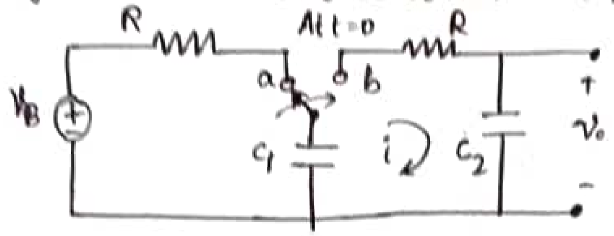
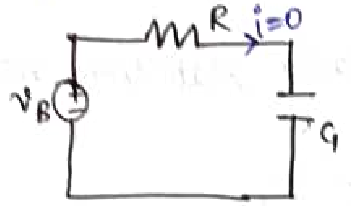
# Assignment -1
## AV213 – Network Analysis
### (Drill Problems Set 1)

SAURABH KUMAR
SC22B146
(ECE)

(1) Switch moves from 'a' to 'b' at $t=0$. Steady state was initially reached at position 'a'. Given $v_o(0^-)=0$. Find $i(t)$ and $v_o(t)$ for $t>0$.

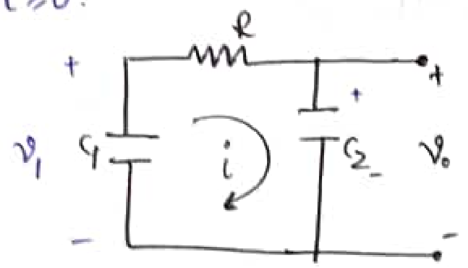



Soln: Initial equivalent circuit:




$$v_{C_1}(0^+) = v_{C_1}(0^-) = v_B = v_1(0^+)$$

For $t \geq 0$.




Using KVL,

$$v_1 = iR + v_2$$

$$\Rightarrow v_1' = i'R + v_2'$$

$$\Rightarrow -\frac{i}{C_1} = i'R + \frac{i}{C_2} \qquad \left(C\frac{dv}{dt} = i\right)$$

$$\Rightarrow 0 = i'R + \left(\frac{1}{C_1}+\frac{1}{C_2}\right)i = i'R + (1/C_{eq})\, i$$

$$\Rightarrow 0 = i' + \frac{1}{\tau} i, \quad \boxed{\tau = RC_{eq}}$$

$$\Rightarrow \frac{di}{dt} = -\frac{1}{\tau} i \Rightarrow \int \frac{di}{i} = -\frac{1}{\tau}\int dt \Rightarrow i = Ke^{-t/\tau}$$

As $i(0) = \dfrac{v_1 - v_o}{R} = K$

$$\therefore i = \frac{v_1 - v_o}{R} e^{-t/\tau}$$

$$\Rightarrow \boxed{i = \frac{v_B}{R} e^{-t/\tau}} \qquad (\because v_1(0) = v_B,\ v_o(0) = 0)$$

As $C_2 \frac{dv_o}{dt} = i = \frac{v_B}{R} e^{-t/\tau}$

$$\Rightarrow v_o = -\frac{v_B}{RC_2} \cdot \frac{e^{-t/\tau}}{1/\tau} + K_1$$

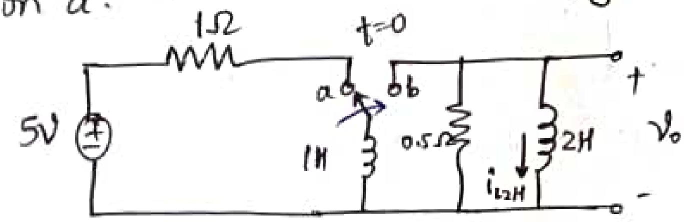
$$= -\frac{v_B C_{eq}}{C_2} e^{-t/\tau} + K_1$$

As $v_o(0^+) = 0$,

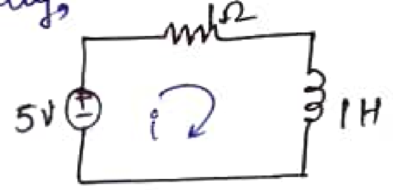
$$K_1 = \frac{v_B C_{eq}}{C_2}$$

$$\therefore \boxed{v_o = \frac{v_B C_{eq}}{C_2}\left(1 - e^{-t/\tau}\right)}.$$

(16) Solve for $v_o(t)$, given $i_{L2H}(0^-) = 0$. Steady state was reached initially at position 'a'.

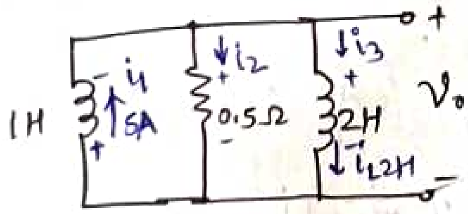



Soln: Initially,




$$i(0^+) = i(0^-) = \frac{5V}{1\Omega} = 5A$$

For $t > 0$,




$$i_1 = i_2 + i_3$$

$$\Rightarrow i_1' = i_2' + i_3'$$

$$\Rightarrow -\frac{v_o}{1} = \frac{v_o'}{0.5} + \frac{v_o}{2}$$

$$\Rightarrow 0 = 2v_o' + \frac{3v_o}{2}$$

$$\Rightarrow 0 = 4v_o' + 3v_o \quad \text{— (i)}$$

$$\Rightarrow 0 = 4\frac{dv_o}{dt} + 3v_o \Rightarrow 4\int \frac{dv_o}{v_o} = -3\int dt$$

