AV491 - Advanced Sensors and Interface Electronics

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SC22B146

Tutorial - 1

B.Tech, 7th Sem

① ⓐ Input signal,

$$V_{in} = 5\sin(200\pi t)$$

$$V_{sig}^2 = V_{rms}^2 = \left(\frac{5}{\sqrt{2}}\right)^2 = \frac{25}{2} = 12.5 \; V^2$$

Noise,

$$V_{noise}^2 = \frac{q^2}{12}, \quad \text{where } q = \frac{V_R}{2^{n-1}} \text{ for bipolar ADC}$$

$$= 1.27\times10^{-4} V^2 \qquad = \frac{5}{2^7} = 0.039$$

$$\therefore SNR_{max} = 10\log\left(\frac{V_{sig}^2}{V_{noise}^2}\right)$$

$$= 10\log\left(\frac{12.5}{1.27\times10^{-4}}\right)$$

$$= 10\log(96,337.92)$$

$$\approx 49.838 \text{ dB.}$$

ⓑ $V_{rms}^2 = \left(\frac{2.5}{\sqrt{2}}\right)^2 V^2 = \frac{6.25}{2} \; V^2$

$$\therefore SNR_{max} = 10\log\frac{(6.25/2)}{(1.27\times10^{-4})}$$

$$= 10\log(24,606.3)$$

$$\approx 43.91 \text{ dB.}$$

② For triangular wave,

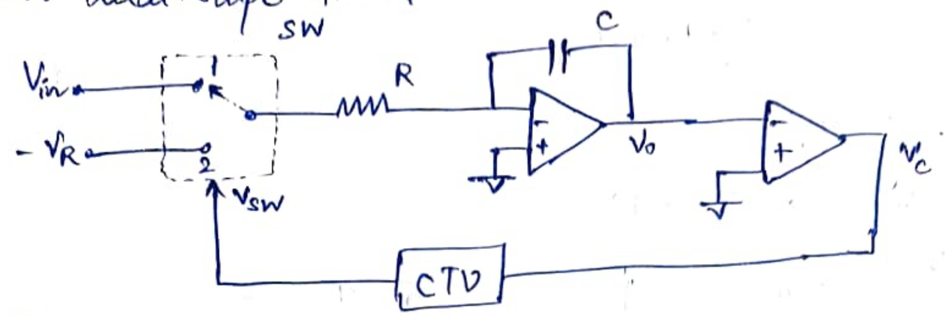
$$V_{sig}^2 = V_{rms}^2 = \left(\frac{V_R}{\sqrt{3}}\right)^2 = \left(\frac{5}{\sqrt{3}}\right)^2 = 8.33 \; V^2$$

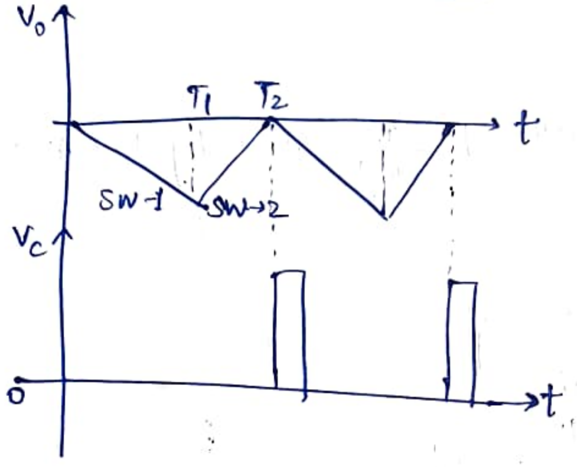
$$V_{noise}^2 = \frac{q^2}{12} = \left(\frac{V_R}{2^{n-1}}\right)\cdot\frac{1}{12} = \left(\frac{5}{2^7}\right)^2\cdot\frac{1}{12} = 1.27\times10^{-4} \; V^2$$

$$\therefore SNR_{max} = 10\log\left(\frac{8.33}{1.27\times10^{-4}}\right)$$

$$= 48.17 \text{ dB.}$$

③ 10-bit dual-slope ADC :





④ $f_{clk} = 1\,MHz \Rightarrow T_{clk} = \frac{1}{f_{clk}} = 1\,\mu s.$

For 10-bit dual-slope ADC,

integration time, $T_1 = 2^n \cdot T_{clk}$

$= 2^{10} \cdot 1\,\mu s$

$= 1024\,\mu s$

$= 1.024\,ms.$

max^m conversion time $= 2T_1$

$= 2.048\,ms.$

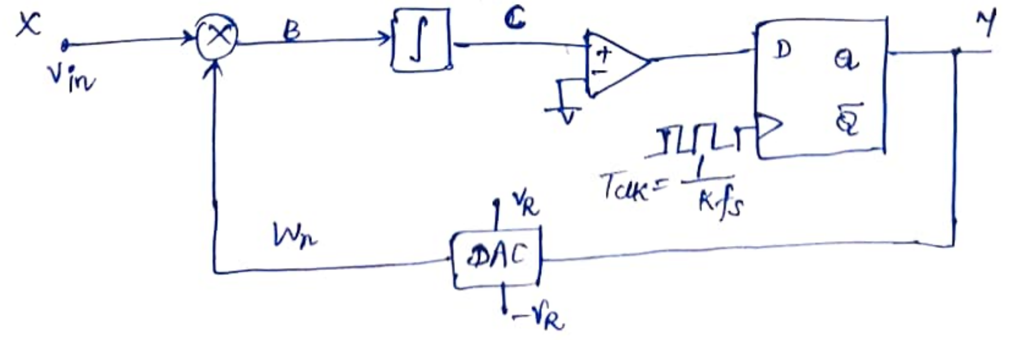
$T_{interference} = \frac{1}{f_{interference}}$

$= 0.02\,s$

Here, T_1 is not integral multiple of $T_{interference}$.

∴ Dual-slope ADC does not reject 50 Hz interference, hence o/p of ADC is not independent from 50 Hz interference.

⑤



$X \in (-V_R, V_R)$, $V_R = 1V$

$\in (-1V, 1V)$

$X = V_{in} = 0.25\,V$

$C_n = C_{n-1} + B_n$, n: sampling instance

$Y_n = \begin{cases} \text{High}(1), & C_n \geq 0 \\ \text{Low}(1), & C_n < 0 \end{cases}$

$W_n = \begin{cases} V_R, & Y_n = 1 \\ -V_R, & Y_n = 0 \end{cases}$

$B_n = X_n - W_{n-1}$

$= X - W_{n-1}$, X should be constant for atleast few clock cycles.

