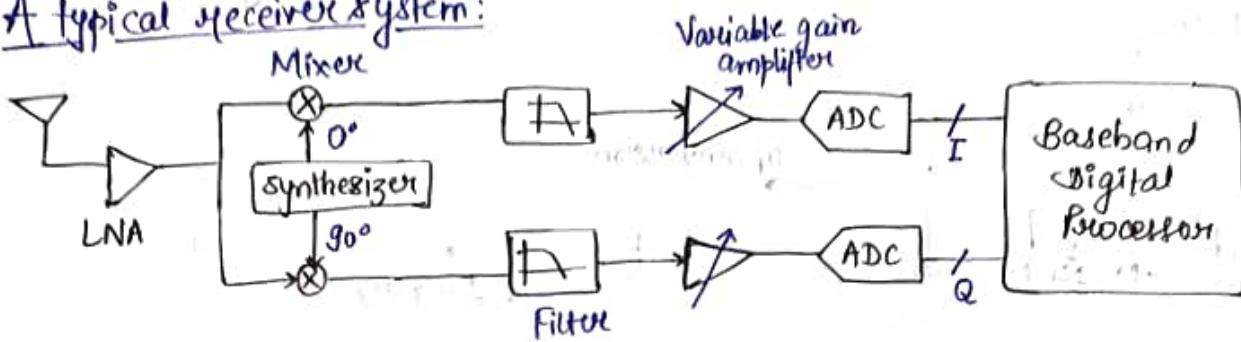


RF and Integrated Circuit

A typical receiver system:



→ CMOS IC design (RFIC)

↳ Technology node: CMOS

↳ Cost effective, size effective.

↳ Extremely good for digital

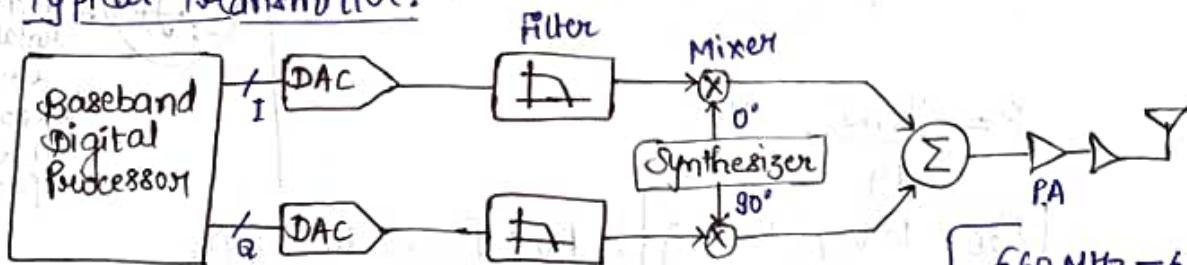
but not much for analog & RF.

↳ Beats other technology in terms of frequency of operation.

Main focus of course: CMOS SoC

- Synthesizer (LO, PLL, ...), Mixers
- LNA (Low Noise Amplifier)
- Filters (RF filters)
- Variable amplifier
- Digital COMMS system.

Typical Transmitter:



→ Matching in Audio amplifier, etc. → Not considered

↳ Audio freq. $\approx 20\text{ kHz}$ → λ much larger than length of wire used

Need of Matching for High frequency:

(when $\lambda <$ length of transmission lines)

In RFIC design, on a chip,

lines are very narrow ($\sim \text{mm}$) $\ll \lambda_{\text{signal}}$

(phase difference
is insignificant
b/w the nodes
of wire)

↳ Matching not necessary in RFIC design \rightarrow saves a lot of time and work

($1\text{GHz} \leftrightarrow 30\text{cm} = \lambda$)

↳ Main difference b/w RFIC & Traditional RF design

Discrete design \rightsquigarrow Not part of course

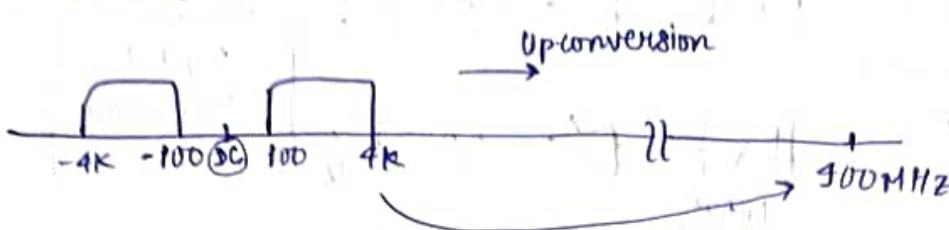
↳ PCB design with RF components

MMIC design \rightsquigarrow Not part of course
(Monolithic
Microwave IC)

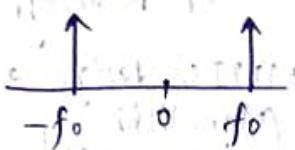
2. Field null

RF (~ 100 MHz): Due to small antenna size, we use high frequency.
for $f = 1$ GHz,

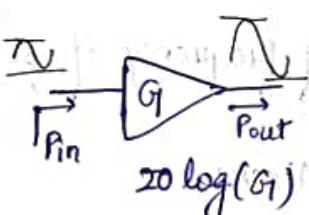
$$\lambda \approx 30\text{cm}$$



$$A \cos(2\pi f_0 t)$$



dBm Unit:



Voltage gain: G

Power gain: P

$$P_{dB} = 10 \log P$$

$$= 10 \log \left(\frac{V_{out}}{V_{in}} \right)^2$$

$$= 20 \log G$$

$$P = \frac{V_{out}^2}{R_L} = \frac{V_{pk}^2}{2R_L}$$

$$P_{dB} = 10 \log \left(\frac{P_w}{1mW} \right), P_w: \text{Power in Watts}$$

$$1mW \Leftrightarrow 0 \text{ dBm}$$

$$1W \Leftrightarrow +30 \text{ dBm}$$

$$1\mu W \Leftrightarrow -30 \text{ dBm}$$

$$1nW \Leftrightarrow -60 \text{ dBm}$$

$$1pW \Leftrightarrow -90 \text{ dBm}$$

$$1fW \Leftrightarrow -120 \text{ dBm}$$

In analog circuit,
we talk about
voltage gain,
not power gain.

$-f \rightarrow$ vector
rotating both
dimn
 \rightarrow Reason for
+ve & -ve
frequencies

Transmitter
design

noise

Transmit power:

For $R_L = 50\Omega$

$$1\text{mW} \rightarrow V_{PK} = 316\text{ mV}$$

$$100\text{mW} \rightarrow V_{PK} = 3.16\text{ V}$$

$$1\text{W} \rightarrow V_{PK} = 10\text{ V}$$

$V_{PK}^2 = 1\text{mW} \times 50 \times 2$

$= 100 \times 10^{-3}$
 $= 10^{-1}$
 $\Rightarrow V_{PK} = 0.316\text{ V}$
 $= 316\text{ mV}$

CMOS V_{DD} :

65nm	1.2V
45nm	1.0V
28nm	0.91
16nm	0.0109

\rightarrow Transmit $V_{PK} > V_{DD}$

\hookrightarrow can push the device to non-linearity
when V_{PK} becomes comparable to V_{DD} .

CE Amplifier:



Every amplifier exhibits non-linear behaviour.

Polynomial Model of Non-linearity



$$y(t) = \alpha x(t) \quad [\text{Linear assumption}]$$

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \dots$$

\rightarrow Memory less polynomial

$$x(t) = A \cos \omega_0 t$$

\hookleftarrow Good model for
memoryless amplifiers

$$y(t) = \alpha_1 A \cos \omega_0 t + \alpha_2 A^2 \cos^2 \omega_0 t + \alpha_3 A^3 \cos^3 \omega_0 t + \dots$$

$$= \alpha_1 A \cos \omega_0 t + \alpha_2 A^2 \left(\frac{1 + \cos 2\omega_0 t}{2} \right) + \alpha_3 A^3 \left(3 \cos \omega_0 t + \cos 3\omega_0 t \right) \frac{1}{4}$$

$$= \alpha_1 A \cos \omega_0 t + \frac{3}{4} \alpha_3 A^3 \cos \omega_0 t$$

$$+ \frac{\alpha_2 A^2}{2} \cos 2\omega_0 t$$

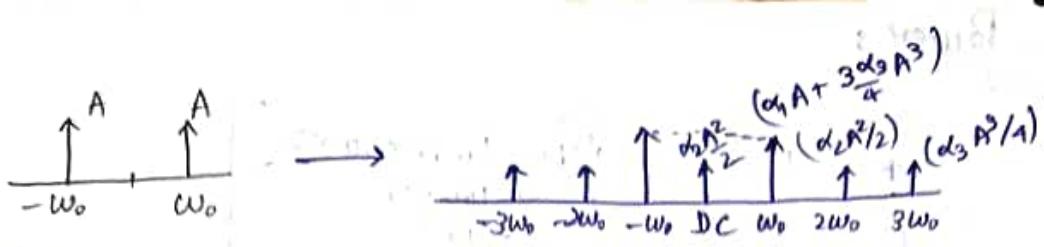
$$+ \frac{\alpha_3 A^3}{4} \cos 3\omega_0 t$$

$$+ \alpha_2 A^2 / 2$$

[~~2nd~~ harmonic]

[~~3rd~~ harmonic]

[DC]

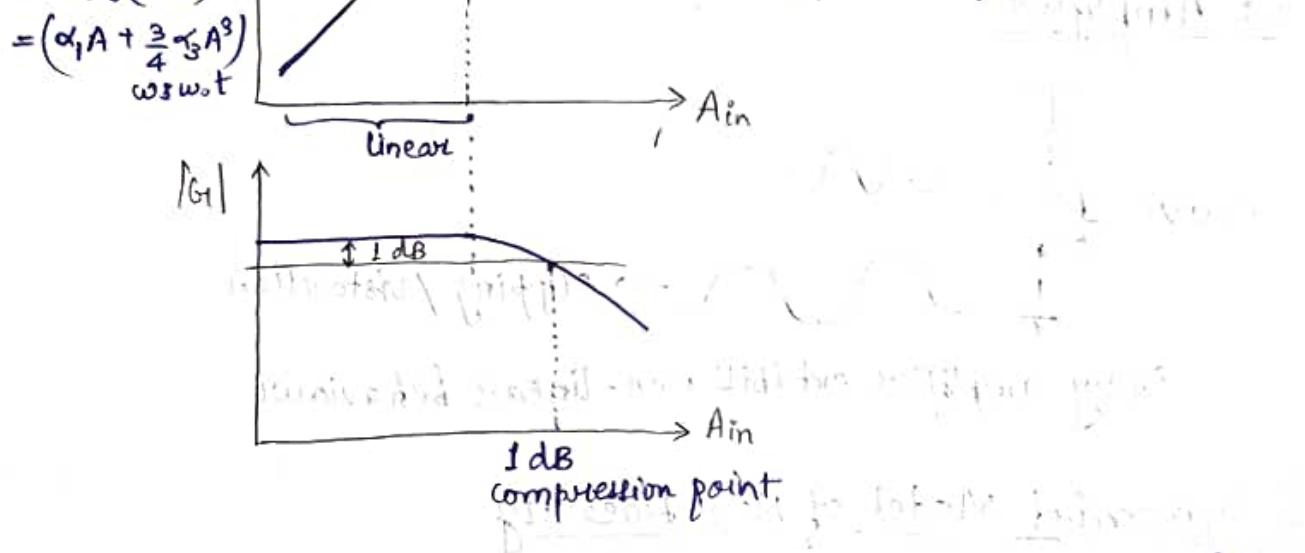
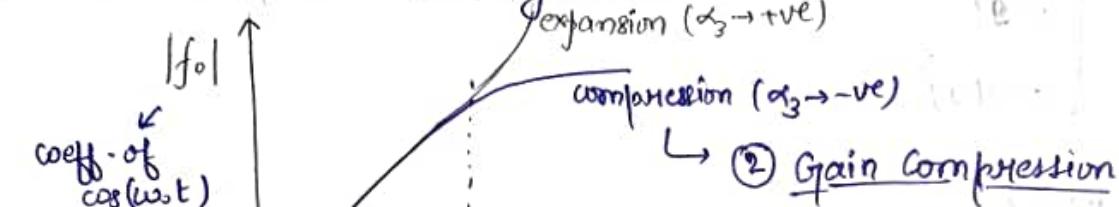


Effects:

① Generation of harmonics: can lead to o/p signal in freq. out of band.

$$f_o = \underbrace{\alpha_1 A \cos \omega_0 t}_{\text{desired}} + \underbrace{\frac{3}{4} \alpha_3 A^3 \cos 3\omega_0 t}_{\text{Extra gain expansion } (\alpha_3 \rightarrow +ve)}$$

[α_3 : usually +ve]



1-dB Compression Point:

$$y = \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \dots$$

$$x[t] = A_{in} \cos \omega_0 t$$

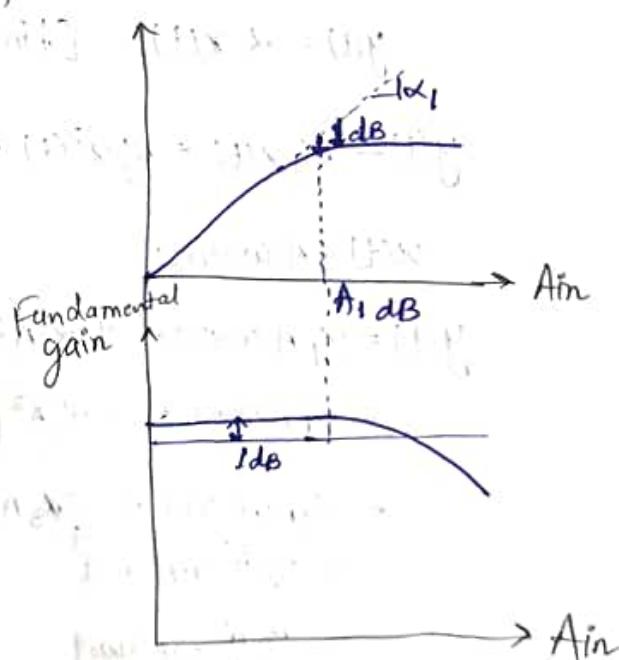
Fundamental freqn:

$$\left[\alpha_1 A_{in} + \frac{3}{4} \alpha_3 A_{in}^3 \right] \cos \omega_0 t$$

Typically $\alpha_3 < 0$

$\approx \alpha_1 A_{in}$ (linear)

21-01-2025



$$V_{out}|_{w_0} = \left[\alpha_1 A_{in} + \frac{3}{4} \alpha_3 A_{in}^3 \right]; V_{in} = A_{in} \cos \omega_0 t$$

$$\text{Gain} = \alpha_1 + \frac{3}{4} \alpha_3 A_{in}^2$$

$$\frac{V_{out}|_{w_0}}{V_{in}} \quad (\text{at } 1 \text{ dB compression})$$

$$= 20 \log \left(\alpha_1 + \frac{3}{4} \alpha_3 A_{1dB}^2 \right) = 20 \log \alpha_1 - 1 \text{ dB}$$

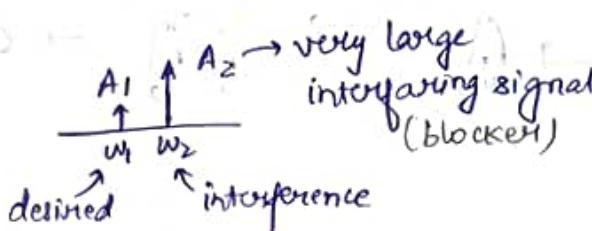
$$\Rightarrow 1 \text{ dB} = 20 \log \left(\frac{\alpha_1}{\alpha_1 + \frac{3}{4} \alpha_3 A_{1dB}^2} \right)$$

$$A_{1dB} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|}$$

(input-referred 1 dB) \rightarrow point at which gain reduces by factor of 10%.

$$x_1(t) = A_1 \cos \omega_1 t; x_2(t) = A_2 \cos \omega_2 t$$

$$A_2 \gg A_1$$



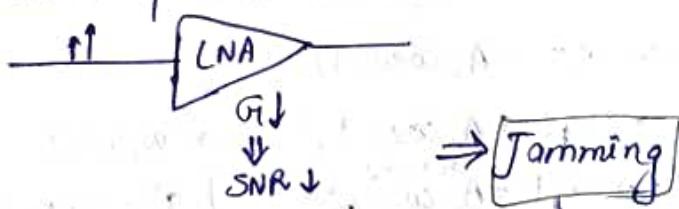
w_1, w_2 : very close
cannot be filtered

$$V_{out} = \alpha_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \frac{3}{4} \alpha_3 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

frequencies at fundamental:

$$\approx \alpha_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \frac{3}{4} \alpha_3 A_2^3 \cos \omega_2 t$$

Due to interference,



\hookrightarrow Receiver Desensitization

CWSS Modulation

$$x_1(t) = A_1 \cos \omega_1 t$$

$$x_2(t) = A_2 (1 + m \underbrace{\cos \omega_m t}_{\text{mod. index}}) \cos \omega_2 t \quad [\text{Amplitude Modulated}]$$

message signal

↳ interfered (strong)

$$(A_1 \ll A_2)$$

$$(\omega_2 + \omega_m)(\omega_3 + \omega_m)$$

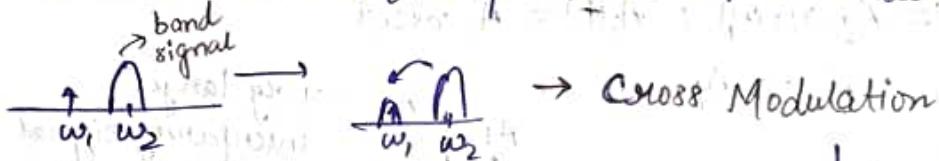
$$y(t) = \alpha_1 x_1(t) + \alpha_2 x_2^2(t) + \alpha_3 x_3^3(t) + \dots$$

fundamental terms:

$$y(t) = \alpha_1 (x_1(t) + x_2(t)) + \frac{3}{4} \alpha_3 \underbrace{(x_1(t) + x_2(t))^3}_{\approx 0}$$

$$\overbrace{x_1^3(t) + x_2^3(t)}^{\approx 0} + 3x_1^2(t)x_2(t) + 3x_1(t)x_2^2(t)$$

Modulation @ $\omega_2 \rightarrow$ transformed to $\omega_1 \rightarrow$ undesired



OFDM



22-01-2025

Intermodulation

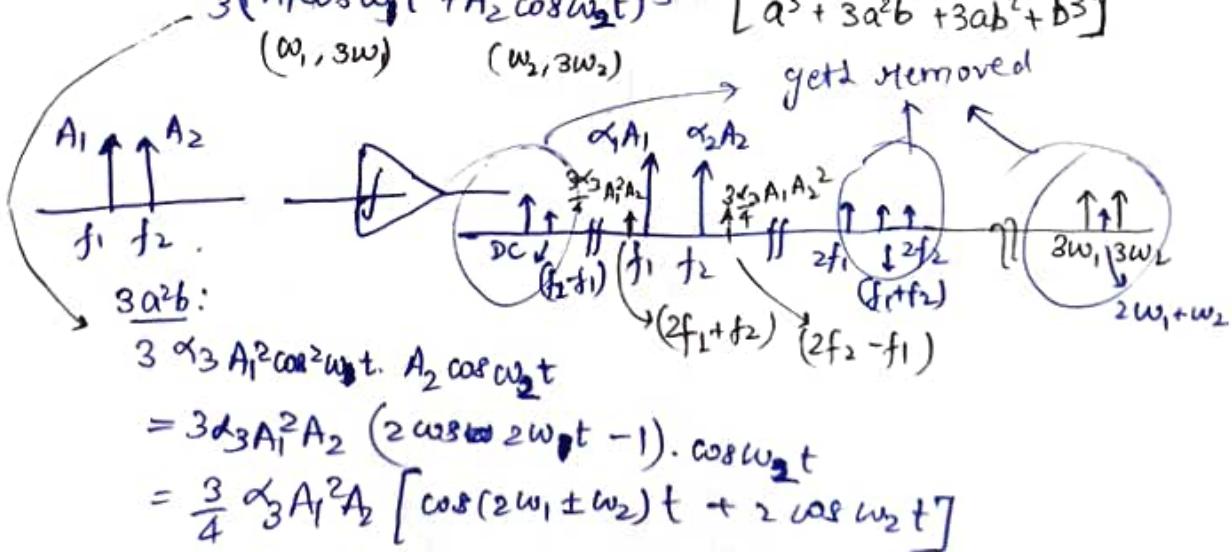
$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \dots$$

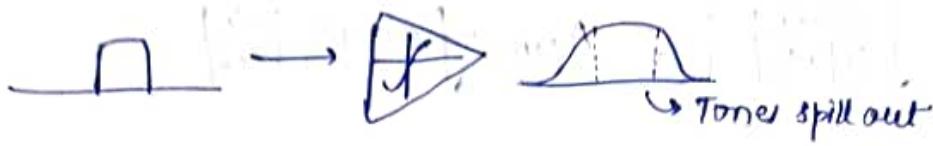
$$= \alpha_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)$$

$$+ \alpha_2 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 \rightsquigarrow (\omega_1 \pm \omega_2)$$

$$+ \alpha_3 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3 \quad [a^3 + 3a^2b + 3ab^2 + b^3]$$

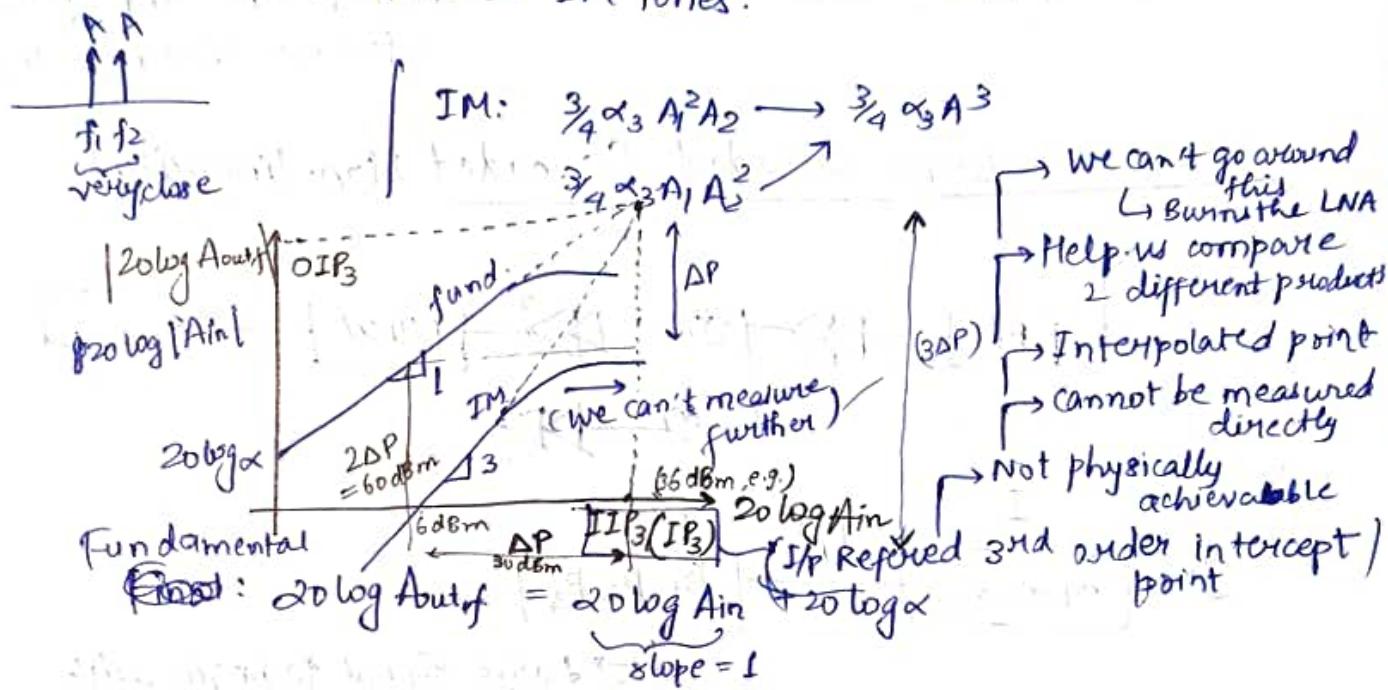


$$\left. \begin{array}{l} 2f_1 - f_2 \\ 2f_2 - f_1 \end{array} \right\} \begin{array}{l} \text{IM (Intermodulation)} \\ \text{tones} \end{array}$$



$$\left. \begin{array}{l} f_1 = 1 \text{ GHz} \\ f_2 = 1.01 \text{ GHz} \\ 2f_1 - f_2 = 990 \text{ MHz} \\ 2f_2 - f_1 = 1.02 \text{ GHz} \end{array} \right\}$$

"Two tone test": Give two tones as i/p to amplifier and measure IM tones.



$$\begin{aligned} \text{IM: } 20 \log A_{\text{out},f} &= 20 \log (\frac{3}{4} \alpha_3 A^3) + 20 \log A_{\text{in}} \\ &= 60 \log (\sqrt{\frac{3}{4} \alpha_3} A) + 60 \log A_{\text{in}} + 20 \log \alpha \end{aligned}$$

$$\text{IM Suppression: } 20 \log \left(\frac{A_f}{A_{\text{IM}}} \right) \quad | \quad (20 \log A_f - 20 \log A_{\text{IM}}) \quad \text{slope of 2}$$

$$\text{IM suppression} = 2 (20 \log IIP_3 - 20 \log A_{\text{in}})$$

$$\left. \begin{array}{l} \text{Eq } IIP_3 = +30 \text{ dBm} \\ P_{\text{in}} = -20 \text{ dBm} \end{array} \right\} 50 \text{ dB}$$

$$P_{\text{IM}} = -120 \text{ dBm}$$

$$\left. \begin{array}{l} IIP_3 = +20 \text{ dBm} \\ P_{\text{in}} = -20 \text{ dBm} \end{array} \right\} 40 \text{ dB}$$

$$P_{\text{IM}} = -100 \text{ dBm}$$

$\rightarrow IIP_3 \uparrow \Rightarrow$ Linearity \uparrow
(better)

Feeding IIP_3 , burns the device.

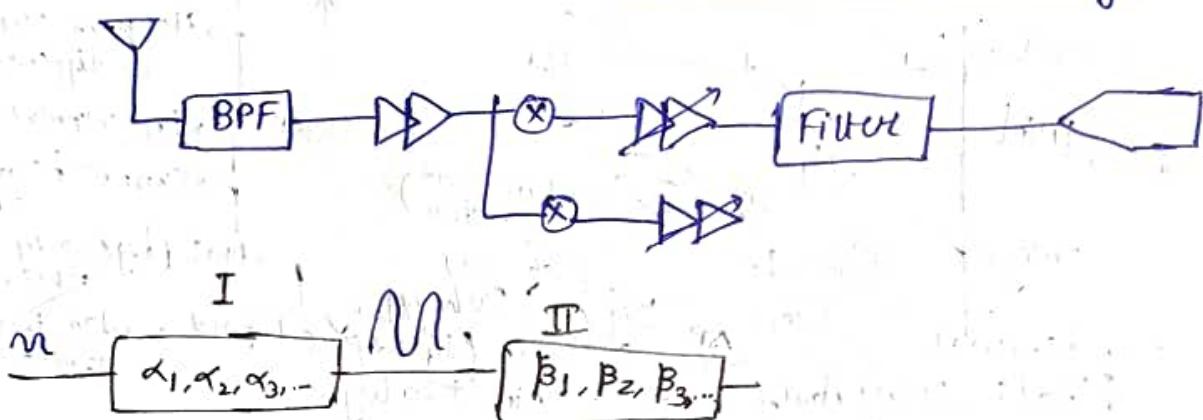
24-01-2025

$$\alpha_1 A_{in} = \frac{9}{4} \alpha_3 A_{in}^3 \quad (\text{Linear & 3rd order IM tone terms become equal at } IIP_3 \text{ point})$$

$$\Rightarrow A_{IIP_3} = \sqrt{\frac{4}{3} \left| \frac{\alpha_1}{\alpha_3} \right|}, \quad A_{LdB} = \sqrt{0.115} \left| \frac{\alpha_1}{\alpha_3} \right|$$

$$20 \log \left(\frac{A_{IIP_3}}{A_{LdB}} \right) = 9.6 \text{ dB} \approx 10 \text{ dB} \rightarrow IIP_3 \text{ point is } 10 \text{ dB higher than } 1\text{-dB comp. point. (for no other non-linearity)}$$

Non-linear terms cascaded (Cascaded Non-linearity)



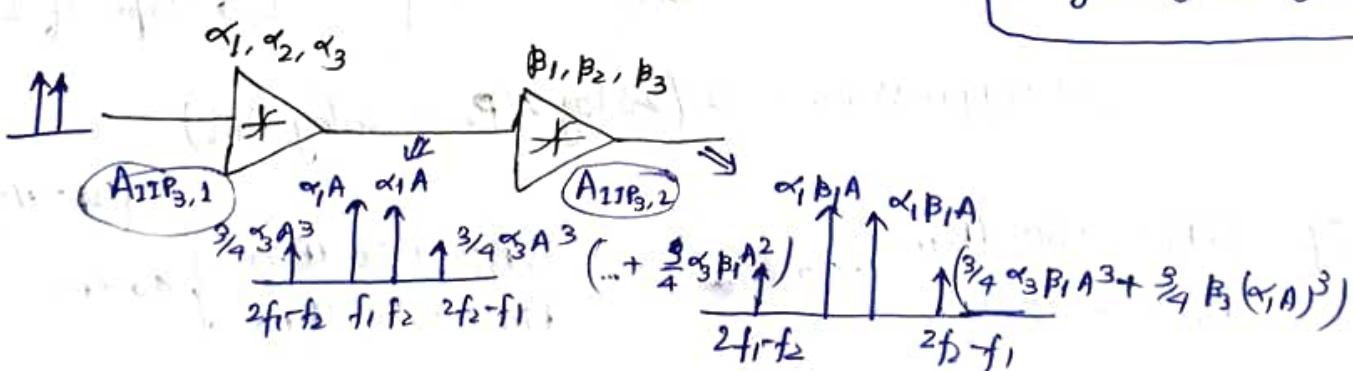
For same II,

$$\textcircled{1}: G_1 > G_2$$

will produce
more non-linearity

Large signal to begin with
Produce more
non-linearity

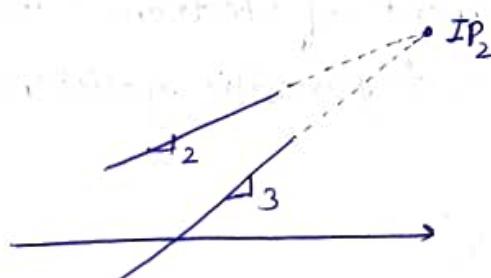
BOOK: RF Microelectronics
2nd ed.
by Behzad Razavi



2nd order distortion:

$$\alpha_2 (A \cos \omega_1 t + A \cos \omega_2 t)^2 = \alpha_2 [A^2 \cos^2 \omega_1 t + A^2 \cos^2 \omega_2 t + 2A^2 \cos \omega_1 t \cdot \cos \omega_2 t]$$

$\downarrow 2\omega_1 \text{ DC}$ $\downarrow 2\omega_2 \text{ DC}$ $\downarrow \omega_1 + \omega_2$



$$\rightarrow \left[\frac{1}{A_{IIP_3, \text{total}}^2} = \frac{1}{A_{IIP_3,1}^2} + \frac{\alpha_1^2}{A_{IIP_3,2}^2} \right] + \left(\frac{\alpha_1^2 \cancel{\beta_1^2}}{A_{IIP_3,3}^2} \right)$$

Noise

↳ Random signal generated by any electronic device.

Thermal Noise:

↳ Anything having non-zero real resistance causes noise, due to brownian movement of electrons (above 0K).

→ Inductors & capacitors, due to parasitic resistances, generate noise.

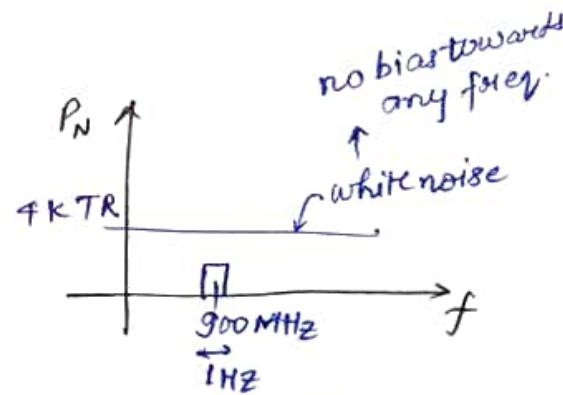
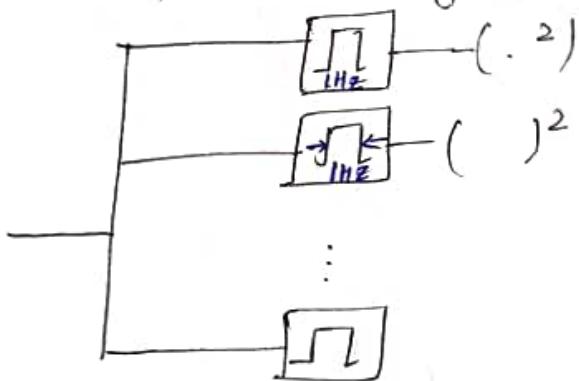


Average, $\mu = 0$
(noise)

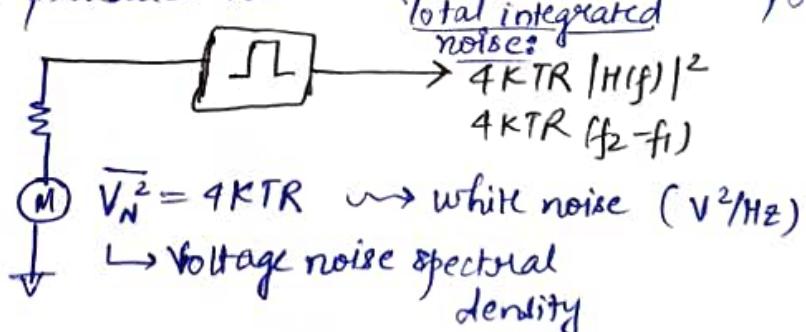
std. deviation or variance, σ^2

↳ gives power
in the noise

Power spectral density (PSD):



→ Resistor produces noise

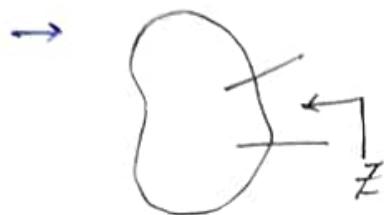
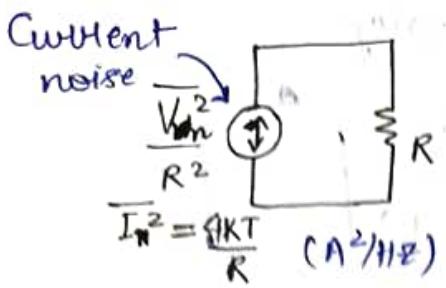


$$\frac{\text{Total integrated noise}}{4KTR |H(f)|^2} = \frac{4KTR}{4KTR (f_2 - f_1)}$$

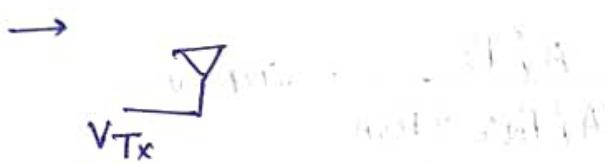
↳ PSD represent
Power of 1Hz
BW at that
freq.

$$X(f) \xrightarrow{[H(f)]} Y(f) = X(f) |H(f)|^2$$

↳ Any random variable ↳ power quantities



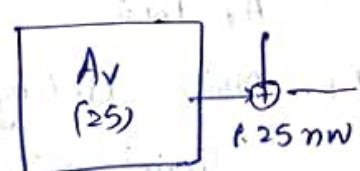
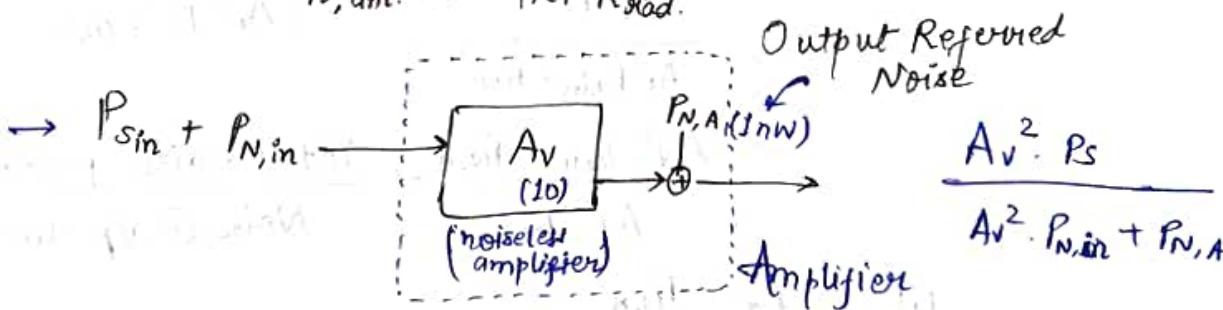
Noise through a part, $\overline{V_n^2} = 4KT \operatorname{Re}\{Z\}$ of any network.



$$P_{\text{Rad}} = \frac{V_{\text{Tx}}^2}{R_{\text{Rad}}} \rightarrow \text{Radiated power}$$

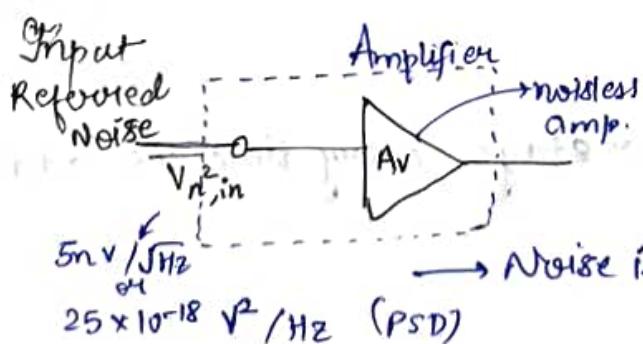
Noise in the antenna,

$$\overline{V_{n,\text{ant.}}^2} = 4KT R_{\text{Rad}}$$



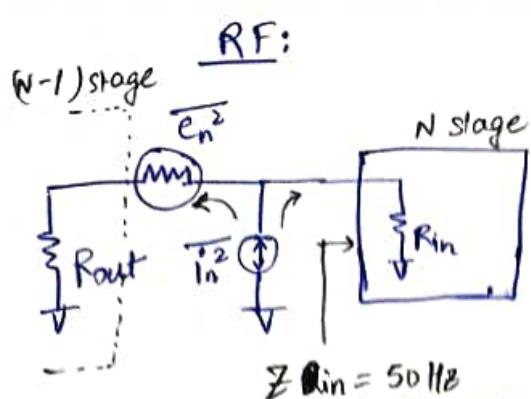
→ less degradation of SNR

→ Signal amplification is much more than the marginal noise added.

