Indian Institute of Space Science and Technology
Modern Signal Processing AVD611
Department of Avionics
M.Tech: Digital Signal Processing/VLSI and Microsystems/RF and Microwave Engineering

End Examination December 2024

11^{th} December 2024, Marks: 100, Time: 9.30 am to 12.30 pm

Answer ALL questions

1. (a) When the input to an LTI system is $x[n] = 5u[n]$ the output is $y[n] = [2\left(\frac{1}{2}\right)^n + 3\left(-\frac{3}{4}\right)^n]u[n]$. [5 Marks]
   (i) Determine the system function $H(z)$. Plot the poles and zeroes and indicate ROC.
   (ii) Write the difference equation that characterize the system.
   (b) A discrete time causal LTI system has the system function $H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{(1+0.81z^{-2})}$. [4 Marks]
   (i) Is the system stable ?
   (ii) $H(z)$ can be decomposed into minimum phase system $H_{min}(z)$ and all pass system $H_{ap}(z)$. Determine $H_{min}(z)$ and $H_{ap}(z)$.
   (c) Consider an LTI system with system function $H(z) = \frac{z^{-2}(1-2z^{-1})}{2(1-0.5z^{-1})}, |z| > 0.5$. Is $H(z)$ an all pass system? Explain. Draw the pole zero plot. [3 Marks]
   (d) If the impulse response of a system is given by $h[n] = \delta[n] - \delta[n-2]$. Determine the type of the FIR system and the group delay. [2 Marks]

2. (a) For the system shown in Figure 1, find the expression for $y[n]$ in terms of $x[n]$. Simplify the expression as much as possible. [2 Marks]

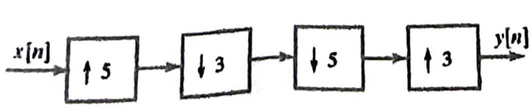

Figure 1:

   (b) Consider the Figure 2, where $H(z)$ is followed by a compressor. Suppose that $H(z)$ has an impulse response given by $h[n] = \begin{cases} (\frac{1}{2})^n, & 0 \le n \le 11 \\ 0, & otherwise \end{cases}$. Draw an efficient polyphase structure for this system with two polyphase components. Specify the filters used. [4 Marks]

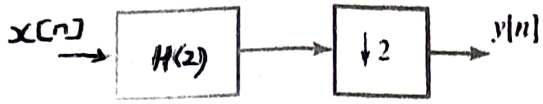

Figure 2:

(c) Determine the analysis filters of a three-channel perfect reconstruction QMF bank with synthesis filters given by $\begin{bmatrix} G_0(z) \\ G_1(z) \\ G_2(z) \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \end{bmatrix}$ [3 Marks]

(d) Plot the signals and their spectrum for rational sampling rate conversion by (i) $I/D = 7/4$ (ii) $I/D = 4/7$. Assume that the spectrum of the input $x(n)$ occupies the entire range $-\pi \leq \omega_x \leq \pi$. [3 Marks]

(e) Consider the following FIR filter transfer function :
$H(z) = -3 + 19z^{-2} + 32z^{-3} + 19z^{-4} - 3z^{-6}$
(i) Does the given system have linear phase. (ii) Show that $H(z)$ is a half band filter. [3 Marks]

(f) Consider the sequence $x(n)$ with $X(e^{j\omega})$ as shown in Figure 3. Let $x(n)$ be obtained at a sampling rate of F_s. Convert the sampling rate of $x(n)$ to $F_s' = \left(\frac{3}{5}\right) F_s$ using multirate building blocks and plot the spectrum of the output signal. [4 Marks]

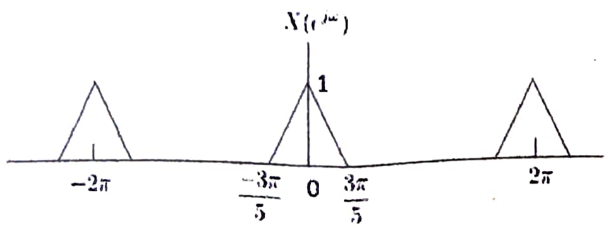

Figure 3:

3. (a) Using Haar wavelets, show that the space spanned by wavelet functions bases are orthogonal among themself. [Explain with suitable equations and figures]. [4 Marks]
(b) A signal is given by $x(t) = e^{-5t}$ for $0 \leq t \leq 1$. Use Haar wavelet family to expand the signal using only scaling function $\phi(2t - k)$. [3 Marks]
(c) Explain how time frequency resolution is achieved in wavelet transform. [2 marks]

4. (a) (i) Show that the variances of the periodogram is not a consistent estimate of power spectrum.
(ii) What was the modification given by Bartlett to improve the periodogram performance? [6 Marks]
(b) What is the quality factor of Bartlett method if $x(n)$ if of $N = 3600$ samples and frequency resolution is $\Delta f = 0.01$? If the frequency resolution is further decreased what happens to the variance? [2 Marks]
(c) Let $s(n)$ be a process with an autocorrelation sequence $r_s(k) = \delta(k) + (0.8)^{|k|} cos\left(\frac{\pi k}{4}\right)$.

the signal is observed in the absence of noise $w(n) = 0$ and $x(n) = s(n)$. Design a three step predictor to predict $s(n+3)$ and compute the MMSE. [6 Marks]

5. (a) Find the power spectrum of a process whose autocorrelation sequence is qiven by $r_x(k) = a^{|k|}$, where $|a| < 1$. [3 Marks]
(b) An AR(2) process is defined by the difference equation $x(n) = x(n-1) - 0.6x(n-2) + w(n)$, where $w(n)$ is a white noise process with variance σ_w^2. Use Yule Walker equation to solve for the values of autocorrelation $\gamma_{xx}(0)$, $\gamma_{xx}(1)$, and $\gamma_{xx}(2)$. [4 Marks]

6. (a) We observe a signal, $x(n)$, in a noisy environment, $y(n) = x(n)+0.8x(n-1)+v(n)$, where $v(n)$ is white noise with variance $\sigma_w^2 = 1$ that is uncorrelated with $x(n)$. Where, $x(n)$ is WSS AR(1) random process with autocorrelation values $r_x = [4, 2, 1, 0.5]^T$. Design a non causal IIR filter that produces the minimum mean square error. [5 Marks]
(b) Let $s(n)$ be an AR(1) process with an autocorrelation sequence $r_s(k) = (0.8)^{|k|}$, and suppose that $s(n)$ is observed in the presence of uncorrelated white noise, $w(n)$, that has a variance of $\sigma_w^2 = 1$, $x(n) = s(n) + w(n)$
(i) Design a first-order FIR Wiener filter to reduce the noise in $x(n)$, that is to estimate $s(n)$.
(ii) Find the MMSE value. [5 Marks]

7. The first three autocorrelation values of a random process consist of a single sinusoid in white noise $r_x(0) = 3, r_x(1) = 1, r_x(2) = 0$. Use PHD method to find the frequency and power of the sinusoid. Also, find the variance of the additive noise. [5 Marks]
(b) Use MUSIC algorithm to determine the frequencies and power levels of the given matrix in part (a). Compare the two approaches and state its significance. [5 Marks]

8. (a) What is the fundamental difference between steepest descent algorithms and LMS in terms of cost function. Derive the set of linear equations for SDA iteration. At what condition does the SDA converge to the Wiener filter. [4 Marks]
(b) Design a two tap Wiener filter for the given autocorrelation matrix $R_x = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$ and cross correlation between observed and desired signal is given by $r_{dy} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$. [4 Marks]
(c) For the data given in part(b) design an adaptive filter using SDA for 3 iteration. [4 Marks]
(d) Compare the results achieved in part(b) and (c) and comment. [2 Marks]
(e) What is the cost function of the RLS adaptive algorithm. State the significance of forgetting factor. How to choose the value of λ in case of non-stationary data. For what type of signal is RLS useful in design of filters. [3 Marks]