मौलिकवैद्युताभियन्त्रणम्

BASIC ELECTRICAL ENGINEERING

13-03-23

CIRCUIT ELEMENTS

Active Elements
↳ Supply energy

Passive Elements
↳ store/consume energy

① Resistor (R): Dissipates (consume) energy
convert to other forms
Resistance unit : Ohm (Ω)

② Inductor (L): Stores electrical energy
↳ kinetic element ($E = \frac{1}{2} L i^2$)
Inductance unit : Henry

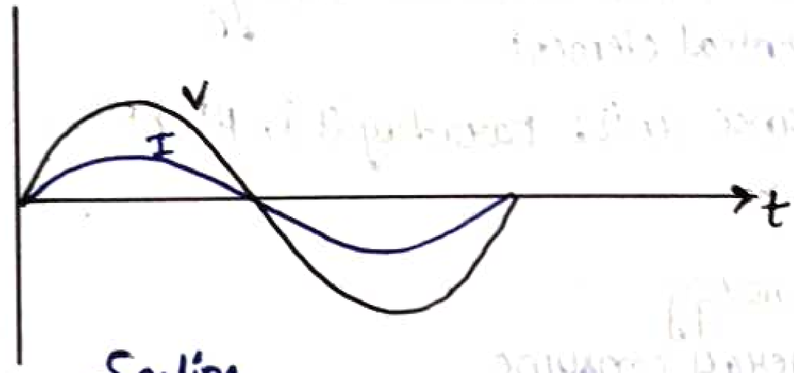
③ Capacitor (C): Stores electrical energy
↳ Potential element
Capacitance unit: Faraday (F), μF, nF

↳ Source of energy
↳ Load – energy consume
(Resistor can be used to represent load)

Element	Basic Relationship (voltage - current relationship)
Resistor	$V = IR$, $I = V/R$
Capacitor	$q = cV$ $i_c = \frac{dq}{dt} = \frac{d}{dt}(cV_c) = c\frac{dV_c}{dt} \Rightarrow i_c = c\frac{dV_c}{dt}$ $\Rightarrow V_c = \frac{1}{c}\int i_c\,dt$
Inductor	$\psi = Li$ $V_L = \frac{d\psi}{dt} = \frac{d}{dt}(Li_L) = L\frac{di_L}{dt} \Rightarrow V_L = L\frac{di_L}{dt}$ $\Rightarrow i_c = \frac{1}{L}\int V_L\,dt$

Resistor

$V = IR$

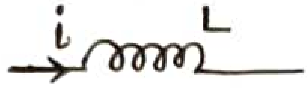


Scaling

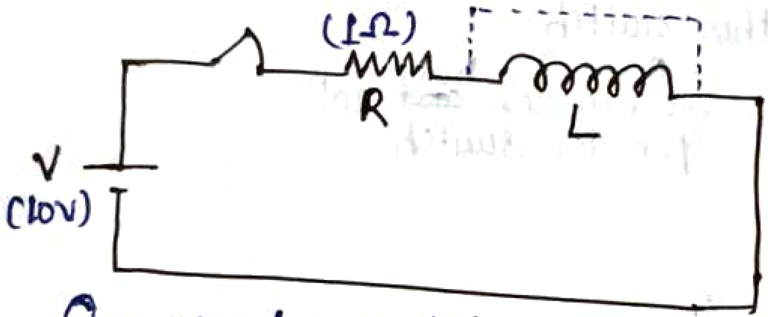
<u>Inductor</u>

$\phi = Li$

$V = \frac{d\phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$

→ 

Current in a fixed inductor system cannot change abruptly.
(as $dt \to 0$, $V \to \infty$)



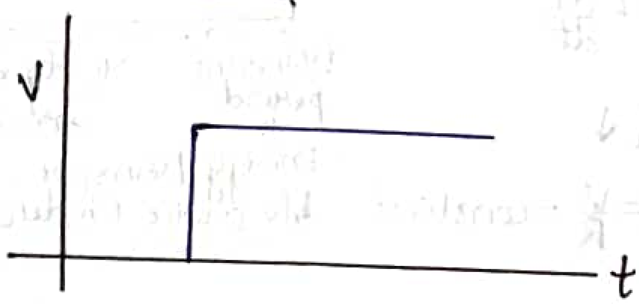
($\frac{V}{R} = 10A$ → steady current)

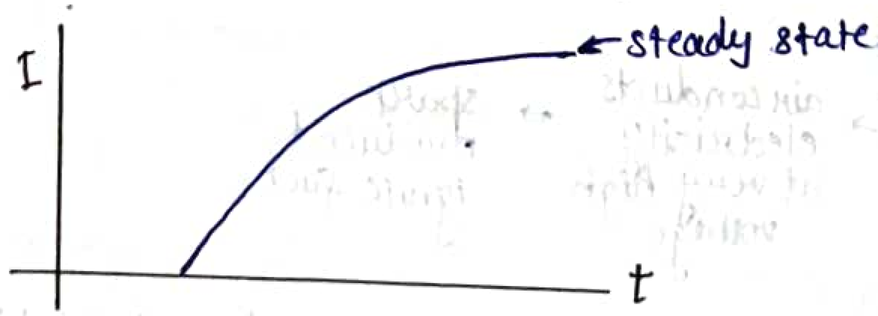
On opening switch,

$V = L\frac{di}{dt} = \frac{1 \times 10}{1 \times 10^{-6}}$ ($dt = 10^{-6}$ for electronic switch)

⇓

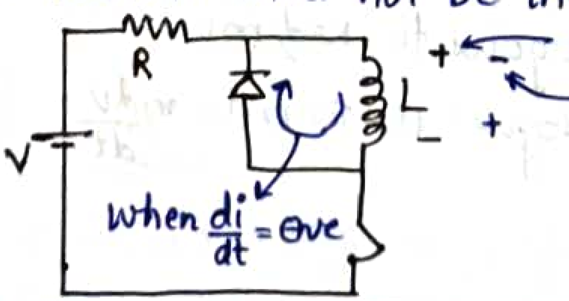
Blast





→ Inductor current shall not be interrupted

$V = L\frac{di}{dt}$



+ − ← when $\frac{di}{dt} = \oplus ve$ ⇒ $v = \oplus ve$

− + ← when $\frac{di}{dt} = \ominus ve$ ⇒ $v = \ominus ve$

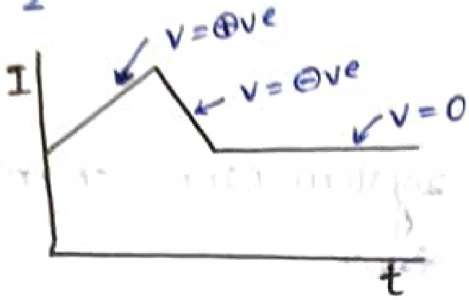
↳ diode → forward biased (current circulates)

14-03-2023

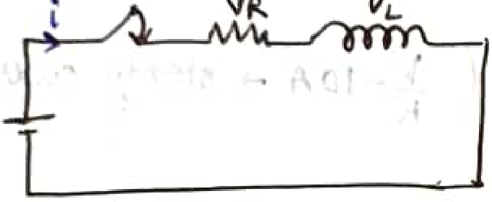
→ i + V_L − ⇐ $V = \oplus ve$ (Absorbing power)

− L + ⇐ $V = \ominus ve$ (Releasing energy)
↓
smoothens/stabilises the current

$E = \frac{1}{2} L i^2$



→ There is no use of this switch



we can close ~~cant~~ but not open the switch

closing switch : $t=0$

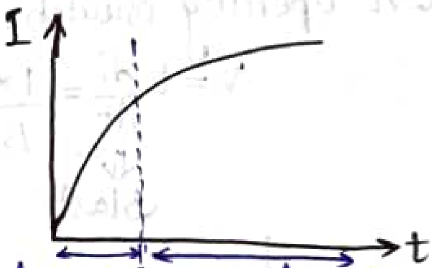
At $t=0^+$, $i=0$

$V_R = 0$

$V_L = V = L\frac{di}{dt}$

After $t=0^+$, $V_R \uparrow$, $V_L \downarrow$

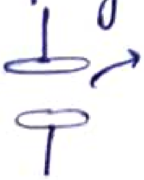
Finally $(t \to \infty)$, $I = \frac{V}{R}$ = constant.



transient period
↓
Energy transfer b/w source & inductor

Steady state
↳ no energy transfer

→ spark plug

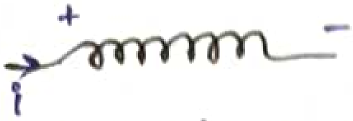


air conducts electricity at very high voltage ⇒ spark produced ignite fuel

→ $\xrightarrow{i}$ ⇒ Inertial/kinetic element ⇒ Exhibits inertia.

$E = \frac{1}{2} L i^2$ analogous to $K = \frac{1}{2} m v^2$

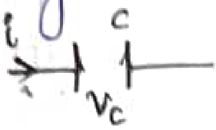
$V = L\frac{di}{dt}$ analogous to $F = ma = m\frac{dv}{dt}$

→ 

Inductor is absorbing power — Inductor is releasing power.

Capacitor

↳ Makes its presence felt in the circuit if there is a change in voltage (otherwise, it'll behave like open circuit).

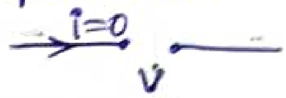


$q = CV$

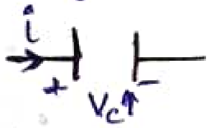
$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$

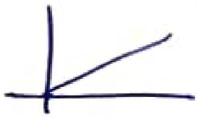
↳ b/c of changing voltage

There is a voltage, but no current: open circuit

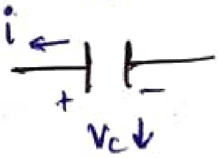


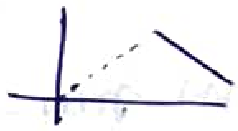
$\frac{dV}{dt} = \oplus ve \Rightarrow i = \oplus ve$

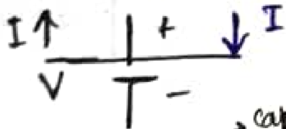




$\frac{dV}{dt} = \ominus ve \Rightarrow i = \ominus ve$
(put same polarity)

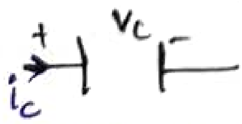


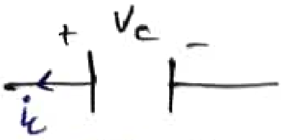




$\ominus VI \Rightarrow$ source is absorbing power

$\oplus VI \Rightarrow$ source is delivering power (capacitor)

→ 



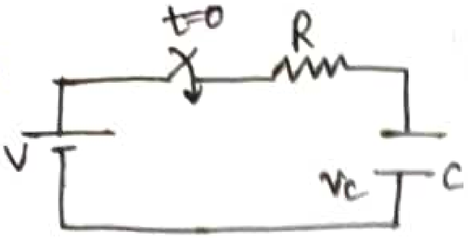
Absorbing/charging power — Releasing power / Discharging

→ Voltage cannot change abruptly/instantaneously, as there will be infinite current.

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$
$(i \to \infty)$ $(dt \to 0)$

→ Capacitor stabilises voltage in a circuit.

15-03-23

→ 

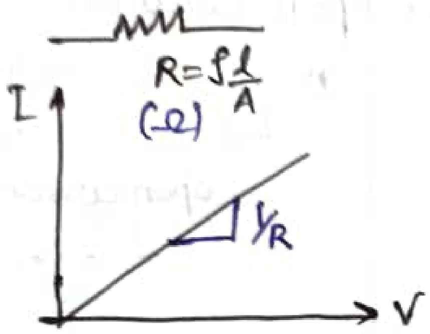
If at $t=0$, $V_c=0$

$t=0^+ \Rightarrow V_c=0$

Uncharged capacitor will act like short circuit.

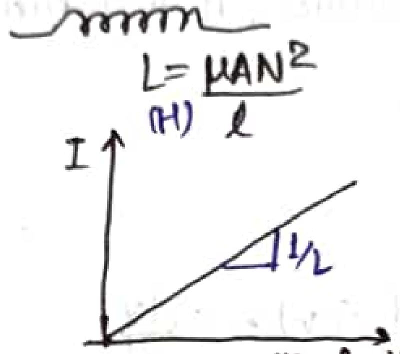
$\Downarrow$

source, capacitor, circuit will fail.

→ 

$R=\rho\frac{l}{A}$ (Ω)

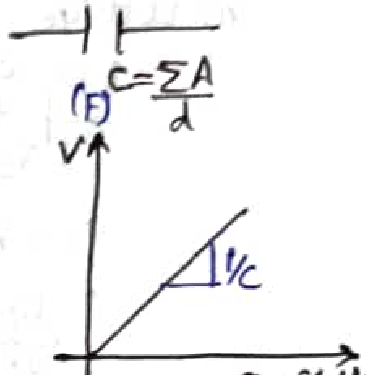
- $P=\int i^2 R\,dt$
- $v=iR$



$L=\frac{\mu A N^2}{l}$ (H)

$\psi=\int v\,dt$

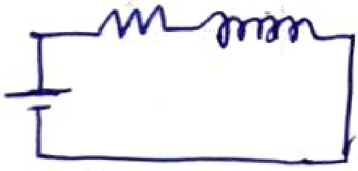
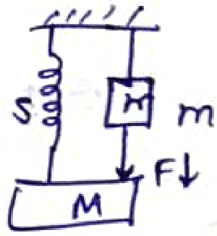
- $E=\frac{1}{2}Li^2$
- $V_L=\frac{L\,di}{dt}$
- $i_c=\frac{1}{L}\int v\,dt$



(F) $C=\frac{\varepsilon A}{d}$

$q=\int i\,dt$

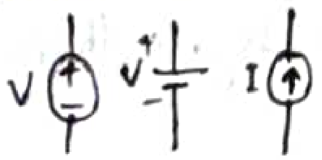
- $E=\frac{1}{2}CV^2$
- $V_c=\frac{1}{C}\int i\,dt$
- $i=C\frac{dV}{dt}$

 → analogous to 

Mass spring Damper

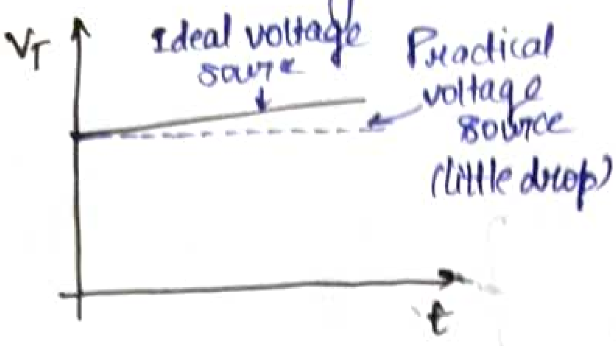
→ Element

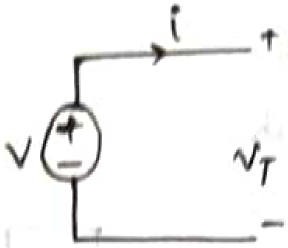
- Active element
  - Voltage Source
  - Current source
- Passive element
  - R L C

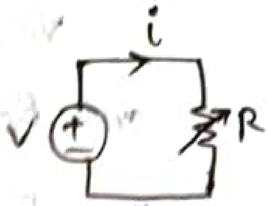


<u>Voltage Source</u>

↳ Terminal voltage remains constant.



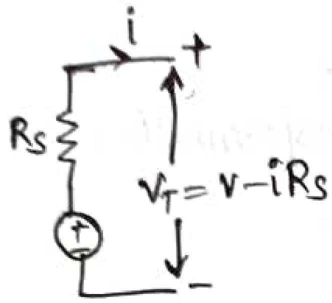




⇓

V : constant

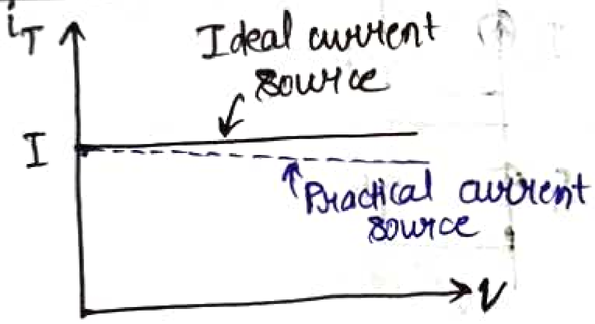
i : changes

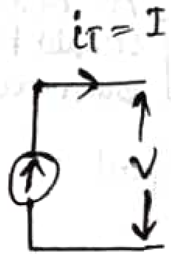


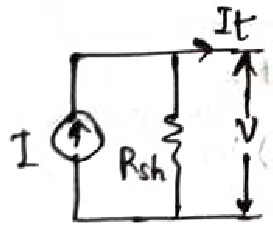
R_s : Internal resistance (of source)

↳ $=0$ (ideally)

<u>Current Source</u>



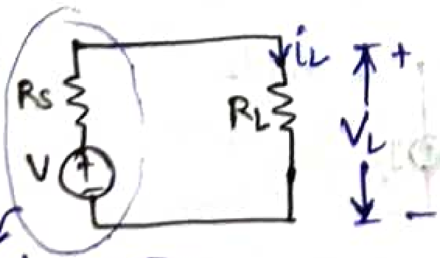




$I_t = I - \frac{V}{R_{sh}}$, R_{sh}: internal resistance of the current source

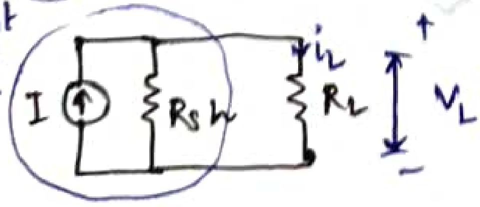
↳ $=\infty$ (ideally)

→



$i_L = \frac{V}{R_s + R_L}$, $V_L = \frac{V}{(R_s + R_L)} R_L = i_L R_L$

equivalent



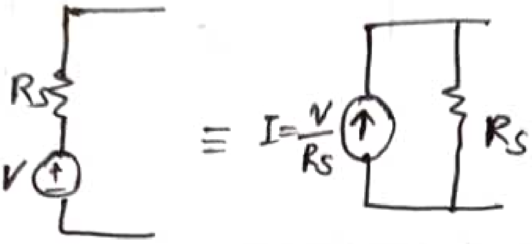
$i_L = \frac{I R_{sh}}{R_{sh} + R_L}$, $V_L = i_L R_L = \frac{I R_{sh} \cdot R_L}{(R_s + R_L)}$

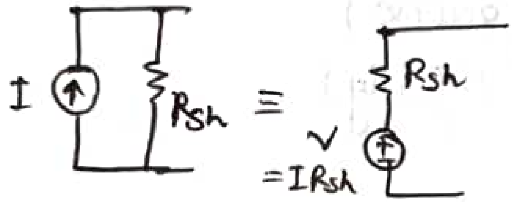
→ The voltage source is equivalent to the given current source if

$V = IR_{sh}$

and $R_s = R_{sh}$

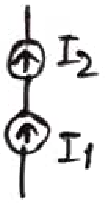
∴





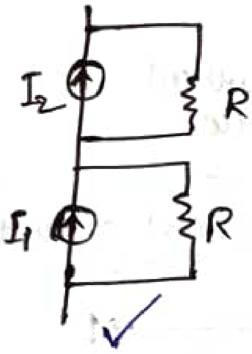
Source transformations

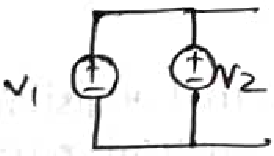
→ (Ideal)



As both should have same current

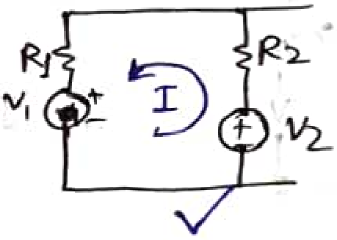
X ⇒ will fail

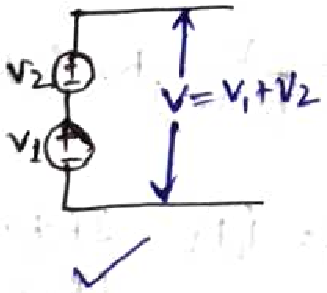


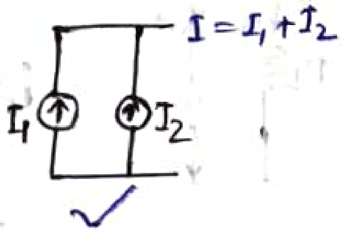


As both the voltages should be same

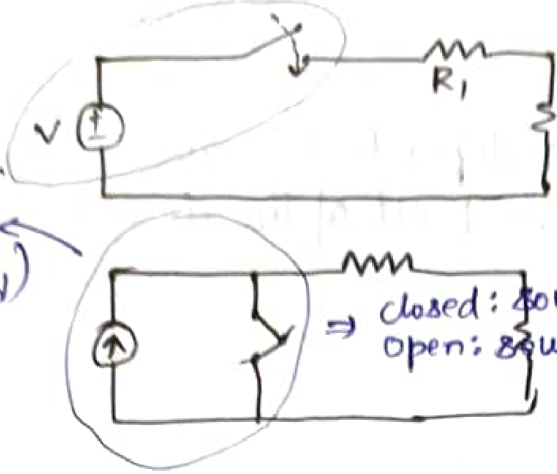
X







→ Turning on/off sources:

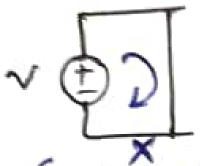


equivalent
(V switch open
I switch closed)

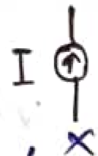
⇒ closed: source off
open: source on

21-03-2023

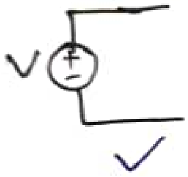
→

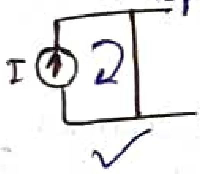


↳ voltage source shouldn't be short-circuit



↳ current source shouldn't be open-circuited



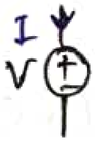


→

Source is delivering power

$P = V \times I$

⇒ Power is ⊕ve

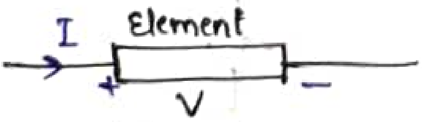


Source is absorbing power

$P = V \times (-I) = -VI$

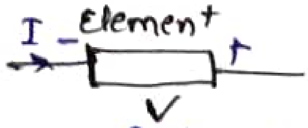
⇒ P is ⊖ve

→



Element is absorbing power

⊕ve power

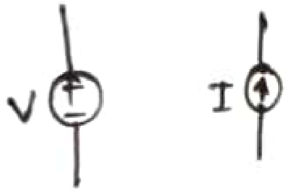


Delivering power

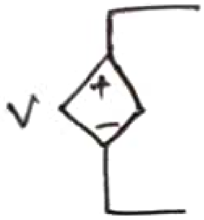
⊖ve power

→ Regenerative Breaking: Load supplies power to the source to stop the vehicle.

→

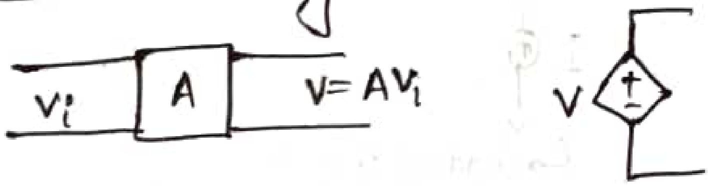


Indepent sources → not depends on current /voltages in other part of the circuit.

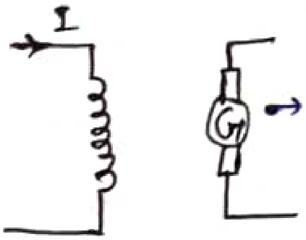


Dependent Source

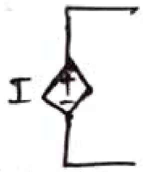
Voltage Controlled Voltage Source (VCVS)



Current Controlled voltage Source (CCVS)

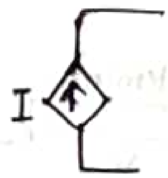


→ voltage changes a/c to I



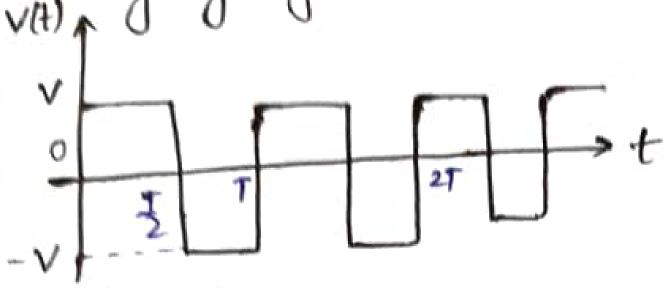
Voltage Controlled current Source (VCCS):

current controlled Current Source (CCCS):



AC CIRCUITS

Time Varying Signal

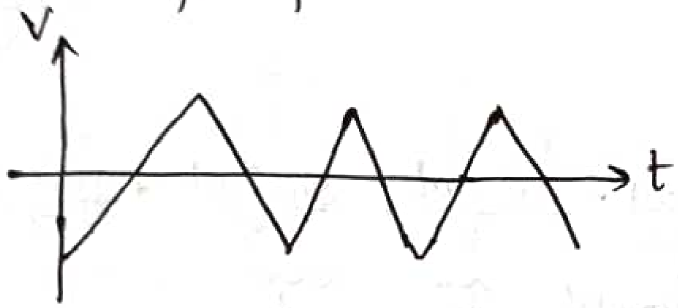


→ Rectangular waveform
→ Periodic wave forms

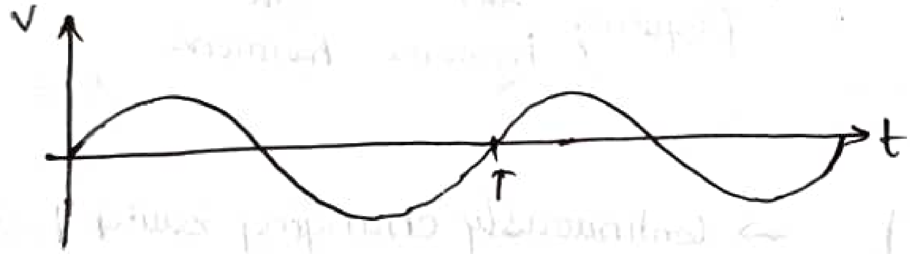
Periodic Signal

f(t) is a periodic function with period 'T' if

$$f(t) = f(t+T)$$



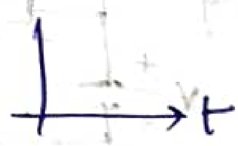
↔ Triangular waveform



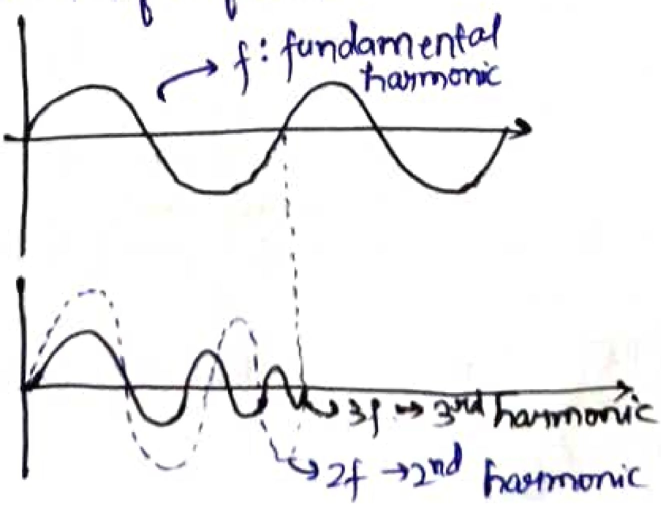
→ Sinosoidal

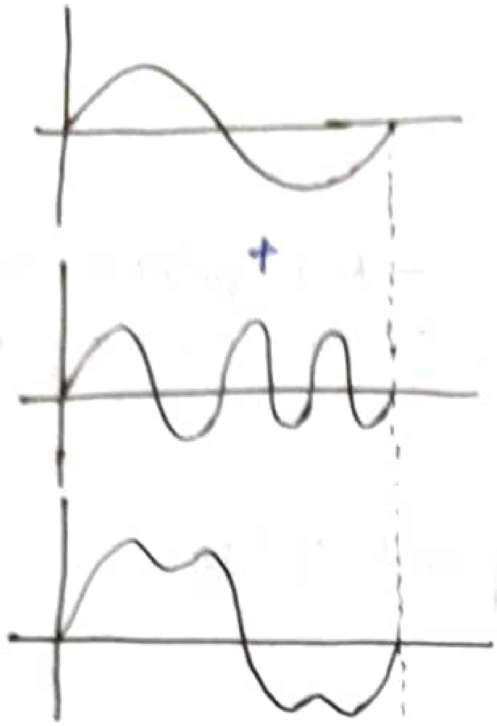
$$\text{frequency} = \frac{1}{T} \rightarrow \text{no. of cycles per second}$$

Waveform: plot of magnitude vs. time.



Fourier Series: Any periodic waveform can be represented as the sum of infinite



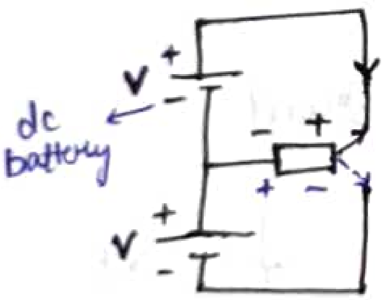


→ Resultant waveform

For square waveform:

$$v(t) = \frac{4V}{\pi}\left[\underbrace{\sin\omega t}_{\text{fundamental frequency}} + \underbrace{\frac{\sin 3\omega t}{3}}_{3^{rd}\text{ harmonic}} + \underbrace{\frac{\sin 5\omega t}{5}}_{5^{th}\text{ harmonic}} + \frac{\sin 7\omega t}{7} + \dots\right]$$

→ Inverter:

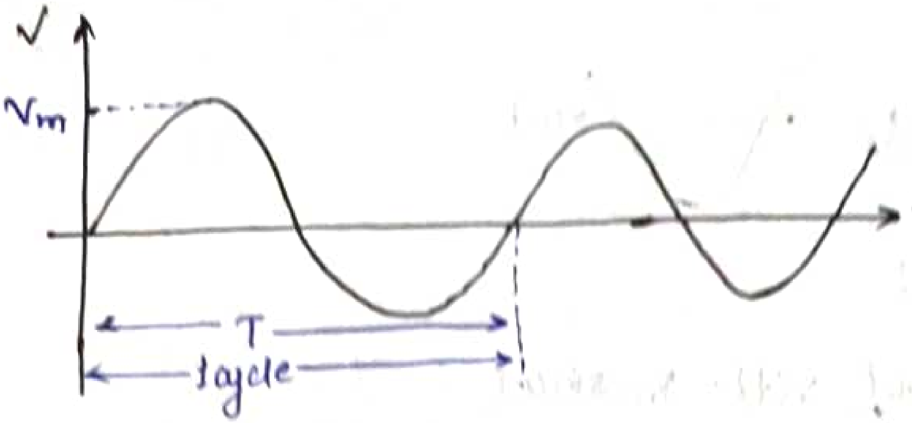


→ Continuously changing switch position will generate rectangular waveform/pulses.

⇓

Doing this 50 times a second will generate a 50 Hz AC current across the element.

22-03-2023



V_m : Peak value

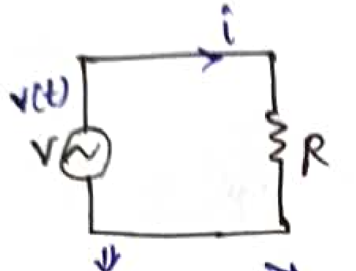
T : period

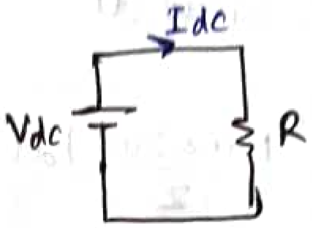
frequency : no. of cycles/second
↓
Hertz (Hz)

Angular frequency, $\omega = 2\pi f$ ↔ rad/sec.

