नियन्त्रण प्रणाली

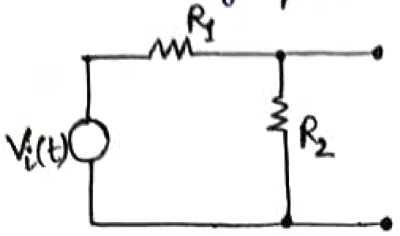
CONTROL SYSTEM

SYSTEM

12-01-2024

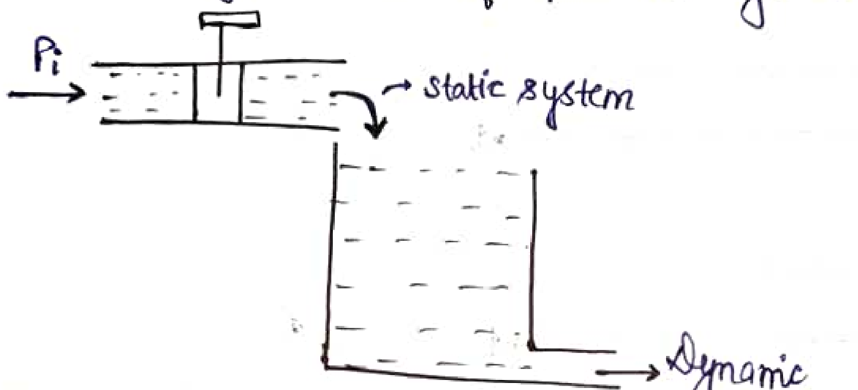
Static System: Present value of output depends only on the present value of input.

eg.



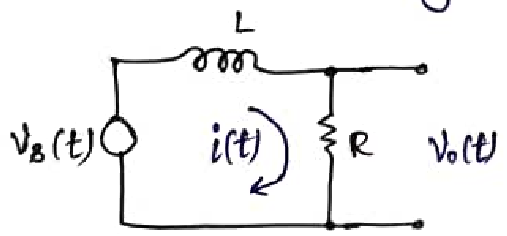
$$V_o(t) = \frac{R_2}{R_1+R_2} V_i(t)$$

Dynamic System: Present value of output depends on the present value as well as the past value of input through storage mechanism.



→ we have to take care of the stability of the system besides the proper output from proper input.

Robustness of the system: System should work fine with the change of parameters by external disturbances.



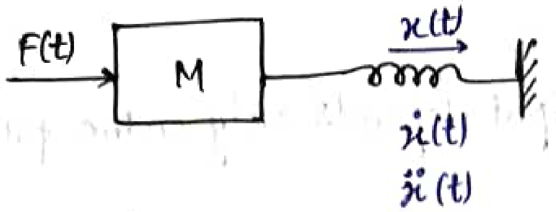
$$V_s(t) = L\frac{di}{dt} + Ri$$

$$V_o(t) = R \cdot i(t)$$

$$L\frac{di}{dt} = V_s(t) - Ri(t)$$

$$\frac{di}{dt} = \frac{V_s}{L}(t) - \frac{R}{L} i(t)$$

Storage of energy : Magnetic energy $= \frac{1}{2}Li^2$

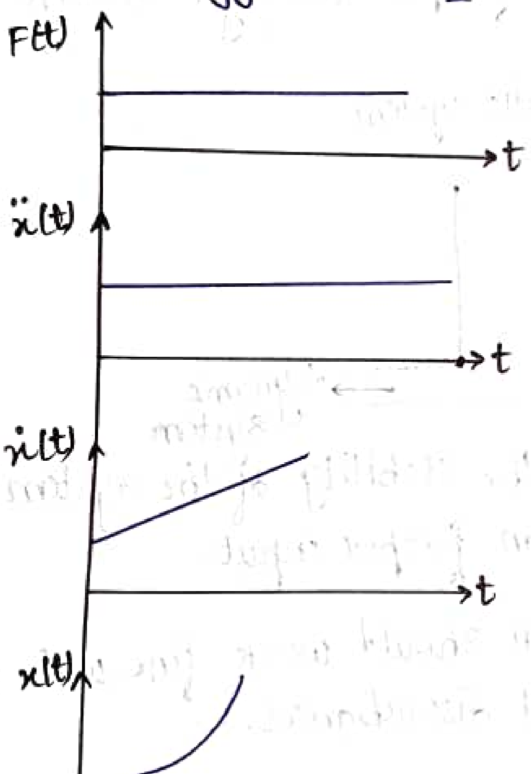


$$\ddot{x}(t) = \frac{1}{M} F(t)$$

$$\dot{x}(t) = \dot{x}(0) + \int_0^t \frac{F(t)}{M}\, dt$$

$$x(t) = x(0) + \int_0^t \dot{x}(t)\, dt$$

Energy stored $= \frac{1}{2} K x^2$



Classification of Dynamic System :

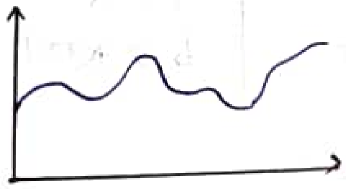
- Proper
- Improper
- Strictly proper

Book
→ Control Systems by M Gopal

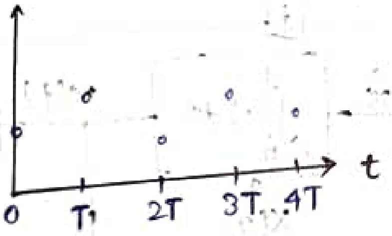
15-1-2024

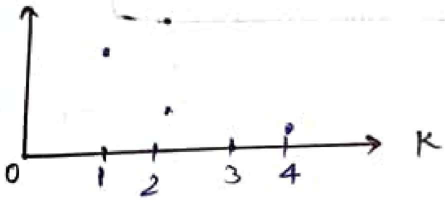
- Classes of Systems
  - Distributed Parametric System ✗
  - Lumped Parametric system
    - Stochastic ✗
      - A probability factor is attached with, say, dist. & time.
    - Deterministic ✓
      - Continuous time
        - Time varying
        - Time Invarient
          - Linear ✓
            - SISO ✓ (single i/p single o/p)
            - MIMO (multi i/p multi o/p)
          - Non-Linear
      - Discrete time

Continuous time :

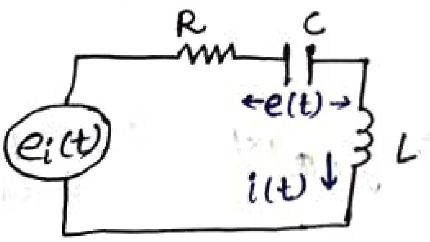


Discrete time :





→



→ e(t) and i(t) cannot be changed suddenly.

→ 2 state variables : e(t), i(t)

$$q(t) = c\,e(t)$$

$$i(t) = c\,\frac{de(t)}{dt}$$

$$e_i(t) = R\,i(t) + e(t) + L\frac{di}{dt}$$

$$\Rightarrow L\frac{di}{dt} = -R\,i(t) - e(t) + e_i(t)$$

Write the rate of change of state variables.

$$\frac{de}{dt} = \frac{1}{c} i(t) \quad \dots ①$$

$$\frac{di}{dt} = -\frac{R}{L} i(t) - \frac{1}{L} e(t) + \frac{1}{L} e_i(t) \quad \dots ②$$

$$\underbrace{\frac{d}{dt}\begin{bmatrix} e \\ i \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} e \\ i \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 1/L \end{bmatrix}}_{b} \underbrace{e_i(t)}_{r(t)}$$

$r(t)$ → i/p signal

$\dot{x} = Ax + br$: single input ~~signal~~ system

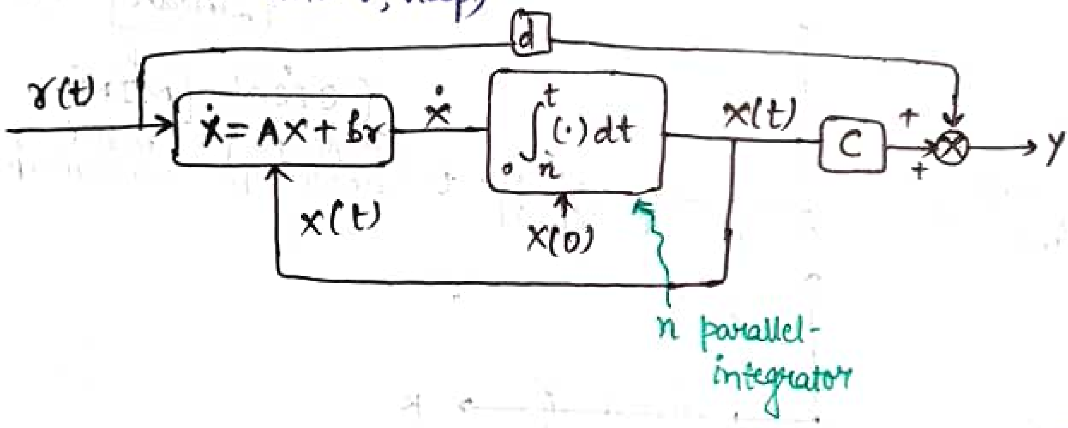
$y = cx + dr$

↳ functions of state variables and external input (x and r, resp.)

$x \in \mathbb{R}^n$

$A \in \mathbb{R}^{n \times n}$

$b \in \mathbb{R}^{n \times 1}$



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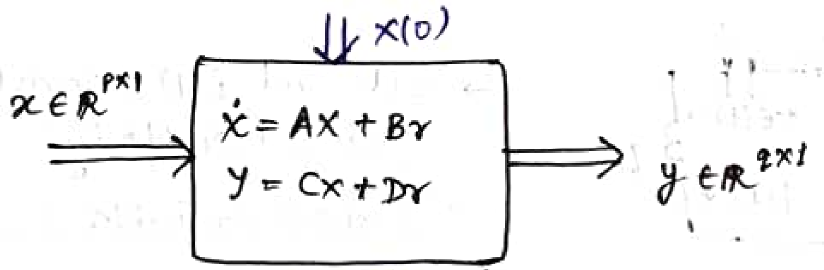


: p-input q-output linear invariant system

(A, B, C, D : indep. of time)

$x \in \mathbb{R}^n$

$A \in \mathbb{R}^{n \times n}$

$B \in \mathbb{R}^{n \times p}$

$C \in \mathbb{R}^{q \times n}$

$D \in \mathbb{R}^{q \times p}$

Time variant : $A(t), B(t), C(t), D(t)$,

Non-linear : A, B, C, D f^n of system state

$\dot{x} = f(x, r, t)$

$y = \phi(x, r, t)$, f, ϕ : non-linear f^n

We'll discuss only SISO LTI (Single I/P single O/P Linear Time Invariant) system.

Linear System: Follows superposition principle.

$$x \rightarrow y$$
$$\Rightarrow \alpha x \rightarrow \alpha y$$

$$x_1 \rightarrow y_1$$
$$x_2 \rightarrow y_2$$
$$\Rightarrow \alpha x_1 + \beta x_2 \rightarrow \alpha y_1 + \beta y_2$$

↳ Can process multiple i/ps and produce output which is the superposition of the results produced one at a time.

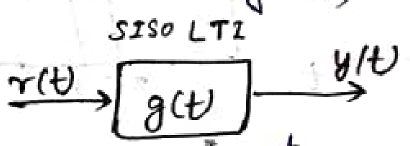
Suppose $X(0)=0$: System is relaxed initially.
(all energies/state variables are zero.)

Eg

$$x(0)=0$$
$$\dot{x}(0)=0.$$

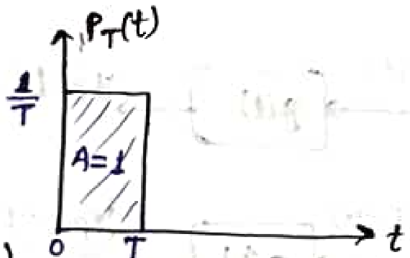
→ **Causal System:** No output before application of input.

→ **Non-causal system:** Output can be predicted before applying input. (Predictive system)

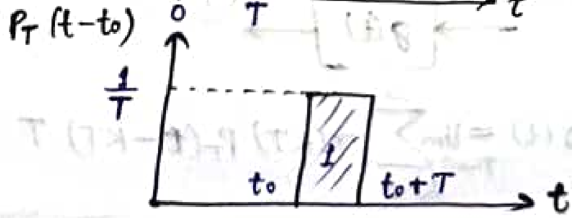
→ 

$$\left.\begin{aligned} y(t) &= \int_0^t r(\tau)\, g(t-\tau)\, d\tau \\ &= \int_0^t r(t-\tau)\, g(\tau)\, d\tau \end{aligned}\right\}$$ convulation integral; both are equal

Unit Pulse Function:



Shifted unit Pulse Function:



$$\lim_{T \to 0} P_T(t) = \delta(t)$$

$$\lim_{T \to 0} P_T(t-t_0) = \delta(t-t_0)$$

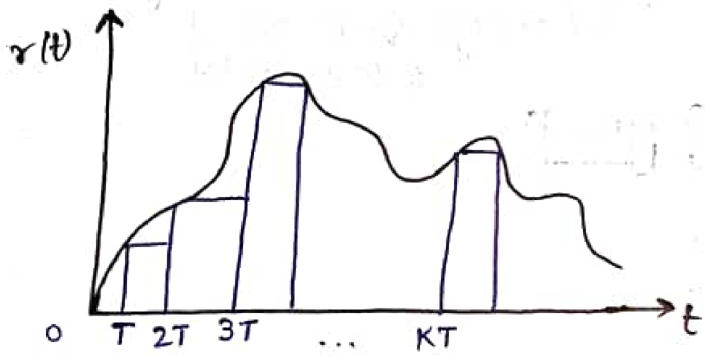
$\delta(t) = 0$ for $t \neq 0$

- $\displaystyle\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

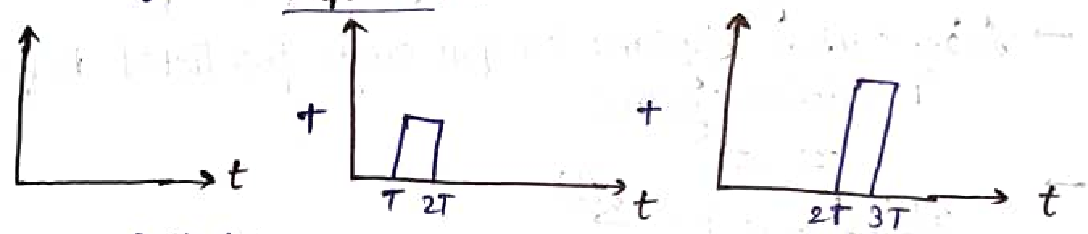
$\delta(t-t_0) = 0$ for $t \neq t_0$

- $\displaystyle\int_{t_0-}^{t_0+} \delta(t-t_0)\,dt = 1$
- $\displaystyle\int_{t_0-}^{t_0+} f(t)\,\delta(t-t_0)\,dt = f(t_0)$

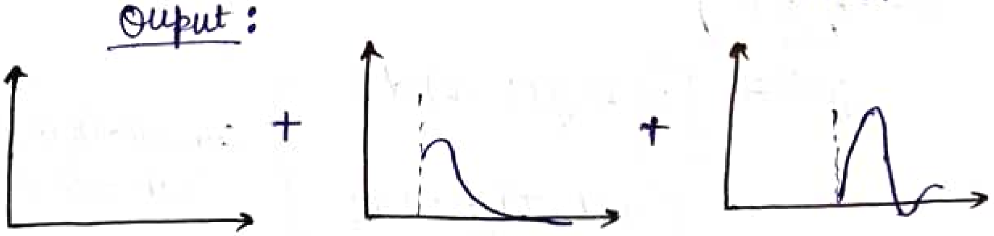
→



Input: $r(t)$



Ouput:



$$P_T(t) \longrightarrow \boxed{g(t)} \longrightarrow y_P(t)$$

$$P_T(t-t_0) \longrightarrow \boxed{g(t)} \longrightarrow y_P(t-t_0)$$

$$r(t) = \lim_{T \to 0} \sum_{K=0}^{\infty} r(KT)\, P_T(t-KT)\, T$$

$$y(t) = \lim_{T \to 0} \sum_{K=0}^{\infty} r(KT)\, y_p(t-KT)\, T$$

When $T \to 0$, $KT = \tau$; $T \to d\tau$

$$P_T(t-KT) \to \delta(t-\tau)$$
$$y_p(t-KT) \to g(t-\tau)$$

Pulse → Impulse
Unit pulse → Unit impulse

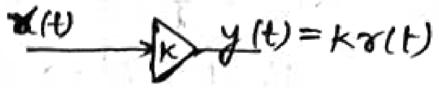
$$y(t) = \int_0^t r(\tau)\, g(t-\tau)\, d\tau$$

For $\tau > t$, $t-\tau = -ve$
$\Rightarrow g(t-\tau) = 0$

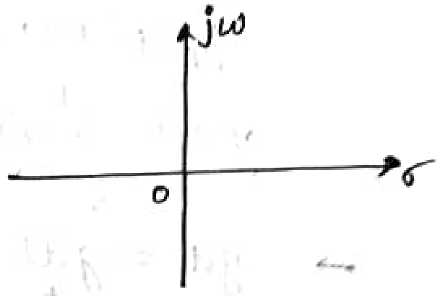
$$\rightarrow \quad y(t) = g(t) * r(t)$$
$$= \int_0^t g(\tau)\, r(t-\tau)\, d\tau$$ ... Convolution integral

19-01-2024

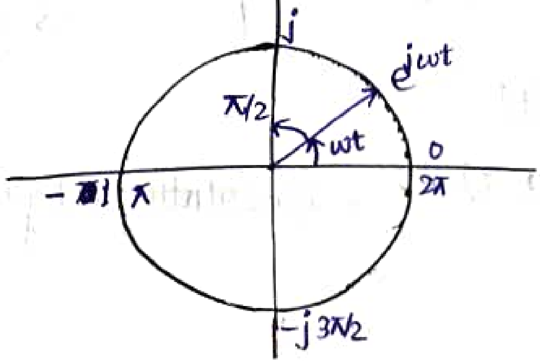
LAPLACE TRANSFORM

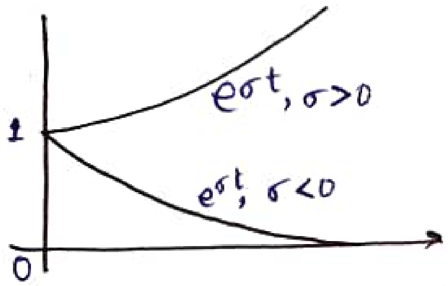


$s = \sigma + j\omega$

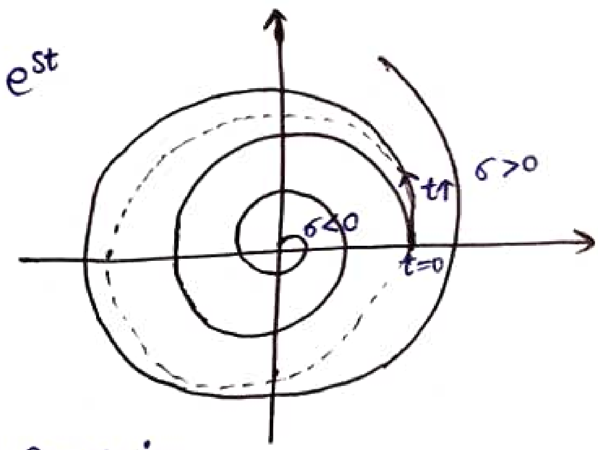


→ $e^{j\omega t} = \cos\omega t + j\sin\omega t$





→ $e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t}$



→ $s = \sigma + j\omega$

$$F(s) = \int_0^{\infty} e^{-st} f(t)\,dt \;;$$

$f(t) = 0$ for $t < 0$.

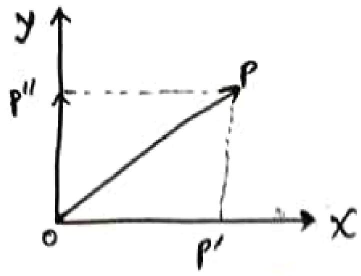
→ $f(t) \rightarrow F(s)$ ... Laplace transform

$f(t) \leftarrow F(s)$ ... Inverse Laplace transform

↳ Reversible transform

↳ Linear transform

→



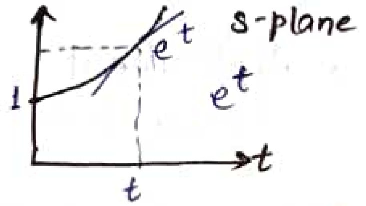
$$F(s_1) = \int_0^{\infty} e^{-s_1 t} f(t)\, dt$$

→ All physical world signals are time-signals.

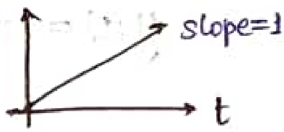
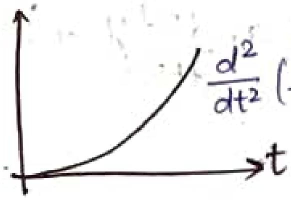
→ Mapping : time-domain → frequency domain

↳ complex frequency signal has properties : cyclicity, convergence or divergence.

↳ Signal 'e^t' has a peculiar property ; it is same as its derivative.



Signals

			LT
→ Unit step : $\mu(t)$		⇝ 0, t=0 ; >0, t→0⁺	→ $1/s$
→ Unit impulse : $\delta(t)$			→ 1
→ Unit Ramp signal : $t\mu(t)$		slope=1	→ $1/s^2$
→ Unit Parabolic : $\frac{t^2}{2}\mu(t)$		$\frac{d^2}{dt^2}\left(\frac{t^2}{2}\mu(t)\right) = 1$	→ $1/s^3$

$$\cos\omega t \longrightarrow \frac{s}{s^2+\omega^2}$$

$$\sin\omega t \longrightarrow \frac{\omega}{s^2+\omega^2}$$

$$e^{-at} \longrightarrow \frac{1}{s+a}$$

$$e^{-at} f(t) \longrightarrow F(s+a)$$

$$e^{-at}\cos\omega t \longrightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

$$\frac{dy(t)}{dt} \longrightarrow sY(s) - y(0)$$

$$\frac{d^2y(t)}{dt^2} \longrightarrow s^2Y(s) - sy(0) - \dot{y}(0)$$

$$\int_0^t y(t)\,dt \longrightarrow \frac{Y(s)}{s}$$

Final value theorem : $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

Convolation Integral

$$y(t) = g(t) * r(t)$$

$$\Rightarrow Y(s) = G(s) \cdot R(s)$$

⇝ Applicable if system is relaxed at $t=0$. } Convolution theorem

$$\Rightarrow G(s) = \frac{Y(s)}{R(s)}$$

→ For static system, $G(s)$ is simply gain.
For dynamic system, $G(s)$ is frequency dependent gain.

Static system : $y(t) = kr(t)$

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1}y}{dt^{n-1}} + \dots + a_{n-1}\frac{dy}{dt} + a_n = b_0 \frac{d^m r}{dt^m} + b_1 \frac{d^{m-1}r}{dt^{m-1}} + \dots + b_{m-1}\frac{dr}{dt} + b_m$$

$n \geq m$

↳ g can be parameterised by the coefficients $a_1, a_2, \dots, a_{n-1}$.

23-01-2023

nth order

$$r(t) \rightarrow \boxed{g(t)} \rightarrow y(t)$$

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1}y}{dt^{n-1}} + \dots + a_{n-1}\frac{dy}{dt} + a_n y(t) = b_0 \frac{d^m r}{dt^m} + b_1 \frac{d^{m-1}r}{dt^{m-1}} + \dots + b_{m-1}\frac{dr}{dt} + b_m r(t)$$

$$y(0)=0, \left.\frac{dy}{dt}\right|_{t=0} = 0, \dots, \left.\frac{d^{n-1}y}{dt^{n-1}}\right|_{t=0} = 0.$$

$$s^n Y(s) + a_1 s^{n-1} Y(s) + \dots + a_n Y(s) = b_0 s^m R(s) + \dots + b_{m-1} s R(s) + b_m R(s)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}, \quad m \le n$$

↳ Proper system : $m \le n$

Improper system : $m > n \rightarrow K_1 s + $ (Proper system)

Strictly proper system : $m < n$.

→ $G(s) = \mathcal{L}\{g(t)\}$

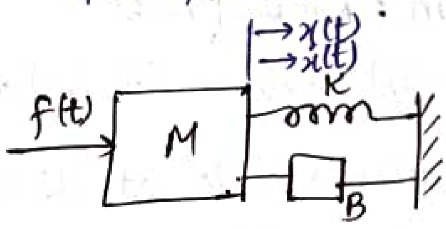
$$\mathcal{L}\{\delta(t)\} \rightarrow \boxed{G(s)} \rightarrow Y(s)$$

If $R(s) = \mathcal{L}\{\delta(t)\}$, $G(s) = Y(s)$.

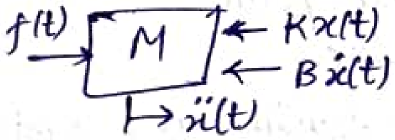
→ $G(s) = \left.\dfrac{\mathcal{L}\{y(t)\}}{\mathcal{L}\{r(t)\}}\right|$ system is relaxed at $t=0$

$G(s) = Y(s)$ when $r(t) = \delta(t)$.

Eg.



"Mathematical Modelling"



$$M\ddot{x}(t) = f(t) - Kx(t) - B\dot{x}(t)$$

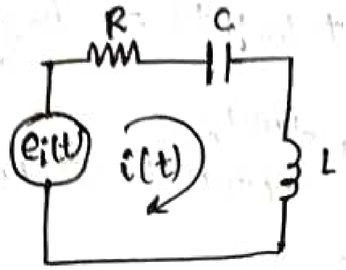
$x(0) = 0$

$\dot{x}(0) = 0$

$$\Rightarrow Ms^2X(s) + BsX(s) + KX(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} = G(s)$$

Eg.



$q = C e_c$

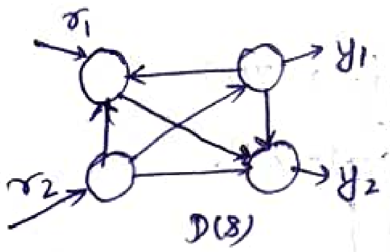
$i = \frac{dq}{dt} = C\frac{de_c}{dt}$

$$e_i(t) = R\,i(t) + L\frac{di}{dt} + e_c$$

→ $$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1}s + b_m}{s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n} = \frac{N(s)}{D(s)} = \frac{N(s)}{\Delta(s)}$$

$D(s) = 0$: Characteristic equation

↳ Characterize the stability properties of the system.



→ MIMO

→ $D(s)$ will be same, only $N(s)$ will change.

Poles of the system : Roots of $D(s)$

$D(s) = (s-P_1)(s-P_2)\dots(s-P_n)$, $P_1, P_2, \dots, P_n$

Zeroes of the system : Roots of $N(s)$

$N(s) = (s-Z_1)(s-Z_2)\dots(s-Z_m)$, $Z_1, Z_2, \dots, Z_m$

→ A system can be stable or unstable at its equilibrium point.

$y(0) = 0$ ⇝ equilibrium point.

On giving a small impulse,

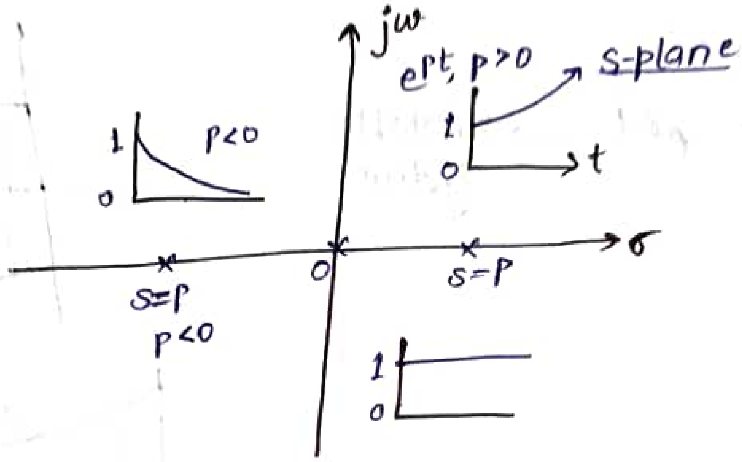
if y diverges from equilibrium : system is unstable.

if y converges to equilibrium : system is stable.

Eg.

$\uparrow \delta(t)$ $\quad R(s) = 1$ $\quad \cancel{\frac{1}{s-p}}$

$$Y(s) = \frac{1}{s-p}, \quad y(t) = e^{pt}$$



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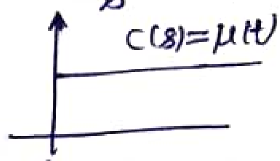
→ $G(s) = \frac{1}{s}$

$R(s) = 1$

$C(s) = G(s)R(s)$

$= \frac{1}{s} \times 1 = 1/s$



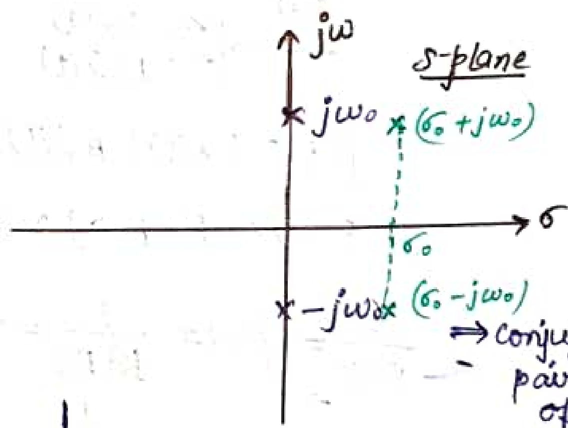
(Unit impulse of disturbance)

→ Imaginary poles:

$$G_1(s) = \frac{1}{2}\left(\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right) = \frac{s}{s^2+\omega_0^2}$$

$r(t) = \delta(t)$

$c(t) = \cos\omega_0 t$

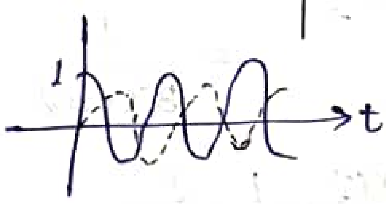


⇒ conjugate pairs of poles

$$G_2(t) = \frac{1}{2j}\left(\frac{1}{s-j\omega_0} - \frac{1}{s+j\omega_0}\right)$$

$$= \frac{1}{2j}\left(\frac{s+j\omega_0 - (s-j\omega_0)}{s^2+\omega_0^2}\right)$$

$$= \frac{\omega_0}{s^2+\omega_0^2}$$



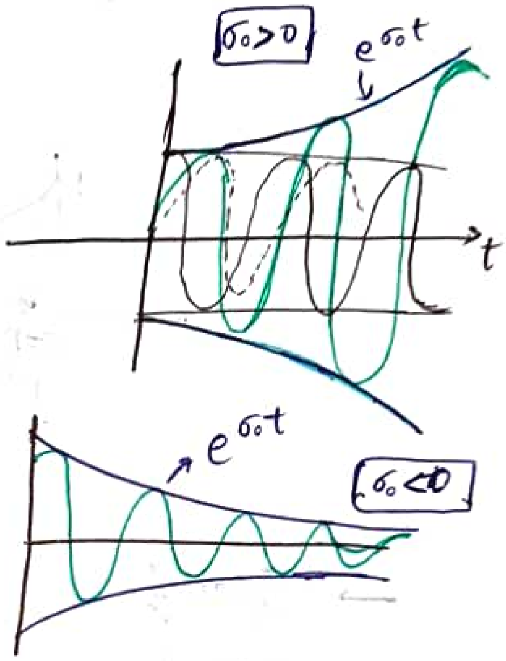
→ Complex poles:

$$G(s) = \frac{1}{2}\left[\frac{1}{s-(\sigma_0+j\omega_0)} + \frac{1}{s-(\sigma_0-j\omega_0)}\right]$$

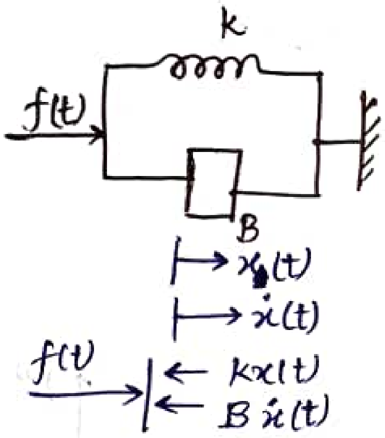
$$= \frac{1}{2}\left[\frac{s-(\sigma_0 - j\omega_0) + s-(\sigma_0 + j\omega_0)}{(s-\sigma_0)^2 + \omega_0^2}\right]$$

$$= \frac{(s-\sigma_0)}{(s-\sigma_0)^2+\omega_0^2}$$

$c(t) = \cos\omega_0 t \cdot e^{\sigma_0 t}$ ⇝ Unstable System



Eg.



Initially relaxed System

$$f(t) = Kx(t) + B\dot{x}(t)$$

$$F(s) = (K + Bs)\,X(s)$$

$$\frac{X(s)}{F(s)} = G(s) = \frac{1}{K+Bs} = \frac{1}{B(s+K/B)} = \frac{1}{K(1+\frac{B}{K}s)} = \frac{1}{K(1+s\tau)}, \quad \tau = B/K.$$

$$= \frac{1}{B(s+\omega_0)}$$

$$\frac{X(s)}{F(s)} = \frac{1}{K}\,\frac{1}{1+s/\omega_0}$$

$$= \frac{1}{K}\left(\frac{\omega_0}{s+\omega_0}\right)$$

Unit Step Response of $\left(\frac{\omega_0}{s+\omega_0}\right)$:

$$c(t) = \mathcal{L}^{-1}\frac{1}{s}\left(\frac{\omega_0}{s+\omega_0}\right)$$

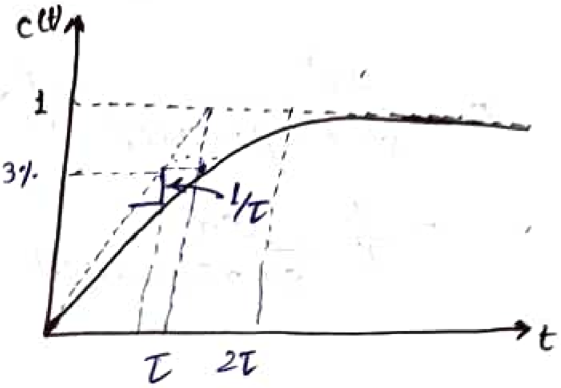
$$= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+\omega_0}\right\}$$

$$= \mu(t) - \mu(t)e^{-\omega_0 t}$$

$$= \mu(t)\left[1 - e^{-\omega_0 t}\right]$$

$$\frac{\omega_0}{s(s+\omega_0)} = \frac{K_1}{s} + \frac{K_2}{s+\omega_0}$$

$$K_1 = 1,\ K_2 = -1$$



$\tau \to 63\%$

$2\tau \to 84\%$

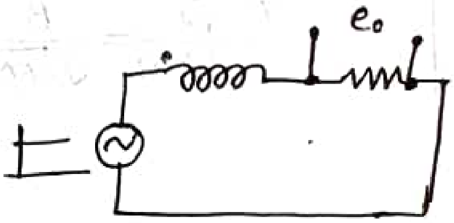
$3\tau \to 95\%$

$4\tau \to 98\%$

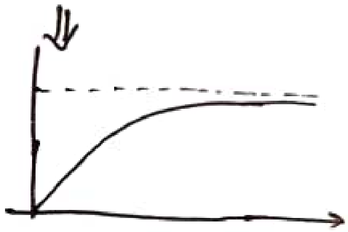
$5\tau \to 99\%$

$$\dot{c}(t)\Big|_{t=0} = \omega_0 e^{-\omega_0 t}\Big|_{t=0} = \omega_0 = \frac{1}{\tau}$$

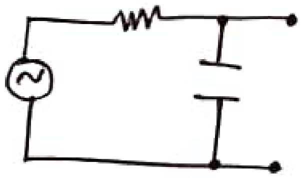
Eg.



