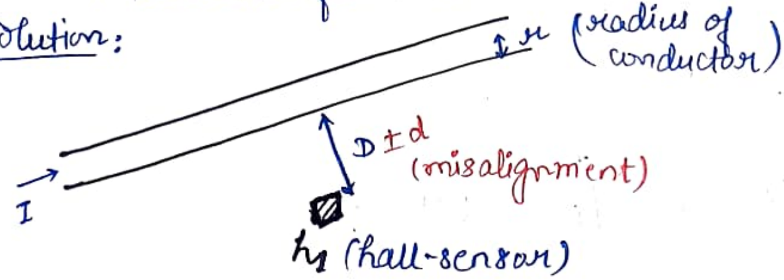


⇒ Clamp-on Hall Probes

↳ 360° measurement

- ↳ Requires full 360° access around the conductor
- ↳ Not suitable for PCB mounted circuit

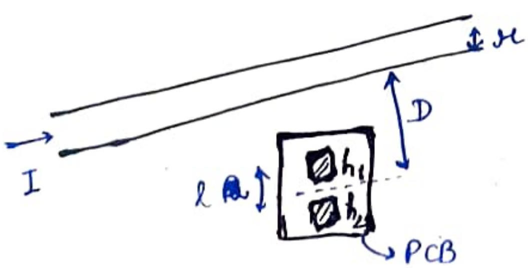
Solution:



Mag. field seen by h_1, $B_1 = \dfrac{\mu_0 I}{2\pi(D \pm d + r)}$

↳ Depends on D, d, r

Solution: Use two hall sensors.



- Hall-sensors h_1, h_2 soldered on the PCB

Fields seen by sensors,

$$B_1 = \frac{\mu_0 I}{2\pi\left[D + r - l/2\right]}$$

$$B_2 = \frac{\mu_0 I}{2\pi\left[D + r + l/2\right]}$$

$$\frac{1}{B_2} - \frac{1}{B_1} = \frac{2\pi}{\mu_0 I} l$$

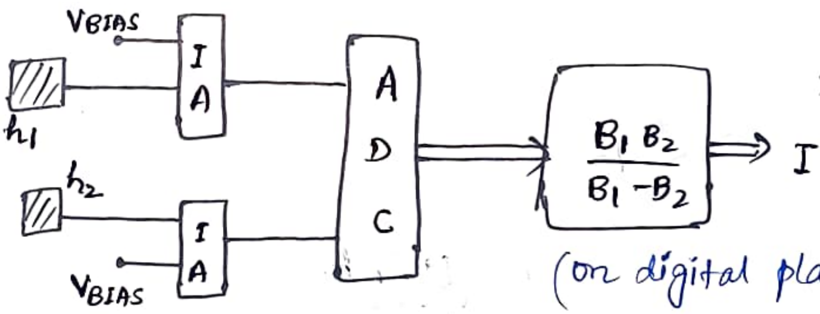
$$\therefore \frac{B_1 B_2}{B_1 - B_2} = \frac{\mu_0 I}{2\pi l}$$

→ free of D, r

↳ depends on l ⇒ decided by us.

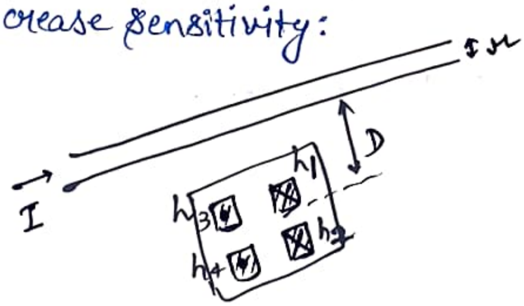
$$V_{H_1} = V_{BIAS}\left[1 + KB_1\right]$$

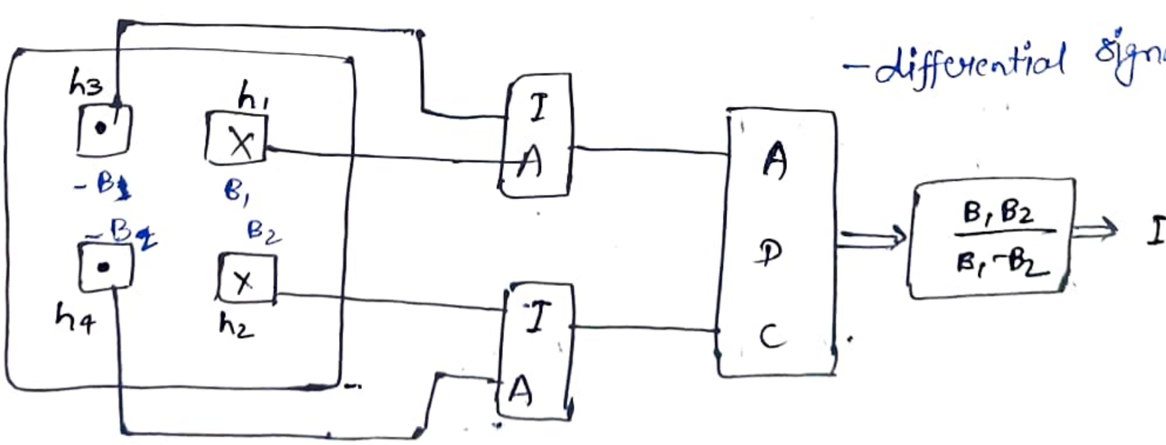
$$V_{H_2} = V_{BIAS}\left[1 + KB_2\right]$$



(on digital platform)

