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INSTRUMENTATION AND MEASUREMENT

INSTRUMENTATION

8-01-2024

Measurement concepts
Devices and circuits for good quality measurement
Analog and digital instruments

Measurement: Process of estimating the value of unknown variable.
measurand

Measurand: temperature, pressure, humidity, displacement speed, acceleration, force, torque, light intensity.

Industrial scenarios: Automotive, aerospace, biomedical, process industry, etc.

	Angle/angular displacement (accuracy)	
Automotive	1°	(By steering wheel)
Aerospace	$1/3600^\circ$	(Solar panel in the dirn of sun)
Biomedical	$1/60^\circ$	(Robotic arm)

Measurand energy form:
- → Mechanical (speed, acceleration, force, torque, etc.)
- → Optical (light intensity)
- → Thermal (temperature)
- → Chemical
- → Radiation
- → Electrical

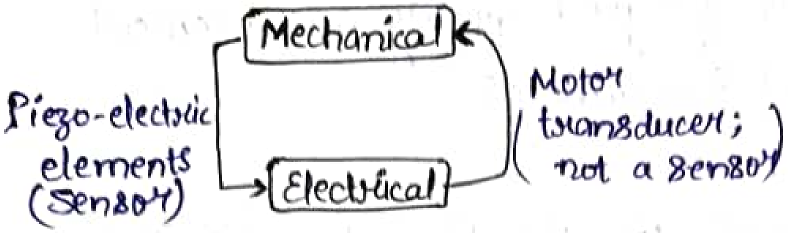
Transducer: Converts one energy form into other form.

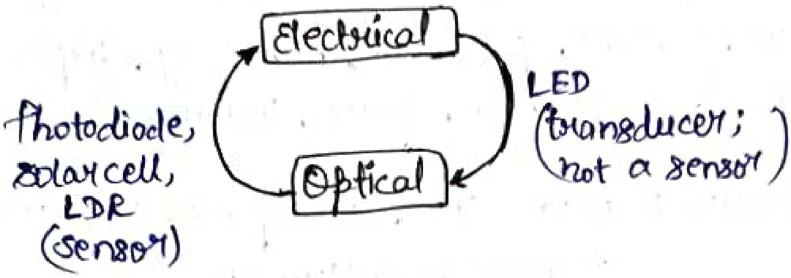
Input energy → $\boxed{\text{Transducer}}$ → Output energy
(with electrical o/p)
Sensors

→ We generally prefer transducer with electrical o/p energy because electrical energy is easy to transmit, amplify, control, display.

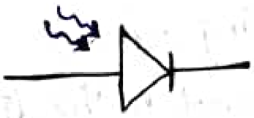
Sensor: Transducer with electrical ~~o/p~~ output energy.

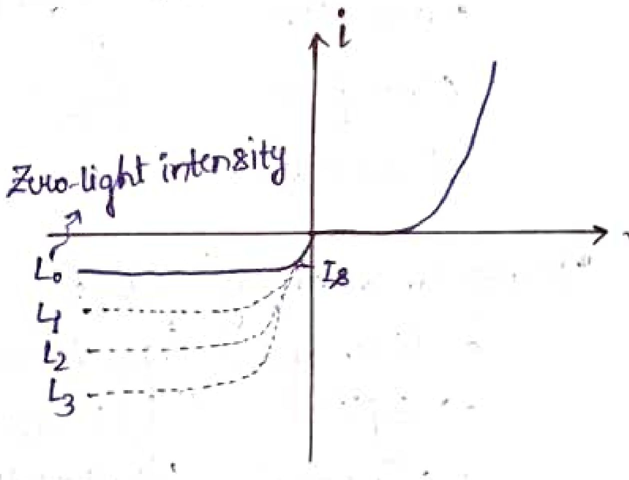
Eg.





Photodiode:





I_s : Reverse saturation current (due to minority carriers)

L : light intensity

→ More light intensity
⇒ More photons
⇒ More minority carriers
⇒ more I_s

$L_3 > L_2 > L_1 > L_0$

$\boxed{I_s \propto L}$

$\Rightarrow I_s = KL$

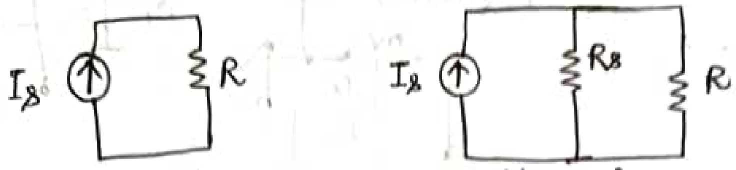
"Current" Output sensor (As current is in o/p)

→ As current is in o/p, we can call photodiode as current-output sensor.

→ We need to get o/p in voltage b/c we measure most of the times in voltage.

→ we can convert current into voltage using resistor R but it will not work as photo-diode has its internal resistance.

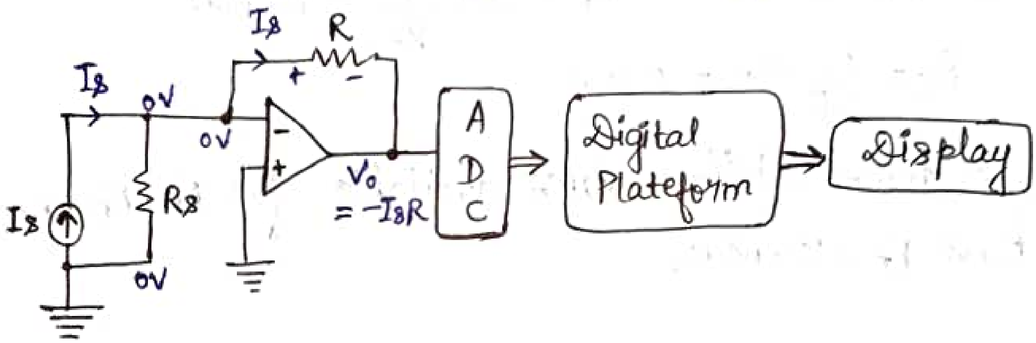
Modelling photodiode :



(Ideal) (Actual)

→ Photodiode is not an ideal current source.

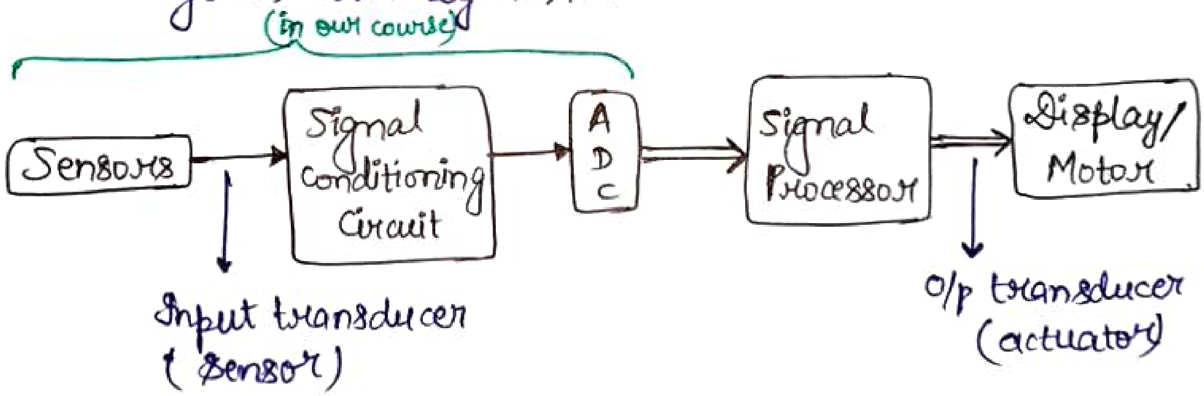
→ We will use opamp instead of resistor because ~~intenal~~ internal resistance would not affect o/p voltage in opamp.



$I_s \propto L$

$I_s = KL$

To get L, divide by R, K.

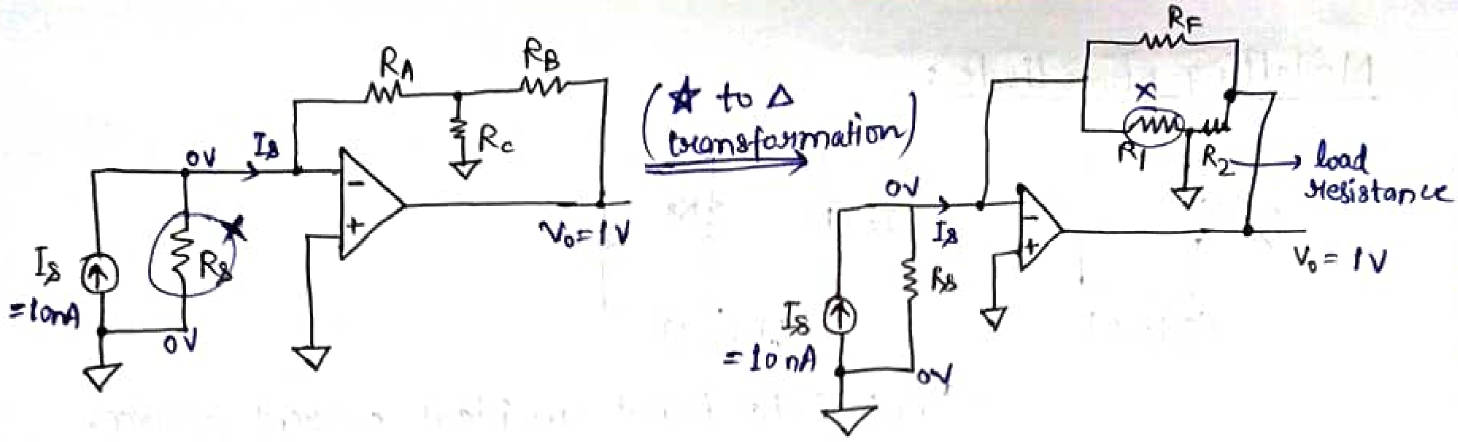


Using T (star) N/w for low current conversion into voltage:

$I_s = 10nA$, $V_o = 1V$,

then we need $R = 100 M\Omega$ by previous formula as $V_o = -I_sR$, but we cannot have such ~~bi~~ large resistance.

The solution of above problem can ~~we~~ be get using T-N/W.



$V_o = -I_s R_F$, $R_F = R_A + R_B + \frac{R_A R_B}{R_C} = 100\,M\Omega$.

If $R_A = R_B = 1\,M\Omega \Rightarrow R_F = 2 + \frac{1}{R_C} = 100\,M\Omega$

$R_C = 1/98\ M\Omega \approx 10\,K\Omega$

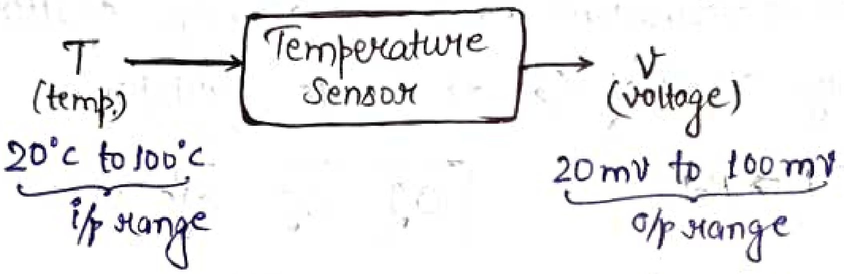
So, we have all the values available for R_A, R_B, R_C and can have $R_F = 100\,M\Omega$.

09-01-2024

Sensors: Sensors are classified on the basis of o/p:

R : Resistive sensor, e.g., RTD, Thermistor
C : Capacitive sensor, e.g., Level detector, Tactile
L : Inductive sensor, e.g., Proximity sensors
I : Current o/ps, e.g., Photodiodes, Ionospheric probes
V : Voltage o/ps, e.g., Thermocouple
Q : Charge o/ps, e.g., Piezoelectric elements.

Characteristics of Instrumentation System



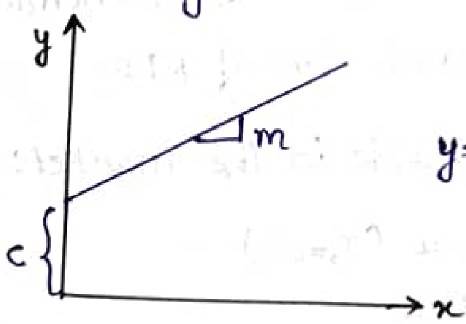
Input span: Difference b/w i/p and min. i/p values.
Here, i/p span = $100°C - 20°C = 80°C$.

Output span: Difference b/w o/p and min. o/p values.
Here, o/p span = $100\,mV - 20\,mV = 80\,mV$.

Linear Sensor:

$y = F(x)$

We want our system to be linear (need not pass through origin)



$y = mx + c$

↳ Eg.: RTD.

Resistance Temperature Detector (RTD): Gives o/p in resistance form.

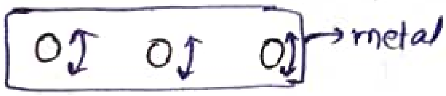
$$\underbrace{R_T}_{\text{Resistance @ T}} = \underbrace{R_0}_{\text{Nominal resistance @ } T_0}\left[1+\underbrace{\alpha}_{\text{Temperature coefficient}}(T-\underbrace{T_0}_{\text{Nominal temperature}})\right]$$

→ R_T has linear relation with T, so RTD is "linear sensor".

→ RTDs are generally made of metal.

→ $T\uparrow \Rightarrow$ Atoms undergo significant motion from equilibrium position (more vibration) $\Rightarrow$ more charge scattering $\Rightarrow$ less avg. speed of e^-s $\Rightarrow$ more resistance.

So, $\boxed{T\uparrow \Rightarrow R\uparrow}$



$$R_T = R_0\left[1+\alpha(T-T_0)+\beta(T-T_0)^2+\gamma(T-T_0)^3\right]$$

If we choose correct metals like Pt, then above relation will become almost linear and sensor will also become linear.

(β, γ, can be removed ~~both~~ as per the choice of metals)

For Pt $\Rightarrow$ $\alpha \gg \beta \gg \gamma$

$\alpha = 0.0039/°C$

$\beta = 5.7\times 10^{-7}/°C^2$

→ we can use copper (high α) but due to chemically inert nature of Pt, it is used for fabrication of RTD.

→ There are 3 RTDs available in the market:

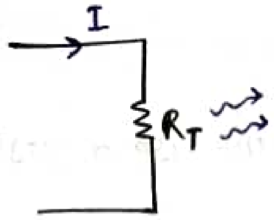
Pt 100 $\Rightarrow R_0 = 100\Omega$ $(T_0 = 0°C)$

Pt 1000 $\Rightarrow R_0 = 1k\Omega$

Pt 10000 $\Rightarrow R_0 = 10k\Omega$.

Optimal working range : [-100°C, 600°C].

$$R_{600} = 100\left[1+0.0039\times 600\right] = 334\Omega.$$



Power dissipation $= I^2R_T \Rightarrow$ Rise in temp. $\Rightarrow$ self heating $\downarrow$ (should be as minm as possible)

Self-heating: Local rise in temperature due to power dissipation

Power dissipation factor (P_D): Power required for RTP to raise the temperature by 1°C.

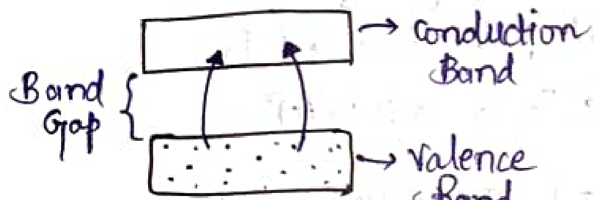
Rise in temperature : $\boxed{\Delta T = \frac{I^2R_T}{P_D}}$

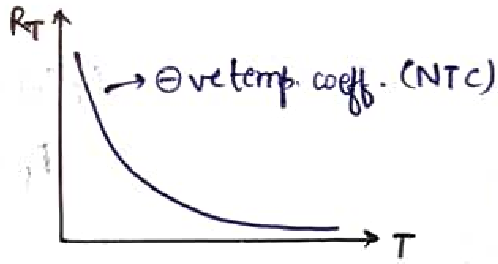
Non-Linear Sensor

↳ Eg. Thermistor

Thermistor : Thermally sensitive resistor.

↳ made of semiconductor materials.

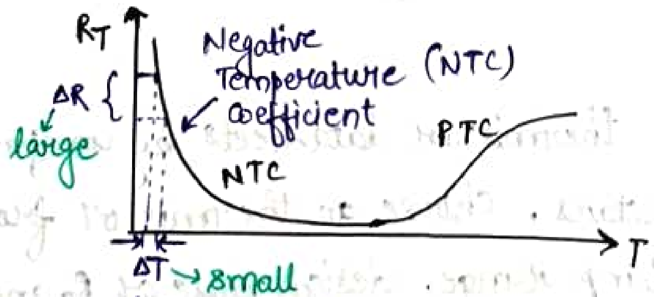




$T\uparrow \Rightarrow$ more free $e^{\ominus}$s $\Rightarrow R\downarrow$

↳ Also called NTC thermistor.

↳ On large no. of e⁻s in conduction band, semiconductor behaves like metal then it becomes PTC thermistor.

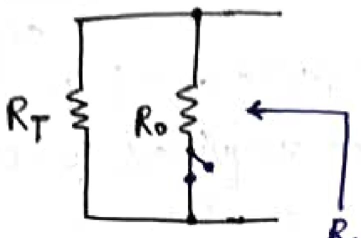


↳ We should use thermistor in high attenuation area because we can detect large R change on minute temperature variation.

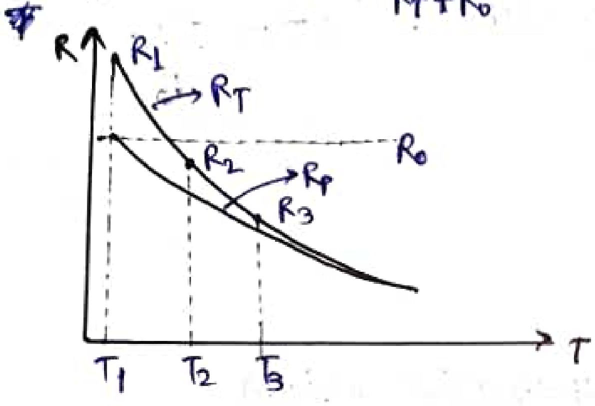
↳ Not preferred in wide span : $R_T = Ae^{\frac{B}{T+\theta}}$

Improving Linearity of thermistor:

↳ To increase linearity / reduce non-linearity, we use a resistance R_0 in $\parallel$ to thermistor.



$$R_p = \frac{R_T R_0}{R_T + R_0}$$



T_1, T_2, T_3 : temp. range of thermistor

T_2 : middle point b/w T_1 & T_3.

So, T_1, T_2, T_3 are in A.P.

$$R_p(T_2) - R_p(T_1) = R_p(T_3) - R_p(T_2)$$

$$R_p = \frac{R_T R_0}{R_T + R_0}$$

R_T @ $T_1 = R_1$

R_T @ $T_2 = R_2$

R_T @ $T_3 = R_3$

Derive R_0 : $$\boxed{R_0 = \frac{R_2(R_1 + R_3) - 2R_1R_3}{R_1 + R_3 - 2R_2}}$$

Assignment: Loop up at thermistor datasheets by company Vishay Semiconductors. Choose an thermistor from the website. Choose temp. range, design value of R_0 and linearize the thermistor.

Derivation of R_0:

$$R_p(T_2) - R_p(T_1) = R_p(T_3) - R_p(T_2)$$

$$\Rightarrow 2R_p(T_2) = R_p(T_1) + R_p(T_3)$$

As $R_p = \dfrac{R_T R_0}{R_T + R_0}$,

$$\Rightarrow 2\frac{R_2 R_0}{R_2 + R_0} = \frac{R_1 R_0}{R_1 + R_0} + \frac{R_3 R_0}{R_3 + R_0}$$

$$\Rightarrow 2R_2(R_1 + R_0)(R_3 + R_0) = [R_1(R_3 + R_0) + R_3(R_1 + R_0)](R_2 + R_0)$$

$$\Rightarrow 2R_1 \cancel{R_2} R_3 + 2R_0^2 R_2 + 2R_0 R_2 (R_1 + R_3) = 2R_1 \cancel{R_2} R_3 + R_0^2 (R_1 + R_3) + 2R_0 R_1 R_3 + R_0 R_2 (R_1 + R_3)$$

$$\Rightarrow 2R_0 R_2 + 2R_2 \cancel{(R_1 + R_3)} = R_0(R_1 + R_3) + 2R_1 R_3 + R_2 \cancel{(R_1 + R_3)}$$

$$\Rightarrow R_2(R_1 + R_3) - 2R_1 R_3 = R_0(R_1 + R_3) - 2R_0 R_2$$

$$\Rightarrow R_0 = \frac{R_2(R_1 + R_3) - 2R_1 R_3}{R_1 + R_3 - 2R_2}$$

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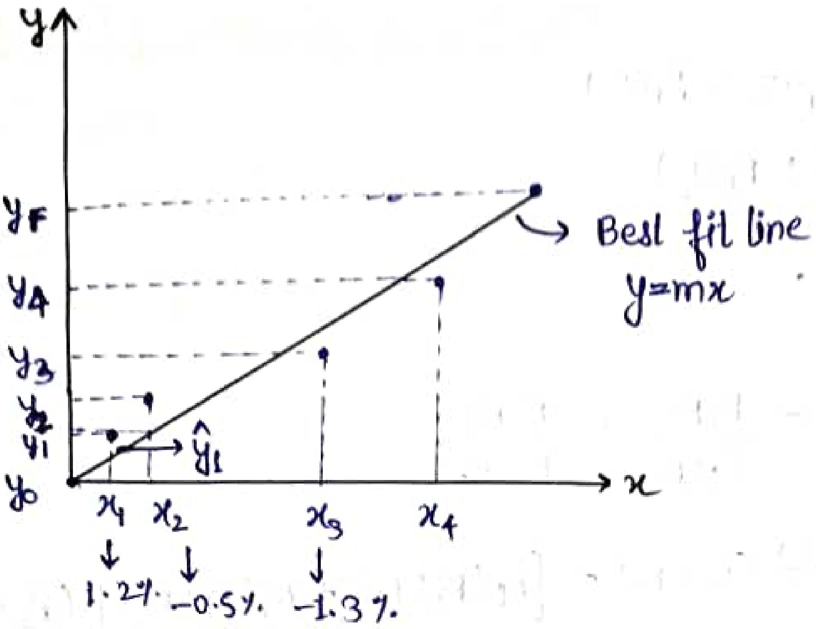


deviation / non-linearity $= y_1 - \hat{y}_1$

y_F : full scale reading

$y_F - y_0$: span (o/p span)

% non-linearity / % deviation $= \dfrac{y_1 - \hat{y}_1}{y_F - y_0} \times 100\,\%$

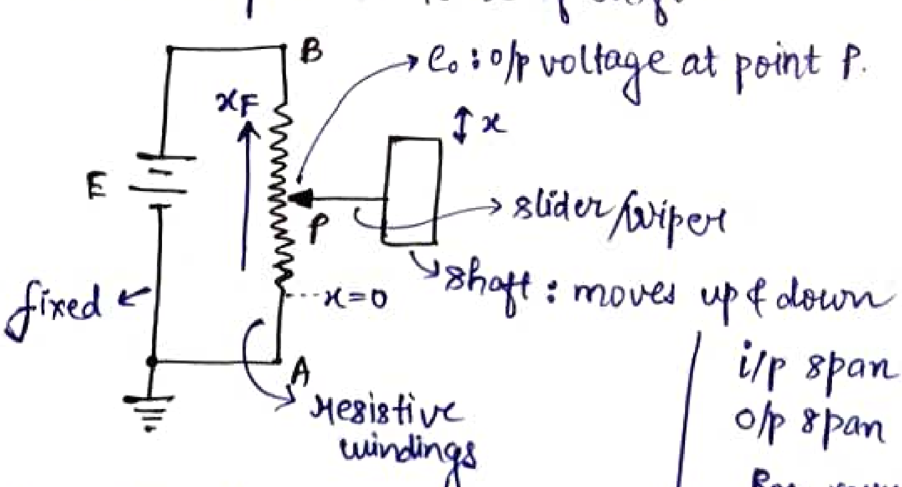
Sensor non-linearity: Worst-case non-linearity, here $= \pm 1.3\%$
(max^m deviation)

Sensitivity (S): Rate of change of output w.r.t. input
$= m =$ slope of characteristic

$S =$ constant, for linear sensor

$S =$ variable, for non-linear sensor.

To measure displacement 'x' of shaft:



i/p span $= x_F$

o/p span $= E$

R_{PA} vary with x.

* resistive displacement sensor

x	e_o
0	0
x_F	E
x	$E\left(\frac{x}{x_F}\right)$

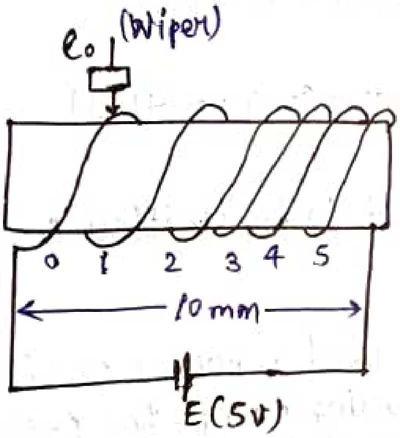
$$e_o = \left(\frac{E}{x_F}\right) x \equiv y = mx \rightsquigarrow \text{linear sensor}$$

$$S = \frac{E}{x_F}$$

Let $E = 5V$, $x_F = 10\,mm$.

Then, $S = \frac{5V}{10\,mm} = 500\,\frac{mV}{mm} = 500\,\frac{\mu V}{\mu m} = 500\,\frac{nV}{nm}$

Writing the units does not mean we can measure upto these level of displacements.



$N = 6$, $x_F = 10\,mm$, $E = 5V$

$e_o = 0V$ at $x = 0$

$e_o \neq 0V$ at $x = 2mm$.

When wiper moves by 2mm, then o/p will change.

Input Resolution (q) $= 2mm = \frac{10mm}{6-1} = \frac{x_F}{N-1}$

↳ Min^m ~~value~~ change in the input that can be detected in output.

$$\boxed{q = \frac{x_F}{N-1}}$$

$N \gg 1 \Rightarrow \boxed{q \approx \frac{x_F}{N}}$ For a potentiometric displacement sensor.

Output Resolution $= sq = 5\,mV$

↳ Min^m^ value of the o/p that the sensor can detect.

Let $N = 1000$, $x_F = 10\,mm$, $E = 5V$,

then $q = 10\,\mu m$,

$S = 500\,\mu V/\mu m$

o/p resolution $= sq = 5\,mV$.

The o/p ~~vo~~ change will come in resolution of 5 mV, like

0, 5mV, 10mV, ...

$\boxed{\text{Sensor}} \longrightarrow y$

Measured value (MV) $\neq$ True Value (TV)

Error = MV − TV (unit: same as output)

$\%\ \text{error} = \dfrac{MV - TV}{\text{ref. value}} \times 100\%$ (unit: unitless)

↳ we want error unit to be unitless so we can divide by ref. value.

Ref value = TV } ⇒ not used as MV, TV can become 0 if reading can be b/w (−50, 50), so high error will come
= MV }
= Span ✓

⇓

Span is used as ref. value.

% error (span) = 1% : Class-1 Instrument

% error (span) = 5% : Class-5 instrument.

Q) Class-1 pressure sensor; range: 0 to 50 bar. Find % error when true value (TV) = 10 bar.

Soln: $\%\ \text{error} = \dfrac{MV - TV}{50} \times 100\% = 1\%$ (span = 50)

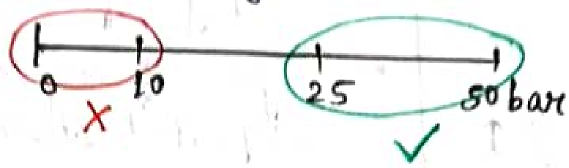
$\Rightarrow MV - TV = 0.5$ bar

when TV = 10 bar, $\%\ \text{error} = \dfrac{0.5\text{ bar}}{10\text{ bar}} \times 100\% = 5\%$

when TV = 1 bar, $\%\ \text{error} = \dfrac{0.5\text{ bar}}{1\text{ bar}} \times 100\% = 50\%$.

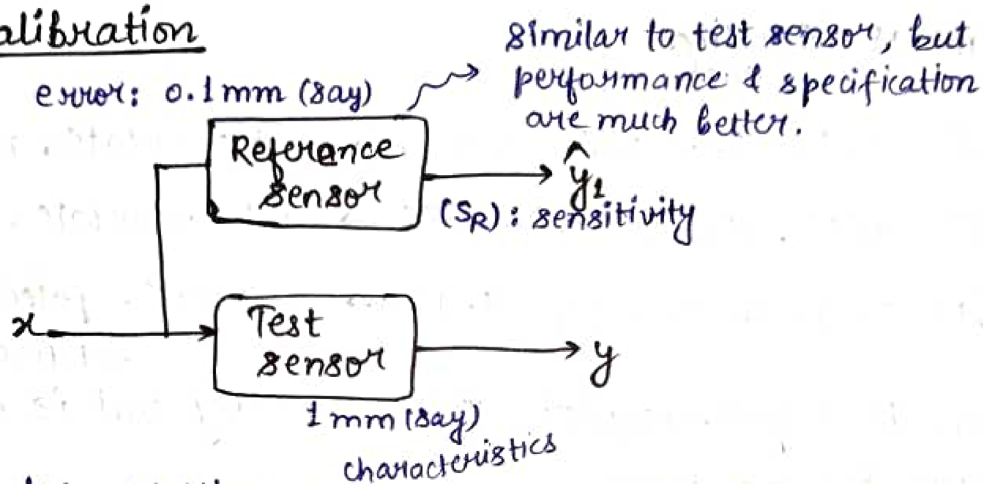
⇒ 1 bar ± 0.5 bar

To reduce the error, we should operate our sensor in high regimes, not low regimes/levels.



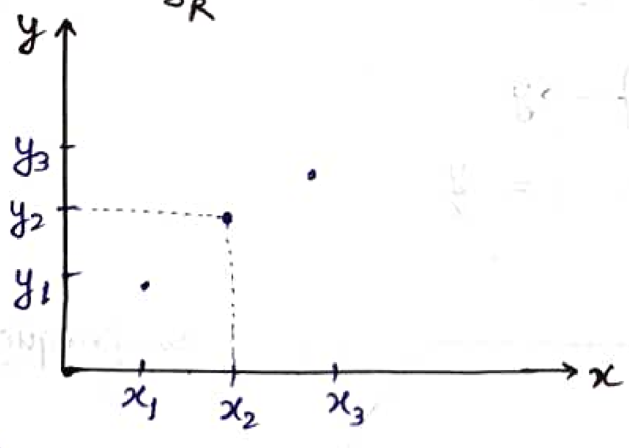
$\boxed{15-01-2024}$

Calibration



While plotting x-$\hat{y}$, y can be found out by looking at the o/p of test sensor; to know x, we use a referance sensor whose properties are well-known.

$$x_1 = \frac{\hat{y}_1}{S_R}$$



In practice, reference sensor can be costlier and bulky, etc. So, in field, we remove reference sensor, get a value, say y_2, from test sensor, and predict the i/p, x_2, using the calibrated data.

Accuracy and Precision

Accuracy	Precision
• Closeness to truth (true value).	• Sharply defined.
• Error ↓ ⇒ Accuracy ↑	• Degree of agreement of repeated results.

Eg. TV = 5 mm

3 instruments : 5 values each

Ⓐ : 4.7, 4.6, 4.6, 4.7, 4.8 ⇝ not accurate, not precise

Ⓑ : 4.74, 4.74, 4.75, 4.75, 4.76 ⇝ not accurate, precise

Ⓒ : 4.99, 5.01, 5.00, 5.01, 4.99 ⇝ accurate, precise

Relative

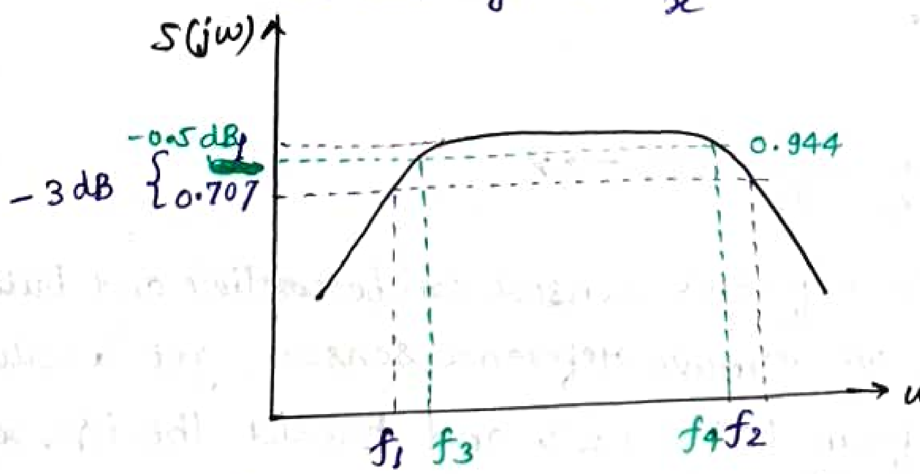
→ Precision is a pre-requisite for accuracy but is not the gurantee for accuracy.

Bandwidth (BW)

↳ Useful range of frequencies where the system/sensor ~~works~~ can be used.

$x \longrightarrow \boxed{\text{sensor}} \longrightarrow y$

Sensitivity, $S = \frac{y}{x}$



ω : frequency of x

$BW = f_2 - f_1$

Ideal value of $S = 1$

At f_1, f_2, actual value of $S = 0.707$

error $\approx -30\%$ $(=-29.3\%)$

↳ Too much error

(Not useful)

So, we use a conservative approach. Take say -0.5 dB, as the frequency.

$$20 \log_{10} S = -0.5$$

$$S = \left(10^{-0.5/20}\right) = 0.944$$

At f_3, f_4, actual value of $S = 0.944$

error $\approx 5\%$

Measurement Errors

Measuring voltages using 2 voltmeters,

$v_1 = 10\,V \pm 0.1\,V \rightarrow$ error $= \frac{0.1}{10} \times 100\% = 1\%$

$v_2 = 1\,V \pm 0.1\,V \rightarrow$ error $= 10\%$

$v_1 + v_2 = 11\,V \pm 0.2\,V \rightarrow$ error $= 1.8\%$

$v_1 - v_2 = 9\,V \pm 0.2\,V \rightarrow$ error $= 2.2\%$

$< 10\%$, so OK!

Now, say

$v_1 = 10\,V \pm 0.1\,V \rightarrow$ error $= 1\%$

$v_2 = 9\,V \pm 0.1\,V \rightarrow$ error $= 1.1\%$

$v_1 - v_2 = 1\,V \pm 0.2\,V \rightarrow$ error $= 20\%$ } Error shoot up!

→ Never take difference b/w two experimental values which are close to each other.

Transformer:

Efficiency, $\eta = 98\%$ or above (typical value)

$$= \frac{P_o}{P_I} \times 100\%$$, P_o : Output power

P_I : Input power

$$\Rightarrow P_o = 0.98\, P_I$$

$Losses = P_I - P_o$

$P_I = 1W \pm 0.01 W$ (say)

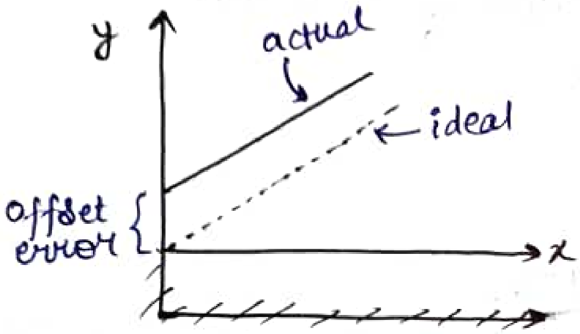
$P_o = 0.98W \pm 0.01W$

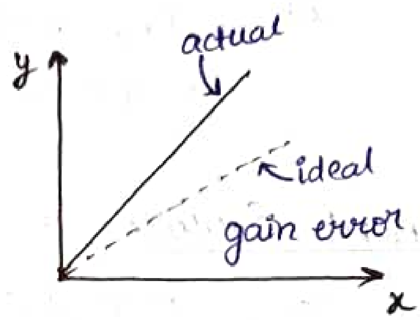
$losses = 0.02W \pm 0.02W \rightarrow 100\%$ error } Never do this!

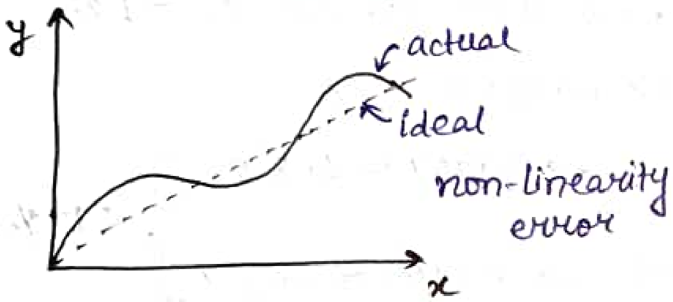
To find losses of a transformer, use techniques like short-circuit test, open-circuit test.

16-01-2024

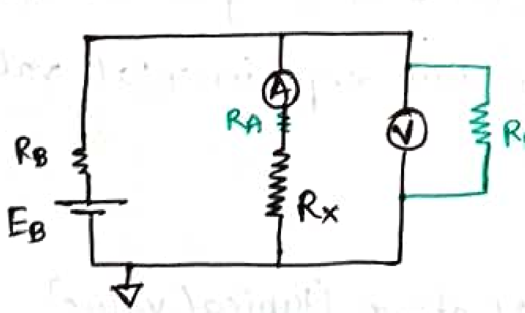
Measurement Errors:







① Identify the error.
② Analyse
③ Compensate



$M.V. = \frac{V}{A} = R_x = T.V.$

Due to error, $M.V. = \frac{V}{A} = R_x + R_A$

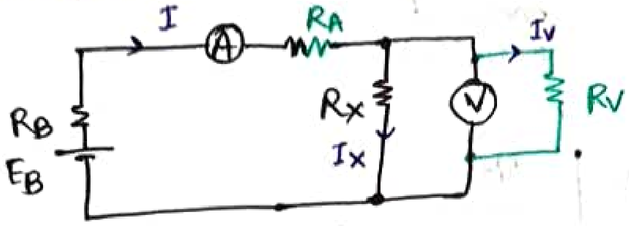
$\%\ error = \frac{R_x + R_A - R_x}{R_x} \times 100\% = \frac{R_A}{R_x} \times 100\%$

↳ does not depend on Battery (battery quality can be compromised)

To reduce the error,

$R_A \ll R_x$: High quality ammeter required;
voltmeter quantity can be compromised
(Independent of voltmeter)
↳ suitable for high R_x estimation

To measure low R_x,



$$M.V. = R_x \parallel R_V$$

$$\% \text{ error} = \frac{\frac{R_x R_V}{R_x + R_V} - R_x}{R_x} \times 100\%$$

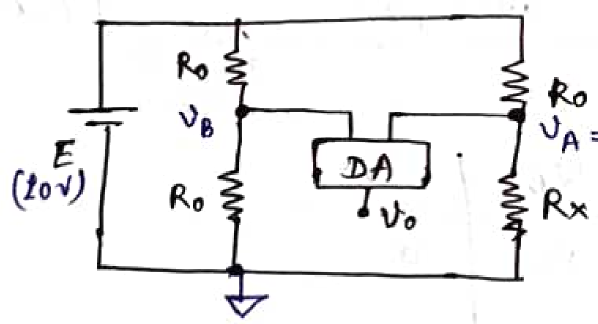
$$= \frac{\frac{\cancel{R_x R_V} - R_x^2 - \cancel{R_x R_V}}{R_x + R_V}}{\cancel{R_x}} \times 100\%$$

$$= \frac{-R_x}{R_x + R_V} \times 100\%$$

To reduce the error,
$R_V \gg R_x$

↳ These setups require both ammeter and voltmeter.
↳ Not suitable for end-user.

To measure the sensor resistance R_x :



$$R_x = R_o [1 + X]$$

$$V_A = \frac{R_x E}{R_x + R_o}$$

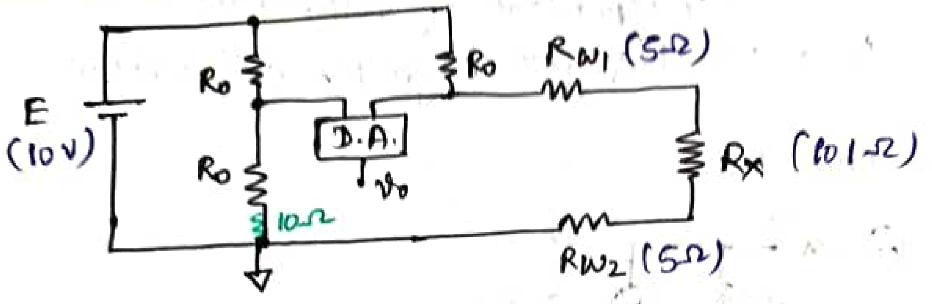
DA: difference amplifier
gain, $G = 1$
high input impedance

$\Rightarrow R_o = 100\,\Omega$, $X \in [0, 0.01]$

X	V_A	V_B	V_o
0	$E/2$	$E/2$	0
0.01	$\frac{0.01E}{2.01}$	$E/2$	24.8 mV

Source of error:

Suppose the electronics are at a harsh, hazardous place. Sensor should be kept far from such places.



x	V_A	V_B	V_o
0	$\frac{110}{210}$ E	E/2	238 mV
0.01	$\frac{111}{211}$ E	E/2	261 mV

} Large offset error

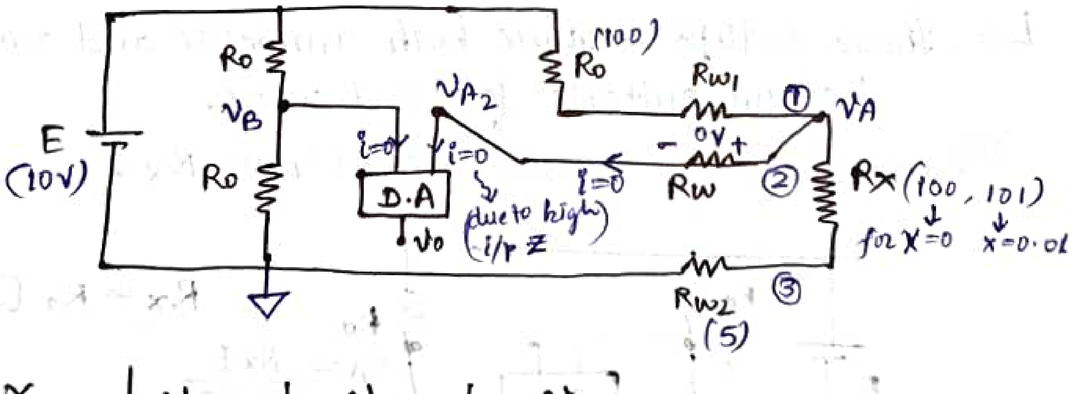
Compensation of error:

1. Substract 238mV from DA using microcontroller.
2. Connect a resistance of 10Ω in series with Ro.

In that way, $V_o(x=0) = 0$

But, these solutions fail miserably b/c R_{w1} & R_{w2} are unknown and changing.

↳ Solution:



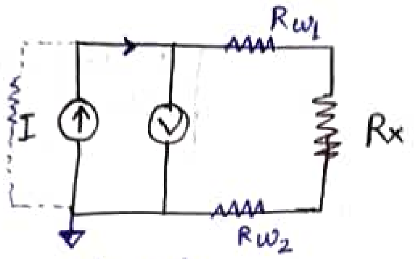
x	V_A	V_B	V_o
0	$\frac{105}{210}$ E	E/2	0
0.01	$\frac{106}{211}$ E	E/2	23.7 mV

In scenario where $R_{w1} \neq R_{w2}$, this model will not work.

