INDIAN INSTITUTE OF SPACE SCIENCE & TECHNOLOGY

B. Tech(I Year) **Physics - I (PH111)** **Assignment**

Answer all questions

1. An insect flies on a trajectory given by: $r = be^{(\omega t)}$, $\theta = \omega t$.

   i. Give a rough sketch for the trajectory of the insect

   ii. Find the velocity and acceleration vectors of the insect at time t.

   iii. What is the angle between these vectors at time t.

2. A honeybee homes in on its hive in a spiral path in such a way that the radial distance decreases at a constant rate: $r = b - ct$, while the angular speed increases at a constant rate: $\omega = kt$. Find the speed of the bee as a function of time.

3. A particle moves on a trajectory given by: $r = \frac{bt}{\tau^2}(2\tau - t)$, $\theta = \frac{t}{\tau}$ $(0\ leqt \leq 2\tau)$. Here b and τ are positive constants. What is the lowest speed of the particle in the gi ven time interval. What is the acceleration of the particle when it achieves its lowest speed.

4. A particle moves along a path given by $r = ae^{b\theta}$, where b is a positive constant of appropriate dimensions. If the angular component of acceleration is known to be zero, then write the angular speed in terms of r and sketch the path.

5. Find the force law for a central force field (i.e., $\mathbf{F} = f(r)\hat{\mathbf{r}}$) that allows a particle to move in a logarithmic spiral orbit given by $r = k\exp(\alpha\theta)$, where k and α are constants. Determine $r(t)$ and $\theta(t)$.

6. Two particles of mass m and M undergo uniform circular motion about each other at a separation R under the influence of an attractive force F. The angular velocity is ω rad/s. Show that $R = F/(\mu\omega^2)$, where $\mu = \dfrac{mM}{m+M}$ is the reduced mass.

7. Spherical polar coordinates (r, θ, ϕ) are related to the cartesian coordiantes by the relations:

   $$x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ r\cos\theta,$$

   and to cylindrical coordinates (s, ϕ, z) by the relations:

   $$x = s\cos\phi,\ y = s\sin\phi,\ z = z,$$

   a) Express the unit vectors $\hat{\mathbf{r}}$, $\hat{\theta}$ and $\hat{\phi}$ in terms of $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$, and conversely, the inverse forumulas for $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ in terms of $\hat{\mathbf{r}}$, $\hat{\theta}$ and $\hat{\phi}$.

b) Express the cylindrical unit vectors $\hat{\mathbf{s}},\ \hat{\phi},\ \hat{\mathbf{z}}$ in terms of $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$, and conversely $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ in terms of $\hat{\mathbf{s}},\ \hat{\phi},\ \hat{\mathbf{z}}$.

8. Express the kinetic energy of a particle in 3 dimensions using spherical polar coordinates $r,\ \theta$ and ϕ, and the corresponding velocities $\dot{r}, \dot{\theta}$ and $\dot{\phi}$.

9. A piece of string of length L, which can support a maximum tension T , is used to whirl a particle of mass m in a circular path. What is the maximum speed with which the particle may be whirled if the circle is
(a) horizontal, i.e. plane of motion parallel to earth's surface;
(b) vertical?

10. A square sheet of sides a has mass density varying as $\sigma = \alpha(1 + x^2)$. Find the location of its center of mass.