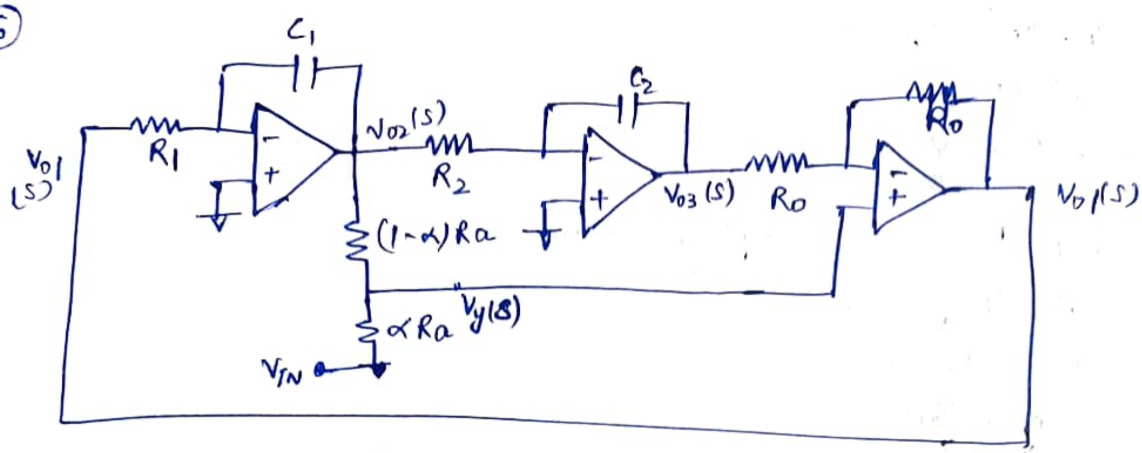
n	X	B	C	Y	W_n
1	0.25	0.25	0.25	1	1
2	0.25	-0.75	-0.5	0	-1
3	0.25	1.25	+0.75	1	1
4	0.25	-0.75	0	1	1
5	0.25	-0.75	0.75	0	-1
6	0.25	1.25	0.5	1	1
7	0.25	-0.75	-0.25	0	-1
8	0.25	1.25	0	1	1
				$\bar{Y} = \frac{5}{8}$	

For 3-bit ADC,

X	D_2	D_1	D_0	
(-1, -0.75)	0	0	0	1/8
(-0.75, -0.5)	0	0	1	2/8
(-0.5, -0.25)	0	1	0	3/8
(-0.25, 0)	0	1	1	4/8
(0, 0.25)	1	0	0	5/8
$0.25 \in$ (0.25, 0.5)	1	0	1	6/8 → ✓
(0.5, 0.75)	1	1	0	7/8
(0.75, 1)	1	1	1	8/8

Digital o/p of the ADC = 101.

(6)



$$V_{02}(s) = -\frac{1}{sR_1C_1} V_{01}(s)$$

$$V_{03}(s) = -\frac{1}{sR_2C_2} V_{02}(s) = \frac{1}{s^2R_1R_2C_1C_2} V_{01}(s)$$

$$V_y(s) = (1-\alpha) V_{IN}(s) + \alpha V_{02}(s)$$

Also, $V_{01}(s) = -V_{03}(s) + 2V_y(s)$

$$\Rightarrow V_{01}(s) = \frac{V_{01}(s)}{s^2R_1R_2C_1C_2} + 2(1-\alpha)V_{IN}(s) - \frac{2\alpha V_{01}(s)}{sR_1C_1}$$

$$\Rightarrow V_{01}(s)\left[1 + \frac{2\alpha}{sR_1C_1} + \frac{1}{s^2R_1R_2C_1C_2}\right] = 2(1-\alpha)V_{IN}(s)$$

$$\Rightarrow V_{01}(s)\left[\frac{s^2R_1R_2C_1C_2 + 2\alpha R_2C_2 s + 1}{s^2R_1R_2C_1C_2}\right] = 2(1-\alpha)V_{IN}(s)$$

$$\Rightarrow \frac{V_{01}}{V_{IN}}(s) = \frac{2(1-\alpha)s^2}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1R_2C_1C_2}} \quad \text{: HPF}$$

$$\frac{V_{02}}{V_{IN}}(s) = \frac{-2(1-\alpha)\frac{s}{R_1C_1}}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1R_2C_1C_2}} \quad \text{: BPF}$$

Comparing with $\frac{2K\xi\omega_0 s}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{K\left(\frac{\omega_0}{Q}\right)s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$,

$$\omega_0 = 2\pi f = \frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi \times 2000 = 12,560 \text{ rad/s.}$$

$$BW = \frac{f_0}{Q} = 5\,Hz$$

$$\Rightarrow Q = \frac{2K}{5} = 400.$$

Take $R_1 = R_2 = R$, $C_1 = C_2 = C$.

$$\Rightarrow \omega_0 = \frac{1}{RC}$$

Take $C = 1nF$.

$$R = \frac{1}{(12560)(1\times10^{-9})} \approx 79.62\,k\Omega.$$

Now, $\frac{W_0}{Q} = \frac{2\alpha}{RC}$

$$\Rightarrow \alpha = \frac{W_0RC}{2Q} = \frac{12560\times 79.62\times 10^{-9}\times 10^3}{2\times 400}$$

$$= 1.25\times10^{-3}.$$

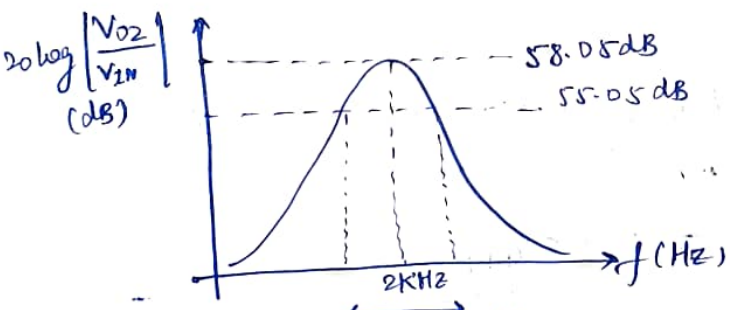
$$K = \frac{1}{\alpha} - 1 = \frac{10^3}{1.25} - 1 = 799 \rightarrow 20\log 799 \approx 58.05\,dB$$

Take $R_a = 200k\Omega$.

$\alpha R_a = 250\Omega$.

$(1-\alpha)R_a = 199.75\,k\Omega$.

Take $R_0 = 100\,k\Omega$.



(7) $$\frac{V_{o3}}{V_{IN}}(s) = \frac{2(1-\alpha)\frac{1}{R_1R_2C_1C_2}}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1R_2C_1C_2}}$$; LPF

Notch filter : $\frac{V_{o1}}{V_{IN}}(s) + \frac{V_{o3}}{V_{IN}}(s)$

$$\equiv \frac{2(1-\alpha)s^2 + 2(1-\alpha)\frac{1}{R_1R_2C_1C_2}}{s^2 + \frac{2\alpha s}{R_1C_1} + \frac{1}{R_1R_2C_1C_2}}$$

comparing with $\frac{K(s^2+w_0^2)}{s^2 + 2\xi w_0 s + w_0^2}$,

$$w_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi\times500 \approx 3140\ rad/s$$

$$K = 2(1-\alpha) = 1 \Rightarrow \alpha = 1/2.$$

$BW = \frac{f_0}{Q} = \frac{2\alpha}{R_1 C_1} \quad (= 2\xi_1 \omega_0)$ $\quad \Big| \quad Q = \frac{f_0}{BW} = \frac{500}{10} = 50.$

Take $C_1 = 10\,\mu F$.