$= \frac{2\pi}{T}$

→





$i = \frac{v(t)}{R}$ ⇒

If both R produce the same power/heat, effective value of i must be I_{dc}.

For dc: $P = I_{eff}^2 R$

If v and i are instantaneous values of voltages and current (in the ac),

instantaneous power, $p = v \times i$
$= i^2 R$

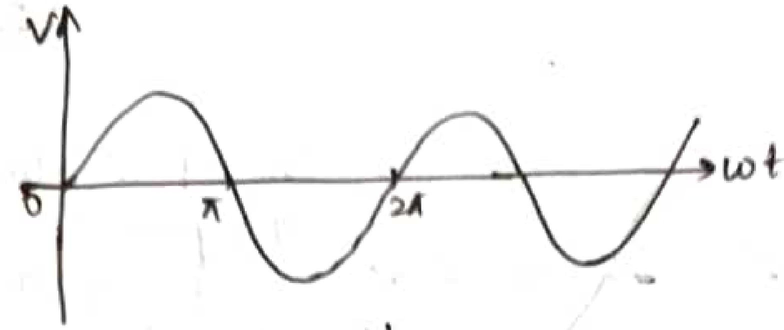
average power, $P_{av} = \frac{1}{T}\int_0^T i^2 R\, dt$

$\therefore I_{eff}^2 R = \frac{1}{T}\int_0^T i^2(t) R\, dt$

$\Rightarrow I_{eff}^2 = \frac{1}{T}\int_0^T i^2(t)\, dt$

$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}}$ → Root Mean Square value (RMS)

→ RMS value of $f(t)$ with period T : $\sqrt{\frac{1}{T}\int_0^T f(t)^2\, dt}$.

→ 

$V = V_m \sin(\omega t)$

RMS value of sinusoid $v(t) = V_m \sin\omega t$:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T V_m^2 \sin^2\omega t\,dt}$$

$$= \sqrt{\frac{V_m^2}{T}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right)dt}$$

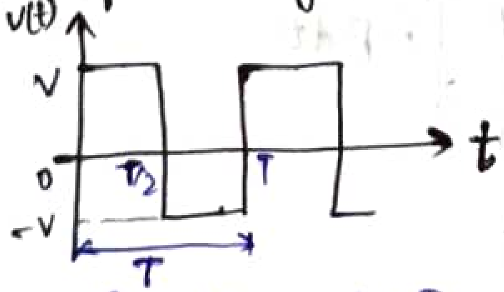
$$= \sqrt{\frac{V_m^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T}$$

$$= \sqrt{\frac{V_m^2}{2T}\left[T - \frac{\sin 2.\frac{2\pi}{T}.T}{2\omega}\right]} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$\therefore \boxed{V_{RMS} = \frac{V_m}{\sqrt{2}} = \frac{\text{peak value}}{\sqrt{2}}}$$

→ RMS value of rectangular voltage waveform:



$$v(t) = \begin{cases} V, & \text{for } 0 < t \le T/2 \\ -V, & \text{for } T/2 < t \le T \end{cases}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} v^2 (dt) + \int_{T/2}^{T} (-v)^2 dt\right]}$$

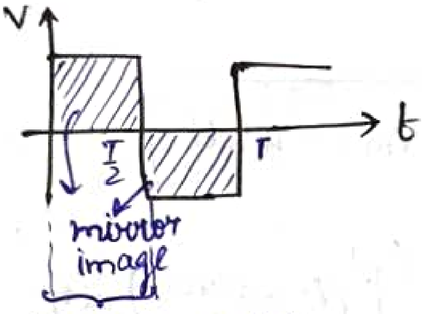
$$= \sqrt{\frac{1}{T}\left[(v^2 t)_0^{T/2} + (v^2 t)_{T/2}^{T}\right]} = \sqrt{\frac{v^2}{T}\left[\frac{T}{2} + T - \frac{T}{2}\right]}$$

$$= \sqrt{v^2}$$

$$\therefore \boxed{V_{rms} = V}$$

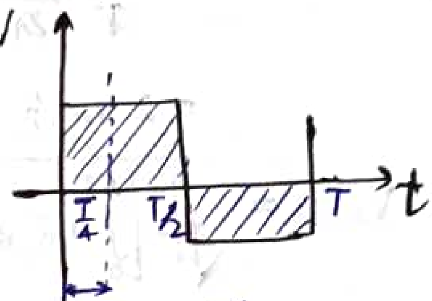
→ Half-wave symmetry:

$f(t) = -f(t - T/2)$



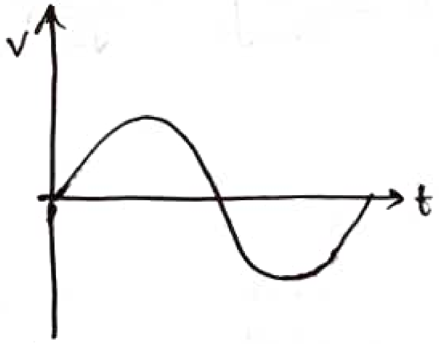
$\underbrace{2\times RMS}_{(\text{for } T/2)} = \underbrace{RMS}_{(\text{for } T)}$

→ Quarter wave Symmetry:

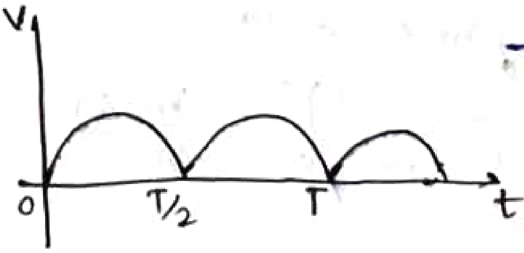


$\underbrace{4\times RMS}_{(\text{for } \frac{T}{4})} = \underbrace{RMS}_{(\text{for } T)}$

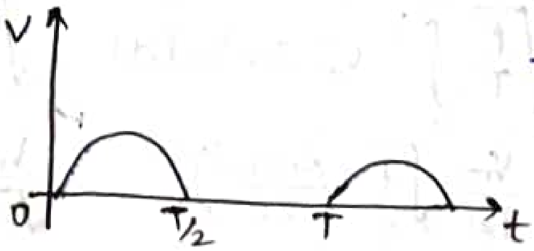
→ $v = v_m \sin \omega t$



M:



→ Full wave rectifier
- → RMS value: $\frac{V_m}{\sqrt{2}}$
- → Average value: $\frac{2V_m}{\pi}$



→ Half wave rectifier
- → RMS value: $\frac{V_m}{2}$
- → Average value: $\frac{V_m}{\pi}$

Full wave rectifier:

$$V_{av} = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T} V_m \sin(\omega t)\,dt + \int_{T/2}^{T} V_m(-\sin\omega t)\,dt\right]$$

$$= \frac{V_m}{T}\left[\left(-\frac{\cos\omega t}{\omega}\right)_0^{T/2} + \left(\frac{\cos\omega t}{\omega}\right)_{T/2}^{T}\right]$$

$$= \frac{V_m}{\cancel{T}}\cdot\frac{\cancel{T}}{2\pi}\left((1+1)+(1+1)\right)$$

$$= 2V_m/\pi$$

$$V_{rms} = \sqrt{\frac{1}{T}\left[\int_0^T V_m^2 \sin^2\omega t\,dt + \int_{T/2}^{T} V_m^2 \sin^2\omega t\,dt\right]}$$

$$= \frac{V_m}{\sqrt{T}}\left(\int_0^T \sin^2\omega t\,dt\right)^{1/2} = \frac{V_m}{\sqrt{T}}\left(\int_0^T\left(\frac{1-\cos 2\omega t}{2}\right)dt\right)^{1/2}$$

$$= \frac{V_m}{\sqrt{2T}}\sqrt{\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T} = \frac{V_m}{\sqrt{2T}}\sqrt{(T-0)}$$

$$= \frac{V_m}{\sqrt{2}}$$

Half wave rectifier:

$$V_{av.} = \frac{1}{T}\int_0^{T/2} V_m \sin(\omega t)\,dt$$

$$= \frac{V_m}{T}\left(-\frac{\cos\omega t}{\omega}\right)_0^{T} = \frac{V_m}{2\pi}(2)$$

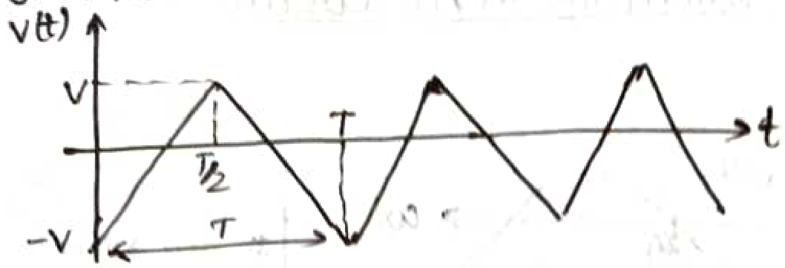
$$= \frac{V_m}{\pi}$$

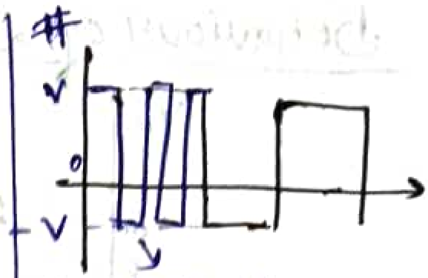
$$V_{rms.} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2\omega t\,dt} = \sqrt{\frac{V_m^2}{T}\int_0^{T/2}\sin^2\omega t\,dt}$$

$$= \frac{V_m}{\sqrt{2T}}\sqrt{\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^{T/2}} = \frac{V_m}{\sqrt{2T}}\sqrt{\frac{T}{2} - 0}$$

$$= \frac{V_m}{2}$$

Q) Find RMS value:





RMS value remains same

Soln: $v(t) = \begin{cases} \left(\frac{4V}{T}\right)t - V, & 0 < t \le T/2 \\ -\left(\frac{4V}{T}\right)t + 3V, & \frac{T}{2} < t \le T \end{cases}$

$$V_{av.} = \frac{1}{T}\left[\int_0^{T/2}\left(\left(\frac{4V}{T}\right)t - V\right)dt + \int_{T/2}^{T}\left(-\left(\frac{4V}{T}\right)t + 3V\right)dt\right]$$

$$= \frac{1}{T}\left[\left(\frac{4V}{T}\cdot\frac{t^2}{2} - Vt\right)_0^{T/2} + \left(-\frac{4V}{T}\frac{t^2}{2} + 3Vt\right)_{T/2}^{T}\right]$$

$$= \frac{1}{\cancel{T}}\left[\left(\cancel{\frac{V\cancel{T}}{2}} - \cancel{\frac{V\cancel{T}}{2}}\right) + \left(-2V\cancel{T} + \frac{V\cancel{T}}{2} + 3V\cancel{T} - \frac{3V\cancel{T}}{2}\right)\right]$$

$$= V - V$$

$$\therefore \boxed{V_{av} = 0}$$

$$V_{rms}^2 = \frac{1}{T}\int_0^{T/2}\left(\frac{4V}{T}t - V\right)^2 dt + \int_{T/2}^{T}\left(-\frac{4V}{T}t + 3V\right)^2 dt$$

$$= \frac{1}{T}\int_0^{T/2}\left(\frac{16V^2}{T^2}t^2 + V^2 - 8\frac{V^2 t}{T}\right)dt + \frac{1}{T}\int_{T/2}^{T}\left(\frac{16V^2}{T^2}t^2 + 9V^2 - \frac{24}{T}V^2 t\right)dt$$

$$= \frac{1}{T}\left(\frac{16V^2}{T^2}\frac{t^3}{3} + V^2 t - \frac{\cancel{8}^{4}V^2}{T}\frac{t^2}{\cancel{2}}\right)_0^{T/2} + \frac{1}{T}\left(\frac{16V^2}{T^2}\frac{t^3}{3} + 9V^2 t - \frac{\cancel{24}^{12}V^2}{T}\frac{t^2}{\cancel{2}}\right)_{T/2}^{T}$$

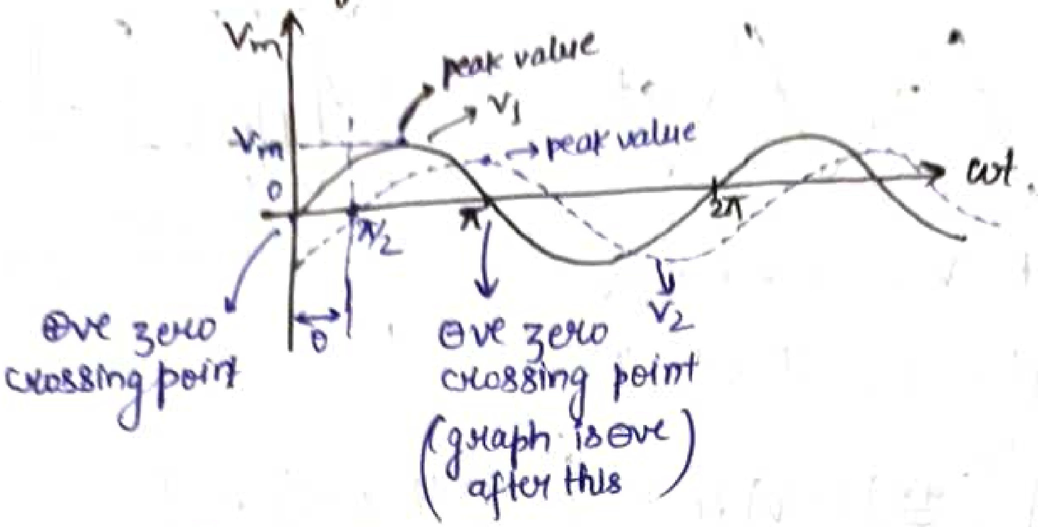
$$= \left(\cancel{\frac{2V^2}{3}} + \frac{V^2}{2} - V^2\right) + \left(\frac{16}{3}V^2 + \cancel{9V^2} - 12V^2\right) - \left(\cancel{\frac{2V^2}{3}} + \frac{9}{2}V^2 - \cancel{3V^2}\right)$$

$$= \left(\frac{16}{3} - 4 - 1\right)V^2 = \left(\frac{16}{3} - 5\right)V^2 = \frac{V^2}{3}$$

$$\Rightarrow \boxed{V_{rms} = \frac{V}{\sqrt{3}}}$$

28-03-2023

Behaviour of Different Elements in AC Circuit



10 $\sin\omega t$ → V_{av}

230V → V_{rms}

(i) phase

(ii) phase difference

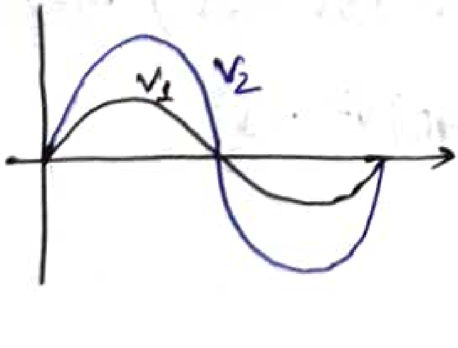
→ There is a phase difference b/w the two waveforms V_1 and V_2.
(peak, +ve zero crossing point, etc. of the two waveforms are happening at the different instant; similar points are not occuring simultaneously.)

→ V_2 is lagging behind V_1,
or V_1 is leading ahead of V_2.

If $V_1 = V_{m_1} \sin\omega t$
$V_2 = V_{m_2} \sin(\omega t - \theta)$
} frequencies need to be same for the two waveforms

If $V_2 = V_{m_2} \sin\omega t$
$V_1 = V_{m_1} \sin(\omega t + \theta)$

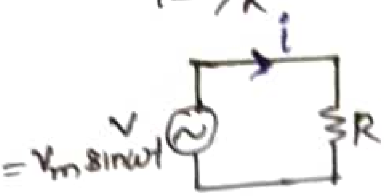
→



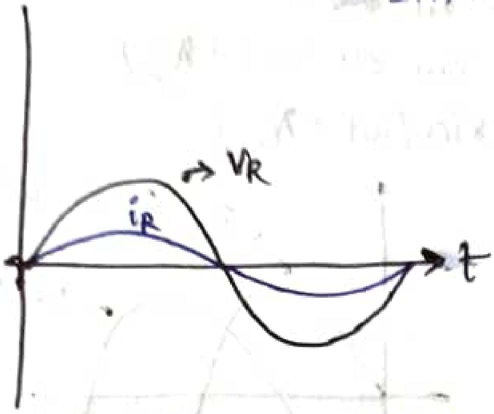
V_1 and V_2 are in phase.

Resistor (R):

$i = V/R$



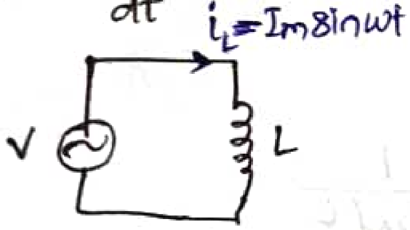
$$i = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$
$$= \left(\frac{V_m}{R}\right) \sin\omega t$$
$$= I_m \sin\omega t$$



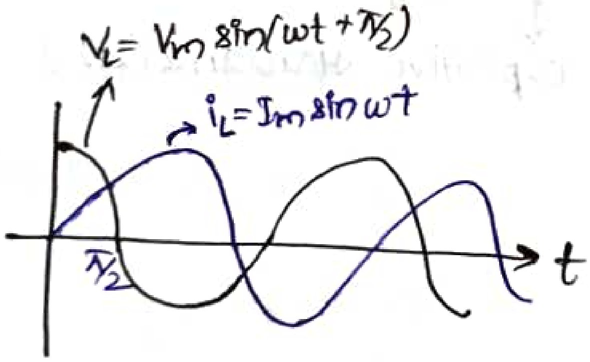
In a resistor, voltage and current are in phase.

Inductor (L):

$V_L = L\frac{di_L}{dt}$

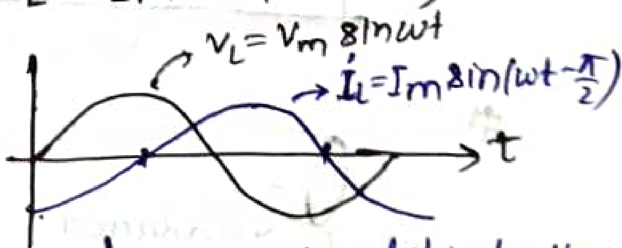


$$V_L = L\frac{di_L}{dt}$$
$$= L\frac{d}{dt} I_m \sin\omega t$$
$$= \omega L I_m \cos\omega t$$
$$= \omega L I_m \sin(\omega t + \pi/2)$$
$$= V_m \sin(\omega t + \pi/2)$$



$V_L = V_m \sin\omega t$

$i_L = I_m \sin(\omega t - \pi/2)$



$V_m = \omega L I_m$

$\frac{V_m}{I_m} = \omega L = 2\pi f L$

- same dimension as R
- resists the flow of current like R
- but doesn't dissipate power

- current lags behind voltage
- OR voltage leads ahead of current by an angle of $\pi/2$

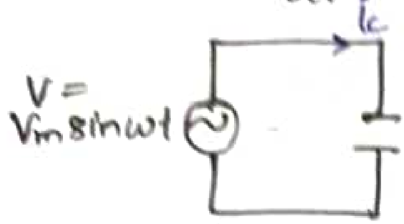
$\Downarrow$

Inductive resistance: $X_L = \omega L = 2\pi f L$

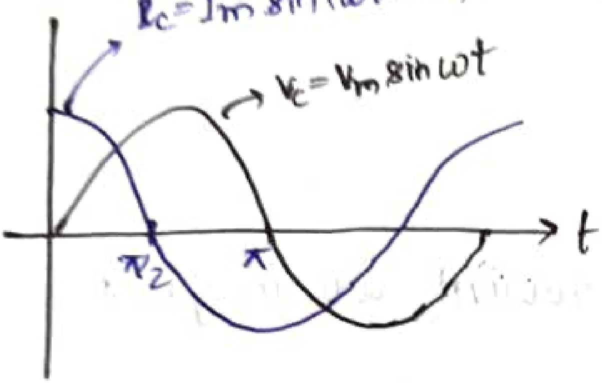
↳ frequency depedent opposition to the flow of current (unlike R)

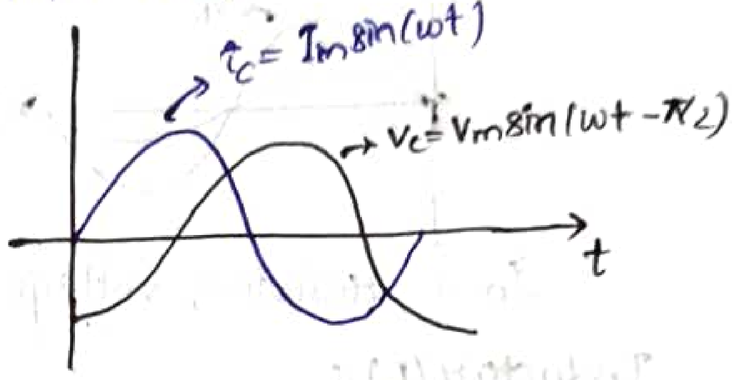
Capacitor (C):

$$i = C\frac{dV}{dt}$$



$$i_c = C\frac{dV}{dt}$$
$$= C\frac{d}{dt} V_m \sin\omega t$$
$$= \omega C V_m \cos\omega t$$
$$= \omega C V_m \sin(\omega t + \pi/2)$$
$$= I_m \sin(\omega t + \pi/2)$$

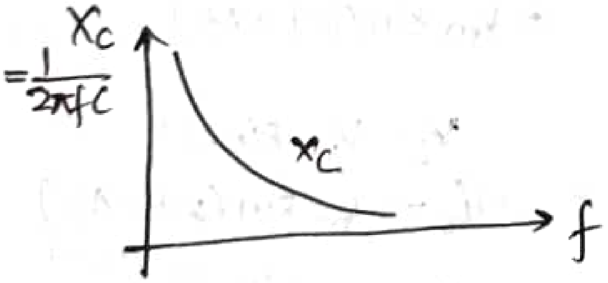




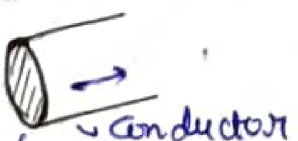
→ voltage lags behind current.
OR
→ current leads voltage by $\pi/2$.

$$I_m = \omega C V_m$$
$$\frac{V_m}{I_m} = \frac{1}{\omega C} = \frac{1}{2\pi f C} \Rightarrow X_c = \frac{1}{2\pi f C}$$

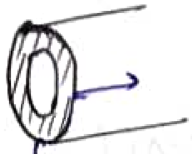
↓ capacitive reactance (Ω)



#



↳ dc flows over the entire conductor

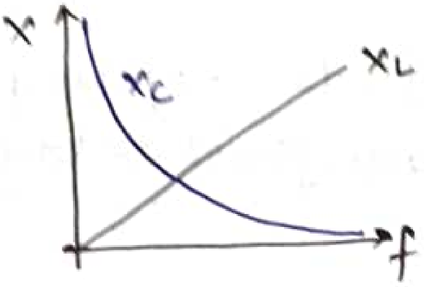


↳ ac only flows on the surface
↳ $f\uparrow \Rightarrow$ ac flows on the surface only.
↳ skin resistance
⇓
negligible effect of f on resistance

29-03-23

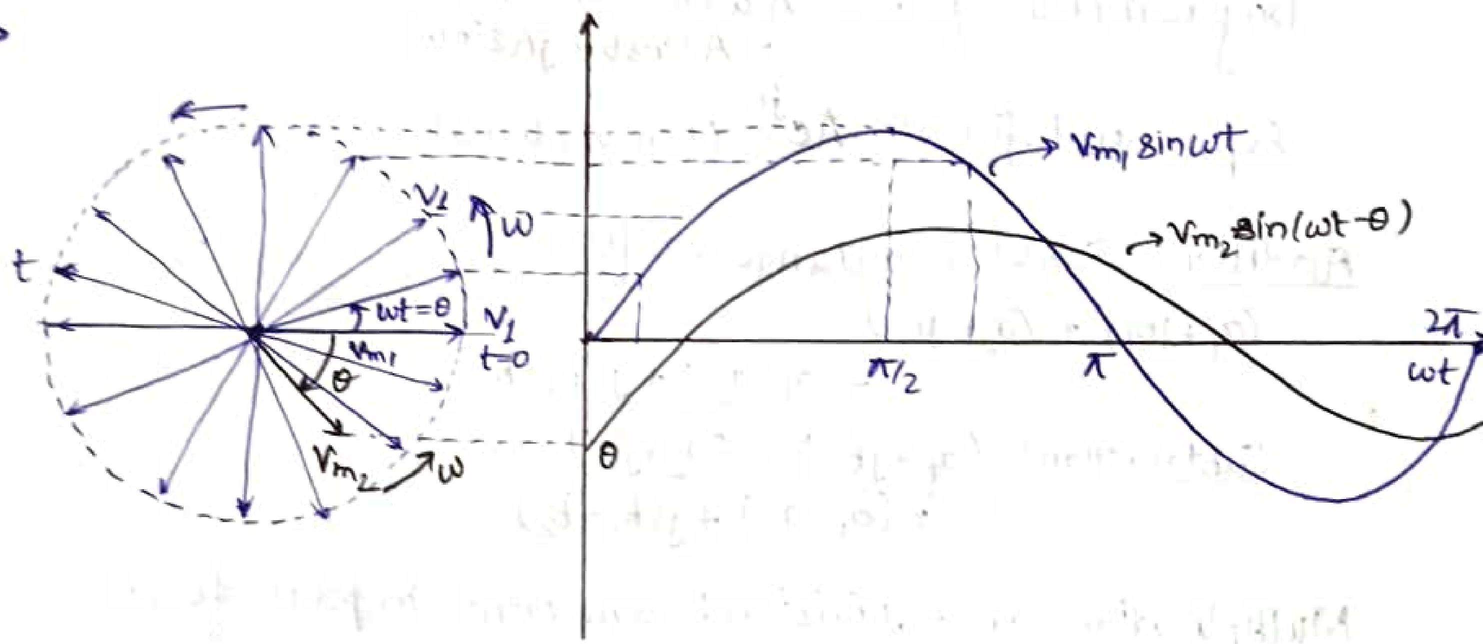
Reactance

- Inductive reactance: $X_L = 2\pi f L \rightarrow f=0 \Rightarrow$ short circuit | $f=0 \Rightarrow DC$
- capacitive reactance: $X_C = \frac{1}{2\pi f C} \rightarrow f=0 \Rightarrow$ infinite resistance

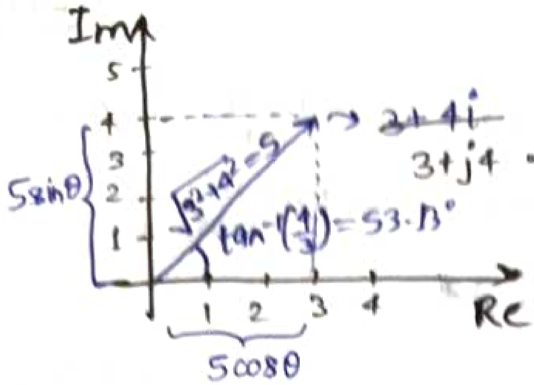


ω = rad/sec
= $\frac{\text{Angle}}{\text{time}}$

→



Rotating vector: Phasor
- represents only sinusoids
- rotates
- constant angular velocity
- can be represent using complex no.



$3+j4 \rightarrow$ Rectangular form

'i' → reserved for current

$5\,\angle 53.13^\circ \rightarrow$ polar form

$5\cos\theta + j5\sin\theta \rightarrow$ trigonometric form

Rectangular form: $a+jb$

Polar form: $A\angle\theta$

Trigonometric form: $A\cos\theta + jA\sin\theta$
$= A(\cos\theta + jA\sin\theta)$

Exponential form: $Ae^{j\theta}$ (θ in radians).

Addition: Easier in rectangular form

$(a_1+jb_1)+(a_2+jb_2)$
$=(a_1+a_2)+j(b_1+b_2)$

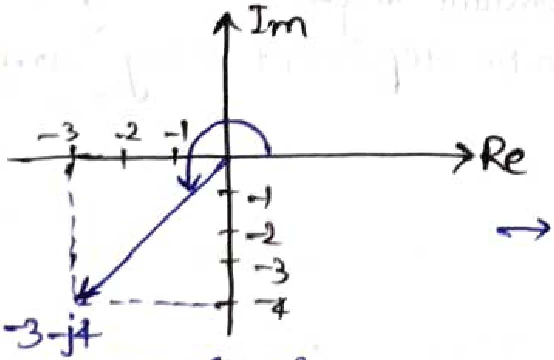
Subtraction: $(a_1+jb_1)-(a_2+jb_2)$
$=(a_1-a_2)+j(b_1-b_2)$

Multiplication and Division: Convenient in polar form

$A_1\angle\theta_1 \times A_2\angle\theta_2 = A_1A_2\angle\theta_1+\theta_2$

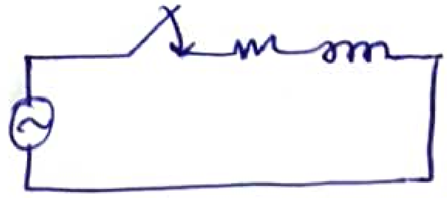
$\frac{A_1\angle\theta_1}{A_2\angle\theta_2} = \left(\frac{A_1}{A_2}\right)\angle\theta_1-\theta_2$

Sinusoids
↓
represented using phasor, not vector



$-3-j4$
$=5\,\angle 53.13^\circ+180^\circ$

↔



↳ After some time, sinusoidal steady state is achieved.

