Tutorial-1

AV222- Instrumentation and Measurement

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SC22B146

① Determine the input and output range, input and output span and sensitivity of the pressure transducer shown in Fig. 1.

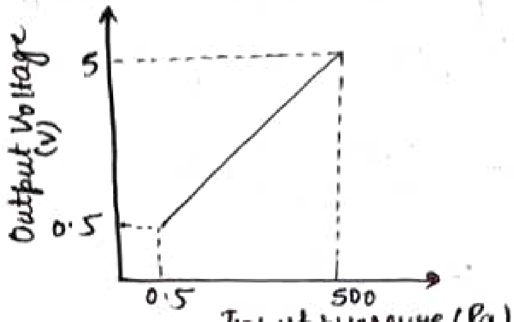


Fig. 1

Soln: Input range = 0.5 — 500 Pa

Output range = 0.5 — 5 V

Input Span = 500 − 0.5 = 499.5 Pa

Output span = 5 − 0.5 = 4.5 V

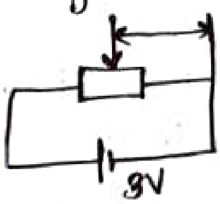
$$\text{Sensitivity} = \frac{5000 - 500}{500 - 0.5}\ \text{mV/Pa}$$

$$= \frac{4500}{499.5}\ \text{mV/Pa}$$

$$\approx 9\ \text{mV/Pa}.$$

② A potentiometric arrangement shown in Fig. 2 is used to sense displacement (X) of range 30 cm. A wire-wound potentiometer of 300 turns is used. Sensitivity of the circuit is 10 mV/mm. Determine the input and output resolution (assume that the wiper can touch only 1 turn at a time) of the sensor.

~~Soln~~



Soln: Range, $x_F = 30$ cm.

Windings, $N = 300$

Sensitivity, $S = 10$ mV/mm.

Input resolution, $q = \frac{x_F}{N-1} \approx \frac{x_F}{N} = \frac{30\ \text{cm}}{300} = 0.1$ cm.

Output resolution $= Sq = 10 \frac{mV}{mm} \times 0.1\,cm$
$= 1\,cV$
$= 0.01\,V$

(3) A voltmeter with a range of 2—10 V reads a voltage 5.2 V. True value of this voltage is 5.4 V. Determine the % error in terms of reading and full-scale span.

Soln: % error (reading) $= \frac{5.2 - 5.4}{5.4} \times 100$
$\approx -3.7\,\%$

% error (span) $= \frac{5.2 - 5.4}{(10-2)} \times 100$
$= -2.5\,\%$

(4) A wire-bound potentiometer of (N=) 101 turns is used to realise a linear displacement sensor of range 0 to 5 mm. The wiper of this potentiometer can touch at most 2 turns at any position. This arrangement is excited by a +5 V DC source. Derive and compute the values of minor and major output resolution pulses as the wiper transits from 50^{th} to 51^{st} turn (give proper reasoning).

Soln: When wiper touches the 50^{th} turn, output voltage $= \frac{5}{101-1} \times (50-1)$
$= 2.45\,V$

When wiper touches the 51^{st} turn, output voltage $= \frac{5}{101-1} (51-1)$
$= 2.5\,V$

When wiper touches both 51^{st} and 50^{th} turn, they get short-circuited and the effective turns reduces to 100.

$\therefore$ Output voltage $= \frac{5}{100-1} \times (50-1) = 2.4747\,V$

Minor output resolution pulse $= 2.4747 - 2.45 = 24.7\,mV$.

Major output resolution pulse $= 2.5 - 2.4747 = 25.3\,mV$.

⑤ A current of 2.5A is to be measured. which one of the following ammeters you would prefer?

ⓐ 0–5A, class-1, ⓑ 0–3A, class 2.

Soln: For class-1 instrument,

% error (span) = 1%

$$\Rightarrow \frac{MV - 2.5}{5} \times 100 = 1\%$$

$$\Rightarrow MV = 2.5 \pm 0.05 \text{ V}$$

For class-2 instrument,

% error (span) = 5%

$$\Rightarrow \frac{MV - 2.5}{3} \times 100 = 5\%$$

$$\Rightarrow MV = 2.5 + \frac{15}{100}$$

$$= 2.5 + 0.15 \text{ V}$$

As measured value (MV) is near to 2.5A more for class-1 instrument, ⓐ is better.

⑥ Voltage (VI), current (I) and resistance (R) associated with a circuit element are measured with limiting error of 1%. Calculate which of the expressions (I^2R, VI, V^2/R) is best for the calculation of power.