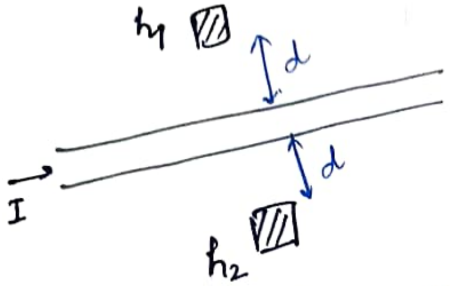
To increase sensitivity:





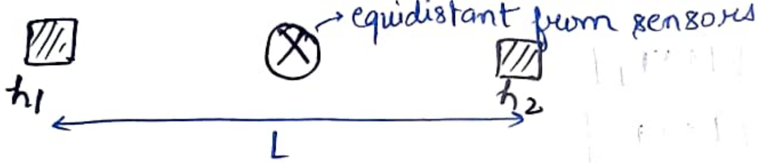
- differential signalling

Anti-differential Current Probes



Current-carrying conductor should be equidistant from the sensors, which is diffcult to achieve ⇒ Misalignment.

⊗ → suppose the conductor is misaligned and is here



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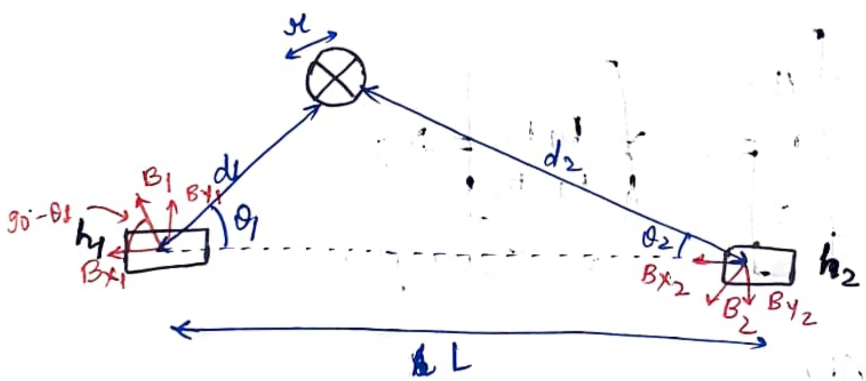


$$(d_1+r)\cos\theta_1 + (d_2+r)\cos\theta_2 = L \quad \ldots (1)$$

$$B_1 = \frac{\mu_0 I}{2\pi(d_1+r)}$$

$$B_2 = \frac{\mu_0 I}{2\pi(d_2+r)}$$

$$(1) \Rightarrow \frac{\mu_0 I}{2\pi B_1}\cos\theta_1 + \frac{\mu_0 I}{2\pi B_2}\cos\theta_2 = L$$

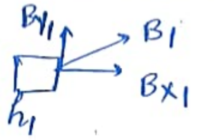
$$\left[\cos\theta_1 = \frac{B_{y1}}{B_1}; \quad \cos\theta_2 = \frac{B_{y2}}{B_2}\right]$$

$$\Rightarrow \frac{\mu_0 I}{2\pi B_1^2} B_{y_1} + \frac{\mu_0 I}{2\pi B_2^2} B_{y_2} = L$$

$$\Rightarrow I = \frac{2\pi L}{\mu_0 \left[\frac{B_{Y2}}{B_{22}} + \frac{B_{Y1}}{B_{12}} \right]}$$

→ Independent of r, d_1, d_2

↳ Dual axis Hall sensors to measure B_{Y_1}, B_{Y_2} (at h_1, h_2)



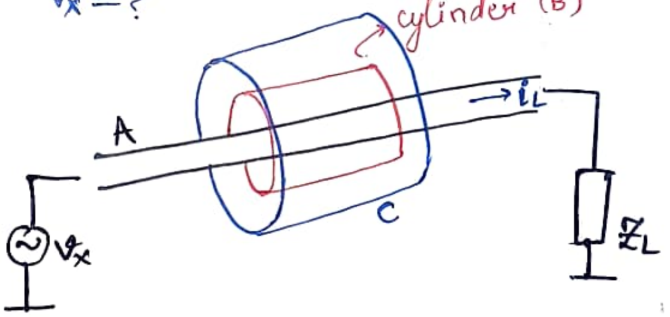
~~Measure~~

Non-contact Voltage Measurement

↳ Measure the voltage (RMS) (typically high) in the insulated conductor without touching it.

$v_x = \sqrt{2} V_x \sin(\omega_x t)$

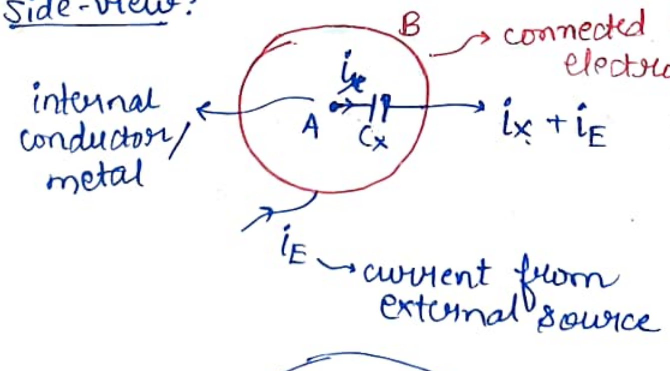
$V_x = ?$



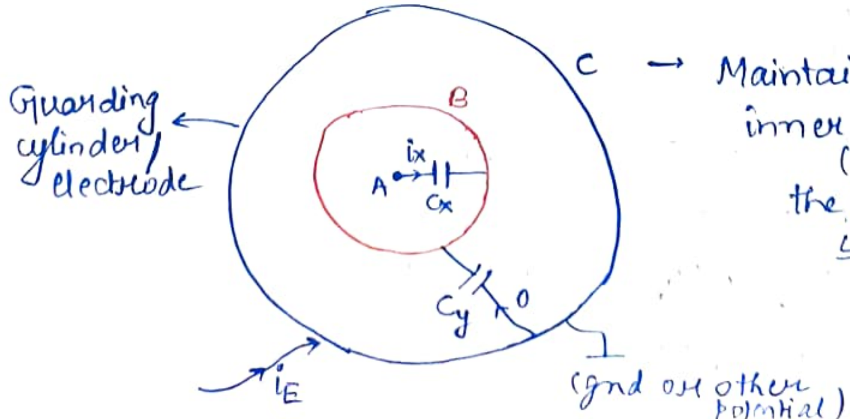
→ very large current
↳ Magnetic principle not ~~applicable~~ possible