$\Rightarrow \quad v_o = ke^{-3/4\, t}$

~~From (i),~~

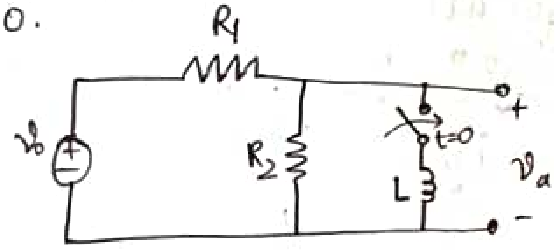
~~$0 = 4v_o'(0+) + 3v_o(0+)$~~

~~$\Rightarrow 0 =$~~

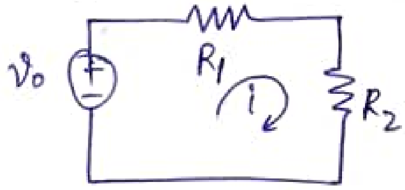
As $v_o(0+) = i \times R = -5 \times 0.5 = -2.5\,V$

$$\therefore v_o = -2.5\, e^{-3/4\, t}$$

(17) $v_o = 3V$, $R_1 = 10\Omega$, $R_2 = 5\Omega$, $L = 0.5\,H$. at $t=0$, switch is closed. Find $v_a(t)$ for $t>0$.

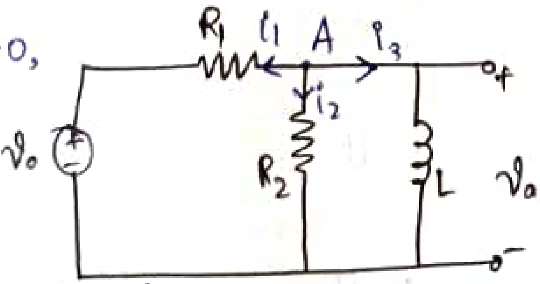



Soln: Initial equivalent circuit:




$$i(0+) = i(0-) = \frac{v_0}{R_1+R_2} = \frac{3}{15}\,A \Rightarrow v_a = \frac{3}{15} \times 5 = 1\,V$$

For $t>0$,




At point A,

$$i_1 + i_2 + i_3 = 0$$

$$\Rightarrow \frac{v_a - v_0}{R_1} + \frac{v_a}{R_2} + i_3 = 0$$

$$\Rightarrow \frac{v_a'}{R_1} + \frac{v_a'}{R_2} + \frac{v_a}{L} = 0 \qquad \text{(differentiating w.r.t. } t)$$

$\Rightarrow v_a'\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = -\frac{v_a}{L}$

$\Rightarrow \frac{v_a'}{v_a} = \frac{-1}{L}\left(\frac{R_1R_2}{R_1+R_2}\right)$

$\Rightarrow v_a = K e^{-\frac{1}{L}\left(\frac{R_1R_2}{R_1+R_2}\right)t}$

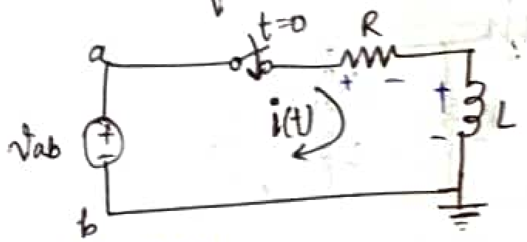
As $v_a(0^+) = 1V$

$\Rightarrow K = 1$

$\therefore v_a(t) = e^{-\frac{2}{0.5}\left(\frac{10\times5}{\frac{15}{3}}\right)t}$

$= e^{-\frac{20}{3}t}$ V

(18) $v_{ab} = V_m \sin\omega_0 t$, solve for $i(t)$.




Soln: Applying KVL (for $t>0$),

$v_{ab} = iR + Li'$

$\Rightarrow V_m \sin\omega_0 t = iR + Li'$

$\Rightarrow \frac{di}{dt} + \frac{i}{\tau_F} = \frac{V_m}{L}\sin\omega_0 t$

$IF = e^{\int \frac{1}{\tau_F}dt} = e^{t/\tau_F}$

$\therefore i e^{t/\tau_F} = \int e^{t/\tau_F}\frac{V_m}{L}\sin\omega_0 t\, dt$

$= \frac{V_m}{L}\underbrace{\int e^{t/\tau_F}\sin\omega_0 t\, dt}_{I}$

Let $I = \int e^{t/\tau_F}\sin\omega_0 t\, dt$

$= -\frac{e^{t/\tau_F}}{\omega_0}\cos\omega_0 t + \int \frac{e^{t/\tau_F}}{\omega_0\tau_F}\cos\omega_0 t\, dt$

$= -\frac{e^{t/\tau_F}}{\omega_0}\cos\omega_0 t + \frac{1}{\omega_0\tau_F}\left[e^{t/\tau_F}\sin\omega_0 t - \underbrace{\int\frac{e^{t/\tau_F}\sin\omega_0 t\, dt}{\tau_F\omega_0}}_{I/\tau_F}\right]$

$$\Rightarrow I\left(1+\frac{1}{\omega_0^2\tau_F^2}\right) = -\frac{e^{t/\tau_F}}{\omega_0}\cos\omega_0 t + \frac{e^{t/\tau_F}}{\omega_0\tau_F}\sin\omega_0 t$$

$$\Rightarrow I = \frac{\omega_0^2\tau_F^2}{1+\tau_F^2\omega_0^2}\cdot\frac{e^{t/\tau_F}}{\omega_0\tau_F}\left[\sin\omega_0 t - \tau_F\cos\omega_0 t\right]$$

$$= \frac{\tau_F e^{t/\tau_F}}{1+\tau_F^2\omega_0^2}\left[\sin(\omega_0 t) - \tau_F\cos\omega_0 t\right]$$

