
Problem 1. *Consider a camera that moves in front of a static scene for which the* ***Brightness Change Constraint Equation (BCCE)***

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0$$

is known to hold. The three derivatives and the two unknown optical flow components u and v in this equation are all functions of the coordinates x, y of the pixels in the image. The x-axis is horizontal in the image and points to the right. The y-axis is vertical in the image and points up. The components u and v of the optical flow are along the x and y axes, respectively.

We say that the camera moves sideways *if the vector that describes the motion of the camera in the world is parallel to the rows (scan lines) of the camera's image sensor. For a camera that moves sideways, all image points move parallel to the x-axis, so that*

$$v(x, y) = 0 \quad \textit{for all } x, y$$

and the BCCE consequently reduces to the following:

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial t} = 0.$$

(a) In one sentence, why is solving for optical flow fundamentally easier if the camera is known to be moving sideways in a static world?

(b) Can the aperture problem arise at all for a camera moving sideways in a static world? Justify your answer.

Problem 2. *Consider two sets of corresponding points:*

$$\{\mathbf{p}_{1i} = (x_{1i}, y_{1i})\}_{i=1}^{n} \quad \textit{and} \quad \{\mathbf{p}_{2i} = (x_{2i}, y_{2i})\}_{i=1}^{n}.$$

Assume that each pair of corresponding points is related as follows:

$$\mathbf{p}_{2i} = \alpha R \mathbf{p}_{1i} - \hat{t} + \boldsymbol{\eta}_i,$$

where:

- *R is an unknown rotation matrix,*
- *$\hat{t}$ is an unknown translation vector,*
- *α is an unknown scalar factor (uniform scale),*
- *$\boldsymbol{\eta}_i$ is an unknown noise vector.*

Explain how you would estimate the rotation matrix R, the scalar factor α and the translation vector $\hat{t}$. Derive all necessary equations. Be sure to:

- *Describe any assumptions you make,*
- *Include the centering step (if any),*
- *Explain how the scale factor can be isolated,*
- *Clearly define all variables and matrices used in your derivation.*

Problem 3. *Consider a picture of a static (possibly non-planar) scene acquired by a camera fixed on a tripod. Now the camera is rotated, but it remains fixed on the tripod without any translation, and another picture of the same scene is acquired.*

Let $\mathbf{p}_1$ and $\mathbf{p}_2$ be the pixel coordinates of the images of some physical point in the scene in the two pictures, respectively. Note that $\mathbf{p}_1$ and $\mathbf{p}_2$ are in different image coordinate systems.

Let R be the rotation matrix that represents the rotation of the camera axes from the first position to the second, and let K_1 and K_2 be the intrinsic parameter matrices of the cameras in the two viewpoints. Note that the intrinsic parameters may differ due to changes in focal length or resolution.

Derive a relation between $\mathbf{p}_1$ and $\mathbf{p}_2$ in terms of R, K_1, and K_2. Clearly state all assumptions and explain the geometric intuition behind your derivation.

Problem 4. *The local spatial correlation captures local image structure and is defined as:*

$$c(x, y, \Delta x, \Delta y) = \frac{1}{|W(x, y)|} \sum_{(x_k, y_k) \in W(x,y)} \left[I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y)\right]^2,$$

where:

- *$I(x, y)$ is an image intensity function,*
- *$W(x, y)$ is a local window or neighborhood of pixels centered at (x, y),*
- *$(\Delta x, \Delta y)$ is a small spatial shift in the image.*

1. *Use the first-order Taylor approximation to approximate $I(x_k + \Delta x, y_k + \Delta y)$ around (x_k, y_k). Recall:*

$$I(x_k + \Delta x, y_k + \Delta y) \approx I(x_k, y_k) + \frac{\partial I}{\partial x}(x_k, y_k)\Delta x + \frac{\partial I}{\partial y}(x_k, y_k)\Delta y.$$

2. *Substitute this approximation into the spatial correlation definition to derive:*

$$c(x, y, \Delta x, \Delta y) \approx \begin{bmatrix}\Delta x & \Delta y\end{bmatrix} A(x, y) \begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix},$$

where $A(x, y)$ is a 2×2 matrix that you need to derive.

3. *Show that the matrix $A(x, y)$ is given by:*

$$A(x,y) = \frac{1}{|W(x,y)|} \sum_{(x_k,y_k)\in W(x,y)} \begin{bmatrix} I_x(x_k,y_k)^2 & I_x(x_k,y_k)I_y(x_k,y_k) \\ I_x(x_k,y_k)I_y(x_k,y_k) & I_y(x_k,y_k)^2 \end{bmatrix},$$

where I_x and I_y are the partial derivatives of the image intensity.

4. *Explain briefly why $A(x, y)$ is symmetric and positive semi-definite.*