Indian Institute of Space Science and Technology Trivandrum

I SEMESTER , **2023**
ExamType: Quiz 2

DEPARTMENT OF AVIONICS
Digital Image Processing
(Time allowed: ONE hours)

NOTE: Read all questions first. **There are questions worth 30 marks. Attempt all questions** If something is missing in a problem description, clearly mention your assumptions with your solution. If require, use sketches to illustrate your findings.

1. Write important differences between image enhancement and image restoration. (2 marks)

2. In this problem we want to derive the mathematical model of the blurring effect caused by motion during image acquisition. This is similar to the model we derive for linear motion. Consider an ideal continuous and infinite-size image. This image undergoes uniform linear motion in the vertical direction for a time $T1$ with a speed a . Then the image undergoes uniform linear motion in the horizontal direction for a time $T2$ with a speed b . Assume direction change, shutter opening and closing are instantaneous. Derive the mathematical expression for the blurring function $H(w1, w2)$. Please note (4 marks)

$$\int_0^T e^{-jwat} dt = \frac{2}{wa} sin(\frac{waT}{2}) e^{(-j\frac{waT}{2})}$$

3. Write Salient differences between Wiener filter and Constrained Matrix Inversion Filter. Mention the mathematical expression for both of these restoration procedure. (4 marks)

4. A degraded image is represented by $G(u, v)$ in the frequency domain and an estimation of its degradation function is

$$H(u, v) = -\sigma\sqrt{2\pi}(u^2 + v^2)e^{-j2\pi^2\sigma^2(u^2+v^2)}$$

Provide the expression in the frequency domain of the restored image using the Wiener filter. Assume that the power spectrum of the undegraded image is ten times the power spectrum of the noise throughout all the image. (4 marks)

5. Fourier transform:Explain qualitatively what effect the following transformations will have on the Discrete Fourier Transform of an image: (12 marks)

(a) random shuffling of the intensity values of the image pixels.

(b) convolution of the image with a Gaussian kernel of very tiny (nearly zero) standard deviation

(c) addition of zero mean Gaussian noise with some standard deviation σ to the image

(d) addition of a sinusoidal pattern $z = Asin(ax + by)$ to the image where a and b are known constants, and x, y are spatial coordinates.

(e) point-wise multiplication of the image with a complex exponential pattern $exp(j2\pi(ax + by))$ where a and b are known constants, and x, y are spatial coordinates.

(f) zero-padding of the image on all four sides (for this part, the DFT is computed on the larger zero-padded image. Your answer should be something over and above the obvious statement that the DFT will be a larger array)

6. Briefly explain (in up to 3-4 sentences each) any two uses of the image Laplacian in image processing. The Laplacian is given by $\Delta f(x, y) = \partial^2 f(x, y)/\partial x^2 + \partial^2 f(x, y)/\partial y^2$. Derive the expression for Laplacian of one dimensional Gaussian function. Hint: We have used Laplacian operator in image processing course for something very recently. (2 marks)

7. Explain Radon Transform. (Fourier slice theorem).Now, Using the Radon transform to obtain the projection of a circular region i.e. $g(\rho, \theta)$. (2 marks)

$$f(x, y) = A; if \;\; x^2 + y^2 \leq r^2 \;\; and \;\; 0 \; otherwise$$

where A is a constant and r is the radius of the object.We assume that the circle is centered on the origin of the xy-plane.

Expected value: $E[aX \pm bY \pm c] = aE[X] \pm bE[Y] \pm c$, where X and Y are random variables, and a, b, and c are constants

Variance: $\mathrm{Var}[aX \pm bY \pm c] = a^2\mathrm{Var}[X] + b^2\mathrm{Var}[Y]$, where X and Y are random variables, and a, b, and c are constants

Motion Blur Model – Cartesian System: $\int_0^T f(x - x_0(t), y - y_0(t))dt$, Polar System: $\int_0^T g(\rho - \rho_0(t), \theta - \theta_0(t))dt$

$$\int_a^b e^{\alpha x}dx = \frac{1}{\alpha}\int_{a/\alpha}^{b/\alpha} e^t dt = \frac{1}{\alpha}(e^{b/\alpha} - e^{a/\alpha})$$