$R_1 = \frac{2(\frac{1}{2})\,50}{10\times10^{-6}\times \cancel{3140}\ \cancel{500}\ 10} = \cancel{1.59\,M\Omega}\ 10^4 = 10\,k\Omega.$

Take $C_2 = 10\,\mu F$.

$\omega_0^2 = \frac{1}{R_1 R_2 C_1 C_2}$

$\Rightarrow R_2 = \frac{1}{R_1 C_1 C_2 \omega_0^2}$

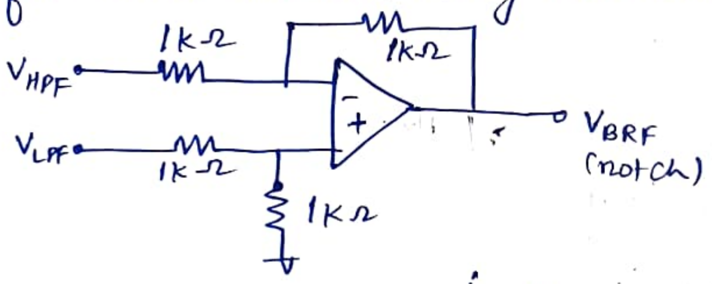
$= \frac{1}{(3140)^2 (10\times10^3)(10\times10^{-6})^2}$

$= 0.10\,\Omega.$

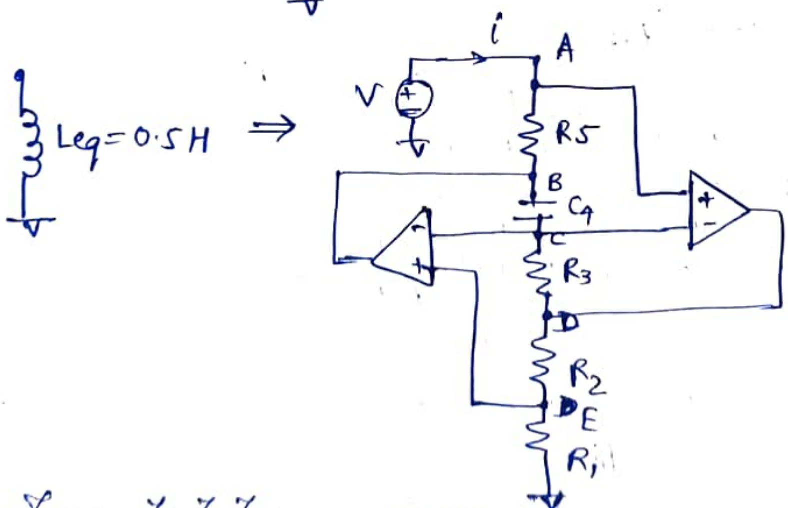
Instead Take $C_2 = 1\,\mu F$.

$R_2 = 1\,\Omega.$

Notch filter can be obtained using ~~notch~~ summing amplifier.



(8) (a)



$L_{eq} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} = \frac{R_1 R_3 R_5}{R_2 (\frac{1}{sC_4})} = s\left(\frac{R_1 R_3 R_5 C_4}{R_2}\right)$

$L_{eq} = \frac{R_1 R_3 R_5 C_4}{R_2} = 0.5H$

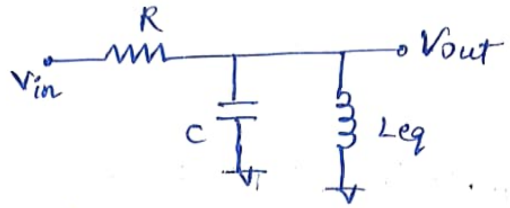
Given that $C_4 = 10nF$, $R_1 = R_3 = R_5 = R_2$.

$$\therefore \frac{R^3 C_4}{R} = 0.5 \Rightarrow R^2 (10 \times 10^{-9}) = 0.5$$

$$\Rightarrow R^2 = 0.05 \times 10^9 = 5 \times 10^7$$

$$\Rightarrow R \approx 7\,k\Omega.$$

(8) BPF :



$$\omega_0 = \frac{1}{\sqrt{L_{eq} C}} = 2\pi \times 5 \times 10^3 = 10000\pi \Rightarrow C = \frac{1}{L_{eq}(31400)^2}$$

$L_{eq} = 0.5$

$$\approx 2\,nF.$$

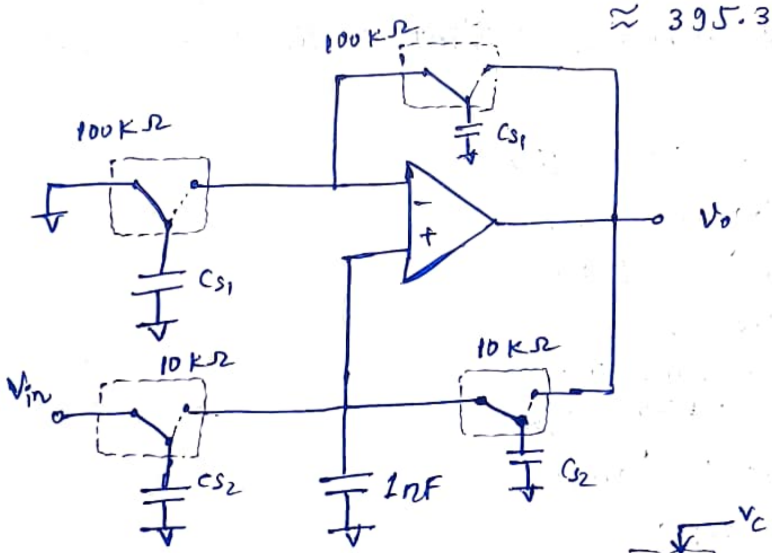
$$Q = R\sqrt{\frac{C}{L_{eq}}} = 25$$

$$\Rightarrow R = 25 \times \sqrt{\frac{0.5}{2 \times 10^{-9}}} \approx 15.8\,k\Omega.$$

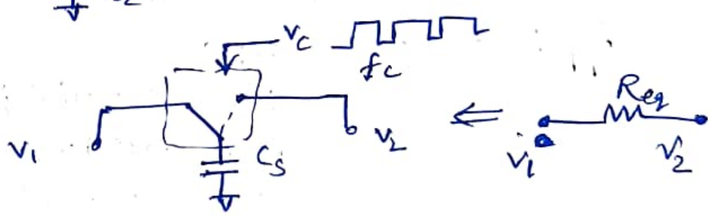
$\times 25$

$$\approx 395.3\,k\Omega.$$

(9)



$$R_{eq} = \frac{1}{f_c C_s}$$



$$C_{S1} = \frac{1}{R_{eq} f_c}$$

$$= \frac{1}{100 \times 10^3 \times 10^6} = 0.01\,nF$$

$$C_{S2} = \frac{1}{10 \times 10^3 \times 10^6} = 0.1\,nF.$$

(10) LF398 is a precision sample-and-hold amplifier. It captures (samples) an analog input voltage and stores (holds) it on a capacitor, providing a constant output voltage even when the input changes.

