परिपथविश्लेषणम्

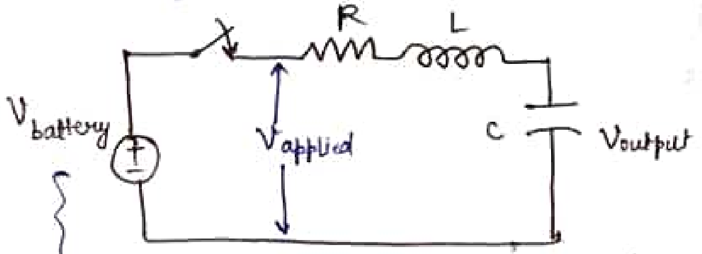
NETWORK ANALYSIS

3-08-2023

NETWORK ANALYSIS

Agenda:

1. What is Network Analysis (NA) all about?
2. Reference directions and conventions.
3. Source characteristics & transformations/rules.
4. Signals and some elementary definition/ideas.

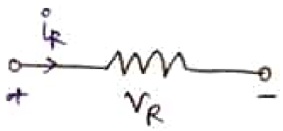


excitation $u(t)$ → Network (N/w) → Response $y(t)$

Network Analysis: Analysis of response from a n/w for given excitation.

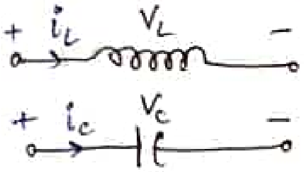
N/w Design: Designing of N/w to get desired response for given excitation.

Reference Directions



→ Load convention
↓ comes from
Instantaneous power
$P_r(t)$

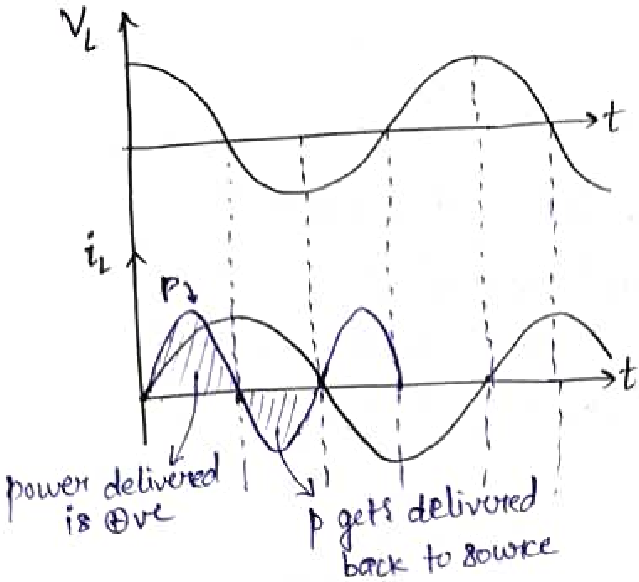
$P_r(t) = V_R(t)\, i_R(t)$
↳ is considered +ve, when load consumes power



↳ also correct; but needs to be consistent

$V_L = V_m \cos t$

i_L (in steady state) $= I_m \sin t$

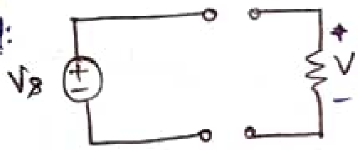


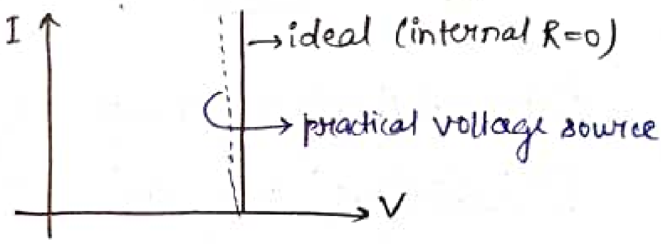
Datasheet

Source Characteristic

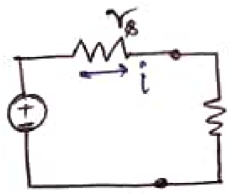
Voltage source:

Ideal:



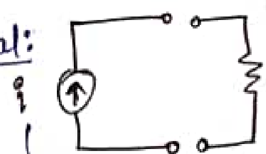


Practical:

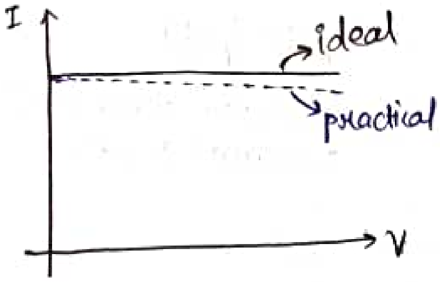


Current source:

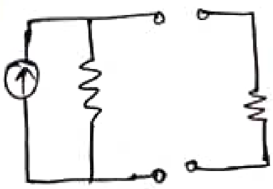
Ideal:



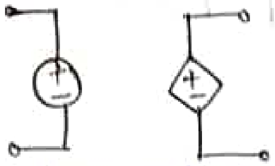
keeps on sourcing current irrespective of voltage across it



Practical:

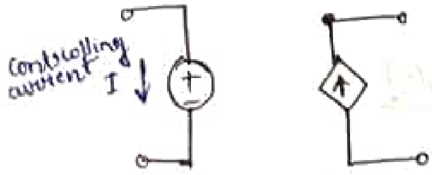


Dependent Sources



Voltage-controlled voltage source (VCVS)

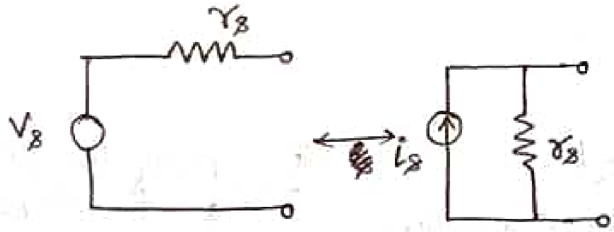
voltage-controlled current source (VCCS)



current-controlled current source (CCCS)

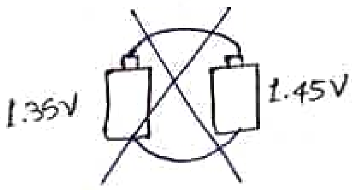
current-controlled voltage source (CCVS)

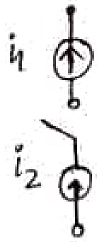
Transformation:



CARDINAL RULES:

- Never connect two Vsources in parallel.
- Never connect two Isources in series.





Signals

↳ Excitation and responses of a network are functions of time — these are known as signals.

Linear Network

↳ A network is linear if the principles of superposition and proportionality are applicable.

$$u_1 \longrightarrow \boxed{N/W\ A^*} \longrightarrow y_1$$

$$u_2 \longrightarrow \boxed{N/W\ A^*} \longrightarrow y_2$$

$$c_1u_1(t)+c_2u_2(t) \longrightarrow \boxed{N/W\ A^*} \longrightarrow c_1y_1(t)+c_2y_2(t)$$

Passive Network

↳ Network which doesn't contain active energy sources.

$\boxed{4\text{-}08\text{-}2023}$

Signals: $v(t)$, $i(t)$
$q(t)$, $\varphi(t)$

Causal System

↳ Response system anticipated
↳ doesn't depend on future excitation
↳ practically all systems are causal.

we'll deal with such systems only.

→ If there's no excitation, there's no response.
→ Response doesn't depend on future excitation.

Idea of Complex Frequency

$$s = \sigma + j\omega$$

TIME DOMAIN ANALYSIS OF 1^{st} ORDER LINEAR NETWORK

1^{st} Order Linear Network or Linear System

↳ where $y(t)$ and $u(t)$ can be related by 1^{st} order linear ODEs.

$$a_0 y + a_1 \frac{dy}{dt} = u \quad -① \quad \text{(Variables are time dependent)}$$

$$\Rightarrow a_0 y + a_1 y' = u$$

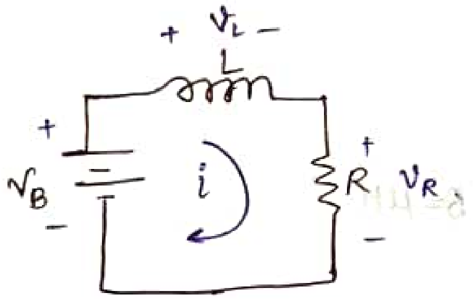
For eqⁿ ① to be linear, a_0 & a_1 should either be constant or functions of independent variable (in this case, t) only.

Eqⁿ ① $\Rightarrow$ $A_0 y(t) + A_1 y'(t) = u(t)$

↳ Linear Time Invariant system (LTI system)

$\rightarrow$ $a_0(t) y(t) + a_1(t) y'(t) = u(t)$

↳ LTV



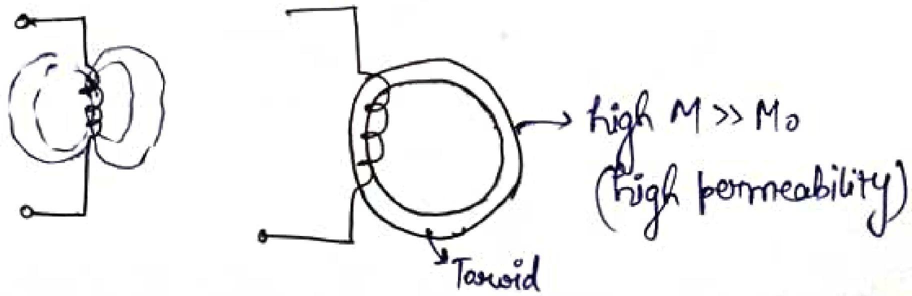
$$v_B = v_L + v_R$$

$$= L\frac{di}{dt} + Ri \rightsquigarrow \text{Linear system} \quad \text{(if } L \text{ \& } R \text{ are current independent)}$$

Brief Note on L and C

Inductance parameter:

↳ Magnetic field chooses path of low inductance

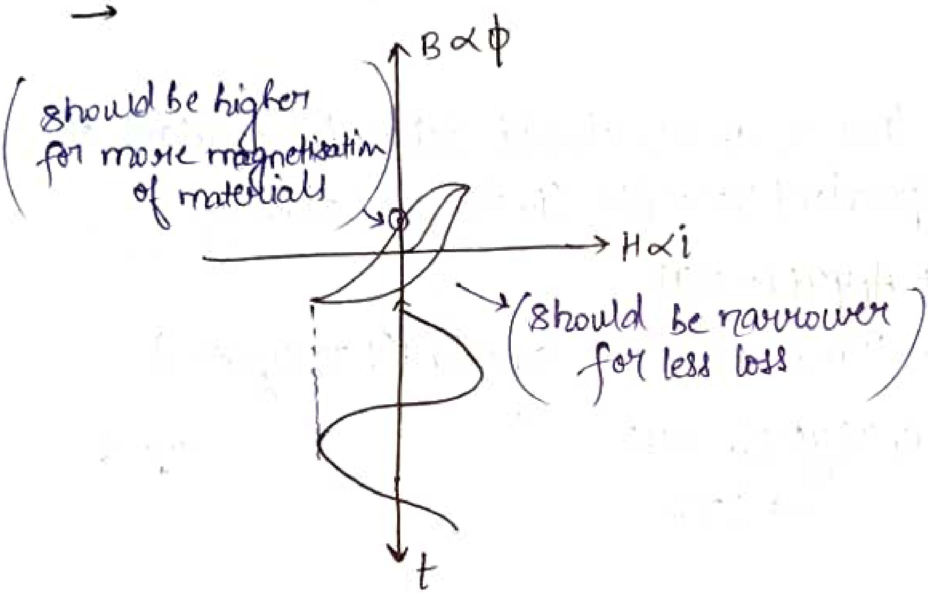


$$v_L = \frac{d\psi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$

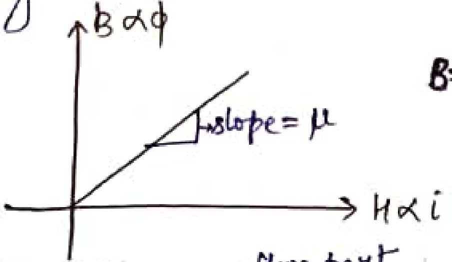
$$\psi = N\phi = Li$$

→ L is constant if the magnetic field is linear
↳ # Assumption for AV213 course.

→



→ In linear system, μ is constant.

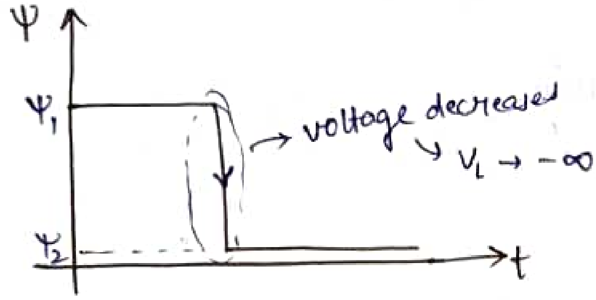


$B = \mu H$

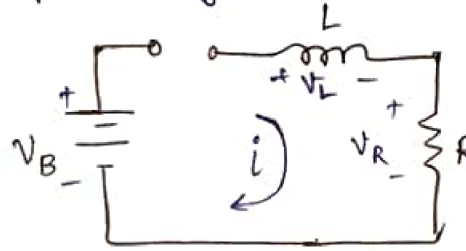
$L = \frac{N^2}{R}$; $R = \frac{l}{\mu A}$ (l → flux part; A → area of core seen by flux)

$= \mu \cdot \underbrace{\frac{A}{l}}_{} N^2$

geometry is fixed (& no. of turns)



If suddenly we cut the wire,



⇒ voltage across L would be ∞
⇓
produces spark.

→ ψ cannot collapse/change significantly instantaneously.
⇓
current through an inductor cannot change suddenly.
⇓
\# conservation of flux

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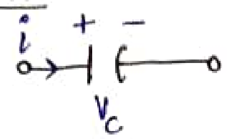
Inductance (L)

↳ i_L cannot change instantaneously.
↳ i stiff element

Capacitance (C)



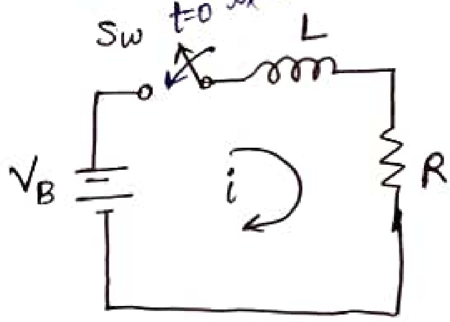
$$q = CV \Rightarrow i = \frac{dq}{dt} = C\frac{dv}{dt}$$

$\frac{dq}{dt} \to \infty \Rightarrow i_{charging} \to \infty$
⇓
not a possibility by design

$q \to$ cannot change suddenly
$\Rightarrow$ C_V cannot change suddenly.

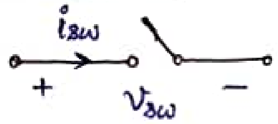
$C \to V$ stiff element.

Time Domain Analysis of 1st Order RL N/w with DC Excitation



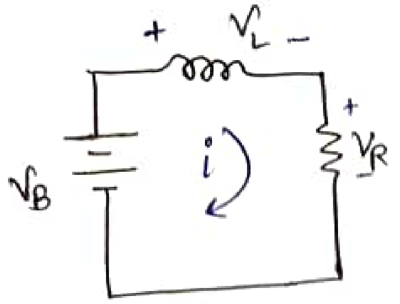
Intially relaxed N/W (no energy stored)

Ideal switch



$i_{sw} = 0$, when sw is open
$V_{sw} = 0$, when sw is closed.

Once switch is closed at $t=0$, the ~~current~~ circuit becomes



$i(0)=0$
$i(0+)=0$

KVL:

$$V_B = V_L + V_R$$
$$= L\frac{di}{dt} + iR$$
$$= Li' + iR$$

Objective: Find and then plot i.

$$\Rightarrow \frac{V_B}{L} = i' + \frac{1}{L/R} i = i' + \frac{1}{\tau} i \quad \dots (e1)$$

where $\boxed{\underbrace{\tau \triangleq L/R}_{\to\, \text{def}^n}}$

Let f: IF.

$$f\frac{V_B}{L} = fi' + \frac{f}{\tau} i = (fi)'$$

So, $f' = f/\tau \Rightarrow \frac{df}{f} = \frac{dt}{\tau} \Rightarrow f = Ke^{t/\tau}$

$$\therefore fi = \int f\frac{V_B}{L} dt$$

or, $Ke^{t/\tau} i = \int Ke^{t/\tau} \cdot \frac{V_B}{L} dt$

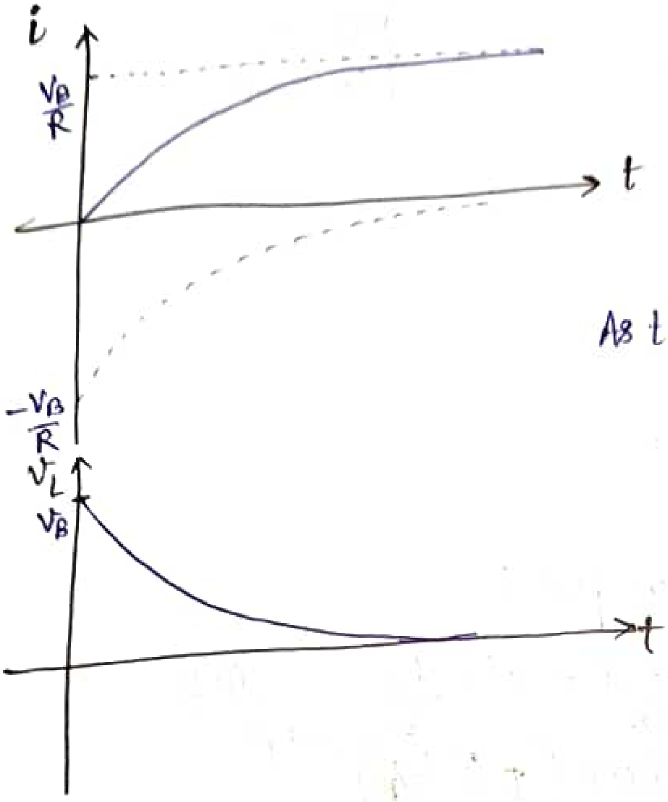
$$\Rightarrow i = \frac{V_B}{L} e^{-t/\tau} \int e^{t/\tau} dt + K_1 e^{-t/\tau}$$
$$= \frac{V_B}{L} e^{-t/\tau} \cdot \tau e^{t/\tau} + K_1 e^{-t/\tau}$$
$$= \frac{V_B}{R} + K_1 e^{-t/\tau}$$

Applying initial condition, $i(0)=0$

$$\Rightarrow K_1 = -\frac{V_B}{R}$$

$$\therefore \boxed{i(t) = \frac{V_B}{R}(1 - e^{-t/\tau})} \Rightarrow \boxed{V_L = V_B e^{-t/\tau}} \quad \left(\because V_L = L\frac{di}{dt}\right)$$
$$(\because L = \tau R)$$

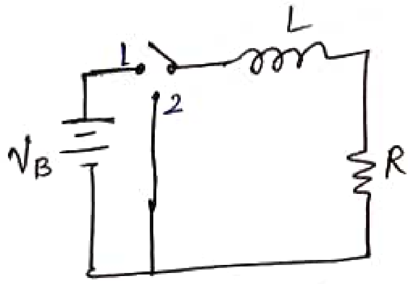
$$i(t) = \frac{V_B}{R}\left(1 - e^{-t/\tau}\right) = \frac{V_B}{R} - \frac{V_B}{R} e^{-t/\tau}$$



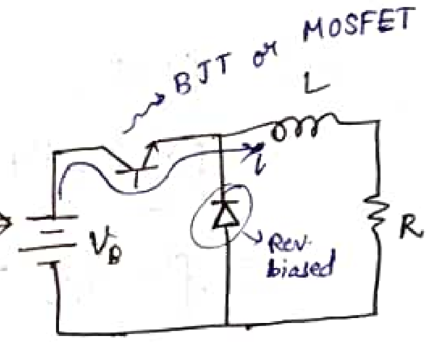
As $t \to \infty$, $I_{steadystate\,(SS)} = \frac{V_B}{R}$

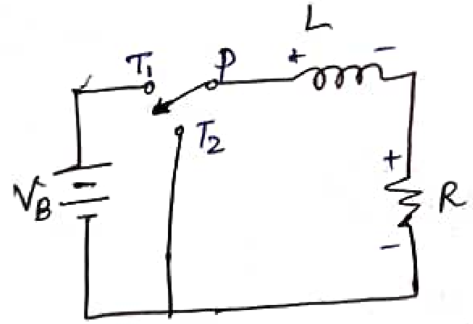
$\underline{V_L = V_B e^{-t/\tau}}$

$V_L \to 0$ in steadystate for DC excitation.



$\Rightarrow$ we want to change to pos②, before disconnecting pos①& without short-circuiting. $\Rightarrow$

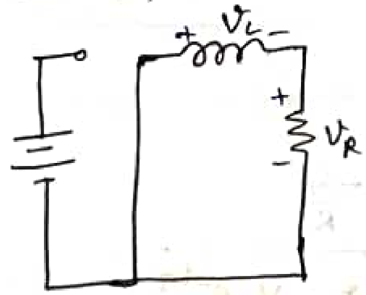




SS was ~~reado~~ reached with Pole pos①.

Then it is toggled to ②.

Once Sw → ②, rel. ~~current~~ circuit becomes

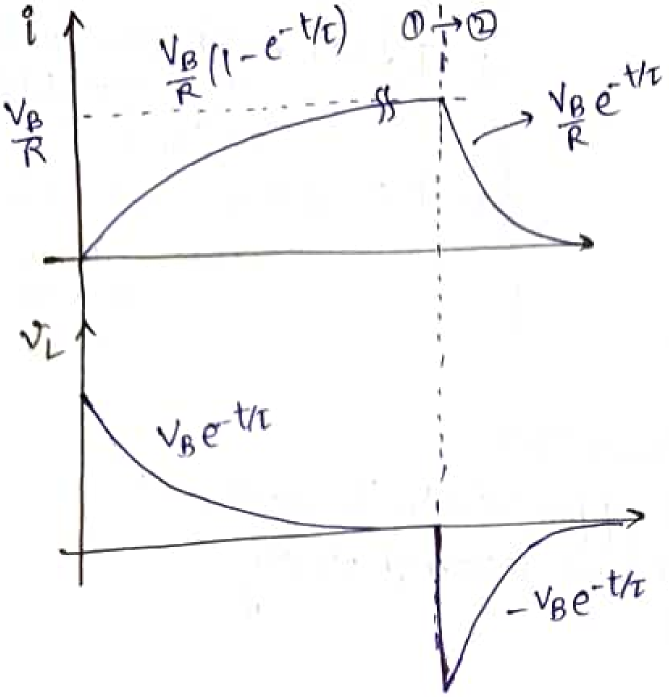


$$i(0) = V_B/R$$

$$0 = V_L + V_R$$

$$= Li' + Ri$$

$$\Rightarrow i = \frac{V_B}{R} e^{-t/\tau}$$



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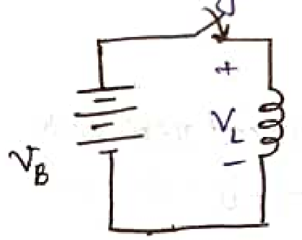
→ we can couple voltage source with stiff element.
↳ But we typically don't apply DC voltage source across L.



$V_B = V_L = L\frac{di}{dt}$

↳ Inductor constitutes magnetic material (around a core).
↳ i keeps on increasing as V=const.
↳ As saturation is reached, μ becomes zero.

$i_L = \frac{V_B}{R}(1-e^{-t/\tau})$ in pos.①

$= \frac{V_B}{R}e^{-t/\tau}$ in pos.②

where, $\tau \triangleq L/R$.

τ → decides the rate of decay or rise of i.

τ : Time constant of the system.

$i_L = I e^{-t/\tau}$

$i_L(1) = I e^{-1/\tau}$

$\tau\uparrow \Rightarrow$ Decay slower

$Ie^{-1/1}$ $\sim 0.3I$ $Ie^{-1/2}$ $\sim 0.6I$

$i_L = I e^{-t/\tau}$

t/τ	i_L/I
0	1
1	~ 0.37
2	~ 0.14
3	~ 0.05
4	~ 0.018

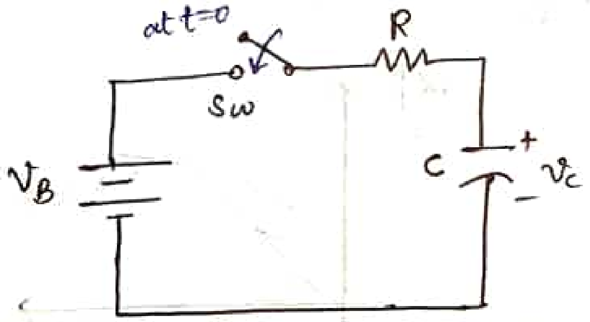
$\rightsquigarrow$ settling time $= (4\tau)$

↳ For all practical applications, the system is assumed to have reached steady state at 4τ.

$\tau \downarrow \Rightarrow L \downarrow \Rightarrow$ Resistive nature $\uparrow$
$\Rightarrow$ current following $V \uparrow$
$\Rightarrow$ lag b/w I & $V \downarrow$

$\tau \uparrow \Rightarrow L \uparrow \Rightarrow$ lag b/w I & $V \uparrow$
$\Rightarrow$ More time to reach steady state.

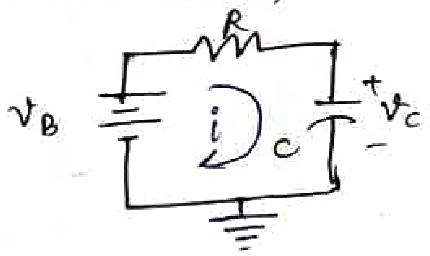
RC Circuit – 1st Order with DC Excitation



Initially relaxed network
$V_c(0^-) = 0$

Objective of Analysis: finding $V_c(t)$ for $t \geq 0$.

Once Sw is closed, the equivalent circuit becomes



$i = C\frac{dV_c}{dt}$
$= C V_c'$

$V_B = iR + V_c$

$\Rightarrow 0 = i'R + i/C \Rightarrow i' + \frac{1}{RC} i = 0$

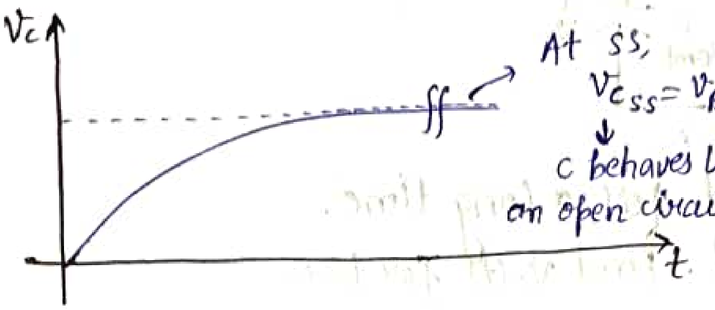
$\boxed{\tau \triangleq RC}$ for RC N/w.

$\Rightarrow i = Ke^{-t/\tau}$

At $t=0$, $i(0) = \frac{V_B - V_c(0)}{R} = \frac{V_B}{R}$

$\therefore \boxed{i = \frac{V_B}{R} e^{-t/\tau}}$.

$\therefore \quad v_C = v_B - iR = v_B(1 - e^{-t/\tau})$

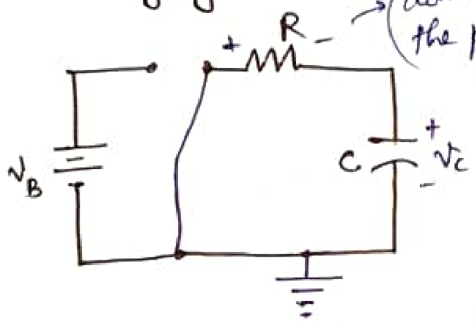


At SS;
$v_{C_{SS}} = v_B$
$\downarrow$
C behaves like an open circuit

At steady state,
- $v_C = v_B$
- no charging current
- open circuit.

Discharging:-

don't change the polarity

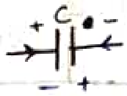


$v_R + v_C = 0$

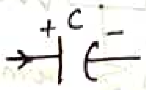
$\Rightarrow iR + v_C = 0$

$\Rightarrow RC v_C' + v_C = 0$

$\Rightarrow v_C = Ke^{-t/\tau}$



: Normal capacitor
↳ can be charged in both dirn
↳ can be used with ac as well as dc voltage.



: Polarised capacitor
↳ can be charged in one dirn only (only dc)
↳ stores large charge in less space.

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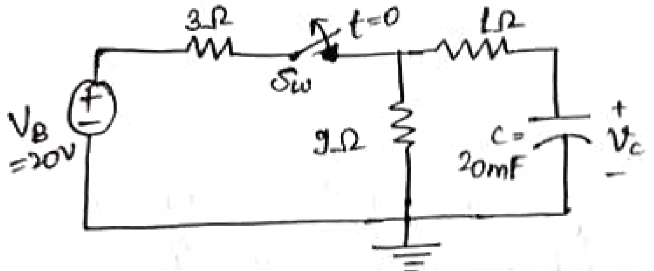
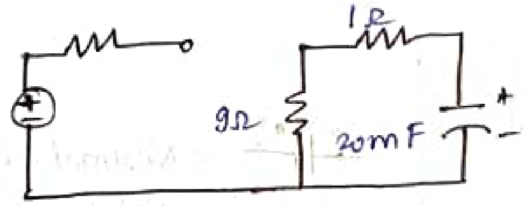
Q)

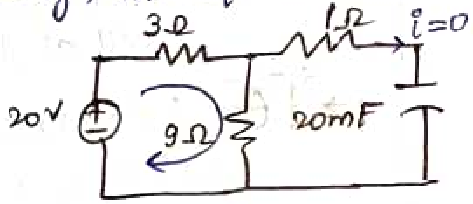


Initially Sw was closed for a long time.
At t=0, Sw was opened. Find $v_c(t)$ for $t \geq 0$.

Soln: For $t \geq 0$, relevant eqv. ckt. is



Initially, the eqv. ckt. was



$$v_c(0^-) = \frac{9}{12} \times 20$$
$$= 15V$$

For $t \geq 0$

$$v_c(0^-) \approx v_c(0^+)$$

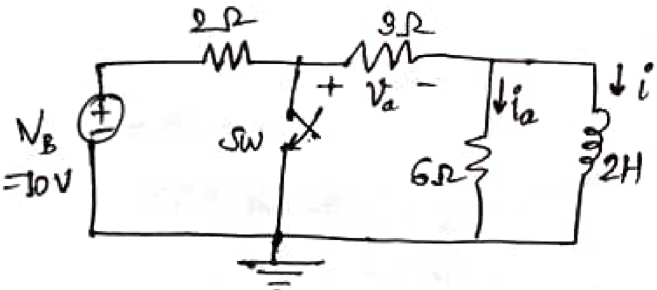
$$v_c(t) = K e^{-t/\tau},$$

where $K = v_c(0^-) = 15V$

$$\tau = RC = 10 \times 20 \times 10^{-3}$$
$$= 0.2$$

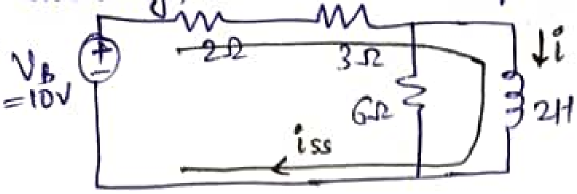
$$\therefore \; v_c(t) = 15 e^{-5t} \; V.$$

Q)



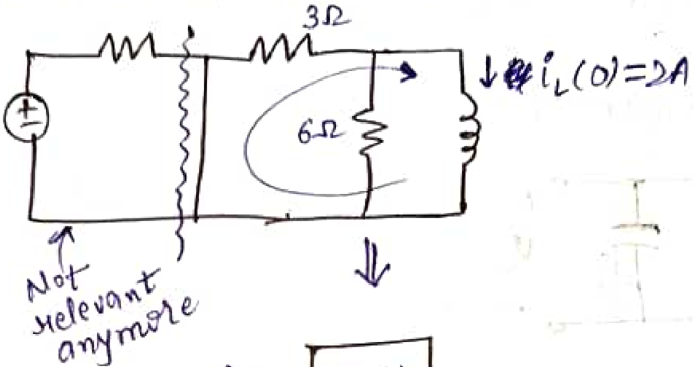
Sw was initially open for a long time. Sw closed at t=0.
Find i, i_a & V_a for $t \geq 0$.

