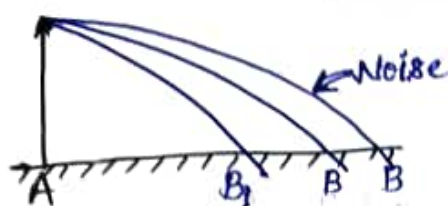


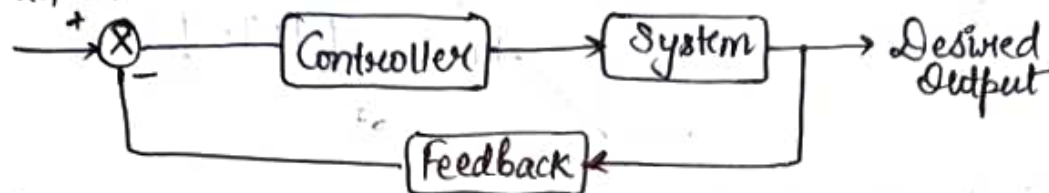
#



Actual Output  
Desired Output

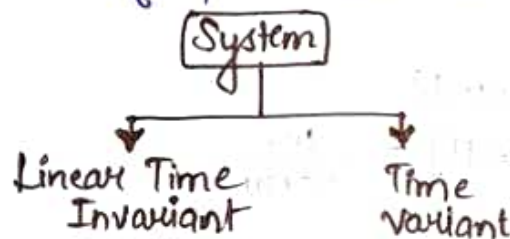
Control system

Refined



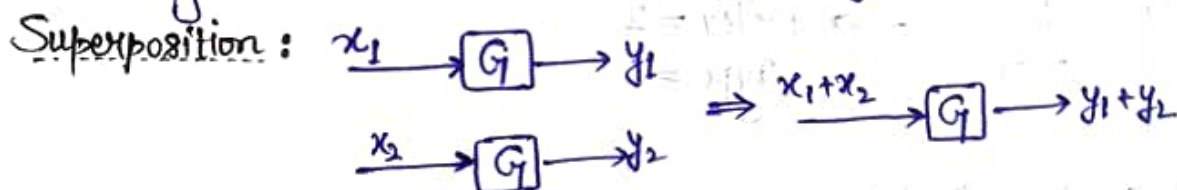
## Transfer Function

↳ Ratio of ~~linear~~ Laplace transform of output to the Laplace transform of input when all initial conditions are zero.

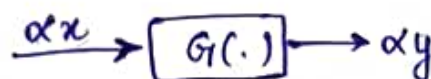


## Linear System

↳ Obeys superposition and homogeneity theorem.



Homogeneity:



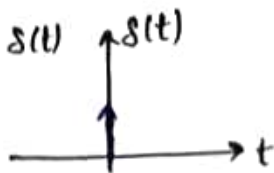
Eg.  $C(s) = \frac{10(s+1)}{(s+2)(s+3)}$

Poles: Makes function infinite  $\rightarrow -2, -3$

Zeros: Makes function zero  $\rightarrow -1$

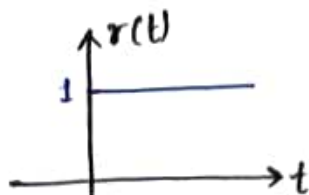
## Signals

① Impulse input:  $\delta(t)$



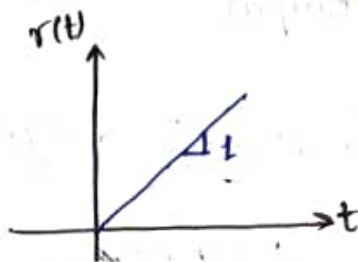
$$\mathcal{L}[\delta(t)] = 1$$

② Unit step input:  $r(t)$



$$\mathcal{L}[1] = \frac{1}{s}$$

③ Ramp/Velocity:  $r(t)=t$



$$\mathcal{L}[t] = \frac{1}{s^2}$$

④ Parabolic/acceleration input:

$$r(t) = \frac{t^2}{2}$$

$$\mathcal{L}[t^2/2] = \frac{1}{s^3}$$

⑤ Sinusoidal input:

$$r(t) = \sin(\omega_0 t)$$

$$\mathcal{L}[\sin(\omega_0 t)] = \frac{\omega_0}{s^2 + \omega_0^2}$$

Order: Total no. of poles.

Type: No. of poles at the origin.

Eg.  $\frac{10}{s(s+2)} \rightarrow \text{Order} = 2$   
 $\text{Type} = 1$

## Second Order System

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow \begin{matrix} \text{order} = 2 \\ \text{Type} = 0 \end{matrix}$$

$\zeta$ : damping ratio

$\zeta = 0$ : Undamped system

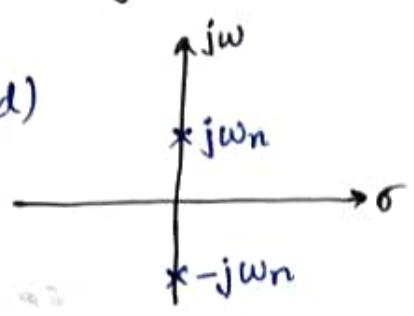
$0 < \zeta < 1$ : Underdamped system

$\zeta = 1$ : Critically damped system

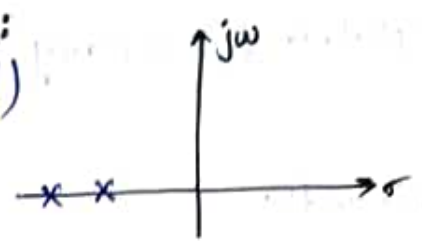
$\zeta > 1$ : Overdamped system

# s-plane ( $\sigma$ - $\omega$ plane):

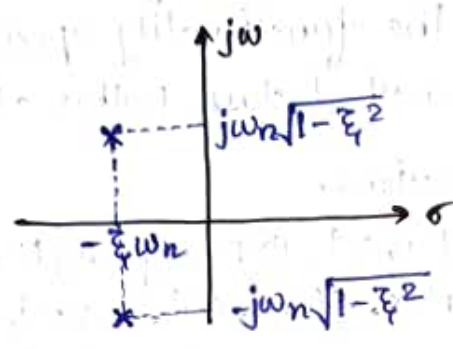
$\xi = 0$ :  
(Undamped)



