AVD 611- Modern Signal Processing

Tutorial-1

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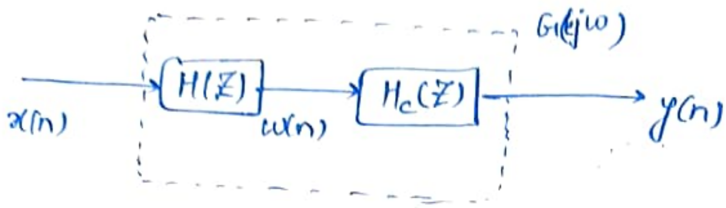


(a) Since minimum phase system and its inverse are stable and causal, and any system can be decomposed into min^m phase system and all-pass filter.

$$H(Z) = H_{ap}(Z) \cdot H_{min\text{-}phase}(Z)$$

$$\Rightarrow H_c(Z) = \frac{1}{H_{min\text{-}phase}(Z)}$$

(b) $$H_c(Z) = \frac{1}{H_{min\text{-}phase}(Z)}$$

and,

$$G(Z) = H(Z)\, H_c(Z)$$
$$= \{H_{ap}(Z) \cdot H_{min\text{-}phase}(Z)\} \cdot \frac{1}{H_{min\text{-}phase}(Z)}$$
$$= H_{ap}(Z)$$

with $|G(z)| = 1$.

(c) $$H(Z) = (1-0.8e^{j0.3\pi}z^{-1})(1-0.8e^{-j0.3\pi}z^{-1})(1-1.2e^{j0.7\pi}z^{-1})(1-1.2e^{-j0.7\pi}z^{-1})$$

$$\Rightarrow H(Z) = H(Z) \cdot \frac{(1-\frac{1}{1.2}e^{j0.7\pi}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})}{(1-\frac{1}{1.2}e^{j0.7}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})}$$

$$= \underbrace{(1-0.8e^{j0.3\pi}z^{-1})(1-0.8e^{-j0.3\pi}z^{-1})(1-\frac{1}{1.2}e^{j0.7\pi}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})}_{H_{min}(Z)} \times \underbrace{\left\{\frac{(1-1.2e^{j0.7\pi}z^{-1})(1-1.2e^{-j0.7\pi}z^{-1})}{(1-\frac{1}{1.2}e^{j0.7\pi}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})}\right.}_{H_{ap}(Z)}$$

$$\therefore H_{min}(Z) = (1-0.8e^{j0.3\pi}z^{-1})(1-0.8e^{-j0.3\pi}z^{-1})(1-\frac{1}{1.2}e^{j0.7\pi}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})$$

$$H_{ap}(Z) = \frac{(1-1.2e^{j0.7\pi}z^{-1})(1-1.2e^{-j0.7\pi}z^{-1})}{(1-\frac{1}{1.2}e^{j0.7\pi}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})}$$

As $H_c(z) = \dfrac{1}{H_{min}(z)}$

$$= \frac{1}{(1-0.8e^{j0.3\pi}z^{-1})(1-0.8e^{-j0.3\pi}z^{-1})(1-\frac{1}{1.2}e^{j0.7\pi}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})}$$

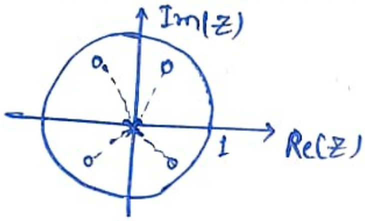
and, $G(z) = H_{ap}(z)$

$$= \frac{(1-1.2e^{j0.7\pi}z^{-1})(1-1.2e^{-j0.7\pi}z^{-1})}{(1-\frac{1}{1.2}e^{j0.7\pi}z^{-1})(1-\frac{1}{1.2}e^{-j0.7\pi}z^{-1})}$$

Pole-zero plot :

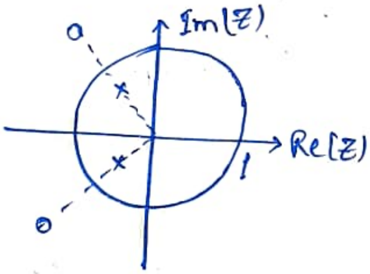
$H_{min}(z)$: Zeros : $0.8e^{\pm j0.3\pi}$, $\frac{1}{1.2}e^{\pm j0.7\pi} = 0.833e^{\pm j0.7\pi}$