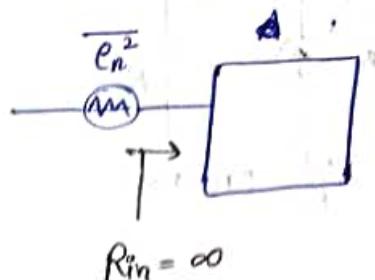


→ Noise is added in series:

$$Av^2 \cdot P_S + Av^2 \cdot P_{N,\text{in}} + Av^2 \overline{V_{n,\text{in}}^2}$$



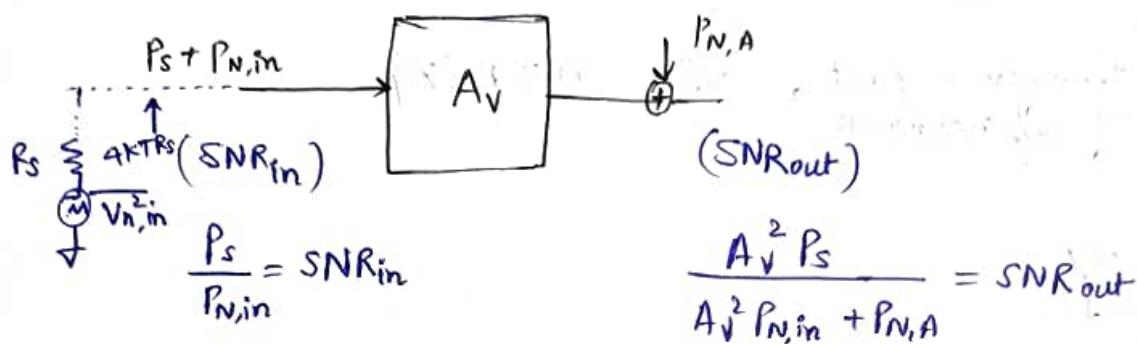
Analog:



$$Z_{in} = 50 \text{ dB}$$

(not \$\infty\$)

↳ Noise voltage as well as noise current : Model



$$\text{Noise figure, } NF = \frac{SNR_{in}}{SNR_{out}}$$

$$= \frac{\frac{P_s}{P_{N,in}}}{\frac{A_v^2 P_s}{A_v^2 P_{N,in} + P_{N,A}}} = \frac{(A_v^2 P_{N,in} + P_{N,A}) P_s}{(A_v^2 P_s) \cdot P_{N,in}}$$

$$= \frac{A_v^2 \cdot P_{N,in} + P_{N,A}}{A_v^2 \cdot P_{N,in}} = \frac{\text{Total noise power@ o/p}}{\text{Noise@ o/p due to i/p noise}}$$

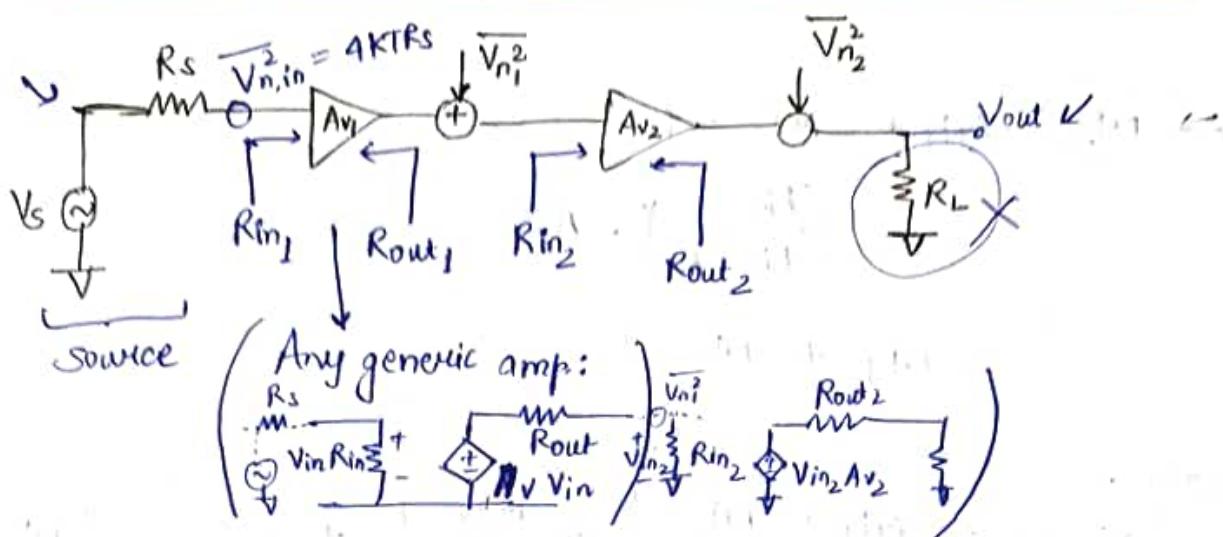
$$NF = 1 + \frac{P_{N,A}}{A_v^2 \cdot P_{N,in}}$$

$$= 1 + \frac{\text{Noise added by amplifier}}{\text{Gain}^2 \cdot \text{noise@ o/p}}$$

$$\rightarrow NF_{dB} = 10 \log \left(\frac{SNR_{in}}{SNR_{out}} \right)$$

29-01-2025

→ SNR degrades stage after stage, every time we add an electrical component.



$$V_{out,s} = V_s \frac{R_{in_1}}{R_{in_1} + R_s} \cdot A_{v1} \cdot \frac{R_{in_2}}{R_{in_2} + R_{out_1}} \cdot A_{v2} \cdot \frac{R_L}{R_L + R_{out_2}}$$

Impedance
voltage
division

$A_o \rightarrow$ overall gain (from \curvearrowleft to \curvearrowright)

$$NF = 1 + \frac{\text{Noise added.}}{\text{Gain}^2 \cdot V_{n,in}^2}$$

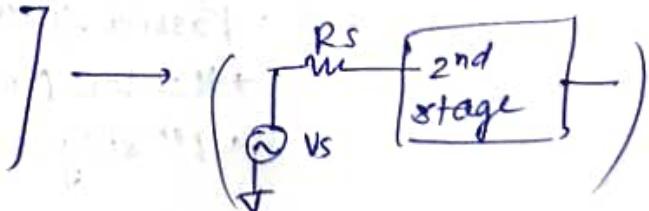
$$V_{out,in}^2 = 4KTR_s \cdot A_o^2 + V_{n1}^2 \left[\frac{R_{in_2}}{R_{in_2} + R_{out_1}} \right]^2 A_{v2}^2 + V_{n2}^2$$

$$V_{n,added}^2 = V_{n1}^2 \cdot \left(\frac{R_{in_2}}{R_{in_2} + R_{out_1}} \right)^2 A_{v2}^2 + V_{n2}^2 \quad \text{noise added by the system}$$

$$\therefore NF = 1 + \frac{V_{n1}^2 \left(\frac{R_{in_2}}{R_{in_2} + R_{out_1}} \right)^2 \cdot A_{v2}^2 + V_{n2}^2}{\left(\frac{R_{in_1}}{R_{in_1} + R_s} \right)^2 A_{v1}^2 \left(\frac{R_{in_2}}{R_{in_2} + R_{out_1}} \right)^2 A_{v2}^2 \cdot 4KTR_s \cdot A_o^2}$$

$$\left[\frac{V_{n1}^2}{\left(\frac{R_{in}}{R_{in} + R_s} \right)^2 A_{v1}^2 \cdot 4KTR_s} = NF_1 - 1 \right] \quad \text{for 1st stage}$$

$$\left[NF_2 = 1 + \frac{V_{n2}^2}{\left(\frac{R_{in_2}}{R_{in_2} + R_{out_1}} \right)^2 A_{v2}^2 + 4KTR_s} \right] \quad \text{for 2nd stage in isolation}$$



$$\Rightarrow NF_{\text{tot}} = NF_1 + \frac{NF_2 - 1}{\left(\frac{R_{\text{in}}}{R_{\text{in}} + R_s} \right)^2 \cdot A_{V1}^2}$$

$$= NF_1 + \frac{NF_2 - 1}{A_1}$$

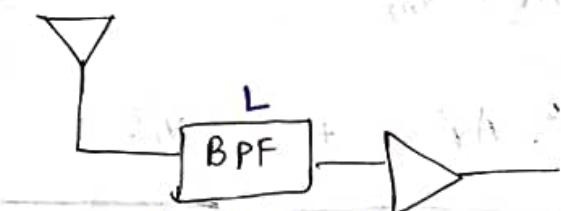
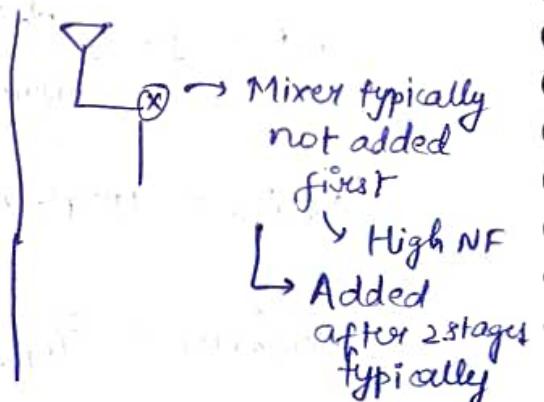
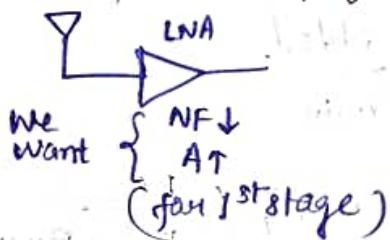
In general,

$$NF_{\text{total}} = NF_1 + \frac{NF_2 - 1}{A_1} + \frac{NF_3 - 1}{A_1 A_2} + \dots$$

directly appears as NF

Very critical

(at or near) Low Noise Amplifier (LNA)



- ↳ Made of:
SAW (Surface/Bulk)
BAW (Acoustic Wave)
- ↳ (not capacitors, T/L)

- ↳ To avoid interference
(avoiding jamming, sensitization, etc.)

- ↳ for fixed band comms.

- ↳ Passive filter.

- ↳ Has loss (=NF)

- ↳ 1st stage

$$NF = NF_L + (NF_{LNA} - 1) L + \underbrace{(NF_{LNA} - 1) L}_{A_{LNA}}$$

$\xrightarrow{\text{loss element (BPF)}}$

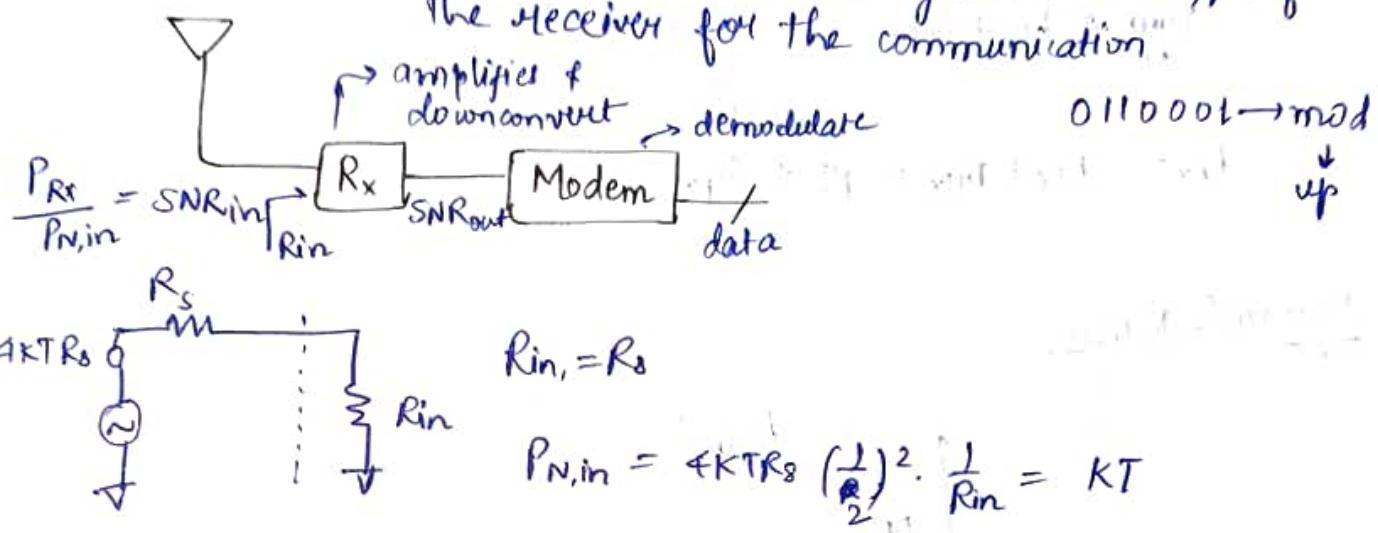
$\xrightarrow{\text{loss} = 1/G}$

Some antennas have attached LNA to avoid loss (as 1st stage).

Every loss element (cable, etc.) has to be modelled.

Sensitivity → Minm power that has to be given to the i/p of the receiver for the communication.

(04-02-2025)



$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{Rx}/KT}{SNR_{out}}$$

$$P_{Rx} = KT \cdot NF \cdot SNR_{out}$$

$$P_{Rx,min} = KT \cdot NF \cdot SNR_{min} \cdot \times (BW)$$

to get total integrated noise

$$\begin{aligned} P_{SNR, dem} &= 10 \log \left(\frac{KT}{f_{mw}} \right) + 10 \log NF + 10 \log SNR_{min} + 10 \log BW \\ &= -174 \text{ dBm/Hz} + NF_{dB} + SNR_{min,dB} + BW_{dB} \end{aligned}$$

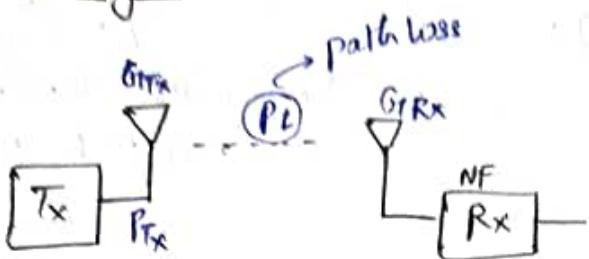
Eg. $NF_{dB} = 7 \text{ dB}$

$$BW = 100 \text{ kHz}$$

$$SNR_{min} = 10 \text{ dB}$$

$$\begin{aligned} \therefore P_{SNR, dem} &= -174 + 50 + 7 + 10 \\ &= -107 \text{ dBm} \end{aligned}$$

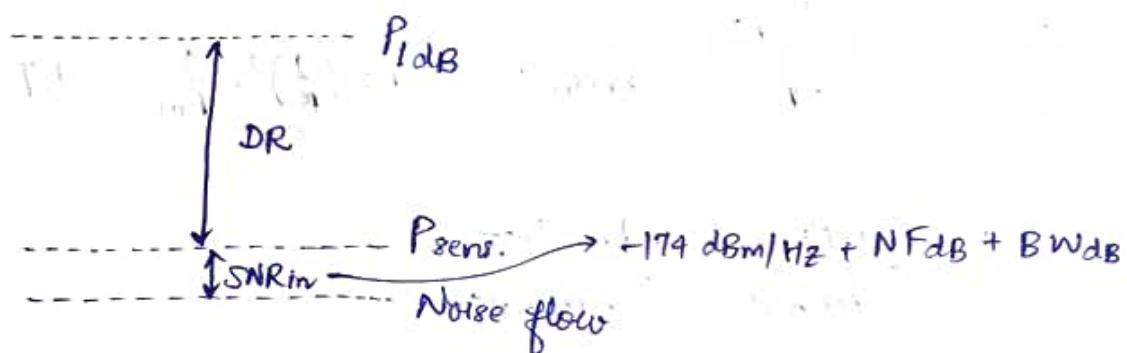
Link Budget



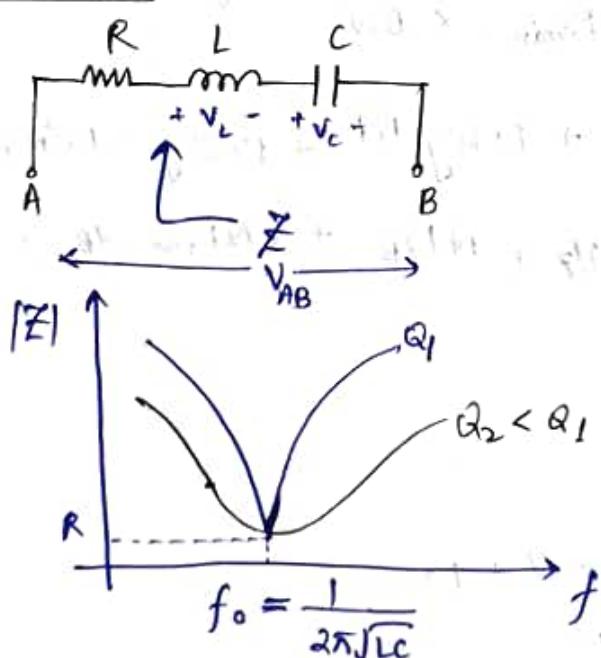
$$P_{\text{out rad.}} = P_{T_x} + G_{T_x}$$

$$P_{R_x} = P_{T_x} + G_{T_x} + \text{Over} + P_L + G_{R_x}$$

Dynamic Range



RLC Circuit:



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$|j\omega_0 L| = \left| \frac{1}{j\omega_0 C} \right|$$

$$V_L |_{\omega=\omega_0} = \frac{V_{AB}}{R} \cdot \omega_0 L$$

$$= V_{AB} \left(\frac{\omega_0 L}{R} \right)$$

$$= V_{AB} \cdot Q \quad \text{"quality factor"}$$

$$V_C |_{\omega=\omega_0} = \frac{V_{AB}}{R} \cdot \frac{1}{\omega_0 C}$$

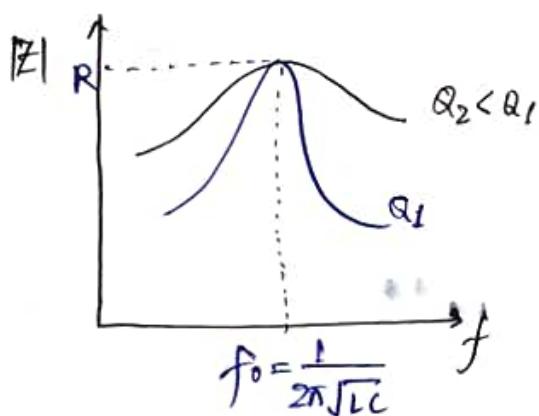
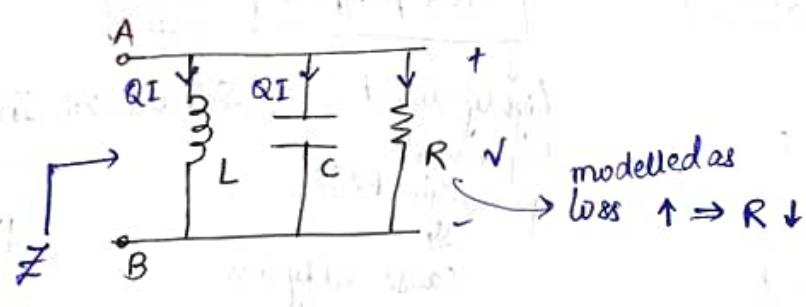
$$= V_{AB} \cdot \frac{1}{\omega_0 R C}$$

$$= V_{AB} \cdot Q$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C}$$

$R \downarrow \Rightarrow Q \uparrow$
 \downarrow
loss

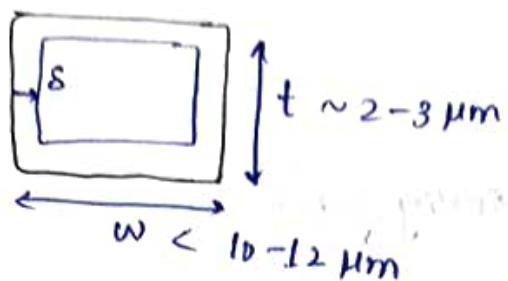
Parallel RLC Network:



$$Q = \frac{R}{\omega_0 L} = \omega_0 R C$$

$R \downarrow \Rightarrow Q \downarrow \Rightarrow \text{loss } \uparrow$

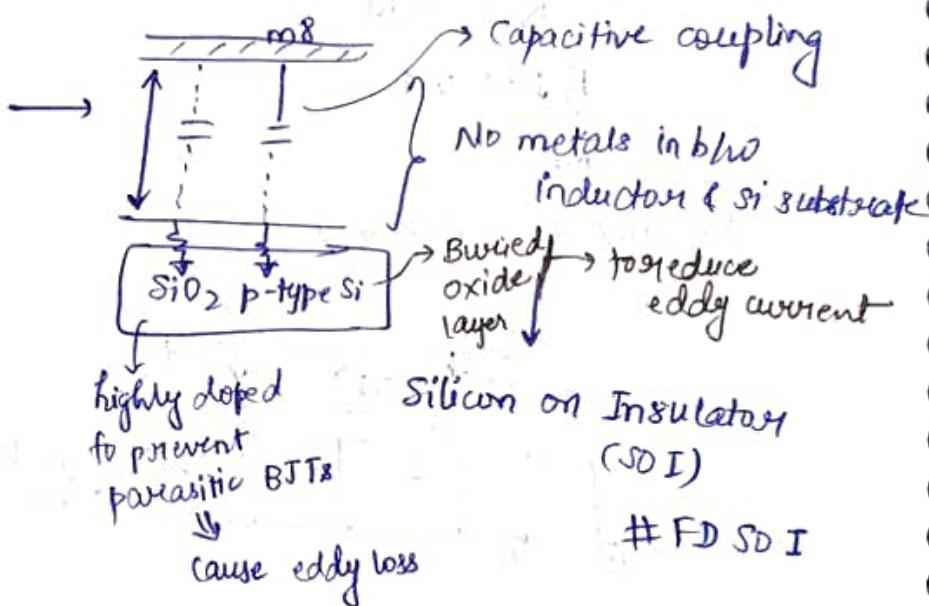
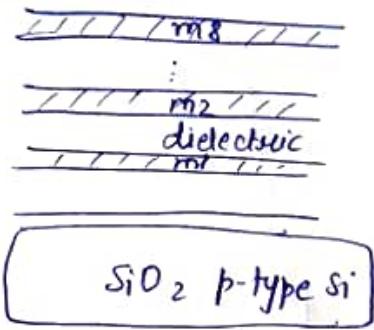
Conductor: Metal in the inductor



Losses:

- ① Ohmic loss
+ Skin effect
- ② Eddy current loss

$$R = \frac{\rho \cdot l}{w \cdot t}$$



Modelling the loss:

- ① $Q = \frac{w_0 L}{R}$

- ② $Q = \frac{R}{w_0 L}$

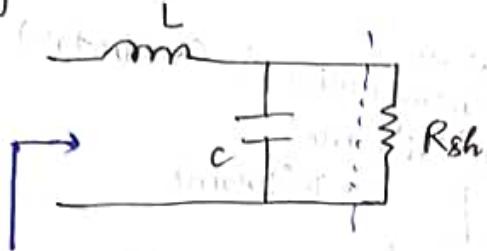
Eg. $f = 1 \text{ GHz}$
 $L = 5 \text{nH}$

$$Q = 10$$

$$\frac{5 \text{nH}}{\text{mm}} \quad \frac{3.14 \Omega}{\text{mm}}$$

$$5 \text{nH} \quad \left\{ \begin{array}{l} \\ \end{array} \right\} 3.14 \Omega$$

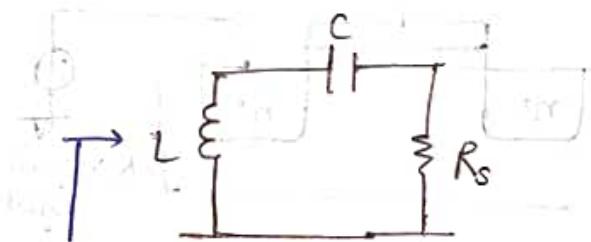
Matching:



23/IV/2014 20M

$$S(L + \frac{1}{SC}) \parallel R_{sh}$$

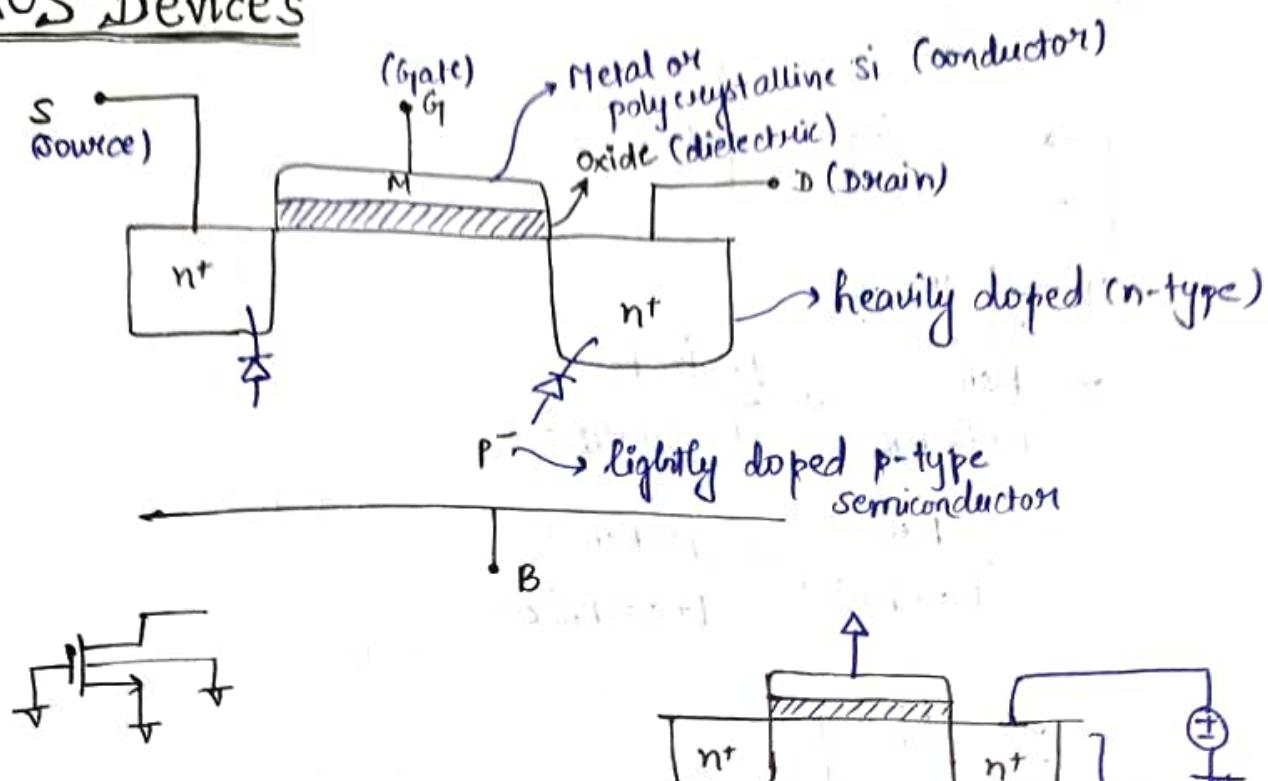
$$\begin{aligned} &= \frac{R_{sh}}{1+j\omega R_{sh} C} \times \frac{1-j\omega R_{sh} C}{1+j\omega R_{sh} C} \\ &= \frac{R_{sh}}{1+\omega^2 R_{sh}^2 C^2} - \frac{j\omega R_{sh}^2 C}{1+\omega^2 R_{sh}^2 C^2} \end{aligned}$$



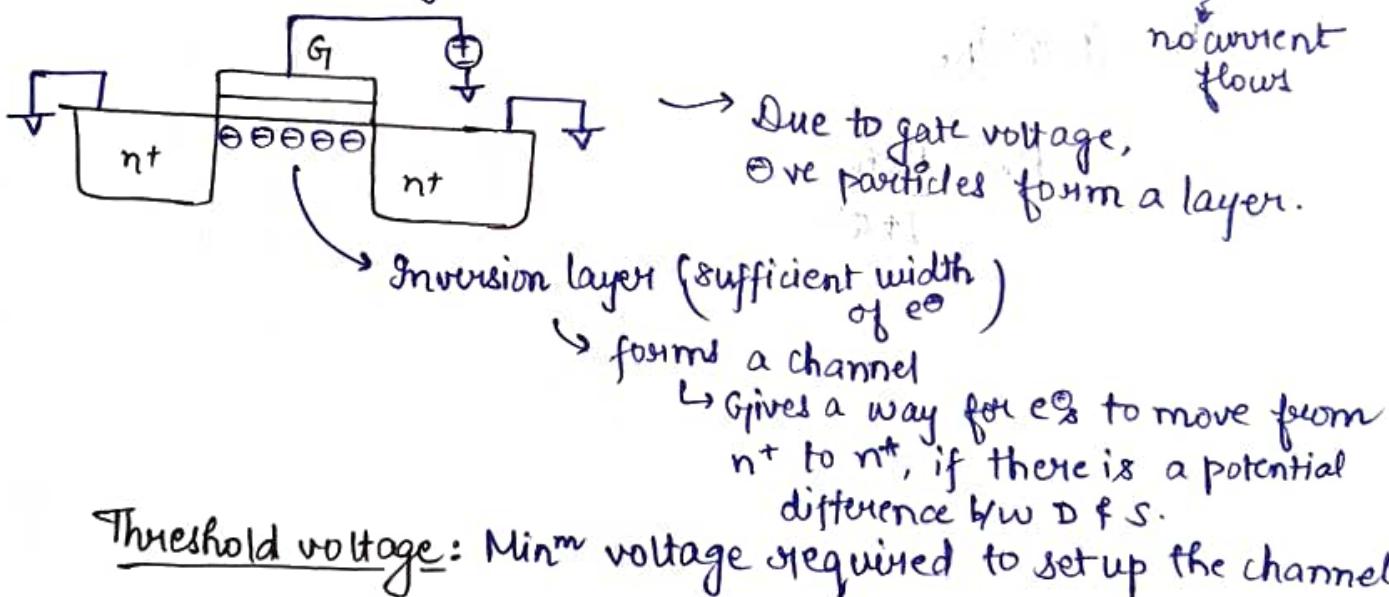
$$R_T = (1+Q^2)R_s$$

$$R_s = \frac{R_{sh}}{1+Q^2}$$

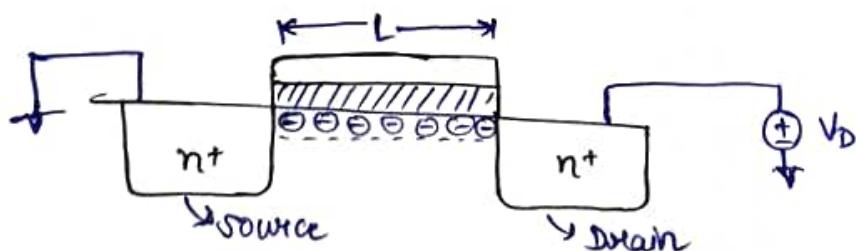
MOS Devices



Put some voltage to gate.



Threshold voltage: Min^m voltage required to set up the channel.



Higher potential : Drain
lower potential : Source
 e^- : $S \rightarrow D$
 I : $D \rightarrow S$

$V_{GS} > V_{TH}$

$V_{DS} > 0$

Current flow

Triode / Linear Region

$$I_D = \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{GS} - V_{TH}) V_{DS}$$

(Small V_{DS})

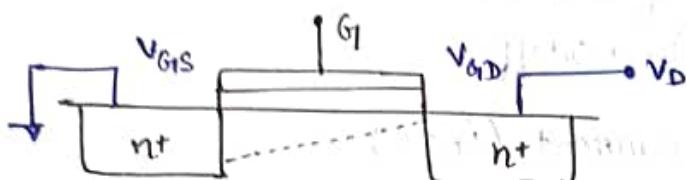
μ_n : Mobility

C_{ox} : Oxide capacitance / unit area

L: Length of the channel

(length e⁻ has to flow through from S to D)

W: Width of the channel
(going inside the plane of paper)



$$V_{GS} > V_{GD} > V_{TH}$$

As $V_D \uparrow$, $V_{GD} \downarrow$ and approaches to V_{TH} \Rightarrow channel starts decreasing on drain side.

After $V_{GD} < V_{TH}$, channel breaks on drain side but

still active on source side \Rightarrow e⁻ able to cross drain side.

But current : constant

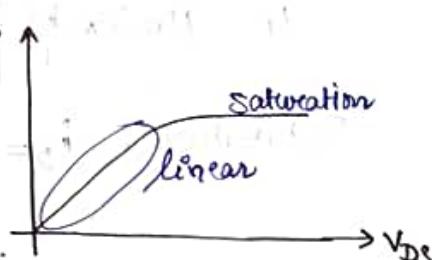
(as in BJT from base to collector)

Saturation

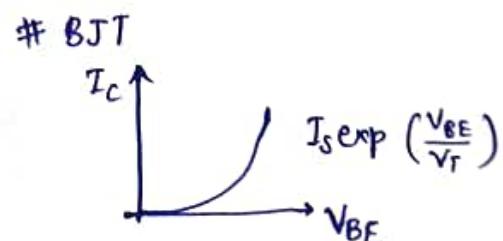
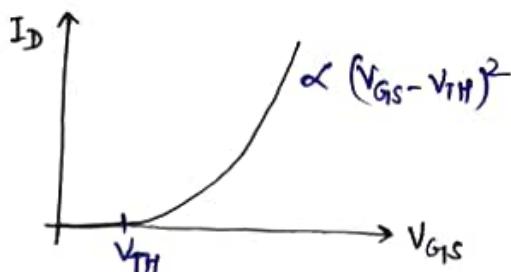
Region

$$I_D = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TH})^2$$

For saturation, $V_{DS} = V_{GS} - V_{TH}$



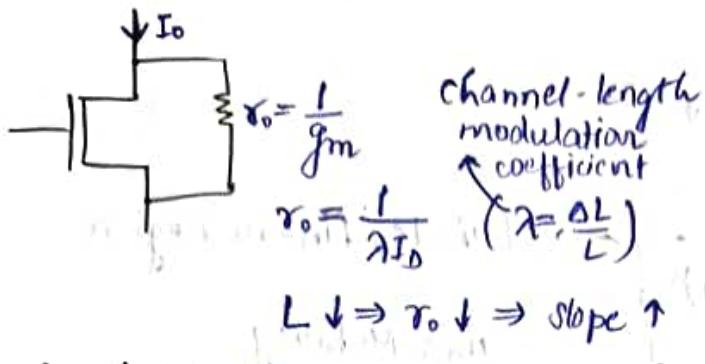
$$I_D = \mu_n C_{ox} \left(\frac{W}{L} \right) \left[(V_{GS} - V_{TH}) V_{DS} - \frac{V_{DS}^2}{2} \right] \quad (\text{large } V_{DS})$$



As channel length \downarrow $\Rightarrow I \uparrow$ } channel length modulation

$$\frac{\partial I_D}{\partial V_{DS}} \Big|_{V_{GS}} = g_{ds} \quad (\text{conductance})$$

$$\gamma_o = \frac{1}{g_{ds}} = \left(\frac{\partial I_{ds}}{\partial V_{ds}} \right)^{-1}$$



Subthreshold: Region where current flows before the formation of channel.

$$(V_{GS} < V_{TH}) \rightarrow \text{cutoff}$$

Cutoff Mode: Drain current ($i_D = 0$)
 $(V_{GS} < V_{TH})$

Linear Mode: Linear drain current

- $V_{DS} \rightarrow \text{small}$

$$i_D = \mu_n C_{ox} \frac{W}{L} [(V_{GS} - V_{TH}) V_{DS}]$$

- \bullet Large V_{DS}

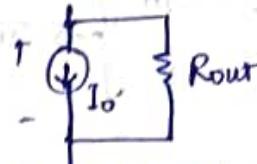
$$i_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$\text{Saturation: } i_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

$$= \frac{\mu_n C_{ox}}{2} \frac{W}{L} V_{DS}^2$$



Current source:

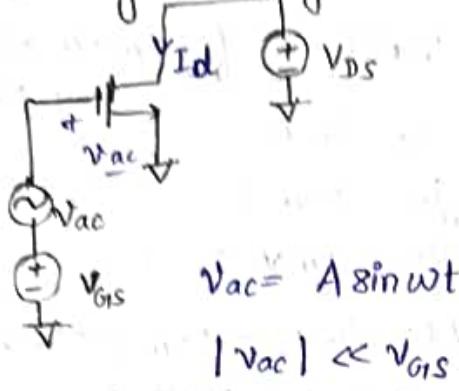


$$I = I_0 + \frac{V_{out}}{R_{out}}$$

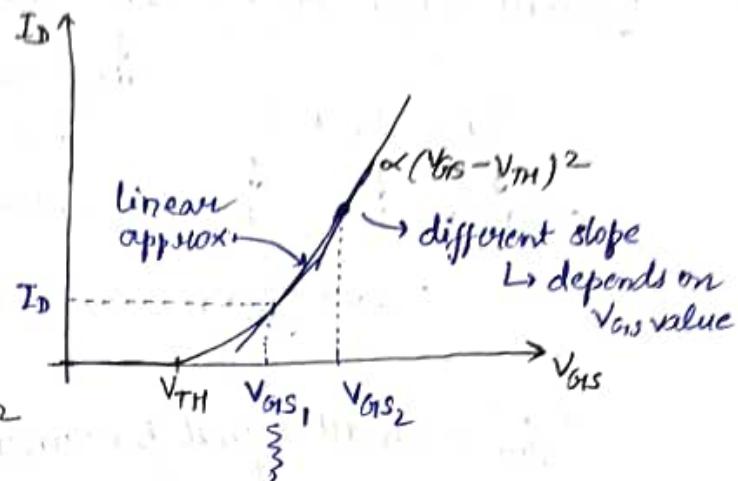
$$\frac{\partial I}{\partial V} = \frac{1}{R_{out}}$$

Small Signal Analysis

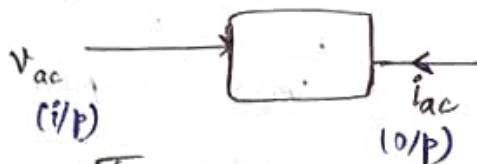
(11-02-2025)



$$I_d = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L} \right) (V_{GS} + V_{ac} - V_{TH})^2$$



(small change \rightarrow for $|V_{ac}| \ll V_{GS}$
around V_{GS})



Transconductance (g_m)

$\left[\begin{array}{l} \text{(Output)} \\ \text{--- Input} \end{array} \right]$ has a unit of conductance, &
the current results around
different terminal

$$I_d = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TH})^2$$

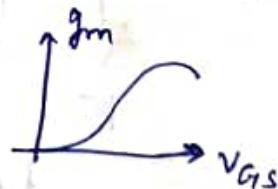
$$\Rightarrow \boxed{\begin{aligned} \frac{\partial I_d}{\partial V_{GS}} &= \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{GS} - V_{TH}) \\ &= \frac{2 I_d}{(V_{GS} - V_{TH})} \\ &= \sqrt{2 I_d \mu_n C_{ox} \left(\frac{W}{L} \right)} \end{aligned}} \quad \left. \right\} g_m$$