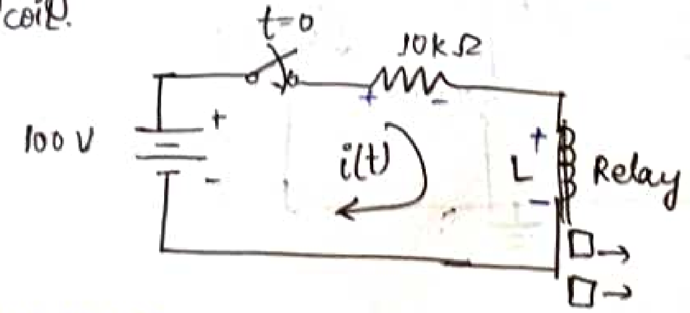
$$\therefore\ i = \frac{\tau_F}{1+\tau_F^2\omega_0^2}\left[\sin\omega_0 t - \tau_F\cos\omega_0 t\right] + K_0 e^{-t/\tau_F}.$$

At $t\to 0^+$, $i=0$.

$$0 = \frac{-\tau_F^2}{1+\tau_F^2\omega_0^2} + K_0 \Rightarrow K_0 = \frac{\tau_F^2}{1+\tau_F^2\omega_0^2}$$

$$\therefore\ i(t) = \frac{\tau_F}{1+\tau_F^2\omega_0^2}\left[\sin\omega_0 t - \tau_F\cos\omega_0 t + \tau_F e^{-t/\tau_F}\right].$$

(19) Relay is an EM switch which constitute of a coil inductance 'L' is having movable parts. The relay is designed and constructed in such a manner that it is activated when current through the coil is 0.008A. After the switch is closed at $t=0$, it is observed that the relay gets activated when $t=0.1$ sec. Find the inductance (L) of the coil.




Soln: Using KVL ($t>0$),

$$100 = 10k\,i + L i'$$

$$\Rightarrow i' + \frac{i}{L}10k = 100$$

$$IF = e^{\int\frac{10k}{L}dt} = e^{10kt/L}$$

$$\therefore\ i e^{10kt/L} = \int e^{10kt/L}\,100\,dt$$

$$= \frac{e^{10kt/L}}{10k/L} + K$$

$$\Rightarrow i = \frac{L}{10K} + Ke^{-10Kt/L}$$

As $i(0+) = 0$,

$$0 = \frac{L}{10K} + K \Rightarrow K = \frac{-L}{10K}$$

$$\therefore i(t) = \frac{L}{10k}\left[1 - e^{-10kt/L}\right]$$

Given that $i = 0.008\,A$ when $t = 0.1$ sec.

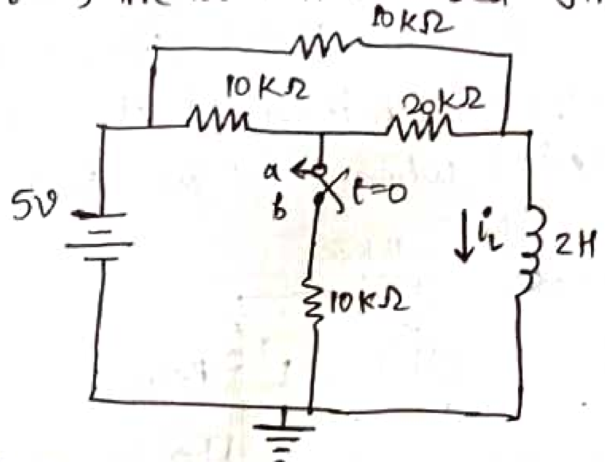
$$\Rightarrow 0.008 = \frac{L}{10K}\left(1 - e^{-10K \times 0.1/L}\right)$$

$$\Rightarrow 80 = L - Le^{-K/L}$$

$$\Rightarrow \ln 80 = \ln\left(L - Le^{-K/L}\right)$$

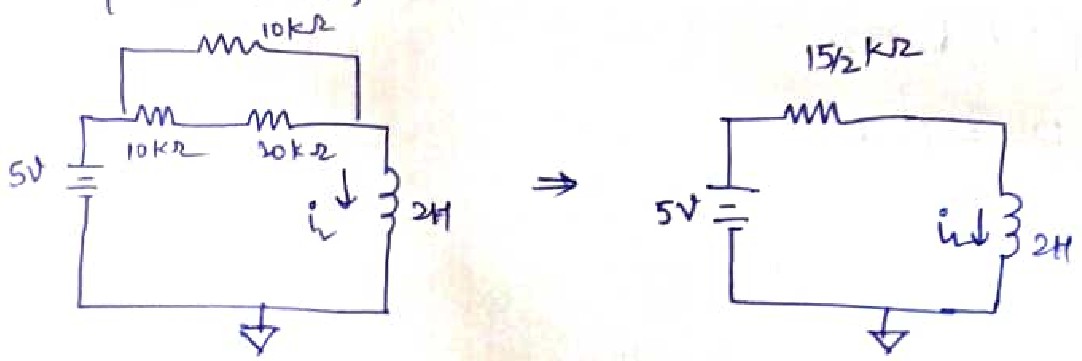
$$\Rightarrow L \approx 621.35\ H.$$

(20) Initially the switch was open with the network reaching steady state. At $t=0$, the switch is closed. Find and plot $i_L(t)$ for $t>0$.