Soln: Power, $P = I^2R$

$$L_P = \left|\frac{\partial I^2R}{\partial I}\right| L_I + \left|\frac{\partial I^2R}{\partial R}\right| L_R$$

$$= (2IR)\,1\% + (I^2)\,1\%$$

Power, $P = VI$

$$L_P = (I)\,\% + (V)\,1\%$$

Power, $P = V^2/R$

$$L_P = \left(\frac{2V}{R}\right) 1\% + \left|-\frac{V^2}{R^2}\right| 1\%$$

As L_P will be smallest for $P = VI$, it is the best method for the calculation of power.

⑦ In Fig.3, Ammeter-A (0-10mA, 100Ω) and voltmeter (0-10V, 100kΩ) is used to measure the value of an unknown resistance R_x. Find the position at which switch needs to be placed to get minimal error when ⓐ $R_x < 100\,\Omega$, ⓑ $R_x > 100\,k\Omega$. Justify your answers.

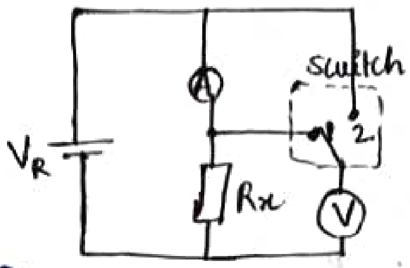


Fig.3

Soln: At position ①,

$$MV = R_x \parallel R_V$$

$$\%\ \text{Error} = \frac{\frac{R_x R_V}{R_x + R_V} - R_x}{R_x} \times 100\%$$

$$= \frac{\cancel{R_x R_V} - R_x^2 - \cancel{R_x R_V}}{(R_x + R_V)\,\cancel{R_x}} \times 100\%$$

$$= \frac{-R_x}{R_x + R_V} \times 100\%$$

If $R_x \ll R_V$, error will be less.

∴ It is suitable for ⓐ $R_x < 100\,\Omega$.

At position ②,

$$MV = R_x + R_A$$

$$\%\ \text{Error} = \frac{\cancel{R_x} + R_A - \cancel{R_x}}{R_x} \times 100\%$$

$$= \frac{R_A}{R_x} \times 100\%$$

∴ It is suitable for ⓑ $R_x > 100\,k\Omega$.

⑧ The following values were obtained from the measurement of the value of a resistor: 147.2 Ω, 147.4 Ω, 147.9 Ω, 148.1 Ω, 147.1 Ω, 147.5 Ω, 147.6 Ω, 147.4 Ω, 147.6 Ω, 147.5 Ω.

Calculate ⓐ arithmetic mean, ⓑ standard deviation and ⓒ probable error of the above measurements.

Soln: (a) $AM = \frac{147.2 + 147.4 + 147.9 + 148.1 + 147.1 + 147.5 + 147.6 + 147.4 + 147.6 + 147.5}{10}$

$= 147.53\ \Omega$

$\approx 147.5\ \Omega$

(b) $\sigma^2 = \frac{1}{9}\left[0.3^2 + 0.1^2 + 0.4^2 + 0.6^2 + 0.4^2 + 0^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0^2\right]$

$= \frac{0.81}{9} = 0.09$

$\Rightarrow \sigma = 0.3\ \Omega$

(c) $n(\bar{x} \pm 0.1) = 6 \geq \frac{10}{2}\ (=5)$

$\therefore P = 0.1$, as atleast 50% of the readings lies b/w $(\bar{x} - P, \bar{x} + P)$.

(9) The value of two resistors was found using a standard measurement technique and repeated measurements. It was found that (1) the mean value of the resistors is 45Ω and 48Ω, (2) the maxm deviation of the resistors from their mean values is 3Ω. Calculate the effective resistance and its limiting error when the above resistors are connected in parallel.

Soln: $R_1 = 45\Omega$, $R_2 = 48\Omega$

$R_{eff.} = \frac{R_1 R_2}{R_1 + R_2} = \frac{45 \times 48}{45 + 48} \approx 23.2\ \Omega$

$L_{R_{eff}} = \left|\frac{\partial R}{\partial R_1}\right| L_{R_1} + \left|\frac{\partial R}{\partial R_2}\right| L_{R_2}$

$= \left[\frac{(\cancel{R_1} + R_2)R_2 - \cancel{R_1 R_2}}{(R_1 + R_2)^2}\right] L_{R_1} + \left[\frac{(R_1 + \cancel{R_2})R_1 - \cancel{R_1 R_2}}{(R_1 + R_2)^2}\right] L_{R_2}$

$= \frac{R_2^2 L_{R_1} + R_1^2 L_{R_2}}{(R_1 + R_2)^2}$

$= \frac{48^2 + 45^2}{(48+45)^2} \times 3 \approx 1.5\ \Omega$.

