INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

Quiz III - April 2015

B. Tech - II Semester

MA121 - Vector Calculus and Differential Equations

Date: 09/04/2015 **Time: 9.00 am - 10.00 am** **Max. Marks: 15**

Attempt all questions

1. (a). Evaluate the following limit with appropriate justification: [2]

$$\lim_{n\to\infty}\int_0^1 \frac{n+e^x}{n+x^2}dx.$$

(b). Check whether the function $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{\cos^n x}{n^3}$ is differentiable on $(-\infty, +\infty)$. Justify your answer. [3]

2. (a) Define directional derivative of a real valued function f defined on a domain $D \subset \mathbb{R}^2$.

[1]

(b) Let $f : \mathbb{R}^2 \longrightarrow R$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^4+y^2} & if\ (x,y) \neq (0,0) \\ 0 & otherwise \end{cases}$$

Suppose $\vec{v} = (v_1, v_2)$ be an vector in $\mathbb{R}^2$ such that $v_1 \neq 0 \neq v_2$. Show that $D_x f|_{(0,0)}$, $D_y f|_{(0,0)}$ and $D_{\vec{v}} f|_{(0,0)}$ exist. Is it possible to express $D_{\vec{v}} f|_{(0,0)}$ in terms of the partial derivatives? Is f is differentiable at $(0,0)$? Justify your answer. [2.5 + 0.5 + 1]

3. For each of following vector fields $\vec{F}$, find a scalar field f, if possible, such that $\nabla f = \vec{F}$ indicating the domain of $\vec{F}$. Justify, if you claim that no such f exists.

(a) $\vec{F}(x,y,z) = (ye^z,\ xe^z,\ xy)$ [2]

(b) $\vec{F}(x,y,z) = (2x + y\sin z,\ y + x\sin z - \sin y,\ xy\cos z)$ [3]