Soln: Initially, at SS with Sw open, the rel. ckt. of interest is:

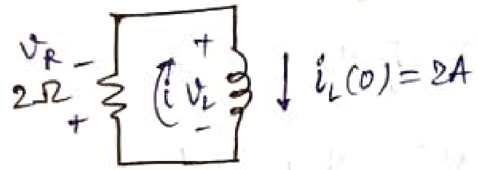


$$i_{ss} = \frac{10}{5} A = 2A$$

When Sw is closed, the relevant ckt. becomes:



$\Downarrow$



$$v_L + v_R = 0$$

$$\Rightarrow Li' + iR = 0$$

$$\Rightarrow i = Ke^{-t/\tau}, \quad \tau = 1, \; K = 2$$

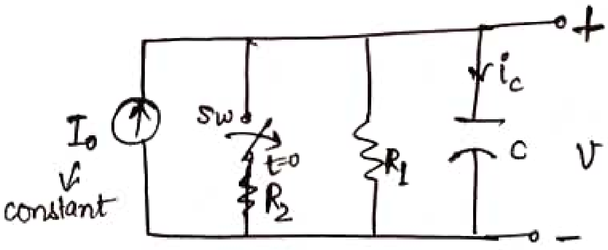
$$\Rightarrow i = 2e^{-t} \text{ A}$$

$$\therefore \; i_a = -\frac{3}{9} \times 2e^{-t} = -\frac{2}{3}e^{-t} \text{ A}$$

and, $v_a = v_L = -Li = -2 \times 2(-1)e^{-t}$

$$\Rightarrow v_a = 4e^{-t} \text{ V}$$

Q1



$R_1 = 1\Omega$, $R_2 = 0.5\Omega$, $C = 0.5F$

SS was reached with Sw closed.

Find $v(t)$ for $t \geq 0$.

Soln: Initially when Sw was closed, ckt. of interest was:

$$R_1 \| R_2 = \frac{1/2}{3/2} = \frac{1}{3}$$

$$V_c(0^-) = I_0 \times \frac{1}{3} V = \frac{I_0}{3} V$$

When switch is opened,

$$V_c(0) = \frac{I_0}{3} V$$

$$I_0 = \frac{V_c}{1} + C V_c' = V_c + \frac{1}{2} V_c'$$

$$\Rightarrow V_c' + 2V_c = 2I_0$$

If f be the IF,

$$f V_c' + 2f V_c = 2f I_0$$

$$f' = 2f \Rightarrow \frac{df}{f} = 2dt$$

$$\Rightarrow f = Ke^{2t}$$

$$\therefore (f V_c)' = 2f I_0$$

$$\Rightarrow f V_c = 2I_0 \int f dt + K$$

$$\Rightarrow V_c = 2I_0 e^{-2t} \int e^{2t} dt + Ke^{-2t}$$

$$= I_0 + Ke^{-2t}$$

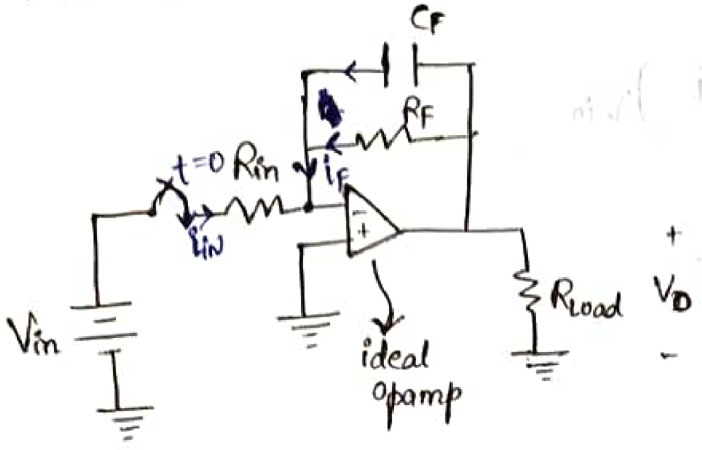
At $t=0$, $V_c = I_0/3 = I_0 + K$

$$\Rightarrow K = -2I_0/3$$

$$\therefore V_c = I_0\left(1 - \frac{2}{3}e^{-2t}\right) V$$

16-08-2023

Q) For $t \geq 0$



$V_o(0) = 0$

$V_o(0^+) = 0$

Find $V_0(t)$ for $t \geq 0$.

$V_- \to 0V$

$V_+ \to 0V$

$V^+ = V^-$ only when we put the negative feedback, i.e., connecting output with $\ominus$ve side

Soln:

$$V_+ = V_- \quad -(1)$$

$$i_{in} + i_F = 0 \quad -(2)$$

$$\begin{cases} i_{in} = \dfrac{V_{in} - 0}{R_{in}} \\ i_F = \dfrac{V_0}{R_F} + C_F \dfrac{dV_0}{dt} \end{cases} \quad -(3)$$

Using (3) in (2),

$$\frac{V_{in}}{R_{in}} + \frac{V_0}{R_F} + C_F V_0' = 0$$

$$\Rightarrow V_0' + \frac{1}{R_F C_F} V_0 = -\frac{1}{R_{in} C_F} V_{in} \quad -(4)$$

Let f be the integrating factor.

$$-f \frac{1}{R_{in} C_F} V_{in} = f V_0' + \frac{f}{R_F C_F} V_0 = (f V_0)'$$

$$\therefore \frac{df}{dt} = \frac{f}{R_F C_F} \Rightarrow f = k e^{t/\tau_F}$$

where $\tau_F \triangleq R_F C_F$

$$\text{Then, } \left(k e^{t/\tau_F} V_0\right)' = -k e^{t/\tau_F} \frac{1}{R_F C_F} V_{in}$$

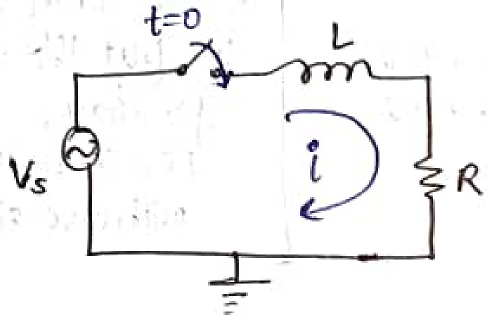
$$\Rightarrow e^{t/\tau_F} V_0 = -\frac{V_{in}}{R_{in} C_F} \int e^{t/\tau_F} dt + k_2$$

$$\Rightarrow V_0 = -\frac{V_{in}}{R_{in} C_F} \cdot \tau_F + k_2 e^{-t/\tau_F}$$

At $t=0$, $k_2 = \frac{R_F}{R_{in}} V_{in}$.

$$\therefore V_0 = -\frac{R_F}{R_{in}}\left(1 - e^{-t/\tau_F}\right) V_{in}.$$

RL with AC Excitation

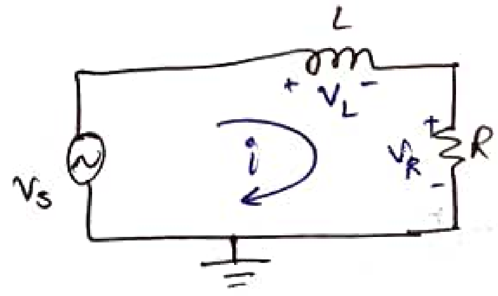


- Initially relaxed N/w.
- Sw closes at $t=0$.

- Objective of analysis : find $i(t)$ for $t \geq 0$.

Given $V_s = V_m \sin(\omega t + \alpha)$

For $t=0$, the relevant circuit is:



Applying KVL, $V_s = Li' + Ri$

$$\Rightarrow V_m \sin(\omega t + \alpha) = Li' + Ri$$

$$\Rightarrow i' + \frac{1}{\tau_F} i = \frac{V_m}{L}\sin(\omega t + \alpha) \quad -①$$

If $f = ke^{t/\tau_F} \longleftrightarrow$ Int. factor

$$\Rightarrow (if)' = \frac{V_m}{L} ke^{t/\tau_F}\sin(\omega t + \alpha)$$

$$\Rightarrow i = \frac{V_m e^{-t/\tau_F}}{L}\int \underbrace{e^{t/\tau_F}\sin(\omega t + \alpha)\,dt}_{I_{ng}} + k_0 e^{-t/\tau_F} \quad -②$$

$$I_{ng} = \int \underbrace{e^{t/\tau_F}}_{\tau_F} \sin(\omega t+\alpha)\,dt$$

$$= \int \tau_F \sin(\omega t+\alpha)\, d(e^{t/\tau_F})$$

$$\int u\,dv = uv - \int v\,du$$

$$= \tau_F \left[e^{t/\tau_F} \sin(\omega t+\alpha) - \int e^{t/\tau_F} \underbrace{d\{\sin(\omega t+\alpha)\}}_{\omega\cos(\omega t+\alpha)} dt \right]$$

$$= \tau_F \left[e^{t/\tau_F} \sin(\omega t+\alpha) - \omega \int e^{t/\tau_F} \cos(\omega t+\alpha)\, dt \right]$$

$$= \tau_F \left[e^{t/\tau_F} \sin(\omega t+\alpha) - \omega \tau_F \int \cos(\omega t+\alpha)\, d(e^{t/\tau_F}) \right]$$

$$= \tau_F \left[e^{t/\tau_F} \sin(\omega t+\alpha) - \omega \tau_F \left\{ e^{t/\tau_F} \cos(\omega t+\alpha) - \int e^{t/\tau_F} \underbrace{d\{\cos(\omega t+\alpha)\}}_{-\omega\sin(\omega t+\alpha)} dt \right.\right.$$

$$= \tau_F e^{t/\tau_F} \left[\sin(\omega t+\alpha) - \omega \tau_F \cos(\omega t+\alpha) \right.$$
$$\left. - \omega^2 \tau_F^2 \int \underbrace{e^{t/\tau_F} \sin(\omega t+\alpha)\, dt}_{I_{ng}} \right]$$

$$\Rightarrow (1+\omega^2\tau_F^2) I_{ng} = \tau_F e^{t/\tau_F} \left[\sin(\omega t+\alpha) - \omega\tau_F \cos(\omega t+\alpha) \right]$$

$$\Rightarrow I_{ng} = \frac{1}{1+\tau_F^2\omega^2} \tau_F e^{t/\tau_F} \left[\sin(\omega t+\alpha) - \omega \tau_F \cos(\omega t+\alpha) \right] \quad -③$$

Using ③ in ②,

$$i(t) = \cancel{e^{-t/\tau_F}} \frac{V_m}{\cancel{K}} \cdot \frac{1}{1+\frac{\omega^2 L^2}{R^2}} \frac{\cancel{K}}{R} \cancel{e^{t/\tau_F}} \left[\sin(\omega t+\alpha) - \omega \tau_F \cos(\omega t+\alpha) \right]$$
$$+ k_0 e^{-t/\tau_F}.$$

$$\Rightarrow i(t) = \frac{V_m}{R} \frac{R^2}{\omega^2 L^2 + R^2} \left[\sin(\omega t+\alpha) - \omega \tau_F \cos(\omega t+\alpha) \right] + k_0 e^{-t/\tau_F}$$

Let $Z \triangleq \sqrt{R^2+\omega^2 L^2}$

$\theta \triangleq \tan^{-1}\left(\frac{\omega L}{R}\right) \rightarrow$ Impedence Angle

& $\omega L \triangleq X$

Then,

$$i(t) = \frac{V_m}{Z}\left[\frac{R}{Z}\sin(\omega t+\alpha) - \underbrace{\frac{R}{Z}\cdot\frac{\omega L}{R}}_{X/Z}\cos(\omega t+\alpha)\right] + k_0 e^{-t/\tau_F}$$

$$\Rightarrow i(t) = \frac{V_m}{Z}\left[\cos\theta\,\sin(\omega t+\alpha) - \sin\theta\cos(\omega t+\alpha)\right] + k_0 e^{-t/\tau_F}$$

$$\Rightarrow i(t) = \frac{V_m}{Z}\sin(\omega t-\theta) + k_0 e^{-t/\tau_F}.$$

17-08-2023

TD Analysis of Linear 2nd Order Network

2nd order ODE: $a_2(t)y''(t) + a_1(t)y'(t) + a_0(t)y(t) = r(t)$ ... (e1)

For (e1) to be linear,

$a_2, a_1, a_0 \rightarrow$ are either constants or, functions of t only.

(e1) $\xrightarrow[(t)]{\text{Dropping the}}$ $y'' + b_1 y' + b_0 y = r_1(t)$ ... (e2)

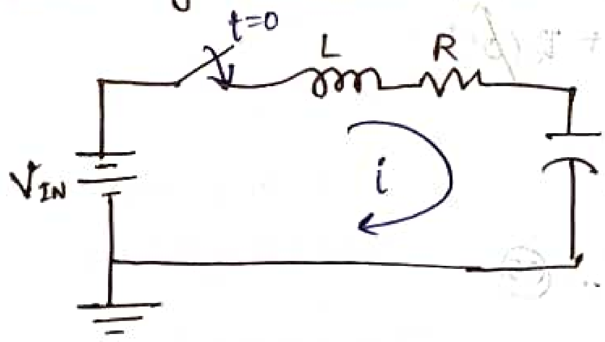
$y(t) = y_p(t) + y_h(t)$

For y_h, $y = ke^{\lambda t}$ trial soln.

Characteristic eqn: $\lambda^2 + b_1\lambda + b_0 = 0$.

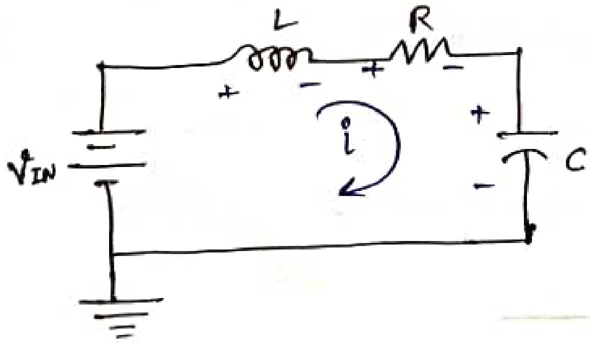
$[\lambda] \rightarrow$ unit of frequency.

Case Study: Series RLC with DC Excitation



- Initially relaxed N/w
- Objective of Analysis : Find & plot $i(t)$ for $t \geq 0$.

For $t \geq 0$, rel. ckt. is,

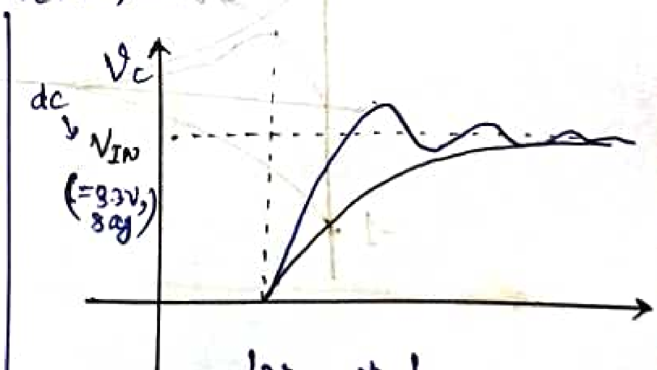


$i(0^-) = 0$; $V_{c,SS} = V_{IN}$

$v_c(0^-) = 0$

$i(0^+) = 0$

$v_c(0^+) = 0$



$\frac{|V_{IN} - V_c|}{V_{IN}} < 0.02$

Using KVL: $V_{IN} = V_L + V_R + V_C$ ...(e1)

Differentiating both sides of (e1)

$0 = v_L' + v_R' + v_C'$

$\Rightarrow 0 = Li'' + Ri' + \frac{1}{C}i$

$$\Rightarrow i'' + \frac{R}{L} i' + \frac{1}{LC} i = 0 \quad \ldots (e2)$$

Let $i = ke^{\lambda t}$ be a trial soln, then the corresponding auxillary eqn is,

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0$$

<u>Note:</u> Notion of τ (R/L or RC) as in 1st order systems is not directly applicable here.

$$\lambda = \frac{1}{2}\left\{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}\right\} \quad \ldots (e3)$$

Eg. (1) $V_{IN} = 1V$, $R = 3\,\Omega$, $L = 1\,H$, $C = \frac{1}{2}F$.

$$\lambda = \frac{1}{2}\left\{-3 \pm \sqrt{9-8}\right\}$$

$$= -1, -2$$

Then, $i(t) = Ae^{-t} + Be^{-2t}$

$$i(0+) = 0 \Rightarrow 0 = A + B \quad \ldots (e4)$$

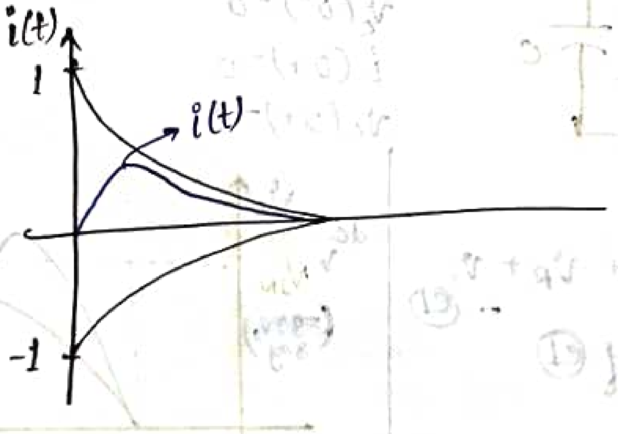
$$(e1) \Rightarrow V_{IN} = \underbrace{v_L(0^+)}_{Li'(0+)} + \cancel{v_R(0^+)} + \cancel{v_C(0^+)}$$

$$\Rightarrow i'(t) = -Ae^{-t} - 2Be^{-2t}$$

At $t = 0^+$, $\underbrace{\frac{V_{IN}}{L}}_{=1} = -A - 2B \quad \ldots (e5)$

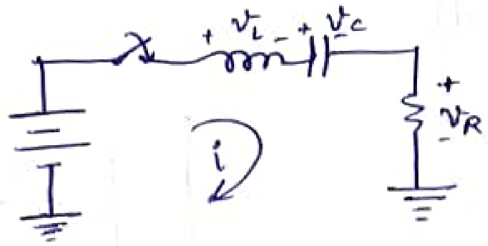
$(e4), (e5) \Rightarrow A = 1, B = -1$

$$\therefore i(t) = e^{-t} - e^{-2t} \; A$$



23-08-2023

RLC (Series) with DC Excitation



$$v_{IN} = v_L + v_R + v_C \quad \cdots (e1)$$

$$0 = i'' + \frac{R}{L} i' + \frac{1}{LC} i \quad \cdots (e2)$$

Characteristic Eq^n/ Auxilliary Eq^n

$$\lambda = \frac{1}{2}\left\{ -\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}} \right\}$$

Eg ① $V_{IN} = 1V$, $R = 2\Omega$, $L = 1H$, $C = 0.5F$.

$$\lambda = \underbrace{-1 \pm j}_{\alpha \pm j\beta}$$

$$i(t) = e^{-t}(A\cos t + B\sin t)$$

$$i(0+) = 0, \quad v_L(0+) = V_{IN}$$

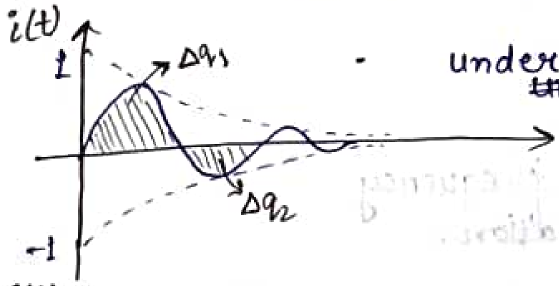
$$i'(0^+) = \frac{V_{IN}}{L} \quad \cancel{CX/I}$$

$$i(t) = B e^{-t} \sin t$$

$$\Rightarrow i(t) = e^{-t} \sin t$$

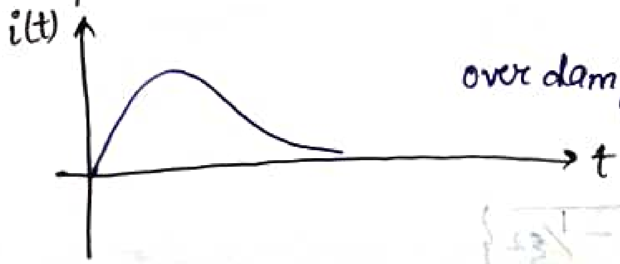
Eg. ①: $i(t) = e^{-t} - e^{-2t}$

Eg ②: $i(t) = e^{-t}\sin t$ $\leq e^{-t}$ damped sine

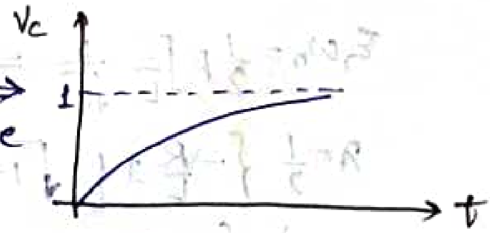


under ~~un~~damped response

Eg①:



over damped response $\Rightarrow$

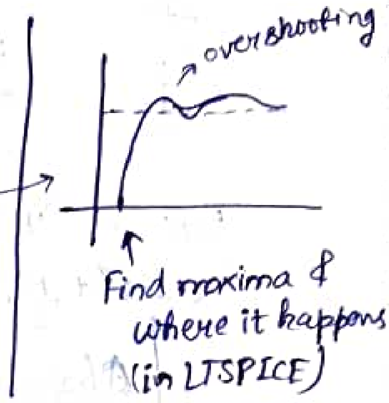


Eg②:

$$v_c(t) = V_{IN} - iR - i'L$$
$$= 1 - 2e^{-t}\sin t - 1\{e^{-t}\cos t - e^{-t}\sin t\}$$
$$= 1 - \underbrace{e^{-t}\{\sin t + \cos t\}}_{\sqrt{2}e^{-t}\sin(t+\pi/4)}$$

↳ can be ⊖ve

↳ means v_c can be $> V_{IN}$
↳ Vinput



$$\rightarrow \lambda = \frac{1}{2}\left\{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}\right\}$$

Hence whether $\frac{R}{L}$ > or = or < $\frac{2}{\sqrt{LC}}$, that decides the nature of transient.

If $R/L \geq \frac{2}{\sqrt{LC}}$ → damped

↳ $\underbrace{\frac{1}{2}R\sqrt{\frac{C}{L}}}$ >/=/< 1

$$\boxed{\xi \triangleq \frac{1}{2}R\sqrt{\frac{C}{L}}}$$

↳ damping ratio / damping factor for series RLC.

ξ	nature of response
>1	overdamped
<1	under damped
=1	critically damped
=0	un damped

We also define, $\boxed{\omega_n \triangleq \frac{1}{\sqrt{LC}}}$

↳ natural frequency of oscillation.

$$\xi\omega_n = \frac{1}{2}R\sqrt{\frac{C}{L}}\frac{1}{\sqrt{LC}} = \frac{1}{2}\frac{R}{L}$$

$$\lambda = \frac{1}{2}\left\{-\frac{R}{L} \pm \frac{R}{L}\sqrt{1 - \frac{4L}{R^2C}}\right\}$$
$$= \frac{1}{2}\left\{-2\xi\omega_n \pm 2\xi\omega_n\sqrt{1 - 1/\xi^2}\right\}$$

$$\Rightarrow \lambda = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

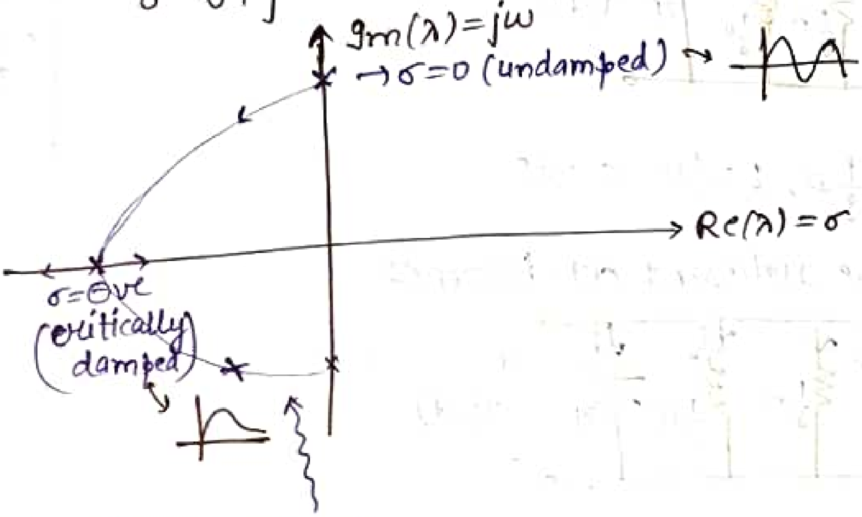
$$\therefore \boxed{\lambda = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}}$$

$[\lambda] \longrightarrow$ frequency

Complex frequency,

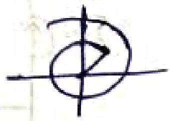
$$s = \sigma + j\omega$$

$|\sigma = -\xi\omega_n$



$\sigma = \oplus ve \Rightarrow \xi = \ominus ve$

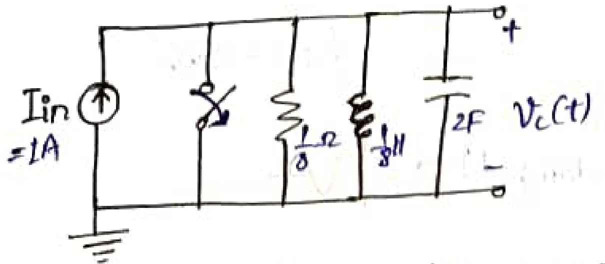
$\hookrightarrow \ominus$ve damped



$$\sigma^2 + \omega^2 = \omega_n^2 \rightsquigarrow \text{circle}$$

24-08-2023

Case Study 2: Parallel RLC with a current source

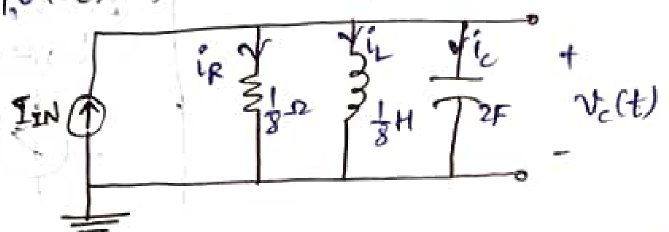


- Initially relaxed N/w.
- Find $v_c(t)$ for $t \geq 0$.

- voltage source shouldn't be short-circuited.
- current source should never be open-circuited.

$I_{IN} = 1A,\ R = \frac{1}{8}\Omega,\ L = \frac{1}{8}H,\ C = 2F$

For $t \geq 0$, the relevant ckt. becomes



Using KCL: $I_{IN} = i_R + i_L + i_c \quad \ldots (e1)$

$$= \frac{v_c}{R} + \frac{1}{L}\int v_c\,dt + C\frac{dv_c}{dt}$$

Differentiating both sides of (e1),

$$0 = \frac{1}{R}v_c' + \frac{1}{L}v_c + C v_c''$$

$$\Rightarrow v_c'' + \frac{1}{RC}v_c' + \frac{1}{LC}v_c = 0 \quad \ldots (e2)$$

$\big\downarrow$ Ch. Eq^n $\quad v = ke^{\lambda t}$

$$\lambda^2 + \frac{1}{RC}\lambda + \frac{1}{LC} = 0 \quad \ldots (e3)$$

$$\Rightarrow \lambda = \frac{1}{2}\left\{-\frac{1}{RC} \pm \sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}}\right\}$$

$$= \frac{1}{2}\left\{-\frac{1}{RC} \pm \frac{1}{RC}\sqrt{1 - \underbrace{4R^2\frac{C}{L}}_{1/\xi^2}}\right\}$$

$$\boxed{\begin{aligned}\xi &\triangleq \frac{1}{2R}\sqrt{\frac{L}{C}} \\ \omega_n &= \frac{1}{\sqrt{LC}}\end{aligned}} \Rightarrow \xi\omega_n = \frac{1}{2RC}$$

$$\Rightarrow \lambda = -\xi\omega_n \pm \xi\omega_n\sqrt{1 - 1/\xi^2}$$

$$\Rightarrow \boxed{\lambda = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}}$$

Useful format for N/w designing
↳ not useful for problem solving

$R=0 \rightarrow$ nothing happens
$R \rightarrow \infty \rightarrow$ under damping

$\xi \propto R$ for series RLC
$\propto 1/R$ for parallel RLC

$$\lambda_{1,2} = \frac{1}{2}\left\{-\frac{1}{1/8\times 2} \pm \sqrt{\left(\frac{1}{1/8\times 2}\right)^2 - \frac{4}{1/8\times 2}}\right\}$$

$$= \frac{1}{2}\left\{-4 \pm \sqrt{16-16}\right\}$$

$$= -2$$

- Current through an L cannot change instant.
- Voltage across a C cannot change instant.

Now,

$$v_c(t) = (A+Bt)\,e^{-2t}$$

$$v_c(0+) = 0 \Rightarrow A = 0$$

$$\therefore v_c(t) = Bte^{-2t}$$

$$I_c(0+) = I_{in} = 1 = Cv_c'(0+) \qquad (\because I_R(0+) = I_L(0^+) = 0)$$

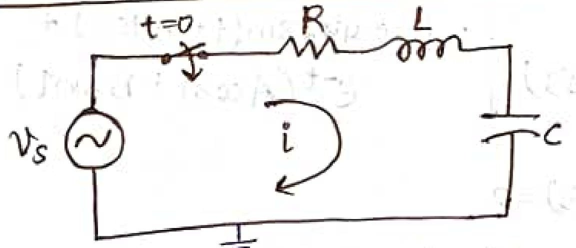
- $i_L(0) = 0$
- $i_R(0^+) = 0$ as, if $i_R \neq 0$, it will change the $v_c(0+)$

$$v_c'(t) = B(e^{-2t} - 2\cdot te^{-2t})$$

$$v_c'(0+) = B = I_{IN}/C = 0.5$$

$$\therefore v_c(t) = 0.5te^{-2t}\ \text{V}.$$

Series RLC with AC Excitation



$$v_s = V_m\cos(\omega t + \alpha)$$

For $t \geq 0$, $v_s = v_R + v_L + v_c$

$$\Rightarrow V_m\cos(\omega t+\alpha) = iR + Li' + \frac{1}{C}\int i\,dt$$

Differentiating both sides,

$$-\frac{V_m\omega}{L}\sin(\omega t+\alpha) = i'' + \frac{R}{L}i' + \frac{1}{LC}i$$

Solution: $i = i_h + i_{nh}$

$$i_{nh} = \frac{V_m}{Z}\cos(\omega t + \alpha - \theta)$$ ⇝ can lag or lead depending on the reactive impedence.

$$\theta = \tan^{-1}\left(\frac{X}{R}\right)$$

$$X = \omega L - \frac{1}{\omega C}$$

Take $R = 2\Omega$, $L = 1H$, $C = 0.5F$, $v_s = \sin t$.

ch. eqn: $\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0$

$$\Rightarrow \lambda = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}$$

$$\Rightarrow \lambda = -\frac{2}{2\times 1} \pm \frac{1}{2}\sqrt{\frac{4}{1} - \frac{4}{1\times\frac{1}{2}}}$$

$$= -1 \pm j1$$

$$\therefore i_h = e^{-t}(A\cos t + B\sin t)$$

$$\therefore i = \frac{1}{Z}\sin(t-\theta) + e^{-t}(A\cos t + B\sin t)$$

where

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{4 + (1-2)^2}$$

$$= \sqrt{5} \approx 2.24$$

$$X = 1 - 2 = -1$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{1}{2}\right) = -0.463$$

$$\therefore i_{ph} = \frac{1}{2.24}\sin(t + 0.463)$$

$$= 0.446\sin(t + 0.463)$$

$$\therefore i = 0.446\sin(t + 0.463) + e^{-t}(A\cos t + B\sin t)$$

and,

$$i(0) = A + 0.446\sin(0.463) = 0$$

$$\Rightarrow A \approx -0.2$$

$$v_L(0+) = v_s(0+) = 0 = Li'(0+)$$

$$i(1) = e^{-1}(-0.2\cos(1) + B\sin(1)) + 0.446\sin(1 + 0.463)$$

$$\rightarrow 1[0.446\cos(+0.463) + e^{0}(-A\sin(0) + B\cos(0)) - e^{0}(A\cos(0) + B\sin(0))] = 0$$

$$\Rightarrow 0.446\cos(0.463) \underset{-A}{+} B = 0$$

$$\Rightarrow B = A - 0.446\cos(0.463) \Rightarrow B = -0.6$$

$$\therefore i(t) = 0.446\sin(t + 0.463) - e^{-t}(0.2\cos t + 0.6\sin t).$$

$$\Rightarrow i(1) = 0.446\sin(1.463) - e^{-1}(0.2\cos 1 + 0.6\sin 1)$$

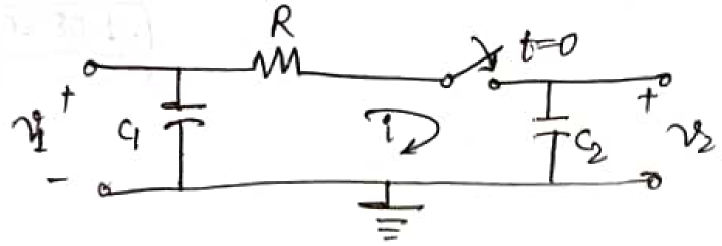
$$= 0.4434 - 0.2255$$

$$= 0.2179\ A$$

$$\approx 0.22A$$

30-08-2023

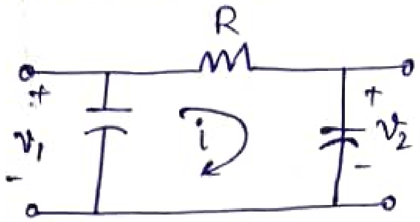
Q1



$v_1(0^-) = V_1$ & $v_2(0^-) = V_2$

Find $i(t)$, $v_1(t)$ & $v_2(t)$ for $t \geq 0$.