We would want a particular 'g_m' specification for LNA.
↳ Depends upon (↑)

- $\left(\frac{W}{L} \right)$, for fixed I_d . ↑

- V_{GS} , for fixed I_d . ↓ →

- $g_m \propto \sqrt{I_d}$



Small signal Model (Low frequency): Valid only for small signals (not DC)

$$I_d = I_D + i_{ac}$$

↓ didn't consider the capacitances
↓ composite DC ac (convention)
 $= I_D + g_m V_{ac}$

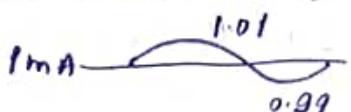
Eq. $I_D = I_m A$

$g_m = 10 \text{ mS}$

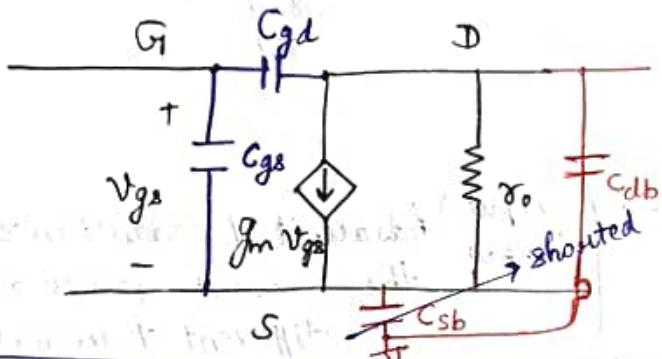
$V_{ac} = 1 \text{ mV } \sin \omega t$

$$I_d = I_m A + 10 \text{ mS} \times 1 \text{ mV } \sin \omega t$$

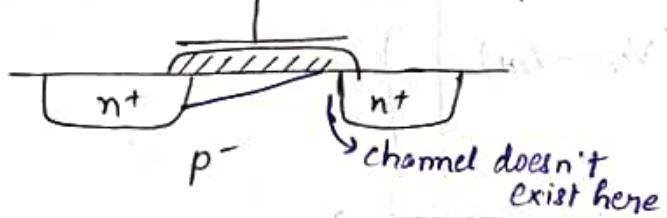
$$= I_m A + 10 \mu\text{A } \sin \omega t$$



' g_m ': small signal parameter



iii) $V_{GDS} > V_{TH}$ & $V_{DS} \geq V_{GDS} - V_{TH}$

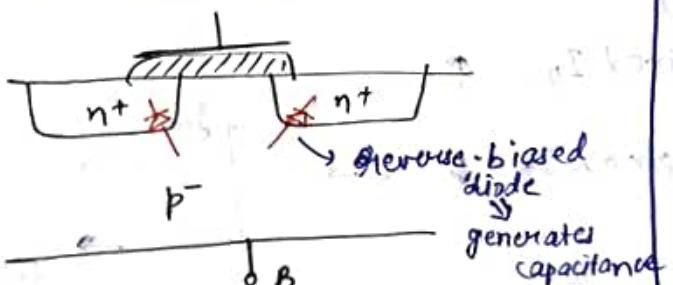


$$C_{gs} = \frac{2}{3} WL C_{ox} + Cov$$

(experimental)

$$C_{gd} = Cov \quad (\text{as channel } \cancel{\text{go}} \text{ doesn't exist})$$

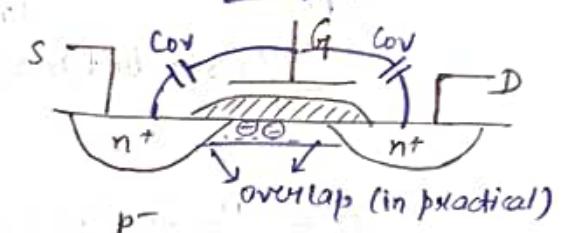
Other capacitances:



$$\frac{1}{V_D} \propto \frac{1}{C} \quad C = f(N)$$

$I_D = g_m \cdot V_{GS} \times$

~~Capacitances:~~



Cov : overlap capacitance

↳ exist even if device is off

↳ due to overlapping of oxide

i) $V_{GDS} < V_{TH}$ ($V_{GDS} = 0$)

ii) $V_{GDS} > V_{TH}$ & $V_{DS} < V_{GDS} - V_{TH}$

↳ channel exists &

in triode region

Triode: $C_{tot.} = WL C_{ox}$

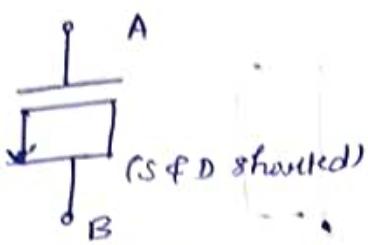
$$C_{gs} = Cov + \underbrace{\frac{1}{2} WL C_{ox}}_{\text{budgeted half for source}} = g_s$$

iii) $V_{GDS} > V_{TH}$ & $V_{DS} \geq V_{GDS} - V_{TH}$ (Saturation)

MOS capacitance: MOSFETs have their own capacitances

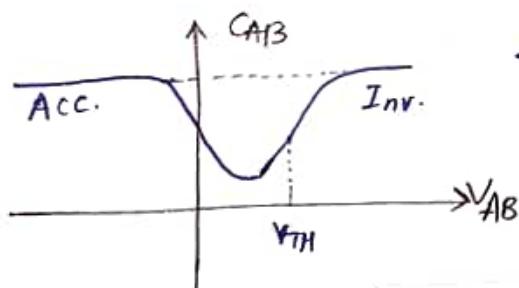
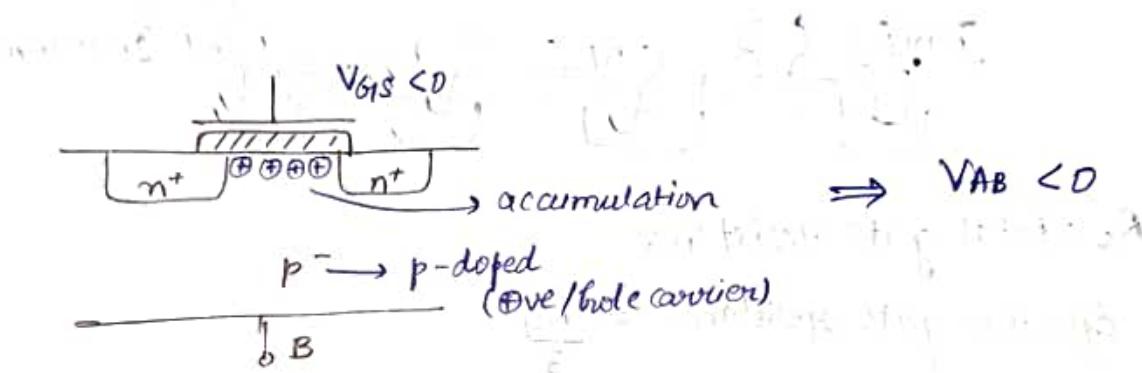
↳ can be used as capacitor

↳ voltage-dependent capacitance

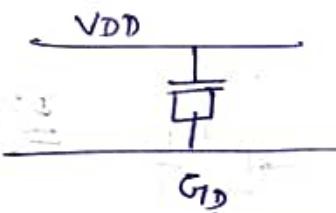


$$V_{AB} > V_{TH}$$

• $C_{tot.} = 2 C_{ov} + WL C_{ox}$

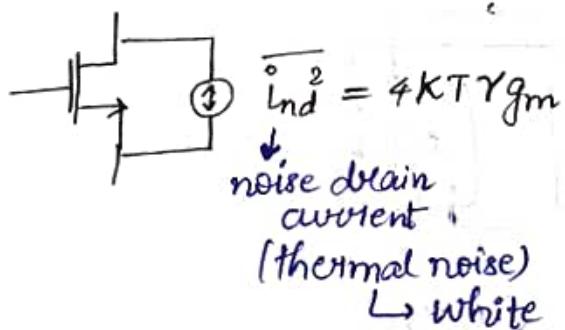


→ Voltage-dependent
↳ Used in supply decoupling capacitor

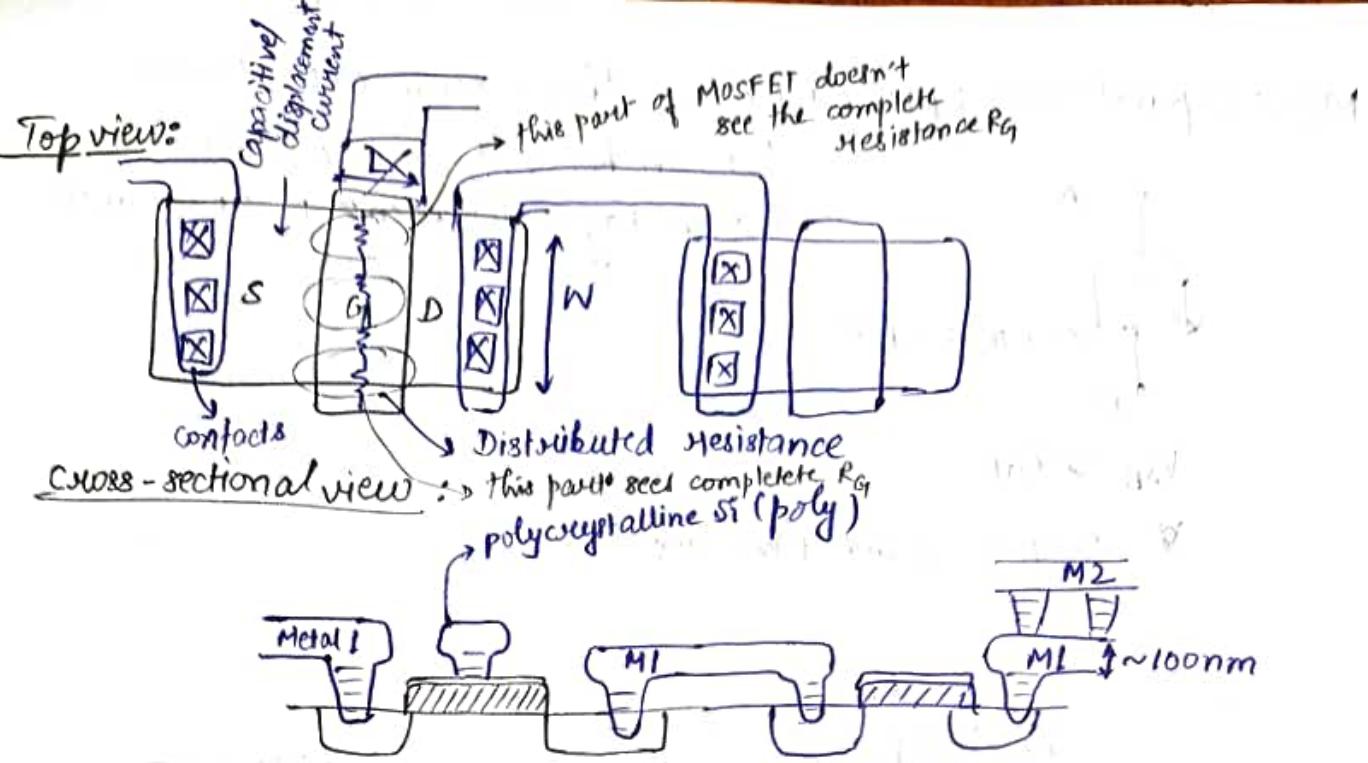


12-02-2025

Noise in a MOSFET



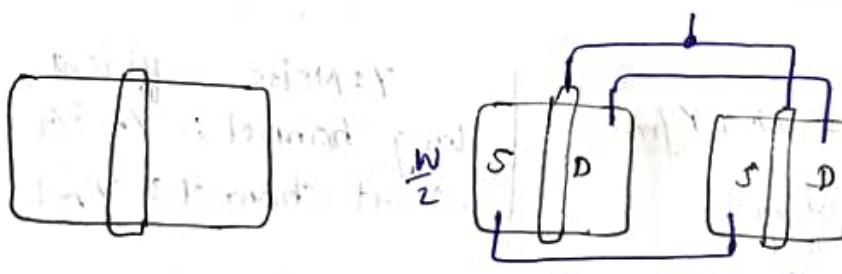
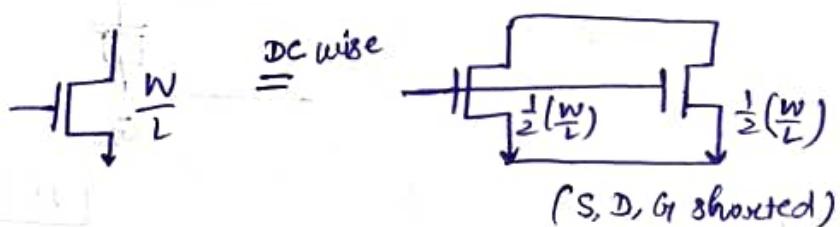
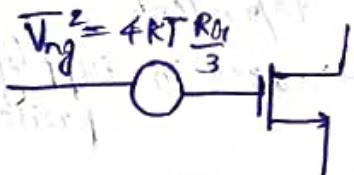
γ : Noise coefficient
long channel: $\gamma \sim 2/3$
short channel: $\gamma \sim 1$



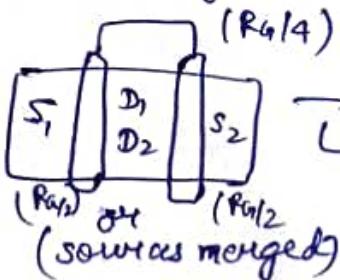
R_g : total gate resistance

$$\text{Effective gate resistance} = \frac{R_g}{3}$$

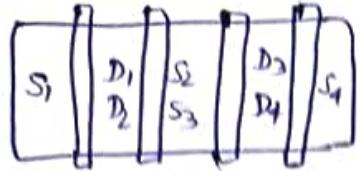
Model:



$$\bar{V_{ng}}^2 = 4KT \frac{R_g}{L_2} \quad (x4 \downarrow)$$



→ No. of fingers = 2
→ Total drain area has reduced.
 $C_{db} \downarrow, C_{sb} \downarrow$



→ 4 fingers

$$\rightarrow \frac{V_{nq}^2}{4} = 4kT \frac{R_G}{L} \quad (\text{X} \downarrow)$$

→ Noise figure ↓

→ Gate noise can be handled by adding no. of fingers.

↳ can be managed.

↳ Routing complexity ↑

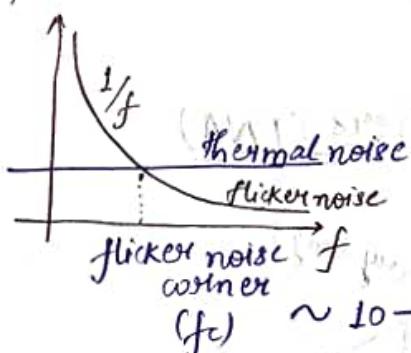
Flicker Noise

↳ source: Dangling bonds capturing charges in the oxide layer. (gate)

$$V_{nf}^2 = \frac{k}{WLCo_x f}$$

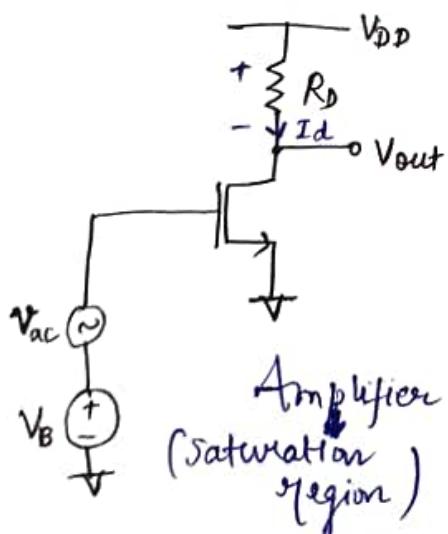
$$\propto \frac{1}{f} \rightarrow \text{Not considerable}$$

for RF. ↳ considered in oscillators, and in analog circuits.



→ No perfect DC
↳ sampled at finite freq.

↳ observing longer
⇒ flicker noise ↑



$$V_B \rightarrow I_D = \frac{MnCo_x}{2} \left(\frac{W}{L} \right) (V_B - V_{TH})^2 \rightarrow V_{out}$$

$$I_d = I_D + g_m V_{ac}$$

$$V_{out} = V_{DD} - I_d R_D$$

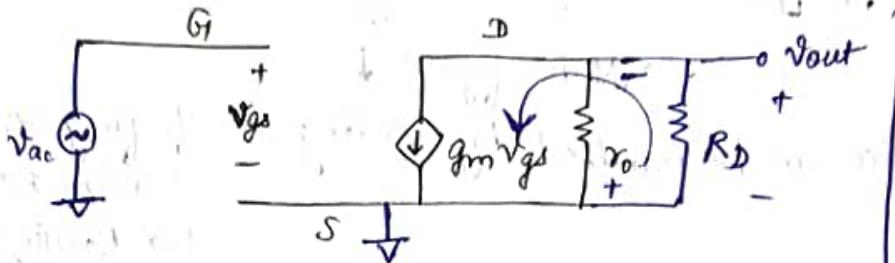
$$= V_{DD} - (I_D + g_m V_{ac}) R_D$$

$$= \underbrace{V_{DD} - I_D R_D}_{\text{DC}} - \underbrace{g_m V_{ac} R_D}_{\text{AC}}$$

$$A_v = \frac{-g_m R_D}{V_{ac}} = -g_m R_D$$

Small-signal equivalent circuit:

Set all DC source = 0



V_{DS} → does not change its value however much current is drawn.
↳ ac ground

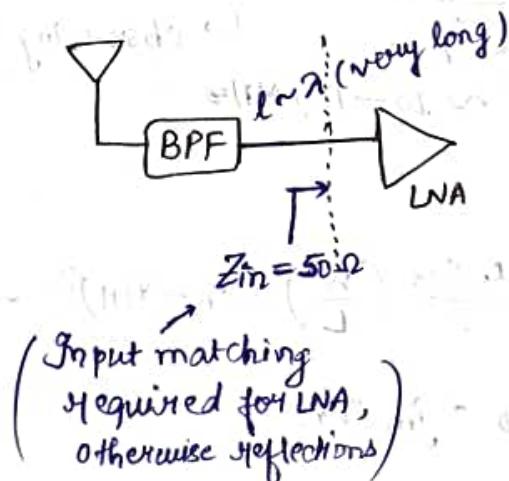
$$V_{out} = -g_m V_{gs} (R_o \parallel R_D)$$

$$V_{gs} = V_{ac}$$

$$\begin{aligned} A_v &= \frac{V_{out}}{V_{ac}} = \frac{-g_m V_{ac} (R_o \parallel R_D)}{V_{ac}} \\ &= -g_m (R_o \parallel R_D) \\ &\approx -g_m R_D \end{aligned}$$

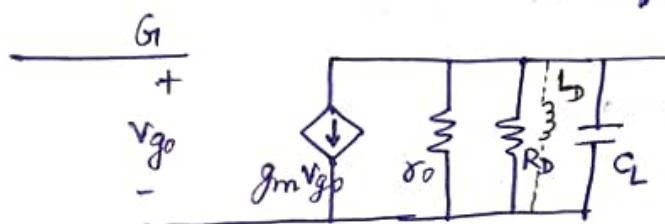
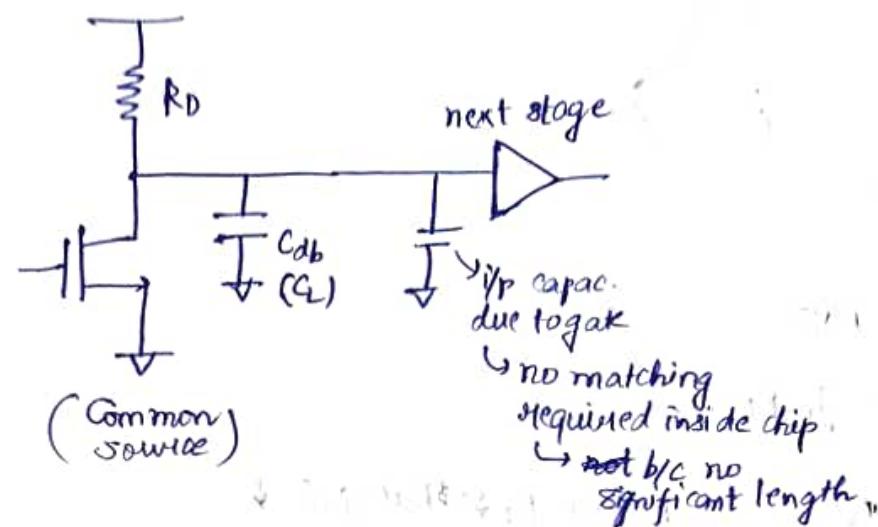
14-02-2025

Low Noise Amplifiers (LNA)

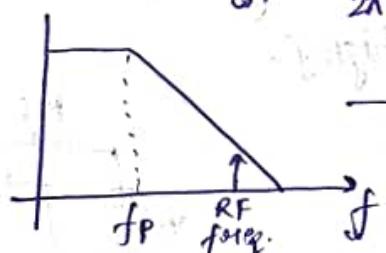


- Input matching
- Noise figure
- Gain
- Power consumption
- Stability → Amplifier should not end up being oscillator
- Risk greater in RF ckt. due to unwanted feedback path

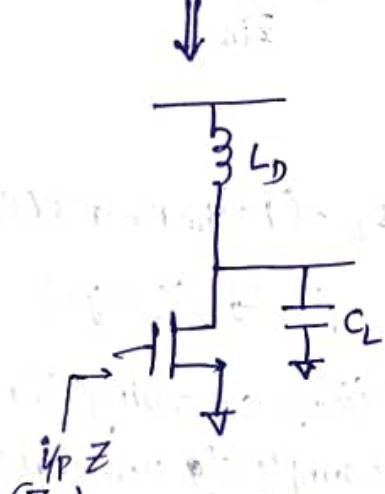
→ Linearity



$$f_P = \frac{1}{2\pi(R_D || r_0) C_L}$$

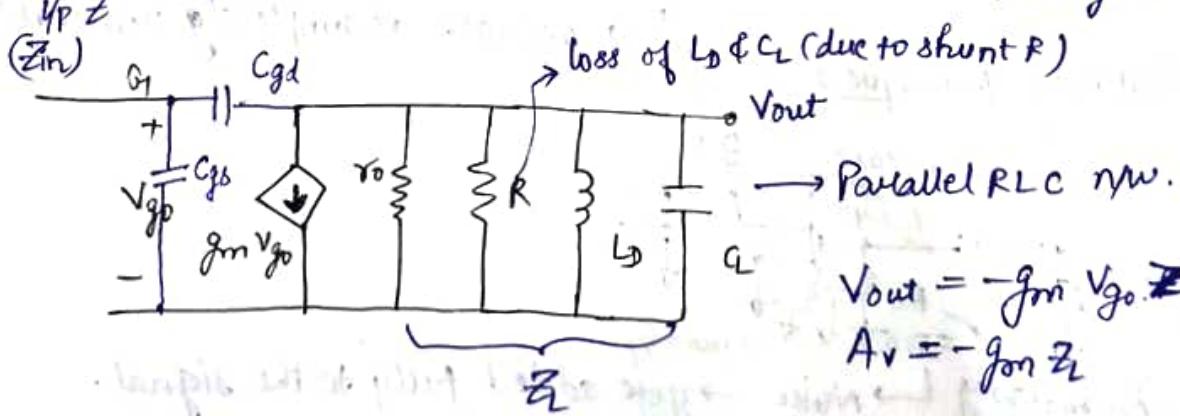


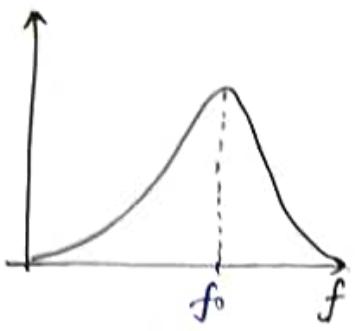
→ Pole location in low frequent. AF freq.
↳ Gets filtered out.
↳ No significant gain.



$$f_0 = \frac{1}{2\pi \sqrt{L_D C_L}}$$

→ Tune for f_0 .
→ Gain Pole location in RF region
↳ Gain limited by r_0 ,
(significant gain),
ideally ∞ .



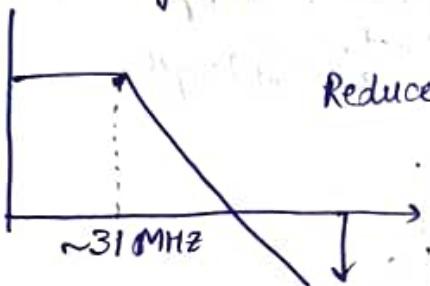


$$\text{Eq. } R_D = 5 \text{ k}\Omega$$

$$C_L = 1 \text{ pF}$$

$$\Rightarrow f_p \approx 31 \text{ MHz}$$

$$Av = g_m R_D$$



Reduce $R_D \downarrow \Rightarrow$ But gain ↓.

Input impedance \rightarrow real + imaginary
low for high f.



Real part due
to R at 0/p

$$Z_{in} = \underbrace{Re\{ \}}_{\text{can become } 0 \text{ v.c.}} + j\{ \}$$

\hookrightarrow can become 0 v.c.

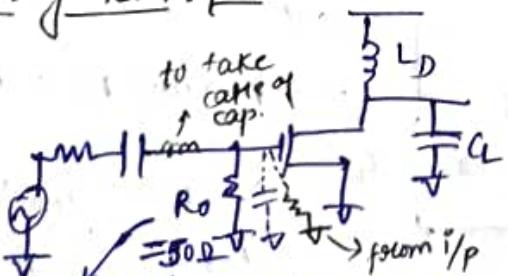
$$\text{beyond } \omega_1^2 = \frac{R_s C_L + g_m L_D - (1 + g_m R_s) R_s (C_L + g_d)}{g_m L_D^2 (C_L + g_d)}$$

oscillation

\hookrightarrow due to 0 v.c. $Re\{Z_{in}\}$ cancelling the loss.

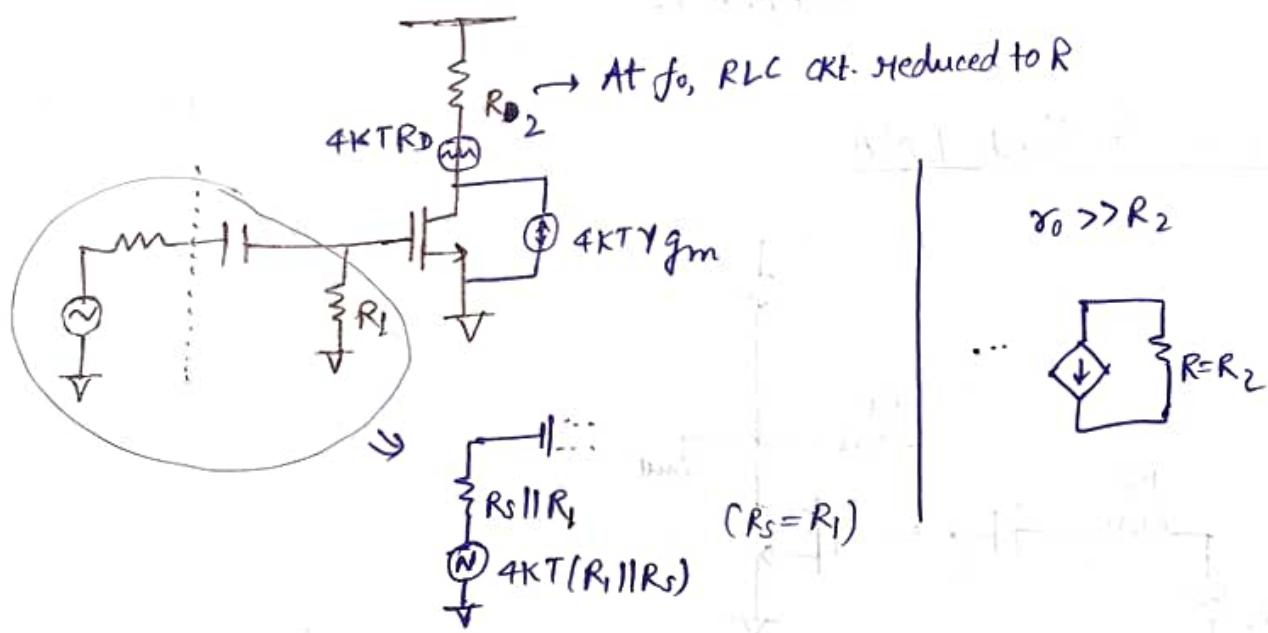
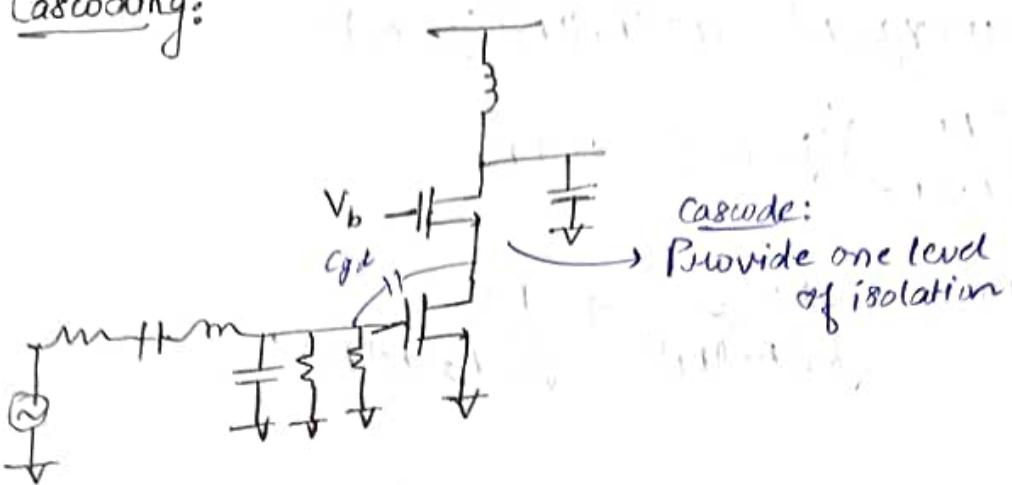
\hookrightarrow oscillates at completely unrelated freq.

Matching Technique:



for matching \hookrightarrow Noise \rightarrow gets added fully to the signal.
 \hookrightarrow straight forward

Cascoding:



Voltage o/p noise power spectral density,

$$\overline{V_{n,o/p}^2} = 4KTR_2 + 4KT\gamma g_m \cdot R_2^2$$

(due to power quantity)

$$+ 4KT(R_1 \parallel R_s)(g_m R_2)^2$$

$$NF = \frac{\text{Total noise @ o/p}}{(G_{\text{gain}})^2 \cdot 4KTR_s}$$

$$\text{Gain} = \left(\frac{R_1}{R_1 + R_s} \right) g_m \cdot R_2$$

$$NF = \frac{4KTR_2 + 4KTYg_m R_2^2 + 4KT(R_1 || R_s)(g_m \cdot R_2)^2}{\left(\frac{R_1}{R_1 + R_s}\right)^2 (g_m R_2)^2 - 4KTR_s}$$

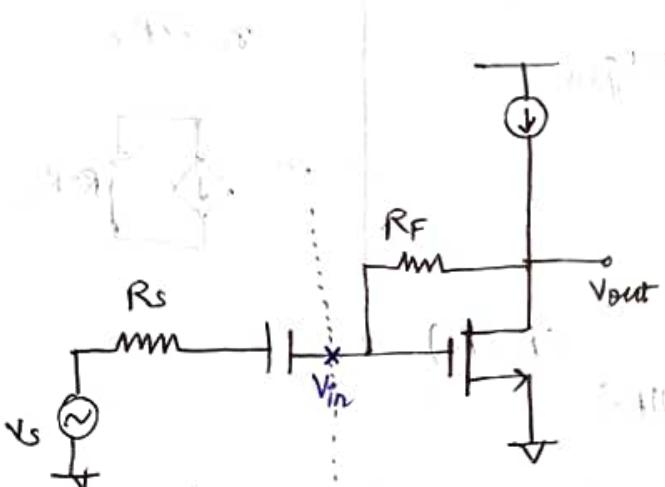
$$= \frac{R_s + R_1}{R_1} + \frac{YR_s}{g_m (R_s || R_1)^2} + \frac{R_s}{g_m^2 R_2 (R_s || R_1)^2}$$

> 2

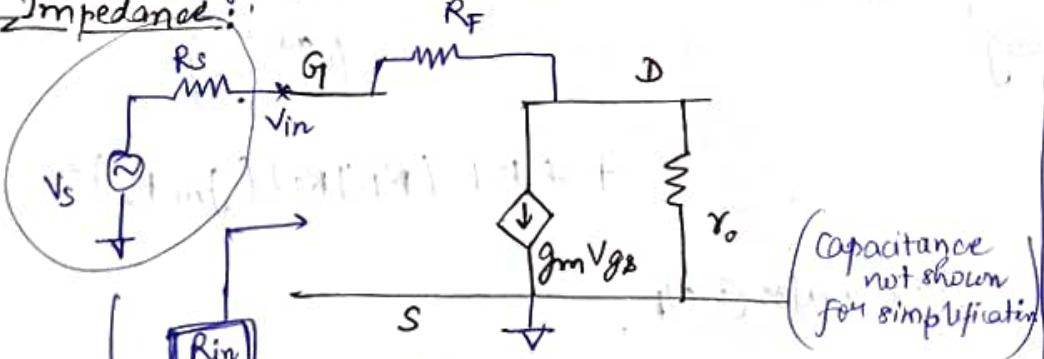
$\therefore NF > 3\text{dB}$. \rightsquigarrow high.
 ↳ Not used

Resistive Feedback LNA

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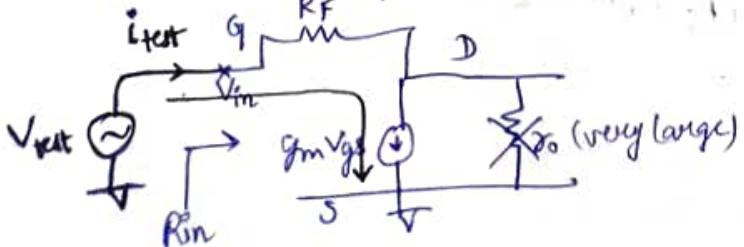


I/P Impedance:



Active equipments (one with supply)
should not be given DC, otherwise
it may get damaged.

To get i/p impedance (of any circuit)
 1) delete previous circuit,
 supply test v and calculate test I.



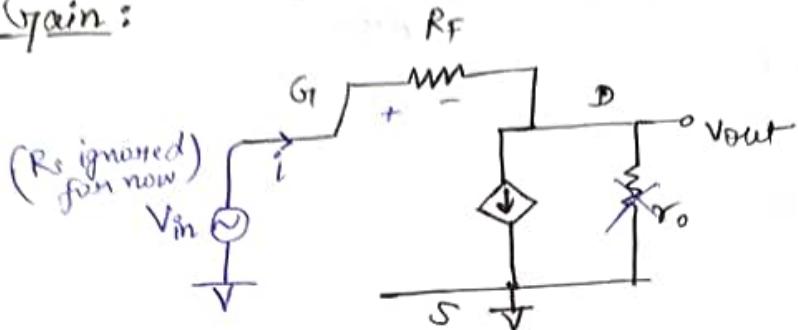
$$I_{test} = g_m V_{in} = g_m V_{test}$$

$$\therefore R_{in} = \frac{V_{test}}{I_{test}} = \frac{1}{g_m}$$

To get $R_{in} = 50\Omega$,

$$g_m = 20 \text{ mS.}$$

Gain:



$$V_{out} = V_{in} - iR_F$$

$$= V_{in} - g_m V_{in} R_F = V_{in} (1 - g_m R_F)$$

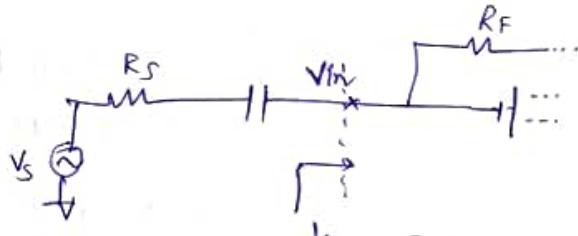
$$\frac{V_{out}}{V_{in}} = 1 - g_m R_F$$

$$= 1 - \frac{R_F \sim 100 - 1000 \Omega}{R_S \sim 100} \quad (\text{To ensure } R_S = \frac{1}{g_m} = 50\Omega)$$

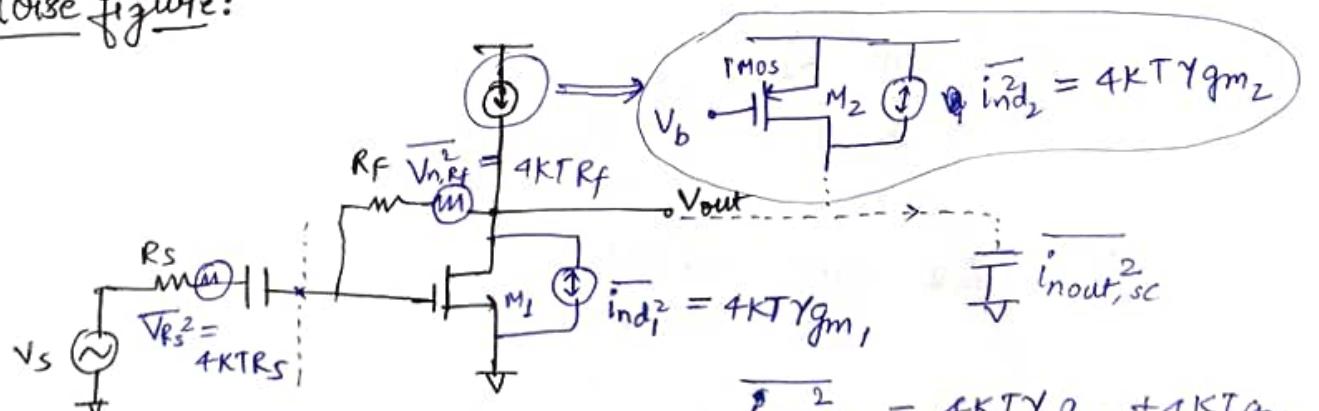
$$\boxed{\text{Gain} \approx -\frac{R_F}{R_S} \approx -g_m R_F}$$

$$\frac{V_{in}}{V_s} = \frac{1}{2}, \text{ if } R_S = \frac{1}{g_m}$$

$$\text{overall gain: } \frac{V_{out}}{V_s} = \frac{1}{2} \left(1 - \frac{R_F}{R_S} \right) \approx \boxed{-\frac{R_F}{2R_S}}$$



Noise figure:



$$NF = 1 + \frac{\text{Noise added}}{\text{Gain}^2 \cdot 4KTR_S}$$

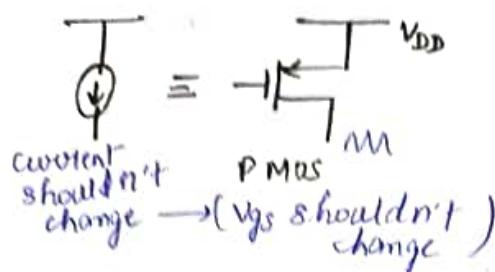
(current noise figure of ckt when load is ∞)

$$\boxed{i_{out,sc}^2 = 4KTg_m1 + 4KTg_m2 + \frac{4KT}{RF}}$$

$$+ \frac{4KT}{RF}$$

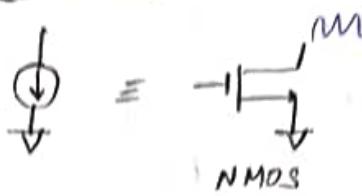
[V_s ignored]

Current source:

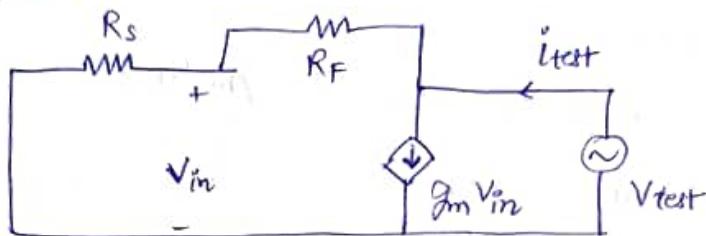


$$V_{n,out}^2 = i_{in,out,sc}^2 \cdot R_{out}^2$$

Current sink:



To find R_{out} :



$$i_{test} = g_m V_{in} + \frac{V_{test}}{R_F + R_s}$$

$$V_{in} = V_{test} \cdot \frac{R_s}{R_s + R_F}$$

$$i_{test} = \frac{g_m \cdot V_{test} \cdot R_s}{R_s + R_F} + \frac{V_{test}}{R_F + R_s}$$

$$= V_{test} \left(\frac{g_m R_s + 1}{R_s + R_F} \right)$$

$$\therefore R_{out} = \frac{V_{test}}{i_{test}} = \frac{R_F + R_s}{1 + g_m R_s}$$

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$$\text{and } g_m = \frac{1}{R_s}$$

$$\Rightarrow R_{out} = \frac{R_s + R_F}{2}$$

$$NF = 1 + \frac{\text{noise added}}{(Gain)^2 \cdot 4KTR_s}$$

$$V_{n,out}^2 = 4KT \gamma (g_{m1} + g_{m2}) \cdot R_{out}^2 + 4KTR_F$$

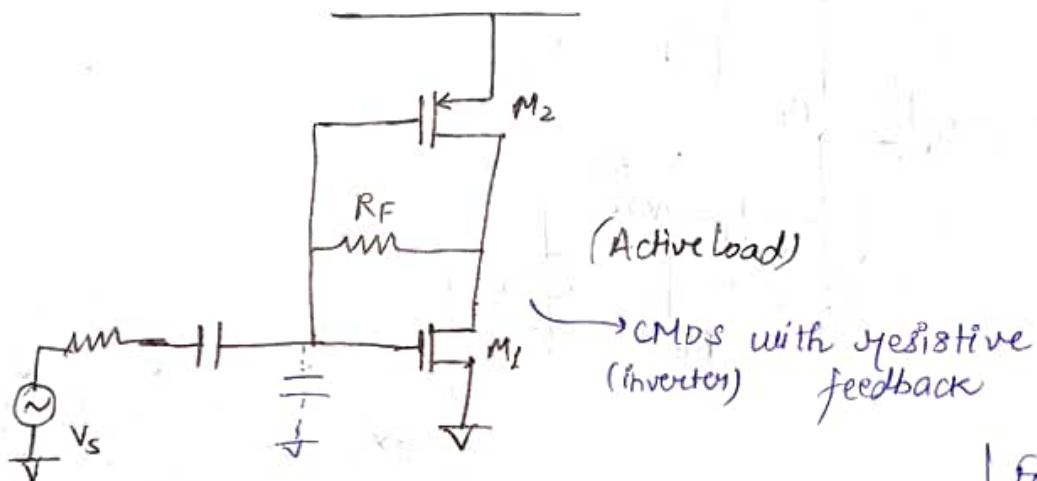
$$\Rightarrow NF = 1 + \frac{4KT \gamma (g_{m1} + g_{m2}) \cdot (R_s + R_F)^2}{\left(\frac{R_F}{2R_s}\right)^2 \times 4KTR_s \times \gamma} + \frac{4KTR_F}{\left(\frac{R_F}{2R_s}\right)^2 \times 4KTR_s \times \gamma}$$

$$= 1 + \frac{\gamma(g_{m1} + g_{m2}) f_{RF} R_S + R_F}{R_F^2 / R_S} + \frac{4}{R_F / R_S} \quad (\because R_S + R_F \approx R_F)$$

$$= 1 + \frac{4 R_S}{R_F} + \underbrace{\gamma(g_{m1} + g_{m2}) R_S}_{\text{very small}}$$

little over 2 (better than the power.)