$\Downarrow$



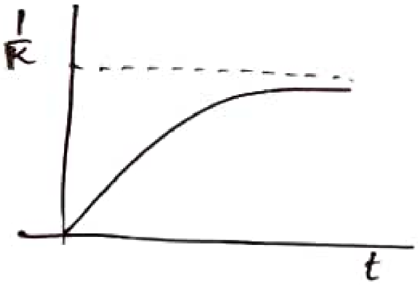
Eg.



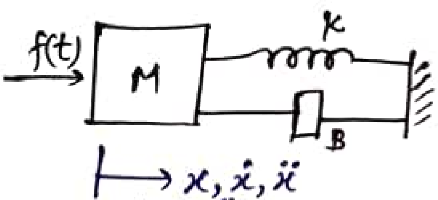
$\Rightarrow$

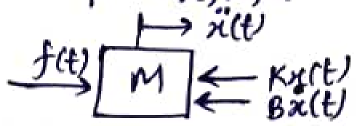
$$\frac{X(s)}{F(s)} = \frac{1}{K}\cdot\frac{1}{1+s/\omega_0}$$

$$= \frac{1}{K}\left(\frac{\omega_0}{s+\omega_0}\right)$$



Eg.





$$M\ddot{x}(t) = f(t) - Kx(t) - B\dot{x}(t)$$

$$\Rightarrow M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = f(t)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} = \frac{1}{M(s^2 + B/M\, s + K/M)}$$

$$\cancel{= \frac{1}{M(s^2 + 2\xi\omega_n}}$$

$$= \frac{1}{M}\left(\frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2}\right)$$, ω_n : undamped natural frequency

$$= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \times \frac{1}{M} \times \frac{M}{K}$$, $2\xi\omega_n = \frac{B}{M}$

$$= \frac{1}{K}\,\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ξ : damping factor or damping ratio

$\xi = 1$: critically damped system

$$\xi = \frac{\frac{B}{M}}{2\omega_n} = \frac{B}{2\sqrt{KM}}$$

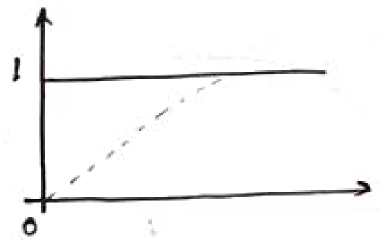
$\boxed{2\text{-}02\text{-}2024}$

2^{nd} Order System :

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{Y(s)}{R(s)}$$

- $\xi = 0$
- $0 < \xi < 1$
- $\xi \geq 1$

$R(s) = \frac{1}{s}$



$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \ldots (1)$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = (s + \xi\omega_n)^2 + \omega_n^2 - \xi^2\omega_n^2$$
$$= (s + \xi\omega_n)^2 + \omega_d^2$$

where $\omega_d^2 = \omega_n^2(1 - \xi^2)$

ω_d : damped natural/oscillation frequency

$$= \frac{K_1}{s} + \frac{K_2 s + K_3}{(s+\xi\omega_n)^2 + \omega_d^2}$$

[Multiply by s and put s=0. $\Rightarrow K_1 = 1$]

$$= \frac{K_1(s^2 + 2\xi\omega_n s + \omega_n^2) + (K_2 s + K_3)s}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$= \frac{(K_1+K_2)s^2 + (2\xi\omega_n K_1 + K_3)s + K_1\omega_n^2}{s(s^2+2\xi\omega_n s + \omega_n^2)} \quad \ldots ②$$

Compare coefficients of s^2, s in ① and ②,

$$K_1 + K_2 = 0 \Rightarrow K_2 = -1$$

$$2\xi\omega_n K_1 + K_3 = 0 \Rightarrow K_3 = -2\xi\omega_n$$

$$\therefore Y(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

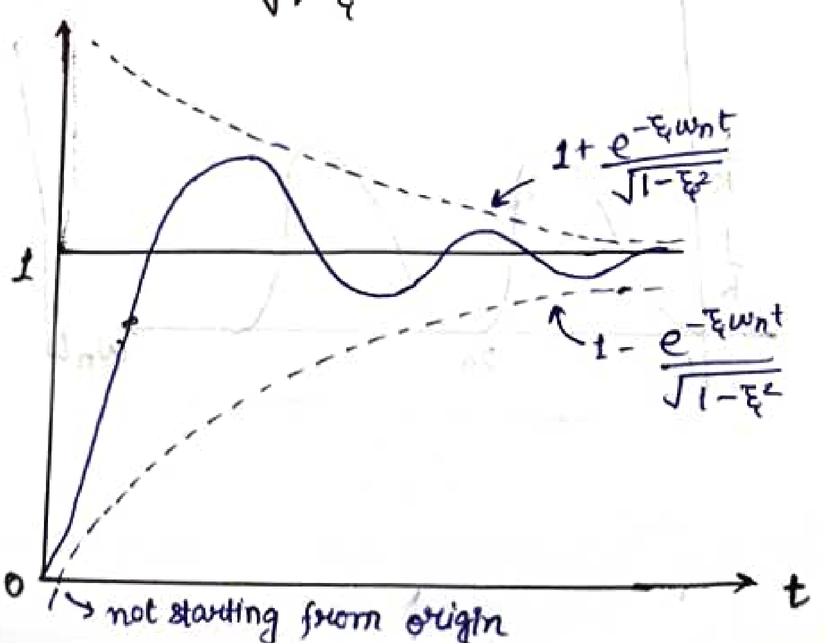
$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n\omega_d}{\omega_d[(s+\xi\omega_n)^2+\omega_d^2]}$$

$$\Rightarrow y(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\Big[\underbrace{\sqrt{1-\xi^2}}_{\substack{\cos\phi \\ \sin\psi}}\cos\omega_d t + \underbrace{\xi}_{\substack{\sin\phi \\ \cos\psi}}\sin\omega_d t\Big]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\Big[\underbrace{\cos(\omega_d t - \phi)}_{[-1,1]}\Big] \quad \text{where } \phi = \tan^{-1}\frac{\xi}{\sqrt{1-\xi^2}}$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \psi) \quad \text{where } \psi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$



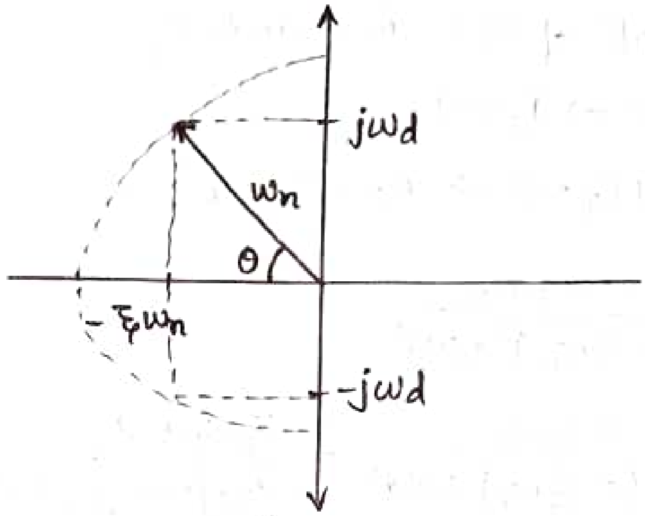
$1 - e^{-t/\tau}$

$$\boxed{\tau = \frac{1}{\xi\omega_n}}$$

→ A state variable cannot change instantaneously.

- 1st order system: state variable → Position (velocity can change instant.)
- 2nd order system: state variables → Position, velocity.

→ Poles: roots → $s = -\xi\omega_n \pm j\omega_d$



ξ : damping factor

$\cos\theta = \xi$

$\theta = 0 : \xi = 1$

$\theta = \frac{\pi}{2} : \xi = 0.$

$\theta > \frac{\pi}{2} : \xi < 0$ (unstable)

$\xi = \frac{B}{2\sqrt{KM}}$

$\xi > 0 \Rightarrow B > 0$

$\xi < 0 \Rightarrow B < 0$

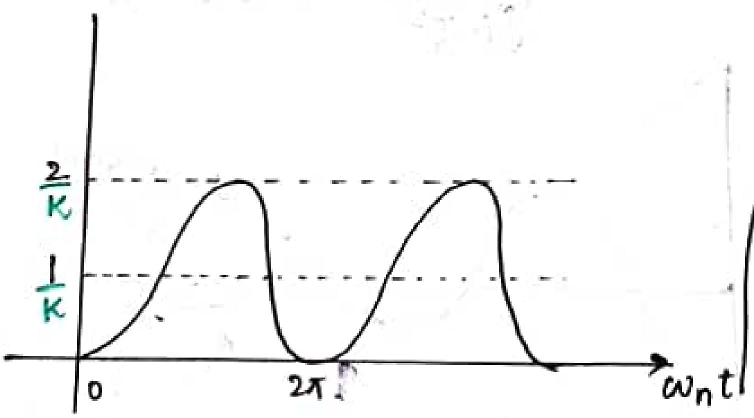
$\xi < 0$: Generative system (unstable)

$\xi > 0$: Dissipative system

$\xi = 0$:

$y(t) = 1 - \frac{e^0}{1}\cos(\omega_n t - 0)$ [$\because \phi = 0$]

$= 1 - \cos\omega_n t$

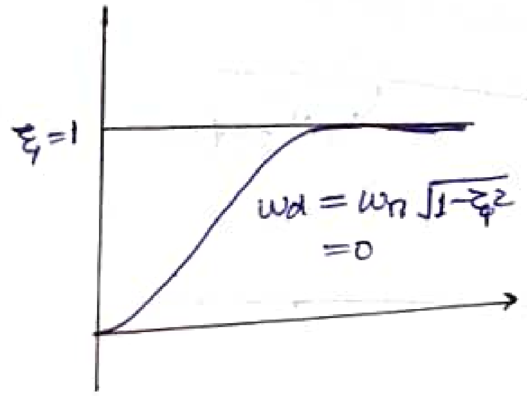


$\frac{1}{K}\left(\frac{\omega_0}{s+\omega_0}\right)$

$\xi = 1$: Critically damped system (oscillation has just vanished).

$$Y(s) = \frac{\omega_n^2}{s(s^2+2\omega_n s+\omega_n^2)} = \frac{1}{s}\frac{\omega_n^2}{(s+\omega_n)^2}$$

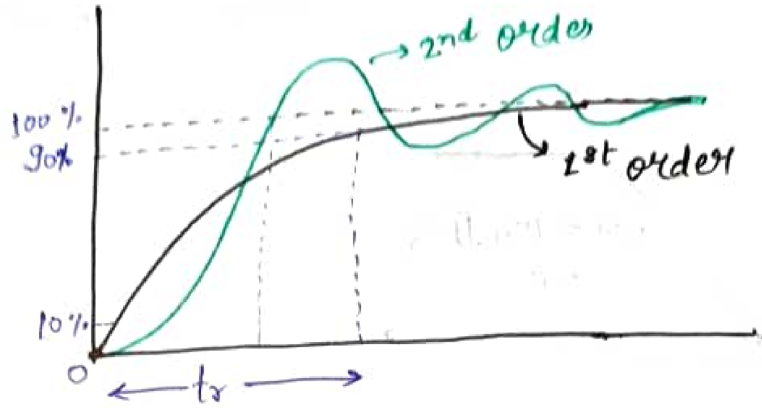
$$= \frac{A_1}{s} + \frac{A_{11}}{s+\omega_n} + \frac{A_{12}}{(s+\omega_n)^2}$$



06-02-2024

Rise Time

Rise time of a dynamical system is defined as the time taken for the output to rise from 10% to 90% for the step command.



1st order system: $y(t) = 1 - e^{-t/\tau}$

$$0.1 = 1 - e^{-t_{10}/\tau}$$

$$0.9 = 1 - e^{-t_{90}/\tau}$$

$$\Rightarrow e^{-t_{10}/\tau} = 0.9 \Rightarrow -t_{10}/\tau = \ln 0.9$$

$$e^{-t_{90}/\tau} = 0.1 \Rightarrow -t_{90}/\tau = \ln 0.1$$

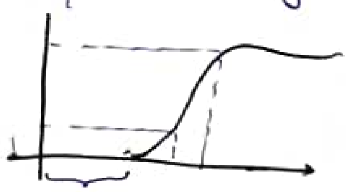
$$t_r = t_{90} - t_{10} \approx \frac{2.2}{\omega_n} = 2.2\tau$$

- Faster the system, smaller the rise time, smaller the time constant, larger the bandwidth.

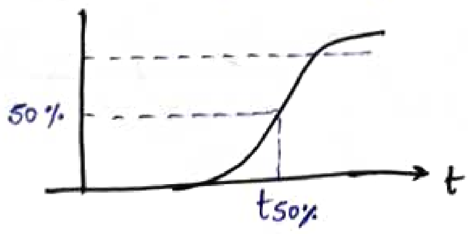
2nd order system: $t_r = t_{100\%} - \cancel{t_0}^{\to 0}$

$$= t_{100\%}$$

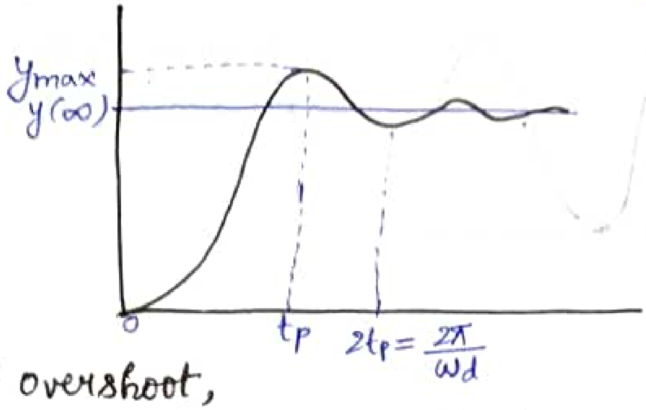
Transportation lag:



Delay Time: Time taken for the response to rise to 50% after the command is given (including command delay/transportation lag).



Overshoot: defined for the underdamped system.



frequency of oscillation : ω_d

Peak overshoot,

$$M_{peak} = \frac{y_{max} - y(\infty)}{y(\infty)} \quad (\neq M_p)$$

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t - \phi), \quad \phi = \tan^{-1}\frac{\xi}{\sqrt{1-\xi^2}}$$

$$\dot{y}(t) = -\left(\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\right)(-\sin(\omega_d t - \phi))\,\omega_d - \cos(\omega_d t - \phi) \times \left(-\frac{\xi\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\right) = 0$$

$$\Rightarrow \sin(\omega_d t - \phi)\,\omega_d + \cos(\omega_d - \phi)\,\xi\omega_n = 0$$

$$\Rightarrow \sin(\omega_d t - \phi)\underbrace{\sqrt{1-\xi^2}}_{\cos\phi} + \cos(\omega_d - \phi)\overbrace{\xi}^{\sin\phi} = 0$$

$$\Rightarrow \sin(\omega_d t - \phi + \phi) = 0$$

$$\Rightarrow \sin(\omega_d t) = 0 \quad \text{when } \dot{y}(t) = 0$$

$$\Rightarrow \omega_d t = 0, \pi, 2\pi, \ldots$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$M_{peak} = \frac{y(t_p) - 1}{1} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t_p - \phi)$$

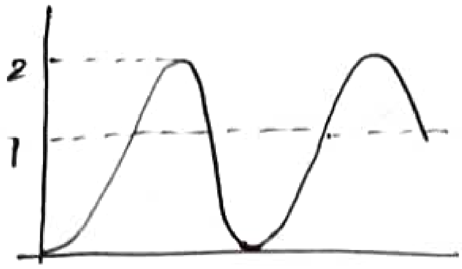
As $\cos(\omega_d t_p - \phi) = \cos\omega_d t_p \cdot \cos\phi + \sin\omega_d t_p \cdot \sin\phi$

$$= -\cos\phi \qquad [\because \omega_d t_p = \pi]$$

$$\Rightarrow M_{peak} = -\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \times -\cos\phi = e^{-\frac{\xi\omega_n\pi}{\omega_n\sqrt{1-\xi^2}}}$$

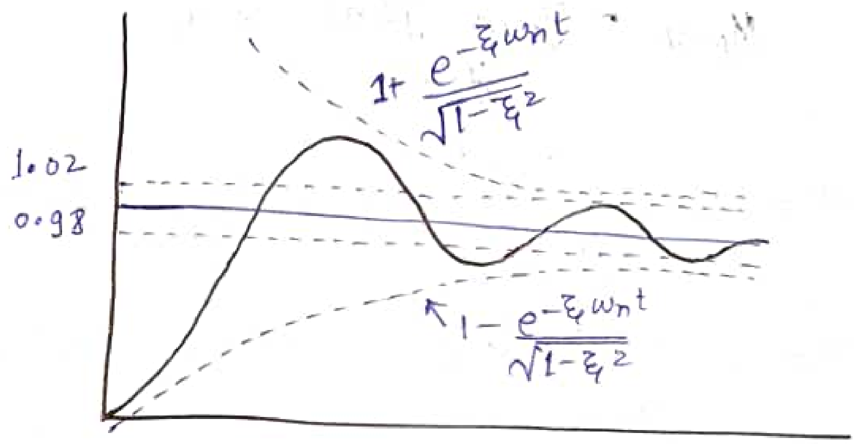
$$\Rightarrow \boxed{M_{peak} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}$$

For $\xi = 0$: $M_{peak} = 1$



→ Pure Oscillation
→ Resonance System

Settling Time



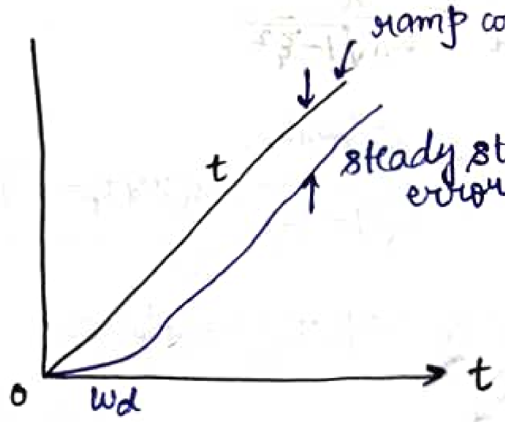
$$\frac{e^{-\xi \omega_n t_s}}{\sqrt{1-\xi^2}} = 0.02$$

$$\Rightarrow \quad e^{-\xi \omega_n t_s} = 0.02\sqrt{1-\xi^2}$$

$$\Rightarrow \quad -\xi \omega_n t_s = \ln 0.02\sqrt{1-\xi^2}$$

$$\Rightarrow \quad t_s = \frac{-\ln 0.02\sqrt{1-\xi^2}}{\xi \omega_n} \approx \frac{4}{\xi \omega_n} \qquad (\xi < 0.9)$$

#



09-02-2024

Sinusoidal Transfer Function

$C(s) = G(s)\,R(s)$

Stable transfer function: $\lim\limits_{t\to\infty} g(t) = 0$.

steady state

Command : $r(t) = A\sin\omega_0 t$, $t \geq 0$

Output : $c(t) = |G(j\omega)|\, A\sin(\omega_0 t + \phi)$, for $t > t_{ss}$.
where $\phi = \angle G(j\omega)$

$G(j\omega) \rightsquigarrow$ sinusoidal transfer function
$\hookrightarrow$ σ effect has died out.

$$r(t) \longrightarrow \boxed{g(t)} \longrightarrow c(t)$$

$$c(t) = g(t) * r(t) = \int_0^t g(\tau)\, r(t-\tau)\, d\tau$$

$r(t) = A\sin\omega t$

$$c(t) = \int_0^t g(\tau)\, A\sin\omega(t-\tau)\, d\tau$$

$$= \mathrm{Im}\left\{ \int_0^t g(\tau)\, A\, e^{j\omega(t-\tau)}\, d\tau \right\}$$

$$= \mathrm{Im}\left\{ A e^{j\omega t} \int_0^t g(\tau) e^{-j\omega\tau}\, dt \right\}$$

Since $\lim\limits_{t\to\infty} g(t) = 0$, we can always find out a $t = t_{ss}$ such that $\displaystyle\int_0^{t_{ss}} g(\tau) e^{-j\omega\tau} d\tau \gg \int_{t_{ss}}^{\infty} g(\tau) e^{-j\omega\tau} d\tau$

Hence, $\displaystyle\int_0^t g(\tau) e^{-j\omega\tau} d\tau \approx \int_0^{\infty} g(\tau) e^{-j\omega\tau} d\tau$, for $t > t_{ss}$

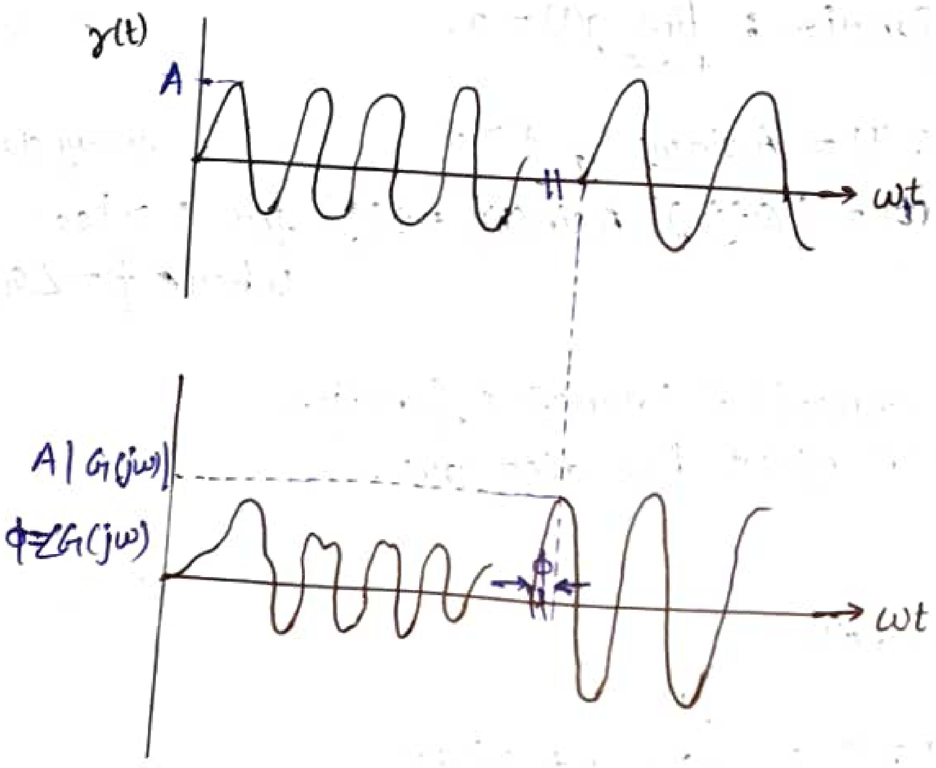
$$\approx G(j\omega).$$

$\therefore$ $c(t) = \mathrm{Im}\left\{ A e^{j\omega t}\, G(j\omega) \right\}$, for $t > t_{ss}$.

$= \mathrm{Im}\left\{ A e^{j\omega t} |G(j\omega)| e^{j\phi} \right\}$, $\phi = \angle G(j\omega)$.

$$= A\,|G(j\omega)|\ \mathrm{Im}\left\{e^{(j\omega t + \phi)}\right\}$$

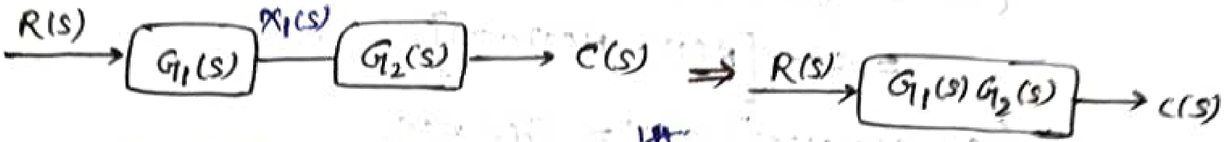
$$= A\,|G(j\omega)|\ \sin(\omega t + \phi)$$



Block Diagram Representation

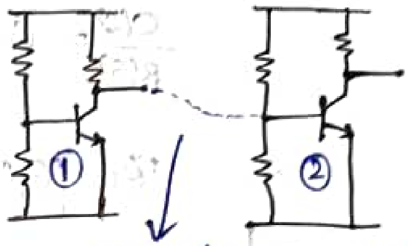
$C(s) = G(s)R(s)$

Representation
- Block diagram
- Signal flow



$C(s) = G_2(s) X_1(s)$
$\quad = G_2(s) G_1(s) R(s)$

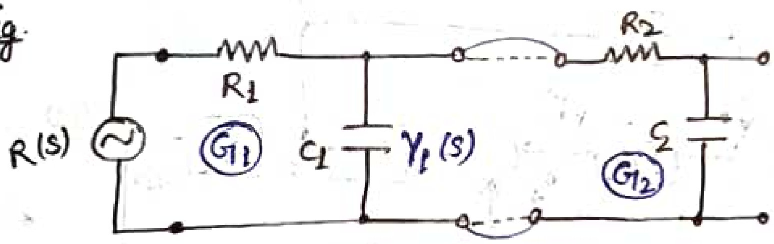
#



Can't be connected directly
⇓
Will change TF of ①
⇓
Connect only if $Z_{1,op} = 0$, $Z_{2,in} = \infty$.

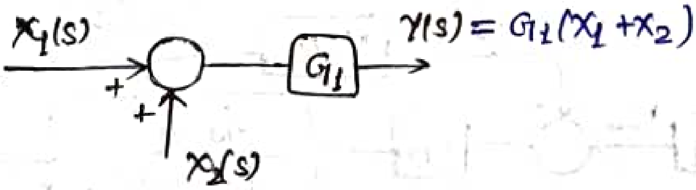
$\boxed{12\text{-}02\text{-}2024}$

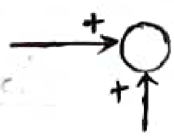
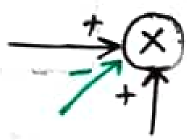
Eg.

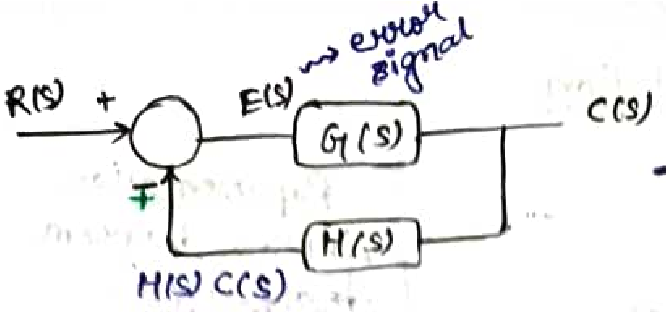


$$G_1 = \frac{Y_1(s)}{R_1(s)}$$

Block Diagram Algebra:



→ Summing Block :  or 



⇝ Closed Loop

$$E(s) = R(s) - H(s)\,C(s)$$

$$C(s) = G(s)\,E(s)$$

$$= G(s)\left[R(s) - H(s)\,C(s)\right]$$

$$\left[1 + G(s)H(s)\right] C(s) = G(s)\,R(s)$$

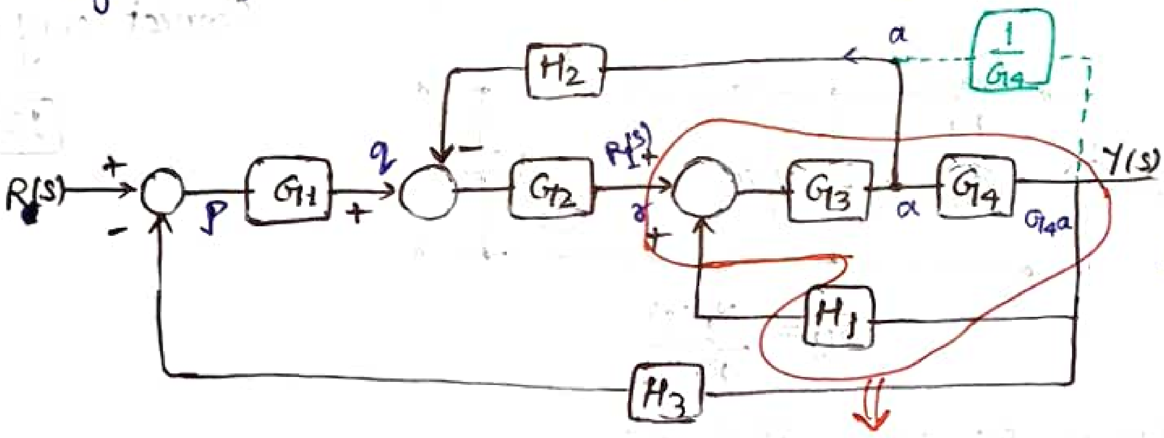
$$\frac{C(s)}{R(s)} = G_{cl}(s) = \frac{G(s)}{1 \pm G(s)\,H(s)}$$

For $|G(s)\,H(s)| \gg 1$, $G_{cl}(s) \approx \frac{1}{H(s)}$ ⇝ Used to take reciprocal

Positive feedback : ~~Stabo~~ Destabalize the system

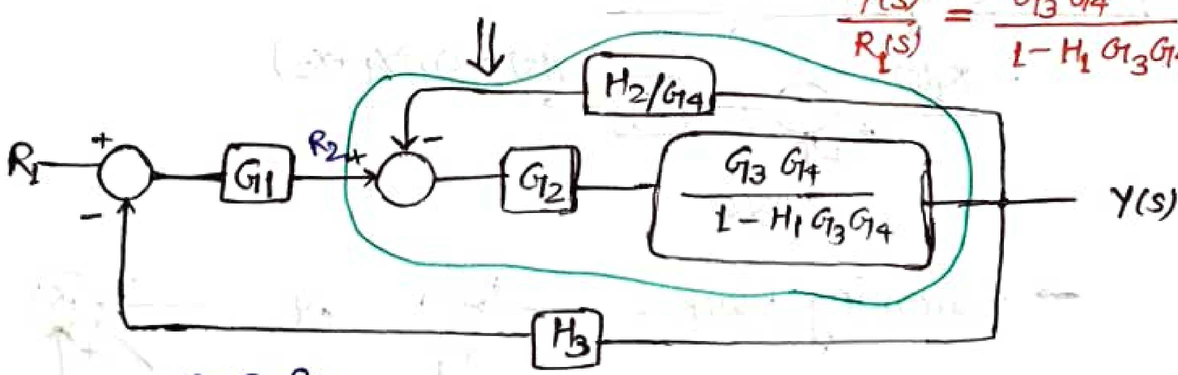
Negative feedback : Stabalize the system.

Eg.



← Two Loops cross-coupled

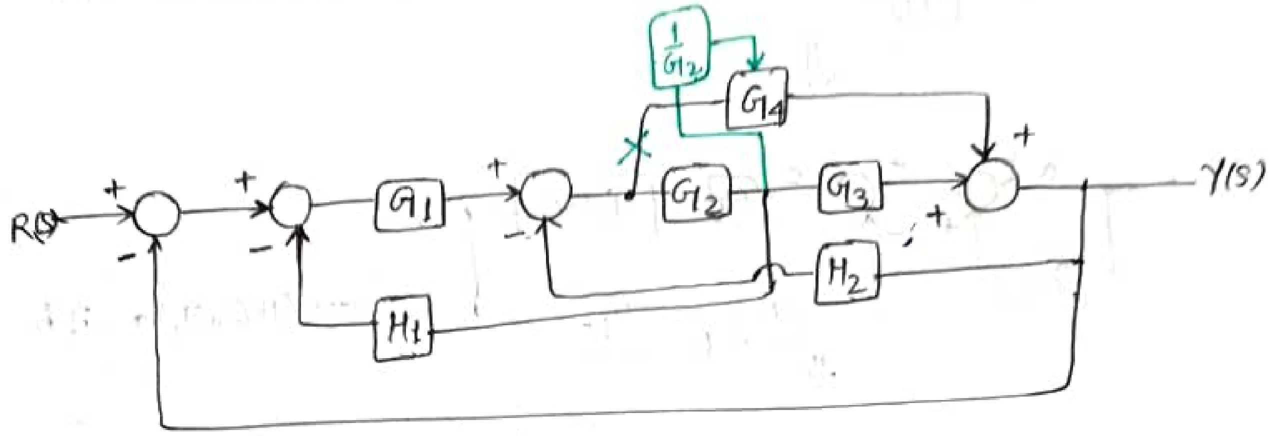
$$\frac{Y(s)}{R_1(s)} = \frac{G_3 G_4}{1 - H_1 G_3 G_4}$$



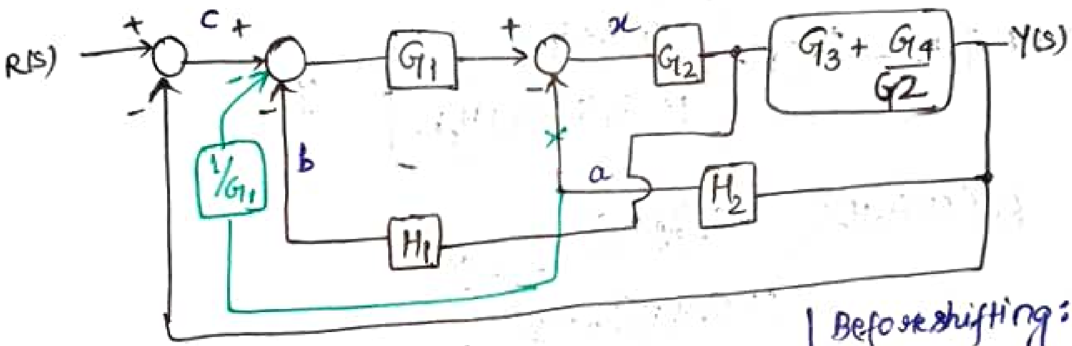
$$\frac{Y(s)}{R_2(s)} = \frac{\dfrac{G_2 G_3 G_4}{1 - H_1 G_3 G_4}}{1 + \dfrac{G_2 G_3 \cancel{G_4}}{1 - H_1 G_3 G_4} \times \dfrac{H_2}{\cancel{G_4}}} = \frac{G_2 G_3 G_4}{1 - H_1 G_3 G_4 + H_2 G_2 G_3}$$

$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3G_4}{1-H_1G_3G_4+H_2G_2G_3}}{1+\dfrac{G_1G_2G_3G_4}{1-H_1G_3G_4+H_2G_2G_3}\times H_3} = \frac{G_1G_2G_3G_4}{1+G_1G_2G_3G_4H_3-H_1G_3G_4+H_2G_2G_3}$$

Eg.



