# AV323 - COMMUNICATION SYSTEMS II

## Assignment-1

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SC22B146

Chapter-2 and 3 problems (Sklar)

Q.1 You want to transmit the word "HOW" using an 8-ary system.

(a) Encode the word "HOW" into a sequence of bits, using 7-bit ASCII coding, followed by an eighth bit for error ~~correction~~ detection, per character. The eighth bit is chosen such that the no. of ones in the 8 bits is an even number. How many total bits are there in the message?

Soln:

| Character | ASCII (Decimal) | ASCII (binary) | Parity (Even) |
|---|---|---|---|
| H | 72 | 1001000 | 0 |
| O | 79 | 1001111 | 1 |
| W | 87 | 1010111 | 1 |

Total no. of bits = $3 \times 8 = 24$.

(b) Partition the bit stream into K = 3 bit segments. Represent each of the 3-bit segments as an octal no. (symbol). How many octal symbols are there in the message?

Soln:

$$\overbrace{100\ \ 100\ \ 00}^{H}\overbrace{1\ \ 001\ \ 111\ \ 1}^{O}\overbrace{10\ \ 101\ \ 111}^{W}$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 100 | 100 | 001 | 001 | 111 | 110 | 101 | 111 |
| octal no. (symbol): | 4 | 4 | 1 | 1 | 7 | 6 | 5 | 7 |

∴ No. of octal symbols = 8.

(c) If the system were designed with 16-ary modulation, how many symbols would be used to represent the word "HOW"?

Soln: 16-ary system → 4 bits per symbol.
$(2^4)$

$$\overbrace{1001\ \ 0000}^{H}\ \ \overbrace{1001\ \ 1111}^{O}\ \ \overbrace{1010\ \ 1111}^{W}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1001 | 0000 | 1001 | 1111 | 1010 | 1111 |
| 16-ary (symbol): | 9 | 0 | 9 | F | A | F |

∴ No. of symbols = 6

(d) If the system were designed with 256-ary modulation, how many symbols would be used to represent the word "HOW"?

Soln: 256 ($2^8$) - ary modulation → 8 ~~sym~~ bits per symbol

$$\underbrace{10010000}_{H} \quad \underbrace{10011111}_{O} \quad \underbrace{10101111}_{W}$$

256-ary (symbol): 144   159   175

∴ No. of symbols = 3.

(2.2) We want to transmit 800 characters/s, where each character is represented by its 7-bit ASCII codeword, followed by an eighth bit for error detection, per character, as in problem 2.1. A multilevel PAM waveform with $M = 16$ levels is used.

(a) What is the effective transmitted bit rate?

Soln: Bits per character = 8

Characters per second = 800

∴ Bits per second

= Bit rate = $8 \times 800$

= 6400 bits per second.

(b) What is the symbol rate?

Soln: 16-level PAM ($2^4$) → $\log_2(16) = 4$ bits per symbol.

∴ Symbol rate = symbols/sec.

$$= \frac{\text{Bits/sec.}}{\text{Bits/symbol}}$$

$$= \frac{6400}{4} = 1600 \text{ symbol per second.}$$

3.1 Determine whether or not $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(-1.5T_2 < t < 1.5T_2)$, where $s_1(t) = \cos(2\pi f_1 t + \phi_1)$, $s_2(t) = \cos(2\pi f_2 t + \phi_2)$, and $f_2 = \frac{1}{T_2}$ for the following cases.

(a) $f_1 = f_2$ and $\phi_1 = \phi_2$.

Soln: $s_1(t)$ and $s_2(t)$ are orthogonal if their inner product over the given interval is zero,

i.e., $$\int_{-1.5T_2}^{1.5T_2} s_1(t) \cdot s_2(t)\, dt = 0.$$

For $f_1 = f_2$, $\phi_1 = f_2$,

$$\int_{-1.5T_2}^{1.5T_2} \cos(2\pi f_1 t + \phi_1) \cdot \cos(2\pi f_2 t' + \phi_2)$$

$$= \int_{-1.5T_2}^{1.5T_2} \cos^2(2\pi f_1 t + \phi_1)\, dt$$

$$= \int_{-1.5T_2}^{1.5T_2} \frac{1 + \cos(4\pi f_1 t + 2\phi_2)}{2}\, dt$$

$$= \frac{1}{2}(3T_2) + \frac{1}{2}\underbrace{\int_{-1.5T_2}^{1.5T_2} \cos(2\pi f_1 t + 2\phi_2)\, dt}_{\to 0 \text{ (symmetric)}}$$

$$= \frac{3}{2} T_2 \neq 0.$$

$\therefore$ Not orthogonal.

