PROBLEMS

10.1 An EM wave propagating in a certain medium is described by

$$\mathbf{E} = 25 \sin (2\pi \times 10^6 t - 6x)\, \mathbf{a}_z \text{ V/m}$$

(a) Determine the direction of wave propagation.
(b) Compute the period T, the wavelength λ, and the velocity u.
(c) Sketch the wave at $t = 0, T/8, T/4, T/2$.

10.2 (a) Derive eqs. (10.23) and (10.24) from eqs. (10.18) and (10.20).
(b) Using eq. (10.29) in conjunction with Maxwell's equations, show that

$$\eta = \frac{j\omega\mu}{\gamma}$$

(c) From part (b), derive eqs. (10.32) and (10.33).

10.3 At 50 MHz, a lossy dielectric material is characterized by $\varepsilon = 3.6\varepsilon_o$, $\mu = 2.1\mu_o$, and $\sigma = 0.08$ S/m. If $\mathbf{E}_s = 6e^{-\gamma x}\, \mathbf{a}_z$ V/m, compute: (a) γ, (b) λ, (c) u, (d) η, (e) $\mathbf{H}_s$.

10.4 A lossy material has $\mu = 5\mu_o$, $\varepsilon = 2\varepsilon_o$. If at 5 MHz, the phase constant is 10 rad/m, calculate

(a) The loss tangent
(b) The conductivity of the material
(c) The complex permittivity
(d) The attenuation constant
(e) The intrinsic impedance

***10.5** A nonmagnetic medium has an intrinsic impedance $240 \angle 30^\circ$ Ω. Find its

(a) Loss tangent
(b) Dielectric constant
(c) Complex permittivity
(d) Attenuation constant at 1 MHz

10.6 The amplitude of a wave traveling through a lossy nonmagnetic medium reduces by 18% every meter. If the wave operates at 10 MHz and the electric field leads the magnetic field by 24°, calculate: (a) the propagation constant, (b) the wavelength, (c) the skin depth, (d) the conductivity of the medium.

10.7 Sea water plays a vital role in the study of submarine communications. Assuming that for sea water, $\sigma = 4$ S/m, $\varepsilon_r = 80$, $\mu_r = 1$, and $f = 100$ MHz, calculate: (a) the phase velocity, (b) the wavelength, (c) the skin depth, (d) the intrinsic impedance.

10.8 In a certain medium with $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$,

$$\mathbf{H} = 12e^{-0.1y} \sin (\pi \times 10^8 t - \beta y)\, \mathbf{a}_x \text{ A/m}$$

find: (a) the wave period T, (b) the wavelength λ, (c) the electric field $\mathbf{E}$, (d) the phase difference between $\mathbf{E}$ and $\mathbf{H}$.

10.9 In a medium,

$$\mathbf{E} = 16e^{-0.05x} \sin (2 \times 10^8 t - 2x)\, \mathbf{a}_z \text{ V/m}$$

find: (a) the propagation constant, (b) the wavelength, (c) the speed of the wave, (d) the skin depth.

10.10 A uniform wave in air has

$$\mathbf{E} = 10 \cos (2\pi \times 10^6 t - \beta z)\, \mathbf{a}_y$$

(a) Calculate β and λ.

(b) Sketch the wave at $z = 0, \lambda/4$.

(c) Find $\mathbf{H}$.

10.11 The magnetic field component of an EM wave propagating through a nonmagnetic medium ($\mu = \mu_o$) is

$$\mathbf{H} = 25 \sin (2 \times 10^8 t + 6x)\, \mathbf{a}_y \text{ mA/m}$$

Determine:

(a) The direction of wave propagation.

(b) The permittivity of the medium.

(c) The electric field intensity.

10.12 If $\mathbf{H} = 10 \sin (\omega t - 4z)\mathbf{a}_x$ mA/m in a material for which $\sigma = 0$, $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, calculate ω, λ, and $\mathbf{J}_d$.

10.13 A manufacturer produces a ferrite material with $\mu = 750\mu_o$, $\varepsilon = 5\varepsilon_o$, and $\sigma = 10^{-6}$ S/m at 10 MHz.

(a) Would you classify the material as lossless, lossy, or conducting?

(b) Calculate β and λ.

(c) Determine the phase difference between two points separated by 2 m.

(d) Find the intrinsic impedance.

***10.14** By assuming the time-dependent fields $\mathbf{E} = \mathbf{E}_o e^{j(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ and $\mathbf{H} = \mathbf{H}_o e^{j(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ where $\mathbf{k} = k_x\mathbf{a}_x + k_y\mathbf{a}_y + k_z\mathbf{a}_z$ is the wave number vector and $\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$ is the radius vector, show that $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ can be expressed as $\mathbf{k} \times \mathbf{E} = \mu\omega\mathbf{H}$ and deduce $\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H$.