- Measure using capacitive principle. [i_x : displacement current]

Side-view:



C_x : cap. b/w main conductor (A) & cylinder B
↳ depends on conductor (wire) thickness, wire insulating material, gap b/w wire & B.
↳ Not in our control (get indep. from it)



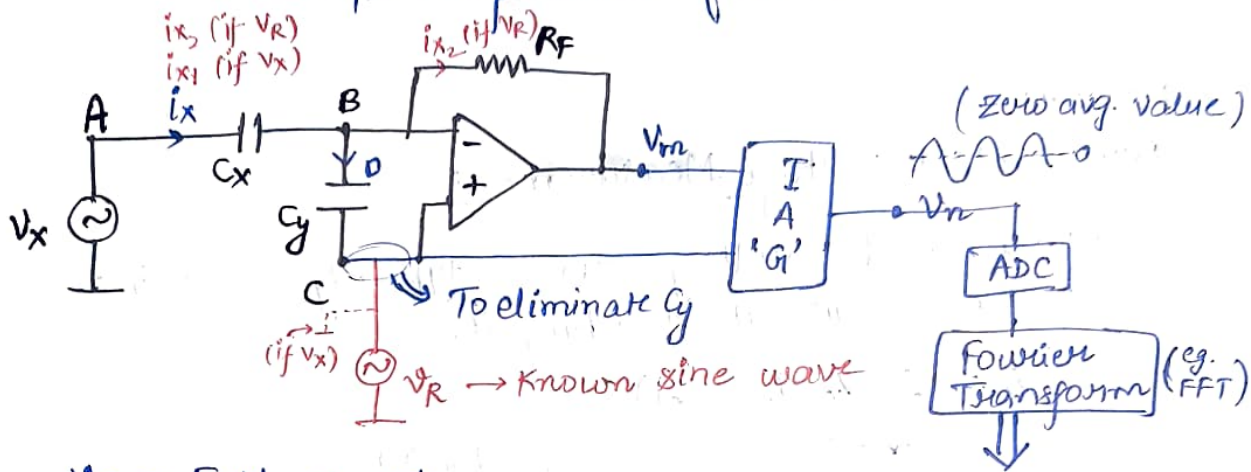
→ Maintain the guard and inner electrode (B & C) at the same potential
↳ Zero current flow b/w them
↳ Inner electrode protected

$i_x \xrightarrow{\text{depends on}} C_x$

↳ Depends on :- thickness of wire
- insulating material of wire
- gap b/w wire & B

↳ Not in our control

↳ Get independence from it



$v_R = \sqrt{2}\, V_R \sin \omega_R t$

$v_x = \sqrt{2}\, V_x \sin \omega_x t$

Using superposition:

$\underline{\underline{V_x}}$

$i_{x_1} = C_x \dot{v}_x$

$v_{m_1} = -i_{x_1} R_F$

$v_{n_1} = G\,[v_{m_1} - 0]$

$\Rightarrow v_{n_1} = -G\, i_{x_1} R_F$

$= -G R_F C_x \dot{v}_x$

$\Rightarrow v_{n_1} = -\underbrace{G R_F C_x \sqrt{2}\, V_x\, \omega_x}\, \sin(\omega_x t + 90^\circ)$

$\Rightarrow v_{n_1} = -V_{N_1} \sin(\omega_x t + 90^\circ)$

$\underline{\underline{V_R}}$

$i_{x_2} = -C_x \dot{v}_R$

$v_{m_2} = v_R - i_{x_2} R_F$

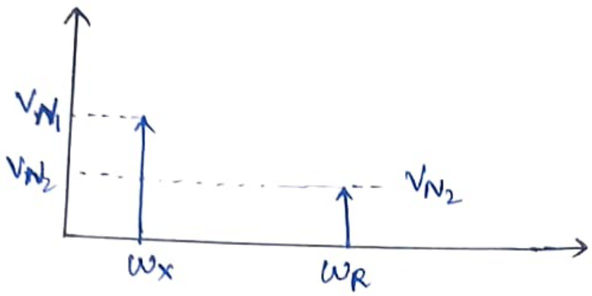
$v_{n_2} = G\,[v_{m_2} - v_R] = -G\, i_{x_2} R_F$

$\Rightarrow v_{n_2} = G\, C_x R_F \dot{v}_R$

$\Rightarrow v_{n_2} = \underbrace{G\, C_x R_F \sqrt{2}\, V_R\, \omega_R}\, \sin(\omega_R t + 90^\circ)$

$\Rightarrow v_{n_2} = V_{N_2} \sin(\omega_R t + 90^\circ)$

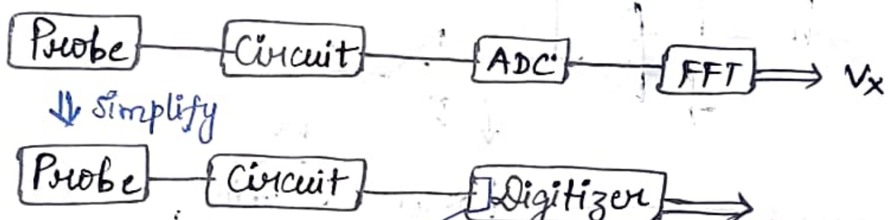
$\therefore v_n = v_{n_1} + v_{n_2}$



$$\frac{V_{N_1}}{V_{N_2}} = \frac{V_x \omega_x}{V_R \omega_R} \Rightarrow V_x = \left(\frac{V_{N_1}}{V_{N_2}}\right) \frac{V_R \omega_R}{\omega_x}$$

→ indep. of thickness, separation of conductors

ω_x : may correspond to power supply.