17-01-2023

Resistive Sensors

- Wheatstone's bridge
  - 2-wire
  - 3-wire
- Current-source
  - 2-wire
  - 3-wire



$R_x = R_0(1+x)$

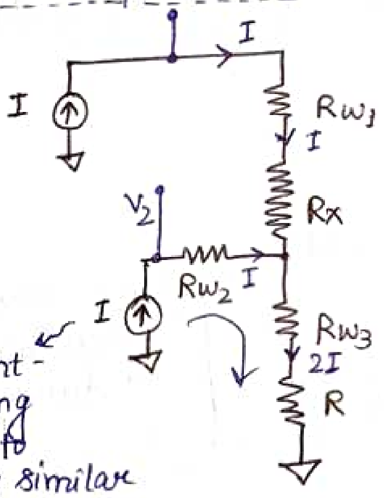
$V = I R_x$

When $x=0$, $V_0 = I R_0$ → finite offset when $x=0$. : Disadvantage

[In wheatstone brige circuit, $V_0 = 0$ for $x=0$]
↳ preferred in some application

$V = I(R_x + R_{w_1} + R_{w_2})$: Error

Lead-wire compensation:



Use current-mirroring circuit to make a similar current source

$$V_1 = I(R_x + R_{w_1}) + 2I(R_{w_3}+R)$$
$$V_2 = I R_{w_2} + \cancel{2I(R_{w_3}+R)}$$

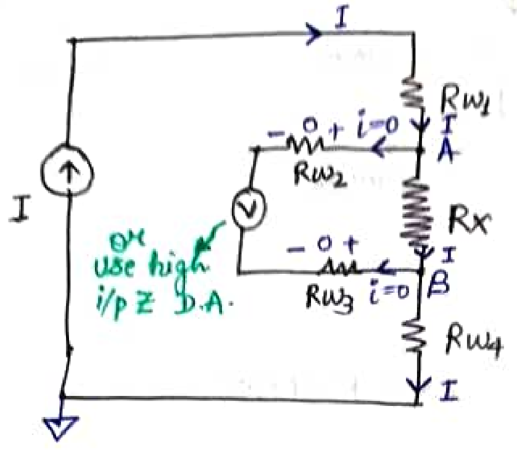
$$V_1 - V_2 = I(R_x + R_{w_1} - R_{w_2})$$

$R_{w_1} = R_{w_2}$ (lead wires should have same resistances)

$\Rightarrow V_1 - V_2 = I R_x$ → Use D.A. to get the difference; not manually.

4-wire connection: Using only one current source.

↳ To get independence from R_{ws} and mismatching.



Offset voltage will be there in any current-source method.

Electronics will be placed far from the sensor.

$$v = v_{AB} = IR_x$$

Q) Is it possible to lead-wire compensation (compensate $R_{w1}, R_{w2}, \dots$) using 2-wire method?

↳ Yes!

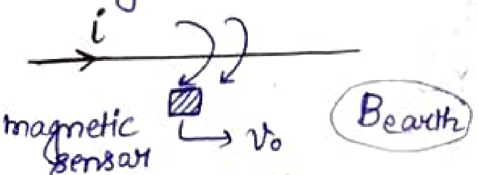
Summary:

Errors → Loading effect of meters; Effect of connecting wires } Instrumental Errors

Ⓐ Instrumental Errors

Ⓑ Environmental Errors

Eg. Measuring current in a wire



Using ammeter
↳ we have to break the wire.
↳ Ammeter may burn due to high i.

$V_o = K_1 B$ ← sensitivity

$B = K_2 i$

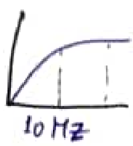
$\Rightarrow V_o = K_1 K_2 i = Ki + K_1 B_{earth}$, $K = K_1 K_2$

B_{earth} ← dc ($f=0$)

Ki ← 50 Hz

$K_1 B_{earth}$: error due to earth's mag. field
↳ constant ⇒ offset

To remove the offset error (dc component), use high-pass filter of $f_c < 50$ Hz.



© Observational Errors

↳ Human errors due to parallex.

↳ Not relevant nowadays as most sensors are digital.

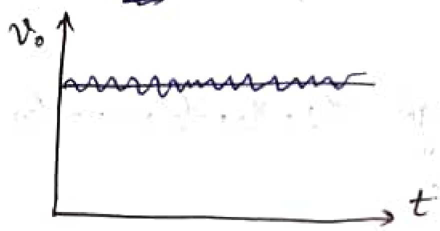
↳ The source of these errors are well-known and ~~can be~~ their effect can be nullified by simple systematic methods.

⇓

Systematic Errors

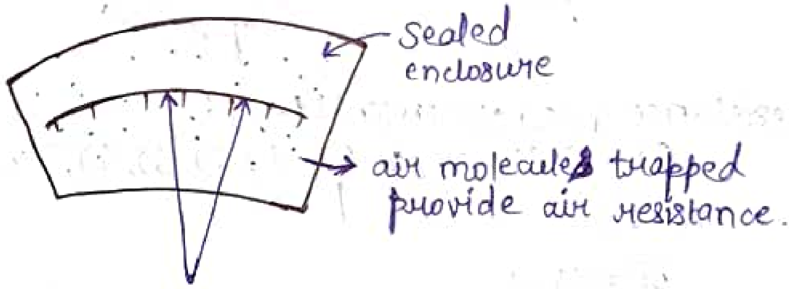
<u>Random Errors</u> (noise) : Fluctuations even when ~~are~~ all systematic errors are compensated.

↳ Eratic, random



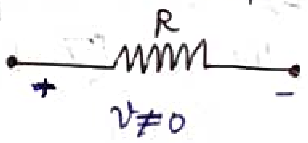
↳ Their source may be known.

Eg. Analog meter :



Eg. Thermal noise ⇝ depends upon R, T, B

$\approx 20nV$



23-01-2024

Measurement Errors

- Systematic
  - Source known
- Random
  - Source may be known or unknown
  - Difficult to remove.

Random Errors:

1. Nullify all systematic errors.
2. Repeated measurements of the physical variable, using same equipment, under the same operating conditions.
3. Average of repeated readings.

x : measurand

Take the readings $x_1, x_2, x_3, \dots, x_n$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\bar{x} \pm \delta$, δ: measure of dispersion

Ammeter:

Zero error

First remove the zero error (systematic).

Eg. Resistance measurements (Ω):

63, 66, 66, 67, 67, 68, 69, 69, 69, 69, 70, 70, 71

$n = 13$

$\bar{x} = 68\,\Omega$

1. Limiting error (L) = 5 ⇝ One of the method to specify δ

   100% readings lie b/w $(\bar{x} - L, \bar{x} + L)$.

   $$68 \pm 5$$

   L: Conservative quantity
   - depends on each and every measurement.

2. Probable Error (P) = 1 ⇝ common in the US.

   50% of the readings lie b/w $(\bar{x} - P, \bar{x} + P)$
   (atleast 50%.)

   $$\frac{13}{2} \approx 7$$

   $P = 1$

   - Mathematical treatment is difficult.
   - More optimistic.

③ Standard Deviation (σ)

Ⓐ $\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$ ✗

Ⓑ $\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$ ✓

In most cases, $n \gg 1$.

Eg. $n = 1$

$x_1 = 63$

$\bar{x} = 63$

$\sigma_A = 0$ ✗

$\sigma_B = \frac{0}{0}$ ✓ [For $n=1$, σ should be un-deterministic.]

Eg. $R_1 = 68 \pm 3\Omega$

$R_2 = 51 \pm 2\Omega$

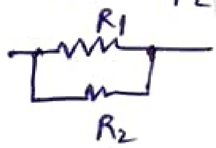
↑ avg. ↑ σ

$\sigma_{R_1} = 3\Omega$

$\sigma_{R_2} = 2\Omega$

Resistors can be combined in any fashion.

σ_R



$\frac{R_1 R_2}{R_1 + R_2}$

Propagation of σ:

Measurands : u, v ⇝ independent variables

↳ value of one doesn't affect the other.

$x = f(u, v)$; Find σ_x when σ_u & σ_v are known.

σ_u: u_1, u_2, $\vdots$, u_n — $\bar{u}$

$\Delta u_1 = u_1 - \bar{u}$

$\Delta u_2 = u_2 - \bar{u}$

$\sigma_u^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\Delta u_i^2)$

σ_v: $\sigma_v^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\Delta v_i^2)$

To find $\sigma_x^2 = \frac{1}{n-1}\sum_{i=1}^{n} \Delta x_i^2$

$\Delta u_i^2 \geq 0$
Δu_i can be $\oplus$ve or $\ominus$ve

$$\Delta x_i = \left(\frac{\partial f}{\partial u}\right)\Delta u_i + \left(\frac{\partial f}{\partial v}\right)\Delta v_i$$

$$\Rightarrow \sum_{i=1}^{n}\Delta x_i^2 = \left(\frac{\partial f}{\partial u}\right)^2\sum_{i=1}^{n}\Delta u_i^2 + \left(\frac{\Delta f}{\partial v}\right)^2\sum_{i=1}^{n}\Delta v_i^2 + 2\frac{\partial f}{\partial u}\frac{\partial f}{\partial v}\underbrace{\sum_{i=1}^{n}(\Delta u_i \Delta v_i)}_{\text{Product of 2 random errors}}$$

[Σ is independent of i]

summing for large n ~ 0

$$\therefore \quad \sigma_x^2 = \left(\frac{\partial f}{\partial u}\right)^2\sigma_u^2 + \left(\frac{\partial f}{\partial v}\right)^2\sigma_v^2$$

Eg. $x = f(w_1, w_2, w_3, \dots, w_m)$

$$\sigma_x^2 = \left(\frac{\partial f}{\partial w_1}\right)^2\sigma_{w_1}^2 + \left(\frac{\partial f}{\partial w_2}\right)^2\sigma_{w_2}^2 + \dots + \left(\frac{\partial f}{\partial w_m}\right)^2\sigma_{w_m}^2$$

Eg. $R_1 = 68 \pm 3\ \Omega \leftarrow \sigma_{R_1}$

$R_2 = 51 \pm 2\ \Omega \leftarrow \sigma_{R_2}$

$R = R_1 + R_2$ (Series connection)

$$\frac{\partial R}{\partial R_1} = \frac{\partial R}{\partial R_2} = 1$$

$$\sigma_R^2 = \left(\frac{\partial R}{\partial R_1}\right)^2\sigma_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2\sigma_{R_2}^2$$

$$\sigma_R = \sqrt{13}\ \Omega$$

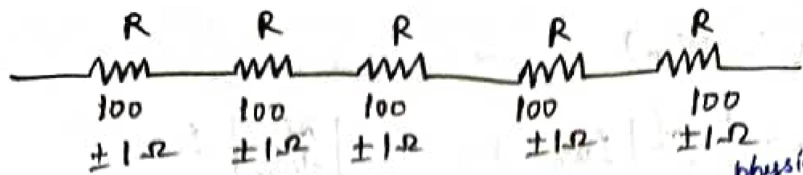
$R = \frac{R_1 R_2}{R_1 + R_2}$ (Parallel connection)

$$\frac{\partial R}{\partial R_1} = \left(\frac{R_2}{R_1+R_2}\right)^2 = \left(\frac{51}{119}\right)^2$$

$$\frac{\partial R}{\partial R_2} = \left(\frac{R_1}{R_1+R_2}\right)^2 = \left(\frac{68}{119}\right)^2$$

$$\sigma_R^2 = \left(\frac{\partial R}{\partial R_1}\right)^2\sigma_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2\sigma_{R_2}^2$$

Eg.



$R_T \neq 5R$ [All ~~R measurements~~ are physically independent of each other]

$= R_1 + R_2 + R_3 + R_4 + R_5$

[In Parallel connection,
$R_T \neq R/5$, $\frac{\partial R}{\partial R_1} \neq \frac{1}{5}$
$R_T = R_1 \| R_2 \| R_3 \| R_4 \| R_5$]

$\boxed{24\text{-}01\text{-}2024}$

Propagation of Limiting Errors

$u \rightarrow u \pm L_u$

$v \rightarrow v \pm L_v$

$x = f(u, v)$

$L_x = ?$

Δu_1

Δu_2

$\vdots$

Δu_3

$L_u = max(|\Delta u_i|)$

Similarly, $L_v = max(|\Delta v_i|)$

$L_x = max(|\Delta x_i|)$

As $|\Delta x_i| = \left|\frac{\partial f}{\partial u}\Delta u_i + \frac{\partial f}{\partial v}\Delta v_i\right|$

$\Rightarrow |\Delta x_i| \leq \left|\frac{\partial f}{\partial u}\right||\Delta u_i| + \left|\frac{\partial f}{\partial v}\right||\Delta v_i|$

$|\Delta x_i|_{max} = \left|\frac{\partial f}{\partial u}\right||\Delta u_i|_{max} + \left|\frac{\partial f}{\partial v}\right||\Delta v_i|_{max}$ (worst case)

$\downarrow$

$L_x = \left|\frac{\partial f}{\partial u}\right| L_u + \left|\frac{\partial f}{\partial v}\right| L_v$

Eg. $x = f(w_1, w_2, w_3, \dots w_m)$

$$L_x = \left|\frac{\partial f}{\partial w_1}\right| L_{w_1} + \left|\frac{\partial f}{\partial w_2}\right| L_{w_2} + \dots + \left|\frac{\partial f}{\partial w_m}\right| L_{w_m}$$

Eg. $R_1 = 68 \pm 3\ \Omega \leftarrow L_{R_1}$
$R_2 = 51 \pm 2\ \Omega \leftarrow L_{R_2}$

$R = R_1 + R_2 = 119 \pm L_R\ \Omega$

$L_R = L_{R_1} + L_{R_2} = 5\Omega$

$$L_x = \left|\frac{\partial f}{\partial u}\right| L_u + \left|\frac{\partial f}{\partial v}\right| L_v$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$L_R = \left|\frac{\partial R}{\partial R_1}\right| L_{R_1} + \left|\frac{\partial R}{\partial R_2}\right| L_{R_2}$$

How many repeated measurements (n) should we take?

x: measurand

Case A	Case B
$n_A = 10$	$n_B = 90$
$\overline{x_{A_1}}$	$\overline{x_{B_1}}$
$\overline{x_{A_2}}$	$\overline{x_{B_2}}$
$\overline{x_{A_3}}$	$\overline{x_{B_3}}$
⇓ More deviation among the means	⇓ Less deviation among the means

using different 10 readings (for $\overline{x_{A_1}}, \overline{x_{A_2}}, \overline{x_{A_3}}$)

⇓ measured using standard deviation among means ($\sigma_{\bar{x}}$)

$$\sigma_{\bar{x}} \propto \frac{1}{\sqrt{n}}$$

$$\frac{\sigma_{\bar{x}_B}}{\sigma_{\bar{x}_A}} = \sqrt{\frac{n_A}{n_B}} = \frac{1}{3}$$

$$\Rightarrow \sigma_{\bar{x}_B} = \frac{\sigma_{\bar{x}_A}}{3}$$ → Precision has improved by a factor of 3.

↳ Need 80 more measurements to get precision
⇒ Not enough return

Eg. $n_c = 1,00,000 \rightarrow$ Long time

$\sigma_{\bar{x}_c} = \dfrac{\sigma_{\bar{x}_1}}{100}$ ↳ Operating conditions need to be the same throughout the time (using same instrument).

→ Mean, $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$; S.D., $\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} y_i^2}$

<u>Grouped Data Approach</u>: Divide raw data into different ranges.

Mean, $\bar{x} = \frac{1}{N}\sum_{\text{for all } R} x_R f(x_R)$

S.D., $\sigma = \sqrt{\frac{1}{N-1}\sum_{\text{for all } R} y_R^2 f(x_R)}$

Sl. No.	Range	x_R	$f(x_R)$	$x_R f(x_R)$	y_R	$y_R^2 \cdot f(x_R)$

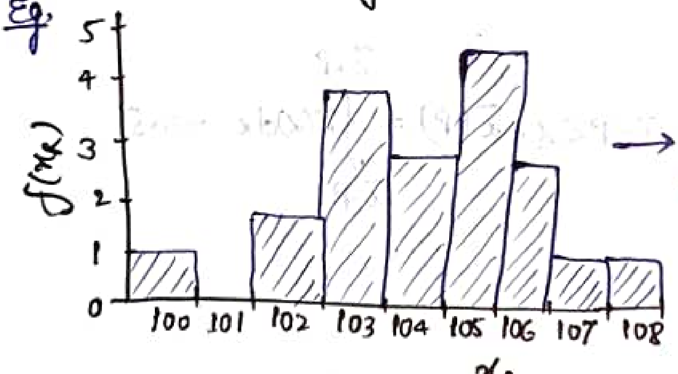
x_R : Midpoint of ranges

$f(x_R)$: no. of measurement points in the range that corresponds to x_R

$y_R = x_R - \bar{x}$

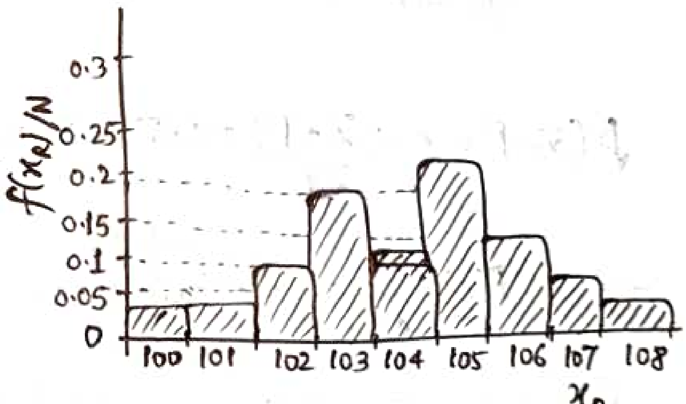
<u>Frequency distribution graph or Histograms:</u>

Eg.



→

<u>Probability Histograms:</u>



$P(x_R) = \dfrac{f(x_R)}{N}$

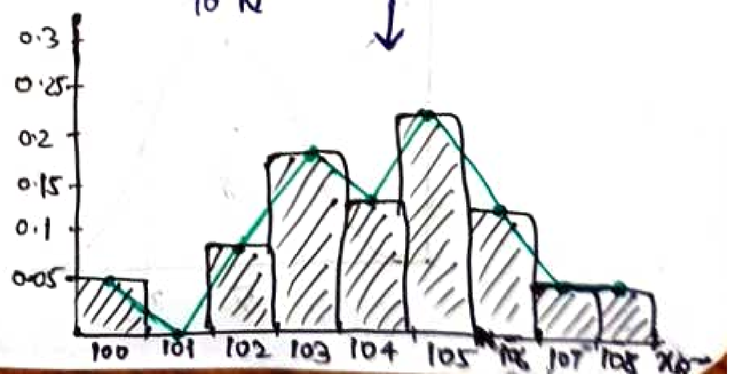
Solution: Normalise $f(x_R)$ with-respect-to N

→ Δx_R: Width of the interval

Solution: Normalize $f(x_R)$ with-respect-to Δx_R

$Z(x_R) = \dfrac{f(x_R)}{N\Delta x_R} = \dfrac{P(x_R)}{\Delta x_R}$

↳ Probability Distribution Function



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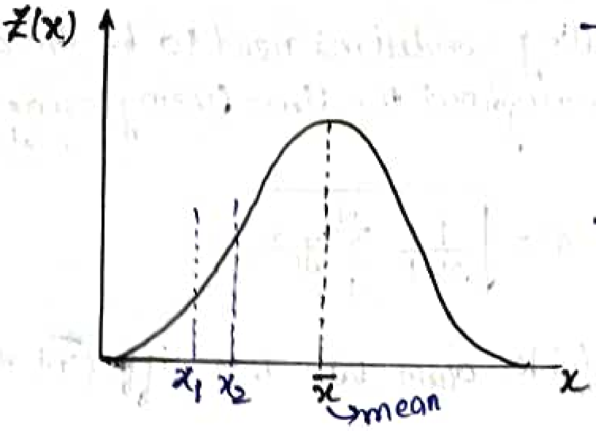


→ In any measurements, the repeated measurements will have highest probability close to mean.

→ Probability of random errors become lesser when we move away from.

→ Even (symmetric) about mean.

$$p(x_1 < x < x_2) = \int_{x_1}^{x_2} Z(x)\,dx$$

$$p(-\infty < x < +\infty) = \int_{-\infty}^{\infty} Z(x)\,dx = 1$$

P : Probable error

p : probability

$$\bar{x} = \frac{1}{N}\sum_{\text{for all } R} x_R \cdot f(x_R) = \sum_{\text{for all } R} x_R \cdot p(x_R) = \sum_{\text{for all } R} x_R \cdot Z(x_R) \cdot \Delta x_R \approx \int_{-\infty}^{\infty} x \cdot Z(x)\,dx$$

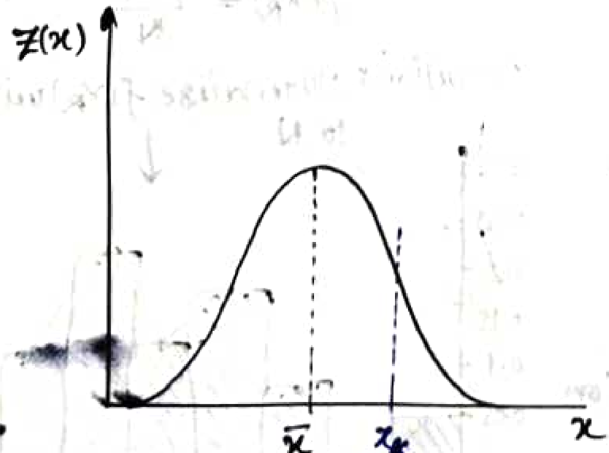
$$\sigma^2 = \frac{1}{N-1}\sum_{\text{for all } R} (x_R - \bar{x})^2 f(x_R) = \sum_{\text{for all } R} (x_R - \bar{x})^2 \cdot p(x_R)$$

$$= \sum_{\text{for all } R} (x_R - \bar{x})^2 \cdot Z(x_R) \cdot \Delta x_R \approx \int_{-\infty}^{\infty} (x - \bar{x})^2 Z(x)\,dx$$

$$p(\bar{x} - P < x < \bar{x} + P) = 0.5 \Rightarrow p(\bar{x} - P < x < \bar{x} + P) = \int_{\bar{x}-P}^{\bar{x}+P} Z(x)\,dx = 0.5$$

$$\rightarrow \quad Z(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

Gaussian Error Curve : Probability Concept



$$Z(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

$$p(\bar{x} < x < x_K) = \int_{\bar{x}}^{x_K} Z(x)\,dx = \int_{\bar{x}}^{x_K} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} dx$$

$y = x - \bar{x}$

$y_K = x_K - \bar{x}$

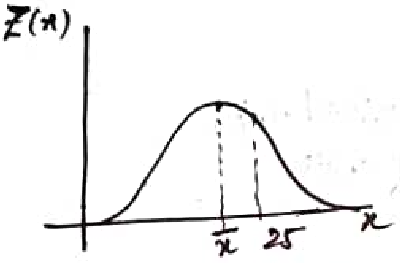
$$\Rightarrow p(0 < y < y_K) = \int_0^{y_K} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} dy$$

$\Big\downarrow \ \frac{y}{\sigma} = t$

$$p(\bar{x} < x < x_K) = p(0 < t < y_K/\sigma) = \int_0^{y_K/\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

Table gives the integral value for some y_K/σ values.

Eg.



$\bar{x} = 21.9$ | $p(x > 25) = ?$

$P = 2.1$ | $N = 100 \Rightarrow n(x > 25) = ?$

Soln: $p(\bar{x} - P < x < \bar{x} + P) = 0.5$

$\Rightarrow p(\bar{x} < x < \bar{x} + P) = 0.25$

$\Rightarrow p(0 < y < P) = 0.25$

$\Rightarrow p(0 < t < P/\sigma) = 0.25$

$P \approx 0.2486$ (from the table)

$\Rightarrow P/\sigma = 0.67$

$\Rightarrow \sigma = \frac{2.1}{0.67}$

$= 3.1$

$\therefore p(x > 25) = 0.5 - p(\bar{x} < x < 25)$

$= 0.5 - p(0 < y < 25 - 21.9)$

$= 0.5 - p(0 < y < 3.1)$

$= 0.5 - p(0 < t < \frac{3.1}{3.1})$

$= 0.5 - p(0 < t < 1)$

$= 0.5 - 0.3413$ (from table)

$= 0.1587$

$\therefore n(x > 25) \approx 16.$

Static Characteristics: Measurements when input is constant.

Long Term Performance
↓ to specify

Reliability (R)

↳ Indicator of whether a sensor or instrument has worked with agreed performance (accuracy, error, etc.) levels, over stated period, under specified conditions.

Eg.

survived

$A \rightarrow 60S,\ 40F$ failed

$B \rightarrow 55S,\ 45F$

$\}$ 100 units of A & B

$\Rightarrow R_A > R_B$

$R = \lim\limits_{N \to \infty} \dfrac{N_S}{N}$, N_S : no. of survived units
N : total no. of units.

$R_A = 0.6$
$R_B = 0.55$

→ $R = 1$: Ideal sensor
$R = 0$: Poor sensor

Unreliability (F)

$F = \dfrac{N_F}{N}$, N_F : No. of failed units.

→ ~~R+F~~ $R(t) + F(t) = 1$

Time Independent Parameter:
Failure Rate (λ):

N_S: no. of survived units at time 't'.
ΔN_F units failed in $(t, t+\Delta t)$.

Probability of failure $= \dfrac{\Delta N_F}{N_S}$

$\lambda = \dfrac{1}{\Delta t} \dfrac{\Delta N_F}{N_S}$ ⇝ normalised with time-span

$= \dfrac{1}{N_S}\left(\dfrac{\Delta N_F}{\Delta t}\right)$

$\Rightarrow \boxed{\lambda = \dfrac{1}{N_S}\left(\dfrac{dN_F}{dt}\right)}$

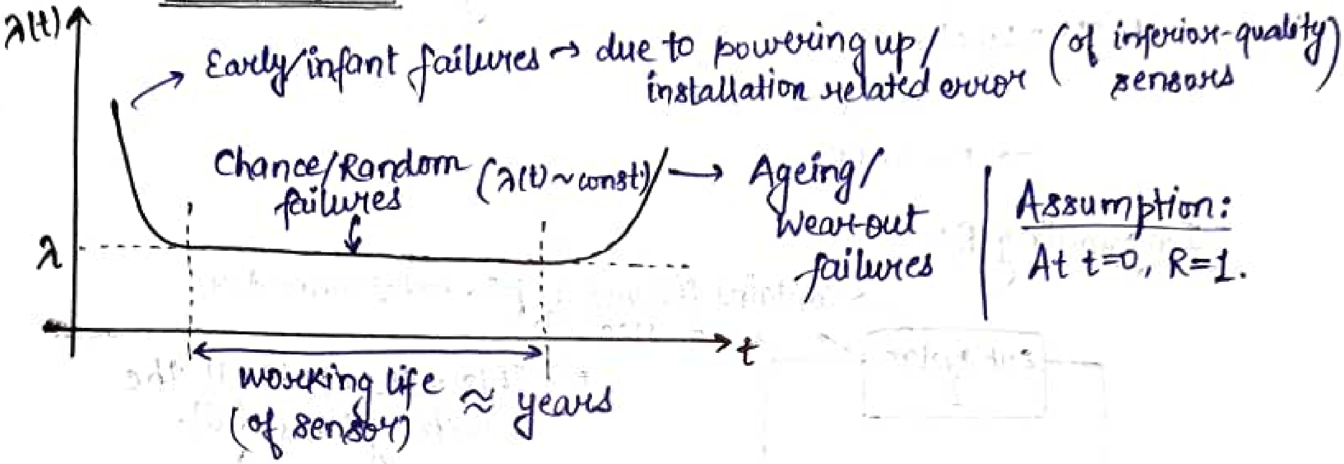
$\rightarrow \quad R = \frac{N_S}{N} = 1 - \frac{N_F}{N}$

$\Rightarrow \frac{dR}{dt} = -\left(\frac{1}{N}\frac{dN_F}{dt}\right)\left(\frac{N_S}{N}\right)$

$\Rightarrow \frac{dR}{dt} = -\lambda R$

$\Rightarrow \boxed{R = e^{-\int \lambda(t)dt}}$

Bath-tub Curve:



Assumption: At $t=0$, $R=1$.

↳ longer for good sensors

$\rightarrow \lambda(t) = \lambda$: working life regime

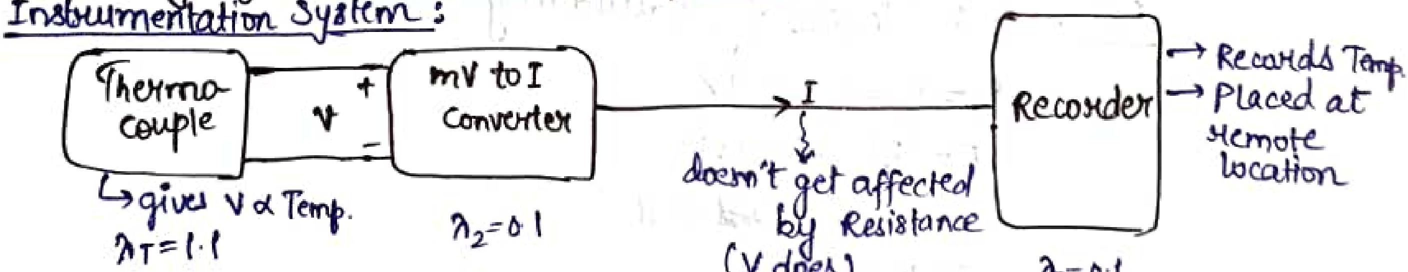
$R = e^{-\lambda t}$

Thermistor: $\lambda = 1\ yr^{-1}$ (say)

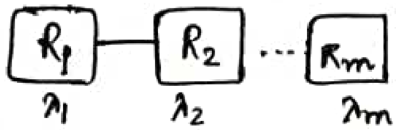
$R(t = 0.5yr) = e^{-0.5}$
$= 0.61$

Reliability of Instrumentation System

Instrumentation System:



Electronics are generally more reliable than sensor components.



$\Rightarrow$ Total system will survive if all the components survive.

$$R = R_1 R_2 R_3 \ldots R_m$$

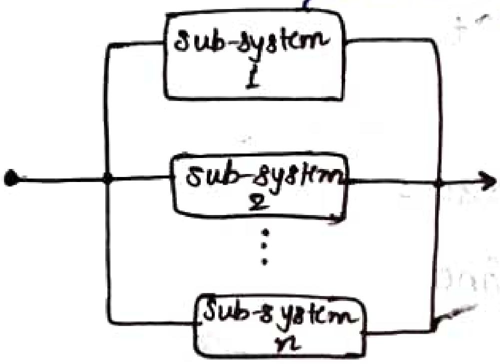
$$\Rightarrow e^{-\lambda t} = e^{-\lambda_1 t} e^{-\lambda_2 t} \ldots e^{-\lambda_m t}$$

$$\Rightarrow \lambda = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_m$$

$\lambda = 1.3$ for the instrumentation system.

$$R(t = 0.5\,yr) = e^{-0.65}$$
$$= 0.52$$
$$\Rightarrow F = 0.48$$

Improving R:



Contains thermo-couple, mV-I converter, & recorder.

$\Rightarrow$ This will fail if all the sub-systems fail.

All are exposed to same physical quantity (e.g., T)

$$F = F_1 F_2 \ldots F_n$$

For $n=3$: 3 levels of redundancy

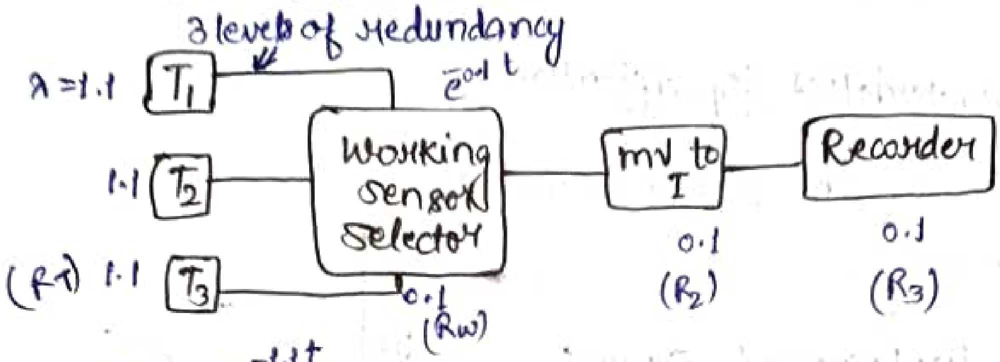
$$F_1 = F_2 = F_3 = 0.48$$
$$F = (F_1)^3 = (0.48)^3$$
$$= 0.11$$
$$\Rightarrow R = 1 - 0.11$$
$$= 0.89$$

Disadvantages:

- Size ↑
- Mass ↑
- Power ↑
- Cost ↑

Improvement:

↳ Bring redundancy to the most critical element.
Here, it is thermo-couple which consumes negligible power.
↳ mV to I converter and Recorder are more reliable.



$R_T = e^{-1.1t}$

$F_T = 1 - e^{-1.1t}$
$= 1 - e^{-0.55}$ $(t = 0.5\ yr.)$
$= 0.423$

F_T of individual $\boxed{T}$ Thermo-couple

F_T of 3 $\boxed{T}$s $= (0.423)^3 = 0.07757$

R_T of 3 $\boxed{T}$s $= 1 - (0.423)^3$
$= 0.9243$

05-02-2024

Reliability of complete system,
$\left[R_W = R_2 = R_3 = e^{-0.05} \right]$

$R = R_T R_W R_2 R_3$
$= 0.9243 \times (e^{-0.05})^3$
$= 0.795$

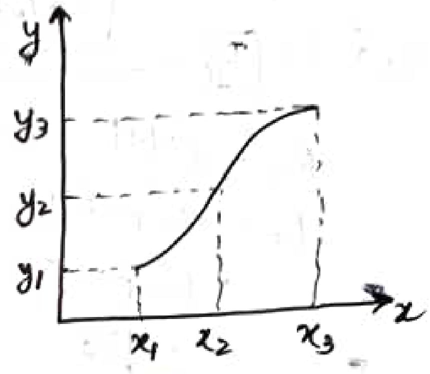
\# 52% initially
↓
89% with high cost, high power, high mass
↓ compromise
79% with low cost

Specification

- static characteristics ⇝ input : constant or slowly changing
- Long-term characteristics → R, F, λ
- Dynamic characteristics

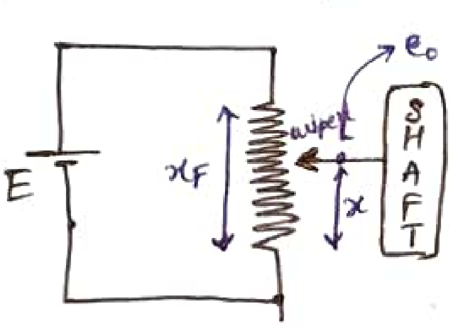
Dynamic characteristics /Specification

↳ Performance of the sensor when input is changing.



↑ constant inputs

Potentiometric displacement sensor:

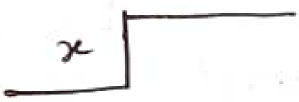


⇒ (we measure the displacement of the shaft with ref. winding)

$$e_0 = \left(\frac{E}{x_F}\right)x = Kx,$$

where sensitivity $K = \left(\frac{E}{x_F}\right)$.

(If i/p changes suddenly) x



⇒ e_0

(If i/p changes continuously) x

⇒ e_0

(output faithfully follows the i/p)

↳ Output is same as the input except for the sensitivity, K.

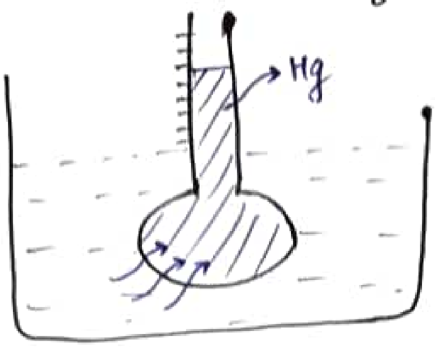
Ideal instrument:

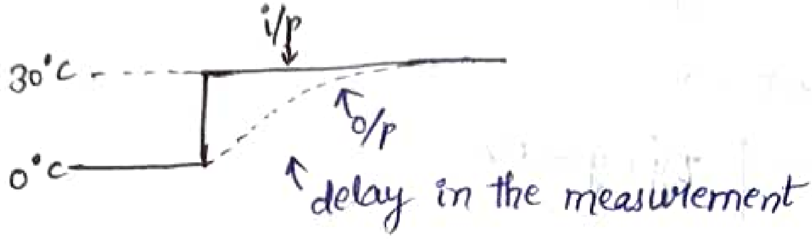
$y(t) = Kx(t)$ → Zero-order instrument

Praticality: Inertial loading, Frictional loading } at the wiper.

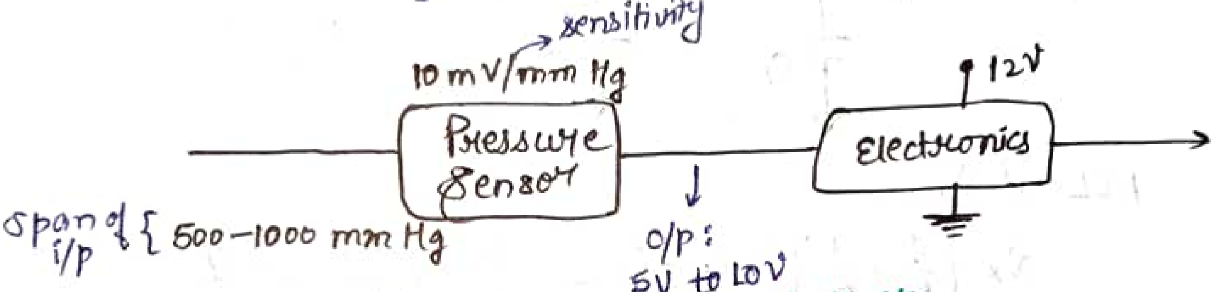
Eg. Measuring temperature of a liquid

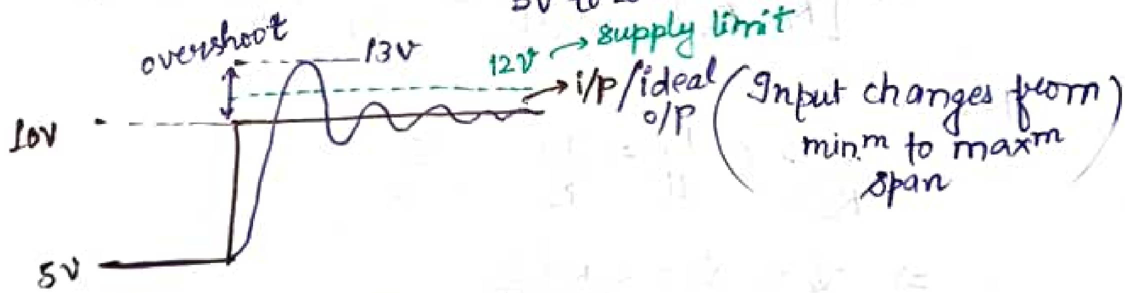
(Hg in glass thermometer)





- Transient error
- Steady-state error





Due to overshoot : • Electronics may get damaged
• Actuator overload.

↓

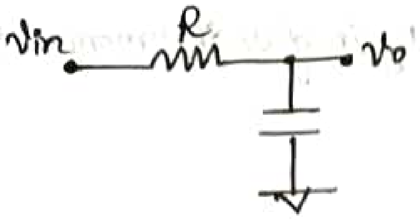
Solution: Redesign the electronics.
or
choose the sensor with less overshoot.

→ $\tau \frac{dy}{dt} + y(t) = Kx(t)$ → 1st order instrument

↓

τ : time-constant Eg Hg in glass thermometer.

Eg: 1^{st} order circuit: Low Pass Filter



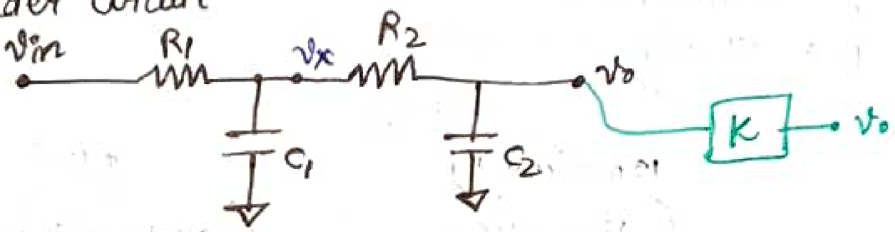
$$\frac{v_{in} - v_o}{R} = C\frac{dv_o}{dt}$$

$$\Rightarrow RC\frac{dv_o}{dt} + v_o = v$$

$\tau = RC$ | $\tau\dot{y} + y = Kx$

$K = 1$

Eg: 2^{nd} order circuit



KCL at v_x,

$$v_x\left[\frac{1}{R_1} + \frac{1}{R_2}\right] + C_1\frac{dv_x}{dt} = \frac{v_{in}}{R_1} + \frac{v_o}{R_2} \quad \cdots (1)$$

KCL at v_o,

$$\frac{v_o}{R_2} + C_2\frac{dv_o}{dt} = \frac{v_x}{R_2} \quad \cdots (2)$$

$$\Rightarrow v_x = v_o + C_2R_2\frac{dv_o}{dt}$$

$$(1) \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(v_o + C_2R_2\frac{dv_o}{dt}\right) + C_1\left(\frac{dv_o}{dt} + C_2R_2\frac{d^2v_o}{dt^2}\right) = \frac{v_{in}}{R_1} + \frac{v_o}{R_2}$$

$$\Rightarrow C_1C_2R_2\frac{d^2v_o}{dt^2} + \left(C_2\frac{R_2}{R_1}\frac{dv_o}{dt} + C_2\frac{dv_o}{dt} + C_1\frac{dv_o}{dt}\right) + \frac{v_o}{R_1} = \frac{v_{in}}{R_1}$$

$$\Rightarrow \frac{d^2v_o}{dt^2} + \frac{dv_o}{dt}\left(\frac{C_1R_1 + R_1C_2 + C_2R_2}{R_1R_2C_1C_2}\right) + \frac{v_o}{R_1R_2C_1C_2} = \frac{v_{in}}{R_1R_2C_1C_2}$$

$$\Rightarrow \frac{d^2v_o}{dt^2} + \left(\frac{R_1C_1 + R_2C_2 + R_1C_2}{R_1R_2C_1C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{R_1R_2C_1C_2} = \frac{v_{in}}{R_1R_2C_1C_2}$$

06-02-2024

Second-Order Instrument

$$x \longrightarrow \boxed{\begin{matrix}\text{2nd order}\\ \text{instrument}\end{matrix}} \longrightarrow y$$

$$\boxed{\frac{d^2y}{dt^2} + (2\xi\omega_n)\frac{dy}{dt} + \omega_n^2 y = K\omega_n^2 x},$$

where ω_n : natural frequency

ξ : damping ratio $\hookrightarrow$ unitless quantity

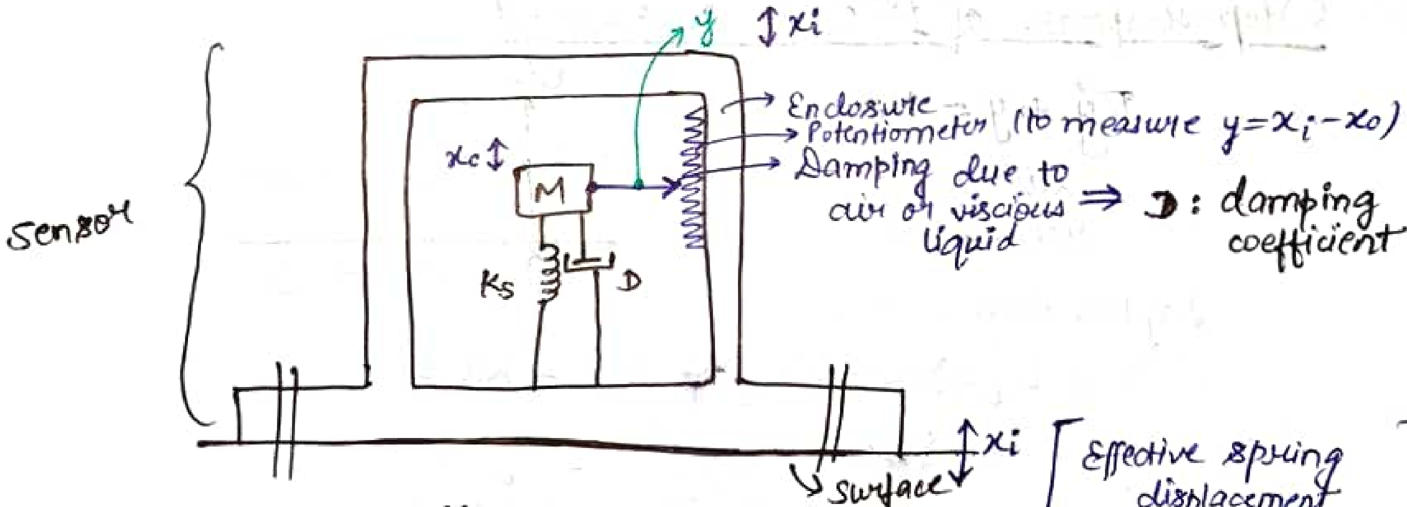
K : sensitivity

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\xi = \frac{R_1C_1 + R_2C_2 + R_1C_2}{2\sqrt{R_1R_2C_1C_2}}$$

2nd Order Sensor

Inertial sensor



[Effective spring displacement $= x_i - x_o$]

- $a = \ddot{x}_i$ (for some spacecraft)
- $K_s(x_i - x_o) + D\frac{d}{dt}(x_i - x_o) = M\frac{d^2x_o}{dt^2}$

Take $y = x_i - x_o \Rightarrow x_o = x_i - y$ $\Rightarrow$ | Extra variable: x_o

$$\therefore K_s y + D\frac{dy}{dt} = M\frac{d^2}{dt^2}(x_i - y)$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \left(\frac{D}{M}\right)\frac{dy}{dt} + \frac{K_s}{M}y = a}$$

- $\omega_n = \sqrt{\frac{K_s}{M}}$; $2\xi\omega_n = \frac{D}{M} \Rightarrow \xi = \frac{D}{2\sqrt{K_s M}} \Rightarrow \boxed{\xi \propto D}$

$\uparrow$ damping ratio

→ $x(t) \rightarrow \boxed{\text{Instrument}} \rightarrow y(t)$

↳ Find the instrument characteristics by observing the output, $y(t)$.