1D Continuous Fourier Transform (1D CFT) in polar coordinates: $G(\omega, \theta) = \int_{-\infty}^{\infty} g(\rho, \theta)e^{-j2\pi\omega\rho}d\rho$

Fourier Slice Theorem: $f(x, y) \leftrightarrow g(\rho, \theta) \quad \Rightarrow \quad F(\omega\cos\theta, \omega\sin\theta) = G(\omega, \theta)$

Translation in space: $\text{1D CFT}\{g(\rho - \rho_0, \theta)\} = G(\omega, \theta)e^{-j2\pi\rho_0\omega}$

Translation in space: $DFT\{f(x + x_0, y + y_0)\} = F(u, v)e^{j2\pi(\frac{x_0 u}{M} + \frac{y_0 v}{N})}$

Translation in frequency: $DFT^{-1}\{F(u + u_0, v + v_0)\} = f(x, y)e^{-j2\pi(\frac{xu_0}{M} + \frac{yv_0}{N})}$

Rotation in space: $DFT\left\{f(x, y)e^{-j2\pi(\frac{xu_0}{M} + \frac{yv_0}{N})}\right\} = F(u + u_0, v + v_0)$

Rotation in frequency: $DFT^{-1}\left\{F(u, v)e^{j2\pi(\frac{x_0 u}{M} + \frac{y_0 v}{N})}\right\} = f(x + x_0, y + y_0)$

2D Scaling and Wavelet Functions: $\varphi(x, y) = \varphi(x)\varphi(y)$, $\psi^H(x, y) = \psi(x)\varphi(y)$, $\psi^V(x, y) = \varphi(x)\psi(y)$, $\psi^D(x, y) = \psi(x)\psi(y)$

$\varphi_{jmn}(x, y) = 2^{\frac{j}{2}}\varphi(2^j x - m, 2^j y - n)$, $\psi_{jmn}^{H|V|D}(x, y) = 2^{\frac{j}{2}}\psi^{H|V|D}(2^j x - m, 2^j y - n)$

1D Haar Scaling and Wavelet Functions:

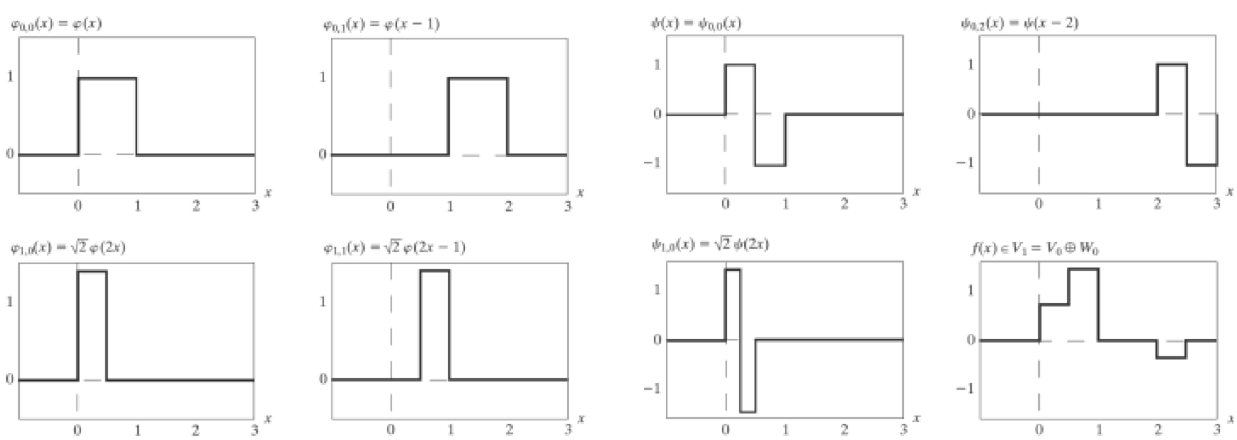

Figure 1: important formulae