Modification:



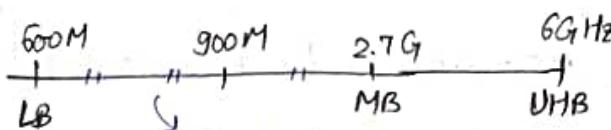
$$R_{in} = \frac{1}{g_{m1} + g_{m2}}$$

For inv,

$$NF = 1 + \frac{4 R_S}{R_F} + \gamma$$

For lower technology
(short channel length),
parasitic resistance
is low
 $S_{11} < 10 \text{ dB}$

5G:



divided into chunks $\sim 40 \text{ MHz}$ (allocated to each user)

20MHz $\begin{cases} 10 \text{ MHz} \rightarrow \text{around } (2.5 \text{ GHz}) \\ 5 \text{ MHz} \rightarrow (720 \text{ MHz}) \\ 5 \text{ MHz} \rightarrow (830 \text{ MHz}) \end{cases}$ } LB (interband) } each handled by separate LNAs

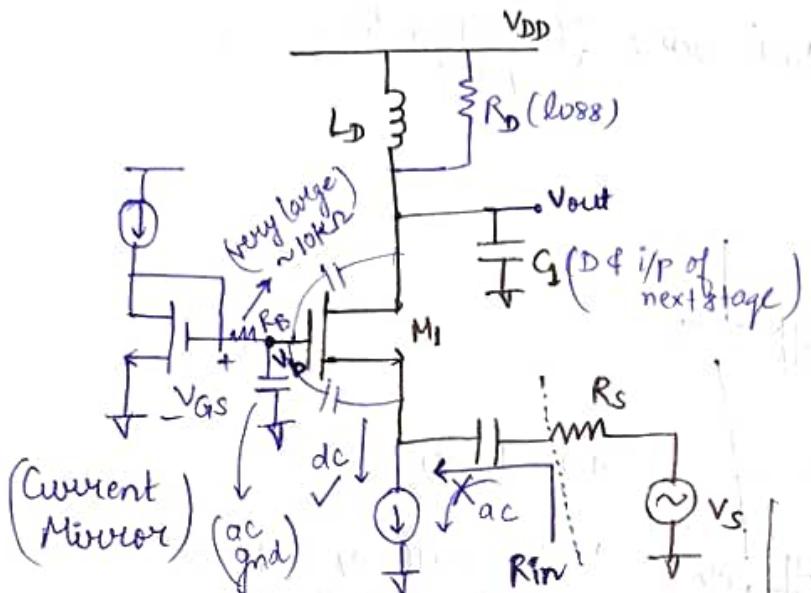
Carrier Aggregation

↳ Interaband and interband CA
within one band (e.g., LB)

Indicator: High size compared to this design.

Signal cancellation (dip) due to interference from two towers.
 ↳ Devices with two LNAs → main and diversity.
 (switch if dip)

Common Gate LNA

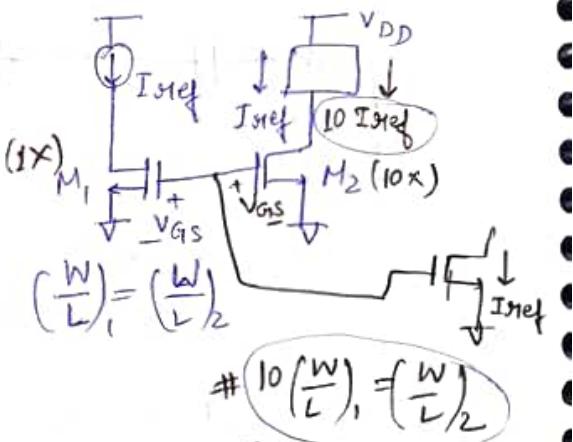


$$f_D = \frac{1}{2\pi\sqrt{L_D C_1}}$$

To resonate at high f,
 if $C_1 \downarrow \Rightarrow L_D \uparrow$
 OR add some capacitance

i/p → S
 o/p → D
 ac GND → G

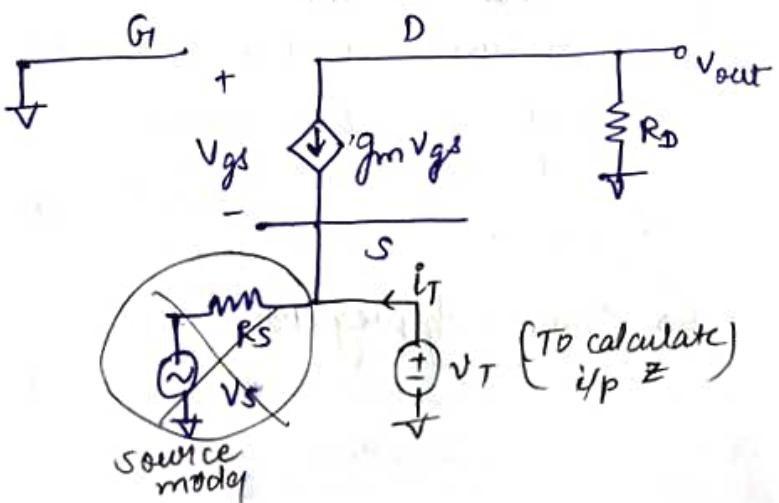
Feeding voltage division of V_{DD} to G_1 will not work, as many factors give mismatch of resistors (fixed division).



$$I_D = \frac{M_n C_{ox}}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TH})^2$$

Small signal model at resonance: $L_D + C_1$ cancel.

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① Input impedance

$$V_{gs} = -V_T$$

$$i_T = -g_m V_{gs}$$

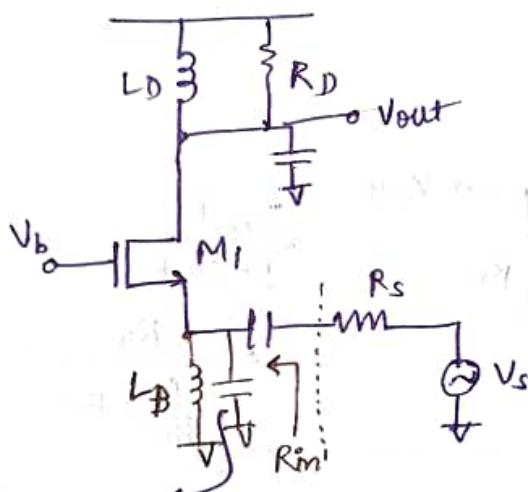
$$= + g_m V_T$$

$$\boxed{R_{in} = \frac{1}{g_m}} = \frac{V_T}{i_T}$$

$$\left[\text{if } g_m = 20 \text{ mS} \Rightarrow \text{Re}\{R_{in}\} = 50 \Omega \right]$$

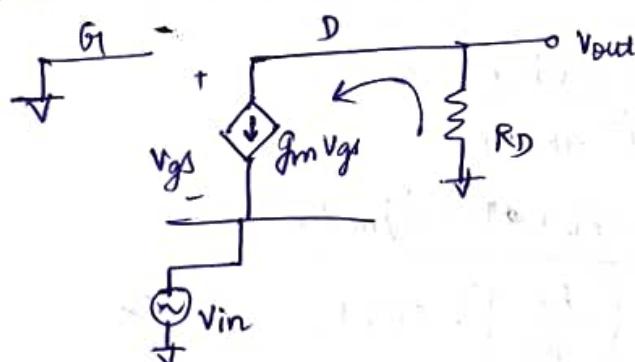
↳ capacitances are not considered.

$$Z_{in} = \frac{1}{g_m} \left(C_{gs} + C_{bs} + \dots \right)$$



(decoupling + pad cap)
routing cap

② Gain



$$V_{gs} = -V_{in}$$

$$\begin{aligned} V_{out} &= -g_m V_{gs} R_D \\ &= g_m V_{in} R_D \end{aligned}$$

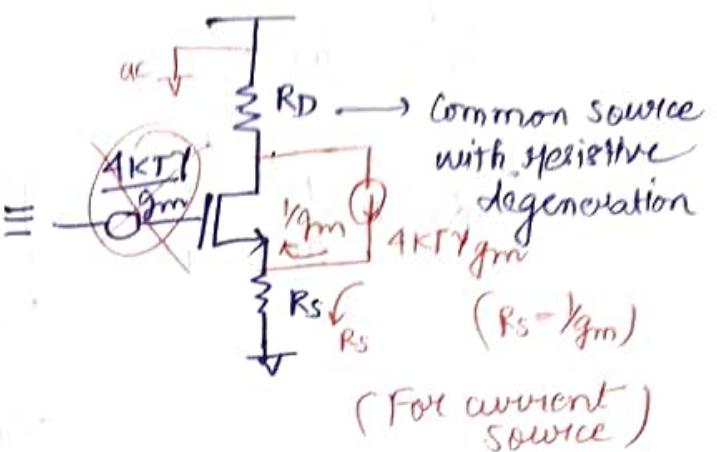
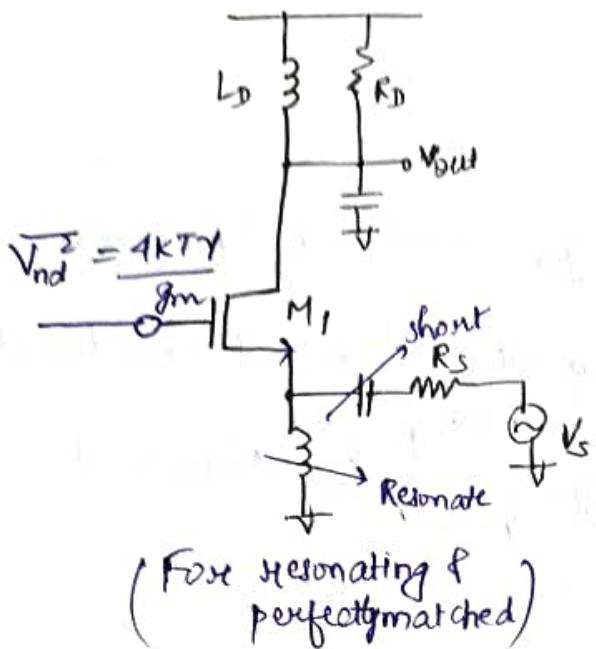
$$\frac{V_{out}}{V_{in}} = g_m R_D$$

$$g_m = \frac{1}{R_s}$$

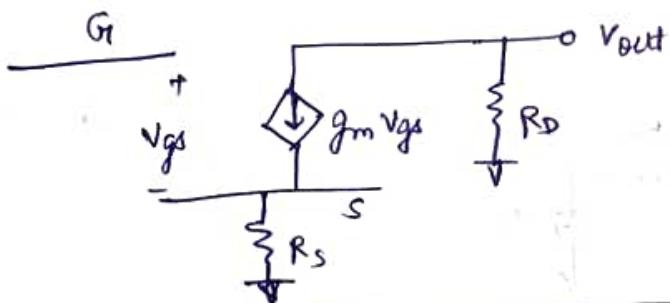
$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_D}{R_s}$$

$$\therefore \boxed{\frac{V_{out}}{V_{in}} = A_v = \frac{R_D}{2R_s}} \quad (\text{Overall gain})$$

③ Noise



$$q_{KT\gamma gm} \frac{1}{4} R_D^2$$



$$\frac{g_m R_D}{1 + g_m \cdot R_S} = \frac{R_D}{\frac{1}{g_m} + R_S} = \frac{R_D}{2R_S}$$

$$\therefore V_{n, \text{out, add}}^2 = 4KTR_D + \underbrace{\frac{4KT\gamma}{g_m} \left(\frac{R_D}{2R_S} \right)^2}_{(KT\gamma g_m R_D^2)}$$

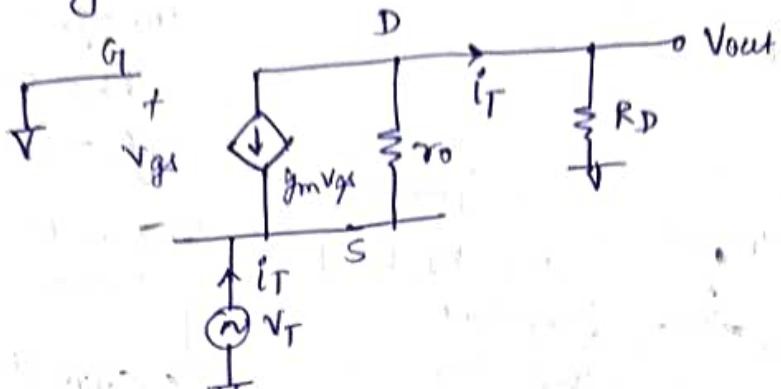
$$④ \text{ NF} = 1 + \frac{\text{Noise added}}{\text{gain}^2 \cdot 4\text{KTR}_S}$$

$$= 1 + \frac{4KTR_D + 4KTg_m R_D^2}{\left(\frac{R_D}{2R_S}\right)^2 \cdot 4KTR_S}$$

$$= 1 + \frac{4 + \gamma g_m R_D}{\left(\frac{R_D}{R_S}\right) \times 4}$$

$$= \boxed{1 + \gamma + 4 \frac{R_s}{R_D}} \quad (\gamma_m = \frac{1}{R_s})$$

Considering τ_0 ,



$$v_{gs} = -v_T$$

$$i_T = -g_m v_{gs} + i_{r_0}$$

$$\Rightarrow i_{r_0} = \frac{v_T - v_{out}}{\tau_0} = \frac{v_T - i_T R_D}{\tau_0}$$

$$i_T = g_m v_T + \frac{v_T - i_T R_D}{\tau_0}$$

$$i_T \left(1 + \frac{R_D}{\tau_0}\right) = v_T \left(g_m + \frac{1}{\tau_0}\right)$$

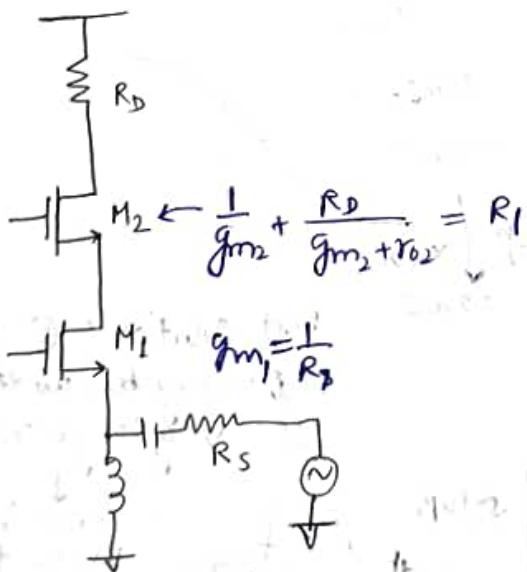
$$R_{in} = \frac{V_T}{i_T} = \frac{1 + \frac{R_D}{\tau_0}}{g_m + \frac{1}{\tau_0}} = \frac{R_D + \tau_0}{1 + g_m \tau_0} \approx \frac{R_D + \tau_0}{g_m \tau_0}$$

$$\approx \left[\frac{1}{g_m} + \frac{R_D}{g_m \tau_0} \right]$$

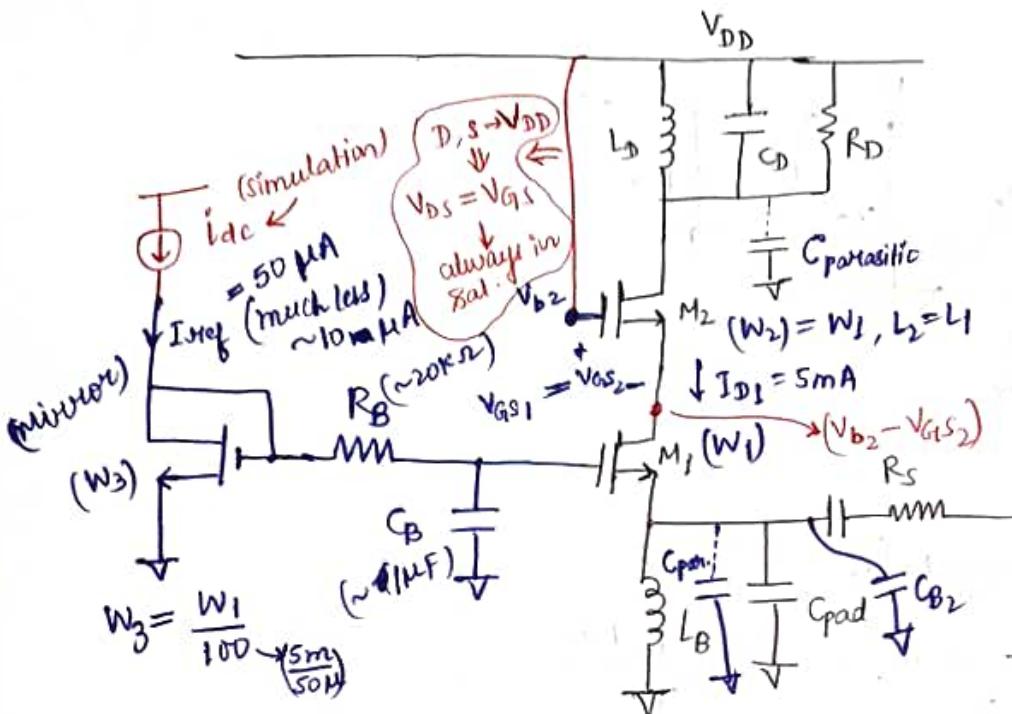
intrinsic
gain
(limit)

↳ determined
by R_D

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$$R_{in} = \frac{1}{g_{m1}} + \frac{1}{g_{m1} \tau_0} + \frac{R_D}{(g_{m1} \tau_0)(g_{m2} \tau_2)} \approx \frac{1}{g_{m1}}$$



$$\frac{1}{g_{m1}} = 50\Omega \Rightarrow g_{m1} = 20\text{ ms}$$

Decide W_1, L_1, V_{bias}

- $L = L_{min} \rightarrow$ To reduce the parasitic capacitances
(Take lowest channel length)

- $W_0 = 1\text{ }\mu\text{m}$ (easier!)

$L_0 = L_{min}$ ← start with

$$\frac{W_0}{L_0} = \frac{1\text{ }\mu\text{m}}{L_{min}}$$

$$g_m = \frac{\partial I_D}{2 V_{GS}}$$

$$g_m = \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{GS} - V_{th})$$

$$\propto \frac{W}{L}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \downarrow 2.4\text{ GHz} \rightarrow \text{For small } C, L \text{ has to be large}$$

can be pushed offchip.

$L_D \rightarrow$ cannot be offchip.

Replace inductor by transistor as current source
OR
Add an extra capacitor along with inductor.
(C_{BS2})

Design:

$$R_{in} = 50\Omega$$

$$f_o = 2.4\text{ GHz}$$

Tech. node = 28nm

$$\hookrightarrow V_{DD} = 1 \text{ or } 0.9\text{ V}$$

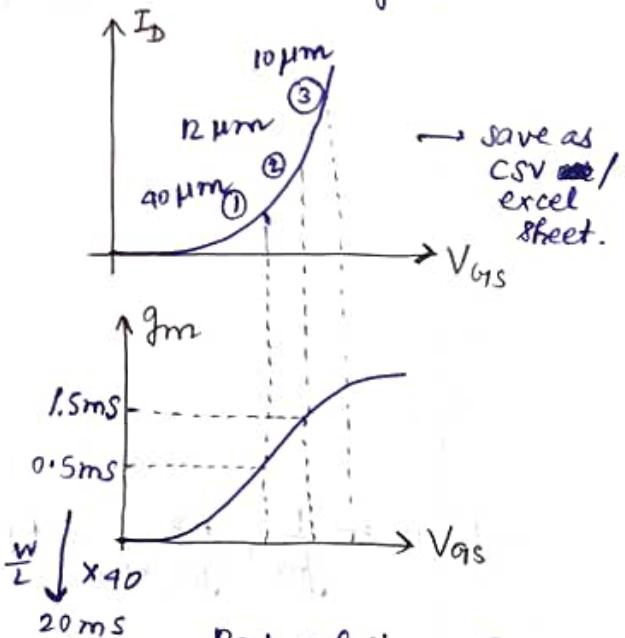
Saturation Mode:

$$V_{DS} > V_{GS} - V_{th}$$

Simulation gives $V_{DS,sat}$.
For $V_{DS} > V_{DS,sat}$, device is in sat.

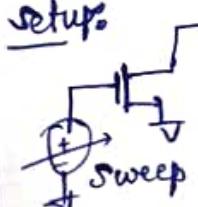
First simulate DC

To check if ckt. is alive.



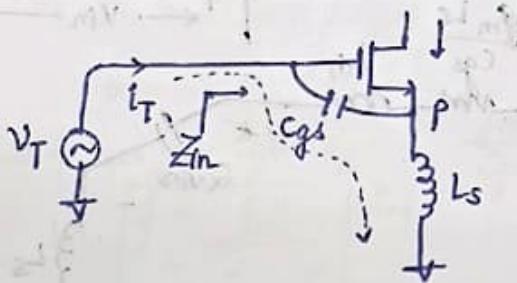
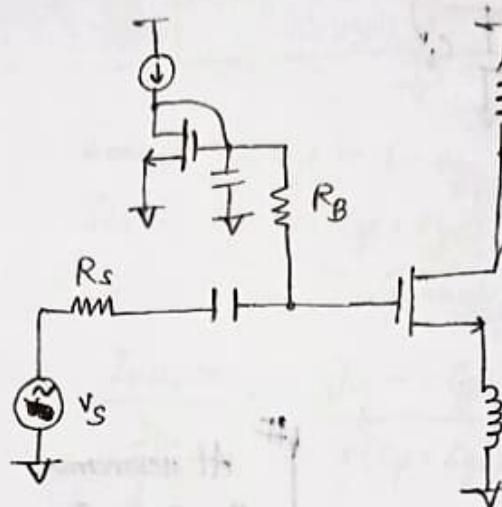
Best solution: ② large
①: very low (~~large~~) current
③: very large current

setup:



Take little more than the saturated value.
 $\sim 80\%$ of sat. value.

Common Source with Inductive Degeneration LNA



$$i_p = s L_s (i_T + g_m v_{gs})$$

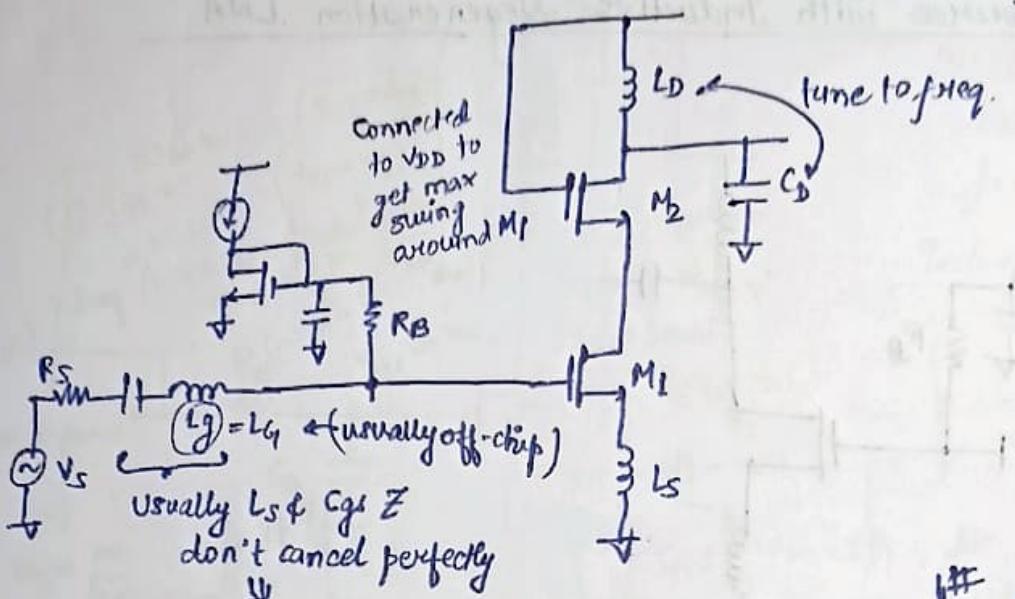
$$v_{gs} = \frac{i_T}{s C_{gs}}$$

$$v_T = \underbrace{\frac{i_T}{s C_{gs}}}_{v_{gs}} + s L_s \left(i_T + \frac{g_m i_T}{s C_{gs}} \right)$$

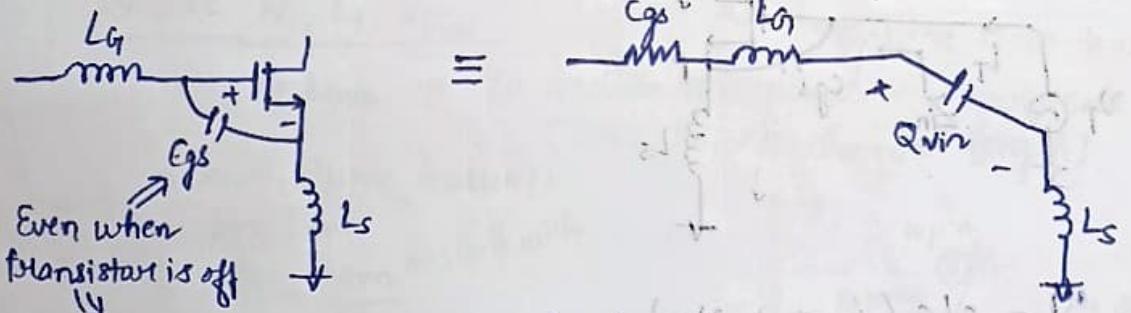
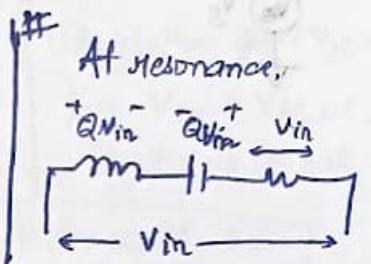
$$\therefore Z_{in} = \frac{v_T}{i_T} = \underbrace{\frac{1}{s C_{gs}}}_{\text{obvious}} + \underbrace{s L_s}_{\text{3 components in series}} + \underbrace{\frac{g_m L_s}{s C_{gs}}}_{\text{Real part}}$$

$\underbrace{(C+L)}_{\text{obvious}}$ $\underbrace{\frac{g_m L_s}{s C_{gs}}}_{\text{Real part}}$

$$\text{Re}\{Z_{in}\} = \frac{g_m L_s}{s C_{gs}}$$



Introduce L_g to
make $\text{Im}\{Z\} = 0$, $\text{Re}\{Z\} = 50 \Omega$.



Provides extra \Rightarrow overall good gain

$$\left[jw(L_s + L_g) - \frac{i}{wC_{gs}} \right] = 0 \quad (\text{at resonance})$$

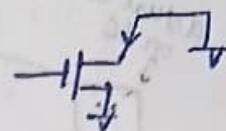
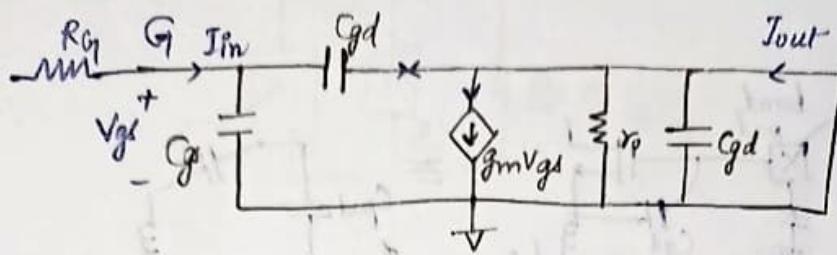
$$w(L_s + L_g) = \frac{1}{wC_{gs}}$$

$$w^2 = \frac{1}{wC_{gs}(L_s + L_g)} \Rightarrow w = \frac{1}{\sqrt{C_{gs}(L_s + L_g)}}$$

f_T : Transition frequency. (for any active device)

\equiv frequency @ current gain = 1

short-circuit [Best current gain]



$$I_{out, sc} = g_m V_{ds} + (-V_{gs} \cdot s(C_{gd}))$$

$$I_{in} = V_{gs} \cdot s(\underbrace{C_{gs} + C_{gd}}_{\text{in parallel}})$$

$$\frac{I_{out, sc}}{I_{in}} = \frac{g_m - sC_{gd}}{s(C_{gs} + C_{gd})} = 1$$

s.c. current gain

$$\Rightarrow g_m = s(C_{gs} + 2C_{gd})$$

$$\omega_T = \frac{g_m}{C_{gd} + 2C_{gs}} \approx \frac{g_m}{C_{gs}}$$

$(\approx C_{gs} + \frac{2}{3}WL\text{Cox}) = C_{bulk}$

$$\therefore f = \frac{g_m}{2\pi C_{gs}}$$

→ Beyond f_T (and around f_T), the device is useless.

→ f_T should be much higher (~5-10 times) than the frequency of interest.

