Fourier series homework
Signals and Systems

Homework learning objectives: By the end of this homework, you should be able to:

- Practice manipulating the complex exponentials
- Analyze periodic signals using the Fourier series
- Synthesize periodic signals using the Fourier series

Question #1: *(2 pts)* How many hours did you spend on this homework?

Question #2: *(8 pts)* Convert the complex exponentials into sines and cosines (and **no complex numbers**) with Euler's Formula:

$$e^{-j\theta} = \cos(\theta) - j\sin(\theta) \quad , \quad e^{+j\theta} = \cos(\theta) + j\sin(\theta)$$

(a) $x_1(t) = e^{-j\pi t} + e^{+j\pi t}$

(b) $x_2(t) = \frac{2}{j}\left[e^{-j(12t-\pi/3)} - e^{+j(12t-\pi/3)}\right]$

(c) $x_3(t) = \left(1 - j\sqrt{3}\right)e^{-j12t} + \left(1 + j\sqrt{3}\right)e^{j12t}$

(d) $x_4(t) = (2+j)e^{+j\pi t} + (2-j)e^{-j\pi t} - (2-3j)e^{+j4\pi t} - (2+3j)e^{-j4\pi t}$

Question #3: *(6 pts)* Convert cosines and sines into complex exponentials with Euler's Formula:

$$\cos(\theta) = \frac{1}{2}\left(e^{+j\theta} + e^{-j\theta}\right) \quad , \quad \sin(\theta) = \frac{1}{2j}\left(e^{+j\theta} - e^{-j\theta}\right)$$

(a) $x_1(t) = \cos(2\pi t)$

(b) $x_2(t) = \cos(2\pi t + \pi/2)$

(c) $x_3(t) = \cos(3\pi t) + 5\sin(6\pi t) + 2\cos(6\pi t)$

Question #4: *(8 pts)* Determine the Fourier Series coefficients for the following signals. For these signals, use the trigonometric (cosine and sine) form of the Fourier series, i.e.,

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt \qquad a_k = \frac{2}{T_0}\int_{T_0} x(t)\cos(k\omega_0 t)\,dt \qquad b_k = \frac{2}{T_0}\int_{T_0} x(t)\sin(k\omega_0 t)\,dt \quad k \geq 1$$

where ω_0 is the fundamental angular frequency of our periodic signal $\omega_0 = 2\pi/T_0$.

(a) $x_1(t) = 3\sin(4\pi t) + 2$

(b) $x_2(t) = 3\sin((1/7)t) + 5\cos((21)t) + 3\pi$

(c) $x_3(t) = 3\sin(\pi t) + 123\cos(\pi t) + 5\cos(3\pi t) + 5\cos(4\pi t) + 6\cos(7\pi t)$

(d) $x_4(t) = 4\cos(3\pi t + \pi/3)$ [hint: you may need to use a trigonometric identity]

Question #5: *(6 pts)* Answer the following problems using the complex exponential form of the Fourier Series.

(a) Show that the real part of c_k is even.

(b) Show that the imaginary part of c_k is odd.

(c) Show that the magnitude of c_k (i.e., $|c_k|$) is even.

Question #6: *(8 pts)* Determine the Fourier Series coefficients c_k for the following signals. For these signals, use the complex exponential form of the Fourier series, i.e.,

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad , \quad c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$$

where ω_0 is the fundamental angular frequency of our periodic signal $\omega_0 = 2\pi/T_0$.

(a) $x_1(t) = 3\sin(4\pi t) + 2$

(b) $x_2(t) = 3\sin((1/7)t) + 5\cos((21)t) + 3\pi$

(c) $x_3(t) = 3\sin(\pi t) + 123\cos(\pi t) + 5\cos(3\pi t) + 5\cos(4\pi t) + 6\cos(7\pi t)$

(d) $x_4(t) = 4\cos(3\pi t + \pi/3)$ [hint: you may need to use a trigonometric identity]

Question #7: *(8 pts)* Given the following Fourier Series coefficients c_k (for the complex exponential form of the Fourier Series), determine the corresponding periodic signal $x(t)$. Write the result as a real-valued function if possible (i.e., not as complex exponentials).

(a) $c_0 = 1,\ c_1 = -2j,\ c_{-1} = 2j$, and all other $c_k = 0$. Assume $\omega_0 = 2\pi$.

(b) $c_0 = 0,\ c_1 = j - 9,\ c_{-1} = -j - 9$, and all other $c_k = 0$. Assume $\omega_0 = 3$.

(c) $c_0 = 10, c_2 = -2j + 1,\ c_{-2} = 2j + 1,\ c_7 = 1,\ c_{-7} = 1$, and all other $c_k = 0$. Assume $\omega_0 = 1/5$.

(d) $c_2 = e^{j\pi/2},\ c_{-2} = e^{-j\pi/2},\ c_3 = \sqrt{2}e^{-j\pi/4},\ c_{-3} = \sqrt{2}e^{j\pi/4}$, and all other $c_k = 0$. Assume $\omega_0 = 2\pi$.

Question #8: *(6 pts)* Consider the following signal

$$x(t) = |\cos(\pi t)|$$

(a) Sketch $x(t)$.

(b) Compute Fourier Series coefficients c_k (for the complex exponential form of the Fourier Series) for the signal $x(t)$. Note: This needs to be solved with the Fourier Series equations.

(c) Sketch c_k for $-5 \le k \le 5$.