11.25/15

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Signal and Systems (AV242/AV223)
Class Test on Laplace and Z transforms
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Marks: 15 Time: 45 Mins

Each question carries 1 mark. For each wrong answer 0.25 mark will be deducted. Put a $\surd$ for the correct answer. All your rough work should be in the provided sheet.

1. If $X(s) = L[x(t)] = \frac{k}{(s+1)(s^2+4)}$ then the final value $x(\infty)$

-0.25

 A. $k/4$.
 B. zero. ✓
 C. infinite.
 D. undefined.

2. The inverse Laplace transform of the function $\frac{s+5}{(s+1)(s+3)}$

 A. $2e^{-t} - e^{-3t}$ ✓
 B. $2e^{-t} + e^{-3t}$
 C. $e^{-t} - 2e^{-3t}$
 D. $e^{-t} + 2e^{-3t}$

3. A LTI system has a impulse response $h(t) = e^{2t}$, for $t > 0$.If initial conditions are zero and the input $x(t) = e^{-3t}$, the output for $t > 0$ is

 A. None of these ✓
 B. $e^{3t} - e^{2t}$
 C. e^{5t}
 D. $e^{3t} + e^{2t}$

4. The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the out output is

 A. $\frac{t^2}{2}u(t)$
 B. $\frac{t(t-1)}{2}u(t-1)$
 C. $\frac{(t-1)^2}{2}u(t-1)$ ✓
 D. $\frac{t^2-1}{2}u(t-1)$

5. The input $x(t)$ and output $y(t)$ of an LTI system are related by the differential equation $\frac{d^2y(t)}{dt} - \frac{dy}{dt} - 6y(t) = x(t)$. If the system is neither causal nor stable, the impulse response $h(t)$ of the system is

 A. $\frac{1}{5}e^{3t}u(-t) + \frac{1}{5}e^{-2t}u(-t)$
 B. $-\frac{1}{5}e^{3t}u(-t) + \frac{1}{5}e^{-2t}u(-t)$ ✓
 C. $\frac{1}{5}e^{3t}u(-t) - \frac{1}{5}e^{-3t}u(-t)$
 D. $-\frac{1}{5}e^{3t}u(-t) - \frac{1}{5}e^{-2t}u(-t)$

6. The Laplace transform of a function $x(t) = \begin{cases} 1 & if a \le t \le b \\ 0 & otherwise \end{cases}$ where a, $b > 0$

A. $X(s) = \frac{e^{-as} - e^{bs}}{s}$

B. $X(s) = \frac{e^{(a-b)}}{s}$

C. $X(s) = \frac{e^{-as} - e^{-bs}}{s}$

D. $X(s) = \frac{a-b}{s^2}$

7. If $X(s) = \frac{2(s+1)}{s^2+4s+7}$ then the initial and final values of $x(t)$ are respectively

A. $0, 2$

B. $2, 0$

C. $0, 2/7$

D. $2/7, 0$

8. A sequence $x(n)$ with z- transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear time invariant system with impulse response $h(n) = 2\delta(n-3)$. The output at $n = 4$ is

A. -6

B. 2

C. *zero*

D. -4

9. Consider a signal $x(t) = e^{-7t}u(t) + e^{-\beta t}u(t)$ and its Laplace transform is denoted by $X(s)$. The ROC of $X(s)$ is $Re\{s\} > -5$. Find the value of β?

A. 7

B. 5

C. -5

D. none of the above

10. The z- transform of a signal is given by $X(z) = \frac{1}{(1-2z^{-1})^2}$ then ROC $|z| > 2$, then $x[2]$ is

A. 0

B. 1

C. 12

D. 4

0.24

11. A causal LTI system is given by the transfer function $H(z) = \frac{2z^2+3}{(z+\frac{1}{3})(z-\frac{1}{3})}$ which of the following statement is/are true

A. The system is stable

B. Final value of the impulse response is 0

C. The initial value of the impulse response is 2

D. all the above

12. A causal LTI system is described by the difference equation $2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$. The system is stable only if

A. $|\alpha| = 2, |\beta| < 2$

B. $|\alpha| > 2, |\beta| > 2$

\ ✓C. $|\alpha| < 2$, any value of β

D. $|\beta| < 2$, any value of α

13. The ROC of the Z-transform of a sequence $(5/6)^n u[n] - (6/5)^n u[-n-1]$

A. $|z| < 5/6$

B. $|z| > 5/6$

\ ✓C. $5/6 < |z| < 6/5$

D. $6/5 < |z| < \infty$

14. A discrete time signal $x[n] = \delta[n-3] + 2\delta[n-5]$ has z transform $X(z)$. If $Y(z) = X(-z)$ is the z transform of another signal $y[n]$, then

A. $y[n] = x[n]$

\ B. $y[n] = x[-n]$

✓C. $y[n] = -x[n]$

D. $y[n] = -x[-n]$

-0.25 15. The ROC of the given DT signal $x[n] = (2)^{|n|}, -\infty < n < \infty$ is ....$1/2 < |z| < 2$......