10.15 Assume the same fields as in Problem 10.14 and show that Maxwell's equations in a source-free region can be written as

$$\mathbf{k} \cdot \mathbf{E} = 0$$

$$\mathbf{k} \cdot \mathbf{H} = 0$$

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = -\omega\varepsilon\mathbf{E}$$

From these equations deduce

$$\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H \qquad \text{and} \qquad \mathbf{a}_k \times \mathbf{a}_H = -\mathbf{a}_E$$

10.16 The magnetic field component of a plane wave in a lossless dielectric is

$$\mathbf{H} = 30 \sin (2\pi \times 10^8 t - 5x)\, \mathbf{a}_z \text{ mA/m}$$

(a) If $\mu_r = 1$, find ε_r.
(b) Calculate the wavelength and wave velocity.
(c) Determine the wave impedance.
(d) Determine the polarization of the wave.
(e) Find the corresponding electric field component.
(f) Find the displacement current density.

10.17 In a nonmagnetic medium,

$$\mathbf{E} = 50 \cos (10^9 t - 8x)\, \mathbf{a}_y + 40 \sin (10^9 t - 8x)\, \mathbf{a}_z \text{ V/m}$$

find the dielectric constant ε_r and the corresponding $\mathbf{H}$.

10.18 In a certain medium

$$\mathbf{E} = 10 \cos (2\pi \times 10^7 t - \beta x)(\mathbf{a}_y + \mathbf{a}_z) \text{ V/m}$$

If $\mu = 50\mu_o$, $\varepsilon = 2\varepsilon_o$, and $\sigma = 0$, find β and $\mathbf{H}$.

10.19 Which of the following media may be treated as conducting at 8 MHz?

(a) Wet marshy soil ($\varepsilon = 15\varepsilon_o$, $\mu = \mu_o$, $\sigma = 10^{-2}$ S/m)
(b) Intrinsic germanium ($\varepsilon = 16\varepsilon_o$, $\mu = \mu_o$, $\sigma = 0.025$ S/m)
(c) Sea water ($\varepsilon = 81\varepsilon_o$, $\mu = \mu_o$, $\sigma = 25$ S/m)

10.20 Calculate the skin depth and the velocity of propagation for a uniform plane wave at frequency 6 MHz traveling in polyvinylchloride ($\mu_r = 1$, $\varepsilon_r = 4$, $\tan \theta_\eta = 7 \times 10^{-2}$).

10.21 A uniform plane wave in a lossy medium has a phase constant of 1.6 rad/m at 10^7 Hz and its magnitude is reduced by 60% for every 2 m traveled. Find the skin depth and speed of the wave.

10.22 (a) Determine the dc resistance of a round copper wire ($\sigma = 5.8 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_r = 1$) of radius 1.2 mm and length 600 m.
(b) Find the ac resistance at 100 MHz.
(c) Calculate the approximate frequency where dc and ac resistances are equal.

10.23 A 40-m-long aluminum ($\sigma = 3.5 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_r = 1$) pipe with inner and outer radii 9 mm and 12 mm carries a total current of $6 \sin 10^6 \pi t$ A. Find the skin depth and the effective resistance of the pipe.

10.24 Show that in a good conductor, the skin depth δ is always much shorter than the wavelength.

10.25 Brass waveguides are often silver plated to reduce losses. If at least the thickness of silver ($\mu = \mu_o$, $\varepsilon = \varepsilon_o$, $\sigma = 6.1 \times 10^7$ S/m) must be 5δ, find the minimum thickness required for a waveguide operating at 12 GHz.

10.26 A uniform plane wave in a lossy nonmagnetic media has

$$\mathbf{E}_s = (5\mathbf{a}_x + 12\mathbf{a}_y)\mathbf{e}^{-\gamma z}, \gamma = 0.2 + j3.4/\text{m}$$

(a) Compute the magnitude of the wave at $z = 4$ m.

(b) Find the loss in dB suffered by the wave in the interval $0 < z < 3$ m.

(c) Calculate the Poynting vector at $z = 4$, $t = T/8$. Take $\omega = 10^8$ rad/s.

10.27 In a nonmagnetic material,

$$\mathbf{H} = 30 \cos (2\pi \times 10^8 t - 6x)\, \mathbf{a}_y \text{ mA/m}$$

find: (a) the intrinsic impedance, (b) the Poynting vector, (c) the time-average power crossing the surface $x = 1, 0 < y < 2, 0 < z < 3$ m.

