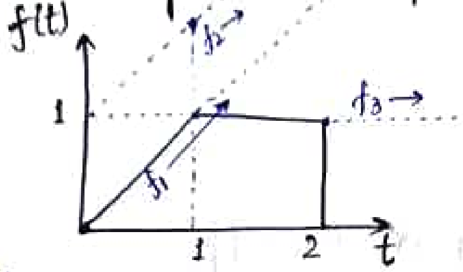
AV 213 - Network Analysis
Assignment-2

Saurabh Kumar
SC22B146

Drill Problems - Set-1

Q Find Laplace Transform of the following:

(1)



Soln:

$$f(t) = f_1(t) - f_2(t) - f_3(t)$$

$$= t(u(t)) - [t\,u(t-1) - u(t-1)] - u(t-2)$$

$$= t\,u(t) - t\,u(t-1) + u(t-1) - u(t-2), \text{ where}$$

$u(t-a) = 1$, for $t \geq a$
$= 0$, otherwise

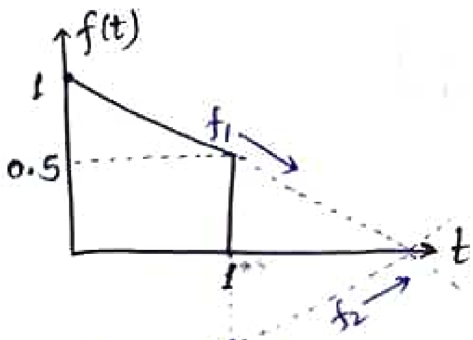
$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t\,u(t)\} - \mathcal{L}\{t\,u(t-1)\} - \mathcal{L}\{u(t-1)\} - \mathcal{L}\{u(t-2)\}$$

$$= \frac{1}{s^2} - \mathcal{L}\{u(t-1)\cdot(t-1)\} - \mathcal{L}\{u(t-2)\}$$

$$= \frac{1}{s^2} - e^{-s}\mathcal{L}\{t\} - \frac{e^{-2s}}{s} \qquad \left[\because \mathcal{L}\{u(t-a)F(t-a)\} = e^{-as}F(s),\ \mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}\right]$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$

(2)



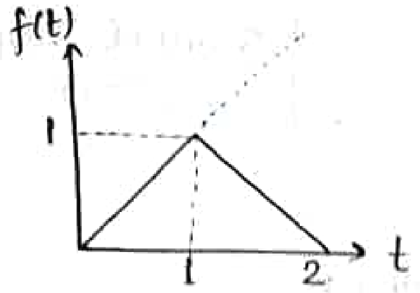
Soln:

$$f(t) = \left[-\frac{t}{2}u(t) + u(t)\right] + \left[\frac{t}{2}u(t-1) - u(t-1)\right] (= f_1 - f_2)$$

$$\mathcal{L}\{f(t)\} = -\frac{1}{2}\mathcal{L}\{t\} + \mathcal{L}\{u(t)\} + \frac{1}{2}\mathcal{L}\{(t-1)u(t-1)\} - \frac{1}{2}\mathcal{L}\{u(t-1)\}$$

$$= -\frac{1}{2s^2} + \frac{1}{s} + \frac{1}{2}\frac{e^{-s}}{s^2} - \frac{1}{2}\frac{e^{-s}}{s} = \frac{e^{-s} - 1 + 2s - s e^{-s}}{2s^2}$$

③



Soln:

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ -(t-2), & 1 \le t \le 2 \\ 0, & t > 2 \end{cases}$$

$$f(t) = t\left[u(t) - u(t-1)\right] - (t-2)\left[u(t-1) - u(t-2)\right]$$

$$= t\,u(t) - 2(t-1)\,u(t-1) + (t-2)\,u(t-2)$$

$$\mathcal{L}\{f(t)\} = \frac{e^{-s}}{s} - 2\frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

Q. Solve for $y(t)$ using Laplace Transform of the following:

④ $y'' + 0.04y = 0.02t^2$; $y(0) = -25$; $y'(0) = 0$.

Soln: $\mathcal{L}\{y''\} + 0.04\,\mathcal{L}\{y\} = 0.02\,\mathcal{L}\{t^2\}$

$$\Rightarrow s^2 Y(s) - s\,y(0) - y'(0) + 0.04\,Y(s) = 0.02 \times \frac{2!}{s^{2+1}}$$

$$\Rightarrow Y(s)\{s^2 + 0.04\} + 25s - 0 = \frac{0.04}{s^3}$$

$$\Rightarrow Y(s) = \left(\frac{0.04}{s^3} - 25s\right)\left(\frac{1}{s^2 + 0.04}\right)$$

$$= \frac{0.04 - 25s^4}{(s^2 + 0.04)s^3}$$

$$= \frac{\frac{1}{25} - 25s^4}{s^3\left(s^2 + \frac{1}{25}\right)}$$

$$= \frac{1 - 25^2 s^4}{s^3(25s^2 + 1)} = \frac{1 - 25s^2}{s^3} = \frac{1}{s^3} - \frac{25}{s}$$

$$L^{-1}Y(s) = L^{-1}\left\{\frac{1}{s^3}\right\} - L^{-1}\left\{\frac{25}{s}\right\}$$

$$\Rightarrow y(t) = \frac{1}{2}t^2 - 25 \qquad \left[\because L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}\right]$$

⑤ $y'' + 3y' + 2.25y = 9t^3 + 64$; $y(0) = 1$; $y'(0) = 31.5$

Soln: $L\{y''\} + 3L\{y'\} + 2.25\,L\{y\} = 9L\{t^3\} + L\{64\}$

$$\Rightarrow s^2Y(s) - s\,y(0) - y'(0) + 3sY(s) - y(0) + 2.25\,Y(s) = 9\left(\frac{6}{s^4}\right) + \frac{64}{s}$$

$$\Rightarrow Y(s)(s^2 + 3s + 2.25) - s - 31.5 = \frac{54}{s^4} + \frac{64}{s}$$

$$\Rightarrow Y(s)\left[(s+1.5)^2\right] = \frac{54}{s^4} + \frac{64}{s} + s + 31.5$$

$$\Rightarrow Y(s) = \frac{54}{s^4(s+1.5)^2} + \frac{64}{s(s+1.5)^2} + \frac{s}{(s+1.5)^2} + \frac{31.5}{(s+1.5)^2}$$

$$= \frac{4s^5 + 138s^4 + 256s^3 + 216}{s^4(2s+3)^2}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{Es+F}{(2s+3)^2}$$

$$\Rightarrow 4s^5 + 138s^4 + 256s^3 + 216$$
$$= As^3(4s^2+12s+9) + Bs^2(4s^2+12s+9) + Cs(4s^2+12s+9) + D(4s^2+12s+9) + (Es+F)s^4$$

$\Rightarrow 4A + E = 4$, $12A + 4B + F = 138$, $9A + 12B + 4C = 256$,
$9B + 12C + 4D = 0$, $9C + 12D = 0$, $9D = 216$

$\Rightarrow$ $D = 24$
$C = -32$
$B = 32$
$A = 0$
$F = 10$
$E = 4$.

$$\therefore Y(s) = \frac{32}{s^2} - \frac{32}{s^3} + \frac{24}{s^4} + \frac{4s+10}{4s^2+12s+9}$$

$$= \frac{32}{s^2} - 16\left(\frac{2}{s^3}\right) + 4\left(\frac{6}{s^4}\right) + \frac{1}{s+1.5} + \frac{1}{(s+1.5)^2}$$

$$\therefore y(t) = \mathcal{L}^{-1}\{Y(s)\} = 32t - 16t^2 + 4t^3 + e^{-3t/2} + te^{-3t/2}$$

⑥ $y'' + 2y' + 5y = 50t - 100$; $y(2) = -4$; $y'(2) = 14$.