Rough Work

$\lim_{n\to 0} h(n) = \lim_{z\to\infty} H(z)$

$\lim_{x\to\infty} x(t) = \lim_{s\to 0} sX(s)$

$\lim_{n\to\infty} = \lim_{z\to 0} (1-z^{-1})X(z)$

$\lim_{z\to 0} \frac{2z^2+3}{(z+\frac{1}{3})(z-\frac{1}{3})}$

$\frac{2z+3z^{-1}}{(z+\frac{1}{3})(z-\frac{1}{3})}$

$\frac{2z^2+3}{z(z+\frac{1}{3})(z-\frac{1}{3})}$

$\sum_{n=-\infty}^{-1} \left(\frac{6}{5}\right)^n z^{-n}$

$-n-1 \geq 0$, $-n \geq 1$, $n \leq -1$

$\frac{1}{1-\frac{5}{6}z^{-1}}$, $|\frac{5}{6}z^{-1}| < 1$, $|z| > 5/6$

$\frac{2s(s+1)}{s^2+4s+7}$

$\frac{2(1+\frac{1}{s})}{1+\frac{4}{s}+\frac{7}{s^2}}$

$\frac{A}{s+1} + \frac{B}{s+3} = \frac{s+5}{(s+1)(s+3)} = \frac{2}{(s+1)} + \frac{-1}{(s+3)}$

$2e^{-t}u(t) - e^{-3t}u(t)$

$A = \frac{4}{2} = 2$

$B = \frac{2}{-2} = -1$

$y = x * h$

$Y = X(s)H(s) = \left(\frac{1}{s-2}\right)\left(\frac{1}{s+3}\right) = \frac{1}{(s-2)(s+3)}$

$= \frac{A\,1/5}{s-2} + \frac{B\,-1/5}{s+3}$

$x(t) = \begin{cases} 1 & [a,b] \\ 0 & \end{cases}$

$= u(t-a) - u(t-b)$

$\frac{e^{-as} - e^{-bs}}{s}$

$2z^{-3}(z^4+z^2-2z+2-3z^{-4})$

$2(z+z^{-1}-2z^{-2}+2z^{-3}-3z^{-7})$

$n=1$

$x(n) = 2^{|n|}$

$X(z) = \sum_{n=-\infty}^{-1} 2^{-n} z^{-n} + \sum_{n=0}^{\infty} 2^n z^{-n}$

$\frac{1}{1-2z^{-1}}$, $|2z^{-1}| < 1$, $|z| > 2$

$= \frac{1}{5}e^{+2t} - \frac{1}{5}e^{-3t}$

$X(z) = z^{-3} + 2z^{-5}$

$Y(z) = X(-z) = -z^{-3} - 2z^{-5}$

$y(n) = -\delta(n-3) - 2\delta(n-5)$

$y(n) = -x(n)$

$\frac{e^{-s}}{s} \cdot \frac{1}{s^2} = \frac{e^{-s}}{s^3}$

$\frac{2z^2+3}{(z+\frac{1}{3})(z-\frac{1}{3})}$

$\frac{1}{s+7} + \frac{1}{s+\beta}$

$Re\{s\} > -7$

$Re\{s\} > -\beta$

$s^2Y(s) - sY(s) - 6Y(s) = X(s)$

$\frac{Y(s)}{X(s)} = \frac{1}{s^2-s-6} = \frac{1}{(s+2)(s-3)}$

$= \frac{A}{s+2} + \frac{B\,1/5}{s-3}$

$-\frac{1}{5}e^{-2t} + \frac{1}{5}e^{3t}$

$2Y(z) = \alpha z^{-2}Y(z) - 2X(z) + \beta z^{-1}X(z)$

$Y(z)\{2-\alpha z^{-2}\} = X(z)\{\beta z^{-1} - 2\}$

$H(z) = \frac{\beta z^{-1} - 2}{2 - \alpha z^{-2}}$

$2 - \frac{\alpha}{z^2} = 0 \Rightarrow 2z^2 - \alpha = 0$

$\Rightarrow z^2 = \frac{\alpha}{2}$

$\Rightarrow z = \pm\sqrt{\frac{\alpha}{2}}$

$\sqrt{\frac{\alpha}{2}} < 1$, $\alpha < 2$