(3-04-2023)

Eg: $3 \rightarrow 3+j0 \rightarrow 3\angle 0$

$j3 \rightarrow 0+j3 \rightarrow 3\angle 90^\circ$

$j = \sqrt{-1}$

$j \times j3 = -3 \rightarrow -3+j0 \rightarrow 3\angle 180^\circ$

$\rightarrow \quad a+jb \rightarrow A\angle\theta$

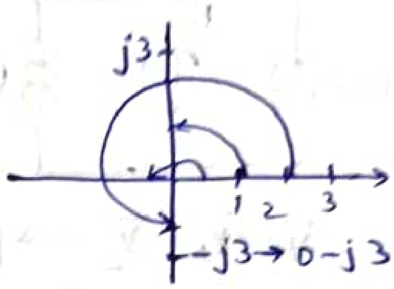
$\Rightarrow \quad A = \sqrt{a^2+b^2}$

and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Eg:

$-3 \rightarrow 3\angle 180^\circ$

$-j3 \rightarrow 3\angle 270^\circ$



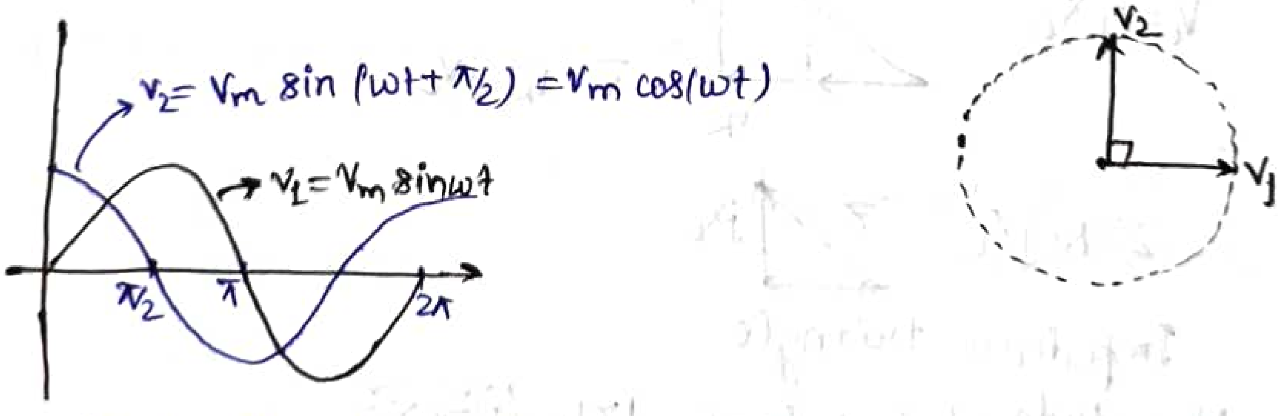
$3 \rightarrow 3+j0 \rightarrow 3\angle 0$

$j3 \rightarrow 0+j3 \rightarrow 3\angle 90^\circ$

$-3 \rightarrow -3+j0 \rightarrow 3\angle 180^\circ$

$-3j \rightarrow 0-j3 \rightarrow 3\angle -90^\circ$

SINUSOIDAL STEADY STATE ANALYSIS OF AC CIRCUIT



$V_1 = V_m \sin\omega t$

$V_2 = V_m \cos\omega t = jV_1 = jV_m \sin\omega t$

$\frac{dv_1}{dt} = \frac{d}{dt} V_m \sin\omega t = \omega V_m \cos\omega t = \omega V_2$

$= j\omega V_1$

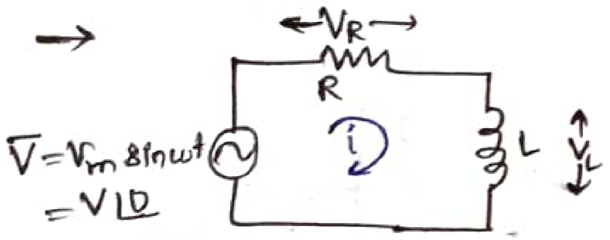
$\therefore \boxed{\frac{dV_1}{dt} = j\omega V_1}$

$\rightarrow V_2 = jV_1 \Rightarrow V_1 = \frac{V_2}{j}$

$$\int v_2\,dt = \int V_m \cos\omega t\,dt = \frac{V_m}{\omega}\sin\omega t = \frac{V_1}{\omega} = \frac{V_2}{j\omega}$$

$$\therefore \boxed{\int v_2\,dt = \frac{V_2}{j\omega}}$$

$\rightarrow$



$\bar{V} = \bar{V}_R + \bar{V}_L \rightsquigarrow$ phases

$V = iR + L\frac{di}{dt}$

Using sinusoidal steady state,

$V = iR + j\omega Li = i(R + j\omega L)$

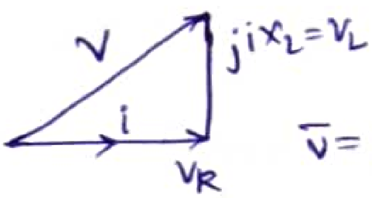
$V = iZ$, $Z \rightarrow$ Impedence

$Z = R + jX_L$

$\omega L = 2\pi f L = X_L$.
$\downarrow$
Inductive reactance

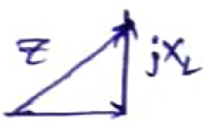
$V_R = iR$

$V_L = ijX_L$



$\bar{V} = V_R + jV_L$

$Z = R + jX_L$

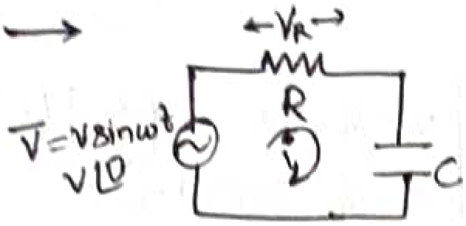


Impedence triangle

Magnitude of Impedence, $|Z| = \sqrt{R^2 + X_L^2}$

Angle of impedence, $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

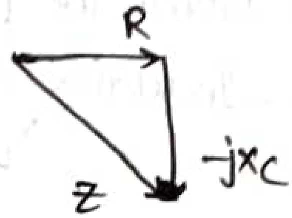
$Z = R + jX_L = |Z|\angle\theta$



$$V = Ri + \frac{1}{C}\int i\,dt$$

Using sinusoidal steady state,

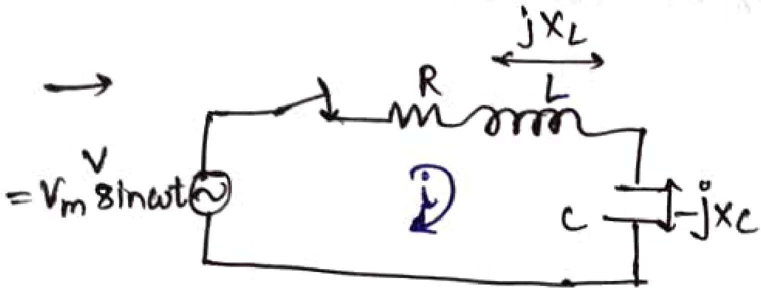
$$V = Ri + \frac{i}{j\omega C}$$
$$= Ri - jX_C i$$
$$= i(R - jX_C)$$
$$= iZ$$



$Z = R - jX_C$

$|Z| = \sqrt{R^2 + X_C^2}$

$\angle Z = \theta = \tan^{-1}\left(-\frac{X_C}{R}\right)$



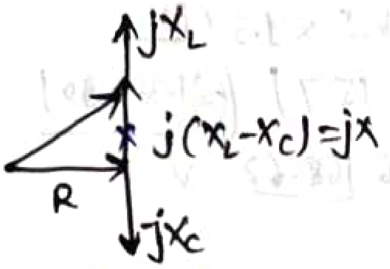
Using KVL,

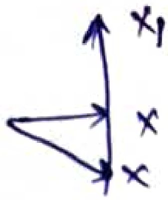
$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Using sinusoidal steady state condition,

$$V = iR + j\omega L i + \frac{i}{j\omega C}$$
$$V = iR + jX_L i - jX_C i$$
$$= iR + ij(X_L - X_C)$$
$$\therefore V = i(R + jX)$$

⇝ capacitor reactance opposes the inductive reactance.

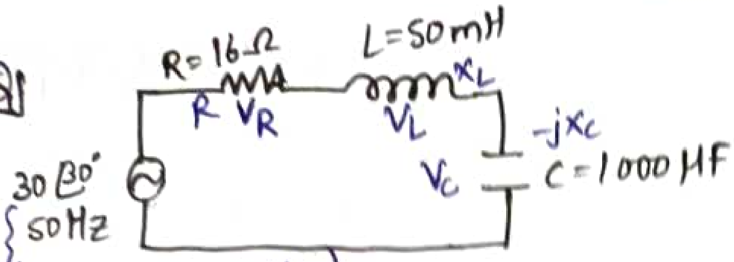




At some time t,

$X_L = X_C$ ⇝ at resonance

Q)



(rms value if not mentioned anything)

$V_1 = V_{m1} \sin \omega t \rightarrow \left(\frac{V_{m1}}{\sqrt{2}}\right) \angle 0^\circ$

$V_2 = V_{m2} \sin(\omega t - 60^\circ)$

$\downarrow$

$\left(\frac{V_{m2}}{\sqrt{2}}\right) \angle -60^\circ$

① Determine the current 'i' in the circuit.

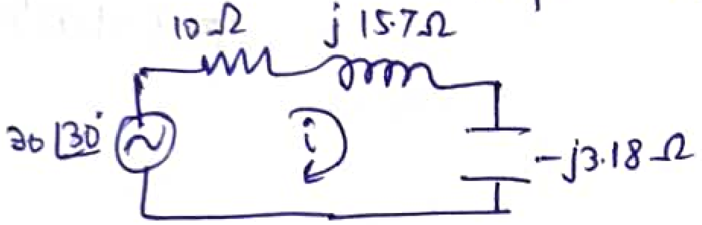
② Draw the phasor diagram representing i, V_R, V_L, V_C, V.

Soln: Inductive reactance,

$$X_L = 2\pi f L = 2\pi \times 50 \times 50 \times 10^{-3}$$
$$= 15.7\,\Omega$$

capacitive reactance,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 1000 \times 10^{-6}} = 3.18\,\Omega$$



$$\text{Current, } i = \frac{V}{Z} = \frac{V}{R + jX_L - jX_C} = \frac{30 \angle 30}{10 + j15.7 - j3.18\,\Omega}$$

$$= \frac{30 \angle 30}{10 + j12.52} = \frac{30 \angle 30}{16.03 \angle 51.38} = 1.87 \angle -21.38 \text{ A}$$

$10 + j12.52$
$= \sqrt{10^2 + 12.52^2}$
$\tan^{-1}\left(\frac{12.52}{10}\right)$

voltage across R,

$$V_R = iR = 1.80 \angle -21.38^\circ \times 10 \angle 0$$
$$= 18.7 \angle -21.38^\circ$$

Voltage across the inductor,

$$V_L = i \times jX_L = 1.87 \angle -21.38 \times j15.7$$
$$= 1.87 \angle -21.38 \times 1.57 \angle 90^\circ$$
$$= (1.87 \times 15.7) \angle (-21.38 + 90)$$
$$= 29.36 \angle 68.62 \text{ V}$$

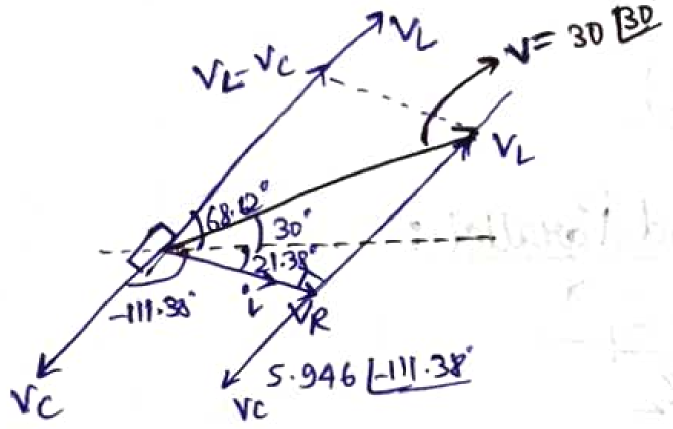
Voltage across capacitor,

$$V_C = i(-jX_C) = 1.87\angle{-21.38} \times -j\,3.18$$

$$= 1.87\angle{-21.38} \times 3.18\angle{-90^\circ}$$

$$= (1.87 \times 3.18)\angle{-21.38-90}$$

$$= 5.946\angle{-111.38}\ V$$



$$\bar{V}_R + \bar{V}_C + \bar{V}_L = 18.7\angle{-21.38} + 29.36\angle 68.62 + 5.946\angle 29.95$$

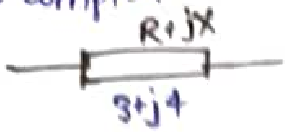
$$= 29.8\angle 29.95$$

$$\approx 30\angle 30^\circ$$

11-04-2023

Impedence (Ω)

↳ complex



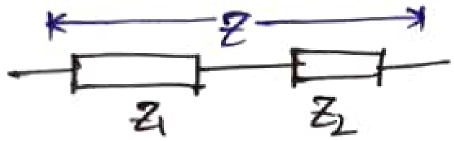
$$Z = R + jX$$

$$|Z| = \sqrt{R^2 + X^2}$$

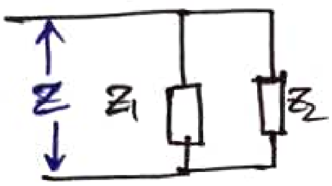
$$\angle(Z) = \tan^{-1}\left(\frac{X}{R}\right)$$

$$R + jX = \sqrt{R^2 + X^2} \angle \tan^{-1}(X/R)$$

→ **Impedence in Series and Parallel:**



$$Z = Z_1 + Z_2$$



$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\Rightarrow Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

→ $Z = R + jX$

$\frac{1}{R}$ → Admittance ($\mho$)

$$\frac{1}{Z} = \frac{1}{R+jX} = \frac{R-jX}{(R+jX)(R-jX)} = \frac{R-jX}{R^2+X^2}$$

$$= \frac{R}{R^2+X^2} - \frac{jX}{R^2+X^2}$$

$$\Rightarrow \boxed{\frac{1}{Z} = G - jB}$$

$G = \frac{R}{R^2+X^2}$ → conductance

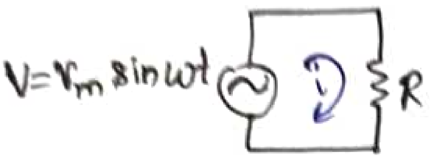
$B = \frac{X}{R^2+X^2}$ → Susceptance

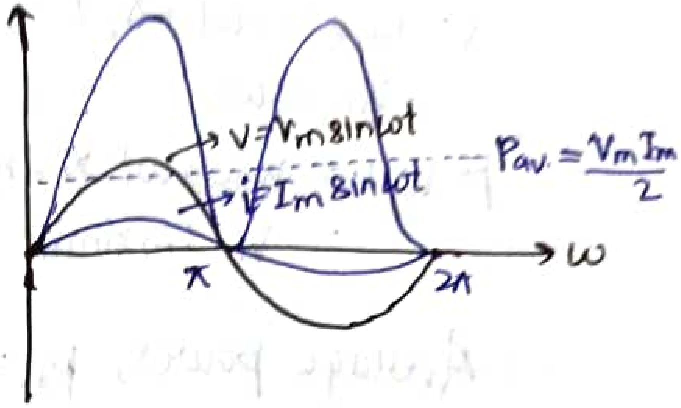
POWER IN AC CIRCUITS

Instantaneous power = Inst. voltage × Inst. current

$$p = v \times i$$

Power in Resistor





$$i = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$

$$= \left(\frac{V_m}{R}\right)\sin\omega t$$

$$= I_m \sin\omega t$$

$$p = v \times i = V_m \sin\omega t \times I_m \sin\omega t$$

$$= V_m I_m \sin^2\omega t = \frac{V_m I_m}{2}(1-\cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

Average power, $P_{av.} = \frac{1}{T}\int_0^T \underbrace{p}_{\text{inst. power}}\, dt$

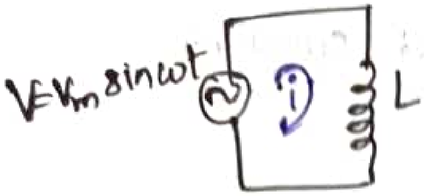
$$= \frac{1}{T}\int_0^T \left(\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right) dt$$

$$= \frac{1}{T}\int_0^T \left(\frac{V_m I_m}{2}\right) dt - \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cancelto{0}{\cos 2\omega t}\, dt$$

$$= \frac{V_m I_m}{2} = \frac{V_m I_m}{\sqrt{2}\sqrt{2}} = V_{rms} \cdot I_{rms}$$

$$\therefore \boxed{P_{av} = V_{rms} I_{rms} = V_{rms}^2/R = I_{rms}^2 R}$$

Power in Inductor



$i = I_m \sin \omega t$

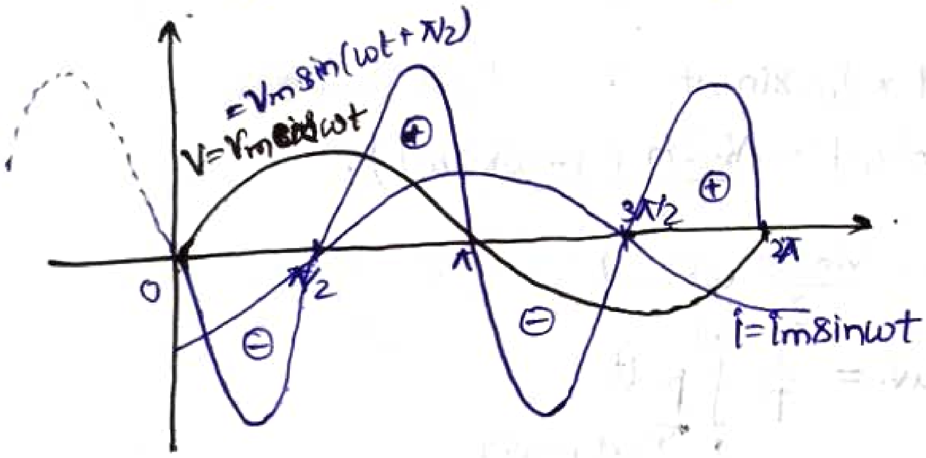
$v = V_m \sin(\omega t + \pi/2)$

$= V_m \cos \omega t$

$p = v \times i = V_m \cos \omega t \times I_m \sin \omega t$

$= V_m I_m \sin \omega t \cos \omega t = \frac{V_m I_m}{2} \sin 2\omega t$

Average power, $p_{av} = \frac{1}{T} \int_0^T p\,dt = \frac{1}{T} \int_0^T \frac{V_m I_m}{2} \sin 2\omega t \, dt = 0$



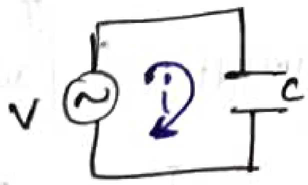
⊖ p ⇒ Inductor is releasing power

⊕ p ⇒ Inductor is absorbing power

⇓

Doesn't consume power

Power in Capacitor



$V = V_m \sin \omega t$

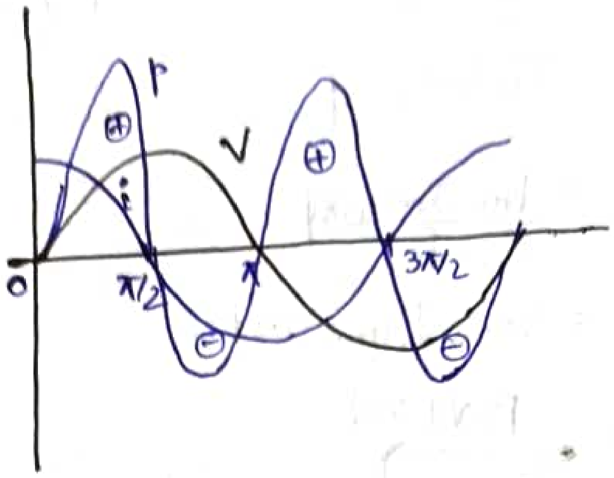
$i = I_m \sin(\omega t + \pi/2)$

$= I_m \cos \omega t$

$p = v \times i = V_m \sin \omega t \times I_m \cos \omega t$

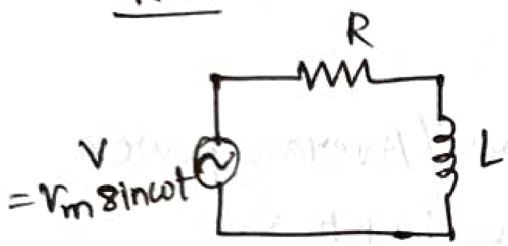
$= V_m I_m \sin \omega t \cos \omega t = \frac{V_m I_m}{2} \sin 2\omega t$

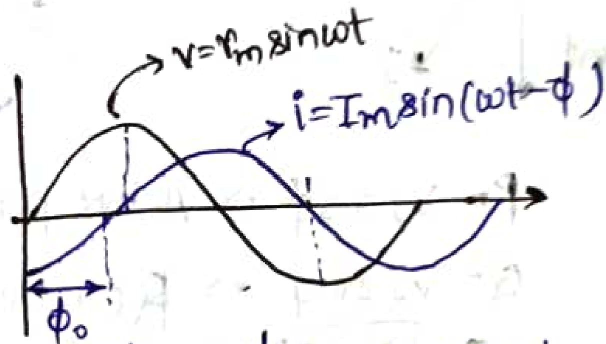
Average power, $p_{av} = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T \frac{I_m V_m}{2} \sin 2\omega t = 0$



12-04-2023

→ R-L





Inst. power = Inst. voltage × inst. current

$v = V_m \sin\omega t$
$i = I_m \sin(\omega t - \phi)$

$\Rightarrow p = v \times i$

$\Rightarrow p = V_m \sin\omega t \times \sin(\omega t - \phi)$

$= V_m I_m \sin\omega t . \sin(\omega t - \phi)$

$= V_m I_m \sin\omega t \left[\sin\omega t \cos\phi - \cos\omega t \sin\phi\right]$

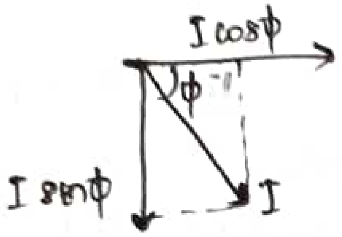
$= V_m I_m \left[\sin^2\omega t \cos\phi - \sin\omega t \cos\omega t \sin\phi\right]$

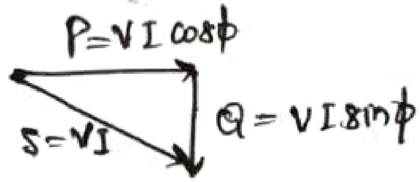
$= V_m I_m \left[\left(\frac{1-\cos 2\omega t}{2}\right)\cos\phi - \frac{\sin 2\omega t}{2}\sin\phi\right]$

$= \frac{V_m I_m}{2}\left[\cos\phi - (\cos 2\omega t \cos\phi + \sin 2\omega t \sin\phi)\right]$

$p = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}(\cos(2\omega t - \phi))$

Average power,
$$P_{av.} = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos\phi\,dt - \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cancel{\cos}(2\omega t - \phi)\,dt$$
$$= \frac{V_m I_m}{2}\cos\phi$$
$$= \frac{V_m}{\sqrt{2}}\cdot\frac{I_m}{\sqrt{2}}\cos\phi$$
$$\therefore P_{av.} = V_{rms} I_{rms} \cos\phi$$





$I = I_{rms}$
$V = V_{rms}$

$P = VI\cos\phi \Rightarrow$ Active power / Real power / Average power

$Q = VI\sin\phi \Rightarrow$ Reactive power ⇝ flows back to source (circulates)

$S = \sqrt{P^2+Q^2} \Rightarrow$ Apparent power

$$P^2+Q^2 = (VI\cos\phi)^2 + (VI\sin\phi)^2$$
$$= (VI)^2[\cos^2\phi + \sin^2\phi]$$
$$= (VI)^2 = S^2$$
$$\therefore S = \sqrt{P^2+Q^2}$$

→ Complex power, $\bar{S} = P + jQ$
$$|\bar{S}| = \sqrt{P^2+Q^2} = S$$

→ Apparent power is the magnitude of the complex power.

→ Active power, $P = S\cos\phi$ → Watts, KW, MW

Reactive power, $Q = S\sin\phi$ → Volt-Ampere Reactive (VAR, KVAR, MVAR)

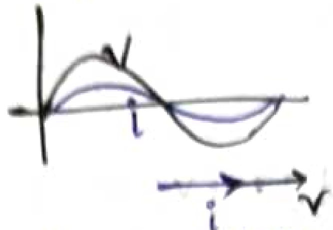
Apparent power, $S = \sqrt{P^2+Q^2}$ → Volt-Ampere (VA, KVA, MVA)

→ Power factor (pf) = $\frac{\text{Active power}}{\text{Apparent power}} = \frac{P}{S} = \cos\phi$

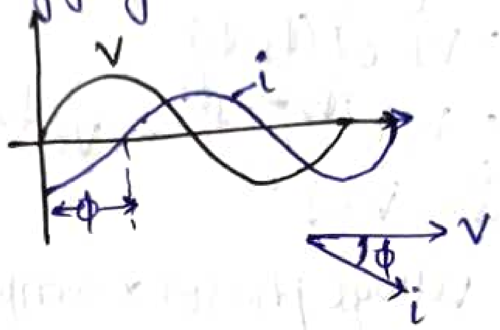
↳ Best pf = 1 when $\cos\phi = 1$ or $\phi = 0$ ⇝ purely reactive.

$\boxed{18\text{-}04\text{-}2023}$

↳ Unity power factor: $\cos\phi = 1$, $\phi = 0$



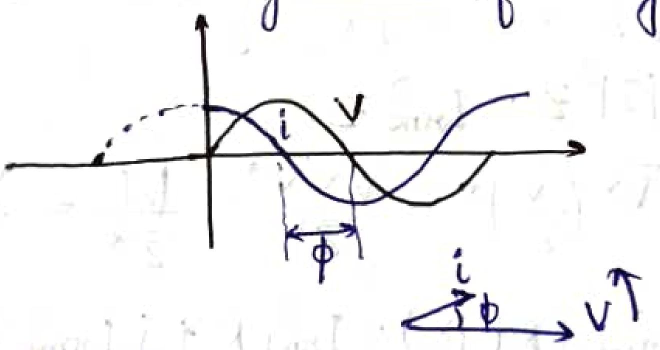
↳ current is lagging behind the voltage



→ lagging power factor

$\cos\phi < 1$

↳ current is leading ahead of voltage



→ Leading power factor

$\cos\phi < 1$

Eg. 0.8 lag } power
0.8 lead } factor

→ $S = P + jQ$ Polar form   Rectangular form

$v = V_m \sin(\omega t + \phi_v) \rightarrow V\angle\phi_v \rightarrow Ve^{j\phi_v}$

$i = I_m \sin(\omega t + \phi_i) \rightarrow I\angle\phi_i \rightarrow Ie^{j\phi_i}$

| V → rms value
| I → rms value
| ϕ_v, ϕ_i → in radians

Active power, $P = VI\cos(\phi_v - \phi_i)$

Reactive power, $Q = VI\sin(\phi_v - \phi_i)$

Complex Conjugate

$3+j4 \longrightarrow 3-j4$

$5\angle \tan^{-1}(4/3) \longrightarrow 5\angle \tan^{-1}(-4/3)$

$A\angle\theta \longrightarrow A\angle{-\theta}$

$Ae^{j\theta} \longrightarrow Ae^{-j\theta}$

Complex power, $\bar{S} = P+jQ$

$= VI\cos(\phi_v-\phi_i) + jVI\sin(\phi_v-\phi_i)$

$= VI\left[\cos(\phi_v-\phi_i) + j\sin(\phi_v-\phi_i)\right]$

$= VI\, e^{j(\phi_v-\phi_i)}$

$= VI e^{j\phi_v} e^{-j\phi_i} = Ve^{\phi_v} \times Ie^{-j\phi_i}$

$\therefore \boxed{\bar{S} = \bar{V}\times\bar{I}^*}$

Comple power = voltage phasor × complex conjugate of the current phasor

$\bar{S} = \bar{V}\times\bar{I}^* = \bar{I}\bar{Z}\times\bar{I}^* = \bar{I}\times\bar{I}^* Z$

$= |I|^2 Z = I_{rms}^2 Z$

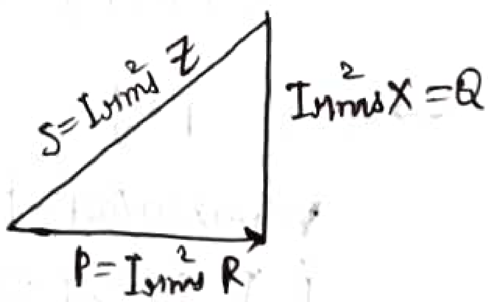
$\bar{S} = \bar{V}\times\bar{I}^* = \bar{V}\times\left(\frac{\bar{V}}{\bar{Z}}\right)^* = \frac{\bar{V}\times\bar{V}^*}{\bar{Z}^*} = \frac{|V|^2}{\bar{Z}^*} = \frac{V_{rms}^2}{\bar{Z}^*}$

$\bar{S} = I_{rms}^2 Z = I_{rms}^2 (R+jX) = I_{rms}^2 R + j I_{rms}^2 X$

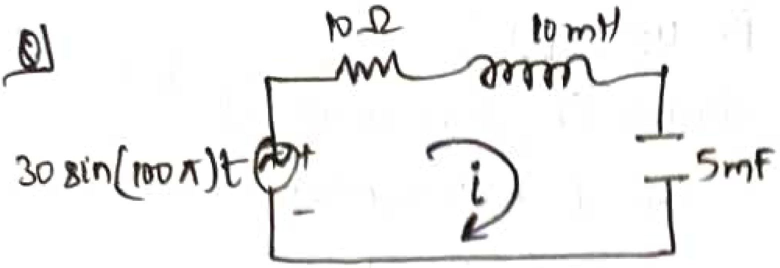
$\bar{S} = P+jQ$

Active power, $P = I_{rms}^2 R$

Reactive power, $Q = I_{rms}^2 X$



Q)



① Determine the current i.

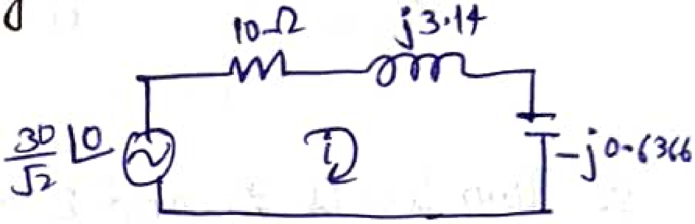
② Evaluate the complex power delivered by the source.

③ Prove that:

ⓐ Active power delivered by the source is equal to the active power consumed by the circuit.

ⓑ Reactive power delivered by the source is equal to the reactive power absorbed by the circuit.

Soln:



$V = V_m \sin \omega t = V_m \sin(2\pi f)t$

$\Rightarrow V = 30 \sin(100\pi)t$

$V_m = 30$, $V_{rms} = 30/\sqrt{2}$

$2\pi f = 100\pi \Rightarrow f = 50\,Hz$.

$X_L = 2\pi f L = 2\pi \times 50 \times 10 \times 10^{-3} = 3.14\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-3} \times 5} = 0.6366$

Total impedence, $Z = 10 + j3.14 - j0.6366$

$= (10 + j2.51)\,\Omega$

Current, $i = \frac{V}{Z} = \frac{30/\sqrt{2} \angle 0}{10 + j2.51} = 2.057 \angle -14.09\ A$

Complex power delivered by the source,

$\bar{S} = V \times |\bar{I}|^*$, $\bar{I} = 2.057 \angle -14.09$

$|\bar{I}|^* = 2.057 \angle +14.09\ A$

$\bar{S} = \frac{30}{\sqrt{2}} \angle 0 \times 2.057 \angle 14.09$

$= 43.6462 \angle 14.09\ VA$

$= 42.33 + j10.625\ VA$

$\hookrightarrow P + jQ$

A.p. delivered by the source, $P = 42.33\,W$

A.p. ~~delivered~~ consumed in the circuit (by $10\,\Omega$ resistor)

$$= I_{rms}^2 R = (2.057)^2 \times 10$$
$$= 42.31\,W$$

R.p supplied by the source, $Q = 10.625$ VAR

R.p. absorbed by the inductor $= I_{rms}^2 \times X_L$

$$= (2.057)^2 \times 3.14 = 13.28 \text{ VAR}$$

R.p absorbed by the capacitor $= (I_{rms})^2 (-X_C)$

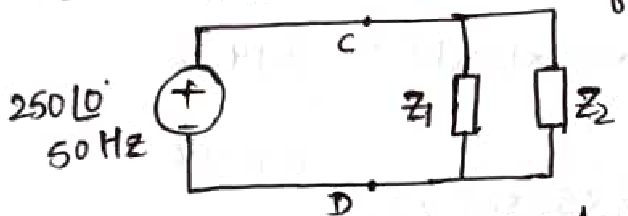
$$= -(2.057)^2 (0.6366)$$
$$= -2.69 \text{ VAR}$$

Net R.p. absorbed by L & C $= 13.28 - 2.69$

$$= 10.59 \text{ VAR}$$

19-04-2023

Q) Two loads Z_1 and Z_2 are connected in parallel across an AC voltage source (G) as shown in fig.



$Z_1 = 4\,KW$ at a p.f. of 0.75 lag

$Z_2 = 2\,KW$ at a p.f. of 0.95 lead.

active power

(a) Evaluate the complex power drawn from the source.

(b) Evaluate the pf at load terminals (C–D).

(c) Evaluate the current drawn from the source (G).