Soln:

Initial equivalent ckt.,



Using KVL and $i(0)=0$,

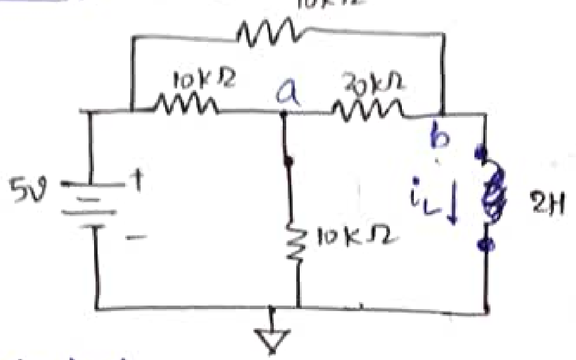
$$i_L(t) = \frac{V}{R}(1-e^{-V_L})$$

$$= \frac{5}{7.5K}\left(1-e^{-t\frac{R}{L}}\right)$$

For $t \to 0^+$,

$$i_L = \frac{5}{7.5K} A = \frac{2}{3} A.$$

For $t>0$, Thevenin's theorem:




Equivalent

At a: $\frac{a-5}{10} + \frac{a}{10} + \frac{a-b}{20} = 0 \Rightarrow 2a-10+2a+a-b \Rightarrow b = 5a-10$

At b: $\frac{b-5}{10} + \frac{b-a}{20} = 0 \Rightarrow 2b-10+b-a=0$

$\Rightarrow 3b-10-a=0$

$\Rightarrow 14a-40=0 \Rightarrow a = \frac{20}{7}$

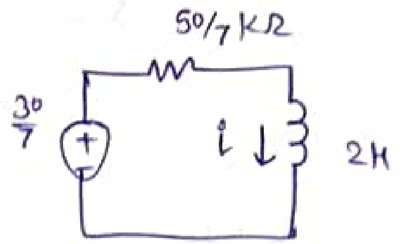
$\Rightarrow b = 5\times\frac{20}{7} - 10 = \frac{30}{7}$

$\therefore$ Thevenin voltage $= \frac{30}{7} = 4.28\,V$.

Thevenin impedance,

$$Z_{th} = 10K\Omega \,||\, 25K\Omega$$

$$= \frac{10\times 25}{35} K = \frac{50}{7} K\Omega.$$




Now,

$$\frac{30}{7} = i\times\frac{50}{7} + Li' \Rightarrow 2i' + \frac{50}{7}i = \frac{30}{7}$$

$$\Rightarrow i' + \frac{25}{7}i = \frac{15}{7}$$

$$IF = e^{\int\frac{25}{7}dt} = e^{\frac{25}{7}t}$$

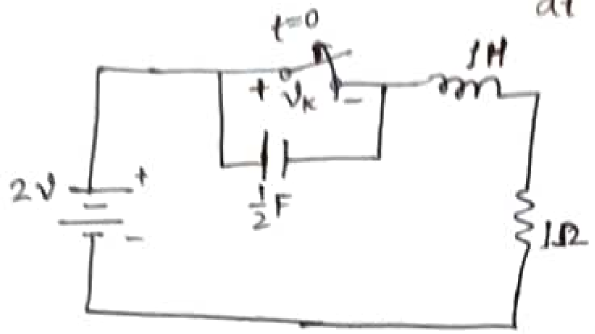
$$\therefore i e^{\frac{25}{7}t} = \int \frac{15}{7} e^{25/7\, t}\, dt$$

$$= \frac{15}{7}\frac{e^{25/7}}{25/7} = \frac{3}{5}e^{25/7\, t} \Rightarrow i = \frac{3}{5} + c\,e^{-25/7\, t}$$

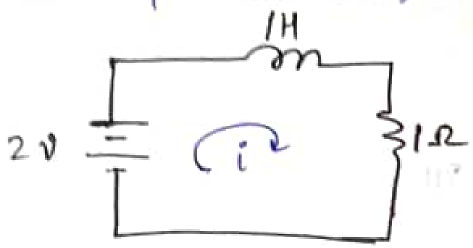
$i(0^-) = \frac{2}{3} A$

$$\Rightarrow \frac{2}{3} = \frac{3}{5} + c \Rightarrow c = \frac{1}{15} \Rightarrow \boxed{i = \frac{1}{15}\left(9 + e^{-25/7\, t}\right)}$$

21. Network was at steady state with Switch $S_w$ closed. At $t=0$, $S_w$ is opened. Determine $v_K$ and $\frac{dv_K}{dt}$ at $t=0^+$.

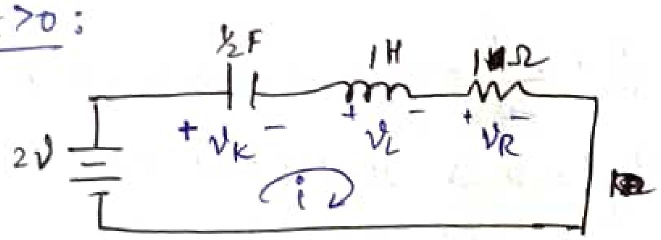



Soln: Initial equivalent ckt.:




for $t \to 0^+$, $i = \frac{2V}{1\Omega} = 2A$.