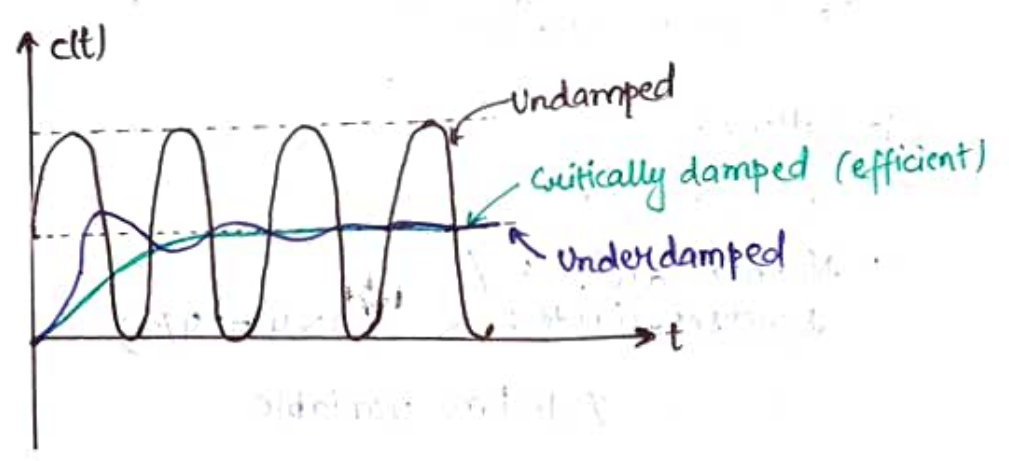
$\xi = 1$ :  
(Critically damped)



$0 < \xi < 1$ :  
(Underdamped)  $s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$



$\xi > 1$ :  
(Overdamped)





# MATLAB

↳ Matrix Laboratory

## Commands:

- **pwd** (present working directory)  
pwd : displays the current working directory.  
s=pwd : returns the current directory in string s.
- **help** : Displays the help text for the functionality specified by name, such as function, method, class, toolbox or variable.
- **help elfun** : Elementary math functions.  
↳ sin, sinh, asin, tan, cot, exp, log, log10, sqrt, abs, angle, complex, floor, ceil, round, mod, rem.
- **lookfor** : Search for keyword in reference page text.  
↳ lookfor transfer

## Operations:

↳ +, -, \*, /, \

↳ Normal divide : /

Reverse divide : \ ( $3 \setminus 4 = 4/3$ )

clc : clears screen.

clear all : clears all workspace variables.

clear abc : clears variable with 'abc' name.

disp : displays word or value

a1=3; % declare variable

a2=4;

r1 = a1/a2;

r2 = a1 \ a2; % = a2/a1

## Plot Graph:

a = 0 : 0.01 : 20;

b = sin(a);

plot(a, b, 'r'); % red (plot y vs x graph)

plot(a, b, '--'); % dashed

plot(a, b, '--r'); % dashed red

xlabel('x'); % label x-axis

ylabel('y'); % label y-axis

title('plot of sine'); % give title of the plot

Two graph on same plot:

plot(x, y1); hold on

plot(x, y2);

legend('sin', 'cos')

↓       ↓  
for y1   for y2

## Subplot:

↳ To plot multiple graphs in same window.

$x = -\pi : \pi/20 : \pi;$

%  $-\pi, -\pi + \pi/20, -\pi + 2\pi/20, \dots, -\pi + 40\pi/20.$

$y_1 = \sin(x);$

$y_2 = \sin \cos(x);$

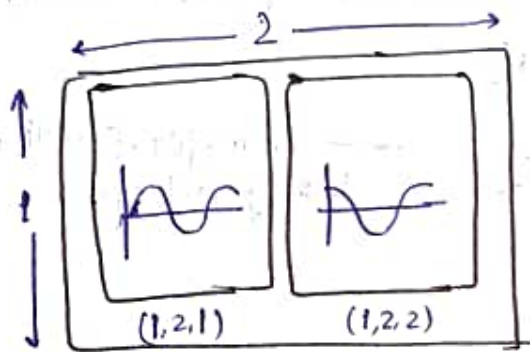
$\text{subplot}(1, 2, 1);$

no. of rows  
no. of columns  
position

$\text{plot}(x, y_1);$

$\text{subplot}(1, 2, 2);$

$\text{plot}(x, y_2);$



## Matrix:

$\text{zeros}(1,8);$  % matrix of  $1 \times 8$  with all values as zero

$\text{ones}(1,8);$  % matrix of  $1 \times 8$  with all values as one

$\text{eye}(4);$  %  $4 \times 4$  identity matrix

$\text{inv}(B);$  % Inverse of matrix B

$\text{det}(B);$  % Inverse of determinant of matrix B

$C = A + B$  % matrix addition

$C = A \cdot B$  % matrix multiplication

$C = A .* B$  % element-by-element matrix multiplication

## Matrix Declaration:

$C = [1 \ 5 \ 3];$  % declare matrix

$\text{size}(C);$  %  $\rightarrow 1 \ 3$

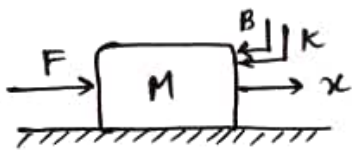
$d = [4 \ 4 \ 7; 4 \ 8; 3 \ 8 \ 0];$

$\text{size}(d);$  %  $\rightarrow 3 \ 3$

Transpose:

$\text{tra} = d';$

Eg. Mass system:



$$F - B\dot{x} - Kx = M\ddot{x}$$

$$\text{LT} \Rightarrow F(s) - BsX(s) - KX(s) = Ms^2X(s) \quad [\text{zero initial conditions}]$$

$$\Rightarrow F(s) = X(s) [Ms^2 + Bs + K]$$

$$\Rightarrow G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} \quad \begin{array}{l} \rightarrow \text{Transfer function} \\ \rightarrow \text{Open-loop} \end{array}$$

MATLAB:

$m = 1;$

$b = 5;$

$k = 7;$

$\text{num} = 1;$

$\text{den} = [m \ b \ k];$

$\text{sys} = \text{tf}(\text{num}, \text{den});$

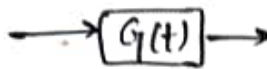
%  $\text{sys} = \frac{1}{s^2 + 5s + 7}$

Standard 1st order system:  $G(s) = \frac{K}{s + \tau} \left[ \lim_{s \rightarrow 0} \frac{K}{s + \tau} = 1 \Rightarrow \text{dc gain} = 1 \right]$

Standard 2nd order system:  $G(s) = \frac{1}{s^2/\omega_c^2 + s/Q\omega_c + 1}$

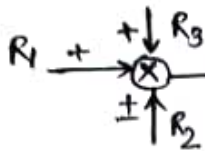


# Modelling a System (Using a block diagram)



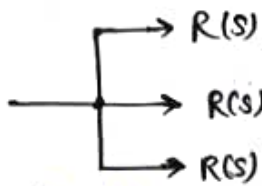
→ : flow of information

□ : transfer function



$$R_1 \pm R_2 + R_3$$

→ Summing point  
or  
Error detector

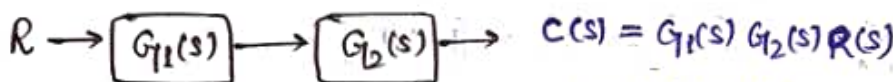


(one i/p, multiple o/p)

→ Takeoff point  
or  
Pickoff point

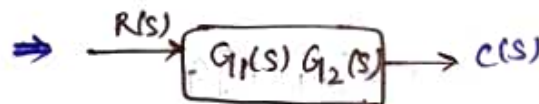
## LTI System:

Eg.