Modes of operation:

(a) Sample Mode:
- The control (sample/hold) pin is logic high.
- The circuit tracks the input signal, the output follows the i/p signal.

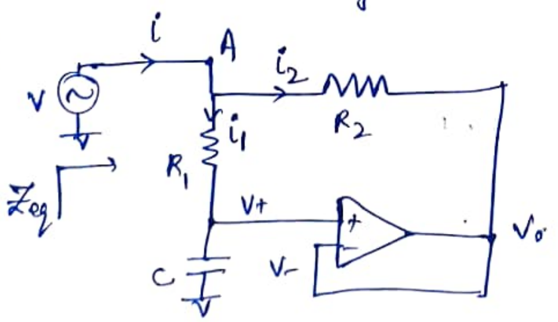
(b) Hold Mode:
- The control pin is logic low.
- The switch opens, isolating the hold capacitor.
- The output remains constant at the last sampled value.

Specifications:

Supply voltage: ±5V to ±18V
I/p offset voltage: ~2mV
Acquisition time: < 10 μs
Hold step: 0.5mV
Droop rate: 0.01 mV/ms
I/p impedance: very high
Output current: ±10mA.

Applications:
- ADC
- Data acquisition and multiplexing systems
- Peak detector
- Digital control system.

(11) (a)



$$V^+ = \frac{1/sC}{R_1 + \frac{1}{sC}} V = V_- = V_o$$

$$i_1 = \frac{V}{R_1 + 1/sC}$$

$$i_2 = \frac{V - V_o}{R_2} = \left[V - \frac{\frac{1}{sC} V}{R_1 + \frac{1}{sC}}\right]\frac{1}{R_2}$$

$$= \frac{V(R_1 + \frac{1}{sC}) - V/sC}{R_2(R_1 + \frac{1}{sC})}$$

$$= \frac{V R_1 + V/sC - V/sC}{R_2 + \frac{1}{sC}} = \frac{V R_1}{R_2(R_1 + \frac{1}{sC})}$$

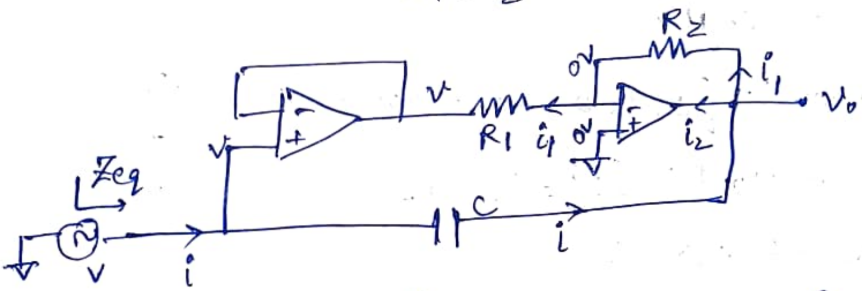
$$i = i_1 + i_2$$

$$= v\left[\frac{1}{R_1 + \frac{1}{sC}} + \frac{R_1}{R_2(R_1 + \frac{1}{sC})}\right]$$

$$= v\left[\frac{R_2 + R_1}{R_2(R_1 + \frac{1}{sC})}\right]$$

$$\therefore Z_{eq} = \frac{v}{i} = \frac{R_2(R_1 + \frac{1}{sC})}{R_1 + R_2}.$$

(b)



$$i = \frac{V - V_o}{\frac{1}{sC}}, \quad i_1 = \frac{V_o}{R_2} = -\frac{v}{R_1} \Rightarrow V_o = -\frac{R_2}{R_1} v$$

$$\Rightarrow i = (V - V_o) sC$$

$$= sC\left(V + \frac{R_2}{R_1} V\right)$$

$$= sC V\left(\frac{R_1 + R_2}{R_1}\right)$$

$$\Rightarrow \frac{V}{i} = Z_{eq} = \frac{1}{sC\left(\frac{R_1 + R_2}{R_1}\right)} = \frac{R_1}{sC(R_1 + R_2)}.$$

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Tutorial - 2

① Input voltage noise:

ⓐ $E_N^2 = \int_{f_L}^{f_H} e_n^2 \, df$

$= \int_{0.1}^{100} e_{nw}^2 \left[1 + \frac{f_{CE}}{f}\right] df$

$= e_{nw}^2 \left[f + f_{CE} \ln f\right] \Big|_{0.1}^{100}$

$= (20\times10^{-9})^2 \left[(100 - 0.1) + 200 \ln\left(\frac{100}{0.1}\right)\right]$

$= (20\times10^{-9})^2 \left[1481.45\right]$

$= 5.9258 \times 10^{-13}$ V

$\therefore E_N = 0.7698\ \mu V.$

ⓑ $E_N^2 = (20\times10^{-9})^2 \left[(20000 - 20) + 200 \ln\left(\frac{20000}{20}\right)\right]$

$= (20\times10^{-9})^2 \left[21361.55\right]$

$= 8.545 \times 10^{-12}$ V

$\therefore E_N = 2.923\ \mu V.$

ⓒ $E_N^2 = (20\times10^{-9})^2 \left[(10^6 - 0.1) + 200 \ln\left(\frac{10^6}{0.1}\right)\right]$

$= (20\times10^{-9})^2 \left[1003223.5\right]$

$= 4.012 \times 10^{-10}$ V

$\therefore E_N \approx 20\ \mu V.$

② ⓐ Thermal noise:

$E_T = \sqrt{4KTBR}$

$= \sqrt{4\times1.38\times10^{-23}\times298\times1\times10^3}$

≈ 4 nV.

ⓑ $E_T = \sqrt{4\times1.38\times10^{-23}\times77\times1\times10^3}$

$= 2.06$ nV

ⓒ $E_T = \sqrt{4\times1.38\times10^{-23}\times4.2\times1\times10^3}$

$= 0.481$ nV

③ Shot noise, $i_{SH}^2 = 2qI_{DC}$

$= 2 \times 1.6 \times 10^{-19} \times 10 \times 10^{-3}$

$= 32 \times 10^{-21}$

$PSD_{SH} \equiv I_{SH}^2 = \cancel{(32 \times 10^{-21}) \times} B \times \hat{i}_{SH}^2$

$= 32 \times 10^{-21} \times 10 \times 10^3$

$= 3.2 \times 10^{-16} \Rightarrow I_{SH} = 1.788 \times 10^{-8} A$

B-E resistance, $r_e = \frac{V_T}{I_E} = \frac{26mV}{10mA} = 2.6\Omega$

$\therefore$ voltage noise, $V_{SH} = I_{SH} r_e$

$= 1.788 \times 10^{-8} \times 2.6$

$= 46.48 nV$.

④ Thermal noise,

$E_T = \sqrt{4KTBR}$

$= \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 10 \times 10^3 \times 2.6}$

$= 2.07 \times 10^{-8} V$

$= 20.7 nV$

⑤ ⓐ Voltage spectral density, $e_T = 4KTR$

$= 4 \times 1.38 \times 10^{-23} \times 300 \times 10 \times 10^3$

$= 1.656 \times 10^{-16} V/\sqrt{Hz}$.