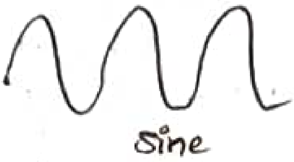
Test Inputs:

step

speed sensor

Ramp

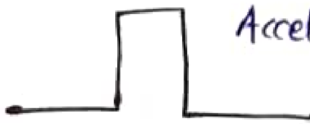
Temperature sensor



Sine

↳ Vibration sensor

↳ Body temperature sensor (changes sinusoidally)

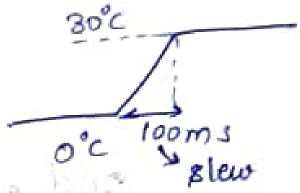


Impulse

Accelerometer

↳ Choice of test inputs depend on the instrument types.

Temp. sensor:

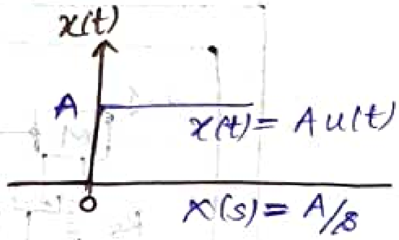


Step-response of 1st order Instrument

$$\tau \dot{y} + y = Kx$$

$x(t) \rightarrow X(s)$

$y(t) \rightarrow Y(s)$



$x(t) = A u(t)$

$X(s) = A/s$

Laplace transform:

$$\tau[sY(s) - y(0)] + Y(s) = KX(s)$$

$$\Rightarrow Y(s) = \frac{KX(s)}{\tau s + 1} + \frac{y(0)}{\tau s + 1}$$

Take $y(0) = 0$.

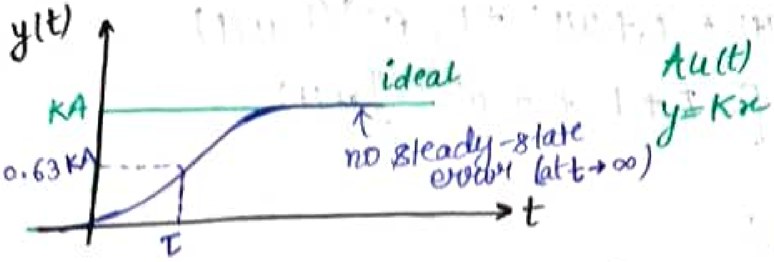
$$\Rightarrow Y(s) = \frac{KX(s)}{\tau s + 1} = \frac{KA}{s(\tau s + 1)} \quad [\because X(s) = A/s]$$

$$= \frac{KA/\tau}{s(s + 1/\tau)}$$

$$\Rightarrow Y(s) = KA\left[\frac{1}{s} - \frac{1}{s + 1/\tau}\right]$$

Inverse LT $\Rightarrow y(t) = KAu(t) - KAe^{-t/\tau}u(t)$

$$\Rightarrow y(t) = KA[1 - e^{-t/\tau}]u(t)$$



t	$y(t)$
0	0
∞	KA ← error = 0
τ	0.63 KA
5τ	0.993 KA ≈ KA

$t = 5\tau \Rightarrow y(t) = KA(1-e^{-5}) = 0.993\ KA$

07-02-2024

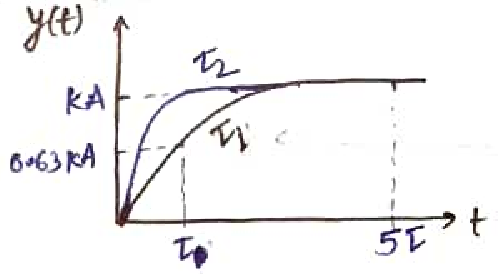
Transient error :

$$e(t) = y(t) - \underbrace{y_I(t)}_{\text{ideal o/p (given by zero error instr.)}}$$

$$= -KA e^{-t/\tau} u(t)$$

$$y_I(t) = KA\ u(t)$$

$\tau \downarrow \Rightarrow \frac{t}{\tau} \uparrow \Rightarrow |e(t)| \downarrow$: τ affects the transient error



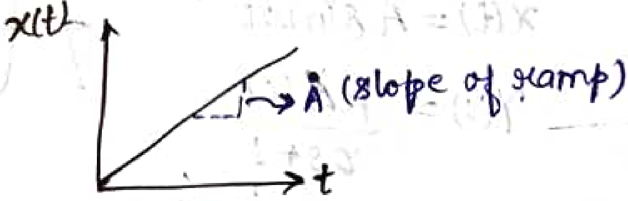
$\tau_1 > \tau_2$
↓
fast response

- τ helps us to find speed of response (or delay) of the system.

$\tau \uparrow \Rightarrow$ Response speed ↓

Ramp Response of 1st Order Instrument

$x(t) = \dot{A}\, t\, u(t)$



$$Y(s) = \frac{KX(s)}{\tau s + 1}$$

$$X(s) = \frac{\dot{A}}{s^2}$$

$$Y(s) = \frac{\dot{A}K/\tau}{s^2(s+1/\tau)} = \frac{K_1}{s} + \frac{K_2}{s^2} + \frac{K_3}{s+1/\tau}$$

$$\Rightarrow K_1 = -K\dot{A}\tau$$
$$K_2 = K\dot{A}$$
$$K_3 = K\dot{A}\tau$$

I-LT $\Rightarrow$ $y(t) = K_1 u(t) + K_2 t\, u(t) + K_3 e^{-t/\tau} u(t)$

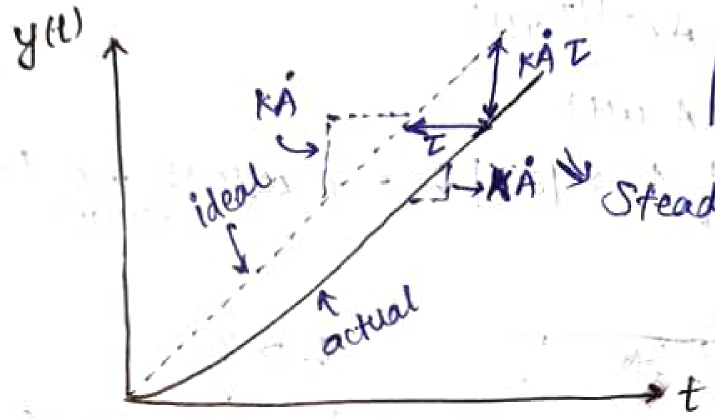
$$y(t) = K\dot{A}\left[-\tau + t + \tau e^{-t/\tau}\right] u(t)$$

t	$y(t)$
0	0
very large (∞)	$K\dot{A}(t-\tau) \neq y_I$

$y_I(t) = K\dot{A}t$

After a long time, the error has not died out.

↳ There's steady state error.



Steady state error $= K\dot{A}\tau$

↳ offset error

↳ time-independent.

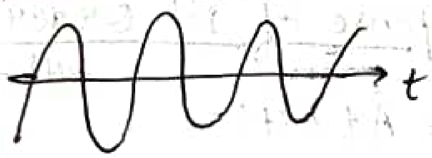
Transient error,

$$e(t) = y(t) - y_I(t)$$

$$= K\dot{A}\tau - K\dot{A}\tau e^{-t/\tau}$$

Sine Response of 1st Order Instruments

$x(t) = A \sin \omega t$



$$Y(s) = \frac{K\dot{A}(s)}{\tau s + 1}$$

$y(t) =$ ~~Transient term~~ $+$ Steady-state term

(as sine is always existing term)

leads to

determines

$s = \cancel{\sigma} + j\omega$

$s = j\omega$

$$\Rightarrow Y(j\omega) = \frac{KX(j\omega)}{j\omega\tau + 1}$$

freq. response of sensor $\rightarrow$ $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)} = \boxed{\dfrac{K}{j\omega\tau + 1}}$

$$x(t) = A\sin(\omega t) \longrightarrow \boxed{\dfrac{K}{j\omega\tau + 1}} \longrightarrow y(t)$$

$$y(t) = A\,|H(j\omega)|\,\sin(\omega t + \angle H(j\omega))$$

$$\text{error in amplitude} = \underset{(\text{ref.})}{AK} - \frac{AK}{\sqrt{\tau^2\omega^2+1}}$$

$$\text{phase error} = \cancel{\angle y(t)}^{\,0} - \angle H(j\omega)$$

$$= 0 - \angle H(j\omega)$$

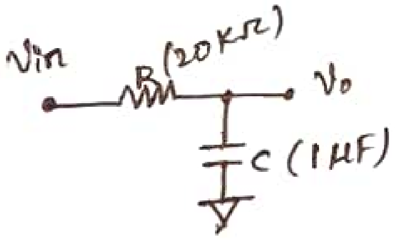
$$= \tan^{-1}(\omega\tau)$$

To reduce the error :

$\omega\tau \ll 1 \Rightarrow$ accurate measurements

or $\boxed{\tau \ll \frac{1}{\omega}}$.

Eg.



→ First Order System

Determine max. frequency that will keep the amplitude error < 5%.

Soln: $$\%\ \text{error in amplitude} = \frac{AK - \frac{AK}{\sqrt{\tau^2\omega^2+1}}}{AK} \times 100\% < 5\%$$

$$\Rightarrow 1 - \frac{1}{\sqrt{\tau^2\omega^2+1}} < \frac{5}{100}$$

$$\Rightarrow \frac{1}{\sqrt{\tau^2\omega^2+1}} > \frac{95}{100}$$

$\tau = RC$
$= (20k\Omega)(1\mu F)$
$= 20\,ms$

$$\Rightarrow \tau^2\omega^2 + 1 < \left(\frac{100}{95}\right)^2$$

$$\Rightarrow \omega^2 < \frac{\left(\frac{100}{95}\right)^2 - 1}{(20m)^2}$$

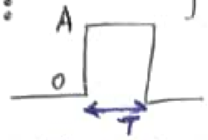
12-02-2024

First-Order Instruments

$\tau, K \rightarrow$ Parameters

Step
Ramp
Sine wave
Pulse:

} Responses

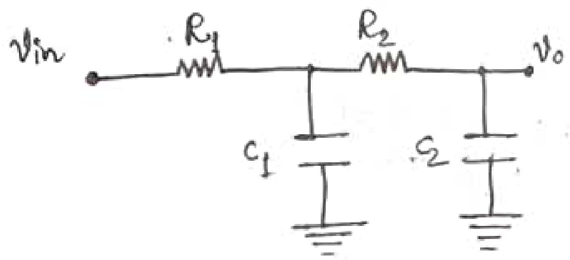


$y(t) = A u(t) - \cancel{A u(t-T)}\ A u(t-T)$

Second-Order Instruments

Parameters: $\xi \quad \omega_n \quad K$

Inertial accelerometer



Step Response:

$$x(t) = A u(t) \Rightarrow X(s) = \frac{A}{s}$$

$$\frac{d^2y}{dt^2} + 2\xi\omega_n \frac{dy}{dt} + \omega_n^2 y = K\omega_n^2 x$$

$$LT \Rightarrow s^2 Y(s) + 2\xi\omega_n s Y(s) + \omega_n^2 Y(s) = K\omega_n^2 X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\Rightarrow Y(s) = \frac{AK\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} \quad \ldots (1)$$

Response $y(t)$ depends on the roots of the quadratic $(s^2 + 2\xi\omega_n s + \omega_n^2)$ in the denominator.

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

Case-1: Real, Unequal roots $\Rightarrow \xi > 1$

$$s_1 = \left[-\xi + \sqrt{\xi^2 - 1}\right]\omega_n$$

$$s_2 = \left[-\xi - \sqrt{\xi^2 + 1}\right]\omega_n$$

Case-2: Real, Equal roots $\Rightarrow \xi = 1$

$$s_1 = s_2 = -\omega_n$$

Case-3: Complex roots $\Rightarrow \xi < 1$

$$s_1 = \left[-\xi + j\sqrt{1-\xi^2}\right]\omega_n$$

$$s_2 = \left[-\xi - j\sqrt{1-\xi^2}\right]\omega_n$$

$\xi > 1$

$$Y(s) = \frac{AK\omega_n^2}{s(s-s_1)(s-s_2)} = \frac{K_0}{s} + \frac{K_1}{s-s_1} + \frac{K_2}{(s-s_2)}$$

$$\Rightarrow K_0 = \frac{AK\omega_n^2}{s_1 s_2} = AK$$

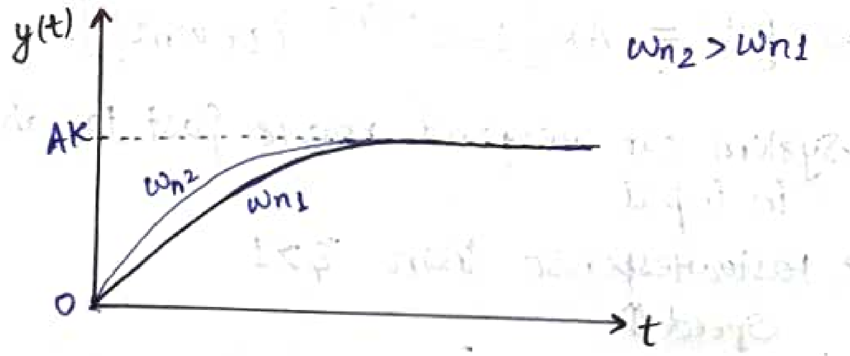
$$K_1 = \frac{AK\omega_n^2}{s_1(s_1-s_2)} = \frac{AK}{[-\xi+\sqrt{\xi^2-1}]\,2\sqrt{\xi^2-1}} = \frac{-AK[\xi+\sqrt{\xi^2-1}]}{2\sqrt{\xi^2-1}}$$

$$K_2 = \frac{AK\omega_n^2}{s_2(s_2-s_1)} = \frac{+AK}{2\sqrt{\xi^2-1}}\left[\xi - \sqrt{\xi^2-1}\right]$$ independent of ω_n

$$\therefore y(t) = \left[K_0 + K_1 e^{s_1 t} + K_2 e^{s_2 t}\right] u(t)$$

$s_1 < 0,\ s_2 < 0$ decaying exponential

$y(t)\big|_{t\to\infty} = K_0 = AK$ (no steady-state error)



- ξ = constant, ω_n vary

$\omega_n \uparrow$ (ω_{n2}) $\Rightarrow$ Exponential decays faster $\Rightarrow$ speed $\uparrow$ $[\because s_1, s_2 \propto \omega_n]$

$\Rightarrow$ Error $\downarrow$

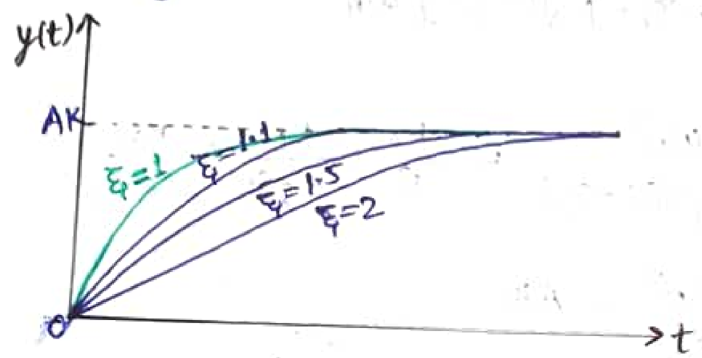
ω_n : determines the speed of response or transient error

- $\xi \uparrow$, ω_n = constant

  $\xi \propto D \rightarrow$ damping $\xi \uparrow \Rightarrow$ speed $\downarrow$

  ↳ System does not respond to sharp changes.

  ↳ system becomes slower.



$\rightarrow \omega_n t$
↳ constant

Assignment:
PDF
① Aim
② code
③ Results
④ Inferences

Plot y(t) vs. t / $\omega_n t$ for different ξ

$A=1$
$K=1$
ω_n

<u>$\xi = 1$</u>

$$Y(s) = \frac{AK\omega_n^2}{s(s+\omega_n)^2}$$

$$= \frac{K_0}{s} + \frac{K_1}{s+\omega_n} + \frac{K_2}{(s+\omega_n)^2}$$

$$\Rightarrow K_0 = AK, \; K_1 = -AK, \; K_2 = -AK\omega_n$$

$$\text{ILT} \Rightarrow y(t) = \left[K_0 + K_1 e^{-\omega_n t} + K_2 t e^{-\omega_n t}\right] u(t)$$

$$\Rightarrow y(t) = AK\left[1 - e^{-\omega_n t}(1+\omega_n t)\right]$$

$\xi = 1$: System can respond more fast to abrupt change in input.

↳ Faster response than $\xi > 1$
Speed $\uparrow$

<u>$\xi < 1$</u>

$$s_1 = \left[-\xi + j\sqrt{1-\xi^2}\right]\omega_n$$

$$s_2 = \left[-\xi - j\sqrt{1-\xi^2}\right]\omega_n$$

$$Y(s) = \frac{AK\omega_n^2}{s(s-s_1)(s-s_2)} = \frac{K_0}{s} + \frac{K_1}{s-s_1} + \frac{K_2}{s-s_2}$$

$$\Rightarrow K_0 = AK$$

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$$K_1 = -\frac{AK[\xi + j\sqrt{1-\xi^2}]}{2j\sqrt{1-\xi^2}}$$

$$K_2 = \frac{AK[\xi - j\sqrt{1-\xi^2}]}{2j\sqrt{1-\xi^2}}$$

$$\therefore y(t) = [K_0 + K_1 e^{s_1 t} + K_2 e^{s_2 t}]u(t)$$

let $\xi = \cos\theta$

$$K_1 = \frac{-AKe^{j\theta}}{2j\sin\theta}$$

$$K_2 = \frac{AKe^{-j\theta}}{2j\sin\theta}$$

$$\therefore y(t) = AK\left[1 - \frac{1}{2j\sin\theta}\left[e^{(s_1 t + j\theta)} - e^{(s_2 t - j\theta)}\right]\right]$$

$$= AK\left[1 - \frac{1}{2j\sin\theta} e^{-\xi\omega_n t}\left(e^{j\underbrace{(\omega_n\sqrt{1-\xi^2}t + \theta)}_{y}} - e^{-j\underbrace{(\omega_n\sqrt{1-\xi^2}t + \theta)}_{y}}\right)\right]$$

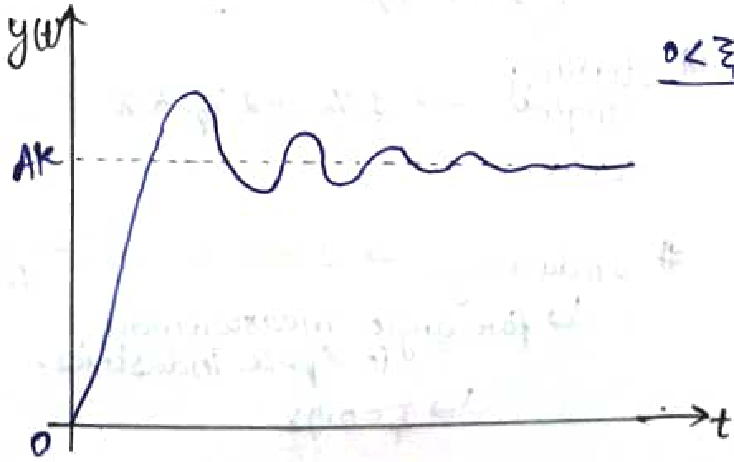
let $\dfrac{e^{jy} - e^{-jy}}{2j} = \sin y$

$$\therefore y(t) = AK\left[1 - \frac{e^{-\xi\omega_n t}}{\sin\theta}\sin(\omega_n\sqrt{1-\xi^2}\,t + \theta)\right]$$

$$y(t) = AK\left[1 - \underbrace{\underbrace{\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}}_{\text{decaying signal}} \times \underbrace{\sin(\omega_n\sqrt{1-\xi^2}t + \cos^{-1}(\xi))}_{\text{sine wave}}}_{\text{decaying sine wave}}\right]$$

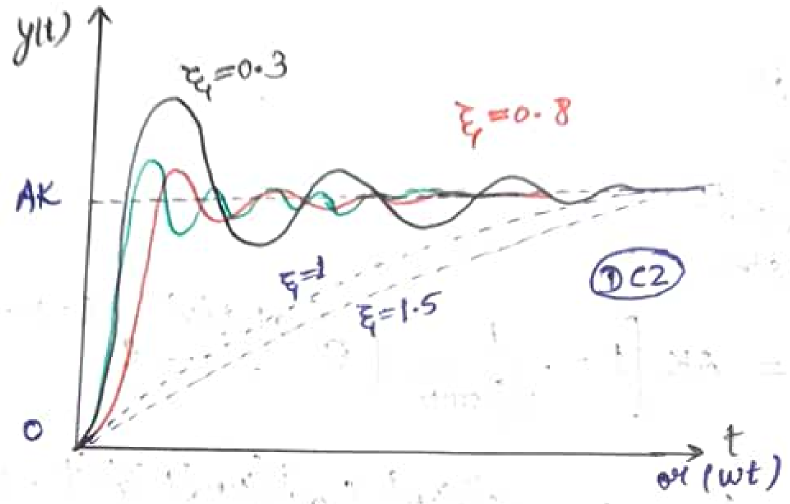
(for $\xi < 1$)

t	$y(t)$
0	0
∞	AK



$0 < \xi < 1$

- ξ = constant ; $\omega_n \uparrow$: Decays faster
  Frequency $\uparrow$
  $\Rightarrow$ Speed $\uparrow$
- ω_n = constant , ξ varies $\rightarrow$ Less overshoot



$\xi > 1$: No overshoots, sluggish (Overdamped)
$\xi = 1$: No overshoots, faster than $\xi > 1$ (critically damped)
$\xi < 1$: overshoots, faster (Underdamped)

we would want our system to be slightly underdamped, i.e., little overshoots.

$\xi = 0.3$ ✗ $\xi = 0.8$ ✓

$\rightarrow$ Ideal value $\rightarrow$ AK

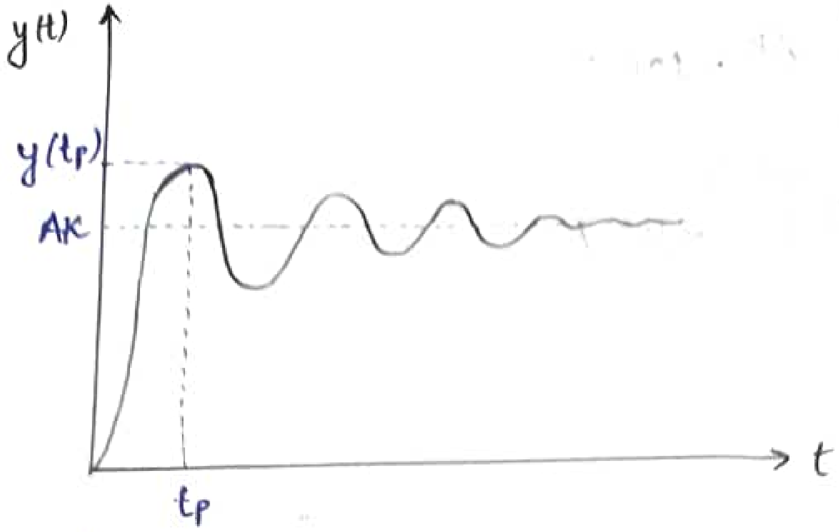
Instrument error $\rightarrow$ 2% (static error) (for all ξ)

$\begin{pmatrix} 0.98\ AK \\ \text{to} \\ 1.02\ AK \end{pmatrix}$ } we want slightly underdamped system.

steering angle sensor $\rightarrow$ 1% $\rightsquigarrow$ $\xi = 0.8$

Inductosyn $\rightarrow$ 1 arc seconds $= \left(\frac{1}{1600}\right)^\circ$
↳ for angle measurement in space industries.
↳ $\xi = 0.98$

$y(t_p)$ calculation :



$$\frac{dy}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}\left[e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}\,t+\theta\right)\right] = 0$$

$$\Rightarrow \sin\left(\omega_n\sqrt{1-\xi^2}\,t+\theta\right)\cancel{e^{-\xi\omega_n t}} \times (+\xi\overset{\cos\theta}{\cancel{\omega_n}})$$

$$\overset{-}{+} \cos\left(\omega_n\sqrt{1-\xi^2}\,t+\theta\right) \times \cancel{\omega_n}\underbrace{\sqrt{1-\xi^2}}_{\sin\theta} \cdot \cancel{e^{-\xi\omega_n t}} = 0$$

$$\Rightarrow \sin\left(\omega_n\sqrt{1-\xi^2}\,t+\cancel{\theta}-\cancel{\theta}\right) = 0$$

$$\Rightarrow \quad t = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$n = 1 \Rightarrow \boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$y(t_p) = AK\left[1 + \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\underbrace{\sin\left(\underbrace{\omega_n\sqrt{1-\xi^2}\,t_p}_{\pi}+\theta\right)}_{-\sin\theta}\right]$$

$$= AK\left[1 - \frac{e^{-\xi\omega_n t_p}}{\cancel{\sqrt{1-\xi^2}}}\cancel{\sin\theta}\right]$$

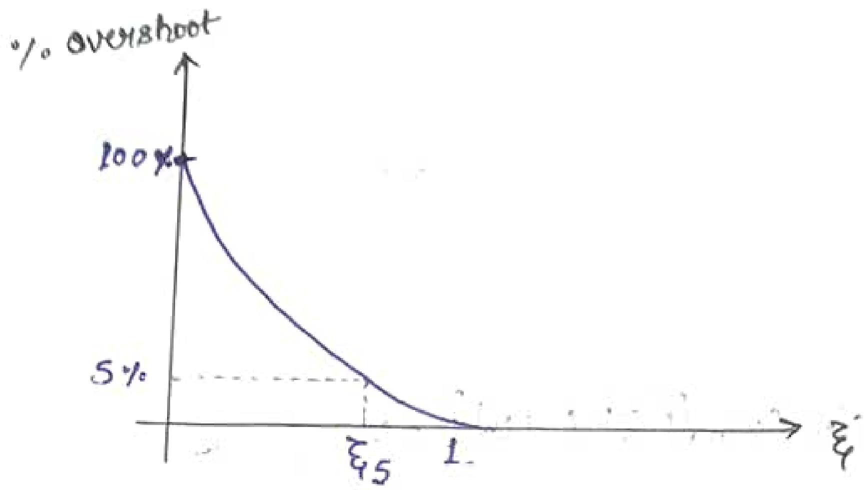
$$= AK\left[1 + e^{-\xi\omega_n t_p}\right]$$

$$\therefore \boxed{y(t_p) = AK\left[1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right]}$$

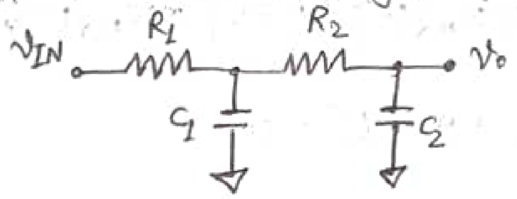
% overshoot (peak)

$$= \frac{y(t_P) - AK}{AK} \times 100\%$$

$$= e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\%$$



Eg. Underdamped 2nd order low pass filter :



$$\xi = \frac{R_1C_1 + R_2C_2 + R_1C_2}{2\sqrt{R_1R_2C_1C_2}} > 1$$

$AM \geq GM$

$$\Rightarrow \frac{R_1C_1 + R_2C_2}{2} \geq \sqrt{R_1R_2C_1C_2}$$

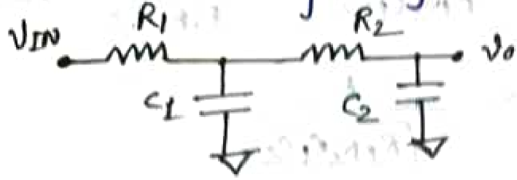
$$\Rightarrow \frac{R_1C_1 + R_2C_2}{2\sqrt{R_1R_2C_1C_2}} \geq 1$$

$$\Rightarrow \frac{R_1C_1 + R_2C_2 + R_1C_2}{2\sqrt{R_1R_2C_1C_2}} \geq 1$$

$\Rightarrow \xi > 1$

$\therefore \xi < 1$ is not possible.

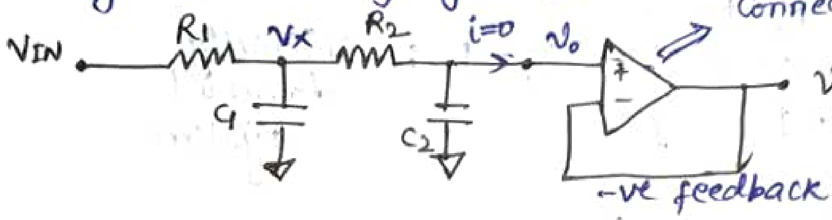
→ Passive RC low pass filter:



$$\xi = \frac{R_1C_1 + R_2C_2 + R_1C_2}{2\sqrt{R_1R_2C_1C_2}}$$

$$\left.\begin{array}{l} R_1 = R_2 = R \\ C_1 = C_2 = C \end{array}\right\} \Rightarrow \xi = 3/2$$

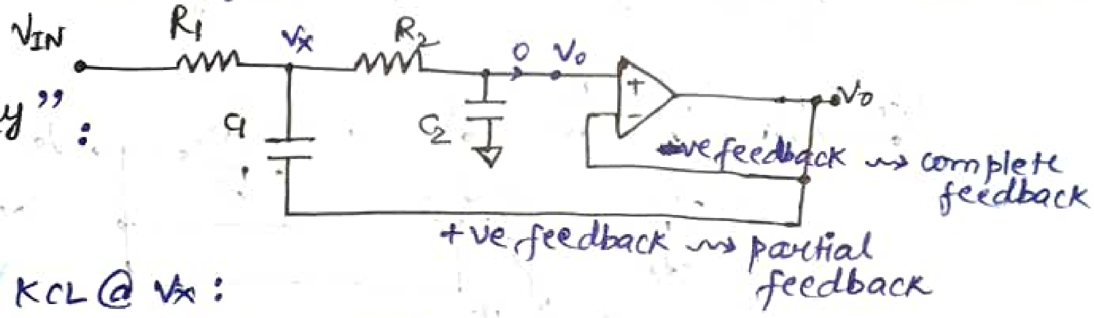
For slight underdamping, $\xi \downarrow$.



⇒ Connected to have zero o/p current for filter

→ Slight underdamping are associated with positive feedback.

"Sallen-Key" : Active LPF



KCL @ V_x:

$$V_x\left[\frac{1}{R_1} + \frac{1}{R_2}\right] + C_1\frac{dV_x}{dt} = \frac{V_{IN}}{R_1} + \frac{V_o}{R_2} + C_1\frac{dV_o}{dt} \quad \ldots (1)$$

KCL @ V_o:

$$\frac{V_o}{R_2} + C_2\frac{dV_o}{dt} = \frac{V_x}{R_2} \quad \ldots (2)$$

(1) and (2) ⇒

$$\left(V_o + C_2R_2\frac{dV_o}{dt}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + C_1\left[\frac{dV_o}{dt} + C_2R_2\frac{d^2V_o}{dt^2}\right] = \frac{V_{IN}}{R_1} + \frac{V_o}{R_2} + C_1\frac{dV_o}{dt}$$

$$\Rightarrow \frac{d^2V_o}{dt} + \frac{dV_o}{dt}\left[\frac{1}{R_1C_1} + \frac{1}{C_2R_2}\right] + \frac{V_o}{R_1R_2C_1C_2} + \cancel{\frac{V_o}{C_1C_2R_2^2}} - \cancel{\frac{V_o}{C_1C_2R_2^2}} - \frac{V_{IN}}{R_1R_2C_1C_2} = 0$$

$$\Rightarrow \frac{d^2V_o}{dt} + \left[\frac{(R_1+R_2)C_2}{R_1R_2C_1C_2}\right]\frac{dV_o}{dt} + \frac{V_o}{R_1R_2C_1C_2} = \frac{V_{IN}}{R_1R_2C_1C_2} \quad \ldots (3)$$

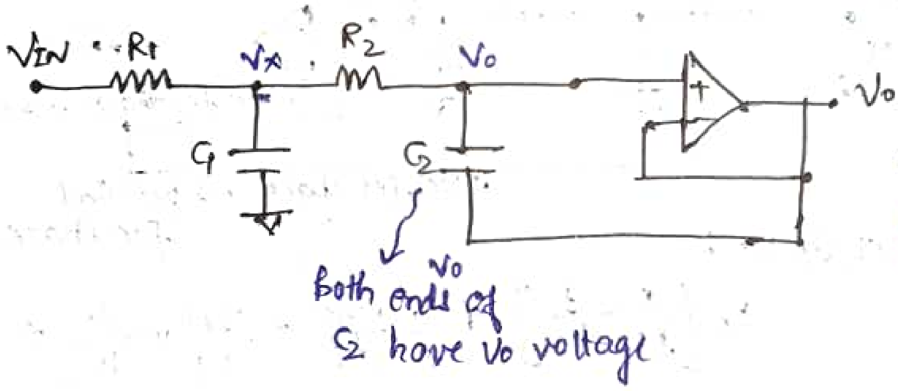
$$\omega_n^2 = \frac{1}{R_1R_2C_1C_2} \Rightarrow \omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$2\xi_1\omega_n = \frac{(R_1+R_2)C_2}{R_1R_2C_1C_2}$$

$$\Rightarrow \xi_1 = \frac{(R_1+R_2)C_2}{2\sqrt{R_1R_2C_1C_2}}$$

$$\left.\begin{array}{l} R_1=R_2=R \\ C_1=C_2=C \\ \xi=1 \end{array}\right\} \Rightarrow \xi_1 = 1 \;:$$ critically damped system

$$\left.\begin{array}{l} R_1=R_2=R \\ C_1=4C \\ C_2=C \end{array}\right\} \Rightarrow \xi_1 = 0.5 \;:$$ Underdamped system



→ Useless circuit ✗

Speed of Response of 2nd Order Instruments:

$\xi < 1$: $y(t) = AK\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n\sqrt{1-\xi^2}\, t + \theta\right)\right]$

(slightly underdamped)

$\rightarrow t = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$

$\rightarrow$ % max overshoot $= e^{\left(\frac{-\xi n\pi}{\sqrt{1-\xi^2}}\right)} \times 100\%$

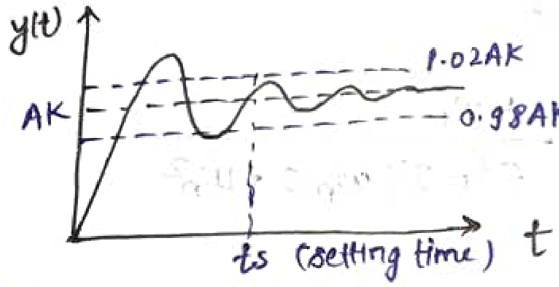
$\Rightarrow y(t) = AK\left[1 - \frac{e^{-\xi\omega_n t}}{\cancel{\sqrt{1-\xi^2}}}\left(\pm \cancel{\sin\theta}\right)\right]$

$\Rightarrow y(t) = AK\left[1 \pm e^{-\xi n\pi/\sqrt{1-\xi^2}}\right]$

% overshoot / undershoot $= \frac{y_{max} - y(\infty)}{y(\infty)} \times 100\%$

$= \pm e^{\left(\frac{-\xi\omega_n t}{\sqrt{1-\xi^2}}\right)} \times 100\%$

Setting Time



Sensors $\longrightarrow$ 2% error

$$e^{-\frac{\xi n\pi}{\sqrt{1-\xi^2}}} \times 100\% < 2\%$$

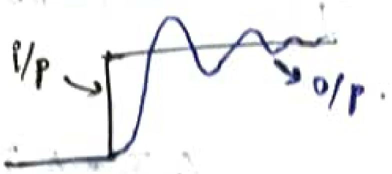
$$-\frac{\xi n\pi}{\sqrt{1-\xi^2}} < -4 \Rightarrow \frac{\xi n\pi}{\sqrt{1-\xi^2}} > 4 \quad \ldots ①$$

$$\boxed{t_s > \frac{4}{\xi\omega_n}}$$

General case : $e^{\frac{-\xi n\pi}{\sqrt{1-\xi^2}}} \times 100\% < M\%$

$$\rightarrow \boxed{t_s > \frac{\ln(100/M)}{\xi\omega_n}}$$

DC 4 : Design a Sallen-Key LPF → for typical values of $\xi < 1$. Simulate the circuit with step signal.



21-02-2024

Ramp Response of 2nd Order Instruments

$x(t) = \dot{A}(t)$

Plot $y(t)$, for $\xi > 1, =1, < 1$.

DC5

Derive and obtain the characteristic of $y(t)$ for $\xi > 1, =1, <1$ and plot.

Frequency response of 2nd order Instruments

$x(t) = A \sin \omega t$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Put $s = j\omega$.

$$\Rightarrow H(j\omega) = \frac{K\omega_n^2}{\omega_n^2 - \omega^2 + 2\xi\omega_n\omega j}$$

$$y(t) = A|H(j\omega)| \sin(\omega t + \angle H(j\omega))$$

$$|H(j\omega)| = \frac{K\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2) + 4\xi^2\omega^2\omega_n^2}}$$

$$= \frac{K}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

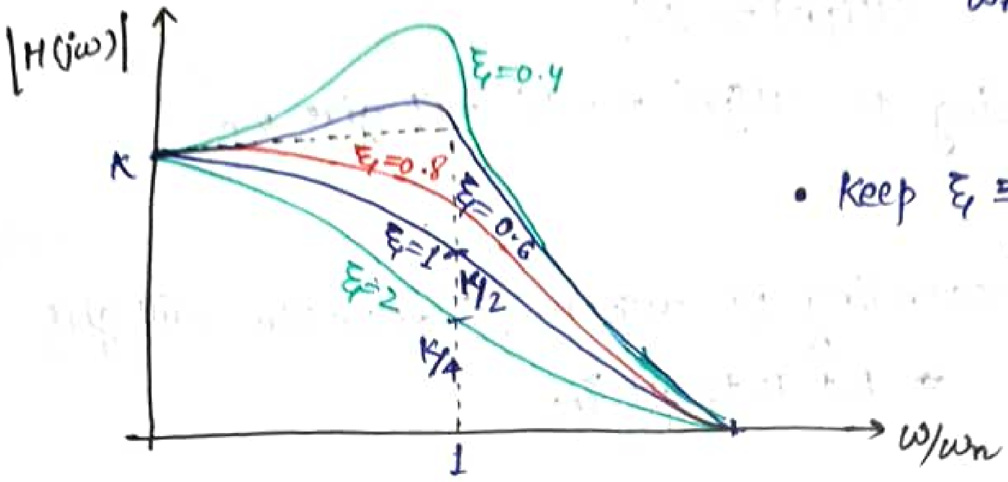
$|H(j\omega)| \approx K$, for a wide span of ω.

$\omega \ll \omega_n \Rightarrow |H(j\omega)| \approx K$.

- $\omega_n \rightarrow$ fixed, ξ vary.

$$\left|H(j\omega)\right|_{\omega=\omega_n} = \frac{K}{2\xi}$$

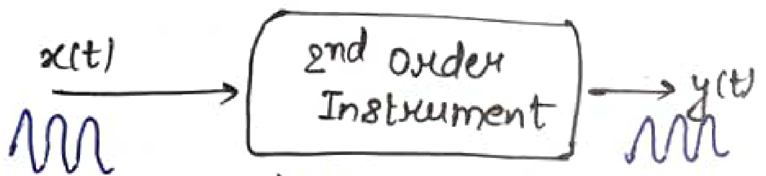
ω	$\lvert H(j\omega)\rvert$
0	K
∞	0
ω_n	$K/2\xi$



- Keep $\xi \equiv 0.6$ to 0.9
  ↳ to keep slightly underdamped.

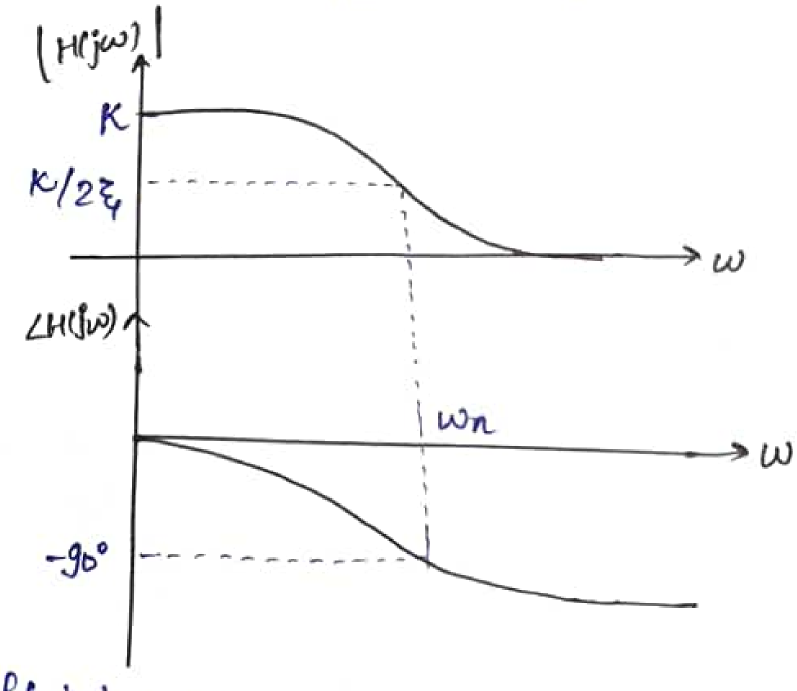
$\xi \downarrow \Rightarrow$ Bandwidth $\uparrow$

Instrument Identification



↳ Parameters: K, ξ, ω_n

→ Obtain the parameters K, ξ, ω_n without looking at the instrument (blackbox) by observing $x(t)$ & $y(t)$.



- Obtaining K:

Output, $y(t)$, corresponding to DC input, $x(t)$, will give K.

Obtaining ω_n:

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{2\xi_1 \omega_n \omega}{\omega_n^2 - \omega^2}\right)$$

$$\omega = \omega_n \Rightarrow \angle H(j\omega) = -90^\circ$$

- ω corresponding to $\angle H(j\omega) = -90^\circ$ will give ω_n.

Obtaining ξ_1:

$|H(j\omega)|$ corresponding to $\angle H(j\omega) = -90^\circ$ or $\omega = \omega_n$ will give $K/2\xi_1$ $\Rightarrow$ Put K to get ξ_1.