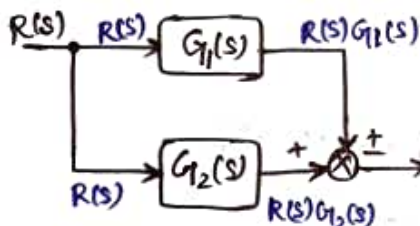


$$C(s) = G_1(s) G_2(s) R(s)$$

⇒ Block in cascade/series

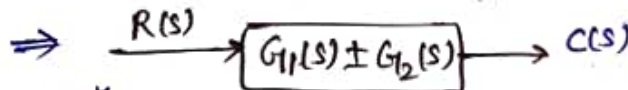


Eg.

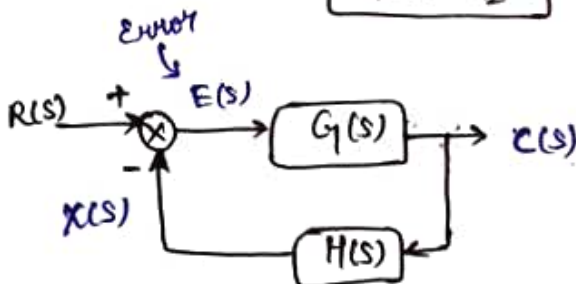


⇒ Blocks in parallel

$$R(s) [G_1(s) \pm G_2(s)]$$



Eg.



⇒ Blocks in feedback

$$E(s) = R(s) - X(s)$$

$$= R(s) - H(s)C(s)$$

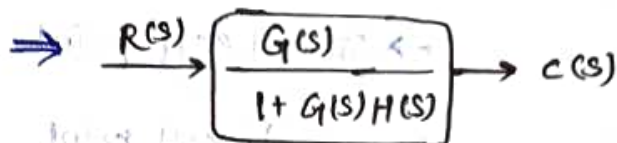
$$C(s) = E(s)G(s)$$

$$= [R(s) - H(s)C(s)]G(s)$$

$$= G(s)R(s) - G(s)H(s)C(s)$$

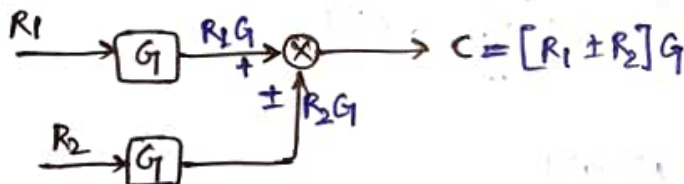
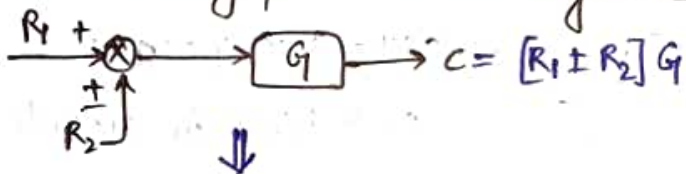
$$\Rightarrow C(s)[1 + G(s)H(s)] = G(s)R(s)$$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}}$$

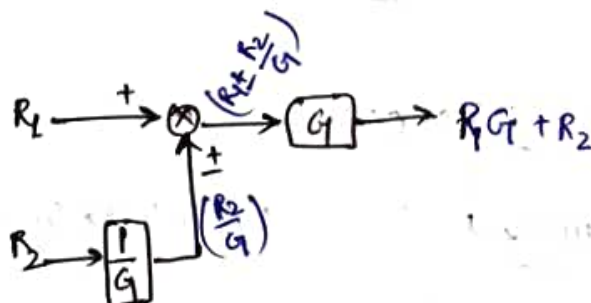
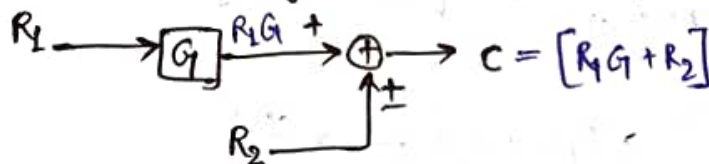


Shifting:

① Shifting the summing point to the right side:



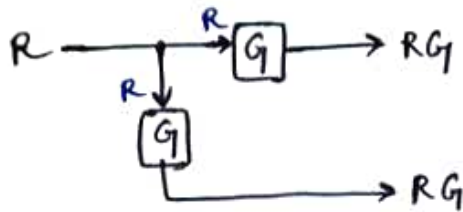
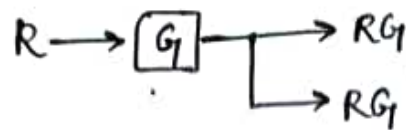
② Shifting the summing point to the left side:



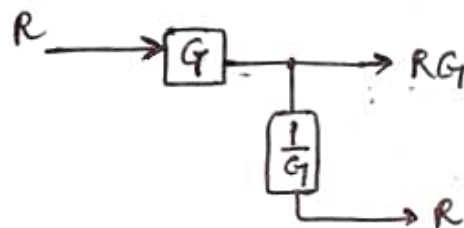
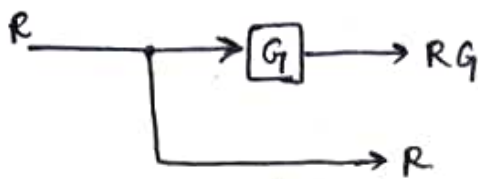


## ③ Shifting the pickoff point:

①



②



## Commonly used Blocks:

- Continuous: Integrator, differentiator, transfer function.
- Discontinuous: Non-linear blocks.
- Math operation: Adder, subtractor, gain.
- Sources: Input signals like step, ramp, sine; from workspace, from spreadsheet, from file.
- Sinks: To see output.

• Scope: Displays output.