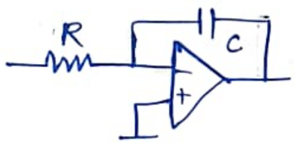


↳ contains dual-slope adc

↳ deintegration count

Adapter

↳ converts output sine (of zero avg. value) (from IA o/p) to a signal having certain avg. value



o/p will be zero if zero avg. value sine is given as i/p.

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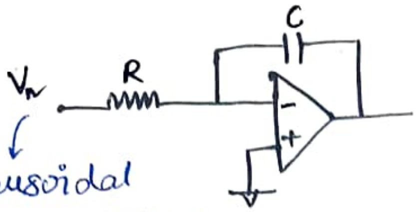
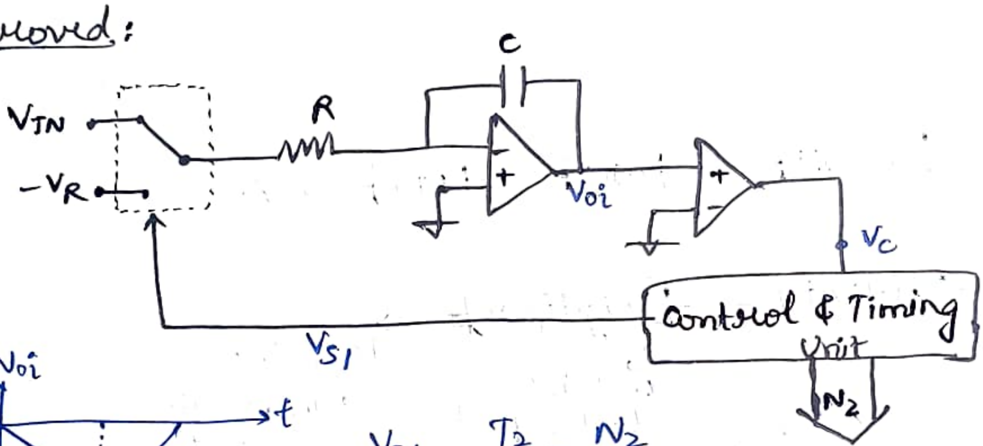
Dual Slope ADC

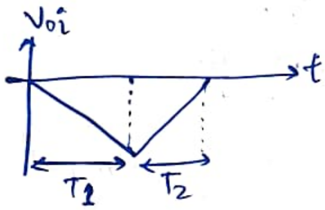


↓
sinusoidal
(zero avg. value)

↳ Digitizer should have an adaptation block that, produces a finite average value.

Improved:

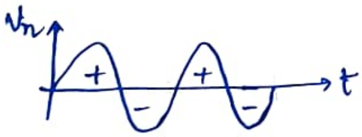




$$\frac{V_{IN}}{V_R} = \frac{T_2}{T_1} = \frac{N_2}{N_1}$$

V_{S1}	
Low	T_1 (integration)
High	T_2 (deintegration phase)

V_{oi} → no meaningful information
[+ve/-ve cycles cancel on integration]



Instead of: Capacitive sensor + Circuit + $\underbrace{\text{ADC + FFT}}_{\text{Digitizer}}$