Soln: Load $Z_1 = 4\,KW$ at a pf 0.75 lag $\rightarrow P_1$

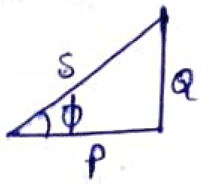
$$\cos\phi_1 = 0.75 \Rightarrow \phi_1 = 41.4096$$

Apparent power of Z_1 $(S_1) = \frac{4000}{0.75} = 5333.33$ VA

Reactive power of $Z_1 = S_1 \cos\phi$

$$= 5333.33 \sin(41.4096) = 3527.668 \text{ VA}$$
(Q_1)

Complex power of $Z_1 = P_1 + jQ_1 = 4000 + j3527.668$ VA



$Q = S\sin\phi$
$P = S\cos\phi$

Load $Z = 2$ kW at pf of 0.95 lead $\rightarrow P_2$

$\cos\phi_2 = 0.95 \Rightarrow \phi_2 = 18.195$

Apparent power of Z_2 $(S_2) = \frac{P_2}{\cos\phi_2} = \frac{2000}{0.95} = 2105.26$ VA

R.P. of Z_2 $(Q_2) = S_2 \sin\phi_2 = 2105.26 \sin 15.195^\circ = 657.367$ VAR

C.P. of $Z_2 = \bar{S_2} = 2000 - j657.367$ VA

Total C.P. drawn from the source $= \bar{S_1} + \bar{S_2}$

$= 4000 + j3527.668 + 2000 - j657.367$

$= 6000 + j2870.3$

$= 6651.2 \angle 25.56$

P.f. at terminal (C–D) $= \frac{\text{Ac.P.}}{\text{Ap.P.}}$

$= \frac{6000}{6651.2} = 0.90209$ lag

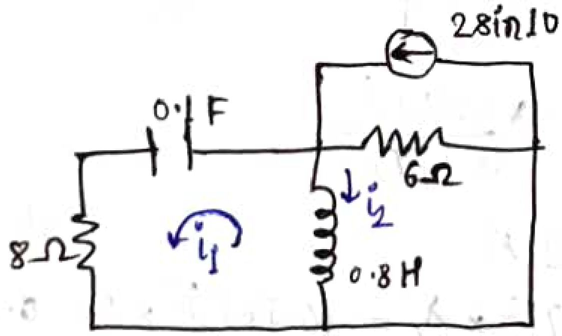
or, $\cos\phi = \cos(25.56) = 0.902$

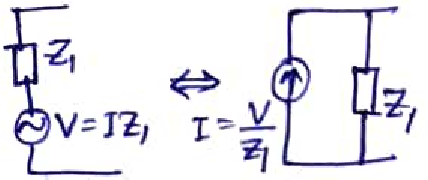
or, $\cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right) = \cos\left(\tan^{-1}\left(\frac{2870.3}{6000}\right)\right) = 0.902$

Ap. P $= V_{rms} \times I_{rms} = 6651.2$

current drawn from the source $= \frac{6651.2}{250} = 26.6$ A

Q)





(a) Evaluate the R.P. associated with 0.1 F capacitor.

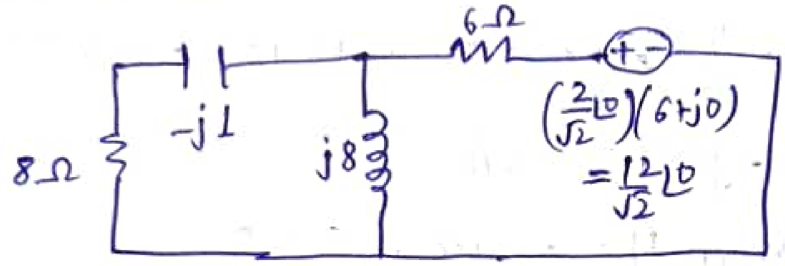
(b) Evaluate the P. consumed by 8 Ω resistor.

Soln: $I_m \sin\omega t = 2\sin 10t$

$\omega = 10\, rad/s$, $I_m = 2A$, $I_{rms} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\angle 0°\, A$

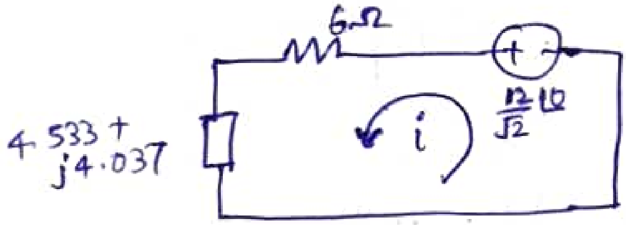
Capacitive reactance of 0.1 F cap. $= \frac{1}{j\omega C} = \frac{1}{j\, 10\times 0.1} = -j1\,\Omega$

Inductive reactance of 0.5H ind. $= j\omega L = j\times 10\times 0.8 = j8\,\Omega$.



Eff. impedence of $(8-j1)\,||\,(j8)$,

$$Z_{eq} = \frac{(8-j1)\,(j8)}{(8-j1)+j8} = 6.07\angle 41.69\,\Omega$$
$$= (4.533 + j4.037)\,\Omega$$



Total current supplied by $\left(\frac{12}{\sqrt{2}}\right)\angle 0$ source

$$i = \frac{\frac{12}{\sqrt{2}}\angle 0}{(6+j0)+4.533+j4.037}$$
$$= \frac{\frac{12}{\sqrt{2}}\angle 0}{10.535+j4.037}$$
$$= 0.752\angle -20.97$$

current flowing through 0.1F cap,

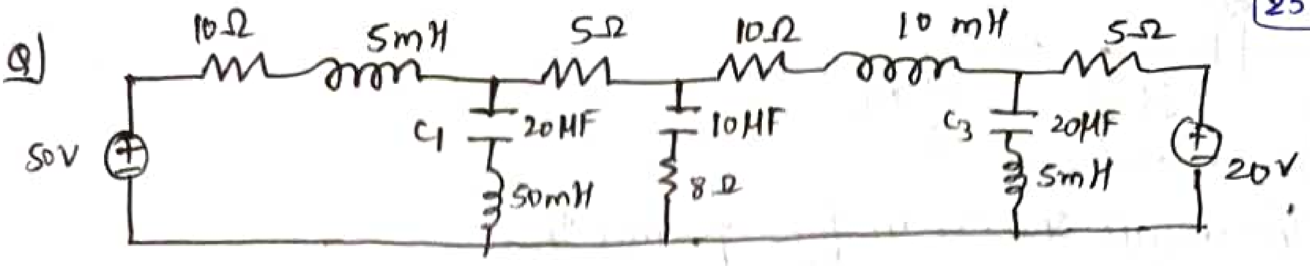
$$i_1 = i\times \frac{j8}{(8-j1+j8)}$$
$$= 0.752\angle -20.97 \times \frac{0+j8}{(8+j7)}$$
$$= 0.565\angle 27.9\, A$$

R.P. associated with 0.1F cap. $= |I|^2 \times X_C = (0.565)^2\times 1 = 0.32$ VAR

Power consumed by 8Ω resistor $= |I|^2\times R = (0.565)^2\times 8 = 2.56W$.

25-04-23

Q)

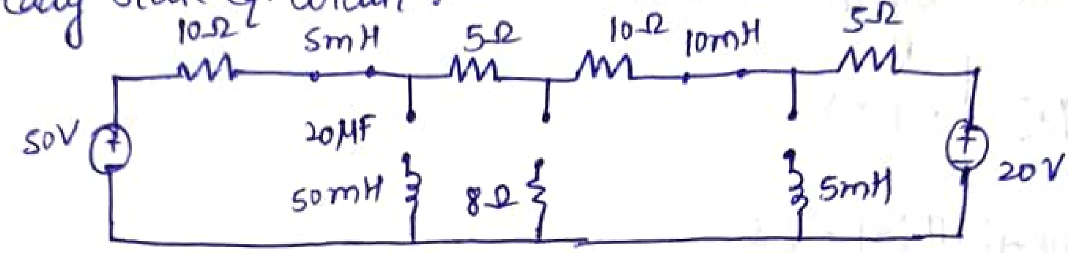


In the circuit shown in fig., evaluate at steady state,

(a) energy stored in the circuit.

(b) power dispersed in the circuit.

Soln: (a) Steady state eq. circuit :



Applying KVL,

$$50-10I-5I-10I-5I-20=0$$

$$\Rightarrow I = 30/30 = 1A$$

Voltage across $C_1 \rightarrow (20\mu F) \Rightarrow 50-10 = 40V$

" $C_2 \rightarrow (10\mu F) \Rightarrow 50-10\times1-5\times1 = 35V$

" $C_3 \rightarrow (20\mu F) \Rightarrow 50-10\times1-5\times1-10\times1 = 25V$

Total energy stored in capacitor $C_1, C_2 \& C_3 \Rightarrow \frac{1}{2}\times20\times10^{-6}\times40^2 + \frac{1}{2}\times10\times10^{-6}\times30^2$

$(\because E = \frac{1}{2}CV^2)$

Total energy stored in inductor $= \frac{1}{2}\times5\times10^{-3}\times1^2 + \frac{1}{2}\times10\times10^{-3}\times1^2$

$\therefore$ Energy stored in circuit $= 35mJ$.

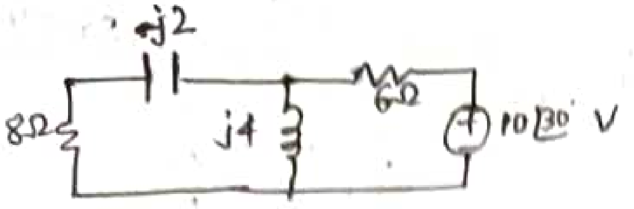
(b) At steady state,

power dissipated in the circuit (resistor)

$= 1^2\times10 + 1^2\times5 + 1^2\times10 + 1^2\times5$

$= 30W$

Q]

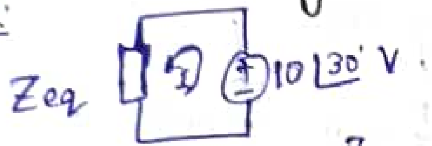


In the circuit shown in fig, prove that

(1) total active power supplied by the source is equal to the total active power consumed.

(2) total r.p. supplied by the source is equal to the total r.p. absorbed by the circuit.

Soln:



$$Z_{eq} = [(8-j2)\,||\,j4] + 6\,\Omega$$

$$= 6 + \frac{(8-j2)\times j4}{8-j2+j4} = 7.8826 + j3.53 = 8.632\angle 24.13\ \Omega$$

Current supplied by the source $= \frac{V}{Z_{eq}} = \frac{10\angle 30}{8.632\angle 24.13} = 1.1581\angle 5.87^\circ\ A$

Complex power delivered by $10\angle 30^\circ$ V source $= V \times I^*$

$$= 10\angle 30 \times 1.1581\angle -5.87$$

$$= 11.581\angle 24.13\ VA$$

$$= \underbrace{10.569}_{P} + \underbrace{j4.73}_{jQ}\ V$$

Current through $(8-j2)\,\Omega$ impedence.

$$= 1.1581\angle 5.87 \times \frac{j4}{8-j2+j4} = 0.562\angle 81.84^\circ\ A$$

$(j4)\,\Omega$ impedence

$$= \frac{1.1581\angle 5.37 \times (8-j2)}{8-j2+j4}$$

$$= 1.1581\angle -22.21\ A$$

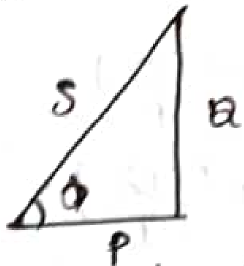
Total active power absorbed $= 1.1581^2 \times 6 + 0.0562^2 \times 8 = 10.57$

Total reactive power consumed $= 0.562^2(-2) + 1.1581 \times 4 = 4.73$

Hence proved.

Power in AC Circuit:

1. Active Power (P) or Real Power/Average Power
2. Reactive Power (Q)
3. Apparent Power (S)



Complex Power, $\bar{S} = \bar{P} + j\bar{Q}$

Apparent Power $= |\bar{S}| \rightarrow$ Magnitude of complex power

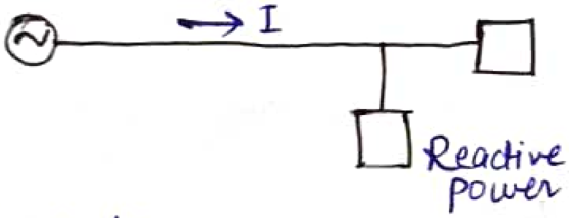
Power factor, $\cos\phi = \dfrac{\text{Active Power}}{\text{Apparent Power}}$

$P = S\cos\phi$

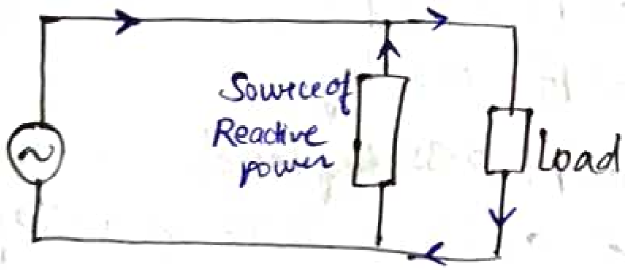
$Q = S\sin\phi$

$\phi = \tan^{-1}(Q/P)$

Schematic representation:



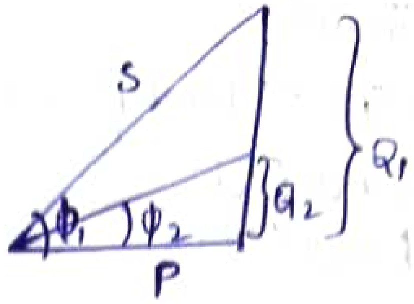
Circuit diagram:



Devices needing magnetic field will need the circulation of some current in the circuit
$\rightarrow$ Needed a reactive power/current.

lag $0.8 \rightarrow \cos\phi_1$,

lag $0.9 \rightarrow \cos\phi_2$



$$\phi_1 = \tan^{-1}\left(\frac{Q_1}{P}\right)$$

$$\phi_2 = \tan^{-1}\left(\frac{Q_2}{P}\right)$$

Reactive power to be compensated,

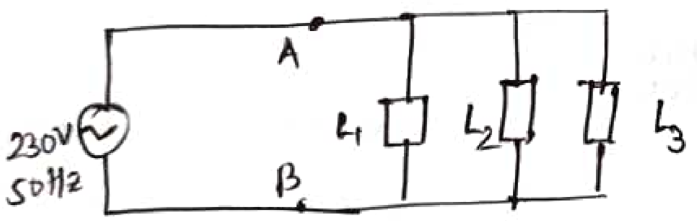
$$Q_1 - Q_2 \rightarrow Q = I^2 X_C = \frac{V^2}{X_C} = \frac{V^2}{\frac{1}{2\pi f C}} = 2\pi f C V^2$$

$$\left(\because X_C = \frac{1}{2\pi f C}\right)$$

$$\boxed{C = \frac{Q}{2\pi f V^2}}$$

Value needs to compensate the reactive power of value Q

Q]



Three loads, L_1 = 10 KW at pf 0.8 lag,

L_2 = 15 KW at pf 0.6 lag,

L_3 = 10 KW at pf 0.95 ~~lag~~ lead,

are connected across a 230 V, 50Hz AC voltage source as shown in figure.

(a) Determine the power factor at load terminals A-B.

(b) Evaluate the capacitance of the capacitor required to be connected across the load terminals (A-B) for improving the power factor to 0.98 lag.

(c) Calculate the current drawn from the source before and after improving the power factor.

Soln: $L_1 \rightarrow 10\,kW$ at pf 0.8 lag $\rightarrow S_1 = \frac{10}{0.8} = 12.5\,kVA$, $Q = S_1 \sin\phi$.

Complex power of $L_1 (S_1)$, $\phi = \cos^{-1}(0.8)$, $\bar{S}_1 = 10 + j\sin\phi$
$= (10 + j7.5)\,kVA$

$L_2 \rightarrow 15\,kW$ at pf 0.6 lag $\rightarrow S_2 = \frac{15}{0.6} = 25\,kVA$

complex power of L_2, $\bar{S}_2 = 15 + j20$

$L_3 \rightarrow 10\,kW$ at pf 0.95 lead, $S_2 = \frac{10}{0.95} = 10.52\,kVA$

complex power of L_3, $\bar{S}_3 = 10 - j\,3.28$

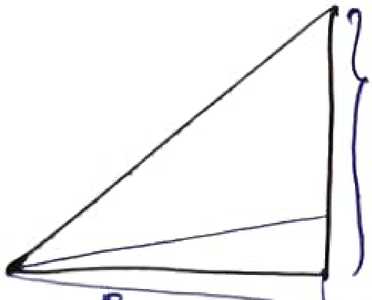
Total complex power supplied by the source,

$$\bar{S} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3$$
$$= 35 + j\,24.213\ kVA$$

Power factor at load terminals

$$= \cos\left(\tan^{-1}\left(\frac{24.215}{35}\right)\right) = 0.822\ lag$$

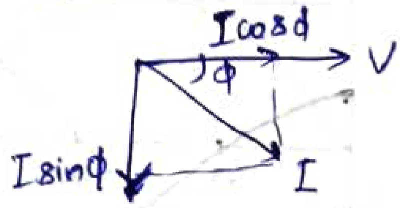
Total apparent power supplied
(before connecting capacitor) $|\bar{S}| = \sqrt{35^2 + 24.213^2}$
$= 42.55\ kVA$

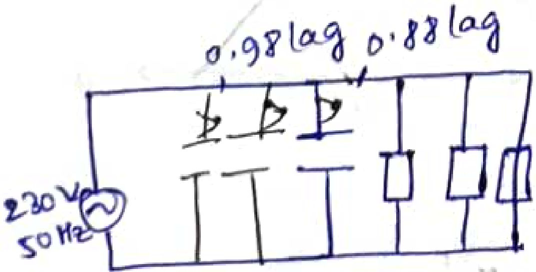


$Q = 24.213 \rightarrow$ pf $= 0.822$ lag

Power factor has to be improved to 0.98 lag

current drawn at pf 0.822 lag $= \frac{S}{V} = \frac{42.55 \times 1000}{230} = 185\,A$





Apparent power drawn at pf 0.98 lag $= \sqrt{35^2 + 7.127^2}$
(after connecting capacitors) $= 35.7\ kVA$

current drawn from the source at pf 0.98 lag $= \frac{35.7 \times 1000}{230} = 155.2\,A$

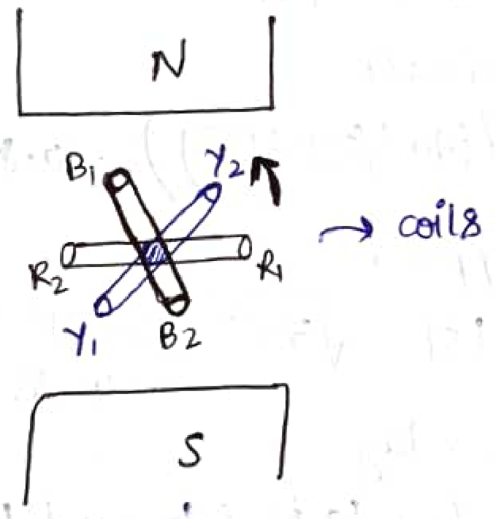
APFC: ~~Active~~ Automatic Power Factor Correction

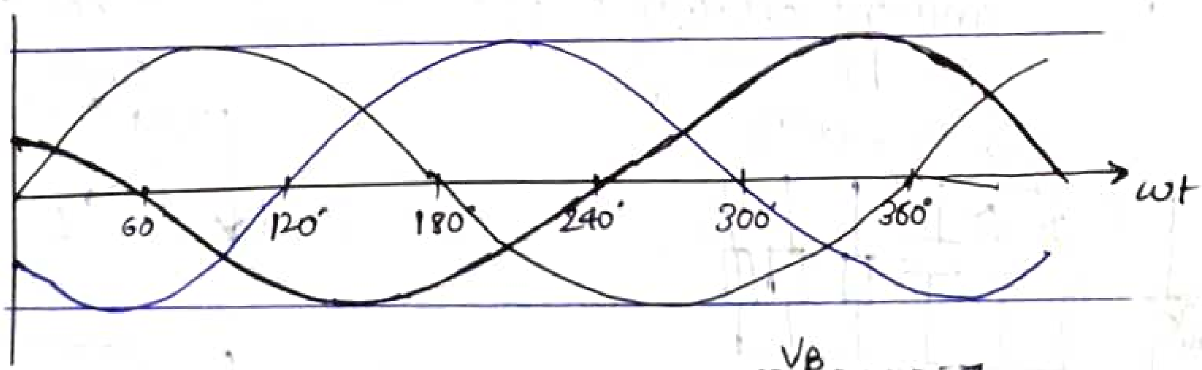
2-05-2023

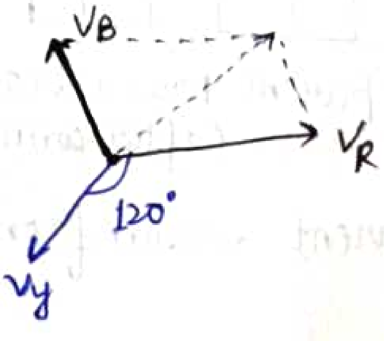
Static VAR Compensator

THREE PHASE AC SYSTEM

- High power factor
- High power density
- constant power
- Reliable
- Provides rotating magnetic field
- Transmission is cheaper







$V_R = V_m \sin\omega t$

$V_y = V_m \sin(\omega t - 120^\circ)$

$V_B = V_m \sin(\omega t - 240^\circ)$

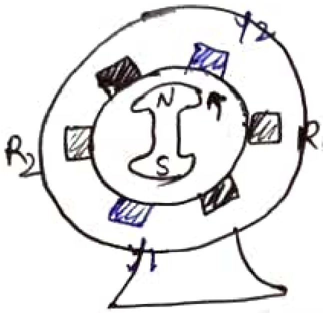
$\Rightarrow V_B = V_m \sin(\omega t + 120^\circ)$

$V_R = V\angle 0^\circ$

$V_y = V\angle -120^\circ$

$V_B = V\angle -240^\circ = V\angle 120^\circ$

→ Actual



→ Magnets rotate instead of the coils

→ Phase Sequence: order in which waveforms attain their peak values.

$\left.\begin{matrix} RYB \\ RBY \end{matrix}\right\}$ Possible phase sequences

RYBRYBYRB

RBY RBY RBY

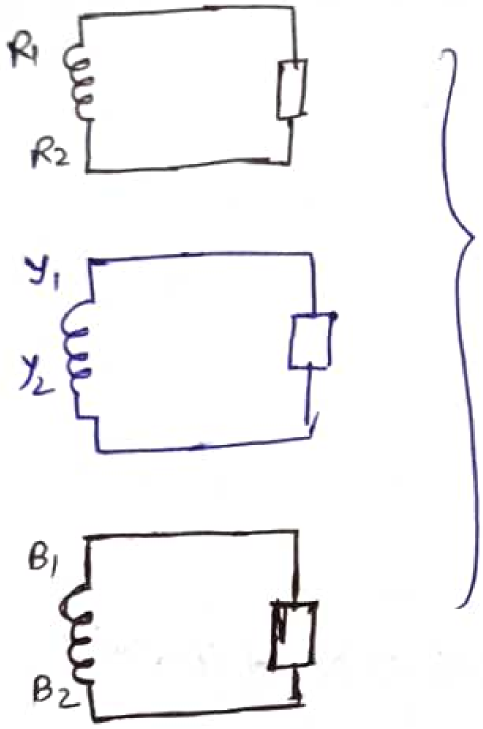
$V_R + V_y + V_B$

$= V_m \sin\omega t + V_m \sin(\omega t - 120^\circ) + V_m \sin(\omega t - 240^\circ)$

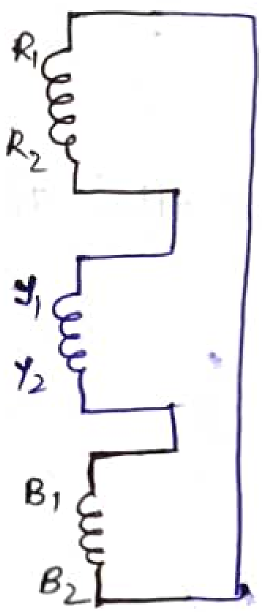
$= V_m [\sin\omega t + \sin\omega t \cos 120^\circ - \cos\omega t \sin 120^\circ + \sin\omega t \cos 240^\circ - \cos\omega t \sin 240^\circ)$

$= V_m [\sin\omega t - \frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t - \frac{1}{2}\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t]$

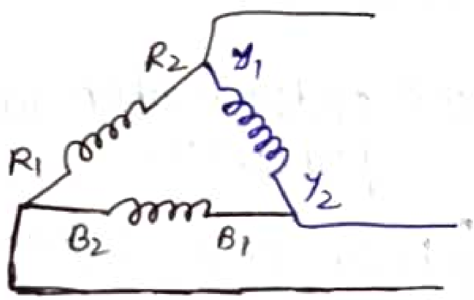
$= 0$



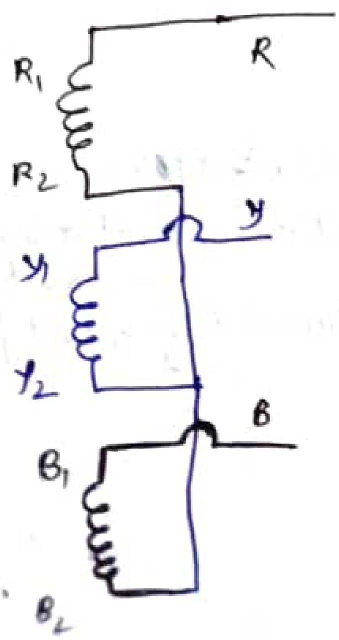
Will be interconnected

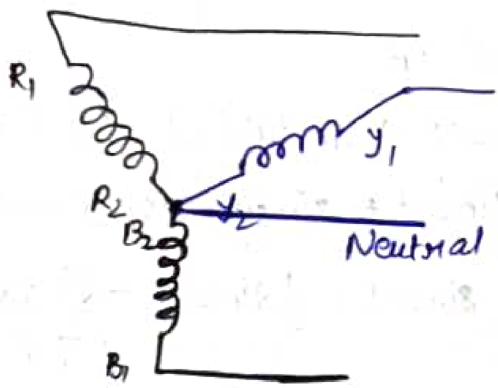


⇒



Delta Connection
(Mesh Connection)

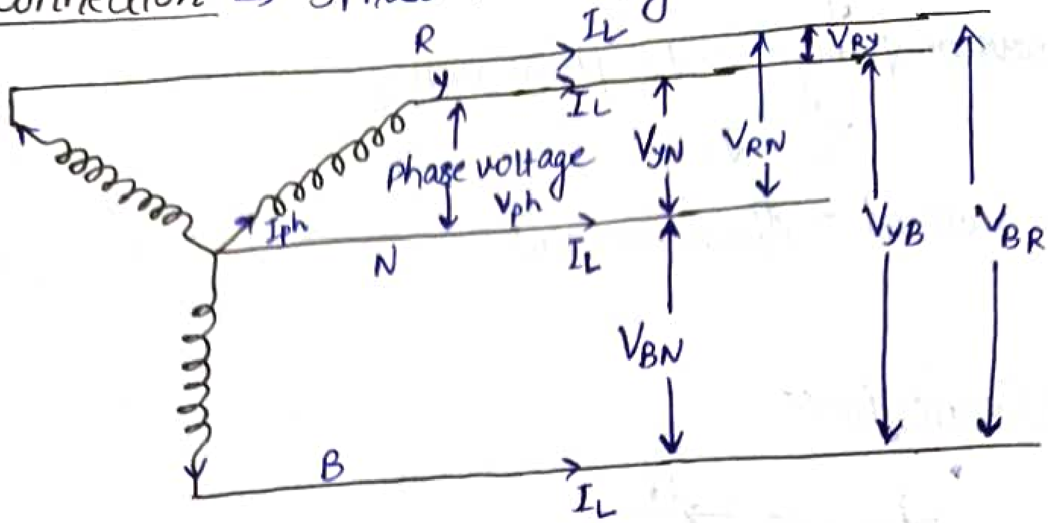




Star Connection
(Y-connection)

8-05-2023

Star Connection → 3Phase 4 wire system



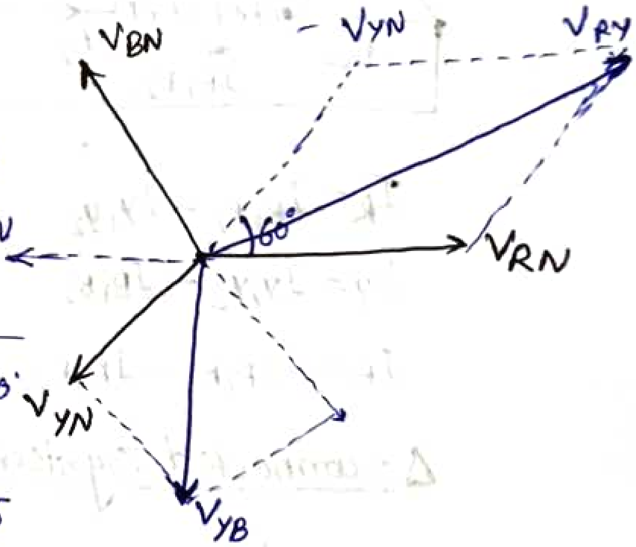
Phase voltages: V_{RN}, V_{YN}, V_{BN}

Line voltages: $V_{RY}, V_{YB}\, V_{BR}$

Line voltage leads the corresponding ph voltage by angle 30°.

$$|V_{RN}| = |V_{YN}| = |V_{BN}| = V_{ph}$$

$$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_L$$



$$\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY}$$
$$= \bar{V}_{RN} - \bar{V}_{YN}$$

$$\bar{V}_{YB} = \bar{V}_{RN} - \bar{V}_{BN}$$
$$V_{BR} = \bar{V}_{BN} - \bar{V}_{RN}$$

$$|V_{RY}| = \sqrt{|V_{RN}|^2 + (|V_{YN}|)^2 + 2V_{RN}\, V_{YN}\cos 60^\circ}$$
$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph} \cdot V_{ph} \times 0.5}$$

$$\therefore \boxed{V_L = \sqrt{3}\, V_{ph}}$$

440V, 3ph, 50 Hz
↓
Line voltage

→ Line current and Phase current

$$\boxed{I_L = I_{ph}}$$

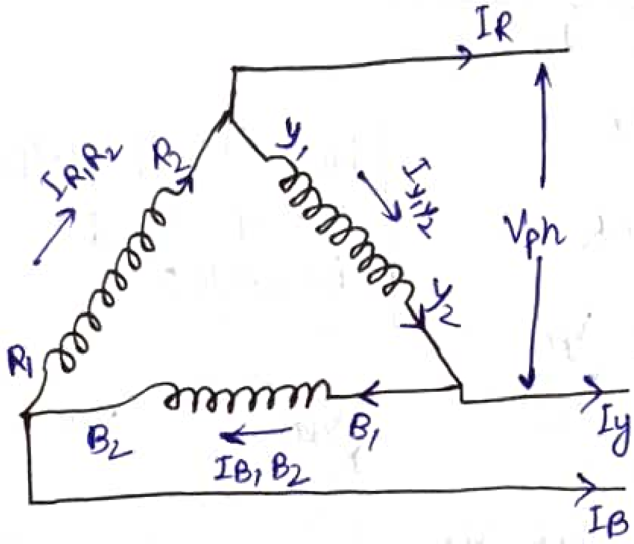
Star (Y)- Connected System:

Line ~~current~~ Voltage = $\sqrt{3}$ · phase voltage

$$V_L = \sqrt{3}\, V_{ph}$$

Line current = Phase current

Delta-Mesh Connection



$$I_R = I_{R_1R_2} - I_{Y_1Y_2}$$

$$I_Y = I_{Y_1Y_2} - I_{B_1B_2}$$

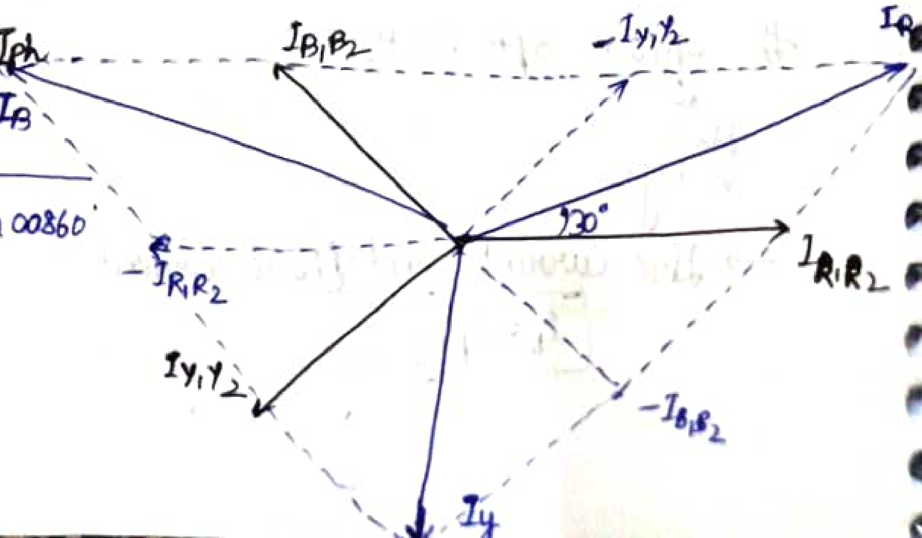
$$I_B = I_{B_1B_2} - I_{R_1R_2}$$

Δ - connected System:

Line voltage = Phase voltage

For a balanced system,

$$|I_{R_1R_2}| = |I_{Y_1Y_2}| = |I_{B_1B_2}| = I_{ph}$$

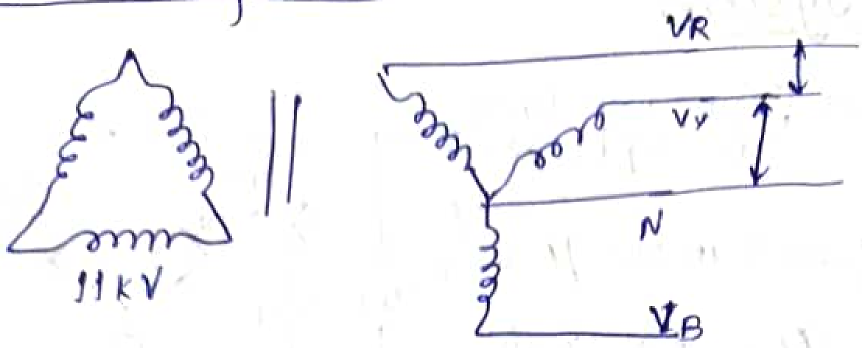


$$I_R = I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} I_{ph} \cos 60}$$

$$= \sqrt{3} I_{ph}$$

$$\therefore \boxed{I_L = \sqrt{3}\, I_{ph}}$$

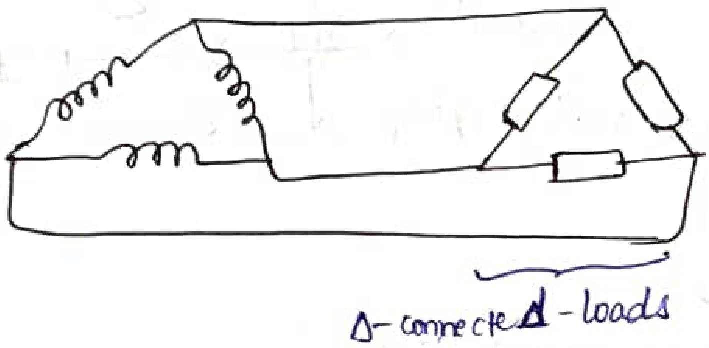
3-phase Transformer

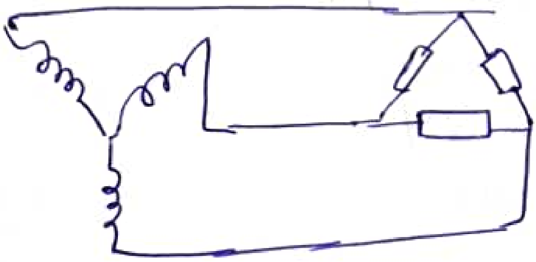


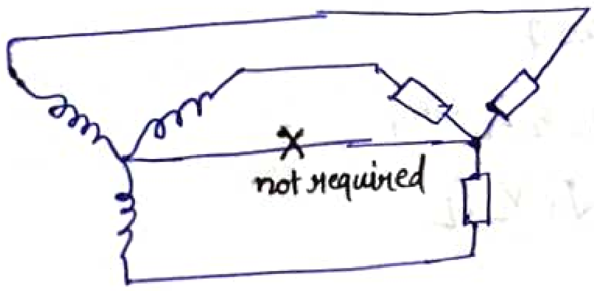
→ Vector phase: Dyn11

Δ Y N — phase shift b/w primary & secondary voltages

→



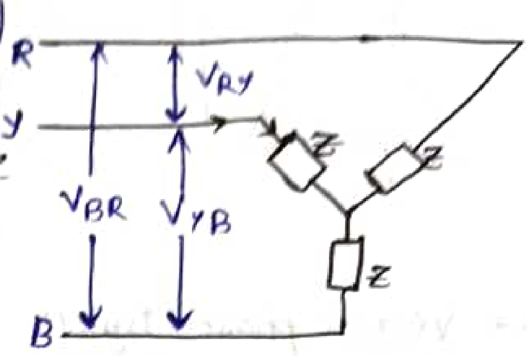




9-05-2023

Power in 3-phase System

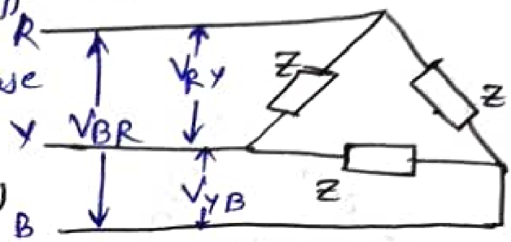
For balanced system (same impedence in all phases, Z)



3-phase power = 3 × power in one phase

3-phase active power (P) = 3 × single phase active power

$$P = 3 V_{ph} \cdot I_p \cos\phi,$$ ϕ: phase diff. b/w phase voltage & phase current



Star connected system:

Line voltage, $V_L = \sqrt{3} \times$ phase voltage (V_{ph})

$$V_L = \sqrt{3}\, V_{ph}$$
$$\Rightarrow V_{ph} = V_L/\sqrt{3}$$

Line current, I_L = phase current (I_{ph})

$$I_L = I_{ph}$$

Active power, $P = 3 \cdot V_{ph} \cdot I_{ph} \cos\phi$

$$= 3 \cdot \frac{V_L}{\sqrt{3}} \cdot I_L \cos\phi$$

$$\therefore P = \sqrt{3}\, V_L I_L \cos\phi$$

Reactive power, $Q = \sqrt{3}\, V_L I_L \sin\phi$

Apparent power, $S = \sqrt{3}\, V_L I_L$

Δ-connected System:

Line voltage (V_L) = phase voltage (V_{ph})

Line current, $I_L = \sqrt{3} \times$ phase current

$$I_{ph} = I_L/\sqrt{3}$$

3-phase active power, $P = 3 \cdot V_{ph} \cdot I_{ph} \cdot \cos\phi$

$$= 3\, V_L \cdot \frac{I_L}{\sqrt{3}} \cos\phi$$

$$\therefore P = \sqrt{3}\, I_L V_L \cos\phi$$

Reactive power, $Q = \sqrt{3}\, V_L I_L \sin\phi$

Apparent power, $S = \sqrt{3}\, V_L I_L$

Complex power, $\bar{S} = P + jQ$

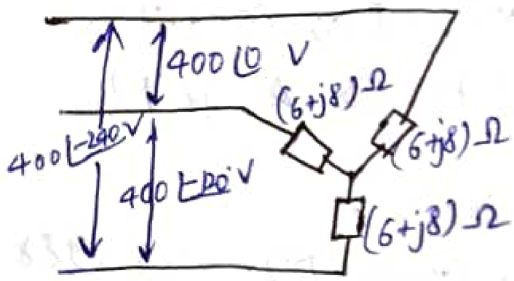
$$\text{Power factor} = \frac{\text{Active power}}{\text{Apparent power}}$$

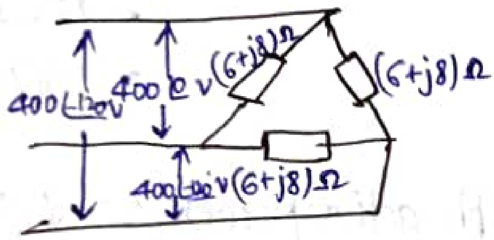
Q) A balanced star connected load of $(6+j8)\,\Omega$ impedence/phase is supplied from a 400V (Line voltage), 3 ph, 50 Hz supply.

ⓐ calculate the active power, reactive power and apparent power drawn from the supply.

ⓑ If the same load is connected in delta, calculate the active power, reactive power, apparent power and power factor.

Soln:





Star connected load:

Active power, $P = \sqrt{3}\, I_L V_L \cos\phi$

$V_L = 400\,V$, $I_{ph} = I_L$

$$\text{phase current, } I_L = \frac{\text{phase voltage}}{\text{impedence}} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L/\sqrt{3}}{Z_{ph}}$$

$$= \frac{400/\sqrt{3}}{(6+j8)} = 23.09 \angle -53.13^\circ = I_L$$

3-phase active power, $P = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 23.09 \cos(53.13^\circ)$

$= 9598.35\ W$

3-phase reactive power, $Q = \sqrt{3}\, V_L I_L \sin\phi$

$= \sqrt{3} \times 400 \times 23.09 \sin(53.13^\circ)$

$= 12797.75\ VAR$

Apparent power, $S = \sqrt{3} \times V_L I_L$

$= \sqrt{3} \times 400 \times 23.09 = 15997.22$ VA

$$\text{power factor} = \frac{\text{Active power}}{\text{Apparent power}} = \frac{9598.35}{15997.22} = 0.6 \text{ lakh}$$

When the same load is connected in Δ:

$V_L = V_{ph}$

$I_L = \sqrt{3} I_{ph}$

$$I_{ph} = \frac{V_{ph}}{Z_{im}} = \frac{400}{6+j8} = 40\angle{-53.13^\circ}$$

$|I_L| = \sqrt{3} I_{ph} = 40\sqrt{3} = 69.28$

3-ph active power, $P = \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 69.28 \cos(53.13^\circ)$

$= \underline{28799.2 \text{ W}}$ ($\gg$ ★ 3-ph P)

3-ph active power, $Q = \sqrt{3} V_L I_L \sin\phi$

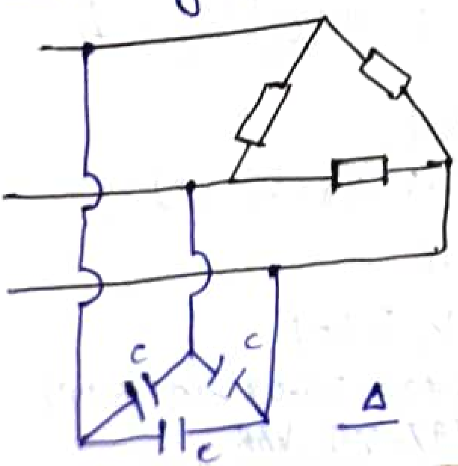
$= \sqrt{3} \times 400 \times 69.28 \sin(53.13^\circ) = -383988$ VAR

Apparent power, $S = \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 69.28$

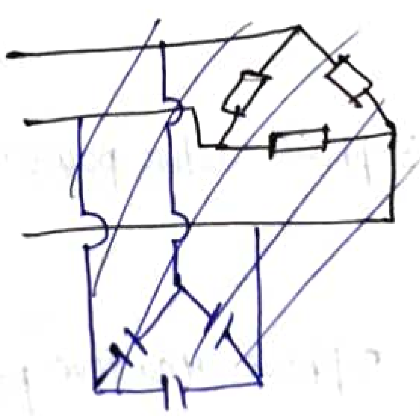
$= 47998.6$ VA

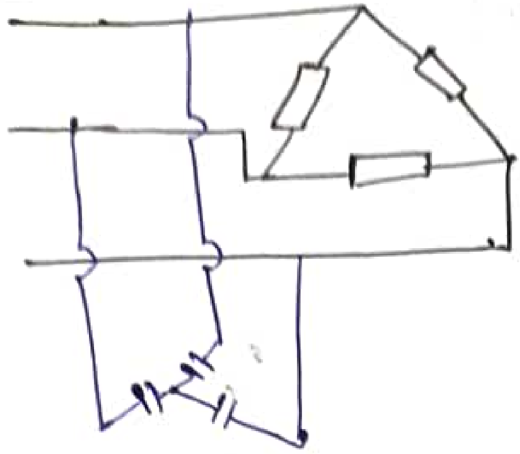
$$\text{Power factor} = \frac{\text{Active power}}{\text{Apparent power}} = \frac{28799.2}{47998.6} = 0.6 \text{ lag}$$

Improving power factor:



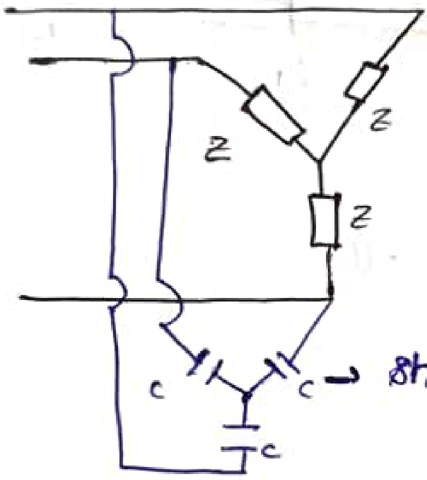
Δ





Reactive power supplied by the capacitor $= \dfrac{V^2}{X_C}$

$$X_C = \frac{1}{2\pi f C}$$

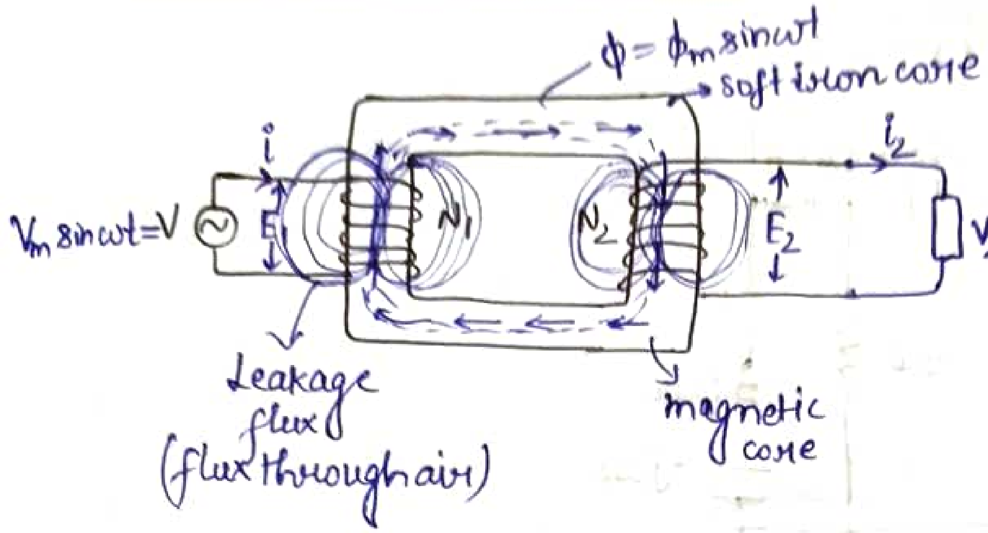


→ should deliver Q

Electrical Machines:

26-06-2023

TRANSFORMER



$$\text{Flux} = \frac{\text{MMF}}{\text{Reluctance}}$$ ⇝ Ohm's law of magnetic circuit
(MMF: Magneto-motive force)

$$\phi = \frac{NI}{S} = \phi_m \sin\omega t$$
S ⇝ Reluctance

By Faraday's Law,

$$\text{EMF}(E) = N\frac{d\phi}{dt}$$

$$E_1 = N_1\frac{d\phi}{dt} \; ; \; E_2 = N_2\frac{d\phi}{dt}$$

$$\Rightarrow \boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$ → Turn's ratio

If $E_2 > E_1 \Rightarrow N_2 > N_1$ → step-up transformer

$E_2 < E_1 \Rightarrow N_2 < N_1$ → step-down transformer

→ $\frac{E_2}{N_2} = \frac{E_1}{N_1}$ ⇝ voltage induced per turn

→ Frequency doesn't change.

→ Power transfer through magnetic medium.

→ Dirⁿ of i_2 is such that it opposes the flux through secondary (Lenz's law).

→ As a result, the flux inside core reduces ⇒ E_1 reduces.

→ As a result, diff. b/w V & E_1 increases, thus more current (i_1) flows. The increase in current nulifies flux due to i_2, hence again orginal flux is established.

Magnetising current (I_M) ⇝ Reactive current (as doesnot constitute to power, ideally)

$\bar{i}_1 = \bar{i}_0 + \bar{i}_2'$ ↳ current reflected from secondary (ie, the increase in current i_1, as discussed above).

Instantaneous power at primary $= E_1 i_1$

$E_1 i_1 = E_2 i_2$ (ideally) $\left[\Rightarrow \cancel{E_1} i_1 = (N_2 \cancel{E_1} i_2)/N_1 \Rightarrow \boxed{N_1 i_1 = N_2 i_2}\right]$

↳ Ampere turns are equal for both ~~sides~~ coils

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{E_2}{E_1}}$$

→ $i N$ = Ampere turn = const.

→ $\frac{E}{N}$ = voltage/turn = const.

Power Loss in Transformer

→ Core Loss: due to { Eddy currents in core ⇝ thus laminating are done. Hysteresis loss

many small loops of currents are formed

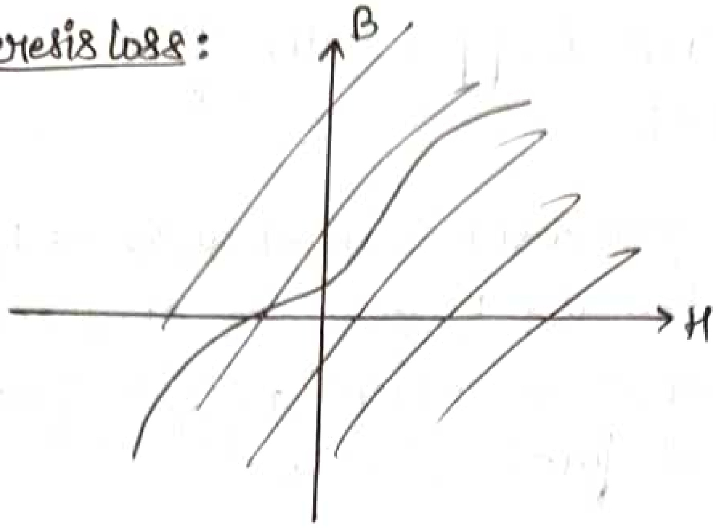
both depends on B & frequency

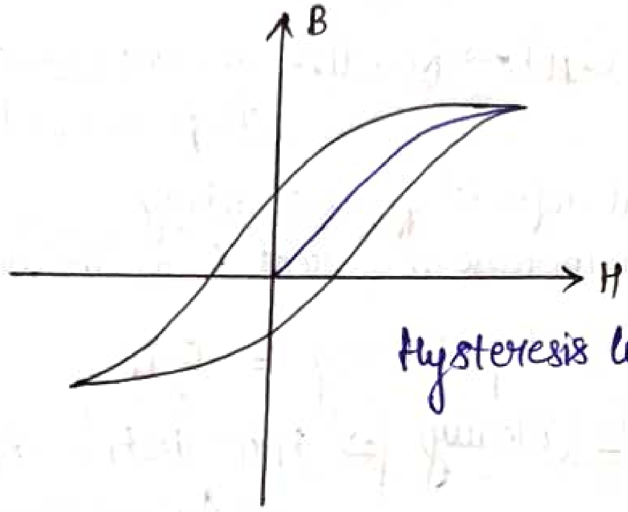
→ Flux density $= \frac{\phi}{A} = B$ (weber/m²)

$M \propto NI$

→ Copper loss: Power loss in windings.

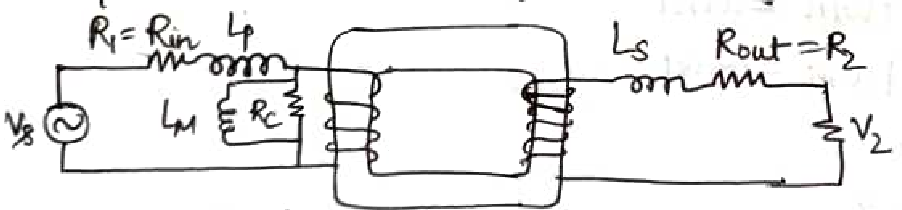
→ Hysteresis loss:

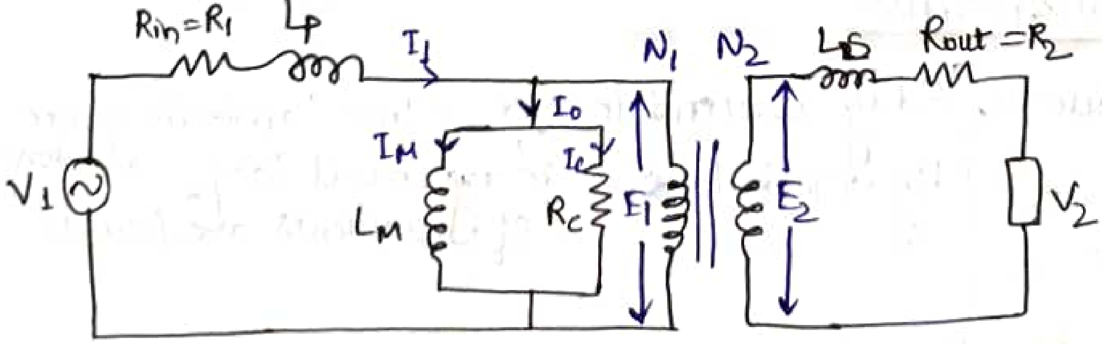




Hysteresis loop

Equivalent Circuit of Transformer:





R_{in} & R_{out} : Copper resistance of 1° & 2° coils

L_p & L_s : leakage inductance of 1° & 2° coils

L_M : Magnetizing inductance (Hysteresis loss) ⇝ represents flux in core

R_c : Resistance of core

↓

(connected in parallel b/c core loss is constant, current through R_c in parallel will be constant if V_1 is constant & does not depend on induced voltage/current due to 2°)

→ Even if load is not there, core losses exist. That's why input to chuties having transformers should be switched off.

→ we don't use capacitor b/c it creates electric field whereas inductor creates magnetic field.

$$\bar{I}_1 = \bar{I}_2' + \bar{I}_0$$

Now, replacing transformer,

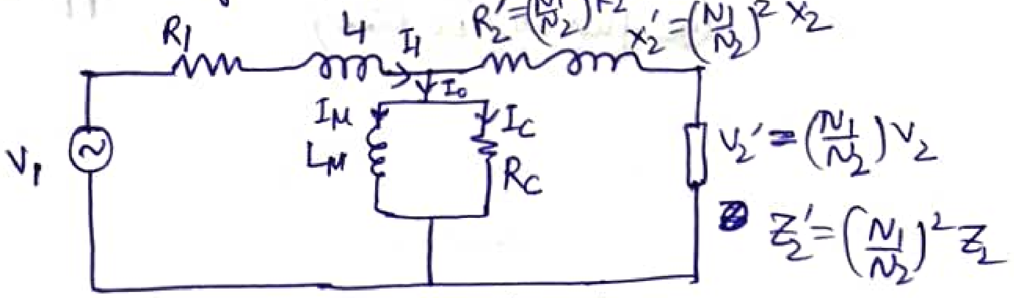
$$I_2^2 R_2 = I_1^2 R_2' \Rightarrow R_2' = \frac{I_2^2}{I_1^2} R_2 = \left(\frac{N_1}{N_2}\right)^2 R_2 \rightarrow \text{Add this to } 1° \text{ side}$$

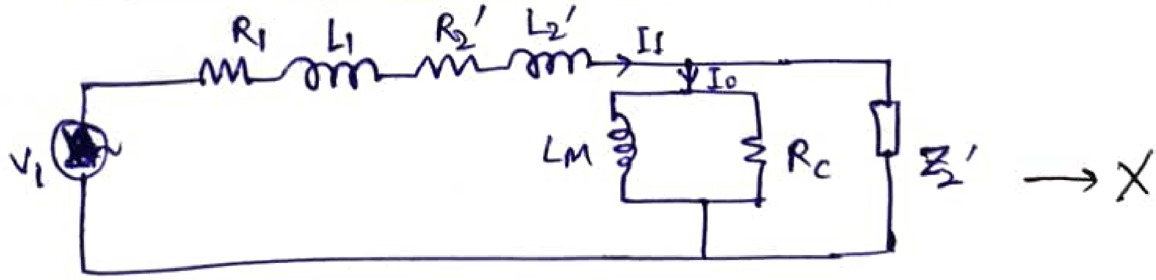
$$\& \ x_2' = \left(\frac{N_1}{N_2}\right)^2 x_2$$

$$I_2^2 x_2 = I_1'^2 (x_2')$$

$$x_2' = \frac{I_2^2 x_2}{I_1'^2} = \left(\frac{N_1}{N_2}\right)^2 x_2$$

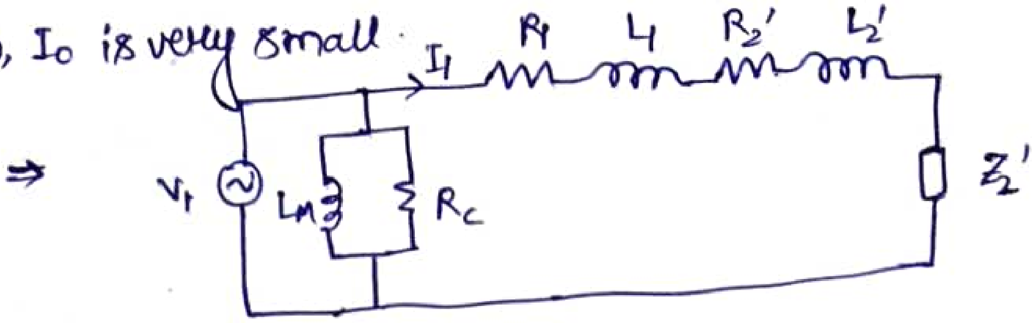
So, equivalent circuit:
(no need of transformer now)



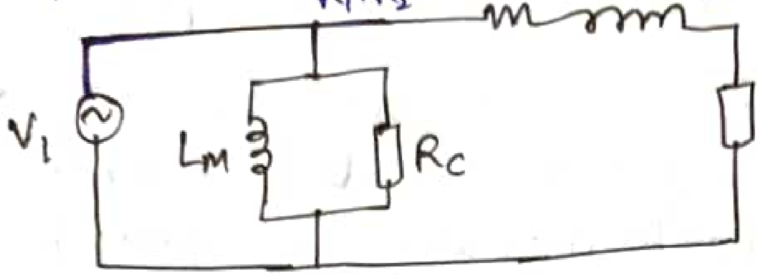


Now, I_0 is very small.

⇒



Further, $R_1 + R_2' = R_{01} = \left(R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2\right)$ $\quad jX_{01} = j(X_1 + X_2') = j\left(X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2\right)$



⇐ Equivalent circuit w.r.t. Primary

↳ We can construct equivalent circuit wrt secondary also.

→ To determine transformer properties:

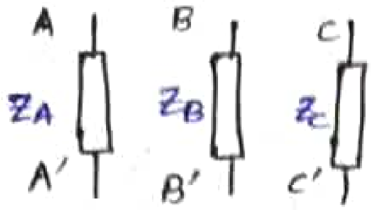
① No load test: Whole I_1 will be in the form of I_0. So, we can (copper loss negligible) measure L_M and R_c. (Core loss)

② Short circuit test: Short circuit the load. (Core loss negligible)

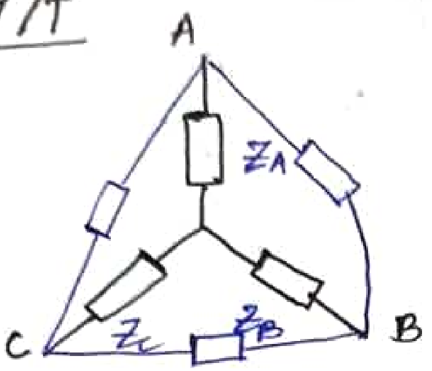
Voltage supply to them be very less.

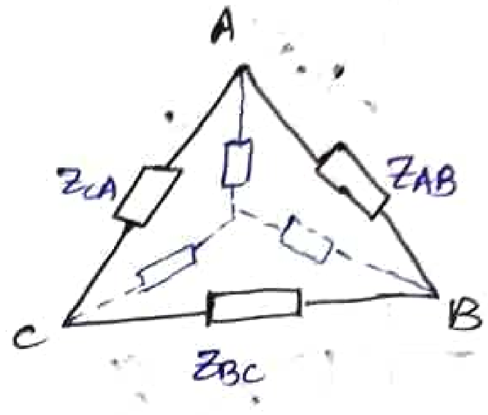
Core lose is negligible; & we can measure copper loss. (b/c flux will be less)

Y/Δ and Δ/Y Transformations



Y/T





Between A and B,

$$Z_A + Z_B = \frac{Z_{AB}\left[Z_{BC} + Z_{CA}\right]}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -(1)$$

Between B and C

$$Z_B + Z_C = \frac{Z_{BC}\left[Z_{AB} + Z_{CA}\right]}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -(2)$$

Between C and A,

$$Z_C + Z_A = \frac{Z_{CA}\left[Z_{AB} + Z_{BC}\right]}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -(3)$$

Δ/Y transformation:

$Z_{AB}, Z_{BC}, Z_{CA} \rightarrow$ Known

$Z_A, Z_B, Z_C \rightarrow$ to be found

(1) + (2) + (3) ⇒

$$Z_A + Z_B + Z_C = \frac{Z_{AB} Z_{CA} + Z_{BC} Z_{AB} + Z_{CA} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -(4)$$

④ − ② ⇒

$$Z_A = \frac{Z_{AB} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -⑤$$

Similarly,

$$Z_B = \frac{Z_{BC} \cdot Z_{AB}}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -⑥$$

$$Z_C = \frac{Z_{CA} \cdot Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -⑦$$

Y/Δ transformation:

$Z_A, Z_B, Z_C \rightarrow$ Known

$Z_{AB}, Z_{BC}, Z_{CA} \rightarrow$ to be found

⑤ × ⑥ + ⑥ × ⑦ + ⑦ × ⑤ ⇒

$$Z_A Z_B + Z_B Z_C + Z_C \cdot Z_A = \frac{Z_{AB} Z_{BC} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}} \quad -⑧$$

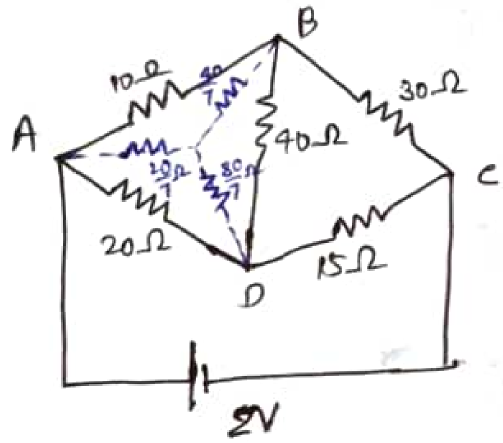
$$\frac{⑧}{⑦} \Rightarrow Z_A + Z_B + \frac{Z_A Z_B}{Z_C} = Z_{AB}$$

Similarly,

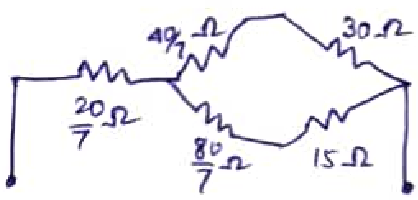
$$Z_{BC} = Z_B + Z_C + \frac{Z_B Z_C}{Z_A}$$

$$Z_{CA} = Z_C + Z_A + \frac{Z_C Z_A}{Z_B}$$

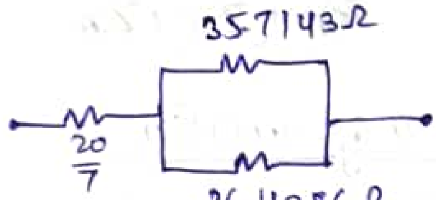
Q) Find the equivalent resistance across the terminals A and C.