→ Configuration parameter: **ctrl + E**

Solver selection Type: Fixed-step  
Solver: ode4

→ Numerical integrator: automatically uses low sample rate

↳ Use fixed step

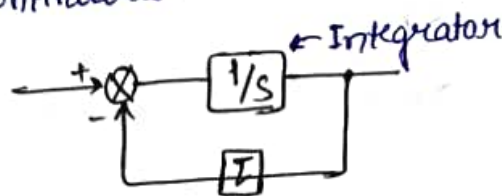
If bandwidth = 50 Hz  $\Rightarrow$  use sampling frequency = 50 Hz x 50  
↳ based on bandwidth.

# Transfer function block:

↳ continuous.

eg.

$$\frac{1}{s+T}$$



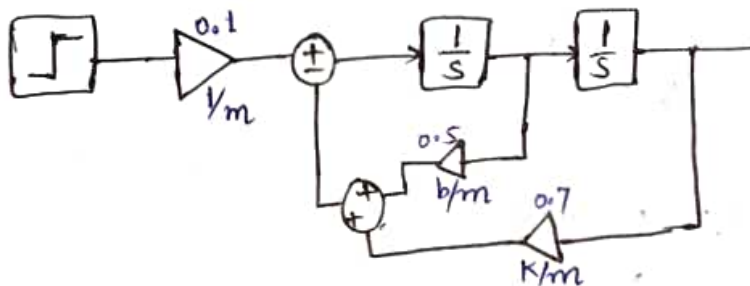
eg.

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

where  $F = M\ddot{x} + B\dot{x} + Kx$

$$\Rightarrow \ddot{x} = \frac{F - B\dot{x} - Kx}{M}$$

$$= \frac{F}{M} - \frac{B}{M}\dot{x} - \frac{K}{M}x$$



# AV241- Instrumentation and Control Lab

## CONTROL LAB-3

①  
01-02-2024

SAURABH KUMAR  
Sc22 B146

$$\rightarrow \frac{C(s)}{R(s)} = \frac{10}{s^2 + 2s + 1} \rightsquigarrow \begin{matrix} \text{Single Input} \\ \text{Single output} \end{matrix}$$

$$\Rightarrow s^2 C(s) + 2sC(s) + C(s) = 10R(s)$$

$$\Rightarrow \ddot{c}(t) + 2\dot{c}(t) + c(t) = 10r(t)$$

$$\text{Let } c(t) = x_1(t)$$

$$\dot{c}(t) = \dot{x}_1(t) = x_2(t)$$

$$\ddot{c}(t) = \ddot{x}_1(t)$$

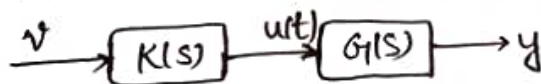
$$\ddot{c}(t) = \dot{x}_2(t) = -2x_2(t) - x_1(t) + 10r(t)$$

$$\underbrace{\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_x + \underbrace{\begin{bmatrix} 0 \\ 10 \end{bmatrix}}_B \underbrace{u(t)}_u$$

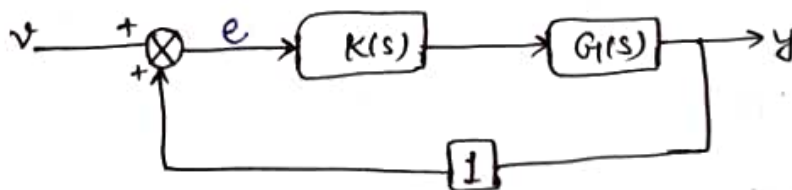
$$y = c(t) = x_1(t)$$

$$= \underbrace{[10]}_C \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_x + \underbrace{[0]}_D \underbrace{u}_u \quad \rightarrow \text{State-space Representation}$$

Open-loop system:



closed-loop (feedback) system:



## MATLAB Commands :

**Complex** → creates complex array

`C = complex(A,B)` %  $A + Bi$

**conj(X)** → complex conjugate of X

**real(X)** → Real part of X

**imag(X)** → Imaginary part of X.

**abs(X)** → absolute value of element X (magnitude of X)

**atan2(y,x)** } four quadrant inverse tangent of x and y.

**atan2d(y,x)** }  
↳ in degrees

Eg. `atan2d(3,4)` % 36.8699

**polyval(P,x)** → Evaluates polynomial P at x

Eg. `P = [3, 2, 1];` %  $p(x) = 3x^2 + 2x + 1$

`x = [5, 7, 9];` % at  $x = 5, 7, 9$

`y = polyval(p,x)` % 86 162 262

**roots(p)** → find polynomial roots

### Arrays:

**ones(x,y)** → x-by-y matrix of ones

**zeros(x,y)** → x-by-y matrix of zeros.

**eye(x,y)** or **eye([x,y])** → x-by-y matrix with 1's on the diagonal and zeros elsewhere.

**ginput** → graphical input from mouse

**ginput(2)** → gets 2 points from current axes and returns x- and y- coordinates.

**norm** → vector and matrix norms

**conv(A,B)** → convolves A and B vectors.

Eg. `x = 0:0.1:10;`

`y = x;`

`plot(y,x);`

`ginput(2);` % → for 2 points



## Transfer Function in Polynomial form

③

$$\% G(s) = [2s^3 + 5s^2 + 3s + 6] / [s^3 + 6s^2 + 11s + 6]$$

$$\text{num} = [2 \ 5 \ 3 \ 6];$$

$$\text{den} = [1 \ 6 \ 11 \ 6];$$

$$\text{sys} = \text{tf}(\text{num}, \text{den});$$

## TF in Partial Fraction / Residue form:

$$[r, p, k] = \text{residue}(\text{num}, \text{den}); \rightarrow \frac{-6}{s+3} + \frac{-4}{s+2} + \frac{3}{s+1} + 2$$

$$\% r = [-6, -4, 3],$$

$$p = [-3, -2, -1],$$

$$k = 2$$

## TF in Pole-Zero form:

$$[z, p, k] = \text{tf2zp}(\text{num}, \text{den}); \quad \% \text{ zero-pole-gain}$$

$$\% z = [-2.3965 + 0i, -0.0518 + 1.1177i, -0.0518 - 1.1177i],$$

$$p = [-3, -2, -1],$$

$$k = 2$$

$$\hookrightarrow \frac{2(s+2.3965)(s+0.0518-j1.1177)(s+0.0518+j1.1177)}{(s+3)(s+2)(s+1)}$$

## Z-P-G format to Polynomial:

$$[\text{num}, \text{den}] = \text{zp2tf}(z, p, k);$$

$$\text{Eg. } z = []; \quad p = [-1+2*j; -1-2*j]; \quad k = 10;$$

$$[\text{num}, \text{den}] = \text{zp2tf}(z, p, k);$$

$$\text{sys} = \text{tf}(\text{num}, \text{den}) \quad \% T(s) = 10 / [s^2 + 2s + 5]$$

## TF to state-space Representation:

$$[A, B, C, D] = \text{tf2ss}(\text{num}, \text{den});$$

## convolution:

$$\text{conv}([1, 10], [1, 4, 16])$$

## state-space to TF:

$$[\text{num}, \text{den}] = \text{ss2tf}(A, B, C, D, \underbrace{iw}_{\text{no. of inputs}});$$

Eq. %A = [0 1; -25 -4]; B = [1 1; 0 1]; C = [1 0; 0 1]; D = [0 0; 0 0]

% obtain the corresponding transfer functions.