$\Downarrow$



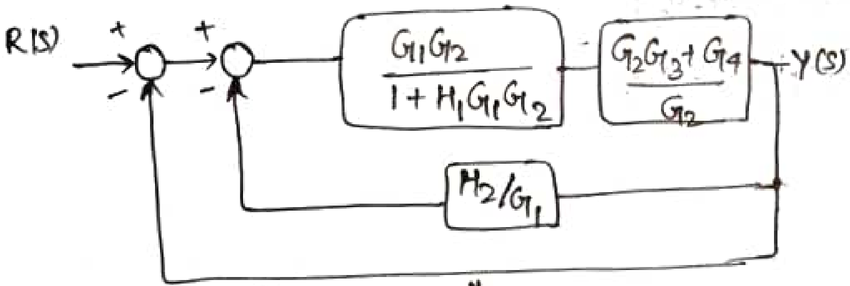
Before shifting:

$x = G_1c - G_1b - a$

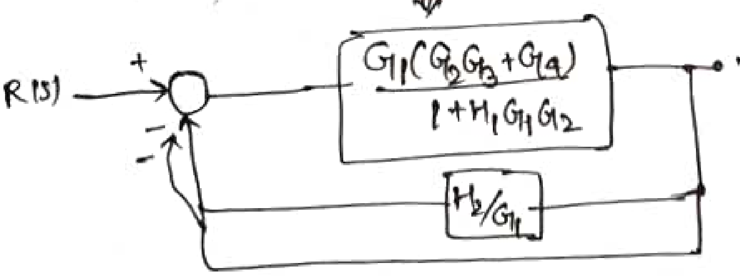
After shifting:

$x = G_1c - G_1b - \frac{a}{\cancel{G_1}} \times \cancel{G_1}$

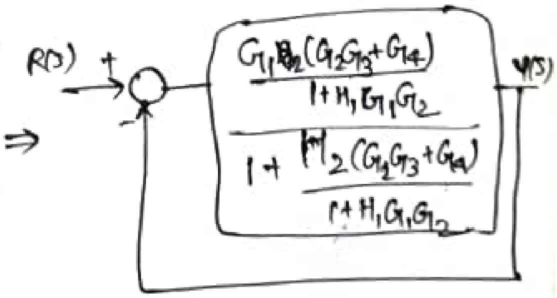
$\Downarrow$



$\Downarrow$



$\Rightarrow$



$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1(G_2G_3+G_4)}{1+H_1G_1G_2+H_2(G_2G_3+G_4)}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+H_1G_1G_2+H_2(G_2G_3+G_4)}} = \frac{G_1(G_2G_3+G_4)}{1+H_1G_1G_2+H_2(G_2G_3+G_4)+G_1(G_2G_3+G_4)}$$

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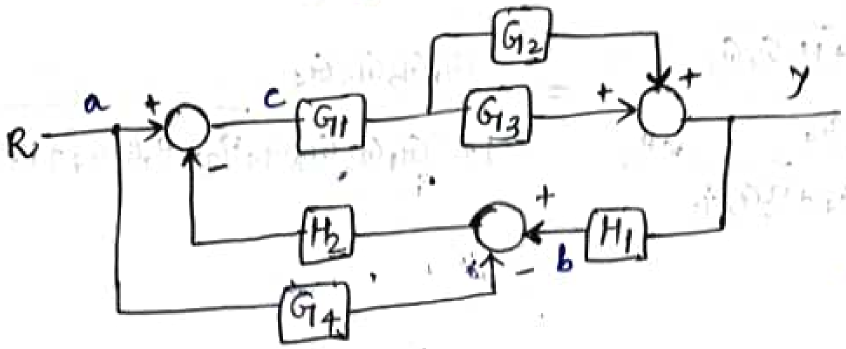
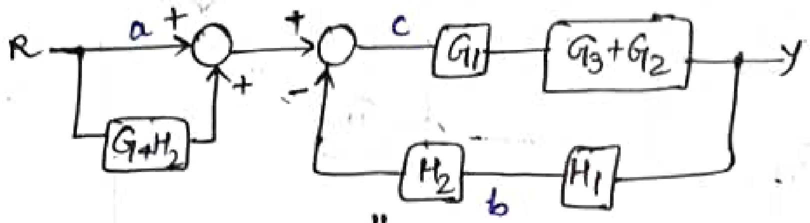
Eg.



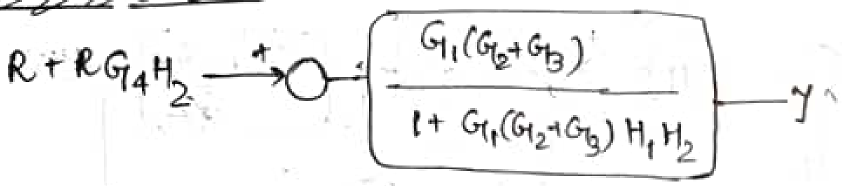
$c = (1+G_4H_2)a - H_2 b$

⇓



$c = (1+G_4H_2)a - H_2 b$

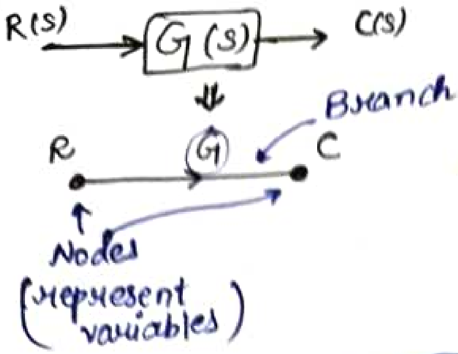
⇓



$$R(1+G_4H_2)\cdot\frac{G_1(G_2+G_3)}{1+G_1H_1H_2(G_2+G_3)} = y$$

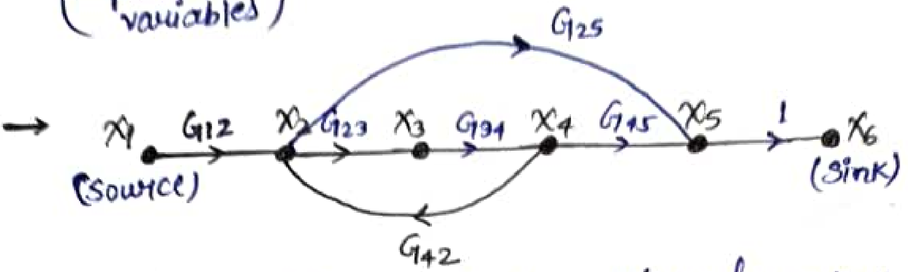
$$\frac{Y}{R} = \frac{G_1(G_2+G_3)(1+G_4H_2)}{1+G_1H_1H_2(G_2+G_3)}$$

Signal Flow Graph



$C(s) = G(s)\,R(s)$

G : Transmittance of branch



<u>Path:</u> Traversal of multiple branches.

↳ No node should be traversed more than once.

↳ Maximum, it can end at the traversed node.

$$x_2 = G_{12}x_1 \pm G_{42}x_4$$

→ Input node, Output node, Mixed node.

Input node (sourcing node) → only outgoing Branch (x_1)

Output node (sink) → only incoming branch (x_6)

Mixed node → Both incoming and outgoing Branch (x_2, x_4, x_5, x_3)

→ <u>Forward path:</u> Path from source to sink (i/p to o/p).

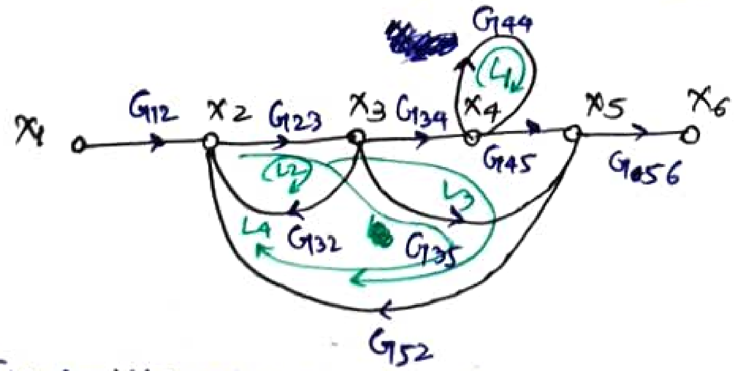
↳ $x_1 \to x_2 \to x_3 \to x_4 \to x_5 \to x_6$

$x_1 \to x_2 \to x_5 \to x_6$

→ <u>Loop:</u> Path starting and ~~and~~ ending at the same node.

↳ $x_2 \to x_3 \to x_4 \to x_2$

Eg.



Nodes: $x_1, x_2, x_3, x_4, x_5, x_6$

I/p node : x_1

O/p node : x_6

Mixed node : x_2, x_3, x_4, x_5

Loops: L_1, L_2, L_3, L_4

Non-touching loops : L_1 & L_2, L_1 & L_4

Transmittance : $G_{12}, G_{23}, G_{34}, G_{44}, G_{45}, G_{56}, G_{32}, G_{35}, G_{52}$

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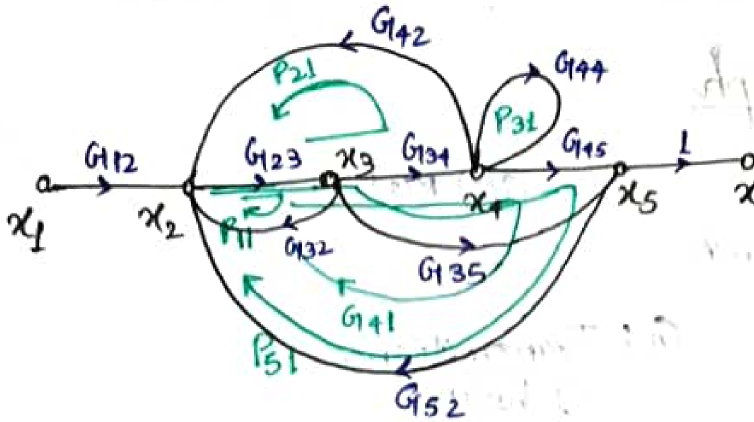
Eg



P_{mr}: Gain product of m^{th} combination of r non-touching loops

P_{m1} : 1 non-touching loop has no meaning

$r=1$: simply gain loop

$M = \frac{X_{out}}{X_{in}}$, M : gain b/w X_{in} and X_{out}

X_{out} : output node variable

X_{in} : Input node variable

N : total no. of forward paths.

$= \frac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}$

P_K : path gain of K^{th} forward path

Δ : determinant of the graph

$\Delta = 1 - \sum_m P_{m1} + \sum_m P_{m2} - \sum_m P_{m3} + \dots$

Δ_K : Value of Δ for the portion of graph not touching K^{th} forward path.

$P_{11} = G_{23} G_{32}$

$P_{21} = G_{23} G_{34} G_{42}$

$P_{31} = G_{44}$

$P_{41} = G_{23} G_{34} G_{45} G_{52}$

$P_{51} = G_{23} G_{35} G_{52}$

$P_{12} = P_{11} P_{31} = (G_{23} G_{32})(G_{44})$

$P_{22} = P_{31} G_{51} = G_{44} (G_{23} G_{35} G_{52})$

$P_{13} = 0$

$P_{23} = 0$

$P_{mr} = 0$, for $r > 2$:

$P_1 = G_{12} G_{23} G_{34} G_{45}$ $[\because G_{56} = 1]$

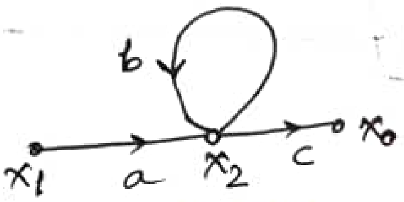
$P_2 = G_{12} G_{23} G_{35}$

$\Delta_1 = 1$ [Removing $x_1 \to x_2 \to x_3 \to x_4 \to x_5 \to x_6$, no graph will be left.
$\Rightarrow \sum_m P_{m_1} = \sum_m P_{m_2} = \dots = 0$]

$\Delta_2 = 1 - G_{44}$ [only P_{31} will remain]

$$\therefore M = \frac{P_1 + P_2(1 - G_{44})}{\Delta}$$

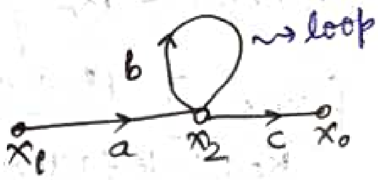
Eg.



$x_2 = a x_1 + b x_2$

$(1-b) x_2 = a x_1 \Rightarrow x_2 = \frac{a}{1-b} x_1$

Eg.

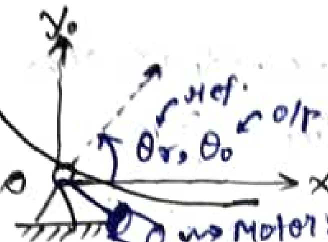


loop (even though dirn of arrow is forward).

$x_2 = a x_1 + b x_2$

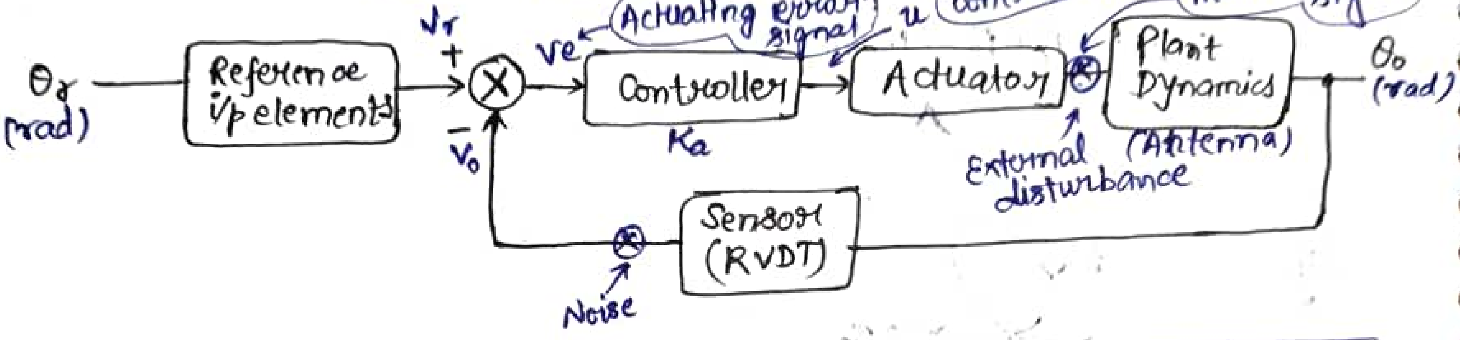
$\Rightarrow x_2 = \frac{a}{1-b} x_1$

Different Components of Control System



→ Motor: High speed, low torque

Reduction Gear → convert to low speed torque



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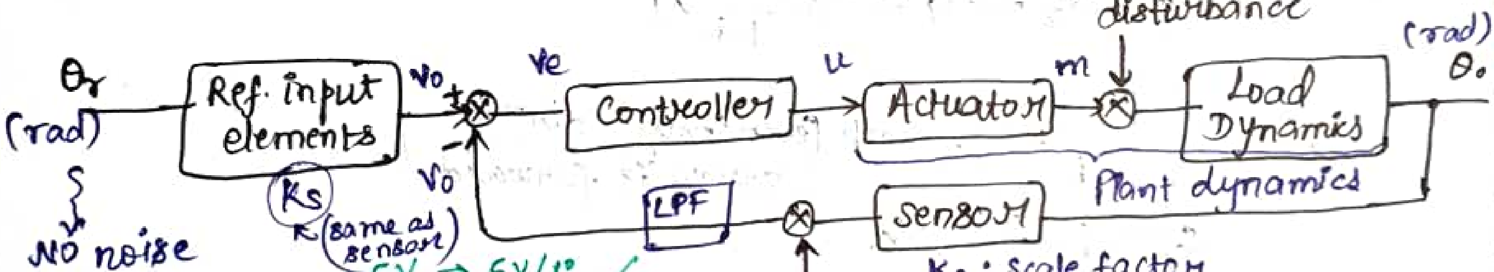


1° → 5V ⇒ 5V/1° ✓

→ 2.5V ⇒ 2.5 V/1° ✗

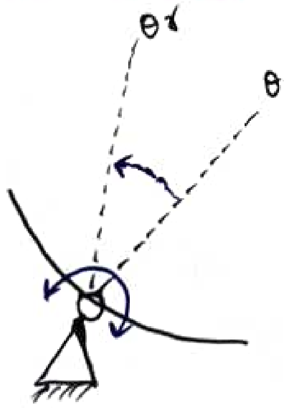
Ks : scale factor

5V/1° (say)

m: manipulating signal

u: control signal

V_e: actuating error signal



→ System dynamics: moment of inertia

$$\tau = \frac{d}{dt}(J\omega) = J\dot{\omega} + \omega \times J\omega$$

→ Due to limit of permanent magnet (limit of flux density), motors are high speed, low torque

↳ But load needs low speed, high torque

↳ Need reduction gear.

Servo motor

↳ low speed, high torque

PMSS Motor

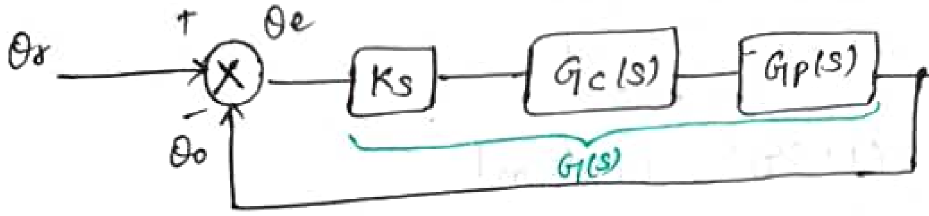
Brushless DC torque motor

→ Actuator contains motors, gear systems.
↳ Generates force : manupulating signal.

→ Disturbances : wind

Controller : Brain
Actuator : Muscle
Load Dynamics : Bones, etc.

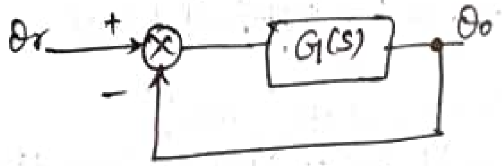
Kalman Filter



$G_p(s) = G_A(s) \cdot G_L(s)$

$$V_e = \theta_r \cdot K_s - \theta_o \cdot K_s$$
$$= K_s(\theta_r - \theta_o)$$
$$= K_s \cdot \theta_e$$

⇓



→ Unity feedback closed loop

$$G(s) = \frac{K(1+s/T_1)\dots(1+s/T_m)}{s^N(1+s/P_1)\dots(1+s/P_n)},$$ ⇝ N poles at origin

$N+n \geq m$

~~$T_i > 0$~~

$P_j > 0$, $K > 0$

$$\lim_{s \to 0} G(s) = \frac{K}{s^N}$$

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→ Without the loss of generality, general control system structure can be drawn as:

$$G(s) = K_S \cdot \underbrace{G_C(s)}_{\text{controller}} \cdot \underbrace{G_A(s)}_{\text{actuator}} \cdot \underbrace{G_L(s)}_{\text{load}}$$

$$G(s) = \frac{K(1+s/\tau_1)\dots(1+s/\tau_m)}{s^N(1+s/p_1)\dots(1+s/p_n)}$$

$N + n \geq m$.

$\underbrace{K>0,\ p_j>0 \text{ for } j\in[1,n]}_{\text{Conditions for stability; poles are on left half plane.}}$

→ How to determine the steady state gain?

↳ we give step signal and wait for a long time, afterwhich the value we get is the gain of the system.

$$Y(s) = G(s) \times \frac{1}{s}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \cancel{s}\frac{G(s)}{\cancel{s}} = \lim_{s\to 0} G(s) = G(0) : \text{steady-state gain}$$

[$s=0$: zero frequency, i.e., steady state]

$$G(0) = \lim_{s\to 0}\left(\frac{K}{s}\right) = \infty$$

$(N=1)$

↳ Type of the system

$$E(s) = \frac{1}{1+G(s)} R(s)$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}.$$

$$r(t) = \frac{t^k}{k!}$$

$k=0$: $r(t) = \frac{t^0}{0!} = \mu(t) = 1$ [Unit Step Command]

$k=1$: $r(t) = t$ [Unit ramp command]

$k=2$: $r(t) = \frac{t^2}{2!}$ [Unit parabolic command]

Unit Step Command

↳ Position Command : $R(s) = \frac{1}{s}$

$$e_{ssp} = \lim_{s\to 0} \frac{s \cdot 1/s}{1+G(s)} = \lim_{s\to 0} \frac{1}{1+G(s)} = \frac{1}{1+K_p}$$

position ↙

$$\therefore \boxed{K_p = \lim_{s\to 0} G(s)}$$

↳ steady-state position error constant.

For type 0 system : $K_p = K$, $e_{ssp} = \frac{1}{1+K}$

For type 1 system : $K_p = \infty$, $e_{ssp} = 0$.

For type 2 system : $K_p = \infty$, $e_{ssp} = 0$.

→ In case of type 1 system, there is an integrator, which accumulates error to ensure zero-steady state for position command.

→ Hydraulic-controlled devices have inbuilt integrator.

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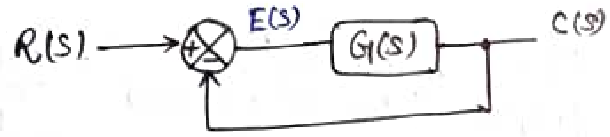
→ 

$$G(s) = \frac{K \prod_{j=1}^{m} \left(\frac{s}{z_j} + 1\right)}{s^N \prod_{i=1}^{n} \left(\frac{s}{P_i} + 1\right)}$$

$r(t) = \frac{t^K}{K!}$; $K=0$: $r(t) = 1$ [unit-step]

$K=1$: $r(t) = t$ [$t \to$ unit ramp or unit velocity]

$N+n \geq m$

$$r(t) = t \longrightarrow \boxed{R(s) = \frac{1}{s^2}}$$

$$E(s) = \frac{R(s)}{1+G(s)}$$

$$e_{ssv} = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} \frac{s \cdot 1/s^2}{1+G(s)} = \lim_{s\to 0} \frac{1}{s+sG(s)} = \lim_{s\to 0} \frac{1}{sG(s)} = \frac{1}{K_v}$$

velocity ↙

$\therefore \boxed{K_v = \lim_{s\to 0} sG(s)}$: Steady-state velocity error constant.

Type 0: $K_v = \lim_{s \to 0} \frac{s \cdot K}{s^0 \cdot 1} = 0$; $\boxed{e_{ssv} = \frac{1}{K_v} = \infty}$

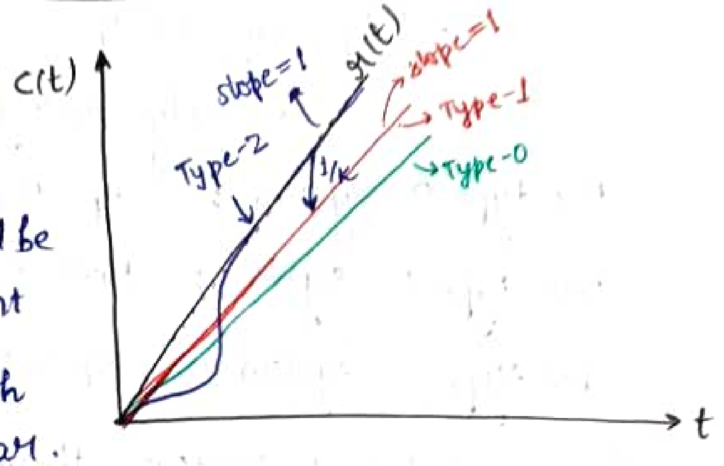
Type 1: $K_v = \lim_{s \to 0} \frac{s \cdot K}{s} = K$; $\boxed{e_{ssv} = \frac{1}{K_v} = \frac{1}{K}}$

Type 3: $K_v = \lim_{s \to 0} \frac{s \cdot K}{s^2} = \infty$; $\boxed{e_{ssv} = \frac{1}{K_v} = 0}$

Type-0 → steady state slopes will be different.

Type-1 → steady state slopes will be same but have different intercepts.

Type-2 → at steady state, both will be collinear.



Unit acceleration :

$r(t) = \frac{t^2}{2}$ [Unit parabolic]

$R(s) = \frac{1}{s^3}$

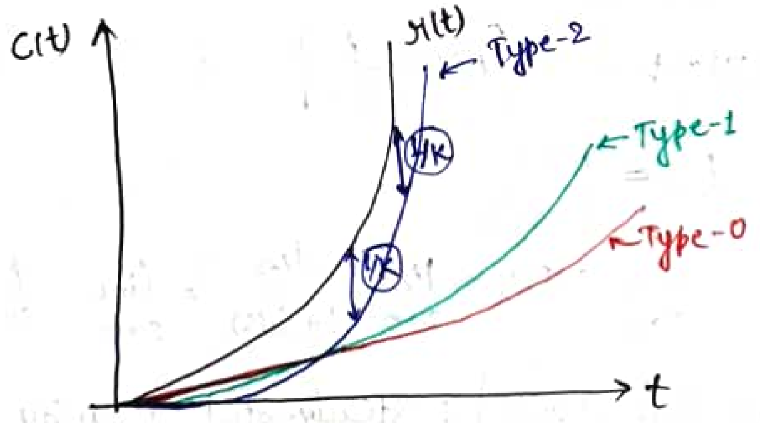
$$e_{ssa} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)} = \lim_{s \to 0} \frac{1/s^2}{1 + G(s)} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)} = \lim_{s \to 0} \frac{1}{s^2 G(s)} = \frac{1}{K_a}$$

$\therefore \boxed{K_a = \lim_{s \to 0} s^2 G(s)}$: steady-state acceleration error constant.

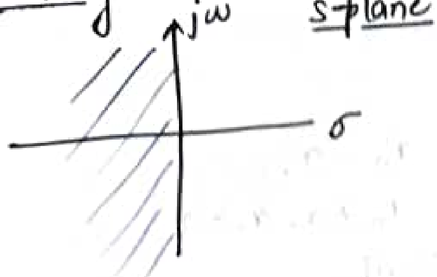
Type 0: $K_a = 0$; $e_{ssa} = \frac{1}{0} = \infty$;)

Type 1: $K_a = 0$; $e_{ssa} = \infty$

Type 2: $K_a = K$; $e_{ssa} = \frac{1}{K}$



Stability:



For stability, all poles should lies on the left-half of s-plane.

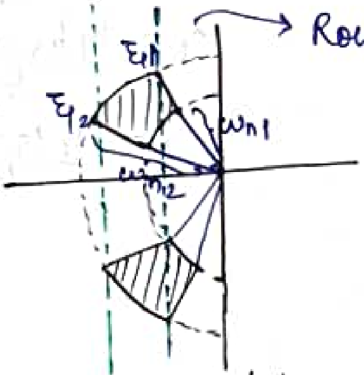
[Rowth Hurwitz]

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{s^n + a_1 s^{n-1} + \dots + a_n}$$

$D(s) = 0$: Characteristic equation of the system.

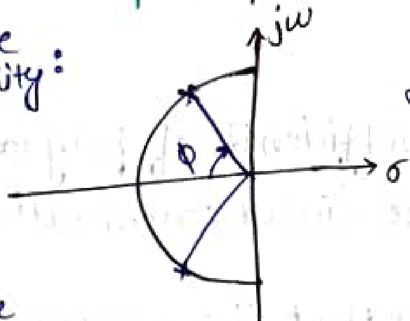
→ Check the no. of poles on left-half, right-half and imaginary axis.

All the poles should be on the left-half of an offset imaginary axis to have a good stability margin : Relative stability.



→ Rowth criteria only tells about a strip (Desired sector).

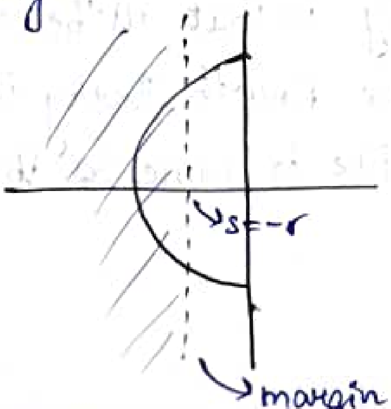
Absolute stability:



$\cos\phi = \xi$ } gives only absolute stability as $\phi \uparrow$, $\xi \downarrow$ (overshoot ↑)

But for actual stability, we should have some max ϕ defined ↳ Margin.

Relative stability:



⇒ Check stability by observing left-side of the margin-line.

→ Damping factor should not also be very high as it makes the system more sluggish. So, we should have a lower and an upper bound of ξ.

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→ $G(s) = \frac{N(s)}{D(s)}$

$D(s) = 0$

$D(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n = 0,$

$a_n \neq 0,\ a_0 > 0$

Necessary condition: $a_j > 0 \ \forall \ j \in [0, n]$.

If $a_j = 0$: ~~imaginary~~ roots on imaginary axis.

$a_j < 0$: roots on right half of plane.

↳ Necessary condition for stability but not sufficient.

Necessary and sufficient condition for stability:

Routh Array

$$
\begin{array}{lllll}
s^n : & a_0 & a_2 & a_4 & \ldots \\
s^{n-1} : & a_1 & a_3 & a_5 & \ldots \\
s^{n-2} : & b_1 & b_2 & b_3 & \ldots \\
s^{n-3} : & c_1 & c_2 & c_3 & \ldots \\
\vdots & & & & \\
s^2 : & * & a_n & & \\
s^1 : & * & 0 & & \\
s^0 : & a_n & & &
\end{array}
$$

$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$

$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$

$c_1 = \frac{b_1 b_3 - a_1 b_2}{b_1}$

$c_2 = \frac{a_1 a_5 - a_1 b_3}{b_1}$

→ First two rows come from coefficients of polynomial and next rows are from the linear combination of first two.

→ Routh Stability criteria states that the necessary and sufficient condition for stability is that all the elements in the first column in the Routh Array is positive.

- No. of zeros on the right side is same as the no. of sign changes.

Eg. $\Delta(s) = s^4 + 8s^3 + 18s^2 + 16s + 5 = 0.$

s^4 :	1	18	5
s^3 :	$\cancel{8}$ 1	$\cancel{16}$ 2	0
s^2 :	$\frac{18-2}{1} = 16$	5	0
s^1 :	$27/16$	0	0
s^0 :	5		

[We can divide a row by some common +ve no.]

$\hookrightarrow$ First column's all elements are positive.

$\Downarrow$

All roots lie on left half of s-plane and a stable system.

Eg. $\Delta(s) = 3s^4 + 10s^3 + 5s^2 + 5s + 2 = 0$

s^4 :	3	5	2
s^3 :	$\cancel{10}$ 2	$\cancel{5}$ 1	0
s^2 :	$7/2$	2	0
s^1 :	$-1/7$	0	0
s^0 :	2	0	0

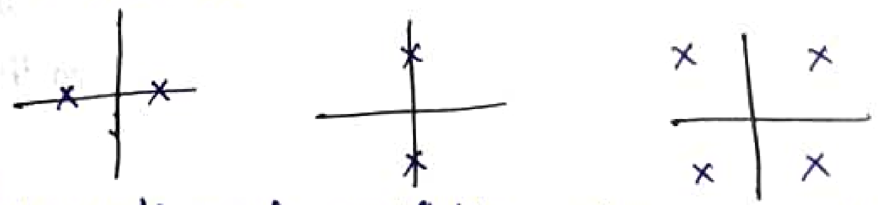
sign change

sign change

Here, first column elements are not all positive $\Rightarrow$ Unstable system.

2 sign changes $\Rightarrow$ 2 characteristic roots lie on right-half of s-plane.

$\hookrightarrow$ Roots can be symmetric w.r.t. real axis, imaginary axis or both axes.



These roots can be available as A(s) or auxillary polynomial (containing even powers only).

$$\boxed{\Delta(s) = \Delta_1(s) \times A(s)}.$$

Eg. $\Delta(s) = s^5 + s^4 + 4s^3 + 24s^2 + 3s + 63 = 0$

$$
\begin{array}{llll}
s^5: & 1 & 4 & 3 \\
s^4: & 1 & 24 & 63 \\
s^3: & \cancel{-20}\ -1 & \cancel{-60}\ -3 & 0 \\
s^2: & \cancel{21}\ 1 & \cancel{63}\ 3 & 0 \\
s^1: & 0 & 0 & \\
s^0: & & &
\end{array}
$$

← Here, s^1 column (first) becomes zero, it means, it contains a factor of even powers.

That even power factor is just above 0,

i.e., $A(s) = s^2 + 3$

Now, $\Delta(s) = A(s) \cdot \Delta_1(s)$

On dividing, $\Delta(s) = \underbrace{(s^2+3)}_{s = \pm j\sqrt{3}} \underbrace{(s^3 + s^2 + s + 21)}_{\Delta_1(s)}$

$$
\begin{array}{lll}
s^3: & 1 & 1 \\
s^2: & 1 & 21 \\
s^1: & -20 & \\
s^0: & 21 &
\end{array}
$$

→ contains –ve terms ⇒ Unstable system

2 sign changes ⇒ 2 roots on right half of s-plane

& 2 on imaginary axis

& 1 on the left half.

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Eg. $\Delta(s) = s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

$$\begin{array}{llll}
s^6 & 1 & 8 & 20 \quad 16 \\
s^5 & \cancel{2}\ 1 & \cancel{12}\ 6 & \cancel{16}\ 8 \\
s^4 & \cancel{2}\ 1 & \cancel{12}\ 6 & \cancel{16}\ 8 \\
s^3 & \cancel{0}\,\cancel{4}\ 1 & \cancel{0}\,\cancel{12}\ 3 & \leftarrow \text{Becomes zero} \\
s^2 & 3 & 8 & \\
s^1 & 1/3 & & \\
s^0 & 8 & &
\end{array}$$

↳ No sign changes ⇒ Roots on left side of Img. axis

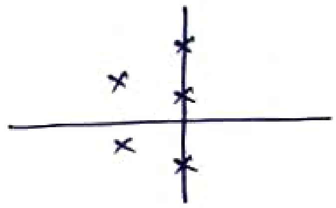
$A(s) = s^4 + 6s^2 + 8$

$\Delta(s) = A(s)\,\Delta_1(s)$

$$\frac{dA(s)}{ds} = 4s^3 + 12s$$

Roots of $A(s)$: $s_{1,2}^2 = \dfrac{-6 \pm \sqrt{36-32}}{2} = \dfrac{-6 \pm 2}{2} = -4, -2$

$s_{1,2,3,4} = \pm j2,\ \pm j\sqrt{2}$

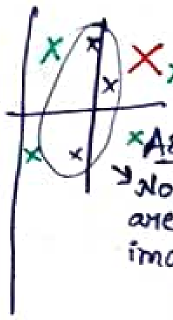


Zero Pivot Element : Case-I

$\Delta(s) = s^5 + 3s^4 + 2s^3 + 6s^2 + 6s + 9$

$$\begin{array}{llll}
s^5 & 1 & 2 & 6 \\
s^4 & \cancel{3}\ 1 & \cancel{6}\ 2 & \cancel{9}\ 3 \\
s^3 & \cancel{0}\ \epsilon & 3 & [\epsilon > 0,\ \epsilon \to 0] \\
s^2 & \frac{2\epsilon - 3}{\epsilon} \to -\frac{3}{\epsilon} & 3 & \\
s^1 & \frac{-9/\epsilon - 3\epsilon}{-3/\epsilon} = 3 & & \\
s^0 & 3 & &
\end{array}$$

↳ Two sign changes
⇓
2 roots on the right side of Img. axis



Assumption: No roots are close to imaginary axis

Zero Pivot Element : Case-II

$$\Delta(s) = s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1$$

s^6	1	3	3	1
s^5	1	3	2	
s^4	$\cancel{0}\,\epsilon$	1	1	
s^3	$\frac{3\epsilon-1}{\epsilon} = -\frac{1}{\epsilon}$	$\frac{2\epsilon-1}{\epsilon} = -\frac{1}{\epsilon}$		
s^2	$\frac{-1/\epsilon + 1}{-1/\epsilon} = 1$	1		
s^1	0			
s^0				

[Assumption: No roots on Img-axis]

$A(s) = s^2+1, \quad \Delta(s) = \Delta_1(s) A(s)$

$$s^2+1 \overline{\smash{\big)}\, s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1} \big(s^4 + s^3 + 2s^2 + 2s + 1$$

$$\begin{array}{l}
s^6 + 0 + s^4 \\
\hline
s^5 + 2s^4 + 3s^3 + 3s^2 + 2s + 1 \\
s^5 \qquad + s^3 \\
\hline
2s^4 + 2s^3 + 3s^2 + 2s + 1 \\
2s^4 \qquad + 2s^2 \\
\hline
2s^3 + s^2 + 2s + 1 \\
2s^3 \qquad + 2s \\
\hline
s^2 + 1 \\
s^2 + 1 \\
\hline
0
\end{array}$$

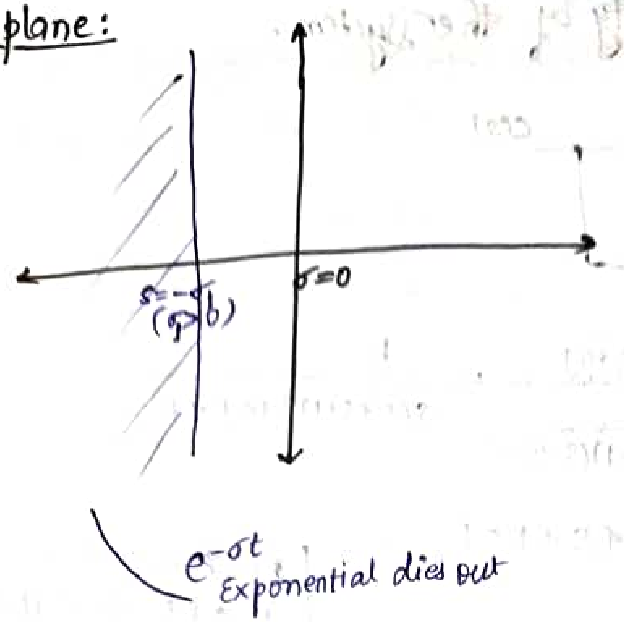
$\therefore \Delta_1(s) = s^4 + s^3 + 2s^2 + 2s + 1$

s^4	1	2	1
s^3	1	2	
s^2	$\cancel{0}\,\epsilon$	1	
s^1	$\frac{2\epsilon-1}{\epsilon}$		
s^0	1		

[No roots on Img-axis]

[2 roots on LHP; 2 roots on RHS; 1 conjugate pair on Img-axis]

S-plane:



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$\hat{s}=s+\sigma_1 \Rightarrow s=\hat{s}-\sigma_1$

$(s+P_1)(s+P_2)=0$

$\Rightarrow (\hat{s}-\sigma_1+P_1)(\hat{s}-\sigma_1+P_2)=0$

$\Rightarrow [\hat{s}+(P_1-\sigma_1)][\hat{s}+(P_2-\sigma_2)]=0$

$\Rightarrow (\hat{s}+\hat{P}_1)(\hat{s}+\hat{P}_2)=0$, $\hat{P}_1=P_1-\sigma_1$, $\hat{P}_2=P_2-\sigma_2$

Roots of $\hat{s}$ plane are $-\hat{P}_1, -\hat{P}_2$

By Routh-criterion, $-\hat{P}_1<0$, $-\hat{P}_2<0$

$\Rightarrow -(P_1-\sigma_1)<0$

$\Rightarrow -P_1+\sigma_1<0$ or $-P_1<-\sigma_1$

Similarly, $-P_2<-\sigma_2$

[$-P_1$ lies to the left of $-\sigma_1$; $-P_2$ to the left of $-\sigma_2$]

Eg. $\Delta(s)= s^3+7s^2+25s+39=0$.