Current spectral density, $I_T = \frac{4KT}{R}$

$= \frac{4 \times 1.36 \times 10^{-23} \times 300}{10 \times 10^3}$

$= 1.656 \times 10^{-24}$ $A/\sqrt{Hz}$.

ⓑ RMS voltage noise,

$e_T = \sqrt{4KTBR}$

$= \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 19980 \times 10 \times 10^3}$

$= 1.818 \mu V$.

⑥ ⓐ $I_{SH}^2 = 2qI_{DC}B$

$= 2 \times 1.6 \times 10^{-19} \times 10^{-6} \times 10^6$

$= 3.2 \times 10^{-19} A^2$

$I_{DC}^2 = 10^{-12} A^2$

$\therefore SNR_{(dB)} = 10 \log\left(\frac{I_{DC}^2}{I_{SH}^2}\right) = 10 \log\left(\frac{10^{-12}}{3.2 \times 10^{-19}}\right)$

$\approx 64.95 dB$.

⑥ $I_{SH}^2 = 2 \times 1.6 \times 10^{-19} \times 10^{-9} \times 10^{6}$

$= 3.2 \times 10^{-22}\ A^2$

$I_{DC}^2 = 10^{-18}$

$\therefore \underset{(dB)}{SNR} = 10 \log\left(\frac{10^{-18}}{3.2 \times 10^{-22}}\right)$

≈ 34.95 dB.

⑦ 2nd order LPF: $H(S) = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$

Noise BW, $B = \frac{1}{|H(0)|^2} \int_0^{\infty} |H(jf)|^2\, df$

$= 1 \cdot \int_0^{\infty} \frac{f_c^2}{f_c^2 + j2\xi f f_c - f^2}\, df$

$= \frac{f_c}{4\xi}(4\xi^2 + 1)$

(i) $\xi = 1 \Rightarrow B = \frac{f_c}{4}(5) = 1.25 f_c$

(ii) $\xi = 2 \Rightarrow B = \frac{f_c}{8}(17) = 2.125 f_c$

$\cancel{|H(j\omega)| = \frac{f_c^2}{f_c^2 + j2\xi f f_c - f^2}}$

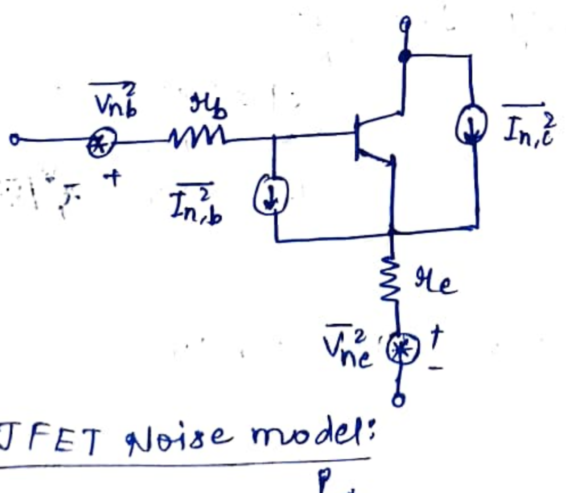
$\cancel{|H(0)| = \frac{f_c^2}{f_c^2} = 1}$

$H(jf) = \frac{f_c^2}{f_c^2 + j2\xi f f_c - f^2}$

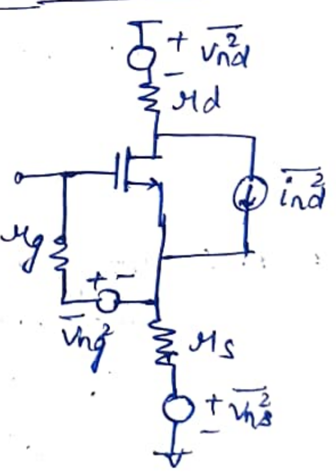
$|H(jf)|^2 = \frac{f_c^4}{(f_c^2 - f^2)^2 + (2\xi f f_c)^2}$

$|H(0)|^2 = 1$

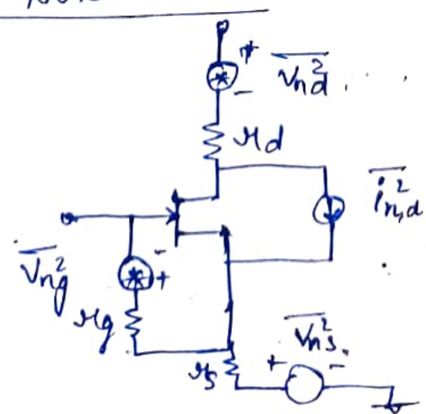
⑧ BJT Noise model:



MOSFET Noise model:



JFET Noise model:



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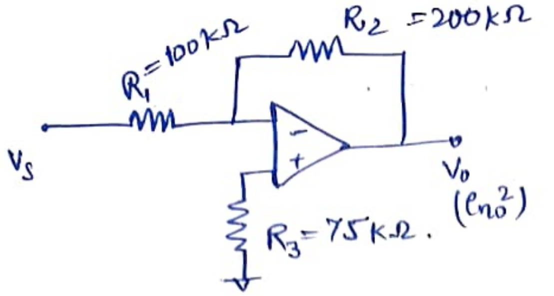


$G=2$
$G_N=3$ | $f_N = \frac{A_0 f_B}{G_N} = \frac{1\,MHz}{3}$

$e_{ni}^2 = e_{nw}^2\left(1+\frac{f_{CE}}{f}\right)$

$i_n^2 = i_{nw}^2\left(1+\frac{f_{CE}}{f}\right)$

$e_{ni}^2 = e_n^2 + 8KTR_3 + 2\,i_n^2 R_3^2$

As R_3 is very large,

$e_{ni}^2 \approx 2\,i_n^2 R_3^2$

$= 2\,i_{nw}^2\left(1+\frac{f_{CE}}{f}\right)R_3^2$

$e_{no}^2 = \frac{G_N^2}{1+\left(\frac{f}{f_N}\right)^2}\,e_{ni}^2$

$\Rightarrow E_{No}^2 = \int_{f_L}^{f_H} e_o^2\,df = 2G_N^2\,i_{nw}^2 R_3^2 \int_{0.1}^{\infty}\frac{(1+f_{CE}/f)}{\left(1+\left(\frac{f}{f_N}\right)^2\right)}\,df$

$= 2G_N^2\,i_{nw}^2 R_3^2\left[f_N\tan^{-1}\left(\frac{f}{f_N}\right) + \frac{f_{CI}}{2}\ln\left(\frac{f^2}{f^2+f_N^2}\right)\right]_{0.1}^{\infty}$

$= 2\times 3^2\times(500\times10^{-15})^2\times(75\times10^3)^2\times$

$\left[\frac{10^6}{3}\left(\frac{\pi}{2}\right) + \frac{2\times10^3}{2}\ln(1)^{\to 0} - \frac{10^6}{3}\left(\frac{0.1}{\frac{10^6}{3}}\right) - \frac{2\times10^3}{2}\ln\left(\frac{0.1}{\frac{10^6}{3}}\right)\right]$