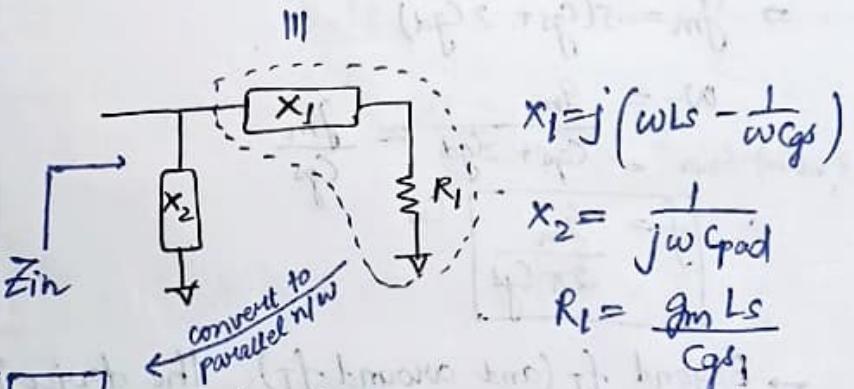
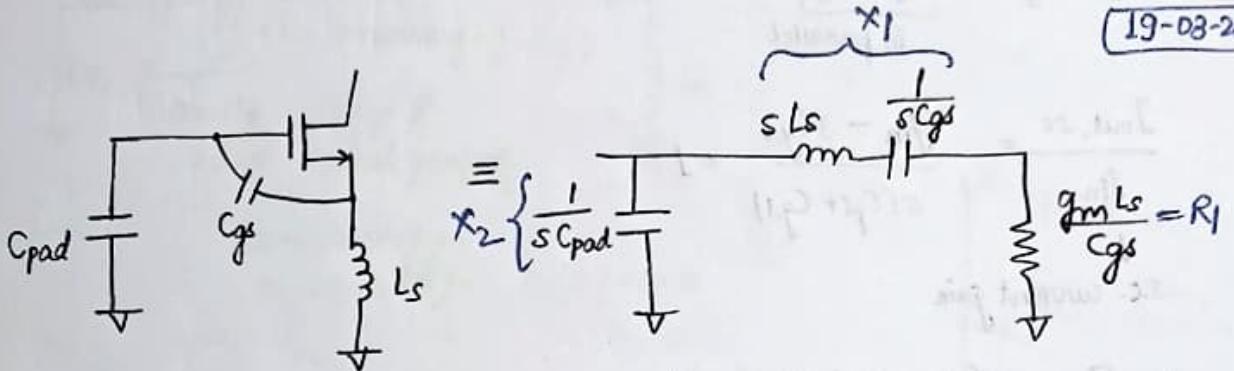
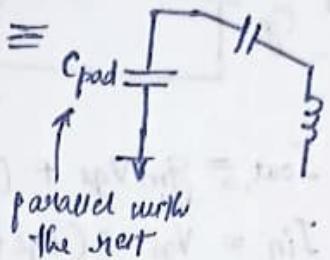
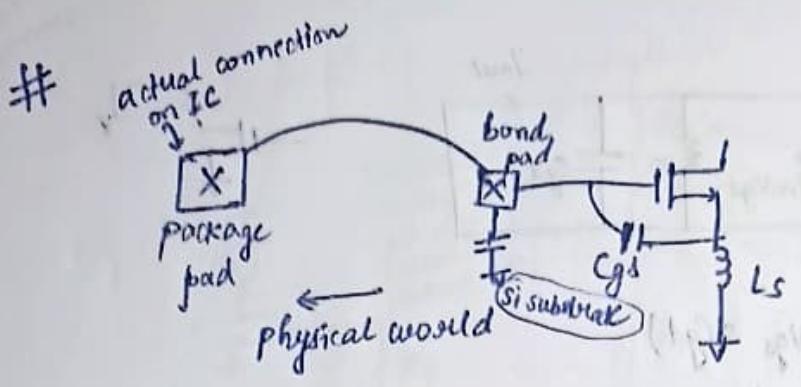
$$\therefore \text{Re}\{Z_{in}\} = \frac{g_m L_s}{C_{gs}}$$

$$= \omega_T L_s$$

f_{max} : frequency @ power gain = 1.

$$f_{max} \approx \frac{f_T}{\sqrt{R_G (g_m \frac{C_{gd}}{C_{gs}}) + (R_G + R_{ch} + R_s) g_{ds}}}$$

\downarrow physical channel resistance
total cap. at gate

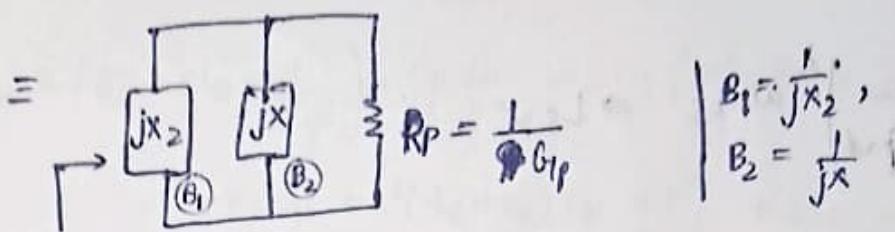


$$(R_I + jX_1) = \frac{R_p - jX}{R_p + jX} = R_I + jX_1$$

$$\frac{jX R_p}{(R_p + jX)} \cdot \frac{(R_p - jX)}{(R_p + jX)} = \frac{X^2 R_p}{R_p^2 + X^2} + j \frac{R_p^2 X}{R_p^2 + X^2}$$

$$\therefore \frac{X^2 R_p}{R_p^2 + X^2} = R_I$$

$$\frac{X R_p^2}{R_p^2 + X^2} = X_1$$



$$Y_{in} \downarrow = G_{IP} + j(B_1 + B_2)$$

$$\Sigma_{in} = \frac{1}{G_{IP} + j(B_1 + B_2)} \cdot \frac{G_{IP} - j(B_1 + B_2)}{G_{IP} - j(B_1 + B_2)}$$

$$\Rightarrow \text{Re}\{\Sigma_{in}\} = \frac{G_{IP}}{G_{IP}^2 + (B_1 + B_2)^2} = \frac{1/R_P}{\left(\frac{1}{R_P}\right)^2 + \left(\frac{1}{X_1} + \frac{1}{X_2}\right)^2} \approx \left(\frac{X_2^{-1}}{X_1 + X_2}\right)^2 \cdot R_L$$

$$X_1 = j\left(\omega L_s - \frac{1}{\omega C_{gs}}\right) \underset{\substack{\text{very small} \\ (\sim 100 \mu\text{H})}}{\sim} \frac{1}{\omega C_{gs}}$$

$$\sim \left(10^9 \times 10^{-12} - \frac{1}{10^9 \times 10^{-15}}\right) \underset{\text{dominate}}{\sim}$$

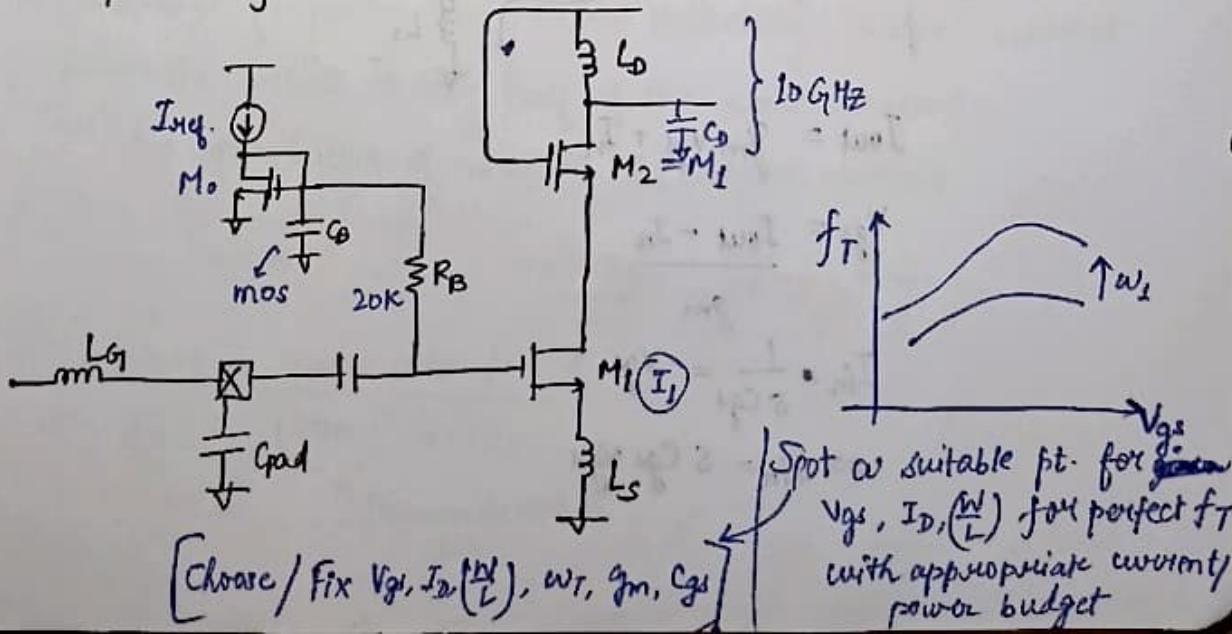
$$\therefore R_{in} = \left(\frac{C_{gs}}{C_{gs} + C_{pad}}\right)^2 \cdot \frac{g_m L_s}{C_{gs}}$$

Eg. $f_0 = 10 \text{ GHz}$

$$65 \text{ nm tech} \Rightarrow L_{min} = 60 \text{ nm}$$

$$I_{req.} = 0.5 \text{ mA}$$

$$C_{pad} = 50 \text{ fF}$$



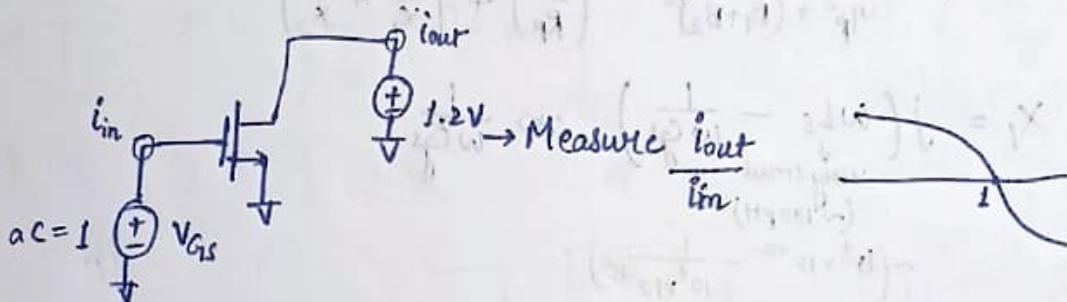
$$(50 \Omega) \frac{\left(\frac{C_g}{(g_d + g_{ad})}\right)^2 \omega r_s}{\omega r_s} \Rightarrow L_s \checkmark$$

L_q : off-chip

↳ To cancel the imaginary part of $R_s - jX_1$

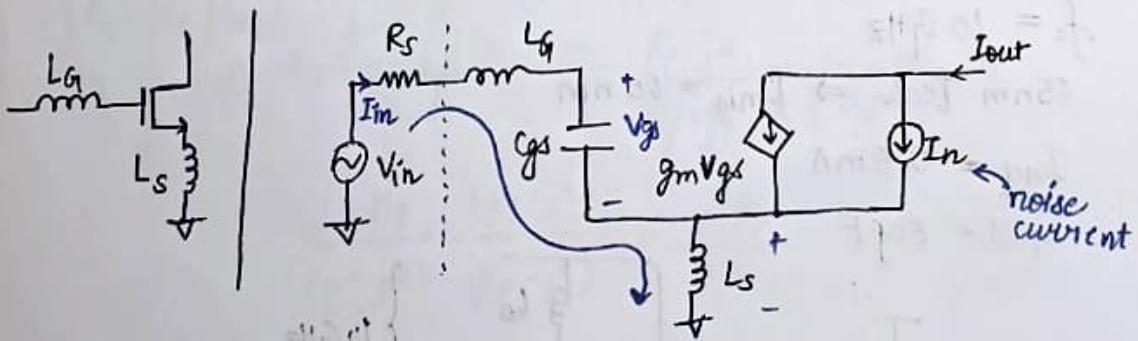
$$|j\omega L_q| = |jX_1|$$

Measuring f_T :



21-03-2025

Noise figure - CS with Inductor Degeneration



$$I_{out} = g_m V_{gs} + I_n$$

$$V_{gs} = \frac{I_{out} - I_n}{g_m}$$

$$I_{in} = \frac{1}{s C_{gs}} = V_{gs}$$

$$\Rightarrow I_{in} = s C_{gs} V_{gs}$$

$$\begin{aligned}
 V_{in} &= \left(R_s + sL_G + \frac{1}{sC_{gs}} \right) sC_{gs}V_{gs} + sL_s(sC_{gs}V_{gs} + I_{out}) \\
 &= V_{gs} \left[sR_s C_{gs} + s^2(L_G + L_s)C_{gs} + 1 \right] + sL_s I_{out} \\
 &= \frac{I_{out} - I_n}{g_m} \left[sR_s C_{gs} + \underbrace{s^2(L_G + L_s)C_{gs} + 1}_{X} \right] + sL_s I_{out}
 \end{aligned}$$

At $\omega = \omega_0 = \frac{1}{\sqrt{(L_G + L_s)C_{gs}}}$ and $s = j\omega$, and $\frac{g_m L_s}{C_{gs}} = R_s$: (matching)

$$\Rightarrow V_{in} = \frac{I_{out} - I_n}{g_m} (s g_m L_s) + sL_s I_{out}$$

$$\Rightarrow V_{in} = I_{out} (2sL_s) - I_n sL_s$$

$$\Rightarrow \left| \frac{I_{out}}{V_{in}} \right| = \left| \frac{1}{2j\omega L_s} \right| = \frac{1}{2\omega_0 L_s} = \boxed{\left(\frac{\omega_T}{\omega_0} \right) \frac{1}{2R_s}} \quad (I_n = 0) \quad (V_{in}, I_{out})$$

(Transconductance)

→ Farther away from the f_T , higher the transconductance and hence the gain.

for $V_{in} = 0$

$$\Rightarrow I_n sL_s = I_{out} (2sL_s)$$

$$\Rightarrow \boxed{I_{out} = \frac{|I_n|}{2}}$$

→ At resonant frequency and perfect matching, output current from the circuit is only half of the noise current.

↳ Contribution of noise in the output current.

$$\therefore \overline{I_{out,n}^2} = \frac{\overline{I_n^2}}{4} = \frac{4kT \gamma g_m}{4} = \boxed{K T \gamma g_m}$$

$$\therefore N.F. = 1 + \frac{\text{noise added}}{(\text{gain})^2 \cdot 4kT R_s}$$

* \uparrow transconductance g_m

$$= 1 + \frac{KTY_{gm}}{\left[\left(\frac{\omega_0}{\omega_T} \right) \frac{1}{2R_s} \right]^2 \cdot 4KTR_s}$$

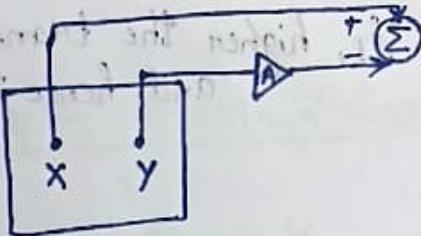
$$= 1 + \left(\frac{\omega_0}{\omega_T} \right)^2 \frac{KTY_{gm} \cdot (4R_s^2)}{4KTR_s}$$

$$\therefore NF = 1 + \left(\frac{\omega_0}{\omega_T} \right)^2 \gamma_{gm} R_s$$

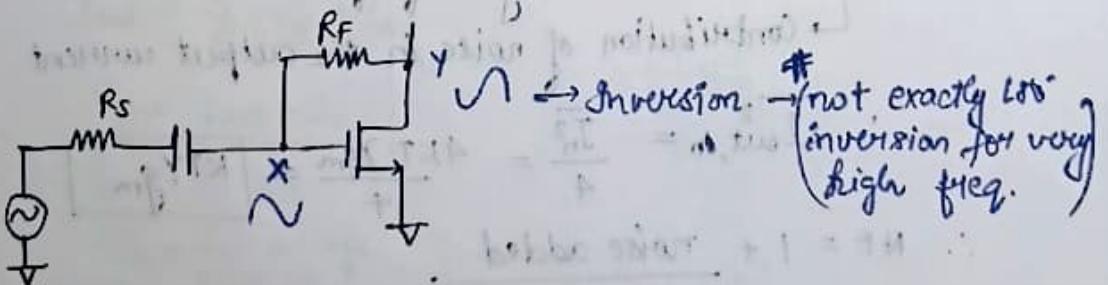
- Moving away from ω_T , we have very good NF.
- Choose design such that diff of dimensions, bias, current such that f_T is maximum (very high).

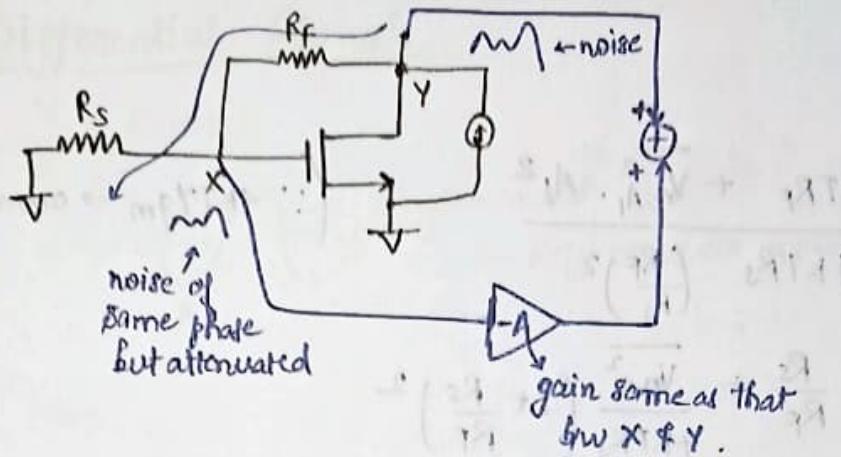
Noise Cancelling LNAs

$$NF = 1 + (\text{due to i/p transistor}) + (\text{due to o/p load})$$

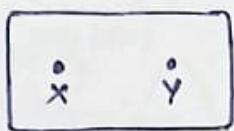


Identify two points in the circuit such that
 signal → out of phase
 noise → in phase





25-03-2025

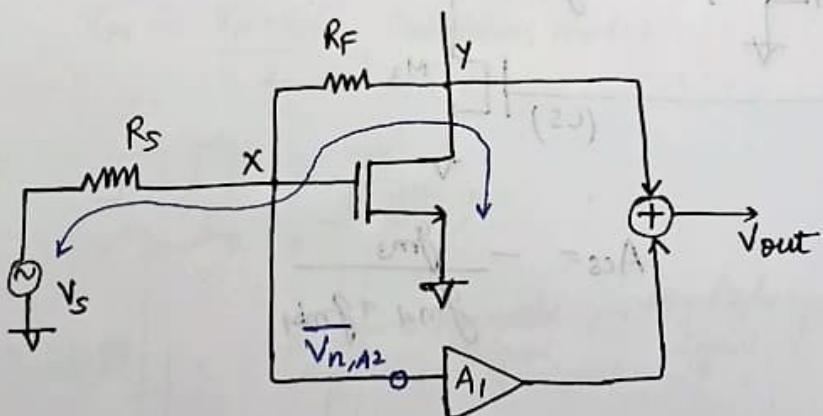


Signal : Out of phase
Noise : In phase

$$\text{(Signal)} \quad \frac{V_{ys}}{V_{xs}} = 1 - g_m R_F = 1 - \frac{R_F}{R_s}$$

$$\approx -\frac{R_F}{R_s}$$

(Noise) : $V_s = 0$



$$V_{xn} = V_{yn} \cdot \frac{R_s}{R_s + R_F}$$

$$\Rightarrow \frac{V_{yn}}{V_{xn}} = 1 + \frac{R_F}{R_s}$$

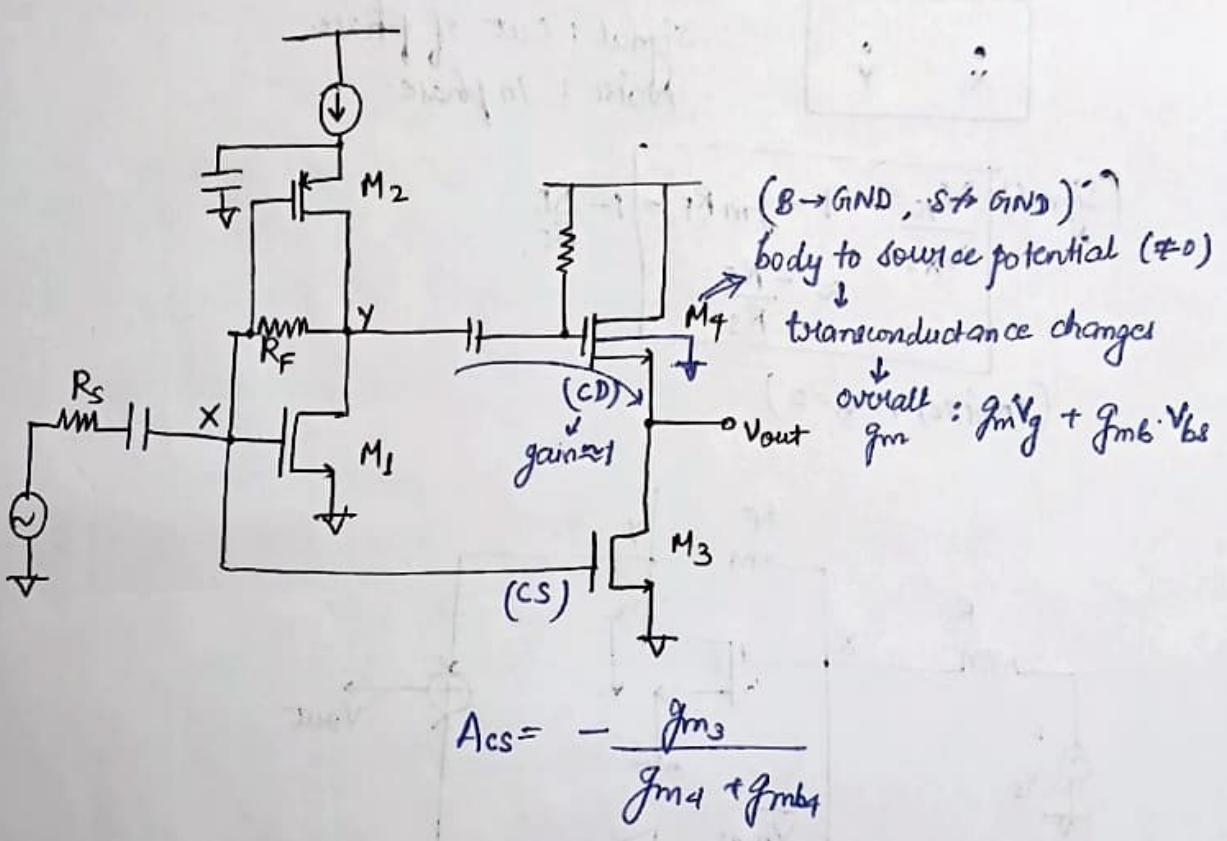
$\therefore A_1 = - \left(1 + \frac{R_F}{R_s} \right)$ [To cancel the noise]

$$\frac{V_{out}}{V_x} = \left(1 - \frac{R_F}{R_s} \right) - \left(1 + \frac{R_F}{R_s} \right) = -\frac{2R_F}{R_s}$$

$$\frac{V_{out}}{V_s} = -\frac{R_F}{R_S}$$

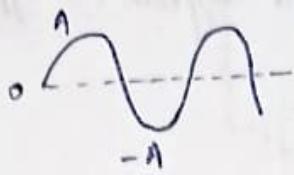
$$NF = 1 + \frac{4KTR_F + \overline{V_{nA_1}}^2 \cdot A_L^2}{4KTR_S \cdot \left(\frac{R_F}{R_S}\right)^2} \quad : \quad (\because 4KTR_F \rightarrow \text{cancelled})$$

$$= 1 + \underbrace{\frac{R_S}{R_F} \frac{R_S}{R_F}}_{\text{Small}} + \frac{\overline{V_{nA_1}}^2}{4KTR_S} \left(1 + \frac{R_S}{R_F}\right)^2$$



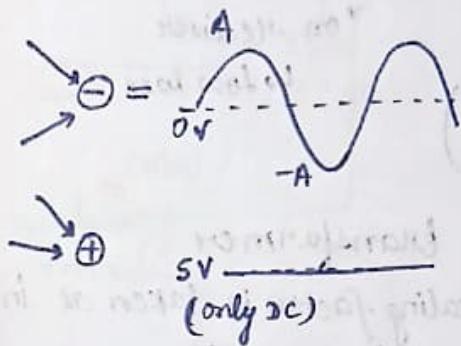
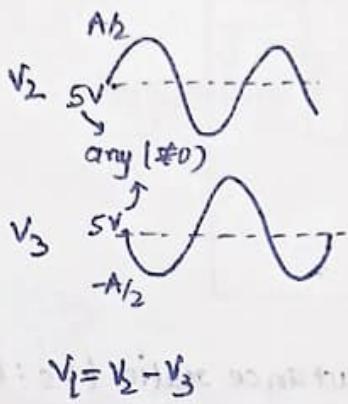
Differential Signals

$$V_1 = A \sin \omega t$$



single-ended

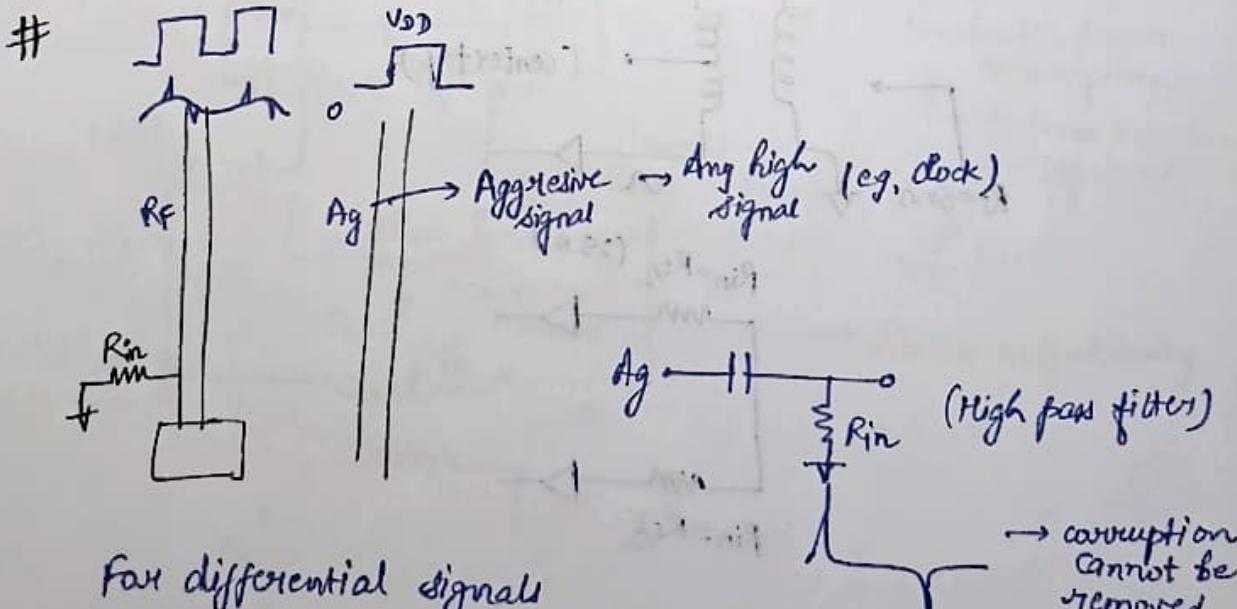
↳ Referenced to ground.



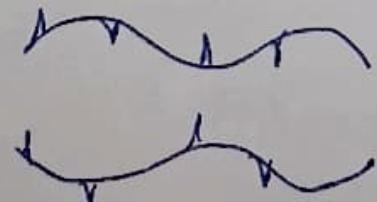
For V_A, V_B ,

$$V_d = V_A - V_B \quad (\text{differential})$$

$$V_{CM} = \frac{V_A + V_B}{2} \quad (\text{common mode})$$



For differential signals



} Both signals corrupted by small amount
 \rightarrow gets eliminated

Balun

↳ Transformer that converts single-ended signal to differential signal.

Balanced — Unbalanced

(Differential) (Single-ended)

↳ On-chip or off-chip

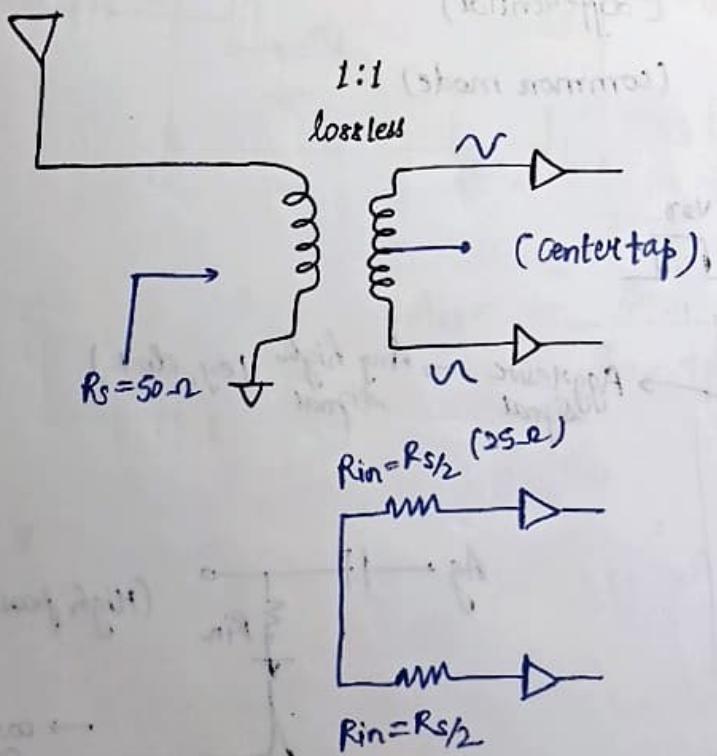
high loss
(used for large no. of receive signals)

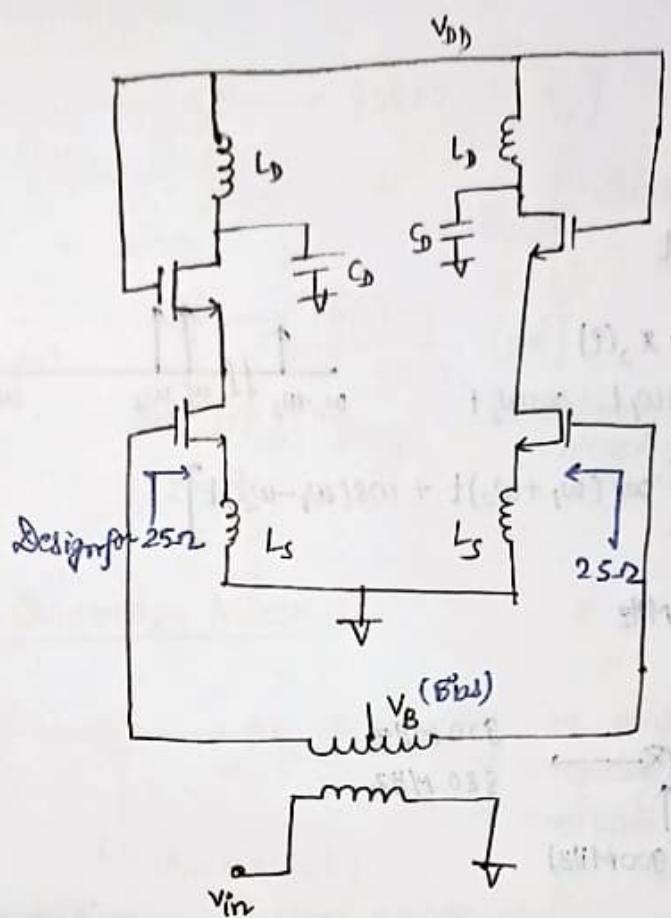
on Receiver

↳ Low loss

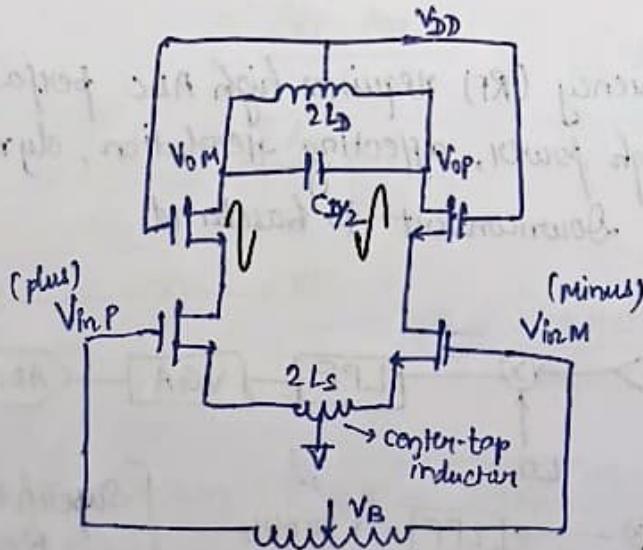
↳ Planar transformer

↳ scaling factor is taken as inductance ratio ($L_s : L_p$), not turns ratio.





$M_{12} = M_{13} = M_{23}$



2 LNAs
→ double power consumption
→ 2-times matching required

→ Double output using

MIXERS

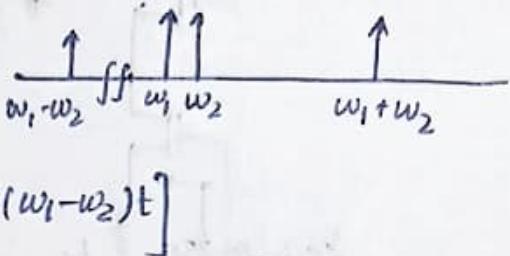
$$x_1(t) = A_1 \cos \omega_1 t$$

$$x_2(t) = A_2 \cos \omega_2 t$$

$$x_{\text{out}}(t) = x_1(t) \times x_2(t)$$

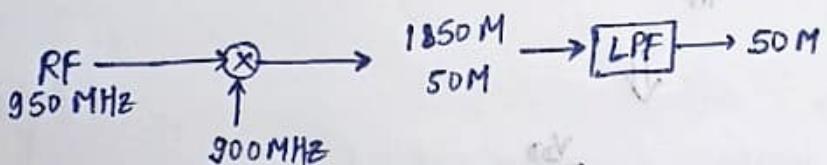
$$= A_1 A_2 \cos \omega_1 t \cdot \cos \omega_2 t$$

$$= \frac{A_1 A_2}{2} \left[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t \right]$$

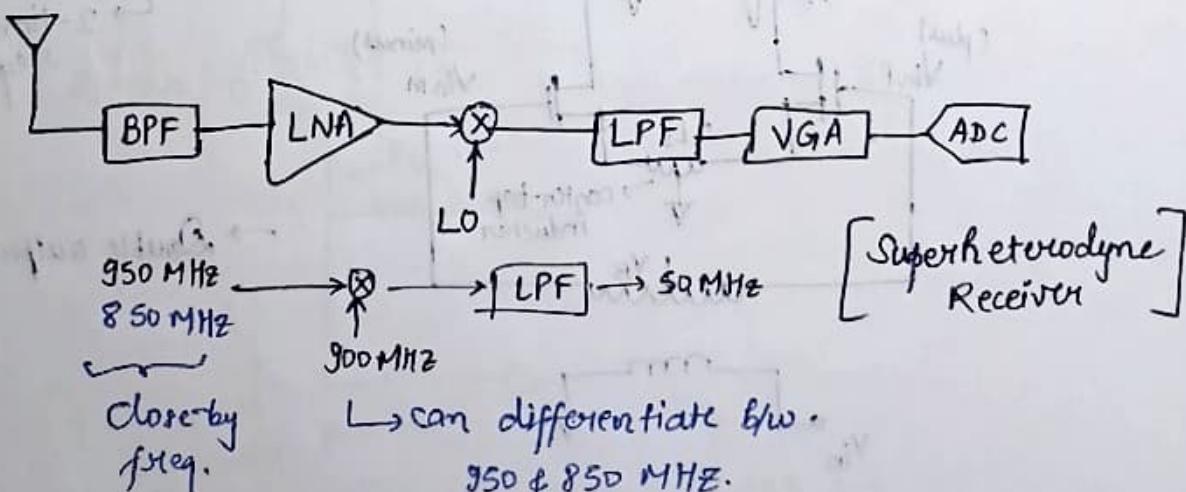


$$\omega_1 = \text{IF} \approx 20 \text{ MHz}$$

$$\omega_2 = 900 \text{ MHz}$$



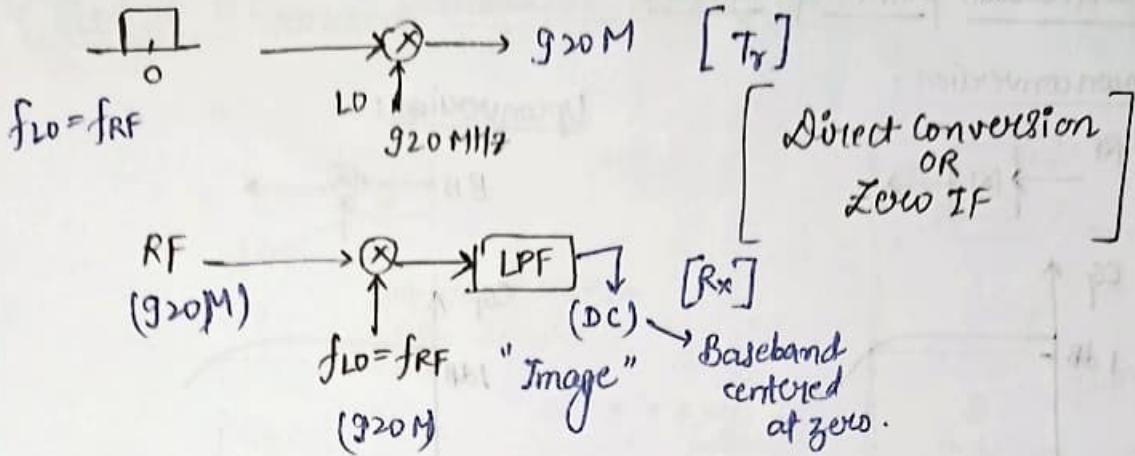
→ Digitizing high frequency (RF) requires high ADC performance and thus high power, affecting resolution, dynamic range.
 ↳ Downconvert to baseband.



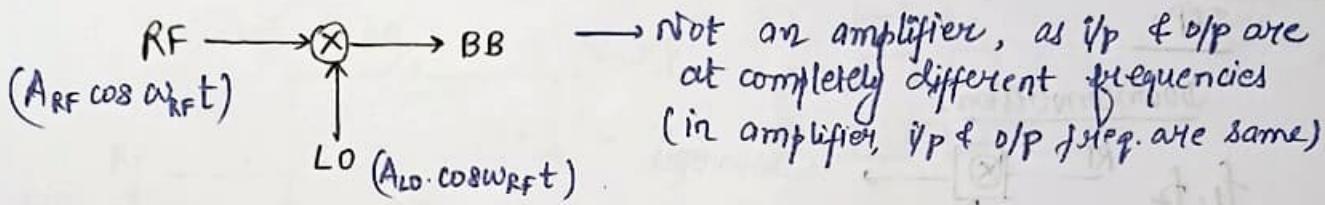
↳ can differentiate B/W.
 950 & 850 MHz.

↳ Produce image frequency.

↳ Use image reject filter.



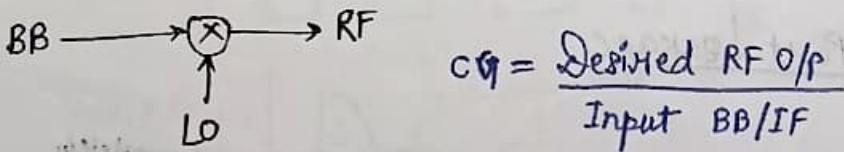
Down Conversion Mixer



$$\text{Conversion gain} = \frac{\text{Desired BB/IF o/p}}{\text{Input RF}}$$

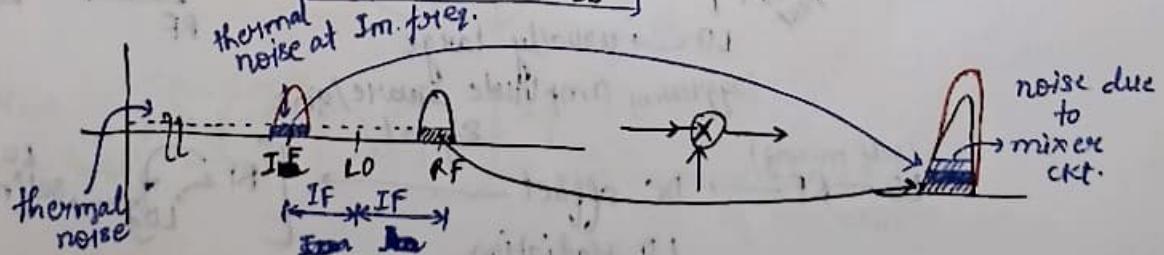
$$= \frac{A_{RF} \cdot A_{LO}/2}{A_{RF}} : \quad (\text{o/p} : \frac{A_{RF} A_{LO}}{2} [\cos(\theta)t + \cos(2\omega_{RFT}t)])$$

$$= \boxed{\frac{A_{LO}}{2}}$$



Downconversion :

$$NF = \frac{SNR_{RF}}{SNR_{BB}}$$



DSB NF (Double sideband NF) → double signal power

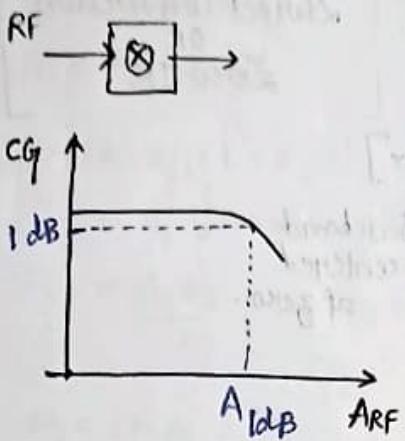
$\rightarrow DSB NF < SSB NF \text{ by } 3dB$

NF → SSB NF (Single sideband NF)

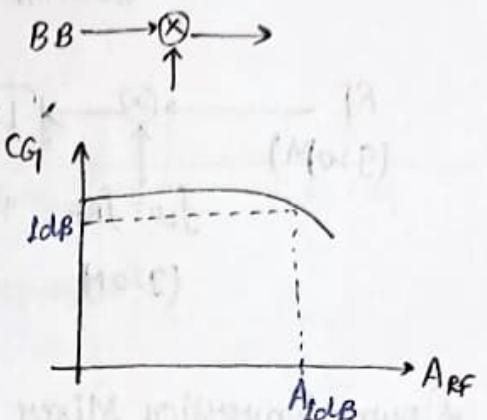
↪ Don't take signal at JF.