A = [0 1; -25 -4];

B = [1 1; 0 1];

C = [1 0; 0 1];

D = [0 0; 0 0];

[num, den] = ss2tf(A, B, C, D, 1);

[num, den] = ss2tf(A, B, C, D, 2);

### Partial Fraction Expansion

% f = 2 / [(s+1)(s+2)]

partfrac(2/((s+1)\*(s+2))) % = 2/(s+1) - 2/(s+2)

### Laplace transform:

syms s % create symbolic variable

f = ilaplace(s/((s+1)\*(s+2))) % inverse laplace transform

% f = 2\*exp(-2\*t) - exp(-t)

pretty(f); % to resemble type-set mathematics

% f = exp(-2t) 2 - exp(-t)

Eq. % obtain the laplace transform of f(t) = sin(3t+45)

syms t

f = sin(3\*t+45);

F = laplace(f);

pretty(F);

% 
$$\frac{3 \cos(45) + s \sin(45)}{s^2 + 9}$$

Eq. % G1 = 10 / [s^2 + 2s + 10]; G2 = 5 / [s + 5]

n1 = 10; d1 = [1 2 10];

n2 = 5; d2 = [1 5];

%% series

[ns ds] = series(n1, d1, n2, d2);

printsys(ns ds) % Print system in pretty format

% num/den = 
$$\frac{5s}{s^3 + 7s^2 + 20s + 50}$$

%. Parallel

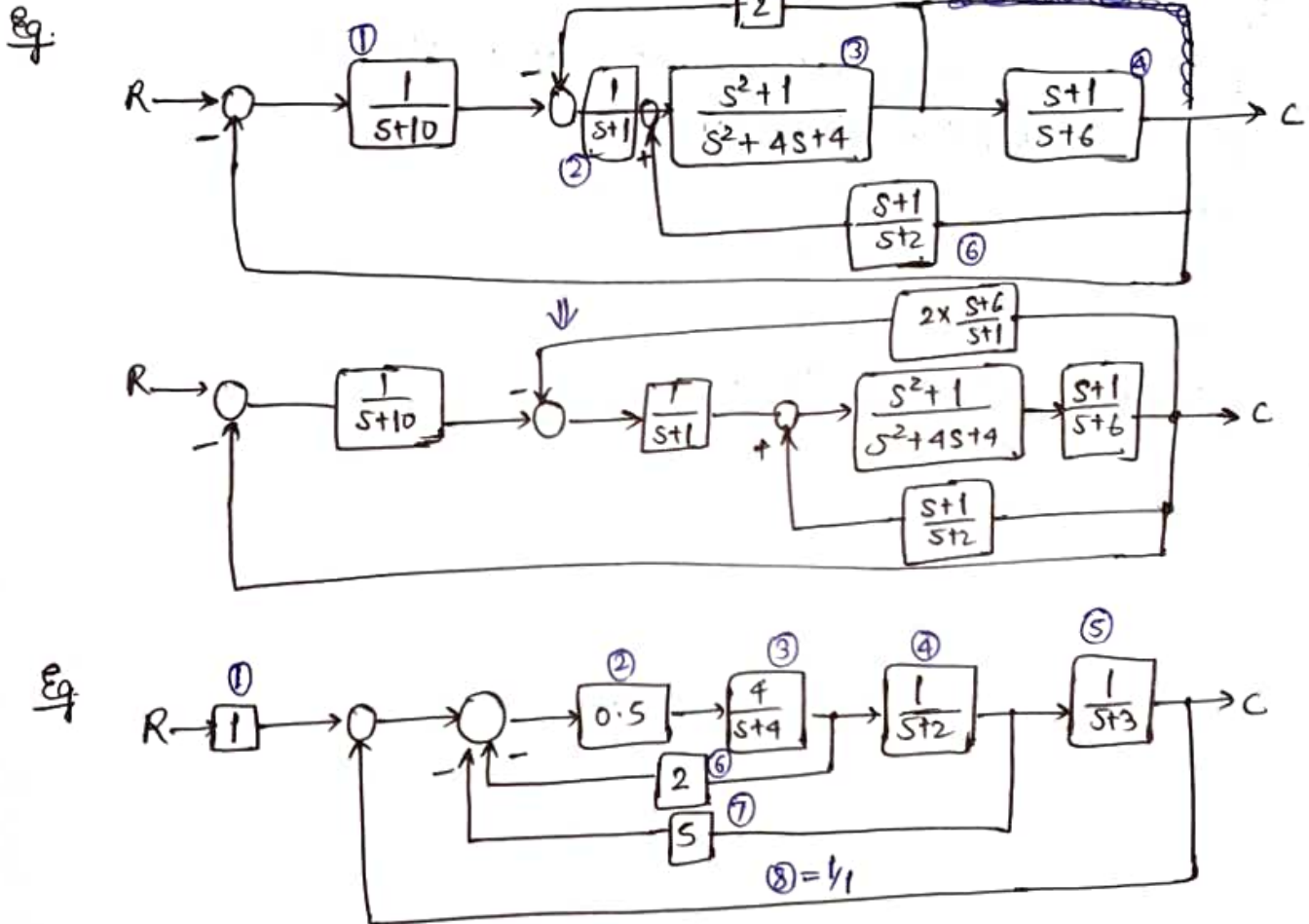
```
[np, dp] = parallel(n1, d1, n2, d2);  
printsys(np, dp)
```

%. Feedback

```
[nf, df] = feedback(n1, d1, n2, d2);
```

%. Positive Feedback

```
[nfp, dfp] = feedback(n1, d1, n2, d2, 1);
```



```
n1=1; d1=1;  
n2=0.5; d2=1;  
n3=4; d3=[1 4];  
n4=1; d4=[1 2];  
n5=1; d5=[1 3];  
n6=2; d6=1;  
n7=5; d7=1;  
n8=1; d8=1;
```

```
nblocks = 8; % no. of blocks
```