Soln:



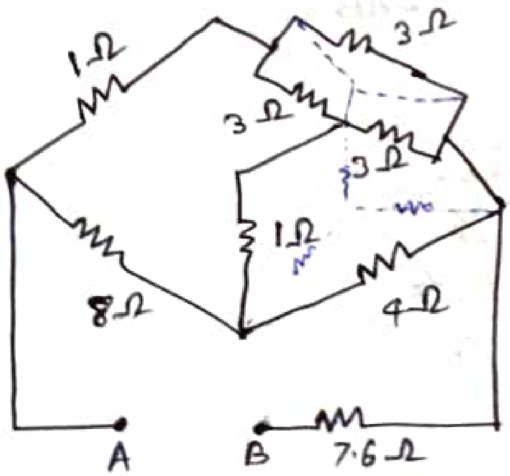
$\Rightarrow$



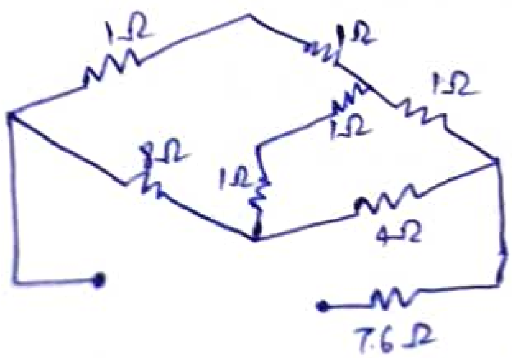
$\Rightarrow$

$R_{eq} = 18.046\ \Omega$

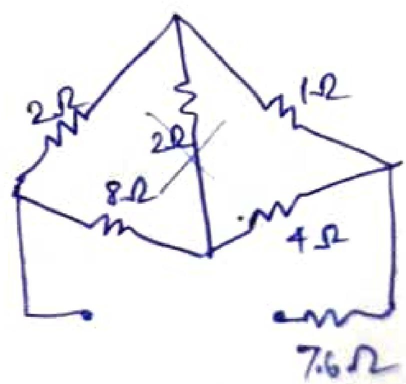
Q) Calculate Eq. resistance b/w A & B.



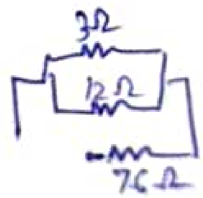
Soln:



$\Rightarrow$

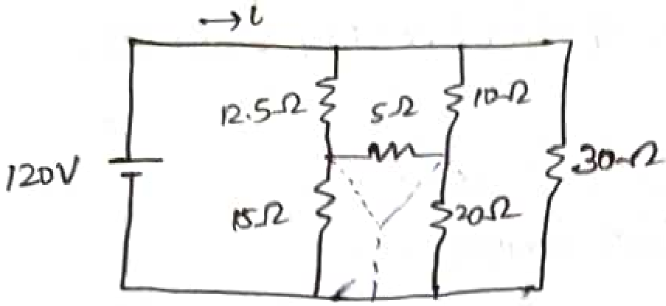


$\Rightarrow$

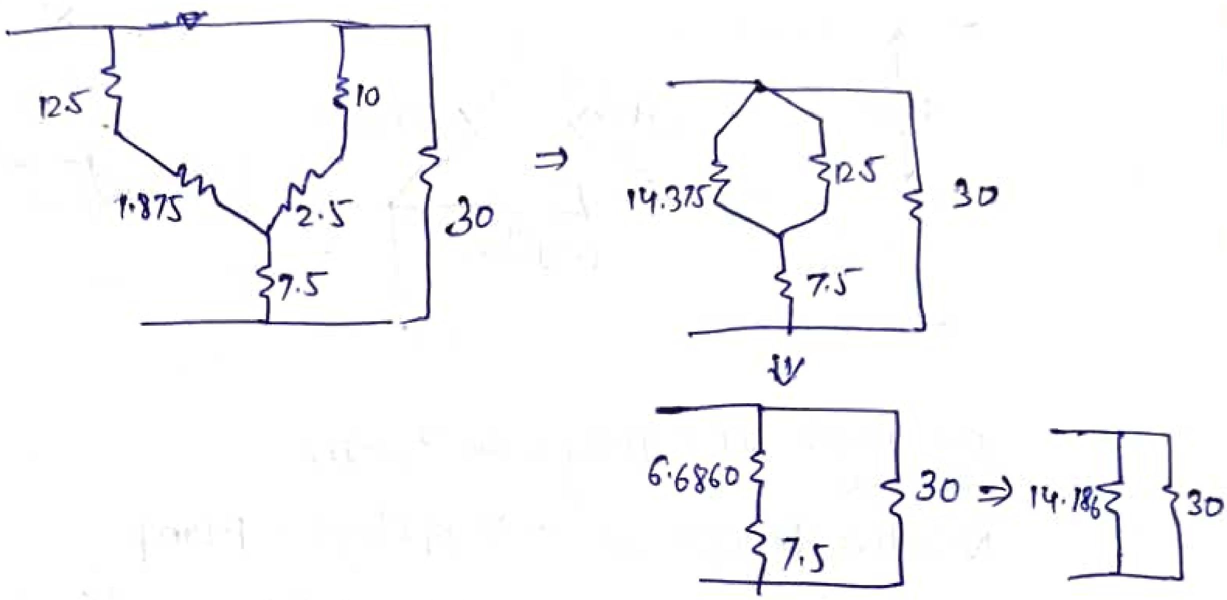


$R_{eq} = 11.7\ \Omega$

Q) Find the current i.



Soln:



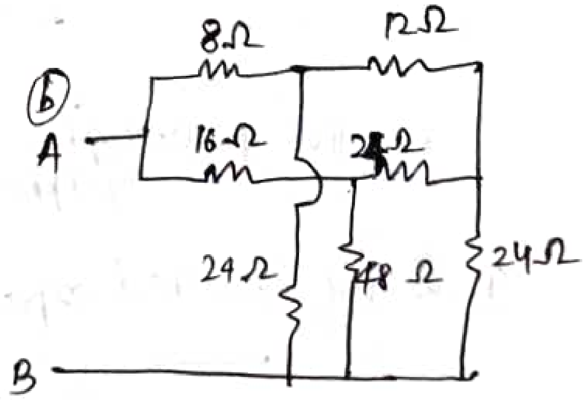
$R_{eq} = 9.6316\,\Omega$

$i = \dfrac{120\text{ V}}{9.6316\,\Omega} = 12.459\text{ A}.$

Q) Find the resistance b/w A and B.

(a)

(b)



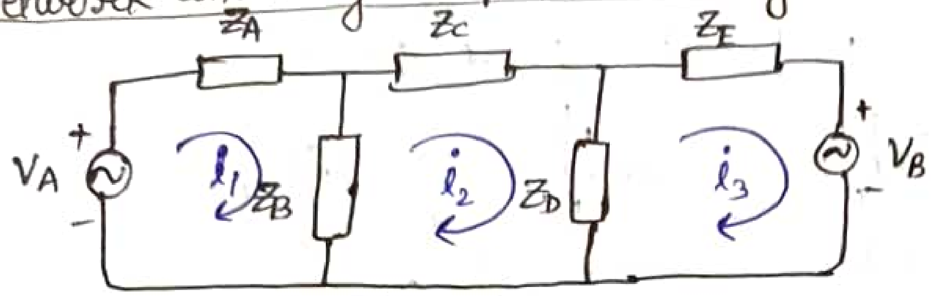
Ans: $R_{AB} = 2\,\Omega$

$R_{eq} = 16\,\Omega$.

(22-05-23)

Mesh/Loop Current Method

(a) Network containing independent voltage sources and impedences



Loop 1:

$$V_A = Z_A i_1 + Z_B(i_1 - i_2) = i_1(Z_A + Z_B) + i_2(-Z_B)$$

Loop 2:

$$0 = Z_C i_2 + Z_D(i_2 - i_3) + Z_B(i_2 - i_1) = i_1(-Z_B) + i_2(Z_B + Z_C + Z_D) + i_3(-Z_D)$$

Loop 3:

$$-V_B = (i_3 - i_2) Z_D + i_3 Z_E$$

$$\Rightarrow -V_B = i_2(-Z_D) + i_3(Z_D + Z_E)$$

$$\begin{bmatrix} V_A \\ 0 \\ -V_B \end{bmatrix} = \begin{bmatrix} Z_A + Z_B & -Z_B & 0 \\ -Z_B & Z_B + Z_C + Z_D & -Z_D \\ 0 & -Z_D & Z_D + Z_E \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$Z_{ii} \rightarrow$ sum of all impedences through which loop current 'I_i' is flowing.

$Z_{ij} \rightarrow$ sum of all impedences common to I_i and I_j (+ve, if both are in same direction, else −ve).

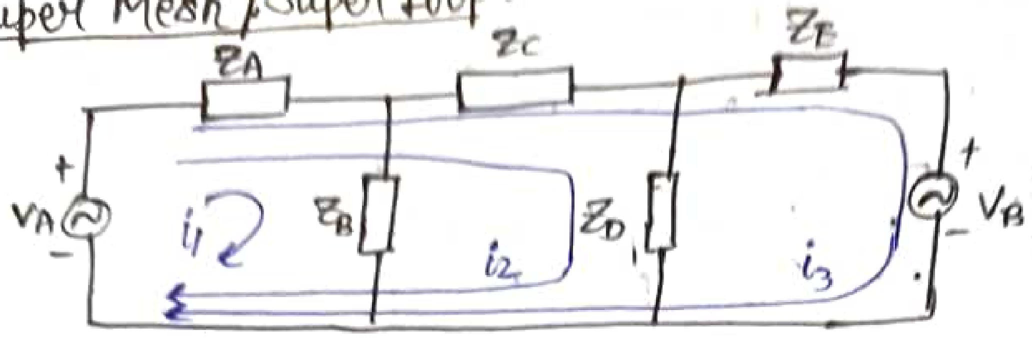
$V_i \rightarrow$ +ve, if voltage source drives the current in the same direction as the loop current, else negative.

$I_1 = \frac{\Delta_1}{\Delta}$, $I_2 = \frac{\Delta_2}{\Delta}$, $I_3 = \frac{\Delta_3}{\Delta}$ where

$$\Delta = \begin{vmatrix} (Z_A + Z_B) & -Z_B & 0 \\ -Z_B & (Z_B + Z_C + Z_D) & -Z_D \\ 0 & -(Z_D) & Z_D + Z_E \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} V_A & -Z_B & 0 \\ 0 & (Z_B + Z_C + Z_D) & -Z_D \\ -V_B & -Z_D & Z_D + Z_E \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} Z_A + Z_B & V_A & 0 \\ -Z_B & 0 & -Z_D \\ 0 & -V_B & (Z_D + Z_E) \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} Z_A + Z_B & -Z_B & V_A \\ -Z_B & (Z_B + Z_C + Z_D) & 0 \\ 0 & -Z_D & -V_B \end{vmatrix}$$

Super Mesh / Super Loop:

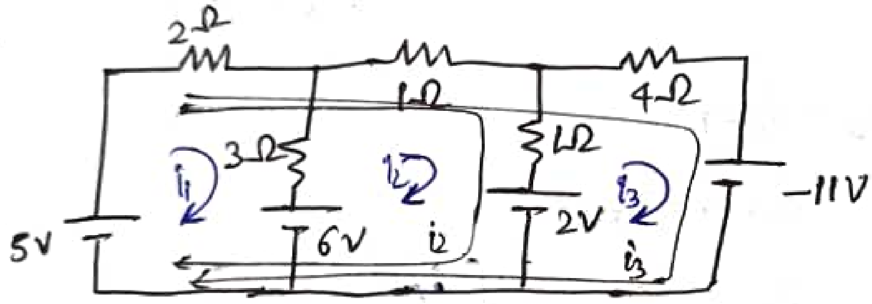


Loop 1: $V_A = [i_1 + i_2 + i_3] Z_A + i_1 Z_B$

Loop 2: $V_A = [i_1 + i_2 + i_3] Z_A + [i_2 + i_3] Z_C + i_2 Z_D$

Loop 3: $V_A - V_B = [i_1 + i_2 + i_3] Z_A + [i_2 + Z_3] Z_C + i_3 Z_E$

Q)



Soln:

$$\begin{bmatrix} -1 \\ 4 \\ 13 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 0 \\ -3 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -3 & 0 \\ -3 & 5 & -1 \\ 0 & -1 & 5 \end{vmatrix} = 24(5) + 3(-15) = 120 - 45 = 75$$

$$\Delta_1 = \begin{vmatrix} -1 & -3 & 0 \\ 4 & 5 & -1 \\ 13 & -1 & 5 \end{vmatrix} = -24 + 99 = 75 \quad ; \quad \Delta_2 = \begin{vmatrix} 5 & -1 & 0 \\ -3 & 4 & -1 \\ 0 & 13 & 5 \end{vmatrix} = 165 - 15 = 150$$

$$\Delta_3 = \begin{vmatrix} 5 & -3 & -1 \\ -3 & 5 & 4 \\ 0 & -1 & 13 \end{vmatrix} = 345 - 117 - 3 = 225$$

$i_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{75}{75} = 1\ A$

$i_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{150}{75} = 2A$

$i_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{225}{75} = 3A$

By super mesh,

$$\begin{bmatrix} -1 \\ 3 \\ 16 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 2 \\ 2 & 4 & 3 \\ 2 & 3 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

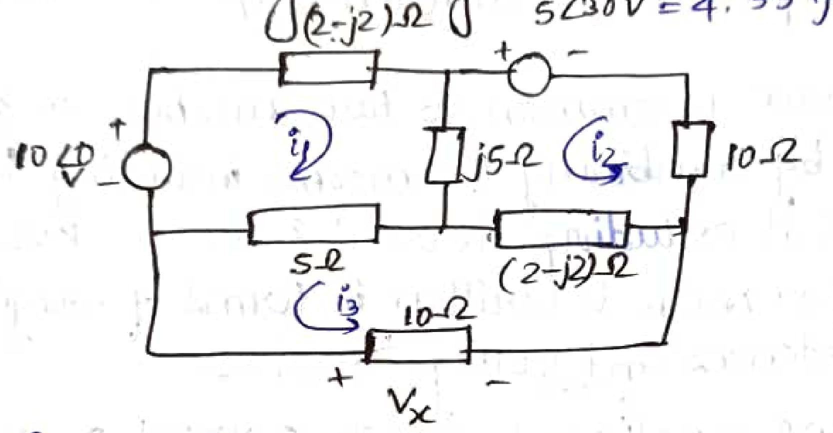
$\Rightarrow i_1 = -1\ A$

$i_2 = -1\ A$

$i_3 = 3A$

Q obtain the voltage using mesh method.

$5\angle 30 V = 4.33 + j2.5$



Soln:

$$\begin{bmatrix} 10 \\ 4.33 + j2.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 7+j3 & j5 & 5 \\ j5 & 12+j3 & -(2-j2) \\ 5 & -(2-j2) & 17-j2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$\Rightarrow i_3 = 0.4353 \angle 165.87^\circ A$

$V_x = 10 i_3$

$= 4.353 \angle 165.87^\circ A$

23-05-2023

⑥ Network of independent current and voltage source and impedances

→ If possible, convert the current source into voltage source.

→ If the current source is common only to one mesh, then the mesh current is given by the source current.

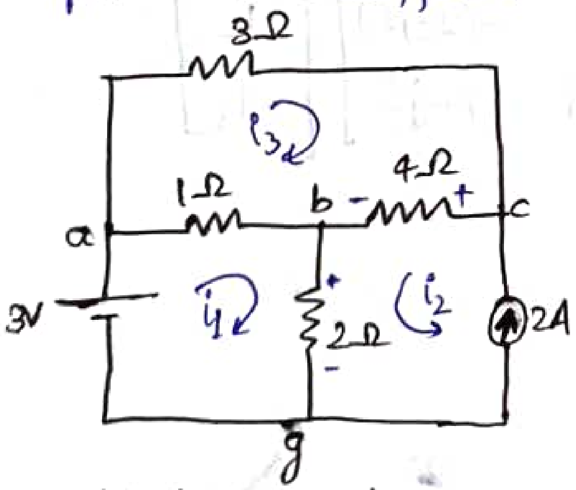
→ otherwise, define voltage across current source and write the mesh equations as if these source voltages were known.

- Augment the set of equations with one equation for each current source expressed as difference b/w two mesh currents, or

OR If a current source is common to two meshes, a super mesh is obtained by combining the meshes including the current source but excluding current source. KVL equation around the super mesh is written in terms of original mesh currents, impedences and voltage sources.

Augment with one equation for each current sources, expressed as difference b/w two mesh current.

Q)



Find I_{ab} and V_{cg}.

Soln: $i_2 = 2A$

$3 = 1(i_1 - i_3) + 2(i_1 + i_2)$

$0 = 3i_3 + 4(i_3 + i_2) + 1(i_3 - i_1)$

Solving,

$\cancel{3 = 1(i_1 - i_3) + 2(i_1 + i_2)}$

$\cancel{0 = 3i_3 + 4(i_3 + i_2) + 1(i_3 - i_2)}$

$3 = i_1 - i_3 + 2i_1 + 4$

$-1 = 3i_1 - i_3$

$0 = 3i_3 + 4i_3 + 8 + i_3 - i_1$

$-8 = 8i_3 - i_1$

$-25 = 23 i_3$

$\Rightarrow i_3 = \frac{-25}{23} = -1.0869\,A$

$i_1 = \frac{i_3 - 1}{3} = -0.6956\,A$

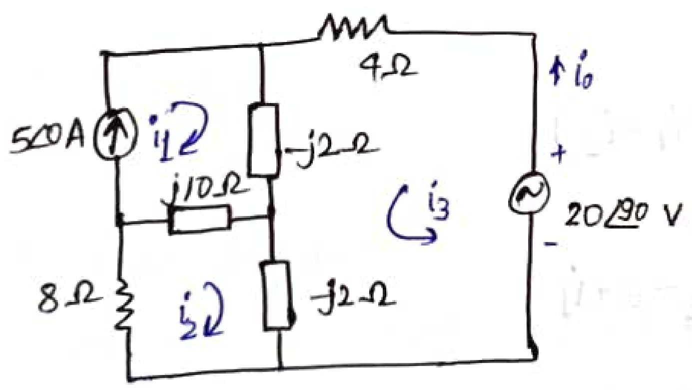
$i_{ab} = 0.3913\,A$

$\therefore V_{cg} = 2(i_1 + i_2) + 4(i_2 + i_3)$

$= 2(2 - 0.6956) + 4(2 - 1.0869)$

$= 6.2612\,V$

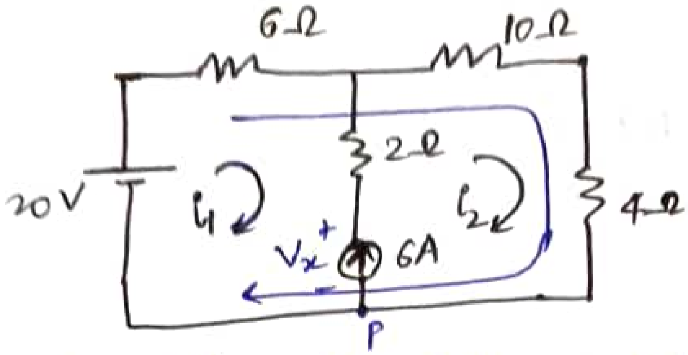
Q)



Ans:

$i_0 = 6.1201 \angle 144.78\,A$

Q) Using mesh current method, obtain i_1 and i_2.



Soln: $20 - V_x = 6i_1 + 2[i_1 - i_2]$ — ①

$V_x = 10i_2 + 4i_2 + 2[i_2 - i_1]$ — ②

Using KCL at P,

$i_2 = 6 + i_1$ — ③

Put ③ & ② in ①

$20 - 6i_1 - \cancel{2i_1} + 2(6 + \cancel{i_1}) = 10 \times (6 + i_1) + 4(6 + i_1) + 2(6 + \cancel{i_1}) - \cancel{2i_1}$

$\Rightarrow \overset{20}{\cancel{32}} - 6i_1 = \overset{40}{\cancel{60}} + 24 + \cancel{12} + 14i_1$

$\Rightarrow 20i_1 = -64 \Rightarrow i_1 = \frac{-64}{20} = -\frac{32}{10} = -3.2A$

and, $i_2 = 6 + i_1 = 2.8A$

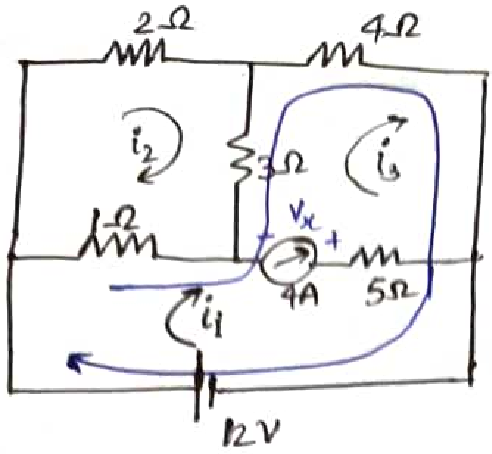
Using super mesh,

$20 - V_x = 6i_1 + 2(i_1 - i_2)$

$20 = 6i_1 + 14i_2$

and using KCL, $i_2 = 6 + i_1$

Q)



Soln: $0 = 2i_2 + 3(i_2 - i_3) + 1(i_2 - i_1)$ - ①

$-V_x = 4i_3 + 5(i_3 - i_1) + 3(i_3 - i_2)$ - ②

$V_x + 12 = 1(i_1 - i_2) + 5(i_1 - i_3)$ - ③

and using KCL, $i_1 = 4 + i_3$

① ⇒ $6i_2 = i_1 + 3i_3$

② ⇒ $V_x + 12i_3 = 5i_1 + 3i_2$

③ ⇒ $V_x + 12 + i_2 = 6i_1 - 5i_3$

⇒ $i_1 = 6A$
$i_2 = 2A$
$i_3 = 2A$

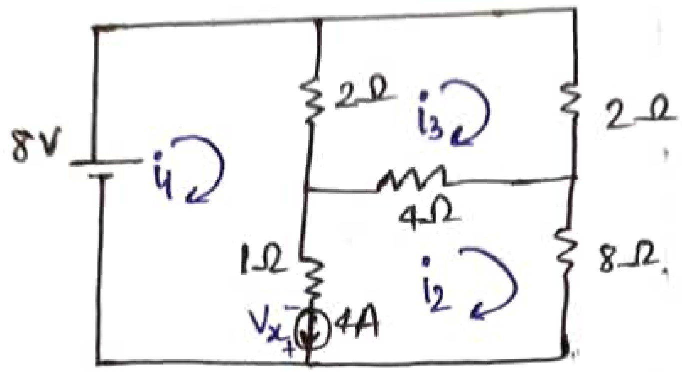
Using super mesh

~~$V_x = 12i_3 - 3i_2 - 5i_1$~~ $6i_2 = i_1 + 3i_3$ - ①

$12 = 1[i_1 - i_2] + 3[i_3 - i_2] + 4i_3$ - ②

KCL: $i_1 = 4 + i_3$ - ③

Q) Find the mesh current.



Soln: Loop 1: $2(i_1 - i_3) + 1(i_1 - i_2) = 8 + V_x$

Loop 2: $-V_x = 1(i_2 - i_1) + 4(i_2 - i_3) + 8i_2$

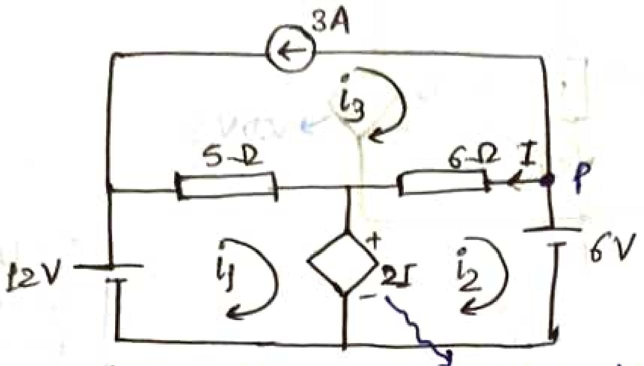
Loop 3: $0 = 2(i_3 - i_1) + 2i_3 + 4(i_3 - i_2)$

and, $4 = i_1 - i_2$

$\Rightarrow i_1 = \frac{88}{19}A,\ i_2 = \frac{12}{19}A,\ i_3 = \frac{28}{19}A,\ V_x = -\frac{28}{19}V.$

Mesh/Loop Analysis

Q) Find I.



| $I = 0.75$ A

Soln: $i_3 = -3A$ — current dependent voltage source

Loop 1 ⇒ $12 - 2I = 5[i_1 - i_3]$

Loop 2 ⇒ $-6 + 2I = 6(i_2 - i_3)$

Loop 3 ⇒ $i_3 = -3A$

KCL ⇒ $I = i_3 - i_2$ (at P)

⇒ $i_1 = -0.9A$

$i_2 = -(15/4)A$

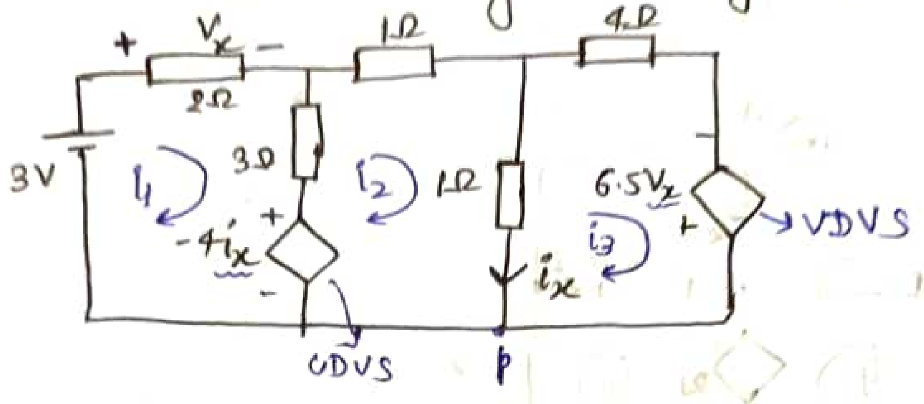
$I = 0.75$ A

© Network containing DEPENDENT sources

When a network contains a dependent source, the mesh equations are written in the usual way..

A dependent source is treated exactly as an independent source in obtaining the mesh eqⁿs. Then an eqⁿ for each dependent source is written in terms of one or more mesh currents. These values are substituted into the main mesh eqⁿs from which mesh currents can be obtained.

Q) Obtain the current i_x and voltage V_x using mesh current method.



$i_1 = 1A$
$i_2 = 2A$
$i_3 = 3A$

Soln:

$$3+4i_x - V_x = 3(i_1 - i_2) \quad -①$$

$$-4i_x = 3(i_2 - i_1) + 1(i_2 - i_3) \quad -②$$

$$6.5V_x = 1(i_3 - i_2) + 4i_3 \quad -③$$

KCL (at p): $i_x = i_2 - i_3 \quad -④ \Rightarrow i_3 = i_2 - i_x$

$2i_1 = V_x \quad -⑤ \Rightarrow i_1 = \frac{V_x}{2}$

① $\Rightarrow 3 + 4i_x - V_x = 3i_1 - 3i_2 \Rightarrow 3 + 4i_x = 5i_1 - 3i_2 \Rightarrow 3 + 7i_2 = 5i_1 + 4i_3$ −(i)

② $\Rightarrow -4i_x = 4i_2 - 3i_1 + i_x - i_2$

$\Rightarrow 4i_x = 3i_1 - 3i_2 - i_x$

$\Rightarrow 5i_x = 3i_1 - 3i_2 \Rightarrow 8i_2 - 5i_3 = 3i_1$ −(ii)

③ $\Rightarrow 6.5V_x = 5i_2 - 5i_x - i_2$

$\Rightarrow 6.5V_x = 4i_2 - 5i_x \Rightarrow 13i_1 = 4i_2 - 5i_2 + 5i_3$

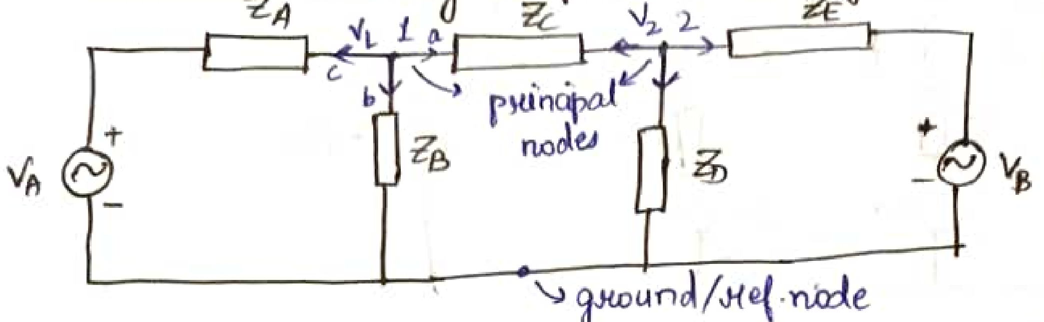
$\Rightarrow i_1 = 1A, i_2 = 2A, i_3 = 3A.$ $\Rightarrow 13i_1 + i_2 = 5i_3$ −(iii)

$\Rightarrow i_x = i_2 - i_3 = -1A$

$V_x = 2i_1 = 2V.$

NODE VOLTAGE METHOD

ⓐ Network containing independent voltage sources and impedences



→ 3 total nodes
→ 2 principal nodes
→ 1 reference node

→ Node generally means principle nodes.

KCL at node 1 ⇒ $\frac{V_1 - V_A}{Z_A} + \frac{V_1 - V_2}{Z_C} + \frac{Z_1}{Z_B} = 0$

Node : junction of 3 or more branches

$$\Rightarrow \left[\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}\right] V_1 + \left[-\frac{1}{Z_C}\right] V_2 = \frac{V_A}{Z_A}$$

KCL at node 2 ⇒ $\frac{V_2 - V_1}{Z_C} + \frac{V_2}{V_D} + \frac{V_2 - V_B}{Z_E} = 0 \Rightarrow \left[-\frac{1}{Z_C}\right] V_1 + \left[\frac{1}{Z_C} + \frac{1}{Z_D} + \frac{1}{Z_E}\right] V_2 = \frac{V_B}{V_Z}$

$$\Rightarrow \begin{bmatrix} \left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}\right) & -\frac{1}{Z_C} \\ -\frac{1}{Z_C} & \frac{1}{Z_C} + \frac{1}{Z_D} + \frac{1}{Z_E} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_A}{Z_A} \\ \frac{V_B}{Z_E} \end{bmatrix}$$

| $\frac{1}{Z} = y$: Admittance

$\frac{1}{Z_{ii}} = Y_{ii}$ → sum of all admittances connected to node i

$\frac{1}{Z_{ij}} = Y_{ij}$ → negative of sum of all admittances connected b/w nodes i and j

I_i → ⊕ve, if the source drives the current into node i, else, ⊖ve.

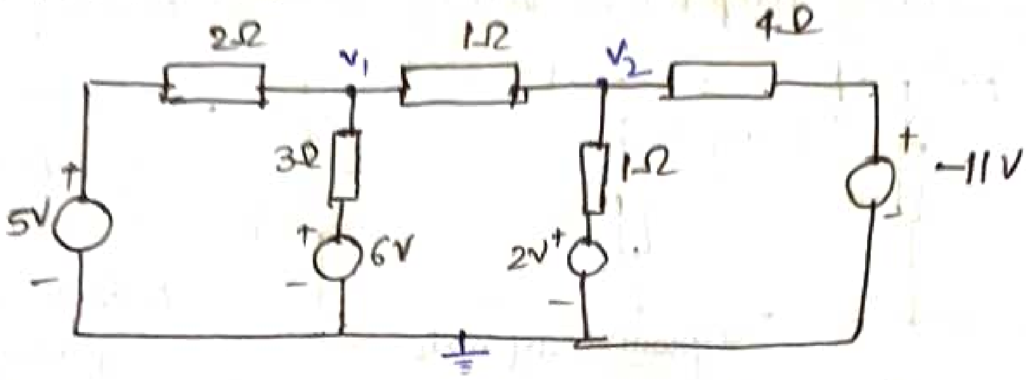
→ If n = total no. of nodes (including ref. node), $(n-1)$ equations have to be solved.