1-dB conversion point of mixer:

Downconversion:

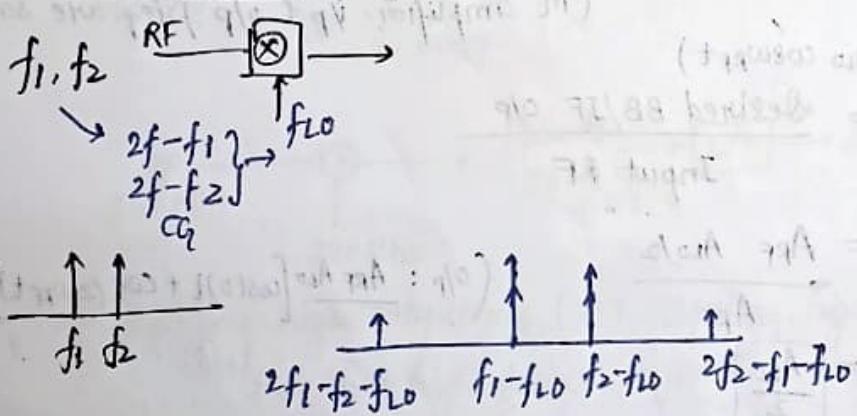


Upconversion:

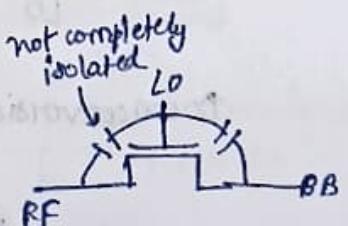
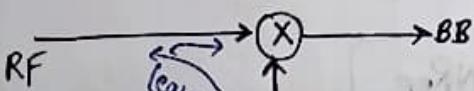


IIP₃

Downconversion:



Feed Through / Port Leakage



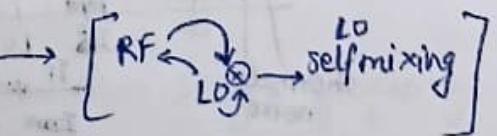
LO → usually large
(aggressive) amplitude square/sinc

(self mixing)

DC offset

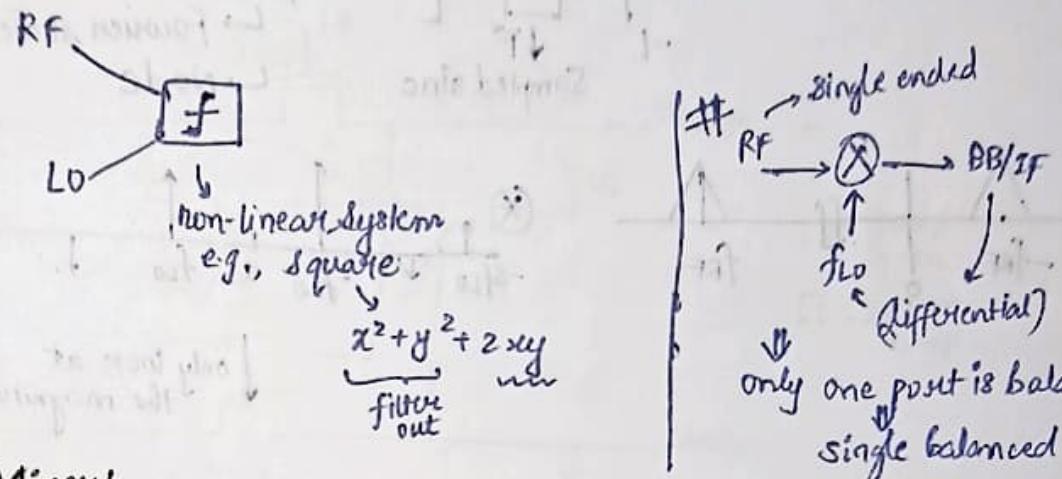
LO radiation

Signal

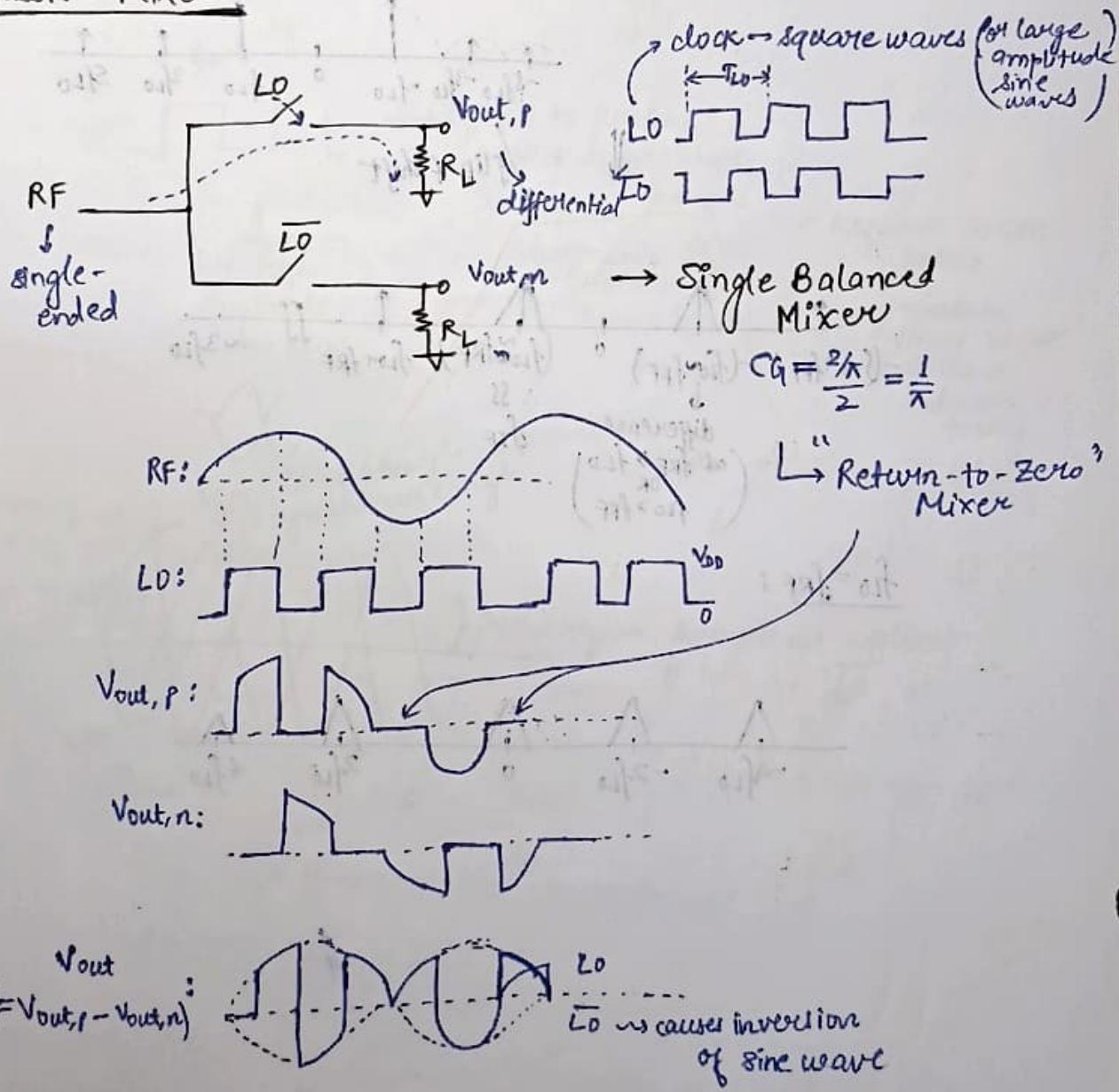


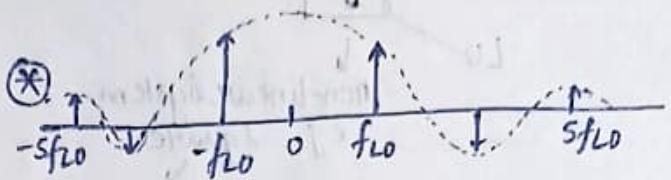
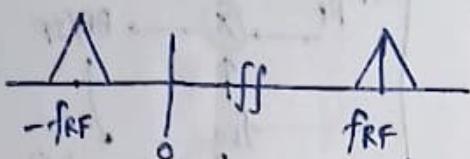
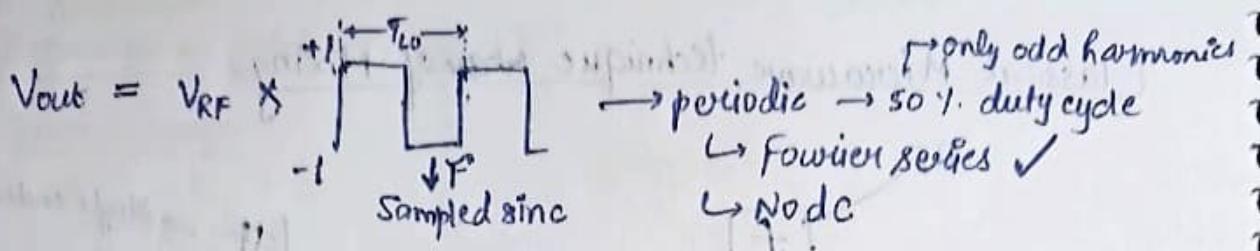
Oscillator pulling → strong RF pull LO

Classical Microwave technique way of Mixing:

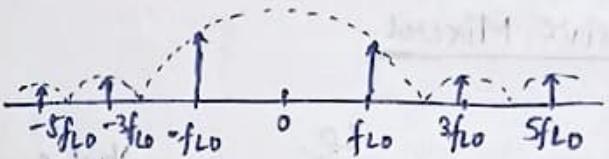


Passive Mixers

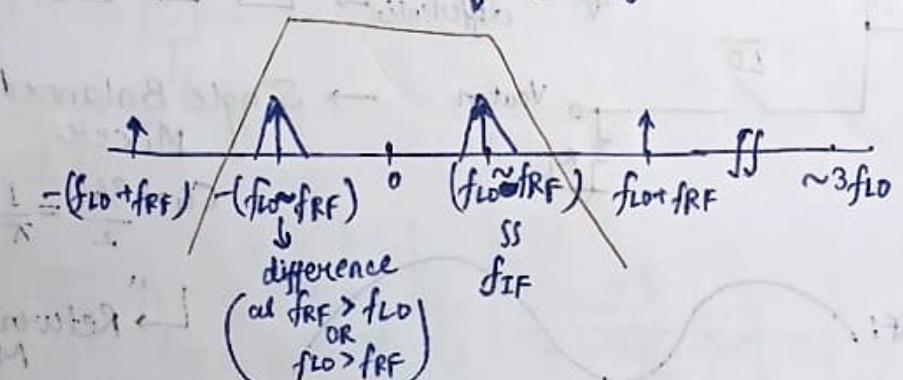




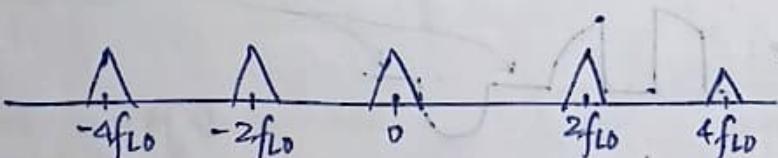
↓ only look at the magnitude (not sign)



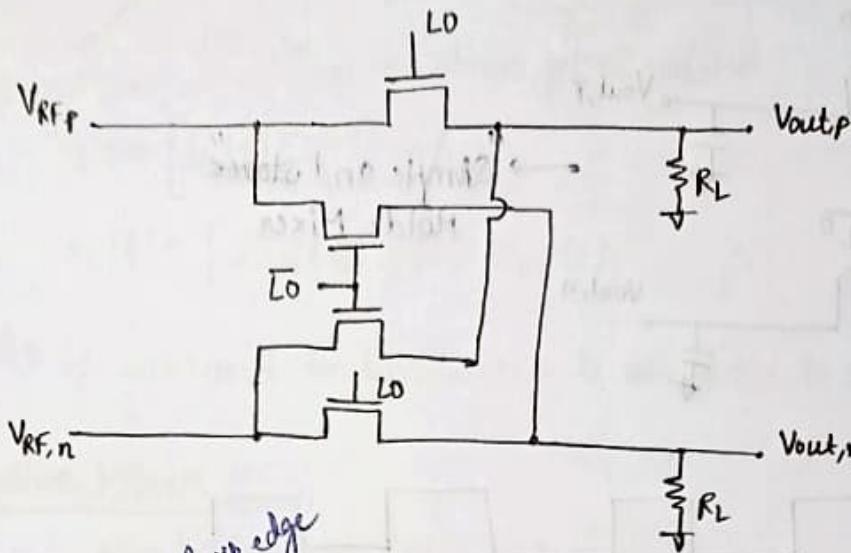
✓ flip & shift



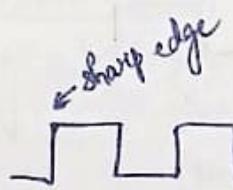
$$f_L0 = f_{RF} :$$



Double Balanced Passive Mixer



LO: $P \rightarrow P$
 $n \rightarrow n$
LO: $P \rightarrow n$
 $n \rightarrow P$

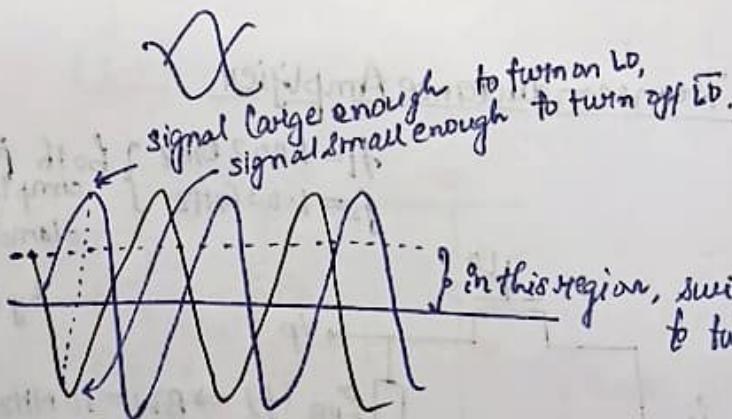


→ difficult to produce perfect square wave.

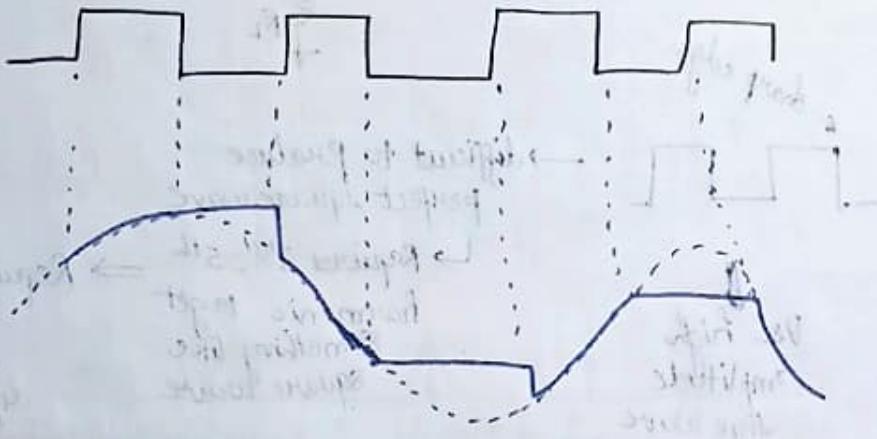
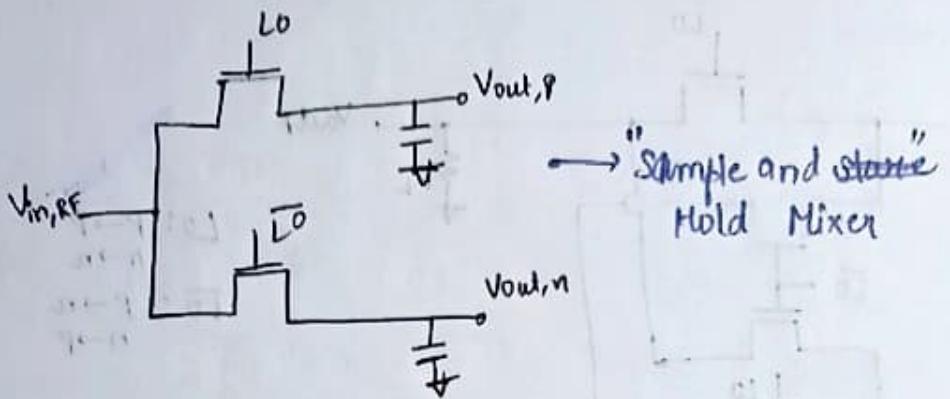
↳ Requires 3rd, 5th harmonic to get something like square wave

→ Required 30 GHz, 50 GHz
↳ difficult to get without inductive tuning

↓
Use high amplitude sine wave



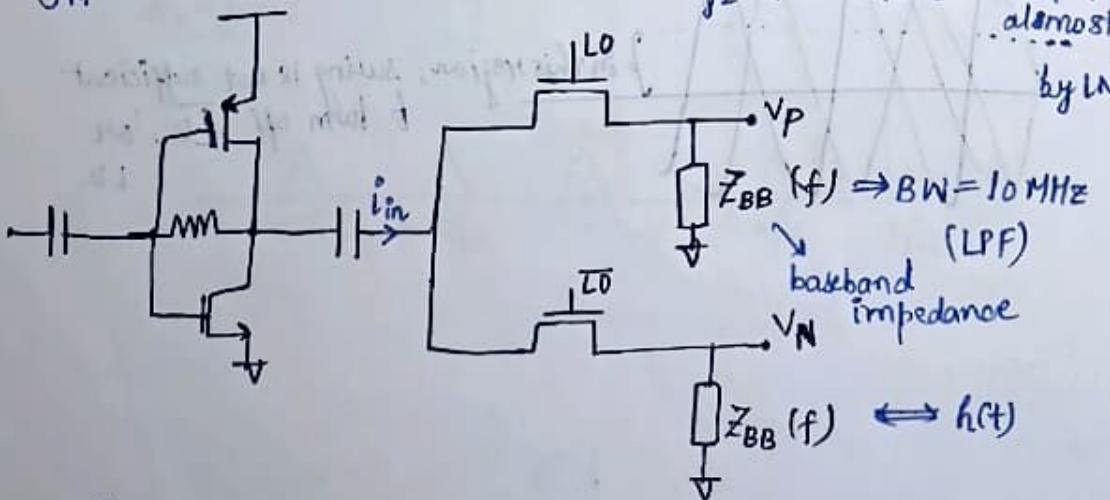
Passive Mixers: Sampling Mixer



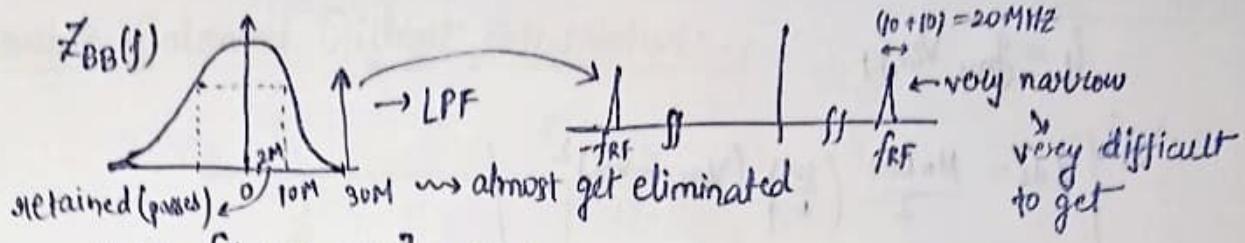
LNTA: Low Noise Transconductance Amplifier

$$f_{RF} = 1 \text{ GHz}$$

$f_1 = 1.002 \text{ GHz}$ } Both passed &
 $f_2 = 1.03 \text{ GHz}$ } amplified by
 almost same
 by LNTA



$$\begin{matrix} +1 \\ -1 \end{matrix} \Rightarrow s(t)$$

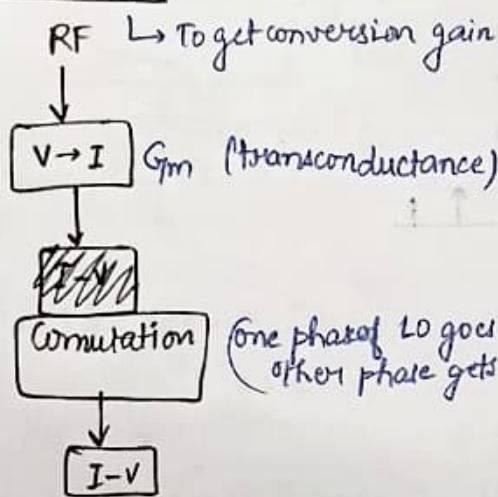


$$V_1(t) = [i_{in}(t) \otimes s(t)] \otimes h(t)$$

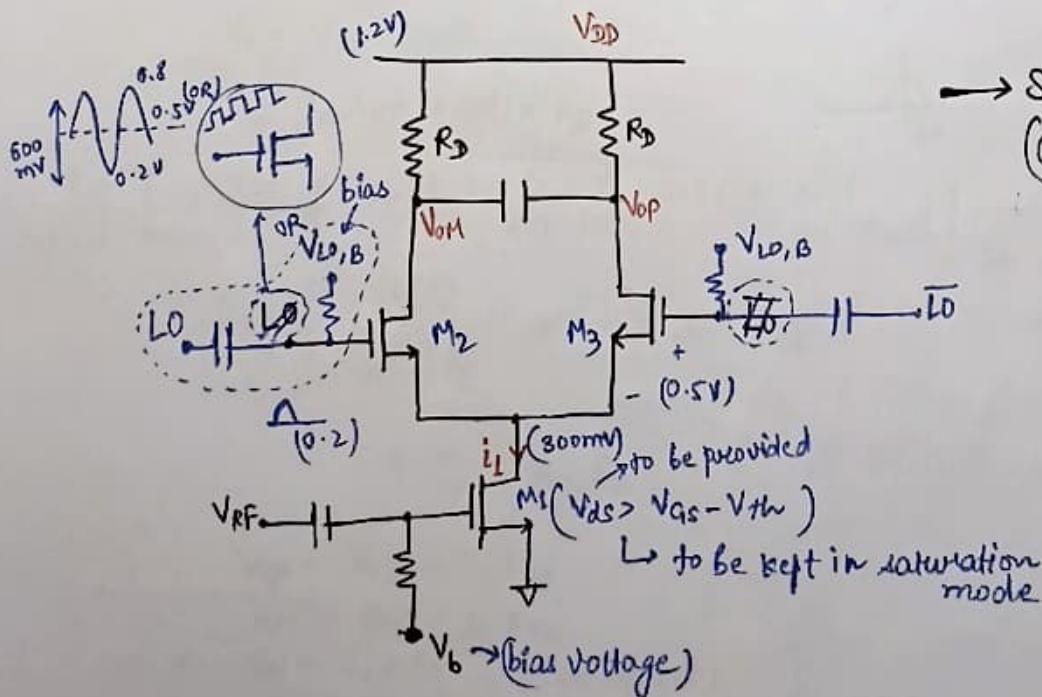
$$V_1(f) = [I_{in}(f) \otimes s(f)] Z_{BB}(f).$$

→ As if baseband bandwidth is converted to RF.

Active Mixer



(one phase of LO goes other, other phase gets inverted) is like taking passive filter, rotating it and fitting it in



$$i_d = g_m V_{in,RF}$$

$$\left[I_{d1} = \frac{M_n K_{ox}}{2} \left(\frac{W}{L} \right) (V_{ds1} - V_{th})^2 \right]$$

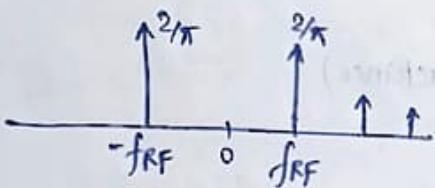
$$V_{op} = V_{OM} = V_{DD} - \frac{I_{d1}}{2} R_D$$

$$v_o = i \times S(t) R_D$$

$$= g_m V_{in,RF} R_D S(t)$$

$$= g_m V_{in,RF} R_D \cdot \frac{4}{\pi} \cos \omega_{RF} t$$

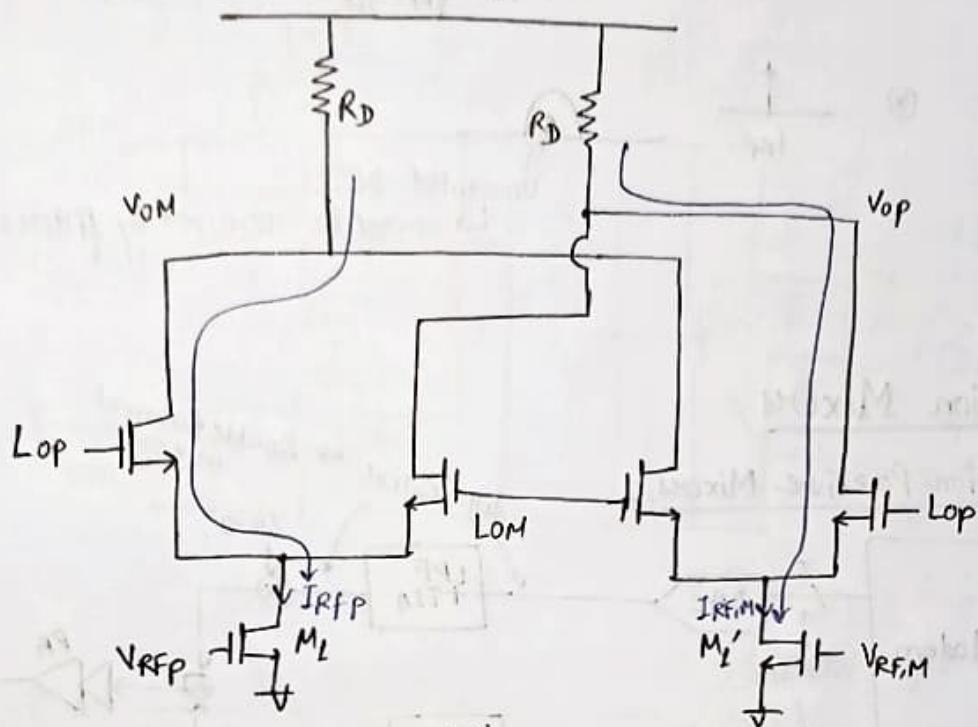
$$S(t) = \begin{cases} +1 & \\ -1 & \end{cases}$$



$$V_{in,RF} = A \cos \omega_{RF} t$$

$$\therefore CG = \frac{2}{\pi} g_m R_D$$

Double Balanced Gilbert Cell Mixer



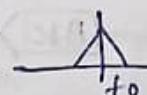
$$I_o + g_m V_{RFP} = I_{RFP}$$

$$V_{RF} = V_{RFP} - V_{RFM}$$

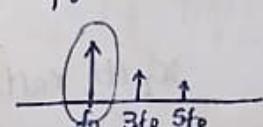
$$I_{RF} = I_{RFP} - I_{RFM} = g_m (V_{RF})$$

$$V_o = V_{op} - V_{oM}$$

$$= g_m V_{RF} \times S(t) \times R_D$$



$$= \frac{4}{\pi} \omega_0 \omega_0 t + \sum_{n=2}^{\infty} \frac{4}{12\pi} \cos(2n-1)\omega_0 t$$



(LPF)

$$= g_m V_{RF} \frac{2}{\pi} R_D$$

$$\therefore C.G. = \frac{2}{\pi} g_m R_D$$

$$V_{op} = V_{DD} - I_o R_D,$$

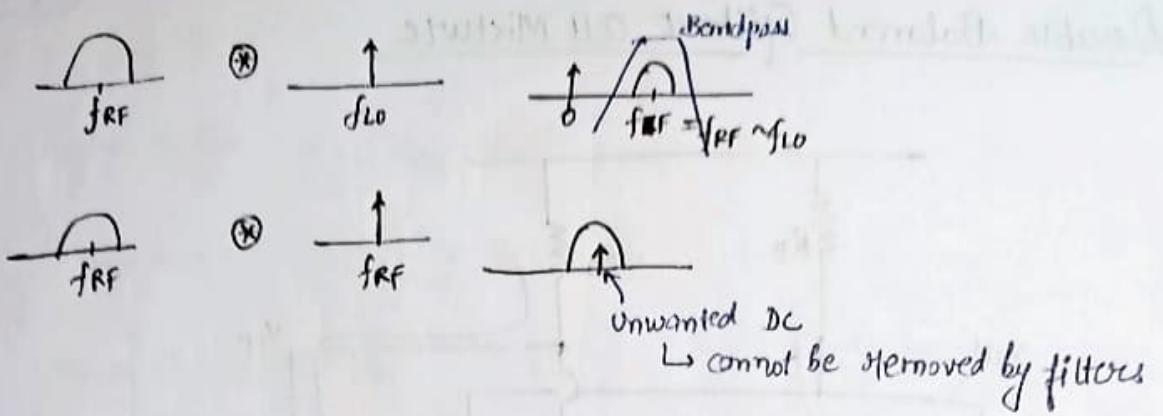
$$V_{oM} = V_{DD} - I_o R_{D2}$$

$$V_o = I_o \Delta R_D. \quad (\text{DC offset})$$

due to mismatch

B/W 2 R_D's

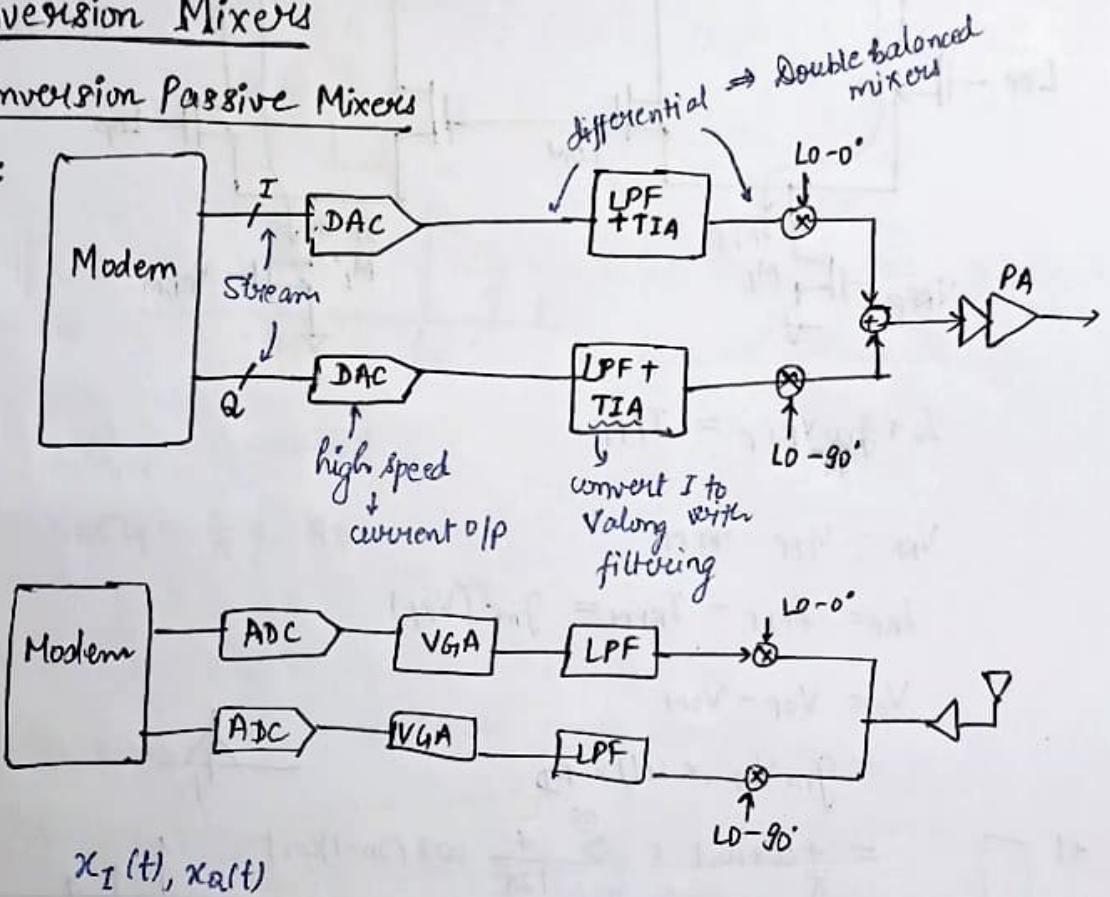
(2 g_m's, 2 I_o's)



Upconversion Mixers

Upconversion Passive Mixers

System:



$$V_{RF,out}(t) = X_I(t) \cos \omega_{Lo} t + j X_Q(t) \sin \omega_{Lo} t$$

$\cos \omega_{Lo} t :$

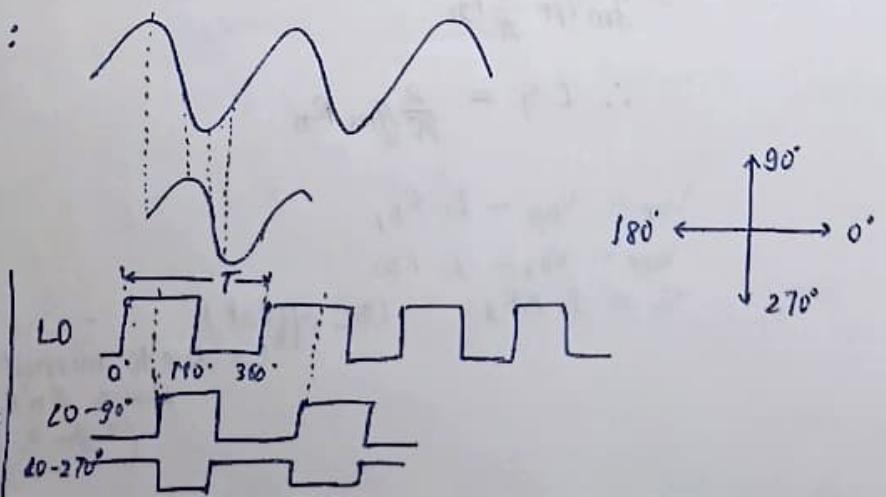
($\cos \theta$)

$$\theta = \frac{t}{T} \times 360^\circ$$

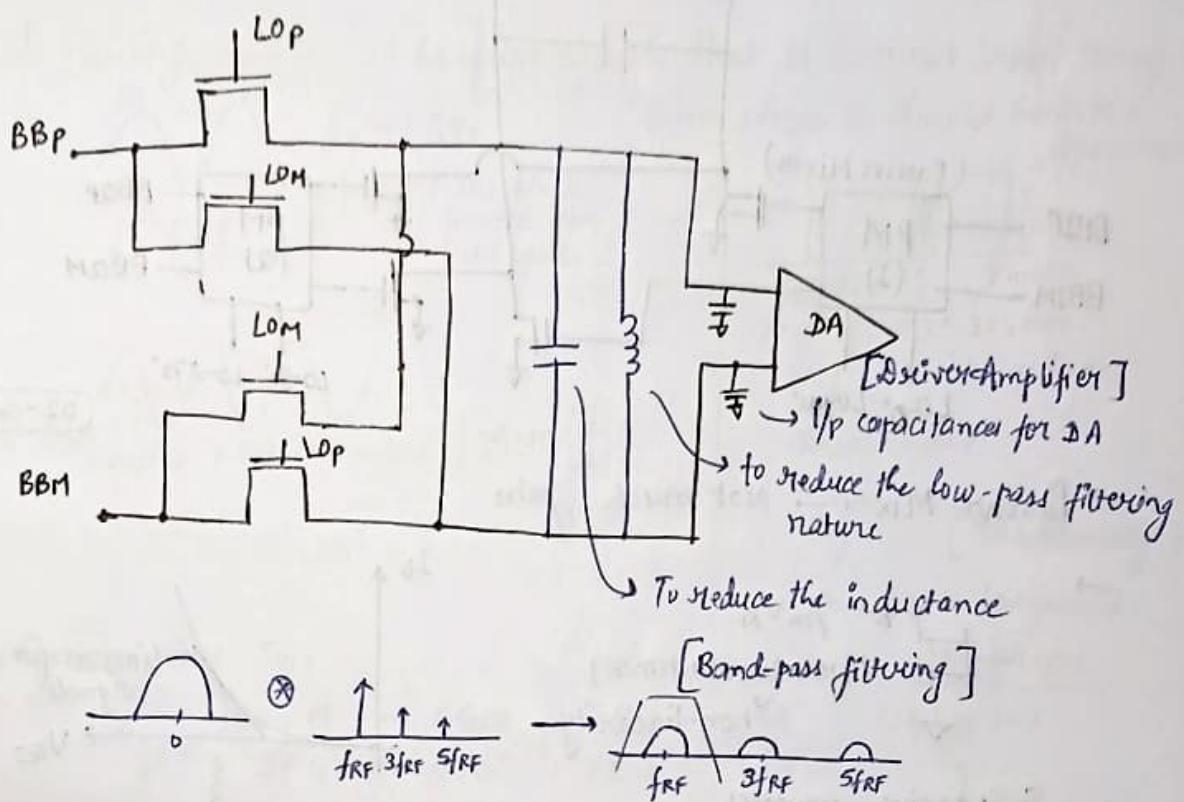
or

$$\frac{t}{T} \times 2\pi$$

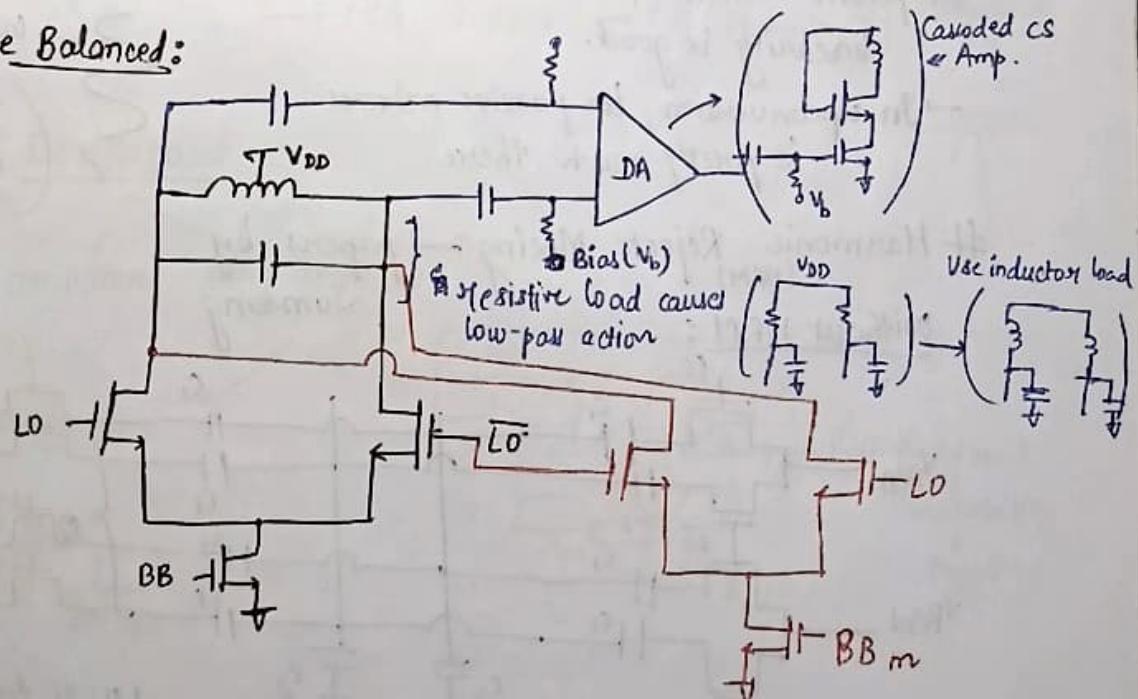
$$90^\circ \Rightarrow t = T/4$$

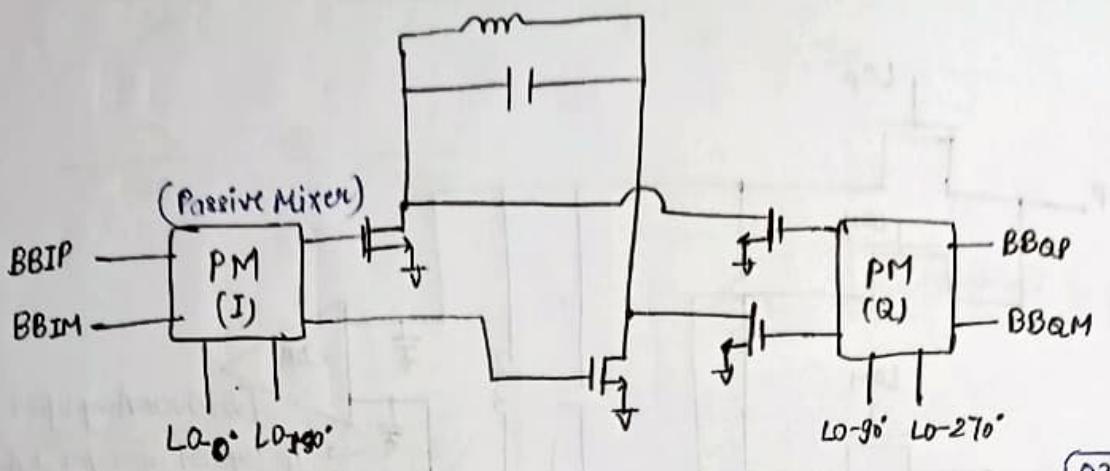


Upconversion Passive Mixer



Single Balanced:





02-04-2025

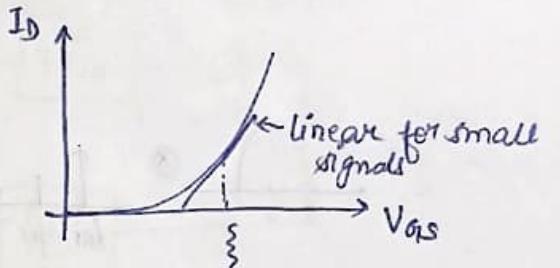
→ Passive Mixer → Not much gain.