***10.28** Show that eqs. (10.67) and (10.68) are equivalent.

10.29 In a transmission line filled with a lossless dielectric ($\varepsilon = 4.5\varepsilon_o$, $\mu = \mu_o$),

$$\mathbf{E} = \frac{40}{\rho} \sin (\omega t - 2z)\, \mathbf{a}_\rho \text{ V/m}$$

find: (a) ω and $\mathbf{H}$, (b) the Poynting vector, (c) the total time-average power crossing the surface $z = 1$ m, 2 mm $< \rho <$ 3 mm, $0 < \phi < 2\pi$.

10.30 (a) For a normal incidence upon the dielectric–dielectric interface for which $\mu_1 = \mu_2 = \mu_o$, we define R and T as the reflection and transmission coefficients for average powers, i.e., $P_{r,\text{ave}} = RP_{i,\text{ave}}$ and $P_{t,\text{ave}} = TP_{i,\text{ave}}$. Prove that

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad \text{and} \quad T = \frac{4n_1n_2}{(n_1 + n_2)^2}$$

where n_1 and n_2 are the reflective indices of the media.

(b) Determine the ratio n_1/n_2 so that the reflected and the transmitted waves have the same average power.

10.31 The plane wave $\mathbf{E} = 30 \cos(\omega t - z)\mathbf{a}_x$ V/m in air normally hits a lossless medium ($\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$) at $z = 0$. (a) Find Γ, τ, and s. (b) Calculate the reflected electric and magnetic fields.

10.32 A uniform plane wave in air with

$$\mathbf{H} = 4 \sin (\omega t - 5x)\, \mathbf{a}_y \text{ A/m}$$

is normally incident on a plastic region with the parameters $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, and $\sigma = 0$. (a) Obtain the total electric field in air. (b) Calculate the time-average power density in the plastic region. (c) Find the standing wave ratio.

10.33 A plane wave in free space with $\mathbf{E} = 3.6 \cos(\omega t - 3x)\,\mathbf{a}_y$ V/m is incident normally on an interface at $x = 0$. If a lossless medium with $\sigma = 0$, $\varepsilon_r = 12.5$ exits for $x \geq 0$ and the reflected wave has $\mathbf{H}_r = -1.2 \cos(\omega t + 3x)\,\mathbf{a}_z$ mA/m, find μ_2.

10.34 Region 1 is a lossless medium for which $y \geq 0$, $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, whereas region 2 is free space, $y \leq 0$. If a plane wave $\mathbf{E} = 5 \cos(10^8 t + \beta y)\,\mathbf{a}_z$ V/m exists in region 1, find: (a) the total electric field component of the wave in region 2, (b) the time-average Poynting vector in region 1, (c) the time-average Poynting vector in region 2.

10.35 A plane wave in free space ($z \leq 0$) is incident normally on a large block of material with $\varepsilon_r = 12$, $\mu_r = 3$, $\sigma = 0$ which occupies $z \geq 0$. If the incident electric field is

$$\mathbf{E} = 30 \cos(\omega t - z)\,\mathbf{a}_y \text{ V/m}$$

find: (a) ω, (b) the standing wave ratio, (c) the reflected magnetic field, (d) the average power density of the transmitted wave.

10.36 A 30-MHz uniform plane wave with

$$\mathbf{H} = 10 \sin(\omega t + \beta x)\,\mathbf{a}_z \text{ mA/m}$$

exists in region $x \geq 0$ having $\sigma = 0$, $\varepsilon = 9\varepsilon_o$, $\mu = 4\mu_o$. At $x = 0$, the wave encounters free space. Determine (a) the polarization of the wave, (b) the phase constant β, (c) the displacement current density in region $x \geq 0$, (d) the reflected and transmitted magnetic fields, and (e) the average power density in each region.

10.37 A uniform plane wave in air is normally incident on an infinite lossless dielectric material having $\varepsilon = 3\varepsilon_o$ and $\mu = \mu_o$. If the incident wave is $\mathbf{E}_i = 10 \cos(\omega t - z)\,\mathbf{a}_y$ V/m, find:

(a) λ and ω of the wave in air and the transmitted wave in the dielectric medium
(b) The incident $\mathbf{H}_i$ field
(c) Γ and τ
(d) The total electric field and the time-average power in both regions

***10.38** A signal in air ($z \geq 0$) with the electric field component

$$\mathbf{E} = 10 \sin(\omega t + 3z)\,\mathbf{a}_x \text{ V/m}$$

hits normally the ocean surface at $z = 0$ as in Figure 10.19. Assuming that the ocean surface is smooth and that $\varepsilon = 80\varepsilon_o$, $\mu = \mu_o$, $\sigma = 4$ mhos/m in ocean, determine

(a) ω
(b) The wavelength of the signal in air
(c) The loss tangent and intrinsic impedance of the ocean
(d) The reflected and transmitted $\mathbf{E}$ field

10.39 Sketch the standing wave in eq. (10.87) at $t = 0$, $T/8$, $T/4$, $3T/8$, $T/2$, and so on, where $T = 2\pi/\omega$.