(10) Few 10 Ω resistors of standard deviation 10 % are available. It is required to realize a 30 Ω resistor using these 10 Ω resistors. Suggest the best connection from the different circuit connections shown in fig. 4 (a) to (d).

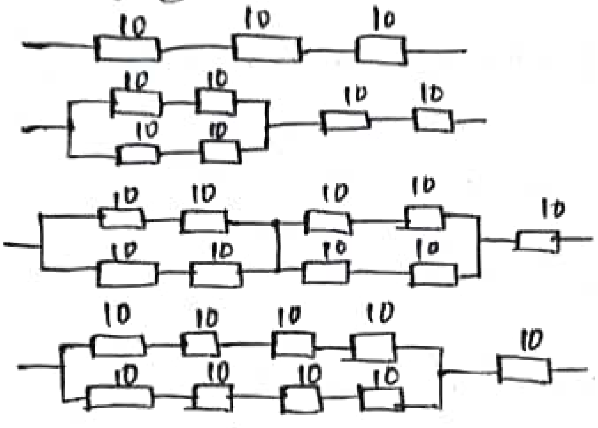


Fig. 4 (a) to (d).

Soln:

(a) → $R_{eff} = 3R$, $R = 10\Omega$.

(b) → $R_{eff} = \frac{2R}{2} + 2R = 3R$

(c) → $R_{eff} = \frac{4R}{2}$

(a) : $R_{eff} = R_1 + R_2 + R_3$ | $\sigma_{R_i} = \frac{10 \times 10}{100} = 1\Omega$

$$\frac{\partial R_{eff}}{\partial R_1} = \frac{\partial R_{eff}}{\partial R_2} = \frac{\partial R_{eff}}{\partial R_3} = 1.$$

$$\therefore \sigma_{R_{eff}}^2 = \sum \sigma_{R_i}^2$$

$$\Rightarrow \sigma_{R_{eff}} = \sqrt{3}\ \Omega = 1.73\ \Omega.$$

(b) : $R_{eff} = \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} + R_5 + R_6$

$$\frac{\partial R_{eff}}{\partial R_1} = \frac{\sum R_i (R_3 + R_4) - (R_1 + R_2)(R_3 + R_4)}{(\sum R_i)^2}$$

$$= \frac{40(20) - 20(20)}{(40)^2}$$

$$= \frac{8 - 4}{16} = \frac{\partial R_{eff}}{\partial R_2} = \frac{\partial R_{eff}}{\partial R_3} = \frac{\partial R_{eff}}{\partial R_4} = \frac{1}{4}.$$

$$\frac{\partial R_{eff}}{\partial R_5} = 1 = \frac{\partial R_{eff}}{\partial R_5}.$$

$$\therefore \sigma_{R_{eff}} = 4 \times \left(\frac{1}{4}\right)^2 \times 1^2 + 2 \times 1^2 \times 1^2$$

$$= \frac{1}{4} + 2 = 2.25\ \Omega.$$

(c): $R_{eff} = \frac{(R_1+R_2+R_3+R_4)(R_5+R_6+R_7+R_8)}{(\Sigma R_1)^2} + R_9$

$$\frac{\partial R_{eff}}{\partial R_1} = \frac{\Sigma R_1 (\sum_5^8 R_i) - 1.(\sum_1^4 R_i)(\sum_5^8 R_i)}{(\Sigma R)^2} \cancel{+R_9}$$

$$= \frac{80\Omega(40\Omega) - (40)40}{(80)^2} \cancel{+10}$$

$$= \frac{32-16}{64}$$

$$= 0.25 = \frac{\partial R_{eff}}{\partial R_2} = \dots = \frac{\partial R_{eff}}{\partial R_8}$$

$$\therefore \sigma_{R_{eff}}^2 = 8 \times (0.25)^2 \times 1^2 + 1^2 \times 1^2$$

$$\Rightarrow \sigma_{R_{eff}} = 1.5\Omega.$$

(d): $R_{eff} = \frac{\sum_1^4 R_i \sum_5^8 R_i}{\sum_1^8 R_i} + R_g$

$$\frac{\partial R_{eff}}{\partial R_1} = \dots = \frac{\partial R_{eff}}{\partial R_8} = \frac{\sum_1^8 R_i (\sum_5^8 R_i) - \sum_1^4 R_i \sum_5^8 R_i}{(\sum_1^8 R_i)^2}$$

$$= \frac{80(40) - (40)(40)}{(80)^2}$$

$$= \frac{1}{4}$$

$$\therefore \sigma_{R_{eff}} = 1.5\Omega$$

Since (c) and (d) has the lowest standard deviation, these are the best.