Ensure that all the roots of $\Delta(s)$ are to the left of $s=-1$.

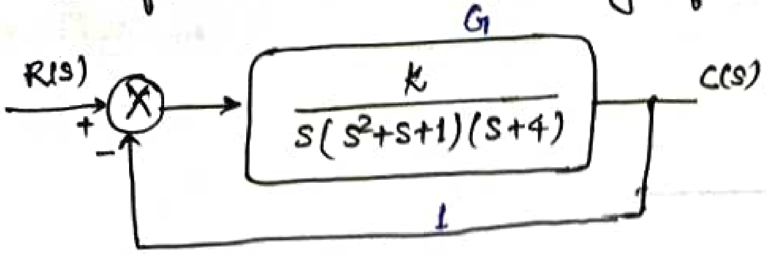
$\sigma_1=1$

$\hat{s}=s+1 \Rightarrow s=\hat{s}-1$

$\Delta(\hat{s})=\hat{s}^3+4\hat{s}^2+14\hat{s}+20$

$\hat{s}^3$:	1	14
$\hat{s}^2$:	~~4~~ 1	~~20~~ 5
$\hat{s}^1$:	9	
$\hat{s}^0$:	5	

Eg Value of K to ensure the stability of the system.



$$G_{CL}(s) = \frac{G}{1+G} = \frac{\frac{K}{s(s^2+s+1)(s+4)}}{1+\frac{K}{s(s^2+s+1)(s+4)}} = \frac{K}{s(s^2+s+1)(s+4)+K}$$

$$\Rightarrow \Delta(s) = s^4 + 5s^3 + 5s^2 + 4s + K = 0$$

Routh criterion can be used to find the PID coefficients.

s^4 :	1	5
s^3 :	5	4
s^2 :	$21/5$	K
s^1 :	$\frac{84/5 - 5K}{21/5}$	
s^0 :	K	

$K > 0$, [If K is negative, make the feedback positive]

$$\frac{84}{5} - 5K > 0$$

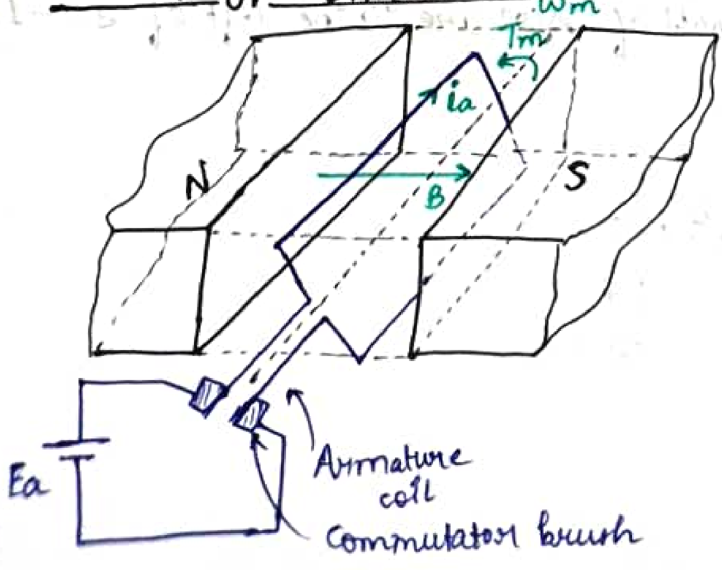
$$\Rightarrow 84 > 25K$$

$$\Rightarrow \frac{84}{25} > K$$

$$\therefore \boxed{0 < K \leq 84/25}$$

If relative order of the system > 2 :
Increasing gain, system becomes unstable.

→ **Brush-Type Armature-Controller DC Torque Motor :**



→ Force dirⁿ given by Fleming's left hand rule :
mid-finger : i
index-finger : B
⇓
thumb : Force

Motor torque, $T_m = K_T i_a$

Back emf, $E_b = K_b \, \omega_m$

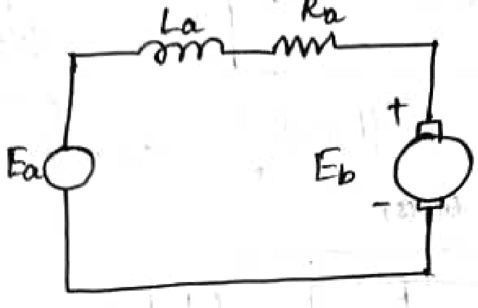
$K_T = K_b$

$\begin{bmatrix} K_T \to N\text{-}m/A \\ K_b \to V/(rad/s) \end{bmatrix}$

PMSM :
↳ gives smooth torque

Equivalent Circuit of Armature Excitation:

Direction / Polarity of E_b is such that it opposes its cause (i.e., current).



J : Mo.I of mechanical load

B : viscous damping coefficient

θ_m : angle of motor shaft

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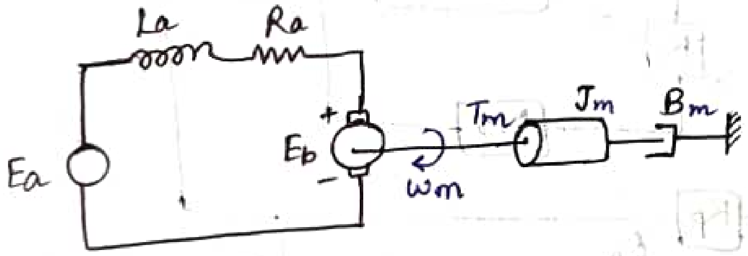


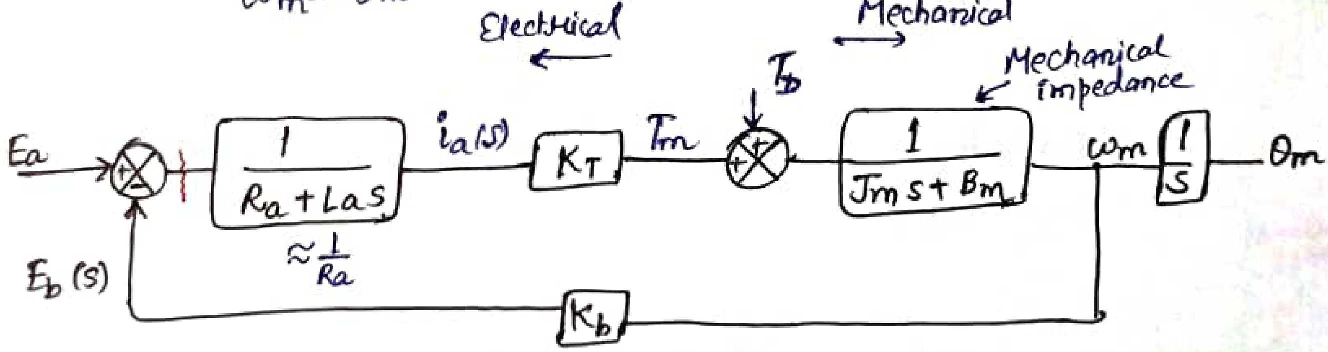
$T_m = K_T i_a$

$E_b = K_b \omega_m$

$E_a = (R_a + L_a s)\, i_a(s) + E_b(s)$

$T_m(s) = [J_m s^2 + B_m s]\, \theta_m(s)$

$\omega_m = \dot{\theta}_m$



$$\frac{\omega_m}{E_a}(s) = \frac{\dfrac{K_T/R_a}{J_m s + B_m}}{1 + \dfrac{K_T/R_a}{J_m s + B_m} \cdot K_b} = \frac{K_T/R_a}{J_m s + \underbrace{\left(B_m + \frac{K_T K_b}{R_a}\right)}_{\text{extra damping}}}$$

(ignore T_D)

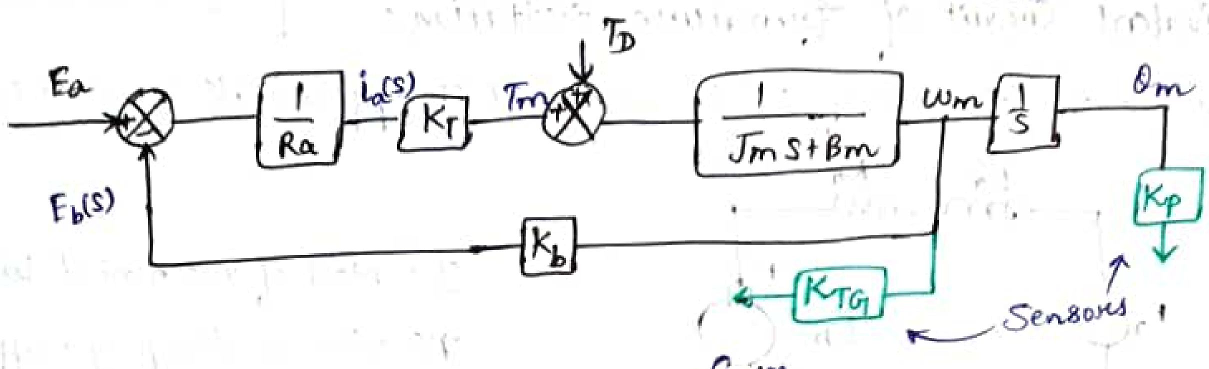
extra damping due to EM } Passive dynamics

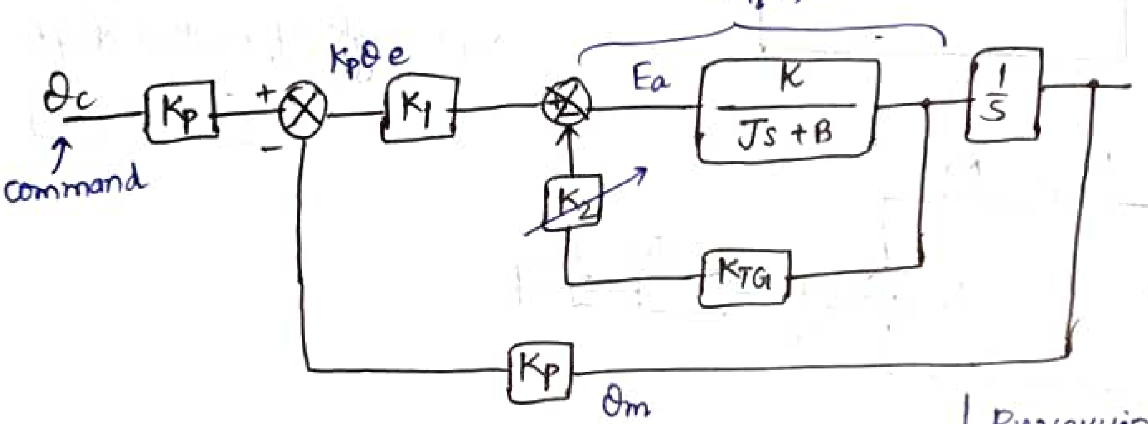
Open loop motor: Moving motor manually (T_D)
⇓
we experience only 'B_m' damping

$$\therefore \frac{\omega_m}{E_a}(s) = \frac{K}{Js+B}, \text{ where } J = J_m,\ K = K_T/R_a$$
$$B = B_m + \frac{K_T K_b}{R_a}$$

$$\frac{\theta_m}{E_a}(s) = \frac{K}{s(Js+B)} \rightarrow \text{Actuator + Load : Type-I system}$$

Actuator → $K_T/R_a = K$, Load → $\frac{1}{Js+B}$





By varying K_2, damping can be changed.

$$G_1(s) = \frac{\frac{K}{Js+B}}{1 + \frac{K K_2 K_{TG}}{Js+B}} = \frac{K}{Js + (B + K K_2 K_{TG})}$$

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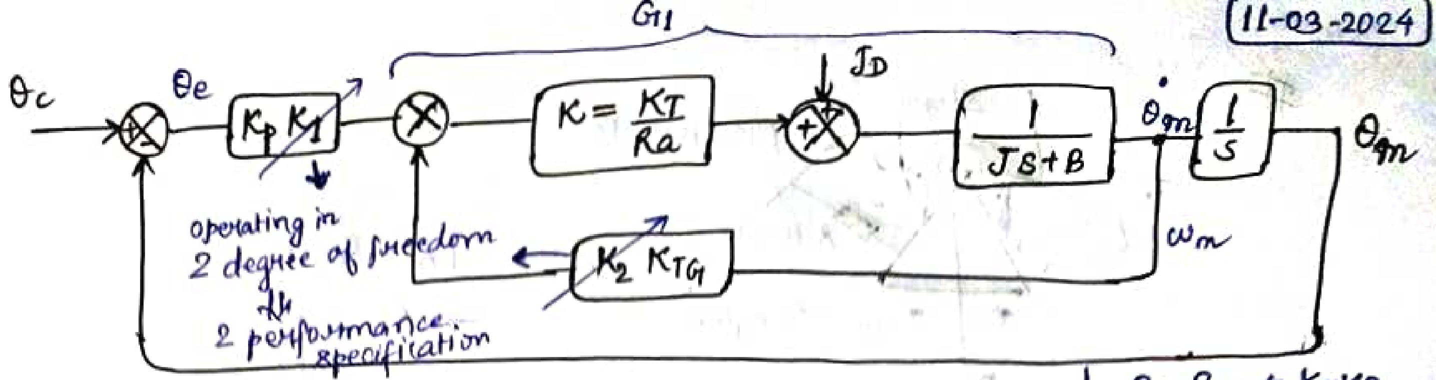


$B = B_m + \frac{K_T K_B}{R_a}$

$$G_1 = \frac{\frac{K}{Js+B}}{1 + \frac{K K_2 K_{TG}}{Js+B}} = \frac{K}{Js + B_{eq}}, \quad B_{eq} = B + K K_2 K_{TG}$$

$$G_{CL} = \frac{\left(\frac{K_p K_1 \cdot K}{Js + B_{eq}}\right)\frac{1}{s}}{1 + \left(\frac{K_p K_1 K}{Js + B_{eq}}\right)\frac{1}{s}} = \frac{K_p K_1 K}{Js^2 + B_{eq}s + K_p K_1 K}$$

$$= \frac{K_p K_1 K / J}{s^2 + \frac{B_{eq}}{J}s + \frac{K_p K_1 K}{J}}$$

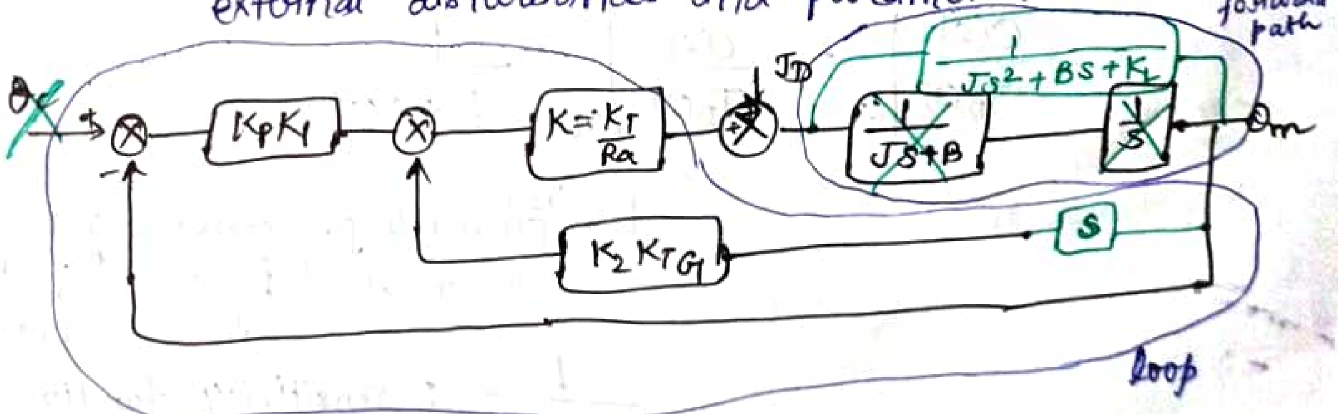
$$= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore \frac{K_p K_1 K}{J} = \omega_n^2$$

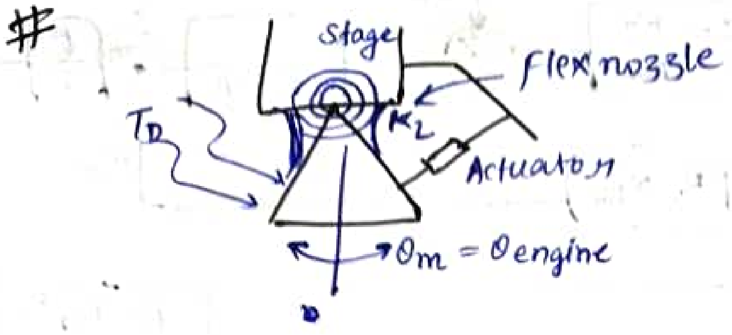
$$\frac{B_{eq}}{J} = 2\xi\omega_n = \frac{B + K K_2 K_{TG}}{J}$$

$\Rightarrow$ Get K_1, K_2.

Robustness: Closed-loop performance is least affected by external disturbances and parameter values.



#



$$T_D = \theta_m \cdot K_L$$

$$\Rightarrow \theta_m = \frac{K_D}{K_L}$$

$$\frac{\theta_m}{T_D}(s) = \frac{\dfrac{1}{Js^2+Bs+K_L}}{1+\dfrac{(K_pK_1+K_2K_{TG}s)s}{Js^2+Bs+K_L}} = \frac{1}{Js^2+Bs+K_L+K_pK_1K + K_2K_{TG}K_S}$$

Steady-state:

$$\lim_{s\to 0}\frac{\theta_m(s)}{T_D} = \frac{1}{K_L + K_pK_1K}, \quad K_L + K_pK_1K : \text{servo stiffness}$$

extra stiffness added to passive stiffness

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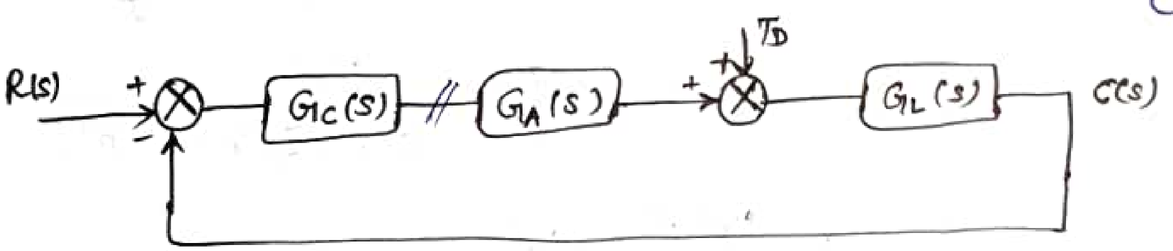


T.F. w.r.t. disturbance input:

$$\left.\frac{C(s)}{T_D(s)}\right|_{R(s)=0} = \frac{G_L(s)}{1+G_L(s)\cdot G_C(s)\cdot G_A(s)}$$

$$= \frac{G_L(s)}{1+L(s)}, \quad L(s) = G_C(s)\,G_A(s)\cdot G_L(s).$$

$$\left.\frac{C(s)}{T_D(s)}\right|_{CL} = \left.\frac{C(s)}{T_D(s)}\right|_{OL} \times \frac{1}{1+L(s)}$$

↳ Using Closed loop, sensitivity to disturbance is reduced by factor $\left[\frac{1}{1+L(s)}\right]$.

$\frac{1}{1+L(s)}$: Sensitivity function
↳ provides rejection to external disturbance

Parameter Sensitivity :

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{\text{Fractional change in } F}{\text{Fractional change in } P}$$

$F(P)$

↳ sensitivity of f^n F w.r.t. parameter P

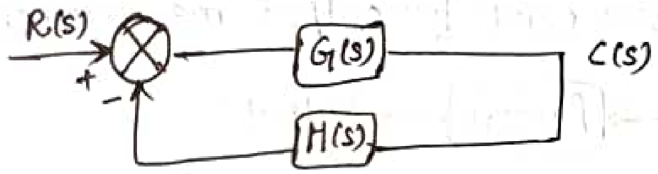
$$= \lim_{\Delta P \to 0} \frac{\Delta F / F}{\Delta P / P}$$

$$= \frac{P}{F} \frac{\partial F}{\partial P}$$

Eg. $F = \overset{\text{changing}}{K_1} K_2 K_3$ [open loop]

$$\frac{\partial F}{\partial K_1} = K_2 K_3$$

$$S_{F:P} = \frac{K_1}{K_1 K_2 K_3} \cdot K_2 K_3 = 1$$



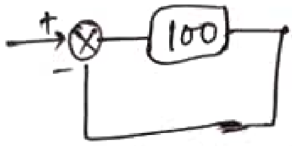
$$\frac{G(s)}{F(s)} \frac{\partial F(s)}{\partial G(s)} = 1 \quad \text{[open-loop]}$$

$$G_{CL} = \frac{G}{1+GH}$$

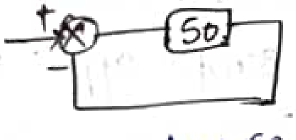
$$\underset{(F)\ \ (P)}{S_{G_{CL}:G}} = \frac{G}{G_{CL}} \frac{\partial G_{CL}}{\partial G}$$

$$= \frac{G(1+GH)}{G} \frac{\partial \frac{G}{1+GH}}{\partial G}$$

$$= (1+GH) \frac{(1+GH) \times 1 - GH}{(1+GH)^2} = \frac{1}{1+L(s)}$$



$$\rightarrow \frac{100}{1+100} = \frac{100}{101}$$



$$\rightarrow \frac{50}{51}$$

$\Rightarrow$ Closed loop ↓ Rejects the variation of parameter in forward path

$100 > 50$

$$\frac{100}{101} \approx \frac{50}{51} \approx 1$$

$$S_{G_{cl}:H} = \frac{H}{\left(\frac{G}{1+GH}\right)} \frac{\partial\left(\frac{G}{1+GH}\right)}{\partial H}$$

$$= \frac{H(1+GH)}{G} \left[\frac{(1+GH)\times 0 - G^2}{(1+GH)^2}\right]$$

$$= -\frac{GH}{1+GH}$$

$$= \frac{-L}{1+L}$$

$$L \to \infty \Rightarrow S_{G_{cl}:H} \approx -1$$

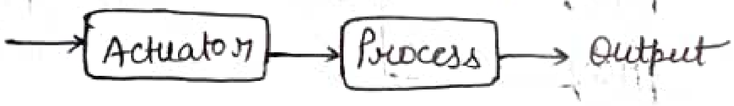
15-03-2024

Open Loop Control

Advantages:

① Minimum no. of subsystems and components, i.e., simple and cheaper.

② Good when no simple sensor exists for output measurement.



③ Can be made as pre-programmable based on general process requirement if the system is stable.

Disadvantages:

① Not useful when the open-loop system is unstable.

② Do not have robustness against parameter variation and disturbance.

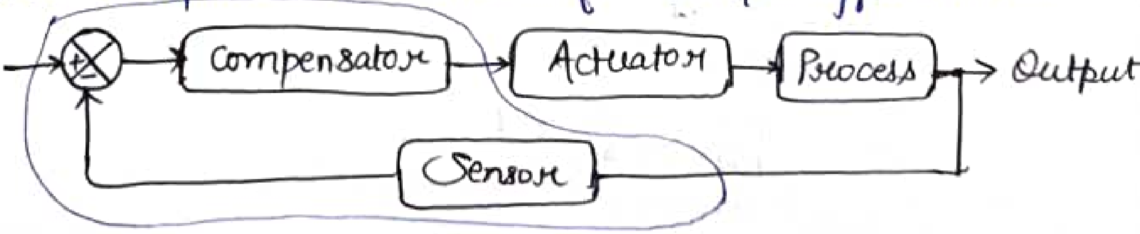
③ Cannot have versatile dynamic performance.

Local closed-loop for subsystems can be used.

Closed Loop Control

Disadvantages:

① More no. of subsystems like sensors, compensators, etc., and hence more complex and costlier for simple application.



Advantages:

1. An unstable process can be stablised by closed-loop feedback control.
2. Robust against external disturbance and parameter variation.
3. Dynamic performance can be tuned using appropriate compensator.
4. Cheaper for high power application by using commercial actuation system.

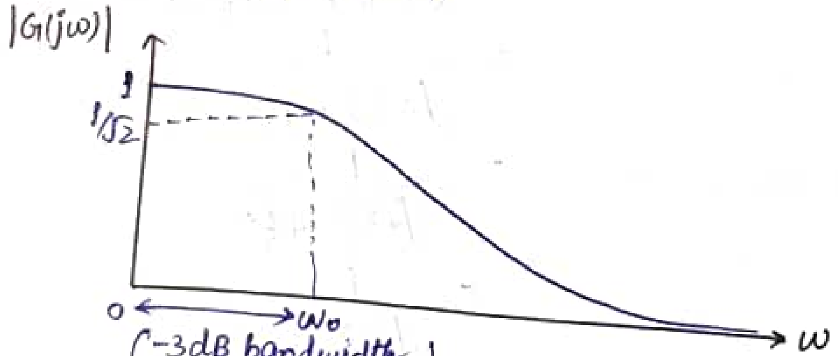
Frequency Response

$G(j\omega) \rightarrow$ "tailor-cut method" to shape compensator to fit with the closed-loop system.

$$G(s) = \frac{\omega_0}{s+\omega_0}$$ ⇝ first-order low pass filter

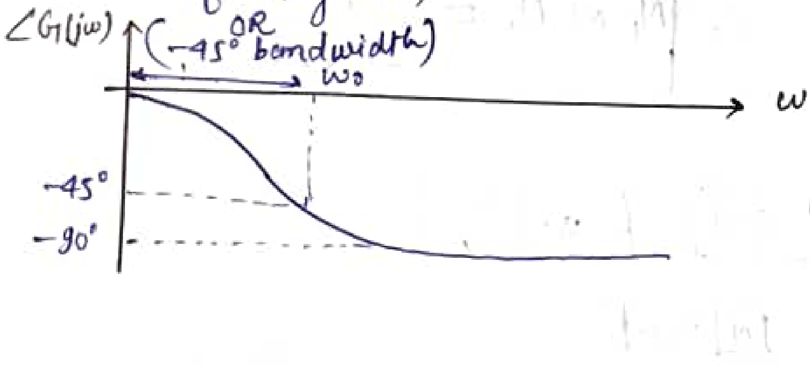
$$G(j\omega) = \frac{\omega_0}{j\omega + \omega_0} = \frac{1}{1+j(\omega/\omega_0)}$$

$$= \frac{1}{\sqrt{1+(\omega/\omega_0)^2}} \angle\left(-\tan^{-1}(\omega/\omega_0)\right)$$



(−3dB bandwidth of the system)

OR



(−45° bandwidth)

$\rightarrow$ $$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$: Second-order low-pass filter

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + \omega_n^2 + j2\xi\omega_n\omega} = \frac{1}{(1-\omega^2/\omega_n^2) + j2\xi\frac{\omega}{\omega_n}}$$

$$= \frac{1}{(1-u^2) + j2\xi u}, \quad u = \omega/\omega_n$$

$\boxed{19\text{-}03\text{-}2024}$

$$G(j\omega) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \left(-\tan^{-1}\frac{2\xi u}{1-u^2} \right)$$

Peak of $|G(j\omega)|$ occurs at the minima of $(1-u^2)^2 + (2\xi u)^2 = f(u)$.

$$\frac{\partial f(u)}{\partial u} = 2(1-u^2)(-2u) + 2(2\xi u)(2\xi) = 0$$

$$\Rightarrow (1-u^2) = 2\xi^2$$

$$\Rightarrow 1 - 2\xi^2 = u^2$$

$$\Rightarrow u = \sqrt{1-2\xi^2}$$

$$\therefore \boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$$

$$\therefore |G(j\omega)|_{peak} : M_p \text{ or } M_r = \frac{1}{\sqrt{(2\xi^2)^2 + 4\xi^2(1-2\xi^2)}}$$

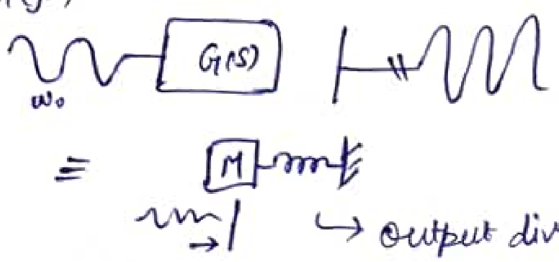
$$= \frac{1}{\sqrt{4\xi^4 + 4\xi^2 - 8\xi^4}}$$

$$= \frac{1}{\sqrt{4\xi^2 - 4\xi^4}}$$

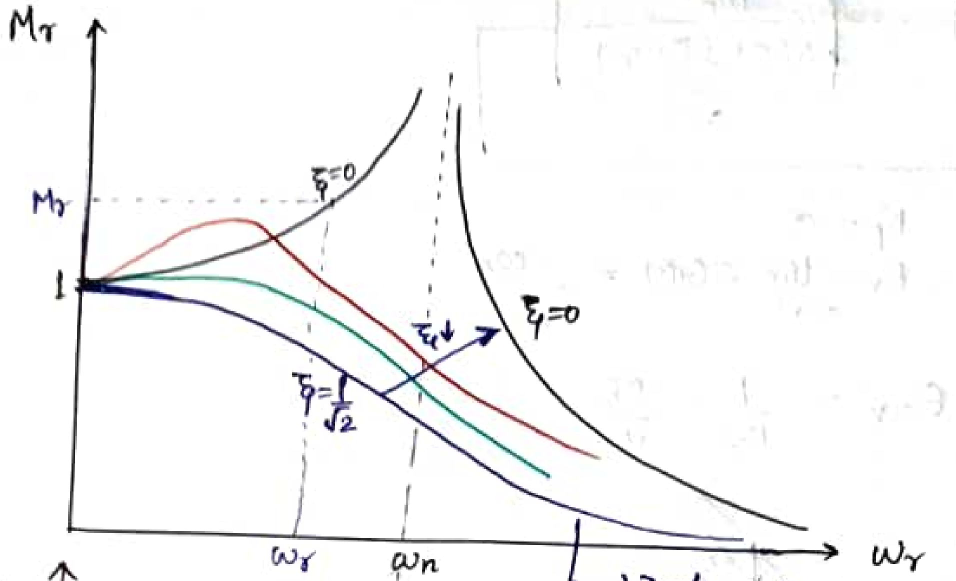
$$= \frac{1}{\sqrt{4\xi^2(1-\xi^2)}}$$

$$\therefore \boxed{M_p \text{ or } M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

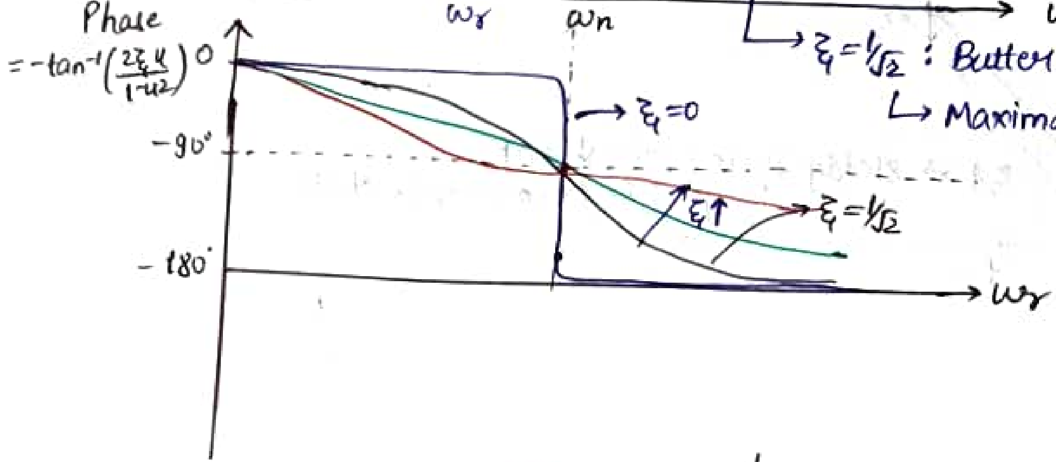
$\xi = 0$: $G(j\omega)$



↪ output diverges till ∞.

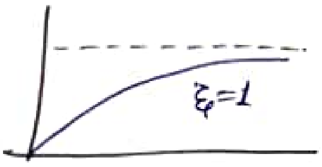


$\hookrightarrow \xi = 1/\sqrt{2}$: Butterworth filter $\xrightarrow{\text{class of filters having}}$
$\hookrightarrow$ Maximally flat response



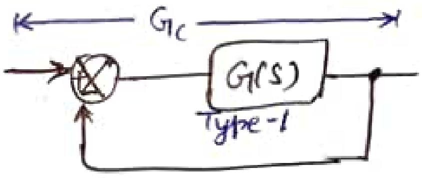
$$M_{peak} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$\xi = 1 \Rightarrow M_{peak} = 0$



Phase lag is more important than Mpeak.