For $t>0$:




Using KVL,

$$2 = v_K + v_L + v_R$$

$$\Rightarrow v_K = 2 - i' - i \quad -①$$

$$\Rightarrow 0 = v_K' + v_L' + v_R'$$

$$\Rightarrow 0 = \frac{i}{1/2} + i'' + i'$$

$$\boxed{v_K(0^+) = v_K(0^-) = 0\ v}$$

$$\Rightarrow 0 = 2i + i'' + i'$$

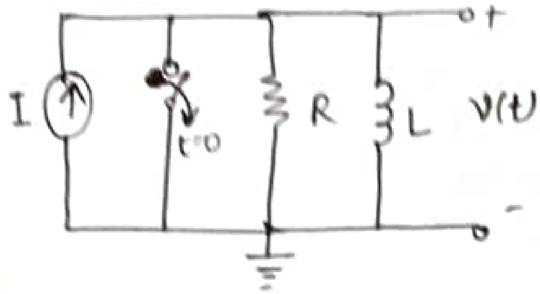
$$\Rightarrow i''(0^+) + i'(0^+) = -2i(0^+) = -2\times2 = 4A$$

$$① \Rightarrow v_K' = -(i'' + i')$$

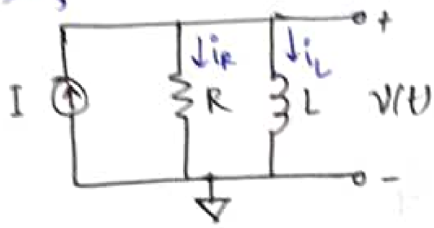
$$\Rightarrow v_K'(0^+) = -(i''(0^+) + i'(0^+))$$

$$\Rightarrow \boxed{v_K'(0^+) = 4\ v\ \text{sec}^{-1}}$$

22. If $I = 2A$, $R = 100\,\Omega$, $L = 1H$, solve for $v$, $\frac{dv}{dt}$, $\frac{d^2v}{dt}$ at $t = 0^+$.




Soln: For $t \geq 0$,




Using KCL,

$$I = i_R + i_L \quad -(i)$$

$$\Rightarrow 2 = \frac{v}{R} + \frac{1}{L}\int v\,dt$$

$$\Rightarrow 0 = \frac{v'}{100} + v \quad -(ii)$$

~~$v(0^+) = 0$~~

~~$\Rightarrow v'(0^+) = 0$~~

$$(i) \Rightarrow I(0+) = i_R(0+) + i_L(0+)$$

$$\Rightarrow 2 = \frac{v(0+)}{100} + 0$$

$$\Rightarrow v(0+) = 200\ v$$

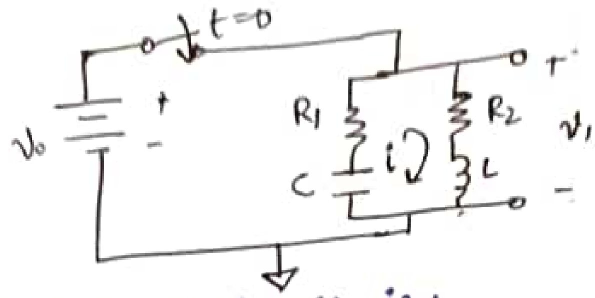
$$(ii) \Rightarrow 0 = v'(0+) + 100 \times 200$$

$$\Rightarrow v'(0+) = -20\,K\ v\,sec^{-1}.$$

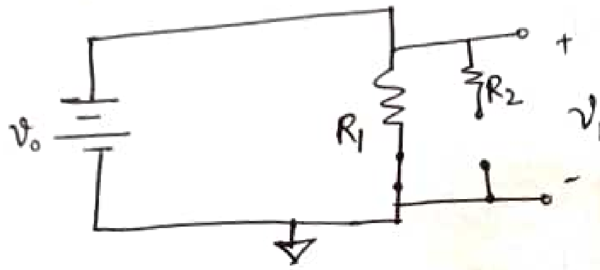
$$(ii) \Rightarrow 0 = v'' + 100\,v'$$

$$\Rightarrow v''(0+) = -2\,M\ sec^{-2}.$$

(23) Network unenergized before $t=0$, determine $v_1, v_1', v_1''$ at $t=0^+$. 




Sol^n: At $t=0^+$, equivalent circuit is:



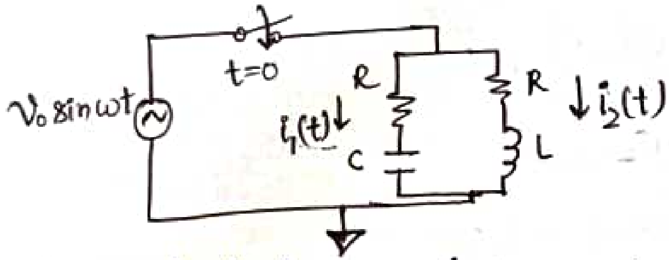

$v_1 = v_0$

$\Rightarrow v_1(0+) = v_0$

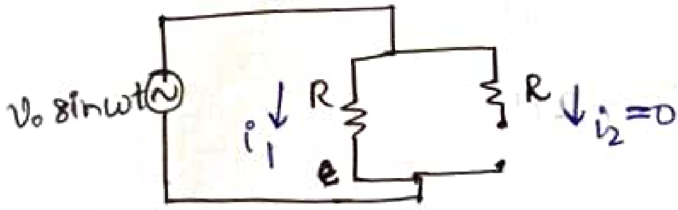
$\Rightarrow v_1'(0^+) = 0$

$\Rightarrow v_1''(0+) = 0.$

(24) Find $\frac{di_1}{dt}$ and $\frac{di_2}{dt}$ at $t=0^+$.