Poles : $z = 0, 0, 0, 0$



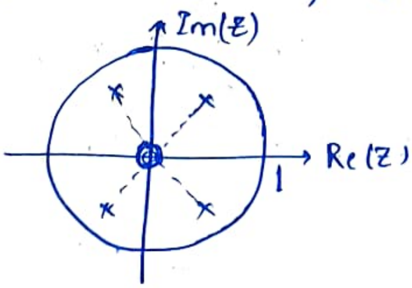
$H_{ap}(z)$ or $G(z)$: Zeros : $1.2e^{\pm j0.7\pi}$

Poles : $0.833e^{\pm j0.7\pi}$



$H_c(z)$: Zeros : $z = 0, 0, 0, 0$

Poles : $0.8e^{\pm j0.3\pi}$, $0.833e^{\pm j0.7\pi}$



(2) Since we don't require causality and stability simultaneously with the linear phase, the statement is false, as zero-phase are properties of the frequency response on the unit circle.

Example: $P(z) = \dfrac{1}{1-0.5z^{-1}}$

$H(z) = P(z)\,P(z^{-1})$

$= \dfrac{1}{(1-0.5z^{-1})(1-0.5z)}$ → Pole at $z = 0.5, 2$

and $\angle H(e^{j\omega}) = 0$.

③ $y(n) - y(n-1) + \frac{1}{4}y(n-2) = x(n)$

$\Rightarrow Y(z) - z^{-1}Y(z) + \frac{1}{4}z^{-2}Y(z) = X(z)$

$\Rightarrow H_1(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1}{1-z^{-1}+\frac{1}{4}z^{-2}}$

$= \dfrac{4}{(z^{-1}-2)^2}$

$H_1(e^{j\omega}) = \dfrac{4}{(e^{-j\omega}-2)^2}$

At $\omega = 0 \Rightarrow H_1(e^{j0}) = \frac{4}{1} = 4$

At $\omega = \pi$, $\Rightarrow H_1(e^{j\pi}) = \frac{4}{9} \approx 0.444 \ll \pi$

↳ Passes low frequency ⇒ LPF.

Now,

$H_2(e^{j\omega}) = H_1(-e^{j\omega}) = H_1(e^{j(\omega+\pi)})$

↳ Frequency shifted-version of H_1 by π ⇒ High-pass Filter

$\therefore S_2 \rightarrow$ HPF.

④ $X(z) = \dfrac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1-\frac{1}{5}z^{-1})}{(1-\frac{1}{6}z^{-1})}$

As $\mathcal{Z}\{\alpha^n x(n)\} = X(z/\alpha) = Y(z)$,

zeros of $X(z)$ (z_0) move to αz_0, when multiplying the sequence by α^n.

$$\therefore \; Y(Z) = X(Z/\alpha) = \frac{\left(1-\frac{\alpha}{2}Z^{-1}\right)\left(1-\frac{\alpha}{4}Z^{-1}\right)\left(1-\frac{\alpha}{5}Z^{-1}\right)}{\left(1-\frac{\alpha}{6}Z^{-1}\right)}$$

For real and min^m phase, all zeros must lie inside the unit circle.

$$\left|\frac{\alpha}{2}\right|, \left|\frac{\alpha}{4}\right|, \left|\frac{\alpha}{5}\right| < 1$$

$$\Rightarrow |\alpha| < 2$$

$$\therefore \boxed{\alpha \in (-2, 2)}.$$

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$$x(n) \longrightarrow \boxed{h(n)} \xrightarrow{w(n)} \boxed{h_i(n)} \longrightarrow y(n)$$

$$h(n) = \delta(n) + 2\delta(n-1)$$

$$\Rightarrow H(Z) = 1 + 2Z^{-1}$$

$$\therefore \; H_i(Z) = \frac{1}{H(Z)} = \frac{1}{1+2Z^{-1}} \rightarrow \text{ROC} : |Z| > 2$$

(a) Since ROC does not contain the unit - circle, it is unstable, but causal.