$= 18\times25\times10^{-26}\times75^2\times10^6\times\left[\frac{\pi}{2}\times\frac{10^6}{3} - 0.1 + 10^3(-15.02)\right]$

$= 2531250\times10^{-20}\left[0.52\times10^6 - \cancel{0.1}^{\approx 0} - \cancel{15.02\times10^3}^{\approx 0}\right]$

$= 1.32\times10^{-8}\ V^2$

$\Rightarrow E_{No} = 1.14\times10^{-4} = 114\ \mu V$

$V_{rms} = 1/\sqrt{2} \Rightarrow G\,V_{rms} = \sqrt{2}$

$\therefore SNR = 20\log\left(\frac{G\,V_{rms}}{E_{No}}\right)$

$= 20\log\left(\frac{\sqrt{2}}{114\times10^{-6}}\right) \simeq 81.87\ dB$

(10) For low effect of Bias current, R_3 has to be very low.

$$e_{ni}^2 = e_n^2 + 8KTR_3 + 2i_n^2 R_3^2$$

$$\approx e_n^2 = e_{nw}^2 \left(1 + \frac{f_{CE}}{f}\right)$$

$$e_o^2 = \frac{G_N^2 e_{nw}^2}{1 + f^2/f_N^2}(1 + f_{CE}/f) = \frac{G_N^2 e_{nw}^2}{4\pi f}\left(\frac{1}{1+f^2/f_N^2} + \frac{f_{CE}/f}{1+f^2/f_N^2}\right)$$

$$\Rightarrow E_{No}^2 = \int_{f_L}^{f_H} e_o^2\, df = G_N^2 e_{nw}^2 \left[\frac{\pi}{2} f_N - f_L - \frac{f_{CE}}{2} \cdot 2\ln\left(\frac{f_N}{f_L}\right)\right]$$

$$= 3^2 \times (20 \times 10^{-9})^2 \left[\frac{\pi}{2}\left(\frac{10^6}{3}\right) - 0.1 - 200 \ln\left(\frac{10^6}{3 \times 0.1}\right)\right]$$

$$= 36 \times 10^{-16} \left[0.52 \times 10^6 - 0.1 - 200 (15.02)\right]$$

$$= 18.72 \times 10^{-10}\ V^2$$

$$E_o = 4.32 \times 10^{-5}\ V = 43.2\ \mu V.$$

$$\therefore SNR = 20 \log\left(\frac{\sqrt{2}}{4.32 \times 10^{-5}}\right) \approx 90.3\ dB$$

$$e_{ni}^2 = 2 i_n^2 R_3^2 = 2 i_{nw}^2 \left(1 + \frac{f_{CI}}{f}\right) R_3^2$$

$$e_o^2 = \frac{G_N^2}{1 + f^2/f_N^2} e_{ni}^2$$

$$\therefore E_{No}^2 = \int_{f_L}^{f_H} e_o^2\, df \doteq 2 G_N^2 i_{nw}^2 R_3^2 \int_{0.1}^{\infty} \frac{1 + f_{CI}/f}{1 + f^2/f_N^2}\, df$$

$$= 2 G_N^2 i_{nw}^2 R_3^2 \left[f_N \tan^{-1}\left(\frac{f}{f_N}\right) + \frac{f_{CI}}{2} \ln\left(\frac{f^2}{f^2 + f_N^2}\right)\right]_{0.1}^{\infty}$$

$$\approx 2 G_N^2 i_{nw}^2 R_3^2 \frac{\pi}{2} f_N.$$

For $E_{No}^2 = (50 \times 10^{-6})^2$,

$$(50 \times 10^{-6})^2 = 2 \times 9 \times (500 \times 10^{-15})^2 \times R_3^2 \times \frac{\pi}{2} \times \frac{10^6}{3}$$

$$\Rightarrow 25 \times 10^{-10} = 3 \times 25 \times 10^{-26} \times R_3^2 \times \pi \times 10^6$$

$$\Rightarrow R_3^2 = \frac{10^{-10}}{3 \times 10^{-20} \times \pi} = 0.106 \times 10^{10}$$

$$\Rightarrow R_3 = 0.326 \times 10^5 \Rightarrow R_3 = 32.6\ K\Omega.$$

Let $R_1 = 400\,K\Omega$.

Then, $R_3 = R_1 \parallel R_2$

$$\Rightarrow 326\,K = \frac{(400\,K)R_2}{R_2 + 400\,K}$$

$$\Rightarrow 326\,K\,R_2 + \overset{130400}{\cancel{326000}}\,M = 400\,K\,R_2$$

$$\Rightarrow 74\,K\,R_2 = 130400\,M$$

$$\Rightarrow R_2 = \frac{130400}{74}\,K$$

$$= 1.76\,M\Omega.$$

$$SNR = 20\log\frac{\sqrt{2}}{50\times10^{-6}}$$

$$\approx 89\,dB$$

(11) $R_3 \to$ very large.

$$e_{ni}^2 \approx 2 i_n^2 R_3^2.$$

$$\therefore E_{No}^2 \approx 2 G_N^2 i_{nw}^2 R_3^2 \frac{\pi}{2} f_N.$$

$$= \cancel{2} \times 9 \times (170\times10^{-15})^2 \times (75\times10^3)^2 \times \frac{\pi}{\cancel{2}} \times \frac{0.6}{2}\times10^6$$

$$= 9\times17^2\times10^{-28}\times75^2\times10^6\times\frac{\pi}{2}\times0.6\times10^6$$

$$= 13{,}786{,}437.94\times10^{-16}$$

$$= 13.79\,\mu V^2$$

$$\therefore E_{No} = 3.71\,\overset{m}{\cancel{\mu}}V.$$

As $E_{No} \propto R$, to reduce the noise by half, take $R_3 = \frac{75}{2}\,K\Omega$.

(12)

$$e_n^2 = e_{nw}^2\left(1 + \frac{f_{CE}}{f}\right) \qquad \begin{array}{|l} f_1 = 10\,Hz \\ f_2 \gg f_{CE} \end{array}$$

$$= A_1 + \frac{A_2}{f}.$$

$$e_n^2(f_1) = 20^2 \times10^{-18}\,V^2/Hz.$$

$$e_n^2(f_2) = 36\times10^{-18}\,V^2/Hz.$$

At $f = f_2$, $e_n^2 = A_1 + \overset{\approx 0}{\frac{A_2}{f_2}} \approx A_1 = 36\times10^{-18}\,V^2/Hz.$

