INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

Assignment I

B.Tech - IV Semester

MA221 - Integral Transforms, PDE and Calculus of Variations

Max. Marks: ~~5~~ 6

1. Let $f : [0,l] \to \mathbb{R}$ be a continuous function such that $f(0) = f(l) = 0$ and $f'(x)$ is piecewise continuous in $(0,l)$.

   Show that for given $k > 0$ and any $t > 0$ and $0 \leq x \leq l$, the series below is convergent.

   $$u(x,t) = \sum_{n=1}^{\infty} a_n e^{-(n\pi/l)^2 kt} \sin\left(\frac{n\pi x}{l}\right)$$

   where,

   $$a_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)\, dx\,.$$

   Hence, show that $u(x,t)$ as defined above satisfies the initial boundary-value problem-

   $$\begin{aligned} u_t &= ku_{xx} & & t > 0\ ,\ \ 0 < x < l \\ u(0,t) &= 0 & & t \geq 0 \\ u(l,t) &= 0 & & t \geq 0 \\ u(x,0) &= f(x) & & 0 \leq x \leq l\,. \end{aligned}$$

   [2 marks]

2. Let $C : (x_0(t), y_0(t))$ be a curve in (xy) - plane with $({x_0}')^2 + ({y_0}')^2 \neq 0$. Consider the following problem: find u such that

   $$\left.\begin{aligned} a(x,y,u)u_x + b(x,y,u)u_y &= c(x,y,u), \\ u(x_0(t), y_0(t)) &= u_0(t). \end{aligned}\right\} \qquad (1)$$

   Now suppose $x_0(t), y_0(t)$ and $u_0(t)$ are continuously differentiable function of t in a closed interval $0 \leq t \leq 1$ and a, b and c are functions of x, y and u with continuous first-order partial derivatives in some domain D in (x,y,u)- space containing the initial curve $\Gamma\ :\ (x_0(t), y_0(t), u_0(t), 0 \leq t \leq 1$, and also satisfy

   $${y_0}'(t)\ a(x_0(t), y_0(t), u_0(t)) - {x_0}'(t)\ b(x_0(t), y_0(t), u_0(t)) \neq 0$$

   Then show that there exists a unique solution of (1) in the neighbourhood of C.

   [2 marks]

3. A thin rectangular homogeneous thermally conducting plate lies in the xy - plane defined by $0 \leq x \leq a, 0 \leq y \leq b$. The edge $y = 0$ is held at the temperature $Tx(x-a)$, where T is a constant, while the remaining edges are held at $0°$. The other faces are insulated and no internal sources and sinks are present. Find the steady state temperature inside the plate.

   (2 Marks) [~~1 mark~~]

END