Soln: $s^2Y(s) - sy(0) - y'(0) + 2sY(s) - y(0) + 5Y(s) = \frac{50}{s^2} - \frac{100}{s}$ $\longrightarrow \times$

Let $y(t) = u(t-2)$ $\Rightarrow \begin{cases} u(0) = -4 \\ u'(0) = 14. \end{cases}$

Let $t = \tau + 2 \Rightarrow y(t) = u(\tau)$.

$\therefore u'' + 2u' + 5u(\tau) = 50(\tau+2) - 100$

$= 50\tau$

$\Rightarrow s^2U(s) - su(0) - u'(0) + 2[sU(s) - u(0)] + 5U(s) = \frac{50}{s^2}$

$\Rightarrow U(s)[s^2 + 2s + 5] + 4s - 14 + 8 = \frac{50}{s^2}$

$$\Rightarrow U(s) = \frac{-4s^3 - 6s^2 + 50}{s^2(s^2+2s+5)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+2s+5}$$

$\Rightarrow -4s^3 + 6s^2 + 50 = As(s^2+2s+5) + B(s^2+2s+5) + (Cs+D)s^2$

$\Rightarrow -4 = A + C \Rightarrow C = 0$

$6 = 2A + B + D \Rightarrow D = 4$

$0 = 5A + 2B \Rightarrow A = -4$

$5B = 50 \Rightarrow B = 10$

$$\therefore U(s) = -\frac{4}{s} + \frac{10}{s^2} + \frac{4}{(s+1)^2 + 2^2}$$

$\Rightarrow u(\tau) = -4 + 10\tau + 2e^{-\tau}\sin(2\tau)$

$\Rightarrow y(t) = u(\tau - 2) = -4 + 10(t-2) + 2e^{-(t-2)}\sin(2(t-2))$

$= 10t - 24 + 2e^{-t+4}\sin(2t-4)$

① $y'' + 3y' + 2y = 4t, \text{ if } 0 \le t < 1, \; y(0) = 0.$
$\qquad\qquad\qquad = 8, \text{ if } t \ge 1, \; y'(0) = 0.$

Soln: $\mathcal{L}\{y''\} + 3\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = 4\int_0^1 t e^{-st}\,dt + 8\int_1^\infty e^{-st}\,dt.$

$$\Rightarrow s^2 Y(s) - s\cancelto{0}{y(0)} - \cancelto{0}{y'(0)} + 3sY(s) - 3y(0) + 2Y(s)$$

$$= \mathcal{L}\{4t\,(u(t) - u(t-1)) + 8\,(u(t-1))\}$$

$$= 4\frac{1}{s^2} - 4\frac{e^{-s}}{s^2} - 4\frac{e^{-s}}{s} + 8\frac{e^{-s}}{s}$$

$$= \frac{4}{s^2} - \frac{4e^{-s}}{s^2} + \frac{4e^{-s}}{s}$$

$$\Rightarrow Y(s)\left[s^2 + 3s + 2\right] = \frac{4 - 4e^{-s} + 4se^{-s}}{s^2}$$

$$\Rightarrow Y(s) = 4\left[\frac{1 + (s-1)e^{-s}}{s^2(s+1)(s+2)}\right]$$

$$= 4\frac{(s-1)}{s^2(s+1)(s+2)}e^{-s} + 4\frac{1}{s^2(s+1)(s+2)} = 4J(s)e^{-s} + 4K(s)$$

$$J(s) = \frac{s-1}{s^2(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+2}$$

$$\Rightarrow (s-1) = As(s+1)(s+2) + B(s+1)(s+2) + Cs^2(s+2) + Ds^2(s+1)$$

$\Rightarrow 3A + B + 2C + D = 0 \Rightarrow 2C + D + 13/4 = 0$

$2A + 3B = 1 \Rightarrow A = 5/4$

$2B = -1 \Rightarrow B = -1/2$

$A + C + D = 0 \Rightarrow \quad C = -2, \; D = 3/4$

$$\therefore J(s) = \frac{s-1}{s^2(s+1)(s+2)} =$$

$$= \frac{5}{4s} - \frac{1}{2s^2} - \frac{2}{s+1} + \frac{3}{4(s+2)}$$

$$\Rightarrow j(t) = \mathcal{L}^{-1}\{J(s)\} = \frac{5}{4} - \frac{t}{2} - 2e^{-t} + \frac{3}{4}e^{-2t}$$

and,

$$K(s)=\frac{1}{s^2(s+1)(s+2)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{D}{s+2}$$

$$\Rightarrow 1= As(s^2+3s+2)+B(s^2+3s+2)+Cs(s+2)+D(s+1)s$$

$$\Rightarrow \frac{1}{2}+C+D=0 \Rightarrow C=-1$$
$$D=\frac{1}{2}$$
$$\frac{3}{2}+2C+D=0$$
$$A=0$$
$$B=\frac{1}{2}$$

$$\therefore K(s)=\frac{1}{s^2(s+1)(s+2)}=\frac{1}{2s^2}-\frac{1}{s+1}+\frac{1}{2(s+1)}=\frac{1}{2s^2}-\frac{1}{2(s+1)}$$

$$\Rightarrow k(t)=\mathcal{L}^{-1}\{K(s)\}=\frac{t}{2}-\frac{e^{-t}}{2}$$

$$\therefore Y(s)=4J(s)e^{-s}+4K(s)$$

$$\Rightarrow y(t)=\mathcal{L}^{-1}\{Y(s)\}=4j(t-1).u(t-1)+4k(t)$$
$$=4u(t-1)\left(\frac{5}{4}-\frac{t-1}{2}-2e^{-(t-1)}+\frac{3}{4}e^{-2(t-1)}\right)+\left(\frac{t}{2}-\frac{e^{-t}}{2}\right)4$$

$$\Rightarrow y(t)=\left(3e^{-2(t-1)}-8e^{-(t-1)}-2t+7\right)u(t-1)+2t-2e^{-t}.$$

(8) $y''+2y'+5y=10\sin t$, if $0\le t<2\pi$, $y(\pi)=1$
$=0$, if $t\geq 2\pi$, $y'(\pi)=2e^{-\pi}-2$.

Soln: Let $\tau=t-\pi \Rightarrow t=\tau+\pi$

$y(t)=y(\tau+\pi)=u(\tau)$ (say)

$\Rightarrow y(t)=u(t-\pi)$

$$\therefore u''+2u'+5u=\underbrace{10\sin(\tau+\pi)}_{=-10\sin\tau}, \text{ if } -\pi\le\tau<\pi$$
$$=0, \text{ if } \tau\geq\pi.$$

$$u(0)=1,\ u'(0)=2e^{-\pi}-2.$$

Taking laplace transform,

$$s^2U(s)-su(0)-u'(0)+2(sU(s))-2u(0)+5U(s)$$
$$=-10\int_0^{\pi}\sin\tau\, e^{-s\tau}d\tau+\int_{\pi}^{\infty}0.e^{-s\tau}d\tau.$$

$$\Rightarrow U(s)\left(s^2+2s+5\right)-(s+2)-2e^{-\pi}+2=-10\,\frac{e^{-\pi s}}{s^2+1}-\frac{10}{s^2+1}$$

$$\Rightarrow U(s)=\frac{s}{s^2+2s+5}+\frac{2e^{-\pi}}{s^2+2s+5}-10\,\frac{e^{-\pi s}}{(s^2+1)(s^2+2s+5)}-\frac{10}{(s^2+1)(s^2+2s+5)}$$

$$=G(s)+H(s)\,e^{-\pi}-10\,I(s)\,e^{-\pi s}-10\,I(s)$$

$$G(s)=\frac{s}{(s+1)^2+2^2}\Rightarrow g(t)=\mathcal{L}^{-1}\{G(s)\}=e^{-t}\cos(2t)$$

