INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY

THIRUVANANTHAPURAM 695 547

Quiz III - April 2016

B. Tech - II Semester

MA121 - Vector Calculus and Differential Equations

Date: 04/04/2016 **Time: 9.00 am - 10.00 am** **Max. Marks: 15**

Attempt all questions

1. (a) Let $f : (1,2)\cup(2,3) \longrightarrow \mathbb{R}$ be a function such that $f'(x) = 0$ for all $x \in (1,2)\cup(2,3)$. Is then f constant? Support your answer. [2.5]

   (b) Corresponding to the given function f, is it possible to get another function $\tilde{f} : (1,3) \longrightarrow \mathbb{R}$ such that $\tilde{f}$ is continuous and $\tilde{f}(x) = f(x)$ for all $x \in (1,2) \cup (2,3)$. Justify your answer. [2.5]

2. (a) Let $\vec{v}_1 = (1,2)$ and $\vec{v}_2 = (-1,2)$ be two vectors in $\mathbb{R}^2$ over $\mathbb{R}$. Show that $L_{\{\vec{v}_1,\vec{v}_2\}} = \mathbb{R}^2$, i.e., any vector $\vec{v} = (x,y) \in \mathbb{R}^2$ can be expresses in terms of linear combination of $\vec{v}_1$ and $\vec{v}_2$. [3]

   (b) Suppose $\vec{v}_1$ and $\vec{v}_2$ be any two vectors in $\mathbb{R}^2$ such that $\vec{v}_1$ and $\vec{v}_2$ make an angle π with the vector $\vec{0}$. Find $L_{\{\vec{v}_1,\vec{v}_2\}}$, with explanation. [2]

3. (a) Let $D \subseteq \mathbb{R}^2$ be an open set and $P_0 \in D$ and $f : D \longrightarrow \mathbb{R}$ a function. Define directional derivative of f at the point P_0 along a non-zero vector $\vec{v}$ in $\mathbb{R}^2$. [1.5]

   (b) Let $f : \mathbb{R}^2 \longrightarrow R$ be given by $f(x,y) = \sqrt{x^2+y^2}$ for all $(x,y) \in \mathbb{R}^2$. Suppose $\vec{v} = (v_1, v_2)$ be an unit vector (i.e., $\sqrt{{v_1}^2 + {v_2}^2} = 1$) in $\mathbb{R}^2$. Check whether $D_{\vec{v}}(f)|_{P_0}$ exists for any point $P_0 = (x_0, y_0) \in \mathbb{R}^2$.
   Hint: Check for $P_0 = (0,0)$ and for $P_0 \neq (0,0)$ [3.5]

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