(b) $h(n) = \delta(n) + \alpha\delta(n-1)$

$$\Rightarrow H(Z) = 1 + \alpha Z^{-1}$$

$$\Rightarrow H_i(Z) = \frac{1}{1+\alpha Z^{-1}} \rightarrow \text{Pole at } z = -\alpha.$$

For causality and stability, pole should be inside the unit circle,

i.e., $|\alpha| < 1$

$$\Rightarrow \boxed{\alpha \in (-1, 1)}.$$

6 $H(Z) = \dfrac{(1+0.2Z^{-1})(1-9Z^{-2})}{(1+0.81Z^{-2})}$

(a) Poles: $Z^2 = -0.81 \Rightarrow Z = \pm j\sqrt{0.81} = \pm j0.9$

$\downarrow$

Inside the unit circle

$\Downarrow$

Stable

6 (b) $H(z) = \dfrac{(1+0.2z^{-1})(1-3z^{-1})(1+3z^{-1})}{(1-0.9z^{-1})(1+0.9z^{-1})} \times \dfrac{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}$

$$= \underbrace{\frac{(1+0.2z^{-1})(1-\frac{1}{9}z^{-1})}{(1-0.81z^{-2})}}_{H_1(z)} \cdot \underbrace{\frac{(1-9z^{-2})}{(1-\frac{1}{9}z^{-2})}}_{H_{ap}(z)}.$$

$$\therefore H_1(z) = \frac{(1+0.2z^{-1})(1-\frac{1}{9}z^{-1})}{(1-0.81z^{-2})}$$

$$H_{ap}(z) = \frac{(1-9z^{-2})}{(1-\frac{1}{9}z^{-2})}.$$

7 $x(n) = \left(\frac{1}{2}\right)^{|n-1|} + \left(\frac{1}{2}\right)^{|n|}$

Let $X(z) = X_1(z) + X_2(z)$

$$X_2(z) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} + \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n z^n$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{2}z}{1-\frac{1}{2}z}$$

$$= \frac{1-\cancel{\frac{1}{2}z}+\frac{1}{2}z(\cancel{1}-\frac{1}{2}z^{-1})}{1+\frac{1}{4}-\frac{1}{2}(z+z^{-1})}$$

$$= \frac{1-\frac{1}{4}}{1+\frac{1}{4}-\frac{1}{2}(z+z^{-1})}$$

$$= \frac{3/4}{5/4-\frac{1}{2}(z+z^{-1})}$$

$$X_1(z) = z^{-1}X_2(z)$$

$$\therefore X(z) = X_2(z)(1+z^{-1})$$

$$\Rightarrow X(e^{j\omega}) = X_2(e^{j\omega})(1+e^{-j\omega})$$

$$= (1+e^{-j\omega})\frac{3/4}{5/4-\frac{1}{2}(e^{j\omega}+e^{-j\omega})}$$

$$= (1+e^{-j\omega}) \times \frac{3/4}{5/4 - \frac{1}{2}(2\cos\omega)}$$

$$= (1+e^{-j\omega}) \left(\frac{3/4}{5/4 - \cos\omega}\right)$$

$$= e^{-j\omega/2} \underbrace{\left(e^{j\omega/2} + e^{-j\omega/2}\right)}_{2\cos(\omega/2)} \left(\frac{3/4}{5/4 - \cos\omega}\right)$$

$$= e^{-j\omega/2} \frac{\left(\frac{3}{2}\cos\frac{\omega}{2}\right)}{\left(\frac{5}{4} - \cos\omega\right)}$$

$$|X(e^{j\omega})| = \frac{\left|\frac{3}{2}\cos\frac{\omega}{2}\right|}{\left|\frac{5}{4} - \cos\omega\right|}, \quad \angle X(e^{j\omega}) = -\frac{\omega}{2}.$$

Group delay,

$$\tau_g = -\frac{d}{d\omega}(-\omega/2) = \boxed{\frac{1}{2}}.$$