At $f = f_1$, $e_n^2 = 20^2\times10^{-18} = 36\times10^{-18} + \frac{A_2}{10}$

$$\Rightarrow A_2 = 364\times10^{-18}\times10$$

$$= 364\times10^{-17}\,V^2.$$

RMS o/p noise,

$$E_{No}^2 = \int_{1m}^{1M} e_n^2\, df = \int_{1m}^{1M} \left(A_1 + \frac{A_2}{f}\right) df$$

$$= A_1 (10^6 - 10^{-3}) + A_2 \ln\left(\frac{10^6}{10^{-3}}\right)$$

$$\simeq A_1 \times 10^6 + A_2 \ln 10^9$$

$$= 36\times10^{-18}\times10^6 + 364\times10^{-17}\times 20.72$$

$$= 36\times10^{-12} + 7543.27\times10^{-17}$$

$$\simeq 36\times10^{-12}\ V^2$$

$$\Rightarrow E_N \approx 6\ \mu V/\sqrt{Hz}$$

(13) TF of amplifier, $H_1(f) = H_2(f) = \dfrac{A_0}{1+jf/f_B} = \dfrac{1}{1+jf/f_B}$

$$\therefore BW = \int_0^\infty |H_1(f)|^2 \cdot |H_2(f)|^2\, df$$

$$= \int_0^\infty \frac{1}{(1+f^2/f_B^2)^2}\, df$$

Take $f/f_B = x \Rightarrow df = f_B\, dx$

$$\Rightarrow BW = \int_0^\infty \frac{1}{(1+x^2)^2} f_B\, dx$$

Take $x = \tan\theta$ $\Rightarrow dx = \sec^2\theta\, d\theta$ | $x=0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$; $x=\infty \Rightarrow \tan\theta=\infty \Rightarrow \theta = \pi/2$

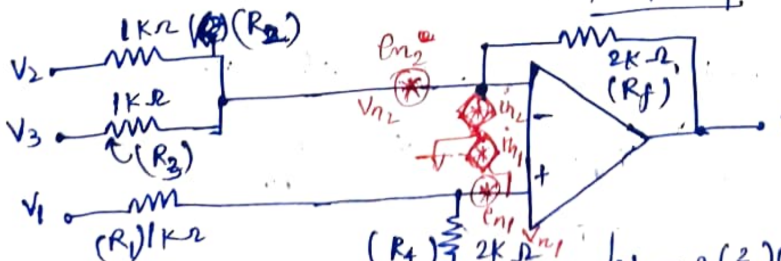
$$\Rightarrow BW = f_B \int_0^{\pi/2} \frac{1}{(1+\tan^2\theta)^2}\sec^2\theta\, d\theta$$

$$= f_B \int_0^{\pi/2} \frac{1}{\sec^2\theta}\, d\theta = f_B \int_0^{\pi/2} \cos^2\theta\, d\theta = f_B \int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\, d\theta\right)$$

$$= \frac{f_B}{2}\left[\theta + \sin 2\theta\right]_0^{\pi/2}$$

$$= f_B/2\,(\pi/2) = \boxed{\frac{f_B}{4}\cdot\pi}$$

(14)



$V_o = 2(V_1 - V_2 - V_3)$

$$V_o = 2\left(\frac{2}{3}\right)\left(\frac{2}{1}+1\right) - V_2\left(\frac{2}{1}\right) - V_3\left(\frac{2}{1}\right)$$

$$= 2V_1 - 2V_2 - 2V_3$$

$$V_o = \left(1+\frac{R_f}{R_2||R_3}\right)\left[V_{n1} - V_{n2} + I_{n1}(R_1||R_4) + I_{n2}(R_1||R_2||R_f)\right]$$

$$G_N = 1 + \frac{R_f}{R_2||R_3}$$

$$e_o^2 = G_N^2 \cdot e_{ni}^2$$

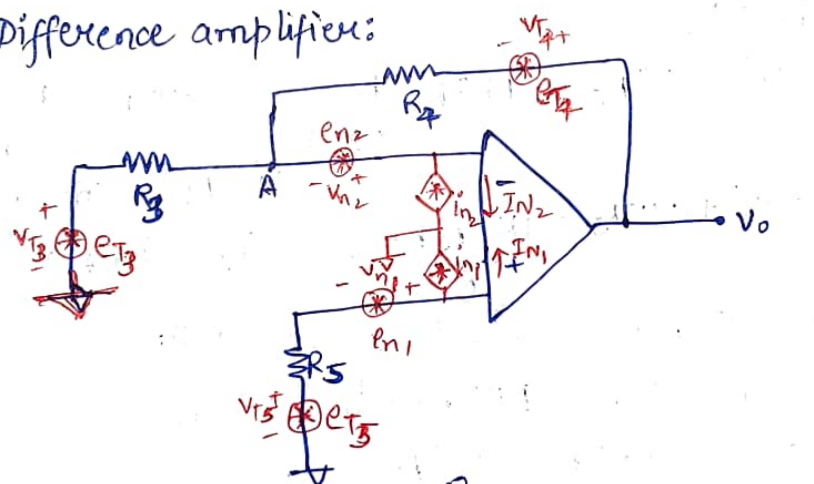
$$E_o^2 = \int_{f_L}^{f_H} e_o^2 \, df = \int_{f_L}^{f_H} G_N^2 \cdot e_{ni}^2 \, df$$

$$e_{ni}^2 = V_{n1}^2 + i_{n1}^2 \left[(R_1||R_4) + R_3||R_2||R_f\right]$$

$$e_{ni}^2 = e_{nw}^2\left(1+\frac{f_{CE}}{f}\right) + i_{nw}^2\left(1+\frac{f_{CE}}{f}\right)\left(R_1||R_4 + R_3||R_2||R_f\right)$$

$$\therefore E_o^2 = \int_{f_L}^{f_H} \frac{\left(1+\frac{R_f}{R_2||R_3}\right)^2}{\left(1+f^2/f_B^2\right)} \left[e_{nw}^2\left(1+\frac{f_{CE}}{f}\right) + i_{nw}^2\left(1+\frac{f_{CI}}{f}\right) \times \left(R_1||R_4 + R_3||R_2||R_4\right)\right] df$$

(15) (a) Difference amplifier:



$$V_p = V_{n1} + V_{T5} - I_{N1}R_5 \quad -(1)$$

$$V_n = V_{n2} + V_A \quad -(2)$$

KCL at node A:

$$\frac{V_A - V_{T3}}{R_3} + \frac{V_A - V_o + V_{T4}}{R_4} + I_{N2} = 0$$

$$\Rightarrow V_A\left[\frac{1}{R_3} + \frac{1}{R_4}\right] - \frac{V_{T3}}{R_3} + \frac{V_{T4}}{R_4} - \frac{V_o}{R_4} + I_{N2} = 0$$

As $V_A = V_n - V_{n2}$,

$$\Rightarrow (V_n - V_{n2})\left(\frac{1}{R_3} + \frac{1}{R_4}\right) - \frac{V_{T3}}{R_3} + \frac{V_{T4}}{R_4} - \frac{V_o}{R_4} + I_{N2} = 0$$

$$V_n = V_{n2} + \left(\frac{V_{T3}}{R_3} - \frac{V_{T4}}{R_4}\right)\left(\frac{R_3R_4}{R_3+R_4}\right) + \left(\frac{V_o}{R_4} - I_{N2}\right)\frac{R_3R_4}{R_3+R_4}$$

$$= V_{n2} + \frac{V_{T3}}{R_3} - \frac{V_{T4}}{R_4} + \frac{V_oR_3}{R_3+R_4} - I_{N2}\frac{R_3R_4}{R_3+R_4} \quad -(3)$$

Now, $v_o = A(v_p - v_n) \Rightarrow v_o/A = v_p - v_n$

If $A \to$ very large (∞), $v_o/A \to 0 \Rightarrow v_p = v_n$.

$$\therefore v_{n_1} + v_{T5} - I_{N_1} I_5 = v_{n_2} + \frac{v_{T_3} R_4}{R_3+R_4} - \frac{v_{T_4} R_3}{R_3+R_4} + \frac{v_o R_3}{R_3+R_4} - I_{N_2}(R_3 \| R_4)$$

$$\frac{v_o R_3}{R_3+R_4} = v_{n_1} - v_{n_2} + v_{T_5} - \frac{v_{T_3} R_4}{R_3+R_4} + \frac{v_{T_4} R_3}{R_3+R_4} - I_{N_2}(R_3 \| R_4) - I_{N_1} R_5$$

Also, $\dfrac{v_o}{1+R_4/R_3} = v_{ni}$

Noise gain, $G_N = 1 + R_3/R_4$.