Sol^n. At $t=0^+$, equivalent ckt. is:




$i_1 = \frac{v_0}{R}\sin\omega t$

$\Rightarrow i_1' = \frac{v_0 \omega \cos\omega t}{R}$

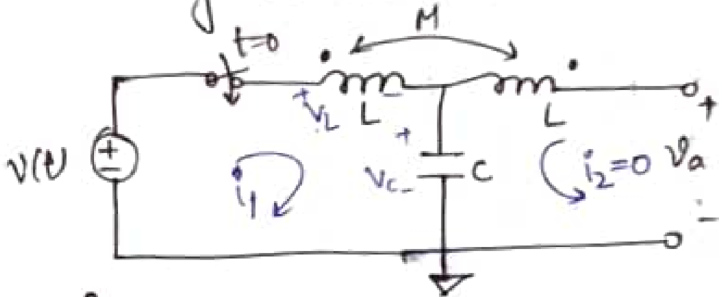
$\Rightarrow i_1'(0+) = \frac{v_0 \omega}{R}$

$i_2 = 0$

$\Rightarrow i_2' = 0$

$\Rightarrow i_2'(0+) = 0.$

(25) Network initially relaxed $v(t) = V_m \sin\left(\frac{t}{\sqrt{MC}}\right)$.




Show that,

(a) $v_a(0+)=0$

(b) $\frac{dv_a}{dt}(0+) = \frac{V_m\sqrt{M}}{L\sqrt{C}}$.

Soln: (a) Using KVL,

$$v = v_L + v_c$$
$$= (L i_1' + M \cancel{i_2'}^{\,0}) + v_c \quad -(i)$$

and, $v_a = \cancel{L i_2'} + M i_1' + v_c \quad -(ii)$

(i) & (ii) $\Rightarrow v - L i_1' = v_a - M i_1'$

$$\Rightarrow v = v_a + (L-M) i_1' \quad -(iii)$$

At $t=0+$,

$$i_1=0,\ v_c=0$$
$$v_L=0,\ v=0 \Rightarrow i_1'=0$$

$\therefore \boxed{v_a(0+)=0}$

(b) (iii) $\Rightarrow v_a = v - (L-M) i_1'$

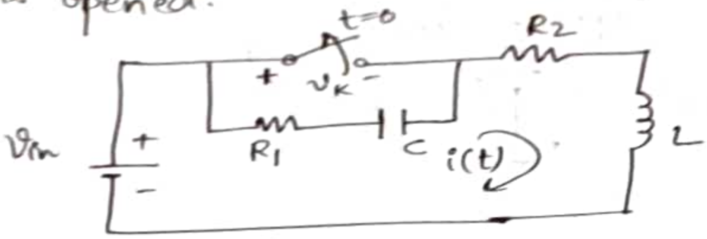
$$v_a' = v' - (L-M) i_1''$$
$$= v_L' + v_c' - (L-M) i_1''$$
$$= v_L' + v_c' - (L-M)\frac{v_L'}{L} \quad -(iv)$$

As $v_L = v - v_c$

$$v_L' = v' - v_c'$$
$$= v' - \frac{i_1}{C}$$
$$v_L'(0+) = v'(0+) - \frac{i_1(0+)}{C}$$
$$= \frac{V_m}{\sqrt{MC}}$$

(iv) $\Rightarrow v_a'(0+) = \frac{V_m}{\sqrt{MC}} - \frac{(L-M)}{L}\frac{V_m}{\sqrt{MC}} = \frac{V_m}{\sqrt{MC}}\cdot\frac{M}{L} = \boxed{\frac{V_m}{L}\sqrt{\frac{M}{C}}}$

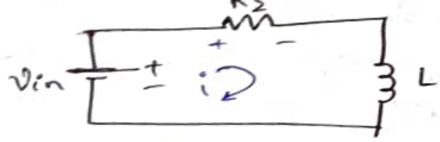
26 Network initially reached steady state with closed switch. At $t=0$, it is opened.




ⓐ Find an expression for the voltage across the switch $v_K$ at $t=0^+$.

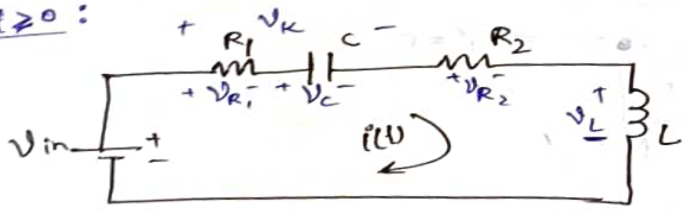
ⓑ If the parameters are adjusted such that $i(0+)=1$ and $\frac{di}{dt}(0+)=-1$, what is the value of $\frac{dv_K}{dt}$ at $t=0+$?

Soln: ⓐ Initial equivalent circuit:




$$i(0+) = \frac{v_{in}}{R_2}$$

For $t \geq 0$:




Using KVL,

$$v_{in} = v_{R_1} + v_C + v_{R_2} + v_L$$

$$\Rightarrow v_{in}(0+) = v_{R_1}(0+) + \overset{0}{\cancel{v_C(0+)}} + v_{R_2}(0+) + v_L(0+)$$

$$\Rightarrow v_{in} = i(0+)R_1 + i(0+)R_2 + L\,i'(0+)$$

$$v_K(0+) = v_{R_1}(0+) + v_C(0+)$$

$$= i(0+)R_1 + 0$$

$$\therefore \boxed{v_K(0+) = \frac{v_{in}}{R_2} \cdot R_1}$$

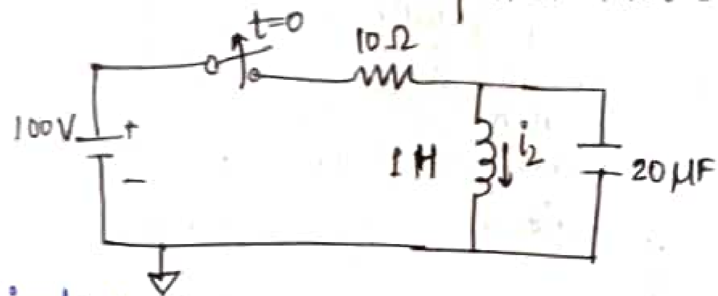
(b) $v_K'(0+) = v_{R_1}'(0+) + v_C'(0+)$