$$H(s)=\frac{2}{s^2+2s+5}=\frac{2}{(s+1)^2+2^2}\Rightarrow h(t)=\mathcal{L}^{-1}\{H(s)\}=e^{-t}\sin(2t)$$

$$I(s)=\frac{1}{(s^2+1)(s^2+2s+5)}=\frac{As+B}{s^2+1}+\frac{Cs+D}{s^2+2s+5}$$

$$\Rightarrow 1=(As+B)(s^2+2s+5)+(Cs+D)(s^2+1)$$

$$\Rightarrow \left.\begin{aligned} &A+C=0\\ &2A+B+D=0\\ &5A+2B+C=0\\ &5B+D=1\end{aligned}\right\}\Rightarrow \begin{aligned} &A=-1/10\\ &B=1/5\\ &C=1/10\\ &D=0\end{aligned}$$

$$\therefore I(s)=\left(-\frac{s}{10}+\frac{1}{5}\right)\frac{1}{s^2+1}+\frac{1}{10}\cdot\frac{1}{(s+1)^2+2^2}$$

$$\Rightarrow i(t)=\mathcal{L}^{-1}\{I(s)\}=-\frac{\cos t}{10}+\frac{\sin t}{5}+\frac{e^{-t}\cos(2t)}{10}$$

$$\therefore u(t)=\mathcal{L}^{-1}\{U(s)\}=g(\tau)+e^{-\pi}h(\tau)-10\,i(\tau-\pi)\,u(\tau-\pi)-10\,i(\tau)$$

$$=e^{-\tau}\cos(2\tau)+e^{-(\tau+\pi)}\sin(2\tau)$$

$$-10u(\tau-\pi)\left(-\cos(\tau-\pi)+2\sin(\tau-\pi)+e^{-(\tau-\pi)}\cos 2(\tau-\pi)\right)$$

$$-\left(-\cos\tau+2\sin\tau+e^{-\tau}\cos 2\tau\right)$$

$$\therefore y(t)=u(t-\pi)=e^{-(t-\pi)}\cos(2t)+e^{-t}\sin(2t)$$

$$-u(t-\pi)\left(-\cos t+2\sin t+e^{-(t-2\pi)}\cos(2t)\right)$$

$$-\cos t+2\sin t-e^{-(t-\pi)}\cos(2t).$$

⑨ $y'' + 4y = \delta(t-\pi)$; $y(0) = 8$; $y'(0) = 0$.

Soln: $s^2 Y(s) - s\,y(0) - y'(0) + 4\,Y(s) = \mathcal{L}\{\delta(t-\pi)\} = 1$

$$\Rightarrow (s^2+4)\,Y(s) - 8s - 0 = 1$$

$$\Rightarrow Y(s) = \frac{1+8s}{s^2+2^2} = \frac{1}{2}\left(\frac{2}{s^2+2^2}\right) + 8\left(\frac{s}{s^2+2^2}\right)$$

$$\therefore y(t) = \mathcal{L}^{-1}\{Y(s)\} = 8\cos(2t) + \frac{\sin(2t)}{2}.$$

⑩ $y'' + 3y' + 2y = 10\,[\sin t + \delta(t-1)]$; $y(0) = 1$; $y'(0) = -1$.

Soln: Taking Laplace transform on both sides,

$$s^2 Y(s) - s\,y(0) - y'(0) + 3s\,Y(s) - 3y(0) + 2Y(s) = \frac{10}{s^2+1} + 10e^{-s}$$

$$\Rightarrow (s^2+3s+2)\,Y(s) - (s+3) + 1 = \frac{10}{s^2+1} + 10e^{-s}$$

$$\Rightarrow (s^2+3s+2)\,Y(s) = \frac{10 + 10e^{-s}(s^2+1) + (s+2)(s^2+1)}{(s^2+1)}$$

$$\Rightarrow Y(s) = \frac{10}{(s^2+1)(s^2+3s+2)} + \frac{10e^{-s}}{(s^2+3s+2)} + \frac{s^3+2s^2+s+12}{(s^2+1)(s^2+3s+2)}$$

$$= 10\,K(s)\,e^{-s} + G(s).$$

$$K(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$\Rightarrow 1 = (s+2)A + (s+1)B$$

$$\Rightarrow \left.\begin{aligned} A+B &= 0 \\ 2A+B &= 1 \end{aligned}\right\} \begin{aligned} A &= 1 \\ B &= -1 \end{aligned}$$

$$\therefore K(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\Rightarrow k(t) = \mathcal{L}^{-1}\{K(s)\} = e^{-t} - e^{-2t}$$

and, $G(s) = \dfrac{s^3+2s^2+s+12}{(s^2+1)(s^2+3s+2)} = \dfrac{As+B}{s^2+1} + \dfrac{C}{s+1} + \dfrac{D}{s+2}$

$$\Rightarrow s^3 + 2s^2 + s + 12 = (As+B)(s^2+3s+2) + (s^2+1)(s+2)C + (s^2+1)(s+1)D$$
$$= (A+C+D)s^3 + (3A+B+2C+D)s^2 + (2A+3B+C+D)s + (2B+2C+D)$$

$$\Rightarrow \left.\begin{array}{l} A+C+D=1 \\ 3A+B+2C+D=2 \\ 2A+3B+C+D=1 \\ 2B+2C+D=12 \end{array}\right\} \Rightarrow \begin{array}{l} A=-3 \\ B=1 \\ C=6 \\ D=2 \end{array}$$

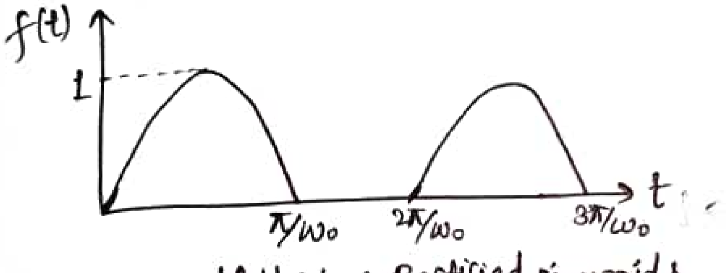
$$\therefore G(s) = -\frac{3s}{s^2+1} + \frac{1}{s^2+1} + \frac{6}{s+1} - \frac{2}{s+2}$$

$$\Rightarrow g(t) = \mathcal{L}^{-1}\{G(s)\} = -3\cos t + \sin t + 6e^{-t} - 2e^{-2t}.$$

$$\therefore y(t) = \mathcal{L}^{-1}\{Y(s)\} = g(t) + 10k(t-1)u(t-1)$$
$$= 6e^{-t} - 2e^{-2t} - 3\cos t + \sin t + 10u(t-1)\left(e^{-(t-1)} - e^{-2(t-1)}\right).$$

Q Find $F(s)$ for the periodic waveforms $f(t)$ in the problems (11) to (14).