$\Rightarrow v_o = G_N v_{ni}$

$e_o^2 = G_N^2 e_{ni}^2$

$$e_{ni}^2 = e_{n_1}^2 + e_{n_2}^2 + e_{T_5}^2 + e_{T_3}^2\left(\frac{R_4}{R_3+R_4}\right)^2 + e_{T_4}^2\left(\frac{R_3}{R_3+R_4}\right)^2 + i_{n_1}^2 R_5^2 + i_{n_2}^2 (R_3 \| R_4)^2$$

Take $e_n^2 = e_{n_1}^2 + e_{n_2}^2$

$i_n^2 = i_{n_1}^2 + i_{n_2}^2$

$$\Rightarrow e_{ni}^2 = e_n^2 + 4KTR_5 + 4KTR_3 \frac{R_4^2}{(R_3+R_4)^2} + 4KTR_4 \frac{R_3^2}{(R_3+R_4)^2} + i_n^2 R_5^2 + i_n^2 (R_3 \| R_4)^2$$

$$= e_n^2 + 4KTR_5 + 4KT\frac{R_3 R_4}{R_3+R_4} + i_n^2\left[R_5^2 + (R_3 \| R_4)^2\right]$$

For i/p current bias compensation, $R_5 = R_3 \| R_4$.

$$\Rightarrow e_{ni}^2 = e_n^2 + 4KTR_5 + 4KTR_5 + i_n^2(R_5^2 + R_5^2)$$

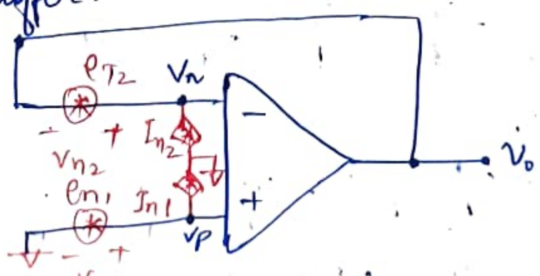
$$= e_n^2 + 8KTR_5 + 2i_n^2 R_5^2$$

Now, $e_o^2 = e_{ni}^2 G_N^2$

$$= G_N^2\left[e_n^2 + 8KTR_5 + 2i_n^2 R_5^2\right]$$

$$E_{No}^2 = \int_{f_L}^{f_H} e_o^2 \, df.$$

(b) Voltage buffer:



$v_p = v_{n_1}$ — (1)

$v_n = v_{n_2} + v_o$ — (2)

$v_o = A(v_p - v_n)$

$A \to \infty$, $\frac{v_o}{A} \to 0 \Rightarrow v_p = v_n$.

$\Rightarrow V_{n_1} = V_{n_2} + v_0$

$\Rightarrow V_0 = \underbrace{V_{n_1} - V_{n_2}}_{V_{ni}}$

$G_N = 1$

$V_0 = G_N V_{ni}$

$e_0^2 = G_N^2 e_{ni}^2$

$e_{ni}^2 = e_n^2 = e_{nw}^2 \left(1 + \frac{f_{CE}}{f}\right)$

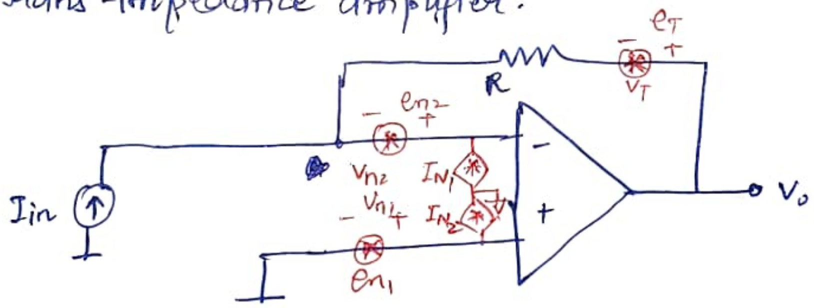
$e_0^2 = G_N^2 e_n^2$

$$E_{NO}^2 = \int_{f_L}^{f_H} e_0^2\, df = \int_{f_L}^{f_H} G_N^2 e_{nw}^2 \left(1 + \frac{f_{CE}}{f}\right) df$$

$$= G_N^2 e_{nw}^2 \int_{f_L}^{f_H} \left(1 + \frac{f_{CE}}{f}\right) df$$

$$= G_N^2 e_{nw}^2 \left[(f_H - f_L) + f_{CE} \ln\left(\frac{f_H}{f_L}\right)\right]$$

(c) Trans-impedance amplifier:



$V_p = v_{n_1}$ —(1)

$V_n = V_{n_2} + V_A$ —(2)

KCL at (A) $\Rightarrow \dfrac{V_A - V_0 + V_T}{R} + I_{N_2} = 0$

$\Rightarrow \dfrac{V_A}{R} - \dfrac{V_0}{R} + \dfrac{V_T}{R} + I_{N_2} = 0$ $\quad [V_A = V_n - V_{n2}$ from (2)$]$

$\Rightarrow \dfrac{V_n - V_{n2}}{R} - \dfrac{V_0}{R} + \dfrac{V_T}{R} + I_{N2} = 0$

$\Rightarrow V_n = V_{n2} + V_0 - V_T + I_{N_2} R$

If $A \to \infty \Rightarrow \frac{V_0}{A} \to 0$.

$\Rightarrow V_p = V_n$

$\Rightarrow V_{n_1} = \underbrace{V_{n_2} + V_0 - V_T + I_{N_1} R}_{V_{ni}}$

$e_0^2 = G_N^2 e_{ni}^2, \quad G_N = 1$

$e_{ni}^2 = e_{n_1}^2 + e_{n_2}^2 + e_T^2 + i_{n_2}^2 R^2$

Take $e_{n_1}^2 + e_{n_2}^2 = e_n^2$; $i_n^2 = i_n^2$

$\Rightarrow e_{ni}^2 = e_n^2 + 4KTR + i_n^2 R^2$

$$E_0^2 = \int_{f_L}^{f_H} e_0^2\, df = \int_{f_L}^{f_H} G_N^2 \left[e_{nw}^2\left(1 + \frac{f_{CE}}{f}\right) + 4KTR + i_{nw}^2\left(1 + \frac{f_{CE}}{f}\right) R^2\right] df.$$