(b) $f_1 = \frac{1}{3} f_2$ and $\phi_1 = \phi_2$

Soln: $$\int_{-1.5T_2}^{1.5T_2} s_1(t) \cdot s_2(t)\, dt = \int_{-1.5T_2}^{1.5T_2} \cos\left(2\pi \frac{f_2}{3} t + \phi_1\right) \cos(2\pi f_2 t + \phi_1)\, dt$$

$$= \frac{1}{2} \int_{-1.5T_2}^{1.5T_2} \Big[\underbrace{\cos\left(2\pi \tfrac{f_2}{3} t - 2\pi f_2 t\right)}_{\int = 0} + \underbrace{\cos\left(2\pi \tfrac{f_2}{3} t + 2\pi f_2 t + 2\phi_1\right)}_{\int = 0}\Big]\, dt$$

$$= 0$$

$\therefore$ Orthogonal.

(c) $f_1 = 2f_2$ and $\phi_1 = \phi_2$

Soln: $\int_{-1.5T_2}^{1.5T_2} \cos(4\pi f_2 t + \phi_1)\cos(2\pi f_2 t + \phi_1)\,dt$

$$= \frac{1}{2}\int_{-1.5T_2}^{1.5T_2} \left[\underbrace{\cos(2\pi f_2 t)}_{\int=0} + \underbrace{\cos(6\pi f_2 t + \phi_2)}_{\int=0}\right] dt$$

$= 0$

$\therefore$ orthogonal.

(d) $f_1 = \pi f_2$ and $\phi_1 = \phi_2$.

Soln: $\int_{-1.5T_2}^{1.5T_2} \cos(2\pi^2 f_2 t + \phi_1)\cdot\cos(2\pi f_2 t + \phi_1)\,dt$

$$= \frac{1}{2}\int_{-1.5T_2}^{1.5T_2} \left[\underbrace{\cos(2\pi f_2 \underbrace{(\pi-1)}_{\text{irrational}} t)}_{\int\neq 0} + \underbrace{\cos(2\pi f_2(\pi+1)t + 2\phi_1)}_{\int\neq 0}\right] dt$$

irrational
↓
not integral multiple of $1/T_2$
↓
not periodic

$\neq 0$

$\therefore$ Not orthogonal.

(e) $f_1 = f_2$ and $\phi_1 = \phi_2 + \pi/2$.

Soln: $\int_{-1.5T_2}^{1.5T_2} \cos(2\pi f_2 t + \phi_2 + \pi/2)\cdot\cos(2\pi f_2 t + \phi_2)\,dt$

$$= -\int_{-1.5T_2}^{1.5T_2} \sin(2\pi f_2 t + \phi_2)\cdot\cos(2\pi f_2 t + \phi_2)\,dt$$

$$= -\frac{1}{2}\int_{-1.5T_2}^{1.5T_2} \underbrace{\sin(4\pi f_2 t + 2\phi_2)}_{\int=0}\,dt$$

$= 0.$

$\therefore$ orthogonal.

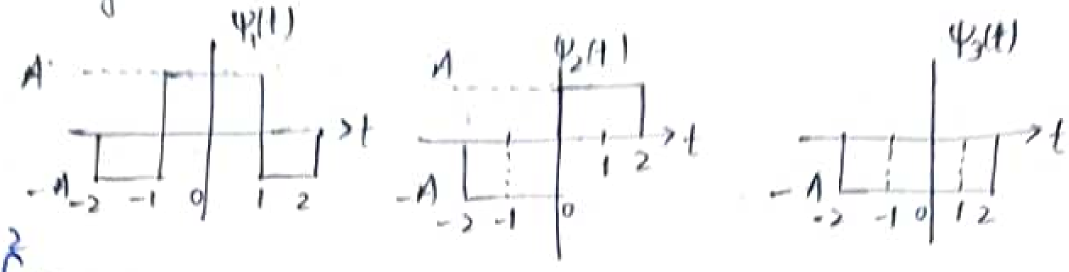
(f) $f_1 = f_2$ and $\phi_1 = \phi_2 + \pi$.

Soln: $\int_{-1.5T_2}^{1.5T_2} \cos(2\pi f_2 t + \phi_2 + \pi)\cdot\cos(2\pi f_2 t + \phi_2)\,dt$

$$= -\int_{-1.5T_2}^{1.5T_2} \cos^2(2\pi f_2 t + \phi_2)\,dt = -\frac{1}{2}\int_{-1.5T_2}^{1.5T_2} (\underbrace{1}_{\int\neq 0} + \underbrace{\cos(4\pi f_2 t + 2\phi_2)}_{\int=0})\,dt$$

$\neq 0.$ $\therefore$ Not orthogonal.