(11) Consider the circuit shown in figure (5). Mean and limiting error of the resistors and input voltage: $R_1 = 100 \pm 5\Omega$, $R_2 = 200 \pm 8\Omega$. Voltage $V_{in} = 1 \pm 0.1V$. Calculate the output and its maximum deviation (assume ideal opamp).

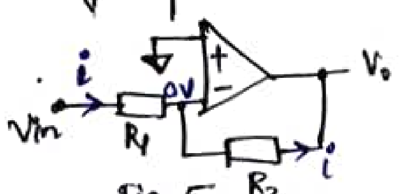


Fig. 5

Soln: $i = \frac{V_{in}}{R_1}$

$$V_0 = -i R_2$$
$$= -V_{in}\left(\frac{R_2}{R_1}\right)$$
$$= -1\left(\frac{200}{100}\right)$$
$$= \boxed{-2V.}$$

$$\sigma_{V_0}^2 = \left|\frac{\partial V_0}{\partial V_{in}}\right|^2 \sigma_{vin}^2 + \left(\frac{\partial V_0}{\partial R_2}\right)^2 \sigma_{R_2}^2 + \left(\frac{\partial V_0}{\partial R_1}\right)^2 \sigma_{R_1}^2$$
$$= \left(\frac{R_2}{R_1}\right)^2 \sigma_{vin}^2 + \left(\frac{V_{in}}{R_1}\right)^2 \sigma_{R_2}^2 + \left(V_{in}\frac{R_2}{R_1^2}\right)^2 \sigma_{R_1}^2$$
$$= (4)\times(0.1)^2 + (0.01)^2\times(8)^2 + \left(\frac{2}{100}\right)5$$
$$= (0.04) + (0.08)^2 + (0.1)^2$$
$$= 0.0564\ \Omega^2$$
$$\Rightarrow \sigma_{V_0} = 0.2375\ \Omega.$$

% ma

$$L_{V_0} = \left|\frac{\partial V_0}{\partial V_{in}}\right| L_{vin} + \left|\frac{\partial V_0}{\partial R_2}\right| L_{R_2} + \left|\frac{\partial V_0}{\partial R_1}\right| L_{R_1}$$
$$= \left(\frac{R_2}{R_1}\right)(L_{vin}) + \left(\frac{V_{in}}{R_1}\right) L_{R_2} + \left(V_{in}\frac{R_2}{R_1}\right) L_{R_1}$$
$$= 2(0.1) + (0.01)(8) + \left(\frac{10}{100}\right)$$
$$= 0.2 + 0.08 + 0.1$$
$$= 0.38$$

% maximum deviation $= \frac{0.38}{2} \times 100\%$

$$= 19\%$$

(12) A resistor manufacturer received a customer's order for 100,000 precision resistors of nominal resistance 10,000 Ω, which were not to exceed 10,025 and not to be less than 9,950 Ω. The manufacturer made a sample batch of 1000 resistors, and it was found that 80 resistors of this batch exceeded 10,025 Ω. Assuming gaussian distribution (a) predict the no. of remaining resistors which will confirm to the specifications, (b) predict the total no. of resistors that the manufacturer needs to make to obtain 100,000 resistors that specify customer's specification. Assume the sample batch of 1000 resistors is representative in your calculations.

Soln: (a) Given that,

$$p(x > 10{,}025) = 0.08$$

$$\Rightarrow p(10K < x < 10{,}025) = 0.5 - 0.08$$

$$\Rightarrow p(0 < y < 25) = 0.42$$

$$\Rightarrow p(0 < t < \tfrac{25}{\sigma}) = 0.42$$

From that table,

$$\frac{25}{\sigma} = 1.41 \Rightarrow \sigma = \frac{25}{1.41} = 17.73.$$

Now, $p(10K < x < 10{,}050)$

$$= p(0 < y < 50)$$

$$= p(0 < t < \tfrac{50}{17.73} \approx 2.82)$$

$$= 0.4976 \quad \text{(from the table)}$$

$\therefore$ $p(9950 < x < 10{,}025)$

$$= p(10K < x < 10K50) + [0.5 - p(x > 10K25)]$$

$$= 0.4976 + [0.5 - 0.08]$$

$$= 0.9176$$

$\therefore$ Total no. of resistors meeting specification ≈ 917.