Block no.  $\rightarrow$  max no. of incoming (internal) signals

$q = [ \begin{matrix} 1 & 0 & 0 & 0 & 0 \end{matrix} ]$

$2 \ 1 \ -6 \ -8 \ -7 \leftarrow$  i/p's coming from blocks 1, 6, 7, 8

$3 \ 2 \ 0 \ 0 \ 0$

$4 \ 3 \ 0 \ 0 \ 0$

$5 \ 4 \ 0 \ 0 \ 0$

$6 \ 3 \ 0 \ 0 \ 0$

$7 \ 4 \ 0 \ 0 \ 0$

How many  
no. of  
blocks

$\rightarrow 8 \ 5 \ 0 \ 0 \ 0$

$i_u = 1; i_y = 5$ , % Input (block 1) to output (block 5)

blkbuild % builds the matrix

$[A \ B \ C \ D] = \text{connect}(a, b, c, d, q, i_u, i_y)$

$[\text{num}, \text{den}] = \text{ss2tf}(A, B, C, D)$

$\text{sys} = \text{tf}(\text{num}, \text{den})$

% Ans =  $2 / [s^3 + 13s^2 + 56s + 80]$

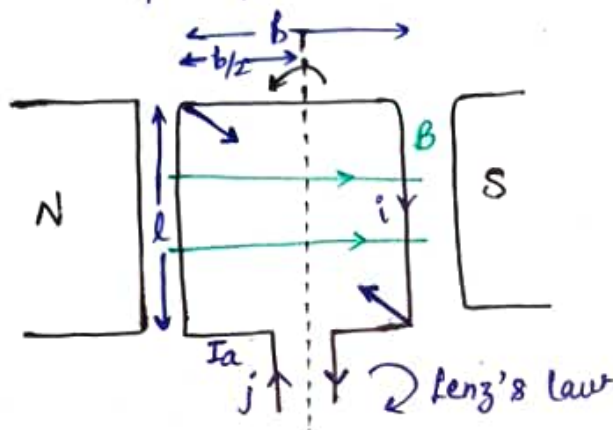


# Mathematical Modelling of Electromechanical Actuation System

Electrical input  $\rightarrow$  Mechanical

$\rightarrow$  Motor-control system  
 $\rightarrow$  Antenna Control system

Motor: Armature-controlled DC (brushed type)  
field: fixed



$\rightarrow$  Direction of force given by:  
Fleming's left hand rule

$\rightarrow$  EMF: back emf ( $e_b$ )

$$F = \sum B I_a l \sin \theta_j$$

$$\text{Total force} = 2F$$

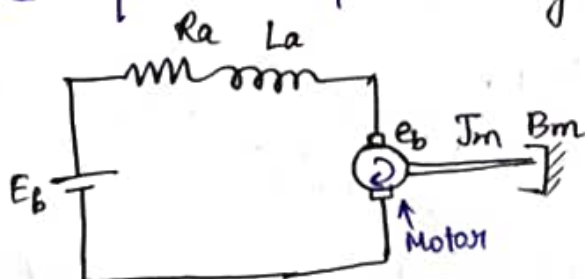
$$\begin{aligned} \text{Torque, } T &= f \times I^{\text{th}} \text{ distance} \\ &= \sum 2B I_a l \sin \theta_j \times b/2 \end{aligned}$$

$$\begin{aligned} T &= 2B I_a l \sin \theta_j \times b/2 \\ &= K_T \times I_a \end{aligned}$$

$$\text{Total torque} = \sum K_T I_a$$

Back emf:  $e_b = K_b \omega_m$  [Due to electromagnetic induction]  
 $\downarrow$  constant

Generator equation:  $E_g = \frac{\phi Z P \omega_m}{60 A}$ ,  $\omega_m$ : speed  
 $Z$ : no. of conductors  
 $P$ : no. of poles



$J_m$ : Moment of inertia of motor

$B_m$ : viscous damping (due to friction)

[Fleming's Right Hand Rule]

Electrical eqn:  $E_a = I_a R_a + L_a \frac{dI_a}{dt} + e_b \dots (1)$

Mechanical eqn:  $T_m = (J_m \ddot{\theta} + B_m \dot{\theta})$  : Torque

$$\Rightarrow T_m = (J_m s^2 + B_m s) \theta(s) \dots (2)$$

$$= K_T I_a$$

LT (1)  $\Rightarrow E_a - e_b = I_a R_a + L_a s I_a$

$$\Rightarrow E_a - e_b = I_a (R_a + L_a s) \dots (1')$$

and,  $e_b = K_b \omega_m \dots (3)$

and,  $\omega_m = \dot{\theta} \Rightarrow \omega_m = s \theta$

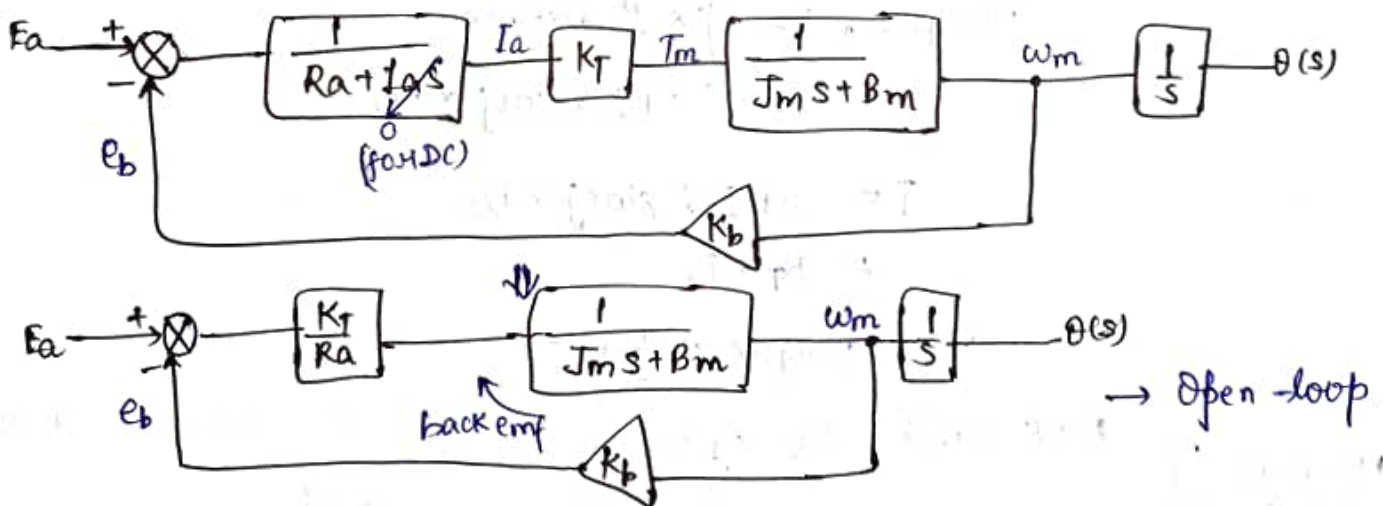
Motor  
I/p:  $E_a$   
O/p:  $\theta$

$$(2) \Rightarrow K_T I_a = (J_m s^2 + B_m s) \theta = \left( \frac{E_a - e_b}{R_a + L_a s} \right) K_T$$

$$\Rightarrow (J_m s^2 + B_m s) \theta = \left( \frac{E_a - K_b \theta s}{R_a + L_a s} \right) K_T$$

$$\Rightarrow \left( J_m s^2 + B_m s + \frac{K_b K_T s}{R_a + L_a s} \right) \theta = \frac{E_a K_T}{R_a + L_a s} \rightarrow \text{for DC}$$

$$\Rightarrow \frac{\theta}{E_a} = \frac{K_T / R_a}{J_m s^2 + (B_m + \frac{K_T K_b}{R_a}) s} \quad [I_a = 0 \text{ for DC}]$$