∴ $V_1 = \frac{\Delta_1}{\Delta}$; $V_2 = \frac{\Delta_2}{\Delta}$ where

replaced by current

$$\Delta = \begin{vmatrix} \left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}\right) & -\frac{1}{Z_C} \\ -\frac{1}{Z_C} & \left(\frac{1}{Z_C} + \frac{1}{Z_D} + \frac{1}{Z_E}\right) \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} \frac{V_A}{Z_A} & -\frac{1}{Z_C} \\ \frac{V_B}{Z_E} & \left(\frac{1}{Z_C} + \frac{1}{Z_D} + \frac{1}{Z_E}\right) \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} \left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}\right) & \frac{V_A}{Z_A} \\ -\frac{1}{Z_C} & \frac{V_B}{Z_E} \end{vmatrix}$$

Q Using node voltage method, obtain the branch currents for the circuit shown below.



Soln:

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{3}+\frac{1}{1} & -\frac{1}{1} \\ -\frac{1}{1} & \frac{1}{1}+\frac{1}{1}+\frac{1}{4} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2}+\frac{6}{3} \\ -\frac{11}{4}+\frac{2}{1} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{11}{6} & -1 \\ -1 & 9/4 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 9/2 \\ -\frac{3}{4} \end{bmatrix}$$

$$\frac{11}{6}V_1 - V_2 = \frac{9}{2}$$

$$-V_1 + \frac{9}{4}V_2 = -\frac{3}{4} \quad \} \times \frac{11}{6}$$

$$\left(-1+\frac{33}{8}\right)V_2 = -\frac{11}{8} + \frac{9\times 4}{2\times 4} = \frac{25}{8}$$

$$\Rightarrow V_2 = \frac{25}{25} = 1\,V$$

and, $V_1 = \frac{9}{4}V_2 + \frac{3}{4} = \frac{12}{4} = 3\,V$

$\Delta = 3.125$

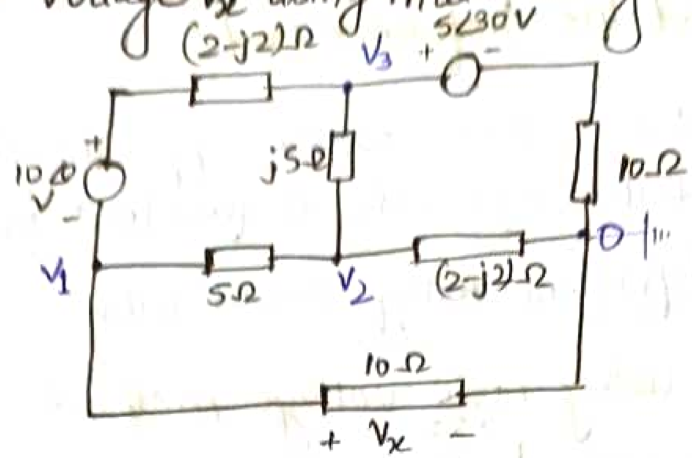
$\Delta_1 = 9.375$

$\Delta_2 = 3.125$

$V_1 = \frac{\Delta_1}{\Delta} = 3\,V$

$V_2 = \frac{\Delta_2}{\Delta} = 1\,V$

Q) Obtain the voltage V_x using nodal voltage method.



Sol:

Node 1: $\dfrac{V_1 - V_2}{5} + \dfrac{V_1 + 10\angle 0 - V_3}{2-j2} + \dfrac{V_1}{10} = 0$

Node 2: $\dfrac{V_2 - V_1}{5} + \dfrac{V_2 - 0}{2-j2} + \dfrac{V_2 - V_3}{j5} = 0$

Node 3: $\dfrac{V_3 - 10\angle 0 - V_1}{2-j2} + \dfrac{V_3 - V_2}{j5} + \dfrac{V_3 - 5\angle 30 - 0}{10} = 0$

$$\Rightarrow \begin{cases} \left(\frac{1}{5} + \frac{1}{2-j2} + \frac{1}{10}\right)V_1 + \left(\frac{-1}{5}\right)V_2 + \left(\frac{-1}{2-j2}\right)V_3 = -\frac{10\angle 0}{2-j2} \\ \left(-\frac{1}{5}\right)V_1 + \left(\frac{1}{5} + \frac{1}{2-j2} + \frac{1}{j5}\right)V_2 + \left(\frac{-1}{j5}\right)V_3 = 0 \\ \left(\frac{-1}{2-j2}\right)V_1 + \left(\frac{-1}{j5}\right)V_2 + \left(\frac{1}{2-j2} + \frac{1}{j5} + \frac{1}{10}\right)V_3 = \frac{10\angle 0}{2-j2} + \frac{5\angle 30}{10} \end{cases}$$

$$\Rightarrow \begin{bmatrix} \left(\frac{1}{5} + \frac{1}{2-j2} + \frac{1}{10}\right) & -\left(\frac{1}{5}\right) & -\left(\frac{1}{2-j2}\right) \\ \left(-\frac{1}{5}\right) & \left(\frac{1}{5} + \frac{1}{2-j2} + \frac{1}{j5}\right) & -\frac{1}{j5} \\ \left(-\frac{1}{2-j2}\right) & -\left(\frac{1}{j5}\right) & \left(\frac{1}{2-j2} + \frac{1}{j5} + \frac{1}{10}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -\frac{10\angle 0}{2-j2} \\ 0 \\ \frac{10\angle 0}{2-j2} + \frac{5\angle 30}{10} \end{bmatrix}$$

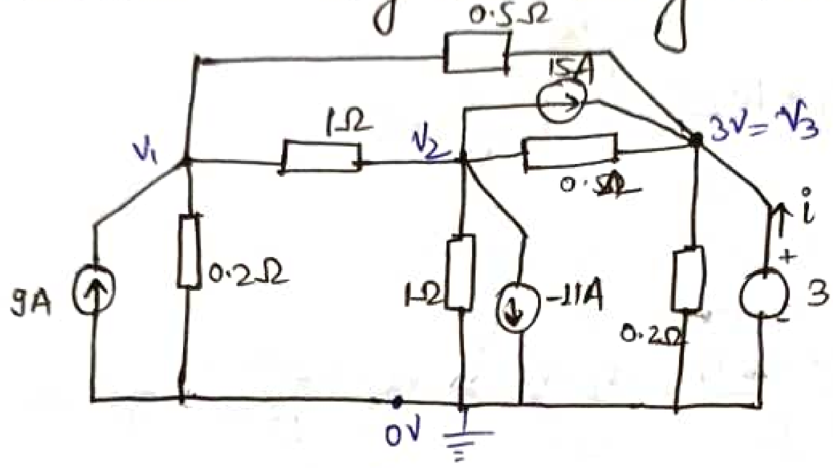
$\Delta = 0.0694 + j0.0324$; $\Delta_1 = -0.3276 - j0.0633$

$\therefore V_x = V_1 = \dfrac{\Delta_1}{\Delta} = 4.3564\angle 165.91\ V$

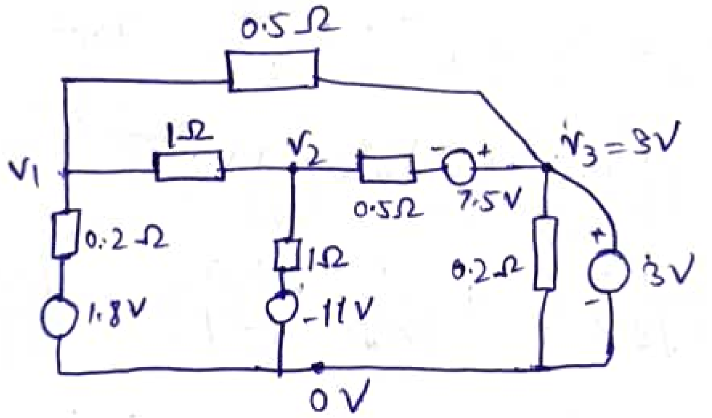
(B) Network containing an independent VOLTAGE source directly b/w a principal node and the reference node

When an independent voltage source appears directly b/w a principal node & the ref. node, it fixes that node voltage to be at the independent voltage source value.

Q] Find the current 'i' using node voltage method.



Soln:



→ X

$V_3 = 3V$
$V_1 = 2V$
$V_2 = 1V$
$i = 6A$

Node 1: $\frac{V_1-0}{0.2} - 9 + \frac{V_1-3}{0.5} + \frac{V_1-V_2}{1} = 0$

Node 2: $\frac{V_2-V_1}{1} + \frac{V_2}{1} - 11A + 15A + \frac{V_2-3}{0.5} = 0$

Node 3: $\cancel{\frac{3-V_1}{0.5} - 15 + \frac{3-V_2}{0.5} + \frac{3}{0.2} + 3/3} = 0$

$V_3 = 3V$

$$\Rightarrow \begin{cases} (5+2+1)V_1 - 1V_2 = 9+6 \\ -1V_1 + 4V_2 = 2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 8 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 8V_1 - V_2 = 15 \;\} \times 4 \\ -V_1 + 4V_2 = 2 \end{cases}$$

$$31V_1 = 62$$

$$\Rightarrow V_1 = 2\text{ V}$$

$$V_2 = 8V_1 - 15 = 16 - 15 = 1\text{ V}$$

<u>At node 3:</u>

$$-i + \frac{3}{0.2} + \frac{3-\cancel{V_2}^{\,1}}{0.5} - 15 + \frac{3-\cancel{V_1}^{\,2}}{0.5} = 0$$

$$\Rightarrow i = \cancel{15} + 6 - \cancel{2} - \cancel{15} + \cancel{6} - \cancel{4}$$

$$= 6\text{A}$$

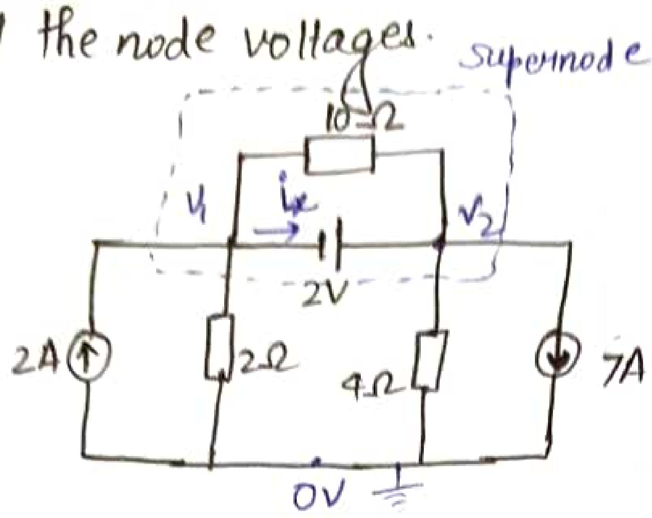
(3) Network containing an independent VOLTAGE source directly b/w principal nodes

When an independent voltage source appears directly b/w two principal nodes, the voltage at one node will be fixed above the other by the independent voltage source. Both KCL and KVL are applied to find the node voltages.

- Define a current through the voltage source and write the nodal eq^ns as if this source is known.

OR The 2 principal nodes b/w which the indp. voltage source is connected form a SUPERNODE. A supernode is formed by enclosing the dep./indep. voltage source connected directly in b/w the principal nodes & any elements connected in ‖ with it. The KCL for the supernode is written in terms of original node voltages and impedances.

8) Find the node voltages.



Soln: $V_1 + 2 = V_2$ (KVL)

Node 1: $-2 + \frac{V_1}{2} + i_x + \frac{V_1 - V_2}{10} = 0$ -①

Node 2: $7 + \frac{V_2}{4} - i_x + \frac{V_2 - V_1}{10} = 0$ -②

$\Rightarrow \cancel{6V_1 - V_2 + 10i_x = 20}$

$\cancel{-2V_1 + 7V_2 - 20i_x = 140}$

$5 + \left(\frac{6V_1}{10} - \frac{V_1}{10}\right) + \left(-\frac{1\times2}{10\times2} + \frac{7}{20}\right)V_2 = 0$

$\Rightarrow 5 + \frac{V_1}{2} + \frac{V_2}{4} = 0$

$\Rightarrow \frac{5\times4}{4} + \frac{V_1\times2}{2\times2} + \frac{V_1+2}{4} = 0$

$\Rightarrow \frac{3}{4}V_1 + \frac{22}{4} = 0$

$\Rightarrow V_1 = \frac{-22}{3} = -7.333\text{ V}$

$V_2 = V_1 + 2 = -5.333\text{ V}$

OR

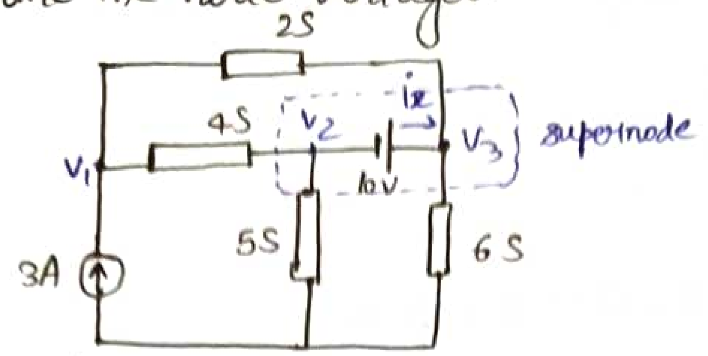
At the supernode,

$2 = \frac{V_1}{2} + \frac{V_2}{4} + 7$ → supernode eqⁿ
↳ (we can get by adding ① & ②)

KVL: $V_2 = 2 + V_1$

30-05-23

Q) Determine the node voltages.



S = Siemens = $\frac{1}{\Omega}$ → unit of admittance

Soln: At node 1:

$3 = (V_1 - V_2)4 + (V_1 - V_3) \times 2$

node 2: $(V_2 - V_1) \times 4 + V_2 \times 5 + i_x = 0$ — ①

node 3: $i_x = V_3 \cdot 6 + (V_3 - V_1)2$ — ② → add to get supernode eqn

KVL: $V_2 + 10 = V_3$

$\Rightarrow V_2 = -5.1818\,V$, $V_3 = 4.8182\,V$
$V_1 = -1.3485\,V$

By supernode

node 1: $3 = (V_1 - V_2)4 + (V_1 - V_3)2$

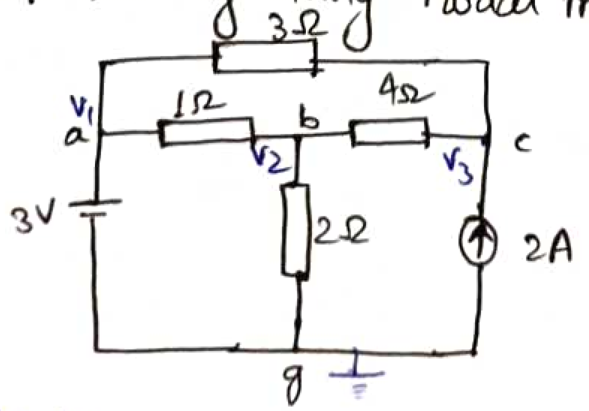
at supernode: $V_2 \cdot 5 + V_3 \cdot 6 + (V_3 - V_1) \cdot 2 + (V_2 - V_1) \cdot 4 = 0$

$\Rightarrow -6V_1 + 9V_2 + 8V_3 = 0$

KVL: $V_2 + 10 = V_3$

$\Rightarrow V_1 = -1.3485\,V$, $V_2 = -5.1818\,V$, $V_3 = 4.8182\,V$.

Q) Find I_{ab} and V_{cg} using nodal method.



Soln: Node 1: $V_1 = 3\,V$

Node 2: $\frac{(V_2 - V_1)}{1} + \frac{V_2}{2} + \frac{V_2 - V_3}{4} = 0$

Node 3: $2 = \left(\frac{V_3 - V_2}{4}\right) + \left(\frac{V_3 - V_1}{3}\right)$

$\Rightarrow V_2 = 2.6088\,V$

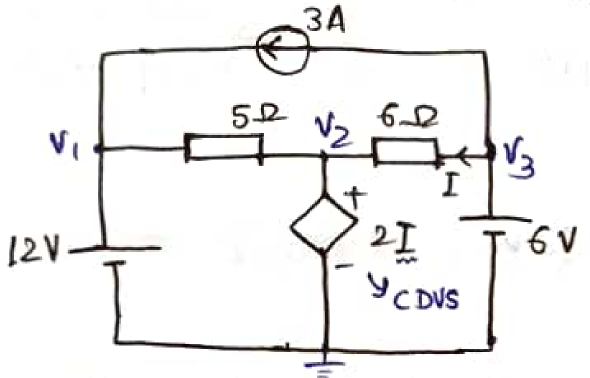
$V_3 = 6.2617\,V$

$V_{cg} = V_3 = 6.2617\,V$

$I_{ab} = \frac{(V_1 - V_2)}{1} = 3 - 2.6088 = 0.3912\,A.$

→ No supernode (b/c we don't have a voltage source connected directly b/w 2 principal nodes).

Q) Determine the current I using nodal method.



Soln:

$V_1 = 12\,V$ (at node 1)

$V_3 = 6\,V$ (at node 3)

At node 2: $\cancel{\left(\frac{V_2 - V_1}{5}\right) + \left(\frac{V_2 - V_3}{6}\right)}\ V_2 = 2I$

$I = \frac{V_3 - V_2}{6} = \frac{6 - 2I}{6}$

$\Rightarrow 8I = 6$

$\Rightarrow I = 3/4 = 0.75\,A$

Superposition Theorem

5-06-2023

The response in any element of a linear active bilateral network containing multiple independent sources is the algebraic sum of the individual responses due to each independent source acting alone with all other sources replaced by their respective internal impedences, i.e., voltage sources are replaced by short circuits and current sources are replaced by open circuits.

~~Q) Obtain the voltage~~

Linear → Superposition prinple
→ Homogeneity property

$$r_1(t) \longrightarrow \boxed{G(\cdot)} \longrightarrow y_1(t)$$

$$r_2(t) \longrightarrow \boxed{G(\cdot)} \longrightarrow y_2(t)$$

$$r_1(t)+r_2(t) \longrightarrow \boxed{G(\cdot)} \longrightarrow y_1(t)+y_2(t)$$

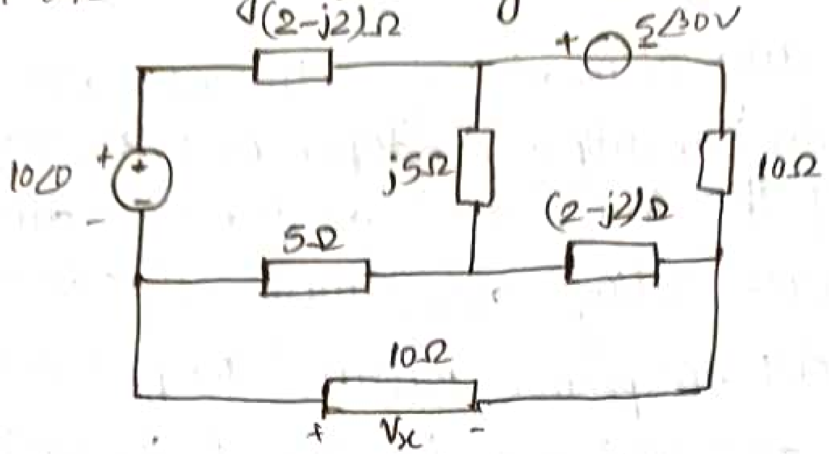
$$r_1(t) \longrightarrow \boxed{} \longrightarrow y_1(t)$$

$$\alpha r(t) \longrightarrow \boxed{} \longrightarrow \alpha y(t)$$ → Homogeneity property

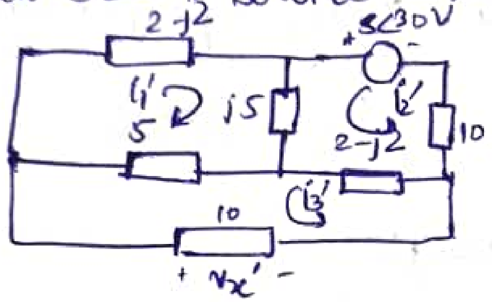
Bilateral Element : ~~Ope~~ property remains same in both the polarities (e.g., resistor)

Unilateral Element : Opposite to bilaterial (e.g., diode)

Q) Obtain the voltage V_x, using Superposition theorem.



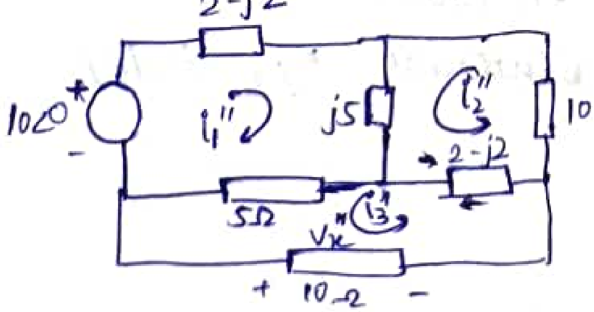
Soln (a) When 5∠30 V source alone is acting



$$\begin{bmatrix} (7+j3) & j5 & 5 \\ j5 & (12+j3) & (-2+j2) \\ 5 & (-2+j2) & (17-j2) \end{bmatrix} \begin{bmatrix} i_1' \\ i_2' \\ i_3' \end{bmatrix} = \begin{bmatrix} 0 \\ 5\angle 30 \\ 0 \end{bmatrix}$$

$$\therefore i_3' = \frac{\Delta_3'}{\Delta} = 0.0855\angle 45.31\text{ A} \Rightarrow V_x' = 10 \times i_3' = 0.855\angle 45.31\text{ V}$$

(b) Now, when 10V source alone is acting.



$$\begin{bmatrix} (7+j3) & j5 & 5 \\ j5 & (12+j3) & (-2+j2) \\ 5 & (-2+j2) & (17-j2) \end{bmatrix} \begin{bmatrix} i_1'' \\ i_2'' \\ i_3'' \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore i_3'' = \frac{\Delta_3''}{\Delta} = 0.4844 \angle 174.61\ A \Rightarrow V_x'' = 10 \times i_3'' = 4.844 \angle 174.61\ V$

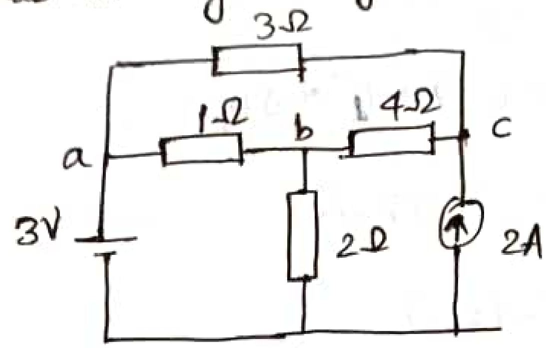
$i_3 = i_3' + i_3'' = 0.0855 \angle 45.31\ A + 0.4844 \angle 174.61\ A$

$= 0.4353 \angle 165.86\ A$

$\therefore V_x = 10 \times i_3 = 4.353 \angle 165.86\ V$

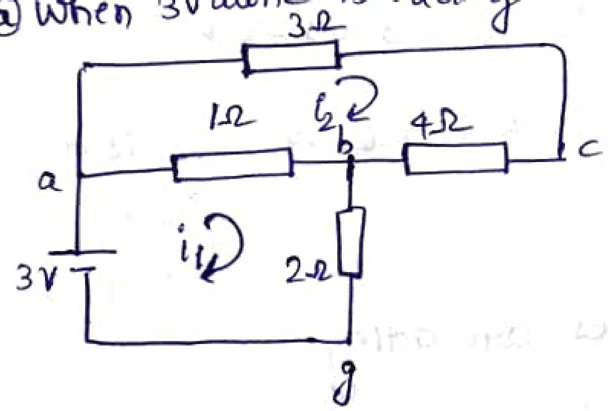
OR $V_x = V_x' + V_x'' = 0.855 \angle 45.3 + 4.844 \angle 174.61 = 4.353 \angle 165.86\ V$

Q Find I_{ab} and V_{cg} using superposition theorem.



• Voltage source ↳ short-circuit
• Current source ↳ open ↳ ~~short circuit~~

Soln: (a) when 3V alone is acting



$$\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$\Delta = 23, \quad \Delta_1 = 24, \Delta_2 = 3$

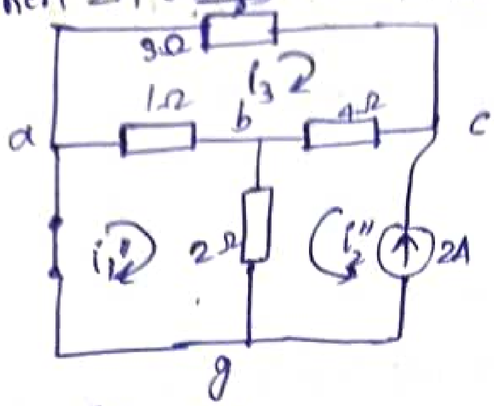
$\therefore i_1 = \frac{\Delta_1}{\Delta} = 1.0435\ A, \quad i_2 = \frac{\Delta_2}{\Delta} = 0.1304\ A$

$\Rightarrow i_{ab}' = i_1 - i_2 = 0.9131\ A$

$V_{cg}' = +4(i_2) + 2(i_1)$

$= 2.6086\ V$

(B) Now, when 2A ~~alone~~ alone is acting



Loop 2 : $i_2'' = 2A$

Loop 1 : $3i_1'' + 2i_2'' - i_3 = 0 \Rightarrow 3i_1'' - i_3 = -4$

Loop 3 : $-i_1'' + 4i_2'' + 8i_3 = 0 \Rightarrow -i_1'' + 8i_3 = -8$

$\Rightarrow i_1'' = -1.7391\,A$; $i_3 = -1.2173A$

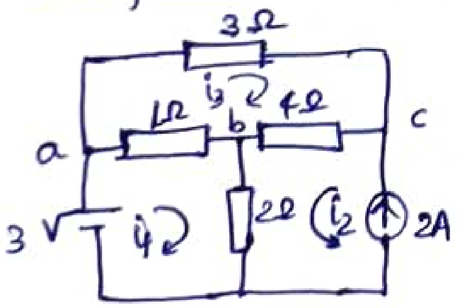
$\therefore i_{ab}'' = i_1'' - i_3 = -0.5218\ A$

$V_{cg}'' = +2(i_1'' + i_2'') + 4(i_2'' + i_3) = 3.6526V$

$\therefore i_{ab} = i_{ab}' + i_{ab}'' = 0.9131\ A\ \cancel{\ldots} - 0.5218\,A$

$V_{cg} = V_{cg}' + V_{cg}'' = 2.6086 + 3.6526 = 6.2612\,V$

Verification: when both sources are acting.



Loop 2 $\Rightarrow$ $i_2 = 2A$

Loop 1 $\Rightarrow$ $3 = 1\times(i_1 - i_3) + 2\times(i_1 + i_2)$

Loop 3 $\Rightarrow$ $0 = 1\times(i_3 - i_1) + 4\times(i_3 + i_2) + 3\times i_3$

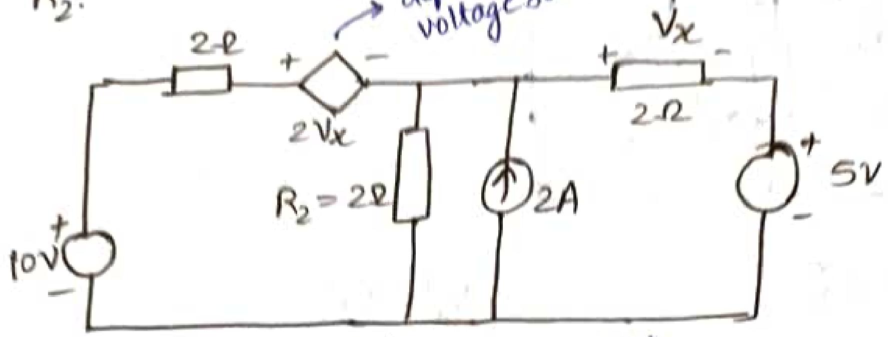
$\Rightarrow i_1 = -0.6956A,\ i_3 = -1.0869\,A$

$\therefore i_{ab} = i_1 - i_3 = 0.3913A$

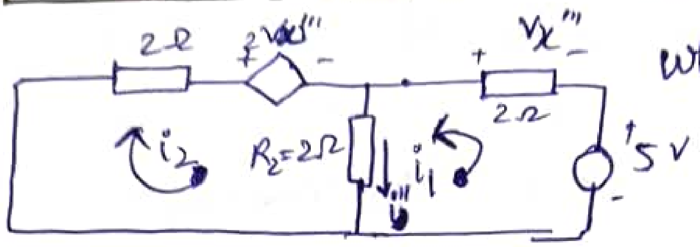
$V_{cg} = 2\times(i_2 + i_1) + 4\times(i_2 + i_3)$

$= 6.2612V.$

Q) Using Superposition theorem, find the power dissipated in R_2.



Soln:



when 5V along is acting

Mesh method: $5 = 4i_1 + 2i_2$ (Loop 1)

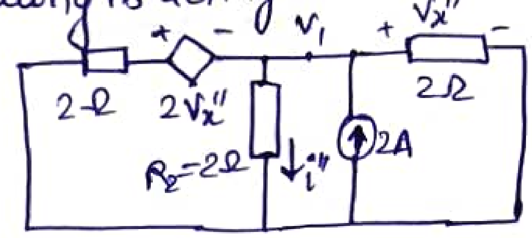
$-2V_x = 4i_2 + 2i_1$ (Loop 2)

$V_x''' = 2i_1$

$\Rightarrow i_1 = 1A,\ i_2 = 0.5A$

$\therefore i''' = i_1 + i_2 = 0.5A$

when 2A along is acting



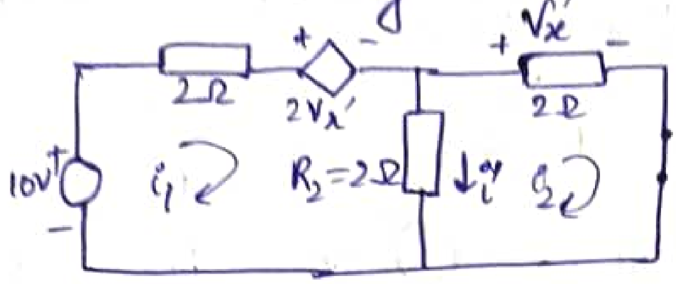
Nodal method: $2 = \frac{(V_1 + 2V_x'')}{2} + \frac{V_1}{2} + \frac{V_1}{2}$

$\Rightarrow \frac{(V_1 + 2V_1)}{2} + V_1 = 2$

$\Rightarrow V_1 = 0.8V$

$\therefore i'' = \frac{V_1}{2} = 0.4A$

When 10V alone is acting



$10-2V_x=4i_1-2i_2$ | $V_x'=2i_2$

$V_x=2i_2$

$0=-2i_1+4i_2$

$\Rightarrow i_1=2A,\ i_2=1A$

$\therefore i'=i_1-i_2=1A$

Hence, total current (i) through $R_2=2\Omega=i'+i''+i'''=2.9A$

Therefore, power dissipated by $R_2=i^2R_2=16.82W$.

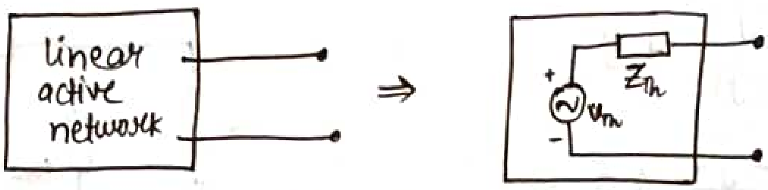
(12-06-2023)

THEVENIN THEOREM

Thevenin theorem states that any linear active network can be replaced by single voltage source (V_{Th}) in series with a single impedence (Z_{Th}).