Eg. $G_c(s) = \dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$ ⇝ ~~ope~~ closed-loop [Unit-step-response]



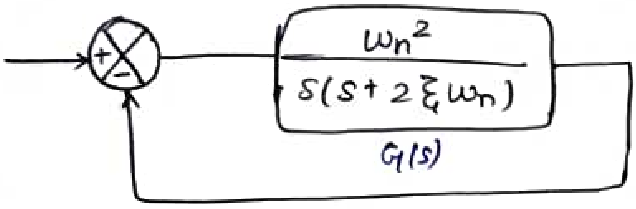
$$G_c = \frac{G}{1+G}$$

$$\Rightarrow G_c(1+G) = G$$

$$\Rightarrow G_c = G(1-G_c)$$

$$\Rightarrow G = \frac{G_c}{1-G_c}$$

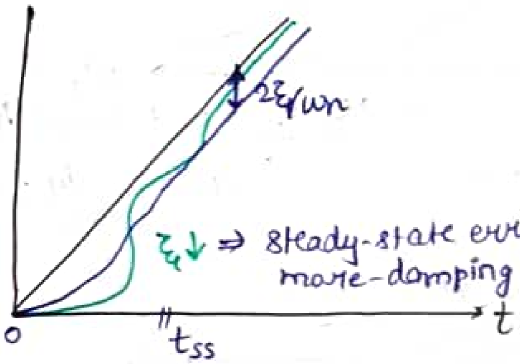
$$\therefore G = \frac{\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}{1-\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$



$$K_p = \infty$$

$$K_v = \lim_{s\to 0} sG(s) = \frac{\omega_n}{2\xi}$$

$$e_{ssv} = \frac{1}{K_v} = \frac{2\xi}{\omega_n}$$



$\xi\downarrow \Rightarrow$ steady-state error $\downarrow$ but more-damping in transient-state

22-03-2024

Bode Plot

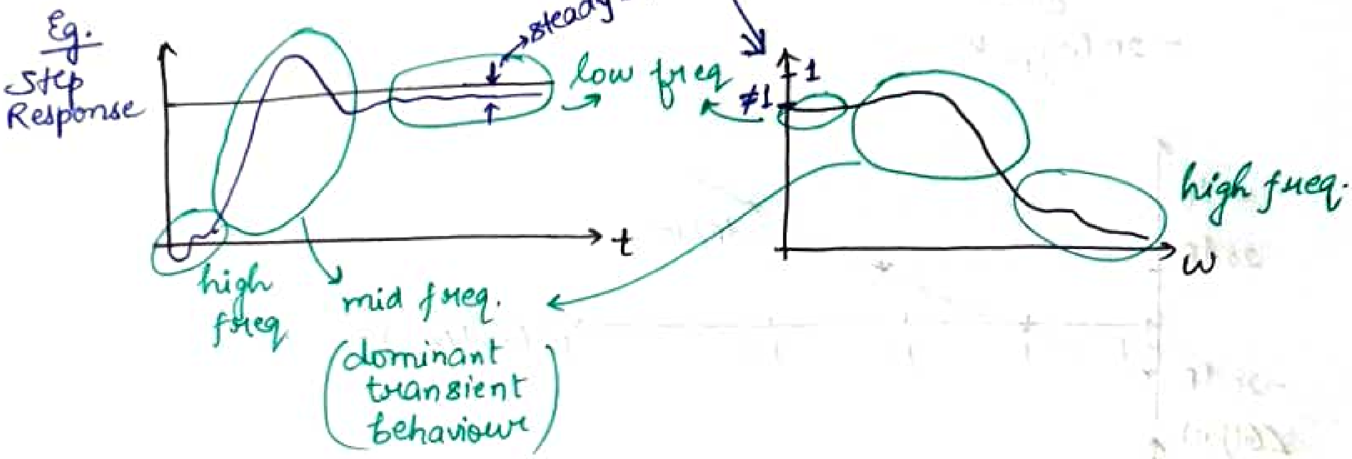
$G(s)$
$\downarrow$
$G(j\omega) = |G(j\omega)| \angle G(j\omega)$

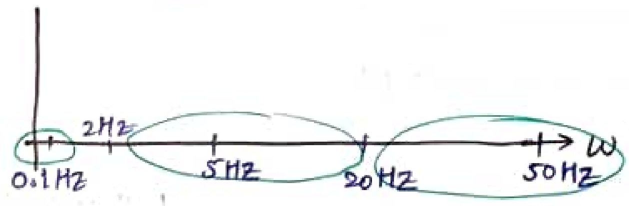
$$Mag = 20 \log_{10} |G(j\omega)|$$

Angle = $\angle G(j\omega)$ in degrees.

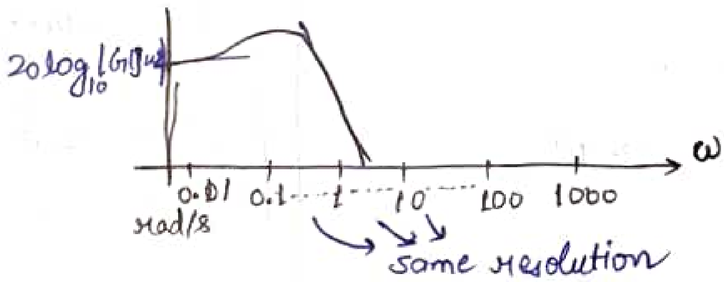
ω will be in log-axis.

Eg.





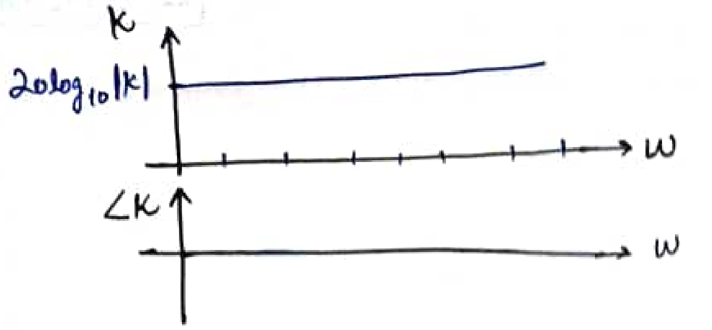
⇝ In bodeplot, all frequencies are important equally.
⇓
Use log-scale.



$$G(j\omega) = \frac{K}{(j\omega)^N} \frac{(1 + j\omega/z_1) \ldots (1 + j\omega/z_m)}{(1 + j\omega/p_1) \ldots (1 + j\omega/p_n)}$$

$$Mag = 20\log_{10}\left|\frac{K}{(j\omega)^N}\right| + 20\log_{10}|1 + j\omega/z_1| \ldots - 20\log_{10}|1 + j\omega/p_1| - \ldots$$

$$Phase = \angle \frac{K}{(j\omega)^N} + \angle(1 + j\omega/z_1) + \ldots - \angle(1 + j\omega/p_1) - \ldots$$

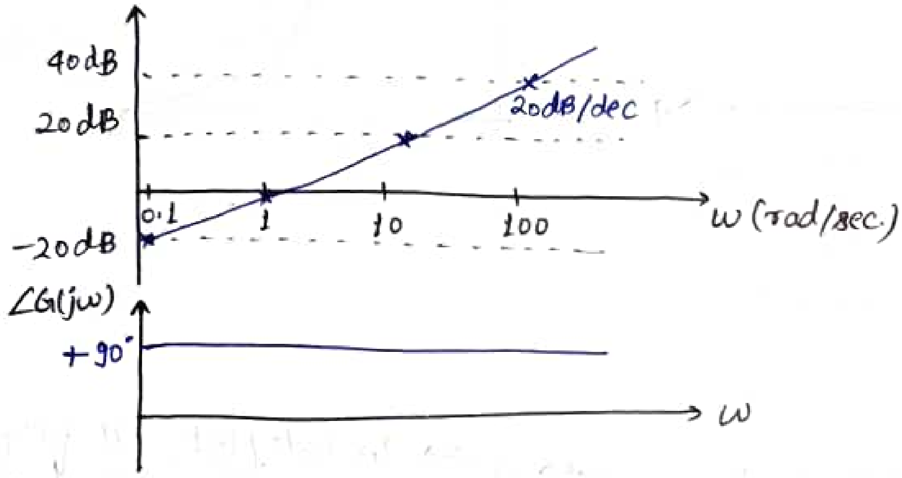


→ $G(s) = s$

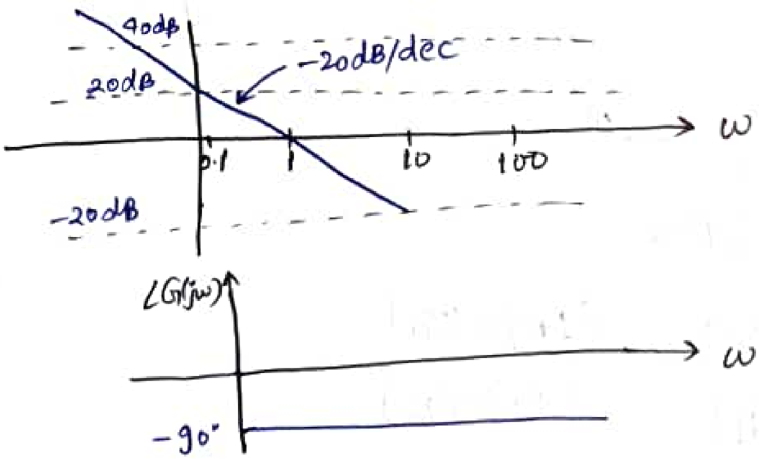
$G(j\omega) = j\omega$

$M = 20\log_{10}|j\omega|$

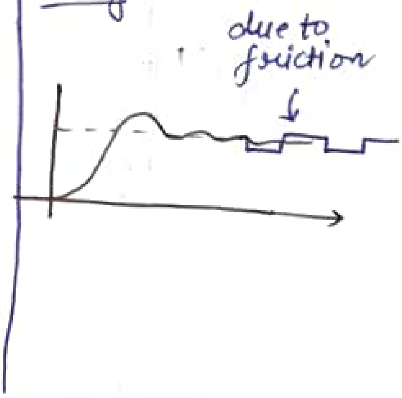
$= 20\log_{10}\omega$



→ $G(s) = \frac{1}{s}$ ⇝ Integrator



Integrator:



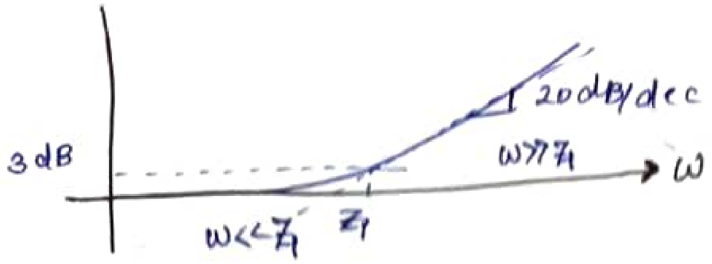
→ $G(s) = (1 + s/z_1)$

$G(j\omega) = |1 + j\omega/z_1| \angle \tan^{-1}(\omega/z_1)$

$Mag = 20\log_{10}\left(\sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}\right)$

$\omega \ll z_1$: $Mag = 0$ dB

$\omega \gg z_1$: $Mag = 20\log_{10}(\omega/z_1) = 20\log_{10}\omega - 20\log_{10}z_1$



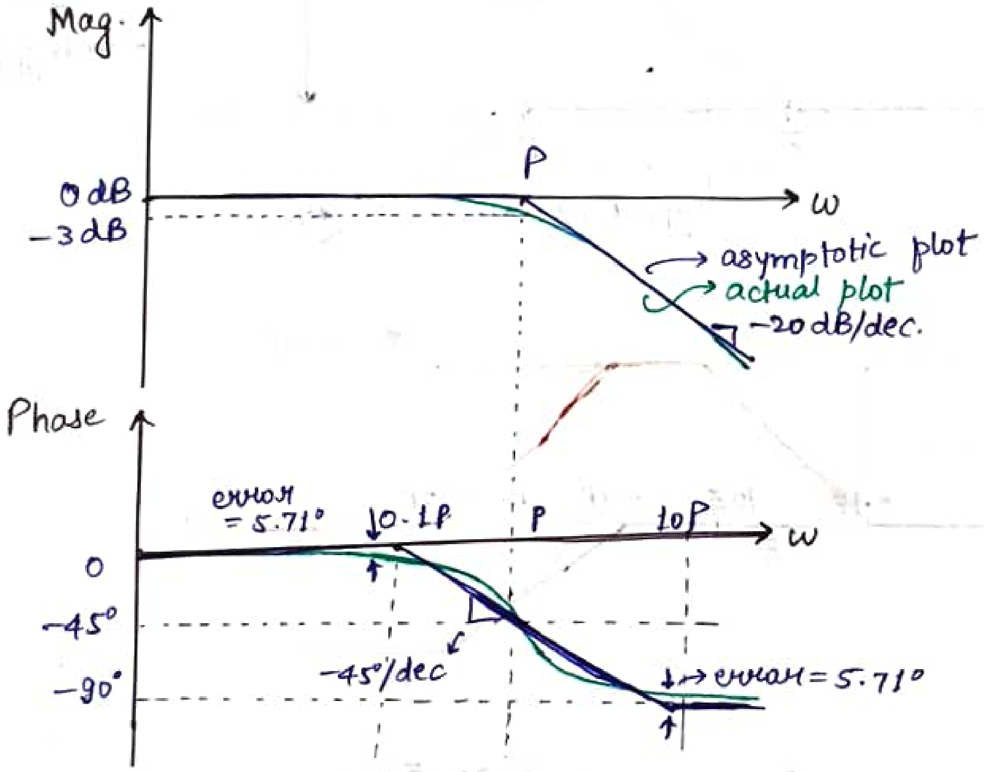
26-03-2024

→ $G(s) = \dfrac{\omega_0}{s+\omega_0}$

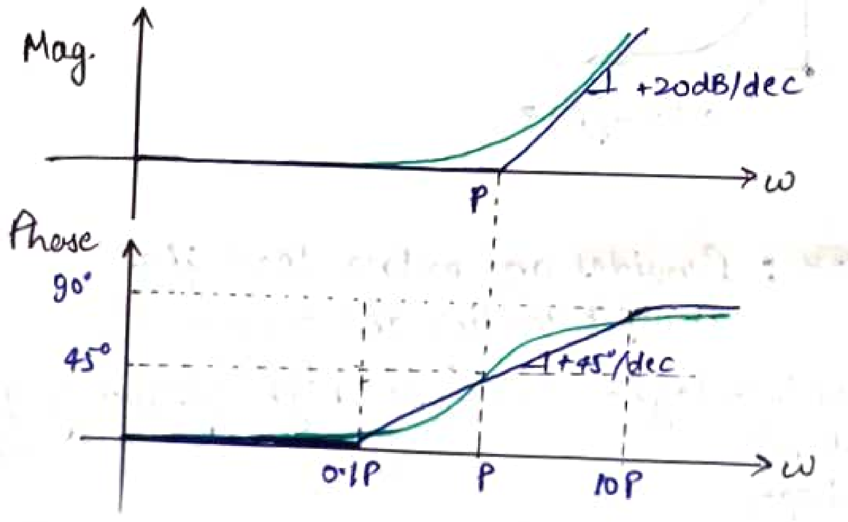
- $G(j\omega) = \dfrac{1}{1+j\omega/p}$, $p = \omega_0$

$$\text{Mag.} = 20\log_{10}\frac{1}{\sqrt{1+(\omega/p)^2}}$$

$$\text{Phase} = -\tan^{-1}(\omega/p)$$



- $G(j\omega) = 1 + j\omega/z$

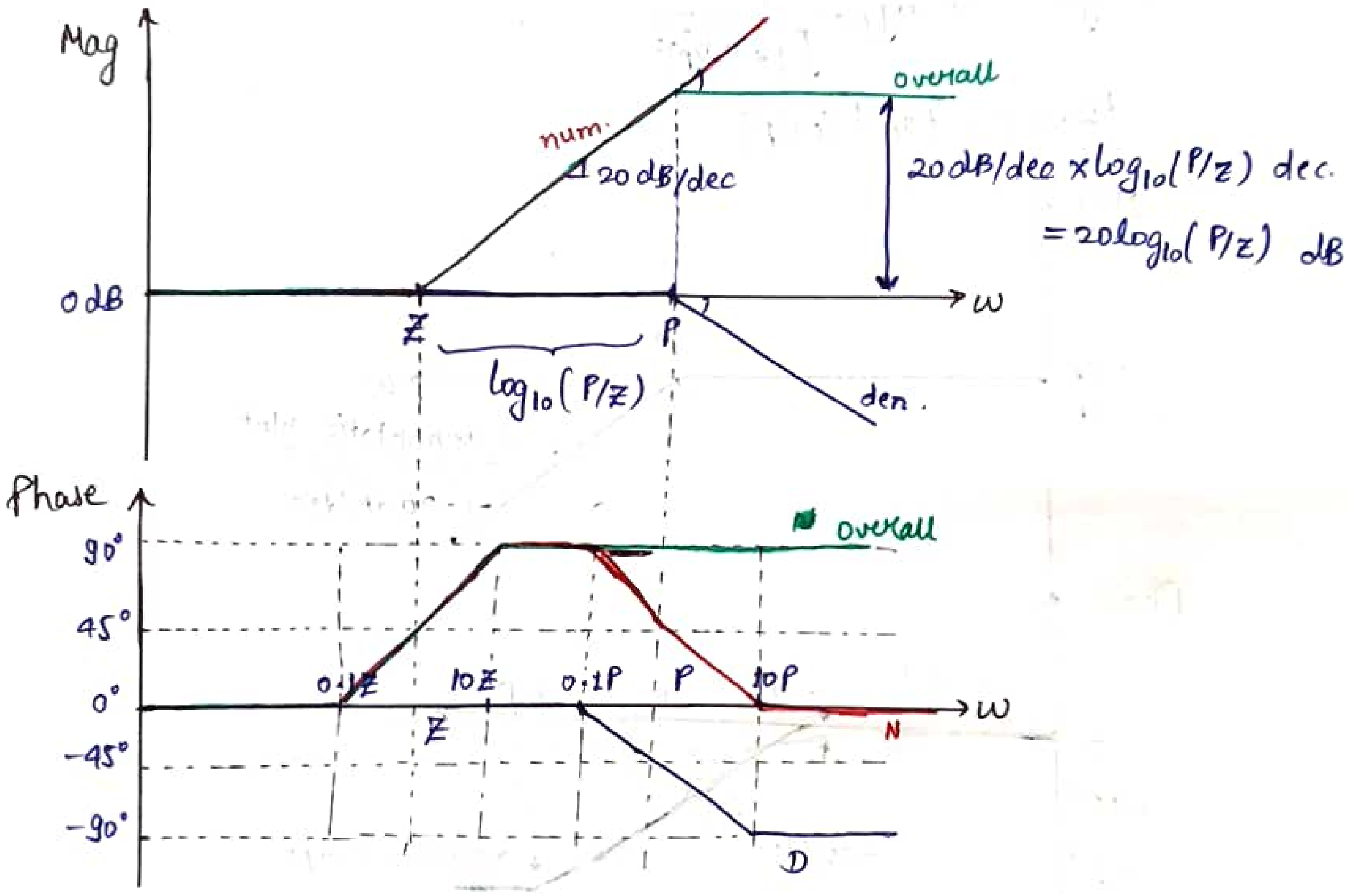


Lead Compensator

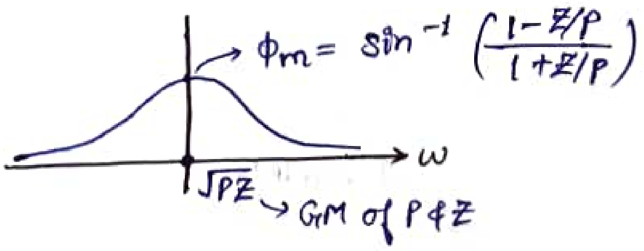
$$G(s) = \frac{1+s/z}{1+s/p}, \quad z<p$$

$$G(j\omega) = \frac{1+j\omega/z}{1+j\omega/p}$$

$$\text{Mag.} = 20\log_{10}|1+j\omega/z| + 20\log_{10}\left|\frac{1}{1+j\omega/p}\right|$$



Actual Phase Plot:



Lead compensator: Provides an extra lead phase [output leading the command]

↳ Disadvantage: Increase high frequency gain. (make noise)

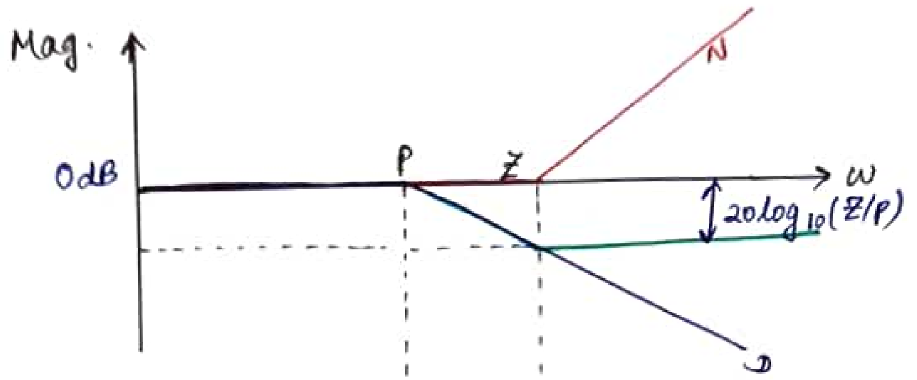
+ve Phase : Lead
-ve Phase : Lag

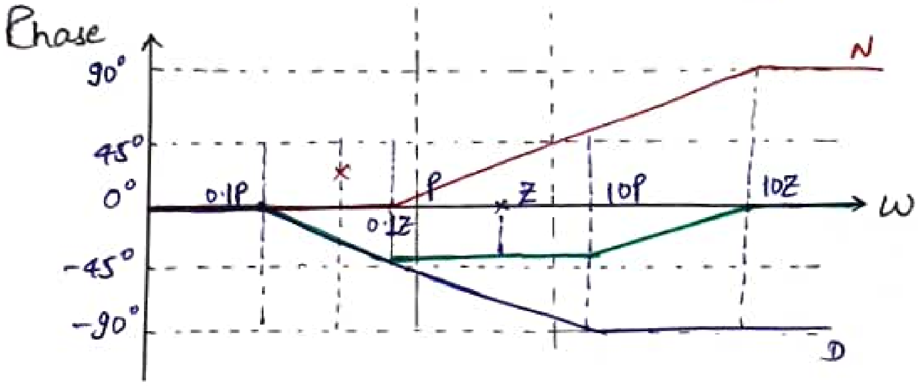
Lag Compensator :

$$G(s) = \frac{1+s/Z}{1+s/P}, \quad P < Z$$

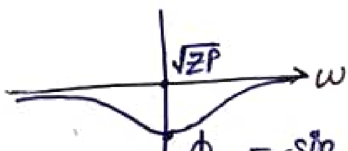
$$G(j\omega) = \frac{1+j\omega/Z}{1+j\omega/P}$$

$$\text{Mag.} = 20\log_{10}|1+j\omega/Z| + 20\log_{10}\left|\frac{1}{1+j\omega/P}\right|$$





Actual Phase plot :



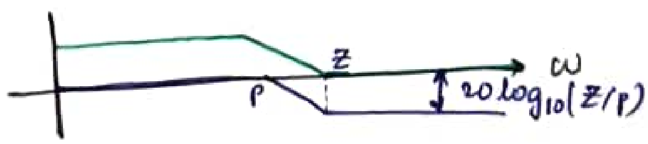
$$\phi_m = \sin\left(\frac{1-Z/P}{1+Z/P}\right)$$

Advantage: Introduce more proportional path gain.
[Reduce high frequency gain.]

Disadvantage: Phase lag.

$$G(s) = K\cdot\left(\frac{1+s/Z}{1+s/P}\right)$$

Take $K = 20\log_{10}(Z/P)$



~~Lead~~ Lag compensator vs. PI Controller

↳ can be tuned

↳ oscillation due to ~~Coulo~~ Coulomb friction

1-04-2024

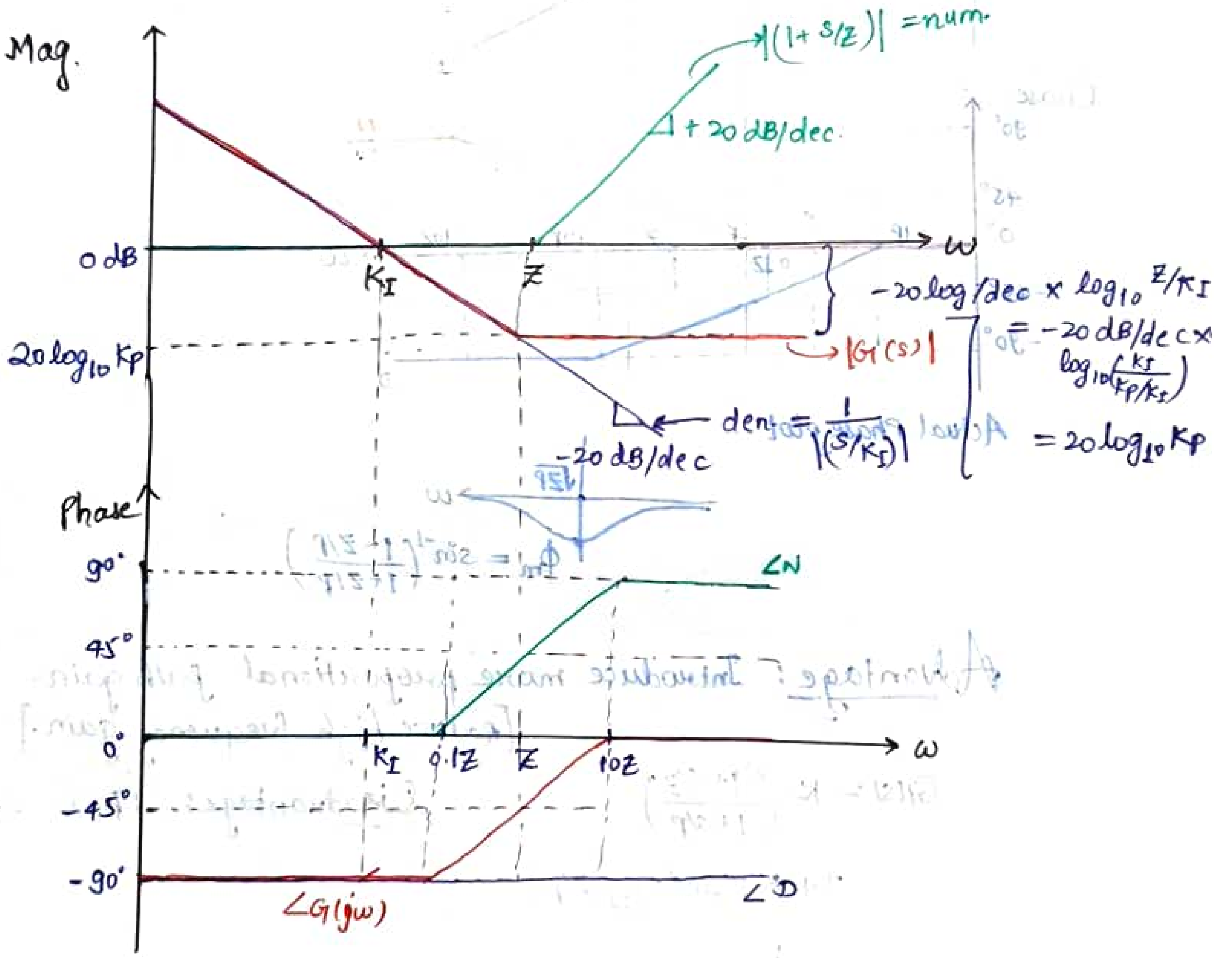
PI Controller

$$G(s) = K_p + \frac{K_I}{s}, \quad K_p > 0, \quad K_I > 0$$

$$= \frac{K_p s + K_I}{s} = \frac{K_I\left(1 + \frac{K_p}{K_I}s\right)}{s}$$

$$= \frac{K_I\left(1 + s/Z\right)}{s}$$

$$= \frac{1 + s/Z}{(s/K_I)}, \quad Z = K_I/K_p$$



↳ **Advantages:** Provides high robustness to the rejection of noise, parameter variation. [very high gain at steady state $(\omega = 0)$].

Disadvantages: Provides phase lag of 90°.

↳ Provides good performance for linear systems.

→ 2nd order System :

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\Rightarrow G(j\omega) = \frac{\omega_n^2}{\omega_n^2-\omega^2+j2\xi\omega_n\omega}$$

$$= \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\frac{\omega}{\omega_n}}$$

$$= \frac{1}{1-u^2+j2\xi u}, \quad u=\frac{\omega}{\omega_n}$$

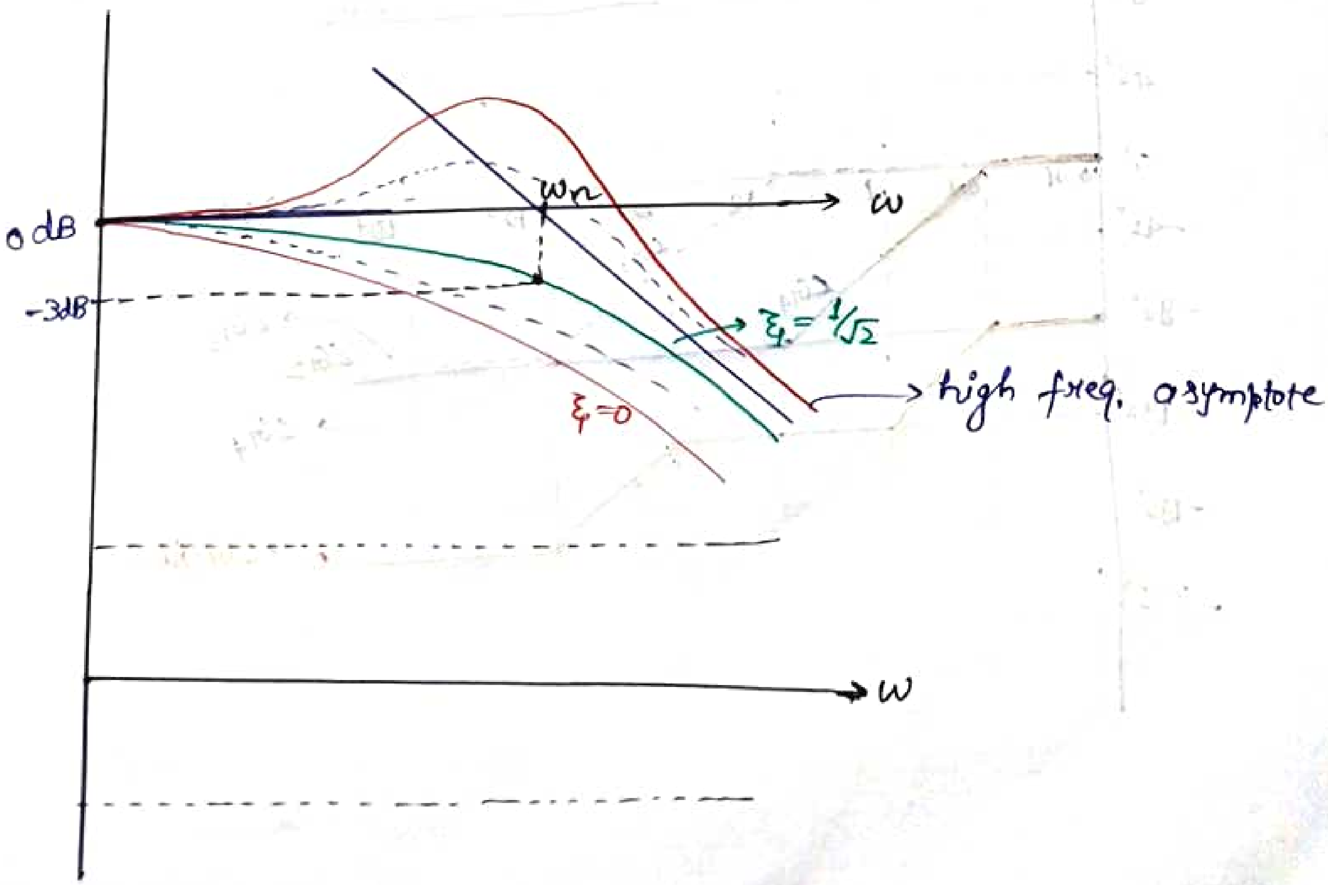
$$= \frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}} \angle -\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)$$

$$Mag = 20\log_{10}\frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}}$$

If $u<<1$, $Mag = 0\,dB$

If $u>>1$, $Mag. = -40\log_{10}u = -40\log_{10}(\omega/\omega_n)$

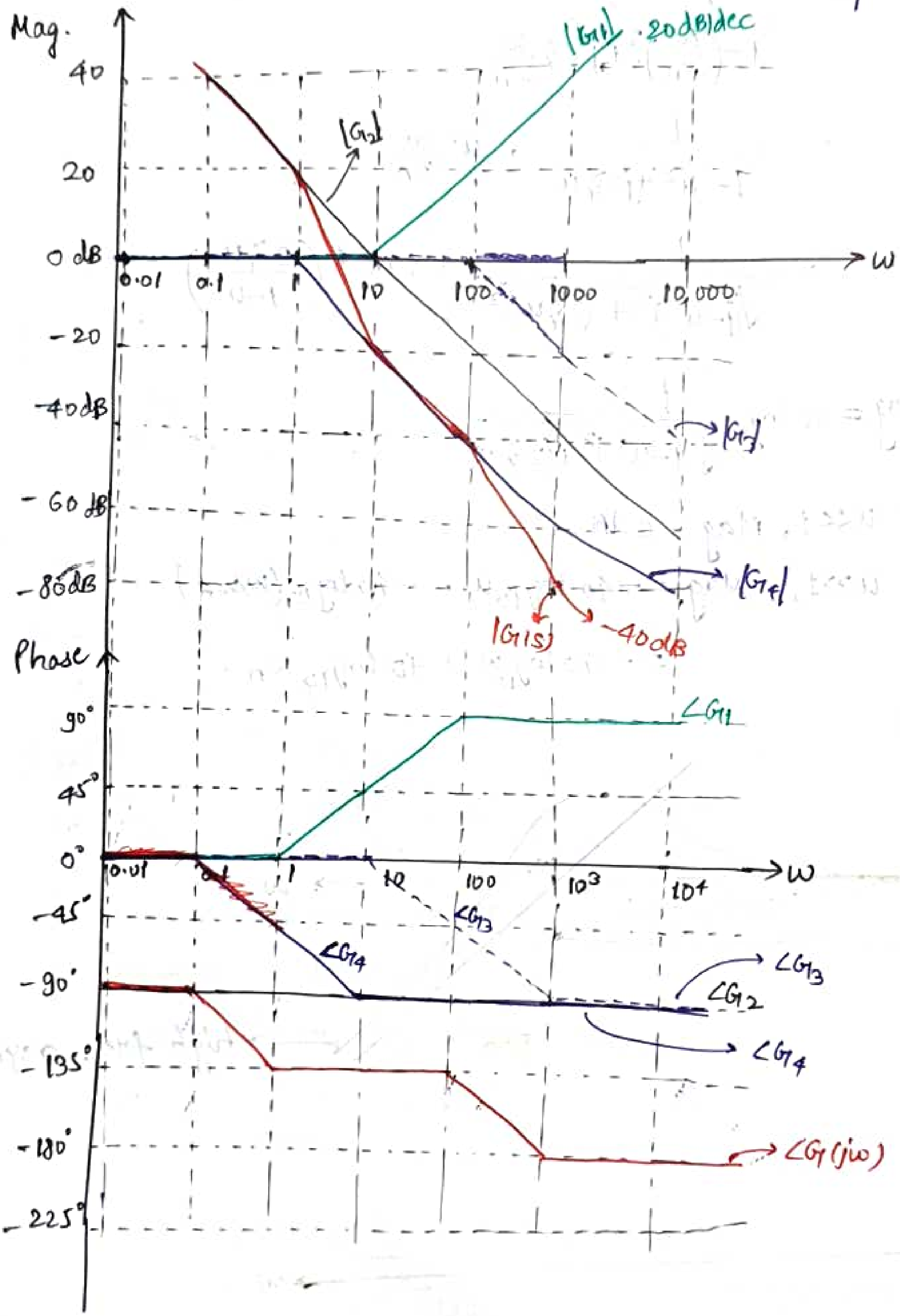
$$= -40\log_{10}\omega + 40\log_{10}\omega_n.$$



$$\rightarrow \quad G(s) = \frac{10\,(1 + s/10)}{s\,(1 + s/100)\,(1+s)}$$

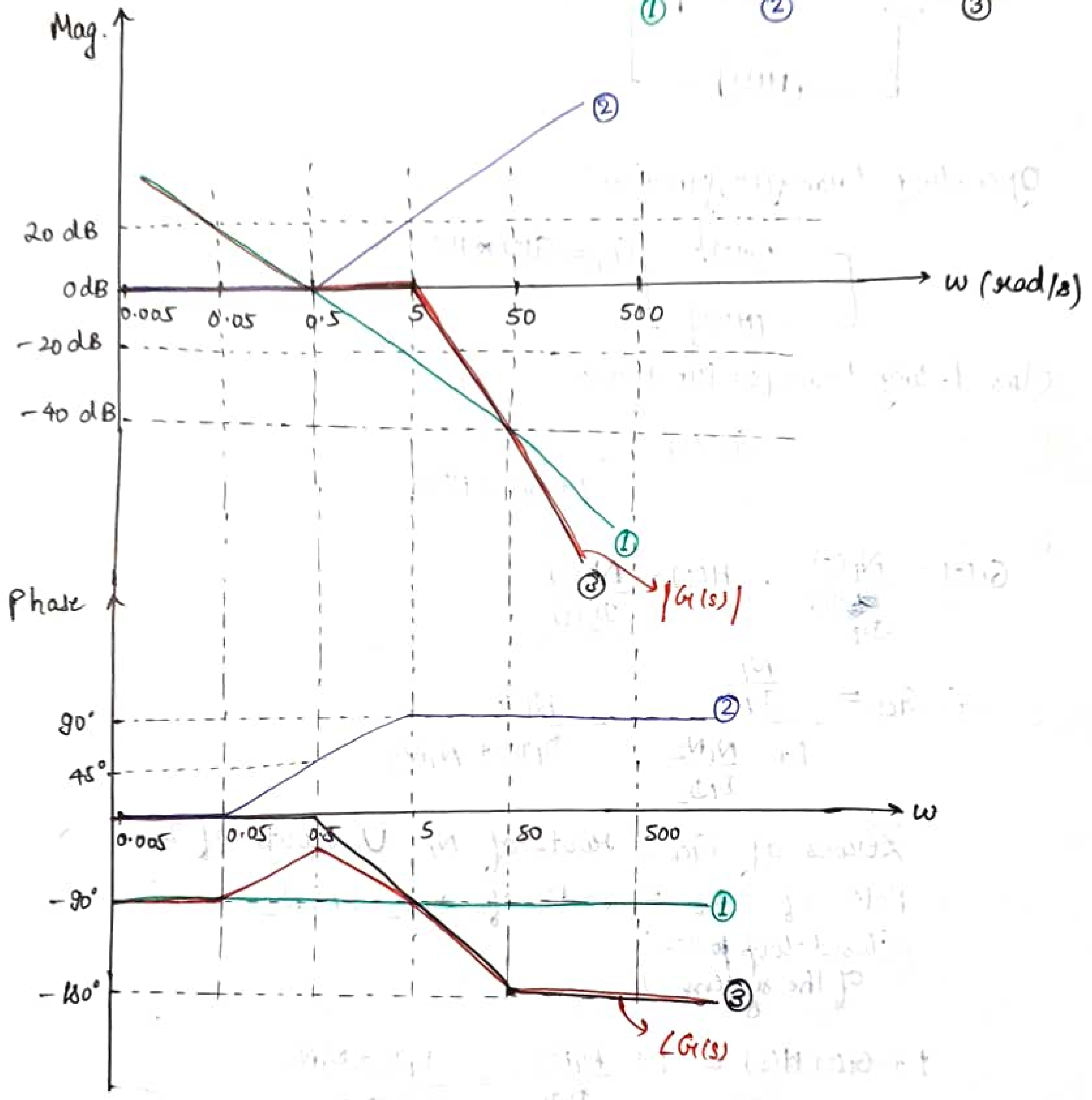
$$G_1 = \frac{1}{1+s/10}, \quad G_2 = \frac{1}{s/10}, \quad G_3 = \frac{1}{1+s/100}, \quad G_4 = \frac{1}{1+s}$$

Take frequencies two decades above & two decades below of each corner frequency.