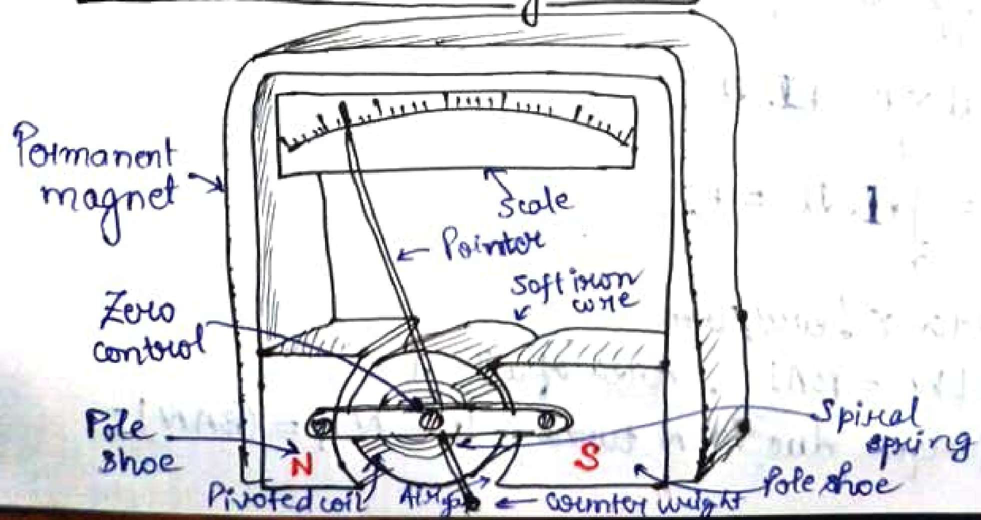
21-02-2024

INSTRUMENTS

- Analog Meters
- Digital Meters

	Analog Meters	Digital Meters
Best possible accuracy	0.1 %	0.005 % or better
Typical Resolution	1/100th of Full scale	1/10000th of Full-scale
Speed	slow (Electro-mechanical principles used)	very fast, depends on clock frequency
Noise and Errors	Human Parallax Errors	Less error-prone
	(Accuracy can be reduced by operating near full-scale/proper range selection)	
Construction	Simple, rugged construction, performed well in harsh conditions	Less rugged instruments, special care is needed for the electronics
Power Requirements	Derives power from Measurand (no need of battery)	Auxiliary Power source needed.
Information Storage	Not possible	Possible (USB, RS232, etc.)
Application	Process control, panel displays, Flowmeters, Pressure gauges, Switch-board meters, etc. (low-cost indication & maintenance, good performance in harsh conditions, less training for operators)	

Most Common Analog Meters



Permanent Magnet Moving Coil (PMMC) Meter
↓
Basic Ammeter

Major Parts:
- Permanent Magnet
- Coil carrying unknown current

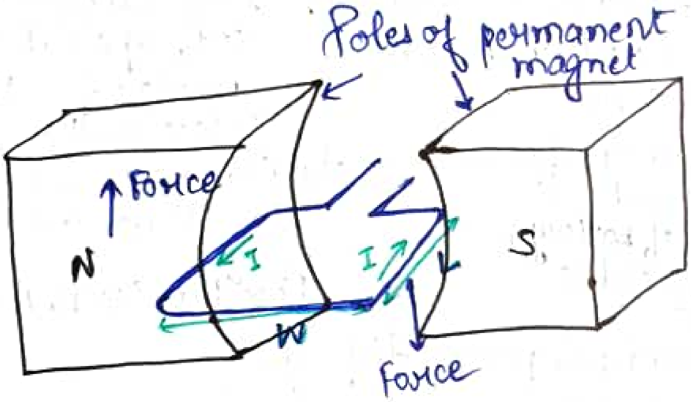
Additional Parts:
- Soft Iron Core
- Counter weight
- Zero Control

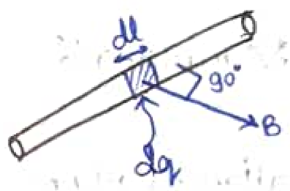
Basic Principle: D'Arsonval Movement

Shoft Iron core : large relative permeability (μ_r)
↳ helps concentrate the flux into the region of coil.

To remove zero error, move screw to move pointer to zero.

PMMC Meter





dl : infinitesimal length
dq : Charge flowing through ds in time dt
B : magnetic field provided by magnet
v : velocity of charge carriers

Force (df) on element ds,

$$df = dq(v \times B)$$

$$Idl = \frac{dq}{dt} . dl = dq . v$$

$$df = I.dl \times B = BI.dl$$

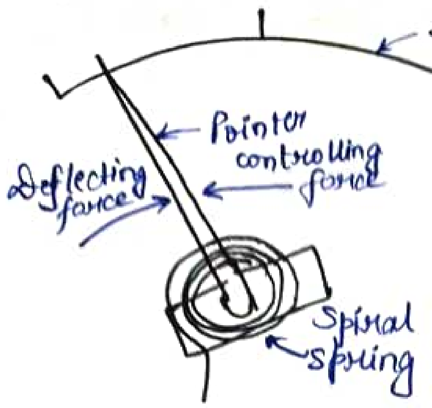
$$\therefore F = \int_0^L df = \int_0^L BI . dl = BIL$$

Torque = Force × Lever Arm

$T_1 = BILW = BAI$, Area of coil = A

Deflecting torque due to N turns, $T_d = NT_1 = (BAN)I$

Restoring torque by Spring:



Current, I

⇓

Torque due to current,

$T_I = NT_I = (BAN)I$

⇓

Coil system rotates

⇓

Spring to wind-up and introduce a restoring (controlling) torque (say, T_R)

At equilibrium, $T_R = T_I$

$$K_s . \theta = (BAN) I$$

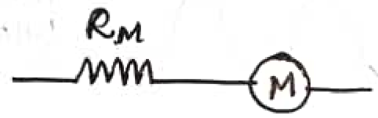
$$\theta = \left(\frac{BAN}{K_s}\right) I = K \times I$$

Deflection of pointer, connected to Spring, can be used to measure I

⇓

Linear Instrument

Equivalent Electrical Model of PMMC Meter:



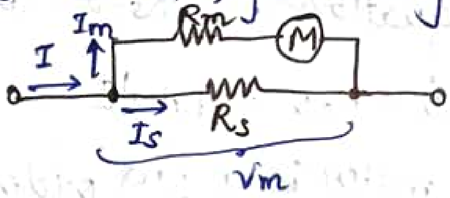
R_M: Resistance of coil in the meter

DC Ammeter using PMMC Principle:

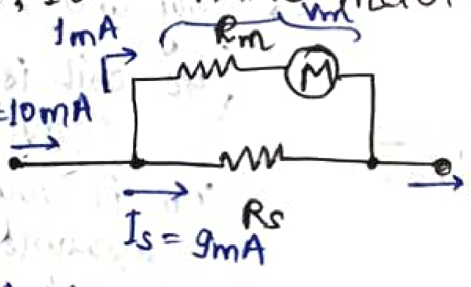
→ Coil has a finite resistance:

↳ Basic PMMC meter can only measure limited current (e.g., <10mA).

→ To measure larger current, place a proper Shunt R.



Eg. Convert a 1mA, 100Ω PMMC meter to an ammeter of 10mA range.

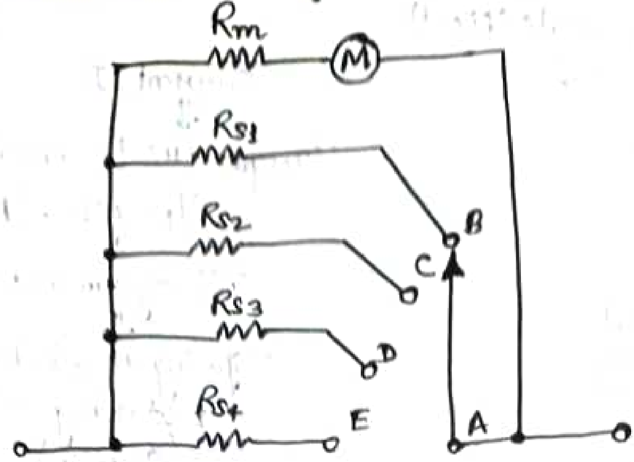


10mA = 9mA × R_s + 1mA × R_m

$$1mA \times R_m = 9mA \times R_s$$

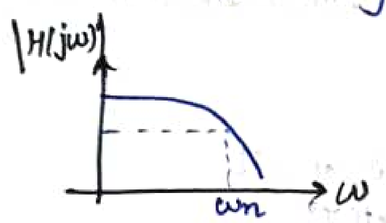
$$\Rightarrow R_s = \frac{R_m}{9}$$

Multi-Range Ammeter using PMMC Principle:

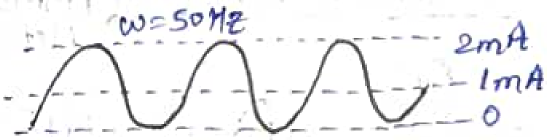


Dynamic Characteristics of PMMC Meter

→ PMMC meter responds to varying current.



$i = [1 + \sin\omega t]$ mA



Meter will display average value = 1mA

→ PMMC meter do not respond to AC currents.
↳ settles to DC value of input.

→ Coil system is rotating.
↳ has a moment of inertia, J.
↳ damping coefficient, D.

Steady state equation : $K_s \cdot \theta = (BAN) I$

$$J\ddot{\theta} + D\dot{\theta} + K_s\theta = (BAN) I$$

∴ PMMC meter is a 2nd order instrument.

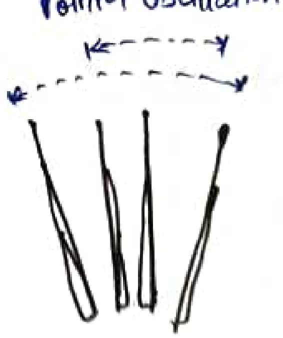
$$\omega_n = \sqrt{\frac{K_s}{J}}$$ ⟶ very low value (<1 rad/s)
(b/c coil is delicate and has significant MOI)

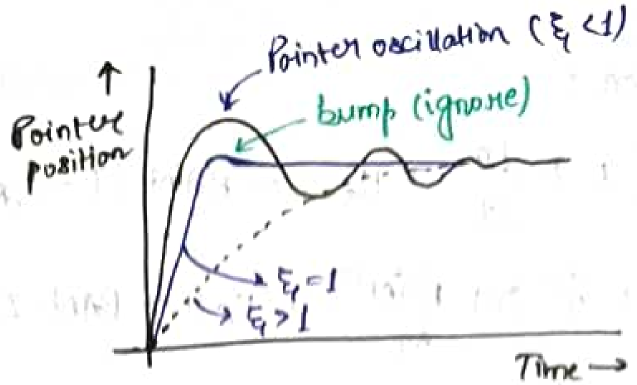
$$\xi = \frac{D}{2\sqrt{K_s \cdot J}}$$

[ω_n is very low, so we can't measure AC signals with that]

$\xi > 1$: Overdamped PMMC
$\xi = 1$: Critically-damped PMMC
$\xi < 1$: underdamped PMMC

ξ or D : can be varied using:
- air-damping
- Eddy current damping (Al frame)
- Motional EMF





$\xi < 1$: We don't want overshoot as it may damage the meter (needle crossing maximum deviation).

- ξ close to 1 is preferred.

$$\xi = \frac{D}{2\sqrt{K_s \cdot J}}, \quad D, K_s, J : \text{magnetic parameter}$$

↳ no control over these

↳ To increase damping, use Al coil, causing eddy current which opposes the motion (causing damping).

Induced emf in transient state, [Transient Motional EMF]

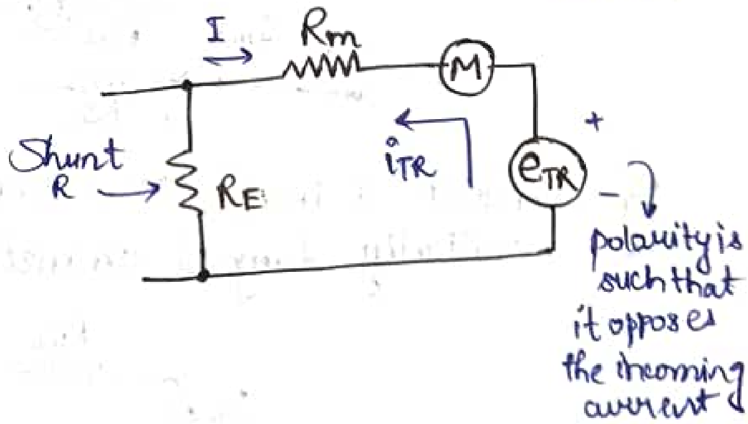
$$e_{TR} = B\,2LN\,V_{TR}, \quad B: \text{magnetic field}$$
V_{TR}: velocity of rotation.

Eliminate V_{TR}: $V_{TR} = \frac{W}{2} \cdot \frac{d\theta}{dt}$

$$\therefore e_{TR} = BAN\dot{\theta}$$

Current due to e_{TR}, $i_{TR} = \frac{BAN}{R_T} \cdot \frac{d\theta}{dt}$, R_T: total resistance in the coil circuit $= R_m + R_E$

$$= \frac{e_{TR}}{(R_m + R_E)}$$



Effective current on meter $= I - i_{TR}$

$$\therefore \boxed{i_{TR} = \frac{BAN}{RT}\,\frac{d\theta}{dt}}$$

Effective expression:

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K_s.\theta = BAN\,(I - i_{TR})$$

$$\Rightarrow J\ddot{\theta} + \dot{\theta}\left(D + \frac{B^2A^2N^2}{R_T}\right) + K_s\theta = BAN.\,I$$

↳ ξ get affected

↳ we have electrical control over ξ.

$$\xi = \frac{\left[D + \frac{B^2A^2N^2}{R_T}\right]}{2\sqrt{K_sJ}}$$

$$= \frac{\left[D + \frac{B^2A^2N^2}{R_M + R_E}\right]}{2\sqrt{K_sJ}}$$

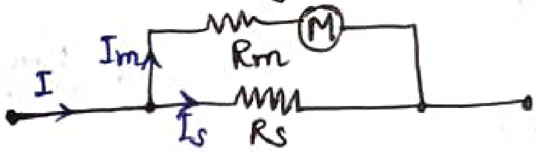
↳ The value of R_E which ensures critical damping : Critical Resistance

[we can get $\xi = 1$]

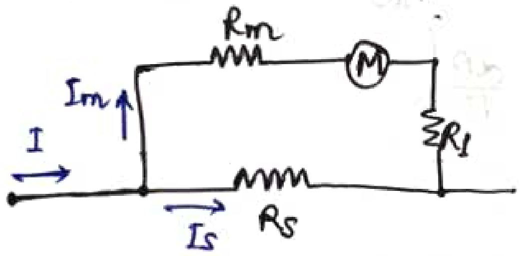
$\xi = 1 \rightarrow R_C = R_E$

26-02-2024

Critically Damped DC ~~Using~~ Ammeter using PMMC Principle



Eg. Convert a 1mA, 100Ω PMMC meter ~~using~~ with $R_C = 200\Omega$ to a critically damped ammeter of 10mA range.



External resistance, $R_E = R_1 + R_s$
$= 200\,\Omega$

$I_m(R_m+R_1) = I_s R_s$

$\Rightarrow I_m(100+R_1) = 9mA \cdot R_s$

$\Rightarrow R_s - R_1 = 100\,\Omega$

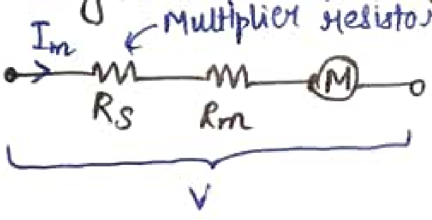
Also, $R_E = R_s + R_1 = 200\,\Omega$

↳ external R.

$\therefore 10R_s = 300\,\Omega \Rightarrow R_s = 30\,\Omega$

and, $R_1 = 200 - 30 = 170\,\Omega$.

DC Voltmeter Using PMMC Principle



I_{FS} : full-scale current

θ_{FS} : full-scale deflection

Eg. Convert a 1mA, 100Ω PMMC meter to a 10V voltmeter.

$$\frac{10}{R_s+R_m} = 1mA \Rightarrow R_s = 9.9\,k\Omega.$$

For an PMMC meter, output is θ;
input is I.

We can define our own sensitivity as

$$S = \frac{\theta}{I}$$

Since, PMMC is linear, $S = \dfrac{\theta_{FS}}{I_{FS}}$

Normalised w.r.t. θ, (since θ is mechanical parameter)

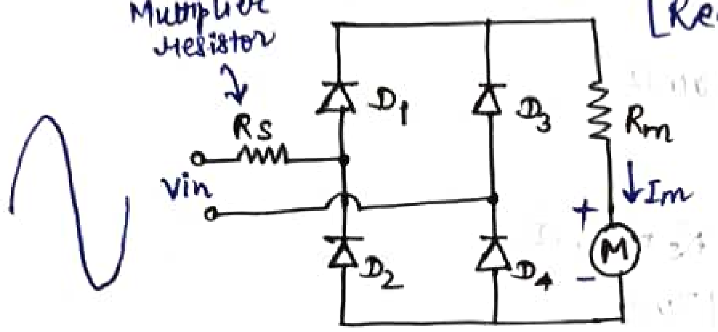
~~Voltmeter~~ Sensitivity

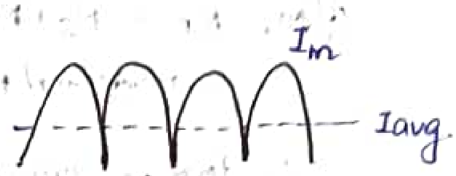
$$\left\{\begin{array}{ll} S = \dfrac{1}{I_{FS}} & : \text{Ammeter sensitivity} \\ \text{or} & \\ S = \dfrac{R_s + R_m}{I_{FS}} & : \text{voltmeter sensitivity} \end{array}\right.$$

AC Voltmeter using PMMC Principle

→ RMS value of the AC input needs to be measured.

[Rectifier-Type PMMC]





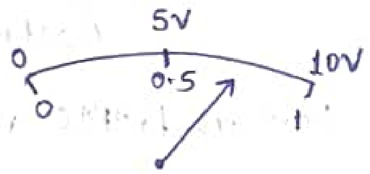
Vin > 0, D_1, D_4 ON
Vin < 0, D_3, D_2 ON

$$\boxed{I_{avg} = \frac{2}{\pi} I_{peak}}$$ (Full wave Rectifier)

$$\boxed{I_{rms} = \frac{I_{peak}}{\sqrt{2}}}$$

Eg. Convert a 1mA, 100 Ω PMMC meter to a 10V AC voltmeter.

RMS value → 10 V
Peak value → $10\sqrt{2}$ V
Peak current → $\frac{10\sqrt{2}}{R_s + R_m}$



Now, $\frac{2}{\pi}\left(\frac{10\sqrt{2}}{R_s + R_m}\right) = 1mA$, $R_m = 100\,\Omega = 0.1\,k\Omega$

$\Rightarrow R_s = 8.9\,k\Omega$

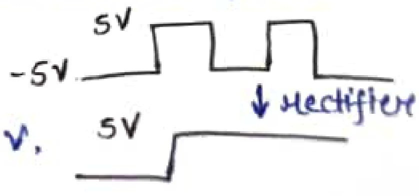
$\therefore$ 10V RMS ⟷ 1mA (meter)

→ Due to non-ideal behaviour of diode (V_D, & R_{on}), there will be error.

→ If we measure square wave with this, then after rectifier this square wave will convert to DC.

V_{avg} : DC value only.

So for 5V → $V_{avg.}$, here, is 5V.



$\therefore \frac{5}{8.9+0.1} = \frac{5}{9} = 0.55\,mA > 0.5\,mA$

Disadvantages (Issues) of PMMC :

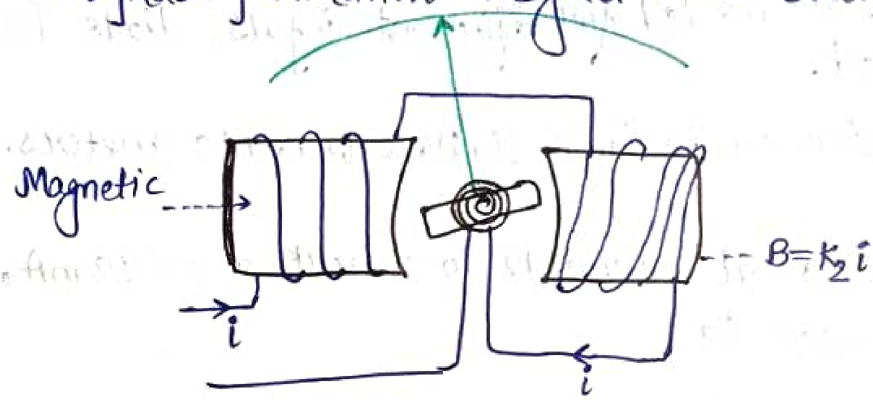
→ Cannot be used for AC measurement without rectifier circuits (Threshold voltage, ON resistance of diodes).

→ Not a generic solution for AC voltage measure

↳ solution: Introduce separate scale for different inputs.

Modified Principle

↳ Replace permanent magnet with electromagnet.



Now, $B = k_2 i$

$$T_1 = (BAN) i$$
$$= (k_2 AN) i^2$$
$$= K_s \cdot \theta$$

$$\boxed{\theta = \left(\frac{k_2 AN}{K_s}\right) i^2}$$

Now, Since i is AC

$$\theta = k i^2$$

↓

θ will also be ac.

$\theta_{avg.} = K \cdot \frac{1}{T} \int_{\langle T \rangle} i^2 dt$, $\boxed{K = \frac{k_2 AN}{K_s}}$

$\boxed{\theta_{avg.} = K I_{rms}^2}$

In case of DC, $\boxed{\theta = K I^2}$

} so, scaling factor is same for AC and DC.

27-02-2024

→ Same current is connected to the coil through the spring.

↳ <u>Electrodynamometer Principle</u> (A modified principle)

→ Two coils: One fixed and one movable; both carry unknown current.

→ Gives the square of rms value of any periodic input.

→ Non-linear scale.

→ Have higher current-rating compared to PMMC.

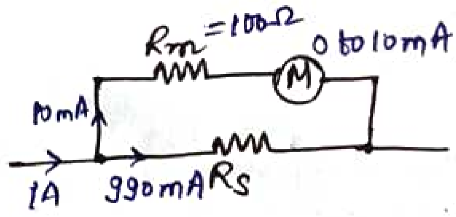
→ Calibration can be done for DC and same instrument can be used for AC measurement.

→ No separate scale needed for different inputs, diode bridge not required.

→ Range extension can be done similar to PMMC meters.

Eg. Design suitable shunt networks to convert a 0–10 mA, 100 Ω electrodynamometer to

ⓐ 0–1 A DC ammeter.



$$10 \times R_m = R_s \times 990 \Rightarrow R_s = \frac{R_m}{99} = \frac{100}{99}$$

$$\Rightarrow R_s = 1.01\,\Omega$$

ⓑ 0–1 A AC ammeter

↳ No change of such R_s, $R_s = 1.01\Omega$.

ⓒ 0–100 A AC ammeter

↳ Approx $R_s = \frac{10 \times 100}{100} = 10\,m\Omega$.

Factors during High current measurement:

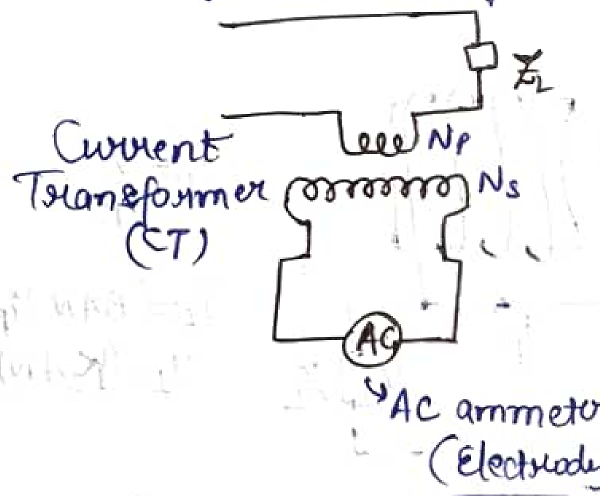
1. Shunt resistance becomes very low.
2. Large power dissipation ($I_s^2 R_s$ is very high).
3. Risk to operator.

Factors during High Voltage measurement:

1. Series resistance becomes very high.
2. Resistance leakage.
3. Large power dissipation.

Solution for high ~~power~~ current measurement:

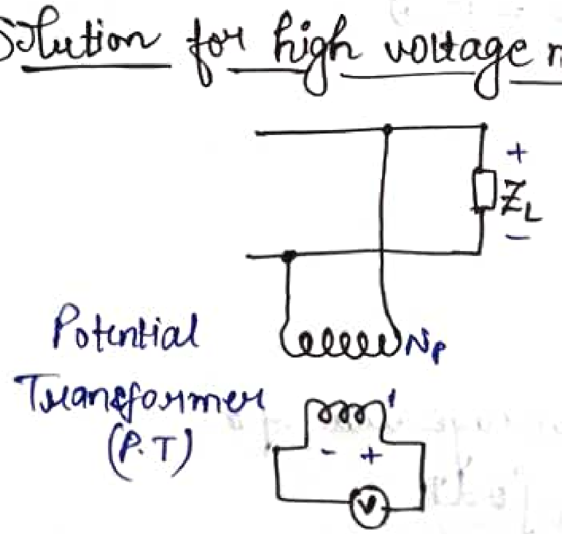
↳ Put a step down transformer.



$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \Rightarrow \boxed{I_p = \left(\frac{N_s}{N_p}\right) I_s}$$

↳ turns ratio, 'n'

Solution for high voltage measurement:



$$\frac{V_s}{V_P} = \frac{N_s}{N_P}$$

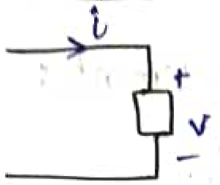
$$\Rightarrow \boxed{V_P = \left(\frac{N_P}{N_s}\right) V_s}$$

↳ Turns ratio, 'n'

Instrument Transformer

- Current Transformer (C.T.)
- Potential Transformer (P.T.)

Power Measurement:



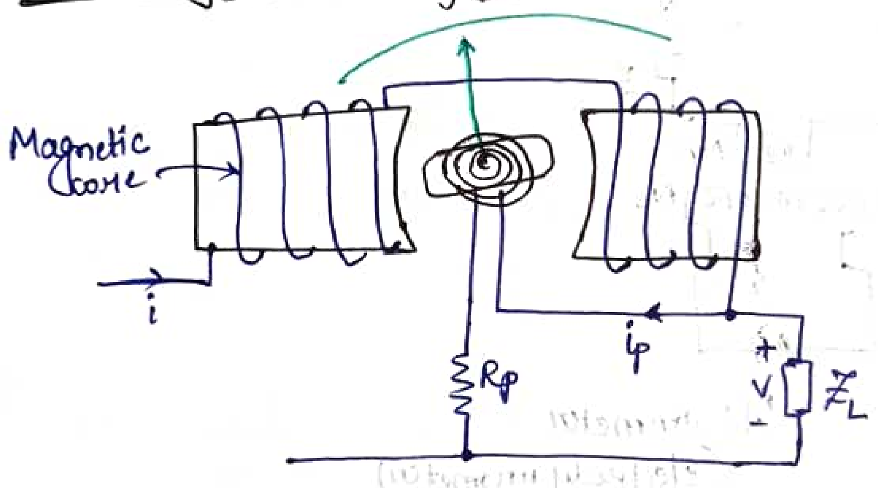
$$P_{avg.} = \frac{1}{T}\int_{<T>} v.i\,dt$$

$$v = \sqrt{2}\, V_{rms} \cos\omega t$$

$$i = \sqrt{2}\, I_{rms} \cos(\omega t - \phi)$$

$$\therefore \boxed{P_{avg.} = V_{rms}.\, I_{rms}.\cos\phi}$$

Electrodyamometer for Wattmeter



$$i_p = \frac{V}{R_p}$$

$$T_I = (BAN)\, i_p$$

$$T_I = (K_2 A N)\, i_p$$

$$T_I = (K_2 A N)\, i \frac{V}{R_p}$$

$$= \left(\frac{K_2 A N}{R_p}\right) Vi = K_S.\theta$$

$$\theta = \underbrace{\left(\frac{K_2 A N}{R_p . K_s}\right)}_{K_p} Vi$$

$$\Rightarrow \theta = K_p.\, Vi$$

→ Meter will only show average value of θ:

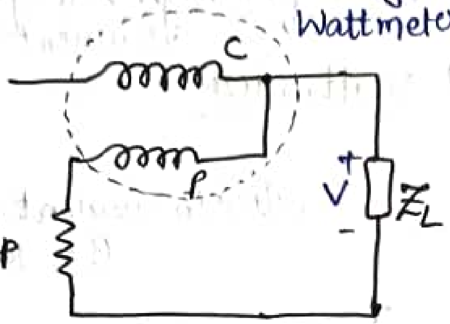
$$\theta_{avg.} = \frac{1}{T}\int_{<T>} \theta\, dT$$

$$\theta_{avg.} = \frac{k_P}{T}\int_{\langle T\rangle} vi\,dt$$

$$\therefore \boxed{\theta_{avg.} = k_P \cdot P_{avg}}$$

Fixed coil : Current coil (C) → Thick wires

Movable coil : Voltage / Pressure coil (P) → Thin wires



→ Gives average power for periodic inputs.

→ Calibration can be done for DC and same calibration will hold good for AC measurement.

$$\theta_{avg} = \frac{k_P}{T}\int_{\langle T\rangle} v_P . i_c\,dt \quad , v_P = v, \quad i_c = i + i_P$$

$$= \underbrace{\frac{k_P}{T}\int_{\langle T\rangle} vi\,dt}\ + \frac{k_P}{T}\int_{\langle T\rangle} v\,i_P\,dt \qquad [v = i_P R_P]$$

$$\Rightarrow \theta_{avg} = \theta_{true} + \frac{k_P}{T}\int_{\langle T\rangle} v\,i_P\,dt$$

$$= \theta_{true} + \underbrace{\frac{k_P R_P}{T}\int_{\langle T\rangle} i_P^2\,dt}_{\text{+ve error}}$$

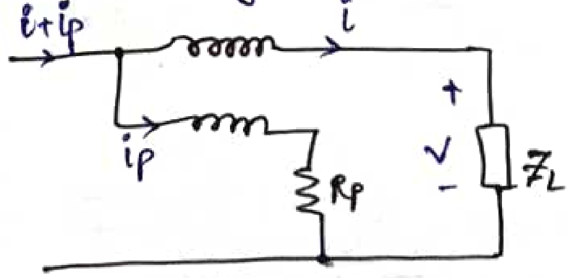
$$\therefore \boxed{\theta_{avg.} > \theta_{true}}$$

Now, to minimise error,

$$i \gg i_P \Rightarrow i \gg \frac{v}{R_P}$$

↳ This instrument ~~will~~ is more suited for high current, low voltage load.

Now,



R_c : Resistance of coil

$$\theta_{avg} = \frac{K_P}{T}\int_{<T>} i(v + iR_c)\,dt$$

$$= \frac{K_P}{T}\int_{<T>} vi\,dt + \frac{K_P}{T}\int_{<T>} i^2 R_c\,dt$$

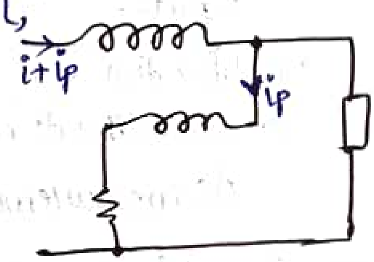
$$\Rightarrow \theta_{avg} = \theta_{true} + \frac{K_P}{T}\int_{<T>} i^2 R_c\,dt$$

$$\therefore \boxed{\theta_{avg.} > \theta_{true}}$$

To minimise error, $v \gg iR_c$: Suited for high voltage, low current

Universal Wattmeter : Compensented Wattmeter

In current coil,



Earlier in current coil,
$B = K(i + i_p)$

We have to remove the effect of i_p in current coil.
So, we can direct i_p wire into same coil in opposite way.

So, net $B = K_p(i + i_p) - K_p i_p$
$= K_p i$

↳ Can be used to detect AC only.
↳ Intrusive measurement.

→ Non-intrusive measurement is using measurement of magnetic field due to current-carrying coil.

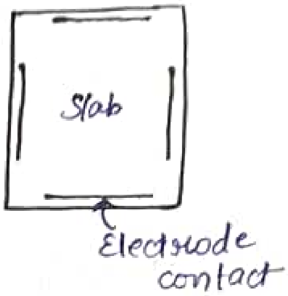
04-03-2024

Hall Effect Sensors

$v_H = K_T B_m$

Sensitivity, $K_T = \dfrac{I}{nq\,d_H} = 0.625\ \dfrac{\mu V}{T}$

$= 0.625\ \dfrac{nV}{mT}$ >> noise



$I = 1mA$

$q = 1.6 \times 10^{-19}\ C$

$n = 10^{27}\ m^{-3}$

$d_H = 10\ \mu m$

Improving K_T:

$I \uparrow$ (not preferable) ↳ more power consumption

$n \downarrow$

$d_H \downarrow$ (already very low)

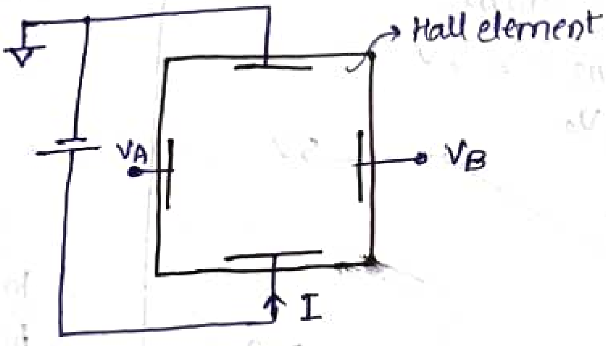
$\Rightarrow n \downarrow$

Take $n = 6 \times 10^{21}\ m^{-3}$ (doped si)

$\therefore K_T = 0.1\ \dfrac{mV}{mT}$

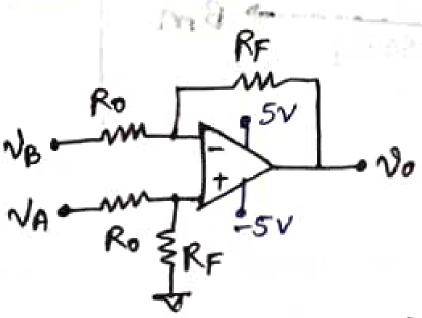
$= 100\ \dfrac{\mu V}{mT}$

Commercially available hall effect sensors are made up of doped semiconductor, and not metal, for this reason.



$v_H = v_A - v_B$

Difference amplifier:

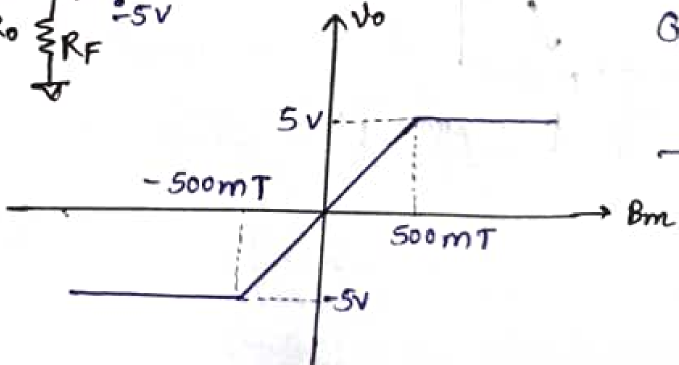


$v_o = G_D(v_A - v_B) = G_D K_T B_m$,

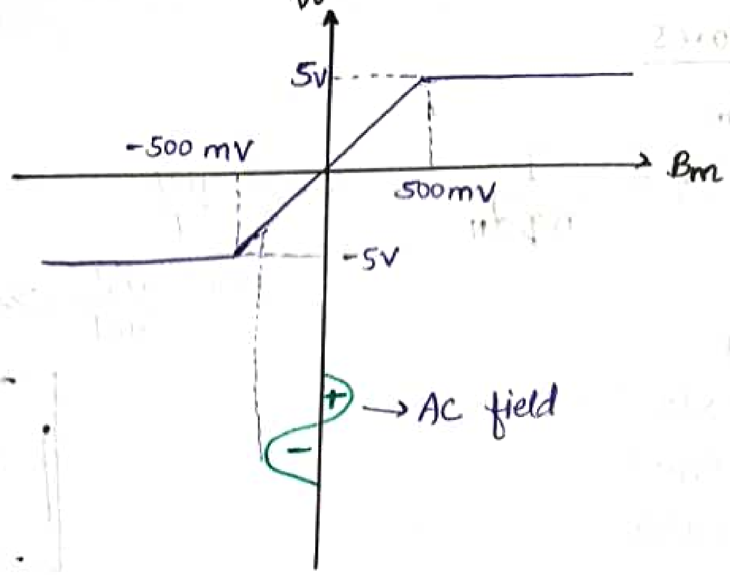
where $G_D = \dfrac{R_F}{R_o} = 100$

$G_D K_T = 10\ \dfrac{mV}{mT}$

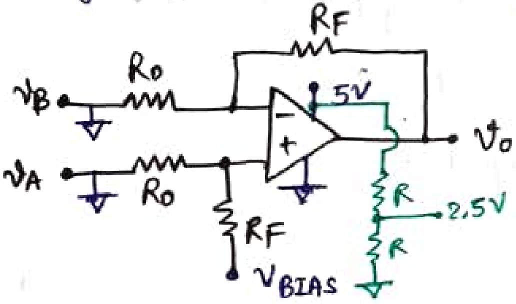
IC:



→ Limitation due to power supply, not hall effect sensor.



- We want only one power supply for the opamp (say, only 5V) while detecting both positive and negative AC fields.



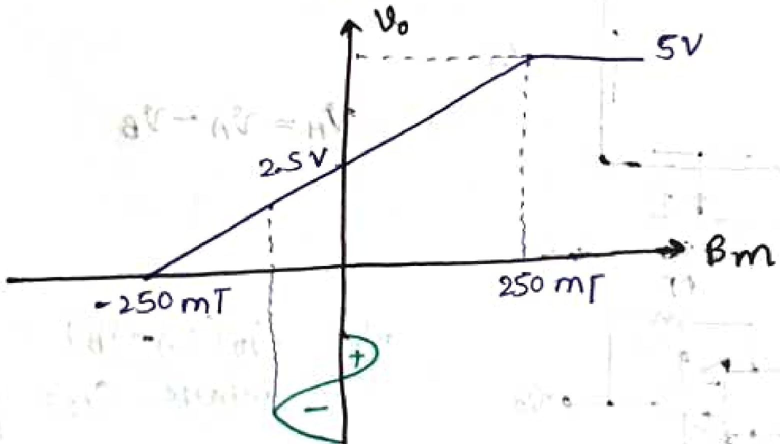
(Superposition principle)

$$v_o = G_D K_T B_m + V_{BIAS} \frac{\cancel{R_o}}{\cancel{R_o + R_F}} \left(\frac{\cancel{R_o + R_F}}{\cancel{R_o}} \right)$$

← gain

$$v_o = G_D K_T B_m + V_{BIAS}$$

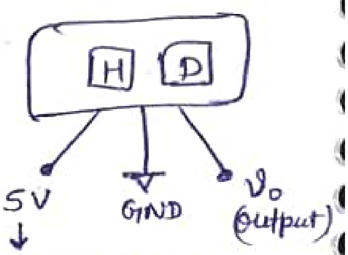
Take $V_{BIAS} = 2.5V$.



We get positive voltage for negative fields.

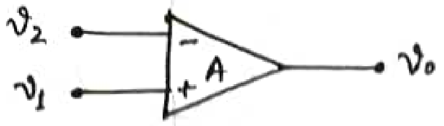
SS495 IC

3-pin solution



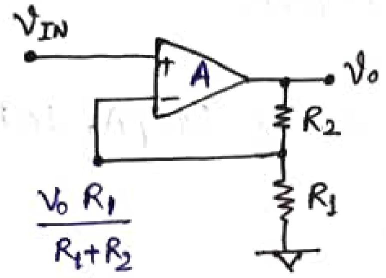
5V ↓ to power both hall-effect sensor and opamp.

Measurement Circuits



A: open loop gain of the opamp

$$v_o = A(v_1 - v_2)$$



$$\frac{V_o R_1}{R_1+R_2}$$

$$v_o = \left(1+\frac{R_2}{R_1}\right) V_{IN}$$

$$v_o = A\left[V_{IN} - v_o \frac{R_1}{R_1+R_2}\right]$$

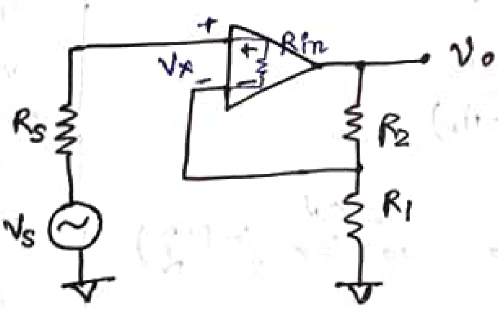
$$= \left[\frac{1+R_2/R_1}{1+\frac{1}{A}(1+R_2/R_1)}\right] V_{IN}$$

$$= \frac{G_0 V_{IN}}{1+\frac{G_0}{A}}, \quad G_0 = 1+\frac{R_2}{R_1}$$

For $G_0 \ll A$ or $\frac{G_0}{A} \ll 1$,

$$v_o = G_0 V_{IN}.$$

→ $G_0 = 2$ (say)



$$V_x = \frac{R_{in} V_s}{R_{in}+R_s}$$

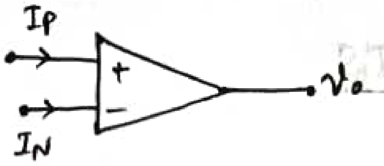
[we want V_s to be directly fed to the opamp for amplification
⇒ R_{in} ~~<<~~ >>]

$R_s \approx 100\,M\Omega$ (very large input impedance)

$R_{in} \gg R_s$ (~~small~~ large input resistance, R_{in})

BJT-based opamp	FET-based opamp
OP07	TL082
LM741	LF0356

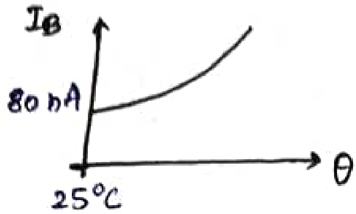
FET-based opamp has higher input impedance
⇒ better than BJT-based-opamp.

→ 

Bias current, $I_B = \frac{I_P + I_N}{2}$

Offset current, $I_{os} = |I_P - I_N|$

LM741
$I_B \approx 80\,nA$
$I_{os} \approx 3\,nA$
$I_{os} < I_B/10$



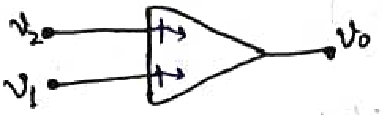
→ Must look for this in the datasheet.

For current measurement in the range (10 pA, 10 μA).

Electrometer ⇝ e.g., OPA 128

$I_B \approx 100\,fA$

5-03-2024

→ 

A_D : difference gain (= open loop gain)

LM741 : CMRR ≈ 80 dB
OP07 : CMRR ≈ 120 dB

Ideal: $V_o = A_D (V_1 - V_2)$

Actual:

$$v_o = A_1 V_1 - A_2 V_2$$
$$= A_D (V_1 - V_2) + \underbrace{A_C}_{\text{common-mode gain}} \underbrace{\left(\frac{V_1 + V_2}{2}\right)}_{V_{CM}\ \text{(common-mode signal)}}$$
$$= A_D V_D + \underbrace{A_C V_{CM}}_{0}$$

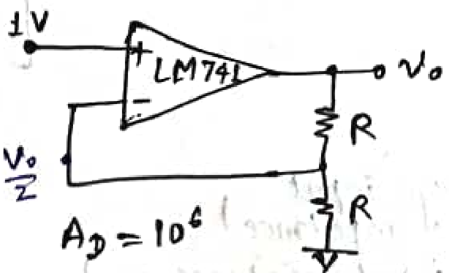
Error ↓, A_C ↓

$$CMRR = \frac{A_D}{A_C} = \frac{A_1 + A_2}{2(A_1 - A_2)}$$

$V_D = V_1 - V_2$
$2V_{CM} = V_1 + V_2$

$V_1 = \frac{V_D}{2} + V_{CM}$
$V_2 = -\frac{V_D}{2} + V_{CM}$

Here, V_{CM} is common in both eqn, that is why it is called common-mode signal

Eg. 