Soln: Once switch is closed, the relevant ckt. becomes



Using KVL,

$$v_1 = iR + v_2 \quad \text{...(e1)}$$

where $C_1 \frac{dv_1}{dt} = -i$ ⇝ discharging

$C_2 \frac{dv_2}{dt} = i$ ⇝ charging } ...(e2)

Diff. (e1),

$$v_1' = i'R + v_2' \quad \text{...(e1)'}$$

Using (e1)' and (e2),

$$\frac{-i}{C_1} = i'R + \frac{1}{C_2} i$$

$$\Rightarrow 0 = i'R + \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}_{1/C_{eq}} i$$

$$\Rightarrow 0 = i' + \frac{1}{\tau} i, \text{ where } \tau = RC_{eq}$$

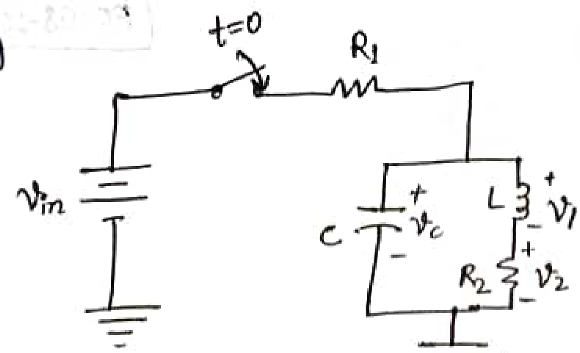
$$\Rightarrow i = Ke^{-t/\tau}$$

As $i(0) = \frac{V_1 - V_2}{R} = K$, (Assuming $V_1 > V_2$)

$$\therefore i = \frac{V_1 - V_2}{R} e^{-t/\tau}.$$

31-08-2023

Q)



Solve for:

ⓐ v_1 & v_2 at $t=0+$

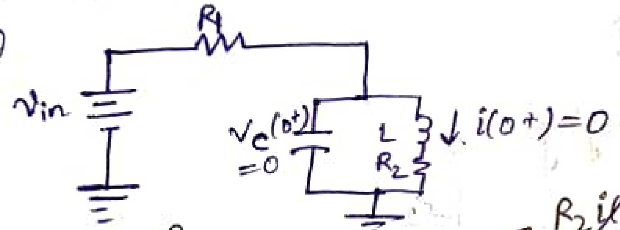
ⓑ v_1 & v_2 as $t \to \infty$

ⓒ $v_1'(0+)$, $v_2'(0+)$

ⓓ $v_2''(0+)$.

Valkenburg Exercise Prob. 5-13

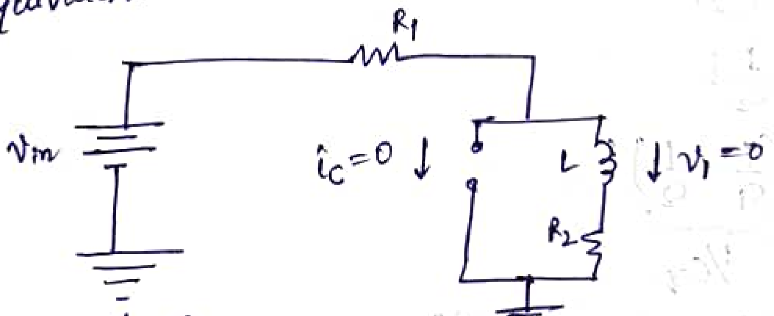
Soln: ⓐ



$$\underbrace{\cancel{v_c(0+)}}_{0} = v_1(0+) + \underbrace{\cancel{v_2(0+)}}_{R_2 \underbrace{i(0+)}_{0}}$$

$$\Rightarrow v_1(0+) = v_2(0+) = 0$$

ⓑ ~~Effective~~ Equivalent circuit as $t \to \infty$



$v_1 = 0$

$$v_2 = \frac{R_2}{R_1+R_2} v_{in}$$

ⓒ $v_c(t) = v_1(t) + v_2(t)$

$\Rightarrow v_c'(t) = v_1'(t) + v_2'(t)$

$$\Rightarrow \underbrace{v_c'(0+)}_{\frac{1}{C} i_c(0+) \,=\, \frac{1}{C}\frac{V_{IN}}{R_1}} = v_1'(0+) + \underbrace{v_2'(0+)}_{R_2 i'(0+) \,=\, R_2 \frac{\cancel{v_1(0+)}^{0}}{L}}$$

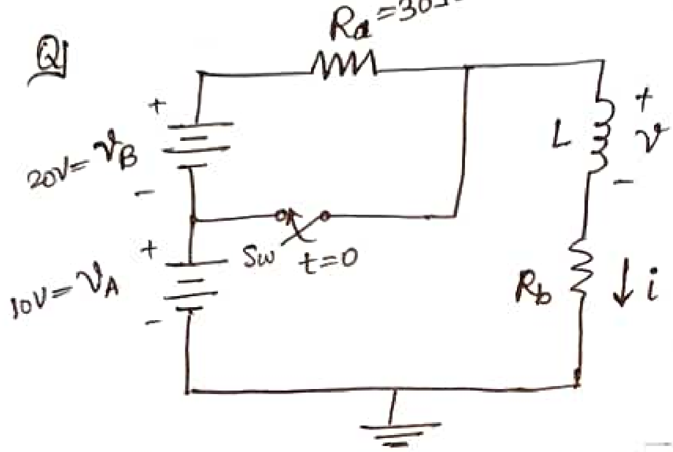
$$\therefore v_2'(0+) = 0 \text{ \& } v_1'(0+) = \frac{V_{IN}}{R_1 C}$$

(d) $v_2''(0+) = i'' R_2$

$= \frac{1}{L} v_1' R_2 \qquad (\because\ i' = \frac{v_1}{L})$

$= \frac{R_2}{L R_1 C} V_{IN} \qquad (\because\ v_1'(0+) = \frac{V_{IN}}{R_1 C})$

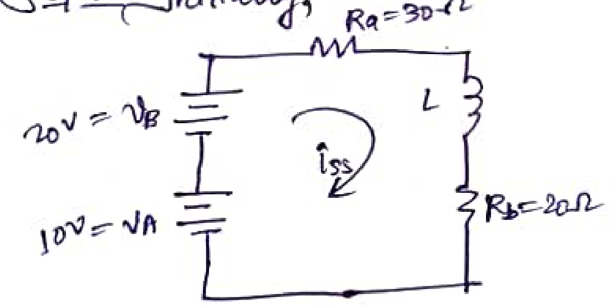
Q)



$L = 0.5H$
$R_b = 20\Omega$.

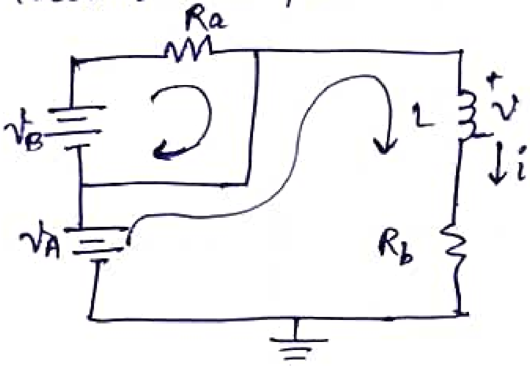
Find and plot $v(t)$ & $i(t)$ for $t \geq 0$.

Soln: Initially,



$i_{ss} = \frac{30V}{50\Omega} = 0.6\ A$

The relevant ckt. for $t \geq 0$:



$V_A = Li' + R_b i$

$\Rightarrow 0 = Li'' + R_b i'$

$\Rightarrow 0 = v' + \frac{R_b}{L} v$

$\Rightarrow \int \frac{dv}{v} = -\frac{R_b}{L} \int dt$

$\Rightarrow v = Ke^{-t/\tau}, \quad \tau = L/R_b$

At $t=0^+$, $v(0^+) = v_A - i(0^+) R_b$

$$= 10 - 0.6 \times 20$$

$$= -2 \text{ V}$$

$$\therefore v = -2e^{-t/\tau}$$

$$= -2e^{-40t} \text{ V.}$$

and, $i(t) = \int \frac{v}{L} \, dt$

$$= \frac{1}{0.5} \int -2e^{-40t} \, dt$$

$$i(t) = -4 \left[\frac{e^{-40t}}{-40} \right] + c$$

$$= \frac{e^{-40t}}{10} + c$$

At $t=0^+$, $i(0^+) = 0.6$ A

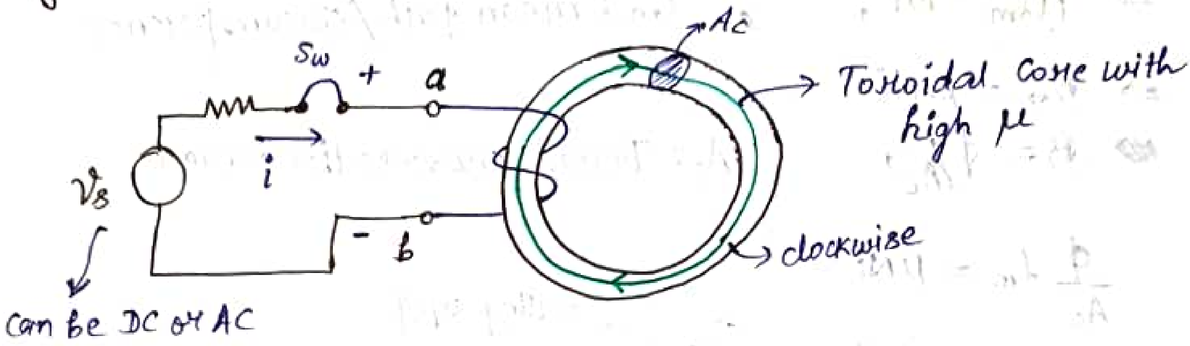
$$\Rightarrow 0.6 = \frac{1}{10} + c$$

$$\Rightarrow c = 0.5$$

$$\therefore i(t) = \left(0.5 + 0.1 e^{-40t}\right) \text{ A.}$$

MAGNETIC CIRCUITS REVIEW

Background



Once Sw is connected, $v_{ab} \approx v_s$

$$i \xrightarrow[\text{up}]{\text{sets}} \text{Magnetic field, } \phi$$

$\phi = \Psi/N$, Ψ: total flux linkage of coil

$$\Psi = \int v_{ab}\, dt \quad \leftarrow \text{Faraday's law}$$
$$= N\phi$$

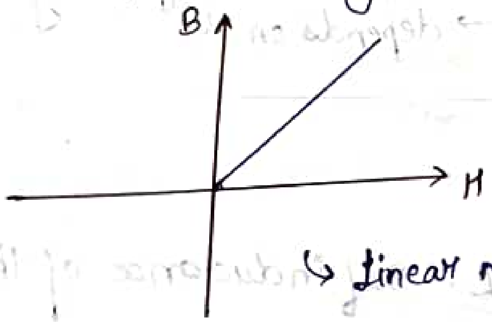
Review of L, Ni, ϕ

$$L \triangleq \Psi/i$$

Assumptions:

#1. Effect of electromagnetic radiation is negligible.

#2.



$B = \mu H$ ← constant

↳ linear magnetic field

#3. Magnetic ϕ path is well-defined.

Ampere's Circuital Law:

$$\oint_l \vec{H}.dl = i_{enclosed} = Ni$$

$\Rightarrow Hl_m = Ni$, $\quad$ l_m : mean path/circumference

$\Rightarrow Bl_m = \mu Ni$

$B = \phi/A_c$, $\quad$ A_c: Toroid cross-sectional area

$$\frac{\phi}{A_c} l_m = \mu Ni$$

$$\Rightarrow \phi = Ni\frac{\mu A_c}{l_m} = \frac{Ni}{\left(\frac{l_m}{\mu A_c}\right)}$$

Ni → setting up ϕ

$\left(\frac{l_m}{\mu A_c}\right) \rightsquigarrow R$ (reluctance) $= \left(\rho\frac{l}{A}\right)$ ← Resistance

$\phi \xleftrightarrow[\text{to}]{\text{analogous}} i$

$Ni \longleftrightarrow e$

$R \longleftrightarrow R$

Inductance,

$$L = \Psi/i = \frac{N\phi}{i}$$

$$= \frac{N}{i}\frac{Ni}{R}$$

$$= N^2/R$$

$$= \frac{N^2 A_c}{l_m}\mu.$$

→ N, l, A_c are fixed by design/geometry

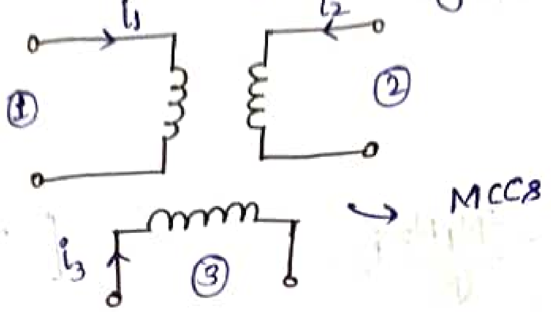
→ depends on μ.

$$V_s = V_{ab} = \frac{d\Psi}{dt} = \frac{d}{dt}(Li)$$

$$= L\frac{di}{dt}, \quad L: \text{self-inductance of the coil}$$

Magnetically Coupled Circuits (MCC)

→ Circuits/coils are said to be magnetically coupled, if 'i' flowing through one leads to flux linkage in another coil.



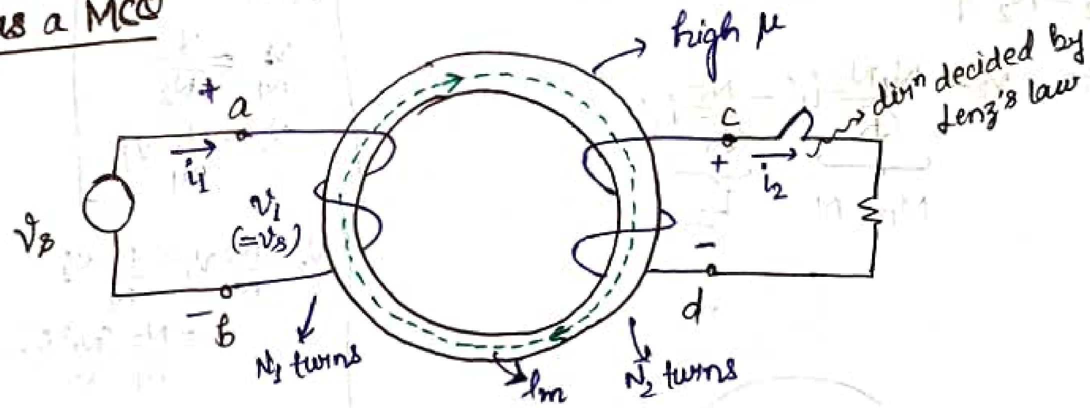
In such case,

ψ_1 : total flux linkage with coil 1.

$$= \underbrace{L_{11} i_1}_{\psi_{11}} + \underbrace{M_{12} i_2}_{\psi_{12}} + \underbrace{M_{13} i_3}_{\psi_{13}} .$$

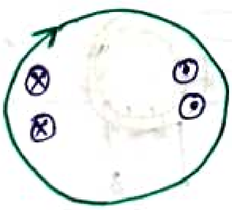
M_{12} : Mutual inductance b/w ① & ② due to current in ②.

2 Winding Transformer (2 wdg. T/f): as a MCC



ϕ → affected by i_1 and i_2.

7-09-2023



$$\oint \vec{H} \cdot d\vec{l} = N_1 i_1 - N_2 i_2$$
$$= H l_m$$
$$\Rightarrow H = \frac{N_1 i_1 - N_2 i_2}{l_m}$$

$$\Rightarrow \phi = BA_c = \mu H A_c = \frac{N_1 i_1 - N_2 i_2}{l_m} \mu A_c$$

$$= \frac{N_1 i_1 - N_2 i_2}{(l_m/\mu A_c)}$$

$$\Rightarrow \phi = \frac{N_1}{\mathcal{R}} i_1 - \frac{N_2}{\mathcal{R}} i_2$$

Then, $\psi_1 = N_1 \phi = \frac{N_1^2}{\mathcal{R}} i_1 - \frac{N_1 N_2}{\mathcal{R}} i_2.$ $[\psi = N\phi]$

$L_{11} \triangleq \frac{N_1^2}{\mathcal{R}}$ → Self inductance of ①

$M_{12} \triangleq \frac{N_1 N_2}{\mathcal{R}}$ → Mutual inductance b/w coil 1 & 2.

Then,

$$\psi_1 = L_{11} i_1 - M_{12} i_2$$

$$v_1 = \psi_1' = L_{11} i_1' - M_{12} i_2'$$

$$\psi_2 = N_2 \phi$$

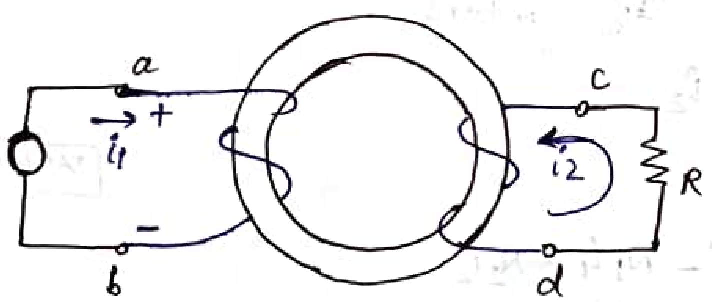
Then, $v_2 = \underbrace{\frac{N_1 N_2}{\mathcal{R}}}_{M_{12}=M} i_1' - \underbrace{\frac{N_2^2}{\mathcal{R}}}_{L_{22}} i_2'$

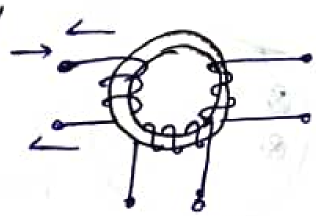
[Also,

$$\frac{v_1}{N_1} = \frac{v_2}{N_2}$$

$$v_2 = \frac{N_2}{N_1} v_1]$$

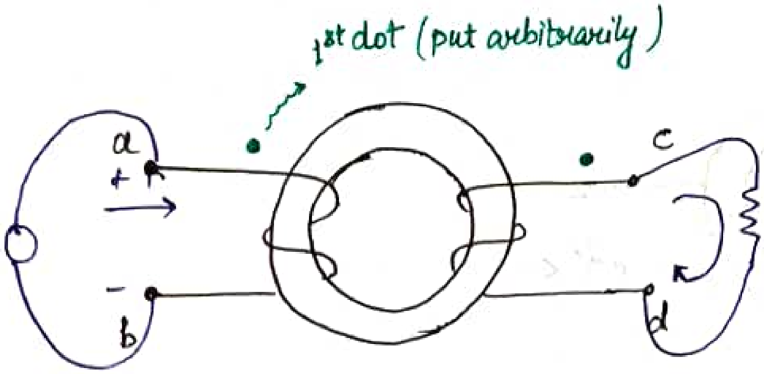
$$\left[v_2 = \frac{N_2}{N_1} v_1 = \frac{N_2}{N_1} \psi_1' = \frac{N_2}{N_1}\left(\frac{N_1^2}{\mathcal{R}} i_1' - \frac{N_1 N_2}{\mathcal{R}} i_2'\right) = \frac{N_1 N_2}{\mathcal{R}} i_1' - \frac{N_2^2}{\mathcal{R}} i_2' \right]$$

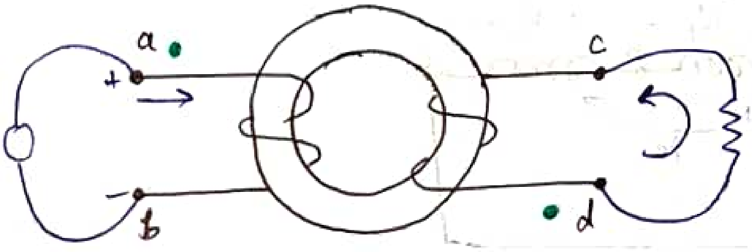




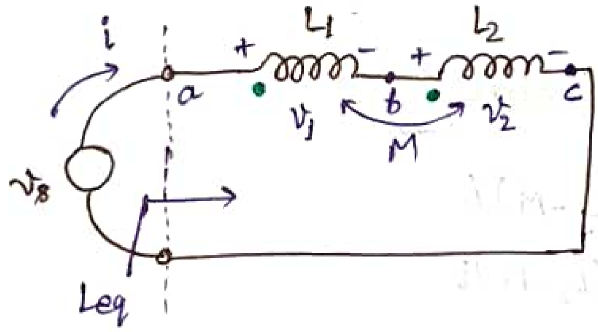
Dot Convention

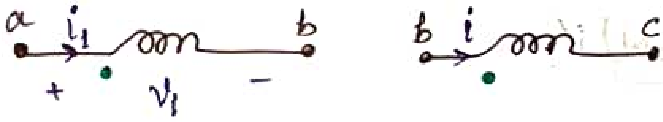
↳ current flowing into 'dot' terminals leads to additive flux.





Eg.





$$v_1 = \Psi_1', \quad \Psi_1 = L_1 i_1 + M i_1$$
$$= (L_1 + M) i_1$$

$$\therefore \; v_1 = (L_1 + M) i_1'$$

$$\Psi_2 = L_2 i + M i$$
$$= (L_2 + M) i$$

$$\Rightarrow v_2 = (L_2 + M) i'$$

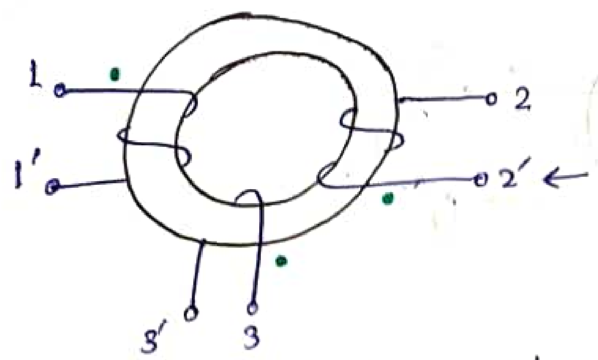
Then,

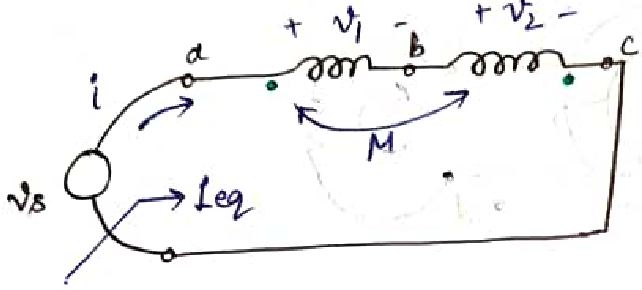
$$v_s = v_1 + v_2$$

$$\Rightarrow v_s = \underbrace{(L_1 + L_2 + 2M)}_{L_{eq}} i'$$

08-09-2023

→





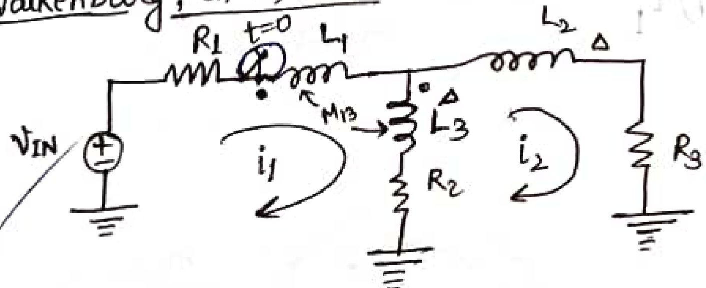
$v_1 = \psi_1'$; $\psi_1 = L_1 i - Mi = (L_1 - M)i$

$v_2 = \psi_2'$; $\psi_2 = L_2 i - Mi = (L_2 - M)i$

$v_s = v_1 + v_2$

$= \underbrace{(L_1 + L_2 - 2M)}_{Leq} i'$

Q) Valkenburg, ch-5, Q.22.



M_{13}
M_{23}
→ 1 & 2 are not coupled

Find $i_1'(0+)$ and $i_2'(0+)$.

Soln:

$$V_{IN} = R_1 i_1 + R_2(i_1 - i_2) + \{L_1 i_1 + M_{13}(i_1 - i_2)\}'$$
$$+ \{L_3(i_1 - i_2) + M_{13} i_1 - M_{23} i_2\}'$$

$$0 = R_2(i_2 - i_1) + R_3 i_2 + \{L_3(i_2 - i_1) - M_{13} i_1 + M_{23} i_2\}'$$
$$+ \{L_2 i_2 + M_{23}(i_2 - i_1)\}'$$

$$\Rightarrow V_{IN} = (R_1+R_2)i_1 - R_2 i_2 + \{L_1 + 2M_{13} + L_3\} i_1' + \{-M_{13} - L_3 - M_{13}\} i_2' \quad \ldots (e1)$$

$$0 = -R_2 i_1 + (R_3+R_2) i_2 + \{-L_3 - M_{13} - M_{23}\} i_1' + \{L_3 + M_{23} + L_2 + M_{23}\} i_2' \quad \ldots (e2)$$

As $i_1(0+) = i_2(0+) = 0$,

$$(e1) \Rightarrow V_{IN} = \{L_1 + L_3 + 2M_{13}\} i_1'(0+) - \{M_{13} + L_3 + M_{23}\}(i_2'(0+)) \quad \ldots (e3)$$

$$(e2) \Rightarrow 0 = -\{L_3 + M_{13} + M_{23}\} i_1'(0+) + (L_3 + L_2 + 2M_{23}) i_2'(0+)$$

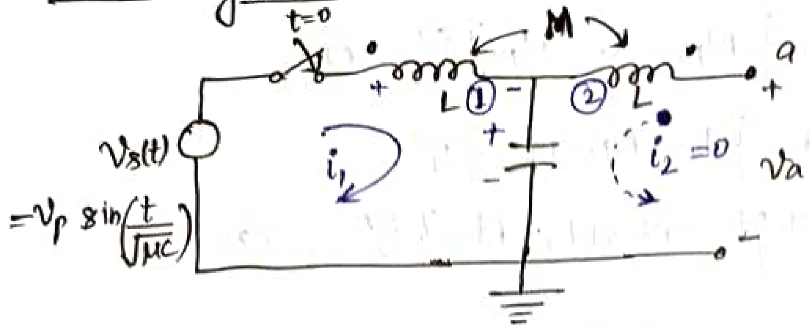
$$\Rightarrow (L_3 + M_{13} + M_{23}) i_1'(0+) = (L_3 + L_2 + 2M_{23}) i_2'(0+) \quad \ldots (e4)$$

Using (e4) in (e3),

$$V_{IN} = (L_1 + L_3 + 2M_{13}) i_1'(0+) - i_1'(0+) (M_{13} + L_3 + M_{23}) \frac{(L_3 + M_{13} + M_{23})}{(L_3 + L_2 + 2M_{23})}$$

$$\Rightarrow i_1'(0+) = \frac{V_{IN}}{(L_1 + L_3 + 2M_{13}) - \frac{(M_{13} + L_3 + M_{23})(L_3 + M_{13} + M_{23})}{(L_3 + L_2 + 2M_{23})}}$$

Q) Valkenburg, Ch-5, Prob. 24



Initially relaxed N/w.

Find $v_a(0+)$, $v_a'(0+)$ and $v_a''(0+)$

Soln: $v_s = v_L + v_c$;

$v_L = \psi_1' = Li_1' + \cancel{Mi_2'}^{0}$

$v_s = (Li_1' + \cancel{Mi_2'}^{0}) + v_c$ ... (e1)

$v_a = (\cancel{Li_2'}_{0} + Mi_1') + v_c$ ... (e2)

$-v_c + v_{L2} + v_a = 0$

$-v_c + \{\cancel{Li_2}_{0} - Mi\}' + v_a = 0$

13-09-2023

$\Rightarrow v_s - Li_1' = v_a - Mi_1'$

$\Rightarrow v_s = v_a + (L-M)i_1'$ ... (e3)

Initial conditions:

$i_1(0+) = 0, \quad v_c(0+) = 0$

$v_L(0+) = 0 \Rightarrow i_1'(0+) = 0 \qquad$ (as $v_s(0+) = 0$)

$\therefore \boxed{v_a(0+) = 0}$

(e3) $\Rightarrow v_s' = v_a' + (L-M)i_1''$

$\Rightarrow v_s'(0+) = v_a'(0+) + (L-M)\frac{v_L'}{L}(0+)$ ...(e4) $\quad (\because v_L = Li_1')$

(e5) $\rightarrow v_s = v_L + v_c$

$\Rightarrow v_s' = v_L' + v_c'$

$= v_L' + \frac{1}{C}i_1$ ... (e5)

$\Rightarrow v_L'(0+) = v_s'(0+) - \frac{1}{C}\cancel{i_1(0+)}^{0}$

$= \frac{v_p}{\sqrt{MC}}$ ... (e6) $\qquad \left(\because v_s = v_p \sin\left(\frac{t}{\sqrt{MC}}\right)\right)$

(e4) $\Rightarrow v_a'(0+) = \frac{v_p}{\sqrt{MC}} - \frac{(L-M)}{L}\cdot\frac{v_p}{\sqrt{MC}}$

$= \frac{M}{L}\cdot\frac{v_p}{\sqrt{MC}} = \boxed{\frac{v_p}{L}\sqrt{M/C}}$

21-09-2023

LAPLACE TRANSFORM (LT)

↓

From a Network Analysis perspective

<u>Agenda</u> #1 Why study LT?

#2 The basic transform

#3 Some properties of LT (2 keys)

Time Domain (TD) $\underset{\text{Inverse LT}}{\overset{\text{LT}}{\rightleftarrows}}$ Frequency Domain (FD)

SISO ⇓ Single Input Single Output

MIMO ⇓ Multiple Input Multiple output

Advantage 1:

$e(t) \longrightarrow$ [N/W] $\longrightarrow r(t)$

↓ Apply relevant laws

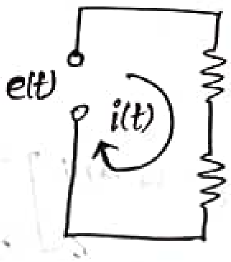
ODEs, ICs (Initial conditions) $\longrightarrow$ f domain Series of algebraic equations

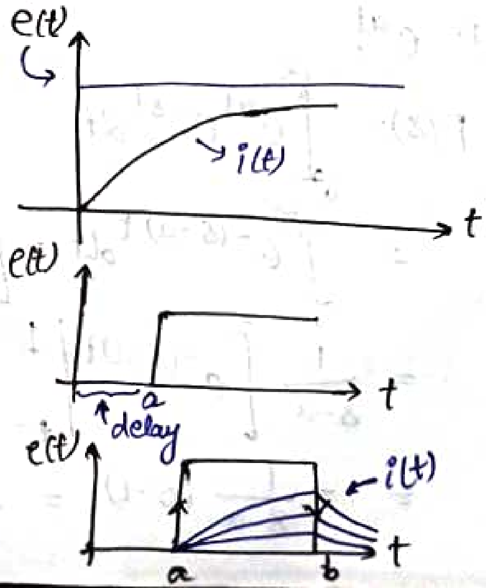
↓ R(s)

$r(t) \xleftarrow{\text{Inv. LT}} R(s)$

Advantage 2:

LT handles ~~exci~~ excitations e(t)s like pulse, impulse very effectively.