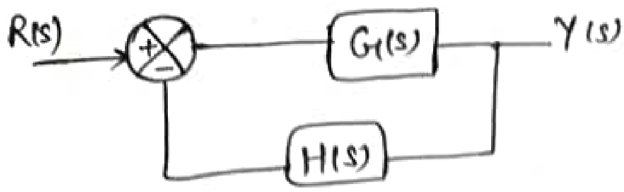


2-04-2024

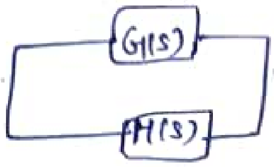
$$\rightarrow G(s) = \frac{0.5\left(1+\frac{s}{0.5}\right)25}{s(s^2+5s+25)} = \underbrace{\frac{1}{s/0.5}}_{①} \cdot \underbrace{\left(1+s/0.5\right)}_{②} \cdot \underbrace{\frac{25}{s^2+5s+25}}_{③}$$



Nyquist Plot



Open-loop transfer function:



$G_{OL} = G(s)\, H(s)$

Closed-loop transfer function:

$$G_{CL}(s) = \frac{G(s)}{1 + G(s)\, H(s)}$$

$$G(s) = \frac{N_1(s)}{D_1(s)}, \quad H(s) = \frac{N_2(s)}{D_2(s)}$$

$$\therefore G_{CL} = \frac{\frac{N_1}{D_1}}{1 + \frac{N_1 N_2}{D_1 D_2}} = \frac{N_1 D_2}{D_1 D_2 + N_1 N_2}$$

Zeroes of G_{CL} = roots of N_1 ∪ roots of D_2

Poles of G_{CL} ≡ roots of $D_1 D_2 + N_1 N_2$
(Closed-loop poles of the system)

$$1 + G(s)\, H(s) = 1 + \frac{N_1 N_2}{D_1 D_2} = \frac{D_1 D_2 + N_1 N_2}{D_1 D_2}$$

roots of $(D_1 D_2 + N_1 N_2)$ = Zeroes of $(1 + G(s)\, H(s)) \equiv \Delta(s)$

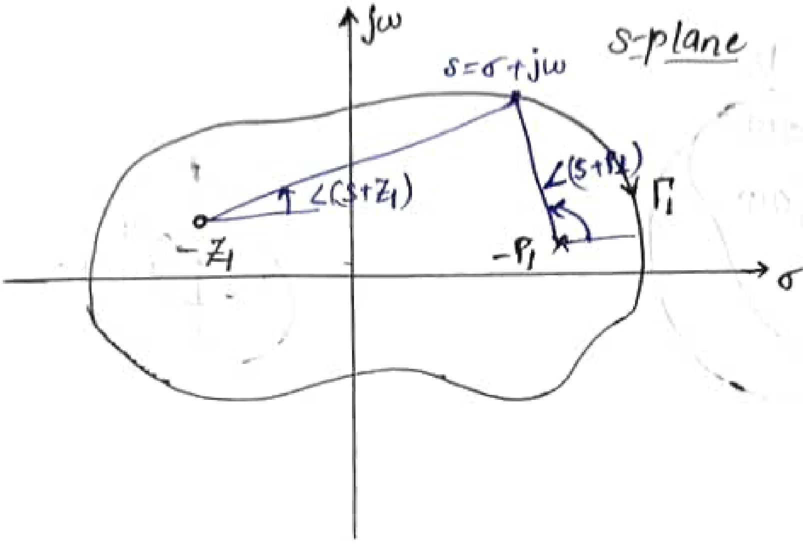
≡ roots of the eqⁿ $\underbrace{1 + G(s)\, H(s) = 0.}_{\text{Characteristic equation}}$

$G(j\omega)\, H(j\omega) \rightarrow$ Open-loop frequency response
$\omega \rightarrow 0 - \infty$

↓ we can find stability

Mapping from s-plane to Q(s) plane:

$$1 + G(s)H(s) = Q(s) = \frac{s + Z_1}{s + P_1}$$



Γ_1 : clockwise loop

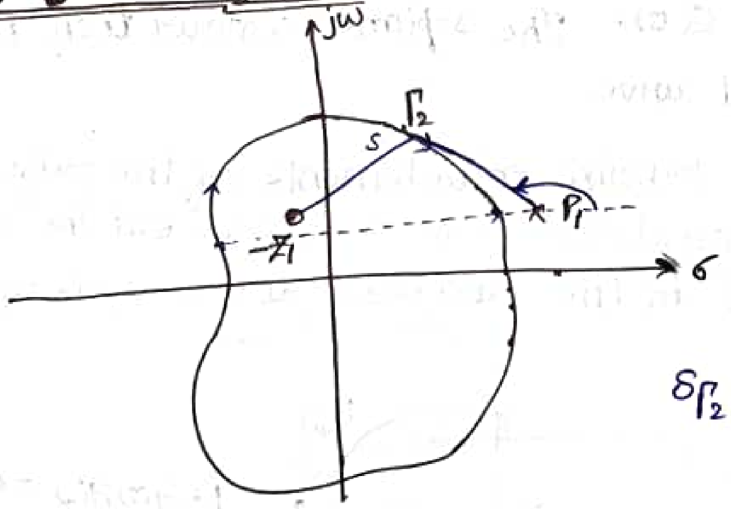
$\delta_{\Gamma_1} \angle(s+Z_1) = -2\pi$ rad [change in $\angle(s+Z_1)$ as the typical point traverses through loop Γ_1]

$\delta_{\Gamma_1} \angle(s+P_1) = -2\pi$ rad. [negative due to clockwise]

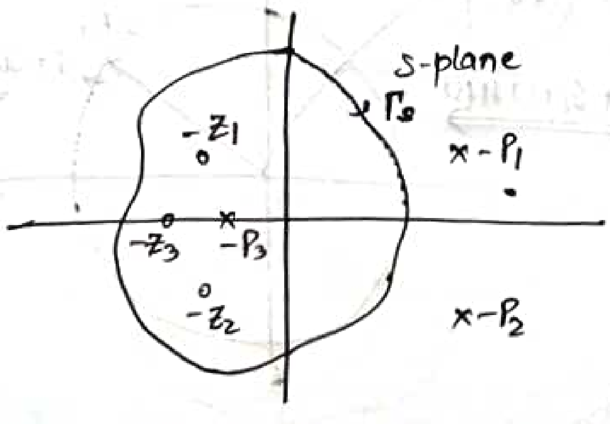
$$\delta_{\Gamma_1} \angle Q(s) = \delta_{\Gamma_1} \angle(s+Z_1) - \delta_{\Gamma_1} \angle(s+P_1)$$
$$= -2\pi - (-2\pi)$$
$$= 0$$

NYQUIST PLOT

05-04-2024



$\delta_{\Gamma_2}(s+Z_1) = -2\pi$ rad

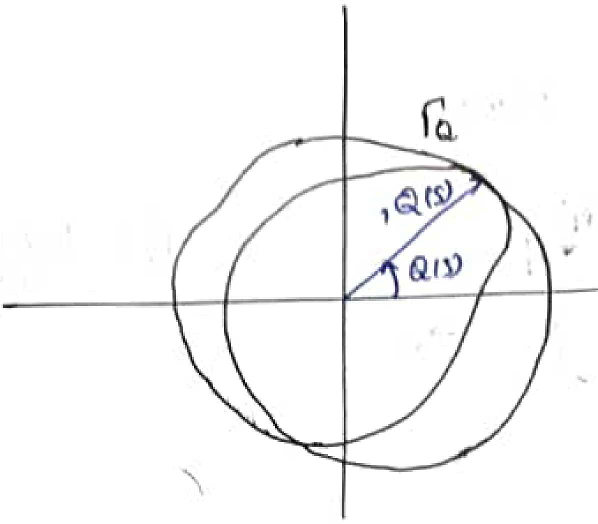


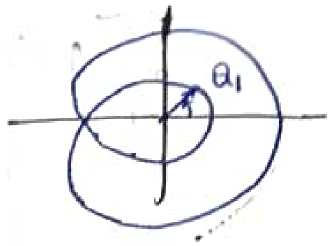
$\delta_{\Gamma_3} \angle Q(s) = -2\pi(Z - P)$ rad.

Z no. of zeroes and P no. of poles of Q(s) within Γ_3 contour.

→ ~~Only~~ zeroes and poles lying inside the contour affect.

$\rightarrow Q(s) = 1 + G(s)H(s)$.



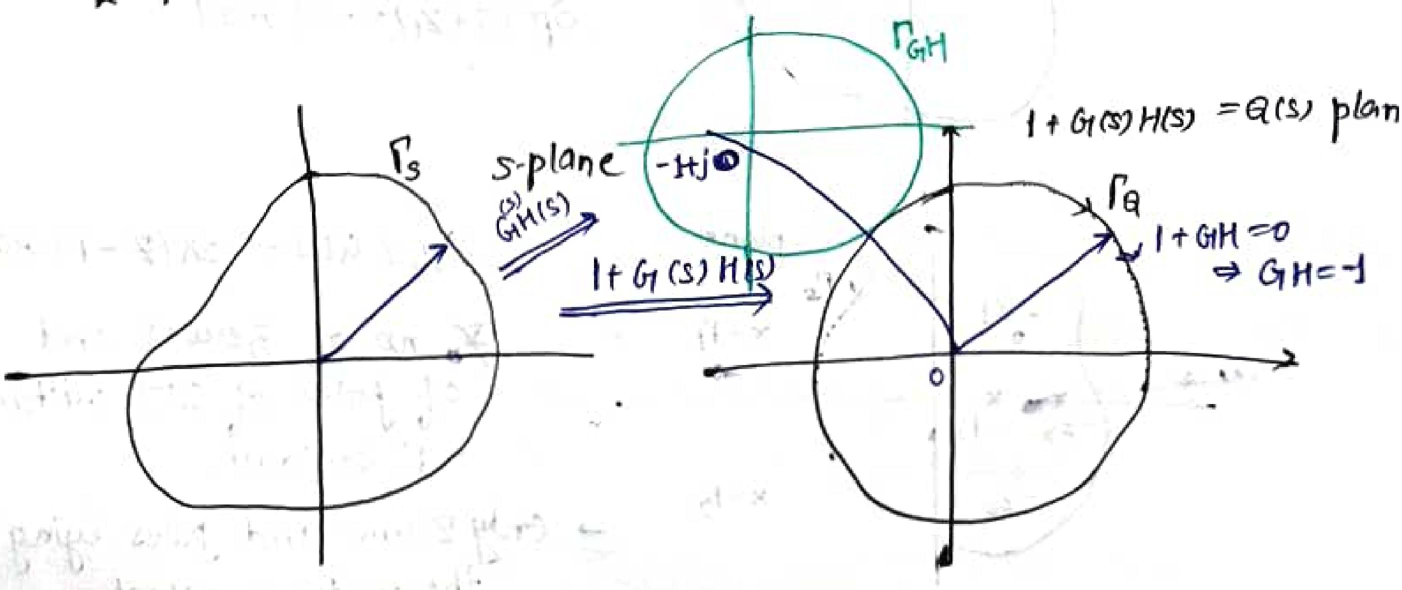


Principle of Argument:

Let Q(s) be a ratio of polynomial in s. Let P be the no. of poles and Z be the no. of zeroes of Q(s) which are ~~inclued~~ enclosed by a simple closed contour in the s-plane.

Let the closed contour be such that it does not pass through any poles or zeroes of Q(s). The s-plane contour then ~~mappes~~ maps into Q(s) contour ~~into~~ as a closed curve.

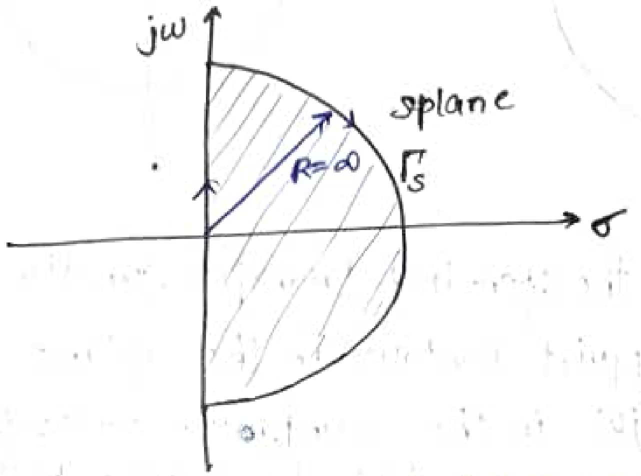
The no. N of ~~of~~ the clockwise encirclements of the origin of Q(s)-plane, as a representative point s traces out the entire contour in the s-plane in the clockwise direction, is equal to Z − P.



→ Encirclement about 0 of the Q(s) plane is same as the no. of encirclement about $-1+j0$ of the Γ_{GH} plane.

→ Ensure that the contour does not pass through any of the zeroes or poles.

Zeros and poles should ~~not~~ lie either inside or outside the contour.

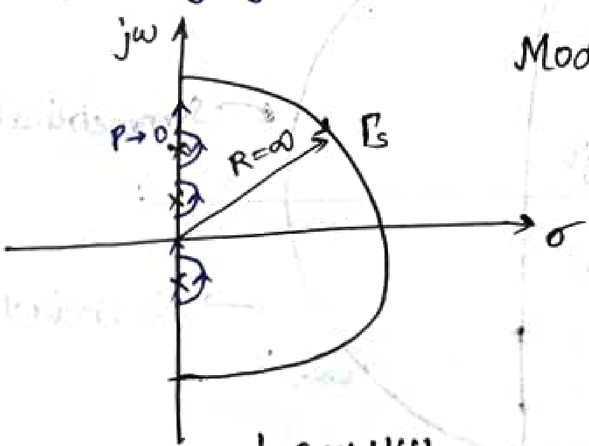


$$\text{Pole}\,(1+G(s)H(s)) = \text{pole}\,(G(s)H(s)).$$

→ P no. of poles of $1+G(s)H(s)$ is same as the P no. of poles of $G(s)H(s)$.

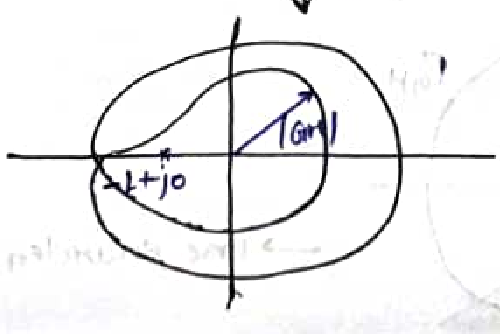
Eg. $G(s)H(s) = \dfrac{s+1}{(s-2)(s-3)}$ → 2 pole of $G(s)H(s)$
⇓
2 poles of $1+G(s)H(s)$.

→ For poles lying on the contour:



Modified Nyquist Contour
⇓
Make infinitesimal small contours around poles. (area to be removed).

↓ $G(s)\,H(s)$



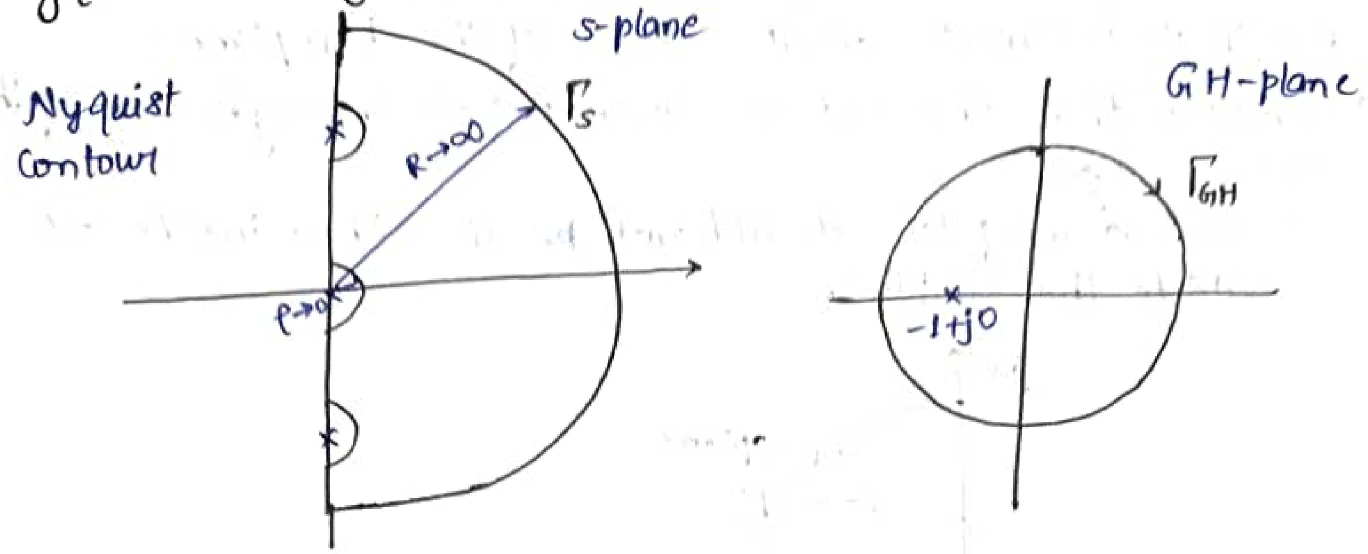
N: No. of zeroes on right-side
P: No. of Poles on right-side

$N = Z - P$

Z should be zero.

$\Rightarrow N = -P$.

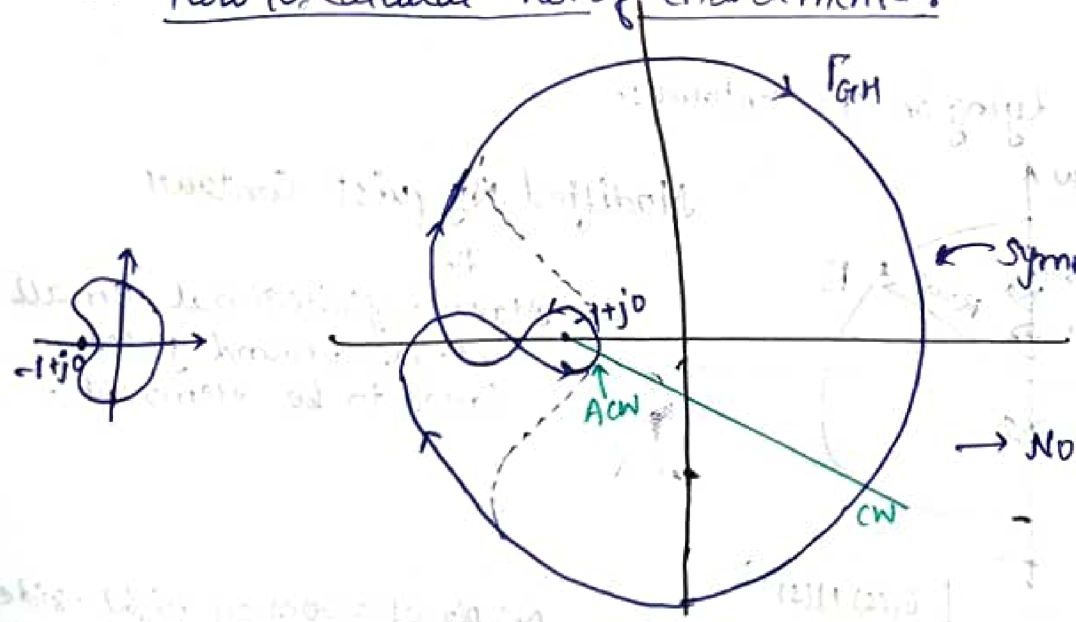
Nyquist (stability) criteria :



If the Nyquist plot of the open-loop transfer function $G(s)H(s)$ corresponding to the Nyquist contour in the s-plane encircles the critical point $(-1+j0)$ in the counter-clockwise direction as many times as the no. of right-hand side poles of $G(s)H(s)$, the closed-loop ~~system~~ system is stable.

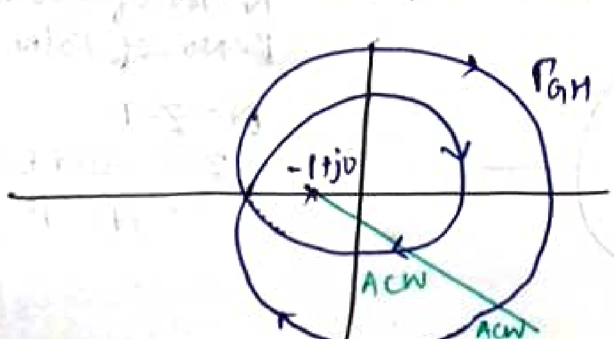
In the commonly occuring case of $G(s)H(s)$ with no poles in the right halfsplane, the closed-loop system is stable if the Nyquist plot of $G(s)H(s)$ doesnot encircle the $-1+j0$ point.

→ How to calculate no. of encirclements ?

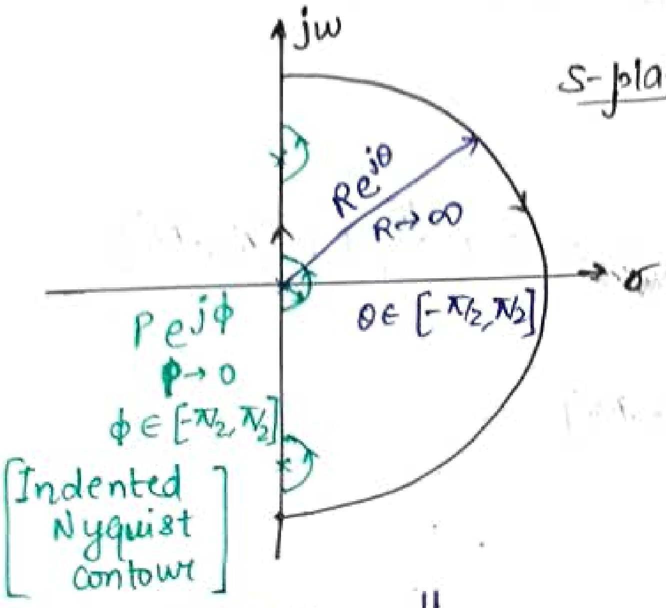


← Symmetrical

→ No encirclement

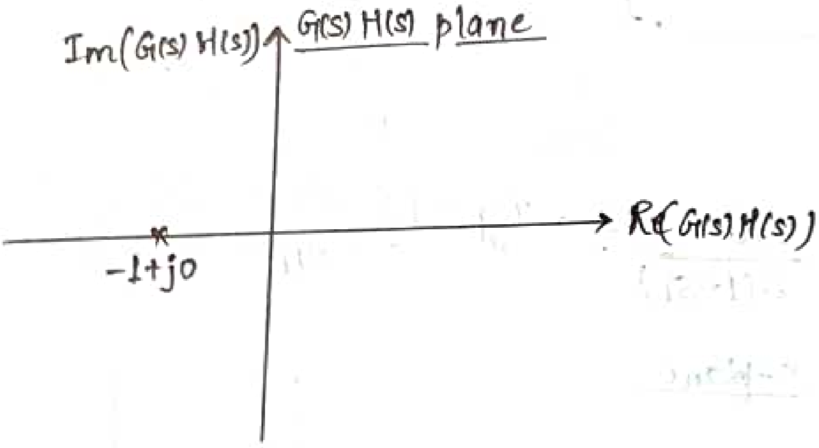


→ One encirclement.

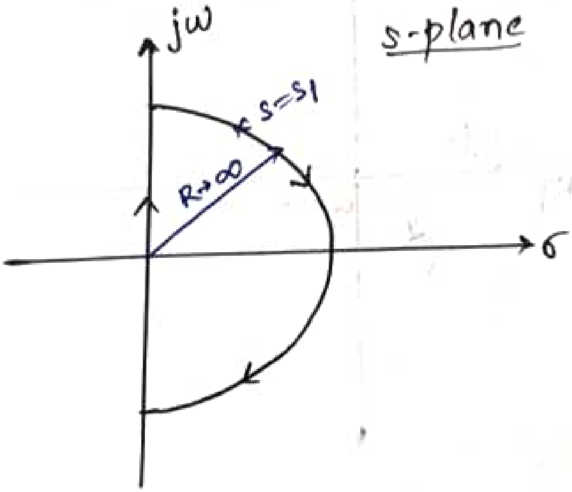


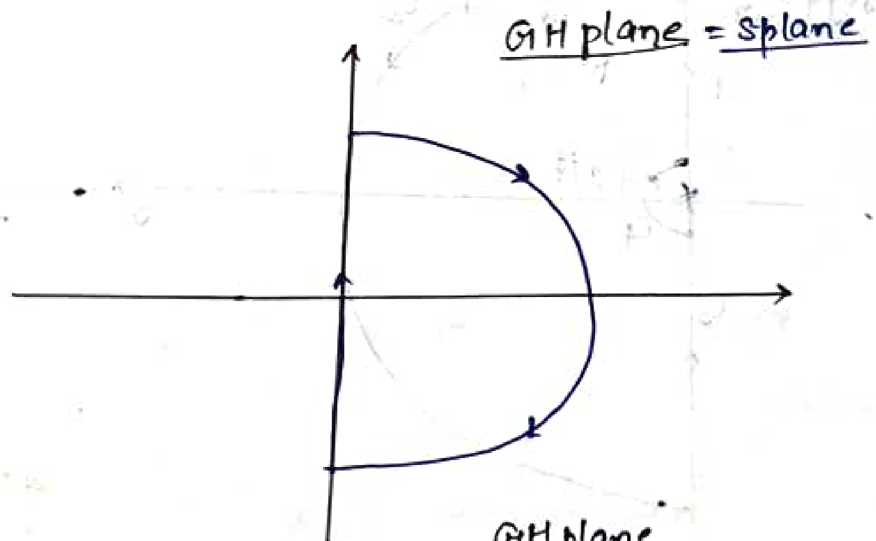
→ Nyquist contour

Mapping in ⇓ GH plane

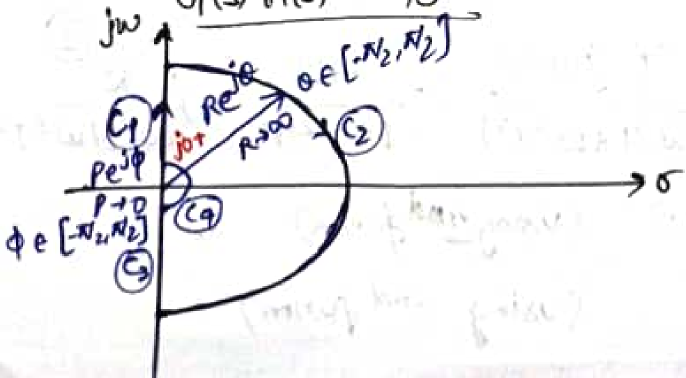


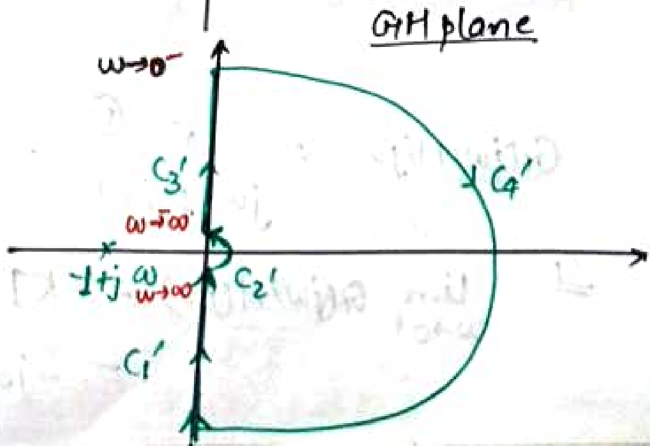
Eg. $G(s)H(s) = s$





$G(s)H(s) = 1/s$





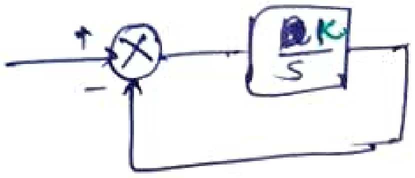
C_1 $\lim_{\omega \to 0^+} G(j\omega) = \lim_{\omega \to 0^+} \frac{1}{j\omega} = \infty \angle -90^\circ$

$\lim_{\omega \to \infty} G(j\omega) = \lim_{\omega \to \infty} \frac{1}{j\omega} = 0 \angle -90^\circ$

C_2 $G(s)H(s)\Big|_{s=Re^{j\theta}} = \lim_{R \to \infty} \frac{1}{Re^{j\theta}} = 0 \cdot e^{-j\theta}, \; \theta \in [\pi/2, -\pi/2]$

C_4 $\lim_{\rho \to 0} \frac{1}{\rho e^{j\phi}} = \infty e^{-j\phi}, \; \phi \in [-\pi/2, \pi/2]$

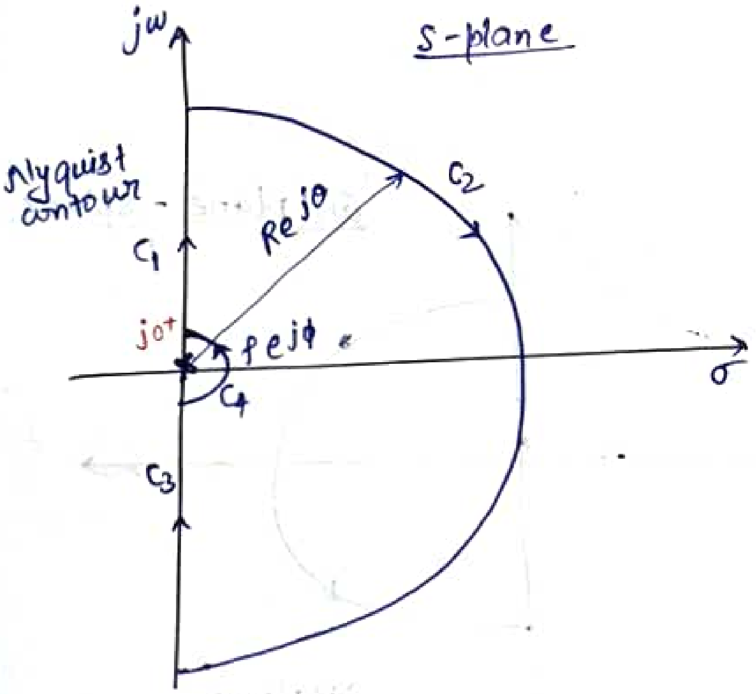
$\phi \in [-\pi/2, \pi/2]$

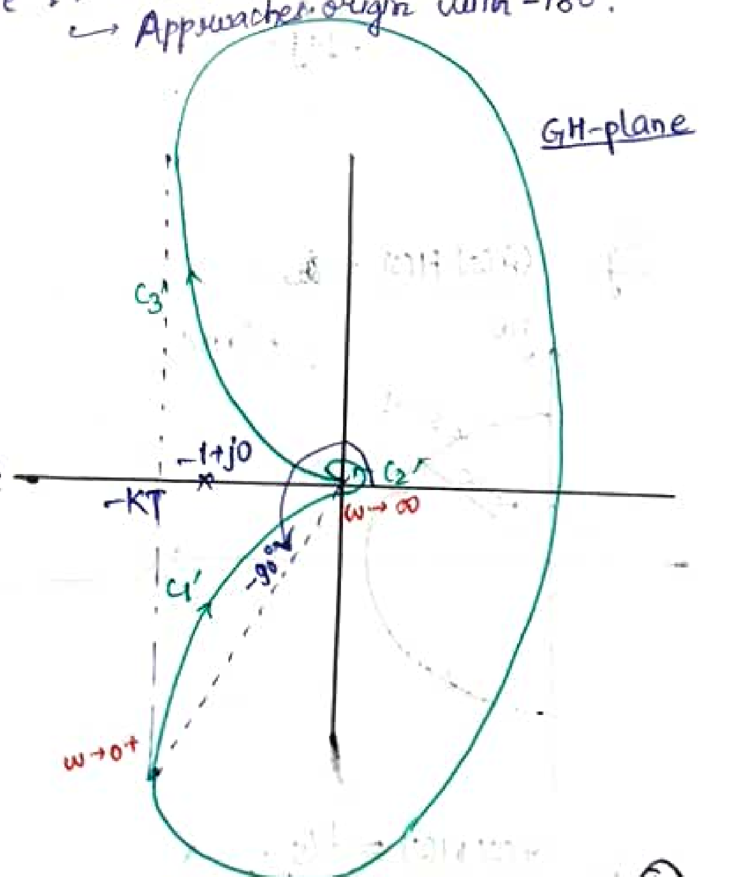


$G(s)H(s) = \frac{1}{s}$ ⟷ stable

$\frac{k}{s+k} = \frac{K/s}{1+K/s}$ → CL gain

Eg. $G(s)H(s) = \frac{K}{s(1+sT)}$ → Type-1: starts with $-\infty$
↳ Approaches origin with -180°.