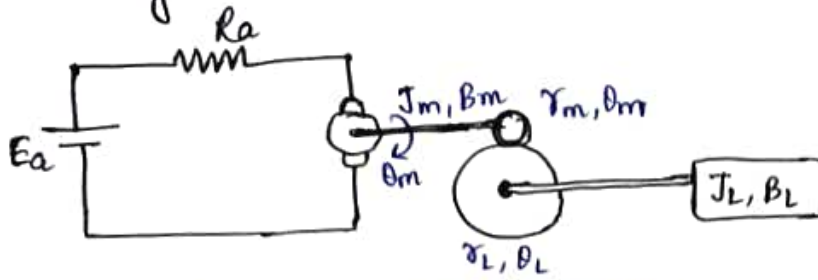
Closed-loop : Speed control using sensor

Open-loop TF :  $\frac{\theta(s)}{E_a(s)} = \frac{K_T / R_a}{J_m s^2 + (B_m + \frac{K_T K_b}{R_a}) s}$  ,  $K_b$  : electromechanical damping

→ Motor : High speed  $\times$  low torque

For mechanical, we need High torque  $\times$  low speed.

# Controlling Motor:



radius of motor  
 $r_m \propto N_m \rightarrow$  no. of teeth  
 $r_L \propto N_L$

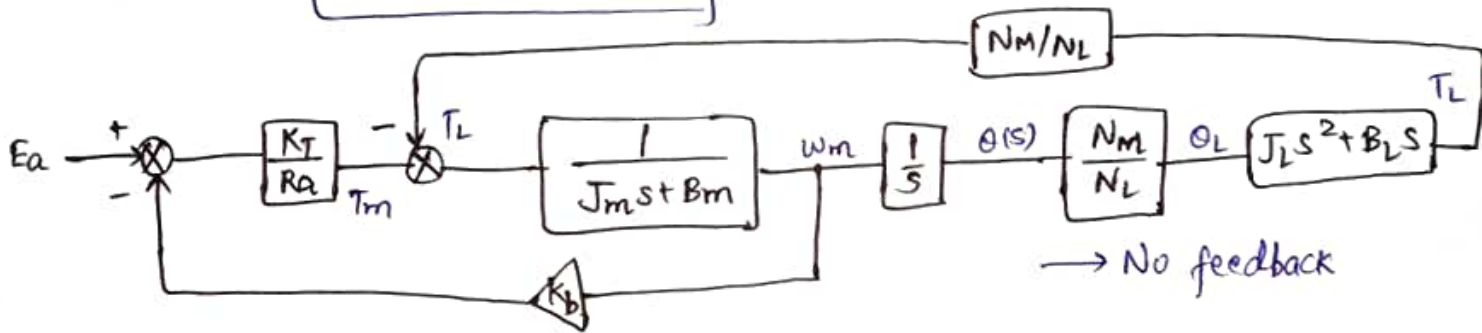
$$\boxed{T_m \theta_m = T_L \theta_L} \rightarrow \text{torque}$$

area covered

$$\frac{T_m}{T_L} = \frac{\theta_L}{\theta_m} = \frac{N_L}{N_m}$$

$$\Rightarrow T_L = T_m \left( \frac{N_m}{N_L} \right)$$

$$\boxed{T_L = (J_L s^2 + B_L s) \theta_L}$$



$$\uparrow I_a = \frac{(E_a - E_b)}{R_a} \downarrow$$

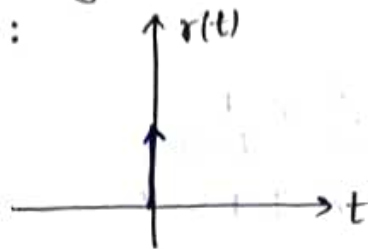
Control LAB-5

16-02-2024

SAURABH KUMAR  
SC22B146

Standard Test Signals

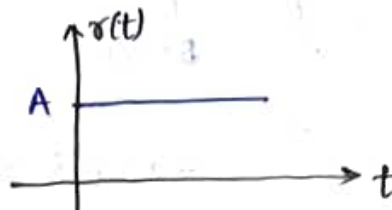
① Impulse input:



$$r(t) = \delta(t)$$

$$\mathcal{L}[\delta(t)] = 1$$

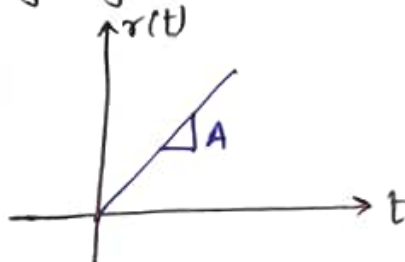
② Step input:



$$r(t) = A, t \geq 0$$

$$R(s) = \frac{A}{s}$$

③ Velocity / Ramp input:

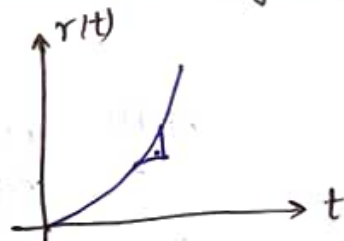


$$r(t) = At, t \geq 0$$

$$= 0, t < 0$$

$$R(s) = \frac{A}{s^2}$$

④ Parabolic / Acceleration input:



$$r(t) = \frac{At^2}{2}, t \geq 0$$

$$R(s) = \frac{A}{s^3}$$

⑤ Sinusoidal input:

$$r(t) = \sin \omega t$$

$$R(s) = \frac{\omega}{s^2 + \omega^2}$$

Type: Number of poles at origin.

Order: Total number of poles.