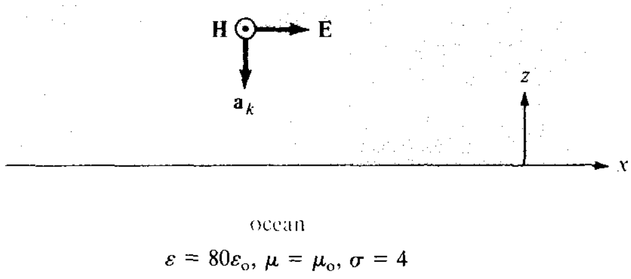

Figure 10.19 For Problem 10.38.

10.40 A uniform plane wave is incident at an angle $\theta_i = 45°$ on a pair of dielectric slabs joined together as shown in Figure 10.20. Determine the angles of transmission θ_{t1} and θ_{t2} in the slabs.

10.41 Show that the field

$$\mathbf{E}_s = 20 \sin(k_x x) \cos(k_y y)\, \mathbf{a}_z$$

where $k_x^2 + k_y^2 = \omega^2 \mu_o \varepsilon_o$, can be represented as the superposition of four propagating plane waves. Find the corresponding $\mathbf{H}_s$.

10.42 Show that for nonmagnetic dielectric media, the reflection and transmission coefficients for oblique incidence become

$$\Gamma_{\parallel} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}, \qquad \tau_{\parallel} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)\cos(\theta_t - \theta_i)}$$

$$\Gamma_{\perp} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}, \qquad \tau_{\perp} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)}$$

***10.43** A parallel-polarized wave in air with

$$\mathbf{E} = (8\mathbf{a}_y - 6\mathbf{a}_z)\sin(\omega t - 4y - 3z) \text{ V/m}$$

impinges a dielectric half-space as shown in Figure 10.21. Find: (a) the incidence angle θ_i, (b) the time average in air ($\mu = \mu_o$, $\varepsilon = \varepsilon_o$), (c) the reflected and transmitted **E** fields.

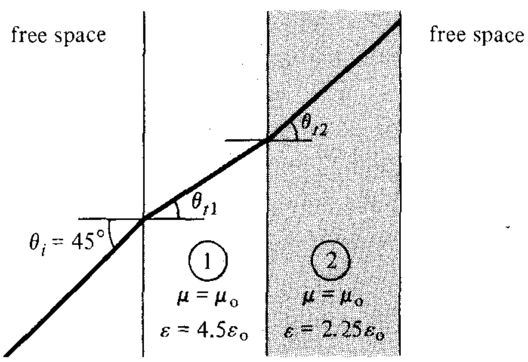

Figure 10.20 For Problem 10.40.

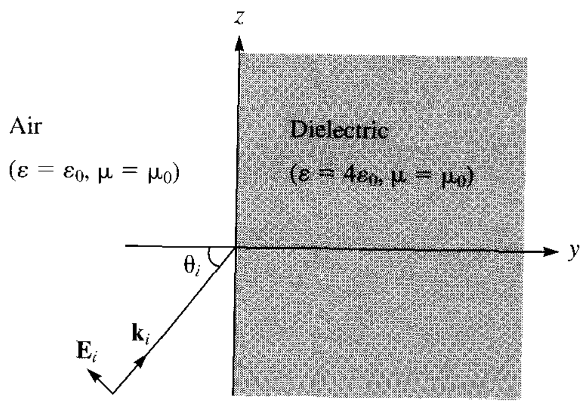

Figure 10.21 For Problem 10.43.

10.44 In a dielectric medium ($\varepsilon = 9\varepsilon_o$, $\mu = \mu_o$), a plane wave with

$$\mathbf{H} = 0.2 \cos (10^9 t - kx - k\sqrt{8}z)\, \mathbf{a}_y \text{ A/m}$$

is incident on an air boundary at $z = 0$, find

(a) θ_r and θ_t

(b) k

(c) The wavelength in the dielectric and air

(d) The incident **E**

(e) The transmitted and reflected **E**

(f) The Brewster angle

***10.45** A plane wave in air with

$$\mathbf{E} = (8\mathbf{a}_x + 6\mathbf{a}_y + 5\mathbf{a}_z) \sin (\omega t + 3x - 4y) \text{ V/m}$$

is incident on a copper slab in $y \geq 0$. Find ω and the reflected wave. Assume copper is a perfect conductor. (*Hint:* Write down the field components in both media and match the boundary conditions.)

10.46 A polarized wave is incident from air to polystyrene with $\mu = \mu_o$, $\varepsilon = 2.6\varepsilon$ at Brewster angle. Determine the transmission angle.