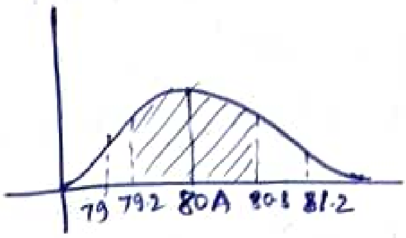
(b) Let manufacturer needs to make z resistors.

$$\therefore z - 0.083z = 10^5$$

$$\Rightarrow z = \frac{10^5}{1 - 0.083} = 1{,}09{,}051.$$

(13) A known current of 80 A was measured by an ammeter. If 40% of the readings are within 0.8 A of the true value, determine the probability that readings can lie b/w 79A and 81.2 A. Determine the deviation (say, ΔI) for which only 0.26 % lie outside $80 \pm \Delta I$.

Soln: Given that,



$p(79.2 < x < 80.8) = 0.4$

$\Rightarrow p(80 < x < 80.8) = 0.2$

$\Rightarrow p(0 < y < 0.8) = 0.2$

$\Rightarrow p\left(0 < t < \frac{0.8}{\sigma}\right) = 0.2$

$\Rightarrow \frac{0.8}{\sigma} \approx 0.52$ [from the table for $p = 0.1985$]

$\Rightarrow \sigma = \frac{0.8}{0.52}$

$= 1.54$

Now, $p(80 < x < 81.2) = p(0 < y < 1.2)$

$= p\left(0 < t < \frac{1.2}{1.54}\right)$

$= p(0 < t < 0.779)$

$= 0.2823$

and, ~~p(80<x<81)~~

$p(79 < x < 80) = p(80 < x < 81)$

$= p(0 < y < 1)$

$= p\left(0 < t < \frac{1}{1.54}\right)$

$= p(0 < t < 0.65)$

$= 0.2422$

$\therefore p(79 < x < 81.2) = 0.2823 + 0.2422$

$= \boxed{0.5245}$

$p(80 - \Delta I < x < 80 + \Delta I) = 1 - \frac{0.26}{100} = 0.9974$

$\Rightarrow p(80 < x < 80 + \Delta I) = 0.9974/2$

$\Rightarrow p(0 < y < \Delta I) = 0.4987$

$\Rightarrow p\left(0 < t < \frac{\Delta I}{\sigma}\right) = 0.4987 \Rightarrow \frac{\Delta I}{\sigma} = 3.01 \Rightarrow \boxed{\Delta I = 4.6354}$

[From the table]

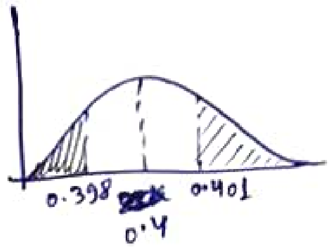
(14) A customer placed an order for 25000 pipes with below specifications to a ~~vendor~~ vendor.

(a) Nominal diameter, $d = 0.4000\,m$.

(b) Error spec : $0.398\,m < d < 0.401\,m$.

After manufacturing the 25000 pipes, according to the above specifications, the vendor did a quality check and found that 2000 pipes were having diameter > 0.401 m. Assuming gaussian distribution, predict the no. of remaining 23000 pipes that will be within customer specifications.

Soln: Given that,



$$p(x > 0.401) = \frac{2000}{25000} \times 100 = 8\% = 0.08$$

$$\Rightarrow p(0.4 < x < 0.401) = 0.5 - 0.08$$

$$\Rightarrow p(0 < y < 0.01) = 0.42$$

$$\Rightarrow p\left(0 < t < \frac{0.01}{\sigma}\right) = 0.42$$

$$\Rightarrow \frac{0.01}{\sigma} = 1.41 \quad \text{(from the table)}$$

$$\Rightarrow \sigma = \frac{0.01}{1.41} = 0.00709$$

Now, $p(0 < x < 0.398) = 0.5 - p(0.398 < x < 0.4)$

$$= 0.5 - p(0.4 < x < 0.402)$$

$$= 0.5 - p(0 < y < 0.02)$$

$$= 0.5 - p\left(0 < t < \frac{0.02}{0.00709}\right)$$

$$= 0.5 - p(0 < t < 2.82)$$

$$= 0.5 - 0.49076$$

$$= 0.0024$$

$$\therefore p(0.398 < x < 0.401) = 1 - (0.08 + 0.0024)$$

$$= 0.9176$$

$$\Rightarrow n(0.398 < x < 0.401) = 0.9176 \times 25000$$

$$= \boxed{22,940}.$$