⇓

Capacitive sensor + circuit + Adapter + Digitizer ⇒ N_2

↓

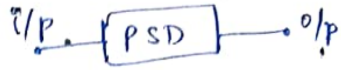
V_n

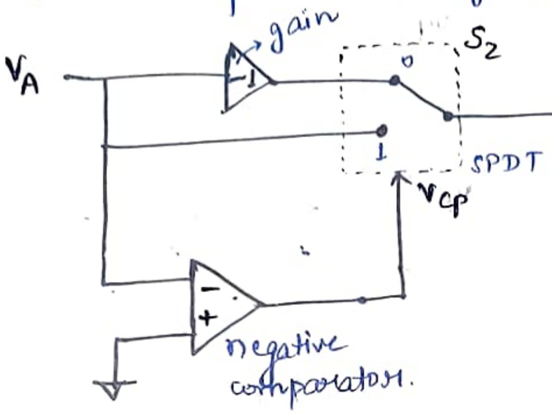
$N_2 \propto V_x$

We need adapter block → PSD

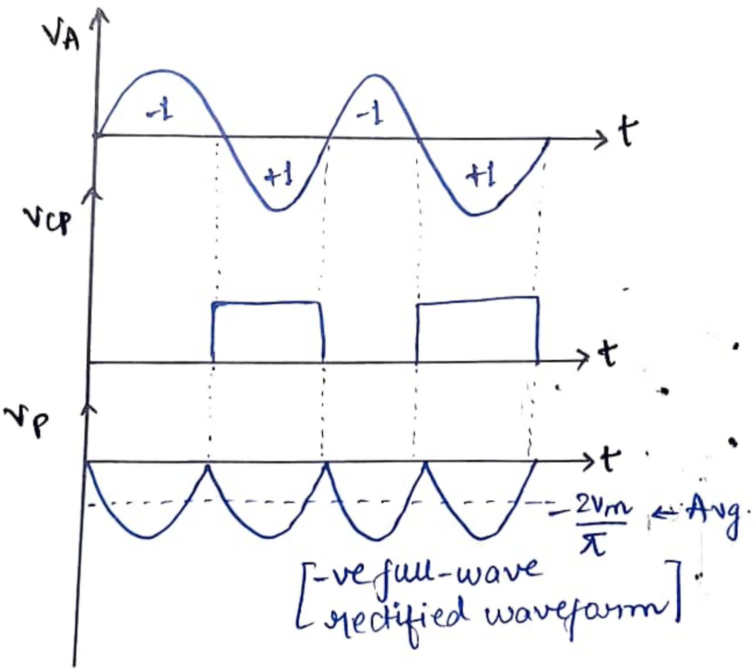
Phase Sensitive Demodulator (PSD)

- Phase of control signal determines the transformation from i/p to o/p.
- O/p is transformation of i/p.





V_A	V_{cp}	S_2	V_P
>0	Low	0	$-V_A$
<0	High	1	V_A



→ signal with finite average value can be processed by dual slope ADC.

Case-1: −ve comparator → Negative Full Wave Rectifier (FWR)

$V_P = -|V_A|$

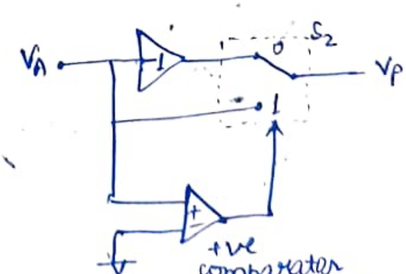
$avg(V_P) = -\frac{2V_m}{\pi}$

→ 90° phase shift
↳ All pass filter.

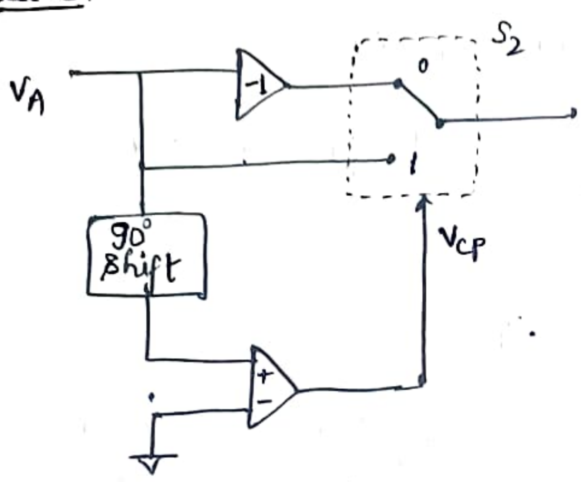
Case-2: +ve comparator → +ve FWR.

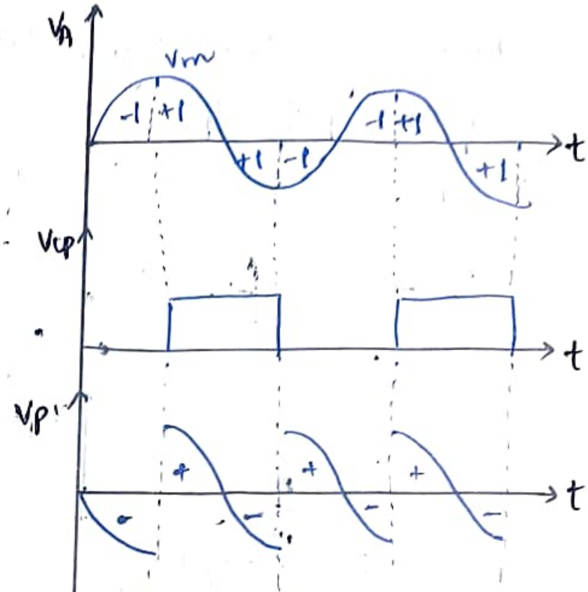
$V_P = |V_A|$

$avg(V_P) = \frac{2V_m}{\pi}$



Case-3:





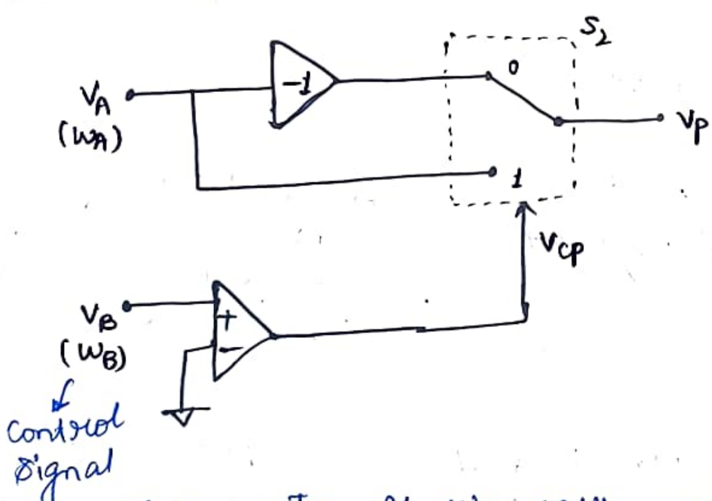
$avg.(V_p) = 0$

↳ avg = 0

Case-4: θ-shift

$$avg(V_p) = \frac{2V_m}{\pi} \cos\theta$$

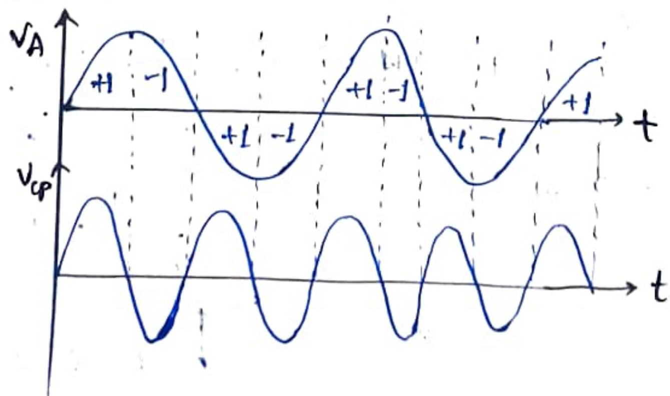
Case-5: $\omega_B = 2\omega_A$



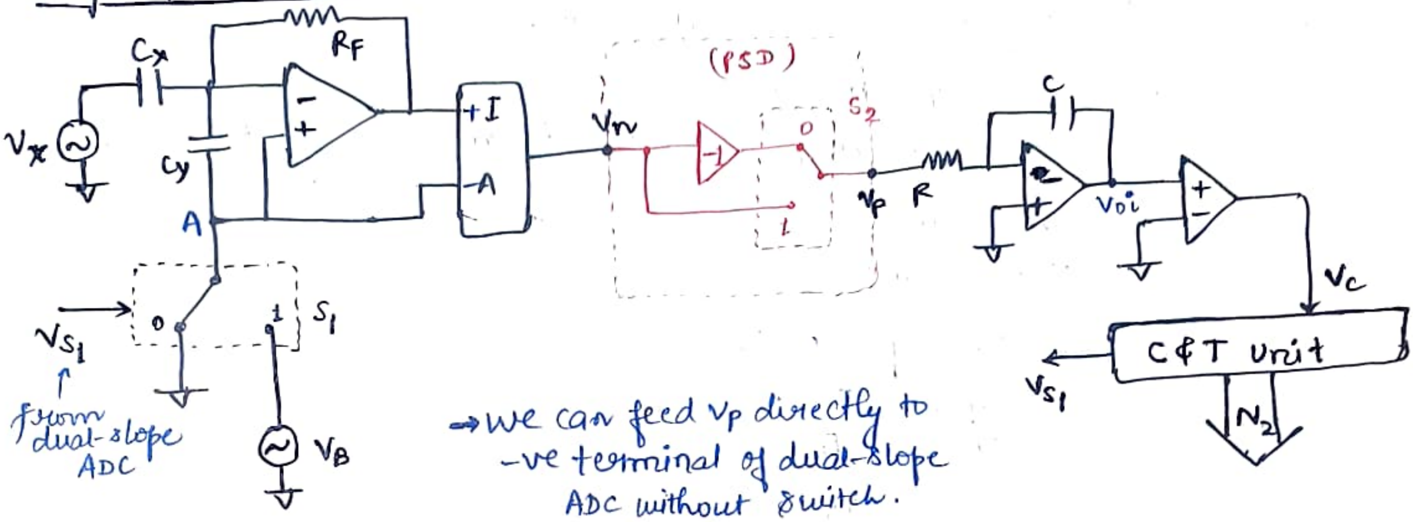
If $T_A = m T_B$ or $\omega_B = m\omega_A$, where m is an integer.

$\Rightarrow avg(V_p) = 0.$

$\omega_B = 2\omega_A$



Capacitive Probe:



→ we can feed V_p directly to −ve terminal of dual-slope ADC without switch.

V_{S_1}		S_1	V_n	S_3	avg (N_p)
Low	T_1	0	V_{n_1}	0	$\cancel{+}\frac{2V_{N_1}}{\pi}$
High	T_2	1	V_{ns}	1	$+\frac{2V_{N_2}}{\pi}$

$V_{n_1} = \sqrt{2}\, G_1 R_F C_x V_x \omega_x$
$V_{n_2} = \sqrt{2}\, G_1 R_F C_x$

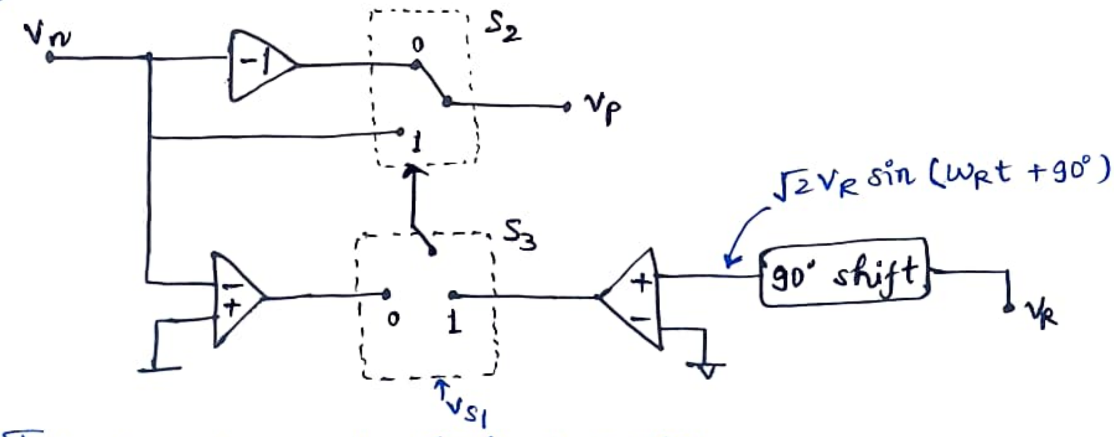
T_1 : $S_1 \to 0$

$$V_n = V_{n_1} = -V_{N_1} \sin(\omega_x t + 90^\circ)$$

T_2 : $S_1 \to 1$

$$V_n = V_{n_2} = V_{N_2} \sin(\omega_R t + 90^\circ) - V_{N_1} \sin(\omega_x t + 90^\circ) = V_{ns}$$