Ideal $V_o = 2V \; (= 1 + \frac{R}{R})$

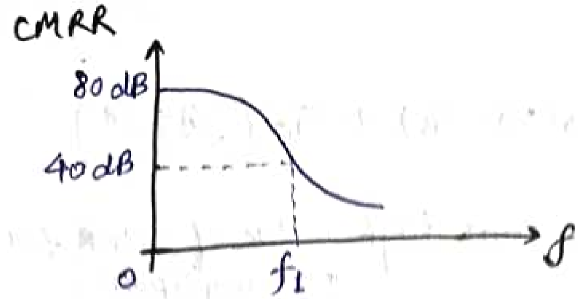
$A_D = 10^6$

CMRR = 80 dB ⇒ 10^4

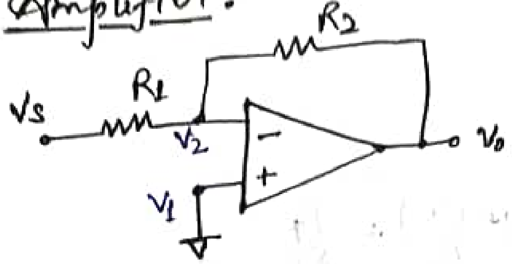
$A_C = 10^2$

$$\left.\begin{aligned} V_D &= 1 - V_o/2 \\ V_{CM} &= \frac{1 + V_o/2}{2} \end{aligned}\right\} \Rightarrow V_o = 2.0002\,V$$

error = 200 μV.



Inverting Amplifier:



$V_1 = 0$

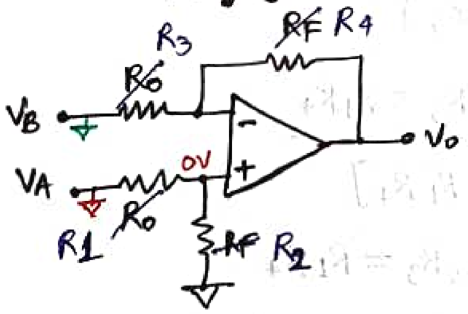
$V_2 \approx 0$ (By virtual short)

$V_{CM} = \frac{V_1 + V_2}{2} \approx 0$

Error ≈ 0

$\therefore$ v_o is very close to the actual value.

Difference Amplifier



Ideal:

$V_o = \frac{R_F}{R_0}(V_A - V_B)$

$= G_D (V_A - V_B)$

$\hookrightarrow$ Issue:

- we need 2 pairs of matched resistances (R_0, R_F).

t: tolerance $\left[\begin{array}{c} 1\% \text{ tolerance} \\ \downarrow \\ t = 0.01 \end{array}\right]$

$R_1, R_3 \in [R_0(1-t), R_0(1+t)]$

$R_2, R_4 \in [R_F(1-t), R_F(1+t)]$

What value to 't' we should use?

$$V_o = \underbrace{\frac{R_2}{R_1+R_2}\left(1+\frac{R_4}{R_3}\right)}_{G_1} V_A - \underbrace{\frac{R_4}{R_3}}_{G_2} V_B$$

(By superposition principle)

$\longrightarrow$ Gain offered to V_A and V_B are different.

Actual:

$$V_0 = G_1 V_A - G_2 V_B = G_D(V_A - V_B) + G_C\left(\frac{V_A + V_B}{2}\right)$$

$CMRR_R = \frac{G_D}{G_C}$ ⇝ we want high value of CMRR for our amplifier

↳ due to tolerance

$$= \frac{G_1 + G_2}{2(G_1 - G_2)}$$

$$= \frac{\frac{R_2}{R_1+R_2}\left(1+\frac{R_4}{R_3}\right) + \frac{R_4}{R_3}}{2\left[\frac{R_2}{R_1+R_2}\left(1+\frac{R_4}{R_3}\right) - \frac{R_4}{R_3}\right]}$$

$$= \frac{R_2(R_3+R_4) + R_4(R_1+R_2)}{2\left[R_2(R_3+R_4) - R_4(R_1+R_2)\right]}$$

$$= \frac{R_1R_4 + R_2R_3 + 2R_2R_4}{2(-R_1R_4 + R_2R_3)}$$

$$\therefore \; CMRR_R = \frac{2R_2R_4 + R_2R_3 + R_1R_4}{2\left[R_2R_3 - R_1R_4\right]}$$

max CMRR $\Rightarrow R_2R_3 = R_1R_4$

min CMRR $\Rightarrow (R_2R_3 - R_1R_4)$ becomes maxm

$$\Rightarrow \begin{cases} R_2 = R_F(1+t) \\ R_3 = R_0(1+t) \\ R_1 = R_0(1-t) \\ R_4 = R_F(1-t) \end{cases}$$

$$\therefore \; CMRR_R = \frac{2R_F^2(1-t^2) + R_FR_0(1+t)^2 + R_FR_0(1-t^2)}{2\left[R_FR_0(1+t)^2 - R_FR_0(1-t)^2\right]}$$

$$= \frac{2R_F(1-t^2) + R_0\left[2+2t\right]}{2R_0\left[4t\right]}$$

$$= \frac{R_F(1-t^2) + R_0(1+t)}{4R_0t}$$

$G_D = R_F/R_0$.

$\therefore CMRR_R = \dfrac{2G_D(1-t^2) + 2(1+t^2)}{2(4t)}$

$= \dfrac{G_D + 1 + t^2(1-G_D)}{4t}$

Now, as t is tolerance (say, 1% ⇒ 0.01)

So, $t^2 \ll 1$

Hence, t^2 term can be neglected.

$\therefore \boxed{CMRR_R = \dfrac{G_D+1}{4t}}$

Eg, $G_D = 10$, $t = 0.05$

$CMRR_R = \dfrac{11}{0.2} = 55 \downarrow 20\log(55) \downarrow 34\,dB$

Even though tolerance was low, $CMRR_R$ is limited.

For $t = 0.01$,

$CMRR_R = \dfrac{11}{0.04} = 275 \xrightarrow{20\log 275} 44\,dB$.

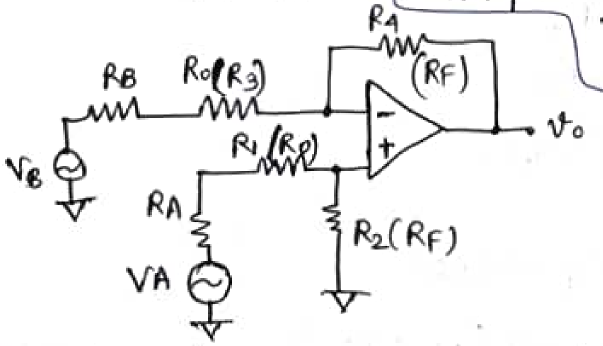
Issue: Differential amplifier has limited CMRR.

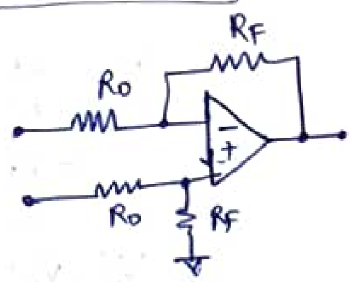
$CMRR_R \ll CMRR_{OA}$ (opamp) $\quad [\because$ CMRR of opamp $= 120\,dB \downarrow 10^{+6}]$

Assume $CMRR_{OA}$ is opamp's, resistive network has $CMRR_R$, then total system has CMRR of $CMRR_T$.

$CMRR_T = F(CMRR_R, CMRR_{OA})$

Obtain F: $\boxed{CMRR_T = \dfrac{CMRR_R \cdot CMRR_{OA}}{CMRR_R + CMRR_{OA}}}$



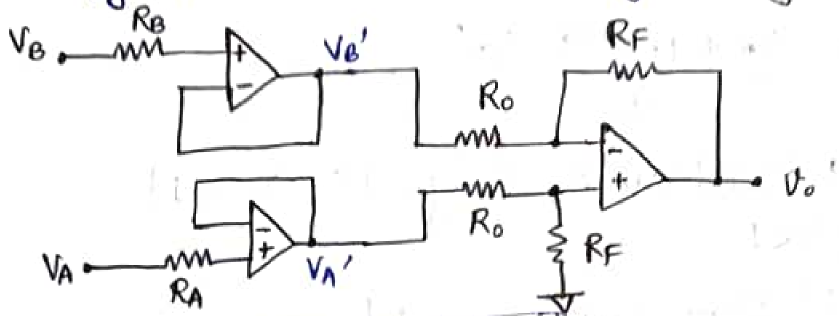


Other issues:

- Effect of i/p resistances: Internal resistances of i/p sources (R_A & R_B) are in series with R_3 & R_1, hence there is more degradation in CMRR. ⇒ Affects gain.

- 2 potentiometers: For gain tuning, we need to change 2 resistors (say, keep 2 rheostats/potentiometers) ⇒ Reduce to 1.

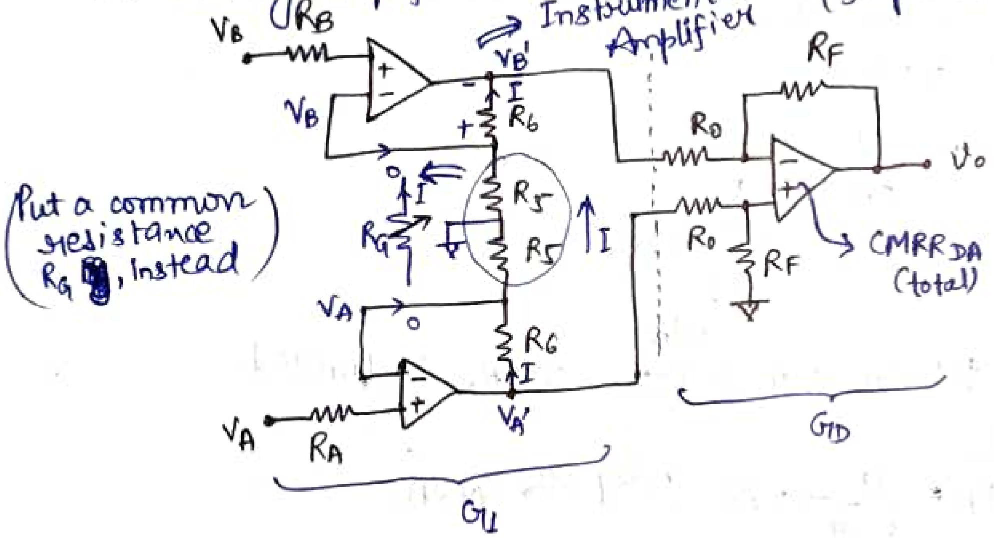
Solution: Nullify effect of R_A and R_B using voltage follower.



$$\boxed{V_0 = \frac{R_F}{R_0}(V_A - V_B)}$$

Make voltage amplifiers as non-inverting amplifiers.

⇒ Instrumentation Amplifier (3-opamp)



$$V_A' = \left(1 + \frac{R_6}{R_5}\right) V_A$$

$$V_B' = \left(1 + \frac{R_6}{R_5}\right) V_B$$

$$\Rightarrow V_0 = \frac{R_F}{R_0}(V_A - V_B)\left(1 + \frac{R_6}{R_5}\right)$$

$$\text{As } I = \frac{V_A - V_B}{R_G}$$

$$V_A' = V_A + IR_6$$

$$V_B' = V_B - IR_6$$

$$\therefore V_0 = \frac{R_F}{R_0}(V_A' - V_B')$$

$$= \frac{R_F}{R_0}(V_A - V_B + 2IR_6)$$

$$V_A' - V_B' = (V_A - V_B)\left(1 + \frac{2R_6}{R_G}\right)$$

$$\therefore V_0 = \underbrace{\frac{R_F}{R_0}}_{G_D}\underbrace{\left(1 + \frac{2R_6}{R_G}\right)}_{G_I}(V_A - V_B)$$

→ Gain tuning can be done through changing R_G.

$G_I = 1 + \frac{2R_G}{R_G}$: gain offered by the 1st stage.

$G_D = \frac{R_F}{R_0}$: gain offered by the 2nd stage.

Hence, $V_0 = G_D \cdot G_I (V_A - V_B)$

$$CMRR_{DA} = \frac{G_D}{G_C}$$

$$V_0 = G_D(V_A' - V_B') + G_C\left(\frac{V_A' + V_B'}{2}\right)$$

As $V_A' = V_A + IR_G$

$V_B' = V_B - IR_G$

$\Rightarrow V_A' - V_B' = G_I (V_A - V_B)$ } Large gain to difference signal

$\frac{V_A' + V_B'}{2} = \frac{V_A + V_B}{2}$ } gain of 1 to common mode signal

Hence, our CMRR will be more as preferential gain to difference signal given..

$$\therefore \boxed{V_0 = G_D G_I (V_A - V_B) + G_C\left(\frac{V_A + V_B}{2}\right)}$$

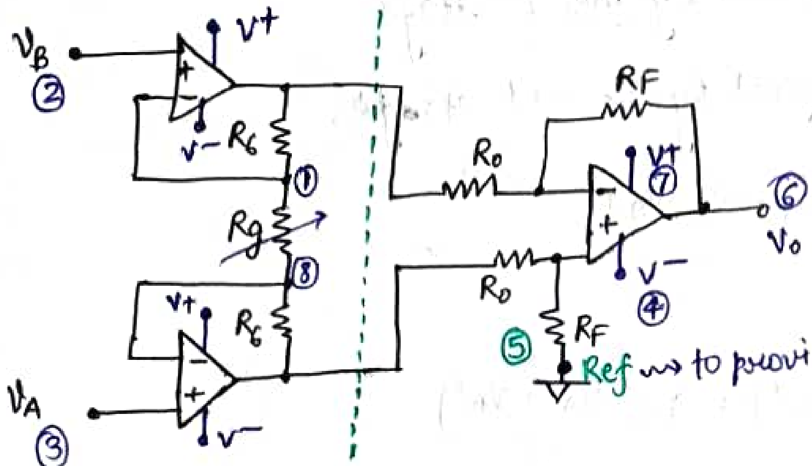
CMRR of Instrumentation amplifier,

$$CMRR_{IA} = \left(\frac{G_D}{G_C}\right) G_I$$

$$\Rightarrow \boxed{CMRR_{IA} = CMRR_{DA} \times G_I}.$$

3-Opamp Instrumentation Amplifier

14-03-2024



Ref ⇝ to provide bias V. to output (clamp up o/p).

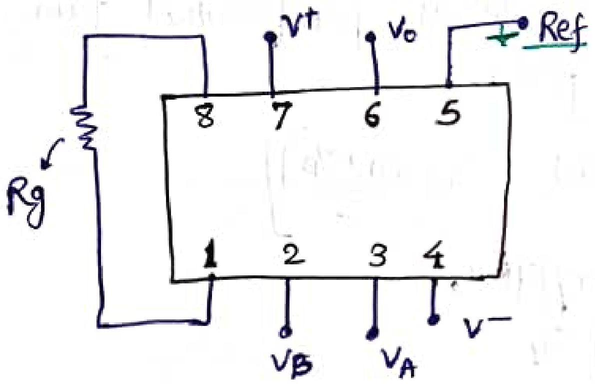
R_g : for tuning

$$G_1 = 1 + \frac{2R_6}{R_g}, \quad R_D = \frac{R_D}{R_o}$$

$$G_{IA} = G_1 \, G_D$$

$$CMRR_{IA} = CMRR + 20 \log G_1$$

ICs: INA 129 (by Texas Instruments), AD 620 (by Analog Devices), INA 101



R_F, R_o
Laser-Trimmed Devices
↳ Type tolerance
↳ made using MOSFET inside IC.

Laser-trimmed devices:

$$R_F = R_o$$

$$\Rightarrow G_D = 1$$

$$G_{IA} = G_1 = 1 + \frac{2R_6}{R_g} = 1 + \frac{50K}{R_g}$$

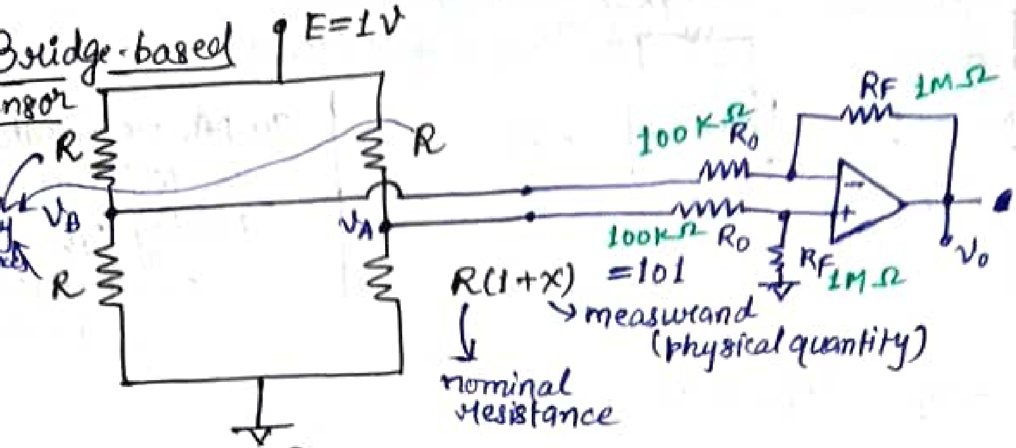
$$R_6 = 25\,K\Omega$$

$$G_{IA} = 10 \Rightarrow R_g = \frac{50K}{9}$$

$$CMRR_{DA} = 80\,dB$$

$$CMRR_{IA} = 80 + 20 \log(10)$$
$$= 80 + 20 = 100\,dB.$$

Eg. Bridge-based sensor



$R = 100\,\Omega$

$x = 0.01$

$$v_A = \frac{101}{201}\,V$$

$$v_B = 1/2\,V$$

$$\Rightarrow v_A - v_B = 2.49\,mV$$

$$\therefore\ v_0 = 10\,(v_A - v_B)$$

$$= 24.9\,mV \quad \text{(Ideal value)}$$

Connecting Difference Amplifier: $R_F = 10\,R_0$

$t = 0.01$ (1% tolerance)

$$CMRR = \frac{11}{0.04} = 275 = \frac{G_D}{G_C}$$

$$G_c = 10/275$$

$$v_0 = G_D v_D + G_C v_{CM}$$

$$v_{CM} = \frac{v_A + v_B}{2} \approx 501\,mV$$

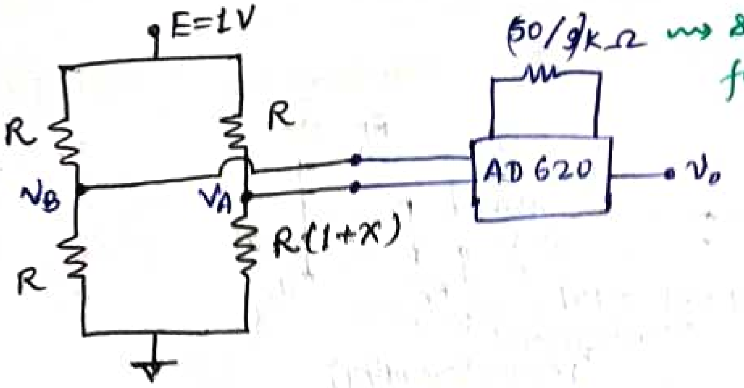
$$\therefore\ v_0 = 43.1\,mV$$

$$\%\ error = \frac{43.1 - 24.9}{24.9} \times 100\,\%$$

$$= 73\,\%$$

↳ very large error for 1% tolerance resistors.

Solution: Use Instrument amplifier.



50/9 kΩ ⇝ single resistor for gain tuning

In DA, we would need 2 resistors for gain tuning ⇓ Create tolerance problem

$G_{IA} = 10 = G_1$ (objective)

$R_g = \frac{50}{9}$ kΩ

$CMRR_{DA} = 10^4 = \frac{G_D}{G_C}$ [$G_D = 1$ for $R_o = R_F$]

$\Rightarrow G_C = 10^{-4}$

$\therefore v_o = G_D G_1 (V_A - V_B) + G_C \left(\frac{V_A + V_B}{2} \right)$

$= (1)(10)(2.49) + 10^{-4}(501)$

$\Rightarrow V_o = 24.95$ mV

% Error $= \frac{24.95 - 24.9}{24.9} \times 100\%$

$= 0.2\%$

73 % (DA)

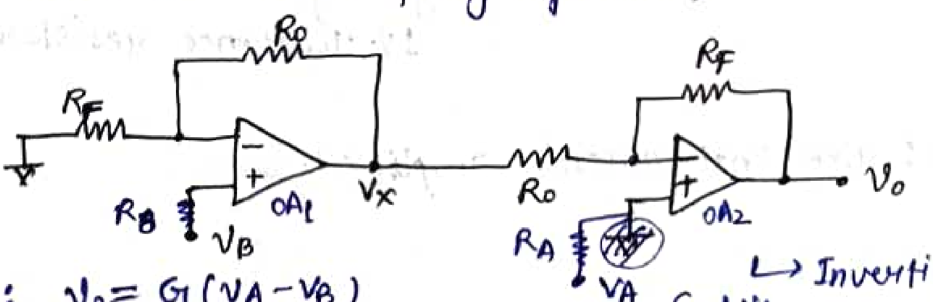
↓

0.2 % (IA)

Laser trimmed resistor
↳ Resistors made from transistors ⇓ W & L are perfectly chosen.

2-Opamp IA:

- ↳ 1 Resistor for gain tuning.
- ↳ High input resistance
- ↳ Lower cost (sacrificing high CMRR)



↳ Inverting amplifier

Feed VA
↳ to get same gain (G) for VA as VB.

To get: $V_o = G(V_A - V_B)$

$V_x = V_B \left[1 + \frac{R_o}{R_F} \right]$

$$v_o = -\frac{R_F}{R_o} v_x = -\left(\frac{R_F}{R_o}+1\right) v_B$$

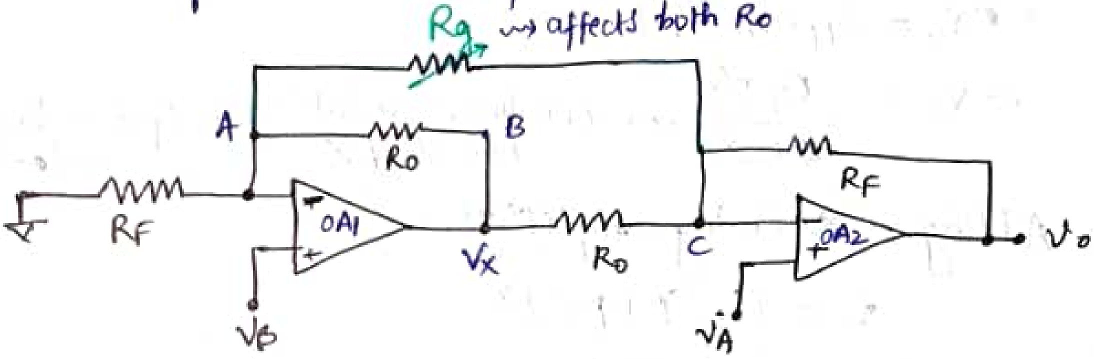
Feed v_A to 2nd opamp : $v_o = \left(1+\frac{R_F}{R_o}\right)(v_A - v_B)$

As v_A and v_B are directly fed to opamps, resistances (internal of source) R_A and R_B will not have much effect.

Objective: Achieve single resistance based gain tuning.

Now, 2 R_F } to be 2 R_o } tuned. + 1R → where?

R_o : Feedback resistance for OA_1 and input resistance for OA_2.



KCL @ V_B :

$$v_B\left[\frac{1}{R_o}+\frac{1}{R_F}+\frac{1}{R_g}\right] = \cancel{\frac{v_x}{R_o}} + \frac{v_A}{R_g} \quad \cdots ①$$

KCL @ V_A :

$$v_A\left[\frac{1}{R_o}+\frac{1}{R_F}+\frac{1}{R_g}\right] = \frac{v_B}{R_g} + \frac{v_o}{R_F} + \cancel{\frac{v_x}{R_o}} \quad \cdots ②$$

① − ②

$$\Rightarrow (v_B - v_A)\left(\frac{1}{R_o}+\frac{1}{R_F}+\frac{1}{R_g}\right) = \frac{v_A - v_B}{R_g} - \frac{v_o}{R_F} \quad \cdots ③$$

$$\Rightarrow v_o = \left(1+\frac{R_F}{R_o}\right)(v_A - v_B)$$

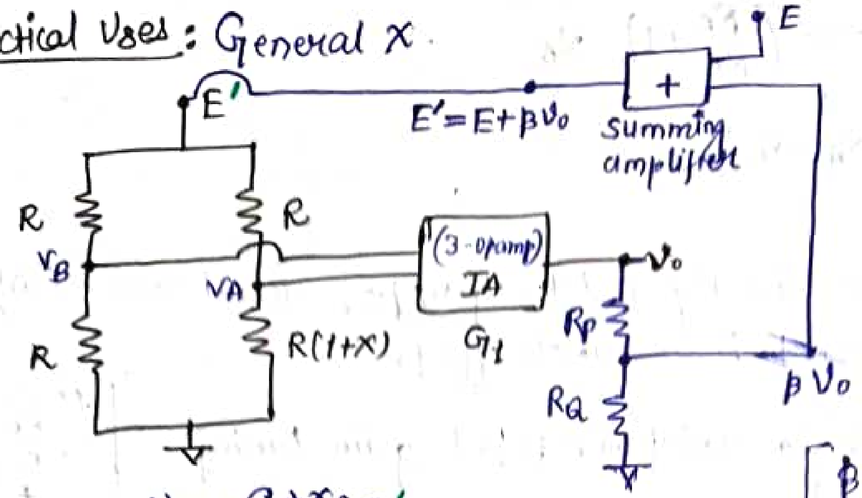
$$\therefore \quad v_o = (v_A - v_B)\left[\frac{R_F}{R_o} + 1 + \frac{2R_F}{\underbrace{R_g}_{\text{tune}}}\right]$$

→ $CMRR_{2\,opamp\ IA} < CMRR_{3\,opamp\ IA}$

↳ lower cost

↳ As 3 opamp IA has spatial amplifier to amplify difference v.

Practical Uses : General x.



$\Rightarrow$ Feedback Compensation

$$\left[\beta = \frac{R_P}{R_P + R_Q}\right]$$

$$V_A = \left(\frac{1+X}{2+X}\right) E'$$

$$V_B = E'/2$$

$$\therefore V_o = G_1 (V_A - V_B)$$

$$\Rightarrow V_o = \frac{G_1 E' X}{4 + 2X}$$ ⇝ non-linear relation b/w V_o & X. $\Rightarrow$ Get a linear releationship

Adding a summing amplifier,

$$V_o = \frac{G_1 (E + \beta V_o) X}{4 + 2X}$$

$$\Rightarrow (4 + 2X) V_o - G_1 \beta X V_o = G_1 E X$$

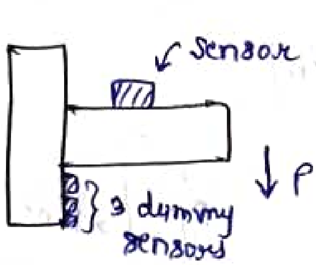
$$\Rightarrow V_o = \frac{G_1 E X}{4 + \underbrace{X(2 - G_1 \beta)}_{0}} \quad \ldots (4)$$

Take $G_1 \beta = 2$. [G_1 and β value can be chosen accordingly.]

$$\therefore V_o = \frac{G_1 E X}{4}$$: Linear relationship

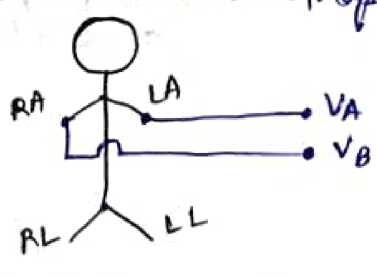
Application :

(1)



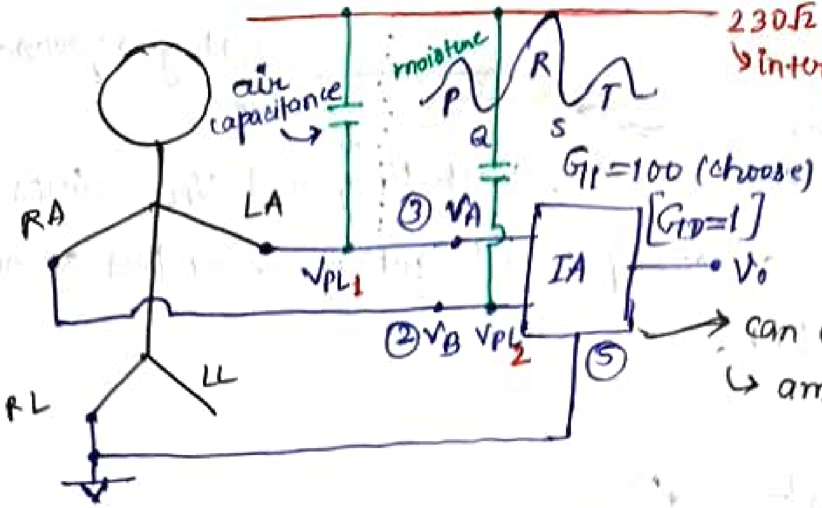
(2) ECG : Electro Cardio Graphy

To measure ECG signal in human body :



Put electrodes plate on the body (potential due to ions in body)
⇓
one electrode inside body ⇓
capacitive coupled V_A, V_B.

→ LA is farthest from RL.



$230\sqrt{2}$ V, 50 Hz (supply line)
↳ interference
≫ V_A, V_B

Typical heart rate
~72 bpm
~60 bpm

→ can only reject CM voltages
↳ amplifies diff. voltages (signal/noise).

$$\left.\begin{aligned} v_A - v_B &\approx 10\,\mu V \ (@\,1\text{ Hz}) \\ \frac{v_A + v_B}{2} &\approx 10\,\mu V \ (@\,1\text{ Hz}) \end{aligned}\right\} \text{Typical order}$$

$$v_o = G_D G_1 (v_A - v_B) + G_C\left(\frac{v_A + v_B}{2}\right) \ \cancel{+10^{-4} \times 10\,\mu V} \ \cancel{10\,nV @ 1\,Hz}$$

$$v_o = 1\text{ mV} @ 1\text{ Hz} + \underbrace{10^{-4} \times 10\,\mu V}_{1\,nV @ 1\,Hz}$$

Practical case:

Due to interference from power supply:

V_{PL} : induced power-line voltage
$V_{PL} \approx 10$ mV

$$v_o = G_D G_1 (v_A - v_B) + G_C\left(\frac{v_A - v_B}{2}\right) + G_C V_{PL}$$

$$= \underset{@\,1\text{ Hz}}{1\text{ mV}} + \underset{@\,1\text{ Hz}}{10^{-4} \times 10\,\mu V} + \cancel{G_C V_{PL}} \ \underset{@\,50\text{ Hz}}{10^{-4} \times 10\text{ mV}}$$

$$= \underset{@\,1\text{ Hz}}{1\text{ mV}} + \underset{@\,1\text{ Hz}}{1\text{ nV}} + \underset{@\,50\text{ Hz}}{1\,\mu V}$$
↳ ≪ 1 mV

Case : $V_{PL_1} \neq V_{PL_2}$

$V_{PL_1} \approx 10.5$ mV
$V_{PL_2} \approx 9.5$ mV

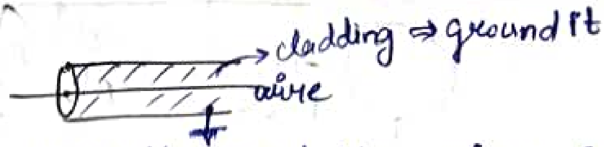
$$\therefore v_o = G_D G_1 (v_A - v_B) + G_C\left(\frac{v_A - v_B}{2}\right) + G_D G_1 \underbrace{(V_{PL_1} - V_{PL_2})}_{\text{amplifies the difference (signal/noise)}} + G_C\left(\frac{V_{PL_1} + V_{PL_2}}{2}\right)$$

$$= \underset{@\,1\text{ Hz}}{1\text{ mV}} + \underset{@\,1\text{ Hz}}{1\text{ nV}} + (100)(1\text{ mV}) + 10^{-4}(10\text{ mV})$$

$$= \underset{@\,1\text{ Hz}}{1\text{ mV}} + \underset{@\,1\text{ Hz}}{1\text{ nV}} + \underset{@\,50\text{ Hz}}{100\text{ mV}} + \underset{@\,50\text{ Hz}}{1\,\mu V}$$
↳ ≫ 1 mV
↳ Too much noise

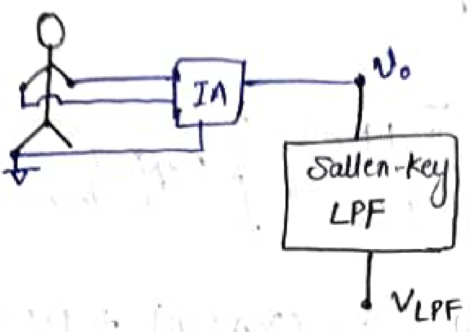
Solution :

↳ Shield the wires



↳(while pointing the circuit), Keep V_{PL_1} and V_{PL_2} wires in as proximity as possiple [to get more or less same V_{PL}s].

↳ Use Sallen-key LPF.



cutoff freq., $f_c = 5\,Hz$.

⇒ To reject common mode voltage.

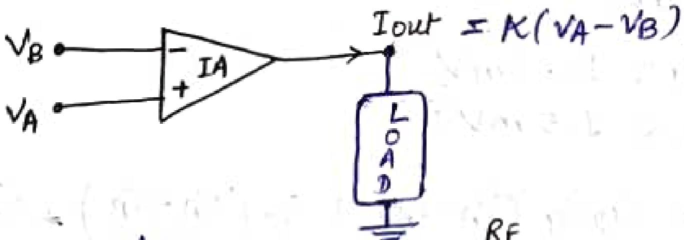
Book:

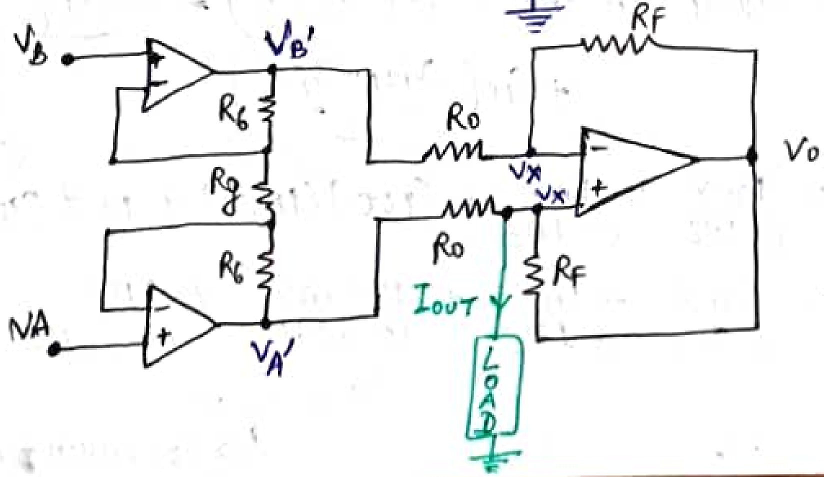
- Operational Amplifier by George Clayton & Steve Winder
- Analog Signal Processing by Ramon Pallas-Areny John G. Webster

19-03-2023

Current-output Instrumentation Amplifier :

For a load at remote location, voltage may get attenuated, so we would want output to be current.





KCL @ +ve terminal of DA:

$$V_x\left[\cancel{\frac{1}{R_0}+\frac{1}{R_F}}\right] + I_{OUT} = \cancel{\frac{V_0}{R_F}} + \frac{V_A'}{R_0} \quad \cdots (1)$$

KCL @ −ve terminal of DA:

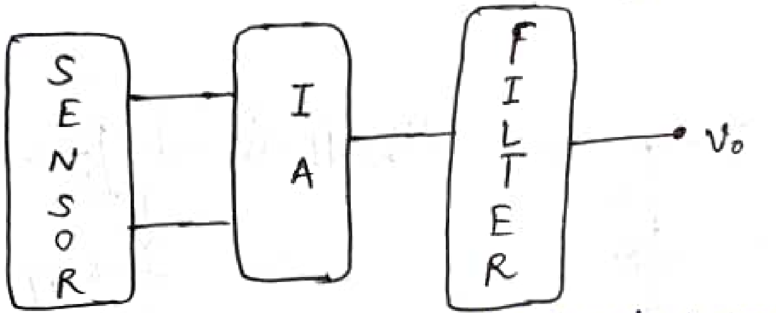
$$V_x\left[\cancel{\frac{1}{R_0}+\frac{1}{R_F}}\right] = \cancel{\frac{V_0}{R_F}} + \frac{V_B'}{R_0} \quad \cdots (2)$$

(1) − (2)

$$\Rightarrow I_{OUT} = \frac{V_A' - V_B'}{R_0}$$

$$\Rightarrow I_{OUT} = \frac{G_I(V_A - V_B)}{R_0}, \quad G_I + \frac{2R_G}{R_g}$$

↳ Doesn't depend on the load.



↳ to reject

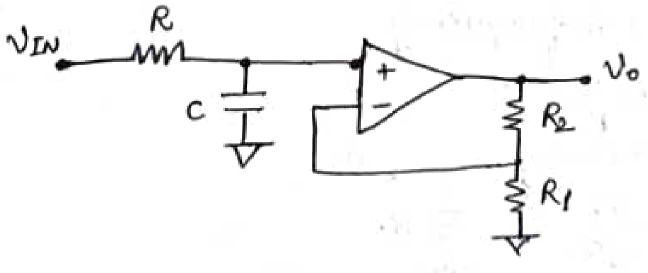
Heart-rate: 60 bpm to 120 bpm
(1 Hz to 2 Hz)

<u>Interferences</u>: (1) Supply: 50 Hz [Power line]
↳ solution: LPF ; $f_{cL} \approx 5$ Hz

(2) Frequency component of motion artefacts and breathing rate (< 0.5 Hz)
↳ Solution: HPF ; $f_{cH} \approx 0.5$ Hz.

→ For powerline very near to the instrument, use band-reject filter [for further filtering]

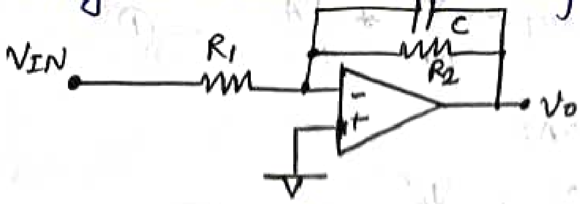
<u>LPF</u>:



$$G = 1 + R_2/R_1$$

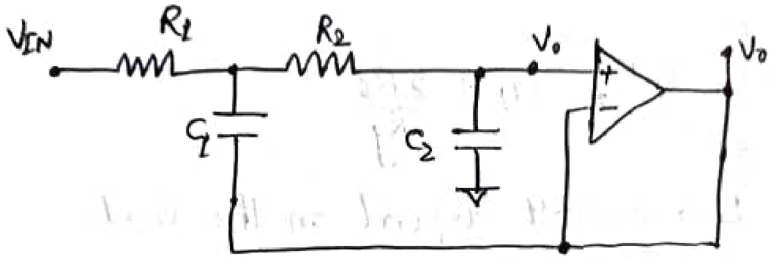
$$f_c = \frac{1}{2\pi RC}$$

Challenge: Use only 2 resistors and a capacitor for the LPF.

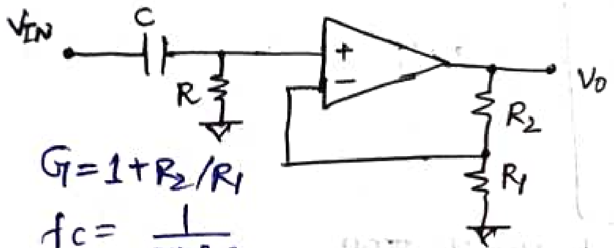


$G = -R_2/R_1$

$f_c = \dfrac{1}{2\pi R_2 C}$

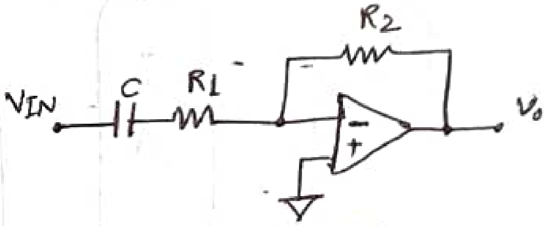


HPF:



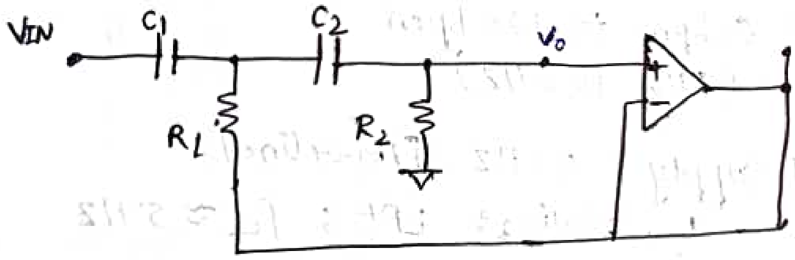
$G = 1 + R_2/R_1$

$f_c = \dfrac{1}{2\pi RC}$



$G = -R_2/R_1$

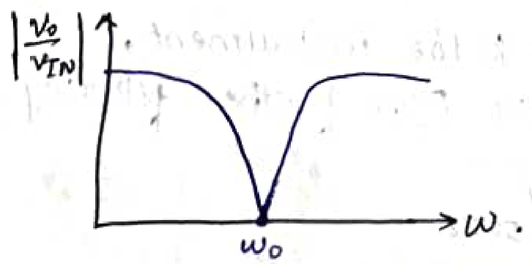
$f_c = \dfrac{1}{2\pi R_1 C}$

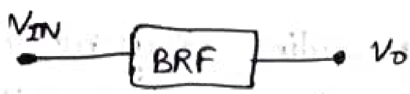


BPF:

↳ LPF + HPF

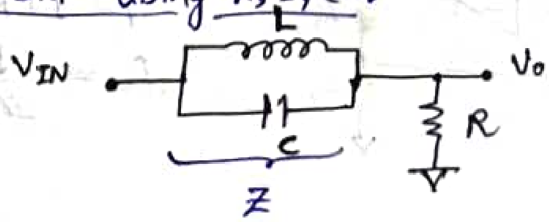
Band-Reject Filter (BRF):





↳ notch frequency

Design BRF using R, L, C:



$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow \underset{(Z_L \parallel Z_C)}{\underline{Z_{high}}} \longrightarrow v_0 = 0$

$\omega \neq \omega_0 \longrightarrow Z_{low} \longrightarrow v_0 = v_{IN}$

Analysis :

$$Z = j\omega L \parallel \frac{1}{j\omega C} = \frac{L/C}{j\omega L + \frac{1}{j\omega C}} = \frac{L/C\,(j\omega C)}{-\omega^2 L/C + 1}$$

$$= \frac{j\omega L}{1 - \omega^2 \frac{L}{C}}$$

$$\frac{v_0}{v_{IN}} = \frac{R}{R+Z}$$

$$= \frac{(j\omega)^2 + \frac{1}{LC}}{(j\omega)^2 + \frac{j\omega}{RC} + \frac{1}{LC}}$$

ω	$\lvert v_0/v_{IN} \rvert$
0	1
∞	1
$\omega_0 = \frac{1}{\sqrt{LC}}$	0

Issue: There is a need of inductor which is bulky and costly, and difficult to miniaturize.

$$\frac{v_0}{v_{IN}} = \frac{(j\omega)^2 + \omega_0^2}{(j\omega)^2 + 2\xi\omega\omega_0 j + \omega_0^2}$$

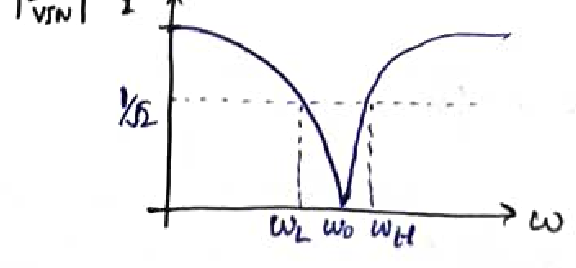
$\left[\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{LC} = \omega_0^2 \right]$

$\left[1/RC = 2\xi\omega_0 \text{ (say)} \right]$

For BRF, we don't use ξ, instead we use

Quality factor, $Q = \frac{1}{2\xi}$ ⇝ more intuitive parameter.

$$\therefore \frac{v_0}{v_{IN}} = \frac{(j\omega)^2 + \omega_0^2}{(j\omega)^2 + \frac{\omega\omega_0}{Q} j + \omega_0^2}$$



Bandwidth (BW) :

$BW = \omega_H - \omega_L$ ⇝ rejection BW

$BW \downarrow \Rightarrow$ Specificity $\uparrow$, Performance $\uparrow$

To get BW from the equation:

At $\omega = \omega_L, \omega_H$

$$\left|\frac{V_o}{V_{IN}}\right| = \frac{1}{\sqrt{2}} \Rightarrow \text{Get 2 solutions } \omega_L, \omega_H.$$

$$BW = \frac{\omega_o}{Q}$$

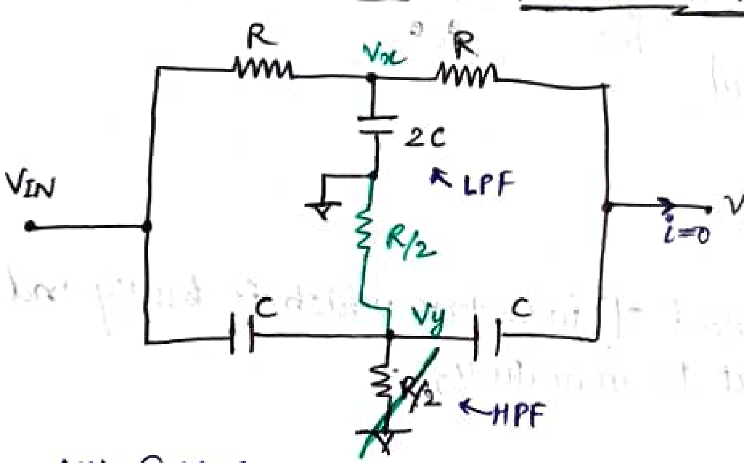
$Q\uparrow \Rightarrow BW\downarrow \Rightarrow$ Better the BRF.