Eg.  $G(s) = \frac{10}{s+1} \rightarrow \text{Type}=0$   
 $\text{order}=1$

$G(s) = 50 \rightarrow \text{Type}=0$   
 $\text{order}=0$

$G(s) = \frac{10}{s^2+s} \rightarrow \text{Type}=1$   
 $\text{order}=2$



## First Order System

②

$$\frac{C(s)}{R(s)} = \frac{1}{1+s\tau} \rightarrow \text{Type}=0, \text{Order}=1$$

Step Response:

$$r(t)=1 \Rightarrow R(s)=\frac{1}{s}$$

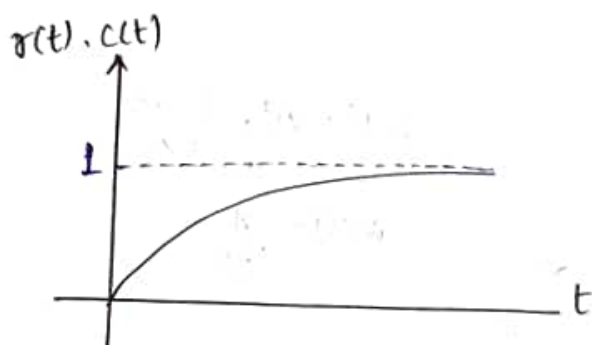
$$C(s) = R(s) \cdot \frac{1}{1+s\tau} = \frac{1}{s} \cdot \frac{1}{1+s\tau} = \frac{A}{s} + \frac{B}{1+s\tau}$$

$$A=1, B=-\tau$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{\tau}{1+s\tau} = \frac{1}{s} - \frac{1}{s+1/\tau}$$

$$\therefore c(t) = 1 - e^{-t/\tau} \rightarrow \text{Unit step response}$$

$\tau$ : time constant



Ramp response:  $R(s) = \frac{1}{s^2}$

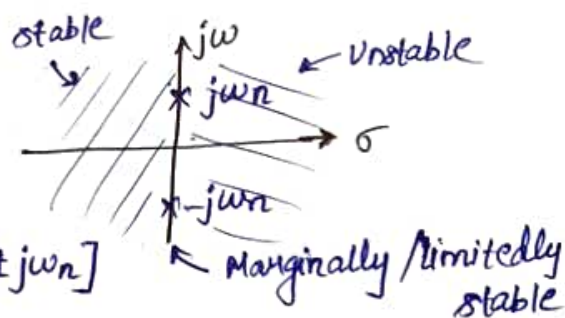
## Second Order System

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\rightarrow \zeta=0$ : Undamped

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

[Poles:  $s = \pm j\omega_n$ ]

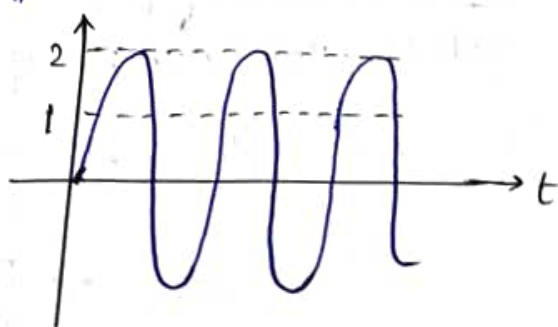


Step Response:  $R(s) = \frac{1}{s} \therefore C(s) = R(s) \cdot \frac{\omega_n^2}{s^2 + \omega_n^2} = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + \omega_n^2} = \frac{A}{s} + \frac{Bs+C}{s^2 + \omega_n^2}$

$$c(t) = 1 - \cos\omega_n t$$

$\downarrow$  Oscillating

# Stability limit



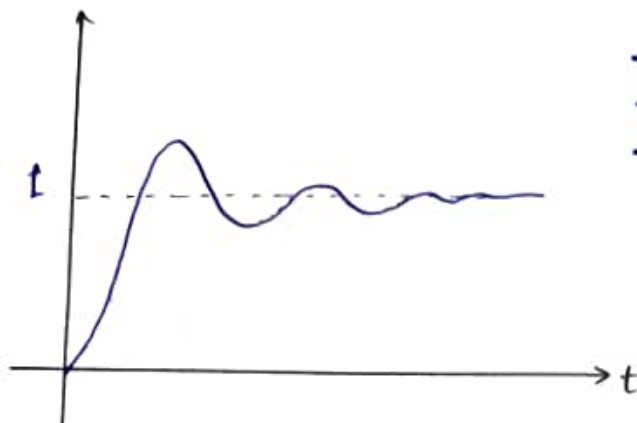
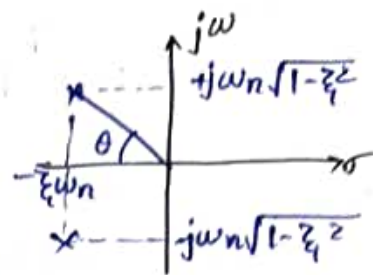
→  $0 < \xi < 1$ : Underdamped

Poles:  $s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$R(s) = 1/s$

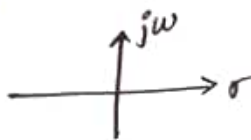
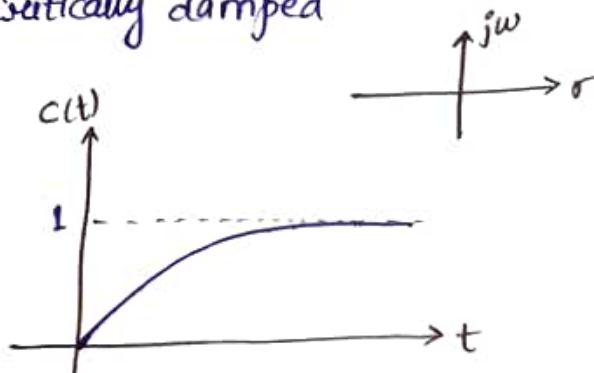
$\Rightarrow c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$ ,

$\theta = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$

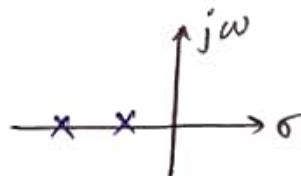


- overshoot
- Peak time
- settling time

→  $\xi = 1$ : Critically damped



→  $\xi > 1$ : Overdamped



Standard second order system:

Eg.  $\frac{25}{s^2 + 25}$

num = 25;

den = [1 0 25];

sys = tf(num, den);

step(sys, 4); % plot step response for 4 sec.

```
t = 0:0.01:10;
```

```
x = ones (size(t)); % Step input
```

```
lsimplot (sys, x, t); % response of linear system to input (arbitrary)  
signal x and t.
```

```
x2 = 5.* ones (size(t));
```

```
lsimplot (sys, x2, t); % plot step response of amplitude 5
```