Indian Institute of Space Science and Technology

Thiruvananthapuram

B.Tech - 2nd Semester, 2017
MA121 - Vector Calculus and Differential Equations

Quiz-I

Date : 13th March, 2017 **Time: 9:00am to 10:00am**

(Answer all questions; each of 3 marks. Total marks 15.)

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1. (a) Define directional derivative of a function $f : D \longrightarrow \mathbb{R}$ at a point $P_0 \in D \subset \mathbb{R}^2$ along a unit vector $\vec{v}$. [1]

   (b) Let $f : \mathbb{R}^2 \longrightarrow R$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2+y^2} & if\ (x,y) \neq (0,0) \\ 0 & otherwise \end{cases}$$

   Suppose $\vec{v} = (v_1, v_2)$ be an unit vector in $\mathbb{R}^2$ such that $v_1 \neq 0 \neq v_2$.

   i. Check whether $D_x f|_{(0,0)}$, $D_y f|_{(0,0)}$ and $D_{\vec{v}} f|_{(0,0)}$ exist.

   ii. If $D_{\vec{v}} f|_{(0,0)}$ exists, is it possible to express $D_{\vec{v}} f|_{(0,0)}$ in terms of the linear combination $D_x f|_{(0,0)}$ and $D_y f|_{(0,0)}$?

   iii. Is f is differentiable at $(0,0)$? Justify your answer.

   [4]

2. (a) Let $C := \gamma(t),\ t \in [a,b]$ be a C^1-type curve. Show that $-C$ is also of C^1-type. [2]

   (b) Let C be a curve given by the cartesian equation

$$x^2 = y,\ z^2 = y,\ x \geq 0,\ z \geq 0.$$

   Parametrize the curve with initial point $(0,0,0)$. Find the arc length function of the given curve with initial point $(0,0,0)$. [3]

3. (a) Let $f : D \longrightarrow \mathbb{R}$ be a continuous scalar field where $D \subset \mathbb{R}^2$. Let $C := \gamma(t),\ t \in [a,b]$ be a C^1-type curve. Show that $\int_C f = \int_{-C} f$. [2]

   (b) Let $\vec{F} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be given by $\vec{F}(x,y,z) = (yz, zx, x^2+y^2)$ for all $(x,y,z) \in \mathbb{R}^3$, and C be a curve given by $C : \gamma(t) = (\cos t, \sin t, 5),\ t \in [0,\pi]$. Does $\int_C \vec{F}$ exists? If yes, find the value of $\int_C \vec{F}$. [3]

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