→ $\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow \frac{\omega_o}{Q} = \frac{1}{RC}$

$$\Rightarrow \boxed{Q = R\sqrt{\frac{C}{L}}}$$

~ By choosing appropriate values of C and L, we can get good Q.

<u>Inductor-free BRF</u> : <u>(Twin-T notch filter)</u>



[Adding extra R & C after 2 filters give better band-reject performance by coupling them]

KCL @ V_x:

$$V_x\left[\frac{2}{R} + j\omega 2C\right] = \frac{V_{IN}}{R} + \frac{V_o}{R}$$

$$\Rightarrow 2V_x[1 + j\omega RC] = V_{IN} + V_o \quad \ldots (1)$$

KCL @ V_y:

$$V_y\left[\frac{1}{R/2} + j\omega 2C\right] = (V_{IN} + V_o)\, j\omega C$$

$$\Rightarrow 2V_y[1 + j\omega RC] = (V_{IN} + V_o)\, j\omega C \quad \ldots (2)$$

(1)/(2) $\Rightarrow$

$$\frac{V_x}{V_y} = \frac{1}{j\omega CR}$$

$$\Rightarrow V_y = (j\omega CR)\, V_x \quad \ldots (3)$$

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KCL @ V_0 :

$$\frac{V_0}{R} + V_0 j\omega C = \frac{V_x}{R} + V_y\, j\omega C$$

$$\Rightarrow V_0[1 + j\omega CR] = V_x + V_y\, j\omega CR \quad \ldots (4)$$

(3) and (4) $\Rightarrow$ $V_0[1+j\omega CR] = V_x + j^2\omega^2 C^2 R^2$

$$\Rightarrow V_x = \frac{V_0[1+j\omega CR]}{1+(j\omega CR)^2} \quad \ldots (5)$$

(1) and (5) $\Rightarrow$ $2\cdot\frac{V_0[1+j\omega CR]}{1+(j\omega CR)^2}\cdot[1+j\omega CR] = V_{IN} + V_0$

$$\Rightarrow 2\frac{V_0}{V_{IN}}\frac{(1+j\omega CR)^2}{1+(j\omega CR)^2} = 1 + \frac{V_0}{V_{IN}}$$

$$\Rightarrow \frac{V_0}{V_{IN}}\left[2\,\frac{\frac{1}{R^2C^2}+(j\omega)^2+\frac{2j\omega}{RC}}{\frac{1}{R^2C^2}+(j\omega)^2}-1\right] = 1$$

$$\Rightarrow \frac{V_0}{V_{IN}} = \frac{(j\omega)^2+\frac{1}{R^2C^2}}{(j\omega)^2+4\frac{j\omega}{RC}+\frac{1}{R^2C^2}} \quad \ldots (6)$$

$\therefore$ $\boxed{\omega_0 = \frac{1}{RC}}$

$\frac{4}{RC} = \frac{\omega_0}{Q} \Rightarrow \boxed{Q = \frac{1}{4}}$: fixed low

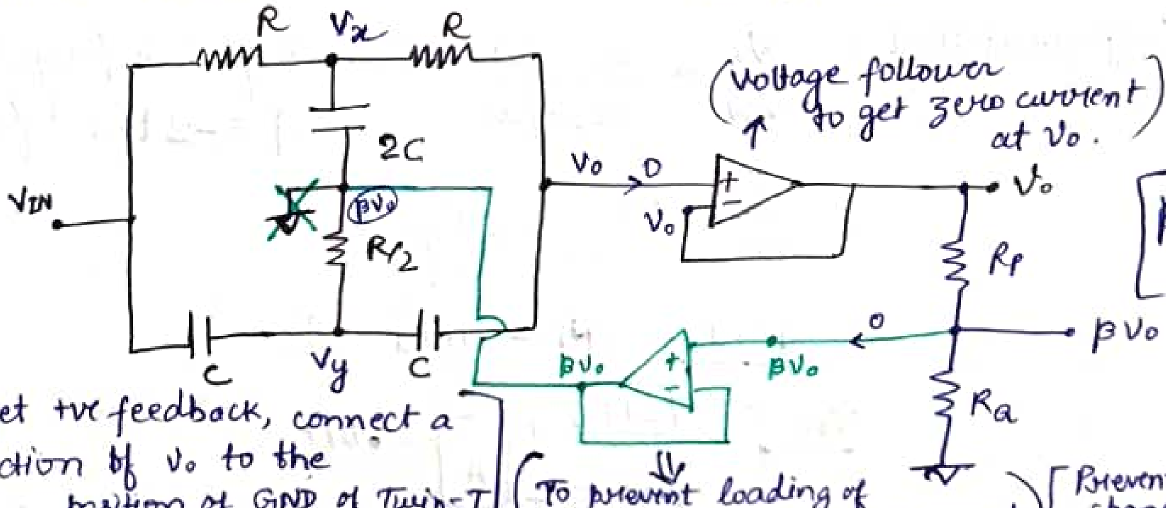
Twin-T Active Notch Filter :

Objective : Improve the value of Q, so that ξ is lowered. (of circuit)

$$Q = \frac{1}{2\xi}$$

[-ve feedback is dominating the +ve feedback (to prevent oscillation)]

$Q\uparrow \Rightarrow \xi\downarrow$ $\Rightarrow$ Bring some positive feedback in the circuit.



$$\left[\beta = \frac{R_Q}{R_P+R_Q}\right]$$

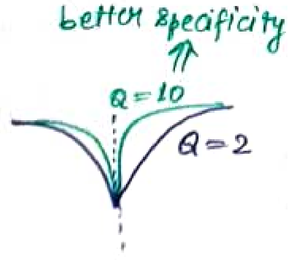
[To get +ve feedback, connect a fraction of V_0 to the position of GND of Twin-T]

(To prevent loading of (R_P & R_Q) on the Twin-T circuit)

[Prevent R/2 from changing due to R_P & R_Q]

Using KCL at V_x, V_y, V_o,

$$\frac{V_o}{V_{IN}} = \frac{(j\omega)^2 + \frac{1}{R^2C^2}}{(j\omega)^2 + \frac{4(1-\beta)}{RC}j\omega + \frac{1}{R^2C^2}}$$



$$\therefore \boxed{\omega_o = \frac{1}{RC}}$$

$$\boxed{Q = \frac{1}{4(1-\beta)}} \text{ : Variable Q}$$

To get $Q=10$,

$$\beta = \frac{39}{40} = \frac{R_Q}{R_P + R_Q}$$: choose R_P & R_Q accordingly.

→ Negative feedback > Positive feedback : Underdamped response

All-Pass Filter (APF)

$$\left|\frac{V_o}{V_{IN}}\right| = 1$$

$$\phi = \angle \frac{V_o}{V_{IN}}$$

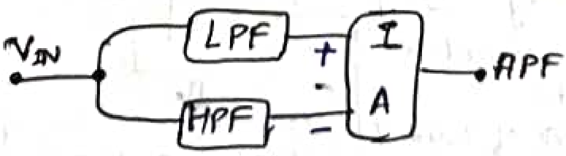
↳ There is a finite value of ϕ (phase). [Used to give phase delay without changing amplitude]

↳ Application : • time-delay
• Synchronization

Implementation : $$\frac{V_o}{V_{IN}} = \frac{\omega_o - j\omega}{\omega_o + j\omega}$$ → obeys both properties $\phi = -2\tan^{-1}\left(\frac{\omega}{\omega_o}\right)$

$$\frac{V_o}{V_{IN}} = \left(\frac{\omega_o}{\omega_o + j\omega}\right) - \left(\frac{j\omega}{\omega_o + j\omega}\right)$$

$$\therefore \boxed{APF = LPF - HPF} \quad \dots (1)$$



⇒ Issue : Requires 2 filters and an IA.

Also, $LPF + HPF = 1$ $\left[\because \frac{\omega_0}{\omega_0 + j\omega} + \frac{j\omega}{\omega_0 + j\omega} = 1\right]$

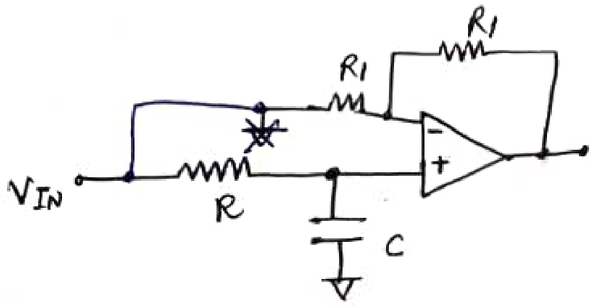
$\Rightarrow HPF = 1 - LPF \quad \cdots ②$

① and ②

$$\Rightarrow \frac{V_o}{V_{IN}} = APF = 2LPF - 1$$

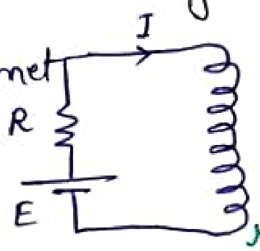
$$\Rightarrow \boxed{V_o = 2LPF(V_{IN}) - V_{IN}} \quad \cdots ③$$

↳ low-pass-filtered version of V_{IN} amplified by 2 times.



Eg. Generate a stable-controlled magnetic field at the point P.
[Permanent magnet is not ~~amplif~~ feasible.]

Solution: Use Electromagnet (solenoid)



$B \propto I$

r_c : coil resistance
↳ depends on operating temperature and conditions.

Ideal : $I = \frac{E}{R}$

Practical : $I = \frac{E}{R + r_c}$

↳ Issue: Becomes uncontrolled due to r_c.

Solution: $R \gg r_c$

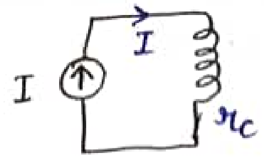
$$\therefore I \approx \frac{E}{R}$$

For $I = 50mA$, $r_c = 50\Omega$

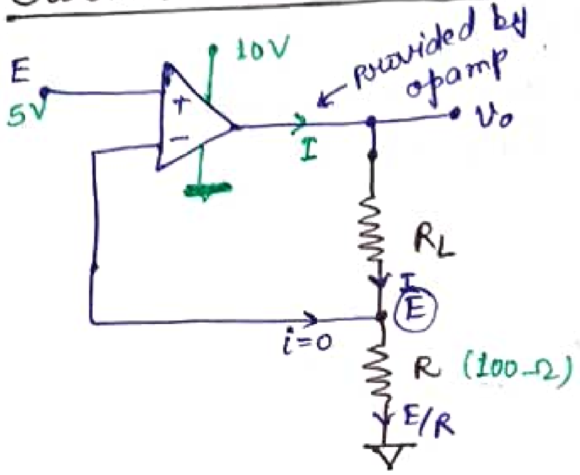
$R = 100\, r_c = 5\,k\Omega$ (say)

Issue: $E = IR = 250\,V$ (Requires ~~too~~ high E)

Solution: Use current-source. 26-03-2024



Current-Source Circuit



$I = \frac{E}{R}$ (provided circuit above R_L forms a closed-path)

R_L: Load resistance (e.g., electromagnet)

R: Extra resistance

To get voltage E with zero current, connect opamp's input to E point.

To get $I = 50mA$,

$E = 5V$, $R = 100\Omega$ } Readily available values.

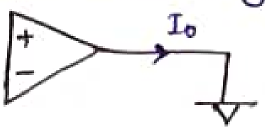
Opamp's output:

$$V_o = \left(1 + \frac{R_L}{R}\right) E$$

$R_L \in (40, 60)\Omega$

R_L	V_o
50	7.5
40	7
60	8

Opamp-specification dealing with opamp's output current:



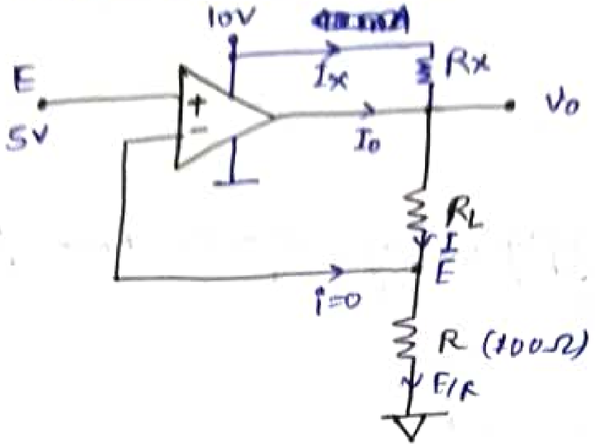
Short-circuit output current

[I_o will be maximum when output is grounded without any load.]

OP07, LM741 } $(I_o)_{max}$ < 25mA

Issue: We need $I_o = 50mA$, but opamp can only provide 25mA.
(or sink)

Solution: Draw current from the supply of 10 V.



$I = \frac{E}{R}$

$V_o = \left(1 + \frac{R_L}{R}\right)E$

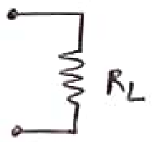
$I = I_o + I_x$

R_L	V_o	I_o	I_x
50	7.5	0	50 mA
40	7	−10 mA	60 mA
60	8	10 mA	40 mA

$R_L = 50\,\Omega$, $V_o = 7.5$: $R_x = \frac{10 - 7.5}{50mA} = 50\,\Omega$ $(\because I_x = 50mA)$

Issue: Both ends of load are at finite voltages.

Floating loads :

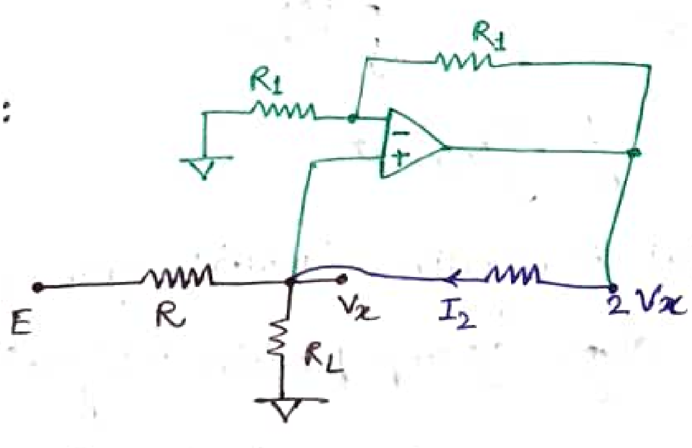


Grounded loads :

(Use grounded load when safety is involved.)

Solution:

Howland's current source :



$I = \frac{E}{R}$

$$I = \underbrace{\frac{E - V_x}{R}}_{I_1} + \underbrace{\frac{2V_x - V_x}{R}}_{I_2 = V_x/R} = \frac{E}{R}$$

↳ To cancel extra $(-V_x/R)$.

$$I = \frac{E}{R} = \frac{5V}{100\Omega}$$

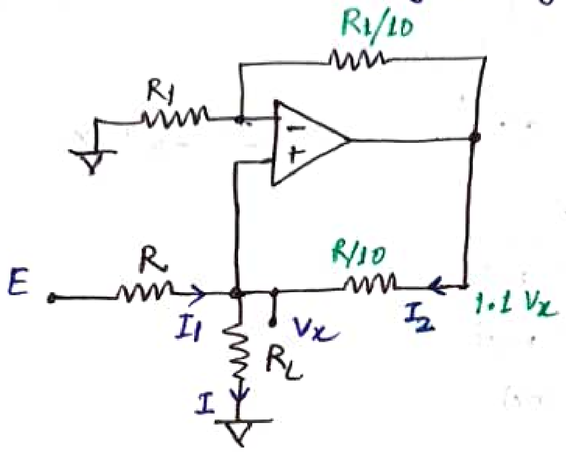
$I = 50mA$

$R_L = 60\Omega$

$$V_x = IR_L$$
$$= 3V$$
$$2V_x = 6V$$

Issue: We would need supply voltage of > 6V (here) for proper operation.

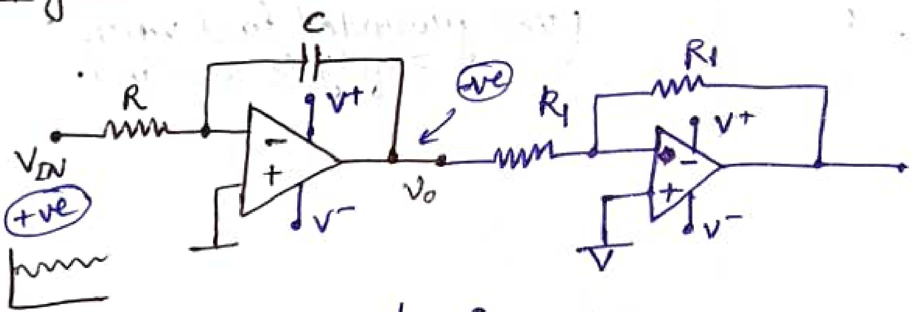
Solution:



$$I = \underbrace{\frac{E - V_x}{R}}_{I_1} + \underbrace{\frac{1.1\,V_x - V_x}{R/10}}_{I_2} = \frac{E}{R}$$

$$1.1\,V_x = 3.3\,V$$
$$< 5V$$

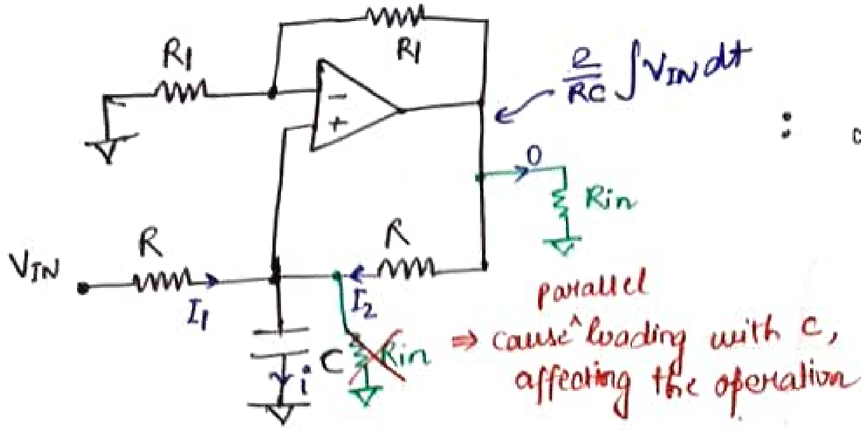
Integrator



$$V_o = -\frac{1}{RC}\int V_{IN}\,dt$$

Issue: Output is negative. [Multiple opamps are not feasible.]

Solution: ~~Solu~~ Get single opamp positive integrator.



: Deboo's Integrator

∟ Howland's Integrator (when used as current source)

$$i = \frac{V_{IN}}{R} \qquad \left(= \frac{V_{IN} - \cancel{V_x}}{R} + \frac{2V_x - V_x}{R} \right)$$

$$\therefore \quad V_x = \frac{1}{C}\int i\,dt = \frac{1}{RC}\int V_{IN}\,dt$$

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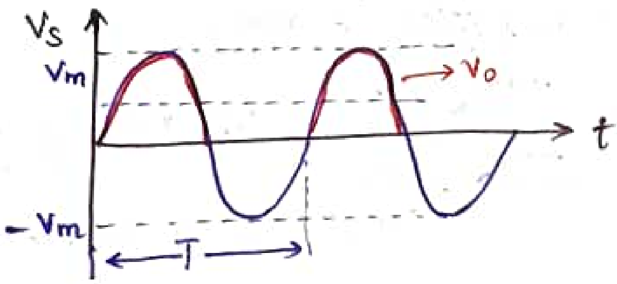
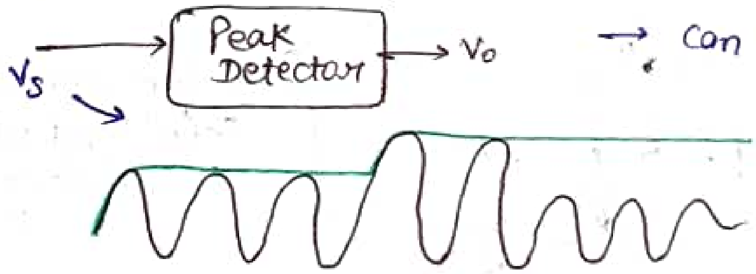
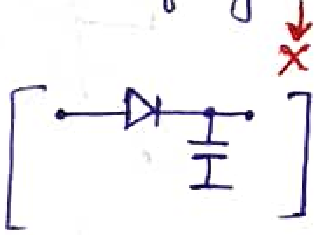
Measure Peak of Signal

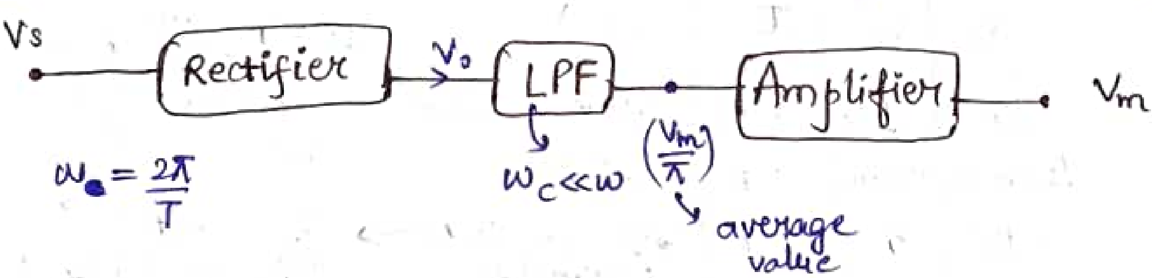




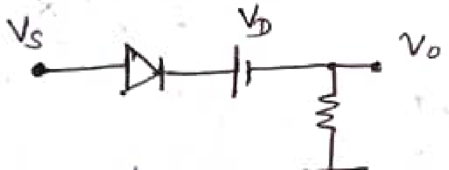
→ can only measure the highest value of signal.



Solution:



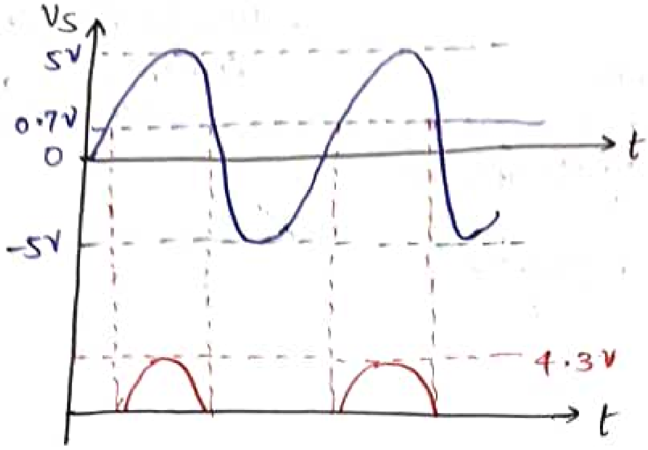
$$\omega = \frac{2\pi}{T}$$



$$V_o = \begin{cases} V_{IN}, & V_{IN} > 0 \\ 0, & V_{IN} < 0 \end{cases}$$

$$V_o = \begin{cases} V_{IN} - V_D, & V_{IN} > V_D \\ 0, & \text{elsewhere.} \end{cases}$$

Suppose $V_m = 5V$
$V_D = 0.7V$.



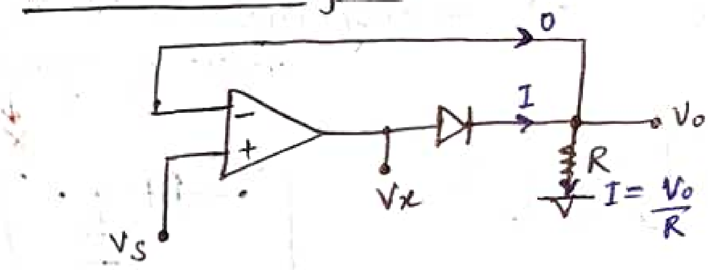
Suppose $V_m = 50mV \longleftrightarrow$ Instrument won't even detect the signal.
$V_D = 0.7V$

<u>Possible solution</u> : ~~and~~ Use amplifier. $\longleftrightarrow$ ×.

$0°C \longrightarrow V_m = 50\,mV \xrightarrow{\times 100} 5V$
$100°C \rightarrow V_m = 500\,mV \xrightarrow{\times 100} 50V$
} V_m can by ~~[illegible]~~ anything.

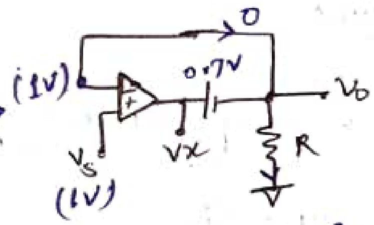
↳ 50V is unhandable.

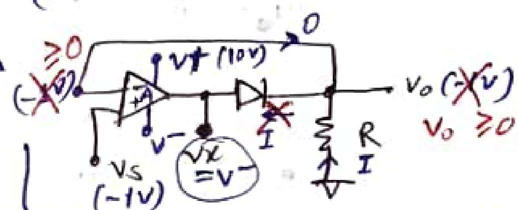
Precision Rectifier



[No fundamental laws should get violated while designing.]

V_s	V_o	D	V_x
1V	1V	F.B	1.7V
100mV	100m	F.B	0.8V
-1V	0	R.B.	V^- (=-10V) (saturates)



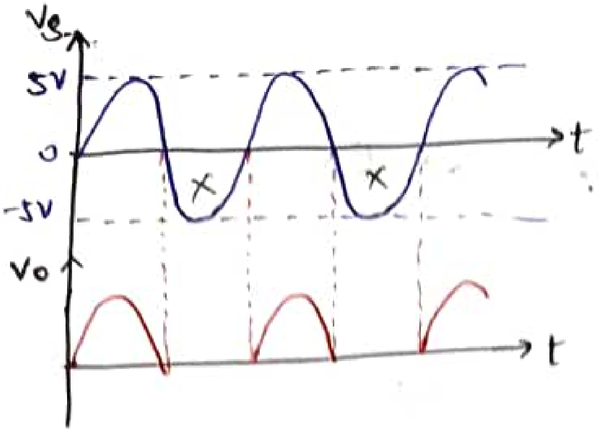


↳ The circuit is independent of cutting voltage of diode.

$\therefore V_o = \begin{cases} V_s, & V_s > 0 \\ 0, & V_s < 0 \end{cases}$

↳ True Rectifier.

Diode won't allow I (C→A).
⇓
I across R would be either ↑ or 0.
⇓
$\boxed{V_o \geq 0}$
⇓
$V_x = A(V_1 - V_2)$ → very large
↳ (saturates to -ve value V^-)

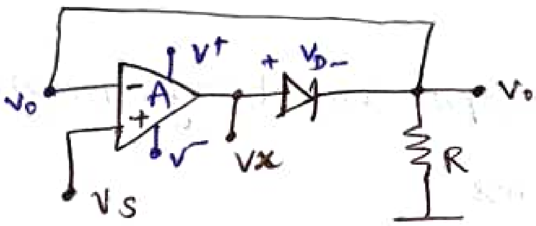


V_S	V_o	D	V_x
>0	V_S	F.B.	$V_o + V_D$
<0	0	R.B	V^- (= $-V_{CC}$)

$V_o \geq 0 \rightarrow$ F.B.

$V_x \geq V_D$

Effect of A (Open Loop Gain):



$V_x = A(V_S - V_o)$

F.B: $V_x = V_o + V_D$

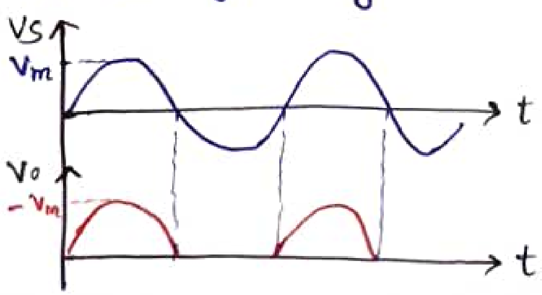
$$\Rightarrow V_o = \frac{A V_S}{A+1} - \frac{V_D}{A+1} \geq 0 \quad \text{(for F.B.)}$$

$A >> 1 \Rightarrow V_o \approx V_S$

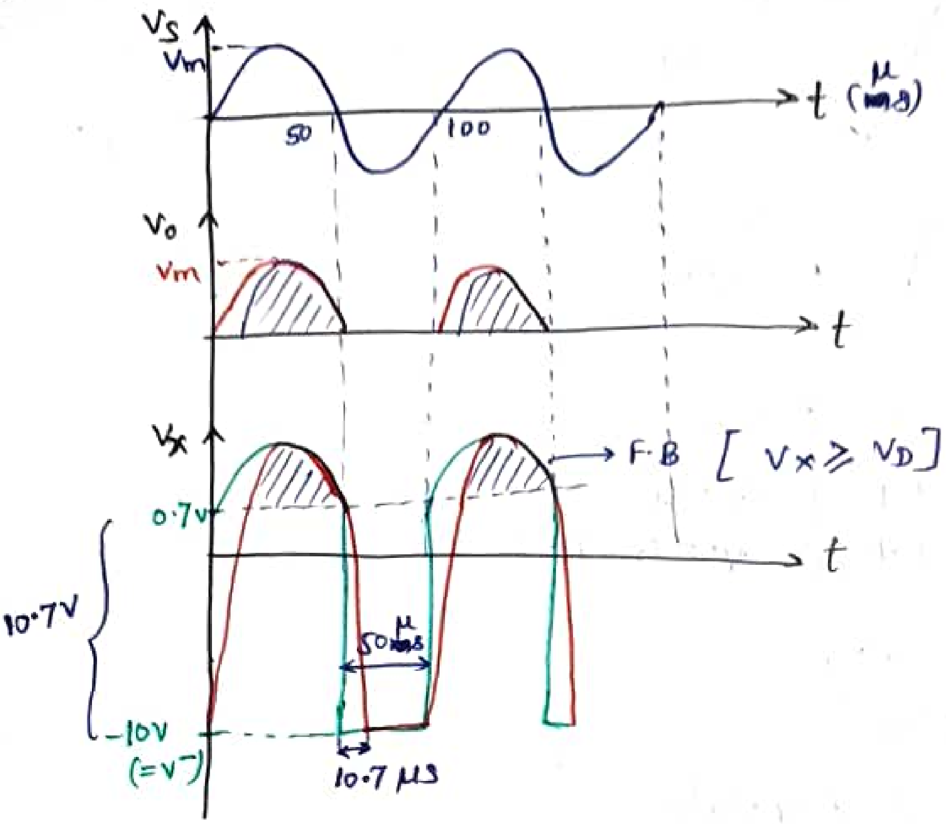
$\Rightarrow V_S \geq \frac{V_D}{A} \rightarrow$ effective cut-in voltage

$\begin{cases} V_D = 0.7V \\ A = 10^6 \end{cases}$

$\therefore$ Cutting voltage is reduced from 0.7V to 0.7 μV.



Effect of Slew-Rate:



$SR = 1V/\mu s$

⇒ In 1 μs, opamp output can change maximum by 1V.

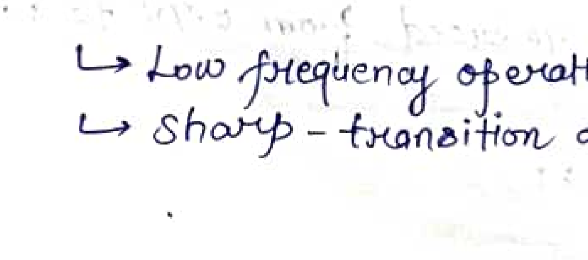
$f = 10 kH$, $\frac{T}{2} = 50$ ms

$10.7 \mu s \longleftrightarrow 10.7 V$

→ The precision rectifier works for low frequency signal.

1-04-2024

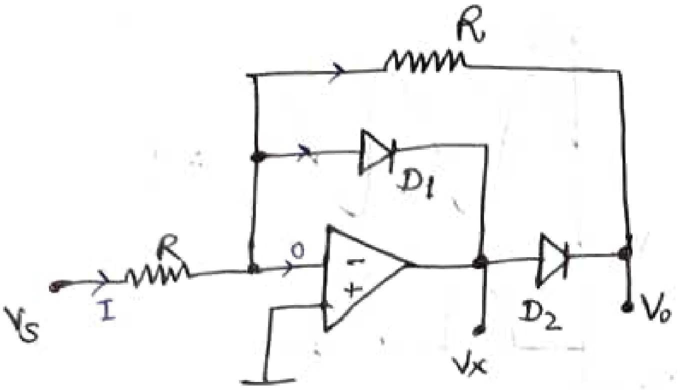
Precision Rectifier



↳ Low frequency operation

↳ Sharp-transition at Vx.
(10.7v).

Improved Precision Rectifier :

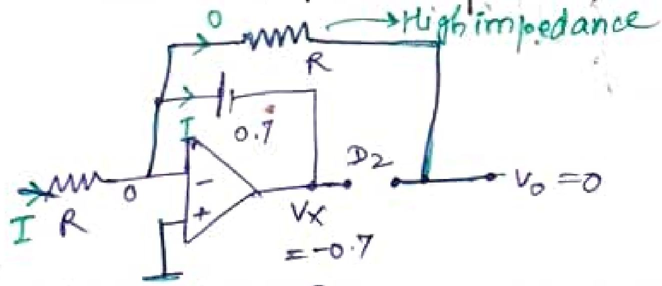


→ Precision Half wave Rectifier
↳ works even for high frequencies.

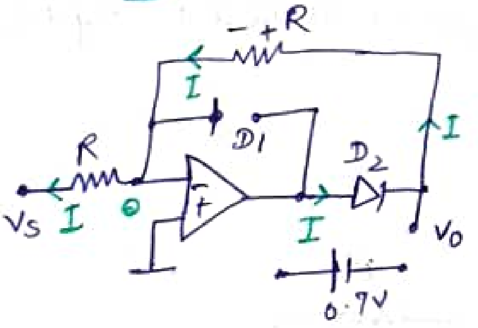
$R \gg r_{ON}$ (diode)
↓
~kΩ

D_2: helps get half wave rectification
D_1: helps reduce slew rate

V_s	D_1	D_2	V_o	V_x
>0	F.B	R.B	0	−0.7 V
<0	R.B	F.B	$-V_s$	V_o+0.7 V



$I = +\frac{V_s}{R}$

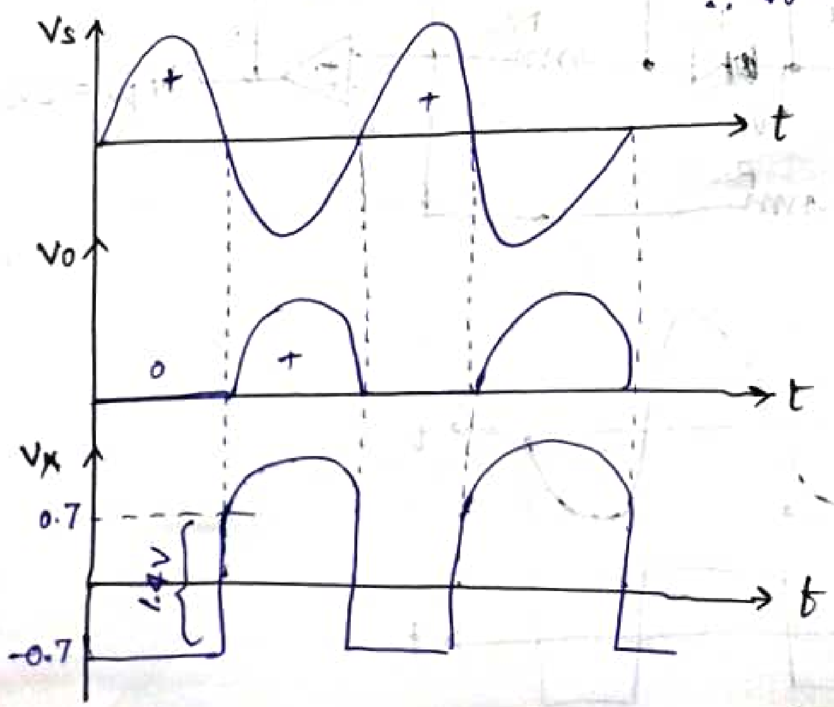


$I = -\frac{V_s}{R}$

$V_o = +IR$

$I = \frac{-V_s}{R}$

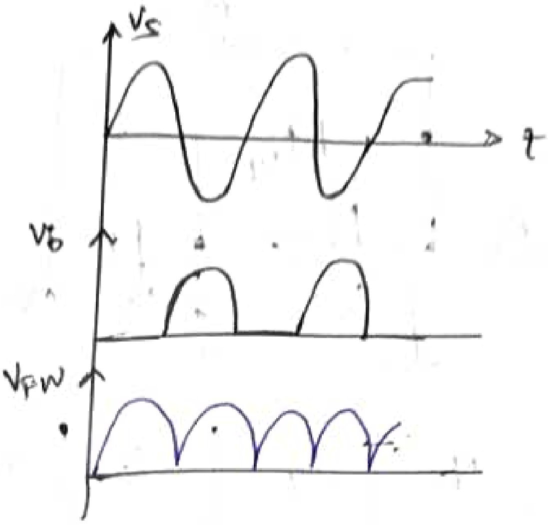
$\therefore V_o = -V_s$

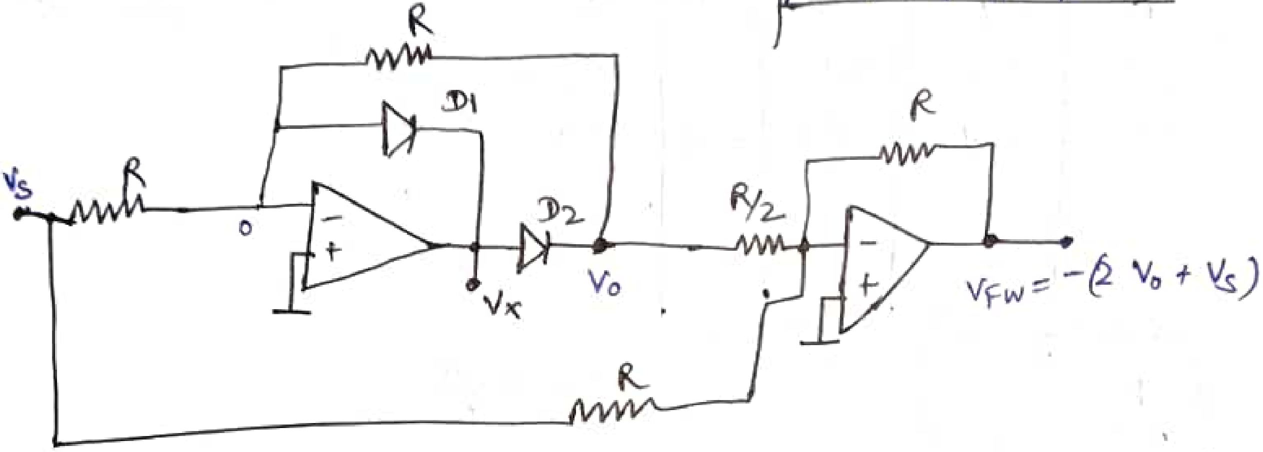


⇒ slew rate = 1.4
↓↓

Precision Full Wave Rectifier

$$V_{FW} = V_s + 2V_o$$

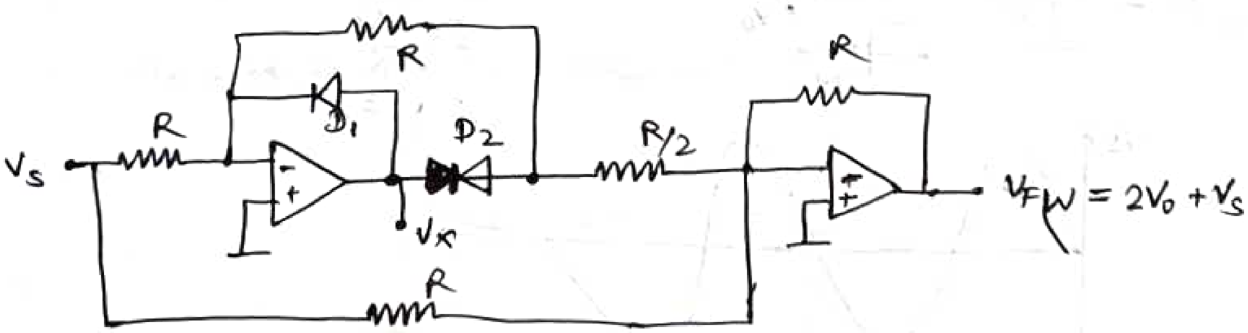




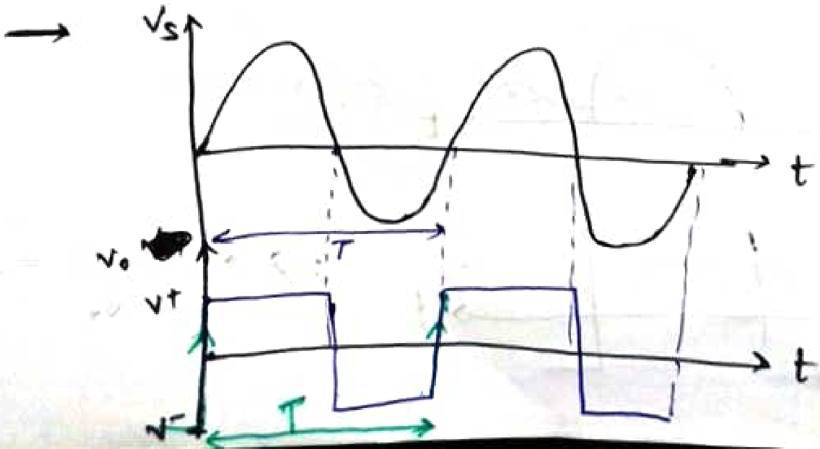
→ Get non-inverted V_{FW}.

[we can't use non-inverting amplifier as it would require 4 resistor components.]

<u>Solution</u> : Invert D_1 and D_2.

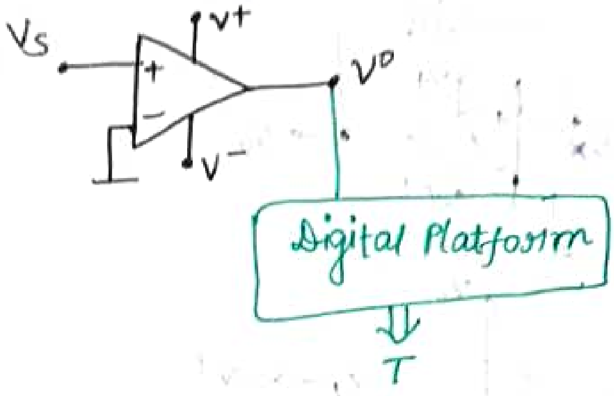


→



Measure the time period T of the sine signal.

<u>Solution</u>: Convert the signal to pulse.