$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T)}$ ① $= \frac{K(1-j\omega T)}{j\omega(1+\omega^2T^2)} = \frac{-KT}{1+\omega^2T^2} - j\frac{K}{\omega(1+\omega^2T^2)}$ ②

C_1 $\lim_{\omega \to 0^+} G(j\omega)H(j\omega) = -KT - j\infty$ (using 2nd form) ✓

$= -j\infty$ (using 2nd form)

$$\lim_{\omega\to\infty} G(j\omega)H(j\omega) = \lim_{\omega\to\infty} \frac{K}{j\omega^2 T} = 0\angle 180^\circ \quad \text{(Using 1st form)}$$

$\underline{C_2}$

$$\lim_{R\to\infty} \frac{K}{Re^{j\theta}(1+Re^{j\theta}T)} = 0\,e^{-j2\theta} \qquad \theta \in [\pi/2, -\pi/2]$$

$\theta \in [\pi/2, -\pi/2]$

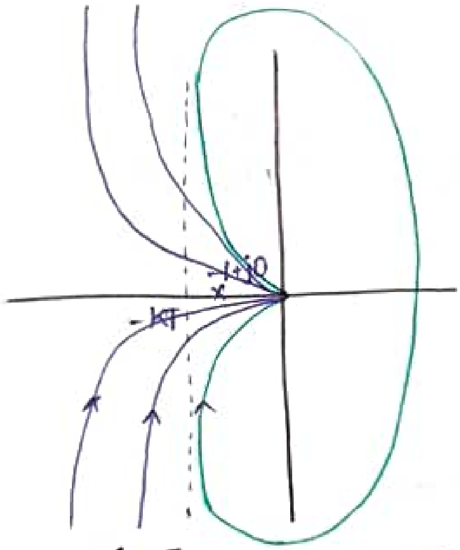
$\underline{C_4}$

$$\lim_{\rho\to 0} \frac{K}{\rho e^{j\phi}[1+\rho e^{j\phi}T]} = \lim_{\rho\to 0} \frac{K}{\rho e^{j\phi}}$$

$\phi \in [-\pi/2, \pi/2]$ $\qquad \phi \in [-\pi/2, \pi/2]$

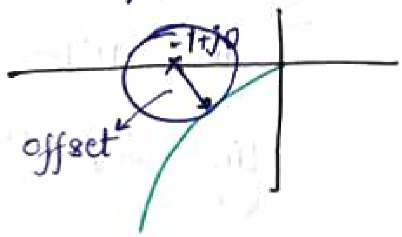
↳ stable.

(12-04-2024)

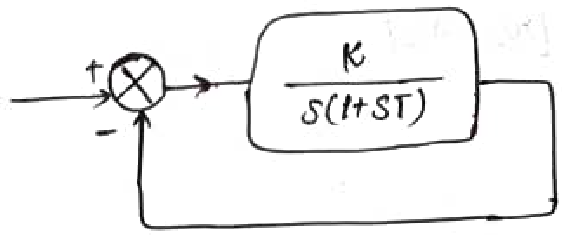


→ Closeness of Nyquist plot with $-1+j0$ determines relative stability

→ An unstable system encloses the critical point.



Increasing KT ⇒ Stability ↓ [Closness of the plot with $-1+j0$ is increasing.
For $KT = \infty$ ⇒ it will pass through the critical point.]

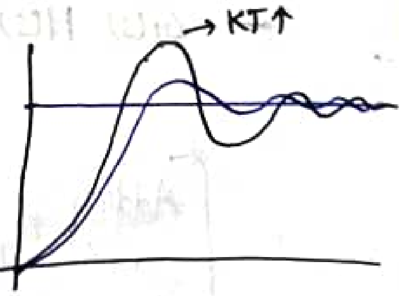
→ 

$$\frac{\frac{K}{S(1+ST)}}{1+\frac{K}{S(1+ST)}} = \frac{K}{S(1+ST)+K}$$

$$= \frac{K}{S^2T+S+K} = \frac{K/T}{S^2+\frac{1}{T}S+\frac{K}{T}}$$

$$[2\xi\omega_n = 1/T] \qquad = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$$

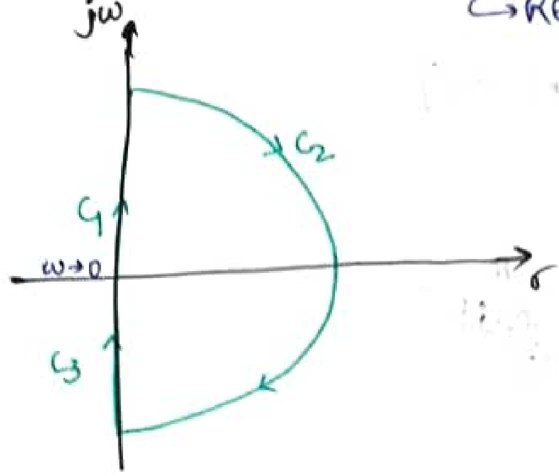
$$\Rightarrow \xi = \frac{1}{2\omega_n T} = \frac{1}{2\sqrt{K/T}\,T} = \frac{1}{2\sqrt{KT}}$$

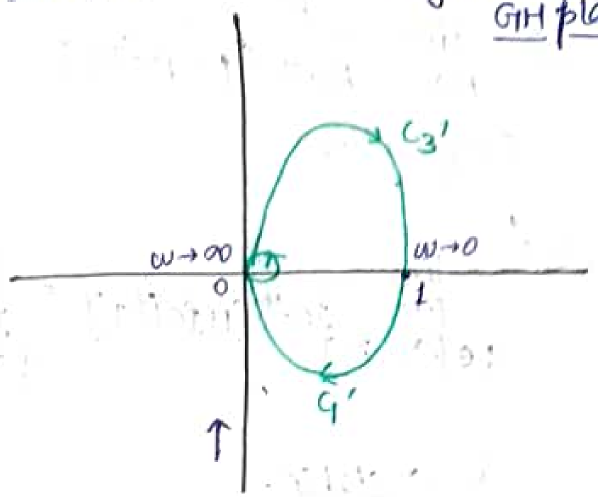


⇒ KT↑ ⇒ ξ ↓

→ $G(s)\,H(s) = \dfrac{1}{1+sT}$ → Type '0' → starts with +ve value on σ-axis.
→ approaches origin with -90°.
↳ Relative order '1'

Nyquist plot
GH plane





$$G(j\omega)H(j\omega) = \frac{1}{1+j\omega T} \quad (1)$$

$$= \frac{1-j\omega T}{1+\omega^2T^2} = \frac{1}{1+\omega^2T^2} - j\frac{\omega T}{1+\omega^2T^2} \quad (2)$$

$\underline{C_1}$

$\lim_{\omega\to 0} GH(j\omega)$

$\lim_{\omega\to\infty} G(j\omega)\,H(j\omega) = 0\angle -90^\circ$
(using exp. (1))

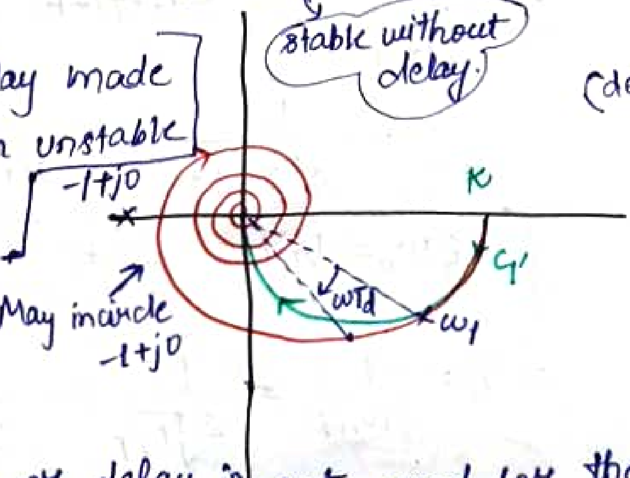
$\underline{C_2}$

$$\lim_{R\to\infty} \frac{1}{1+Re^{j\theta}} = 0\,e^{-j\theta}$$
$\theta \in [\pi/2, -\pi/2]$ $\quad \theta \in [\pi/2, -\pi/2]$
(Use exp. (1))

→ $G(s)\,H(s) = \dfrac{Ke^{-T_d s}}{1+sT}$ → Linear → 1st order → stable without delay.

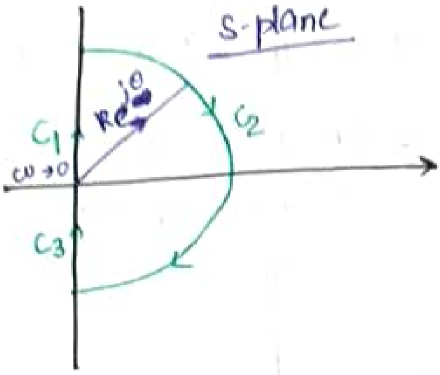
$GH(j\omega) = \dfrac{K}{1+j\omega T} \cdot e^{-j\omega T_d}$
mag. = 1
(delay) phase lag = $\omega_1 T_d$ (for $\omega = \omega_1$)

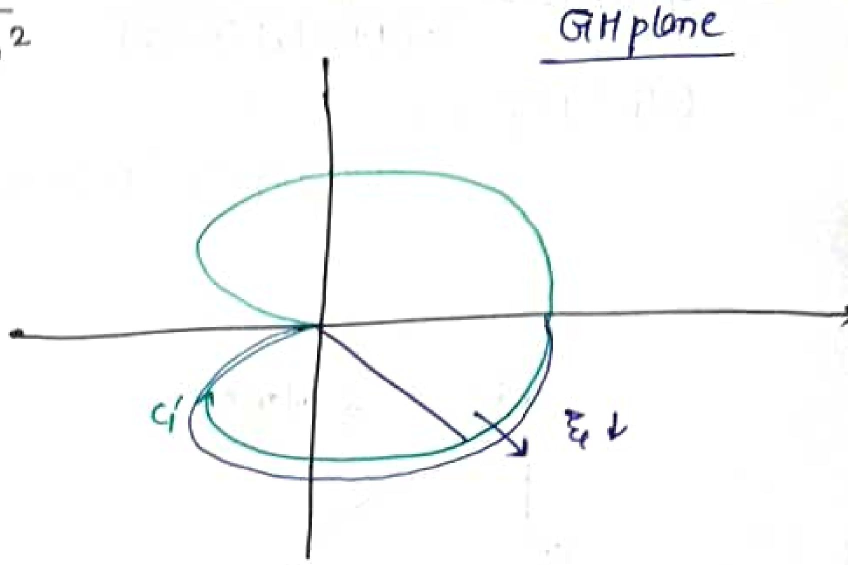
[Adding a time-delay made a 1st order system unstable beyond a value of K.]



In general, lag or delay is not good for the system.

$\rightarrow G(s)H(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$





$\underline{C_1}$ $\omega \to 0 \Rightarrow \lim_{\omega \to 0} G.H(j\omega) = 0$

$\omega \to \infty \Rightarrow \quad -180^\circ$

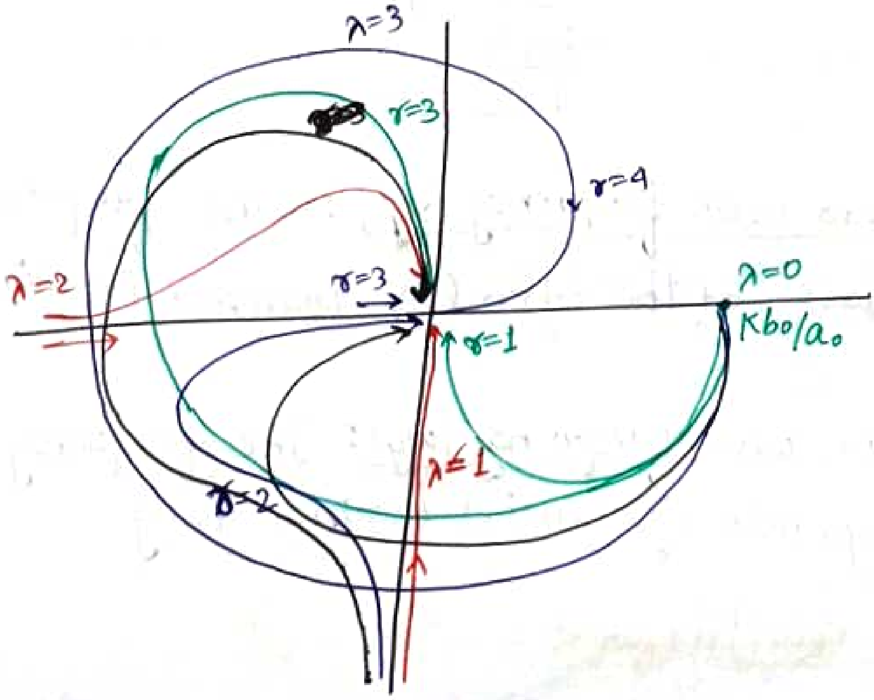
$G(j\omega) = \dfrac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\xi\omega_n\omega}$

↳ verify the Nyquist plot by checking the values for $\omega \to 0$ and $\omega \to \infty$, by checking the type and relative order of the system.

$$G(s)H(s) = \frac{K(b_m s^m + b_{m-1}s^{m-1} + \ldots + b_0)}{s^\lambda(a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0)}$$

Relative order : $\gamma = (\lambda + n) - m \geq 0$

Type : λ



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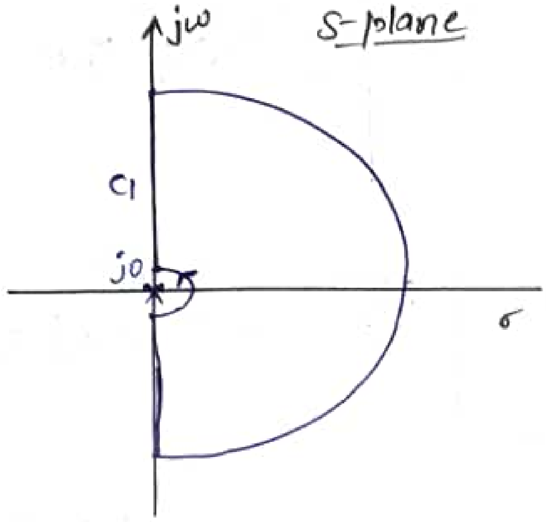
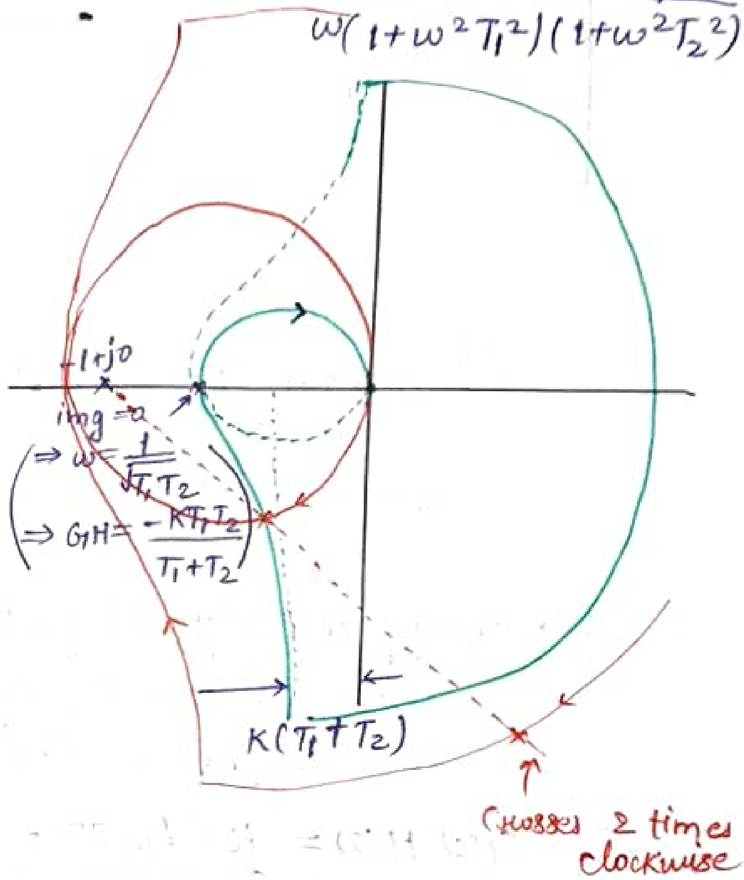
$\rightarrow$ $$G(s)H(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)} = \frac{-K(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$-j\frac{K(1-\omega^2T_1T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$





On real-~~line~~ axis,

$1-\omega^2T_1T_2 = 0$

$\Rightarrow \omega = \frac{1}{\sqrt{T_1T_2}}$

$$\therefore G(j\omega)H(j\omega) = \frac{-K(T_1+T_2)}{\left(1+\frac{T_1}{T_2}\right)\left(1+\frac{T_2}{T_1}\right)}$$

$$= \frac{-KT_1T_2}{T_1+T_2}$$

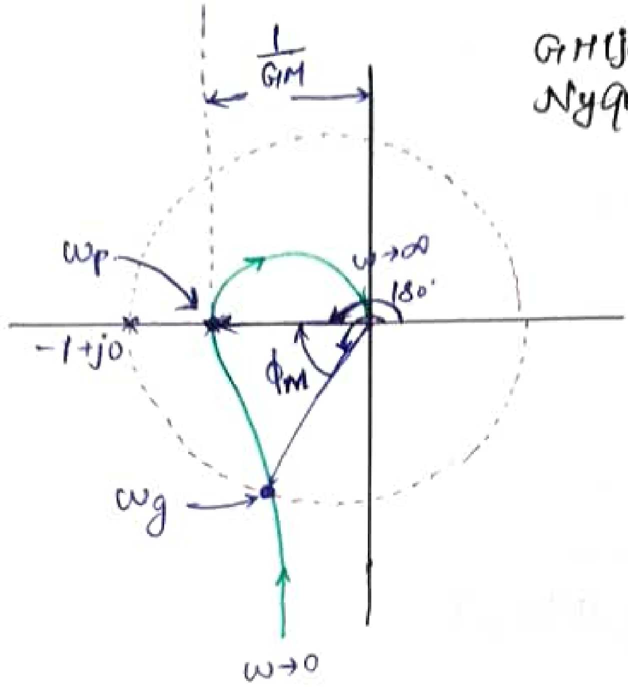
$N = Z - P$

$P = 0$

$N = Z = 2$

$\rightarrow$ Phase cross over frequency (ω_p): The frequency at which the phase of the open loop transfer function, $G(j\omega)H(j\omega)$, becomes 180°.

$\rightarrow$ Gain cross over frequency (ω_g): The frequency at which the magnitude of $GH(j\omega)$ becomes unity.



$GH(j\omega)$
Nyquist plot

ϕ_m : phase margin
↳ Giving a phase of ϕ_m rotates the Nyquist plot by that angle.

Gain margin : Reciprocal of $\frac{1}{GM}$.
↳ Gain after which the system becomes unstable.

→ Gain Margin :
↳ The factor by which the open loop gain, $|G(j\omega) H(j\omega)|$, is to be increased at the phase cross-over frequency, $\omega = \omega_p$, to drive the close-loop system to the ~~vinture~~ verge of instability.

→ Gain Margin w.r.t. Bode plot :
↳ The amount in dB by which the open-loop gain $20\log_{10}|G(j\omega) H(j\omega)|$ is to be increased at the phase-cross frequency to drive the close loop system, to the verge of instability.

$$GM = \frac{1}{|G(j\omega) H(j\omega)|}$$

→ Phase Margin : The additional phase lag in degree that has to be added to the open-loop phase, $\angle G(j\omega) H(j\omega)$, at the gain-cross over frequency to drive the close loop system to the verge of instability.

$$\phi_M = 180° + \angle G(j\omega_g) H(j\omega_g)$$ → should be +ve for stability

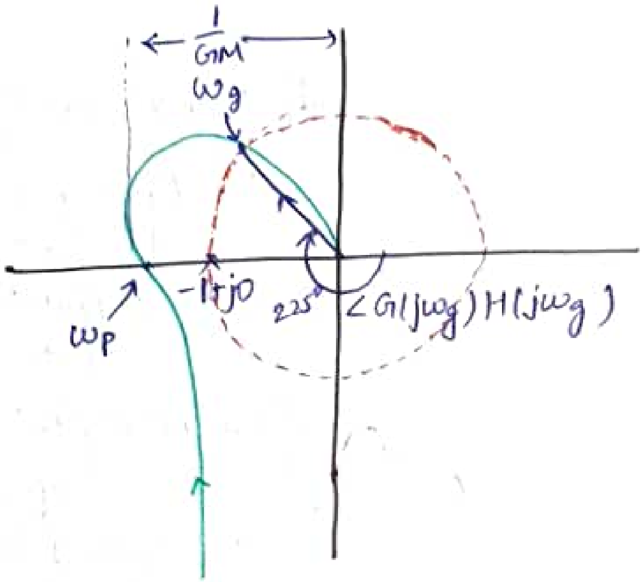
For stability,

$\phi_M > 0$

$GM > 1$

OR > 0 (in dB)

Eg



$$\phi_M = 180^\circ + (-225^\circ)$$
$$= -45^\circ$$ → Unstable

$GM < 1$

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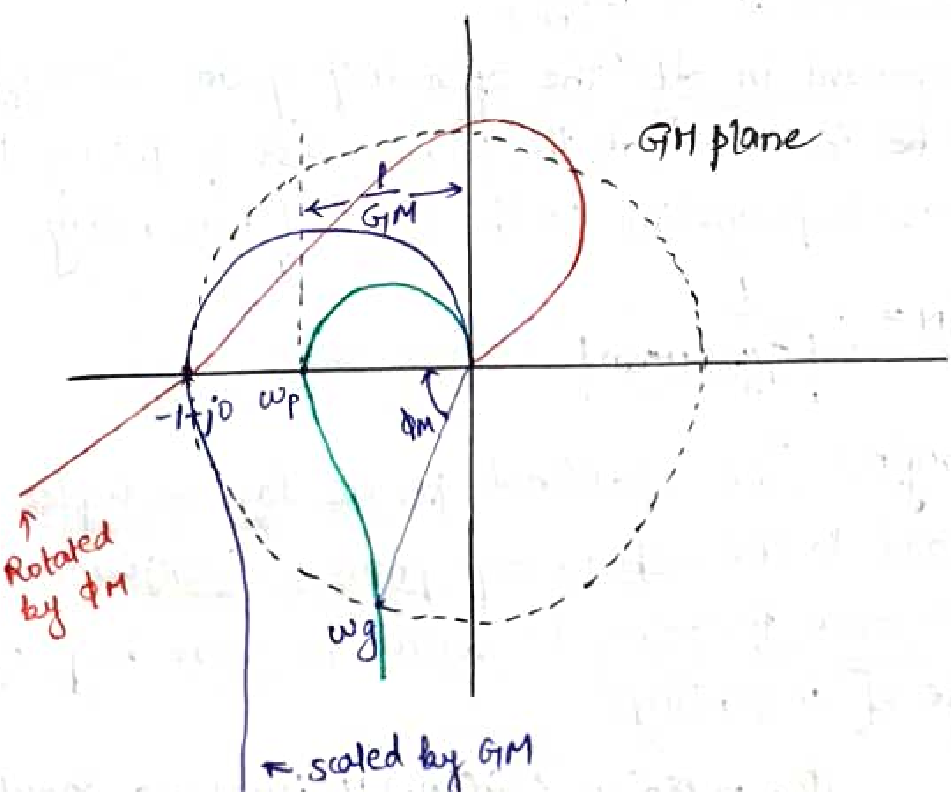
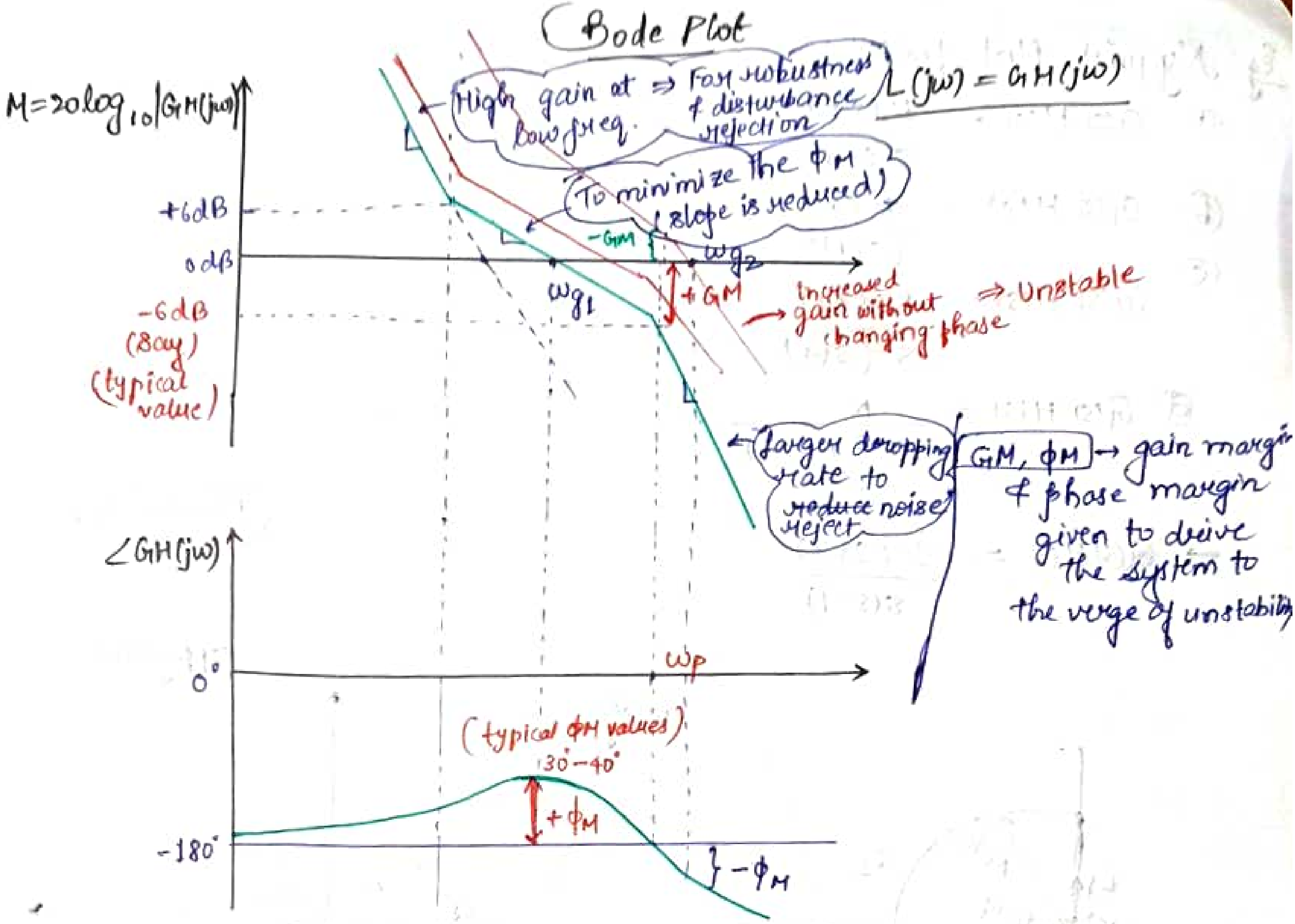
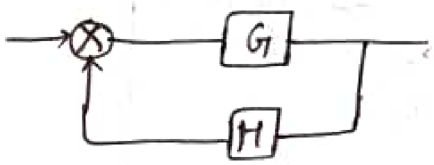
→ $N = Z - P$, $P = 0$



Bode Plot



→



$L = GH$

$$S_{G_{CL}:G} = \frac{1}{1+L}$$

$$S_{G_{CL}:H} = \frac{-L}{1+L}$$

Nichols Chart
Quantitative Feedback Theory (QFT)

Eg. Nyquist plot for:

ⓐ $G(s)H(s) = \dfrac{K(s+a)}{s(s-1)}$

ⓑ $G(s)H(s) = \dfrac{K(s-2)}{(s+1)^2}$

ⓒ $G(s)H(s) = \dfrac{K(s+2)}{s^2(s+4)}$

ⓓ $G(s)H(s) = \dfrac{K}{s(s^2+s+4)}$.

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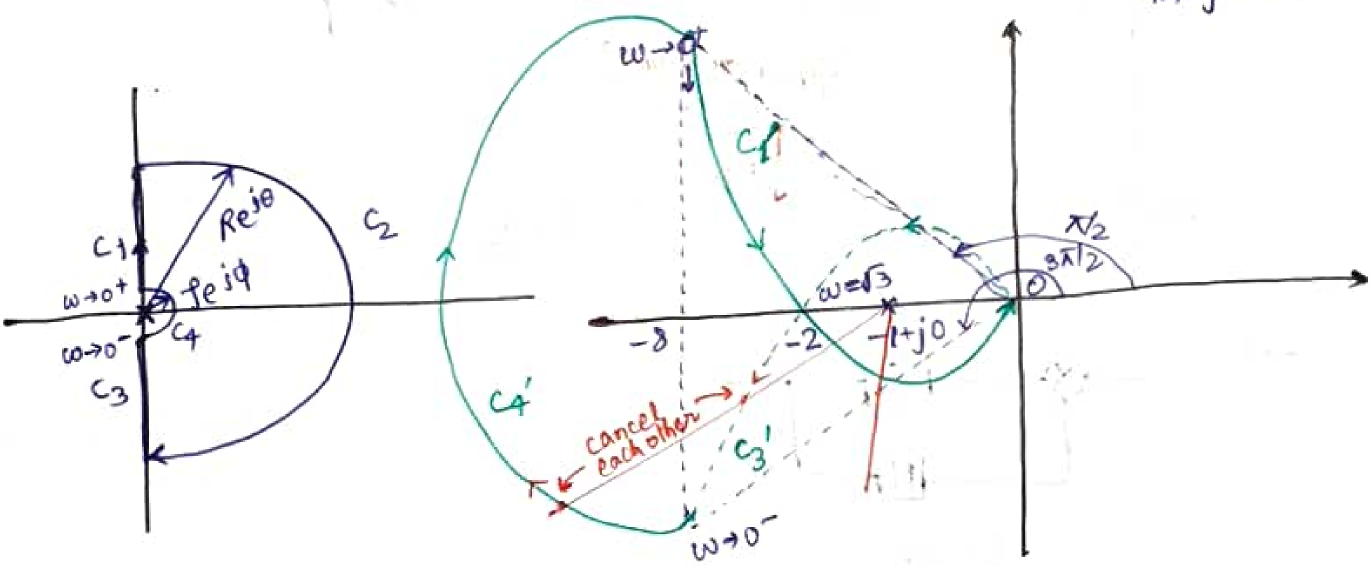
→ $G(s)H(s) = \dfrac{2(s+3)}{s(s-1)}$



$$G(j\omega)H(j\omega) = \underset{①}{\frac{-2(3+j\omega)}{j\omega(1-j\omega)}} = \frac{-2(3+j\omega)(1+j\omega)}{j\omega(1+\omega^2)} = \underset{②}{-\frac{8}{1+\omega^2} + j\,\frac{6-2\omega^2}{\omega(1+\omega^2)}}$$

C_1 $\lim\limits_{\omega\to0^+} G(j\omega)H(j\omega) = -8 + j\infty$ (2nd form)

$\lim\limits_{\omega\to\infty} GH(j\omega) = \dfrac{-2\times j\omega}{j\omega\times(-j\omega)} = 0\angle-90^\circ$ (1st form)

Intersection with real axis:

$6-2\omega^2=0 \Rightarrow 2\omega^2=6 \Rightarrow \omega=\pm\sqrt{3}$

C_4 $\lim\limits_{\substack{\rho\to0\\ \phi\in[-\pi/2,\pi/2]}} G(s)H(s) = \lim\limits_{\substack{\rho\to0\\ \phi\in[\pi/2,\pi/2]}} \dfrac{2(\rho e^{j\phi}+3)}{\rho e^{j\phi}(\rho e^{j\phi}-1)} = \lim\limits_{\substack{\rho\to0\\ \phi\in[-\pi/2,\pi/2]}} -\dfrac{6}{\rho}e^{-j\phi}$

$= \infty\, e^{j(\pi-\phi)}$

$\phi \in [-\pi/2, \pi/2]$

C_4 in s-plane (ϕ) : $\omega \to 0^-$ to $\omega \to 0^+$ anticlockwise

C_4' in GH-plane $(-\phi)$: $\omega \to 0^-$ to $\omega \to 0^+$ anticlockwise.

(OR)

phase $\equiv \pi - \phi$, $\phi \in [-\pi/2, \pi/2]$

$\equiv 3\pi/2$ to $\pi/2$ clockwise.

[Z : right-half zeroes of $1 + G(s)H(s)$.
P : right-half poles of $1 + G(s)H(s)$
$\equiv$ right-half poles of $G(s)H(s)$.]

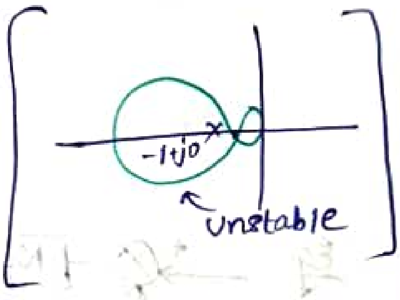
$N = -1$

$P = 1$

$N = Z - P$

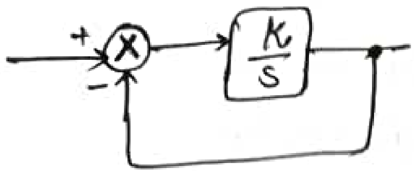
$\Rightarrow Z = N + P = -1 + 1 = 0$ (closed-loop)

↳ system is stable.

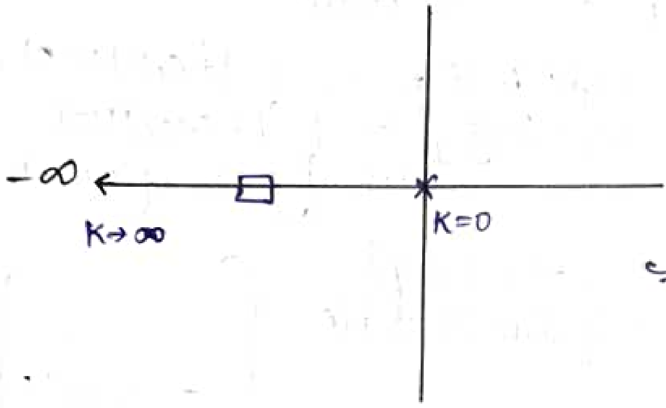


Root Locus Plot

Eg.



$$G_{CL} = \frac{K/s}{1+K/s} = \frac{K}{s+K}$$

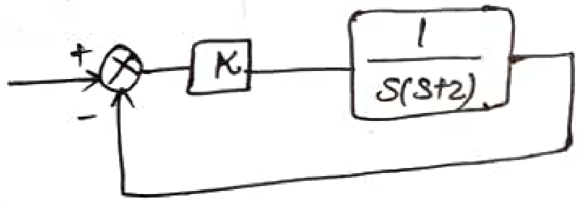


× → open-loop pole

□ → closed-loop pole

⇒ As K ↑, CL pole ~~shifts~~ migrates away from the OL pole, in this case.

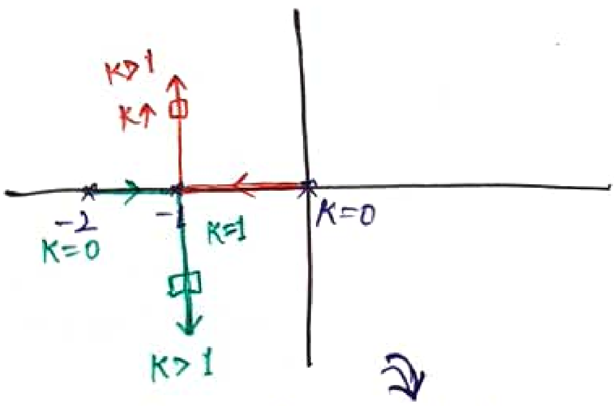
Eg.



$$G_C = \frac{\frac{K}{s(s+2)}}{1+\frac{K}{s(s+2)}} = \frac{K}{s^2+2s+K}$$

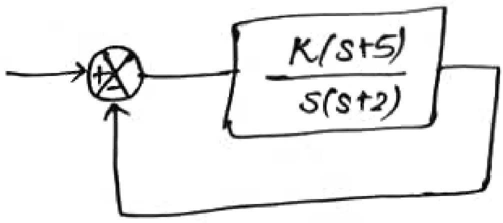
$$s_{1,2} = \frac{-2 \pm \sqrt{4-4K}}{2}$$

$$= -1 \pm \sqrt{1-K}$$

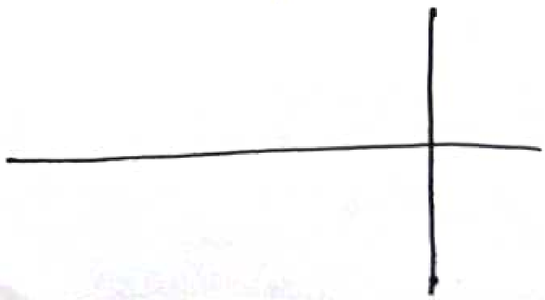


⇒ As K ↑ from 0 to 1, CL poles migrates from OL poles, −2 & 0, towards each other (towards −1), after which they move away from each other with constant real part.