(11)



Half wave Rectified sinusoids

Period $= 2\pi/\omega_0$

$$\mathcal{L}\{f(t)\} = \frac{\int_0^{2\pi/\omega_0} e^{-st} f(t)\,dt}{1-e^{-(2\pi/\omega_0)s}} = \frac{F_1(s)}{1-e^{-(2\pi/\omega_0)s}}$$

Soln: $f_1(t) = \begin{cases} \sin(\omega_0 t), & 0 \le t < \pi/\omega_0 \\ 0, & \pi/\omega_0 \le t < 2\pi/\omega_0 \end{cases}$

$$= \sin(\omega_0 t)\left[u(t) - u(t-\pi/\omega_0)\right]$$

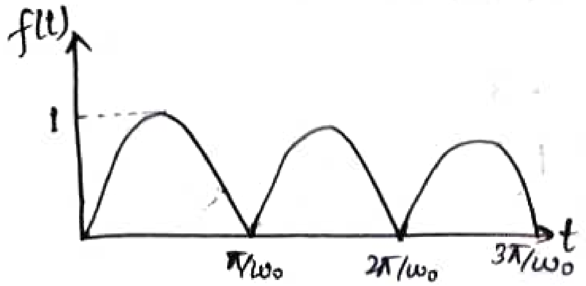
$$= \sin(\omega_0 t)u(t) + \sin\left(\omega_0\left(t-\frac{\pi}{\omega_0}\right)\right)u\left(t-\frac{\pi}{\omega_0}\right) \quad \left[\because \sin\left(\omega_0\left(t-\frac{\pi}{\omega_0}\right)\right) = -\sin(\omega_0 t)\right]$$

Taking laplace transform,

$$F_1(s) = \frac{\omega_0}{s^2+\omega_0^2} + e^{-(\pi/\omega_0)s}\frac{\omega_0}{\omega_0^2+s^2}$$
$$= \frac{\omega_0}{s^2+\omega_0^2}\left(1+e^{-(\pi/\omega_0)s}\right)$$

$$\therefore \; F(s) = \frac{\omega_0}{s^2+\omega_0^2} \cdot \frac{1+e^{-(\pi/\omega_0)s}}{1-e^{-(2\pi/\omega_0)s}}$$

(12)



Soln: Period $= \pi/\omega_0$.

$$f_1(t) = \sin(\omega_0 t), \quad 0 \le t \le \pi/\omega_0$$

$$= \sin(\omega_0 t)\left[u(t) - u(t-\pi/\omega_0)\right]$$

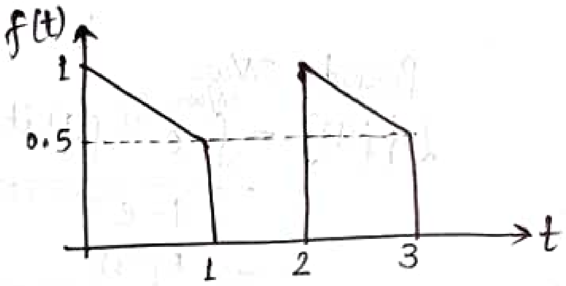
$$= \sin(\omega_0 t)\, u(t) + \sin\left(\omega_0 (t-\pi/\omega_0)\right) u(t-\pi/\omega_0)$$

Taking LT on both sides,

$$F_1(s) = \frac{\omega_0}{s^2+\omega_0^2}\left(1+e^{-(\pi/\omega_0)s}\right)$$

$$\therefore \; F(s) = \frac{\omega_0}{s^2+\omega_0^2} \cdot \frac{1+e^{-(\pi/\omega)s}}{1-e^{-(\pi/\omega_0)s}}$$

(13)



Soln: Period $= 2$.

$$f_1(t) = \begin{cases} 1 - t/2, & 0 \le t < 1 \\ 0, & 1 \le t \le 2. \end{cases}$$

$$F_1(s) = \int_0^1 \left(1-\frac{t}{2}\right) e^{-st}\,dt + \int_1^2 \cancel{e^{-st}\,0}\,dt$$

$$= \int_0^1 e^{-st} - \frac{1}{2}\int_0^1 t e^{-st}\,dt$$

$$= \left.\frac{e^{-st}}{-s}\right|_0^1 - \frac{1}{2}\left(\frac{-te^{-st}}{s} - \frac{e^{-st}}{s^2}\right)\Bigg|_0^1$$

$$= \frac{e^{-s}}{-s} + \frac{1}{s} - \frac{1}{2}\left(-\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + 0 + \frac{1}{s^2}\right)$$

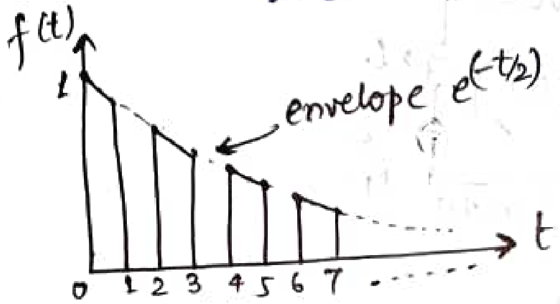
$$= -\frac{e^{-s}}{2s} + \frac{e^{-s}}{2s^2} + \frac{1}{s} - \frac{1}{2s^2}$$

$$= \frac{e^{-s}(1-s) + (2s-1)}{2s^2}$$

$$= \frac{(e^{-s}-1)(1-s) + s}{2s^2}$$

$$\therefore F(s) = \frac{F_1(s)}{1-e^{-2s}} = \frac{(e^{-s}-1)(1-s)+s}{2s^2}.$$

(14)



Soln: Period = 2

$$f_1(t) = \begin{cases} e^{-t/2}, & 0 \le t < 1 \\ 0, & 1 \le t \le 2. \end{cases}$$

$$\therefore F_1(s) = \int_0^1 e^{-t/2} e^{-st}\,dt + \int_1^2 0\, e^{-st}\,dt \to 0$$

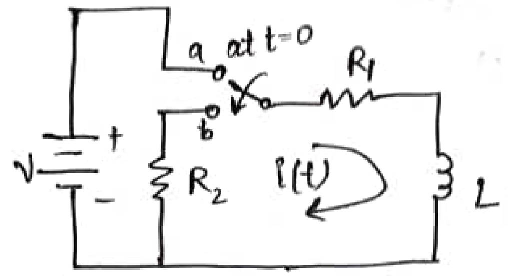
$$= \left.\frac{e^{-(s+1/2)t}}{-(s+1/2)}\right|_0^1$$

$$= \frac{-e^{-(s+1/2)}}{s+1/2} + \frac{1}{s+1/2}$$

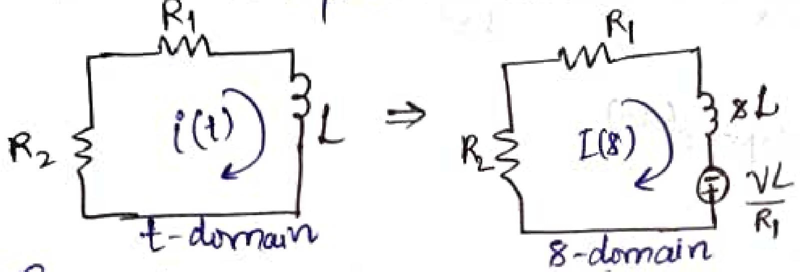
$$= \frac{1-e^{-(s+1/2)}}{s+1/2}$$

$$\therefore F(s) = \frac{F_1(s)}{1-e^{-2s}} = \frac{1}{s+1/2} \cdot \frac{1-e^{-(s+1/2)}}{1-e^{-2s}}$$

(30) Steady state was initially reached with switch at position 'a'. Solve for $i(t)$ using Laplace transform method.