The Basic Transform

$$\underbrace{\mathcal{L}}_{\text{LT operator}}\{\underbrace{f(t)}_{\substack{\text{Time Domain}\\ (TD)}}\} = \underbrace{F(s)}_{\substack{\text{Frequency Domain}\\ (FD)}}$$

s : complex frequency

$s = \sigma + j\omega$

Book → Kreyszig

$$\mathcal{L}\{f(t)\} = F(s) \triangleq \int_0^\infty e^{-st} f(t)\,dt$$

↳ $F(s) = \int_{0^-}^{\infty} e^{-st} f(t)\,dt$, for function for which $f(0^-) \neq f(0^+)$ (like hammering something & lifting quickly)

for $F(s)$ to exist,

$$\int_{0^-}^{\infty} e^{-\sigma t} |f(t)|\,dt \to \text{finite}$$

If $f(t) = A$,

$$F(s) = \int_{0^-}^{\infty} A e^{-st}\,dt$$

$$= A\left[-\frac{1}{s} e^{-st}\right]_{0^-}^{\infty}$$

$$= \frac{A}{s}\left\{-e^{-st}\Big|_{\infty} + \cancel{e^{-st}}\Big|_0^{1}\right\}$$

$= A/s$ for $\mathrm{Re}\{s\} \geq 0$.

If $f(t) = e^{at}$,

$$F(s) = \int_0^\infty e^{at} e^{-st}\,dt$$

$$= \int_0^\infty e^{-(s-a)t}\,dt = \left[-\frac{1}{s-a} e^{-(s-a)t}\right]_0^\infty$$

$$= \frac{1}{s-a}\left[e^{-(s-a)t}\Big|_{t=0}^{1} - e^{-(s-a)t}\Big|_{t\to\infty}\right]$$

$$= -\frac{1}{s-a}(0-1) = \frac{1}{s-a}, \text{ for } \mathrm{Re}\{s\} \geq a.$$

a : abscissa of convergence

$$f(t) \underset{\text{Inverse LT}}{\overset{LT}{\rightleftarrows}} F(s)$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_1 - j\omega}^{\sigma_1 + j\omega} F(s)\, e^{st}\, ds$$

Properties :

① Uniqueness : $f(t) \underset{\text{unique}}{\overset{\text{mapping is}}{\rightleftarrows}} F(s)$

② Linearity : $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$

Then, $K_1 \mathcal{L}\{f(t)\} + K_2 \mathcal{L}\{g(t)\} = K_1 F(s) + K_2 G(s)$

$\boxed{22\text{-}09\text{-}2023}$

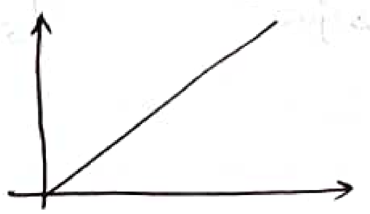
Laplace Transform (LT)

$$\mathcal{L}\{f(t)\} = F(s) = \int_{0^-}^{\infty} e^{-st} f(t)\, dt$$

LT $\begin{cases} \text{Uniqueness} \\ \text{Linearity} \end{cases}$

→ $\mathcal{L}\{1\} = 1/s$, $\mathcal{L}\{K_0\} = \frac{K_0}{s}$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$



$f(t) = At$ ← Ramp function

Eg. $f(t) = t$

$$\mathcal{L}\{f\} = \int_{0^-}^{\infty} t e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} t\left(\frac{-1}{s}\right) \frac{d}{dt}(e^{-st})\, dt = -\frac{1}{s} \int_{0^-}^{\infty} t\, d(e^{-st})$$

$$= -\frac{1}{s}\left[t e^{-st} - \int e^{-st} dt\right]_{0^-}^{\infty}$$

$$= -\frac{1}{s}\left[\frac{1}{s}e^{-st} + t e^{-st}\right]_{0^-}^{\infty}$$

$$= -\frac{1}{s^2} e^{-st}\Big|_{0}^{\infty} - \frac{1}{s}\left[t e^{-st}\right]_{0^-}^{\infty}$$

$$= \frac{1}{s^2}$$

So, $\mathcal{L}\{t\} = 1/s^2$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

Eg. $f(t) = \cos\omega_0 t = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$

Then,

$$F(s) = \int_{0^-}^{\infty} \cos\omega_0 t \; e^{-st} dt$$

$$= \frac{1}{2}\int_{0^-}^{\infty}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{-st} dt$$

$$= \frac{1}{2}\int_{0^-}^{\infty}\left[e^{-(s-j\omega_0)t} + e^{-(s+j\omega_0)t}\right] dt$$

$$= \frac{1}{2}\left[-\frac{1}{s-j\omega_0} e^{-(s-j\omega_0)t}\Big|_{0^-}^{\infty} - \frac{1}{s+j\omega_0} e^{-(s+j\omega_0)t}\Big|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right]$$

$$= \frac{s}{s^2+\omega_0^2}$$

$$\therefore \mathcal{L}\{\cos\omega_0 t\} = \frac{s}{s^2+\omega_0^2}$$

Similarly, $\mathcal{L}\{\sin\omega_0 t\} = \frac{\omega_0}{s^2+\omega_0^2}$

Eg., $f(t) = e^{at}$,

$\mathcal{L}\{1\} = \frac{1}{s}$, $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

$\mathcal{L}\{K_0 e^{at}\} = \frac{K_0}{s-a}$

$$\mathcal{L}\left\{e^{at}\frac{1}{2}\left(e^{j\omega_0 t}+e^{-j\omega_0 t}\right)\right\} = \mathcal{L}\left\{e^{at}\cos\omega_0 t\right\}$$

$$= \int_{0^-}^{\infty} e^{-st}\frac{e^{at}}{2}\left(e^{j\omega_0 t}+e^{-j\omega_0 t}\right)dt$$

$$= \frac{1}{2}\int_{0^-}^{\infty}\left[e^{(-s+a+j\omega_0)t} + e^{(a-s-j\omega_0)t}\right]dt$$

$$= \frac{1}{2}\left[\frac{e^{(a-s+j\omega_0)t}}{a-s+j\omega_0} + \frac{e^{(a-s-j\omega_0)t}}{a-s+j\omega_0}\right]_{0^-}^{\infty}$$

$$= \frac{1}{2}\left[\frac{e^{-(s-a-j\omega_0)t}}{a-s+j\omega_0} + \frac{e^{-(s-a+j\omega_0)t}}{a-s+j\omega_0}\right]_{0^-}^{\infty}$$

$$= \frac{1}{2}\left[\frac{1}{s-a-j\omega_0} + \frac{1}{s-a+j\omega_0}\right]$$

$$\therefore \mathcal{L}\{e^{at}\cos\omega_0 t\} = \frac{s-a}{(s-a)^2+\omega_0^2}$$

Similarly, $\mathcal{L}\{e^{at}\sin\omega_0 t\} = \frac{\omega_0}{(s-a)^2+\omega_0^2}$.

S-shifting Theorem

If $\mathcal{L}\{f(t)\} = F(s)$, then

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a).$$

Q) Find the laplace inverse of

$$F(s) = \frac{3s - 137}{s^2 + 2s + 401}.$$

Soln: $$F(s) = \frac{3(s+1) - 140}{(s+1)^2 + 20^2}$$

$$= 3 \cdot \frac{(s+1)}{(s+1)^2 + 20^2} - 7 \cdot \frac{20}{(s+1)^2 + 20^2}$$

$$f(t) = L^{-1}\{F(s)\} = 3\, L^{-1}\left\{\frac{s+1}{(s+1)^2 + 20^2}\right\} - 7\, L^{-1}\left\{\frac{20}{(s+1)^2 + 20^2}\right\}$$

$$= 3\, e^{-t} \cos 20t - 7 e^{-t} \sin 20t.$$

27-09-2023

Some commonly used transforms so far:

$f(t)$	$\mathcal{L}\{f(t)\}$
1	$1/s$
t	$1/s^2$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos\omega_0 t$	$\frac{s}{s^2+\omega_0^2}$
$\sin\omega_0 t$	$\frac{\omega_0}{s^2+\omega_0^2}$

- Shifting by s:
  If $\mathcal{L}\{f(t)\} = F(s)$,
  then $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$.

- $$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \int_{0^-}^{\infty} e^{-st}\frac{df(t)}{dt}\,dt \qquad \Big|\ \mathcal{L}\{f(t)\} = F(s)$$

$$= \int_{0^-}^{\infty} e^{-st}\,df$$

$$= \left[e^{-st}f(t) - \int f(t)\,d(e^{-st})\right]_{0^-}^{\infty}$$

$$= \left[e^{-st}f(t)\right]_{0^-}^{\infty} + s\int_{0^-}^{\infty} f(t)e^{-st}dt - s\int f(t)e^{-st}dt$$

$$= e^{-st}\cancel{f(t)}^{\to 0}\Big|_{t\to\infty} - e^{-st}\cancel{f(t)}^{\to f(0^-)}\Big|_{t\to 0^-} + sF(s)$$

$$\therefore \boxed{\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-)}$$

$$\mathcal{L}\{f''(t)\} = sF'(s) - f'(0)$$

$$\downarrow$$

$$\frac{df'(t)}{dt} \qquad = s^2F(s) - sf(0^-) - f'(0).$$

- $$\boxed{\begin{aligned} \mathcal{L}\{f'(t)\} &= sF(s) - f(0^-) \\ \mathcal{L}\{f''\} &= s^2F(s) - sf(0^-) - f'(0) \end{aligned}}$$

- $\mathcal{L}\left\{\int_0^t f(t)\,dt\right\}$

Let $\int_0^t f(t)\,dt = g(t) \longrightarrow f(t)dt = d\{g(t)\}$

Then, $\mathcal{L}\left\{\int_0^t f(t)\,dt\right\}$

$$= \int_{0^-}^{\infty} g(t)e^{-st}\,dt = \int_{0^-}^{\infty} g(t)\left(\frac{-1}{s}\right)\frac{d}{dt}(e^{-st})\,dt$$

$$= -\frac{1}{s}\int_{0^-}^{\infty} g(t)\,d(e^{-st})$$

$$= -\frac{1}{s}\left[g(t)e^{-st} - \int e^{-st}\,d\{g(t)\}\right]_{0^-}^{\infty}$$

$$= -\frac{1}{s}\left[g(t)e^{-st} - \int e^{-st}f(t)\,dt\right]_{0^-}^{\infty}$$

$$= -\frac{1}{s}\left[g(t)e^{-st}\right]_{0^-}^{\infty} + \frac{1}{s}\underbrace{\int_{0^-}^{\infty} e^{st}f(t)\,dt}_{F(s)}$$

$$= \frac{1}{s}\cancel{g(0^-)}^{\,0} + \frac{1}{s}F(s).$$

$$\therefore \boxed{\mathcal{L}\left\{\int_0^t f(t)\,dt\right\} = \frac{1}{s}F(s)}$$

Eg.① Solving an ODE. (Find y(t)).

$$y'(t) + 40y(t) = 20 \rightarrow (e1)$$
$$\& \; y(0) = 0.6$$

Soln: Taking LT of (e1),

$$\mathcal{L}\{y'(t) + 40y\} = \mathcal{L}\{20\}$$

$$\Rightarrow \mathcal{L}\{y'\} + 40\mathcal{L}\{y\} = \mathcal{L}\{20\}$$

$$\Rightarrow \cancel{s\,y(s) - y(0) + 40\,y(s) = \frac{20}{s}}$$

$$\Rightarrow \cancel{(s+40)Y(s) =}$$

$$\Rightarrow s\,Y(s) - y(0) + 40Y(s) = 20/s$$

$$\Rightarrow (s+40)Y(s) = 0.6 + 20/s$$

$$\Rightarrow Y(s) = \frac{0.6}{s+40} + \frac{20}{s(s+40)}$$

$$= \frac{0.6}{s+40} + 0.5\,\frac{s+40-s}{s(s+40)}$$

$$= \frac{0.6}{s+40} + \frac{0.5}{s} - \frac{0.5}{s+40}$$

$$= \frac{0.5}{s} + 0.1\,\frac{1}{s+40}$$

$$\therefore \; y(t) = 0.5 + 0.1\,e^{-40t}$$

Eg.② Solving an ODE

$$\left.\begin{array}{l} y'' + 7y' + 12y = 21\,e^{3t} \;\ldots (e1) \\ y(0) = 3.5, \quad y'(0) = -10. \end{array}\right\} \text{solve for } y(t).$$

Soln: Taking LT of (e1),

$$\mathcal{L}\{y''\} + 7\mathcal{L}\{y'\} + 12\mathcal{L}\{y\} = 21\,\mathcal{L}\{e^{3t}\}$$

$$\Rightarrow s^2Y(s) - s\,y(0) - y'(0) + 7\{sY(s) - y(0)\} + 12\,Y(s) = \frac{21}{s-3}$$

$$\Rightarrow \underbrace{(s^2+7s+12)}_{(s+3)(s+4)} Y(s) - (s+7)y(0) - y'(0) = \frac{21}{s-3} \rightarrow \text{Pole}=3 \rightarrow \oplus\text{ve}$$

$\searrow$ diverge

$$\Rightarrow Y(s) = \frac{s+7}{(s+3)(s+4)} y(0) + \frac{y'(0)}{(s+3)(s+4)} + \frac{21}{(s-3)(s+3)(s+4)}$$

$$= \frac{s+7}{(s+3)(s+4)} \frac{7}{2} - \frac{10}{(s+3)(s+4)} + \frac{21}{(s-3)(s+3)(s+4)}$$

$$= \frac{3.5s}{(s+3)(s+4)} + \frac{14.5}{(s+3)(s+4)} + \frac{21}{(s-3)(s+4)(s+3)}$$

$$= 3.5s \cdot \frac{(s+4)-(s+3)}{(s+4)(s+3)} + 14.5 \cdot \frac{(s+4)-(s+3)}{(s+4)(s+3)} + \frac{21}{(s-3)(s+3)(s+4)}$$

$$= \frac{3.5s}{(s+3)} - \frac{3.5s}{(s+4)} + \frac{14.5}{(s+3)} - \frac{14.5}{(s+4)} + \frac{21/42}{(s-3)} + \frac{(-7/2)}{(s+3)} + \frac{3}{(s+4)}$$

$$= \frac{3.5s}{s+3} - \frac{3.5s}{s+4} + \frac{11}{s+3} - \frac{11.5}{s+4} + \frac{0.5}{s-3}$$

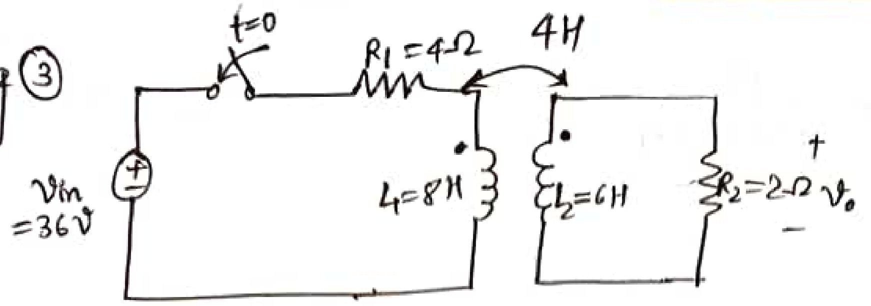
$$= \left[\cancel{3.5} - \frac{10.5}{(s+3)}\right] + \left[-\cancel{3.5} + \frac{14}{(s+4)}\right] + \frac{11}{s+3} - \frac{11.5}{s+4} + \frac{0.5}{s-3}$$

$$= \frac{0.5}{s+3} + \frac{2.5}{s+4} + \frac{0.5}{s-3}$$

$$\therefore y(t) = 0.5e^{-3t} + 2.5e^{-4t} + 0.5e^{+3t}$$

29-09-2023

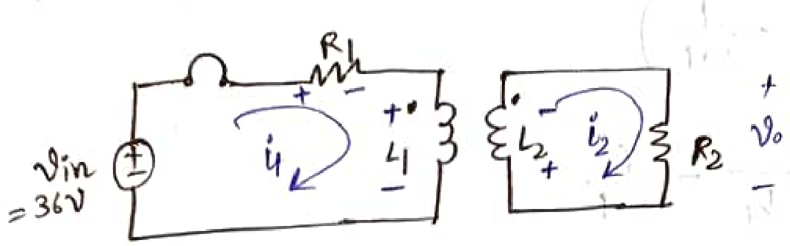
Eg ③



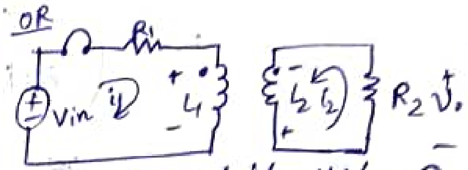
Initially relaxed N/w.

Find $v_0(t)$ for $t \geq 0$.

Soln: For $t \geq 0$, the ~~equi~~ relevant circuit is,



OR



$$v_{IN} = R_1 i_1 + L_1 i_1' - M i_2' \quad \ldots (e1)$$

$$0 = L_2 i_2' - M i_1' + i_2 R_2 \quad \ldots (e2)$$

$$V_{in} = R_1 i_1 + L_1 i_1' + M i_2' \quad \ldots ①$$

$$V_{L2} = L_2 i_2' + M i_1$$

$$v_0 = -i_2 R_2$$

$$\therefore L_2 i_2' + M i_1' + i_2 R_2 = 0 \quad \ldots ②$$

Taking LT of (e1),

$$\frac{V_{in}}{s} = R_1 I_1(s) + L_1 \{s I_1(s) - \cancel{i_1(0)}^{0}\} - M\{s I_2(s) - \cancel{i_2(0)}^{0}\}$$

$$\Rightarrow \frac{V_{in}}{s} = R_1 I_1(s) + s L_1 I_1(s) - s M I_2(s) \quad \ldots (e3)$$

Taking LT of (e2),

$$0 = s L_2 I_2(s) - s M I_1(s) + I_2(s) R_2 \quad \ldots (e4)$$

Using the parametric values,

$$0 = 6s I_2(s) - 4s I_1(s) + 2 I_2(s)$$

$$\Rightarrow I_1(s) = \frac{1}{4s}[6s+2] I_2(s) \quad \ldots (e5)$$

(e3) $\Rightarrow$ $$\frac{36}{s} = 4 I_1(s) + 8s I_1(s) - 4s I_2(s)$$

$$= (4+8s) I_1(s) - 4s I_2(s)$$

Substituting $I_1(s)$ from (e5),

$$\frac{36}{s} = \left[(4+8s) \cdot \frac{1}{4s}(6s+2) - 4s\right] I_2(s)$$

$$= \left[\frac{2(1+2s)(3s+1)}{s} - 4s\right] I_2(s)$$

$$= \frac{12s^2 + 10s + 2 - 4s^2}{s} \cdot I_2(s)$$

$$\Rightarrow I_2(s) = \frac{36}{8s^2+10s+2} = \frac{18}{4s^2+5s+1}$$

$$= \frac{18}{(4s+1)(s+1)}$$

$$= 6 \cdot \frac{4(s+1)-(4s+1)}{(4s+1)(s+1)}$$

$$= 6\left(\frac{4}{s+1} - \frac{1}{s+1}\right)$$

$$= \frac{6}{s+1/4} - \frac{6}{s+1}$$

$$\Rightarrow i_2(t) = 6e^{-t/4} - 6e^{-t}$$

$$= 6\left(e^{-t/4} - e^{-t}\right).$$

(e5) $$\Rightarrow I_1(s) = \frac{1}{4s}(6s+2)\left[\frac{18}{(4s+1)(s+1)}\right]$$

$$= \left(\frac{3s+1}{s}\right)\left(\frac{9}{(4s+1)(s+1)}\right)$$

$$= \left(3+\frac{1}{s}\right)\left(\frac{9}{(4s+1)(s+1)}\right)$$

$$= \frac{27}{(4s+1)(s+1)} + \frac{9}{s(4s+1)(s+1)}$$

$$= \frac{27/4}{3/4}\left[\frac{1}{(s+1/4)} - \frac{1}{(s+1)}\right] + 9\left[\frac{1}{s} + \left(\frac{-16}{3}\right)\frac{1/4}{\left(s+\frac{1}{4}\right)} + \frac{1/3}{s+1}\right]$$

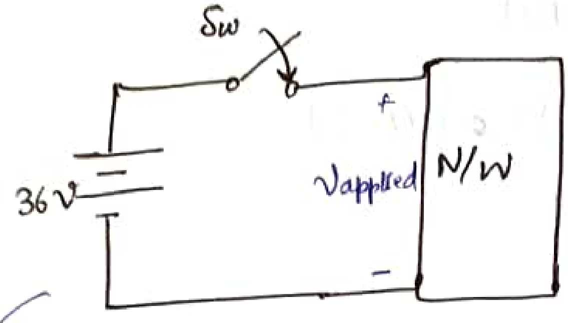
$$\Rightarrow i(t) = 9\left[e^{-t/4} - e^{-t}\right] + 9\left[1 - \frac{4}{3}e^{-t/4} + \frac{1}{3}e^{-t}\right]$$

$$= 9 - 3e^{-t/4} - 6e^{-t}.$$

Unit Step

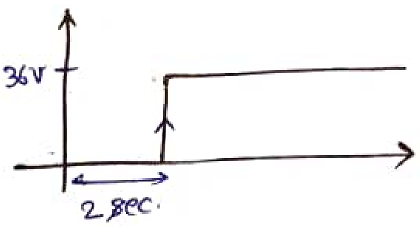
Unit step function,

$$u(t) = 1, \text{ for } t \geq 0$$
$$= 0, \text{ otherwise.}$$



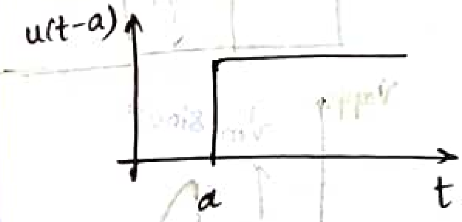
$V_{app} = 36\,u(t)$

$$u(t-2) = 1, \text{ for } t \geq 2$$
$$= 0, \text{ otherwise.}$$

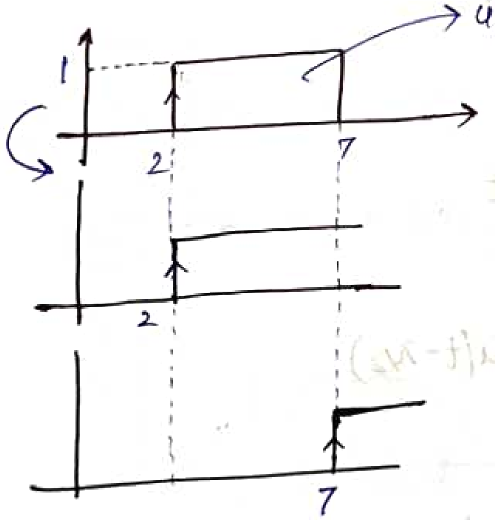


$36\,u(t-2)$

$$u(t-a) = 1, \text{ for } t \geq a$$
$$= 0, \text{ otherwise}$$

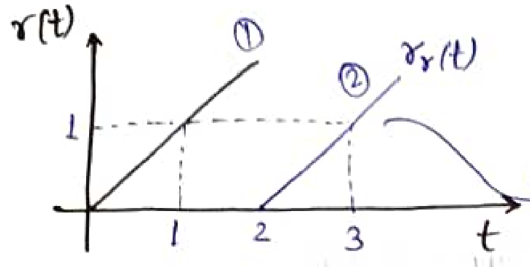


$u(t-2) - u(t-7)$



Unit Ramp

$r(t) = t$, for $t \geq 0$
$= 0$, otherwise

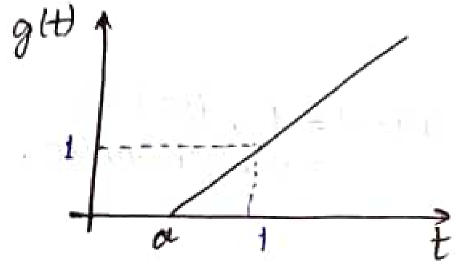


① $t\,u(t)$

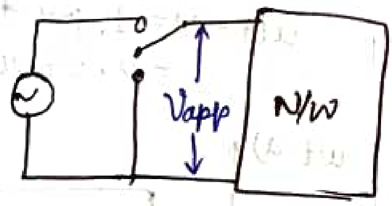
② $(t-2) \times u(t-2)$

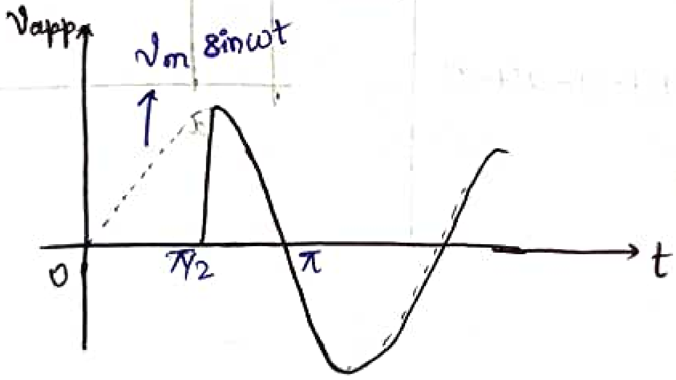
04-10-2023

Delayed ramp:

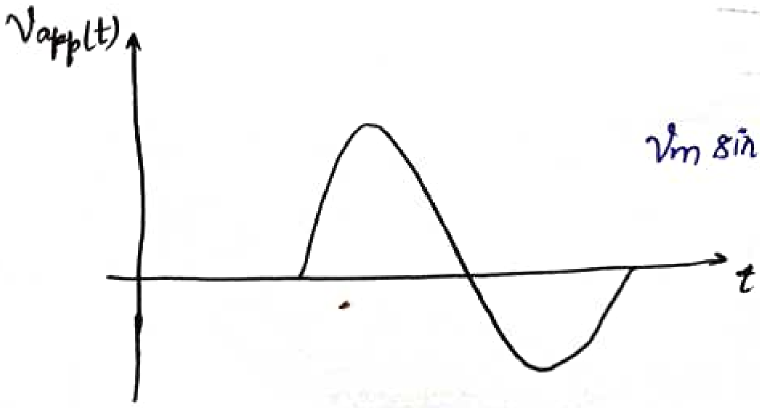


$g(t) = (t-a)\,u(t-a)$





$v_{app}(t) = V_m \sin t \; u(t - \pi/2)$



$V_m \sin(t-\pi)\,u(t-\pi)$

↳ General: $f(t-a)\,u(t-a)$

$$\mathcal{L}\{u(t)\} = \int_{0^-}^{\infty} 1 e^{-st}\,dt = \frac{1}{s}.$$

$$\mathcal{L}\{u(t-a)\} = \int_{a}^{\infty} 1 \cdot e^{-st}\,dt = \frac{e^{-as}}{s}$$

$\mathcal{L}\{f(t)\} = F(s).$

$$\underbrace{\mathcal{L}\{u(t-a)\,f(t-a)\}}_{F_{sh}(s)} = \int_{a}^{\infty} f(t-a)\,e^{-st}\,dt$$

Let $\theta = t-a \Rightarrow d\theta = dt$

When $t \to \infty, \quad \theta \to \infty$

$t \to a, \quad \theta \to 0.$

$$F_{sh}(s) = \int_{0}^{\infty} f(\theta)\,e^{-s(\theta+a)}\,d\theta = e^{-as}\,F(s). \quad \text{... s-shifting Theorem}$$

(second - shifting theorem)

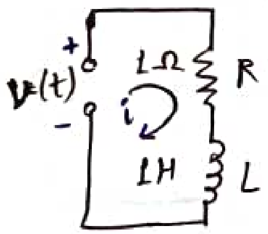
$\therefore \quad \mathcal{L}\{u(t)\} = \frac{1}{s}.$

$\mathcal{L}\{u(t-a)\} = e^{-as}/s.$

$\mathcal{L}\{u(t-a)\,f(t-a)\} = e^{-as}F(s)$

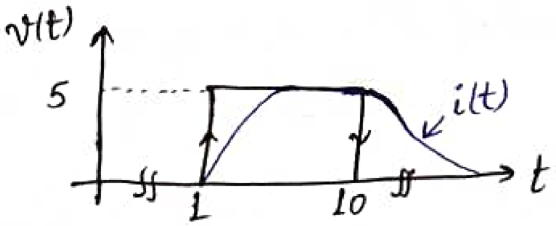
where $\mathcal{L}\{f(t)\} = F(s).$

Eg.



Initially relaxed N/w.

Find $i(t)$ for $t \geq 0$.



Soln: For $t \geq 0$,

$$v(t) = Ri(t) + Li'(t)$$
$$= i(t) + i'(t) \quad \text{... (e1)}$$

$$v(t) = 5\{u(t-1) - u(t-10)\} \quad \text{... (e2)}$$

Using (e1) & (e2),

$$5\{u(t-1) - u(t-10)\} = i(t) + i'(t) \quad \text{... (e3)}$$

Taking LT for both sides of (e3)

$$5\left\{ \frac{e^{-s}}{s} - \frac{e^{-10s}}{s} \right\} = I(s) + sI(s)$$

$$\Rightarrow I(s) = \frac{1}{s+a} \cdot 5\left\{ \frac{e^{-s}}{s} - \frac{e^{-10s}}{s} \right\}$$

$$= 5\left\{ e^{-s} \frac{1}{s} \frac{1}{s+a} - \frac{1}{s} \frac{1}{s+a} e^{-10s} \right\}$$

$$= 5\left[\left(\frac{1}{s} - \frac{1}{s+1}\right) e^{-s} - \left(\frac{1}{s} - \frac{1}{s+1}\right) e^{-10s} \right]$$

$$\Rightarrow i(t) = 5 \left[\mathcal{L}^{-1}\left\{\frac{1}{s}e^{-s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}e^{-s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}e^{-10s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+1}e^{-10s}\right\} \right]$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}e^{-s}\right\} = u(t-1)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1}e^{-s}\right\} = u(t-1)\,e^{-t} \quad \Big\} f(t) = e^{-t}.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}e^{-10s}\right\} = u(t-10)$$

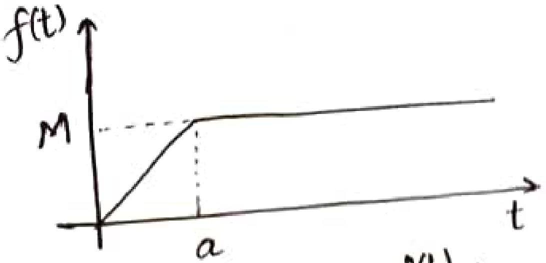
$$\mathcal{L}^{-1}\left\{\underbrace{\frac{1}{s+1}}_{F(s)}\underbrace{e^{-10s}}_{e^{-as}}\right\} = e^{-t}\,u(t-10).$$

$$\therefore i(t) = 5\left[u(t-1)(1-e^{-t}) - u(t-10)(1-e^{-t}) \right]$$

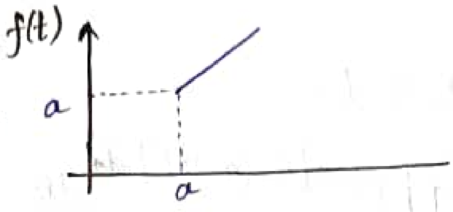
$e^{-(t-1)}$
$t=1, t=2$
rise by 63%.