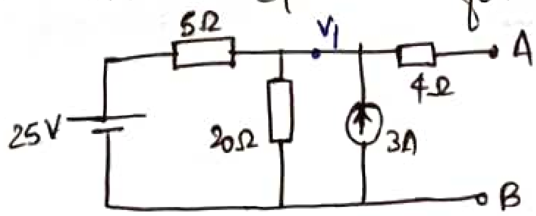
The Thevenin voltage (V_{Th}) is the open circuit voltage measured across the load terminals.

Thevenin impedence (Z_{Th}) is the impedance of the network at the load terminals when all the sources are replaced by their respective internal impedances.



Thevenin Equivalent Circuit

Q) obtain the Thevenin Equivalent for the given network.

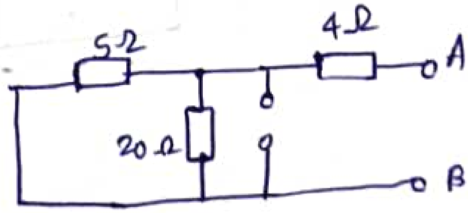


Soln: To find V_{Th}:

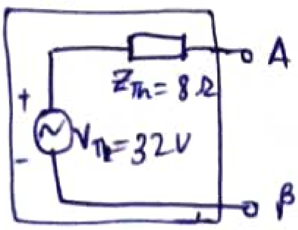
$$\frac{V_1 - 25}{5} + \frac{V_1}{20} = 3 \Rightarrow V_1 = 32V$$

To find R_{Th}:

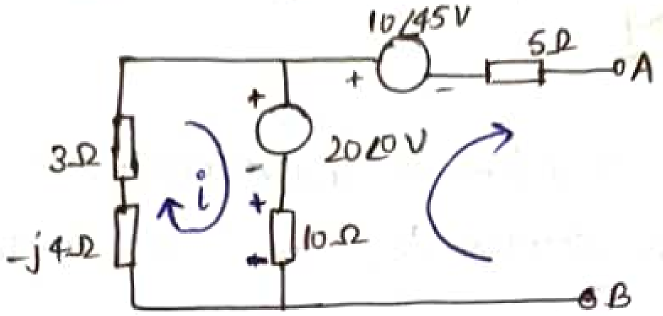
$$R_{Th} = \frac{20 \times 5}{20+5} + 4 = \frac{100}{25} + 4 = 8\Omega$$



Thevenin Eq.:



Q] Obtain the Thevenin Equivalent for the network given below.

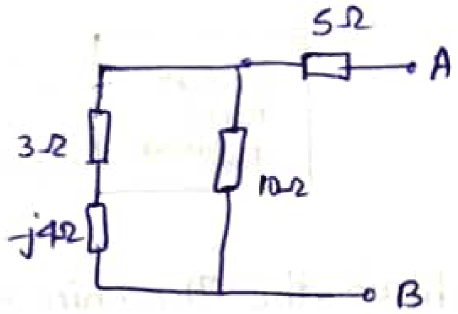


Soln: To find Thevenin voltage $V_{Th} = V_{AB}$

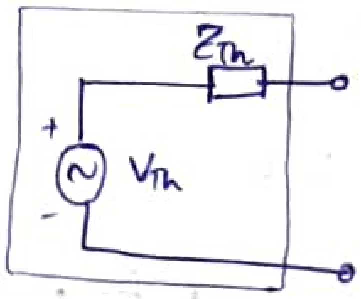
$$i = \frac{-(20\angle 0)}{13 - j4} = 1.4704 \angle -162.89 \text{ A}$$

$$V_{Th} = +10(1.4704\angle -162.89) + (20\angle 0) - (10\angle 45)$$
$$= 11.4524 \angle -95.63 \text{ V}$$

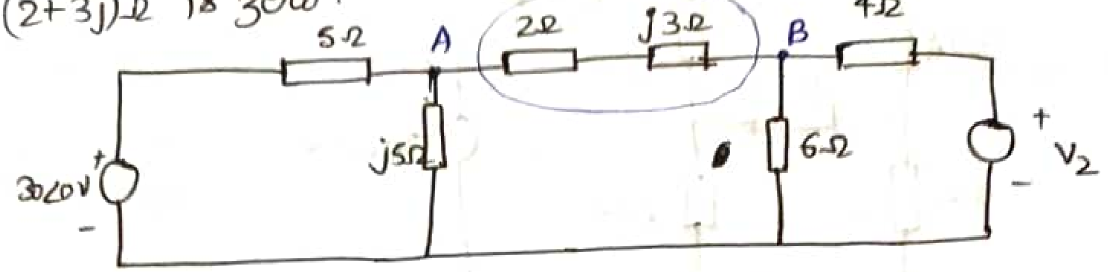
$$Z_{Th} = \frac{10 \times (3 - j4)}{10 + 3 - j4} + 5$$
$$= \frac{30 - j40}{13 - j4} + 5$$
$$= 7.9730 - j\,2.1622$$



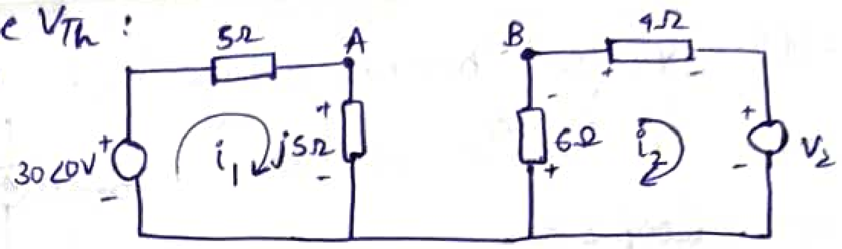
Thevenin eq.:



Q Using Thevenin theorem, find V_2 such that current through $(2+3j)\Omega$ is zero.



Soln: To find Thevenin voltage V_{Th}:

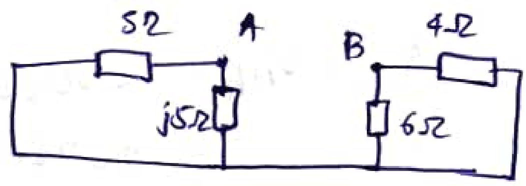


$$i_1 = \frac{30\angle 0}{5+j5} = 4.2426\angle -45\,A$$

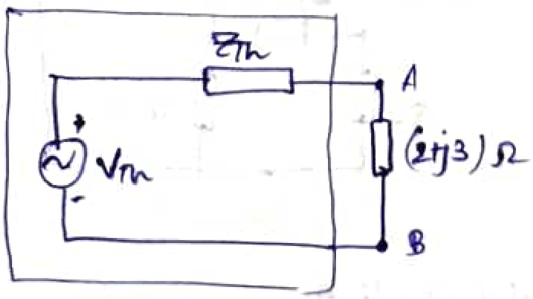
$$i_2 = -\frac{V_2}{6+4} = -\frac{V_2}{10}$$

$$V_{Th} = V_{AB} = +(6i_2) + (j5\, i_1)$$
$$= -0.6V_2 + 21.213\angle 45$$

To find Thevenin impedence Z_{Th}:



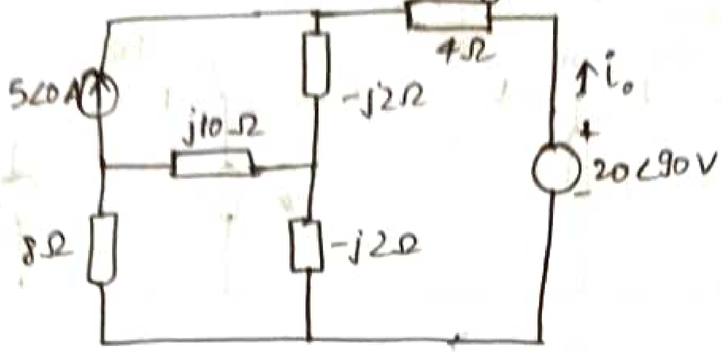
$$Z_{Th} = \frac{6\times 4}{6+4} + \frac{j5\times 5}{5+j5}$$
$$= 2.4 + \frac{j25}{5+j5}$$
$$= (4.9 + j2.5)\Omega$$



For $i_{AB}=0$, $V_{Th}=0$

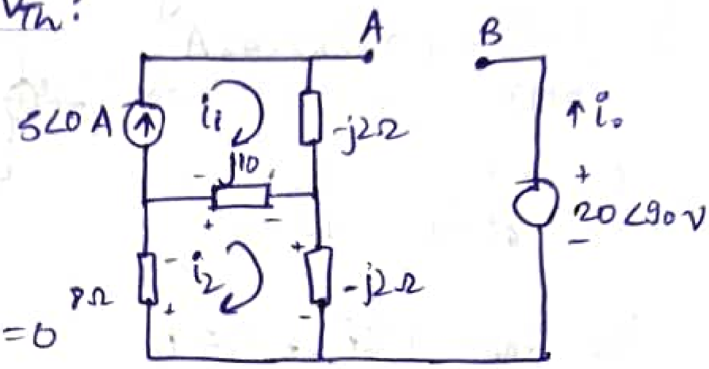
$$\Rightarrow -0.6V_2 + 21.213\angle 45^\circ = 0$$
$$\Rightarrow V_2 = 35.355\angle 45^\circ\, V.$$

Q) Determine the current i_o using Thevenin Theorem.



Soln: To find Thevenin voltage V_{Th}:

$i_1 = 5\angle 0 A$



Loop 2 ⇒

$$(-j2)i_2 + (8)i_2 + (j10)i_2 - (j10)i_1 = 0$$

$$\Rightarrow j8\, i_2 + 8 i_2 = (-j10)5\angle 0 = (5\angle 0) * (10\angle 90)$$

$$\Rightarrow i_2 = \frac{50\angle 90}{8+j8} = 4.4194\angle 45 \text{ A}.$$

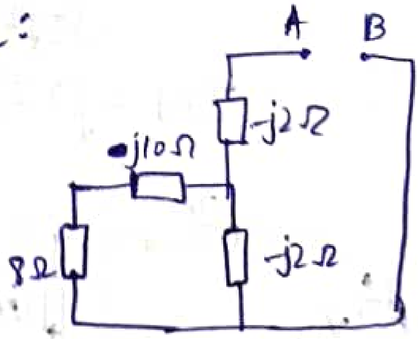
$$V_{Th} = V_{AB} = -20\angle 90 + (-j2)4.4194\angle 45 + (-j2)5\angle 0$$

$$= 36.7848\angle -80.21 \text{ V}$$

To find Thevenin impedence Z_{Th}:

$$Z_{Th} = -j2 + \frac{(8+j10)(-j2)}{(8+j8)}$$

$$= 4.2573\angle -86.63\,\Omega$$



Current through 4Ω,

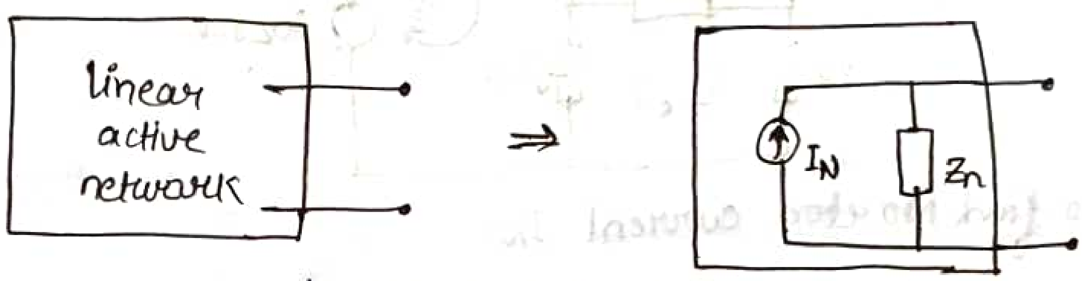
$$i_o = \frac{V_{Th}}{Z_{Th} + 4} = 6.1201\angle -35.21 \text{ A}$$

13-06-2023

NORTON THEOREM

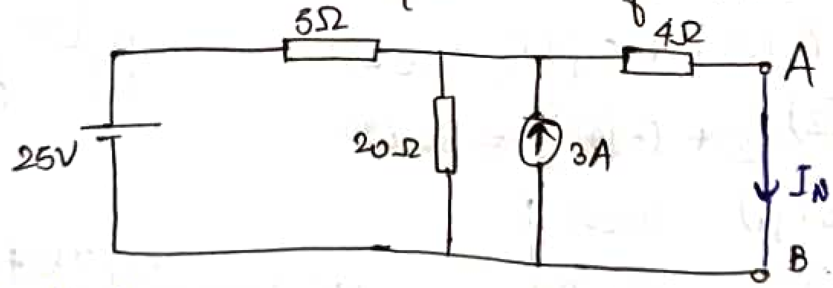
Norton theorem states that any linear active network can be replaced by single current source (I_N) in parallel with a single impedence (Z_N).

The Norton current (I_N) is the short circuit current measured at the load terminals and the Norton impedence (Z_N) is the impedence of the network at the load terminals when all the sources are replaced by their respective internal impedences.



Norton Equivalent Circuit.

Q] Obtain the Norton equivalent for the network given.



Soln: To find Norton current I_N:

By nodal method,

$$\frac{V_1 - 25}{5} + \frac{V_1}{20} + \frac{V_1}{4} = 3$$

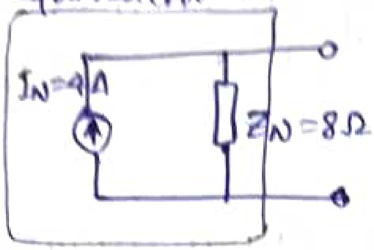
$$\Rightarrow V_1 = 16V$$

$$\therefore I_{sc} = I_N = \frac{16}{4} = 4A$$

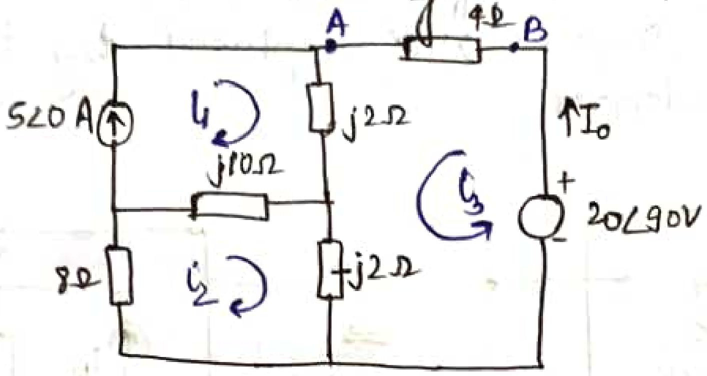
To find Norton Resistance, R_N:

$$R_N = \frac{20\times5}{25} + 4 = 8\,\Omega$$

Norton Equivalent:



Q) Determine the current I_0 using Norton theorem.



Soln: To find Norton current I_N:

$$(8+j8)i_2 - j10\,i_1 - j2\,i_3 = 0 \quad ; \; i_1 = 5\angle0 \text{ A}$$

$$\Rightarrow (8+j8)\,i_2 - j2\,i_3 = 50\angle90$$

$$\begin{bmatrix} (8+j8) & -j2 \\ -j2 & -j4 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50\angle90 \\ 30\angle90 \end{bmatrix}$$

$$i_3 = \frac{\Delta_2}{\Delta}$$

$$(-j2)i_2 + (-j2)i_1 + (-j4)i_3 = 20\angle90$$

$$\Rightarrow (-j2)\,i_2 + (-j4)\,i_3 = 30\angle90$$

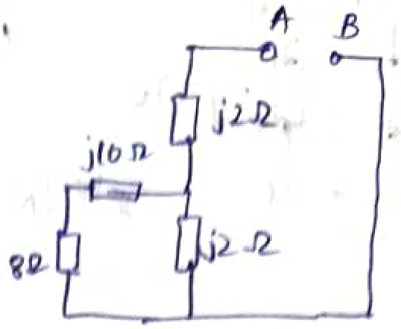
$$i_3 = \frac{\begin{vmatrix} (8+j8) & 50\angle90 \\ -j2 & 30\angle90 \end{vmatrix}}{\begin{vmatrix} (8+j8) & -j2 \\ -j2 & -j4 \end{vmatrix}} = \frac{416.1730\angle44.7824}{48.1663\angle-41.6335}$$

$$\therefore I_N = I_{sc} = i_3 = 8.6403\angle186.41\,A$$

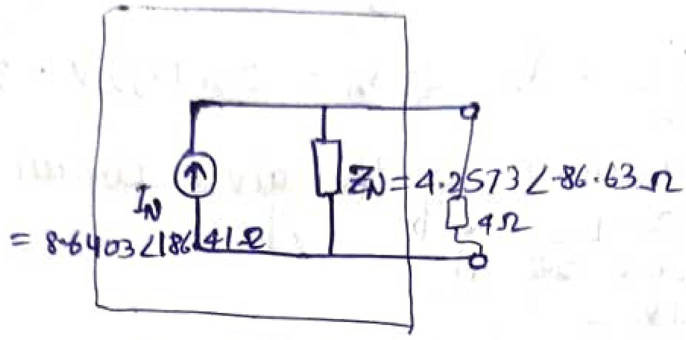
To find Norton impedence Z_N :

$$Z_N = -j2 + \frac{(8+j10)(-j2)}{(8+j8)}$$

$$= 4.2573\angle -86.63\,\Omega$$



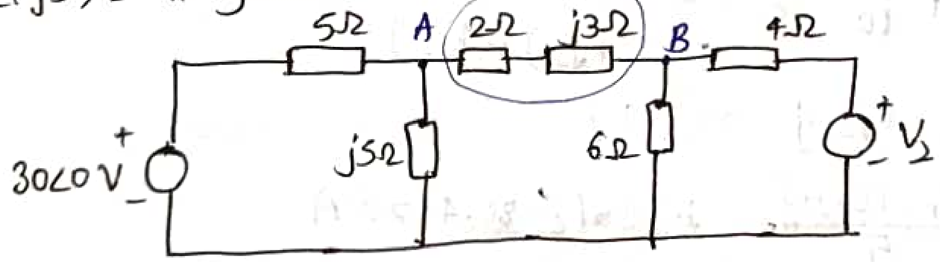
Norton Equivalent :



Current through $4\Omega = I_N \times \frac{Z_N}{(Z_N+4)}$

$$= 6.1199\angle 144.77\,A$$

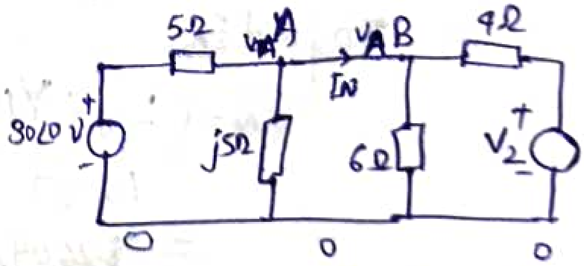
Q] Using Norton theorem, find V_2, such that current through $(2+j3)\Omega$ is zero.



Soln: To find I_N :

$$\frac{V_A - 30}{5} + \frac{V_A}{5j} + \frac{V_A}{6} + \frac{V_A - V_2}{4} = 0$$

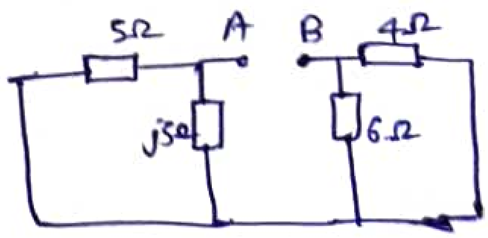
$$\Rightarrow I_N = \frac{V_A}{6} + \frac{V_A - V_2}{4}$$



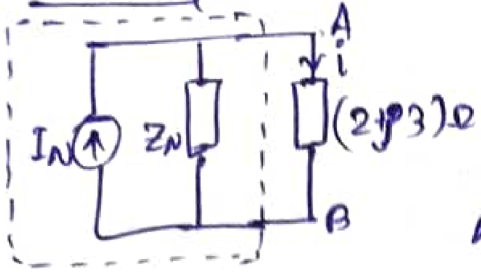
To find Z_N :

$$Z_N = \frac{5\times j5}{5+j5} + \frac{4\times 6}{4+6}$$

$$= (4.9 + j2.5)\,\Omega$$



Norton Eq^n:



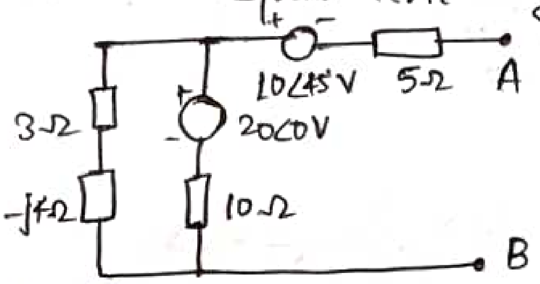
$$i = \frac{I_N \times Z_N}{Z_N + (2+j3)} = 0$$

$$\text{As } I_N = 0 \Rightarrow \frac{V_A - V_2}{4} + \frac{V_A}{6} = 0 = -\left(\frac{V_A - 30}{5} + \frac{V_A}{j5}\right)$$

$$\Rightarrow \frac{V_A(1-j)}{5} = 6 \Rightarrow V_A = 15(1+j)\ V$$

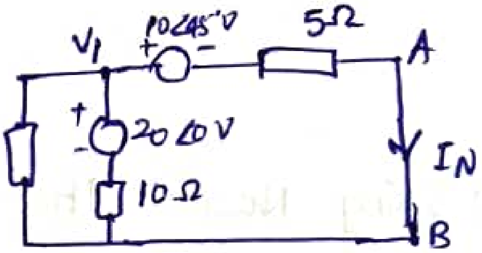
$$10V_A = 6V_2 \Rightarrow V_2 = \frac{5}{3}V_A = 25(1+j)\ V = 35.3553\angle 45^\circ\ V$$

Q) Obtain Norton equivalent of the given circuit.



Soln: To find I_N:

$$\frac{V_1}{3-j4} + \frac{V_1 - 20}{10} + \frac{V_1 - 10\angle 45^\circ}{5} = 0$$



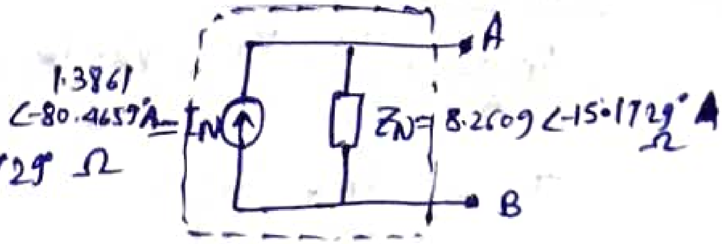
$$\Rightarrow \frac{V_1(3+j4)}{25} + \frac{V_1}{10} + \frac{V_1}{5} = 2\angle 45^\circ + 2$$
$$= (2+\sqrt{2}) + (j\sqrt{2})$$

$$\Rightarrow V_1 = (8.2190 + j0.2361)\ V$$

$$I_N = \frac{V_1 - 10\angle 45^\circ}{5} = 1.3861\angle -80.4659^\circ\ A$$

To find Z_N:

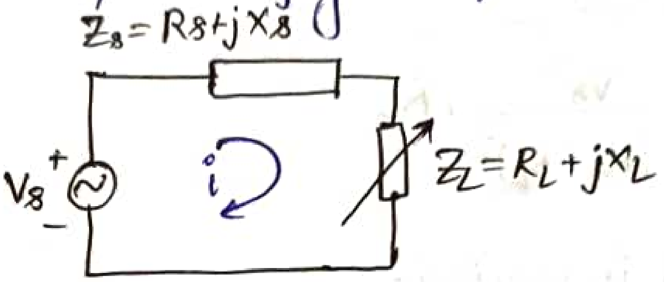
$$Z_N = \frac{(3-j4)(10)}{13-j4} + 5$$
$$= 8.2609\angle -15.1729\ \Omega$$



Norton Eq^n.

MAXIMUM POWER TRANSFER THEOREM

In a linear bilateral active network, the maximum power is transferred from the source to the load if the load impedence is complex conjugate of source impedence.



$$i = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{(R_s + jX_s) + (R_L + jX_L)}$$

$$= \frac{V_s}{(R_s + R_L) + j(X_s + X_L)}$$

$$|i| = \frac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

Since no power is consumed by the load reactance, the power dissipated in the load is due to the resistive part ~~path~~ (R_L) of the load impedence. Hence, the power will be dissipated by R_L only.

$$P_L = \frac{V_s^2}{(R_s + R_L)^2 + (X_s + X_L)^2} \cdot R_L \quad -①$$

① Let R_L be fixed and X_L be variable.

$$\frac{d}{dX_L}[P_L]=0$$

$\Rightarrow$

$\Rightarrow X_s + X_L = 0$

$\Rightarrow \boxed{X_L = -X_s}$

$$\therefore P_L = \frac{V_s^2}{(R_s+R_L)^2} \cdot R_L$$

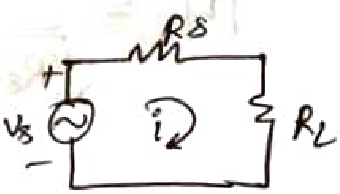
② when R_L is varied:

$$\frac{d}{dR_L}[P_L]=0$$

$\Rightarrow \boxed{R_s = R_L}$

$$Z_L = Z_s^* = R_s - jX_s$$

Efficiency, $\eta = \dfrac{\text{Output}}{\text{Input}}$



$$= \frac{i^2 R_L}{i^2(R_L+R_s)}$$

$$= \frac{R_L}{R_s+R_L} = \frac{1}{\left(\frac{R_s}{R_L}+1\right)}$$

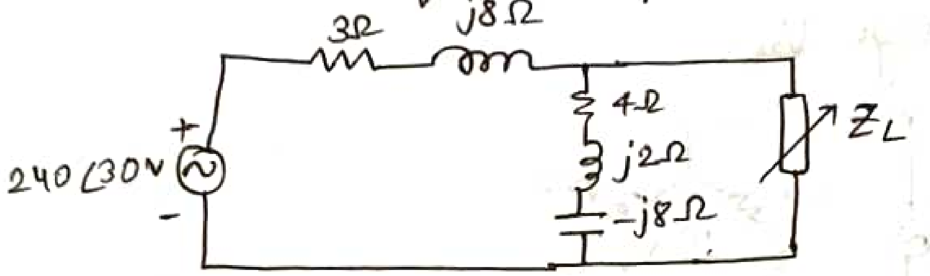
If $R_L = R_s$, $\eta = \frac{1}{2} = 50\%$

If $R_L \to \infty$, or $R_s \to 0$, $\eta = 1 = 100\%$

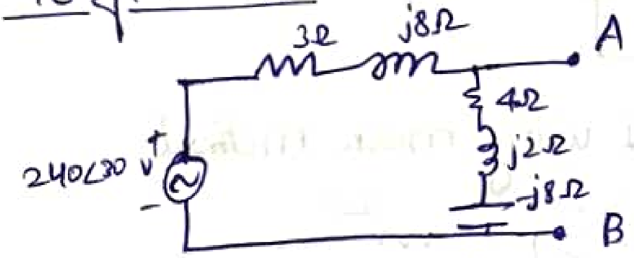
If $R_L \to 0$, $\eta = 0$.

→ Efficiency is only 50% when maximum power transfer is achieved, but approaches 100% as $R_L \to \infty$ or $R_{Th} \to 0$. Efficiency is 0% if $R_L \to 0$.

Q Determine the load impedence Z_L that absorbs max^m power and hence find this power.



Soln: To find V_{Th}:

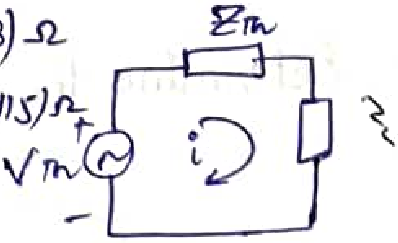


$$V_{Th} = \frac{240\angle 30\,(4-j6)}{(7+j2)}$$

$$= 237.7248\angle -42.25\ V$$

$$Z_{Th} = \frac{(3+j8)(4-j6)}{(7+j2)} = 8.463\angle -2.8113\ \Omega$$

$$= (8.4528 - j0.415)\ \Omega$$



$$Z_L = (8.4528 + j0.415)\ \Omega$$

$$i = \frac{V_{Th}}{2R_{Th}} = \cancel{\frac{28.16899\angle -38.4387}{2}\ A}$$

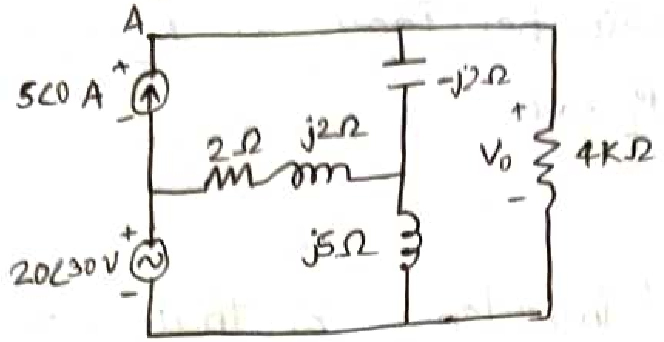
$$P_{max} = I^2 R = \frac{V_{Th}^2}{4R^2} \cdot R = \frac{V_{Th}^2}{4R}$$

$P_{max} = 1671.4307\ W$

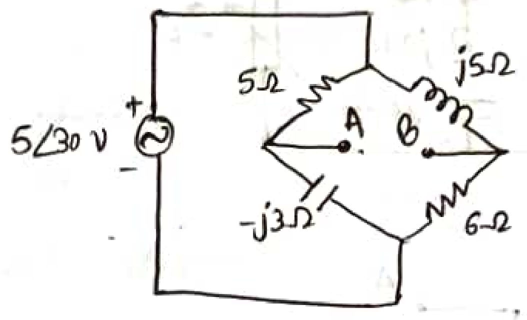
Assignment

20-06-2023

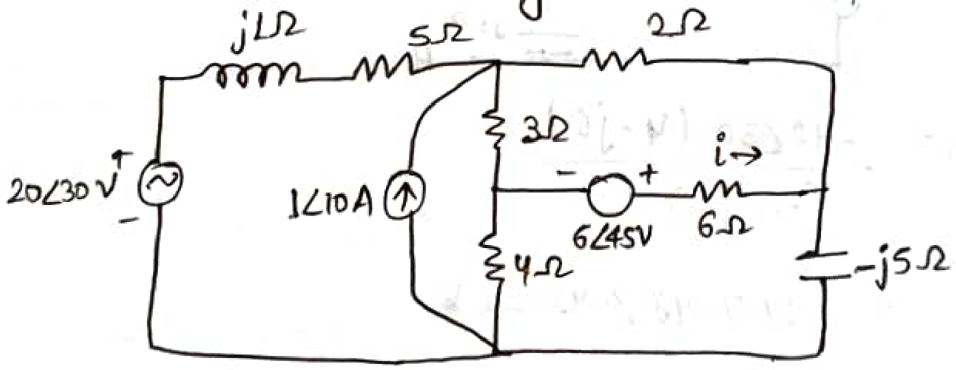
Q) ① Find V_o using mesh and nodal method.



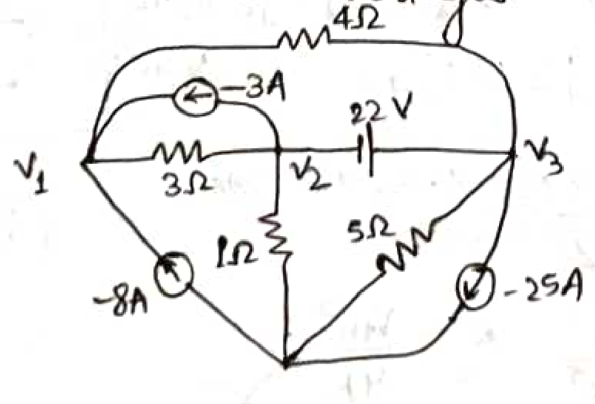
② Find the voltage V_{ab}.



③ Determine current I using mesh method.



④ Determine the node voltages.



5. Using TT and NT, find V_o.