Eg



$$1 + G(s)H(s) = 1 + K$$



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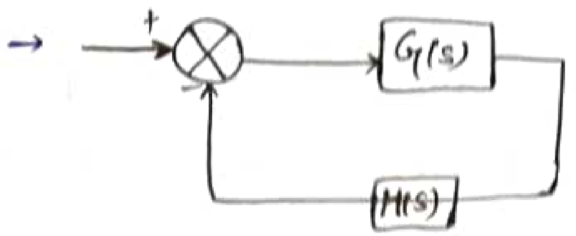
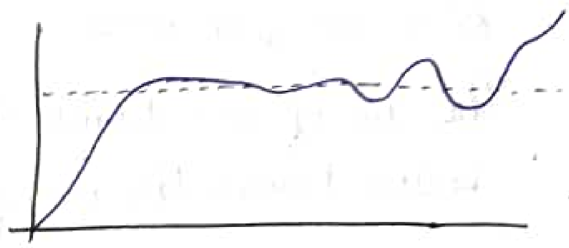


$$G_{cl} = \frac{G(s)}{1+G(s)H(s)}$$

$$\Delta(s) = 1+G(s)H(s) = 1 + K\frac{N_1(s)\,N_2(s)}{D_1(s)\,D_2(s)} = 0$$

$$\Rightarrow D_1(s)D_2(s) + KN_1(s)N_2(s) = 0.$$



$$\Delta(s) = 1 + F(s) = 0$$

$$\boxed{F(s) = G(s)H(s) = \frac{K\prod_{i=1}^{m}(s+Z_i)}{\prod_{j=1}^{n}(s+P_j)}}, \quad m \le n.$$

Magnitude and Angle conditions:

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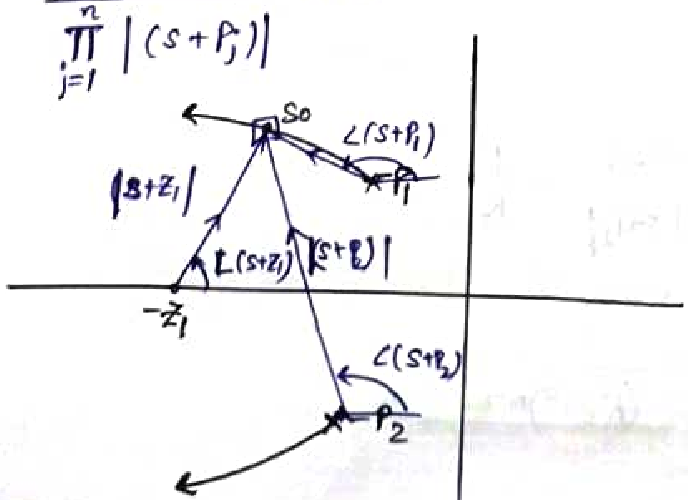
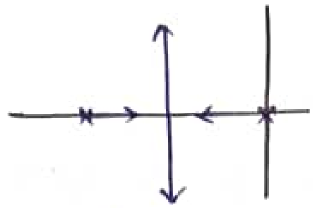
$$\Rightarrow F(s) = -1$$

$$\Rightarrow \begin{cases} \sum_{i=1}^{m}\angle(s+Z_i) - \sum_{j=1}^{n}\angle(s+P_j) = \pm(2q+1)\,180^\circ, \quad q=0,1,2,\ldots \\ \text{and,} \quad \dfrac{K\prod_{i=1}^{m}|(s+Z_i)|}{\prod_{j=1}^{n}|(s+P_j)|} = 1 \end{cases}$$



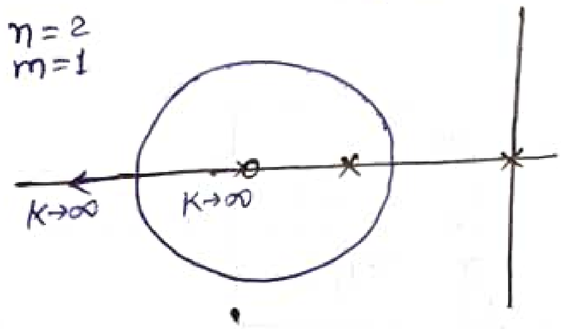
Rules for constructing Root Loci:

Rule: ① The number of root loci : The root locus plot consists of n root loci as $K \to 0$ to ∞. The loci are symmetric w.r.t. real axis.



Rule: ② Starting and terminal points of root loci : As K increases from 0 to ∞, each root locus originates from an open-loop pole with $K=0$ and terminates either on open loop zero or on ∞ with $K \to \infty$.

The no. of loci terminating on ∞ equals the no. of poles ~~munes~~ minus the no. of zeroes.



$$F(s) = \frac{K \prod (s+z_i)}{\prod (s+p_j)} = -1$$

$$\Rightarrow K \prod_{i=1}^{m} (s+z_i) + \prod_{j=1}^{n} (s+p_j) = 0$$

(I) (II)

As $K \to \infty$, (I) will dominate.

$$\frac{\prod |s+z_i|}{\prod |s+p_j|} = \frac{1}{K}$$

As $s \to \infty$

$$\frac{1}{(Re^{j\theta})^{n-m}} = \frac{1}{K}$$

Rule:

③ Asymptotes of root loci: The $(n-m)$ root loci which tends to ∞, do so ~~around~~ along straight-line asymptotes radiating out from a single point, $s = -\sigma_A$, on the real axis (called centroid)

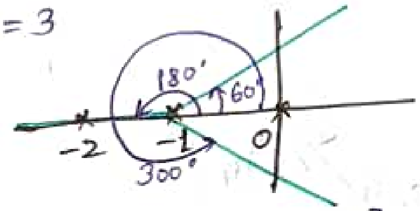
where
$$-\sigma_A = \frac{\sum \text{real parts of open-loop poles} - \sum \text{real parts of open-loop zeroes}}{(n-m)}$$

The $(n-m)$ asympotes will have angle

$$\phi_A = \frac{(2q+1)180^\circ}{n-m}, \quad q = 0, 1, 2, \ldots$$

Eg
$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

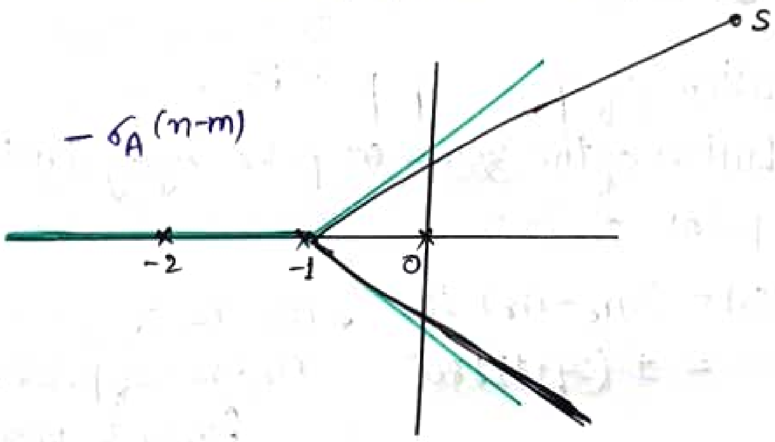
$n-m=3$



$$-\sigma_A = \frac{[0+(-1)+(-2)]-0}{3} = -1$$

$$\phi_A = \frac{(2q+1)180^\circ}{3}, \quad q = 0, 1, 2, \ldots$$

$$= 60^\circ, 180^\circ, 300^\circ$$



$$1 + \frac{K \prod_{i=1} (s+z_i)}{\prod_{j=1}^{m} (s+p_j)} \approx 1 + \frac{K}{(s+\sigma_A)^{n-m}} \quad \text{as } s \to \infty$$

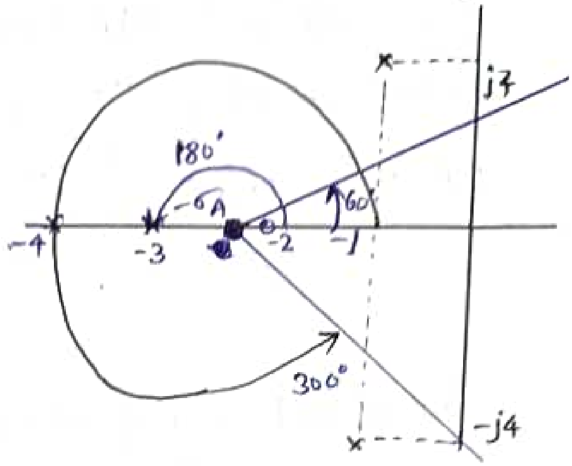
$$= \frac{1}{(s+\sigma_A)^{n-m}}.$$

Eg $F(s) = \dfrac{K(s+2)}{(s+1+j4)(s+1-j4)(s+3)(s+4)}$

$-\sigma_A = \dfrac{[-1+(-1)+(-3)+(-4)]-2}{4-2} = -\dfrac{7}{3}$

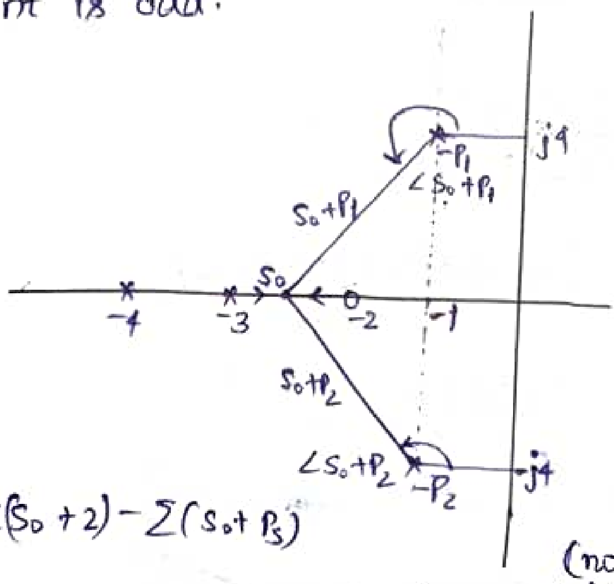
$\phi_A = \pm\dfrac{(2q+1)180}{4-1} = \pm\dfrac{180}{3}(2q+1)$

$= \pm 60(2q+1)$



Rule-4: On locus segments of the real axis:

A point on the real axis ~~of the number~~ lies on the locus if the no. of open loop poles plus zeros on the real axis of this point is odd.



→ We need to worry only about the open loop zeros lying right of s_0.

$F(s_0) = \angle(s_0+2) - \sum(s_0+P_s)$

(not on real axis)

↳ Angle contribution of open-loop pole is 0 or 360°

↳ Angle contribution of the zeros or poles lying left to a typical point s_0 is 0.

$\angle F(s_0) = (m_r - n_r)\,180°$, m_r: no. of zeros right of s_0

$= \pm(2q+1)180°$ $\quad n_r$: no. of poles right of s_0.

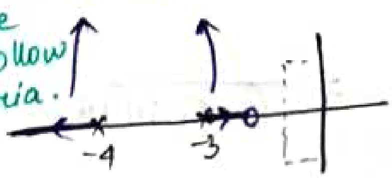
(Only OL zeros & poles considered)

$m_r - n_r$ should be odd.

$m_r - n_r + 2n_r$ should be odd.

$m_r + n_r$ should be odd.

only these parts follow the criteria.



[we are finding the part of real axis which will be in root locus.

→ not all are on root locus.]

Rule (5): On locus points of the imaginary axis:

· The intersection (if any) of the root locus with the imaginary axis can be determined by the use of Routh criteria.

Eg. $$F(S) = \frac{K(S+2)}{(S+1+j4)(S+1-j4)(S+3)(S+4)}$$

$$1+F(S)=0$$

$$\Rightarrow 1 + \frac{K(\quad)}{(\quad)} = 0$$

$$\Rightarrow (S+1+j4)(S+1-j4)(S+3)(S+4) + K(S+2) = 0$$

$$\Rightarrow S^4 + 9S^3 + 43S^2 + (143+K)S + (204+2K) = 0$$

S^4:	1	43	$204+2K$
S^3:	9	$143+K$	
S^2:	$\frac{244-K}{9}$	$204+2K$	
S^1:	$\frac{18368-61K-K^2}{244-K}$		
S^0:	$204+2K$		

For stability:

$244-K>0 \Rightarrow 244>K$, — (i)

$18368-61K-K^2>0$, — (ii)

$204+2K>0$

$\Rightarrow K>-102$ — (iii)

(ii) $\Rightarrow$ $$K_{1,2} = \frac{-61 \pm \sqrt{61^2+4\times18368}}{2}$$

$= 108.41, -164.41$

$(K-K_1)(K-K_2)<0$

+ | − | +

K_2 (−169.41) K_1 (108.41)

$\Rightarrow K_2 < K < K_1$

$\Rightarrow -169.41 < K < 108.41$ — (iv)

From (i), (iii) & (iv) $\Rightarrow$ $-102 < K < 108.41$ → for stability

$K = 108.41$ – crossing criteria of stability.

Auxiliary eqⁿ:

$$A(s) = \frac{244 - 108.41}{9} s^2 + (204 + 2 \times 108.41) = 0$$

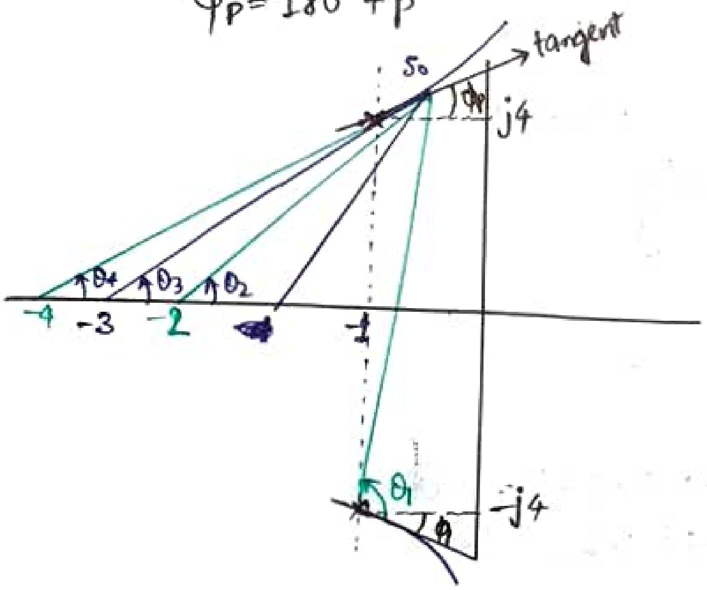
$$\Rightarrow 15.06 s^2 + 420.82 = 0$$

$$\Rightarrow s_{1,2} = \pm j5.28.$$

Rule-⑥: Rule of Departure:

The angle of departure, ϕ_p, of a locus from a complex open loop pole is given by $\phi_p = 180° + \phi$, where ϕ is the ~~angle~~ net angle contribution at this pole of all other open loop poles or zeros.

$$\phi_p = 180° + \phi$$



If $s_0 \to p$

$$\theta = \theta_2 - (\theta_1 + \theta_3 + \theta_4)$$

$$= 76° - (90° + 63° + 53°)$$

$$= -130°$$

$$F(s_0) = \pm 180°(2q+1)$$

$$\Rightarrow \phi - \phi_p = \pm 180°$$

$$\phi_p = 180° + \phi$$

$$= 180° - 130°$$

$$= 50°$$

At $-1-j4$,

$$\phi_p = -50°$$

Rule ⑦: Angle of arrival at complex zeros:

The angle of arrival (ϕ_z) ~~at complex~~ of a locus at a complex zero is given by

$$\phi_z = 180° - \phi,$$

where ϕ is the net angle contribution at this zero by all other open loop poles and zeros.

Eg. $1 + F(s) = 1 + \frac{K(s^2+1)}{s(s+2)} = 0$:

$\phi_z + \phi = \pm 180°$

$\phi_z = 180° - \phi$

$\phi_p = 180° + \phi$ (departure)

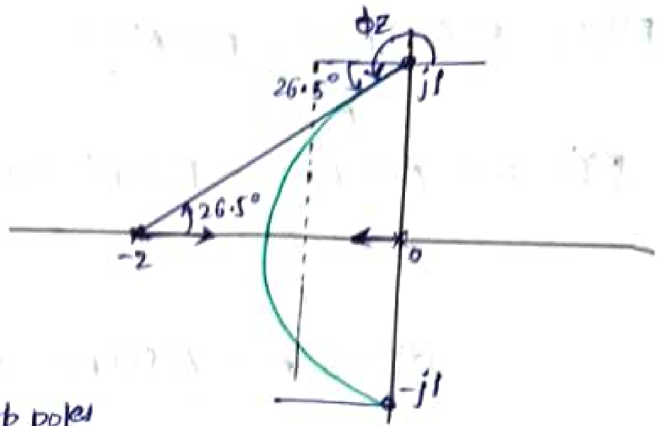
$\phi = 90° - (90° + 26.5°)$

open-loop zeros   open-loop poles

$= -26.5°$

$\phi_z = 180° - \phi = 180° + 26.5°$

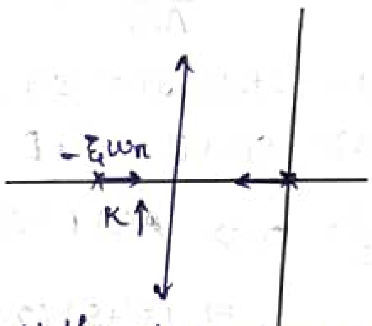
$= 206.5°$



Rule-⑧: Locus of multiple roots:

Eg. $\frac{K}{s(s+\xi w_n)}$

$\frac{dK}{ds} = 0$



Points at which multiple roots of the characteristic equation occur (breakaway points of root loci) are the solutions of

$$\frac{dK}{ds} = 0, \quad \text{where } K = \frac{-\prod_{j=1}^{n}(s+P_j)}{\prod_{i=1}^{m}(s+Z_i)} \quad (n \geq m)$$

$$1 + F(s) = 0$$

$$F(s) = \frac{K\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+P_j)} = -1, \quad n \geq m$$

$$1 + F(s) = (s-s_0)^r M(s), \quad r \geq 2$$

$$\frac{dF(s)}{ds} = r(s-s_0)^{r-1} M(s) + (s-s_0)^r \frac{dM(s)}{ds}$$

$$= (s-s_0)^{r-1}\left[r M(s) + (s-s_0) M'(s)\right], \quad r \geq 2.$$

$$\text{At } s = s_0 \Rightarrow \frac{dF}{ds_0} = 0$$

$$F(s) = K\frac{B(s)}{A(s)}$$

$$F'(s) = \frac{K\, A(s)\, B'(s) - B(s)\, A'(s)}{A^2(s)}$$

$$F'(s) = 0 \Rightarrow A(s)\, B'(s) - B(s)\, A'(s) = 0$$

$$K = -\frac{A(s)}{B(s)}$$

$$\frac{dK}{ds} = 0 \Rightarrow \frac{-A'(s)\, B(s) + A(s)\, B'(s)}{B^2(s)}$$

$$\therefore \boxed{F'(s) = 0 \Rightarrow \frac{dK}{ds} = 0}$$

Eg $$1 + F(s) = 1 + \frac{K(s+2)(s+3)}{s(s+1)}$$

$$= 1 + K\frac{B(s)}{A(s)} = 0$$

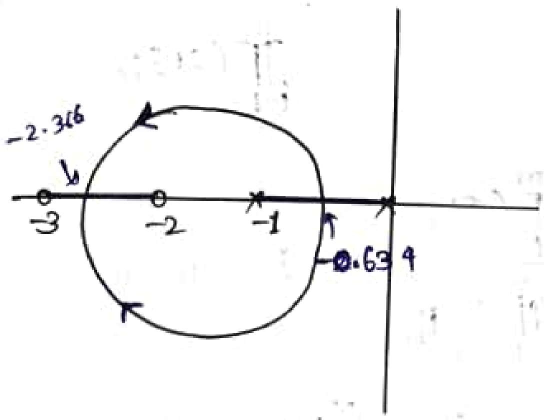
$$B(s) = (s+2)(s+3) = s^2 + 5s + 6$$

$$A(s) = s(s+1) = s^2 + s$$

$$\frac{dK}{ds} = 0 \Rightarrow A(s)\, B'(s) - A'(s)\, B(s) = 0$$

$$\Rightarrow (s^2+s)(2s+5) - (s^2+5s+6)(2s+1) = 0$$

$$\Rightarrow s_{1,2} = -0.634, -2.366$$



(29-04-2024)

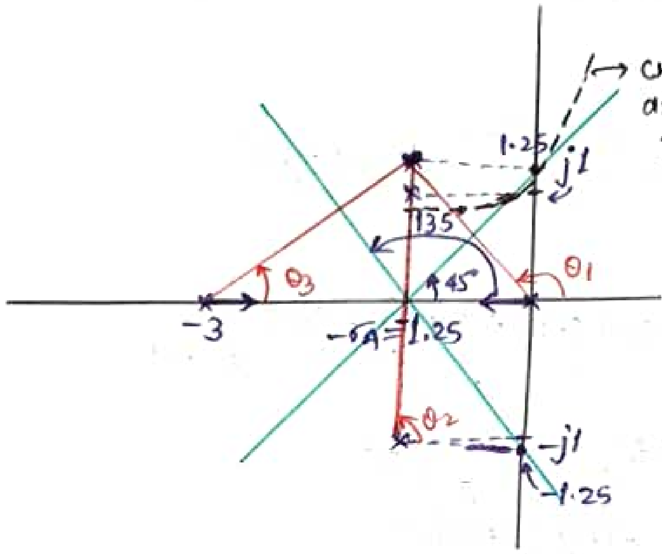
Eg. Characteristic eqn: $1 + \frac{K}{s(s+2)(s^2+2s+2)} = 0$, $K \geq 0$.

$s = 0, -3, -1 \pm j1$

centroid, $-\sigma_A = \frac{[0+(-3)+(-1)+(-1)] - [0]}{4} = -\frac{5}{4} = -1.25$

$\phi_A = \frac{(2q+1)}{n-m} 180^\circ$, $q = 0, 1, 2, \dots$; $n-m = 4$.

$= 45^\circ, 135^\circ, 225^\circ, 315^\circ$.



$\theta_1 = 135^\circ$
$\theta_2 = 90^\circ$
$\theta_3 = 26.6^\circ$

Intersection with Img.-axis:

$s^4 + 5s^3 + 8s^2 + 6s + K = 0$

s^4 : 1 8 K

s^3 : 5 6

s^2 : 34/5 K

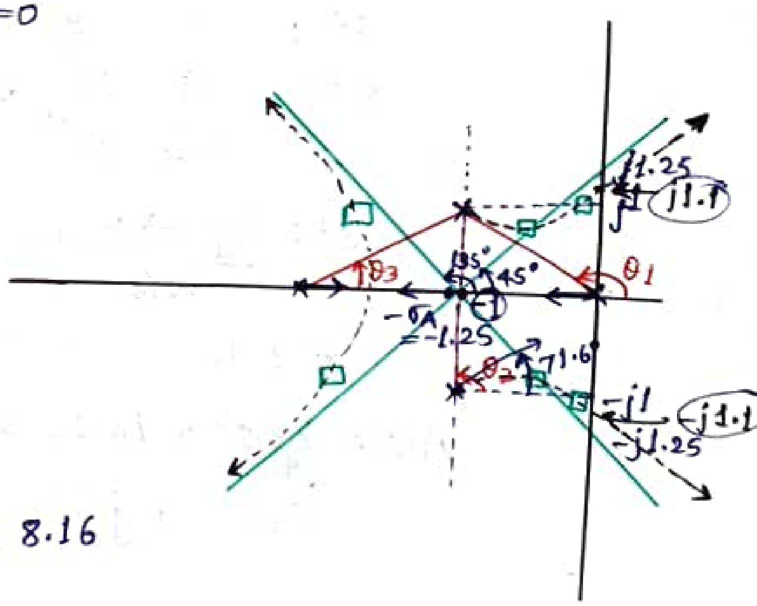
s^1 : $\frac{204/5 - 5K}{34/5}$

s^0 : K

$\Rightarrow \frac{204}{5} - 5K = 0 \Rightarrow K = 8.16$

$A(s) = \frac{34}{5}s^2 + 8.16 = 0$

$s_{1,2} = \pm j1.1$



Angle of departure:

$\phi_p = 180^\circ + \phi$

$\phi = (-135^\circ - 90^\circ - 26.5^\circ)$

$\Rightarrow \phi_p = 180^\circ + \phi = -71.6^\circ$

Breakaway Point: $\frac{dK}{ds} = 0$

$$K = -s(s+3)(s^2+2s+2)$$

$$\frac{dK}{ds} = 0 \Rightarrow s^3 + 3.75s^2 + 4s + 1.5 = 0$$

roots: $-2.3, \; -0.725 \pm j0.365$

Eg. $$F(s) = \frac{K}{s(s+2)(s^2+2s+5)}, \quad K \geq 0.$$

Open-loop poles:

$$s = 0, \; -2, \; \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j2$$

Centroid:

$$-\sigma_A = \frac{[0+(-2)+(-1)+(-1)]-[0]}{4} = -1$$

$$\phi_A = 45°, 135°, 225°, 315°.$$

Intersection with Img.-axis:

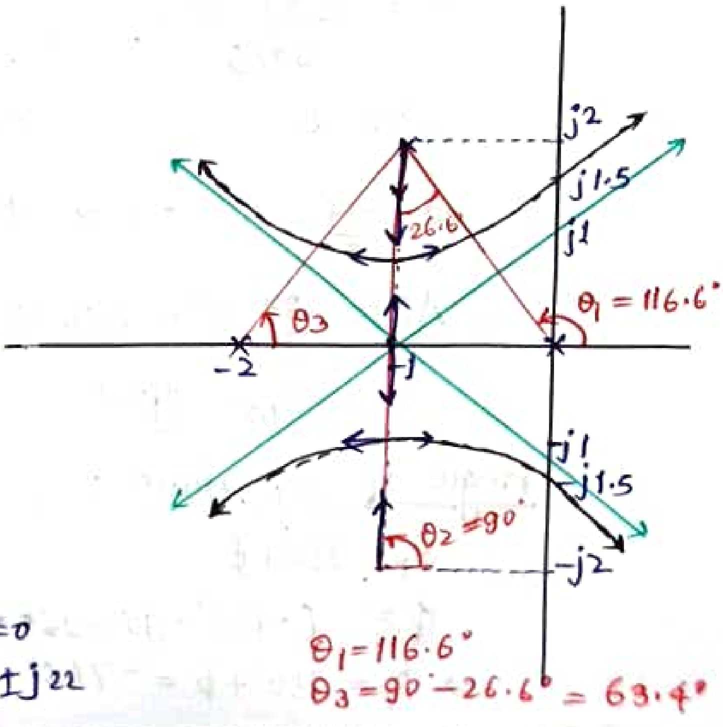
Ch. eqn: $s^4 + 4s^3 + 9s^2 + 10s + K = 0$

s^4:	1	9	K
s^3:	4	10	
s^2:	26/4	K	
s^1:	$\frac{\frac{260}{4} - 4K}{26/4}$		
s^0:	K		

s^1: $\frac{\frac{260}{4} - 4K}{26/4} = 0 \Rightarrow K = 16.25$

$$A(s) = \frac{26}{4}s^2 + 16.25 = 0$$

$$s_{1,2} = \pm j1.5$$



Angle of departure:

$$\phi = -116° - 90° - 63.4° = -270°$$

$$\phi_p = 180° + \phi = -90°$$

Breakaway points:

$$K = -s(s+2)(s^2+2s+5)$$

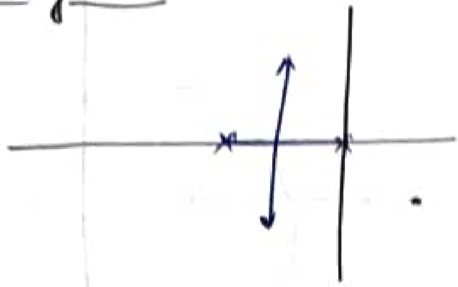
$$\frac{dK}{ds} = s^3 + 3s^2 + 4.5s + 2.5 = 0$$

$$\Rightarrow s = -1, \; -1 \pm j\sqrt{2}$$

$\theta_1 = 116.6°$

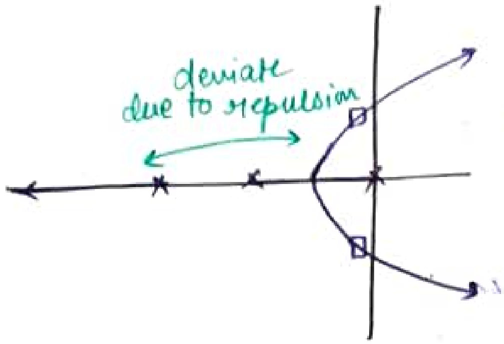
$\theta_3 = 90° - 26.6° = 63.4°$

2nd order system :

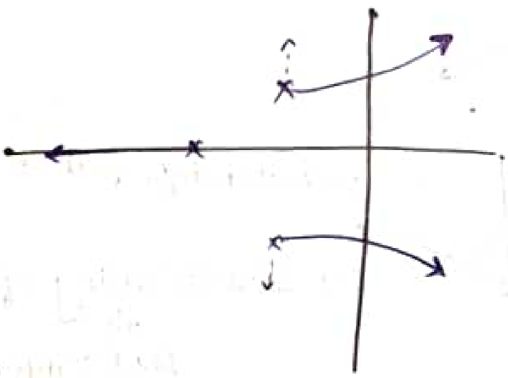


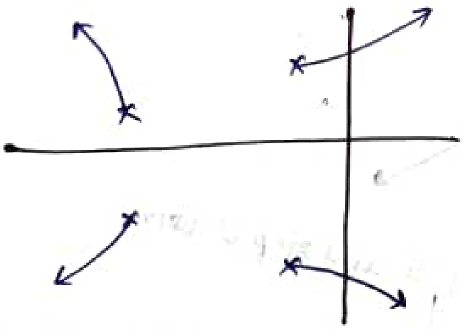
⇒ Always stable

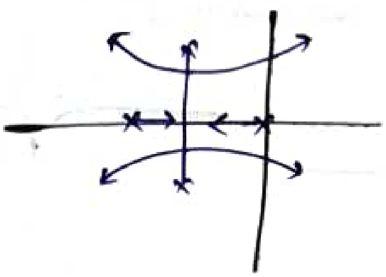
3rd order system :

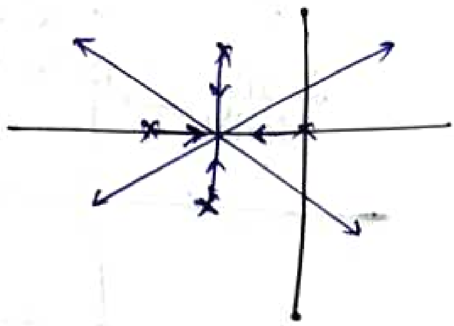


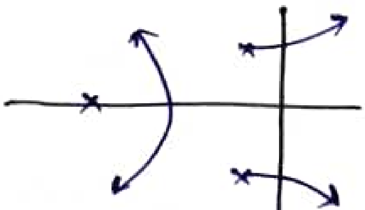
⇒ Unstable after some point

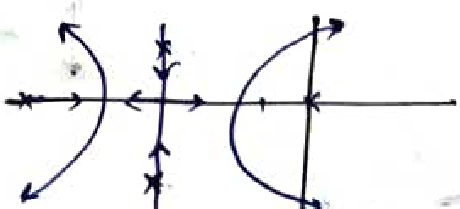


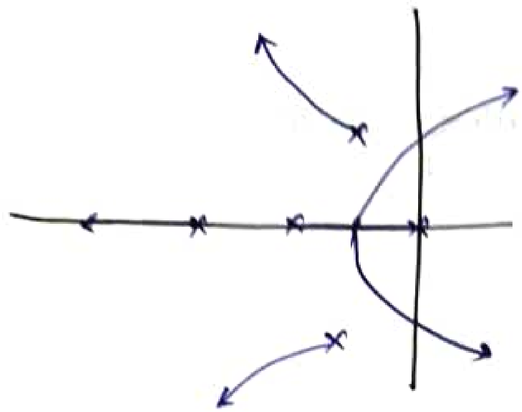


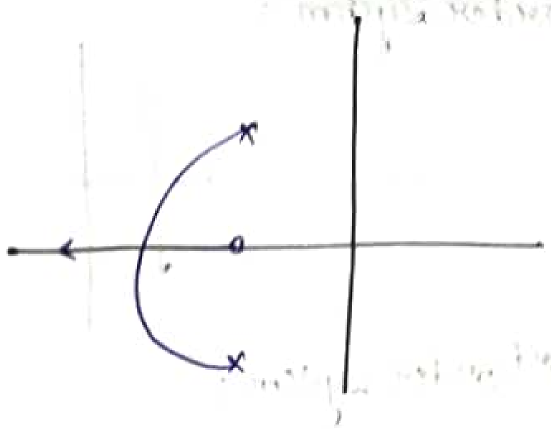


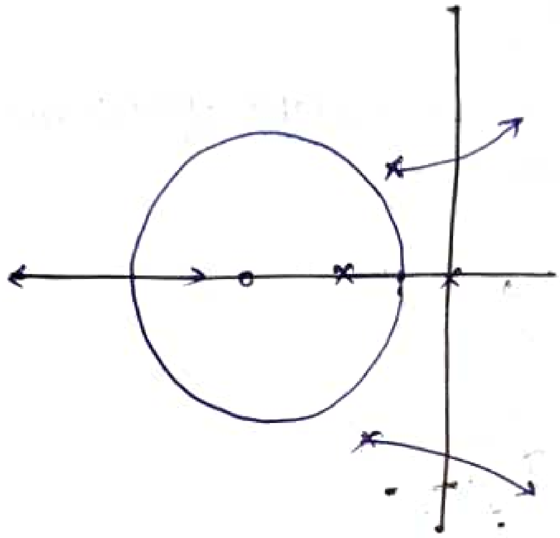


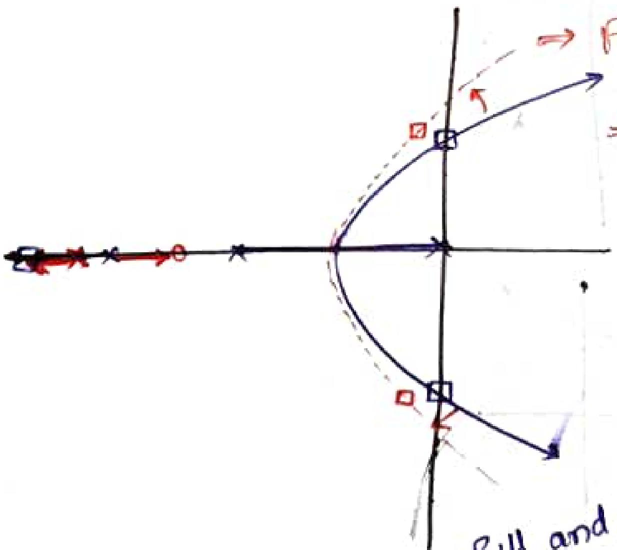












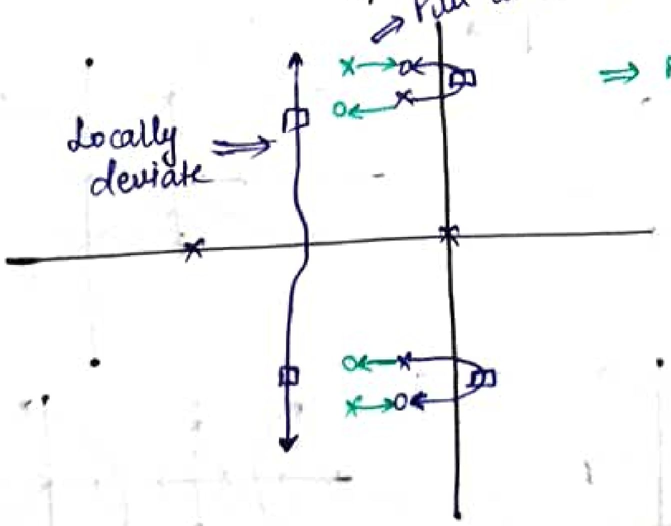
⇒ For stability, pull it

⇒ Zero is coming after pole

⇓

Lead compensator

⇒ Pull and stop U-turn



⇒ Notch filter