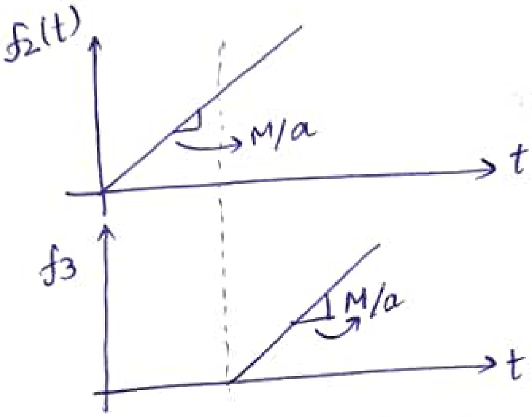
05-10-2023

Eg.



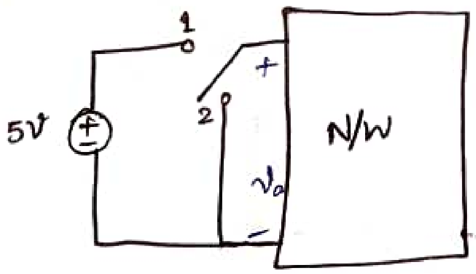
$tu(t-a) \longrightarrow$

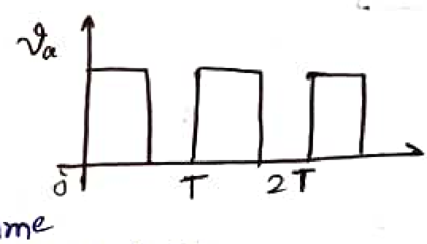




$$f(t) = f_2(t) - f_3(t)$$
$$= \frac{M}{a} t\, u(t) - \frac{M}{a}(t-a)\, u(t-a)$$

→





If $f(t)$ is periodic with period T, then $f(t) = f(t+T)$.

LT of a periodic function $f(t)$

$$\mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt.$$

$$= \int_{0^-}^{T} f(t) e^{-st} dt + \int_{T}^{2T} f(t)\, e^{-st} dt + \dots + \underbrace{\int_{(n-1)T}^{nT} f(t) e^{-st} dt}_{I_n}$$

$$+ \dots = \sum_{n=1}^{\infty} I_n$$

$$I_n = \int_{(n-1)T}^{nT} f(t) e^{-st} dt.$$

Let $\theta = t-(n-1)T$,

then $d\theta = dt$

and as $t \to (n-1)T$

$\theta \to 0$

and when $t = nT$, $\theta = T$.

Then, $$I_n = \int_0^T \underset{\downarrow\; f(\theta)}{f[\theta+(n-1)T]}\, e^{-s[\theta+(n-1)T]}\, d\theta$$

$$= \underbrace{\int_0^T f(\theta) e^{-s\theta} d\theta}_{I_1}\; e^{-(n-1)Ts}$$

$$= e^{-(n-1)Ts}\, I_1$$

$\mathcal{L}\{f(t)\}$

$$= I_1 + e^{-sT} I_1 + e^{-2sT} I_2 + \dots$$

$$\Rightarrow \boxed{\mathcal{L}\{f(t)\} = \frac{I_1}{1-e^{-sT}}}$$

06-10-2023

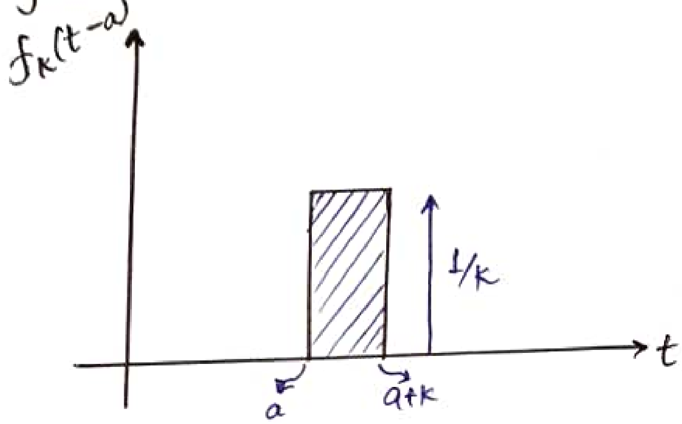
Agenda:

① Impulse function and its LT

② Rel^n b/w unit step unit impulse and unit ramp.

③ Final and Initial value Theorems.

Impulse Function

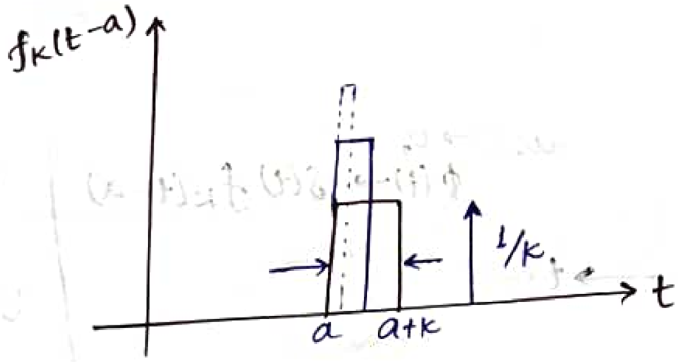


Let $f_K(t-a) = 1/K$, for $a \le t \le a+K$ [K→ a small +ve number]

$= 0$, otherwise.

$$\int_{0^-}^{\infty} f_K(t-a)\, dt = 1$$

As we reduce K,



As $K \to 0$,

$$\lim_{K\to 0} f_K(t-a) \to \infty \text{ for } t=0$$

$= 0$, otherwise

$\triangleq \delta(t-a)$

Unit Impulse:

$\triangleq \delta \rightarrow \infty, \; t=0$
$\quad = 0, \text{ otherwise.}$

- $$\int_{0^-}^{\infty} \delta(t)\,dt = 1$$

- If $g(t)$ is continuous,

$$\int_{0^-}^{\infty} g(t)\,\delta(t)\,dt = g(0).$$

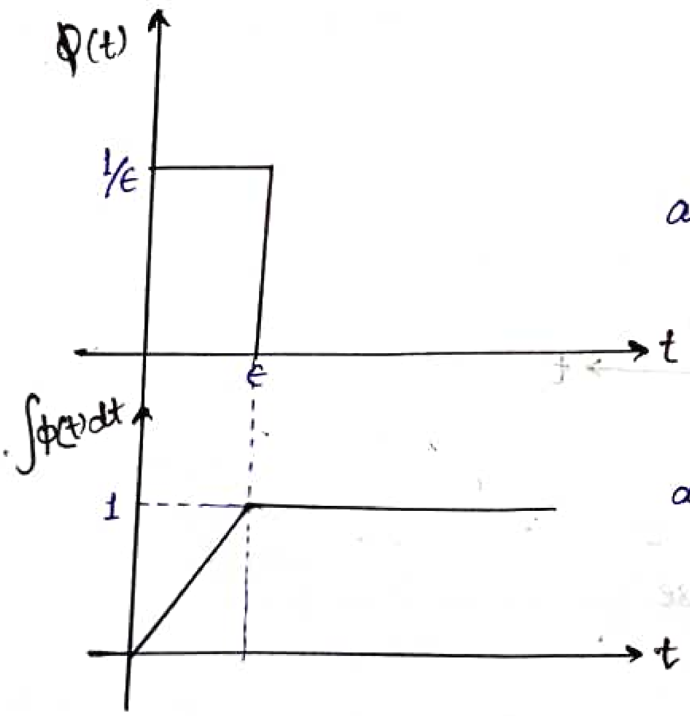
LT of $\delta(t)$:

$$\mathcal{L}\{\delta(t)\} = \int_{0^-}^{\infty} \underbrace{e^{-st}}_{g(t)} \delta(t)\,dt$$

$$= e^{-s(0)}$$

$$= 1.$$

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}$$



as $\epsilon \rightarrow 0$,
$\phi(t) \rightarrow \delta(t) \; f_K(t-a)$

as $\epsilon \rightarrow 0$,
$\int \phi(t)dt \rightarrow u(t)$

Unit Impulse fn
↑ diff.
Unit step fn
↓ Integrate
Unit ramp fn

Initial and Final Value Theorem

↳ Getting $f(0)$ and $\lim_{t\to\infty} f(t)$ directly from $F(s)$.

Initial Value Theorem:

$$f(0) = \lim_{s\to\infty} sF(s)$$

$$\lim_{s\to\infty}\int_{0^-}^{\infty} e^{-st} f'(t)\,dt = \lim_{s\to\infty} sF(s) - f(0) \qquad \left[\mathcal{L}\left\{\frac{df}{dt}\right\} = sF(s) - f(0)\right]$$

$$\Rightarrow \int_{0^-}^{\infty} \cancel{\lim_{s\to\infty} e^{-st}}^{\,0} f'(t)\,dt = \lim_{s\to\infty} sF(s) - f(0)$$

$$\Rightarrow 0 = \lim_{s\to\infty} sF(s) - f(0)$$

$$\Rightarrow \lim_{s\to\infty} sF(s) = f(0)$$

Final value Theorem:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$\mathcal{L}\{f'(s)\} = sF(s) - f(0)$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left(\mathcal{L}\{f'(s)\} + f(0)\right)$$

$$= \int_{0^-}^{\infty} f'(t) \lim_{s\to 0} e^{-st}\,dt + f(0)$$

$$= \int_{0^-}^{\infty} f'(t)\,dt + f(0)$$

$$= f(\infty) - f(0) + f(0)$$

$$= \lim_{t\to\infty} f(t)$$

11-10-2023

→ If $\mathcal{L}\{f(t)\}$, $\mathcal{L}\{f_1(t)\}$ and $\mathcal{L}\{f_2(t)\}$ are $F(s)$, $F_1(s)$ and $F_2(s)$, respectively, and

$$\boxed{F(s) = F_1(s) F_2(s)} \quad \dots (e1)$$

$$\Rightarrow \mathcal{L}^{-1}\{F\} \neq \mathcal{L}^{-1}\{F_1\} \mathcal{L}^{-1}\{F_2\}$$

$$f(t) \neq f_1(t) f_2(t)$$

not necessarily

Then,

$$f(t) = \int_0^t f_1(\tau) f_2(t-\tau)\, d\tau \rightarrow \text{Convolution Integral (CI)} \quad \dots (e2)$$

$$= \int_0^t f_1(t-\tau) f_2(\tau)\, d\tau$$

$$= \boxed{f_1(t) * f_2(t)} \leftarrow \text{convolution}$$

Convolution Integral

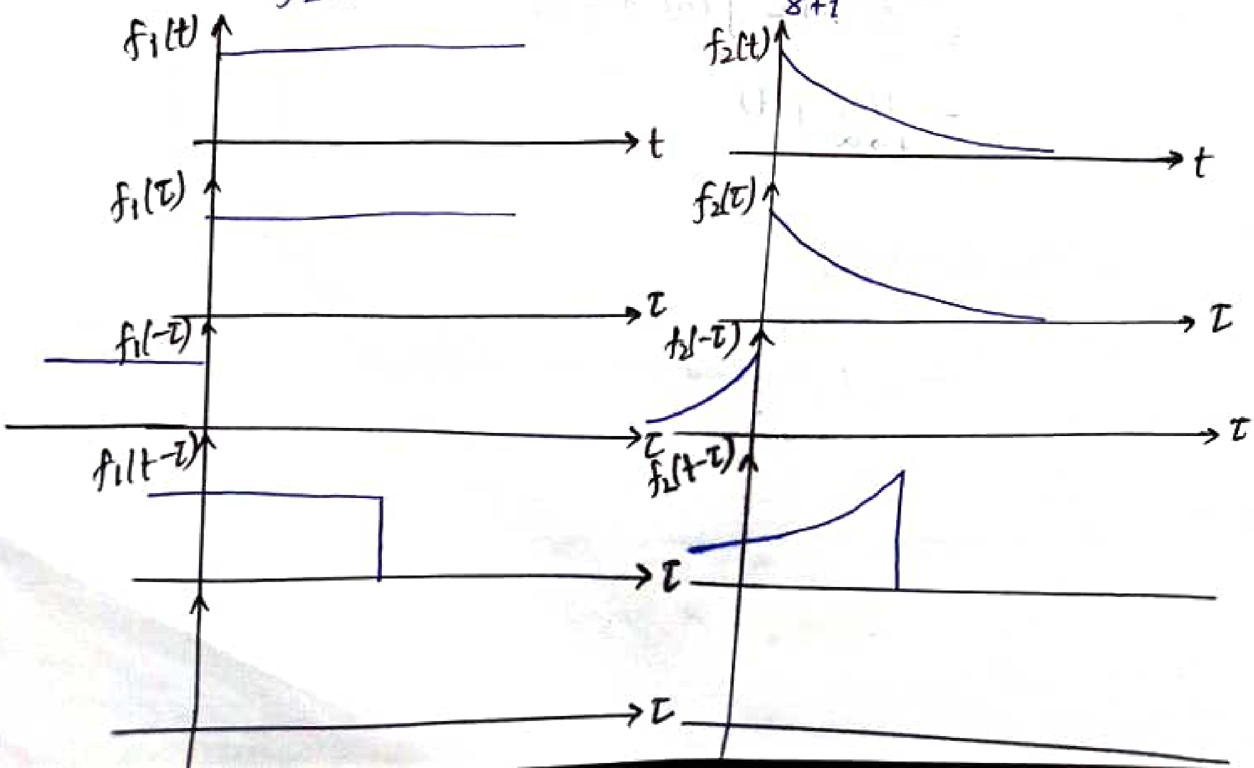
CI: Involves 4 steps.

1. Reverse/fold one of the signals.
2. Shift the folded signal.
3. Multiply 2 signals
4. Integration.

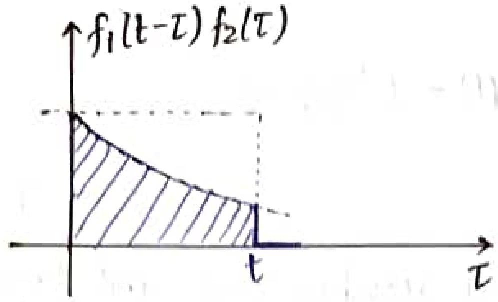
Eg.

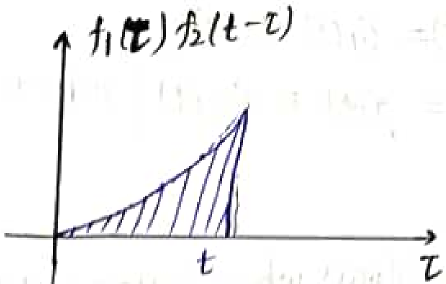
$$f_1(t) = u(t) \longrightarrow F_1(s) = 1/s$$

$$f_2(t) = e^{-t} u(t) \longrightarrow F_2(s) = \frac{1}{s+1}.$$



$f_1(t) * f_2(t)$





$$\int_0^t \underbrace{f_1(t-\tau)}_{1} \underbrace{f_2(\tau)}_{e^{-\tau}} d\tau$$

$$= \int_0^t e^{-\tau} d\tau$$

$$= -e^{-\tau}\Big|_0^t$$

$$= \boxed{1 - e^{-t}}$$

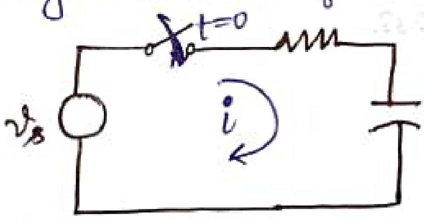
$$\int_0^t f_1(\tau) f_2(t-\tau) d\tau$$

$$= \int_0^t e^{-(t-\tau)} d\tau$$

$$= -e^{-(t-\tau)}\Big|_0^t$$

$$= \boxed{1 - e^{-t}}$$

→ CI allows us to compute the response of a N/w to any arbitrary excitation if the impulse response is known.



Initially relaxed System.

$$v_s(t) = Ri(t) + \frac{1}{C}\int i(t)dt$$

$$LS \Rightarrow \quad V_s(s) = RI(s) + \frac{1}{sC} I(s)$$

$$V_s(s) = \frac{sCR + 1}{sC} \cdot I(s)$$

$$\Rightarrow I(s) = \frac{sC}{sCR+1} \cdot V(s) \quad \ldots \text{(ea)}$$

If $v_s(t) = \delta(t)$

then, $V_s(s) = 1$

Then, $I(s) = \frac{sC}{sCR+1} \longrightarrow G(s) \quad \ldots \text{(eb)}$

↓ Impulse response of the system

$$\text{(ea)} \Rightarrow \quad \frac{I(s)}{V(s)} = \frac{sC}{sRC+1} \triangleq G(s) \quad \ldots \text{(ec)}$$

↳ Transfer function → Impulse response of the system

Using (ea) & (ec),

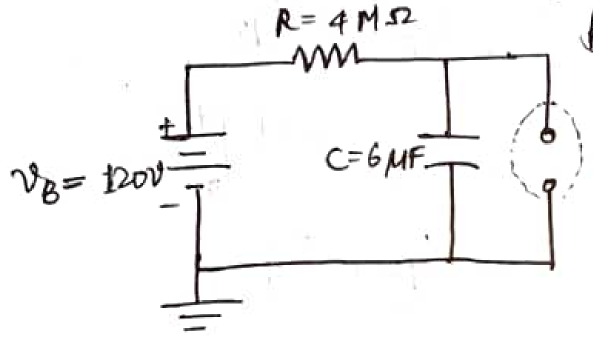
$$I(s) = G(s)\, V_s(s)$$

$$\Rightarrow \boxed{i(t) = g(t) * v_s(t)} \quad \text{where } g(t) = \mathcal{L}^{-1}\{G(s)\}.$$

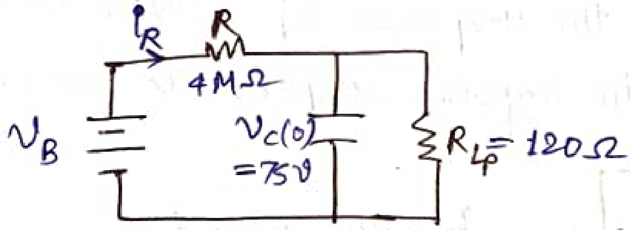
[13-10-2023]

Quiz-1 Q.3

Q1 The lamp glows when voltage across it reaches 75V and turns off when the voltage drops to 30V. The lamp resistance is 120 Ω when glowing and infinite when turned off. Initial transients while connecting the battery can be neglected. Find the time duration for which the lamp glows.



Soln: For $30V < v \le 75V$,



$$\begin{cases} v_B = R i_R + v_c \\ i_R = C v_c' + v_c / R_{LP} \end{cases}$$

$$\hookrightarrow v_B = RC v_c' + \frac{R}{R_{LP}} v_c + v_c$$

$$= RC v_c' + \left(\frac{R}{R_{LP}} + 1\right) v_c$$

$$\Rightarrow \frac{v_B}{RC} = v_c' + \frac{1}{C} \frac{R + R_{LP}}{R R_{LP}} v_c$$

$$= v_c' + \frac{1}{\tau_P} v_c \quad , \quad \tau = C\,(R_{LP} \parallel R)$$

$$\Rightarrow v_c = v_B \underbrace{\frac{(R_{LP} \parallel R)}{R}}_{\ll 1} + K_0 e^{-t/\tau}$$

At $t=0$, $v_c(0) = 75V$, $75 \approx K_0$.

$$\therefore v_c = \frac{v_B (R_{LP} \parallel R)}{R} + 75\, e^{-t/\tau}$$

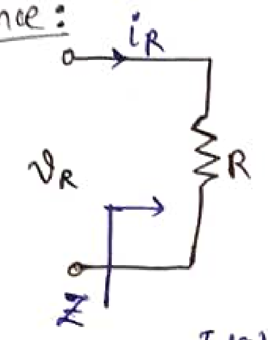
Say, time taken for glowing is t_g,

$$v_c(t_g) = 30V = v_B \frac{(R_{Lp} \| R)}{R} + 75\, e^{-t_g/\tau}$$

Transform Impedance

LT : 't' domain to 's' domain.

Resistance :



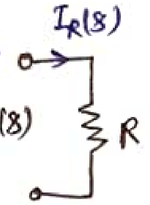
$$v_R(t) = R\, i_R(t)$$

$$\Rightarrow v_R(s) = R\, I_R(s)$$

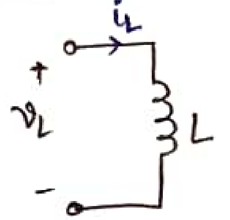
$$\Rightarrow \frac{V_R(s)}{I_R(s)} = R$$

(Terminal voltage and current must be retained)

s domain:



Inductance:

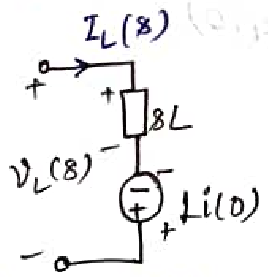


$$v_L(t) = L \frac{di(t)}{dt}$$ → Assumption: Magnetic field is linear. (even while designing)

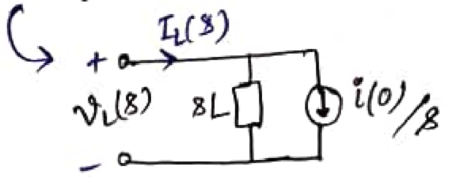
$\downarrow$ LT

$$v_L(s) = L\{sI(s) - i(0)\}$$

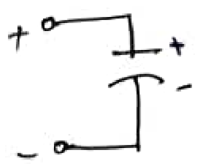
$$= sLI(s) - Li(0)$$

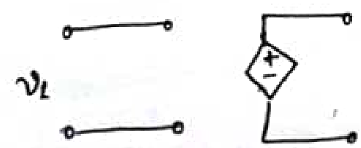


$$\Rightarrow I_L(s) = \frac{v_L(s)}{sL} + \frac{i(0)}{s}$$



capacitance:

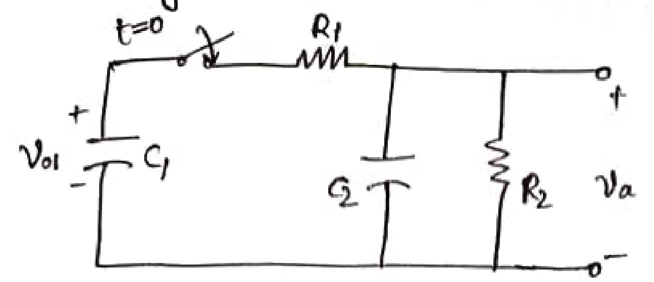




$$v_2(t) = \alpha v_1(t)$$
$$\downarrow$$
$$v_2(s) = \alpha v_1(s)$$

Valkenburg Prob. 7-42

Q)



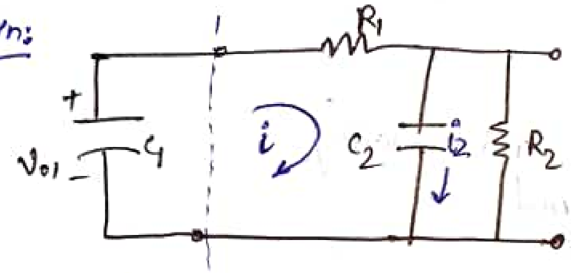
Find $v_a(t)$ for $t \geq 0$.

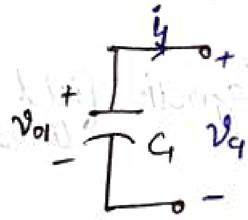
$V_{01} = 10V$

$R_1 = 1\Omega$, $R_2 = 0.8\Omega$

$C_1 = \frac{1}{3}F$, $C_2 = \frac{3}{4}F$.

Soln:



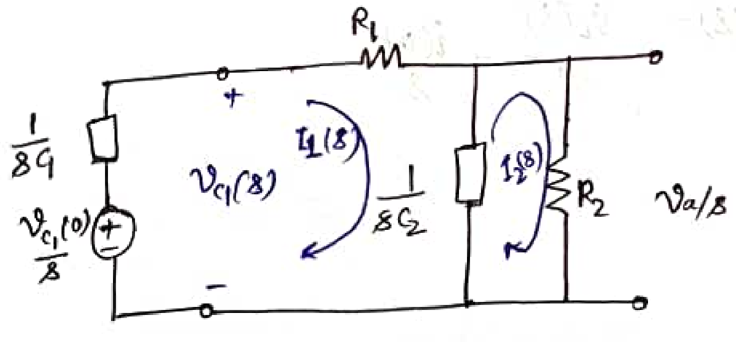


$i_1 = -C_1 \frac{d v_{C1}}{dt}$

$\Rightarrow I_1(s) = -C_1 \left[s V_{C1}(s) - V_{C1}(0) \right]$

$\Rightarrow I_1(s) = -sC_1 V_{C1}(s) + C_1 V_{C1}(0)$

$\Rightarrow V_{C1}(s) = -\frac{1}{sC_1} I_1(s) + \frac{1}{s} V_{C1}(0)$



$I_1: \quad \frac{V_{C1}(0)}{s} = \frac{I_1}{sC_1} + R_1 C_1 + \frac{1}{sC_2}(I_1 - I_2) \quad \ldots (e1)$

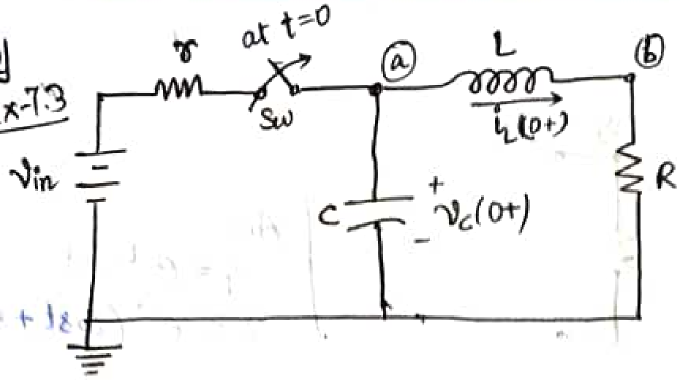
$I_2: \quad \frac{1}{sC_2}(I_2 - I_1) + I_2 R_2 = 0 \quad \ldots (e2)$

$(e1) + (e2) \Rightarrow \frac{I_1}{sC_1} + R_1 I_1 + I_2 R_2 - \frac{V_{C1}(0)}{s} = 0.$

$\Rightarrow I_2 = \frac{1}{R_2} \left[\frac{V_{C1}(0)}{s} - I_1 R_1 - \frac{I_1}{sC_1} \right]$

Q) Kuo Ex-7.3

18-10-2023



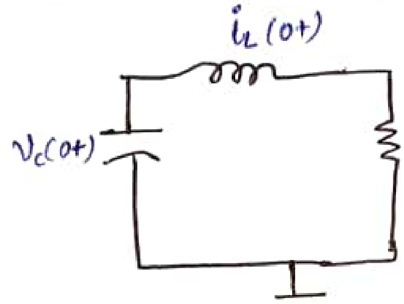
Initially ss was reached with Sw closed.

Ans:
$v_1 = e^{-t}\cos t$
$v_2 = e^{-t}(\cos t + \sin t)$

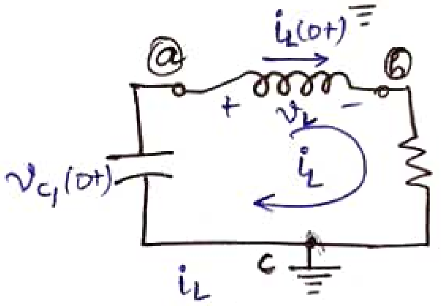
$v_{IN} = 1V$, $L = 0.5H$, $C = 1F$, $R = 1\Omega$, $r = 0.01\,\Omega$.

Find $v_a(t)$ and $v_b(t)$ for $t \geq 0$.

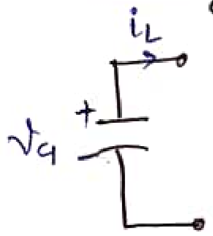
Soln: For $t \geq 0$, the relevant circuit is



$v_c(0+) \approx 1V$
$i_L(0+) \approx 1/1 = 1A \quad \left(\frac{v_{in}}{r+R} = \frac{1}{1.01} \approx 1\right)$



: t-domain

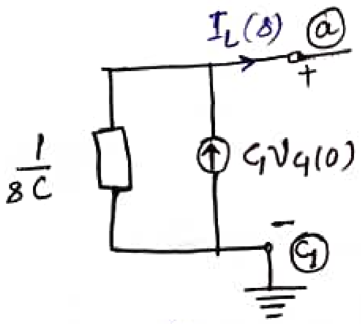


: s-domain

$$C_1 \frac{dv_{c_1}}{dt} = -i_L$$

$$\xrightarrow{LT} C_1\left[s v_{c_1}(s) - v_{c_1}(0)\right] = -I_L(s)$$

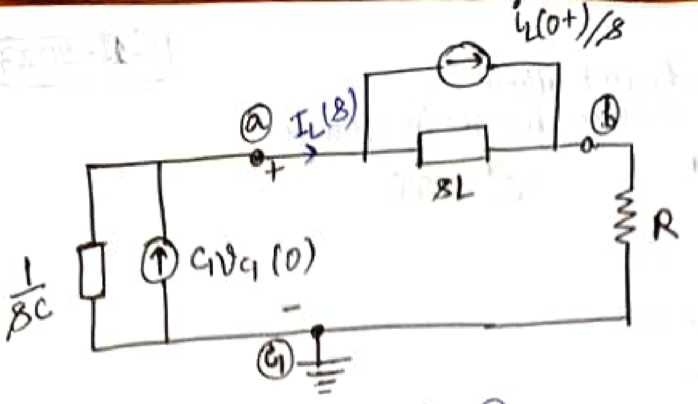
$$\Rightarrow I_L(s) = -sC v_c(s) + C_1 v_{c_1}(0)$$



Taking LT of $v_L = L\frac{di}{dt}$

$$v_L(s) = L\left\{sI_L(s) - i_L(0)\right\}$$

$$\Rightarrow I_L(s) = \frac{1}{sL} v_L(s) + \frac{i_L(0+)}{s}$$



: s domain

Applying KCL at node ⓐ,

$$I_L(s) = -sC V_a(s) + C_1 V_{C_1}(0+) = \frac{1}{sL}\{V_a(s) - V_b(s)\} + i_L(0+)/s = \frac{V_b(s)}{R}$$

Soln: $I_L(s) = -sV_a(s) + 1 = \frac{2}{s}\{V_a(s) - V_b(s)\} + 1/s$

$= V_b(s)$

$\Rightarrow V_b(s) + sV_a(s) = 1$

$-\frac{2}{s}\{V_a(s) - V_b(s)\} + \frac{1}{s} = V_b(s) = 1/s$

$$\begin{bmatrix} s & 1 \\ -\frac{2}{s} & \frac{2+s}{s} \end{bmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 1 \\ 1/s \end{bmatrix}$$

$$\left[\begin{array}{cc|c} s & 1 & 1 \\ -\frac{2}{s} & \frac{2+s}{s} & 1/s \end{array}\right] \rightarrow \begin{bmatrix} 1 & 1/s & 1/s \\ -2 & 2+s & 1 \end{bmatrix}$$

$\Rightarrow -2V_a(s) + (2+s)V_b(s) = 1$

$\Rightarrow -2s V_a(s) + (2s + s^2)V_b(s) = s$

$\quad 2s V_a + 2V_b = 2$

$\Rightarrow (s^2 + 2s + 2)V_b = s + 2$

$\Rightarrow V_b = \frac{s+2}{s^2+2s+2}$

& $sV_a = 1 - \frac{s+2}{s^2+2s+2} = \frac{s^2+s}{s^2+2s+2}$

$\Rightarrow V_a = \frac{s+1}{s^2+2s+2} = \frac{s+1}{(s+1)^2+1}$

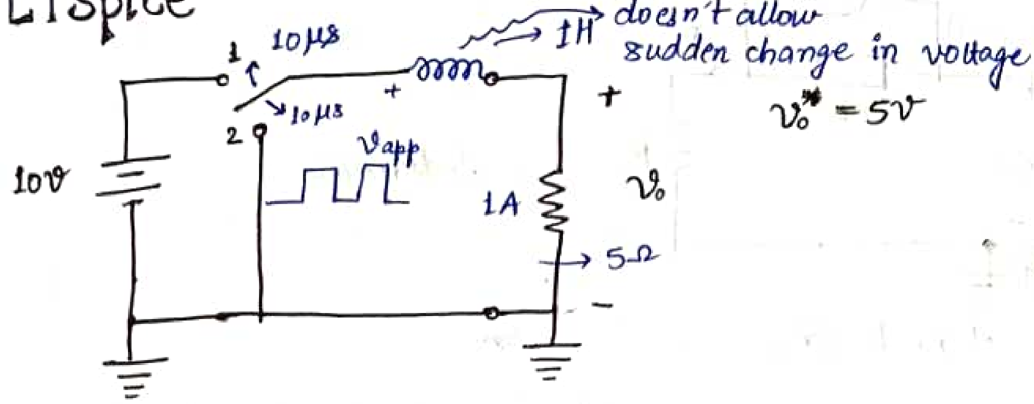
$\Rightarrow V_a(t) = e^{-t}\cos t.$

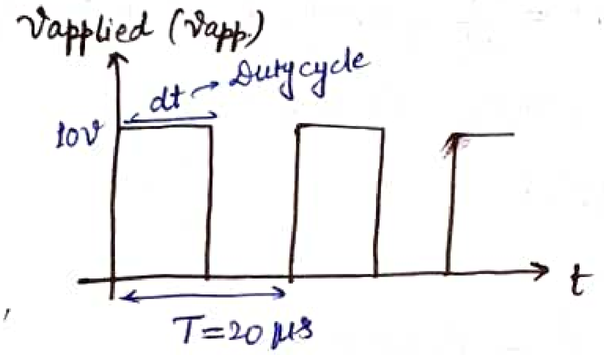
and, $V_b(s) = \frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}$

$\Rightarrow v_b(t) = e^{-t}\cos t + e^{-t}\sin t = e^{-t}(\cos t + \sin t).$

#LTSpice

2-11-2023





$\langle v_{app} \rangle = \frac{1}{T}\int v_{app}\, dt = 5V.$

50% duty cycle → 5V
33% duty cycle → 3.3 V

If we don't use inductor, then around load we get pulsive current.
To avoid this, we can use inductor.
Start with L = 1H.