Soln: At $t=0^+$, the equivalent circuit is



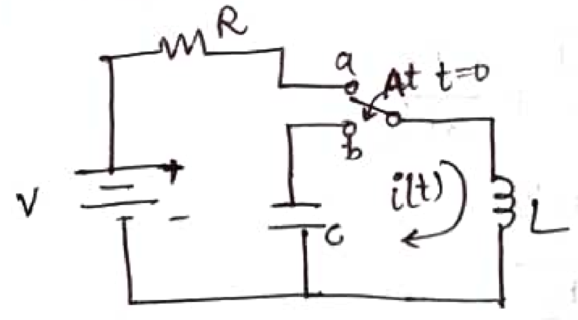
By applying KVL,

$$I(s)\left[R_1 + sL + R_2\right] = \frac{VL}{R_1}$$

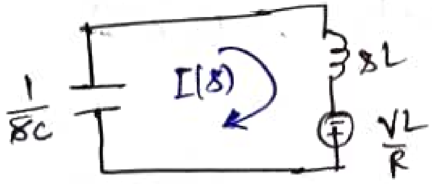
$$\Rightarrow I(s) = \frac{V}{R_1\left(\frac{R_1+R_2+s}{L}\right)}$$

$$\therefore i(t) = \mathcal{L}^{-1}\{I(s)\} = \frac{V}{R_1} e^{-\left(\frac{R_1+R_2}{L}\right)t}$$

(31) Steady state was initially reached at position 'a'. Solve for $i(t)$ using LT method.



Soln: At $t=0^+$, the equivalent ckt. becomes



As $i(0^-) = i(0^+) = \frac{V}{R}$.

Applying KVL,

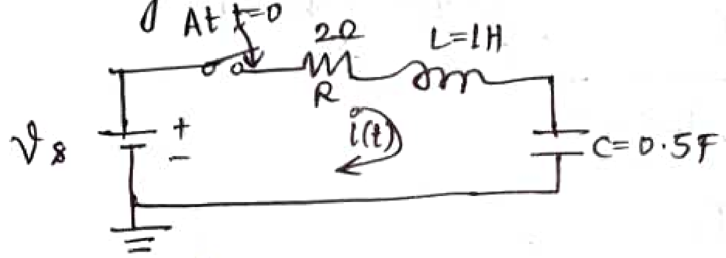
$$I(s)\left[sL+\frac{1}{sC}\right]=\frac{VL}{R}$$

$$\Rightarrow I(s)=\frac{VL}{R}\cdot\frac{1}{\left(sL+\frac{1}{sC}\right)}$$

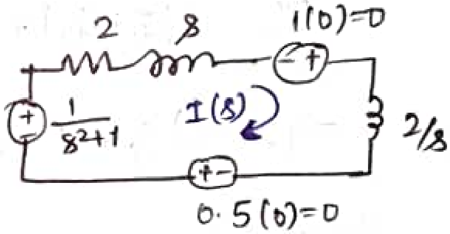
$$=\frac{V}{R}\cdot\frac{s}{s^2+\left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$\therefore i(t)=\mathcal{L}^{-1}\{I(s)\}=\frac{V}{R}\cos\left(\frac{t}{\sqrt{LC}}\right).$$

(32) $v_s=\sin t$, initially relaxed network. Find $i(t)$ for $t>0$.



Soln: Initially, network becomes:



$i(0^-)=0$

Applying KVL,

$$I(s)\left(2+s+\frac{2}{s}\right)=\frac{1}{s^2+1}$$

$$\Rightarrow I(s)=\frac{s}{(s^2+1)(s^2+2s+2)}$$

$$=\frac{As+B}{s^2+1}+\frac{Cs+D}{s^2+2s+2}$$

$$\Rightarrow s=(As+B)(s^2+2s+2)+(Cs+D)(s^2+1)$$

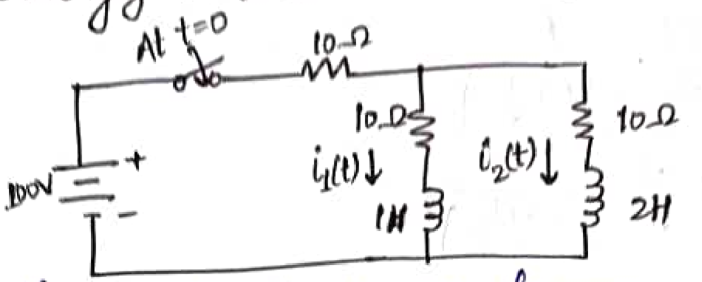
$$\Rightarrow \left.\begin{aligned}&A+C=0\\&2A+B+D=0\\&2A+2B+C=1\\&2B+D=0\end{aligned}\right\}\Rightarrow \begin{aligned}&A=1/5\\&B=2/5\\&C=-1/5\\&D=-4/5\end{aligned}$$

$$\therefore I(s)=\frac{\left(\frac{1}{5}s+\frac{2}{5}\right)}{s^2+1}+\frac{\left(-\frac{1}{5}s-\frac{4}{5}\right)}{(s+1)^2+1}$$

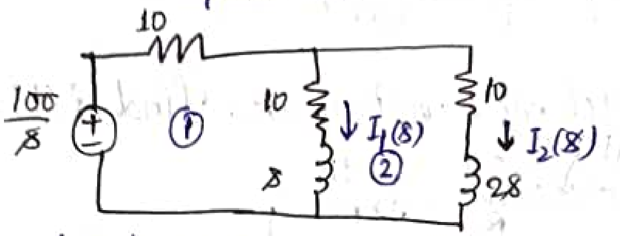
$$=\frac{1}{5}\left[\frac{s}{s^2+1}+\frac{2}{s^2+1}-\frac{(s+1)}{(s+1)^2+1}+\frac{1}{(s+1)^2+1}\right]$$

$$\Rightarrow i(t) = \mathcal{L}^{-1}\{I(s)\} = \frac{1}{5}\left(\cos t + 2\sin t - e^{-t}\cos t - 3e^{-t}\sin t\right).$$

(33) Initially unenergized network. Find $i_1(t), i_2(t)$ for $t>0$.



Soln: At $t=0^+$, the equivalent network becomes



$i_1(0^-) = i_2(0^-) = 0$

In loop-(1),

$$\frac{100}{s} = 10(I_1 + I_2) + I_1(10+s)$$

$$\Rightarrow (20+s)I_1 + 10 I_2 = \frac{100}{s} \quad \text{(1)}$$

In loop-2,

$$I_2(10+2s) - I_1(10+s) = 0$$

$$\Rightarrow I_2 = \frac{10+s}{10+2s} I_1$$

From (1),

$$I_1\left(20+s+\frac{100+10s}{10+2s}\right) = \frac{100}{s}$$

$$\Rightarrow I_1 = \frac{100(5+s)}{s(s^2+30s+150)}$$

$$\Rightarrow I_2 = \frac{100(5+s)(10+s)}{s(10+2s)(s^2+30s+150)}$$

$$\frac{5+s}{s(s^2+30s+150)} = \frac{A}{s} + \frac{Bs+C}{s^2+30s+150}$$

$$\Rightarrow 5+s = (s^2+30s+150)A + s(Bs+C)$$

$$\Rightarrow 30A + C = 1 \quad \Rightarrow C = 0$$
$$A + B = 0 \quad \Rightarrow B = -1/30$$
$$150A = 5 \quad \Rightarrow A = 1/30$$

$$\therefore I_1 = \frac{100}{30}\left(\frac{1}{s} - \frac{s}{(s+15)^2-75}\right)$$

$$= \frac{100}{30}\left(\frac{1}{s} - \frac{(s+15)}{(s+15)^2-75} + \frac{15}{\sqrt{75}} \cdot \frac{\sqrt{75}}{(s+15)^2-75}\right)$$

$$\therefore \Rightarrow i_1(t) = \mathcal{L}^{-1}\{I_1\} = \frac{10}{3}\left(1 - e^{-15}\cosh(\sqrt{75}\,t) + \frac{15}{\sqrt{75}} e^{-15}\sinh(\sqrt{75}\,t)\right)$$

and,

$$\frac{(s+5)(s+10)}{s(10+2s)(s^2+30s+150)} = \frac{A}{s} + \frac{B}{10+2s} + \frac{Cs+D}{s^2+30s+150}$$

$$\Rightarrow s^2+15s+50 = (10+2s)(s^2+30s+150)A + s(s^2+30s+150)B + (Cs+D)(s(10+2s))$$

