Indian Institute of Space Science and Technology

Thiruvananthapuram

B.Tech - 4^{th} Semester, 2023

MA221 -Integral Transforms, PDE and Calculus of Variations

Quiz 1

Date : 13^{th}Feb, 2023 **Time: 9.00 am to 10.00 am**

(Maximum marks 15)

1. Let $a > 0$. Determine the Fourier transform of the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = e^{-ax^2}.$$

[3 marks]

2. Compute the convolution $(f * g)(t)$ for the following pair of functions

$$f(x) = \begin{cases} 2, & \text{for } 0 \le x \le 1/2, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x, & \text{for } -1/4 \le x \le 1/4, \\ 0, & \text{otherwise.} \end{cases}$$

[3 marks]

3. Assume that $f, g \in L_1(\mathbb{R})$ and suppose $F_T(f) = \hat{f}(w)$ and $F_T(g) = \hat{g}(w)$. Prove that

$$F_T(f * g) = \hat{f}(w)\hat{g}(w),$$

where F_T denotes the Fourier transform. [3 marks]

4. Find the inverse Laplace transform of the function $\hat{f}(s)$ given by

$$\hat{f}(s) = \frac{2s+1}{s^2+2s+4}.$$

[3 marks]

5. The charge $Q(x)$ on a capacitor in an inductive circuit is governed by the differential equation

$$\frac{dQ}{dx} - Q = e^{(x-1)}H(x-1), \quad x > 0,$$

and it is known that $Q(x)$ is zero when $x = 0$. Find Q using the Laplace transform. [4 marks]

*** End ***