→

$$y i_{lo} = g_m v_{rf}$$

(transconductance)

non-linearity



In passive mixers,

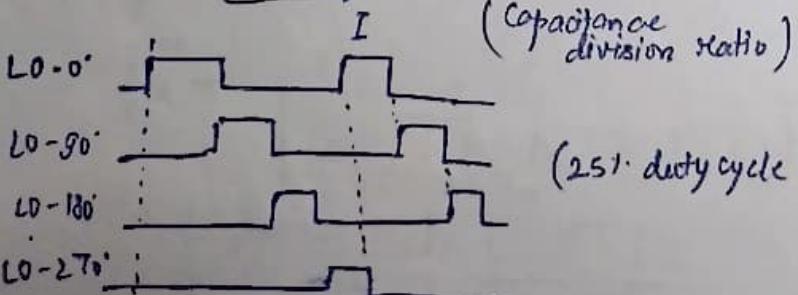
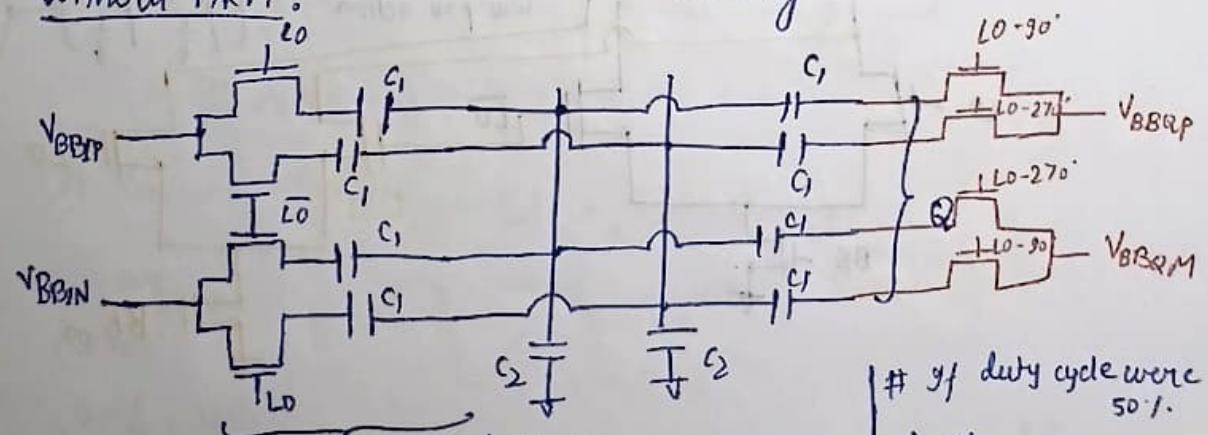
linearity is good.

→ In upconversion, ~~the~~ passive mixer is pretty much there.

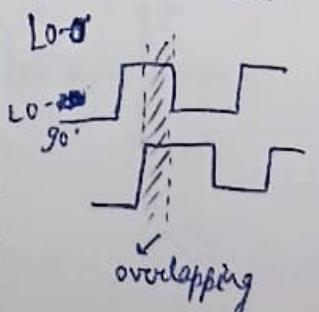
large signal
Not linear
due to inherent
behaviour
of the device

Harmonic Reject Mixing — papers by Samsung

Without HRM :

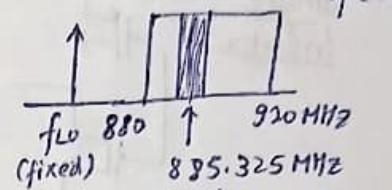
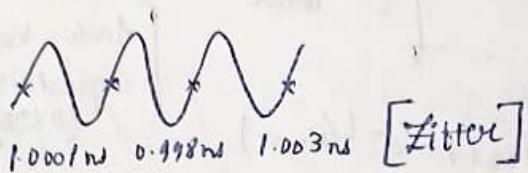
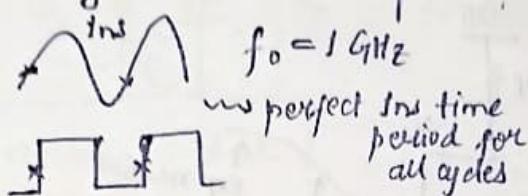


if duty cycle were 50%.



Synthesizer

① Purity → should be pure enough that it does not drift away from where it should be in the spectrum



↓ Downconvert

↓ Digitize [Traditionally]

↓ NO long used

↓ Freq. Mixers

Commercial SGS:

560 M → 6 GHz : FR 1 → 4G {square LO}

2f G → 40 G : FR 2 → 5G

7G → 15G : FR 3 → 6G (Upcoming) } Sinusoid LO

↓ Large

(RF) Oscillators

LC oscillators Ring oscillators

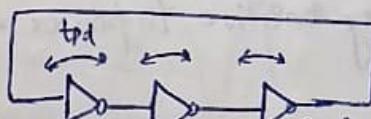
N stage RO :

$$2N T_{pd} = T$$

$$\Rightarrow f_o = \frac{1}{2N T_{pd}}$$

Even-no.-stage oscillator

↳ Possible with differential amp.



0:

t_pd:

2t_pd:

3t_pd:

invertors (odd no.)

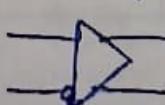
instability

↓ toggling

RO

→ Tiny

→ All n-stages available.

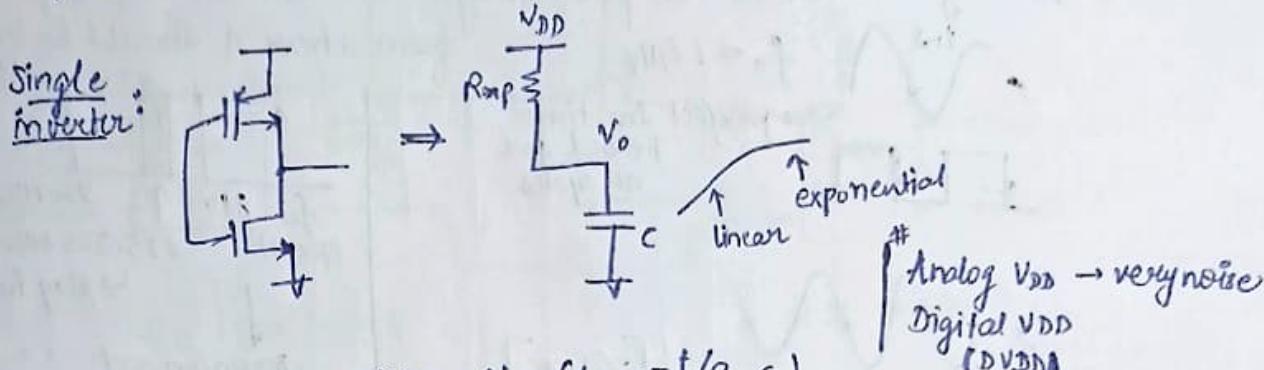


→ Direct square wave o/p

[Very Attractive]

$R_O \rightarrow$ still not used in RF, due to:

- ① Zitter noise \uparrow
- ② Supply sensitive



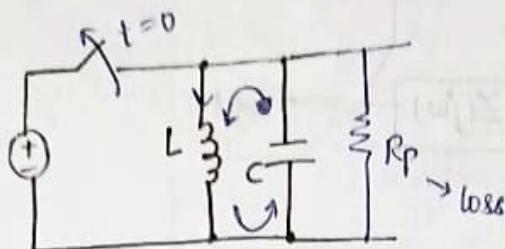
$V_{DD_2} > V_{DD_1}$ If V_{DD} changes $\Rightarrow t_{pd}$ changes

- Power supply induced Zitter (\hookrightarrow very small) \rightarrow Bad phase noise in freq. domain
 \hookrightarrow Due to noise in supply [Fluctuable Zitter]
 \downarrow
changes t_{pd}
 \downarrow
changes f_0

Dynamic power: CV^2f [Dynamic voltage & frequency scaling]
 \downarrow \downarrow \hookrightarrow to reduce power
(Reduce voltage) (Reduce freq.)

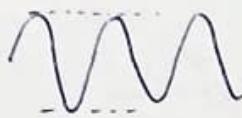
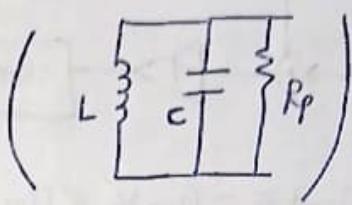
$\rightarrow R_O \rightarrow$ very sensitive to power supply.

LC Oscillation



(L charges C and C charges L)

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$



→ same o/p. amplitude
for no loss



→ decays if there is a loss R_p
(decay ↑ for $R_p \uparrow$)

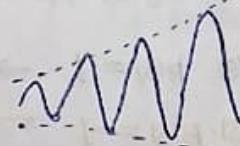
$$Q = \frac{R_p}{\omega_0 L}$$

$Q \uparrow, R_p \uparrow \rightarrow$ underdamped,
prolonged ringing

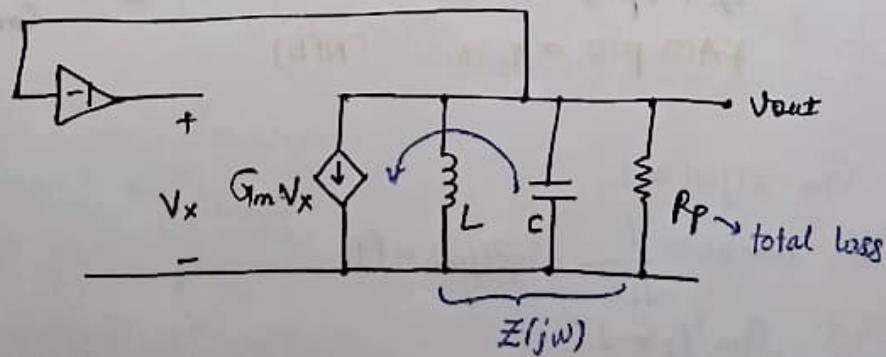
$Q \downarrow, R_p \downarrow \rightarrow$ quick decay

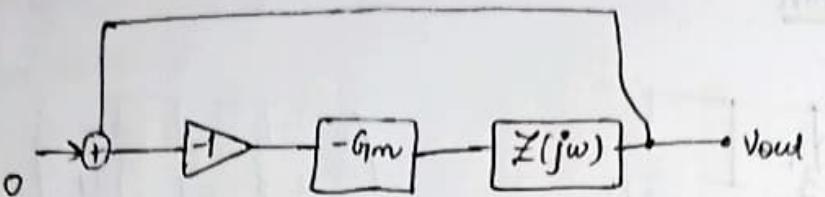
Sustained oscillation → if energy feed to the system to compensate
for the loss

If more energy
is given
↓
overdamped



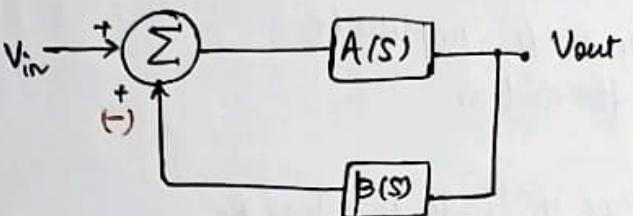
Feed energy → using transistor → +ve feedback





$$V_{out} = -G_m V_x Z(j\omega)$$

$$\text{Loop gain} = G_m \cdot Z(j\omega)$$



$$V_{out} = [V_{in} + \beta(s) \cdot V_{out}(s)] A(s)$$

$$V_{out}(s) [1 - A(s) \beta(s)] = V_{in} \cdot A(s)$$

$$\frac{V_{out}(s)}{V_{in}} = \frac{A(s)}{1 - A(s) \beta(s)} \quad (\text{Positive feedback})$$

$$\frac{V_{out}(s)}{V_{in}} = \frac{A(s)}{1 + A(s) \beta(s)} \quad (\text{Negative feedback})$$

For the system to become oscillator,

$$D^r = 0 \Rightarrow TF \rightarrow \infty$$

$$A(s) \cdot \beta(s) = 1 \rightarrow (\text{PFB})$$

$$A(s) \cdot \beta(s) = -1 \rightarrow \text{NFB}$$

$$|A(s) \beta(s)| = 1$$

$$\cancel{A(s) \beta(s) = 0, 2\pi, \dots} \quad (\text{PFB}) \quad : \text{"Barkhausen's Condition"}$$

$$\cancel{A(s) \beta(s) = \pi, 3\pi, \dots} \quad (\text{NFB})$$

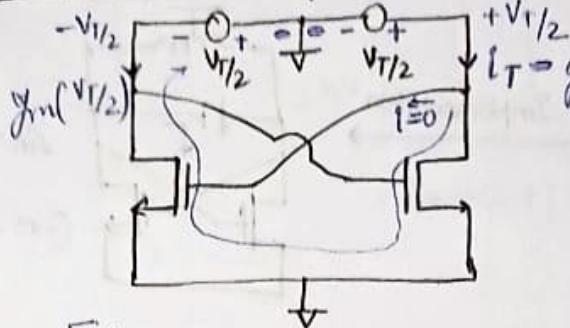
$$G_m \cdot Z(j\omega) = 1$$

$$\text{At } \omega = \omega_0 = \frac{1}{\sqrt{LC}}, \quad Z(j\omega) = R_p$$

$$G_m \cdot P_p = 1$$

$$\Rightarrow \boxed{G_m = \frac{1}{R_p}} \rightarrow \text{More energy requirement for more loss in the system}$$

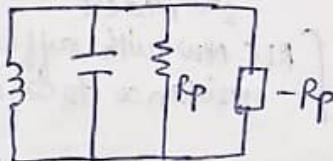
Differential Crossover-Coupled Pair:



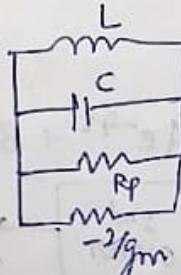
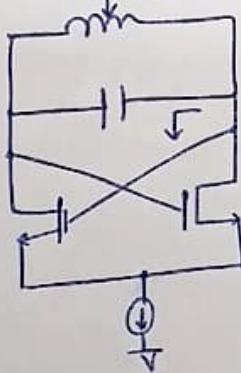
$I_T = g_m(-V_{1/2}) \rightarrow$ D f G_T are cross-connected

[Assumption: Transistors are already biased]

$$Z_{in} = \frac{V_T}{g_m V_{T/2}} = -\frac{2}{g_m} \quad (\text{negative impedance})$$

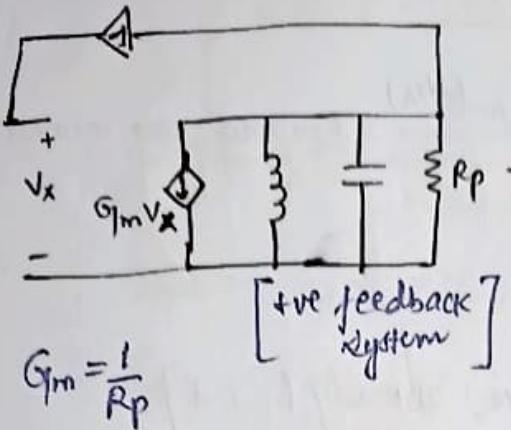


Actual implementation:

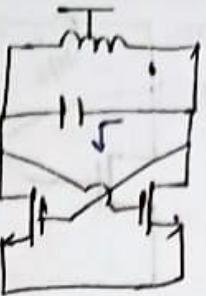


[If R_p & $-1/g_m$ cancel each other perfectly, we get L, C oscillation]

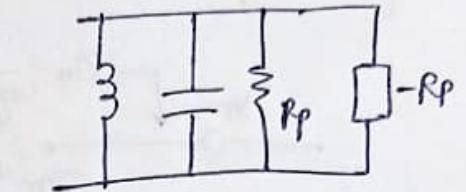
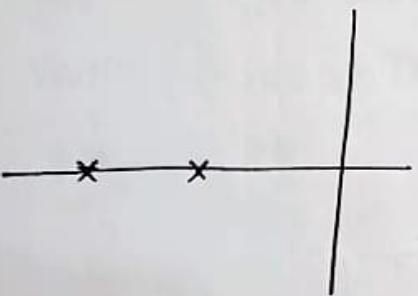
↳ cancels the loss of the system



Implementation:

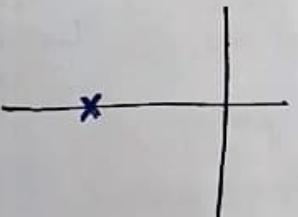
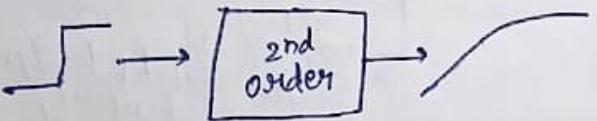
 $G_m R_p < 1$: Dying oscillation $G_m R_p > 1$: Exponentially increasing oscillation

$$G_m Z_m(j\omega) = 1$$



2nd order \rightarrow 2 poles
 [RLC NW with sufficiently large -ve resistance to cancel the loss]

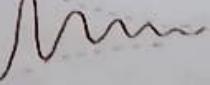
\rightarrow 2 poles on (Real)
 LHP of the Z-plane

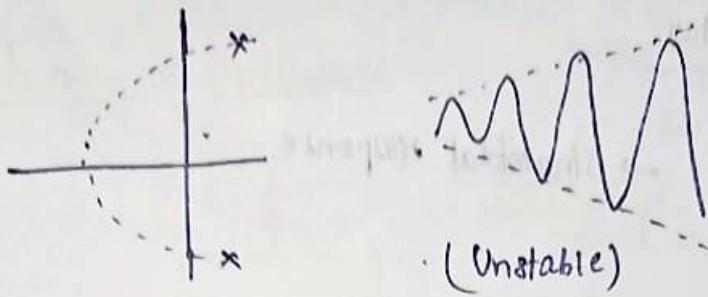
time-domain soln: $A e^{-\alpha t} + B e^{-j\omega t}$  \rightarrow complex conjugate polesAs poles move towards jω axis
poles: $-\alpha \pm j\omega$

$$\text{Soln: } e^{(-\alpha \pm j\omega)t} \Rightarrow \alpha \downarrow$$

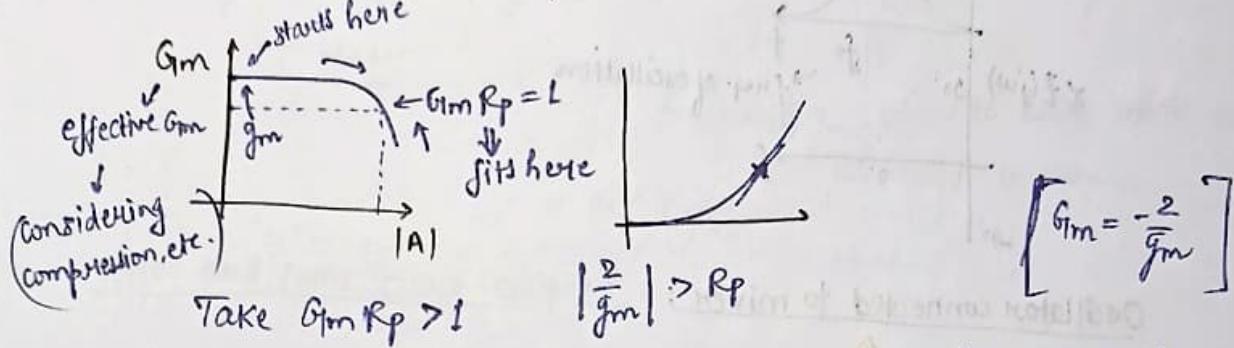
slow dying

peaky response





Undesirable properties of the device → Non-linearity



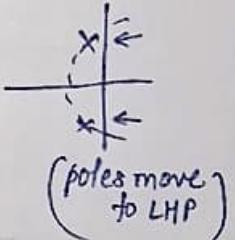
$$\text{Take } G_m R_p > 1 \quad \left| \frac{2}{g_m} \right| > R_p$$

↳ Oscillator → infinite response (no i/p required)

↳ Starts with noise in the device

↳ Poles on RHP.

(Temp. variation \Rightarrow) $G_m \downarrow \Rightarrow R_p \downarrow \Rightarrow G_m R_p \text{ becomes } < 1$
 \Rightarrow Signal amplitude \downarrow