PSD:



T_1 : $V_n = V_{n_1} = -V_{N_1} \sin(\omega_x t + 90^\circ)$

$$\text{avg}(V_p) = -\frac{2V_{n_1}}{\pi}$$

T_2 : $V_n = V_{n_S} = +V_{N_2} \sin(\omega_R t + 90^\circ) - V_{N_1} \sin(\omega_x t + 90^\circ)$

$$\text{avg}(N_p) = \frac{2V_{N_2}}{\pi}.$$

→ To avoid dependence on V_{N_1} term,

choose, $\omega_R = m\omega_x$

or $T_x = mT_R$ ⇒ 2nd term averages to zero.

Eg. $T_x = 20\,ms$

$T_R = 0.2\,ms$

From dual-slope ADC,

$$\frac{V_1}{V_2} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{\frac{2V_{N1}}{\pi}}{\frac{2V_{N2}}{\pi}} = \frac{T_2}{T_1} \Rightarrow \frac{V_{N1}}{V_{N2}} = \frac{T_2}{T_1} = \frac{V_x \omega_x}{V_R \omega_R}$$

can be ~~known~~ calculated

$$\Rightarrow \boxed{\frac{V_x}{V_R} = \frac{T_2}{T_1} * \frac{T_x}{T_R}}$$

$\hookrightarrow m$

T_2 : measured by cf. T unit

T_1 : present integration time

T_x, T_R : known

T_1, V_R : known

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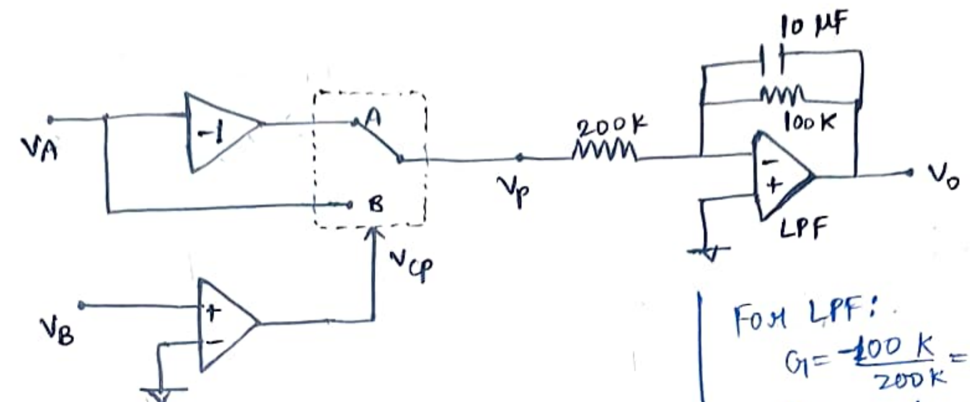
Eg.



For LPF:

$G = \frac{-100\,K}{200\,K} = \frac{-1}{2}$

$f_c = \frac{1}{2\pi(100K \times 10\mu)} = 0.15\,Hz \ll 50\,Hz$

Find V_o for these cases:

ⓐ $V_A = V_B = \sin(100\pi t)$

$V_o = avg\,(V_p) \times G$

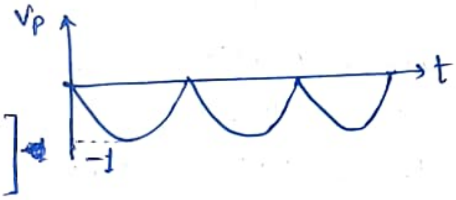
$= -\frac{2}{\pi}\left(-\frac{1}{2}\right) = \frac{1}{\pi}\,V$

V_A	V_{cp}	~~V_B~~ SW	V_p
>0	High	A	$-V_A$
<0	Low	B	$+V_A$

ⓑ $V_A = \sin(100\pi t)$

$V_B = \cos(100\pi t)$

$V_o = 0$ [Since V_A & V_B are phase-shifted by 90°]

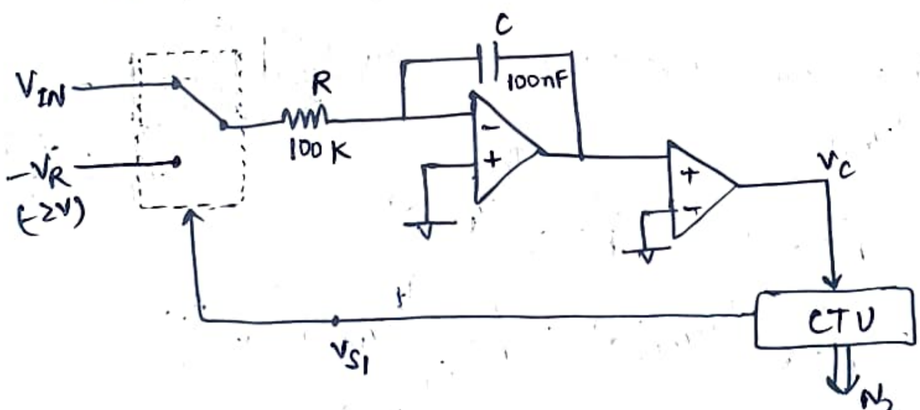


ⓒ $V_A = \sin(100\pi t)$

$V_B = \sin(300\pi t)$

$V_o = 0$

Eg.