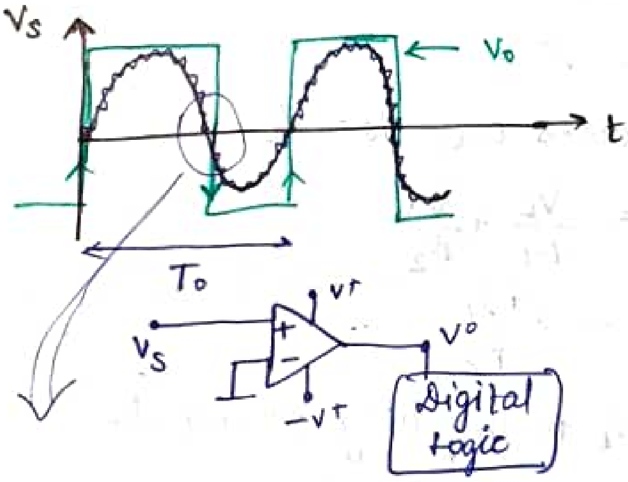
Then, use a digital plateform (like micro controller) to measure the time difference between two rising pulse.

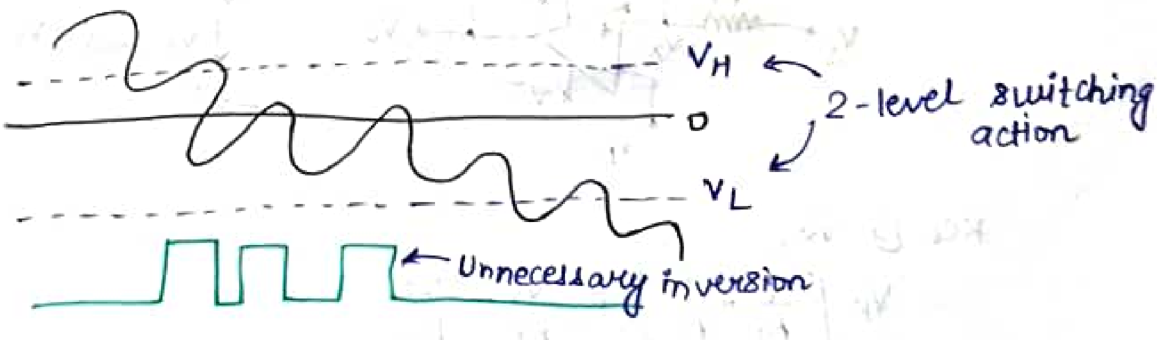
<u>Issue</u>: There may be noise to Vs which can unnecessarily invert the pulse V_o.

<u>Solution</u>: Use Schmidt Trigger.

2-04-2024

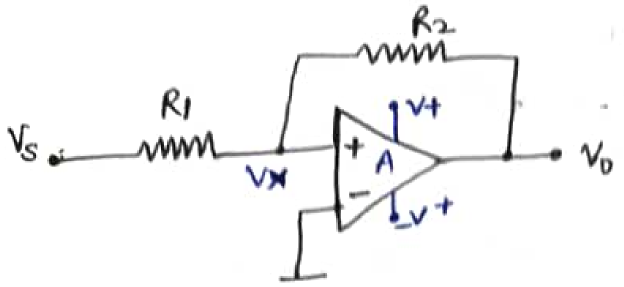
<u>Noisy Signal</u>:





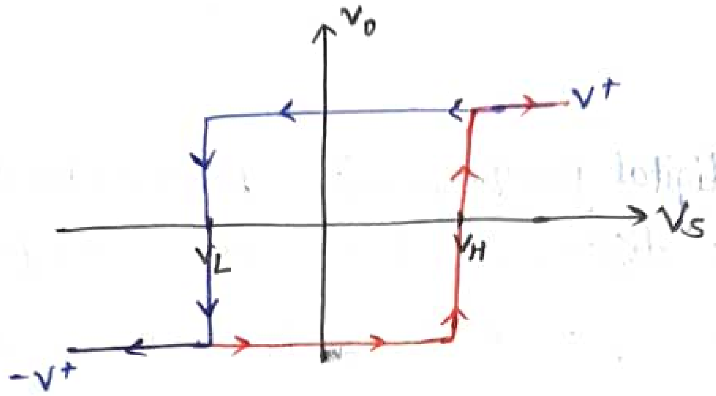
$V_S < V_L \Rightarrow V^+ \xrightarrow{V_0} -V^+$

$V_S > V_H \Rightarrow -V^+ \xrightarrow{V_0} V^+$



$V_0 = A V_x$

$V^+ \downarrow V^-$ $\quad 0^+ \downarrow 0^-$



$|V_L| = V_H.$

KCL @ V_x,

$$V_x\left[\frac{1}{R_1}+\frac{1}{R_2}\right] = \frac{V_S}{R_1}+\frac{V_0}{R_2}$$

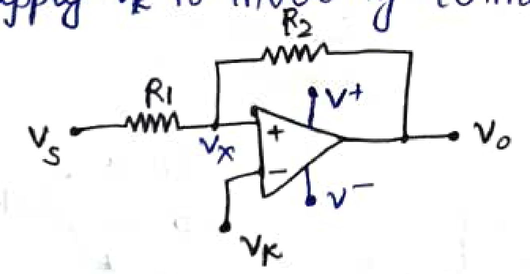
$V_0 = V^+$

V_x cross 0 @ $V_S = V_L$.

$$0 = \frac{V_L}{R_1}+\frac{V^+}{R_2}$$

$$V_L = -\frac{V^+ R_1}{R_2}$$

Now, apply V_K to inverting terminal.



$(V_K = 1V, \text{ say})$

$V^+ \downarrow V_0 -V^+ \Rightarrow V_x \quad V_K^+ \ (1.01) \downarrow V_K^- \ (0.99)$

KCL @ V_x,

$$V_K\left[\frac{1}{R_1}+\frac{1}{R_2}\right] = \frac{V_S}{R_1}+\frac{V_0}{R_2}$$

$V_0 = V^+$

V_x cross V_K @ $V_S = V_L$

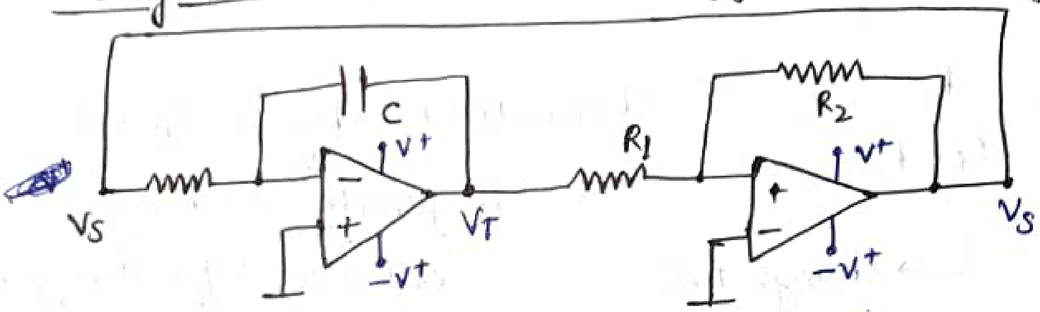
$|V_L| \neq V_H$

$\hookrightarrow$ Assymetrical schmidt Trigger.

$$V_L = -\frac{V^+ R_1}{R_2} + V_K\left(1 + \frac{R_1}{R_2}\right)$$

$$V_H = \frac{V^+ R_1}{R_2} + V_K\left(1 + \frac{R_1}{R_2}\right).$$

→ <u>Integrator with Schmidt-Trigger:</u> [Triangular and square wave generator]

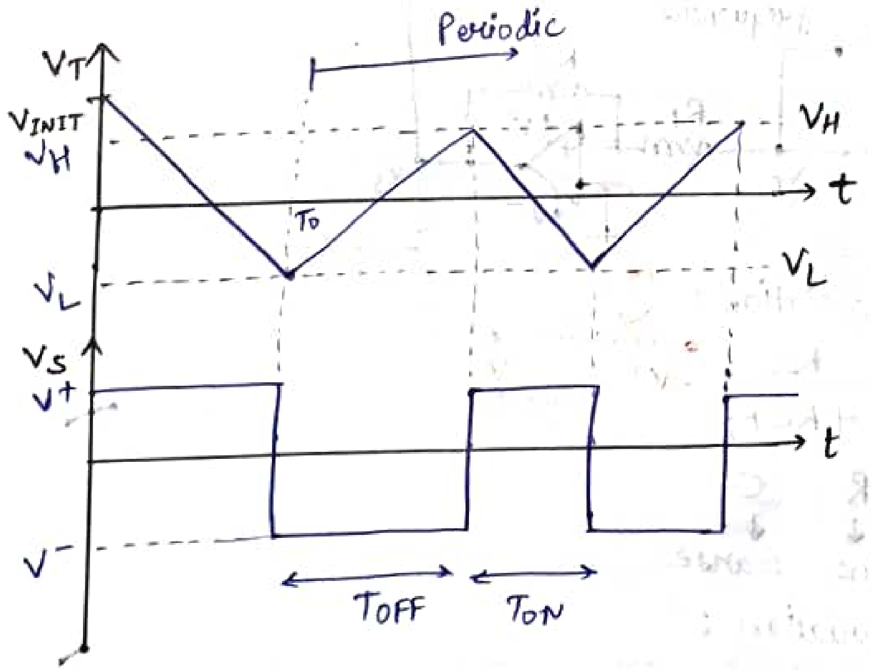


→ Function generator
↓
generates square and triangular waves

→ Capacitor (C) is having some residual voltage, V_{INIT} (initial voltage).
↳ Initial value of V_T.

$$V_T = V_{INIT} - \frac{1}{RC}\int \cancel{V_s}_{\,V^+}\, dt \qquad \Bigg|\; V_L = -V^+\frac{R_1}{R_2}\;;\; V_H = V^+\frac{R_1}{R_2}$$

$$V_T = V_{INIT} - \frac{1}{RC}V^+ t$$



$$T = T_{ON} + T_{OFF}$$

$$f = \frac{1}{T}$$

$$V_T(T_0) = V_L$$

$$V_T = V_L + \frac{V^+}{RC}(t - T_0) \qquad \left[V_T = V_L + \frac{1}{RC}\int_{T_0}^{t} V^+ dt\right]$$

$$\cancel{T_{OFF} = \frac{(V_H - V_L)RC}{V^+}}$$

$$V_T(T_o + T_{OFF}) = V_H$$

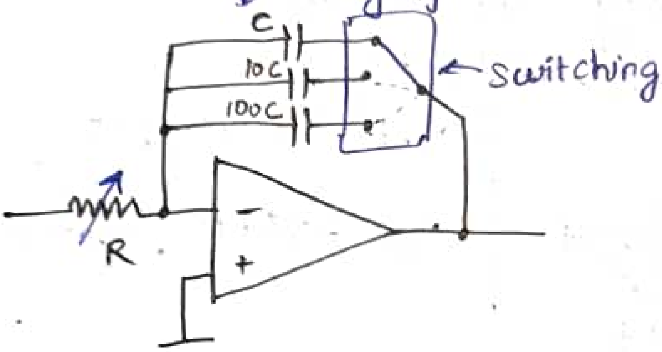
$$\Rightarrow T_{OFF} = \frac{(V_H - V_L)RC}{V^+}$$

$$T_{OFF} = 2RC\frac{R_1}{R_2} = T_{ON}$$ [As charging levels are same for T_{ON} & T_{OFF}]

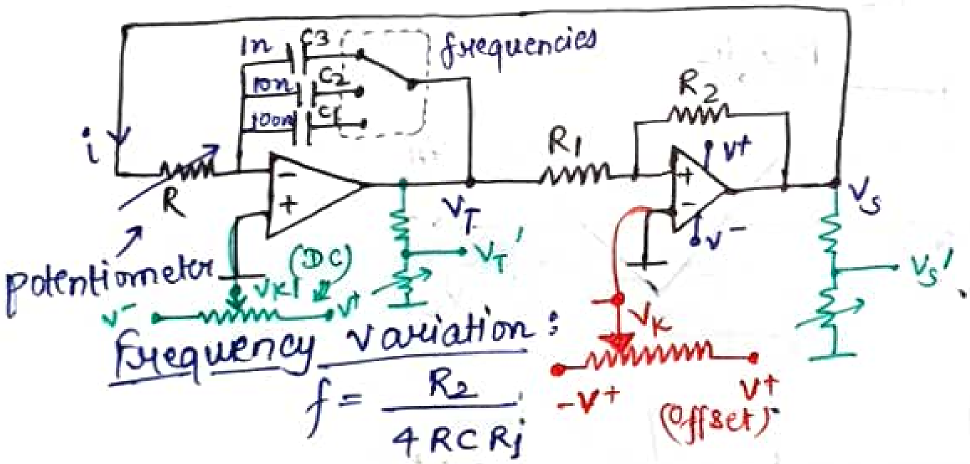
$$\therefore T = 4RC\frac{R_1}{R_2}$$

$$\Rightarrow f = \frac{R_2}{4R_1 RC}$$, Changing R_1 & R_2 changes V_H & V_L
⇓
Amplitude will change
⇓
Not a good function generator.

↳ change R, C to change f.



(08-04-2024)



<u>Frequency variation</u>:

$$f = \frac{R_2}{4RCR_1}$$

R → fine
C → coarse

<u>Amplitude variation</u>:

$$V_H = V^+\frac{R_1}{R_2}$$

→ Cannot be done by changing R_1, R_2 or V^+ (as there are limitations to supply V^+, V^-).

→ done using voltage divider → Get V_T', V_s'.

Offset :

↳ To get $V_H \neq |V_L|$

↳ Use assymetrical schmitt trigger → Voltage to inverting terminal. (V_K)

Duty-cycle:

$T_{ON} = T_{OFF}$ (for the basic circuit). ⇒ Duty cycle = 50%.

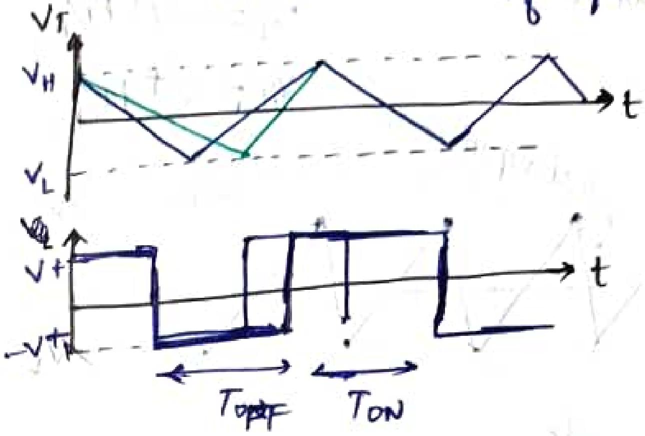
$T_{ON} \neq T_{OFF}$ → Rate of charging ≠ rate of discharging of capacitor.

~~Add a potentia~~

Charging current,

$$i = \frac{V_S - V_{K1}}{R}$$

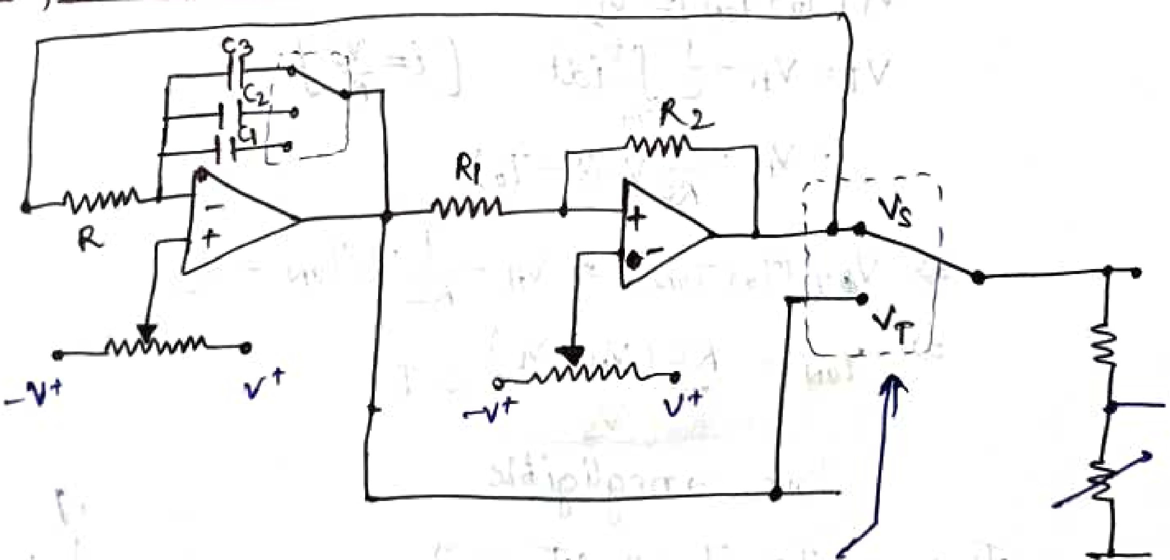
→ Add a potential to non-inverting terminal of integrator.



OFF time : $i_{CHARGE} = \frac{-V^+ - V_{K1}}{R}$

ON time : $i_{DISCHARGE} = \frac{V^+ - V_{K1}}{R}$

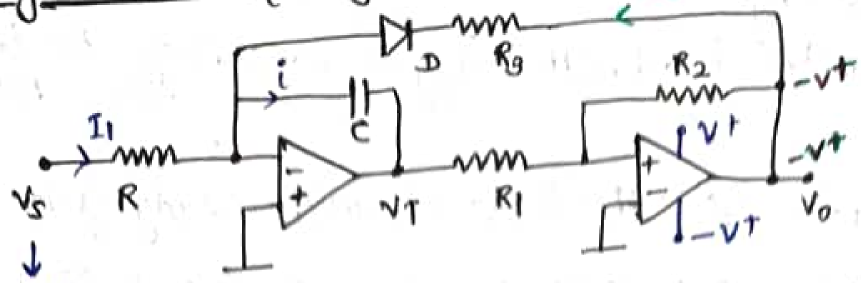
Waveform Selection :



V_S : square wave (Single divider)

V_T : Triangular wave

Voltage-to-Frequency Converter (Using integrator and schmidtt Trigger)

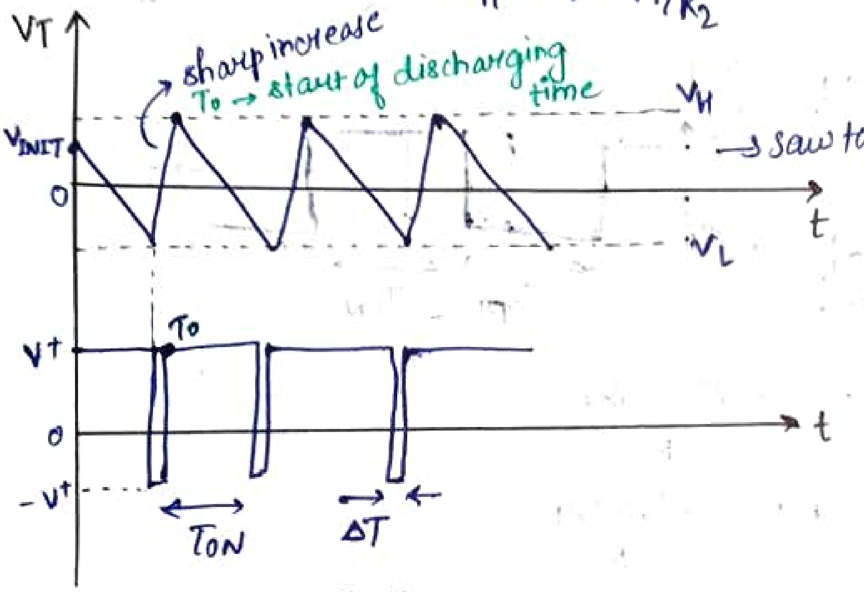


$(R_3 << R)$

V_S → +ve, constant

$$V_L = -V^+ R_1/R_2$$

$$V_H = V^+ R_1/R_2$$



Initially: V^+

$D \Rightarrow R.B.$

$i = I_1 = \frac{V_S}{R}$

$V_T = V_{INIT} - \frac{1}{RC} V_S t$ ↳ discharge

$-V^+$

$D \Rightarrow F.B.$

$I_2 = \frac{-V^+}{R_3}$

$|I_2| >> I_1 \quad [\because R_3 << R]$

$$V_T(T_0) = V_H$$

$$V_T(T_0 + T_{ON}) = V_L$$

$$V_T = V_H - \frac{1}{C}\int_{T_0}^{t} i\,dt \qquad \left[i = \frac{V_S}{R}\right]$$

$$= V_H - \frac{1}{RC} V_S (t - T_0)$$

$$\Rightarrow V_T(T_0 + T_{ON}) = V_H - \frac{1}{RC} V_S T_{ON} = V_L$$

$$\Rightarrow T_{ON} = \frac{RC(V_H - V_L)}{V_S} \approx T$$

T_{OFF} ⇝ negligible

Time period, $T = T_{ON} + \Delta T \approx T_{ON}$

Fundamental frequency, $f = \frac{1}{T} = \frac{V_S}{\left(2RCV^+ \frac{R_1}{R_2}\right)}$

$$\Rightarrow \boxed{f = K V_S}$$

↳ Voltage-to-frequency converter

V → f → Digital units → High-resolution

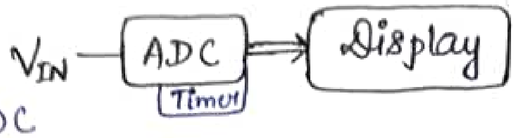
⇓

V, I can be measured with very high resolution

DIGITAL INSTRUMENTATION

09-04-2024

Digital Voltmeter

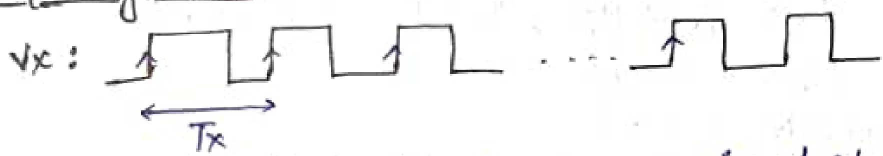


ADC
- → Speed (samples/sec.) of ADC can be slow.
- → Precise and accurate.
- → Independent of component non-idealities.

} based on Time Measurement [includes time module/timer]

Timer
↳ Required for ADC as well as time-measurement of events.

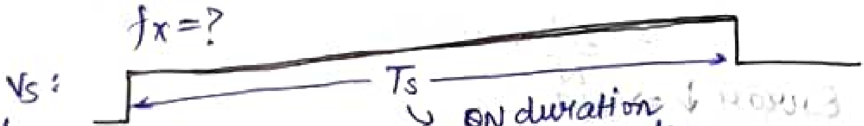
Frequency Meter



Estimate frequency of unknown signal, V_x :

$f_x = \frac{1}{T_x}$

$f_x = ?$



↳ standard pulse (Known)

↳ ON duration of std. pulse

$T_s = 1$ (say)

$f_x = 100$ Hz

$T_x = 0.01s$

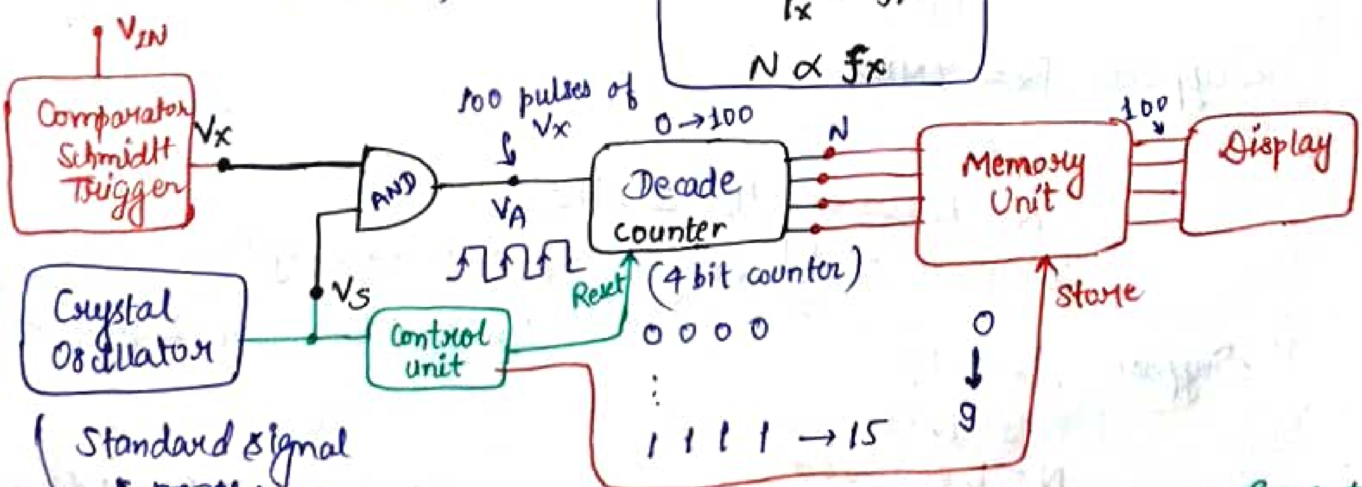
$N = 100 = f_x$

~~cont~~ count the no. of pulses of V_x in 1s (T_s) → N

⇓

count no. of rise.

$$\boxed{N = \frac{T_s}{T_x} = f_x T_s \qquad N \propto f_x}$$



Standard signal repeats:

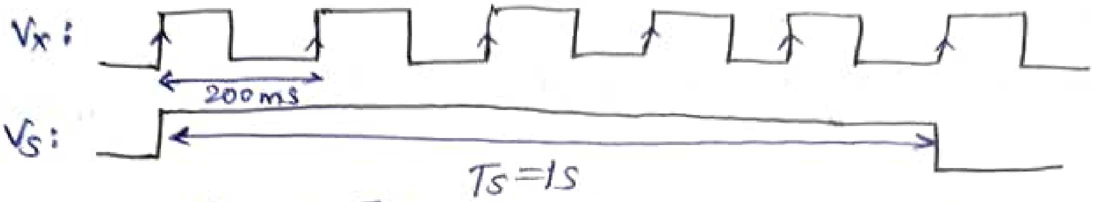
→ Reset to 0 when Vs is low

→ To get a stable count of 100 (not fluctuating) ⇒ Add a memory unit.

→ The input signal can be square / triangle, etc.
⇓
Add comparator Schmidtt Trigger to the i/p side.

Sources of Error :

Suppose $f_x = 5\,Hz \Rightarrow N = 5$
$T_S = 1\,s$



$N_m = 6$ [There can be a chance that there are 6 transitions]

Error in count $= \Delta N = N_m - N = +1$

Suppose $f_x = 4.44\,Hz$, $T_S = 1\,s \Rightarrow N = 4.44$ (desired)
$N_m = 4$ (say)
$\Rightarrow \Delta N = -0.44$

$\therefore\ \Delta N = \pm 1$ (worst case error).

$\%\ error = \frac{\Delta N}{N} \times 100\,\%$

$= \frac{\pm 1}{N} \times 100\,\%$

Error ↓ ⇒ N ↑

Suppose $T_S = 1\,s \longrightarrow T_S = 1000\,s$ — interpret as mHz to get 100 Hz of f_x on display

$f_x = 100\,Hz$

$\therefore\ N = f_x T_S = 100\,(Hz) \pm 1\,(Hz) \longrightarrow 100\,000\,mHz \pm 1\,mHz$

$\Rightarrow$ Error $= \pm 1\,Hz \longrightarrow \pm 1\,mHz$

Resolution $= 1\,Hz$ (count) $\longrightarrow 1\,mHz$

Error ↓ ⇒ T_S ↑

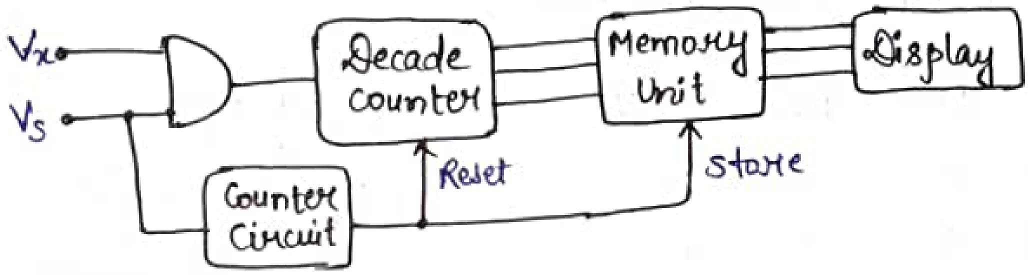
Suppose $f_x = 1\,MHz$
$T_S = 1\,s$
$N = 1{,}000{,}000\,Hz \pm 1\,Hz$

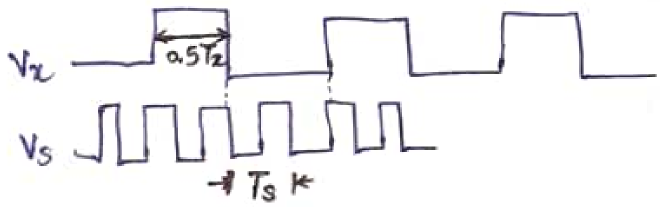
$\%\ error = \frac{\pm 1}{10^6} \times 100\,\% = 10^{-4}\,\%$ → very low error if we wait for 1s.
↳ suitable for high freq.

Suppose $f_x = 1\,Hz$
$T_S = 10^6\,s$
$N = 1{,}000{,}000$
$\%\ error = 10^{-4}\,\%$ → but we have to wait for 11 days to get o/p
↳ Not suitable for low freq.

15-04-2024

Frequency Measurement (Digital Frequency Meter)





$$N = \frac{0.5T_x}{T_s}$$

Eg. $f_x = 1\,Hz$, $T_x = 1s$

$T_s = 0.5\,\mu s \Rightarrow N = 1{,}000{,}000\,(\mu s) \pm 1\,(\mu s)$

$\%\ error = \frac{1}{10^6} \times 100 = 10^{-4}\,\%$ → we have to wait for only $0.5\,T_x$ time (not $10^6 s$) and have low error.

Frequency, $f_x = \frac{1}{N}$.

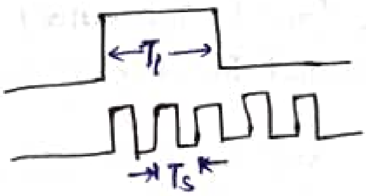
Now,

$T_x = 0.9s$

$N = 9{,}00{,}000\,(\mu s)$

$freq. = \frac{1}{N}$.

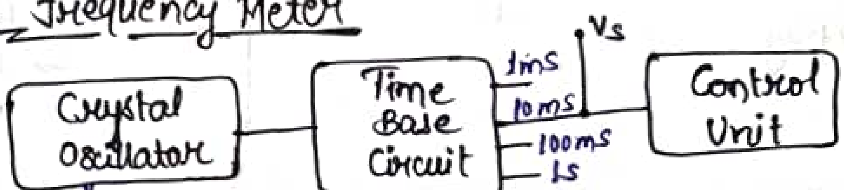
Time Period Mode of Operation : Timer



$$N = \frac{T_1}{T_s}$$

$$\Rightarrow T_1 = N\,T_s$$

Digital Frequency Meter



gives single frequency

frequency divider circuit ⇒ flipflops

→ Single digit decade counter.

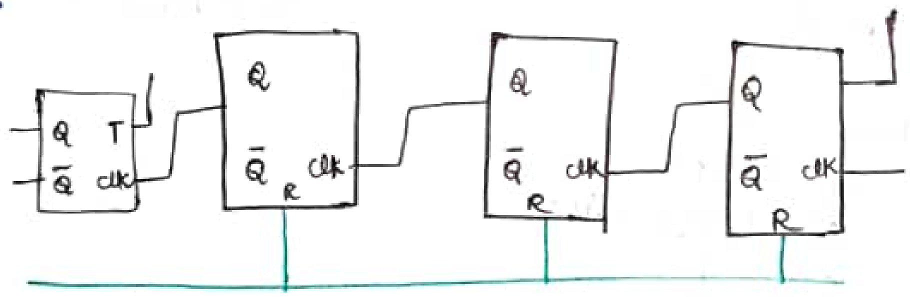
→ n-digit decade counter

↳ cascade single digit meters

T	DC3	DC2	DC1
0	0	0	0	0
1	0	0	0	1
2	0	0	0	2
⋮				
9	0	0	0	9
10	0	0	1	0
11	0	0	1	1
⋮				
999	0	9	9	9
1000	1	0	0	0
1999	1	9	9	9

⇒ 3 full-digit + 1 partial digit = 3.5 digit meter

↳



⇒ TFF : doubles the count.

→ Significance of 1/2 digit : To measure 2 resistors of the same batch.

eg. 1 kΩ

Result : 999 Ω or 1002 Ω (due to tolerance)

↳ cannot be display in 3 digit meter

$\underline{9}\ \underline{9}\ \underline{9}$

3½ digit: $\underline{1}\ \underline{0}\ \underline{0}\ \underline{2}$ ⇒ can be displayed in 3.5 digit meter.

→ LCD or LED is used for display.

R : 1 kΩ 2 kΩ 10 kΩ 100 kΩ
C : 1 nF 10 nF 100 nF
↳ commonly available values for R & C

eg. $f_x = 500\,Hz$

$T_s = 1\,s$

$N = f_x T_s = 500$

$\_\ \underline{5}\ \underline{0}\ \underline{0}$

3.5 meter (max) $\underline{1}\ \underline{9}\ \underline{9}\ \underline{9}$ $\approx 2\,KHz$

$\nearrow 2\,KHz$

Error = 1 Hz

Resolution = 1 Hz

Now, $f_x = 5000\,Hz$

$T_s = 100\,ms$

$N = f_x T_s = 500$

Turn ON the decimal display of 7 segment $\Rightarrow$ $\_\ \underline{5}.\underline{0}\ \underline{0}$

$\underline{1}\ \underline{9}.\underline{0}\ \underline{0}$ $\approx 20\,KHz$

$\rightarrow 2\,KHz$

$\rightarrow 20\,KHz$

Now, $f_x = 5000\,Hz$

$T_s = 10\,ms$

$N = 500$

$\_\ \underline{5}.\underline{0}\ \underline{0}$

$\underline{1}\ \underline{9}.\underline{0}\ \underline{0}$ $\approx 200\,KHz$

16-04-2024

$f_x = 500\,Hz$	$f_x = 5\,kHz$	$f_x = 50\,kHz$
$T_s = 1\,s$	$T_s = 100\,ms$	$T_s = 10\,ms$
$\_\ \underline{5}\ \underline{0}\ \underline{0}$ Hz	$\_\ \underline{5}.\underline{0}\ \underline{0}$ KHz	$\_\ \underline{5}\ \underline{0}.\underline{0}$ KHz
error = resolution = 1 Hz	error = resolution = 10 Hz	error = resolution = 100 Hz

$\nearrow$ • 2K

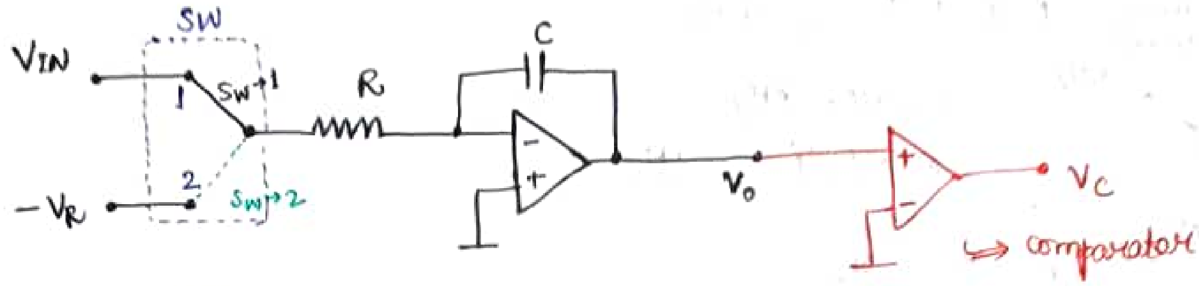
• 20K

• 200K

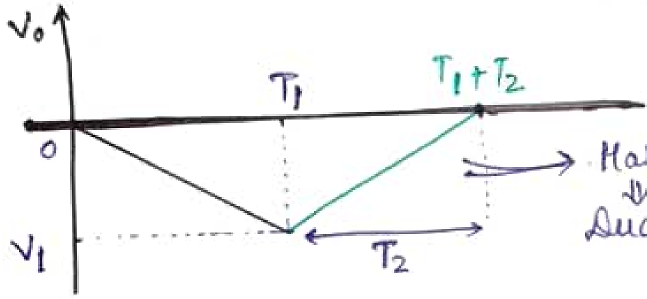
→ Modern meters have auto-ranging facility.

Digital Voltmeter [Dual Slope ADC]

ADC $\xrightarrow{\text{should be}}$
- slow
- precise
- independent of circuit



$$V_o = -\frac{1}{RC}\int V_{IN}\,dt$$
$$= -\frac{1}{RC}V_{IN}\,t$$



→ Has 2 slopes
⇓
Dual slope ADC

$$V_1 = -\frac{1}{RC}V_{IN}T_1 \quad \ldots ③$$

[T_1 : integration period (fixed)]
↳ set before usage

SW→2

Charging : $V_o(t) = V_1 + \frac{V_R}{RC}(t - T_1) \quad \ldots ①$

$V_o @ (T_1+T_2) = 0 \quad \ldots ②$

[V_R : reference voltage ↳ constant
T_2 : deintegration time]

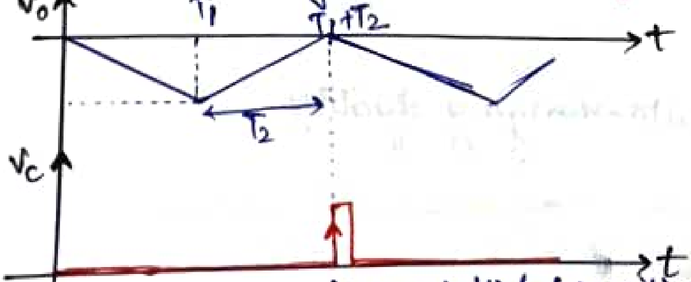
$① \,\&\, ② \Rightarrow V_1 + \frac{V_R}{RC}T_2 = 0$

Using ③ $\Rightarrow -\frac{1}{RC}V_{IN}T_1 + \frac{V_R}{RC}T_2 = 0$

$$\Rightarrow \boxed{\frac{V_{IN}}{V_R} = \frac{T_2}{T_1}}$$

As T_1, V_R : constant $\Rightarrow T_2 \propto V_{IN}$

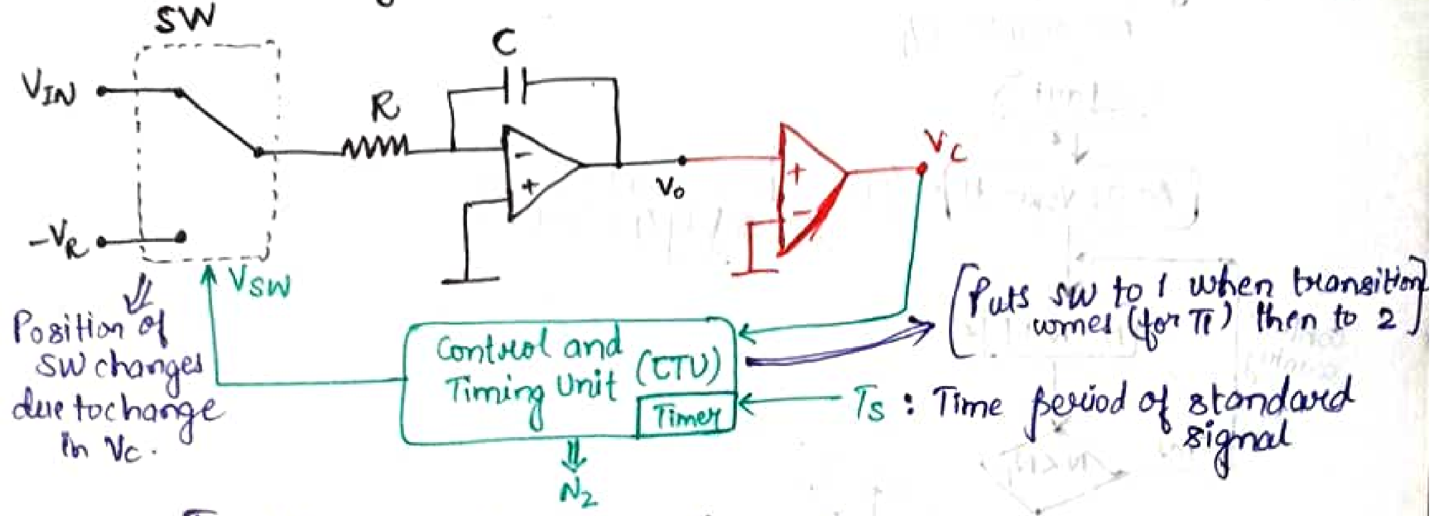
For detecting $T_1+T_2 \Rightarrow$ comparator is required



↳ Low to High transition helps identify (T_1+T_2).

For T_1 : SW→1 : $V_{SW} \equiv$ High (H)
T_2 : SW→2 : $V_{SW} \equiv$ Low (L)
V_{SW} : control signal
V_c has
H to L transition

Control and timing unit (CTU) : Timer



$T_1 = N_1 T_S$, N_1 : integration count (count during integration phase)

$T_2 = N_2 T_S$ N_2 : deintegration count

$$\frac{V_{IN}}{V_R} = \frac{T_2}{T_1} = \frac{N_2}{N_1}$$

As V_R, N_1 : constant $\Rightarrow N_2 \propto V_{IN}$, N_2 : digital count

↳ digital indication of i/p voltage

Eg. $N_1 = 2000$

$f_S = 10\text{KHz}$, $V_R = 2V$

$\Rightarrow T_S = 100\,\mu s$

$T_1 = 200\,ms$

$$\frac{V_{IN}}{V_R} = \frac{T_2}{T_1} = \frac{N_2}{N_1} \Rightarrow \frac{V_{IN}}{2} = \frac{N_2}{2000}$$

$\Rightarrow N_2 = 1000\, V_{IN}$

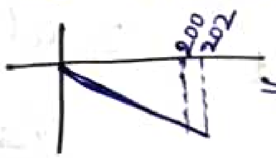
↳ i/p voltage in unit of (mV)

Suppose after 6 months,

$T_S = 101\,\mu s$

$T_1 = 202\,ms$

$N_2 \rightarrow$ remains the same.

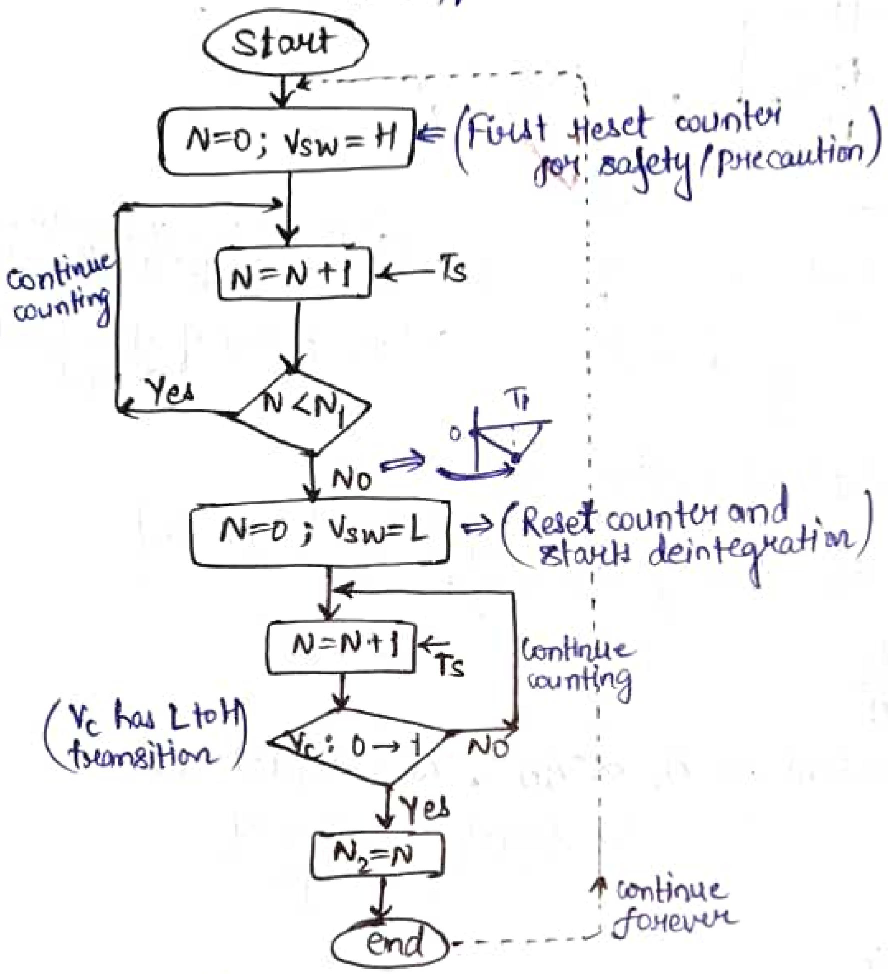


$\Rightarrow$ We're integrating for few more time.

17-04-2024

CTU Logic Flowchart:

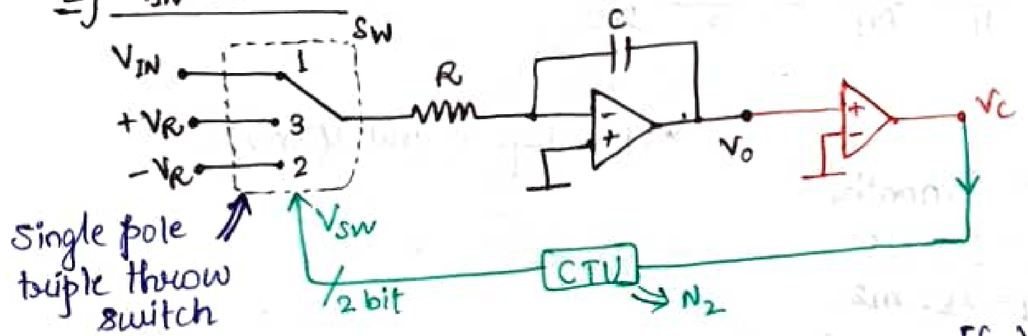
N: counter o/p

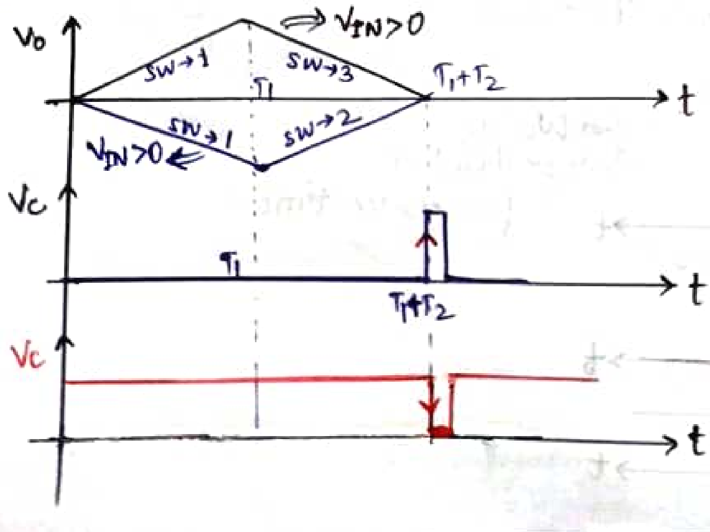


Assumption : Input, $V_{IN} > 0$

↳ (Unipolar DC voltmeter)

If $V_{IN} < 0$ also:





If $V_C(T_1) \equiv$ High $\Rightarrow$ sw→3

If $V_C(T_1) \equiv$ low $\Rightarrow$ sw→2

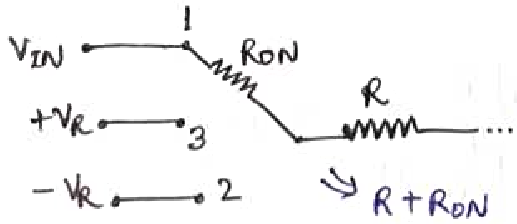
Vsw	sw
0 0	1
0 1	2
1 0	3
1 1	X

V_{SW_2} V_{SW_1} → separate waveforms

→ $\frac{V_{IN}}{V_R} = \frac{T_2}{T_1} = \frac{N_2}{N_1}$ ⇒ output (N_2) is independent of T_S, R, C, R_{ON}, V_{os}

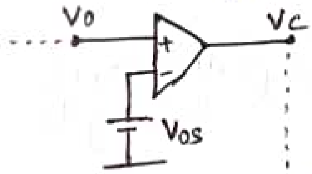
$\underbrace{\quad}$ tolerance & drift don't affect the system

→ Ideal switches have zero ON resistance.
Practical switches have ON resistance.

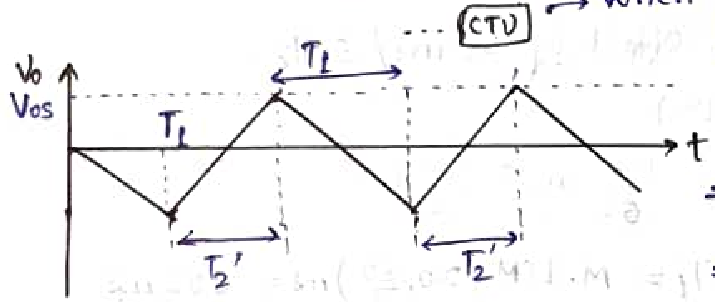


→ As R doesn't have any effect on o/p, R_{ON} also won't have.

→ V_{os}: i/p offset voltage of comparator
↳ modelled as voltage source at ⊖ve terminal.



→ when V_O crosses V_{os}, ↑ (L to H transition)

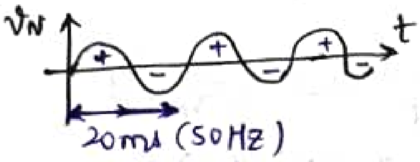


$T_2' \neq T_2$

⇒ Error in deintegration count
⇒ waveform shifted upward (ignoring initial 1st value)
⇓
No effect on o/p as time durations are constant.

→ Unipolar Voltmeter:

Sources of error: Powerline interference ↳ can affect i/p signal
⇓



$v_N = V_N \sin(100\pi t)$ (∵ 50 Hz pulse)

∴ $v_{IN} = V_{IN} + v_N$ [V_N: amplitude of powerline]

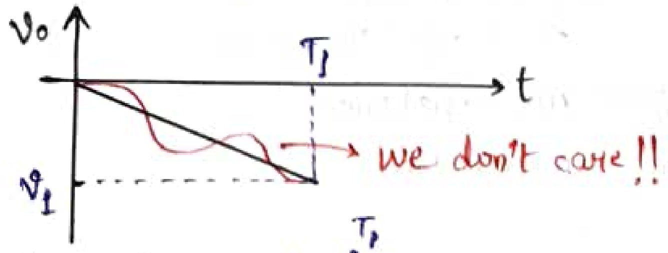
$-V_R$: Ref. voltage (generated by voltage regulator circuit)
↳ doesn't affect i/p

$v_{IN} = V_{IN} + v_N$ (T_1) → SW at 1 for T_1

∴ $-V_R = -V_R$ (T_2) → SW at 2 for T_2

$$v_o = -\frac{1}{RC}\int v_{IN}\,dt$$

$$\Rightarrow v_o = -\frac{1}{RC}\int (v_{IN} + v_N)\,dt$$



[T_I : integration time]

$$v_I = v_o(T_I) = -\frac{1}{RC}\int_0^{T_I}[v_{IN} + v_N]\,dt$$

$$= -\frac{1}{RC}T_I - \cancel{\frac{1}{RC}\int_0^{T_I} v_N\,dt} \rightarrow 0 \text{ (for each cycle of } v_N\text{)}$$

↳ Choose T_I as integral multiple of 20 ms.

$\boxed{T_I = M(20\,ms)}$ → for Indian condition

eg., $T_I = 60\,ms$ ↳ integration vanishes for 3 cycles.

For universal voltmeter,

nullify the effect of 20 ms / 50 Hz.

eg 60 Hz (US)

$$T_N = \frac{1}{f} = \frac{1000}{60}\,ms = \frac{50}{3}\,ms$$

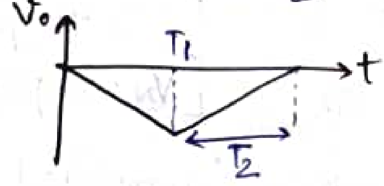
Take $T_I = M \cdot LCM\left(20, \frac{50}{3}\right) ms = 100\,ms$

↳ Reject disturbance by properly choosing T_I instead of filter at V_{IN}.

ADC Speed:

Conversion time (T_C) ↓ ⇒ speed ↑

$= T_1 + T_2$



$$\frac{V_{IN}}{V_R} = \frac{T_2}{T_I} \Rightarrow T_2 = \left(\frac{V_{IN}}{V_R}\right)T_I$$

$$\therefore T_C = T_I\left(1 + \frac{V_{IN}}{V_R}\right)$$

Typical $V_{IN} \in (0, V_R)$

↳ $T_{C_{max}} = 2T_I$

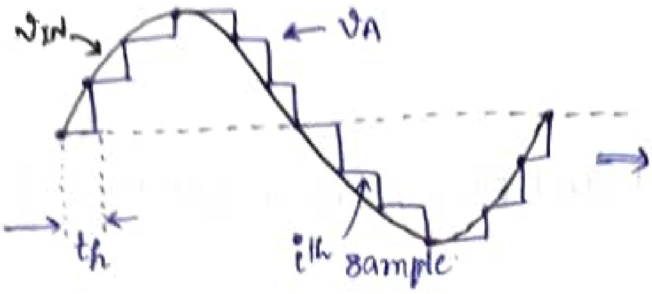
we need to provide $2T_I$ to ADC but T_I has other restriction:

Choose lower T_I and satisfy

$T_I = M \cdot LCM(20, 50/3)$.

22-04-2024

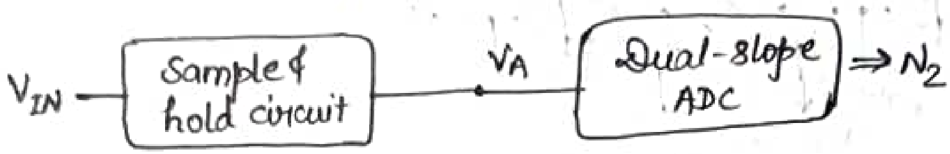
AC Voltmeter



$\Rightarrow$ Sampling which needs to be digitized.

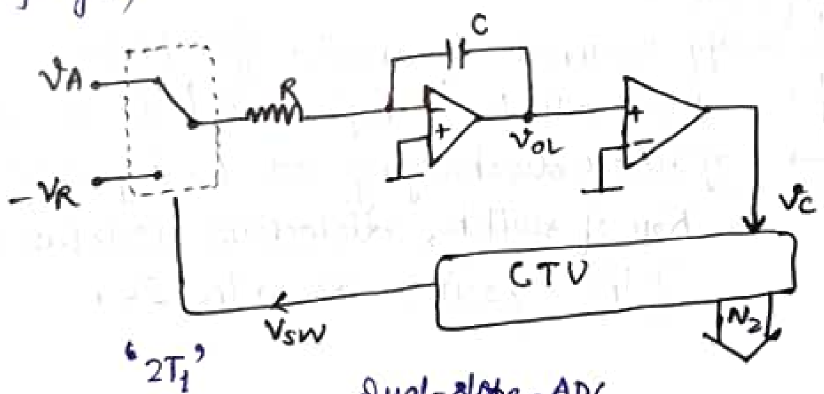
t_h : Hold time

$t_h \geq 2T_1$ [Input should be stable for atleast '$2T_1$']



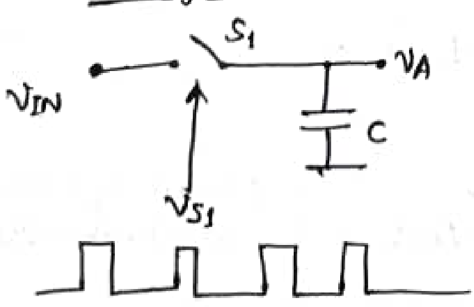
i^{th} sample $\rightarrow N_{2i}$

RMS value (to be displayed) $= \sqrt{\frac{1}{M}\sum_{i=1}^{M} N_{2i}^{\,2}}$, M samples in 1 cycle.
↳ (Approximate formula)

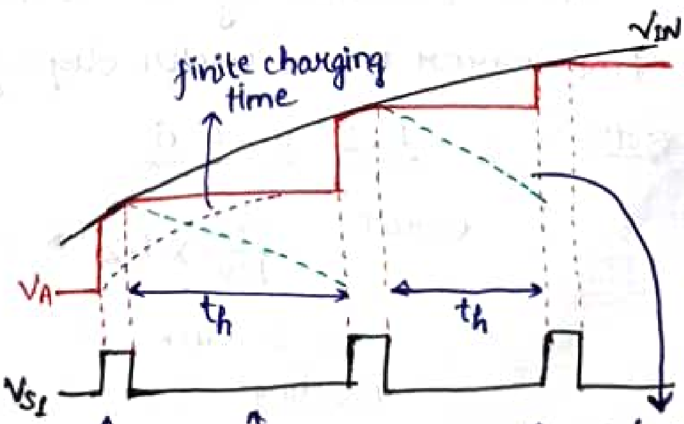


Dual-slope-ADC

Sample and Hold circuit:



V_{S_1} = High $\Rightarrow S_1$ = closed
= Low $\Rightarrow S_1$ = Open

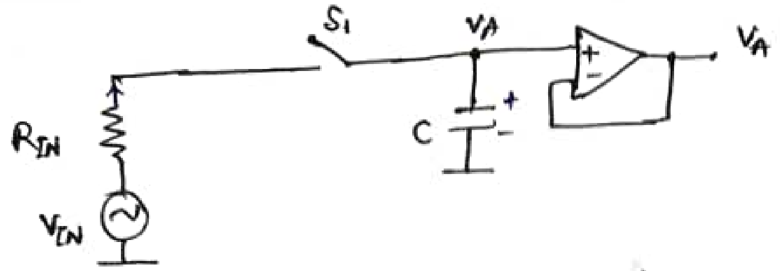


C charges when $S_1 \rightarrow$ closed

C holds voltage when $S_1 \rightarrow$ open

Use voltage follower

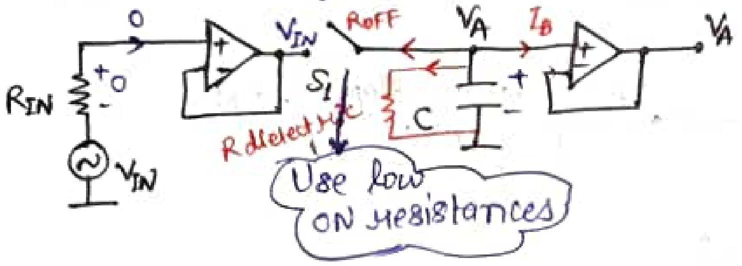
When V_A is connected to ADC, it will discharge capacitor when $S_1 \rightarrow$ open



⇓ Due to R_{IN} (effect of R and C), capacitor will take finite time for charging.

⇓

Use one more voltage follower.



Digital Multimeter (DMM)

↳ Has high errors in AC mode than DC mode

Sources of error:

- → Approximate formula for RMS.
- → Errors due to sample and hold circuit.
- → Capacitor discharging due to input offset current (I_B), R_{OFF} of switch, dielectric resistance ($R_{dielectric}$) of the capacitor or in the IC.

Eg. 4½ DMM

DC: 0.2% of reading + 2 counts

AC: 1% reading + 5 counts

Find error when meter displays 1.2 V DC.

Solⁿ: DC: 1.2 0 0 0

$$\text{error} = \frac{0.2}{100} \times 1.2 + 2 \times 0.1\,\text{mV}$$

[1 count of error ≡ 0.1 mV error]

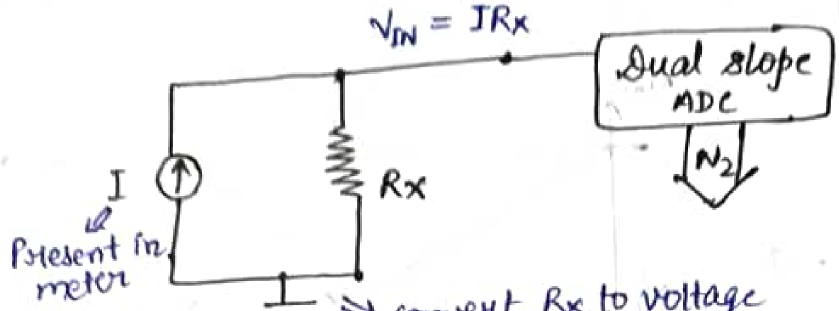
$= 2.6$ mV

Reading: $1.2\,\text{V} \pm 2.6\,\text{mV}$

AC: $\text{error} = \frac{1}{100} \times 1.2 + 5 \times 0.1\,\text{mV}$

$= 12.5$ mV.

Resistance Measurement



⇒ convert R_x to voltage using standalone current source.

$$\frac{V_{IN}}{V_R} = \frac{N_2}{N_1}$$

$$\Rightarrow \frac{IR_x}{V_R} = \frac{N_2}{N_1}$$

$$\Rightarrow N_2 \propto R_x$$

For $I = 1mA$
$N_1 = 2000$
$V_R = 2V$

$$\frac{1mA \; R_x}{2} = \frac{N_2}{2000}$$

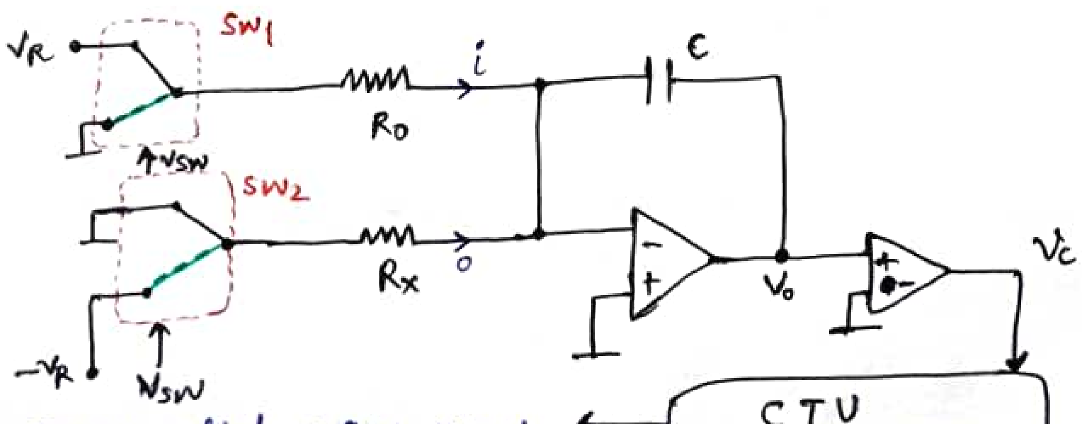
$$\Rightarrow \boxed{N_2 = R_x \, (\Omega)}$$

↳ N_2 is representing the resistance in the units of ohm.

Demerits :
- One grounded current source is needed.
- One separate ADC is needed.
- The circuit is cascaded, adding up the errors.

Solution: Use R_x as the input resistance of ADC, instead.

Resistance to Digital converter :



$T_1 \Rightarrow V_{SW} = high \Rightarrow SW_1, SW_2 \rightarrow 1$

$T_2 \Rightarrow V_{SW} = low \Rightarrow SW_1, SW_2 \rightarrow 2$

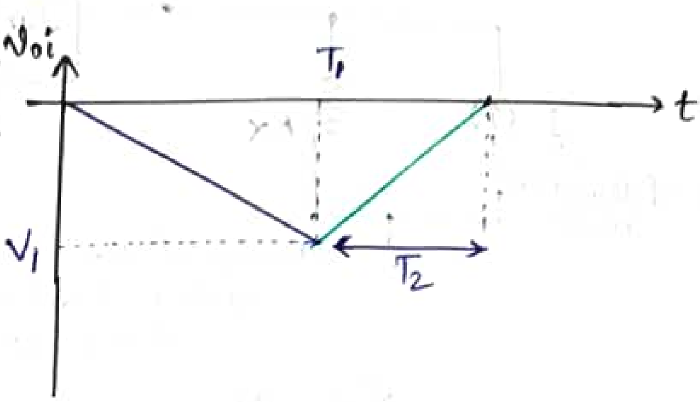
Duving T_1

R_x is inactive.

$$i = \frac{V_R}{R_0}$$

$$V_{oi} = -\frac{1}{R_0 C} V_R t$$

$$\Rightarrow V_1 = -\frac{1}{R_0 C} V_R T_1 \quad -①$$



Duving T_2

R_0 is inactive

$$i = -\frac{V_R}{R_x}$$

$$V_0 = V_1 + \frac{V_R}{R_x C}(t - T_1) \quad -②$$

$$V_0(T_1 + T_2) = 0 \quad -③$$

①, ② & ③ $\Rightarrow$ $$V_0 = -\frac{1}{R_0 \cancel{C}} \cancel{V_R} T_1 + \frac{\cancel{V_R}}{R_x \cancel{C}} T_2 = 0$$

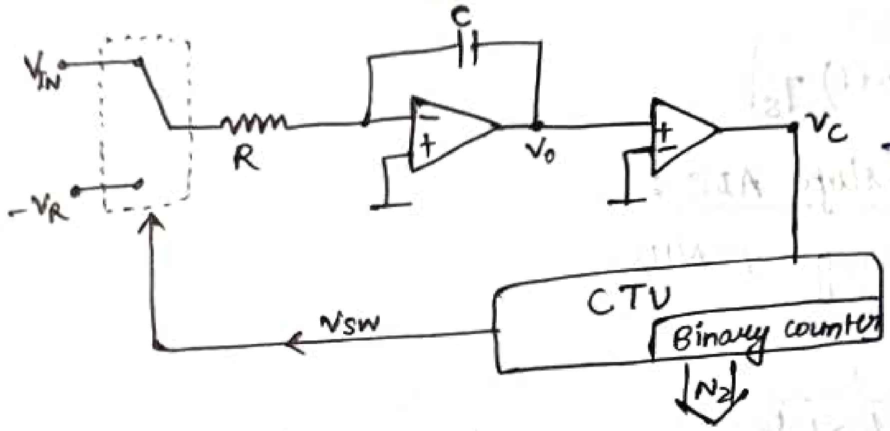
$$\Rightarrow \boxed{\frac{T_2}{T_1} = \frac{R_x}{R_0} = \frac{N_2}{N_1}}$$

Capacitance to Digital Converter

Generic Dual-slope ADC

	Binary counter	→	Decade counter
0	0000		0000
1			
2	⋮		
⋮			
			1001
9	1111		1010 → 0000

↓
Use complete binary counting (not limiting to '9' for display) } for generic use



→ No change in analog part

3-bit dual-slope ADC:

V_{IN}	D_{OUT}
(0,1)	0 0 0
(1,2)	0 0 1
(2,3)	0 1 0
(3,4)	0 1 1
(4,5)	1 0 0
(5,6)	1 0 1
(6,7)	1 1 0
(7,8)	1 1 1

$V_R = 8V$

$V_{IN} \in (0, 8)$

$$\text{Resolution} = 1V = \frac{8V}{2^3}$$

$$n\text{-bit ADC : Resolution} = \frac{V_R}{2^n}$$

↑ Unipolar ADC

Bipolar ADC:

$V_{IN} \in (-V_R, V_R)$

$$\text{Resolution} = \frac{2V_R}{2^n} = \frac{V_R}{2^{n-1}}.$$

For $N_1 = 8 = 2^3$ ✓ [Unipolar]

$$\frac{V_{IN}}{V_R} = \frac{N_2}{N_1}$$

$$\Rightarrow \frac{V_{IN}}{8} = \frac{N_2}{8}$$

$$\Rightarrow N_2 = V_{IN} \checkmark$$

n-bit ADC: [Unipolar]

$$N_1 = 2^n$$

$$T_1 = N_1 T_s$$

$$T_{cmax} = 2T_1$$

$$= 2N_1 T_s$$

$$\therefore \boxed{T_{cmax} = (2^{n+1})\, T_s}$$

Eg 16-bit Dual-slope ADC:

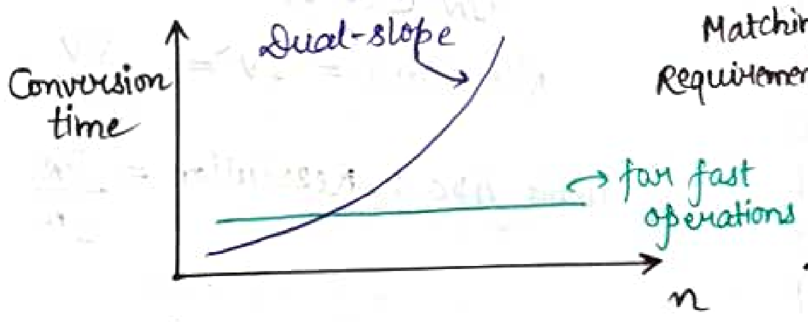
clock frequency = 10 MHZ

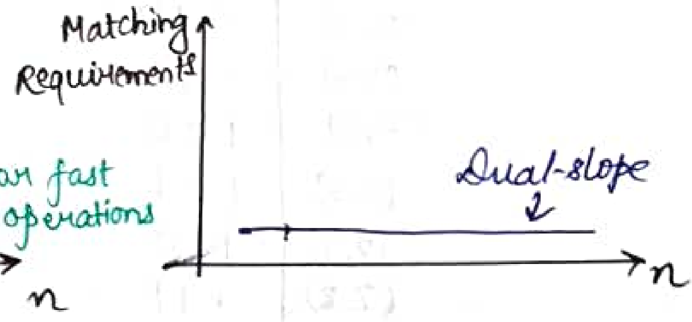
$$n = 16$$

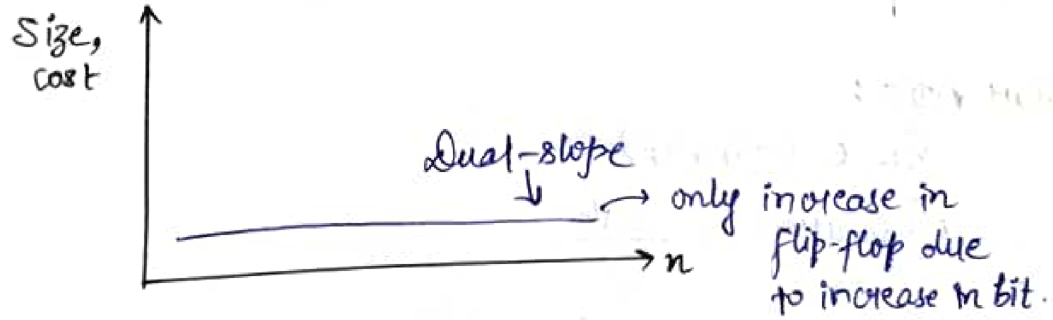
$$T_s = \frac{1}{10 \times 10^6}$$

$$\therefore T_{cmax} = 2^{17} \times 10^{-7} = 13\ ms$$

$$\therefore \text{no. of conversions/sec} = \frac{1}{13\ ms} \approx 76.$$



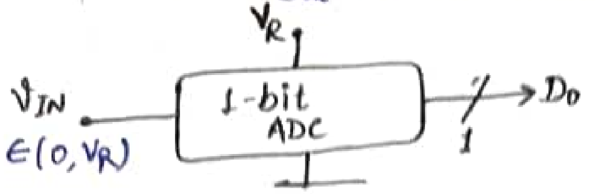




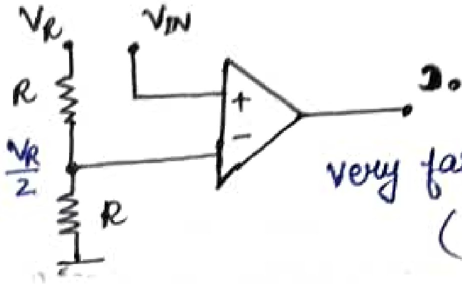
24-04-2024

Flash ADC:

→ DSO
→ Satellite communication

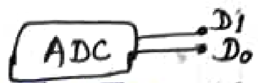


V_{IN}	D_0
$(0, V_R/2)$	0
$(\frac{V_R}{2}, V_R)$	1



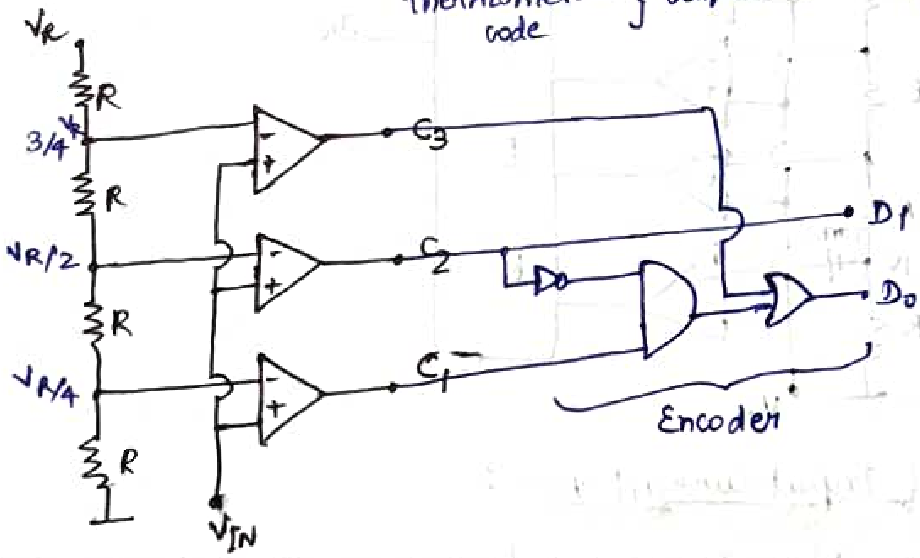
very fast/instantaneous output ($\approx 10ns$)

2-bit Flash ADC:



V_{IN}	D_1	D_0	C_3	C_2	C_1	$\overline{C_2}$	$\overline{C_2}C_1$
$(0, V_R/4)$	0	0	0	0	0	1	0
$(V_R/4, V_R/2)$	0	1	0	0	1	1	1
$(V_R/2, 3/4 V_R)$	1	0	0	1	1	0	0
$(3/4 V_R, V_R)$	1	1	1	1	1	0	0

$\underbrace{C_3 \; C_2 \; C_1}_{\text{thermometer code}}$ } outputs become progressively high

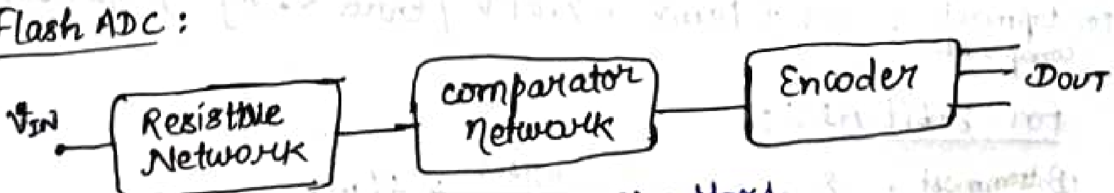


→ Only delay due to propagation delay of comparators and logic gates
very low

$D_1 = C_2$

$D_0 = C_3 + \overline{C_2} C_1$

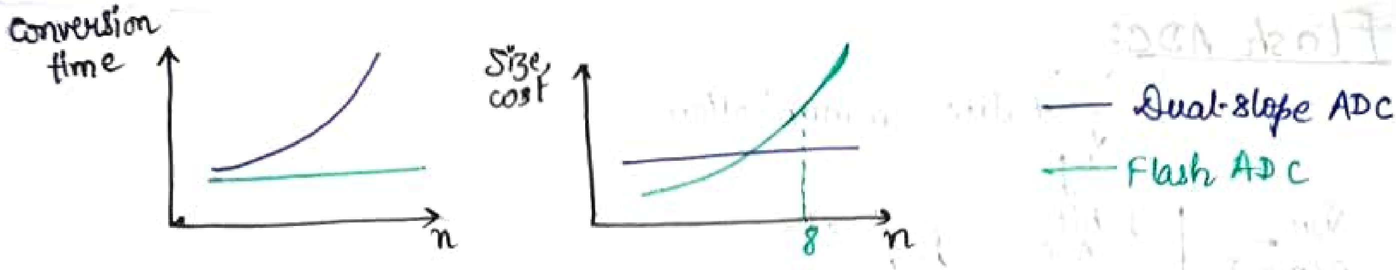
Flash ADC:

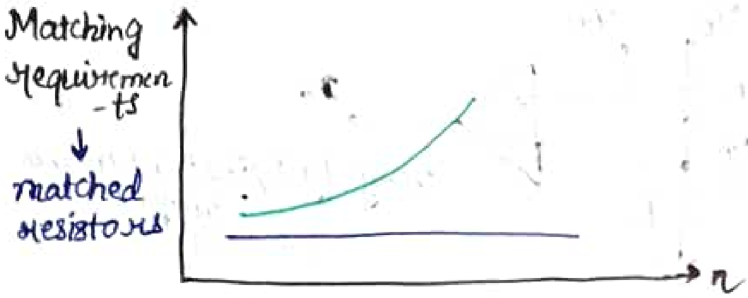


2 bit Flash ⟶ 4R, 3 comparators

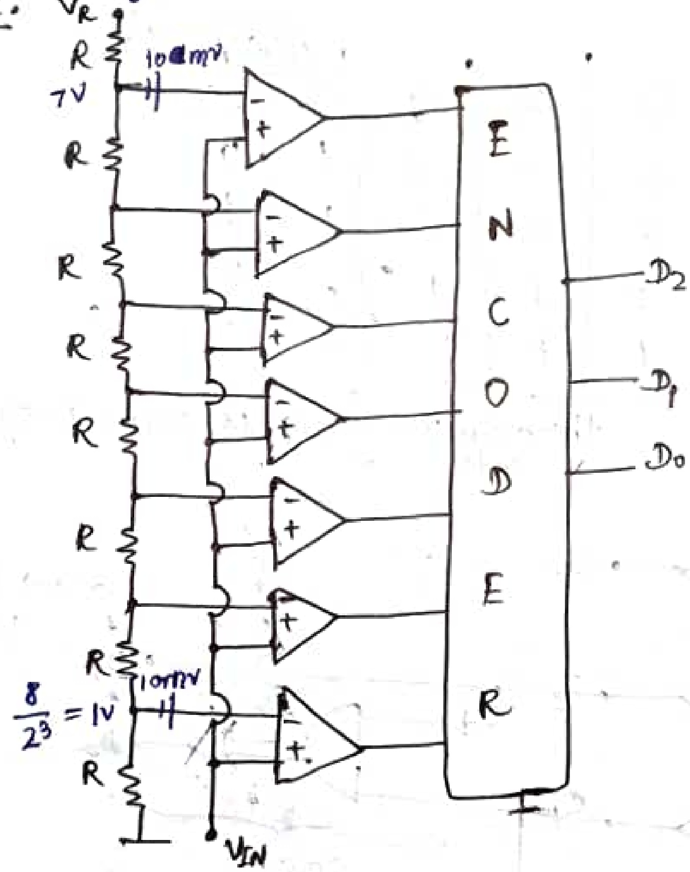
n-bit Flash ADC ⟶ $2^n R$, $(2^n - 1)$ comparators

↳ $n \leq 8$ (available)





3-bit ADC: $V_R = 8V$



Effect of Input Current Bias:

Suppose $V_{os} = 10mV$ → doesn't affect Dual-slope ADC

for bottommost comparator: $1V + 10mV = 1.01V$ [Error = 1%]

for topmost comparator: $7V + 10mV = 7.01V$ [Error < 1%]

} Error is highest for bottommost comparator

For 8-bit ADC:

Bottommost comparator: $\frac{8}{2^8} = 31mV \xrightarrow{\text{offset}} 41mV$

[Error $= \frac{10}{31} \times 100\%$] → very large

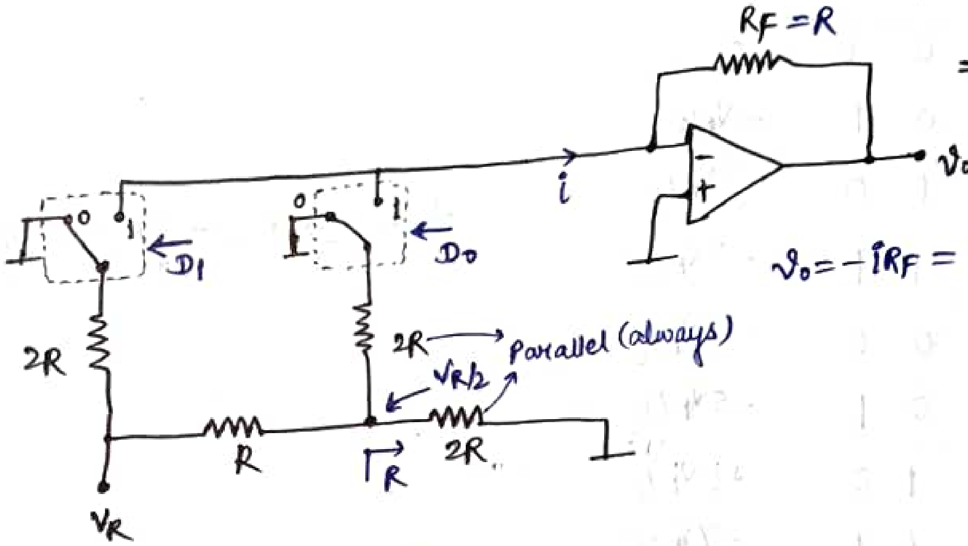
DAC (Digital to Analog Converter)

↳ Present within ADC

2-bit DAC : (V_R)

D_1	D_0	v_o
0	0	0
0	1	$-V_R/4$
1	0	$-V_R/2$
1	1	$-3/4\ V_R$

⇒ R-2R ladder DAC



$v_o = -iR_F = -iR$

$D_1 = 0, D_0 = 0$: $i = 0$

$D_0 = 0, D_1 = 1$: $i = \dfrac{V_R/2}{2R} = \dfrac{V_R}{4R}$

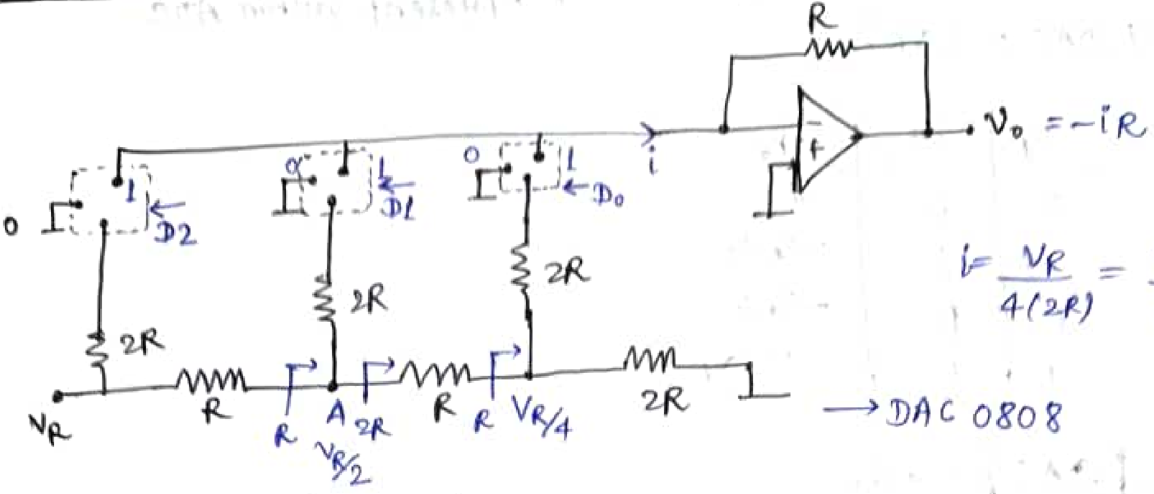
$$v_o = -\frac{V_R R_F}{4R} = -\frac{V_R}{4}$$

$D_0 = 1, D_1 = 0$: $i = \dfrac{V_R}{2R}$, $v_o = -V_R/2$

$D_0 = D_1 = 1$: $v_o = -\dfrac{V_R}{4} - \dfrac{V_R}{2} = -\dfrac{3}{4} V_R$

3-bit DAC :

25-04-2024



$i = \frac{V_R}{4(2R)} = \frac{V_R}{8R}$

→ DAC 0808

D_2	D_1	D_0	v_o
0	0	0	0
0	0	1	$-V_R/8$
0	1	0	$-2V_R/8$
0	1	1	$-3V_R/8$
1	0	0	$-4V_R/8$
1	0	1	$-5V_R/8$
1	1	0	$-6V_R/8$
1	1	1	$-7V_R/8$

$$v_o = -\frac{V_R}{8} D_0 - \frac{V_R}{4} D_1 - \frac{V_R}{2} D_2$$

4-bit DAC:

$$v_o = -\frac{V_R}{16} D_0 - \frac{V_R}{8} D_1 - \frac{V_R}{4} D_2 - \frac{V_R}{2} D_3$$

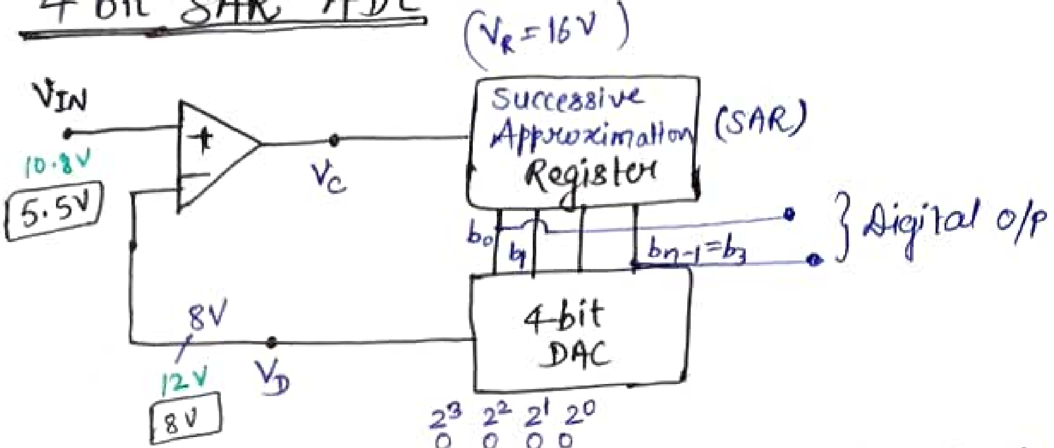
n-bit DAC:

$$v_o = -\frac{V_R}{2^n} D_0 - \frac{V_R}{2^{n-1}} D_1 \dots - \frac{V_R}{4} D_{n-2} - \frac{V_R}{2} D_{n-1}$$

→ Dual slope ADC: $(2^n - 1) T_s$

Flash: Instantaneous

4 bit SAR ADC



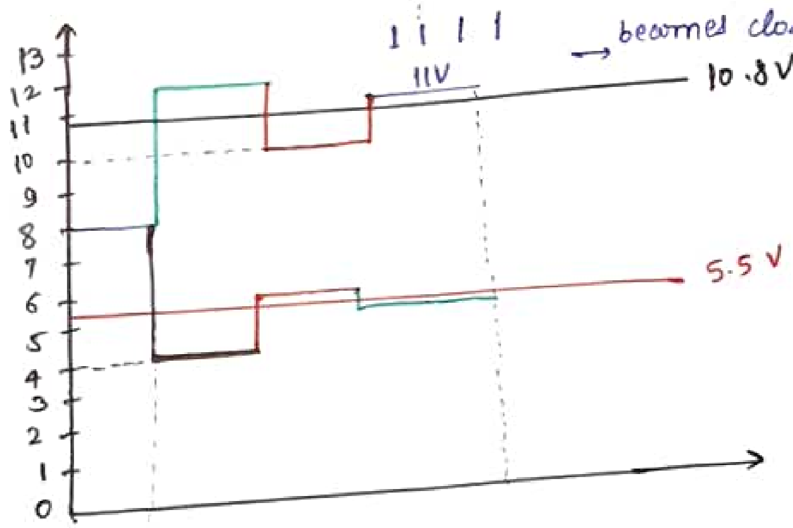
2^3 2^2 2^1 2^0
0 0 0 0

1 0 0 0 ← Initial ⇒ $V_0 = -\frac{V_R}{2}$ $(\because D_0 = D_1 = D_2 = 0)$

2nd clock cycle : 1 1 0 0

1 0 1 0

1 1 1 1 → becomes close to 10.8 V



1 0 0 0
0 1 0 0
0 1 1 0
0 1 0 1