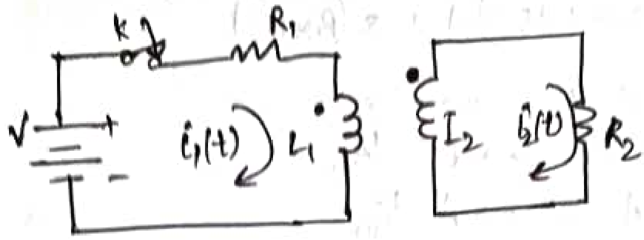
$$\left.\begin{aligned} &\Rightarrow 600A + 150B + 10D = 15 \\ &70A + 30B + 10C + 2D = 15 \\ &2A + B + 2C = 0 \\ &1500A = 50 \Rightarrow A = 1/30 \end{aligned}\right\} \Rightarrow \begin{aligned} B &= 14/5 \\ C &= 41/30 \\ D &= 22 \end{aligned}$$

$$\therefore I_2 = \frac{10}{3}\left[\frac{1}{s} + \frac{42}{s+5} + 41\frac{(s+15)}{(s+15)^2-75} - \frac{615}{\sqrt{75}}\frac{\sqrt{75}}{(s+15)^2-75} + \frac{660}{\sqrt{75}} \cdot \frac{\sqrt{75}}{(s+15)^2-75}\right]$$

$$\therefore i_2(t) = \mathcal{L}^{-1}\{I_2\}$$

$$= \frac{10}{3}\left[1 + 42e^{-5t} + \frac{41}{\sqrt{75}} e^{-15t}\cosh(\sqrt{75}\,t) + \frac{45}{\sqrt{75}} e^{-15t}\sinh(\sqrt{75}\,t)\right]$$

(34) Initially unenergized network. $L_1 = 1H$, $L_2 = 4H$, $M = 2H$, $R_1 = R_2 = 1\Omega$, $V = 1V$. Find $i_1(t)$, $i_2(t)$.



Sol^n: $i_1(0^+) = i_2(0^+) = 0$

Applying KVL,

$$V = R_1 i_1 + L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$0 = R_2 i_2 + L_2 \frac{di_2}{dt} - M\frac{di_1}{dt}$$

Applying LT,

$$\frac{V}{s} = R_1 I_1(s) + \cancel{L_1} s I_1(s) - L_1 \cancel{i_1'(0)} - Ms I_2(s) + M\cancel{i_2(0)}$$

$$\Rightarrow \frac{1}{s} = I_1(s) + sI_1(s) - 2s I_2$$

and,

$$0 = R_2 I_2(s) + L_2 I_2(s) s - L_2 \cancel{i_2'(0^-)} - MsI_1(s) + M\cancel{i_2(0^-)}$$

$$\Rightarrow 0 = I_2(s) + 4s I_2 - 2s I_1(s)$$

$$\Rightarrow I_1(1+s) + I_2(-2s) = 1/s$$

$$I_1(-2s) + I_2(1+4s) = 0$$

$$\Rightarrow I_1 = \frac{4s+1}{s(5s+1)}, \quad I_2 = \frac{2}{5s+1}$$

$$\Rightarrow I_1 = \frac{4\cancel{s}}{\cancel{s}(5s+1)} + \frac{1}{s(5s+1)}$$

$$= \frac{4}{5(s+\frac{1}{5})} + \frac{1}{s(s+\frac{1}{5})}$$

$$= \frac{4/5}{s+1/5} + \frac{1/5}{1/5}\left(\frac{1}{s} - \frac{1}{s+1/5}\right)$$

$$i_1(t) = \mathcal{L}^{-1}\{I_1\} = \frac{4}{5}e^{-t/5} + 1 - e^{-t/5}$$

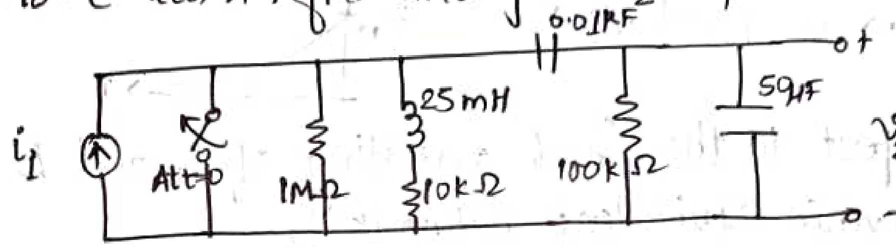
$$= 1 - \frac{1}{5}e^{-t/5}.$$

and, $I_2 = \frac{2}{5s+1}$

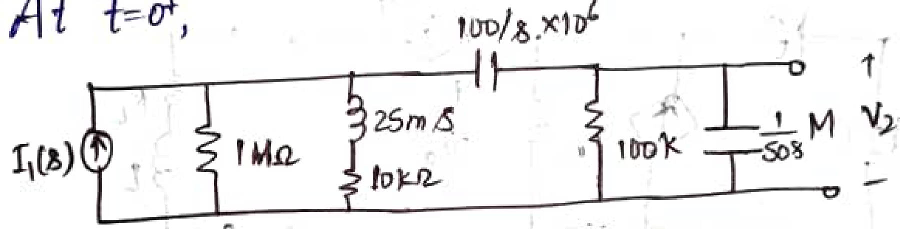
$$= \frac{2}{5\left(\frac{1}{5}+s\right)}$$

$$\therefore i_2(t) = \mathcal{L}^{-1}\{I_2(s)\} = \frac{2}{5}e^{-t/5}$$

(35) Initially unenergized network. Switch opened at $t=0$. If $i_1(t) = 10^{-3}e^{-t}u(t)$ A, find and plot $v_2(t)$ for $t \geq 0$.



Soln: At $t=0^+$,



$$100\times10^3 \parallel \frac{20\times10^3}{s} = \frac{100\times10^3 \times 20\times\frac{10^3}{s}}{\left(100+\frac{20}{s}\right)\times10^3}$$

$$= \frac{2\times10^6\times s}{(2+10s)10} = 10^5\frac{s}{5s+1}$$

$$\left(\frac{1000}{s}+\frac{s}{5s+1}\right)\times10^5 = \left(s^2+5000s+1000\right)10^5$$

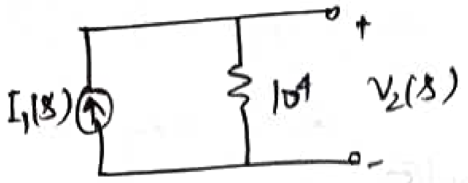
$$\approx (5s+1)10^8$$

$$10\times10^3 + \frac{25s}{1000} = \frac{25s+10^7}{1000} \approx 10^4$$

$$10^4 \parallel (5s+1)10^8 = \frac{(5s+1)10^{12}}{(5s+1)10^8+10^4} \approx 10^4$$

$$10^4 \parallel 10^6 = \frac{10^{10}}{101\times10^4} = \frac{10^6}{101} \approx 10^4$$

Approximate circuit:



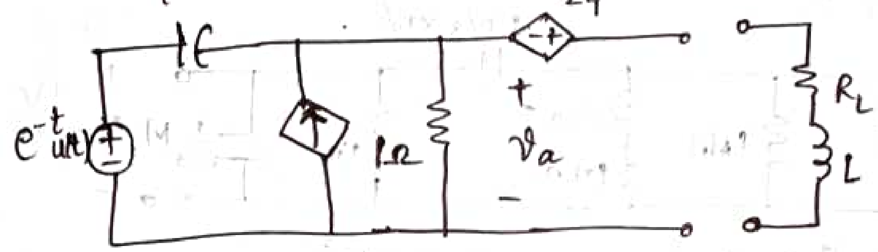
$V_2(s) = 10^4 \, I_1(s)$

Taking L Inverse both sides,

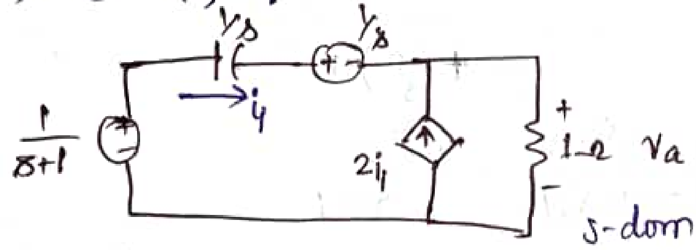
$v_2(t) = 10^4 i_1(t) = 10^4 \times 10^{-3} e^{-t} u(t)$

$\Rightarrow v_2 = 10 e^{-t} u(t).$

(36) With respect to the load consisting of R_L and L determine the Thevenin equivalent network.