$V_{IN} = 1 + 0.01 \sin 100\pi t$.

ⓐ Find T_1 to nullify interference.

ⓑ Find the value of integrator o/p at the end of integration time.

ⓒ Design dual-slope ADC for capacitive sensor probe.

Soln: ⓐ Integration period, $T_1 = K_1 T_N$ to nullify +ve & -ve halves equally (50 Hz component removed)

$V_{IN} = 1 + 0.01 \sin 100\pi t$

↓

$T_N = 20\,ms$

Take $K_1 = 3$.
$\Rightarrow T_1 = 60\,ms$.

Taking higher K,
$T_1 \uparrow$, $(T_1 + T_2) \uparrow$
$\Rightarrow$ conversion time $\uparrow$
$\therefore$ speed $\downarrow$

(b) $i = V_{IN}/R$

$$\therefore V_{oi} = -\frac{1}{C}\int_0^{T_1} i\,dt = -\frac{V_{IN}}{RC}T_1$$

$$= -\frac{1}{100K \times 100n} \times 60m = -6V.$$

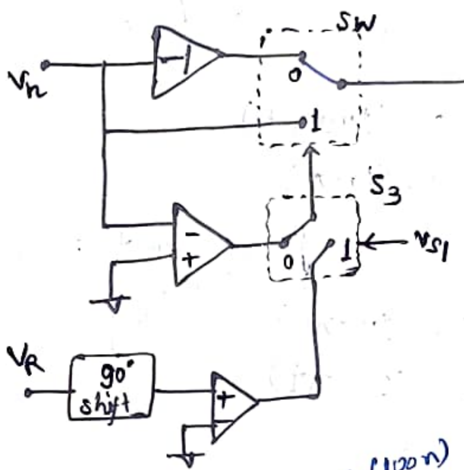
Opamp is giving $-6V$, so power supply should be 'atleast' $\pm 8V$.

(c) $V_R = \sqrt{2}V_R \sin \omega_R t$
$V_x = \sqrt{2}V_x \sin \omega_x t$

Unknown signal $\left\{ \begin{array}{l} \omega_x = 100\pi\, rad/s \\ f_x = 50\,Hz,\ T_x = 20\,ms \end{array} \right.$

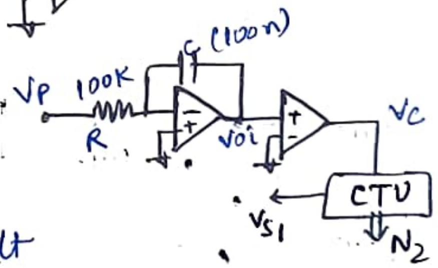
$f_R = 1000\,Hz$ (20 times)
$\omega_R = 2000\pi\, rad/s$

$\omega_R = K_2 \omega_x$



$\underline{T_1}$:
$V_n = V_{n_1} = -V_{N_1}\sin(\omega_x t + 90°)$

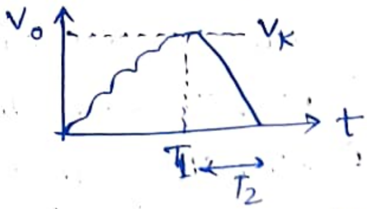
$V_p = -|V_{n_1}|$



$$V_o = -\frac{1}{RC}\int V_p\,dt = \frac{1}{RC}\int_0 |V_{n_1}|\,dt$$

$$V_K = \frac{1}{RC}\int_0^{T_1} |V_{n_1}|\,dt = \frac{1}{RC}\left(\int_0^{T_x} |V_{n_1}|\,dt\right) \times \left(\frac{T_1}{T_x}\right)$$

$$= \frac{1}{RC}T_1 \cdot \frac{2}{\pi}V_{N_1} \quad \ldots (1)$$



$$\left[\because \frac{1}{T_x}\int_0^{T_x} |V_{n_1}|\,dt = \frac{2}{\pi}V_{N_1}\right] \text{(avg)}$$

Power supply to opamp should be more than this.

Design Conditions:

1. $T_1 = K_2 T_x$
2. $T_x = K_1 T_R$

$\underline{T_2}$:

$$V_{ns} = \overbrace{V_{N_2} \sin(\omega_R t + 90^\circ)}^{V_{n2}} - \underbrace{V_{N_1} \cancel{\sin(\omega_x t + 90^\circ)}}_{\text{No role since } \omega_R = k_1 \omega_x}$$

$$V_0 = V_K - \frac{1}{RC} \int_{T_1}^{T_1+T_2} |V_{n_2}| \, dt$$

$$V_0(T_1 + T_2) = 0$$

$$\Rightarrow V_K = \frac{1}{RC} \int_{\langle T_2 \rangle} |V_{N_2}| \, dt$$

$$= \frac{1}{RC} \frac{T_2}{T_R} \int_{\langle T_R \rangle} |V_{N_2}| \, dt$$

$$= \frac{1}{RC} T_2 \cdot 2 \frac{V_{N_2}}{\pi}$$

$$\therefore V_K = \frac{2 V_{N_2}}{\pi} \frac{T_2}{RC} \quad \cdots ②$$

Equating ① and ②,

$$T_1 V_{N_1} = T_2 V_{N_2}$$

$$\Rightarrow \frac{V_{N_1}}{V_{N_2}} = \frac{T_2}{T_1} = \frac{N_2}{N_1} = \frac{V_x \omega_x}{V_R \omega_R}$$

$$\Rightarrow \frac{V_x}{V_R} = \frac{N_2}{N_1} \frac{\omega_R}{\omega_x} = \frac{N_2}{N_1} K_2$$

$$\therefore K_2 = \frac{\omega_R}{\omega_x}$$