$= i'(0+) R_1 + \dfrac{i_1(0+)}{C}$

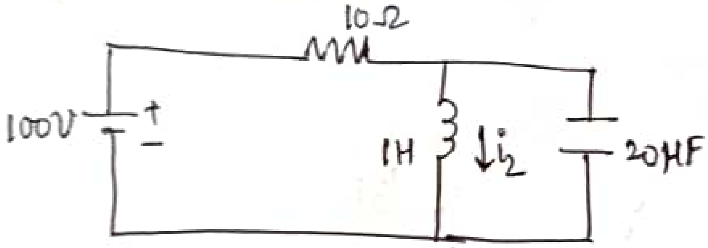
$= (-1) R_2 + \dfrac{1}{C}$

$\therefore \boxed{v_K'(0+) = -R_2 + \frac{1}{C}}$

(27) Initially switch was closed and steady state was reached in the network. At t=0, the switch is opened. Find expression for $i_2(t)$.




Soln: Initial equivalent circuit:




$i_2 = \dfrac{100}{10} = 10\,A$

$\therefore i_2(0+) = 10A$

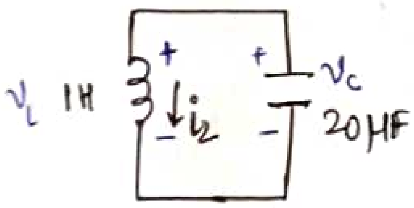
$v_C = 100 - 10\times 10$
$= 0\,V$

$\Rightarrow v_L = v_C = 0$

$\Rightarrow L i_2' = 0$

$\Rightarrow i_2'(0+) = 0$

For $t \geq 0$,




$v_L = v_C$

$\Rightarrow i_2' = \dfrac{1}{C}\int i_2\,dt$

$\Rightarrow i_2 = C\, i_2'' \Rightarrow i_2'' - \frac{1}{C} i_2 = 0$

Characteristics eqn: $\lambda^2 - \frac{1}{C} = 0$

$\Rightarrow \lambda = \pm \frac{1}{\sqrt{C}}$

$\therefore i_2(t) = k_1 e^{\frac{1}{\sqrt{C}}t} + k_2 e^{-\frac{1}{\sqrt{C}}t}$

$\Rightarrow i_2' = \frac{K_1}{\sqrt{c}} e^{\frac{1}{\sqrt{c}}t} - \frac{K_2}{\sqrt{c}} e^{-\frac{1}{\sqrt{c}}t}$

$i_2'(0) = 0 \Rightarrow 0 = \frac{K_1}{\sqrt{c}} - \frac{K_2}{\sqrt{c}} \Rightarrow K_1 - K_2 = 0$

$\Rightarrow K_1 = K_2$

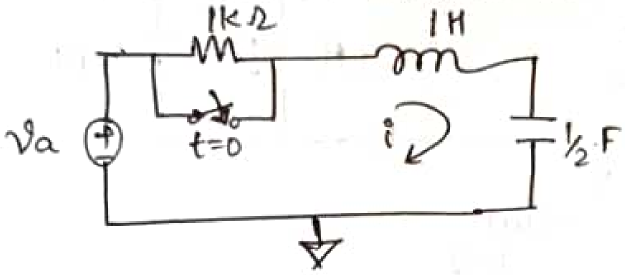
and,

$i_2(0) = 10$

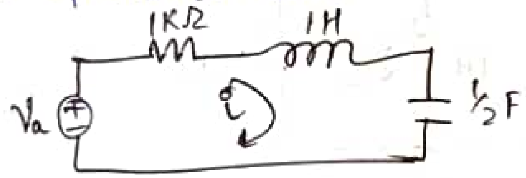
$\Rightarrow 10 = K_1 + K_1 \Rightarrow K_1 = 5A.$

$\therefore \boxed{i_2 = 5e^{\frac{1}{\sqrt{c}}t} + 5e^{-\frac{1}{\sqrt{c}}t}}.$

(28) Steady state was reached with switch open. $v_a = 100 \sin 377t$. At $t=0$, switch is closed. Determine $i(t)$ for $t \geq 0$.

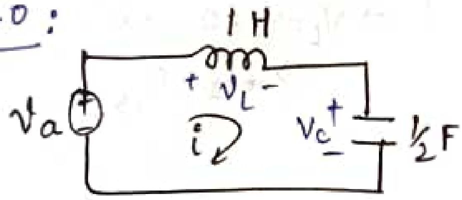



Soln: Initial equivalent ckt.:




$i(0+) = 0, \ v_L(0+) = 0 \Rightarrow Li' = 0 \Rightarrow i'(0+) = 0.$

For $t \geq 0$:




Using KVL,

$v_a = v_L + v_c$

$\Rightarrow v_a = Li' + \frac{1}{c}\int i\,dt$

$\Rightarrow Li'' + \frac{i}{c} = 100(\cos 377t)\,377 = 37700 \cos 377t$

$\Rightarrow i'' + \frac{i}{LC} = 37700 \cos 377t$

Char. eqn: $\lambda^2 + \frac{1}{LC} = 0 \Rightarrow \lambda^2 = -2$

$\Rightarrow \lambda = \pm \frac{1}{\sqrt{LC}} = \pm \sqrt{2}i$

$\therefore i_h(t) = K_1 e^{\frac{1}{\sqrt{LC}}t} + K_2 e^{-\frac{1}{\sqrt{LC}}t}$

$i'(0) = 0 \Rightarrow 0 = \frac{k_1}{\sqrt{LC}} - \frac{k_2}{\sqrt{LC}} \Rightarrow k_1 = k_2$

$i(0) = 0 \Rightarrow 0 = 2k_1 \Rightarrow k_1 = 0$

$\therefore i_h(t) = 0$

and,

$y_p(t) = A \sin 377t + B \cos 377t.$

$\Rightarrow 377^2 A \sin(377t) + (377)^2 \cos(377t) + A$

$i_{nh} = \frac{100}{377} \sin(377t - \pi/2)$

$\therefore i = c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t + \frac{100}{377} \sin(377t - \pi/2)$

As $i(0^-) = i(0^+) = 0.066\sqrt{2} \sin(-0.360)$

$V_L = iX_L = 0.66\sqrt{2} \angle -0.360 \; (377 \angle \pi/2)$

$= 351.88 \angle 1.210$

$\therefore V_L(0^+) = 351.88 \angle 1.210 = 351.88 \sin(1.210)$

$\therefore i(0^+) = -0.66\sqrt{2} \sin(0.36)$

$c_1 + \frac{100}{377}(-1) = -0.328$

$\Rightarrow c_1 = -0.062$

For $c_2$:

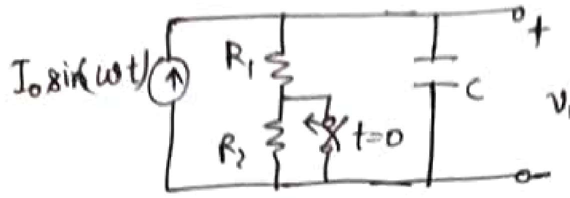
$V_L = Li' = -c_1\sqrt{2} \sin\sqrt{2}t + c_2\sqrt{2} \cos\sqrt{2}t + 100 \cos(377t - \pi/2)$

$V_L(0^+) = c_2\sqrt{2} = 329.224$

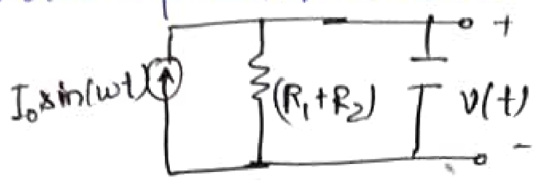
$c_2 = 232.796.$

$\therefore i(t) = -0.062 \cos\sqrt{2}t + 232.796 \sin\sqrt{2}t$
$+ 0.2652 \sin(377t - \pi/2)$

(29) Network initially was operating in steady state with 'k' opened. At $t=0$, 'k' is closed. Find expression for $v(t)$ for $t \geq 0$.



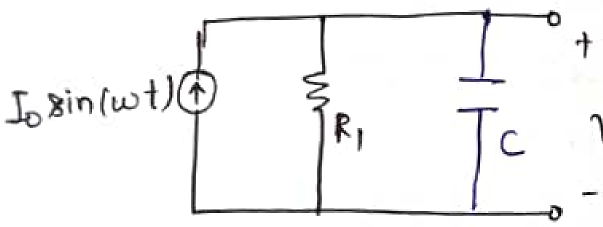

Soln: Initial equivalent circuit:




$$v(t) = I_0 \sin(\omega t) \left[(R_1+R_2) \parallel \frac{1}{\omega C}\right]$$

$$v(0^-) = 0, \quad v(0^+) = 0$$

For $t \geq 0$:




$$I_0 \sin \omega t = \frac{v(t)}{R} + i_c$$

$$\Rightarrow \frac{I_0}{C} \sin \omega t = \frac{v}{RC} + v'$$

$$IF = e^{t/RC}$$

$$\therefore v \cdot e^{t/RC} = \int \frac{I_0}{C} \sin \omega t \cdot e^{t/RC} dt = \frac{I_0}{C} \int \underbrace{\sin(\omega t)\, e^{t/\tau} dt}_{I}$$

$$I = \tau \sin \omega t\, e^{t/\tau} - \int \omega \cos \omega t \cdot e^{t/\tau}\, \tau\, dt$$

$$= \tau \sin \omega t\, e^{t/\tau} - \tau\omega \left[\cos \omega t\, e^{t/\tau} \cdot \tau - \int \sin \omega t \cdot \omega\, e^{t/\tau}\, \tau\, dt\right]$$

$$= \tau \sin \omega t\, e^{t/\tau} - \tau^2 \omega \cos(\omega t)\, e^{t/\tau} + \omega^2 \tau^2 \underbrace{\int \sin \omega t\, e^{t/\tau} dt}_{I}$$

$$\Rightarrow I = \frac{\tau e^{t/\tau}}{1+\omega^2\tau^2}\left[\sin \omega t - \omega\tau \cos \omega t\right]$$

and, $v_c(0^+) = 0$

$$\therefore v = \frac{\tau}{1+\omega^2\tau^2}\left[\sin \omega t - \omega\tau \cos \omega t\right] + \frac{\omega\tau^2}{1+\omega^2\tau^2} e^{-t/\tau}$$