(3.2) (a) Show that the three functions illustrated in figure are pairwise orthogonal over the interval $(-2, 2)$.




Soln:

$$\int_{-2}^{2} \psi_1(t)\,\psi_2(t)\,dt$$

$$= \int_{-2}^{-1} (-A)(-A)\,dt + \int_{-1}^{0} (A)(-A)\,dt + \int_{0}^{1} (A)(A)\,dt + \int_{1}^{2} (-A)(A)\,dt$$

$$= A^2 t\Big|_{-2}^{-1} - A^2 t\Big|_{-1}^{0} + A^2 t\Big|_{0}^{1} + (-A^2)\,t\Big|_{1}^{2}$$

$$= A^2 - A^2 + A^2 - A^2$$

$$= 0$$

$\psi_1(t), \psi_2(t) \rightarrow$ orthogonal.

$$\int_{-2}^{2} \psi_2(t)\,\psi_3(t)\,dt = \int_{-2}^{0} (-A)(-A)\,dt + \int_{0}^{2} (A)(-A)\,dt$$

$$= A^2 t\Big|_{-2}^{0} - A^2 t\Big|_{0}^{2}$$

$$= 2A^2 - 2A^2 = 0$$

$\psi_2, \psi_3 \rightarrow$ orthogonal.

$$\int_{-2}^{2} \psi_1(t).\,\psi_3(t)\,dt = \int_{-2}^{-1} (-A)(-A)\,dt + \int_{-1}^{0} (A)(-A)\,dt + \int_{0}^{1} (A)(-A)\,dt + \int_{1}^{2} (-A)(-A)\,dt$$

$$= A^2 t\Big|_{-2}^{-1} + (-A^2)\,t\Big|_{-1}^{0} + (-A^2)\,t\Big|_{0}^{1} + A^2 t\Big|_{1}^{2}$$

$$= A^2 - A^2 - A^2 + A^2$$

$$= 0$$

$\psi_1, \psi_3 \rightarrow$ orthogonal.

$\therefore$ $\psi_1, \psi_2, \psi_3$ are pairwise orthogonal.

(b) Determine the value of the constant, A, that makes the set of functions in part (a) an orthonormal set.

Soln: Orthonormal set of signals $\Rightarrow$ { (i) Signals are orthogonal to each other. ✓
(ii) Signals have unit norm (unit energy).

$$\int_{-2}^{2} \psi_1^2(t)\,dt = \int_{-2}^{2} \psi_2^2(t)\,dt = \int_{-2}^{2} \psi_3^2(t)\,dt$$

$$= \int_{-2}^{2} A^2\,dt$$

$$= A^2 t \Big|_{-2}^{2} = 4A^2$$

For orthonormality, $4A^2 = 1$

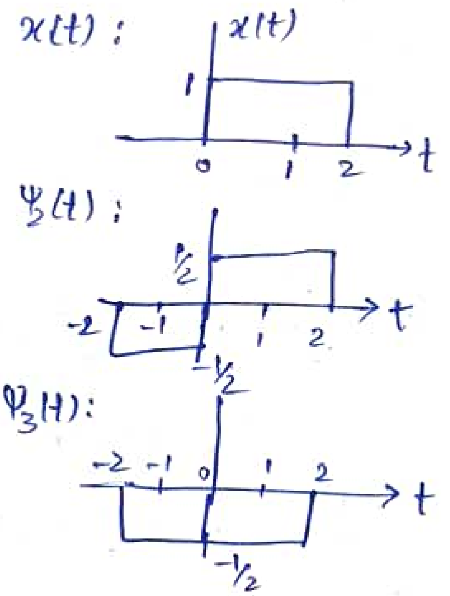
$$\Rightarrow A^2 = 1/4$$

$$\Rightarrow \boxed{A = 1/2}.$$

(c) Express the following waveform, $x(t)$, in terms of the orthonormal set of part (b).