Soln: $v_a(0^+) = e^{-0} u(0) = 1.$



s-domain circuit

Applying KVL,

$$-\frac{1}{s+1} + I_1\left(\frac{1}{s}\right) + \frac{1}{s} + 3I_1 = 0$$

$$\Rightarrow I_1\left(\frac{1}{s} + 3\right) = \frac{1}{s+1} - \frac{1}{s}$$

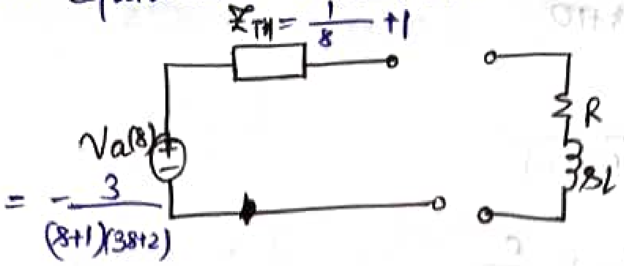
$$\Rightarrow I_1 = -\frac{1}{(s+1)(3s+1)}$$

$$V_a(s) = 3 I_1(s) \times 1$$

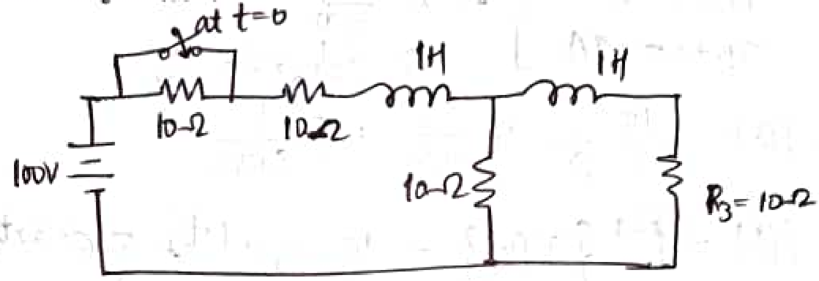
$$= -\frac{3}{(s+1)(3s+1)} = V_{TH}$$

$$Z_{TH} = 1/s \parallel 1 = \frac{1/s \times 1}{\frac{1}{s}+1} = \frac{1}{s+1}$$

Thevenin Equivalent circuit in s-domain:

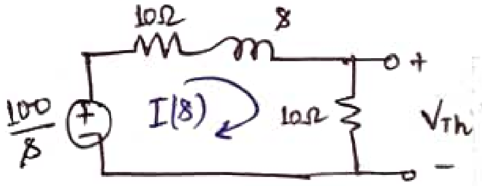


(37) Find the current in R_3 using (a) Thevenin's theorem (b) Norton's theorem.



Soln: (a) $i(0^-) = 0$

At $t=0$,



Applying KVL,

$$\frac{100}{s} = (10 + s + 10)\, I(s)$$

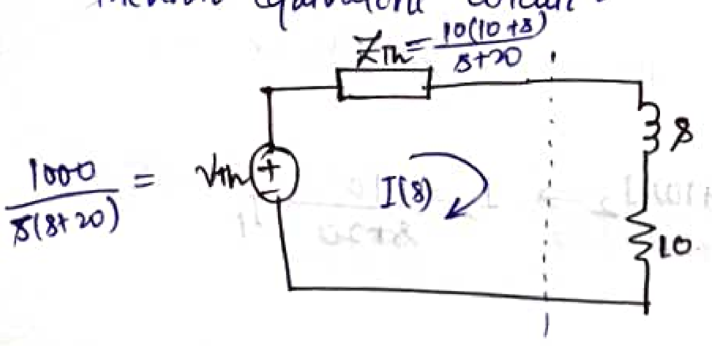
$$\Rightarrow I(s) = \frac{100}{s(s+20)}$$

$$V_{Th} = 10\, I(s) = \frac{1000}{s(s+20)}$$

$$Z_{Th} = 10 \,||\, (10+s)$$

$$= \frac{10(10+s)}{10+(10+s)} = \frac{100+10s}{s+20}$$

Thevenin equivalent circuit:



$$I(s) = \frac{V_{Th}}{Z_{Th}+s+10} = \frac{100/s\,(s+20)}{\frac{10(10+s)}{s+20}+s+10}$$

$$= \frac{1000}{s(s+10)(s+30)}$$

$$\frac{1}{s(s+10)(s+30)} = \frac{A}{s}+\frac{B}{s+10}+\frac{C}{s+30}$$

$$\Rightarrow 1 = (s+10)(s+30)A + s(s+30)B + s(s+10)C$$

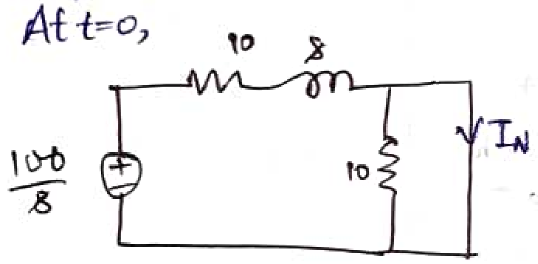
$$\Rightarrow \left.\begin{aligned} B+C &= -A \\ 3B+C &= -4A \end{aligned}\right\} \Rightarrow A = \frac{1}{300},\quad B = -\frac{1}{200},\ C = \frac{1}{600}$$

$$\therefore I(s) = \frac{10}{3}\cdot\frac{1}{s} - 5\cdot\frac{1}{s+10} + \frac{5}{3}\,\frac{1}{s+30}$$

$$\Rightarrow i(t) = \mathcal{L}^{-1}\{I(s)\} = \frac{10}{3} - 5e^{-10t} + \frac{5}{3}e^{-30t}.$$

(B) $i(0^+)=0$

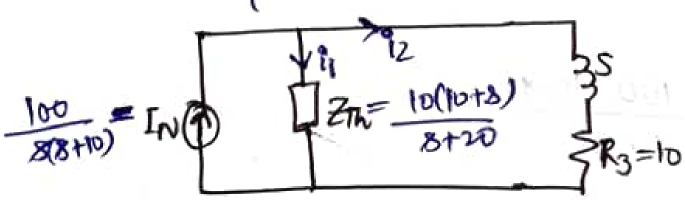
At $t=0$,



$$I_N = \frac{100/s}{10+s} = \frac{100}{s(s+10)}$$

$$Z_N = 10\,\|\,(10+s) = \frac{10(10+s)}{10+(10+s)} = \frac{10(10+s)}{s+20}.$$

Nortan Equivalent circuit:



$$I_1 + I_2 = \frac{100}{s(s+10)}$$

$$\frac{10(s+10)}{s+20}\,I_1 = (s+10)\,I_2 \Rightarrow I_2 = \frac{10}{s+20}\,I_1$$

