INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

B.Tech (Electronics and Communication Engg)-End Semester Exam, May 2023

AV224 – Control Systems
(Answer all questions)

Time: 3 Hours **Date: 15/05/2023** **Max. Marks: 100**

1. Derive the expression for the resonant frequency, ω_r and resonant peak gain, M_r as a function of ω_n and ζ for the second order system transfer function given by

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \text{for} \qquad 0 < \zeta < \frac{1}{\sqrt{2}}$$

(10 marks)

2. Derive the expression for the break away points and angle of departure from complex poles for the root locus plot of an LTI negative feedback system with open loop transfer function, $G(s)H(s)$.

(3+3 marks)

3. Sketch the root locus plot with all relevant details for an LTI system with open loop transfer function,

$$G(s)H(s) = \frac{K}{s(s^2 + 6s + 25)}$$

for positive values of gain, K. State the rules used for each step of the root locus plot.

(12 marks)

4. (a) Derive expressions for the static position, velocity and acceleration error constants for a stable, unity feedback closed loop system with open loop transfer function, $G(s)$.

(6 marks)

(b) A pressure measurement system having the transfer function,

$$G(s) = \frac{c}{s^2 + a.s + b} \ V/Pa, \ c \neq b, \text{and } a, b, c > 0$$

is used for measuring the pressure inside a rocket fuel storage tank having a monotonic pressure decrease of $-1000\ Pa/s$. Find the steady state error in the measured pressure using the appropriate static error constant.

(7 marks)

5. The open loop transfer function of a unity negative feedback control system is given below.

$$G(s) = \frac{K}{s(s^5 + 2s^4 + 8s^3 + 12s^2 + 20s + 16)}$$

Using Routh criterion, determine the value of $K > 0$ which will cause sustained oscillation in the closed loop system at two distinct frequencies. What are the corresponding oscillation frequencies ?

(7 marks)

6. (a) Bring out the difference between first order lead, lag and PI controllers based on their Bode plots.

(9 marks)

(b) Draw the Bode plot of the transfer function, by showing the magnitude and phase characteristics of each factors separately.

$$G(s) = \frac{10(1 + 0.1s)}{s(1 + 0.01s)(1 + s)}$$

(8 marks)

7. Using Nyquist plot, find the stability of the closed loop control system with open loop transfer function,

$$G(s)H(s) = \frac{2(s + 3)}{s(s - 1)}$$

(10 marks)

8. (a) Find the solution of the homogeneous and non-homogeneous parts of the state space equation for a MIMO LTI system given by,

$$\dot{X} = AX + Bu, X \in \mathbb{R}^{n\times 1}, A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, u \in \mathbb{R}^{m\times 1}$$
$$y = CX + Du, y \in \mathbb{R}^{p\times 1}, C \in \mathbb{R}^{p\times n}, D \in \mathbb{R}^{p\times m}$$

with an initial condition of $X(0) = X_0$.

(10 marks)

(b) Find the expression for the input-output transfer function matrix of the above state space system assuming $X_0 = 0 \in \mathbb{R}^{n\times 1}$. Also prove that the transfer function is invariant under similarity transformation of the state equation.

(5 marks)

9. Derive the transfer function $G(s) = \frac{X_2(s)}{X_1(s)}$ for the vehicle suspension system given in Figure 1. Assume that each tyre can be modelled as a spring of stiffness, K_t while the ground is applying a total vertical reaction force of F(t) onto the vehicle through the tyres. Neglect the mass of the tyres and the axial rod interconnecting the tyres. Justify the correctness of $G(s)$ based on its steady state value.

(10 marks)

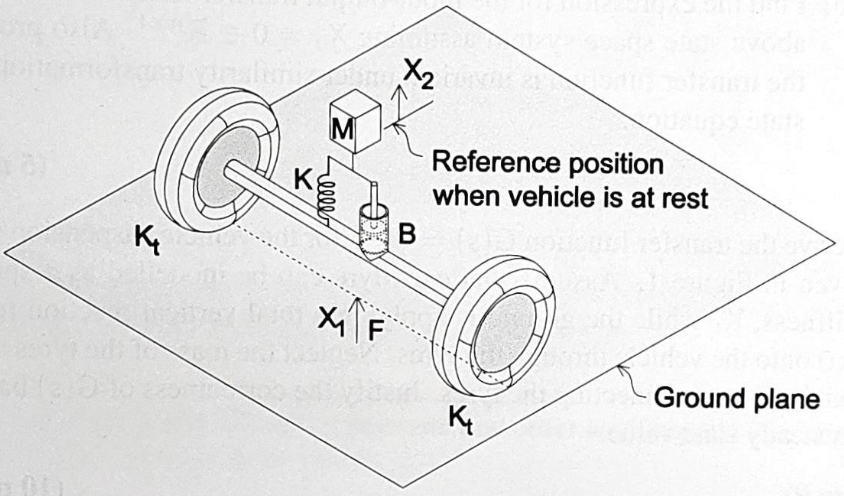

Figure 1: Vehicle suspension system

Hint. *Assume the upward ground reaction force,* $\mathsf{F}(\mathsf{t})$ *as the input excitation to the mass-spring-damper model of vehicle while the vehicle moving forward with a finite velocity over the uneven ground plane. Assume that* $\mathsf{F}(\mathsf{t}) = \mathsf{Mg}$ *when the ground plane is perfectly flat and smooth.*