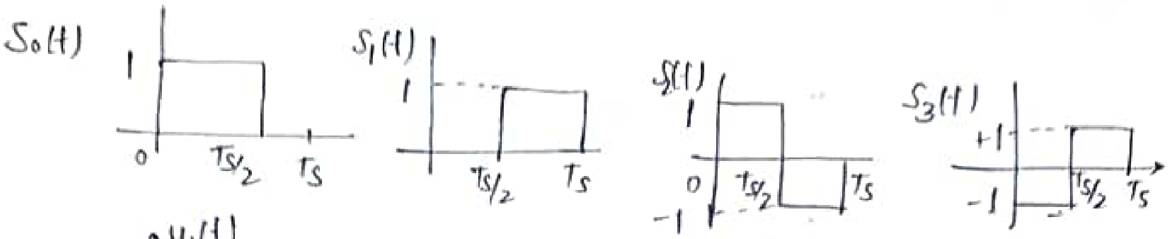
$$x(t) = \begin{cases} 1; & \text{for } 0 \le t < 2 \\ 0, & \text{otherwise.} \end{cases}$$

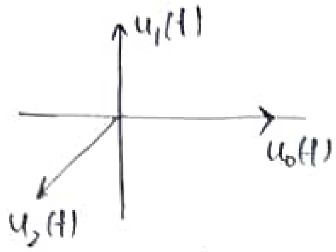
Soln:




$\therefore\ \psi_2(t) - \psi_3(t) = x(t)$.

(Additional question) : Class Test ques.



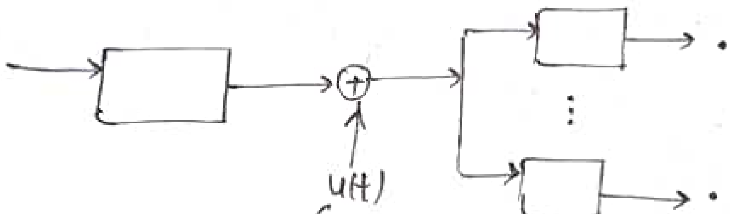




Given the baseband scheme,

(i) Find the ~~vector~~ unit vectors in the signal space.

(ii) Plot the signal ~~cos~~ constellation.

(iii)

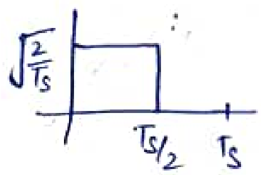



IID → unif $(-1/4, 1/4)$

Find the output signal constellation.

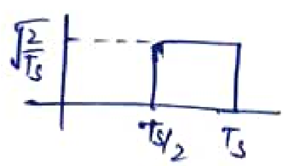
Soln: (i) By Gram-Schmidt procedure,

$$\bar{u}_0 = \frac{\bar{S}_1}{\|\bar{S}_1\|} =$$




$$\bar{U}_1 = \frac{\bar{S}_1 - \bar{u}_0 \cancel{\langle \bar{S}_1, \bar{u}_0 \rangle}^{0}}{\| \bar{S}_1 - \bar{u}_0 \cancel{\langle \bar{S}_1, \bar{u}_0 \rangle}^{0} \|}$$

$$\Rightarrow \bar{u}_1 = \frac{\bar{S}_1}{\|S_1\|} =$$



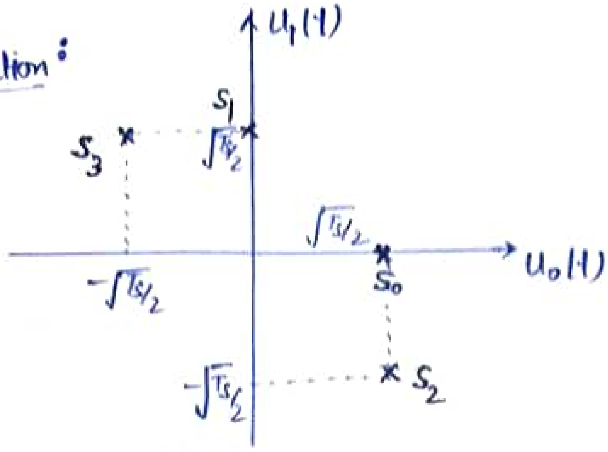

(ii) $S_0(t) = u_0(t) \cdot \sqrt{T_s/2}$

$S_1(t) = u_1(t) \cdot \sqrt{T_s/2}$.

$S_2(t) = \sqrt{T_s/2} \cdot u_0(t) - \sqrt{T_s/2}\, u_1(t)$

$S_3(t) = -\sqrt{T_s/2}\, u_0(t) + \sqrt{T_s/2} \cdot u_1(t)$.

Signal constellation:




(iii) $N(t) \sim U(-\frac{1}{4}, \frac{1}{4})$

$$\mathbb{E}(N) = \frac{-\frac{1}{4}+\frac{1}{4}}{2} = 0$$

$$\text{Var}(N) = \frac{(b-a)^2}{12} = \frac{(\frac{1}{4}+\frac{1}{4})^2}{12} = \frac{\frac{1}{4}}{12} = \frac{1}{48}.$$

Uniform spread within a square of side $\frac{1}{4}+\frac{1}{4} = \frac{1}{2}$, with center as the signal, $s_i$.

Receiver constellation: