MA211- Linear Algebra
Assignment-1

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Q.① Using Gauss-Jordan elimination method, find solutions of:

a (i)
$$\begin{aligned} 2x - y + z &= 3 \\ x - 3y + z &= 3 \\ -5x - 2z &= -5 \end{aligned}$$

Soln:

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & -3 & 1 \\ -5 & 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix}$$

Augmented matrix

$$= \begin{pmatrix} 2 & -1 & 1 & 3 \\ 1 & -3 & 1 & 3 \\ -5 & 0 & -2 & -5 \end{pmatrix} \xrightarrow{R(i) \to R(i) - 2R(ii)} \begin{pmatrix} 0 & 5 & -1 & -3 \\ 1 & -3 & 1 & 3 \\ 0 & -15 & 3 & 10 \end{pmatrix}$$

$$\xrightarrow[\substack{R(i) \to R(ii) \\ R(iii) \to 3R(ii) + R(iii)}]{R(ii) \to R(i)} \begin{pmatrix} 1 & -3 & 1 & 3 \\ 0 & 5 & -1 & 3 \\ 0 & 0 & 0 & 19 \end{pmatrix} \xrightarrow[R(i) \to R(i) + R(ii)]{R(iii)\frac{1}{19}} \begin{pmatrix} 1 & -8 & 2 & 0 \\ 0 & 5 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

~~R(ii) → R(ii) − 3R(iii)~~

$$\xrightarrow[R(i) \to R(i) + 8R(ii)]{R(ii) \to \frac{R(ii)}{5}} \begin{pmatrix} 1 & 0 & \frac{2}{5} & \frac{24}{9} \\ 0 & 1 & \frac{-1}{5} & \frac{3}{9} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow[R(ii) \to R(ii) - \frac{3}{9}R(iii)]{R(i) \to R(i) - \frac{24}{9}R(iii)} \begin{pmatrix} 1 & 0 & 2/5 & 0 \\ 0 & 1 & -1/5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From here,

$$\left.\begin{aligned} x - \frac{2}{5}z &= 0 \\ y - \frac{z}{5} &= 0 \\ 0 &= 1 \end{aligned}\right\} \Rightarrow \text{No solution exists.}$$

(ii) $3x_1 + 6x_3 = 6$

Q. 3 (c) $x_1 + x_2 + 5x_3 + x_4 = 9$

$2x_2 + 4x_3 + 2x_4 = 10$

Soln: Augmented matrix

$$= \begin{pmatrix} 3 & 0 & 6 & 0 & 6 \\ 1 & 1 & 5 & 1 & 9 \\ 0 & 2 & 4 & 2 & 10 \end{pmatrix} \xrightarrow{R(i) \to R(i) - 2R(ii)} \begin{pmatrix} 1 & -2 & -4 & -2 & -12 \\ 1 & 1 & 5 & 1 & 9 \\ 0 & 2 & 4 & 2 & 10 \end{pmatrix}$$

$$\xrightarrow[R(i) \to R(i) + R(iii)]{R(ii) \to R(ii) - R(i)} \begin{pmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 5 & 1 & 11 \\ 0 & 2 & 4 & 2 & 10 \end{pmatrix}$$

$$\xrightarrow{R(iii) \to 2R(ii) - R(iii)} \begin{pmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 5 & 1 & 11 \\ 0 & 0 & 6 & 0 & 12 \end{pmatrix} \xrightarrow{R(iii)/2} \begin{pmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 5 & 1 & 11 \\ 0 & 0 & 1 & 0 & 2 \end{pmatrix}$$

$$\xrightarrow{R(ii) \to -5R(iii) + R(ii)} \begin{pmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 \end{pmatrix}$$

From here,

$x_1 = -2$

$x_2 + x_4 = 1.$

$x_3 = 2$

Put $x_4 = \alpha$.

$$\left.\begin{array}{l} \therefore\ x_1 = -2,\ x_2 = 1-\alpha, \\ x_3 = 2,\ x_4 = \alpha. \end{array}\right\}$$ Infinitely many solutions exist.

Q(2) Test consistency and solve the system

Q. 4 (a)

$x + y + z = 6$

$x + 2y + 3z = 14$

$x + 4y + 7z = 30$

Soln: Augmented matrix

$$= \begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{pmatrix} \xrightarrow[R(iii) \to R(iii) - R(i)]{R(ii) \to R(ii) - R(i)} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{pmatrix}$$

$$\xrightarrow[R(i) \to R(i) - R(ii)]{R(iii) \to R(iii) - 3R(ii)} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

From here

$$x - z = -2$$
$$y + 2z = 8$$

Put $z = \alpha$.

$$\therefore \left.\begin{aligned} x &= \alpha - 2 \\ y &= 8 - 2\alpha \\ z &= \alpha \end{aligned}\right\}$$ The system is consistent and has infinitely many solutions.

Q. ③ Find the values of a for which the system

$$x + 2y + 3z = ax$$
$$3x + y + 2z = ay$$
$$2x + 3y + z = az$$

has non-trivial solution.

Soln: Rewritting the equations:

$$(1-\alpha)x + 2y + 3z = 0$$
$$3x + (1-a)y + 2z = 0$$
$$2x + 3y \quad (1-a)z = 0$$

$$\therefore A = \begin{bmatrix} 1-a & 2 & 3 \\ 3 & 1-a & 2 \\ 2 & 3 & 1-a \end{bmatrix}$$

For non-trivial solution,

$\det(A) = 0$

$$\Rightarrow (1-a)\left[(1-a)^2 - 6\right] - 2\left[3(1-a) - 4\right] + 3\left[9 - 2(1-a)\right] = 0$$

$$\Rightarrow (1-a)(1+a^2-2a-6) - 2(3-3a-4) + 3(9-2+2a) = 0$$

$$\Rightarrow (1-a)(a^2-2a-5) - 2(-3a-1) + 3(2a+7) = 0$$

$$\Rightarrow a^2-2a-5-a^3+2a^2+5a+6a+2+6a+21 = 0$$

$$\Rightarrow a^3-3a^2-15a-18=0$$

$$\Rightarrow (a-6)(a^2+3a+3)$$

$$\Rightarrow \boxed{a=6}$$

④ Q.⑧ Find the values of a and b for which the system

$$3x+2y+z=6$$
$$3x+4y+3z=a$$
$$6x+10y+bz=a$$

has (i) unique sol^n, (ii) no sol^n, (iii) infinitely many solutions.

Sol^n: Augmented matrix $[A:B]$

$$= \begin{pmatrix} 3 & 2 & 1 & : & 6 \\ 3 & 4 & 3 & : & a \\ 6 & 10 & b & : & a \end{pmatrix} \underset{R(iii)\to R(iii)-2R(i)}{\overset{R(ii)\to R(ii)-R(i)}{\sim}} \begin{pmatrix} 3 & 2 & 1 & 6 \\ 0 & 2 & 2 & a-6 \\ 0 & 6 & b-2 & a-12 \end{pmatrix}$$

$$\overset{R(iii)\to R(iii)-3R(ii)}{\sim} \begin{pmatrix} 3 & 2 & 1 & 6 \\ 0 & 2 & 2 & a-6 \\ 0 & 0 & b-8 & a-30 \end{pmatrix} \overset{R(i)\to R(i)-R(ii)}{\sim} \begin{pmatrix} 3 & 0 & -1 & a-12 \\ 0 & 2 & 2 & a-6 \\ 0 & 0 & (b-8) & a-3 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 3 & 4 & 3 \\ 6 & 10 & b \end{pmatrix} \sim \begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & b-8 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 1 \\ 0 & 0 & b-8 \end{pmatrix}$$

Here, $n=3$.

(i) For unique sol^n, rank(A) = rank $[A:B] = 3 \Rightarrow \boxed{b \neq 8}$

(ii) For no sol^n, rank$(A) <$ rank $[A:B] \Rightarrow \boxed{b=8,\ a\neq 3}$

(iii) For infinitely many sol^{n}s, rank$[A]$ = rank $[A:B] < 3$

$$\Rightarrow \boxed{b=8,\ a=3}$$