09-11-2023

Filter Design

- Specification
- Frequency component of excitation
- Network analysis + Frequency Response (FR).

Eg.

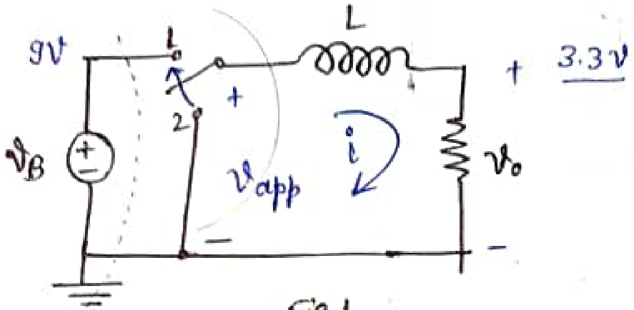
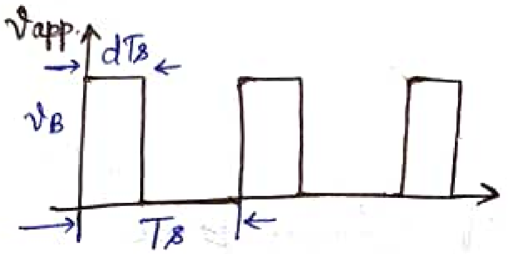
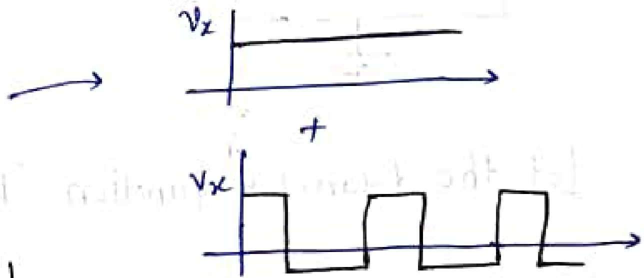


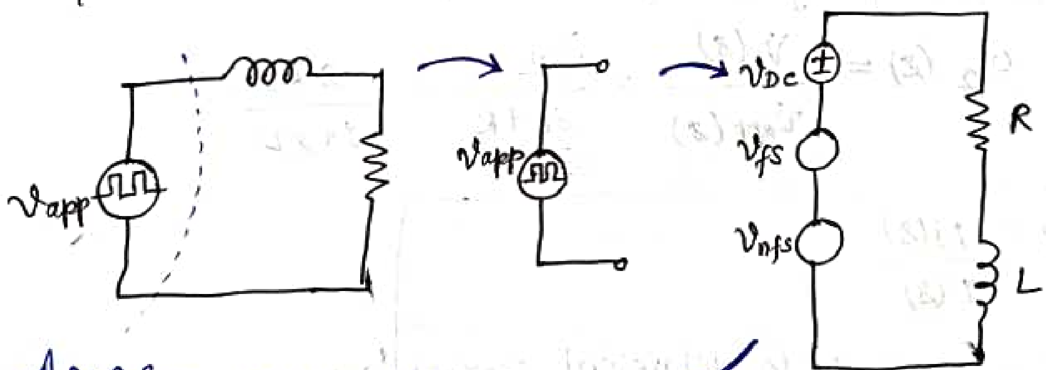
Fig.1



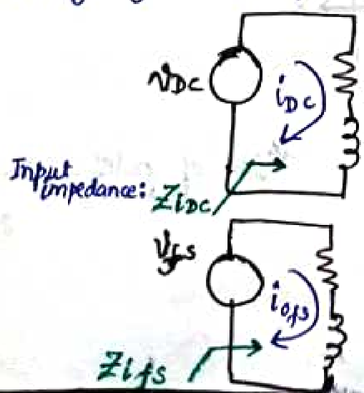


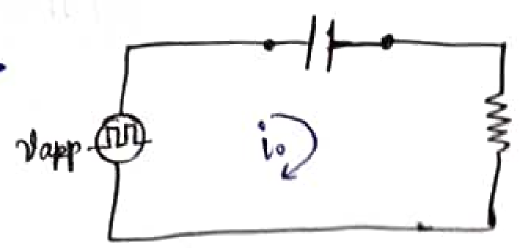
$$\langle v_{app} \rangle = d v_B \quad \left(\frac{dT_s \, v_B}{T_s} \right)$$

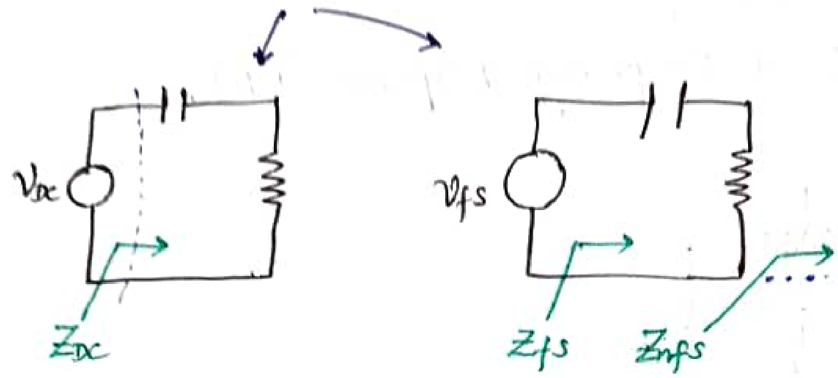
Equivalent circuit of Fig. 1 :

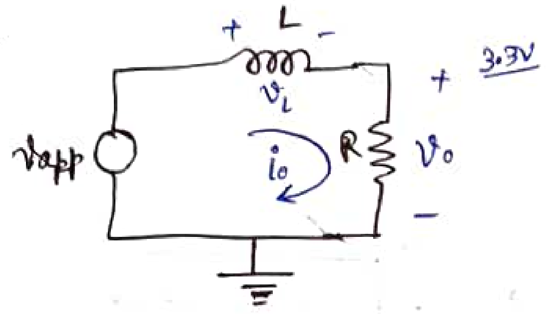


Applying superposition,









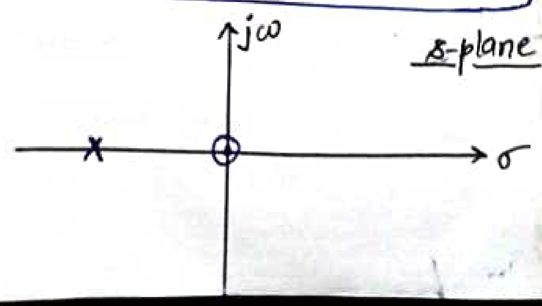
Let the transfer function (TF) of $\frac{v_o(s)}{v_{app}(s)}$ be

$$G_1(s) = \frac{v_o(s)}{v_{app}(s)} = \frac{R}{R+sL} = \frac{1}{1+s\tau}$$

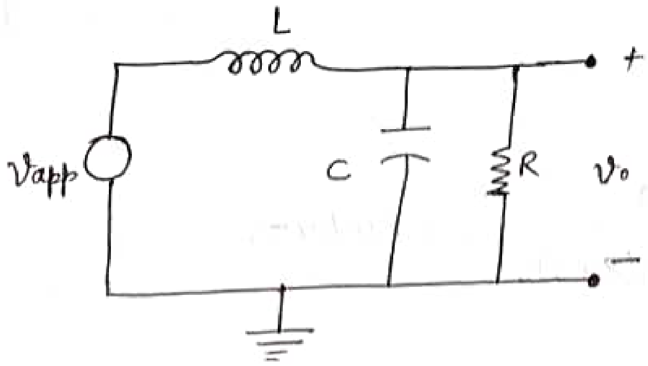
Another transfer function,

$$G_2(s) = \frac{v_L(s)}{v_{app}(s)} = \frac{sL}{sL+R} = \frac{s\tau}{1+s\tau}$$

$$\boxed{\begin{array}{l} G(s) = \dfrac{N(s)}{D(s)} \\ N(s) = 0 \rightarrow \text{roots represent zeroes} \rightarrow \text{o} \\ D(s) = 0 \rightarrow \text{roots represent poles} \rightarrow \times \end{array}}$$



No. of poles = Order of system

→ 

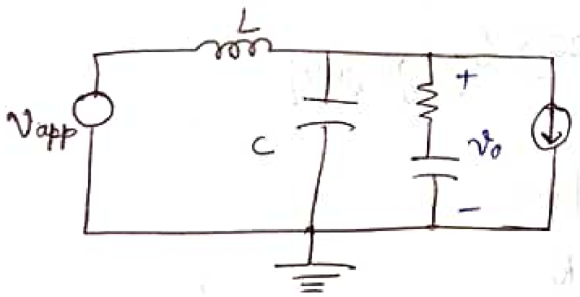
→ 2nd order

$$G_3(s) = \frac{V_o(s)}{V_{app}(s)} = \frac{R \,||\, 1/sC}{sL + R\,||\,1/sC}$$

$$= \frac{\frac{R}{sRC+1}}{\frac{R}{sRC+1} + sL}$$

$$= \frac{R}{R + s^2LCR + sL}$$

$$= \frac{1}{s^2LC + sL/R + 1}$$

→ 

→ 3rd order
(3 independent components)
↓
energy storing element

→ Any G(s) can be represented as

$$G(s) = K \frac{\prod (s+z_r) \prod (s^2 + a_z s + b_z)}{s^m \prod (s+p_r) \prod (s^2 + a_p s + b_p)} \rightsquigarrow \boxed{\text{Pole-zero form}}$$

$$= K_T \frac{\prod (\tau_{zr} s + 1) \prod (c_z s^2 + b_z s + 1)}{s^m \prod (\tau_{pr} s + 1) \prod (c_p s^2 + b_p s + 1)}$$

For filter, we only take $j\omega$ part of s. (Steady state) ⇝ $\underline{\sigma = 0}$.

$$G(j\omega) = K_T \frac{\prod (\tau_{zr} j\omega + 1) \prod (c_z (j\omega)^2 + b_z (j\omega) + 1)}{s^m \prod (\tau_{pr} j\omega + 1) \prod (c_p (j\omega)^2 + b_p j\omega + 1)}$$

$$\therefore \boxed{\begin{aligned} G(s) &= K G_1(s) G_2(s) \ldots G_n(s) \\ G(j\omega) &= K_1 G_1(j\omega) G_2(j\omega) \ldots G_n(j\omega) \end{aligned}}$$

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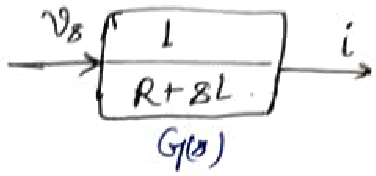
Frequency Response

$$G(s) = \frac{(s+4)(s+5)}{s^2(s^2+2s+2)(s+3)} \rightsquigarrow \text{order}=5$$

$$= \prod_{1}^{5} g_i(s)$$



Putting $\sigma=0$

$$G(j\omega) = \prod g_i(j\omega)$$

$$= |G(j\omega)| \angle G(j\omega)$$

$$= \underbrace{\prod |g_i(j\omega)|}_{G\text{magnitude}} \; \underbrace{\angle \Sigma \angle g_i}_{\angle G}$$

$\searrow |G(j\omega)|$

Then, $\log|G(j\omega)| = \Sigma \log|g_i(j\omega)|$

$$\Rightarrow \underbrace{20\log|G(j\omega)|}_{\text{Gain in decibel}} = 20\Sigma\log|g_i(j\omega)|$$

(dB) $\to |G|_{dB}$

$$G(s) = \frac{K}{s} \qquad \frac{K}{s^2} \qquad \frac{1}{s^n}$$

$$= 10 \cdot \frac{1}{s} \quad (\text{for } K=10)$$

$\uparrow$ G_1 $\quad$ $\uparrow$ G_2

$G_1(s) = 10 \Rightarrow |G_1|_{dB} = 20\log 10 = 20\text{ dB}$

$G_2(s) = \frac{1}{s} \Rightarrow$ ~~[illegible]~~ $G_2(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega}\angle -90^\circ$

$|G_2|_{\omega=1} = 1 \Rightarrow |G_2|_{dB} = 0\text{ dB}$

At, 'one decade above $\omega=1$' $\Rightarrow \omega = 10$

$|G_2|_{\omega=10} = 0.1 \Rightarrow |G_2|_{dB} = 20\log(0.1) = -20\text{ dB}$

$Q/10$

$\uparrow$ 1 decade below

Quantity, Q

$\downarrow$ 1 decade above

$Q \times 10$

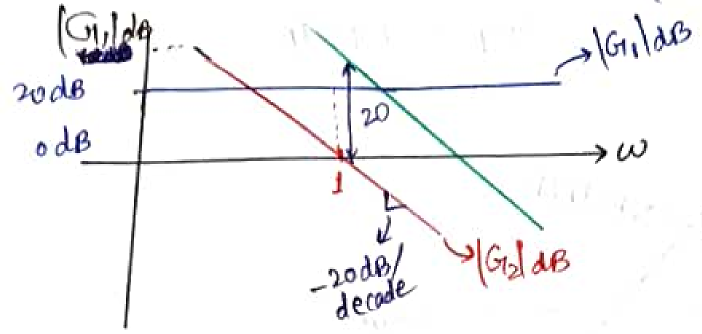
At, 'one decade below' $\omega = 1$,

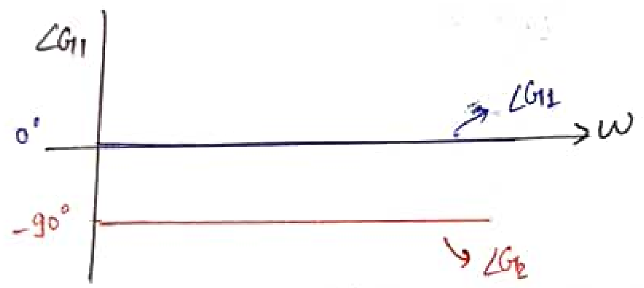
$$|G_2|_{\omega=0.1} = \frac{1}{0.1} = 10 \Rightarrow |G_2|_{dB} = 20\,dB$$

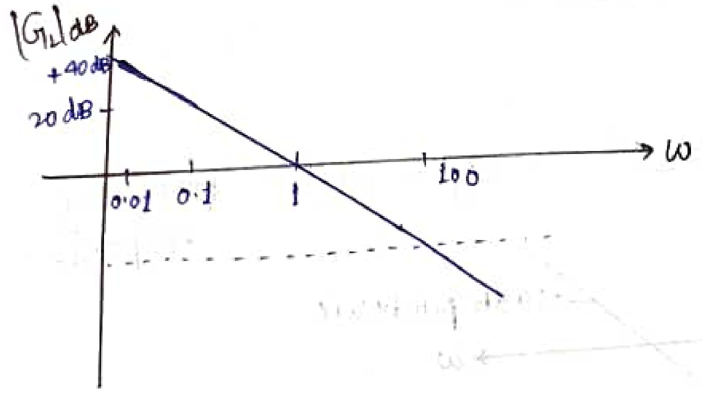
$$|G_2|_{\omega=100} = 0.01 \Rightarrow |G_2|_{dB} = -40\,dB \quad \text{(2 decades above } \omega=1\text{)}$$

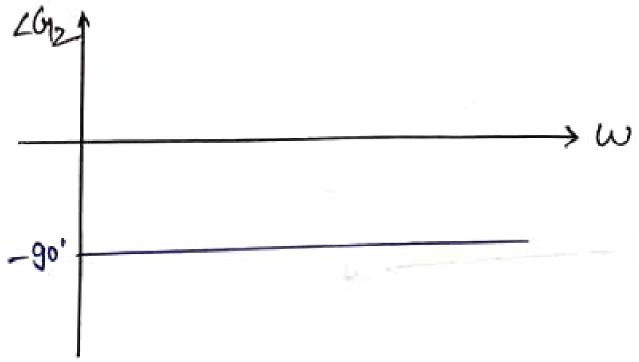
corner frequency

$$|G_2|_{0.01=\omega} = 100 \Rightarrow |G_2|_{dB} = 40\,dB. \quad \text{(2 decades below } \omega=1\text{)}$$





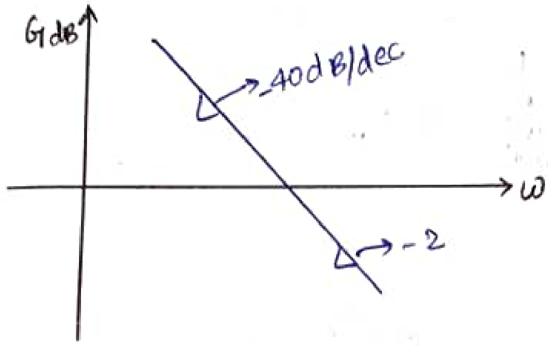


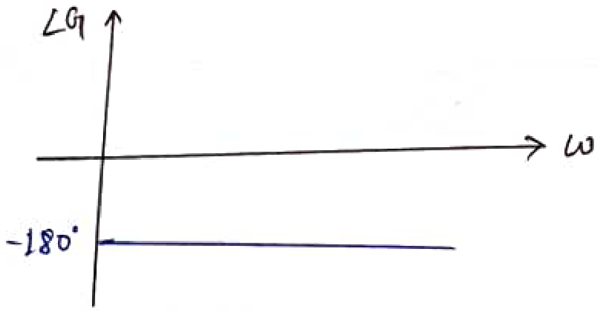


→ $G(s) = \frac{1}{s^2} \Rightarrow G(j\omega) = -\frac{1}{\omega^2} = \frac{1}{\omega^2} \angle -180°$

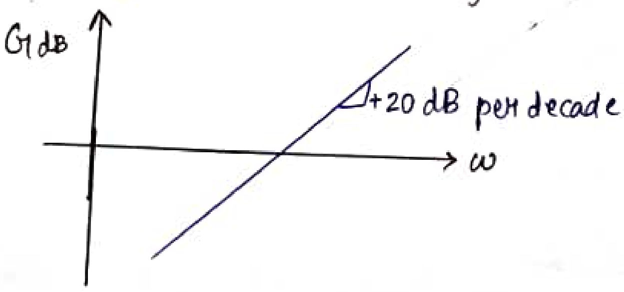
$$\left.\begin{aligned} |G|_{\omega=1} &= 1 \Rightarrow G_{dB} = 0\,dB \\ |G|_{\omega=10} &= 0.01 \Rightarrow G_{dB} = -40\,dB \end{aligned}\right\} -40 \text{ db per decade}$$

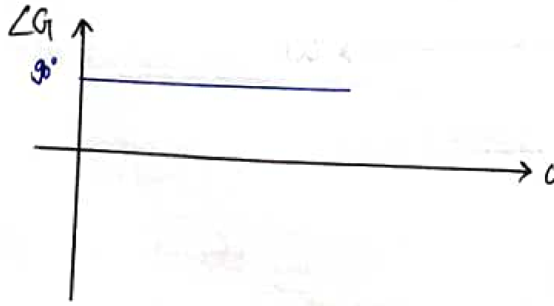
$|G|_{\omega=0.1} = 100 \Rightarrow G_{dB} = +40\,dB$





→ $G(s) = s \Rightarrow G(j\omega) = \omega \angle 90°$: Differentiator frequency response





$\rightarrow G(s) = \dfrac{1}{1+s\tau}$

$\Rightarrow G(j\omega) = \dfrac{1}{1+j\omega\tau} = \dfrac{1}{[1+\omega^2\tau^2]^{1/2}} \angle -\tan^{-1}\omega\tau$

At, $\omega\tau = 1$, $\omega = \frac{1}{\tau}$ $\rightsquigarrow$ Corner frequency (ω_c)

$G_{dB}\Big|_{\omega=\frac{1}{\tau}} = 20\log\dfrac{1}{(1+1)^{1/2}} \approx -3 \text{ dB}$

$\angle G\Big|_{\omega=\frac{1}{\tau}} = -45°$

At $\omega \ll \omega_c$, $|G| \approx 1$

$G_{dB} = 0 \text{ dB}$

$\angle G \approx 0°$

At 1 decade below ω_c,

$|G|_{\omega=\frac{1}{10\tau}} = -0.995$

$\angle G|_{\omega=\frac{1}{10\tau}} = -5.7°$

At 1 decade above ω_c,

$|G|_{\omega=\frac{10}{\tau}} = -20.04 \text{ dB}$

$\angle G|_{\omega=\frac{10}{\tau}} = -84.9°$

At $\omega \gg \omega_c$, i.e., $\omega\tau \gg 1$

$|G| \approx \dfrac{1}{\omega\tau} \rightarrow |G|_{\omega=\frac{10}{\tau}} = 0.1 \xrightarrow{G_{dB}} -20 \text{ dB}$

$\angle G \longrightarrow -90°$

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$\rightarrow \quad G(s) = \dfrac{1}{1+s\tau}$ $\qquad | \; G(s) = g_1(s).g_2(s)\ldots g_n(s).$

$$G_1(j\omega) = \frac{1}{1+j\omega\tau} = \frac{1}{[1+\omega^2\tau^2]^{1/2}} \angle -\tan^{-1}\omega\tau$$

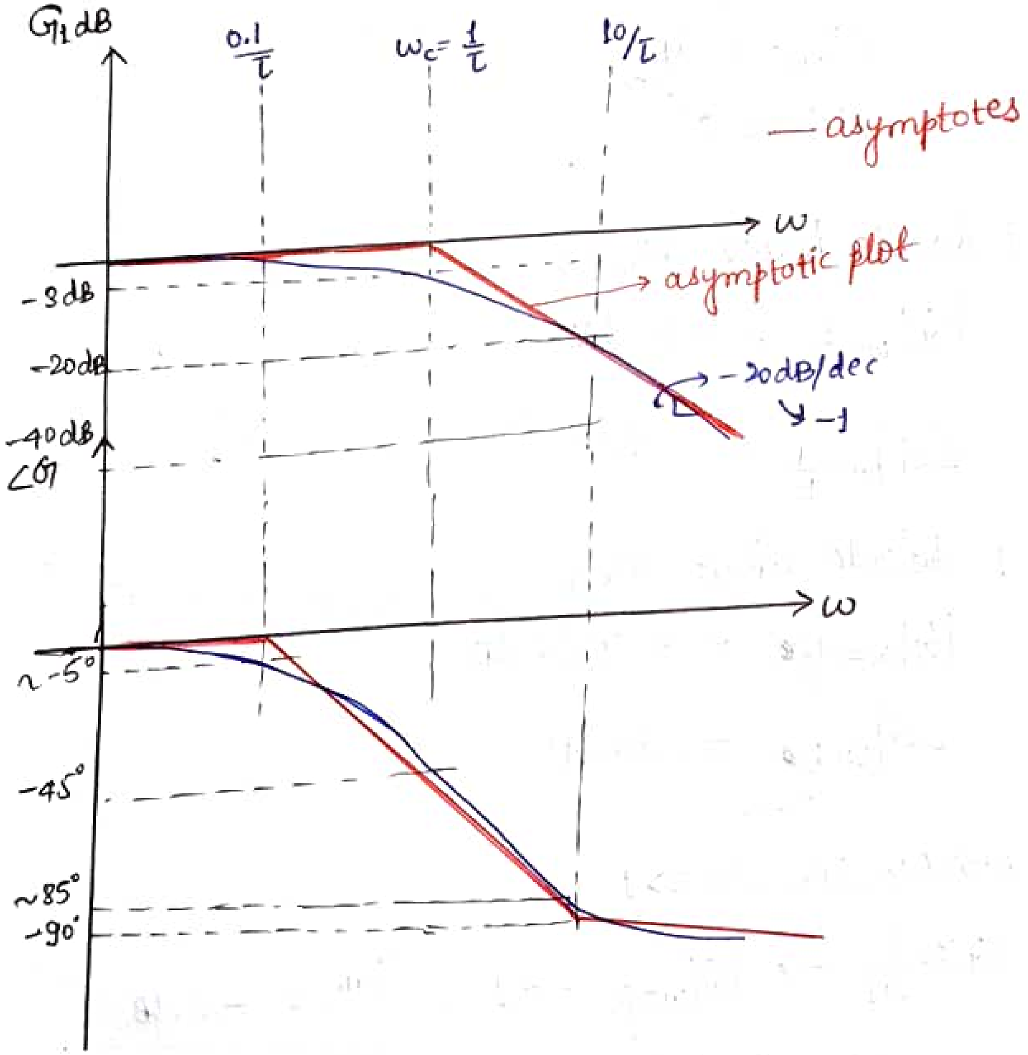
$$G_{1\,dB} = 20\log|G_1(j\omega)|$$

For $\omega \ll \omega_c$,

$$|G| \approx 1$$

for $\omega \gg \omega_c$,

$$|G| \approx \frac{1}{\omega\tau}$$



$\rightarrow G(s) = 1 + \tau s \xleftarrow{} 1/\omega_c$

$G(j\omega) = 1 + j\omega\tau$

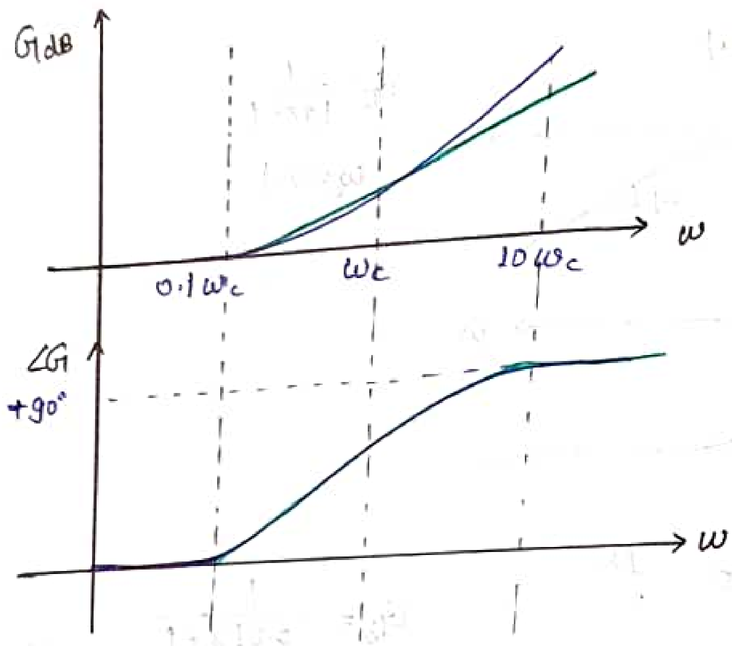
$= [1 + \omega^2\tau^2]^{1/2} \angle \tan^{-1}\omega\tau$

At $\omega = \frac{1}{\tau} = \omega_c$, $|G| \approx 1.414 \Rightarrow G_{dB} = 3\,dB$

$\angle G = 45^\circ$

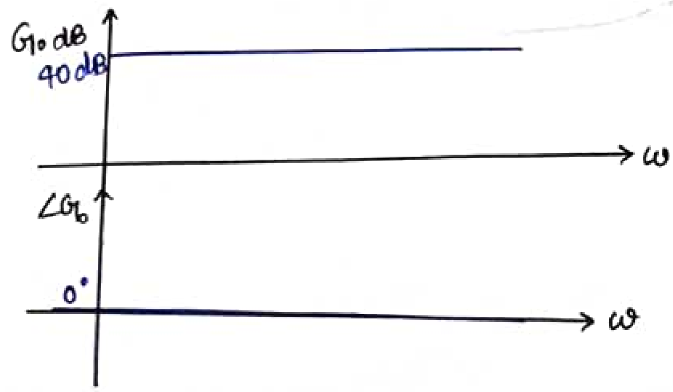
For $\omega \ll \omega_c \Rightarrow \omega\tau \ll 1$, $G_{dB} = 0\,dB$ $(|G| \approx 1)$

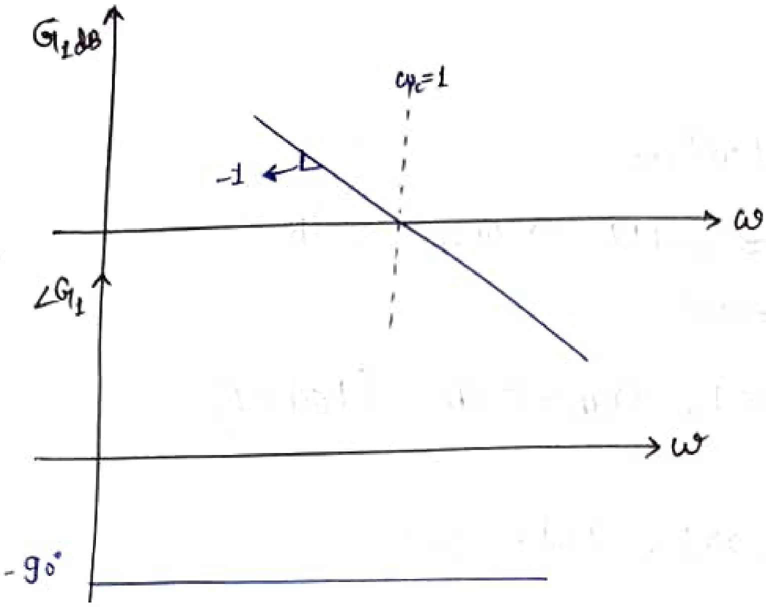
For $\omega \gg \omega_c \Rightarrow \omega\tau \gg 1$, $|G| \approx \omega\tau$



Eg. $G(s) = \dfrac{\overset{\rightarrow g_0}{100}}{\underbrace{s}_{g_1}\underbrace{(10s+1)}_{g_2}\underbrace{(0.01s+1)}_{g_3}}$

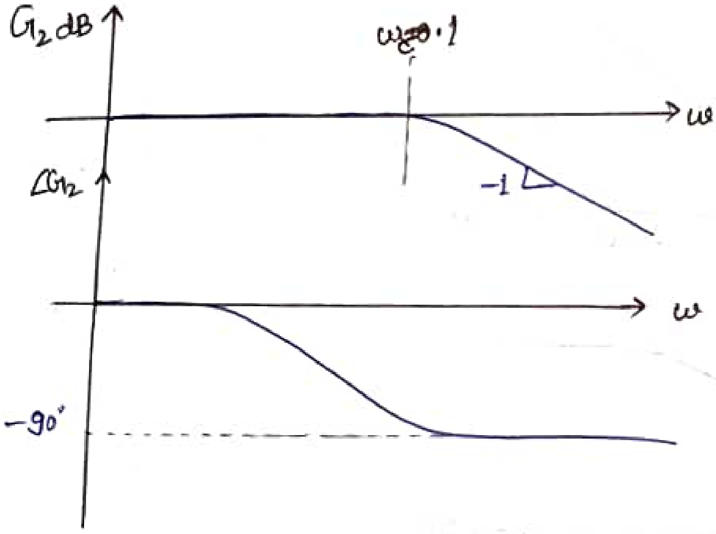
$G_1 \leftrightarrow g_1$





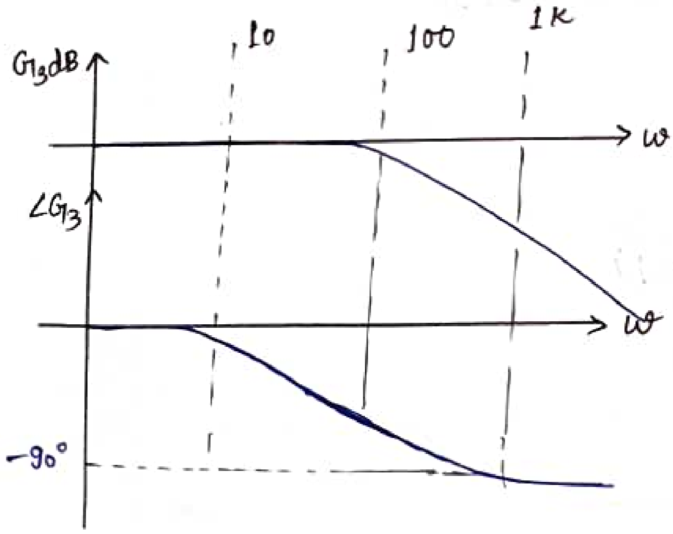
$G_1 = \frac{1}{s}$

$\omega_c = 1$



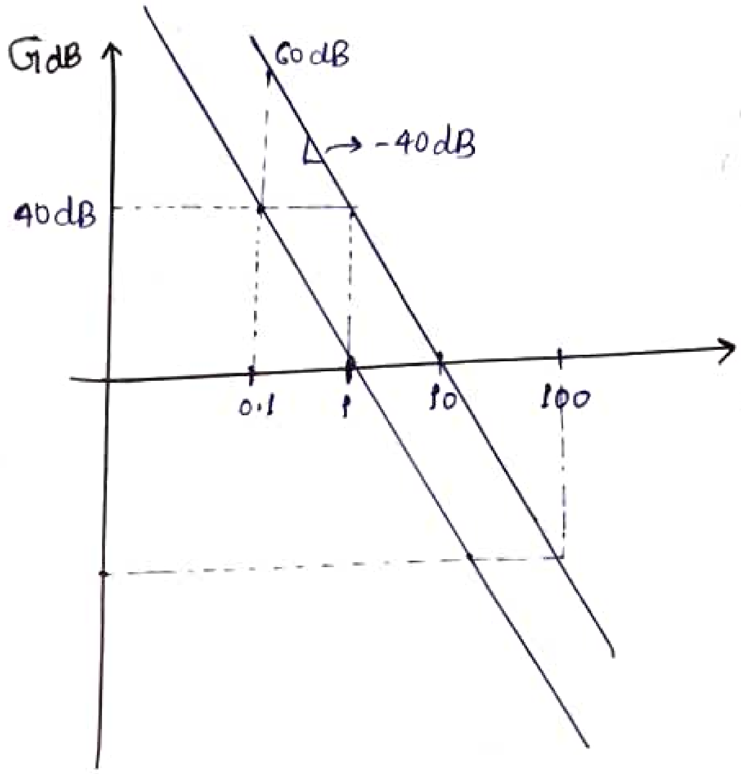
$G_2 = \frac{1}{10s+1}$

$\omega_c = 0.1$



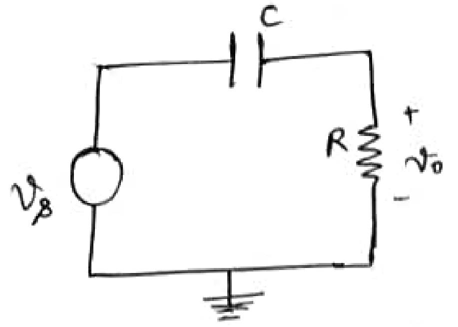
$G_3 = \frac{1}{0.01s+1}$

$\omega_c = 100$



GdB
60dB
-40dB
40dB
0.1
1
10
100

Eg.