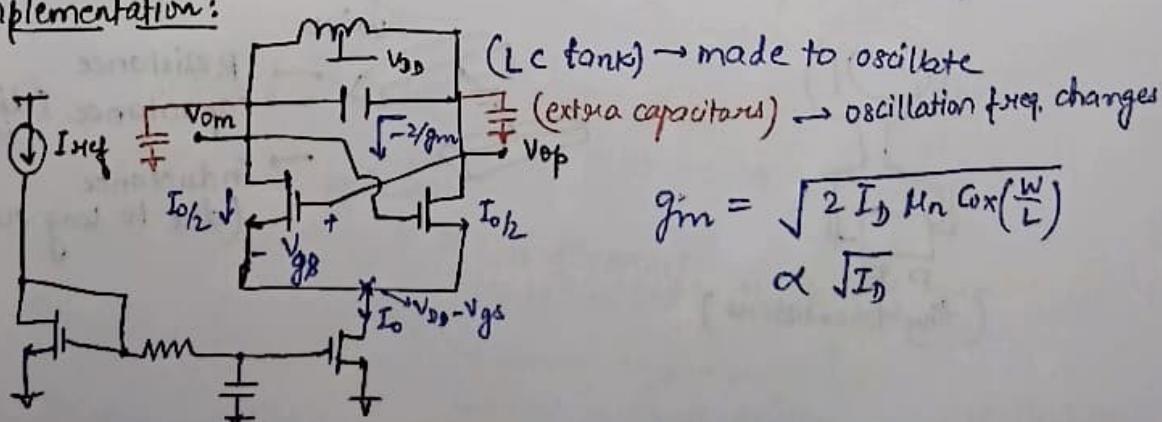
~~~~~ (oscillation  $\downarrow$ )

$G_m$  tries to go to the linear region.

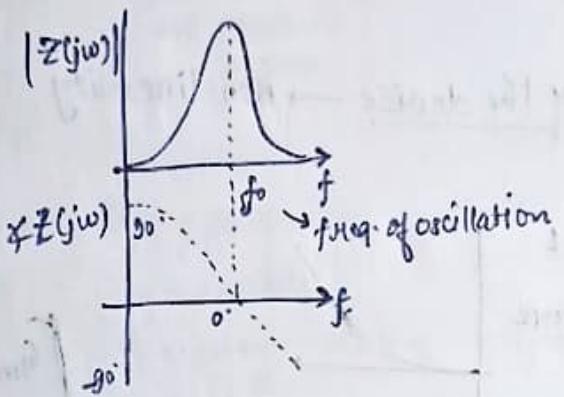
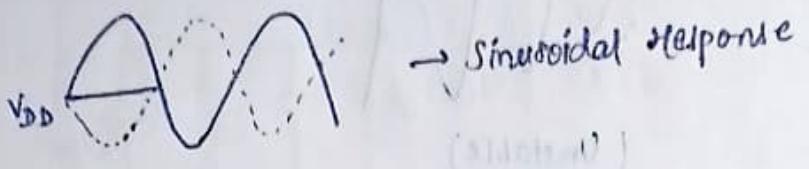
↳ fits to  $G_m R_p = 1$  (due to feedback)

$$\left| \frac{2}{g_m} \right| \times R_p > 1 \quad (\text{to start the oscillations})$$

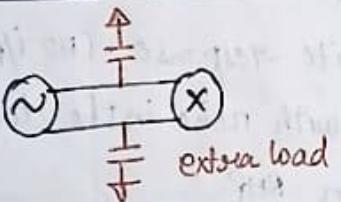
Implementation:



## Diff. Cross-coupled Oscillator



Oscillator connected to mixer:



freq. shifts are to load

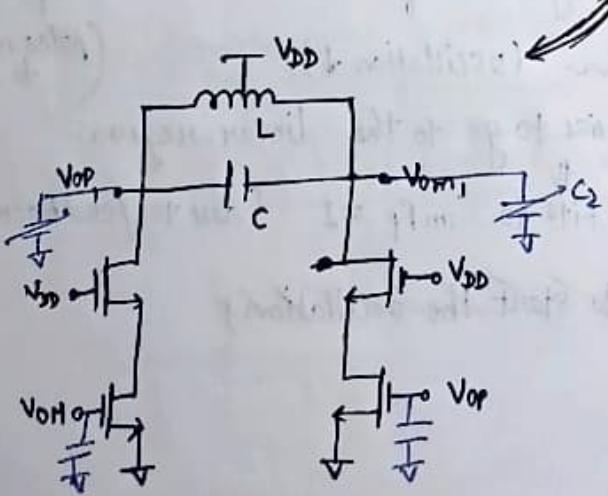
→ Never connect oscillator to any load directly

↓  
Add buffer

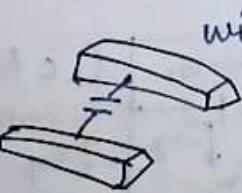
Always design oscillator with a buffer

(use cascading)

↳ can connect to anything.



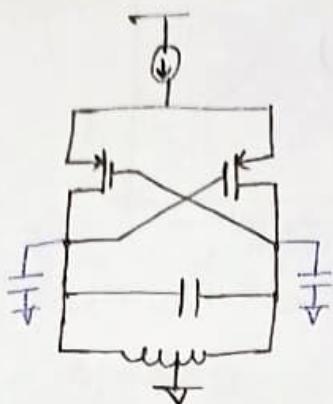
(tiny transistors)



wire

- Resistance
- capacitance (dielectric in b/w)
- Inductance (due to long routing)

## Tuning of Oscillators



→ For PMOS:

$$n_p < n_e \text{ (mobility)}$$

Required larger width ↑

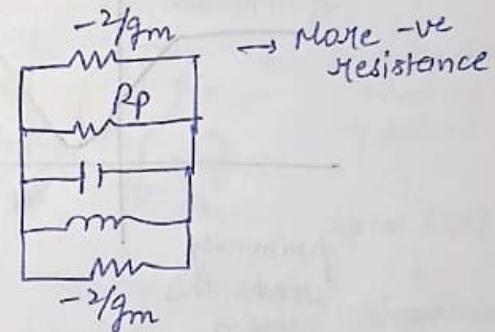
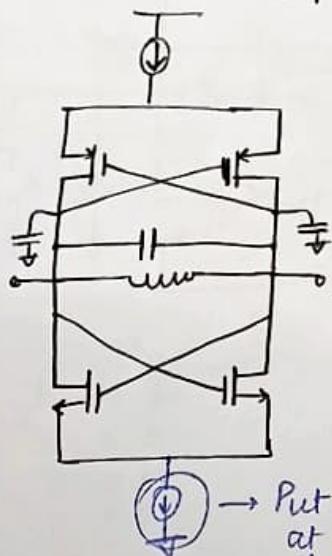
$$g_m = \mu C_{ox} \left( \frac{W}{L} \right) (V_{ds} - V_{th})$$

For same  $g_m$ , needs to burn more current.

or ~~XX~~

↳ More capacitance

### NMOS and PMOS cross-coupled:



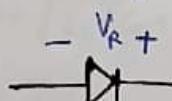
→ More -ve resistance

→ Put either  
at top or at  
bottom

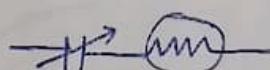
### Digital/Electronic Tuning: (Not manual)

↳ Look for voltage / bits - controller capacitance.

Varactor:



↳ R.B. diode

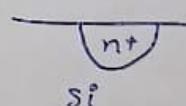


↳ Loss of varactor  
gets added to the loss of tank

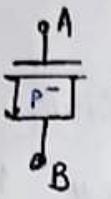
↳ Power consumption ↑

↳ Not a good solution → Use MOS cap.

Diode:



## MOS Capacitor (or MOS Varactor):



→ MOS (short S and D)

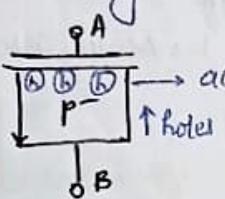
To turn ON,

$$V_{AB} > V_{TH}$$

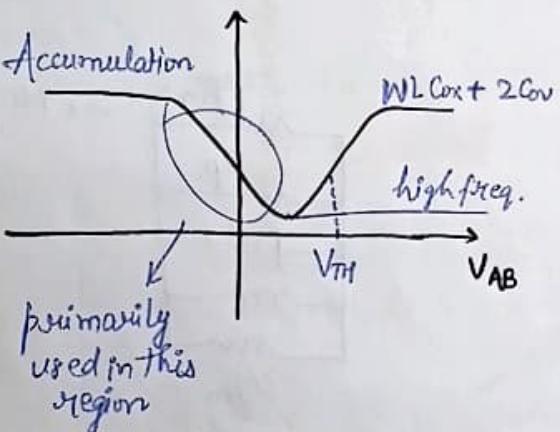
$$WL\ Cox + 2\ Cox$$



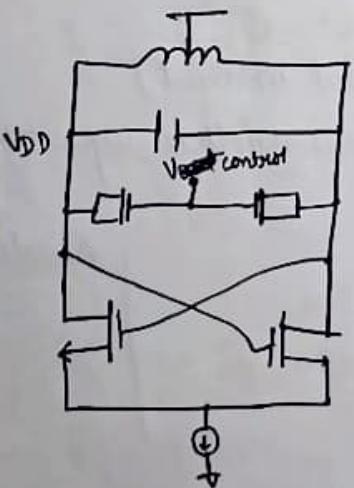
When +ve voltage is applied,

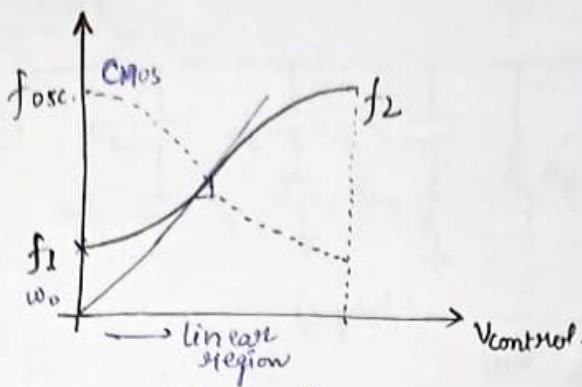


→ accumulation → same cap.



## VCO (Voltage - Controlled Oscillator)

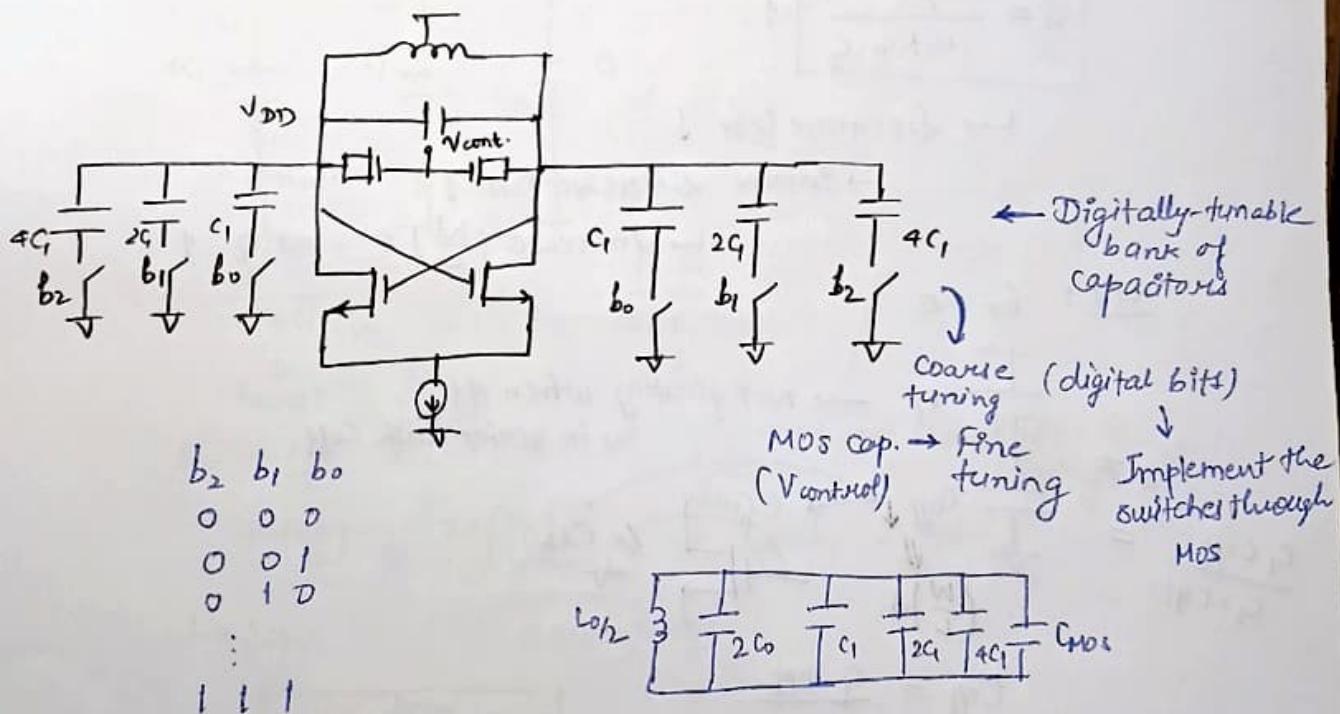




$$\omega_{\text{out}} = \omega_0 + K_{VCO} \cdot V_{\text{cont.}}$$

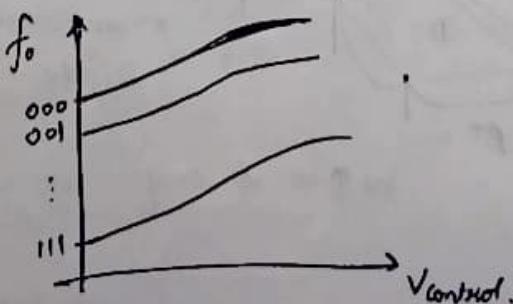
$$\Delta f = f_2 - f_1$$

### Digital Tuning:



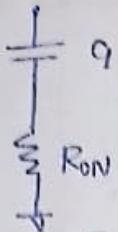
$$@ 000: f_0 = \frac{1}{2\pi \sqrt{L_0 (C_0 + C_{\text{MOS}})}}$$

$$@ 001: f_0 = \frac{1}{2\pi \sqrt{L_0 (C_0 + C_{\text{MOS}} + 2C_1)}}$$

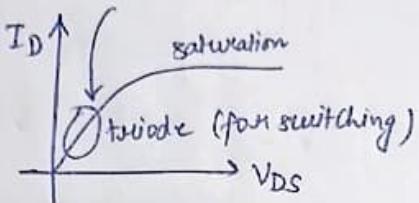


### MOSFET Switch:

ON:  $b_0 = 1$



$$= \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) (V_{DD} - V_{TH})} = (\text{slope})$$



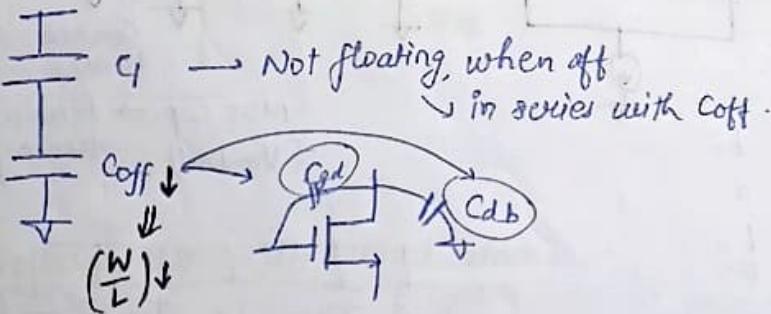
$$Q = \frac{1}{w_0 R_{ON} \cdot g}$$

→ Decrease loss ↓

! → ~~Decrease~~ Decrease  $R_{ON}$  ↓

→ Increase  $\left(\frac{W}{L}\right) \uparrow \Rightarrow Q \uparrow$

OFF:  $b_0 = 0$

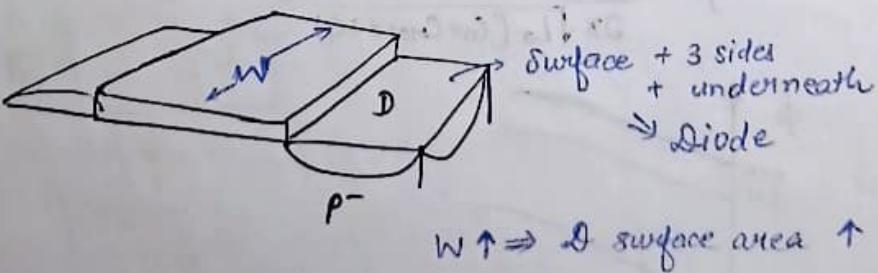


$$\frac{C_i C_{off}}{C_i + C_{off}} = 0 \neq$$

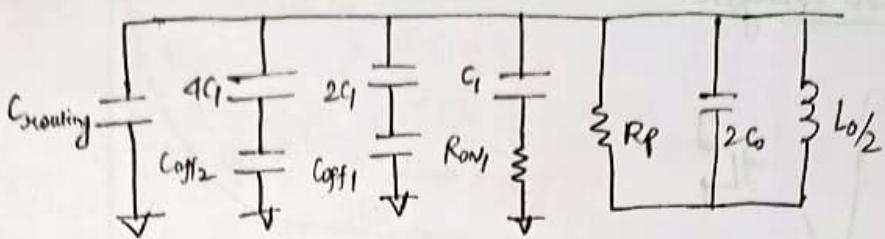
$$C_{off} = \frac{C_{off}}{C_i + C_{off}}$$

$\left(\frac{W}{L}\right) \rightarrow$  needs tradeoff.

08-04-2025

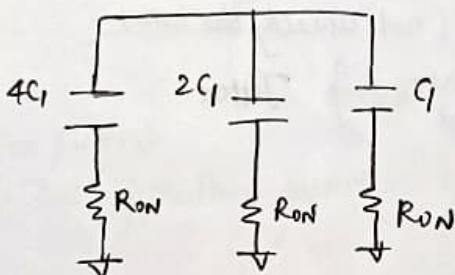


001



$$f_o = \frac{1}{2\pi \sqrt{\frac{L_o}{2} \left[ 2G + Q + \frac{2G}{2G + C_{off1}} + \frac{4G}{4G + C_{off2}} + C_{cooling} \right]}}$$

To increase  $f_o \uparrow \Rightarrow 2G \uparrow$



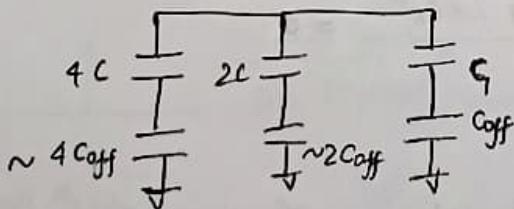
$$Q: \frac{1}{w_o \cdot 4C_1 \cdot R_{on}} \quad \frac{1}{w_o \cdot 2C_1 \cdot R_{on}} \quad \frac{1}{w_o \cdot C_1 \cdot R_{on}}$$

↓ lowest                                                                  ↓ highest

$$R_{on} \rightarrow \frac{R_{on}}{4} \quad R_{on} \rightarrow \frac{R_{on}}{2} \quad \left(\frac{w}{L}\right)_{switch}$$

⇒ Constant-Q  
Sizing

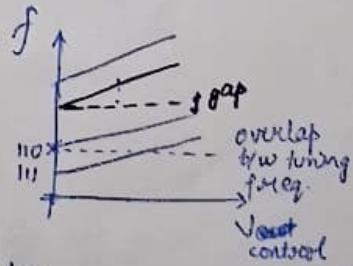
↳ Scale the MOS accordingly.



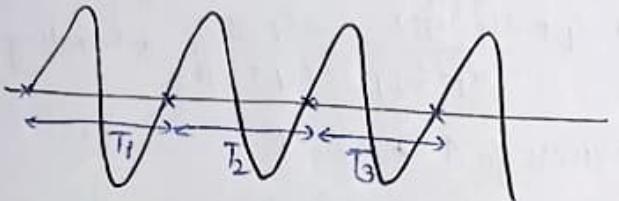
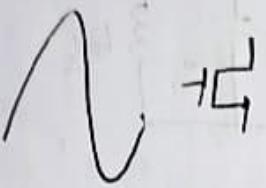
there should not be gap → otherwise that  $f$  cannot be tuned

↳ there should be overlap

↳ Limited tuning range

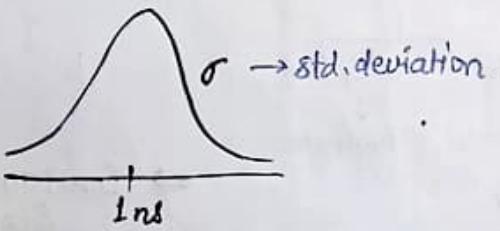
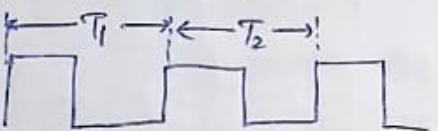


## Phase Noise Analysis

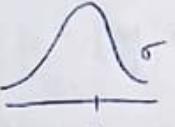


$T_1 = T_2 = T_3 = \dots = T_N = 1\text{ ns}$  (not usually the case)

Something like  $1.0003\text{ ns}$  } Jitter  
 $0.9999993\text{ ns}$



$$T_1 - \mu = \epsilon_1 \\ T_2 - \mu = \epsilon_2 \\ \vdots$$



$$\text{RMS Jitter} = \sqrt{\frac{\epsilon_1^2 + \epsilon_2^2 + \dots + \epsilon_N^2}{N}} = \sigma$$

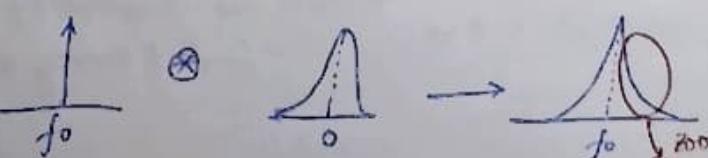
↳ Noise in the phase:

$$A \cos(\omega_0 t + \phi_n(t)) \rightarrow \text{Phase noise}$$

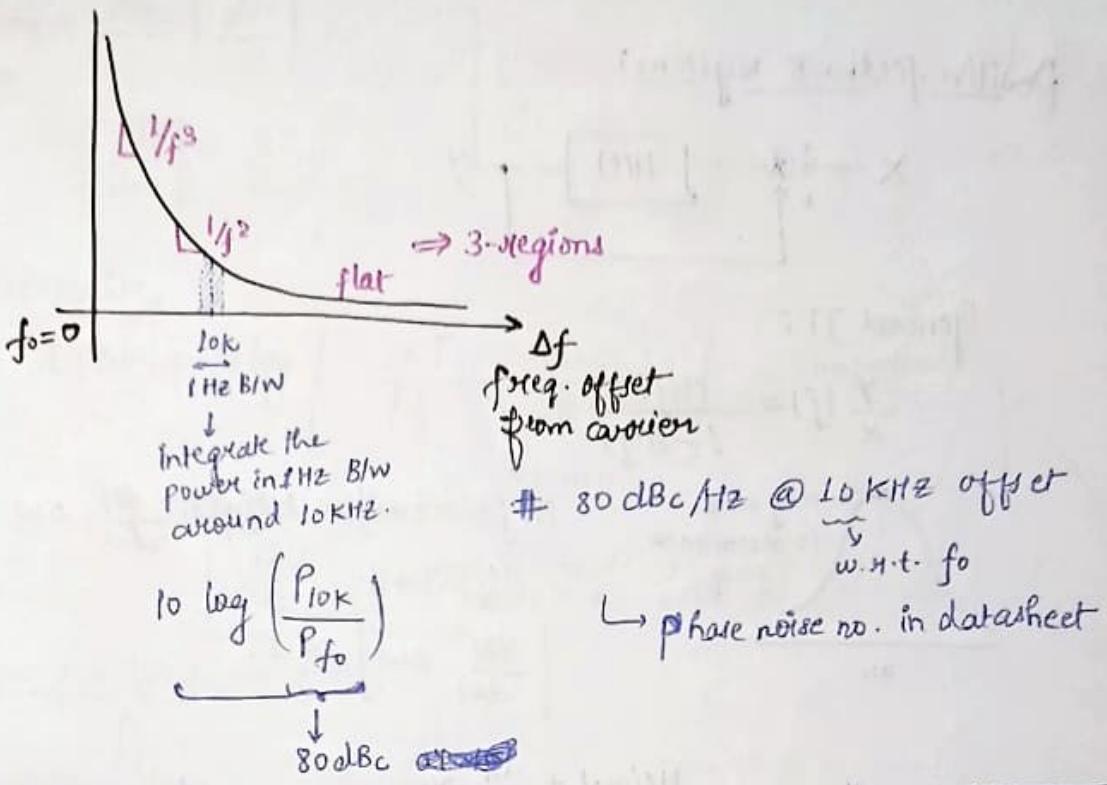
(or Jitter in digital application)

$$= A [\cos \omega_0 t \cdot \cos \phi_n(t) - \sin \omega_0 t \cdot \sin \phi_n(t)]$$

$$\approx A \cos \omega_0 t - A \phi_n(t) \cdot \sin \omega_0 t \quad [\phi_n(t) \ll 1]$$



# Vector Spectrum Analyzer (VSA)  
 To phase noise

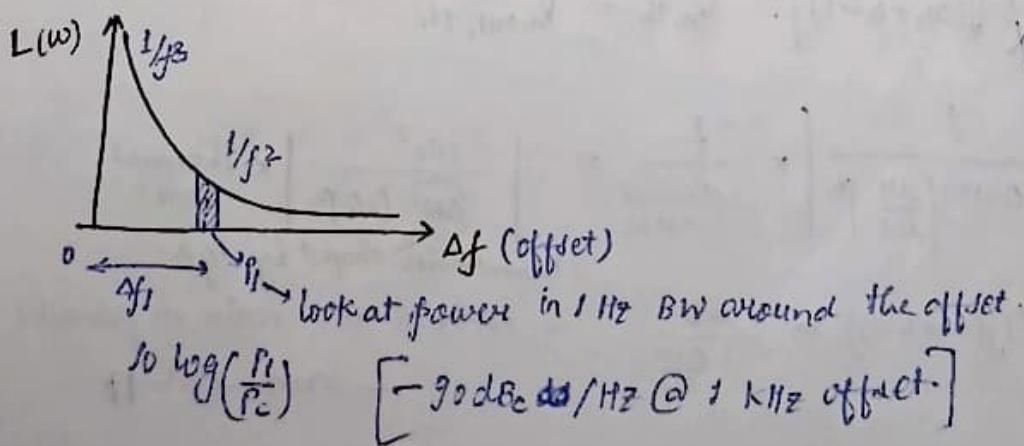
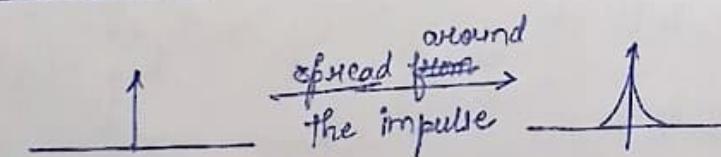


# Analysis for project  
DC → DC operating points

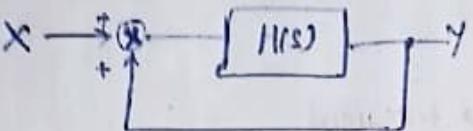
AC  
S-parameter analysis → S-parameters  
h<sub>b</sub> / pss analysis → P<sub>1dB</sub>, I<sub>IP3</sub>  
↓  
harmonic balance periodic steady state  
(preferred)

15-04-2025

### Phase Noise:

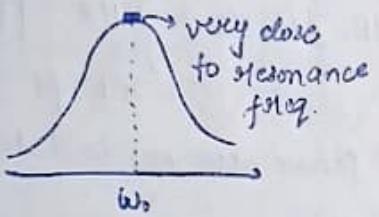


## Positive feedback system:



General TF:

$$\frac{Y}{X}(f) = \frac{M(f)}{1 - H(f)}$$

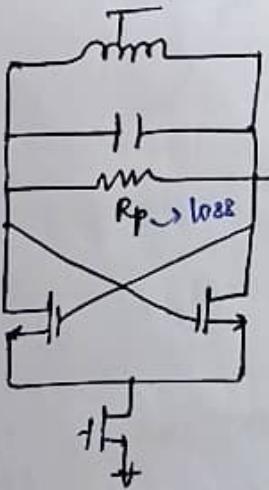


$$M(j(\omega_0 + \Delta\omega)) = M(j\omega_0) + \frac{dM}{d\omega} \cdot \Delta\omega$$

$$M(j\omega_0) = 1$$

$$\left| \frac{dM}{d\omega} \cdot \Delta\omega \right| \ll 1.$$

$$\therefore \frac{Y}{X}(j(\omega_0 + \Delta\omega)) = \frac{M(j\omega_0) + \frac{dM}{d\omega} \cdot \Delta\omega}{1 - M(j\omega_0) - \Delta\omega \frac{dM}{d\omega}} \approx \frac{-1}{\Delta\omega \frac{dM}{d\omega}}$$



→ Noise is shaped by the transfer function of the system (+ve feedback)

$$S_y(f) = S_x(f) \cdot |H(f)|^2$$

$$\left| \frac{Y}{X}(j(\omega_0 + \Delta\omega)) \right|^2 \cdot \frac{V_{n,th}^2}{V_{n,out,th}^2} = \frac{V_{n,th}^2}{V_{n,out,th}^2}$$

$$\left| \frac{1}{\Delta\omega^2 \left| \frac{dM}{d\omega} \right|^2} \right| \times \text{thermal noise} = \left| \frac{\omega_0^2}{(\Delta\omega)^2 (2\pi)^2} \right| \times \text{thermal noise}$$

thermal noise shaped by  $1/f^2$

$$Z(j(\omega_0 + \Delta\omega)) = j \frac{\omega_0 L}{2 \frac{\Delta\omega}{\omega_0}}$$

→ derived for series or parallel RLC

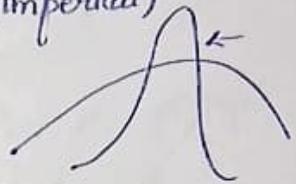
$$Q = \frac{\omega_0}{2} \left| \frac{dH}{dw} \right|$$

$$\Rightarrow \left| \frac{dH}{dw} \right| = \frac{2Q}{\omega_0}$$

Phase noise,

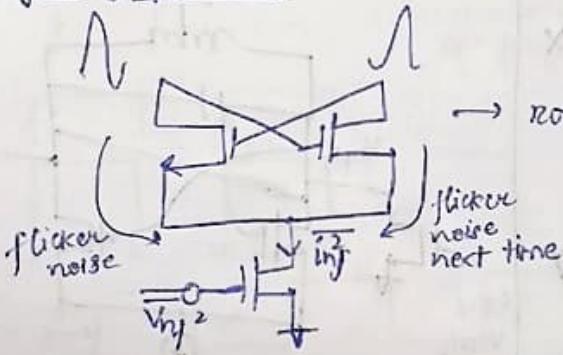
$$L(\Delta\omega) = 10 \log \left[ \frac{2kT}{P_{avg}} \cdot \left( \frac{\omega_0}{2Q\Delta\omega} \right)^2 \right]$$

(empirical)



$L(\Delta\omega) \uparrow \frac{1}{f^3}$ , due to  $(\frac{1}{\Delta\omega})^2$  factor  
 ✓ general thermal noise taken over

Reason for  $\frac{1}{f^3}$  factor:



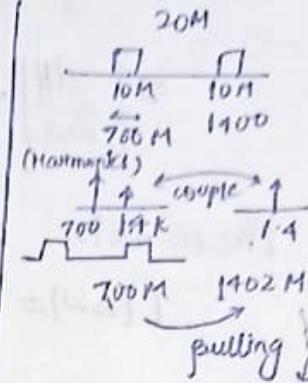
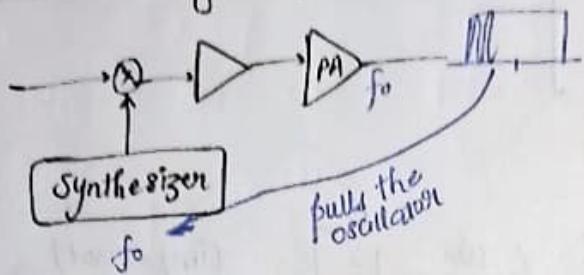
$\Delta f$

→ non-linear oscillator

Including flicker noise factor,

$$L(\Delta\omega) = 10 \log \left[ \frac{2FKT}{P_{avg}} \left( 1 + \left( \frac{\omega_0}{2Q\Delta\omega} \right)^2 \right) \left( 1 + \frac{\Delta\omega_{ny}}{\Delta\omega_0} \right) \right]$$

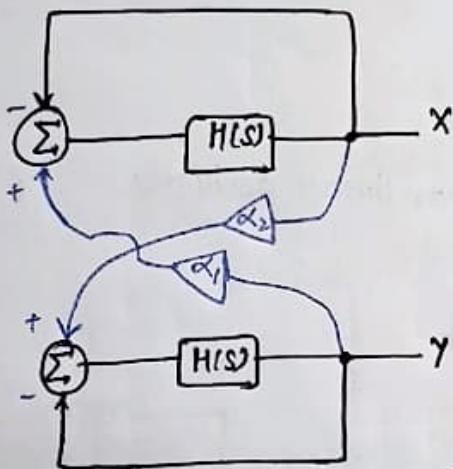
## Oscillator Pulling (Injection Pulling)



was supposed to be 1.4 K

# We can lock bad oscillator with a good oscillator

## Quadrature Coupling



[Two oscillators cross-coupled]

$$x = (\alpha_1 y - \dot{x}) H(s) \quad \{ x \propto \alpha_2 x$$

$$y = (\alpha_2 x - \dot{y}) H(s) \quad \} y \propto \alpha_1 y$$

$$\left\{ \begin{array}{l} \alpha_2 x^2 = (\alpha_1 \alpha_2 xy - \alpha_2 x^2) H(s) \\ \alpha_1 y^2 = (\alpha_1 \alpha_2 xy - \alpha_1 y^2) H(s) \end{array} \right.$$

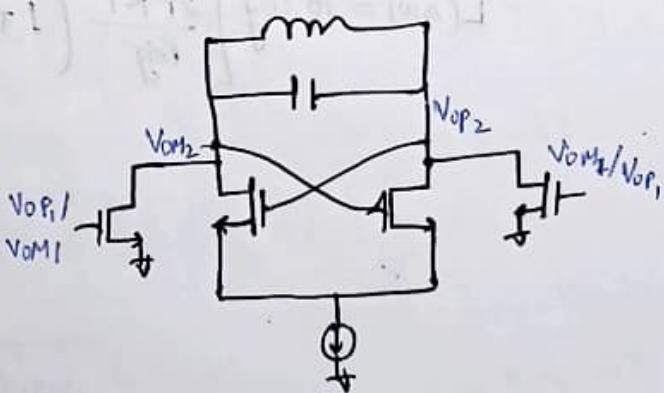
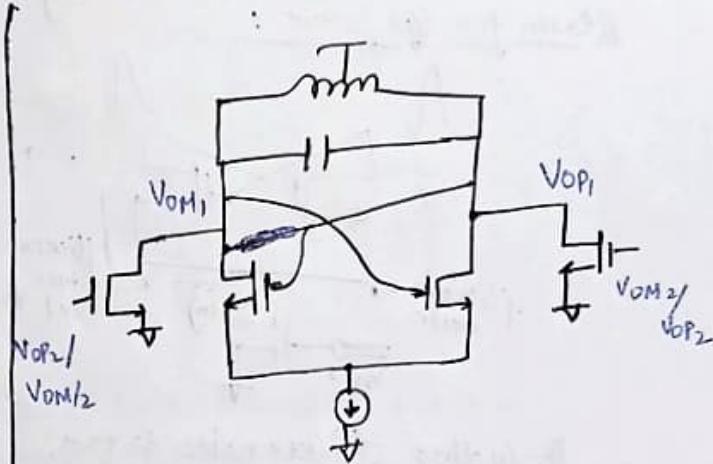
$$\Rightarrow \alpha_2 x^2 - \alpha_1 y^2 = -(\alpha_2 x^2 - \alpha_1 y^2) H(s)$$

$$\Rightarrow (\alpha_2 x^2 - \alpha_1 y^2) [1 + H(s)] = 0$$

$$\text{For } \alpha_2 x^2 - \alpha_1 y^2 = 0 \quad [1 + H(s)] = 0$$

$$\Rightarrow \alpha_1 = \alpha_2 \quad \xrightarrow{\text{not possible}} \text{not possible}$$

$x = y$  : Direct coupling



Coupling → Oscillate at same freq.

Cross-coupling

$$\text{For } \alpha_1 = -\alpha_2 \\ x^2 = -y^2$$

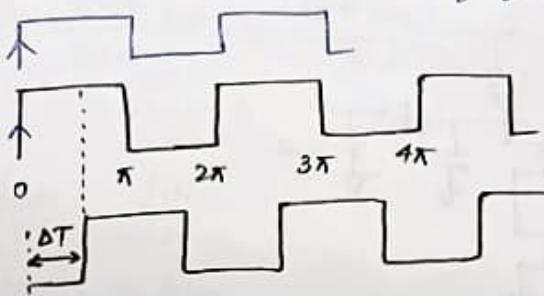
$\Rightarrow x = \pm jy \rightarrow 90^\circ$  phase shift [one directly coupled, other cross-coupled]  
 ↳ Quadrature coupling  
 ↳ Quadrature oscillator

## Phase Locked Loop

$\omega$ : angular frequency

$$\omega = \frac{d\phi}{dt}$$

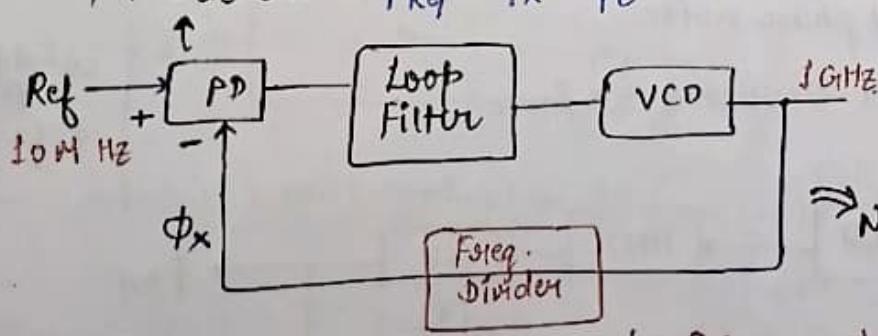
$\phi = \int \omega + K$  : accumulation of freq.



$$\frac{\Delta T}{T} \times 2\pi = \Delta\phi$$

Locked signals: same freq.  
 + same phase: rising edges are aligned.

Phase detector:  $\phi_{\text{Ref}} - \phi_x = \phi_e \xrightarrow{\text{VDD}}$



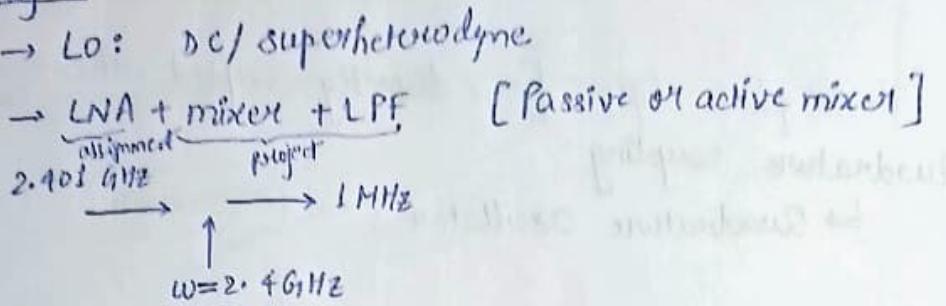
Look at the phase error and ↑/↓ the freq.  
 ↳ using VCO

Not actually used

↳ Integer-end divider  $\Rightarrow$  Integer-end synthesizer.  
 Fractional-end divider  $\Rightarrow$  Fractional-end synthesizer

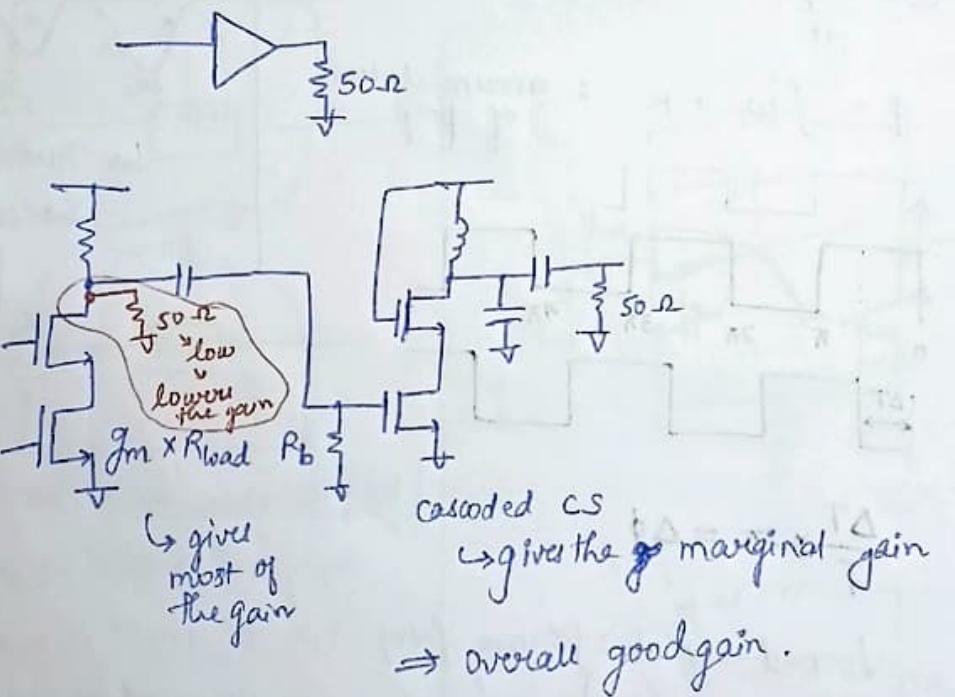
Crystal oscillator  
 ↳ very precise  
 But at low freq (Ref.)

## # Project:



To show:

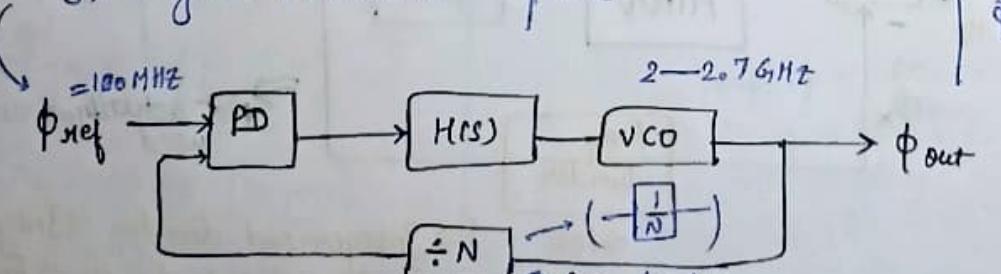
- DC results
- Overall gain
- Overall NF



## PLL:

Oscillator → bad phase noise

(Quartz) Crystal Oscillator → pure



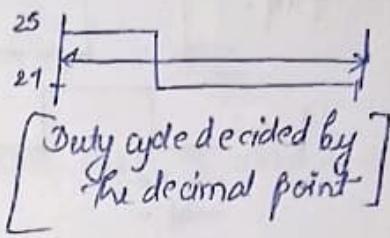
$$\left. \begin{aligned} \omega &= \frac{d\phi}{dt} \\ \phi &= f \omega t \end{aligned} \right\}$$

2.4 GHz :  $N = \frac{2400}{100} = 24$  (division ratio)  
 $\hookrightarrow \div 3 \rightarrow \div 2 \div 2 \rightarrow \div 2$

2.5 GHz :  $N = 25$ .

$$2.425 \text{ GHz} : N = 24.25$$

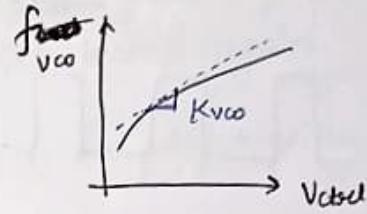
$\hookrightarrow$  75% of time  $\rightarrow N=24$   
 25% of time  $\rightarrow N=25$



$$\omega_{VCO} = 2\pi \cdot K_{VCO} \cdot V_{ctrl}$$

$$f_{VCO} = K_{VCO} \cdot V_{ctrl}$$

$$\phi_{out} = \int 2\pi \cdot K_{VCO} \cdot V_{ctrl}$$



$$K_{VCO} = \frac{\partial f_{VCO}}{\partial V_{ctrl}}$$

$$\Rightarrow \frac{\phi_{VCO}}{V_{ctrl}}(s) = \frac{2\pi K_{VCO}}{s}$$

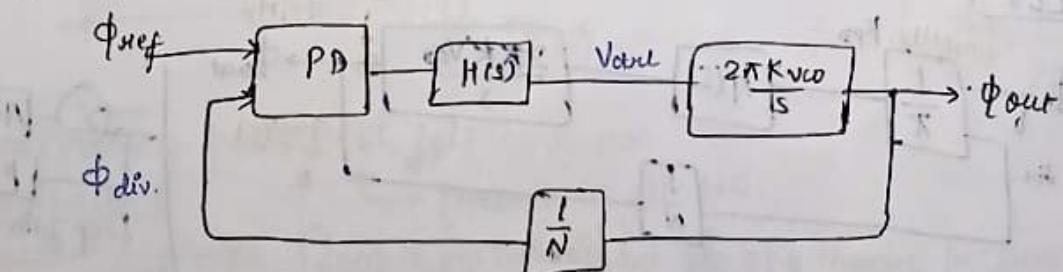
↗  $\boxed{\text{VCO}} \rightarrow$   
block

$$f_{out} = \frac{f_{in}}{N}$$

$$\omega_{out} = \frac{\omega_{in}}{N}$$

$$\phi_{out} = \frac{1}{N} \int \omega_{in} = \frac{1}{N} \phi_{in}$$

Hence,



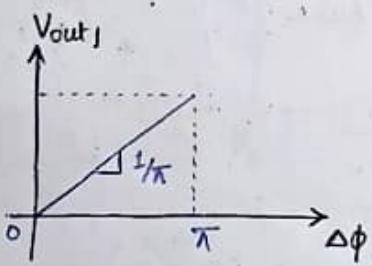
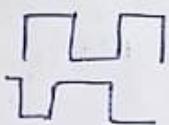
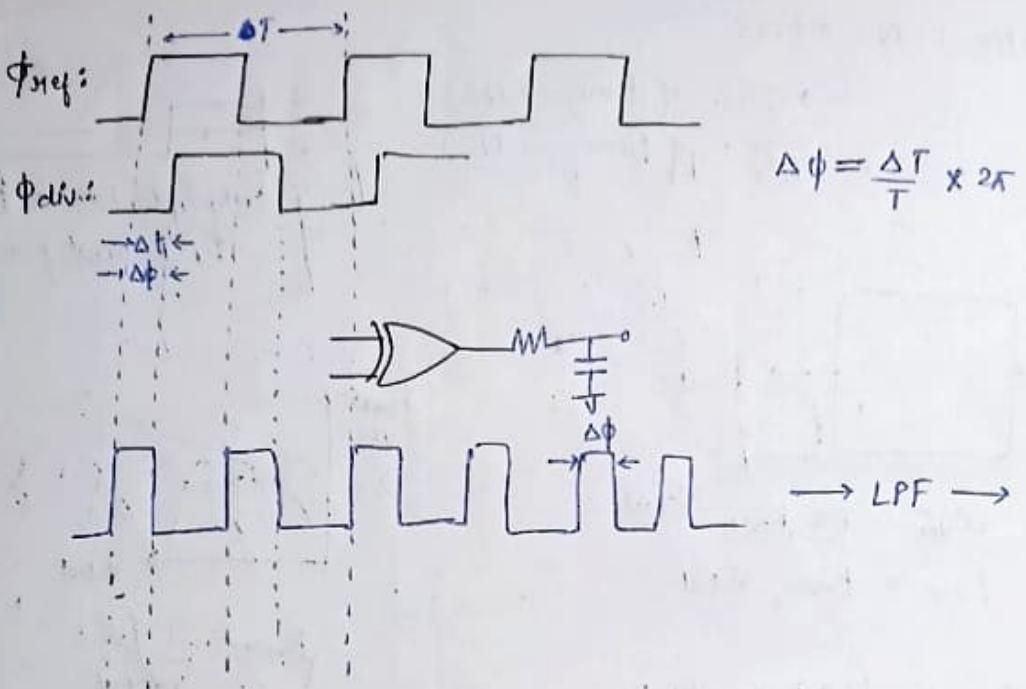
$$\rightarrow A \cos \omega t \times B \cos (\omega t + \phi) = \frac{AB}{2} [\cos(2\omega t + \phi) + \cos \phi]$$

↓ LPF

$$\frac{AB}{2} \cos \phi$$

$\hookrightarrow$  A multiplier works as phase detector  
 $\hookrightarrow$  cos of phase difference.

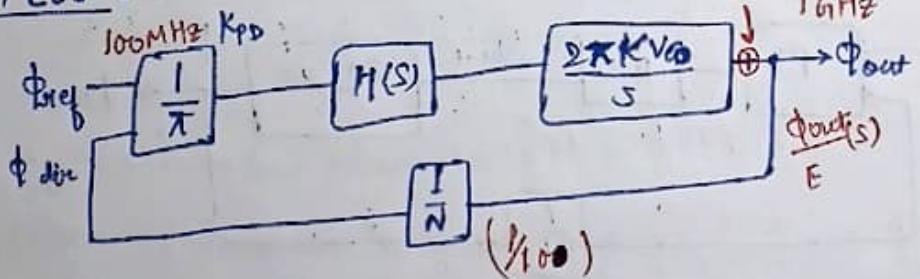
$\hookrightarrow$  Use XOR gate instead



phase noise of the oscillator (Model)  
E(s)

25-04-2025

PLLs:



Loop gain =  $\frac{A(s)}{1 + A(s)P(s)}$

$$\frac{\Phi_{\text{out}}(s)}{\Phi_{\text{ref}}} = \frac{\frac{1}{\pi} \cdot H(s) \cdot \frac{2\pi K_{VCO}}{s}}{1 + \frac{H(s) \cdot 2\pi K_{VCO}}{\pi \cdot s \cdot N}}$$

[ Assume  $H(s) = 1 \Rightarrow$  1st order T.F. ]

$$= \frac{2 H(s) K_{VCO}}{s + 2 H(s) \cdot \frac{K_{VCO}}{N}}$$

PLL BW  $\Rightarrow$  lock if the ratio is within the storage of PLL  
Low pass

Low pass: Litter at the i/p appears at the o/p of PLL.  
 ↳ We cannot have  $\phi_{out}$  better than  $\phi_{ref}$ .

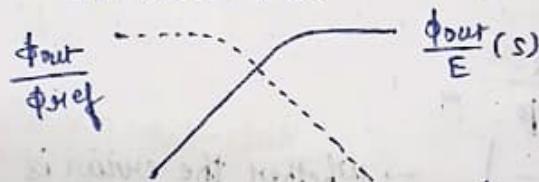
Capture Range: Range of freq. that the PLL can capture and lock the signal before the i/p is given.

Lock Range: If the PLL is already locked and given a perturbation, the perturbation it can sustain to get locked again.

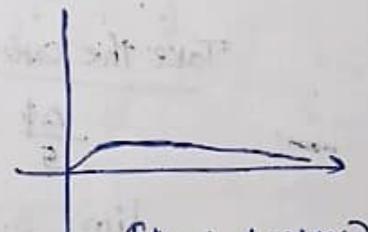
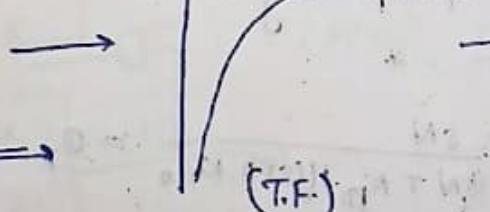
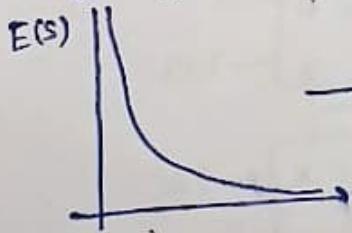
$$\phi_{out}(s) = \left(0 - \frac{\phi_{out}}{N}\right) \cdot \frac{1}{\pi} \cdot H(s) \cdot \frac{2\pi K_{VCO}}{s} + E(s) \quad [\text{superposition, } \phi_{out}=0]$$

$$\Rightarrow \phi_{out}(s) \left[ 1 + \frac{H(s) 2\pi K_{VCO}}{N\pi s} \right] = E(s)$$

$$\Rightarrow \frac{\phi_{out}}{E}(s) = \frac{SN}{SN + 2H(s) \cdot K_{VCO}} \rightarrow \text{High-pass response}$$



Profile of  $E(s)$ :



→ Anything put in PLL gets corrected

Perturbations:  $\Delta\omega$  steps [Loop is perturbed by ~~the~~ step change in freq.]  $\xrightarrow{\text{Present in all MC}}$

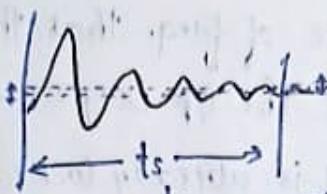
Case-I:  $\Delta\omega$  steps [Loop is perturbed by ~~the~~ step change in freq.]

$$\frac{\Delta\omega}{s} \xrightarrow{\text{phase}} \frac{\Delta\phi}{s^2}$$

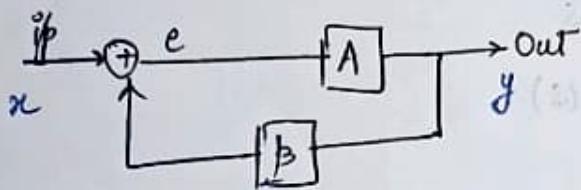
$\Delta\omega$  should be within the BW range of PLL to be corrected

Case-II: Glitch in power supply, etc.

↳  $\Delta\phi$  change in phase ( $\phi_{ref} \rightarrow \text{some}$ )



↳ settling time ( $t_s$ ) of PLL  
 ↳ critical parameters  
 (regulated in ICs)



residual error: error ( $e$ ) when the system has settled.

$$e = x - y \beta$$

$$= x - \frac{Ax\beta}{1 + AB}$$

$$= x \left( \frac{1}{1 + AB} \right) \rightarrow \text{whether the error is zero once settled?}$$

Take the case - ②:  $\Delta\phi$  ~~step~~ change.

$$\frac{\Delta\phi}{s} = T.F.$$

$$\lim_{s \rightarrow 0} s \cdot \frac{\Delta\phi}{s} \cdot \frac{sN}{sN + K_D \cdot H(s) \cdot 2\pi K_{VCO}} = 0,$$

∴ Error  $\rightarrow 0$

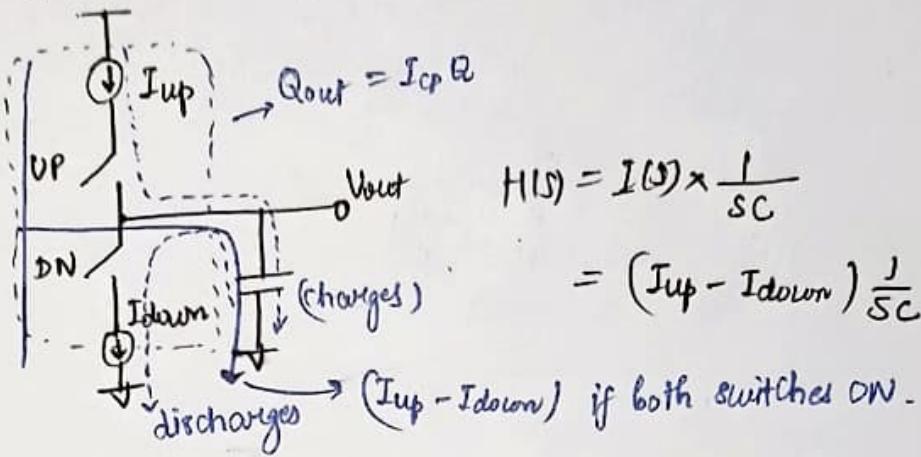
Case - ①:  $\frac{\Delta\phi}{s^2}$  [Type - I PLL]

$$\lim_{s \rightarrow 0} s \cdot \frac{\Delta\phi}{s^2} \cdot \frac{sN}{sN + K_D \cdot H(s) \cdot 2\pi K_{VCO}} \neq 0$$

↳ Almost a residual error.

↳ Never able to correct.

## Charge Pump Integrator



## Lead/Lag Filter:

$$H(s) = \frac{1 + \frac{s}{\omega_p}}{s(C_1 + C_2)(1 + \frac{s}{\omega_z})}$$

$$\omega_z = \frac{1}{R_1 C_2} \quad \omega_p = \frac{C_1 + C_2}{R_1 R_2}$$

## Phase-Frequency Detector