$$\Rightarrow I_1\left(1+\frac{10}{s+20}\right)=\frac{100}{s(s+10)}$$

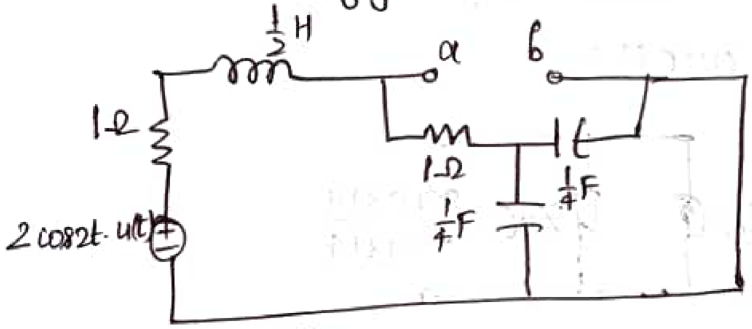
$$\Rightarrow I_1=\frac{100(s+20)}{s(s+10)(s+30)}$$

$$I_2=\frac{10}{s+20}I_1 \Rightarrow I_2=\frac{1000}{s(s+10)(s+30)}$$

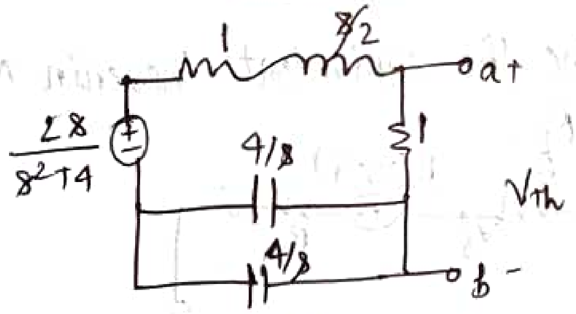
$$=\frac{10}{3}\cdot\frac{1}{s}-5\cdot\frac{1}{s+10}+\frac{5}{3}\cdot\frac{1}{s+30}$$

$$\therefore i_2(t)=\mathcal{L}^{-1}\{I_2(s)\}=\left(\frac{10}{3}-5e^{-10t}+\frac{5}{3}e^{-30t}\right).$$

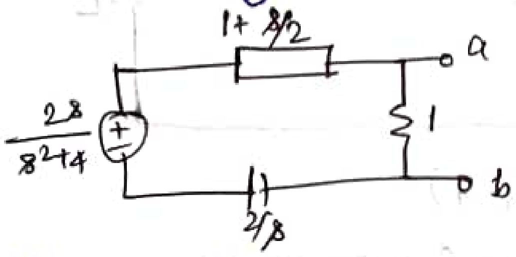
(38) Find both Thevenin's and Nortan's equivalent n/w for the terminals a-b in the figure for zero initial conditions.



Soln:



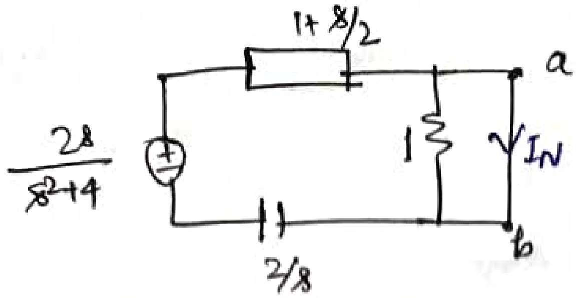
$$\Downarrow$$



$$\left[\because \mathcal{L}\{\cos 2t\}=\frac{s}{s^2+4}\right]$$

$$V_{Th}=\frac{2s/s^2+4}{1+\frac{s}{2}+1+\frac{2}{s}}\times 1 \Rightarrow V_{Th}=\frac{4s^2}{(s^2+4)(s+2)^2}$$

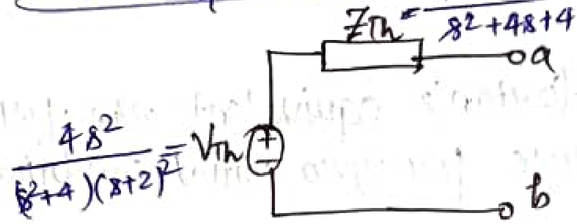
$$Z_{Th}=\left(1+\tfrac{s}{2}+\tfrac{2}{s}\right)\|1=\frac{s^2+2s+4}{s^2+4s+4}$$



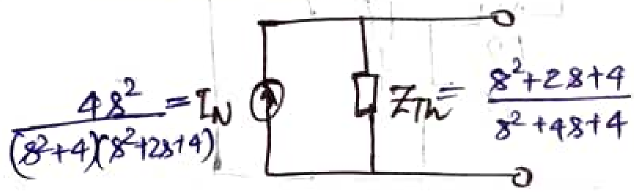
$$I_N = \frac{2s/(s^2+4)}{1+\frac{s}{2}+\frac{2}{s}} = \frac{4s^2}{(s^2+4)(s^2+2s+4)}$$

$$Z_N = Z_{Th} = \frac{s^2+2s+4}{s^2+4s+4}.$$

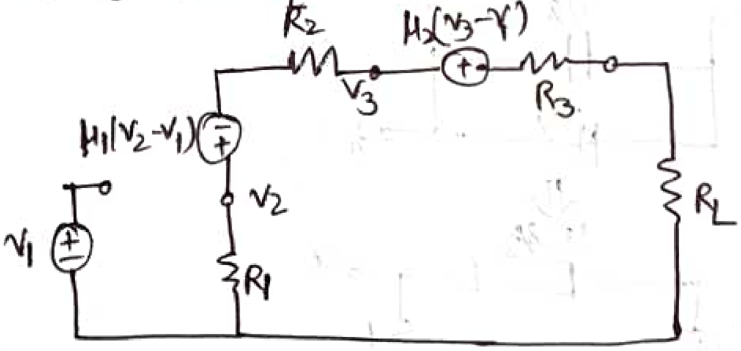
Thevenin Eqⁿ circuit :



Norton eqⁿ circuit :



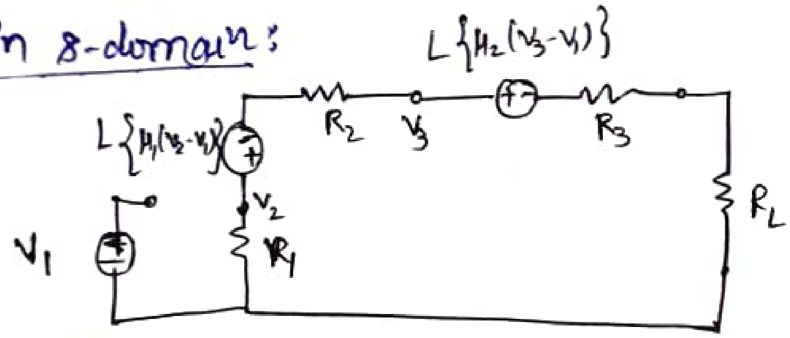
(9) R_L is the load. Determine the equivalent Thevenin N/w to find the current in load R_L.



Soln: $v_{th}(t) = \mu(v_2 - v_1) + \mu(v_3 - v_1)$

$V_{Th}(s) = L\{\mu(v_2 - v_1)\} + L\{\mu_2(v_3 - v_1)\}$

In s-domain:



$$Z_{Th} = R_1 + R_2 + R_3$$

$$I_L = \frac{V_{Th}}{Z_{Th}} = \frac{\mathcal{L}\{\mu_1(v_2-v_1)\} + \mathcal{L}\{\mu_2(v_3-v_1)\}}{R_1+R_2+R_3}$$

$$\Rightarrow i(t) = \mathcal{L}^{-1}\{I_L(s)\} = \frac{\mu_1(v_2-v_1) + \mu_2(v_3-v_1)}{R_1+R_2+R_3}.$$