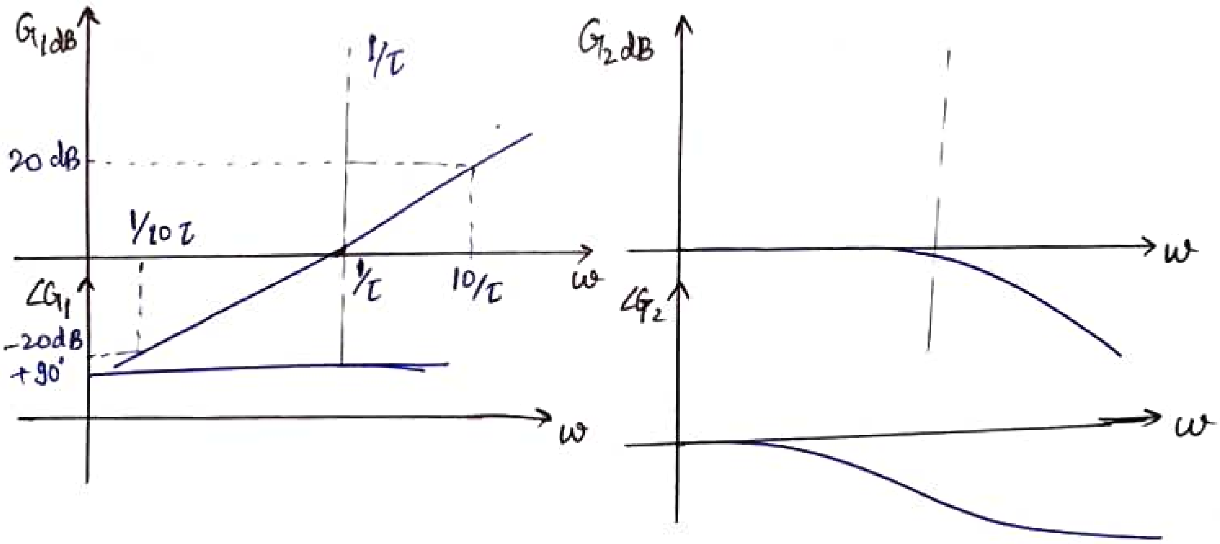


$$\frac{v_o(s)}{v_s(s)} = \frac{\overset{\to G_1}{s\tau}}{\underbrace{1+s\tau}_{\to G_2}}$$

$$= G(s)$$

$$G(j\omega) = \frac{j\omega\tau}{1+j\omega\tau}$$

$$G(s) = \underbrace{s\tau}_{G_1} \cdot \frac{1}{\underbrace{1+s\tau}_{G_2}}$$



Eg. $G(s) = \dfrac{s+10}{(s+100)(s+0.1)}$

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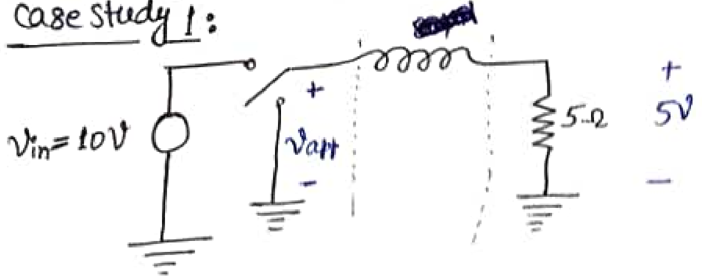
→ $s^{\pm m}, \; (1+\tau s)^{\pm m}, \; K$

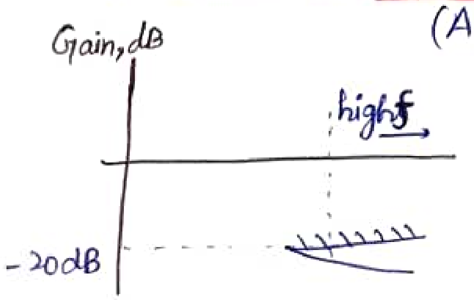
Case Study 1:

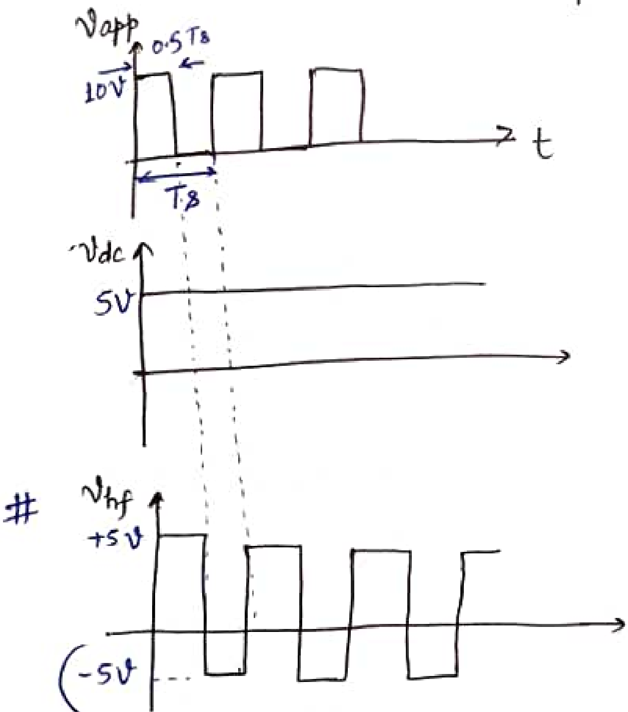


$v_{in} = 10V, \; v_o^* = 5V, \; R = 5\Omega.$

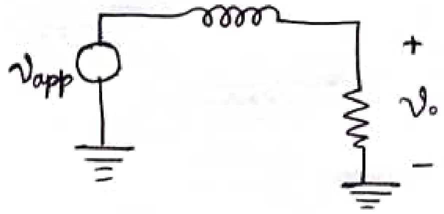
- All high fs ($f_s = 100kHz$) in v_o should be attenuated by 20 dB. (A_t)

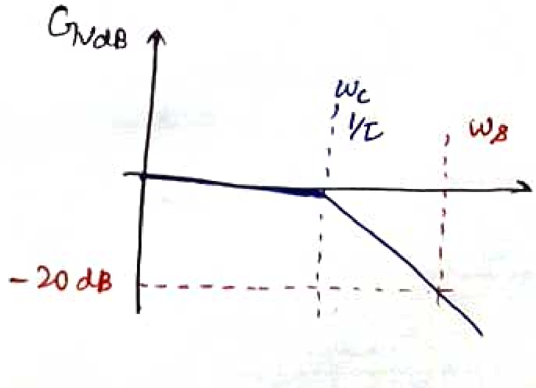
$$A_t = \frac{1}{gain}$$





$$\frac{4 \times 5}{\pi} \sum \frac{\sin n\omega_c t}{n}$$





$$G_v(s) = \frac{V_o(s)}{V_{app}(s)} = \frac{1}{1+s\tau}$$

$$G_{v\,dB}\Big|_{\omega \gg \omega_c} \approx \frac{1}{\omega\tau} = \frac{\omega_c}{\omega}$$

Gain at $\omega_s \le -20\,dB$

$\Rightarrow 20\log\frac{\omega_c}{\omega_s} \le -20\,dB$

$\Rightarrow \frac{\omega_c}{\omega_s} \le -1$

$\Rightarrow \omega_s \ge 10\,\omega_c$

$\Rightarrow \omega_s \ge \frac{10}{\tau} \Rightarrow \omega_s \ge \frac{10R}{L}$ or $L \ge \frac{10\times R}{2\pi f_s}$

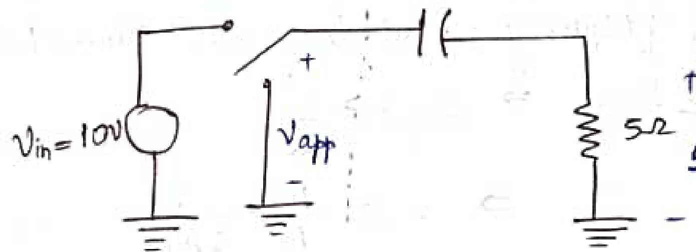
$\Rightarrow L \ge \frac{\cancel{10}^{5}\times 5}{\cancel{2}\pi\times 10^5}$

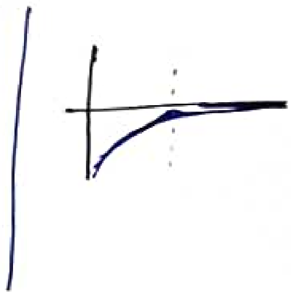
$\Rightarrow L \ge \frac{25\times 10^{-5}}{\pi}$

$\Rightarrow L \sim 7.4\times 10^{-5}$

Case Study 2 :

Block the dc component and pass the high f components with a phase lag/lead < 5° by using minimum size of capacitance.

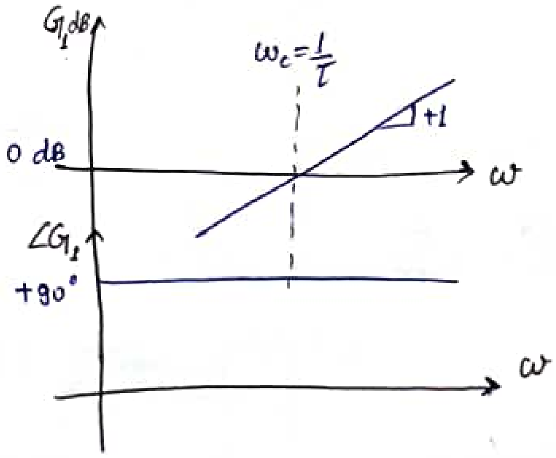




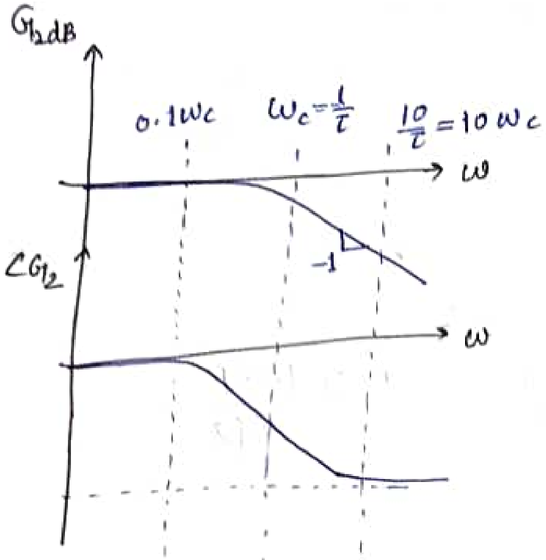
$$G_v(s) = \frac{V_o(s)}{V_{app}(s)} = \frac{s\tau}{1+s\tau}$$

$$= \underbrace{s\tau}_{G_1} \cdot \underbrace{\frac{1}{1+s\tau}}_{G_2}$$

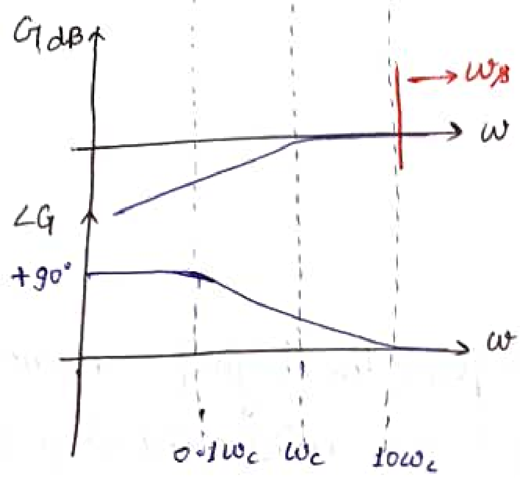
$G_1(s) = s\tau \rightarrow$



$G_2 \rightarrow$



$G_{v\,dB} = (G_{1\,dB} + G_{2\,dB}) \rightarrow$



Switching frequency should be greater than 1 decade above ω_c.

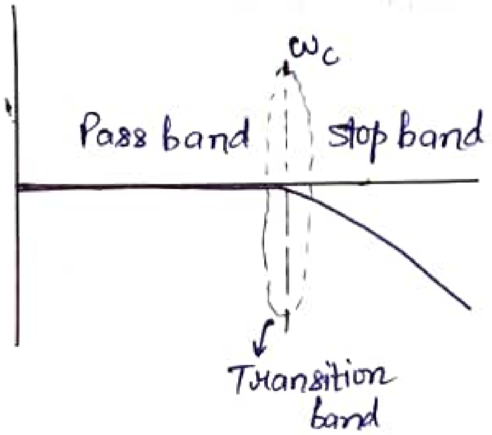
$$\omega_s \geq 10\omega_c \Rightarrow 2\pi f_s \geq \frac{10}{RC}$$

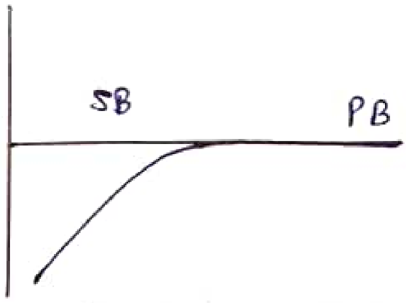
$$\Rightarrow C \geq \frac{10}{R} \times \frac{1}{2\pi f_s}$$

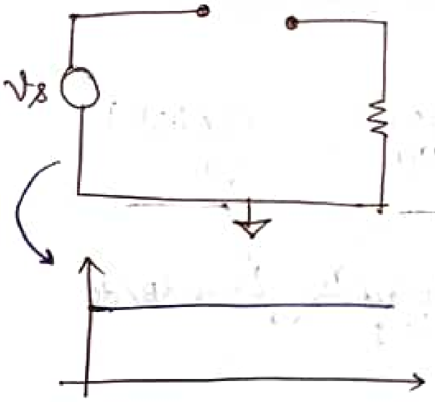
$$\sim \frac{10}{5} \times \frac{1}{6\times 10^5}$$

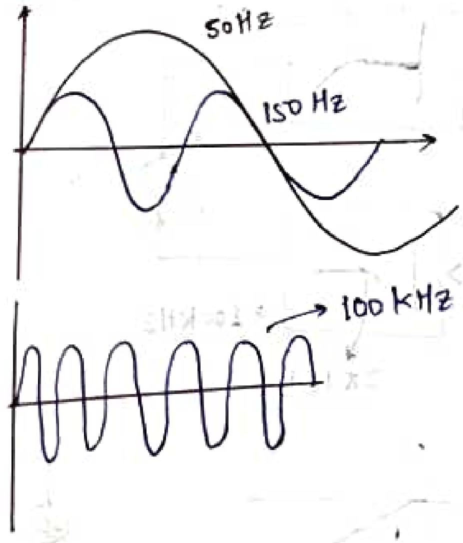
$$\Rightarrow C \geqq 3.18 \times 10^{-6}\ F$$

Pass Band Filter :



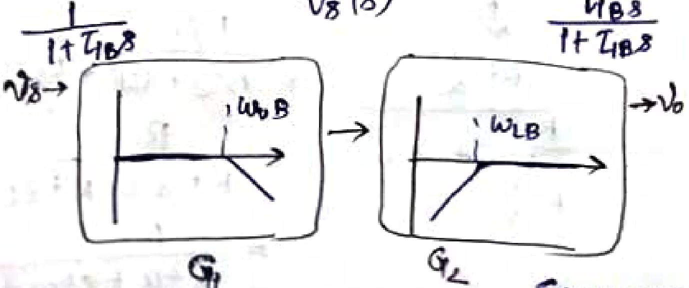


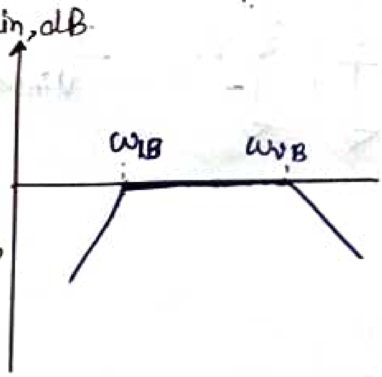




Q) Design a band pass filter that passes 50 Hz and 150 Hz components in v_s with unity gain and attenuates DC and 100 kHz components by atleast 20dB.

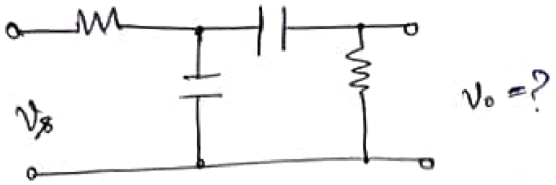
Soln: $G(s) = \dfrac{V_o(s)}{V_s(s)}$



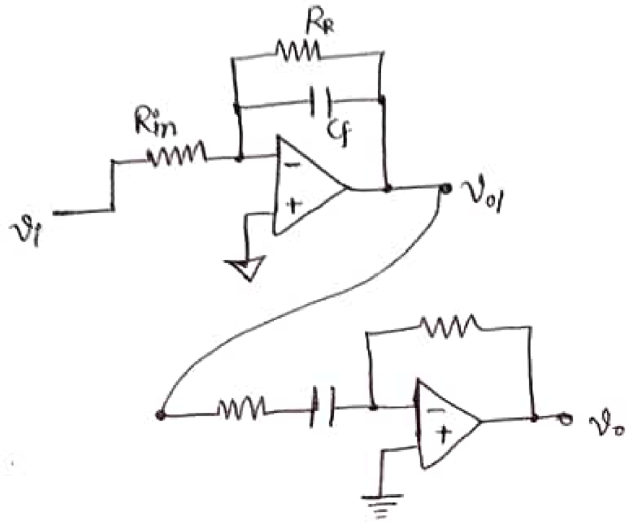


Sequence doesn't matter

$\omega_{UB} \rightarrow 150\,Hz \rightarrow 1.5\,kHz$
$\omega_{LB} \rightarrow 50\,Hz \rightarrow 5\,kHz$

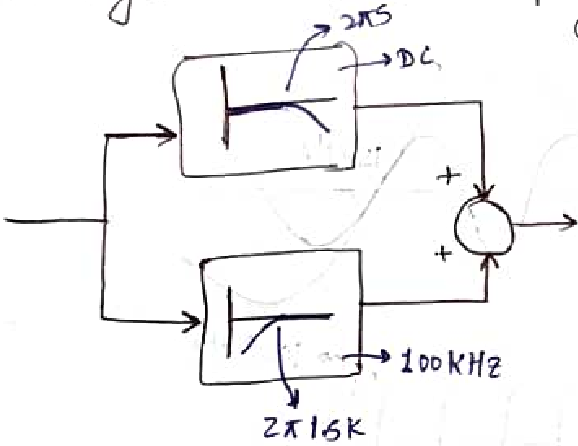


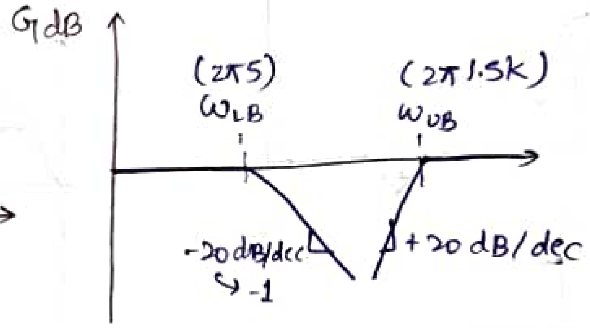
At high frequency → Output = 0
DC → Output = 0

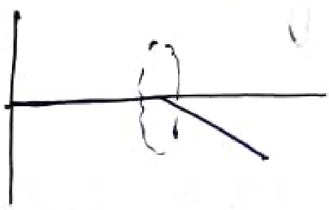


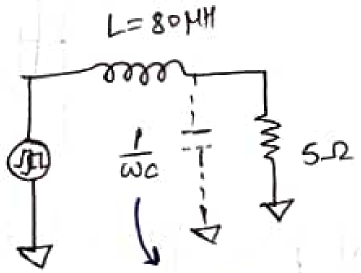
Eg. Design a band stop filter that attenuates 50 Hz and 150 Hz components in v_s by atleast 20dB and pass DC and 100 KHz.

Soln:

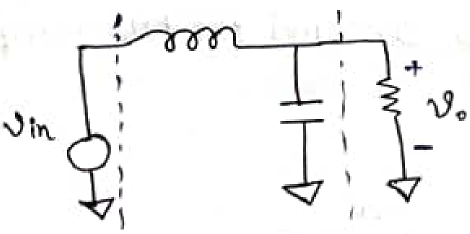








• 100 kHz and above to be attenuated by 20dB.



$$\frac{v_o(s)}{v_{in}(s)} = \frac{R \parallel \frac{1}{sC}}{R \parallel \frac{1}{sC} + sL}$$

$$= \frac{\frac{\frac{R}{sC}}{R + \frac{1}{sC}}}{\frac{\frac{R}{sC}}{R+\frac{1}{sC}} + sL}$$

$$= \frac{R/sC}{R/sC + sLR + L/C}$$

$$= \frac{R}{R + s^2LCR + sL}$$

$$= \frac{1}{s^2LC + sL/R + 1}$$

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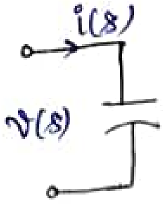
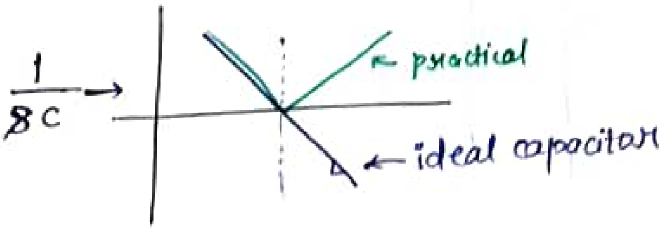
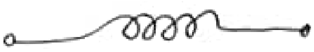
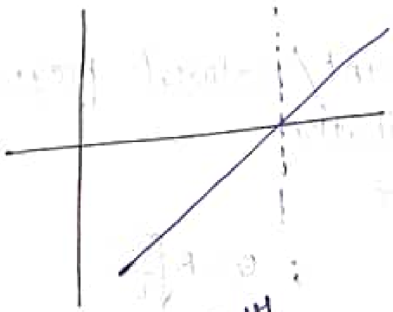
→ 

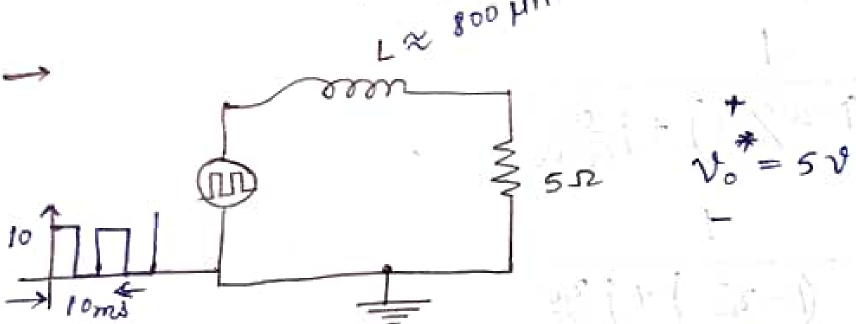


$$V(s) = \frac{1}{sC} I(s)$$

$$\Rightarrow \frac{I(s)}{V(s)} = sC$$

→ 

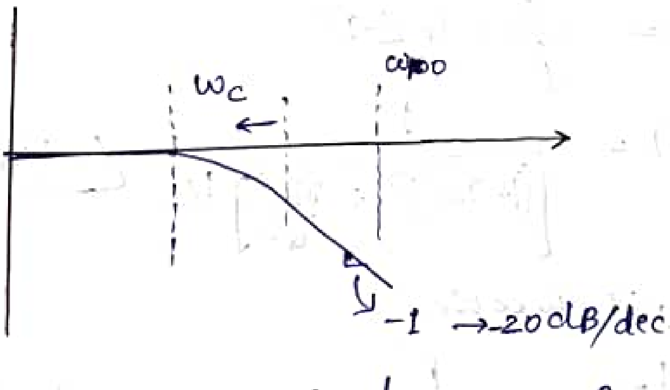


→ 

$V_o^* = 5V$

$G = \frac{1}{1+s\tau}$

Atleast 40 dB attenuation to frequency at 100 kHz and above.

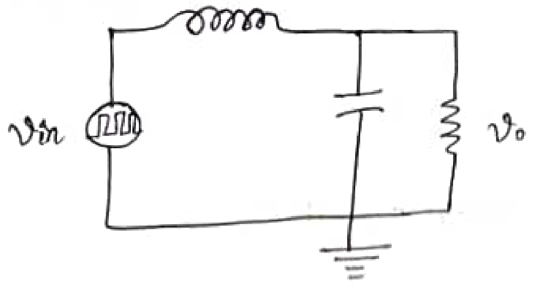


$$\omega_c \leq \frac{1}{100}\omega_{100} \Rightarrow \frac{R}{L} \leq \frac{1}{100} \times 2\pi \times 100K$$

$\frac{R}{L}$

$$\Rightarrow L \geq \frac{100 \times 5}{2\pi \times 100k} = 7.96 \times 10^{-4} H$$

$$\Rightarrow L \approx 800 \mu H.$$

→ 

$$\frac{v_o(s)}{v_{in}(s)} = \frac{1}{LCs^2 + s\frac{L}{R} + 1} = G_v(s)$$

Standardized form of representation :

$$G_v(s) = \frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}$$

where,

ω_c : corner/resonant/natural frequency of oscillation

Q : Quality factor

$\propto \frac{1}{\xi}$; $Q = R\sqrt{\frac{C}{L}}$

$$G_v(j\omega) = \frac{1}{(1-\omega^2/\omega_c^2) + j\frac{\omega}{Q\omega_c}}$$

$$= \frac{1}{(1-r_\omega^2) + j\frac{r_\omega}{Q}}$$

where, $\boxed{r_\omega \triangleq \omega/\omega_c}$.

$$G_v\Big|_{r_\omega} = \frac{Q}{j} = Q\angle{-90^\circ}$$

$$G_v(j\omega) = \frac{1}{\left[(1-r_\omega^2)^2 + \frac{r_\omega^2}{Q^2}\right]^{1/2}} \angle -\tan^{-1}\frac{r_\omega}{Q(1-r_\omega^2)}$$

for $r_\omega \ll 1$, i.e., $\omega \ll \omega_c$,

$$|G_v| = \frac{1}{\left[(1-r_\omega^2)^2 + \frac{r_\omega^2}{Q^2}\right]^{1/2}} \approx 1 \Rightarrow G_{v\,dB} = 0$$

$$\angle G = \angle -\tan^{-1}\frac{r_\omega}{Q(1-r_\omega^2)} \approx \angle \tan^{-1}\frac{r_\omega}{Q} \approx 0^\circ.$$

for $r_\omega \gg 1$, i.e., $\omega \gg \omega_c$,

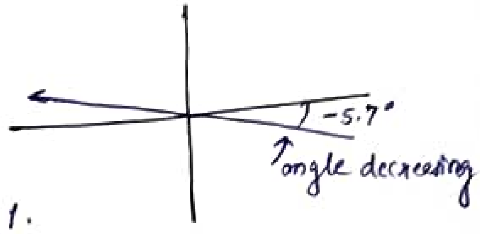
$$|G_v| = \frac{1}{\left[(1-r_\omega^2)^2 + r_\omega^2/Q^2\right]^{1/2}} \approx \frac{1}{r_\omega^2} = \frac{\omega_c^2}{\omega^2}$$

$$\angle G = \angle \tan^{-1} \frac{r_\omega}{Q(1-r_\omega^2)} \approx \angle \tan^{-1}\left(-\frac{1}{Q r_\omega}\right)$$

Suppose $Q = 1$, $r_\omega = 10$

$\Rightarrow \angle G = 5.7$

$r_\omega = 100 \Rightarrow \angle G = 0.57^\circ$

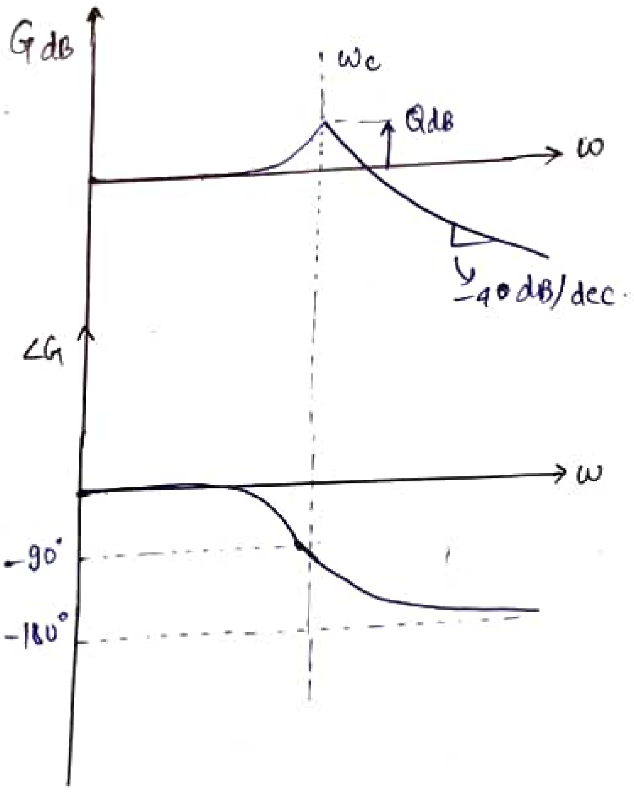


$\angle G$ approaches -180° as $r_\omega \gg 1$.

$$G_v\Big|_{r_\omega=10} = \frac{1}{100} \rightarrow 1\text{dec above } \omega_c : G_{v_{dB}} = -40\,dB$$

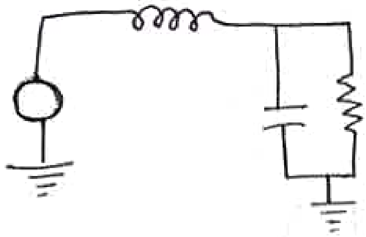
$$G_v\Big|_{r_\omega=100} = \frac{1}{10^4} \rightarrow G_{v_{dB}} = -80\,dB$$

$$G(s) = \frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}$$



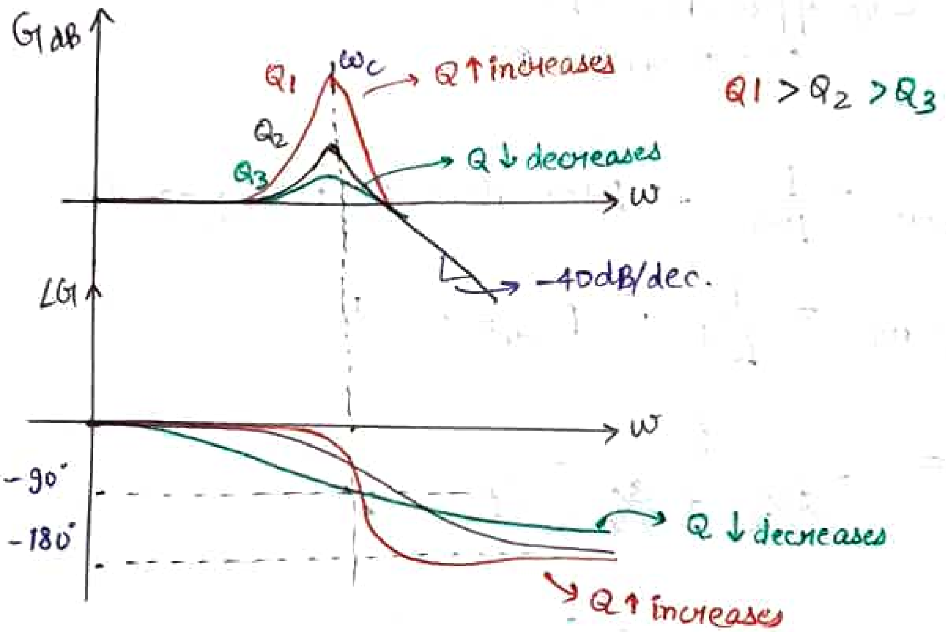
Review:

20-11-2023

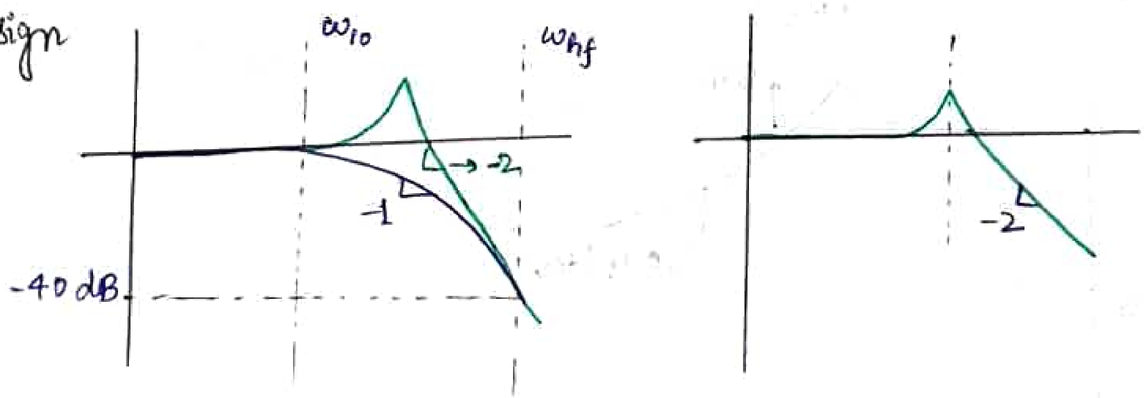


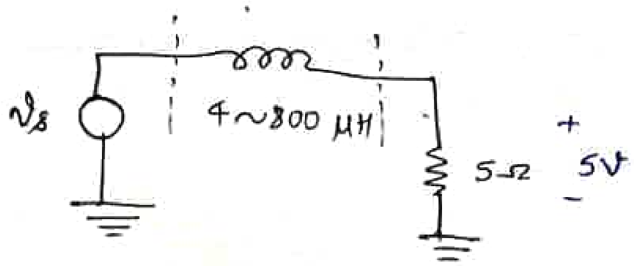
$$G(s) = \frac{1}{LCs^2 + sL/R + 1}$$

$$= \frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}$$



Eg. Design





40 dB attenuation to 100 KHz.

At $\omega \gg \omega_c$,

$$|G|_{\omega \gg \omega_c} \approx \frac{\omega_c^2}{\omega^2}$$

$$20 \log \frac{\omega_c^2}{\omega^2} \leq -40\,dB$$

$$\Rightarrow \omega_c \leq 0.1\,\omega_{100}$$

$$\Rightarrow \frac{1}{\sqrt{LC}} \leq \frac{1}{10} \times 2\pi \times 10^{5}$$

$$\Rightarrow LC \geq \left(\frac{1}{2\pi \times 10^4}\right)^2$$

$$\Rightarrow LC \geq \frac{1}{36 \times 10^8}$$

Let's choose $C = 10\,\mu F$ (63 V)

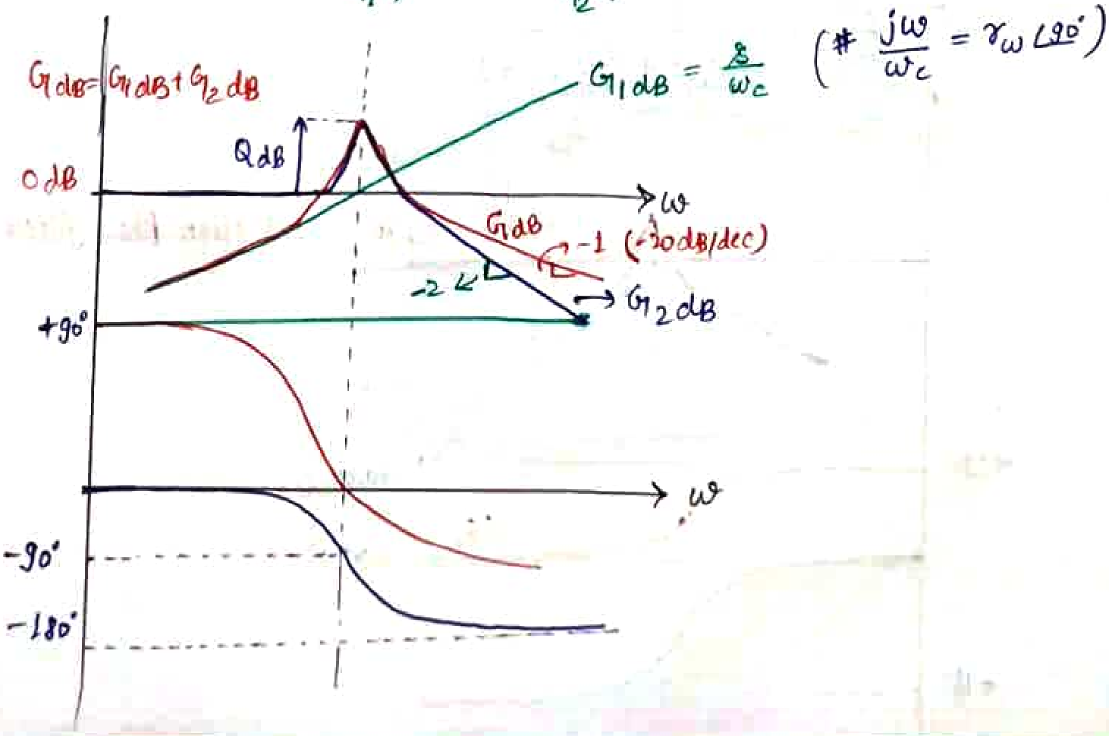
then, $L \geq \frac{10^6}{10} \times \frac{1}{36 \times 10^8}$

$$\Rightarrow L \geq 27\,\mu H$$

Adding a zero at the origin to the transfer function,

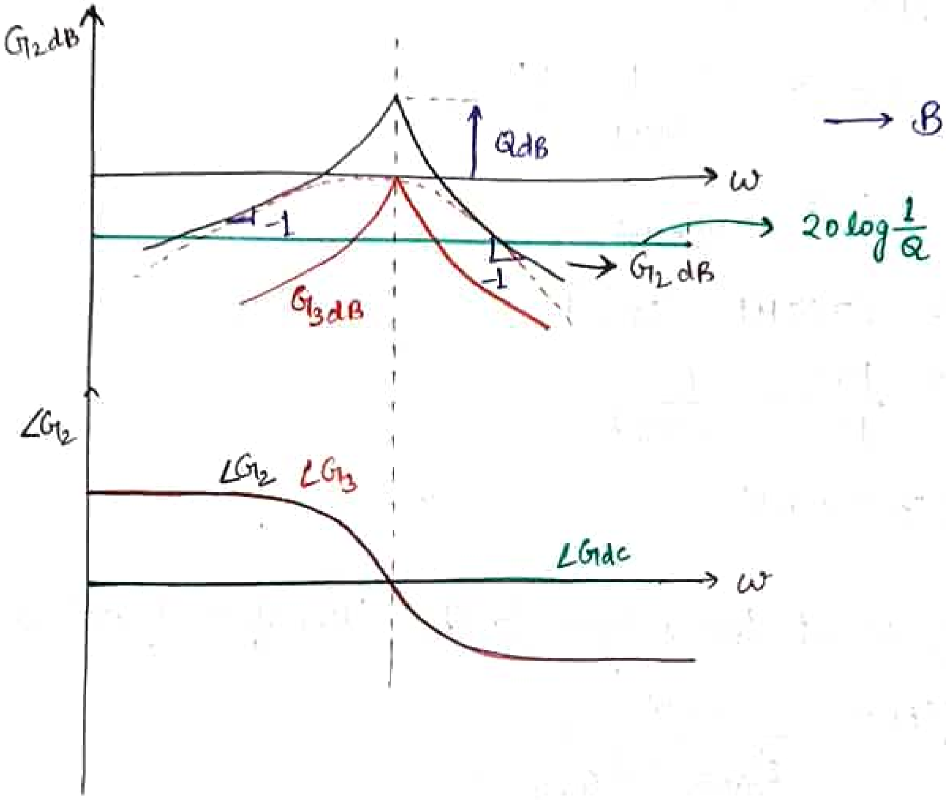
$$G(s) = \frac{s/\omega_c}{s^2/\omega_c^2 + s/Q\omega_c + 1}$$

$$= \underbrace{\frac{s}{\omega_c}}_{G_1(s)} \cdot \underbrace{\frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}}_{G_2(s)}$$



$$G_3(s) = \frac{s/Q\omega_c}{s^2/\omega_c^2 + s/Q\omega_c + 1} \quad \text{(Introducing } 1/Q \text{ factor)}$$

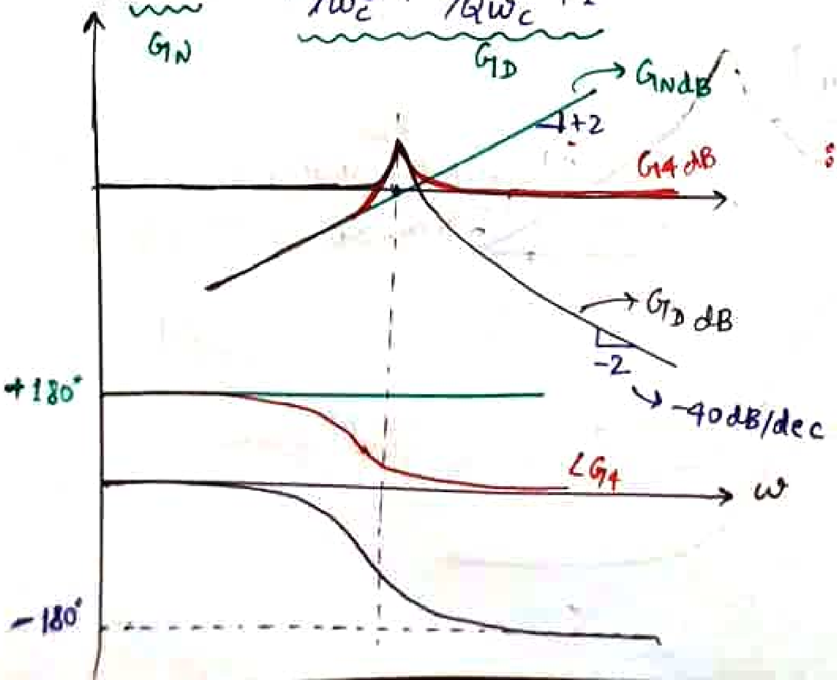
$$= \underbrace{\frac{1}{Q}}_{G_{dc}} \; \underbrace{\frac{s}{\omega_c} \; \frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}}_{G_2}$$



→ Band Pass Filter

$$G_4(s) = \frac{s^2/\omega_c^2}{s^2/\omega_c^2 + s/Q\omega_c + 1}$$

$$= \underbrace{\frac{s^2}{\omega_c^2}}_{G_N} \cdot \underbrace{\frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}}_{G_D}$$



: High Pass Filter

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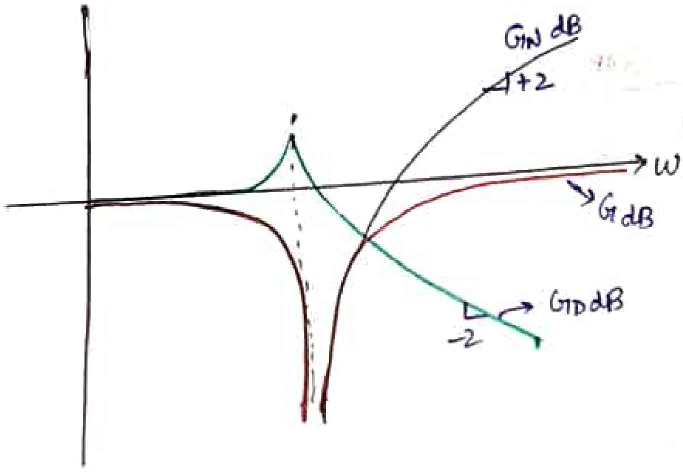
Eg. $G(s) = \dfrac{s^2/\omega_c^2 + 1}{s^2/\omega_c^2 + s/Q\omega_c + 1}$

$G_N(s) = s^2/\omega_c^2 + 1$

$G_N(j\omega) = 1 - \omega^2/\omega_c^2$

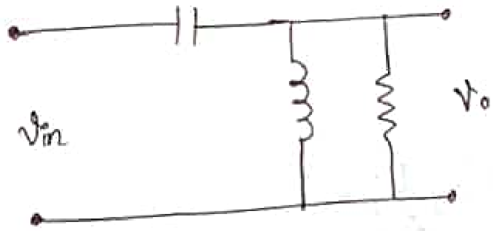
$= 1 - \gamma_\omega^2$

$G_N(j\omega)\big|_{\omega=\omega_c} = 0$



: Noise Filter

High Pass

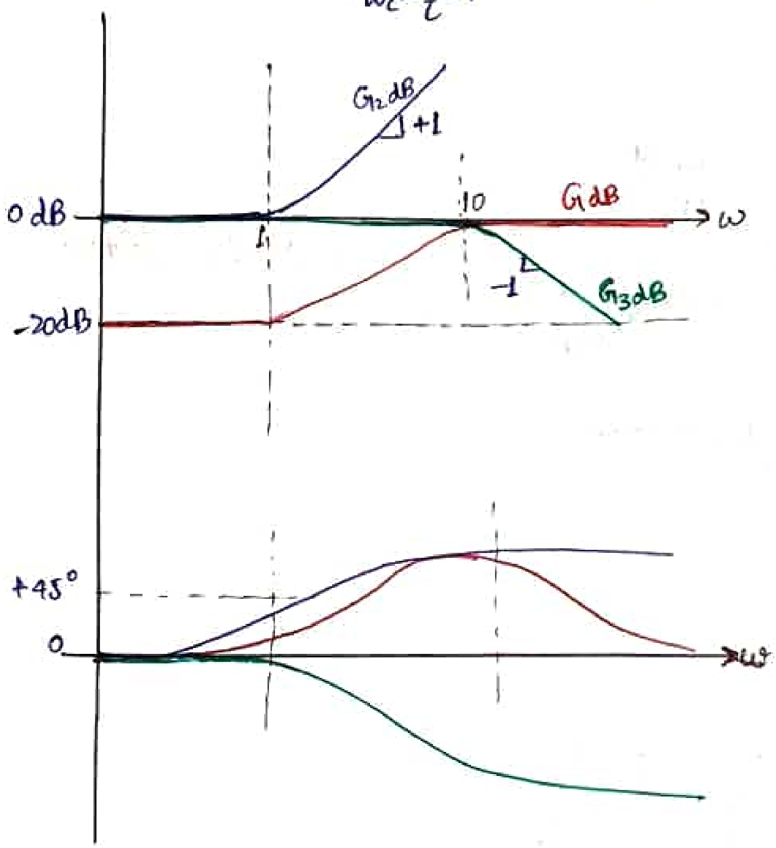


Eg. Drill Prob set 2 ① ⓑ

$$\frac{s+1}{s+10} = \frac{1}{10}\,\frac{s+1}{0.1s+1} \qquad \left(\frac{1+\tau_1 s}{1+\tau_2 s}\text{ form}\right)$$

$$= \underbrace{\frac{1}{10}}_{G_1}\cdot \underbrace{(s+1)}_{G_2}\cdot \underbrace{\frac{1}{(0.1s+1)}}_{G_3}$$

G_2: $\omega_c = \frac{1}{\tau} = 1$ $\qquad$ G_3: $\omega_c = \frac{1}{\tau} = 10$



Eg. D.P. ① ⓕ set 2

$$\frac{s^2+s+1}{s} = \underbrace{(s^2+s+1)}_{G_N}\cdot\underbrace{\left(\frac{1}{s}\right)}_{G_D}$$

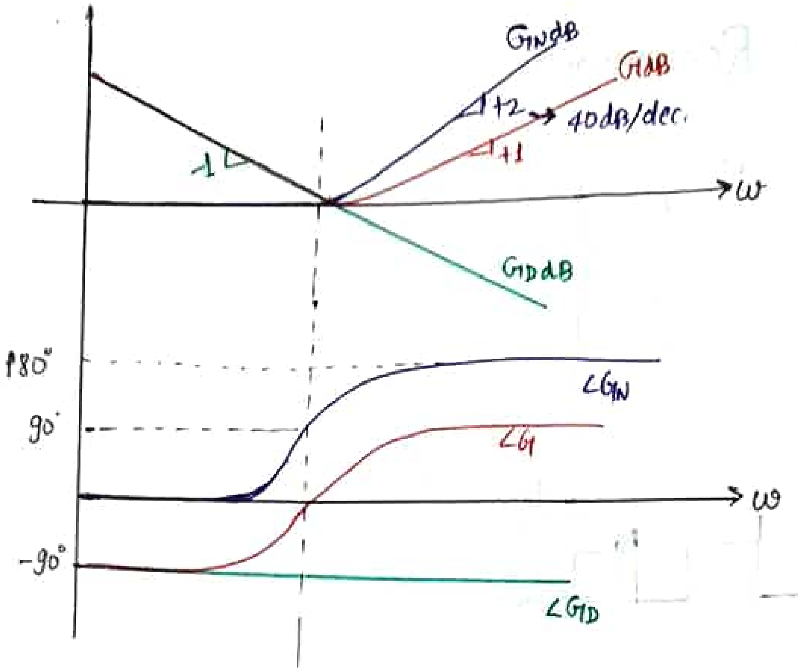
$$\left(s^2/\omega_c^2 + s/Q\omega_c + 1\right)$$

$G_N(j\omega) = (1-\omega^2) + j\omega$

For $\omega = 1$, $G_N(j\omega) = 1 \angle +90°$

$\omega \gg 1$, $|G_N(j\omega)| \approx \omega^2$

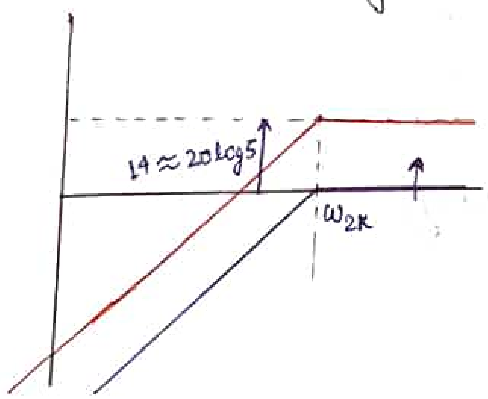
$\omega \ll 1$, $|G_N(j\omega)| = 1$.

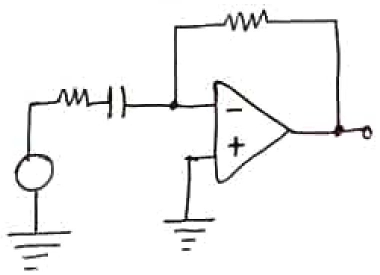


Q Design ~~a passive and~~ an active 1st order high pass filter which passes components ≥ 2KHz with a gain of 5.

D.P. set-2 ④

Soln:





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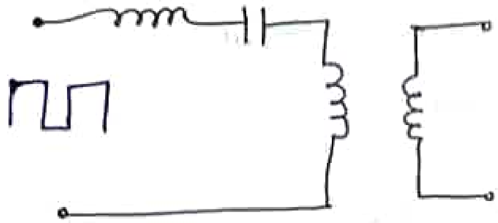
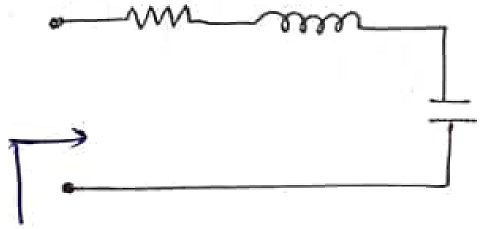
Brief Note on Resonance



Series RLC



Z_{in}

At $\omega = \omega_c$ (resonance frequency)

$$Z_{in} = R$$

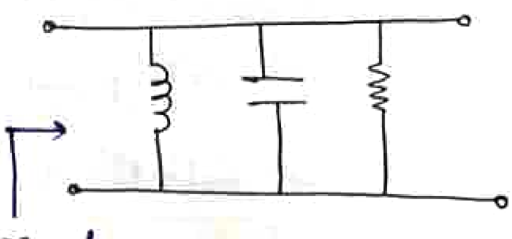
$$\Rightarrow \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \omega_c = \frac{1}{\sqrt{LC}}$$

$$\xi \propto R$$

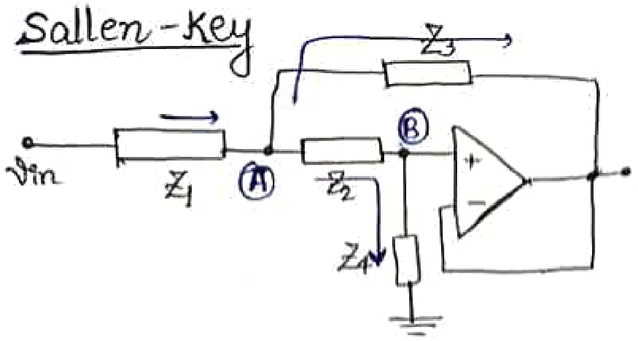
$$Q = \frac{\omega_c L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\frac{s^2}{\omega_c^2} + \frac{s}{Q\omega_c} + 1$$



$$\xi \propto \frac{1}{R}$$

Sallen-Key



$$\frac{v_{in}-v_A}{Z_1} = \frac{v_A-v_o}{Z_2} + \frac{v_A-v_o}{Z_3}$$

and, $v_o = \frac{Z_4}{Z_2+Z_4} v_A \Rightarrow v_A = \frac{Z_2+Z_4}{Z_4} v_o$

$$\Rightarrow \frac{v_{in}}{Z_1} - \frac{Z_2+Z_4}{Z_1Z_4}v_o = \frac{Z_2+Z_4}{Z_2Z_4}v_o - \frac{v_o}{Z_2} + \frac{Z_2+Z_4}{Z_3Z_4}v_o - \frac{v_o}{Z_3}$$

$$\Rightarrow \frac{1}{Z_1} - \frac{Z_2+Z_4}{Z_1Z_4}G(s) = \frac{Z_2+Z_4}{Z_2Z_4}G(s) - \frac{G(s)}{Z_2} + \frac{Z_2+Z_4}{Z_3Z_4}G(s) - \frac{G(s)}{Z_3}$$

$$\Rightarrow \cancel{\frac{Z_4+(Z_2+Z_4)G(s)}{Z_1Z_4}} \quad \cancel{Z_2^2Z_3 + Z_2Z_3Z_4} \quad \frac{Z_1Z_2Z_3 + Z_1\cancel{Z_3}Z_4 - Z_1\cancel{Z_3}Z_4 + Z_1Z_2^2 + Z_1\cancel{Z_2}Z_4 - Z_1\cancel{Z_2}Z_4}{\cancel{Z_1}Z_2Z_3Z_4}(G(s)) = \frac{1}{\cancel{Z_1}}$$

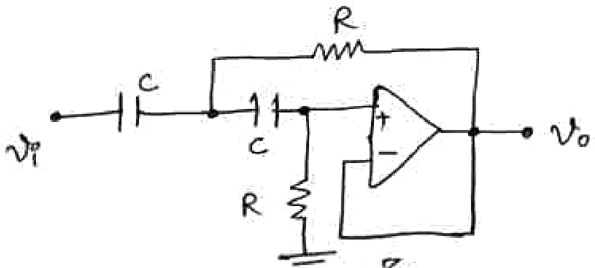
$$\Rightarrow \frac{Z_2Z_3+Z_3Z_4+Z_1Z_3+Z_1Z_2}{Z_3Z_4}G(s) = 1$$

$$\Rightarrow \boxed{G(s) = \frac{Z_3Z_4}{Z_1Z_2+Z_2Z_3+Z_1Z_3+Z_3Z_4}}$$

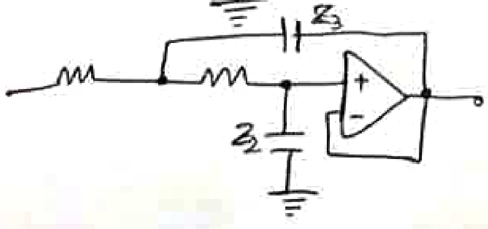
$$\frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}$$

$Z_3Z_4 \rightarrow R$ (C)

$Z_1, Z_2 \rightarrow C$ (R)



: High Pass



: Loss Pass

$$\frac{Z_4 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_3 Z_2 + Z_3 Z_4}$$

$$= \frac{\frac{1}{sC_4} \cdot \frac{1}{sC_3}}{R_1 R_2 + \frac{1}{sC_3}(R_1 + R_2) + \frac{1}{s^2 C_3 C_4}}$$

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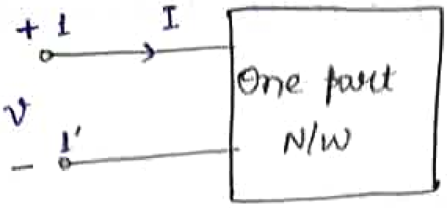
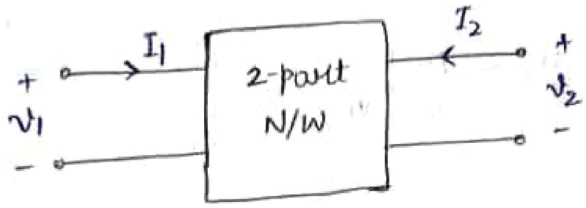
Two Port Parameter





Z-parameters : (Impedance Parameters)

$$v_1 = Z_{11} I_1 + Z_{12} I_2$$
$$v_2 = Z_{21} I_1 + Z_{22} I_2$$

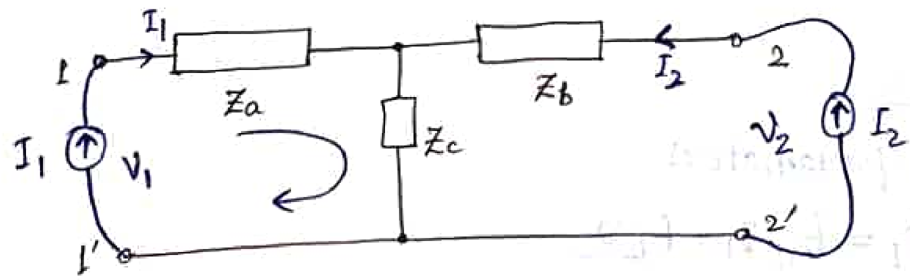
I_1, I_2 : independent

v_1, v_2 : dependent

$$Z_{11} = \frac{v_1}{I_1}\bigg|_{I_2=0}, \quad Z_{12} = \frac{v_1}{I_2}\bigg|_{I_1=0}$$

$$Z_{21} = \frac{v_2}{I_1}\bigg|_{I_2=0}, \quad Z_{22} = \frac{v_2}{I_2}\bigg|_{I_1=0}$$

Eg.



$$Z_{11} = \frac{v_1}{I_1}\bigg|_{I_2=0} = Z_a + Z_c$$

$$Z_{21} = \frac{v_2}{I_1}\bigg|_{I_2=0} = Z_c$$

$$Z_{22} = \frac{v_2}{I_2}\bigg|_{I_1=0} = Z_b + Z_c$$

$$Z_{12} = \frac{v_1}{I_2}\bigg|_{I_1=0} = Z_c$$

Y-Parameters : (Admittance Parameters)

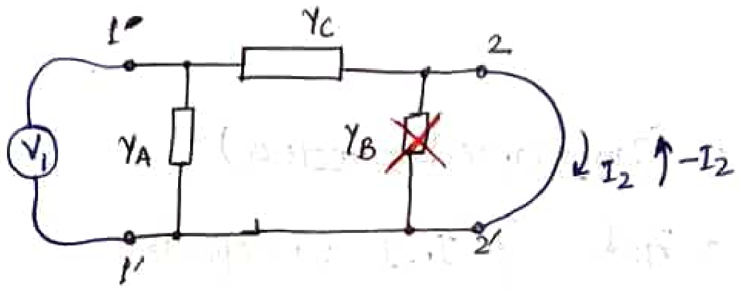
$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = v_{21} V_1 + y_{22} V_2$$

V_1, V_2 : independent variables

$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}, \quad y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$
$$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}, \quad y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

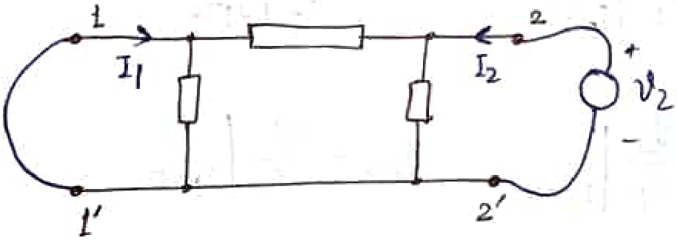
Short-circuit parameters

Eg.



y_{11} : $V_1(Y_A + Y_C) = I_1 \Rightarrow y_{11} = y_A + y_C$

y_{21} : $V_1 Y_C = -I_2 \Rightarrow y_{21} = -Y_C$



h-Parameters

↳ Hybrid-parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

ABCD : Transmission line parameter

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$