

स्पन्दविस्तारपरिवर्तन - शक्तिपरिवर्तकानि अनुप्रयोगाच

PWM POWER CONVERTERS
AND APPLICATIONS

Power Electronics

→ Processing of electrical power using electronic components.
 ↓
 Control or conversion

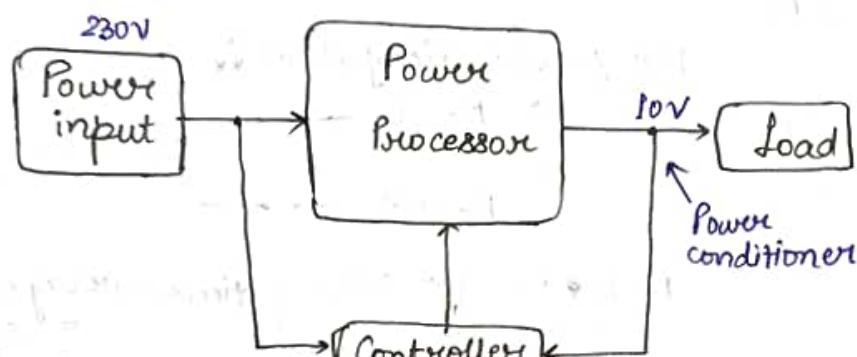
→ Power Electronic converters:

① AC to DC (Rectifiers)

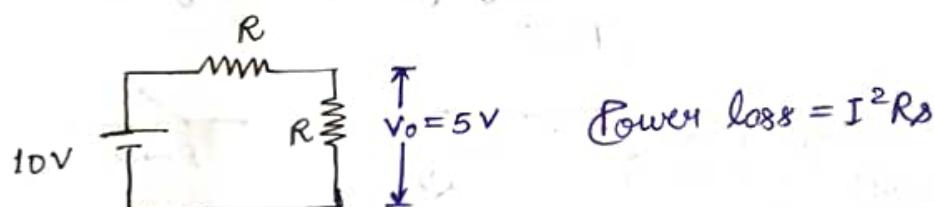
② DC to DC

③ DC to AC (Inverters) [DC to AC con. normally also converts AC to DC]

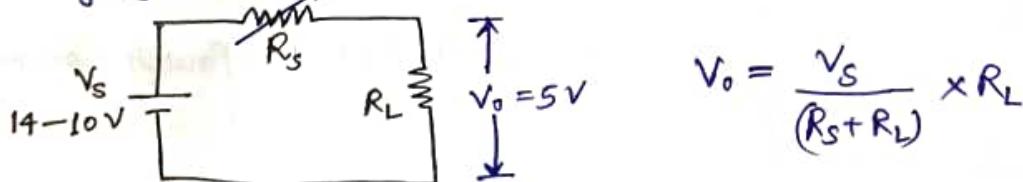
④ AC to AC. (matrix converter) [converts voltage & frequency]



DC to DC Converter
 Eg. Resistor-divider Network (e.g. μC, μP (digital processor))

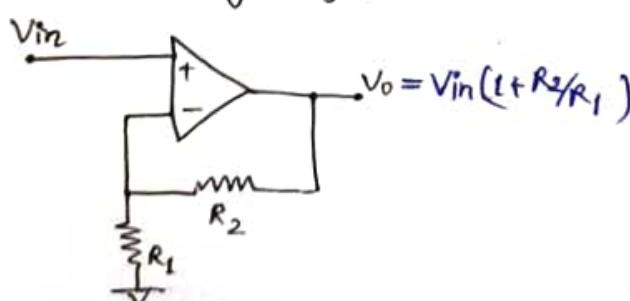


Changing supply:

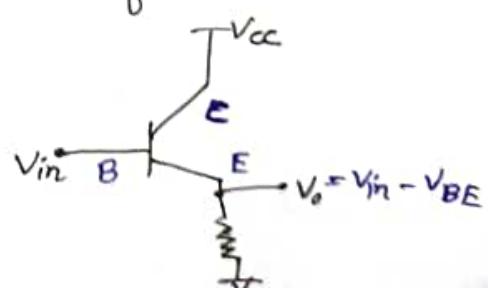


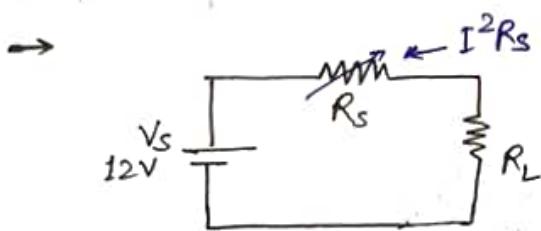
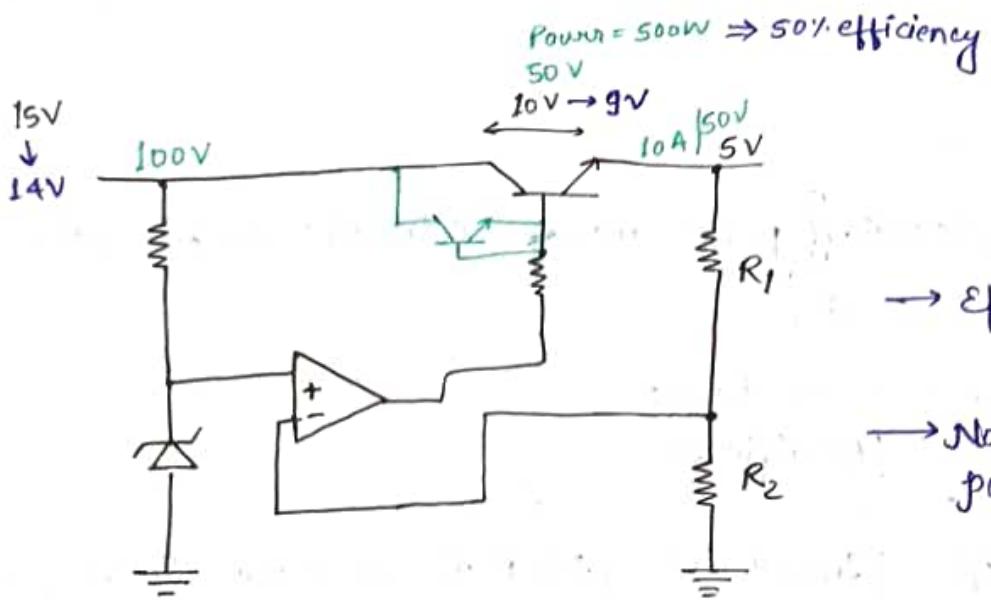
Automatic R_s control:

Non-inverting amplifier:



Emitter follower:





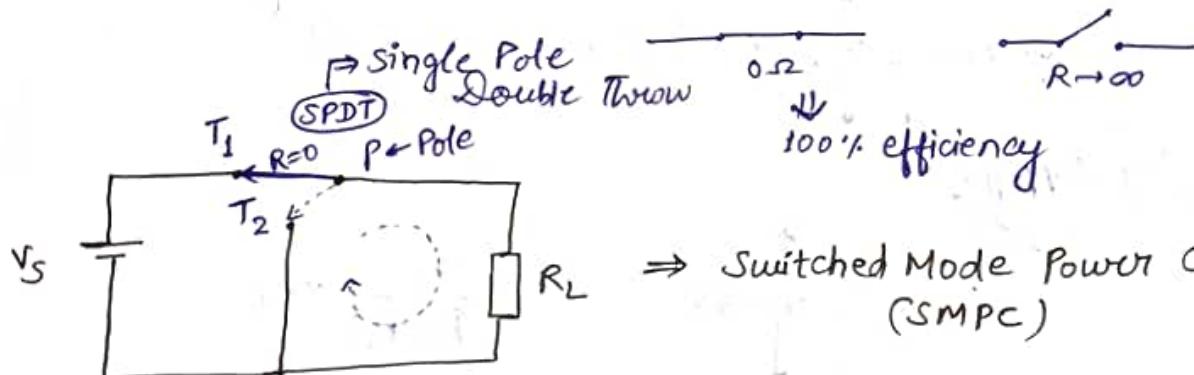
for power dissipation \downarrow ,

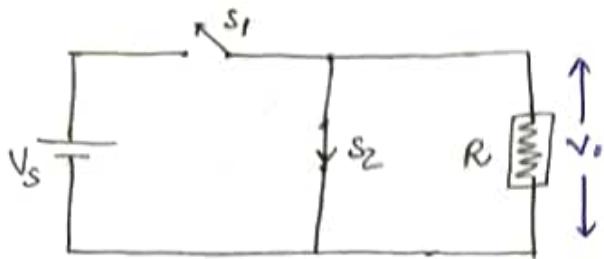
$$\begin{aligned} \Rightarrow R_s &= 0 \\ \text{or} \\ R_s &\rightarrow \infty \end{aligned} \quad \left. \begin{array}{l} \text{SPST} \\ \text{switch} \end{array} \right\}$$

$$R_s = 0 \Omega \text{ for } 0.5s \quad \left. \begin{array}{l} \text{time average for } 1s \\ = 0 \Omega \text{ for } 0.5s \end{array} \right\} = 5 \Omega$$

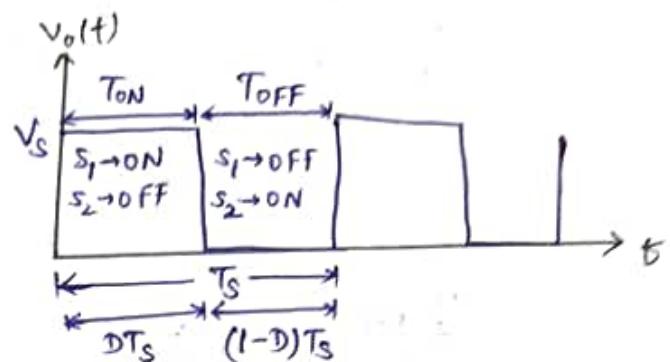
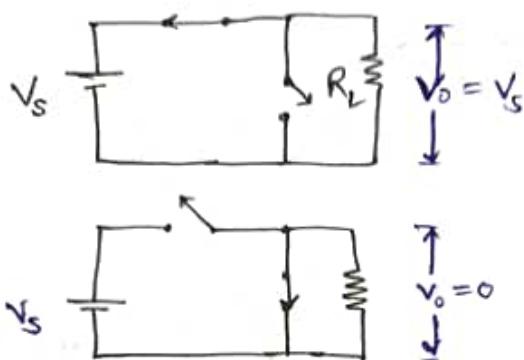
\downarrow
Switch to any average value b/w 0 and ∞ by controlling the time.

SPST:





$\rightarrow S_1$ & S_2 will work as SPDT provided, they work in complementary operation, i.e., when one is open, the other is closed.



Duty ratio = $\frac{\text{Period during which } S_1 \text{ is ON}}{\text{Total switching period}}$

$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}} = \frac{T_{ON}}{T_S}$$

$$T_{ON} = DT_S$$

\hookrightarrow DC but changing \Rightarrow Pulsating DC

$$\begin{aligned} V_0(\text{av.}) &= \frac{1}{T_S} \int_{0}^{T_S} V_0(t) dt \\ &= \frac{1}{T_S} \left[\int_0^{DT_S} V_S dt + \int_{DT_S}^{T_S} 0 dt \right] \end{aligned}$$

$$\therefore \boxed{V_0(\text{av.}) = DV_S}$$

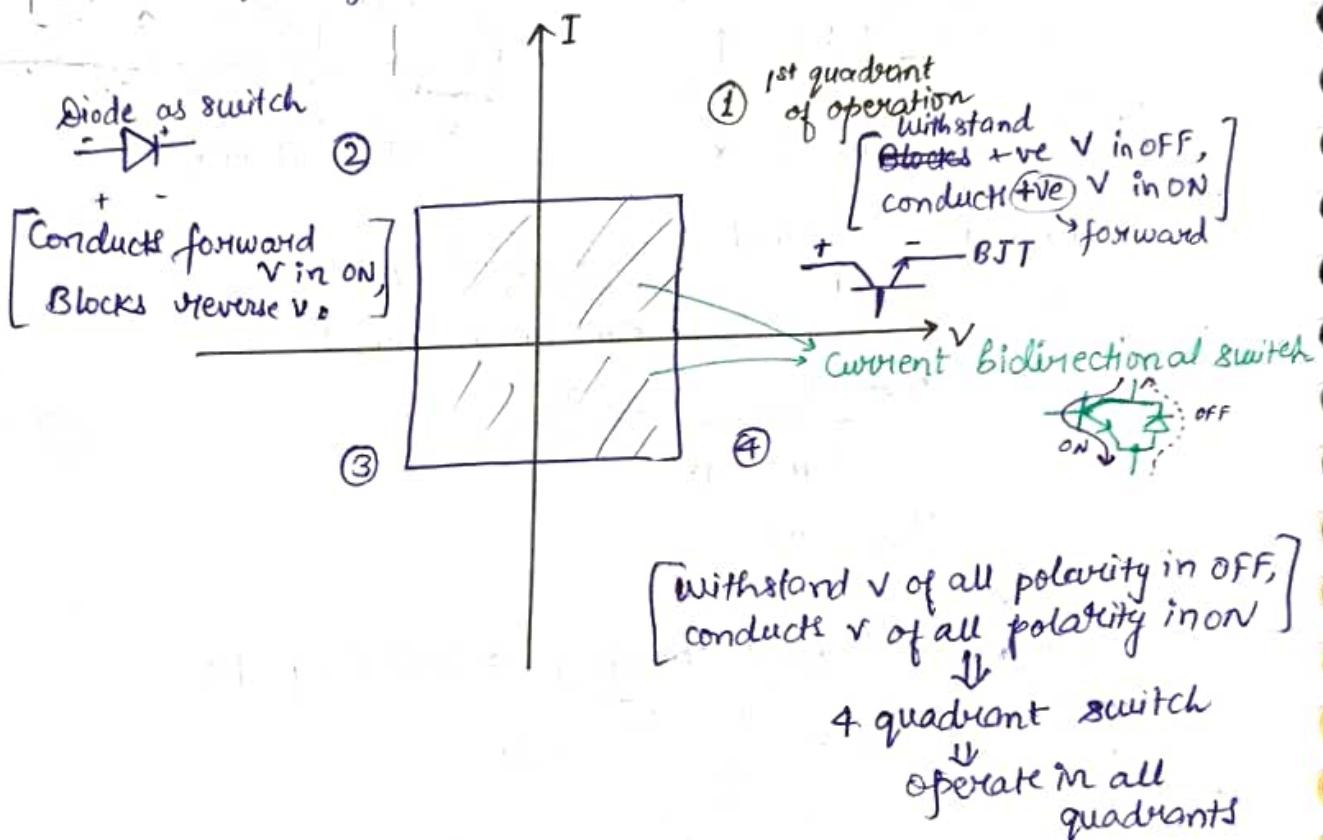
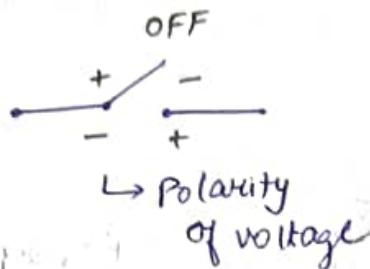
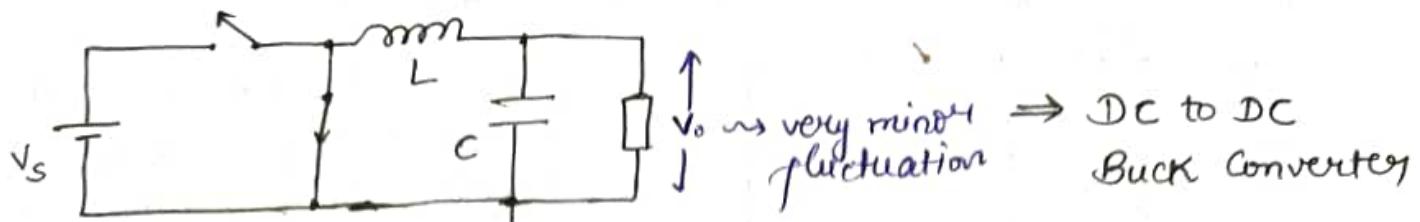
\rightarrow can be changed by changing D.

Max. D = 1, Min. D = 0

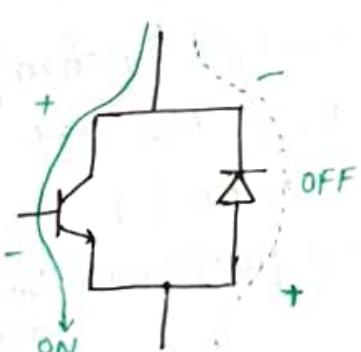
$$D \rightarrow 0 \text{ to } 1$$

\hookrightarrow But fluctuating V_0 is not acceptable.

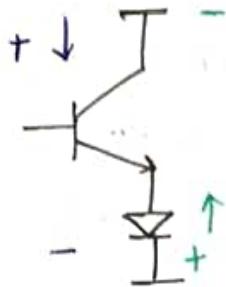
\Downarrow
Introduce energy-storing elements,
i.e., capacitor or inductor.



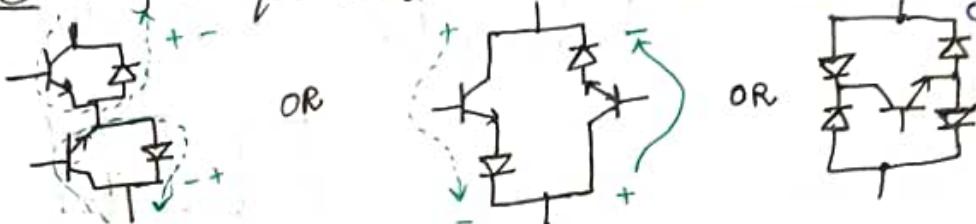
① + ④ : Current Bidirectional Two Quadrant Switch \rightarrow For voltage source converter



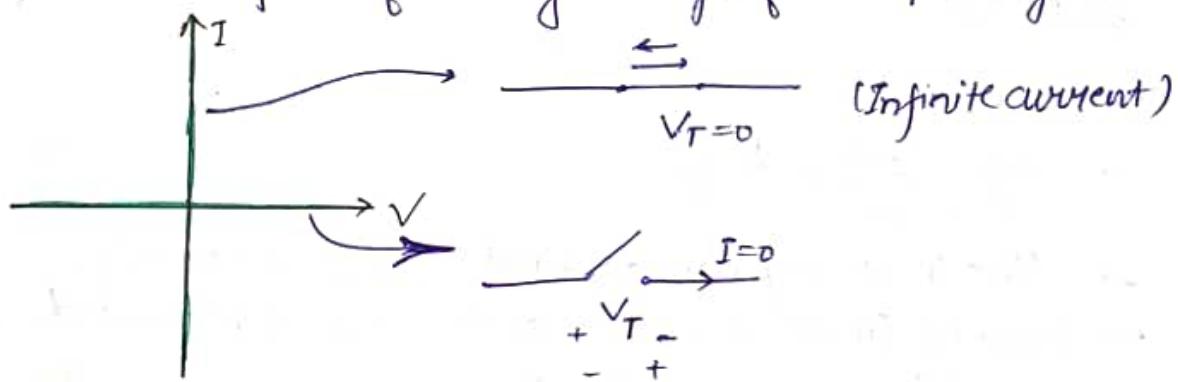
① + ②: Voltage Bidirectional Two Quadrant Switch → for current source converter



① + ② + ③ + ④: four quadrant switch → for AC to AC matrix converter



Ideal switch: Capable of conducting current of both dirⁿ in ON state, capable of blocking voltage of both polarity in OFF state



Basic Elements

Element

Resistor

Basic Relationship

$$V = IR$$

Voltage-Current relationship

$$V = IR, I = V/R$$

Capacitor

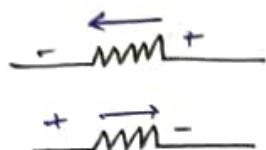
$$q = CV, \text{Energy} = \frac{1}{2}CV^2 \quad i_C = C\frac{dV}{dt}, V_C = \frac{1}{C} \int i_C dt$$

Inductor

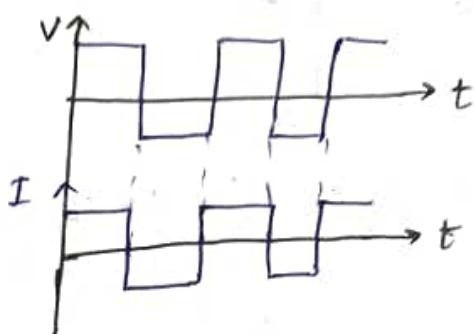
$$\Psi = Li, \text{Energy} = \frac{1}{2}Li^2$$

$$V_L = L \frac{di}{dt}, i_L = \frac{1}{L} \int V_L dt$$

Resistor



Energy $\propto I^2$

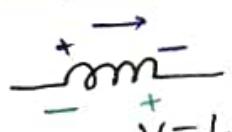


Inductor

$$\Psi = li$$

$$V = \frac{d\Psi}{dt} = \frac{dli}{dt} = L \frac{di}{dt}$$

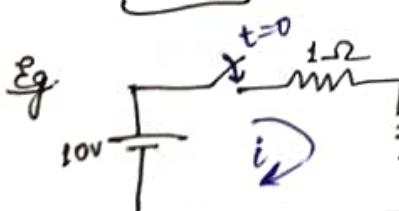
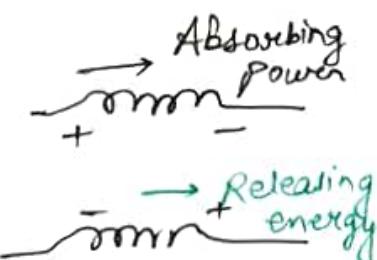
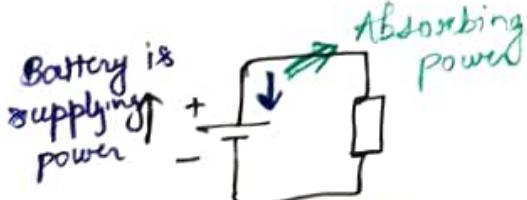
- Flux in a system does not change suddenly.
- current in an inductor shall not be interrupted.



$$V = L \frac{di}{dt} \Rightarrow \text{Current} \rightarrow \text{not changing} \Rightarrow V = 0$$

- Inductor makes its presence felt in the circuit only if the current changes.

$$\text{Energy} = \frac{1}{2} Li^2$$

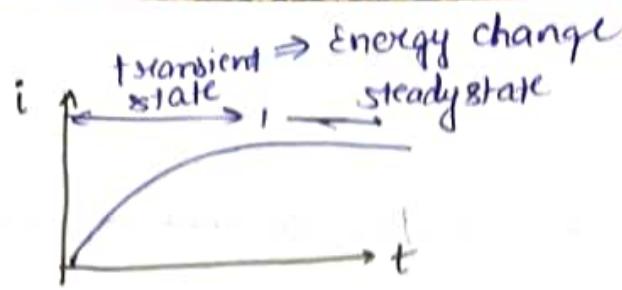


$$V = \begin{cases} 10V & t=0^- \Rightarrow i=0 \\ 0V & t=0^+ \Rightarrow i=0 \end{cases} \Rightarrow \frac{di}{dt} \neq 0$$

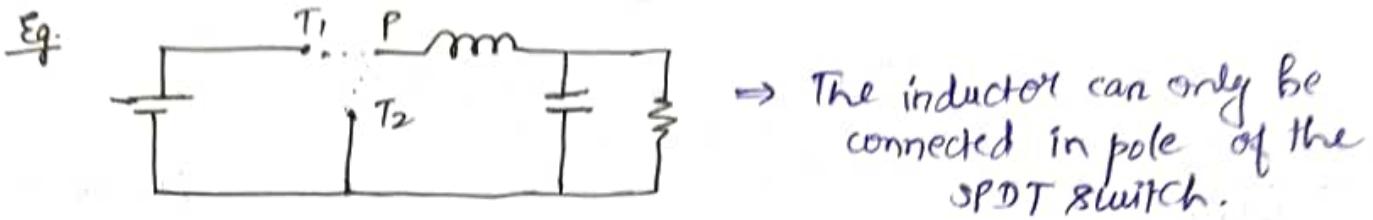
$$V = L \frac{di}{dt} = 10V$$

→ Cannot be charged thereafter

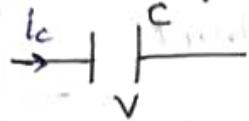
$$V = I \times \frac{(0-0)}{10 \times 10^{-6}} \text{ (say)} \\ = 1MV \Rightarrow 8 \text{ mho}$$



→ A switch shall not be connected in series with an inductor.



Inductor: Capacitor:



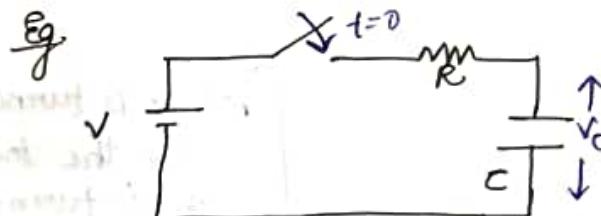
$$i_L = L \frac{dv}{dt} \rightarrow \text{No change in voltage} \Rightarrow \text{No current}$$

→ An ~~inductor~~ ^{capacitor} makes its presence felt in the circuit if there is a change in voltage.

$$i = \frac{dq}{dt} = \frac{d(cv)}{dt} = C \frac{dv}{dt}$$



- charge in a system cannot change suddenly.
- Switch should not be connected across a capacitor



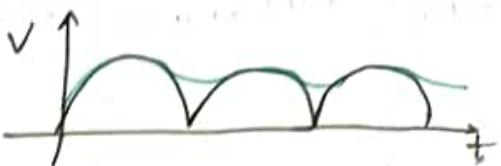
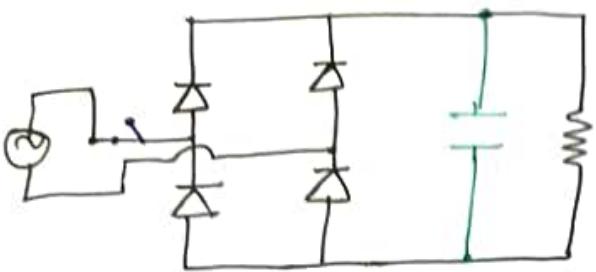
$$t=0^- \Rightarrow i=0 \Rightarrow v_c=0$$

$$t=0^+ \Rightarrow v_c=0, i \checkmark \Rightarrow \text{short circuit}$$

charging \leftarrow

i → discharging

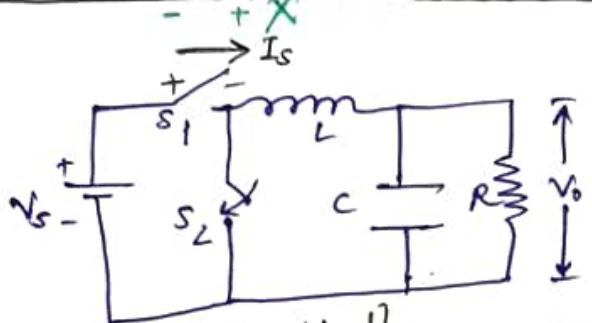
Eg Diode-bridge rectifier



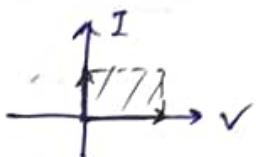
↳ Pre-charge capacitor & then switch

DC to DC Buck Converter

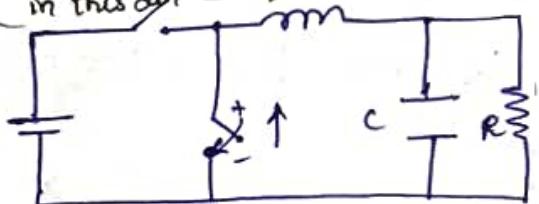
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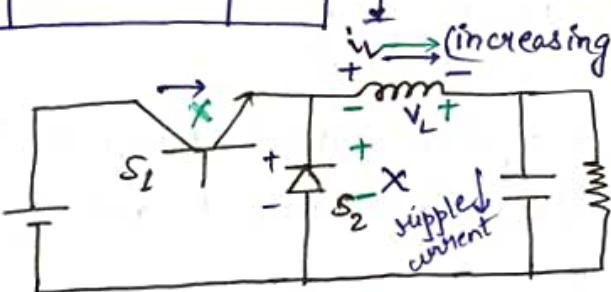
⇒ 1st Quadrant
⇒ BJT



(current is established in this direction)



⇒ 2nd Quadrant
⇒ BJT

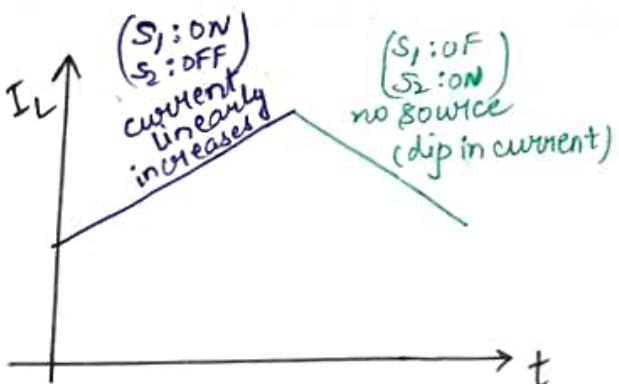


$$I_o = \frac{V_o}{R} \quad (\text{dc current})$$

$\Rightarrow S_1: \text{ON}$
 $S_2: \text{OFF}$
 $\Sigma: \text{ON}$

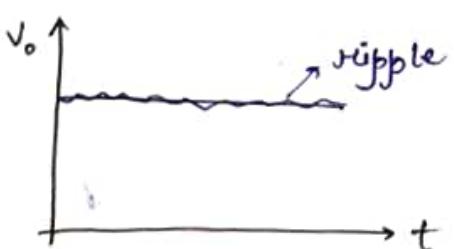
$$V_L = L \frac{di_L}{dt}$$

$\begin{cases} +ve \\ -ve \end{cases}$



Diode is turned off by the inductor & is turned on by the source

⇒ DC to DC Buck converter



$$V_o = V + \Delta V$$

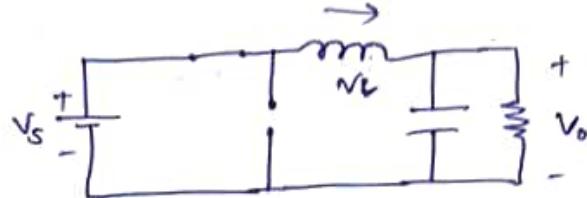
$$i_o = \frac{V_o}{R} = \frac{V_o}{R} + \frac{\Delta V_o}{R}$$

$$\approx \frac{V_o}{R}$$

Equivalent Circuit when S_1 is ON :

$$D = \frac{T_{ON}(S_1) \xrightarrow{\text{active switch}}}{T_{ON}(S_1) + T_{OFF}(S_1)} = \frac{T_{ON}}{T_S} \quad , \quad 0 \leq t \leq DT_S$$

$(T_{ON} = DT_S)$



$$V_S - V_L - V_0 = 0$$

$$\Rightarrow V_L = V_S - V_0$$

$$V_L = L \frac{di_L}{dt}$$

$$i_L = \frac{1}{L} \int V_L dt = \frac{1}{L} \int_0^{DT_S} (V_S - V_0) dt = \frac{1}{L} (V_S - V_0) t$$

$$\text{slope} = \frac{V_S - V_0}{L}$$

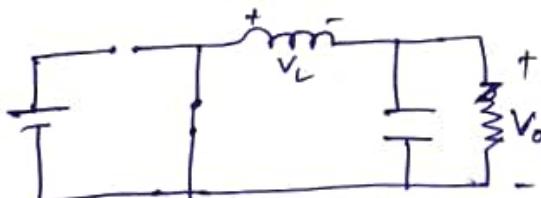
Equivalent Circuit when S_1 is OFF:

$$T_{OFF} = T_S - T_{ON}$$

$$= T_S - DT_S$$

$$= (1-D)T_S$$

$$DT_S \leq t \leq T_S$$



$$-V_L - V_0 = 0$$

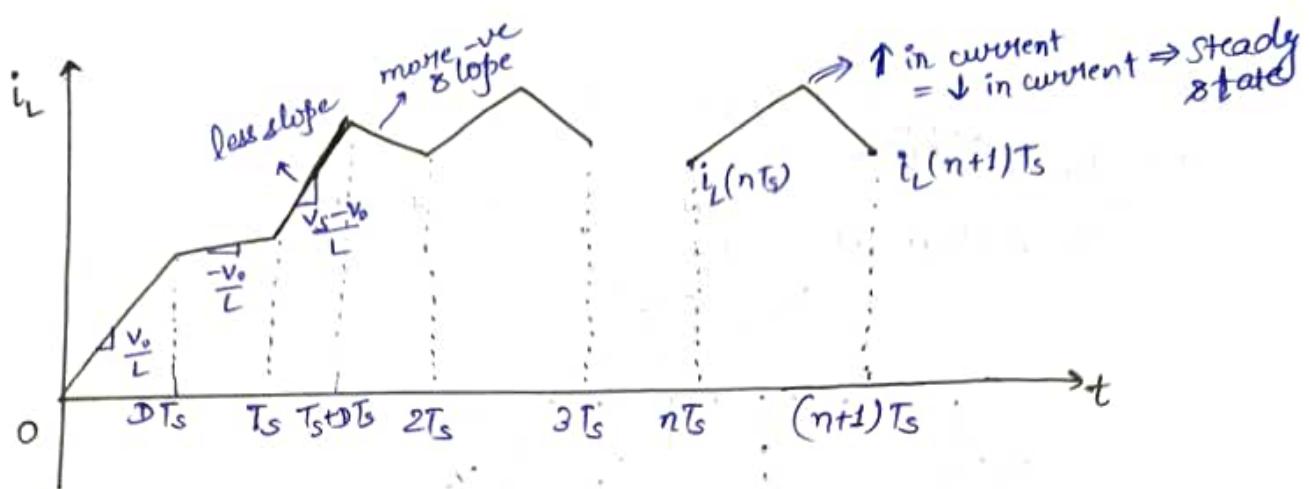
$$\Rightarrow V_L = -V_0$$

$$V_L = L \frac{di_L}{dt} = -V_0$$

$$\frac{di_L}{dt} = \frac{-V_0}{L}$$

$$V_0 \approx 0 \Rightarrow \text{slope} \approx 0$$

19-08-2024



At steady state,

$$i_L(n+1)T_s = i_L(nT_s)$$

$$V_L = L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_L}{L}$$

$$\Rightarrow \int_0^{T_s} di_L = \int_0^{T_s} \frac{V_L}{L} dt$$

$$\Rightarrow i_L(T_s) - i_L(0) = \frac{1}{L} \int_0^{T_s} V_L dt$$

At steady state,

$$i_L(T_s) - i_L(0) = 0$$

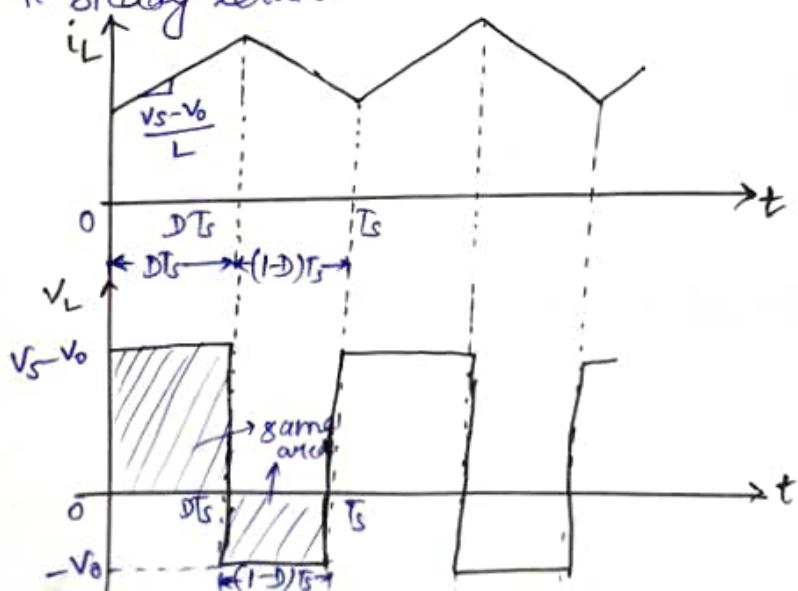
$$\Rightarrow \frac{1}{L} \int_{T_s}^{T_s} V_L dt = 0$$

$$\Rightarrow \int_0^{T_s} V_L dt = 0$$

$$\Rightarrow \frac{1}{T} \int_0^T V_L dt = 0$$

Average value (DC component) of inductor voltage must be zero.

At steady state.



$$(V_s - V_o)DT_s + (-V_o)(1-D)T_s = 0$$

↳ Principle of Volt-second Balance

(Principle of flux balance) $\Rightarrow V = \frac{Nd\phi}{dt}$ (Faraday's Law) $\Rightarrow Vdt = \text{flux}$

$$\Rightarrow V_s DT_s - V_o DT_s + (-V_o)(1-D T_s) = 0$$

$$\Rightarrow \boxed{V_o = D V_s}$$

For capacitor:

Charge accumulates \Rightarrow voltage ~~fails~~ keeps building up
 \Rightarrow converter fail.

$$i_c = C \frac{dV_c}{dt}$$

$$\Rightarrow dV_c = \frac{1}{C} i_c dt$$

$$\Rightarrow \int_0^{T_s} dV_c = \frac{1}{C} \int_0^{T_s} i_c dt$$

$$\Rightarrow V_c(T_s) - V_c(0) = \frac{1}{C} \int_0^{T_s} i_c dt$$

At steady state,

$$V_c(T_s) - V_c(0) = 0$$

$$\Rightarrow \frac{1}{C} \int_0^{T_s} i_c dt = 0$$

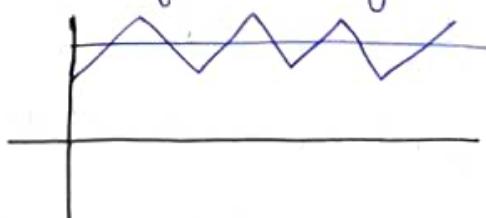
$$\Rightarrow \frac{1}{T_s} \int_0^{T_s} i_c dt = 0$$

\Rightarrow Average value (dc component) of capacitor current must be zero.

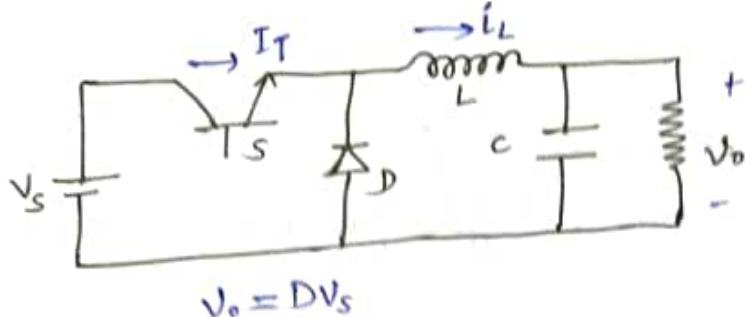
↳ Principle of Ampere-second Balance.

\rightarrow dc current through inductor = Load current $[I_o = \frac{V_o}{R}]$

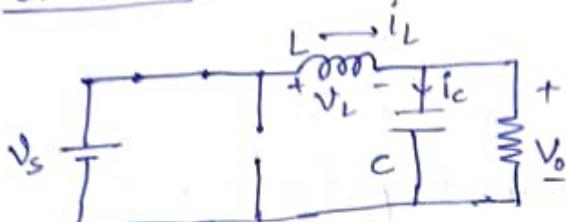
\rightarrow Ripple current flows through the capacitor.



\rightarrow We cannot allow dc current to flow through the capacitor and dc voltage to appear across inductor.



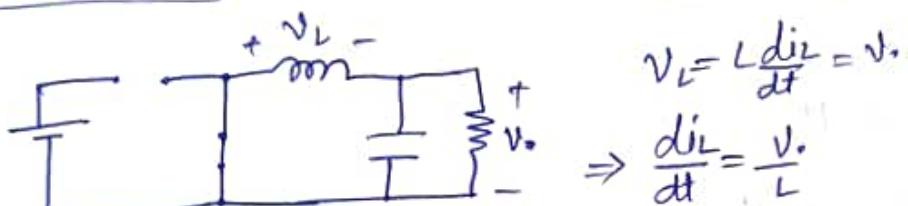
$0 < t \leq DT_S$:



$$V_L = L \frac{di_L}{dt} = V_s - V_o$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

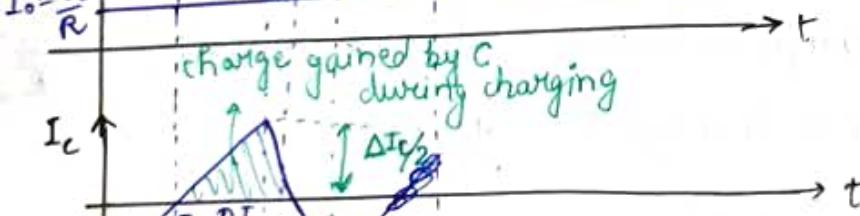
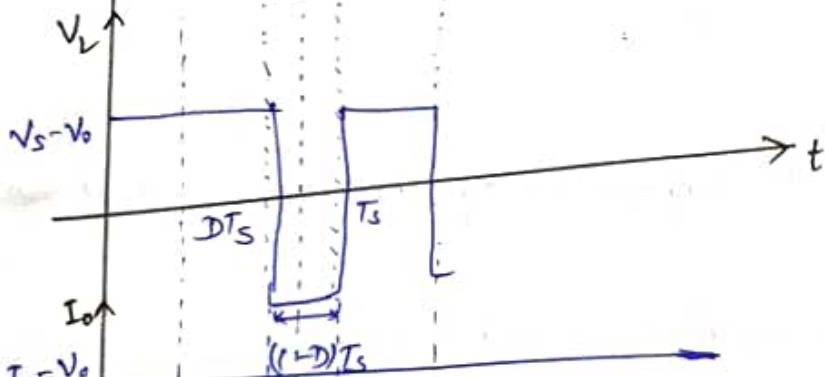
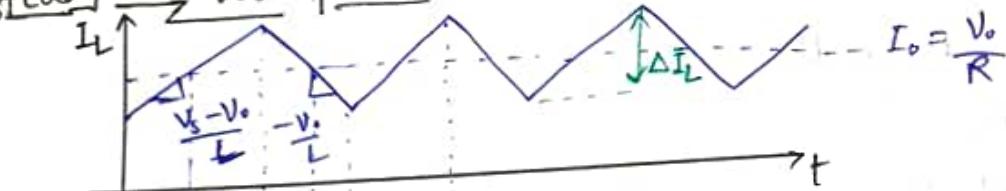
$DT_S < t \leq T_S$:



$$V_L = L \frac{di_L}{dt} = V_o$$

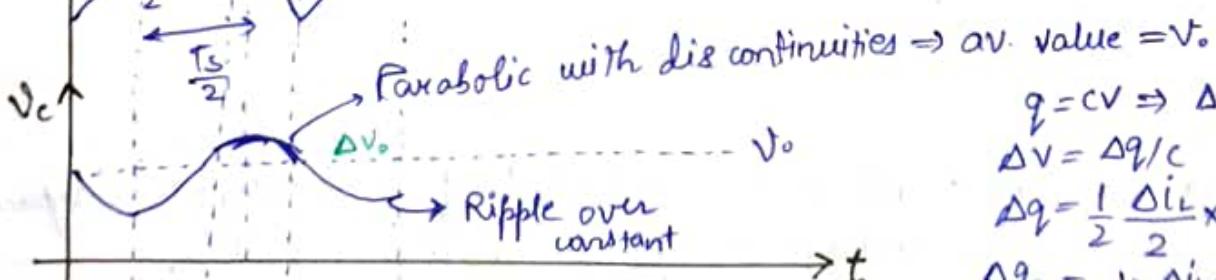
$$\Rightarrow \frac{di_L}{dt} = \frac{V_o}{L}$$

Steady State Waveforms:



$$I_C = C \frac{dv_C}{dt}$$

$$\Rightarrow v_C = \frac{1}{C} \int I_C dt$$



$$q = CV \Rightarrow \Delta q = C \Delta V$$

$$\Delta V = \Delta q / C$$

$$\Delta q = \frac{1}{2} \frac{\Delta i_L}{2} \times \frac{T_S}{2} \quad [\text{Area}]$$

$$\Rightarrow C = \frac{\Delta q}{\Delta V} = \frac{V_2 \Delta i_L \times T_S / 2}{\Delta V}$$

$$\text{Peak-to-peakripple in the inductor current} = \left(\frac{V_s - V_o}{L} \right) D T_s$$

$$\text{or, } \Delta I_L = -\frac{V_o}{L} (1-D) T_s$$

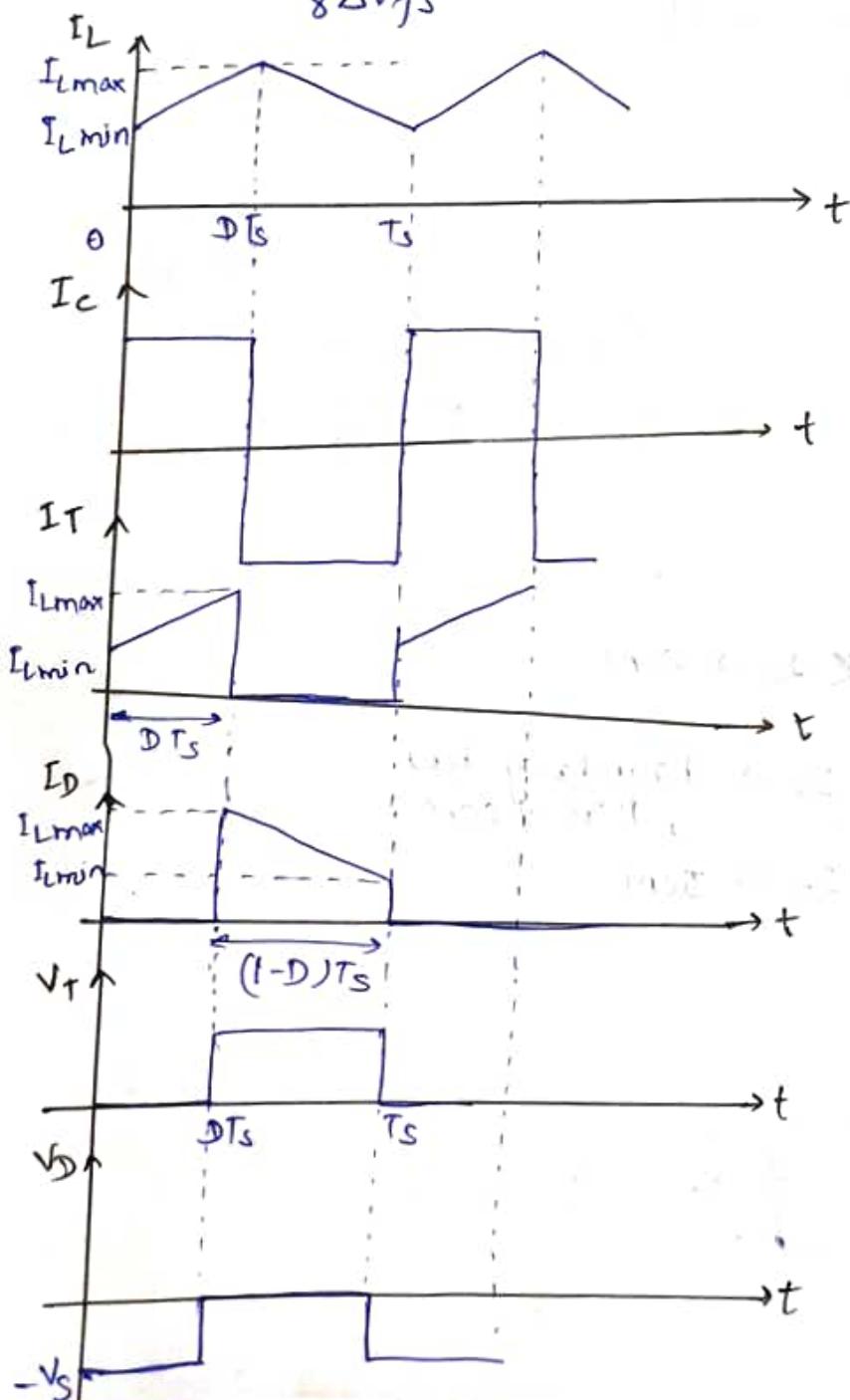
$$\Delta I_L = \left(\frac{V_s - V_o}{L} \right) D T_s$$

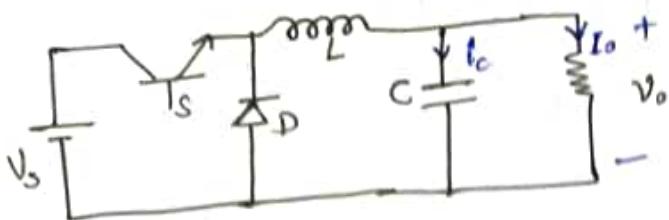
$$\Rightarrow L = \frac{V_s - V_o}{\Delta I_L} D T_s$$

$$= \frac{V_s - V_o}{\Delta I_L f_s} \quad (\because T_s = \frac{1}{f_s})$$

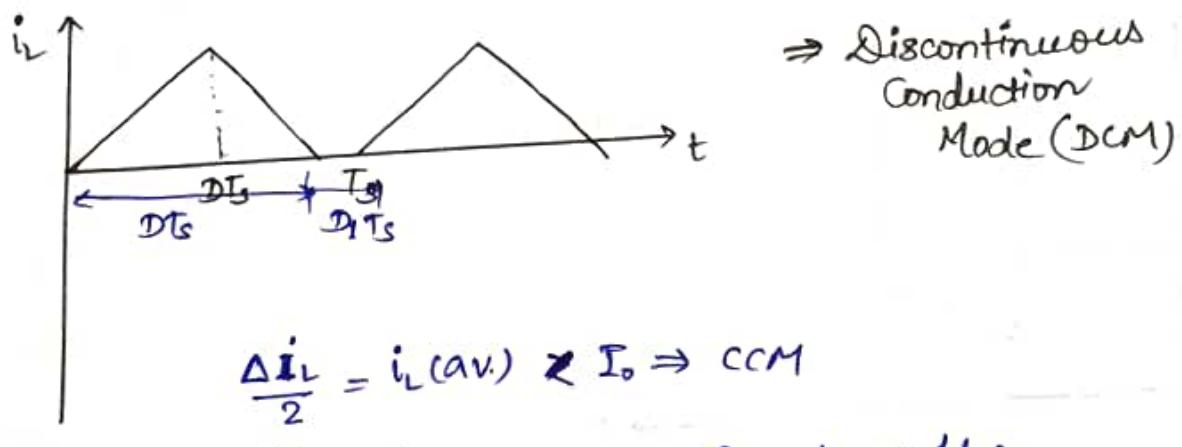
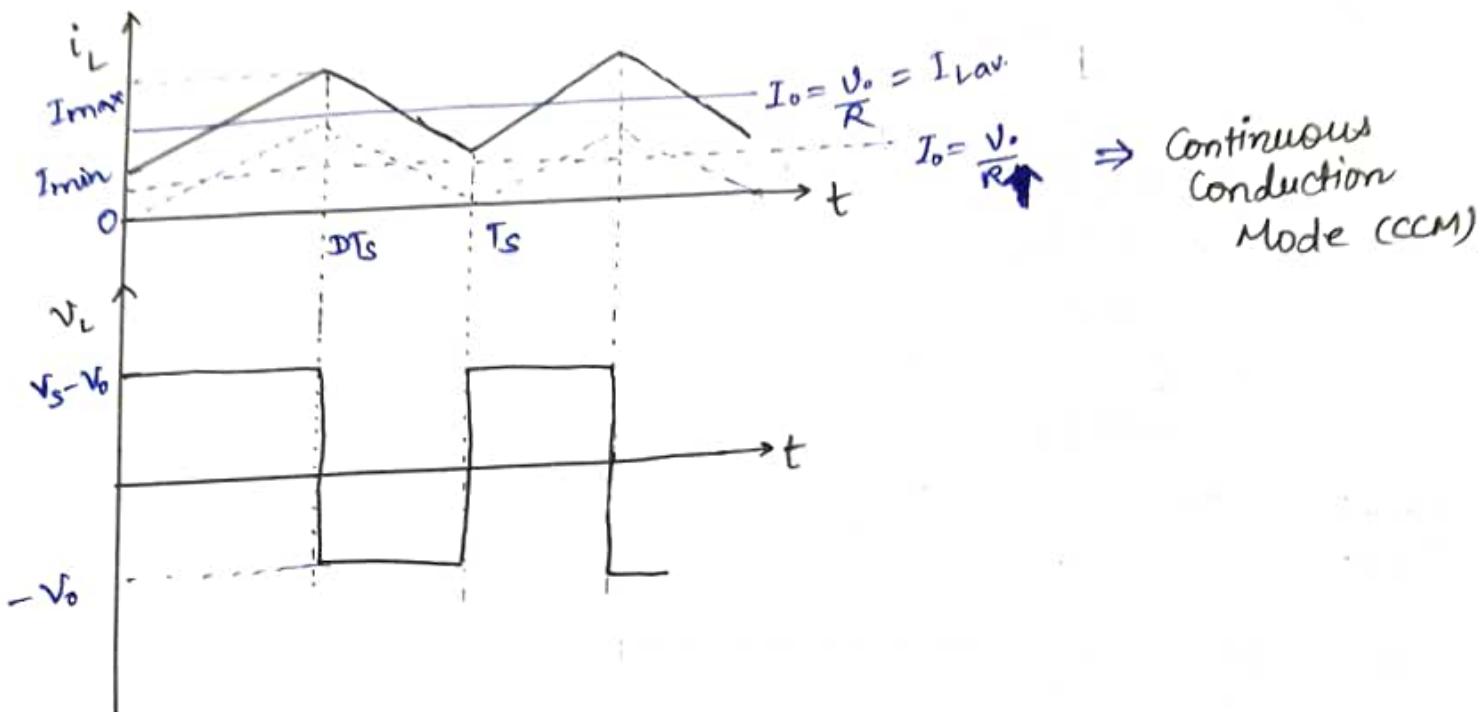
$$C = \frac{\Delta I_L \times T_s / 8}{\Delta V}$$

$$= \frac{\Delta I_L}{8 \Delta V f_s}$$





$$V_0 = D V_S \Rightarrow \text{for COM}$$

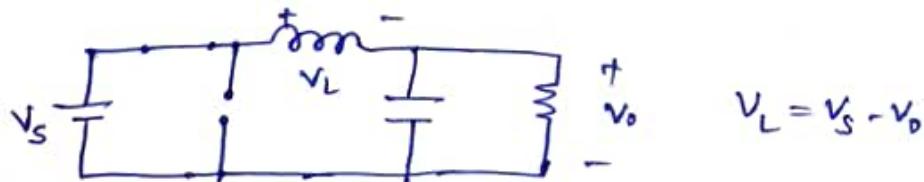


$$\frac{\Delta i_L}{2} = i_L(\text{av.}) > I_0 \Rightarrow \text{CCM}$$

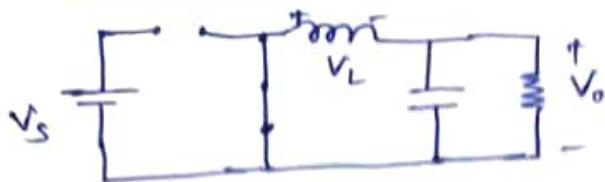
$$\frac{\Delta i_L}{2} = i_L(\text{av.}) = I_0 \Rightarrow \text{Boundary b/w DCM \& CCM}$$

$$\frac{\Delta i_L}{2} = i_L(\text{av.}) < I_0 \Rightarrow \text{DCM}$$

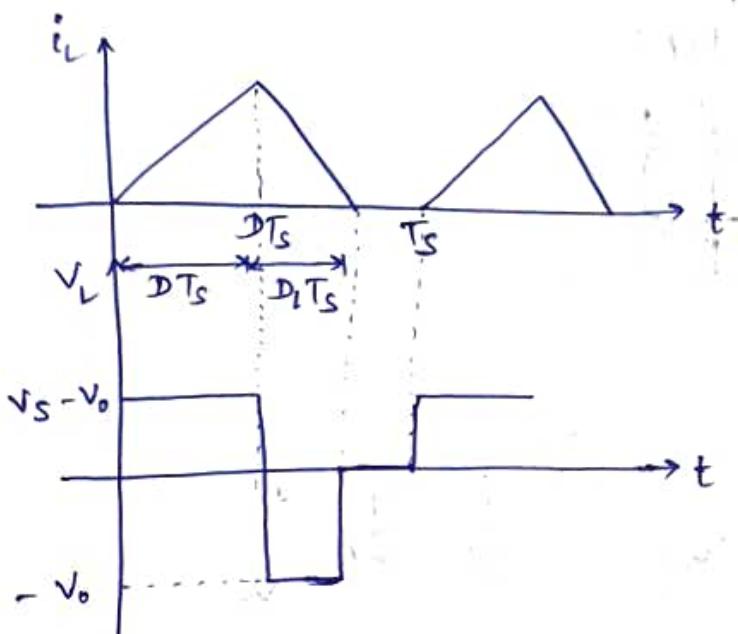
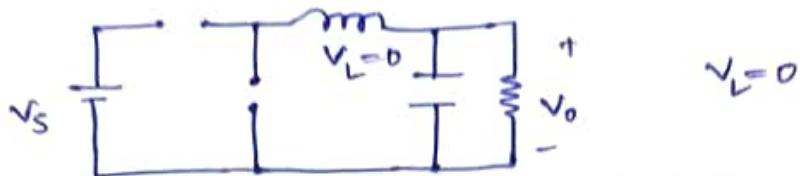
$0 < t \leq D T_S$:



$D T_S < t < (D+D_1) T_S$:



$(D+D_1) T_S < t \leq T_S$:



$$(V_S - V_o) D T_S + (-V_o) D_1 T_S + (0)(t - (D+D_1)) T_S = 0$$

$$\Rightarrow (V_S - V_o) D T_S - V_o D_1 T_S = 0$$

$$\Rightarrow V_S D T_S - V_o D T_S - V_o D_1 T_S = 0$$

$$\Rightarrow D V_S - V_o (D + D_1) = 0$$

$$\Rightarrow \boxed{V_o = \frac{V_S D}{D + D_1}}$$

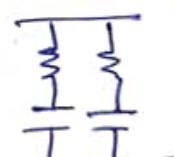
$$\Rightarrow \boxed{V_o = D V_S}$$

Non-ideal capacitor:

$\begin{cases} ESL \Rightarrow \text{Eq. Series Inductance} \\ ESR \Rightarrow \text{Eq. Series Resistance} \end{cases}$

T_C

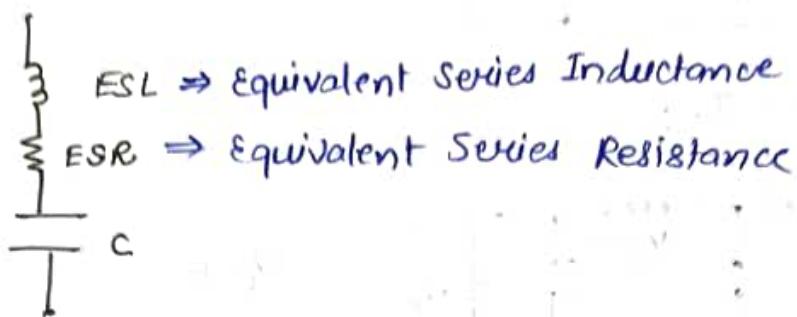
$ESR \uparrow \Rightarrow \text{Ripple} \uparrow$
To decrease ESR:



\hookrightarrow capacitor provides voltage to the load
Ripple increases.

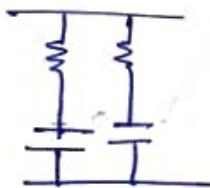
Non-idealities

Non-ideal Capacitor:

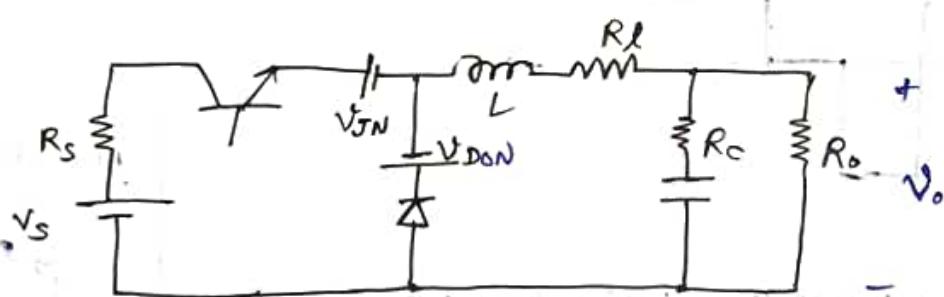


ESR $\uparrow \Rightarrow$ Ripple \uparrow

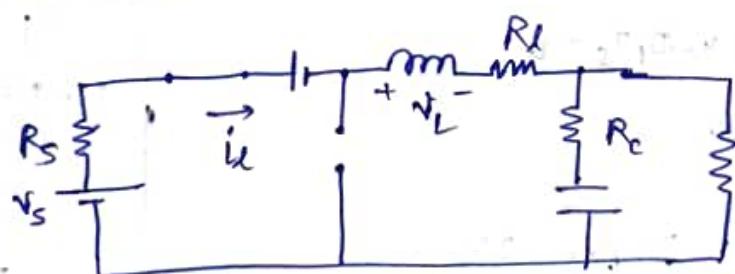
To decrease ESR:



Other non-idealities:



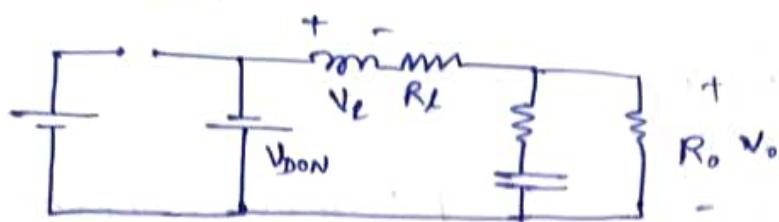
$0 < t \leq DT_s$:



$$V_s - i_L R_s - V_{JN} - V_L - i_L R_L - V_o = 0$$

$$\Rightarrow V_L = V_s - i_L (R_s + R_L) - V_{JN} - V_o$$

$D T_S < t \leq T_S$:



$$-V_{D0N} - V_L - i_L R_L - V_0 = 0$$

$$\Rightarrow V_L = (V_0 + V_{D0N} + i_L R_L)$$

$$(V_S - i_L (R_S + R_L) - V_{D0N} - V_0) D T_S - (V_0 + V_{D0N} + i_L R_L) (1-D) T_S = 0$$

$$i_L = i_L (\text{av.}) = I_0 = V_0 / R$$

$$V_S - \frac{V_0}{R} (R_S + R_L) - V_{D0N} - V_0$$

DCM:

27-08-2024

$$\frac{\Delta i_L}{2} = \frac{V_0}{2L} (1-D T_S) = \frac{V_0}{R} = I_0$$

↑ Operation on the boundary
b/w CCM & DCM

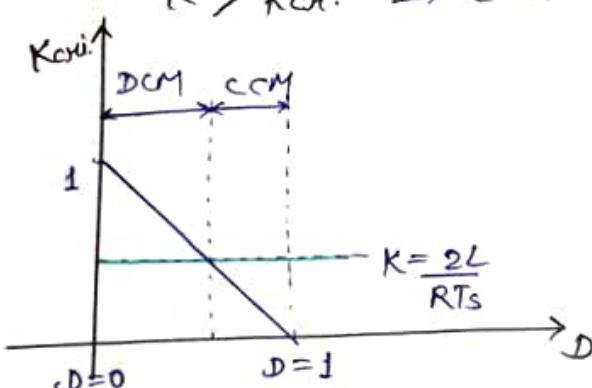
$$\left| \begin{array}{l} \Delta i_L = \frac{(V_S - V_0)}{L} D T_S \\ \Delta i_L = \frac{V_0}{L} (1-D) T_S \end{array} \right.$$

$$\frac{2L}{R T_S} = (1-D) = K : \text{Conduction parameter}$$

$$K_{\text{critical}} = (1-D)$$

$$K < K_{\text{critical}} \Rightarrow \text{DCM}$$

$$K > K_{\text{crit.}} \Rightarrow \text{CCM}$$

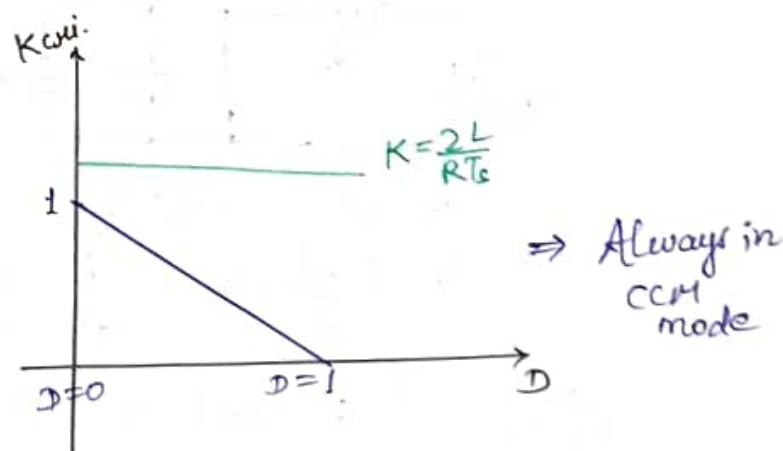


$$\frac{2L}{R T_S} = (1-D)$$

$$R_{\text{crit.}} = \frac{2L}{(1-D) T_S}$$

$$R < R_{\text{crit.}} \rightarrow \text{CCM}$$

$$R > R_{\text{crit.}} \rightarrow \text{DCM}$$



$$R = \frac{2L}{T_S}$$

For Buck converter:

$$V_o = D V_s$$

$$V_o I_o = V_s I_s \quad (\text{Input power} = \text{Output Power})$$

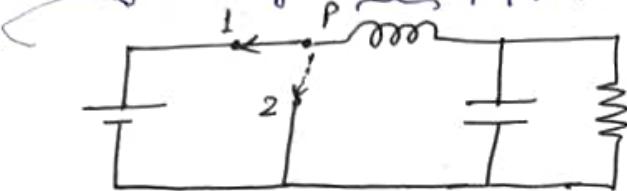
$$D V_s I_o = V_s I_s \Rightarrow D I_o = I_s$$

$$\Rightarrow I_o = \frac{I_s}{D} \quad (\text{Output current} > \text{Input current})$$

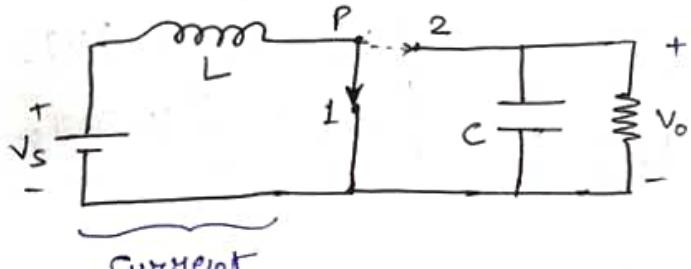
Bucking the voltage \Rightarrow Boost the current

For boosting the voltage \Rightarrow buck the current.

Buckling voltage: Inductor b/w P & output



Inductor b/w P & input



Current source

[Current doesn't change across L]

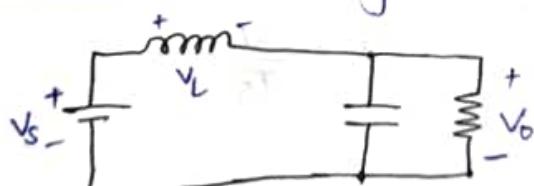
$0 < t \leq DT_s$: Switch in position ①



$$V_s - V_L = 0 \Rightarrow V_L = V_s$$

$$V_L = L \frac{dI_L}{dt} = V_s \quad \frac{dI_C}{dt} = \frac{V_s}{L}$$

$DT_s \leq t < T_s$: Switch in position ②



$$V_s - V_L - V_o = 0$$

$$\Rightarrow V_L = \frac{L di}{dt} = V_s - V_o$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

Applying volt-sec balance,

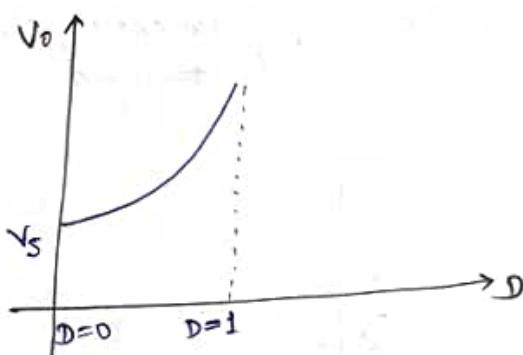
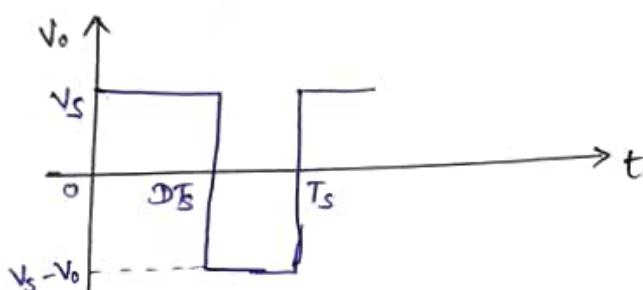
$$V_s D T_s + (V_s - V_o)(1-D)T_s = 0$$

$$\Rightarrow V_s D + V_s(1-D) - V_o(1-D) = 0$$

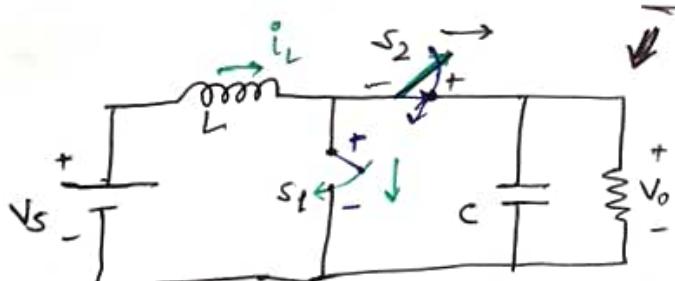
$$\Rightarrow V_s - V_o(1-D) = 0$$

$$\Rightarrow V_o = \frac{V_s}{(1-D)}, \quad D \neq 1$$

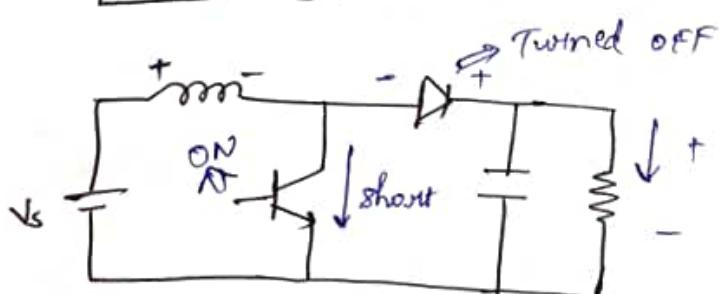
$$D=0 \rightarrow V_o = V_s$$



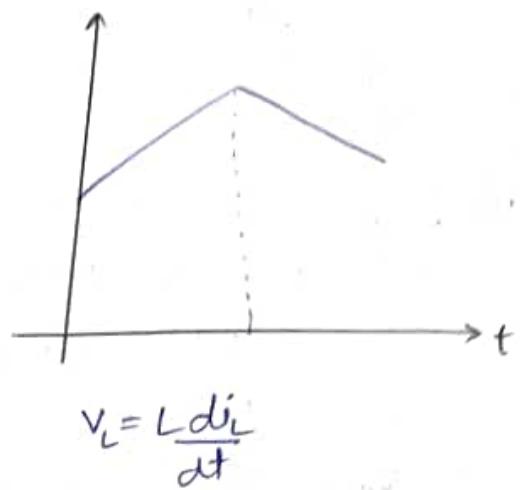
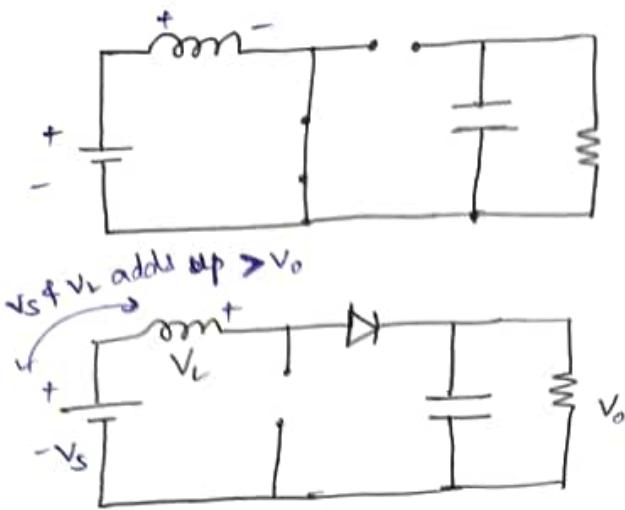
DC to DC Boost Converter



\$S_1\$: 1st quadrant
\$S_2\$: 2nd quadrant

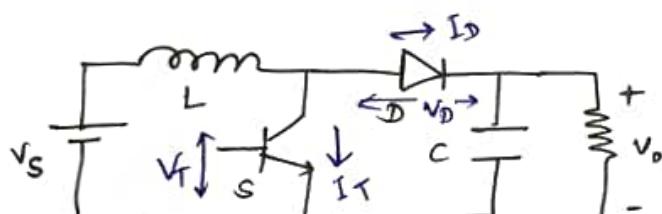


\$\downarrow + \Rightarrow\$ When \$s_1\$ is ON, \$s_2\$ turns OFF



30-08-2024

DC-DC Boost Converter



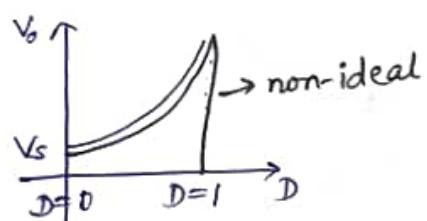
$$V_o = \frac{V_s}{1-D}, D \neq 1 \quad [D = \frac{T_{on}}{T_s} \Rightarrow D = 1 : s' \text{ permanently closed} \Rightarrow L \text{ damaged fails}]$$

Assuming 100% efficiency,

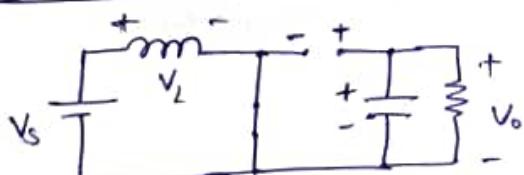
$$V_o I_o = V_s I_s$$

$$\Rightarrow I_s = \frac{V_o I_o}{V_s} = \frac{V_o I_o}{(1-D)V_s} = \frac{I_o}{(1-D)}$$

$$\Rightarrow I_o = I_s (1-D)$$

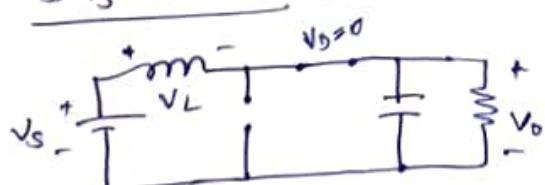


$0 < t \leq DT_s$:

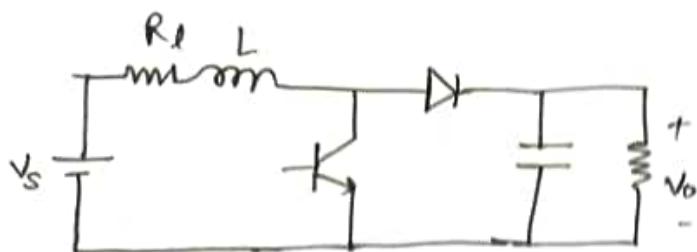


$$\begin{aligned} V_s - V_c &= 0 \\ V_L &= L \frac{di_L}{dt} = V_s \\ \frac{di_L}{dt} &= \frac{V_s}{L} \end{aligned}$$

$DT_s < t \leq T_s$:



$$\begin{aligned} V_s - V_L - V_o &= 0 \\ V_L &= L \frac{di_L}{dt} = V_s - V_o \\ \frac{di_L}{dt} &= \frac{V_s - V_o}{L} \end{aligned}$$

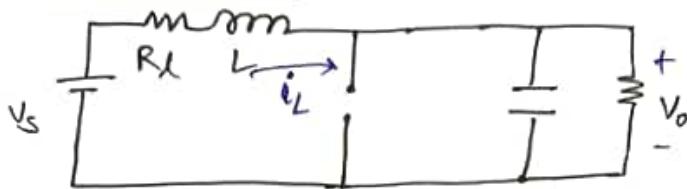


$0 < t \leq DT_s$:



$$V_s - i_L R_L - V_L = 0 \Rightarrow V_L = V_s - i_L R_L$$

$DT_s < t \leq T_s$:



$$V_s - i_L R_L - V_0 - V_L = 0$$

$$\Rightarrow V_L = V_s - i_L R_L - V_0$$

Applying principle of volt-sea. Balance,

$$(V_s - i_L R_L) D T_s + (V_s - i_L R_L - V_0) (1-D) T_s = 0$$

$$\Rightarrow V_s D + V_s (1-D) - i_L R_L (D + (1-D)) - V_0 (1-D) = 0$$

$$\Rightarrow V_s - i_L R_L - V_0 (1-D) = 0$$

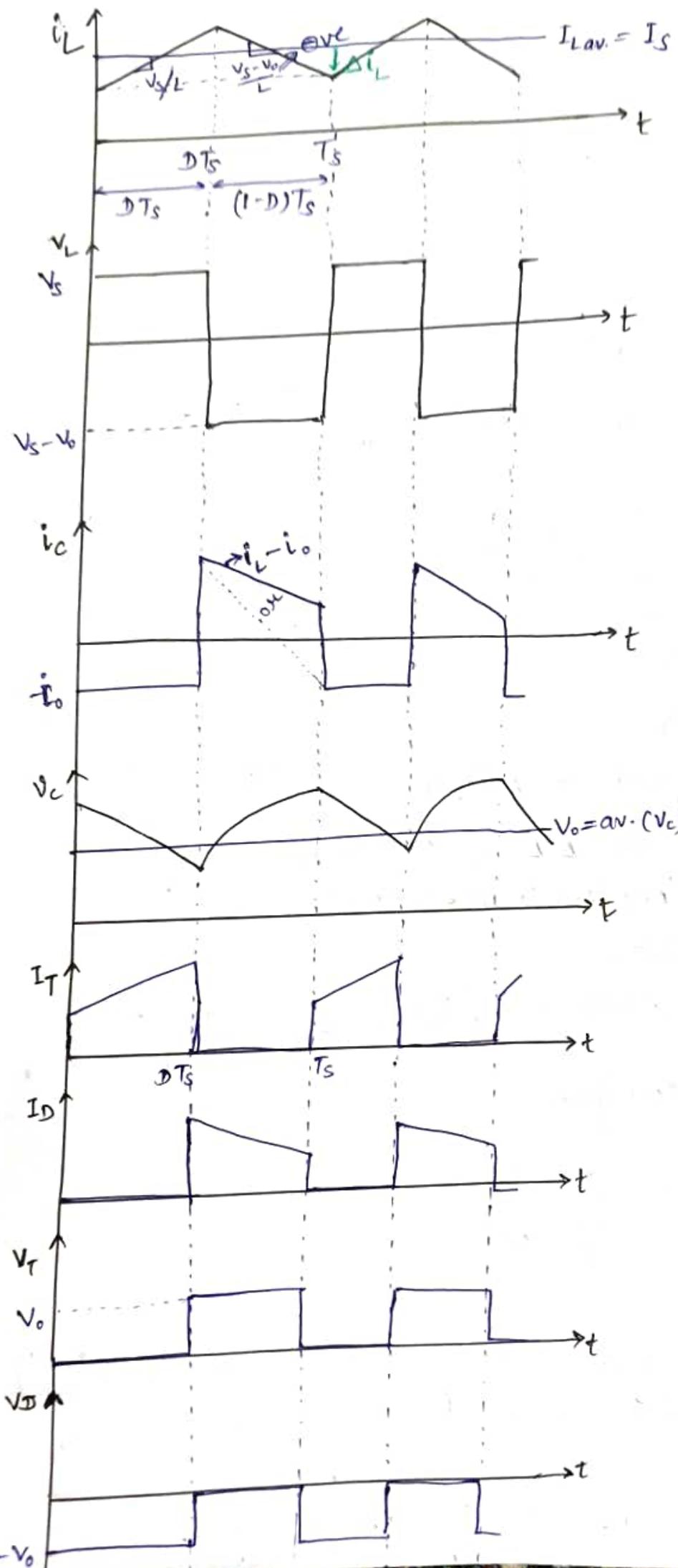
$$\Rightarrow V_s - \frac{V_0}{R(1-D)} R_L - V_0 (1-D) = 0 \quad \left[I_L = I_S = \frac{I_0}{(1-D)} = \frac{V_0}{R(1-D)} \right]$$

$$\Rightarrow V_s - V_0 \left[\frac{R_L}{R(1-D)} + (1-D) \right] = 0$$

$$V_0 = \frac{V_s}{\frac{R_L}{R(1-D)} + (1-D)} = \frac{V_s (1-D)}{\frac{R_L}{R} + (1-D)^2}$$

As $D \rightarrow 1$, $V_0 = 0$.

[Always operate the boost converter with a load, otherwise the voltage builds up and blasts.]



$$\Delta i_L = \frac{V_S}{L} D T_S \\ = \frac{V_S - V_D}{L} (1-D) T_S$$

$$v_c = \frac{1}{C} \int i_c dt$$

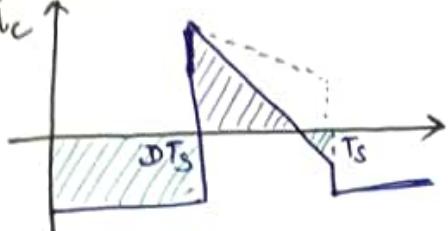
Book: Power Elec by
Daniel W Hart

(02-09-2024)

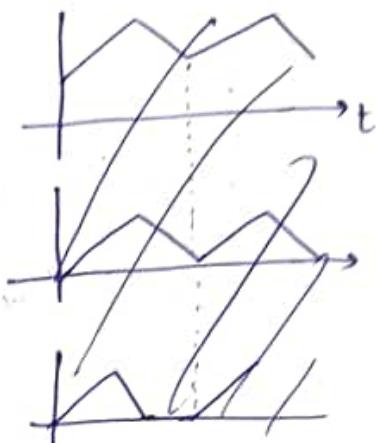
Peak-to-peak ripple in the inductor current:

$$\Delta i_L = \frac{V_s}{L} D T_S = \left(\frac{V_s - V_o}{L} \right) (1-D) T_S$$

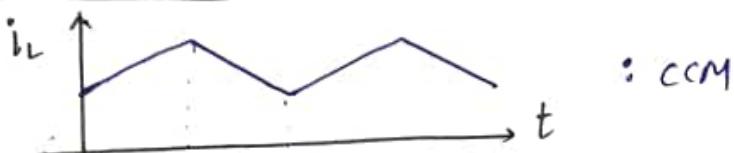
Capacitor current:



$$\Delta V = \frac{\Delta Q}{C}$$

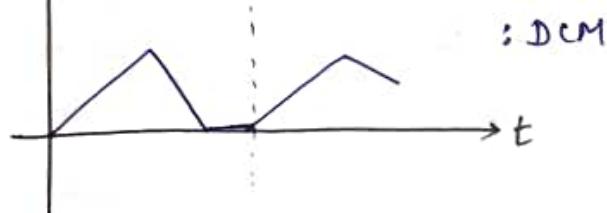


Inductor Current:



: CCM

: operating on the boundary b/w CCM & DCM



: DCM

Assuming 100% efficiency,

$$V_s I_s = V_o I_o, \quad V_o = \frac{V_s}{1-D}$$

$$I_s = \frac{V_o I_o}{V_s} = \frac{V_s I_o}{(1-D)V_s} = \frac{I_o}{1-D}$$

$$I_o = \frac{V_o}{R}$$

$$I_{L(av)} = I_s = \frac{V_o}{R(1-D)} = \frac{V_s}{R(1-D)^2}$$

On the boundary b/w CCM & DCM:

$$I_{L(av)} = \frac{\Delta i_L}{2}$$

$$\frac{V_s}{R(1-D)^2} = \frac{V_s}{2L} (D T_S)$$

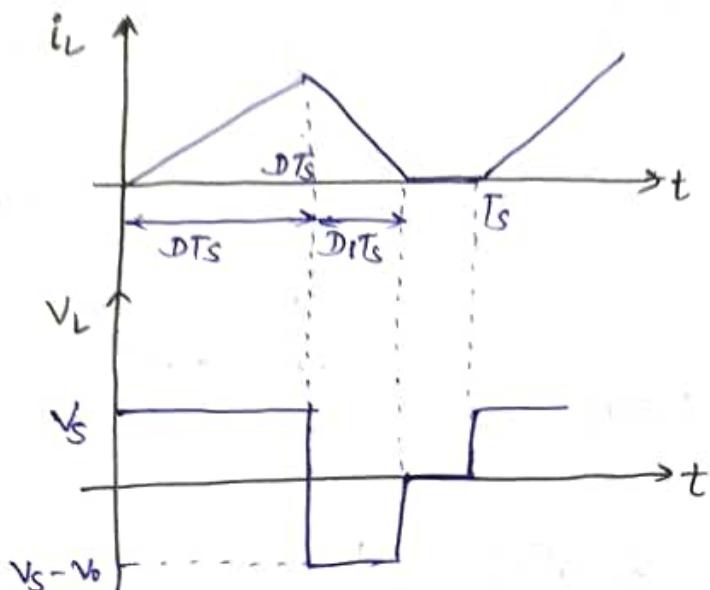
$$K_{ext.} = \frac{2L}{R T_S} = D(1-D)^2$$

| K: conduction parameter

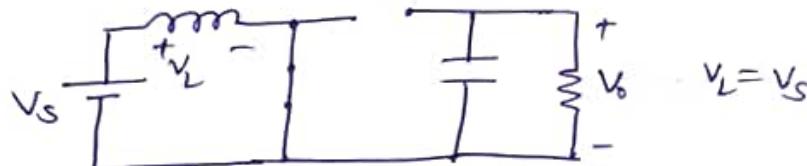
$$I_L(av) > \frac{\Delta i_L}{2} \rightarrow CCM$$

$$I_L(av) < \frac{\Delta i_L}{2} \rightarrow DCM$$

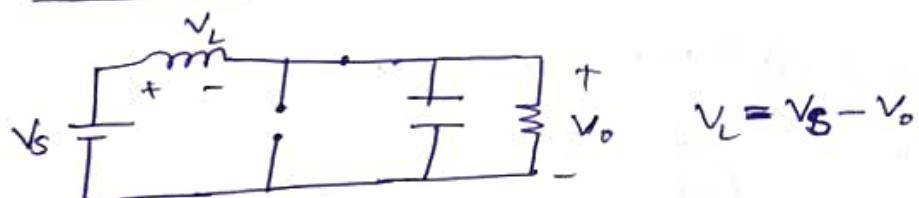
DCM of



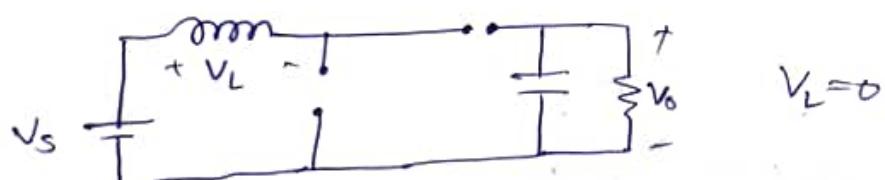
$0 < t \leq D TS$:



$D TS < t \leq (D + D_1) TS$:



$(D + D_1) TS < t \leq T_S$:



Applying volt-second balance,

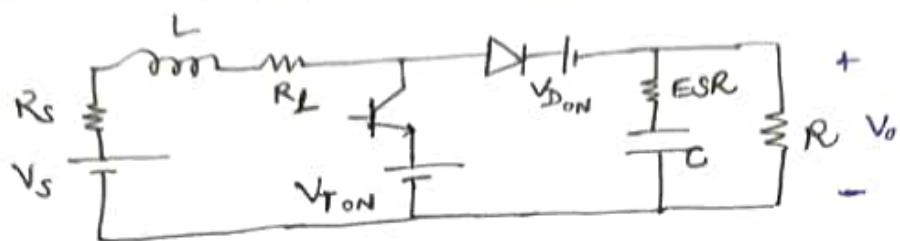
$$v_s D TS + (v_s - v_o) D_1 TS + 0 (1 - (D + D_1)) TS = 0$$

$$\Rightarrow v_s D + (v_s - v_o) D_1 = 0$$

$$\Rightarrow v_s (D + D_1) = v_o D_1$$

$$\Rightarrow v_o = \frac{v_s (D + D_1)}{D_1} = v_s \left(1 + \frac{D}{D_1} \right)$$

Boost Converter with non-idealities :



R_s : Internal resistance of the source

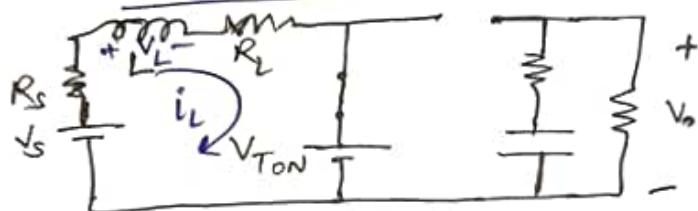
R_L : Parasitic resistance of the inductor

V_{TON} : ON-state voltage drop of the active switch

V_{DON} : ON-state voltage drop of the diode

ESR : Equivalent series resistance of the capacitor.

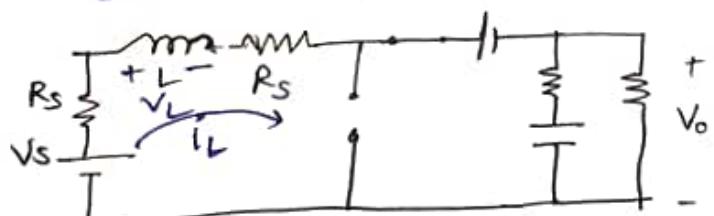
$0 < t \leq D T_S$:



$$V_s - i_L R_s - V_L - i_L R_L - V_{TON} = 0$$

$$\Rightarrow V_L = V_s - i_L (R_s + R_L) - V_{TON}$$

$D T_S < t \leq T_S$:



$$V_s - i_L R_s - V_L - i_L R_L - V_{DON} - V_o = 0$$

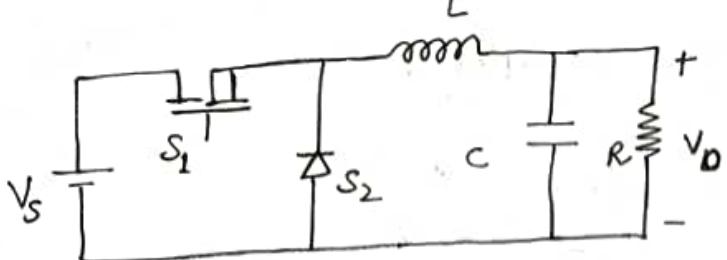
$$\Rightarrow V_L = V_s - i_L (R_s + R_L) - V_{DON} - V_o$$

Applying volt-sec. balance,

$$(V_s - i_L (R_s + R_L) - V_{TON}) D T_S + (V_s - i_L (R_s + R_L) - V_{DON} - V_o) (1-D) T_S =$$

$$\Rightarrow i_L = i_{L(\text{avg})} = I_S = \frac{I_o}{1-D} = \frac{V_o}{R(1-D)}$$

Q1



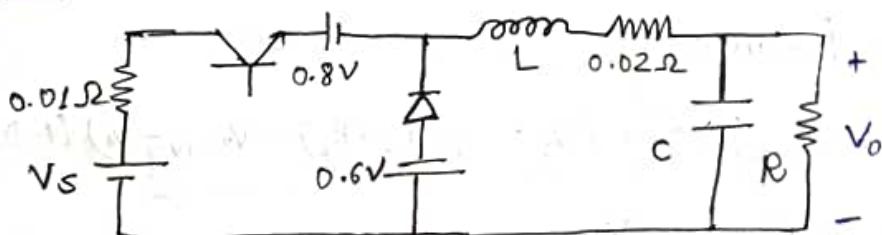
A buck converter operating in CCM mode is shown in figure.

$$V_s = 30V, R = 20\Omega, L = 0.4 \text{ mH}$$

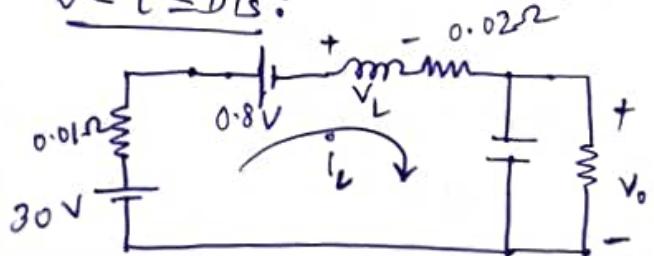
The inductor has a parasitic resistance of 0.02Ω and the input source has an internal resistance of 0.01Ω . The ON-state voltage drop of the switches S_1 and S_2 are $0.8V$ and $0.6V$, respectively. The switching frequency is 25kHz and the operating duty ratio (D) is 0.8 .

- ① Evaluate the output voltage (V_0).
- ② Evaluate the efficiency of the converter.
- ③ Evaluate the peak-to-peak ripple in inductor current.
- ④ Sketch the waveform of inductor current and mark all salient points.
- ⑤ Sketch the waveform of the capacitor current and mark all salient points.
- ⑥ Evaluate the capacitance (C) required to keep the ripple in the output voltage less than 1% of the average output voltage.
- ⑦ Estimate the conduction loss in the switches S_1 and S_2 .

Soln: Buck converter with non-idealities :

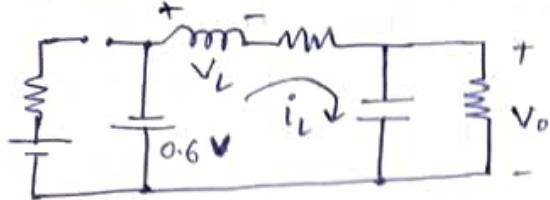


$$0 < t \leq D T_s :$$



$$V_L = (30 - 0.8 - 0.01 i_L - 0.02 i_L) - V_0$$

$DT_S < t \leq T_S$:



$$V_L = -(0.6 + 0.02 i_L + V_o)$$

① Applying volt-second balance,

$$(30 - i_L(0.01 + 0.02) - 0.8 - V_o) DT_S + -(V_o + 0.6 + 0.02 i_L)(1-D) T_S = 0$$

$$\left[i_L = I_o = \frac{V_o}{R}, \quad R = 20\Omega, \quad D = 0.8 \right]$$

$$\Rightarrow \left(30 - \frac{V_o}{20} (0.01 + 0.02) - 0.8 - V_o \right) 0.8 - \left(V_o + 0.6 + 0.02 \frac{V_o}{20} \right) 0.2 = 0$$

$$\Rightarrow V_o = \frac{23.24}{1.0014} = 23.2075 \text{ V}$$

$$\text{Output current, } I_o = \frac{V_o}{R} = \frac{23.2075}{20} = 1.1604 \text{ A}$$

Input source current, $I_S = D I_o = 0.8 \times 1.1604$
 $= 0.928 \text{ A}$

[current conversion ratio is not affected
 by series non-idealities.]

$$\begin{aligned} V_o I_o &= V_S I_S \\ D X_S I_o &= Y_S I_S \\ I_S &= D I_o \end{aligned}$$

② Efficiency of the converter = $\frac{\text{Output power}}{\text{Input power}}$

$$= \frac{V_o I_o}{V_S I_S} = \frac{23.2075 \times 1.1604}{30 \times 0.928} = 0.9669$$

③ Peak-to-peak ripple in the inductor current,

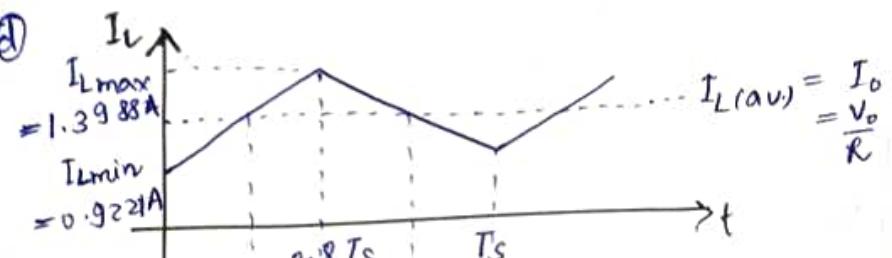
$$\Delta i_L = \frac{V_L}{L} (1-D) T_S =$$

For $DT_S < t \leq T_S$:

$$\begin{aligned} V_L &= 23.2075 + 0.02 \times 1.1604 + 0.6 \\ &= 23.833 \text{ V} \end{aligned}$$

$$T_S = \frac{1}{f_S} = \frac{1}{25 \times 10^3} = 40 \times 10^{-6} \text{ s}$$

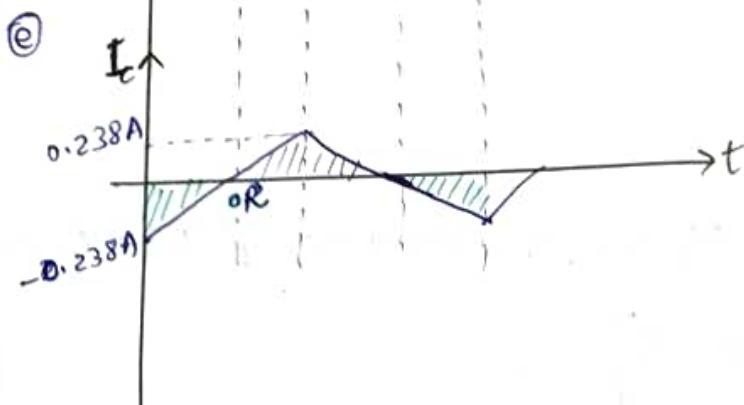
$$\therefore \Delta i_L = \frac{23.833 \times 0.2 \times 40 \times 10^{-6}}{0.4 \times 10^{-3}} = 0.4767 \text{ A.}$$



$$I_{L\min} = I_{L(\text{av.})} - \frac{\Delta i_L}{2}$$

$$= 1.1604 - \frac{0.4767}{2}$$

$$= 0.9221 \text{ A}$$



$$I_{C\max} = I_{L(\text{av.})} + \frac{\Delta i_L}{2}$$

$$= 1.1604 + \frac{0.4767}{2}$$

$$= 1.3988 \text{ A}$$

③ $C = \frac{\Delta Q}{\Delta V}$

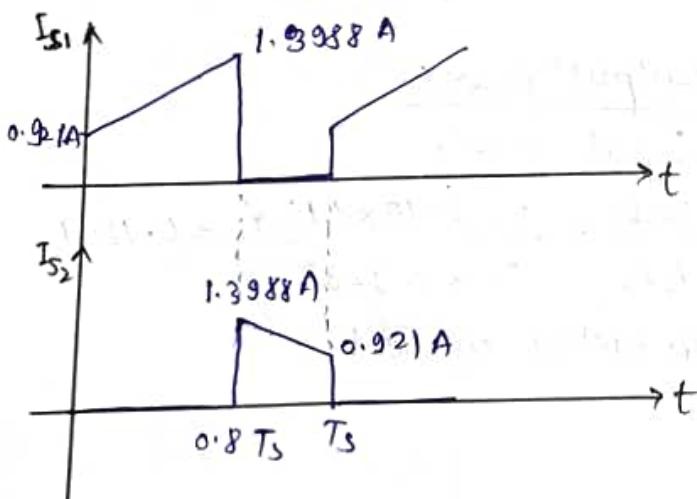
$$\Delta V = 1.1 \cdot 0.8 = 23.275$$

$$= 0.23275$$

$$\Delta Q = \frac{1}{2} \times 0.238 \times \frac{40 \times 10^{-6}}{2}$$

Capacitance, $C = \frac{\frac{1}{2} \times 0.238 \times 40 \times 10^{-6}}{0.23275}$

$$= 10.24 \times 10^{-6} \text{ F}$$



④ Average conduction loss = $\frac{1}{T} \int_0^T V_T \times I_T dt = \frac{1}{T} \int_0^T V_{TON} I_T dt$

$$= V_{TON} \times \frac{1}{T} \int_0^T I_T dt$$

$$= \boxed{V_{TON} \times I_T(\text{av.})}$$

$$V_T = V_{TON} = 0.8 \text{ V for } S_1$$

Average conduction loss in switch S_1

$$= \text{Average current through } S_1 \times \text{ON-state voltage drop}$$
$$= \left(\frac{1.3988 + 0.9221}{2} \right) DT_S \times 0.8 = 0.7427 \text{ W}$$

$$\text{Av. conduction loss in diode} = \left(\frac{1.3988 + 0.9221}{2} \right) (1-D) T_S \times 0.6$$
$$= 0.25208 \times 0.6$$
$$= 1.392 \text{ W}$$



Time for 1/2 P

Half cycle of current

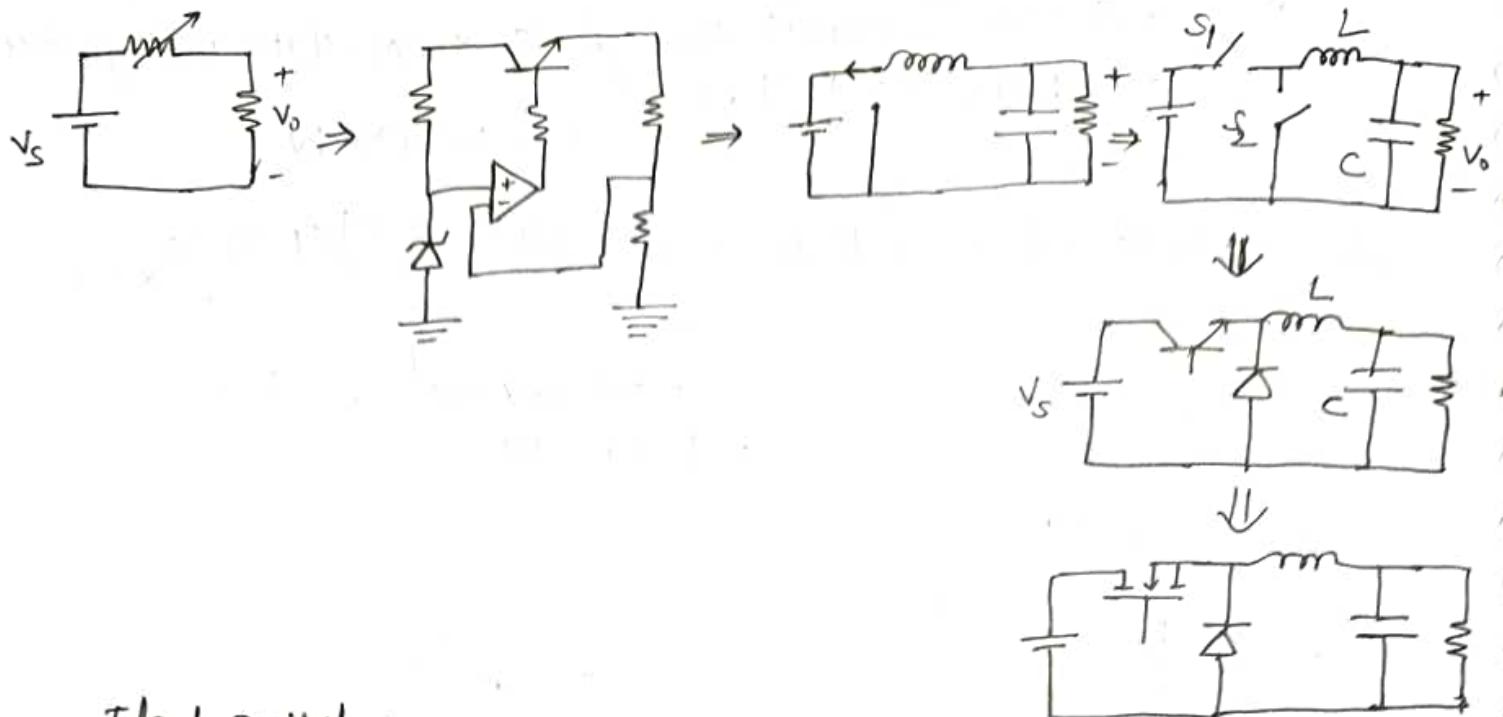
Time for 1/2 P
without consideration
of turn off losses



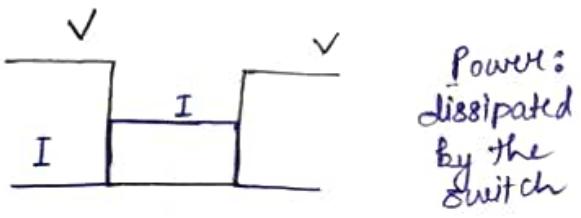
At D = 0.6, current starts to fall from zero at 0.2 sec. At the end of 0.6 sec, current is zero.

Turn off losses are negligible.





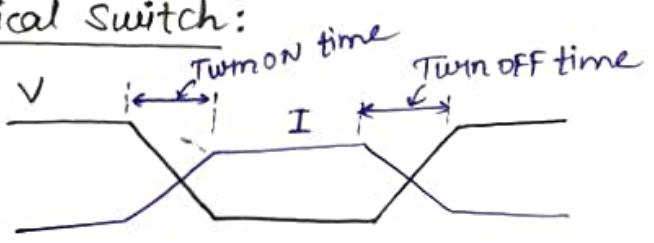
Ideal switch:



Power:
dissipated
by the
switch

+ v - (offset
current)
 ≈ 0
 $v=0$ (offset
voltage)

Practical switch:



OFF to ON
Transition
Turn ON

ON to OFF
Transition
Turn OFF



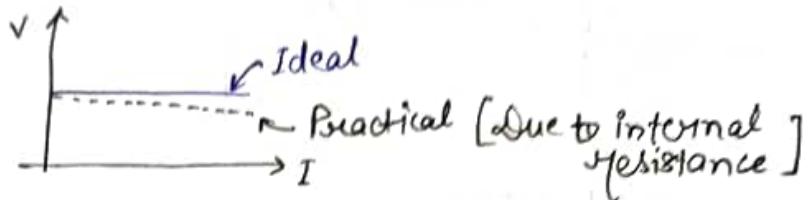
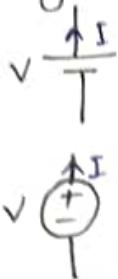
Soft switching: Lossless switching where either V or I is zero.

- High switching frequency.
- Zero voltage switching (ZVS): Maintain $V=0$ while switching.
- Zero current switching (ZCS): Maintain $I=0$.

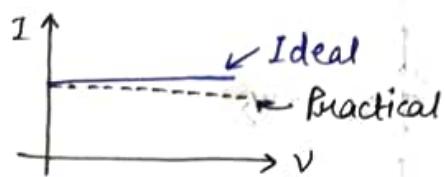
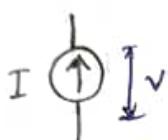
DC to AC Converters

DC Sources:

Voltage Source:



Current source:



DC to AC Converters (Inverters):

Voltage Source Converter (Inverter)
[VSI]

Current Source Converter (Inverter)
[CSI]

Most commonly used

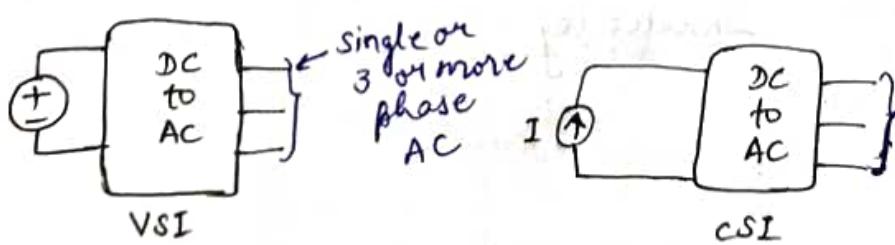
Sine波浪: Give high efficiency

↳ Mathematical advantages: differentiation or Integration gives sine waves.

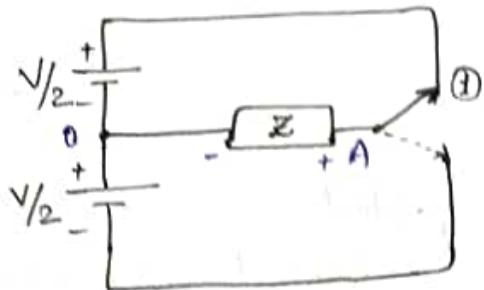
↳ Any periodic wave can be represented as the sum of infinite sine waves.

[Ensure that average value of o/p V is zero.]

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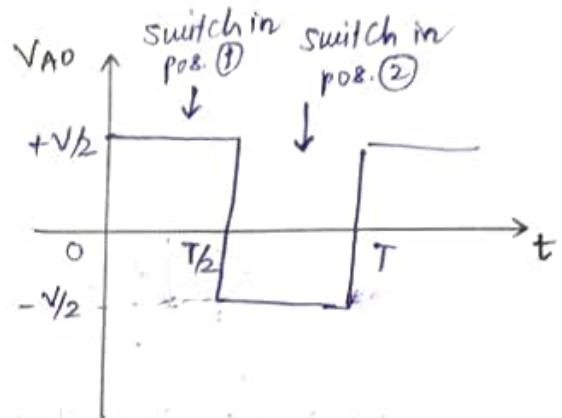
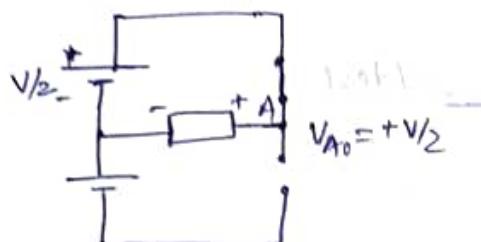


Voltage Source Inverters (VSI): Half-Bridge Inverter

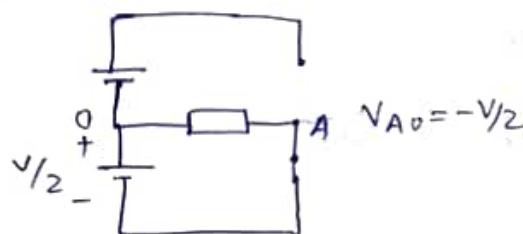


$$0 < t \leq T/2:$$

switch in position ①:



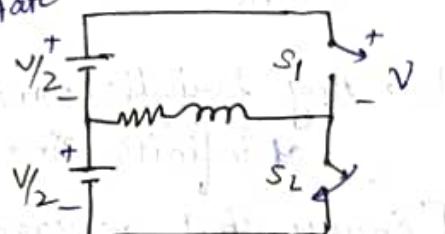
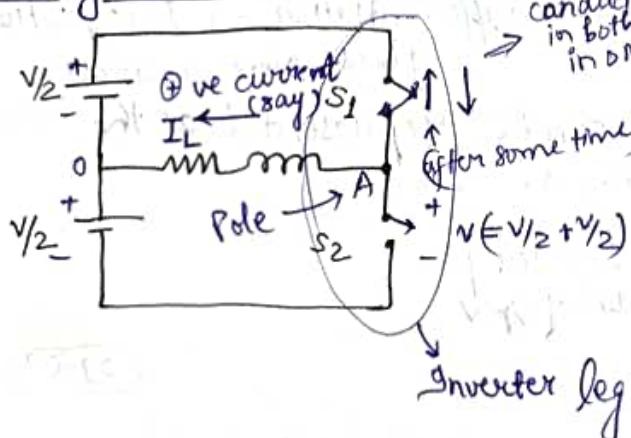
$$T/2 \leq t < T:$$

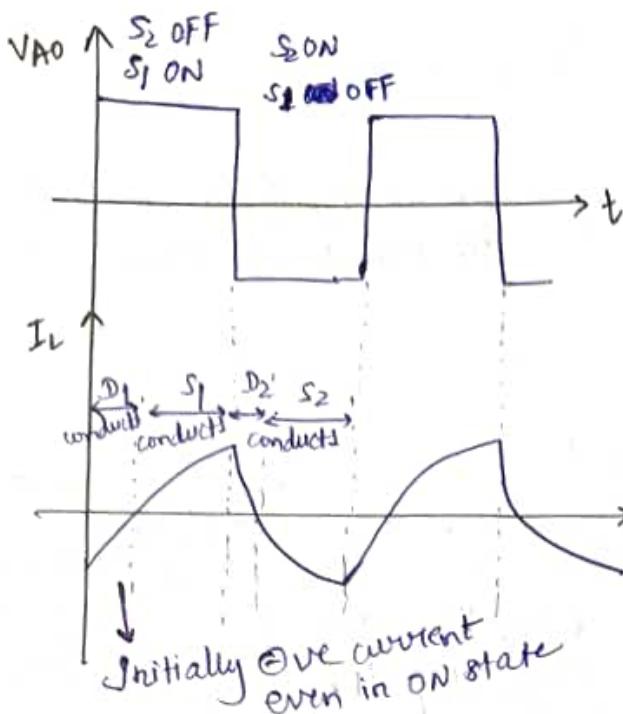


AC Loads: R-L (or R-L-E)

(in general) [Complementary Operation] Current-bidirectional (2 quadrant switch)

Using 2 SPST switches: [conducts current in both dirn in ON state]



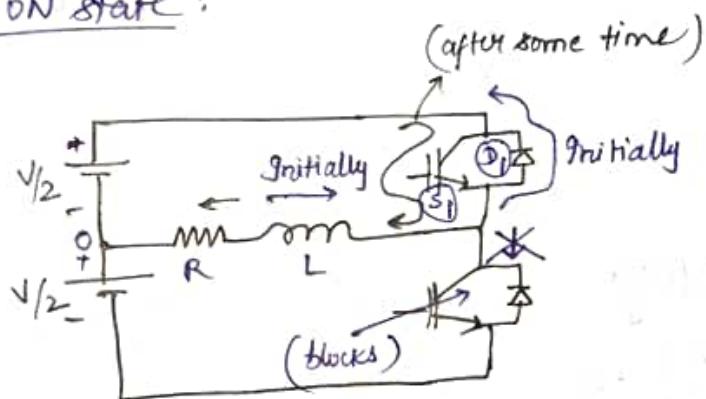


In R-L circuit :

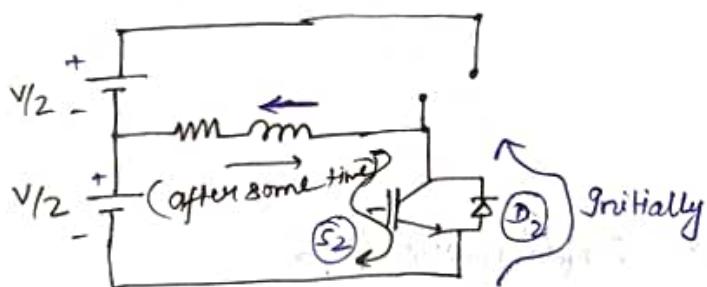
$$i = \frac{V}{R} e^{-\frac{Rt}{L}}$$

[In steady state]
↓
(After some switchings)

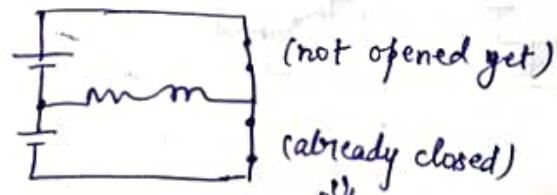
In ON state :



In OFF state :

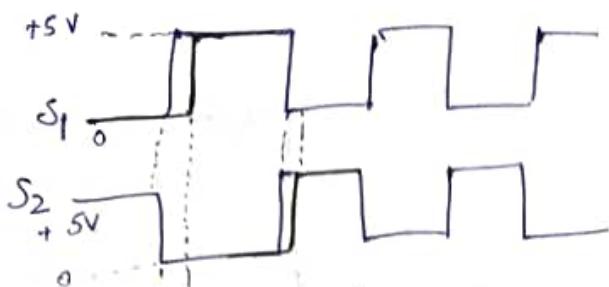


- Switching time is decided by R-L.
- Without reverse diodes, it will blast.
- There is a finite switching ON and switching OFF time.
- Ensure that when one switch is ON, other is OFF.



⇒ short-circuit current
↓
Damages the inverter

Gate pulses of S_1 and S_2 :



→ Ensure T_{ON} (turn ON) & T_{OFF} (turn off time) are same for S_1 & S_2 .

→ Ensure that S_2 is fully opened

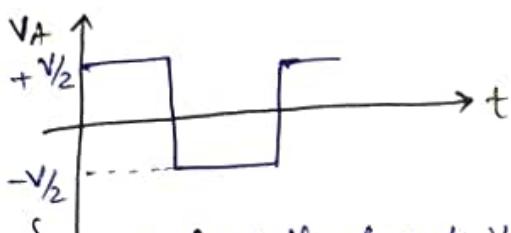
before closing S_1 → Give some time delay to S_1

dead time / dead band

→ Ensure to give certain dead time to the pulses, before turning ON the inverter.

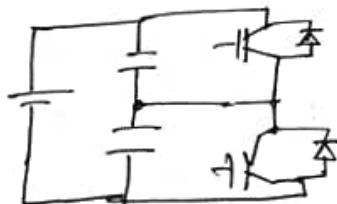
[only delay the rising edges of pulse]

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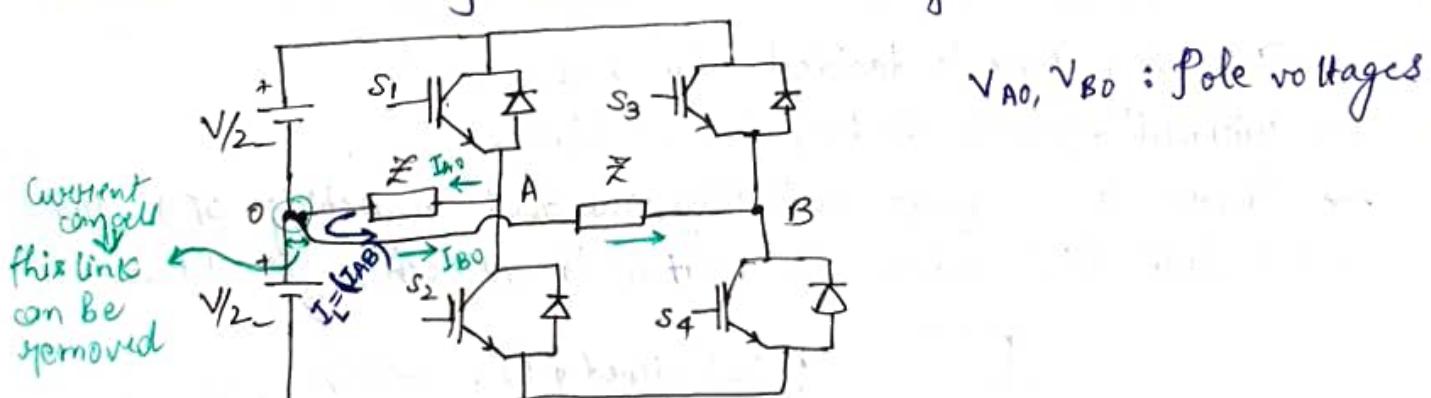


→ only half the input Voltage

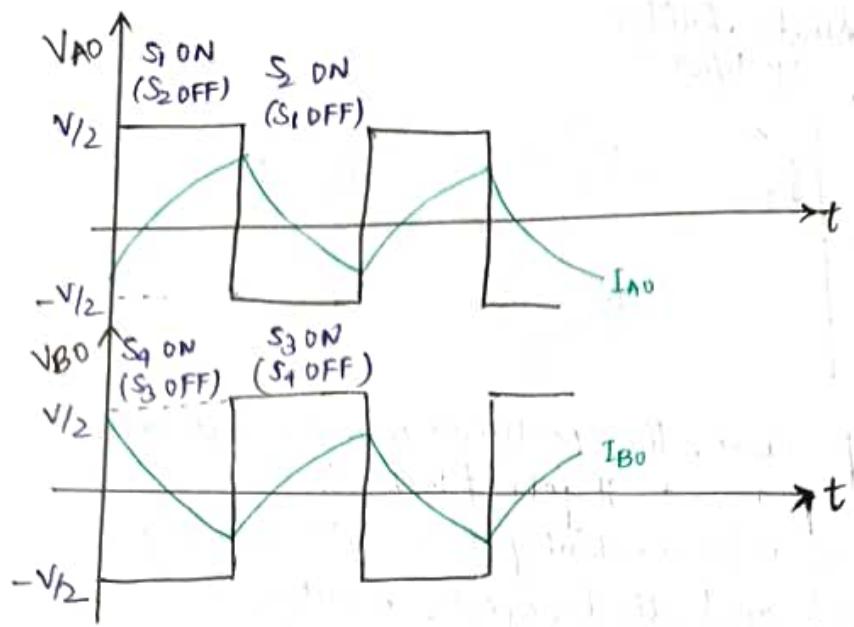
Using only one voltage source:



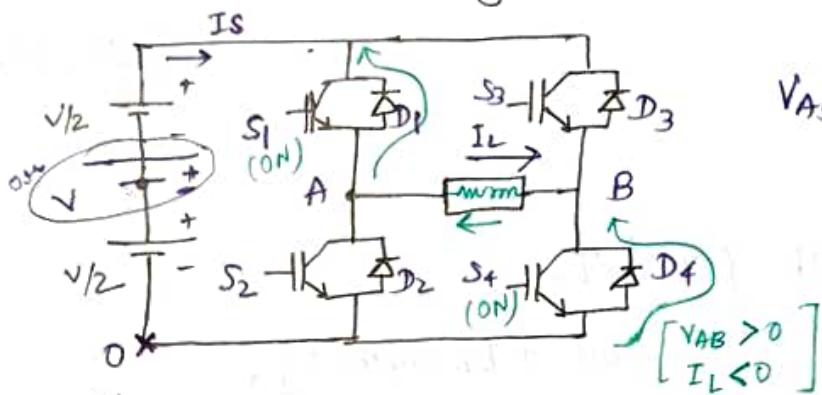
To increase V_A amplitude: Add more legs



V_{A0}, V_{B0} : Pole voltages

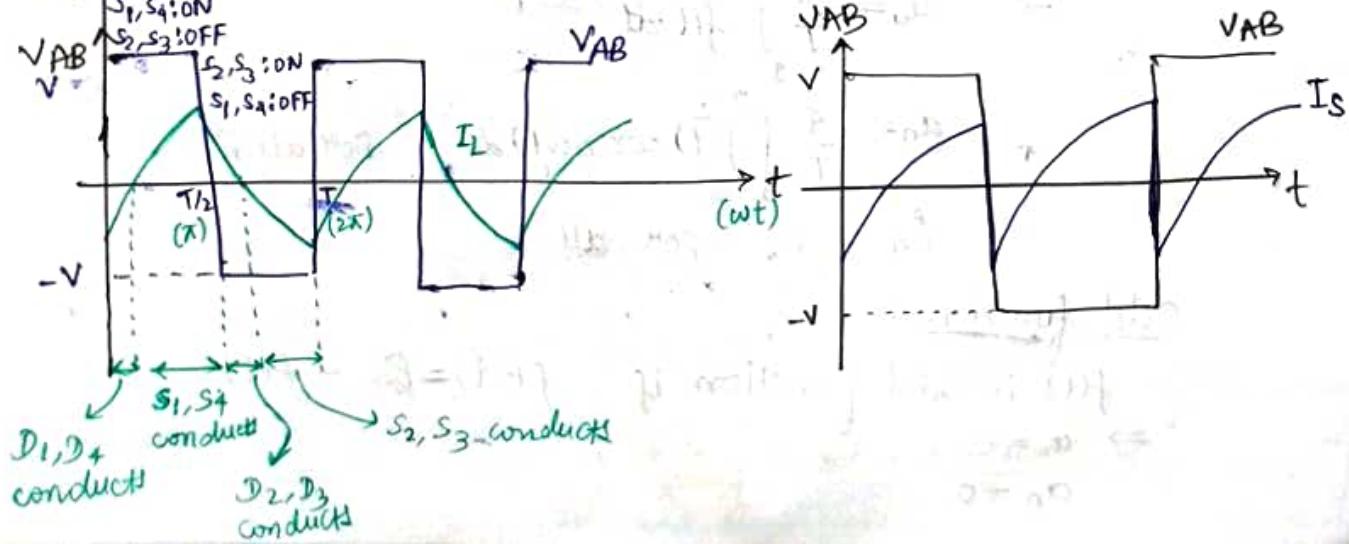
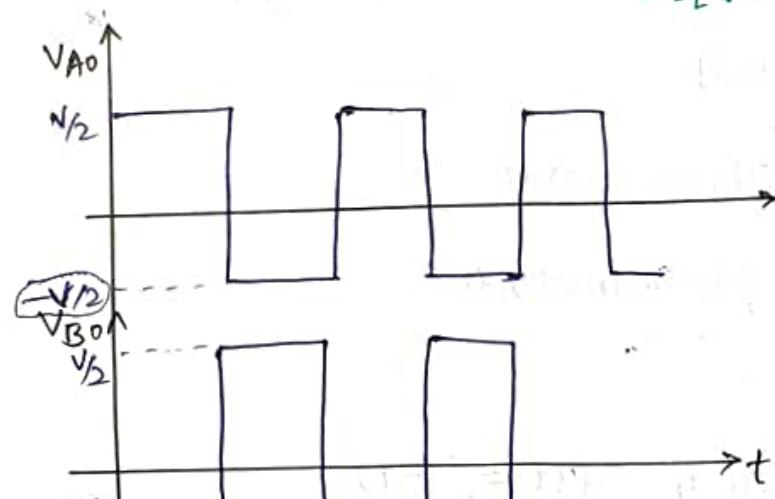


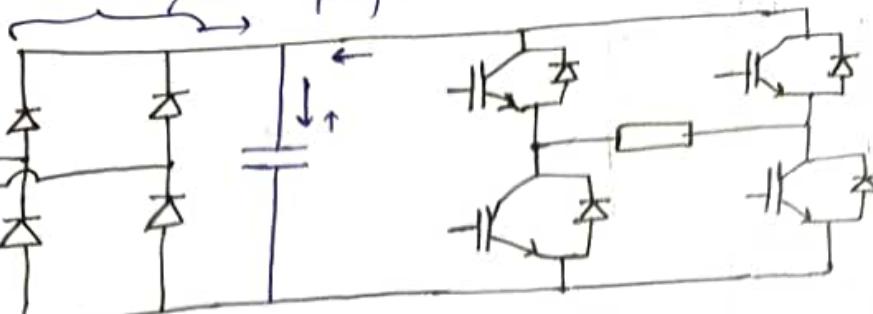
\Downarrow Full Bridge Inverter (H-Bridge Inverter)



$$V_{AB} = V_{AO} + V_{BO}$$

$$= V_{AO} - V_{BO}$$



With AC source: 

without the capacitors, the rectifier won't work as in reverse current, it gets blocked.
with capacitor, it is constantly & simultaneously getting charged and discharged, resulting in ripple which has to be considered.

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Fourier Series

If $f(t)$ is periodic with period T .

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)],$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t) dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t) dt$$

Even function:

$f(t)$ is even function if $f(t) = f(-t)$.

$$\Rightarrow a_0 = \frac{2}{T} \int_0^{T/2} f(t) dt$$

$$a_n = \frac{4}{T} \int_0^{T/2} f(t) \cos(n\omega t) dt, \text{ for all } n$$

$$b_n = 0, \text{ for all } n$$

Odd function:

$f(t)$ is odd function if $f(-t) = -f(t)$.

$$\Rightarrow a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin(n\omega t) dt, \text{ for all } n.$$

Half-wave Symmetry:

$f(t)$ has half-wave symmetry if

$$f(t) = -f(t - T/2).$$

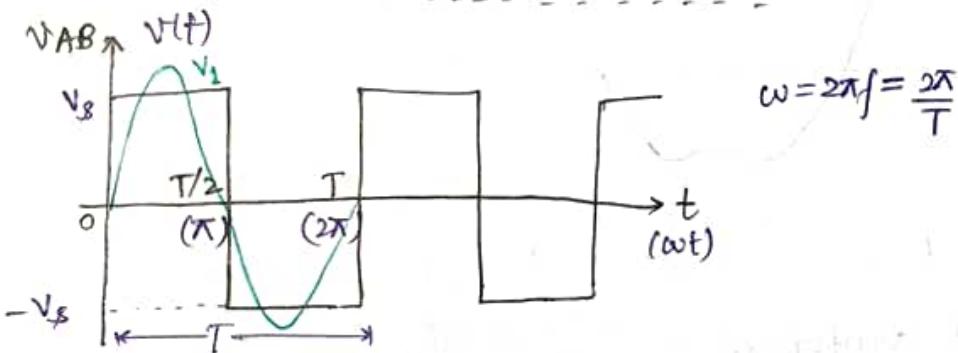
If $f(t)$ is an odd function with half-wave symmetry.

$$a_0 = 0$$

$$a_n = 0, \text{ for all } n$$

$$\text{and, } b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin(n\omega t) dt, \text{ for } n=\text{odd } (n=1, 3, 5, \dots)$$

$$= 0, \text{ for } n=\text{even}.$$



$$V(t) = \begin{cases} +V_s & \text{for } 0 < \theta < \pi \\ -V_s & \text{for } \pi < \theta < 2\pi \end{cases}$$

↪ Odd function with half-wave symmetry.

$$\Rightarrow a_0 = 0, a_n = 0, \text{ for all } n$$

$$b_n = \frac{4}{2\pi} \int_0^{\pi} V_s \sin(n\omega t) d(\omega t), \text{ for } n=1, 3, 5, \dots$$

$$= \frac{4V_s}{2\pi} \int_0^{\pi} \sin(n\omega t) d(\omega t)$$

$$= \frac{4V_s}{2\pi} \left[-\frac{\cos n\omega t}{n} \right]_0^{\pi}$$

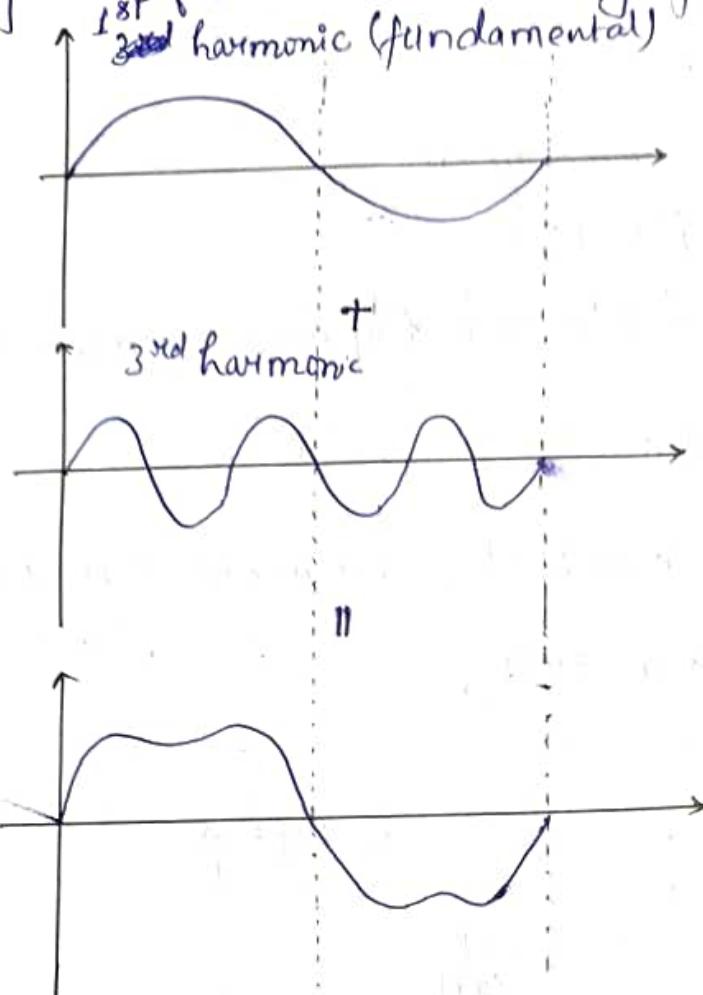
$$= -\frac{4V_s}{2\pi n} [-1 - 1] = \frac{8V_s}{2\pi n}$$

$$= \frac{4V_s}{n\pi} \text{ for } n=1, 3, 5, \dots$$

$$\therefore V(\omega t) = \sum_{n=1}^{\infty} b_n \sin n\omega t = \frac{4V_s}{\pi} \left[\sin \omega t + \underbrace{\frac{\sin 3\omega t}{3}}_{3^{\text{rd}} \text{ harmonic}} + \underbrace{\frac{\sin 5\omega t}{5}}_{5^{\text{th}} \text{ harmonic}} + \dots \right]$$

only odd harmonics

→ Magnitude of harmonics is inversely proportional to the harmonic.



$$\text{Fundamental component} = \frac{4V_s \sin \omega t}{\pi} \quad (V_L)$$

$$\text{Peak value of fund. component} = 4V_{ip} = \frac{4V_s}{\pi} \quad (> V_s)$$

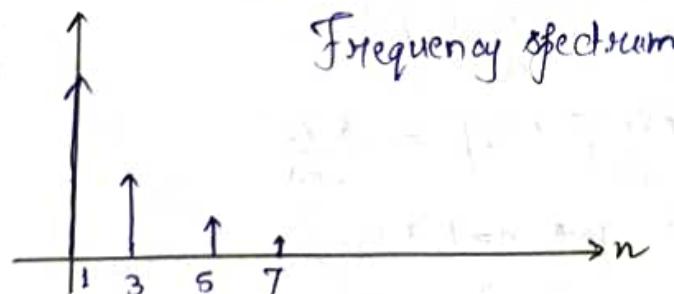
$$\text{RMS value of fund. component} = V_{irms} = \frac{4V_s}{\sqrt{2}\pi}$$

To get fundamental component

↳ Low Pass filter is not desirable as it changes its frequency.

↳ Also, it has high cost and bigger than the inverter itself.

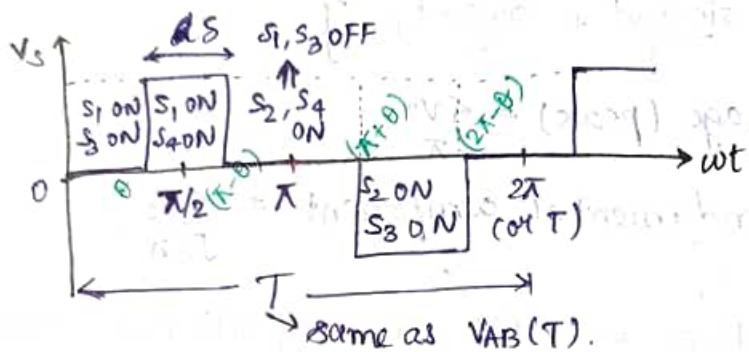
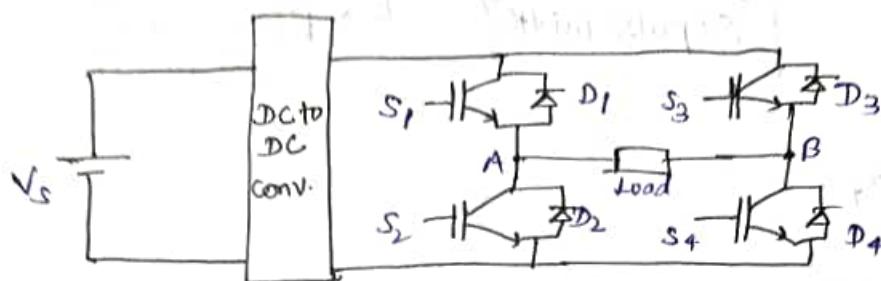
$$\frac{4V_s}{\pi}$$



Frequency spectrum:

To control the magnitude of fundamental component:

↪ change $v_s \rightarrow$ changing / variable DC source is hard to design
 ↓
 Introduce DC to DC converter (Buck converter).



Quasi-Square Wave

→ By changing the value of δ , peak/RMS value of fund. comp. can be changed (by changing v_s).

→ To get $v_s=0 \Rightarrow$ we can't just take $v_s=0 \Rightarrow$ instead just make 2 above or 2 below switches ON.

$\because v_s$ shall not be interrupted suddenly

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$$a_0 = 0, a_n = 0, \text{ for all } n.$$

$$\begin{aligned} b_n &= \frac{8}{2\pi} \int_{0}^{\pi/2} v(t) \sin(n\omega t) d(\omega t) \quad \left[\because V_1 = \frac{4v_s}{\pi} \right] \\ &= \frac{8}{2\pi} \left[\int_0^0 \sin(n\omega t) d(\omega t) + \int_{\pi/2}^{n\pi/2} v_s \sin(n\omega t) d(\omega t) \right] \\ &= \frac{8v_s}{2\pi} \left[-\frac{\cos(n\omega t)}{n} \right]_0^{\pi/2} = \frac{4v_s}{n\pi} \left[-(\cos n\pi/2 - \cos n\theta) \right] \\ &= \frac{4v_s}{n\pi} \cos n\theta \end{aligned}$$

$$\therefore v(t) = \sum_{n=1,3,5}^{\infty} b_n \sin(n\omega t)$$

$$= \frac{4v_s}{\pi} \left[\cos \theta \sin \omega t + \frac{\cos 3\theta \sin 3\omega t}{3} + \frac{\cos 5\theta \sin 5\omega t}{5} + \dots \right]$$

$\therefore n=1, 3, 5, \dots, \cos n\pi/2 = 0$

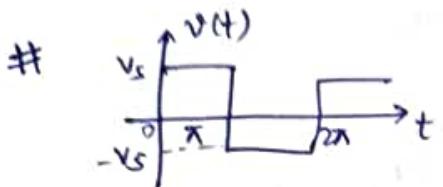
$$\therefore v(t) = \frac{4V_s}{\pi} \left[\cos \theta \sin \omega t + \frac{\cos 3\theta \sin 3\omega t}{3} + \frac{\cos 5\theta \sin 5\omega t}{5} + \frac{\cos 7\theta \sin 7\omega t}{7} + \dots \right]$$

Peak value of fundamental component

$$= \frac{4V_s}{\pi} \cos \theta = \frac{4V_s}{\pi} [\cos(\pi/2 - \delta/2)] = \frac{4V_s}{\pi} \sin \delta/2$$

$\boxed{\delta: \text{pulse width}}$

Pulse width modulation



$$v(t) = \frac{4V_s}{\pi} \left[\sin \omega t + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \dots \right]$$

$$\text{Fundamental voltage (peak)} = \frac{4V_s}{\pi}$$

$$\text{RMS value of fundamental component} = \frac{4V_s}{\sqrt{2}\pi}$$

$$\text{Total Harmonic Distortion (THD)} = \frac{\text{RMS value of all harmonics}}{\text{RMS value of fundamental component}}$$

For a rectangular waveform

$$THD = \sqrt{V_s^2 - \left(\frac{4V_s}{\sqrt{2}\pi} \right)^2} = \sqrt{V_s^2 - \frac{8V_s^2}{\pi^2}} = \sqrt{\pi^2 - 8} \cdot \frac{4V_s}{\sqrt{2}\pi} = \frac{4V_s}{\sqrt{2}\pi} = 0.483$$

$$\therefore THD = 48.3\%$$

\hookrightarrow (Recommended value $\sim 5\%$)
 \hookrightarrow Reducing/eliminating lower order harmonics significantly reduces THD.

$$\theta = \pi/6 \Rightarrow \text{No 3rd harmonic}$$

$$\theta = \pi/10, 0.45^\circ \Rightarrow \text{No 5th harmonic}$$

In general, for $\theta = \frac{90^\circ}{n}$, n th harmonic can be eliminated.

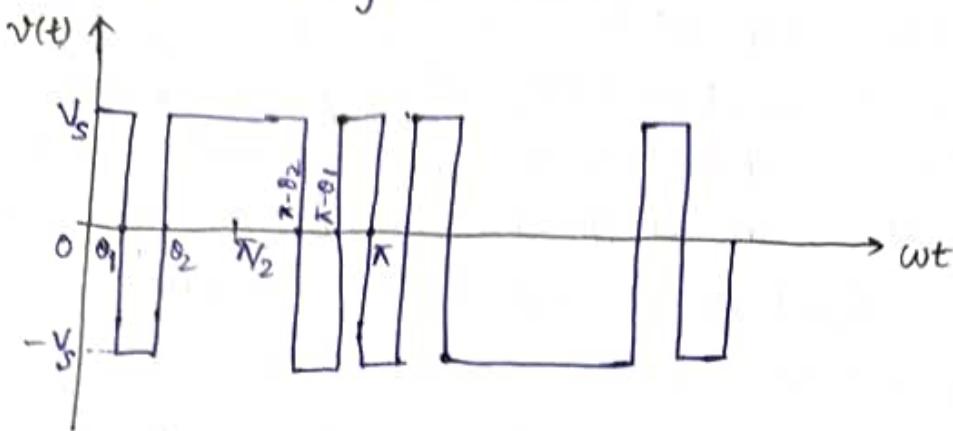
\hookrightarrow Selective Harmonic Elimination (SHEPWM)

\rightarrow We can either control the magnitude of fundamental component or eliminate a harmonic, or control a harmonic magnitude.

\rightarrow If a harmonic is causing troublesome, eliminate it.

$$[\theta = \sin^{-1} \frac{2}{n}, \dots, 2.61^\circ = 15^\circ]$$

To control two parameters:



→ Odd function with half-wave and quarter-wave symmetry.

$$a_0 = 0, a_n = 0 \text{ for all } n.$$

$$b_n = \frac{8}{2\pi} \int_0^{\pi/2} v_s(wt) d(wt) \sin(nwt)$$

$$= \frac{4}{\pi} \left[\int_0^{\theta_1} v_s \sin(nwt) d(wt) + \int_{\theta_1}^{\theta_2} (-v_s) \sin(nwt) d(wt) + \int_{\theta_2}^{\pi/2} (v_s) \sin(nwt) d(wt) \right]$$

$$= \frac{4v_s}{\pi} \left[-\frac{\cos nwt}{n} \Big|_0^{\theta_1} + \frac{\cos nwt}{n} \Big|_{\theta_1}^{\theta_2} - \frac{\cos nwt}{n} \Big|_{\theta_2}^{\pi/2}, n=1,3,5,\dots \right]$$

$$= \frac{4v_s}{n\pi} \left[-\cos n\theta_1 + \cos 0 + \cos n\theta_2 - \cos n\theta_1 - \cos n\frac{\pi}{2} + \cos n\theta_2 \right]$$

$$\Rightarrow b_n = \frac{4v_s}{n\pi} \left[1 - 2\cos n\theta_1 + 2\cos n\theta_2 \right]$$

$$b_1 = \frac{4v_s}{\pi} \left[1 - 2\cos \theta_1 + 2\cos \theta_2 \right] = x \quad \begin{array}{l} \text{[To control magnitude of]} \\ \text{fundamental component]}\end{array}$$

$$b_3 = \frac{4v_s}{3\pi} \left[1 - 2\cos 3\theta_1 + 2\cos 3\theta_2 \right] = 0 \quad \begin{array}{l} \text{[To eliminate 3rd]} \\ \text{harmonic]} \end{array}$$

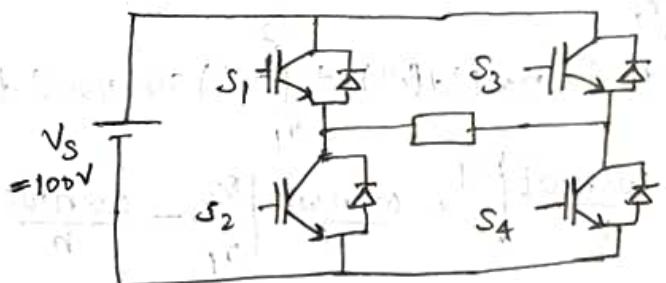
Ensure that there is no DC component, otherwise there will be a DC offset.

$$b_0 = \frac{4v_s}{\pi} \left[1 - 2\cos 0 + 2\cos 0 \right] = 0 \quad \begin{array}{l} \text{[No DC component]} \\ \text{[DC offset]} \end{array}$$

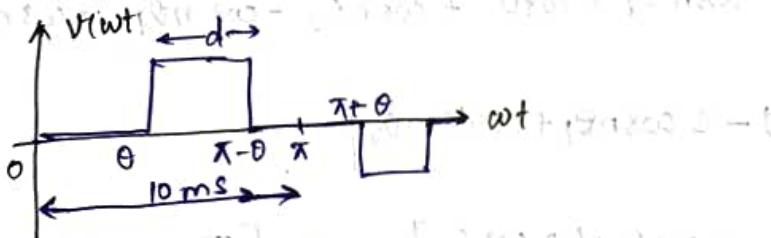
→ This is the improved technique of the previous one.

$$b_0 = \frac{4v_s}{\pi} \left[1 - 2\cos 0 + 2\cos 0 \right] = 0$$

- Eg. A pulse width modulated H-bridge inverter is shown in figure. The frequency of the fundamental component of the voltage across the load is 50 Hz. It has to be ensured that even harmonics are absent in the inverter output voltage.
- ② Evaluate the RMS value of the fundamental voltage realizable if 3rd harmonic has to be eliminated from the output voltage by employing selective harmonic elimination PWM.
- ③ If the required RMS value of the fundamental voltage is 85V, evaluate the pulse width required in a half cycle of the inverter output voltage (quasi square wave) without focusing on elimination of any odd harmonics.



Soln:



$$v(t) = \frac{4V_S}{\pi} \left[\cos \omega t + \cos 3\omega t + \frac{\cos 5\omega t}{3} + \dots \right]$$

② If 3rd harmonic has to be eliminated,

$$3\theta = 90^\circ \Rightarrow \theta = \frac{90^\circ}{3} = 30^\circ$$

$$\begin{aligned} \text{Fundamental component (peak value)} &= \frac{4V_S \cos 30^\circ}{\pi} = \frac{4 \times 100 \cos 30^\circ}{\pi} \\ &= 110.26 \text{ V} \end{aligned}$$

RMS value of the fundamental component realizable

$$= \frac{110.26}{\sqrt{2}} = 77.97 \text{ V}$$

(B) RMS value of fundamental voltage = 85 V
 Peak " " = $85\sqrt{2} = 120.21$ V

$$\frac{4V_s}{\pi} \cos \theta = \frac{4 \times 100}{\pi} \cos \theta = 120.21 \text{ V}$$

$$\Rightarrow \cos \theta = 0.944$$

$$\Rightarrow \theta = 19.25^\circ$$

Pulse width = 141.25°

$$= \frac{141.25}{180} \times 10 \times 10^{-3} = 7.86 \text{ ms}$$

Total harmonic distortion of voltage = $\frac{\text{RMS value of all harmonic}}{\text{RMS value of fundamental harmonic}}$

For elimination of 3rd harmonic, $\theta = 30^\circ$

$$\text{RMS value} = \sqrt{\frac{1}{T} \int_0^T f(t)^2 dt}$$

$$= \sqrt{\frac{1}{2\pi} \int_{0}^{2\pi} v(\omega t)^2 d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi} \left(\int_{\pi/6}^{5\pi/6} V_s^2 d(\omega t) + \int_{5\pi/6}^{11\pi/6} (-V_s)^2 d(\omega t) \right)}$$

$$= \sqrt{\frac{V_s^2}{2\pi} \left(\frac{4\pi}{6} + \frac{4\pi}{6} \right)}$$

$$= \sqrt{V_s^2 \left(\frac{1}{3} + \frac{1}{3} \right)}$$

$$= \sqrt{\frac{2}{3}} V_s$$

$$\text{THD} = \frac{\sqrt{\left(\sqrt{\frac{2}{3}} \times 100\right)^2 - (77.97)^2}}{77.97} = 0.31 \rightarrow 31\%$$

→ Current across load also changes with harmonic

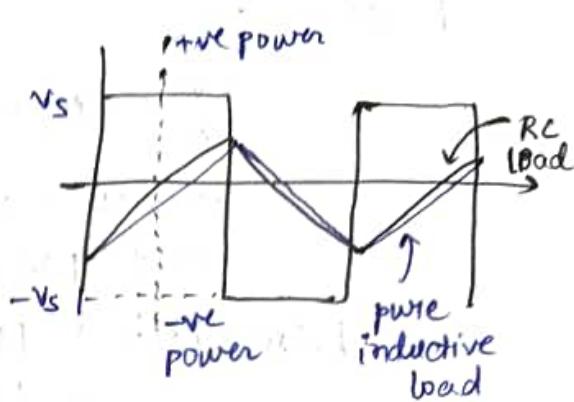
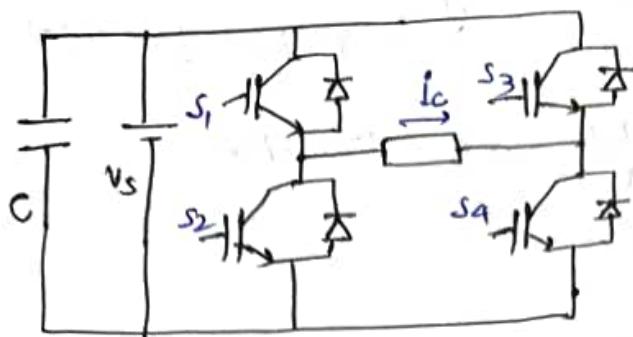
$$Z_L = R + j\omega L \Rightarrow |Z_L| = \sqrt{R^2 + (\omega L)^2}$$

$$Z_3 = R + j3\omega L \Rightarrow |Z_3| = \sqrt{R^2 + (3\omega L)^2}$$

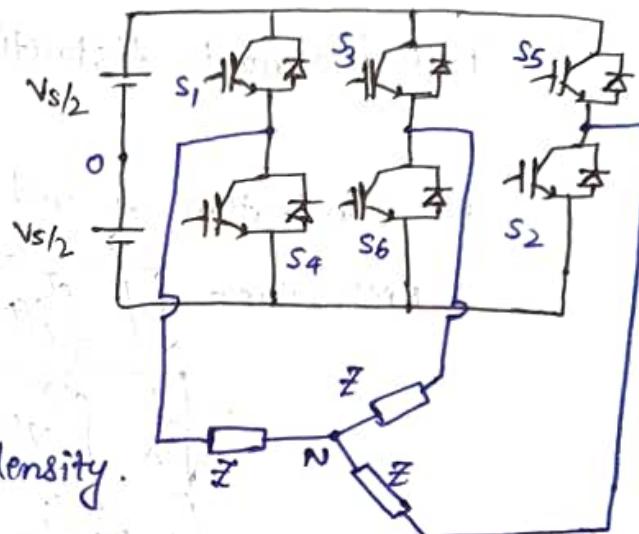
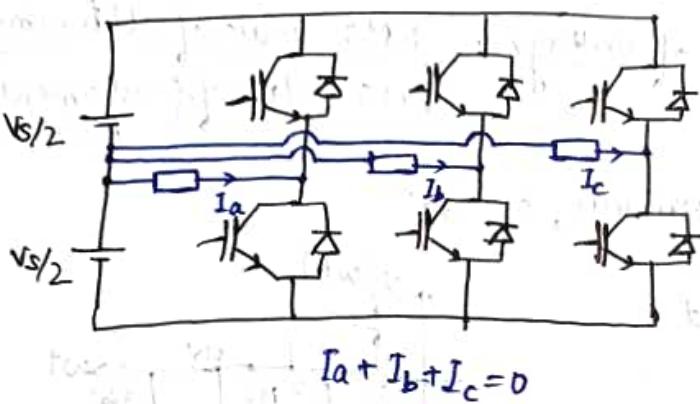
$$I_3 = \frac{V_s}{Z_3}, I_S = \frac{V_s}{Z_S}$$

A8 n↑ $\Rightarrow V_n \downarrow, Z_n \uparrow$
 $\Rightarrow I_n \downarrow$

$$I_{\text{stems}} = \sqrt{I_1^2 + I_3^2 + I_5^2 + \dots}$$

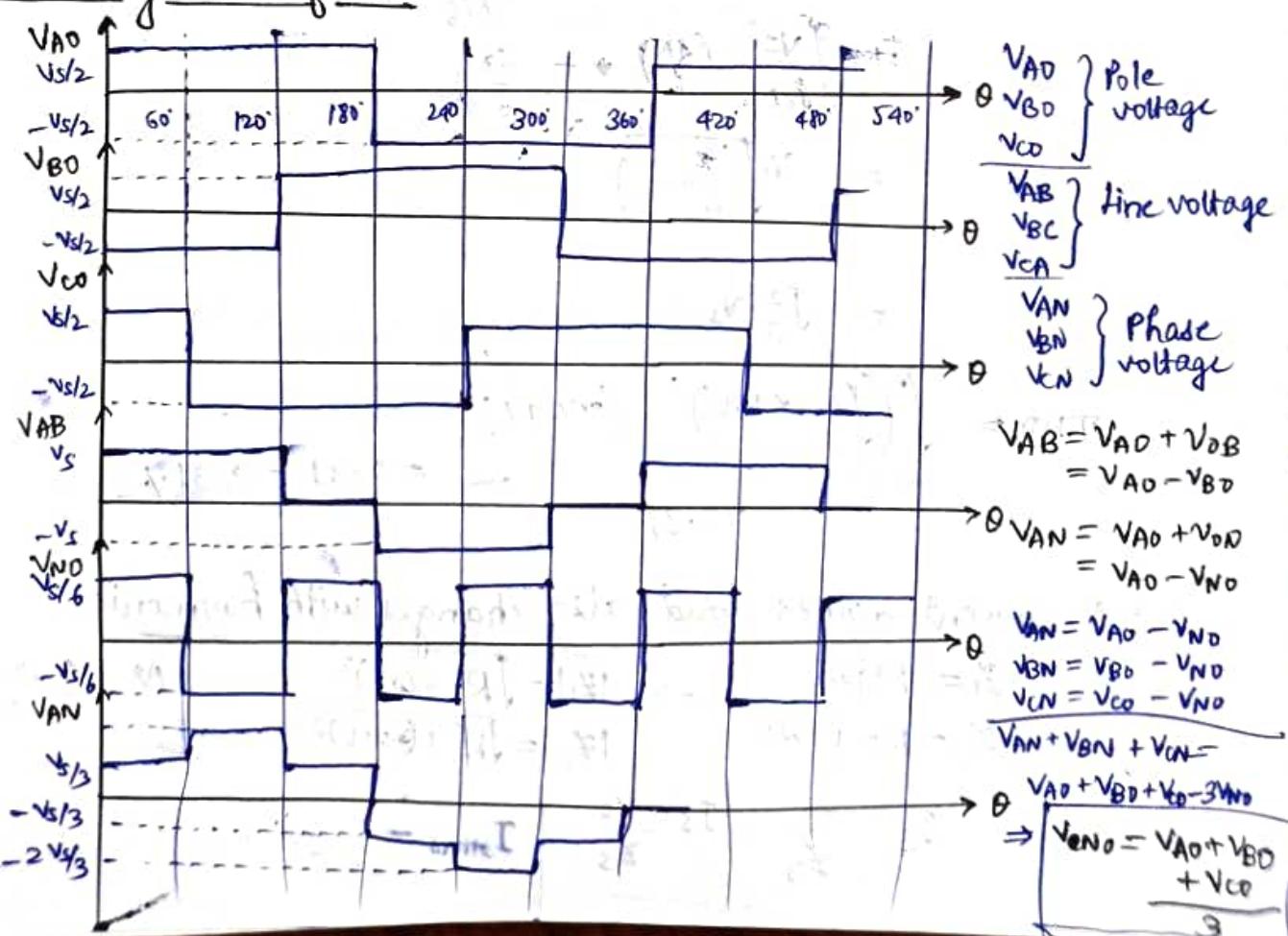


Three Phase Voltage Source Inverter

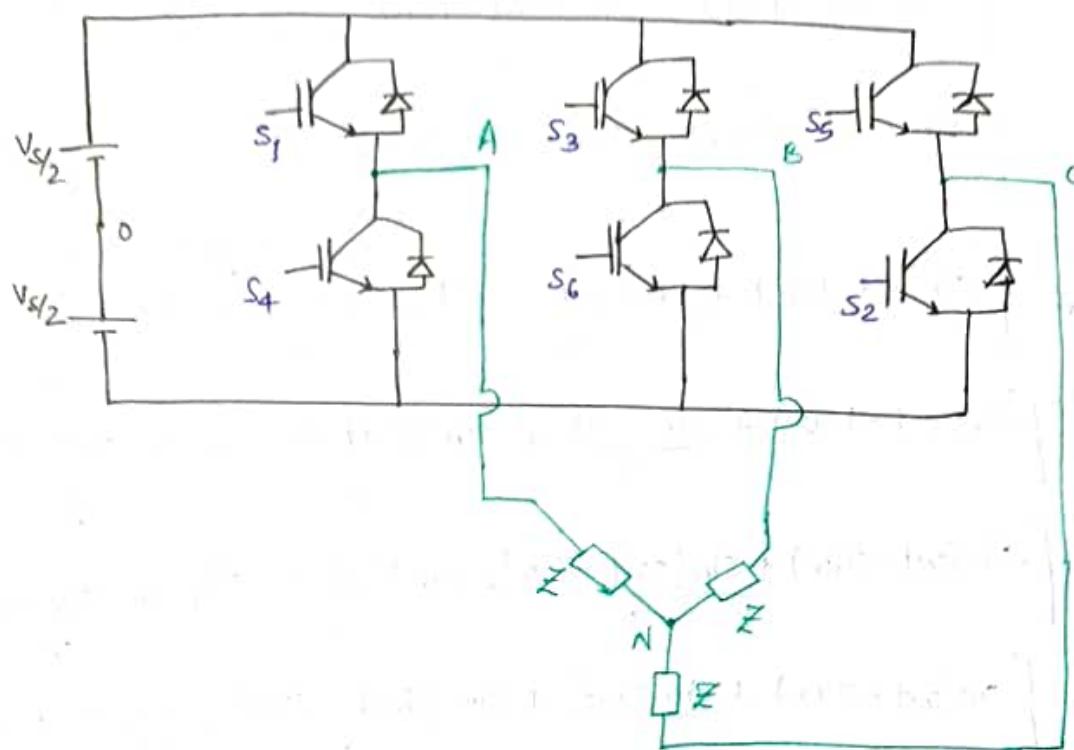


→ 3-phase system will increase power density.
 ↳ Higher efficiency
 ↳ Constant DC voltage.

Pole Voltage Waveform:



Three Phase Voltage Source Inverters



Pole voltages: V_{AO} , V_{BO} , V_{CO}

Line voltages: V_{AB} , V_{BC} , V_{CA}

Phase voltages: V_{AN} , V_{BN} , V_{CN}

$$V_{AN} = V_{AO} - V_{NO}$$

$$V_{BN} = V_{BO} - V_{NO}$$

$$V_{CN} = V_{CO} - V_{NO}$$

Common mode voltage, $V_{NO} = \frac{V_{AO} + V_{BO} + V_{CO}}{3}$

→ No phase shift in cm.v.



It cannot circulate current

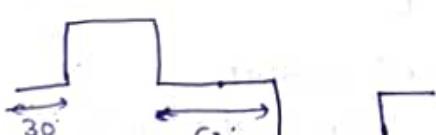
Will circulate if N is connected to ground



Huge current shall not be connected (Useless current; only adds to loss)

In 3-phase circuit, 3rd harmonic won't be present.

Fundamental component (3 sine waves, phase shifted by 120°) won't flow through N



$$V_{AO} = \frac{4V_s/2}{\pi} \left[\sin \omega t + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \frac{\sin 7\omega t}{7} + \dots \right]$$

$$V_{BO} = \frac{4V_s/2}{\pi} \left[\sin(\omega t - 120^\circ) + \frac{\sin 3(\omega t - 120^\circ)}{3} + \frac{\sin 5(\omega t - 120^\circ)}{5} + \dots \right]$$

$$V_{CO} = \frac{4V_s/2}{\pi} \left[\sin(\omega t - 240^\circ) + \frac{\sin 3(\omega t - 240^\circ)}{3} + \frac{\sin 5(\omega t - 240^\circ)}{5} + \dots \right]$$

$$\Rightarrow V_{BO} = \frac{4V_s/2}{\pi} \left[\sin(\omega t - 120^\circ) + \frac{\sin(3\omega t - 360^\circ)}{3} + \frac{\sin(\omega t - 600^\circ)}{5} + \frac{\sin(7\omega t - 840^\circ)}{7} + \dots \right]$$

$$= \frac{4V_s/2}{\pi} \left[\sin(\omega t - 120^\circ) + \frac{\sin 3\omega t}{3} + \frac{\sin(5\omega t - 240^\circ)}{5} + \frac{\sin(7\omega t - 120^\circ)}{7} + \dots \right]$$

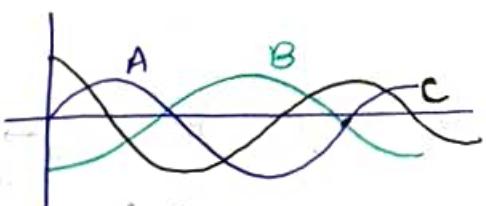
$$V_{CO} = \frac{4V_s/2}{\pi} \left[\sin(\omega t - 240^\circ) + \frac{\sin(3\omega t - 720^\circ)}{3} + \frac{\sin(5\omega t - 1200^\circ)}{5} + \frac{\sin(7\omega t - 1680^\circ)}{7} + \dots \right]$$

$$= \frac{4V_s/2}{\pi} \left[\sin(\omega t - 240^\circ) + \frac{\sin 3\omega t}{3} + \sin \frac{(5\omega t - 120^\circ)}{5} + \frac{\sin(7\omega t - 240^\circ)}{7} + \dots \right]$$

(08-10-2024)

Line voltage, $V_{AB} = V_{AO} - V_{BO}$ \rightarrow no 3rd or any odd harmonics

Phase voltage, $V_{AN} = V_{AO} - \underbrace{V_{BO}}_{3\text{rd harmonic}}$



Possible phase sequences:

A B C

A C B

Fundamental phase sequence : ABC $\rightarrow \left\{ \begin{array}{l} \sin \omega t \\ \sin(\omega t - 120^\circ) \\ \sin(\omega t - 240^\circ) \end{array} \right\}$ +ve phase sequence

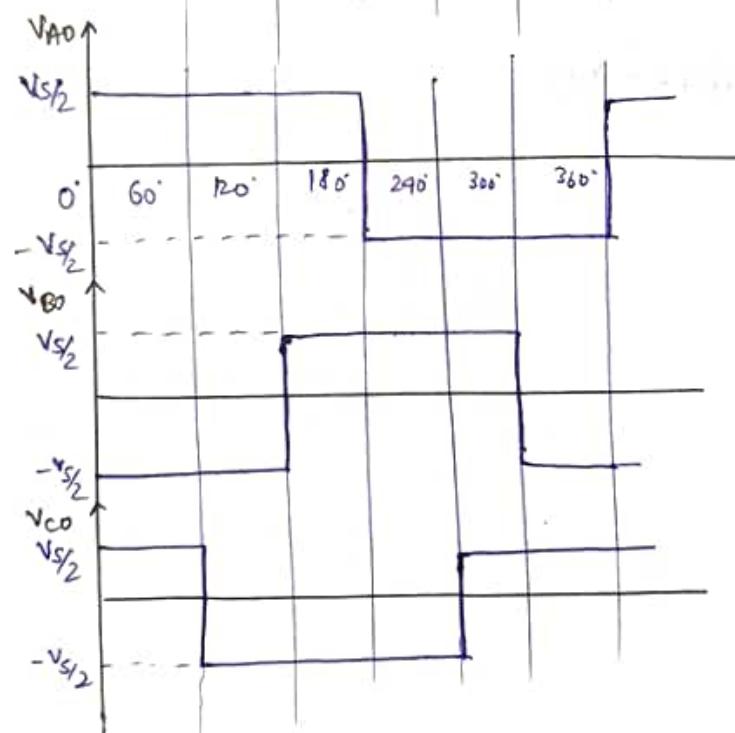
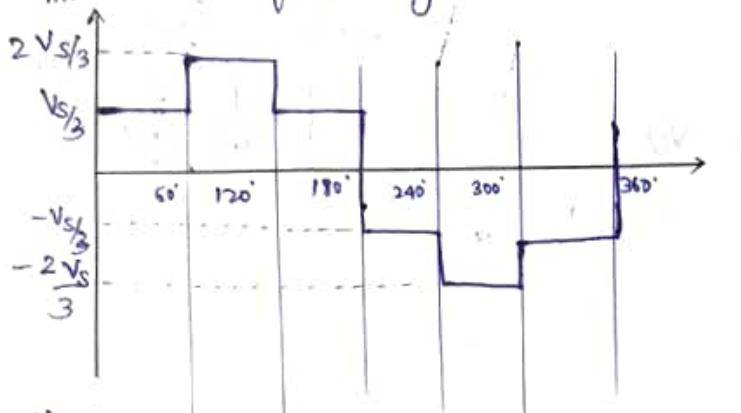
5th harmonic phase sequence : ACB $\rightarrow \left\{ \begin{array}{l} \sin 5\omega t \\ \sin(5\omega t - 240^\circ) \\ \sin(5\omega t - 120^\circ) \end{array} \right\}$ -ve phase sequence

7th harmonic $\rightarrow \left\{ \begin{array}{l} \sin 7\omega t \\ \sin(7\omega t - 120^\circ) \\ \sin(7\omega t - 240^\circ) \end{array} \right\}$ +ve sequence

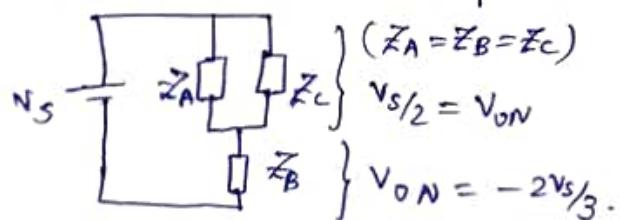
Harmonics: $6n+1 \rightarrow +ve$ sequence
 $6n-1 \rightarrow -ve$ sequence. $[n=1, 2, \dots]$

Feeding this voltage to a motor, fundamental component rotates it in one dirn, 5th harmonic rotates it in opposite direction : Torque pulsation.

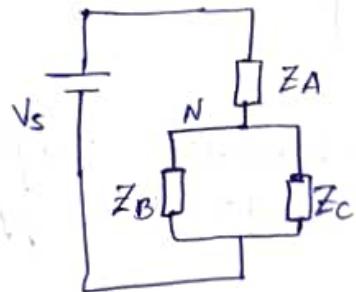
Effective frequency in ACB : 6ω .



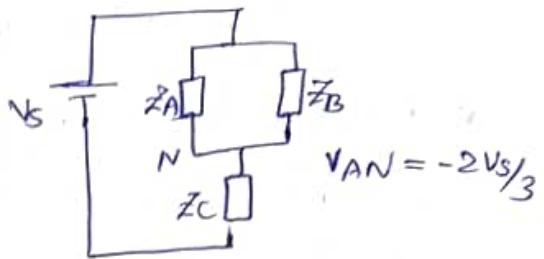
0 - 60° : $S_1 \rightarrow ON, S_4 \rightarrow OFF | S_3 \rightarrow OFF, S_6 \rightarrow ON | S_5 \rightarrow ON, S_2 \rightarrow OFF$.



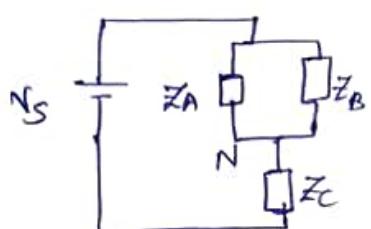
$60^\circ - 120^\circ$



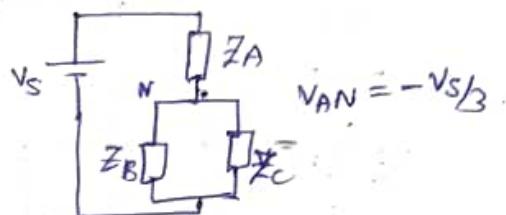
$240^\circ - 300^\circ$



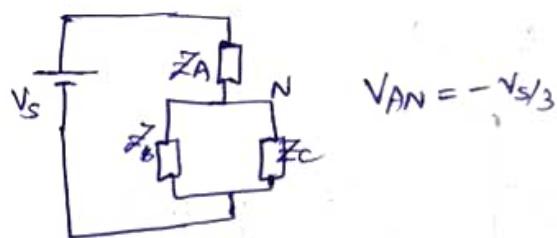
$120^\circ - 180^\circ$



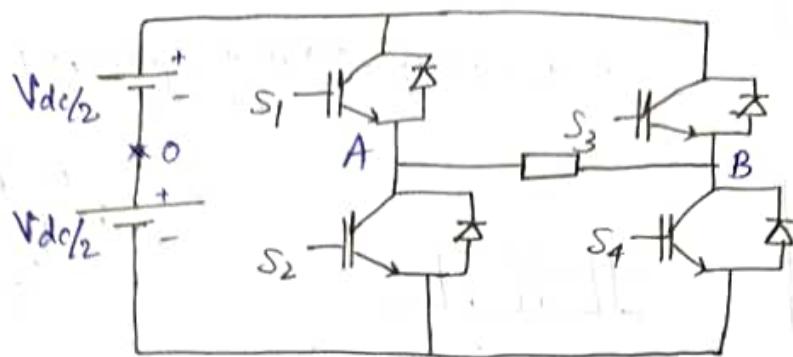
$300^\circ - 360^\circ$



$180^\circ - 240^\circ$

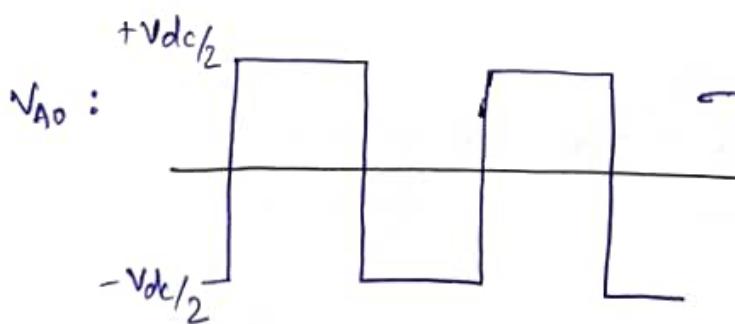
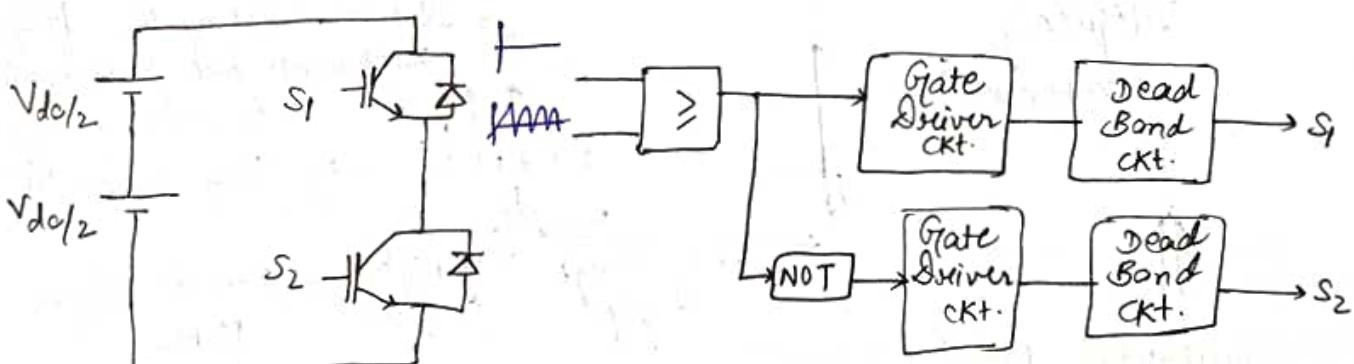
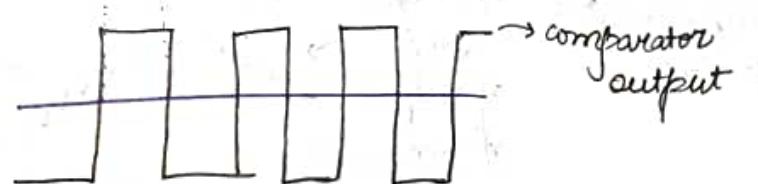
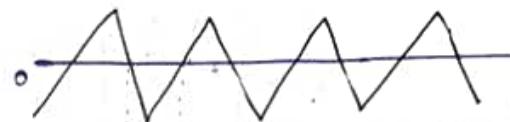
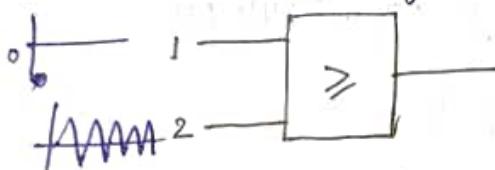


PWM Techniques for Voltage Source Inverter



Comparator:

→ Analog comparator: an opamp

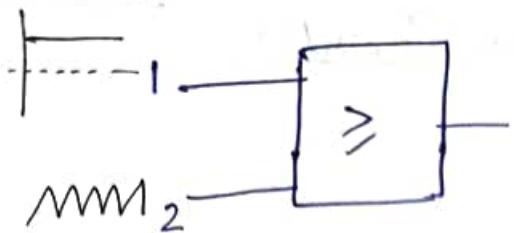


→ Frequency same as freq. of

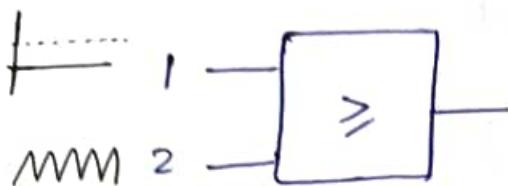
= fundamental frequency of sinusoids.

↳ Average value = 0.

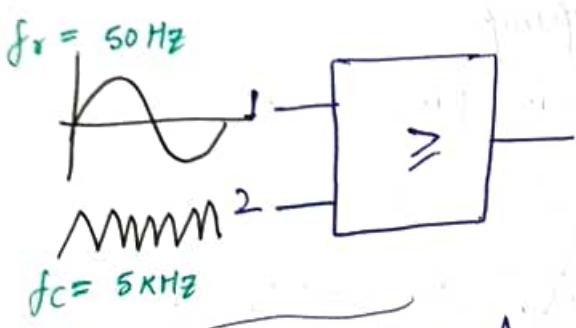
↳ odd harmonics.



→ Rectangular \rightarrow positive average value

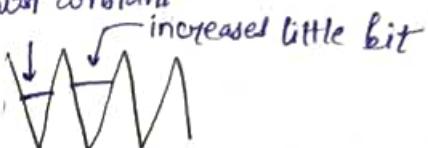


No change in frequency component



→ negative average value \rightarrow No change in frequency component.

almost constant



Average value of output of comparator



Sinusoid steps

Frequency components:



Not eliminating the frequency components but pushing them to a high value so that they can be eliminated

very large gap very high frequencies

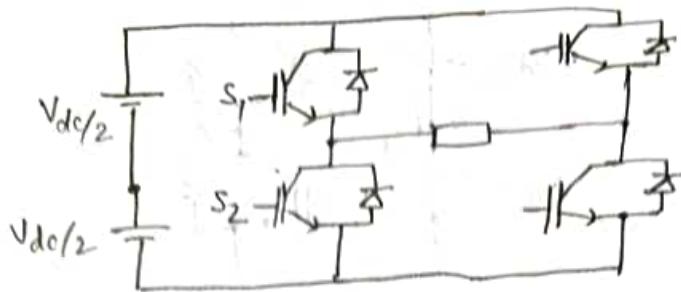
f_c \rightarrow Easy to design filter

Sinusoidal PWM (SPWM)

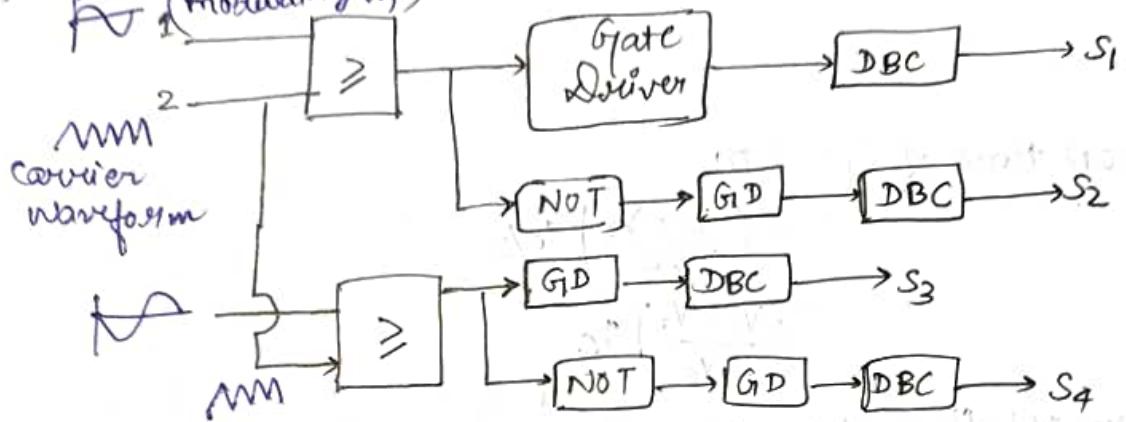


→ smaller gap \rightarrow minimum THD

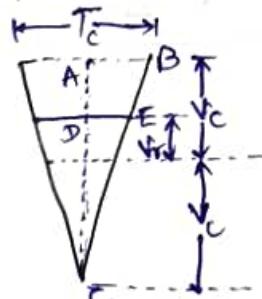
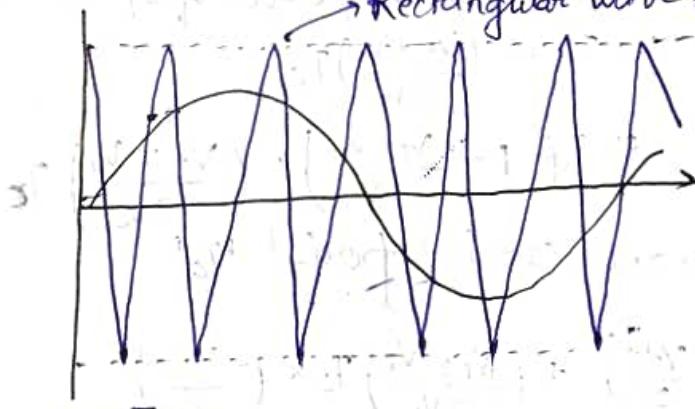
Sinusoidal PWM



Reference waveform
(modulating wf)



Rectangular wave but changing sinusoidally



f_c : frequency of triangular carrier waveform

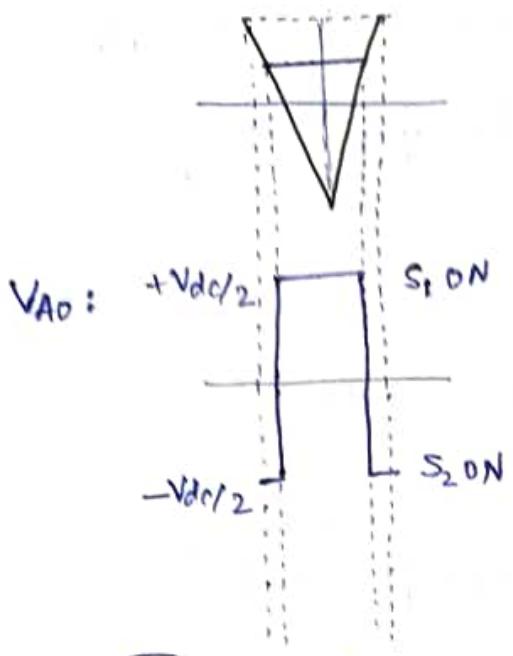
$T_0 = \frac{1}{f_c}$: Period of triangular waveform

Consider similar Δ ABC & DEC

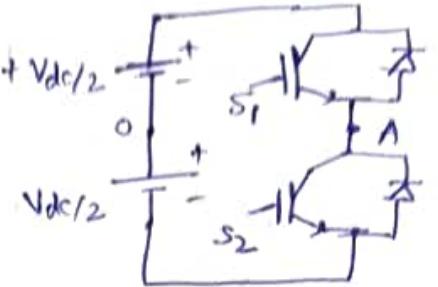
$$\frac{DE}{AB} = \frac{DC}{AC}$$

$$\Rightarrow \frac{DE}{AB} = \frac{V_c + V_r}{2V_c}$$

$$\Rightarrow DE = \left(\frac{V_c + V_r}{2V_c} \right) AB = \left(\frac{V_c + V_r}{2V_c} \right) T_0 / 2$$



$$VAO: \quad +V_{dc}/2 \quad S_1 \text{ ON} \\ -V_{dc}/2 \quad S_2 \text{ ON}$$



Turn ON time of $S_1 = 2\Delta E$

$$= 2 \left(\frac{V_c + V_r}{2V_c} \right) T_c/2 \\ = \left(\frac{V_c + V_r}{2V_c} \right) T_c$$

Turn ON time of $S_2 = T_c - \text{Turn-on period of } S_1$

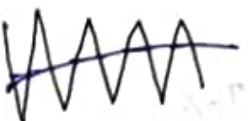
$$= T_c - \left(\frac{V_c + V_r}{2V_c} \right) T_c \\ = T_c \left(1 - \frac{V_c + V_r}{2V_c} \right) = \left(\frac{V_c - V_r}{2V_c} \right) T_c$$

Average pole voltage in one carrier period 'Tc'

$$= \overline{\left(\frac{V_c + V_r}{2V_c} \right) T_c \times \left(\frac{V_{dc}}{2} \right) + \left(\frac{V_c - V_r}{2V_c} \right) T_c \times \left(-\frac{V_{dc}}{2} \right)} \\ = \frac{V_{dc}}{2} \left[\left(\frac{V_c + V_r}{2V_c} \right) - \left(\frac{V_c - V_r}{2V_c} \right) \right]$$

$$VAO (\text{av.}) = \left(\frac{2V_r}{2V_c} \right) \frac{V_{dc}}{2} = \left(\frac{V_r}{V_c} \right) \frac{V_{dc}}{2}$$

$$VAO (\text{av.}) \propto V_r \\ \propto V_m \sin \omega t$$



carrier frequency \uparrow

\Rightarrow Output: more sinusoidal
gap \uparrow \Rightarrow very high
reactance
no impact

$$\text{Amplitude Modulation Ratio} = \frac{\text{Peak value of reference sine waveform}}{\text{Peak value of triangular carrier waveform.}}$$

(Index) [M]



when sine wave touched the peak \Rightarrow Modulation index = 1.

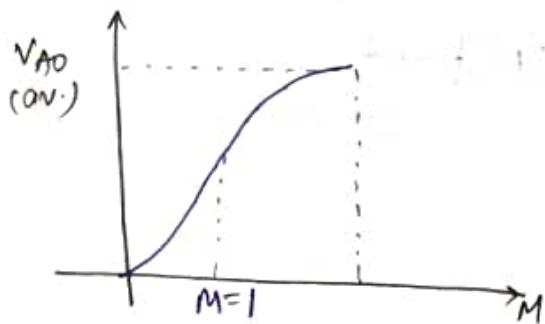
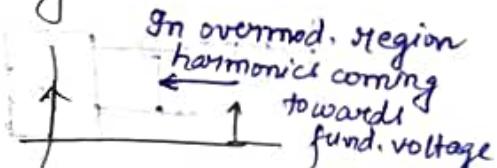
$$\text{Frequency Modulation Ratio} = \frac{\text{Frequency of carrier wave}}{\text{Frequency of reference wave}}$$

(Index)

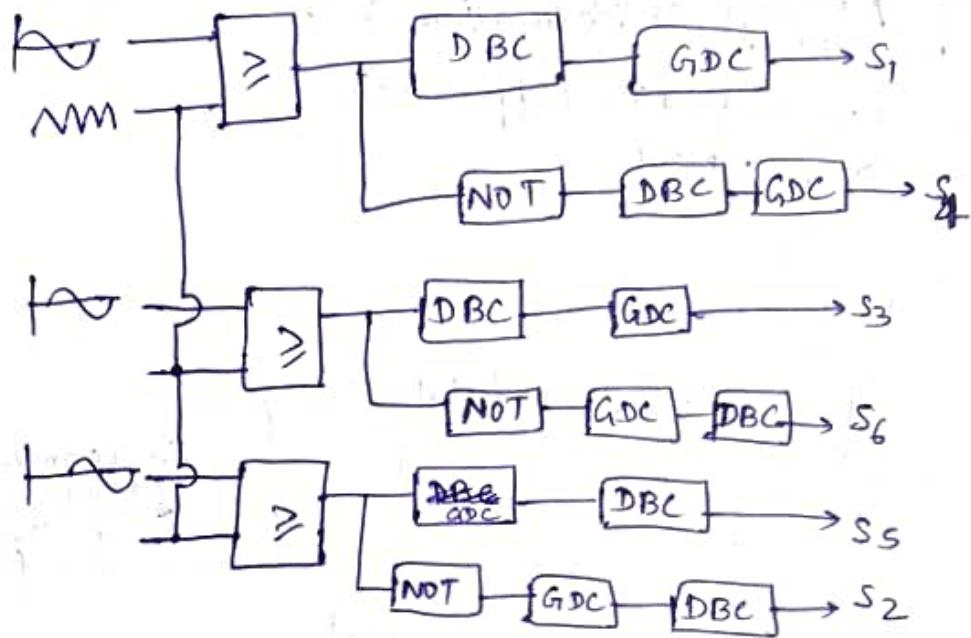
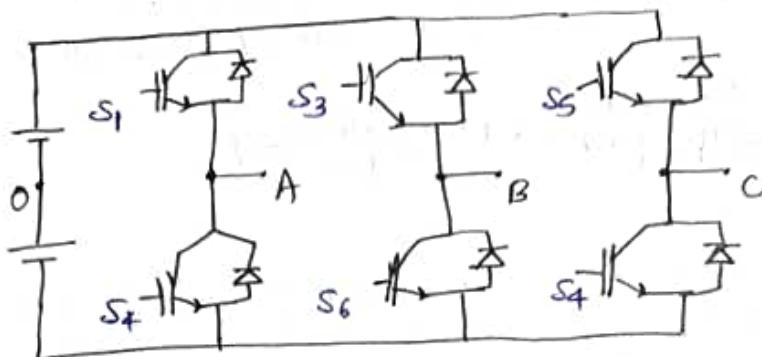
Avg. value of o/p voltage \propto Modulation index.



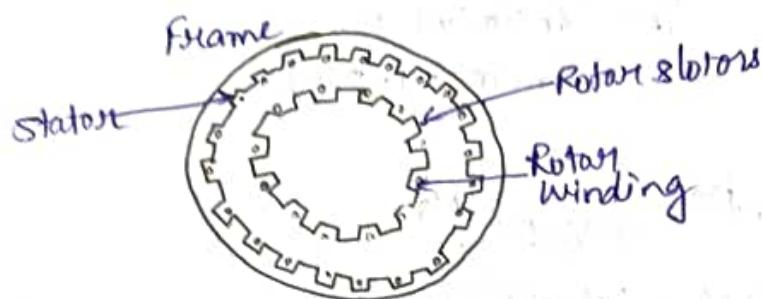
: Overmodulation Region



Sinusoidal PWM :



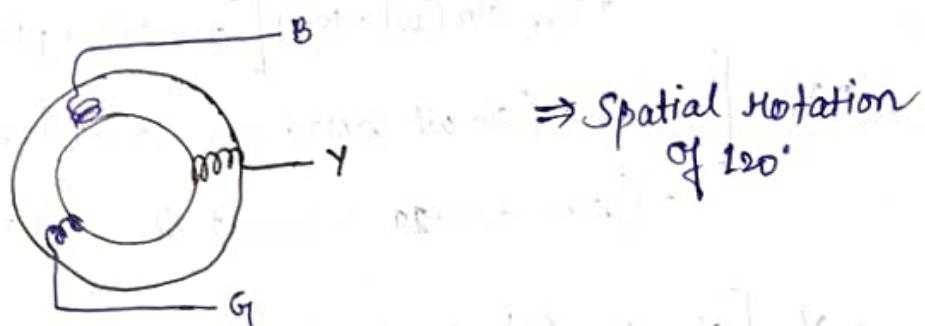
Constructional Features of 3-phase Induction Motor (IM)



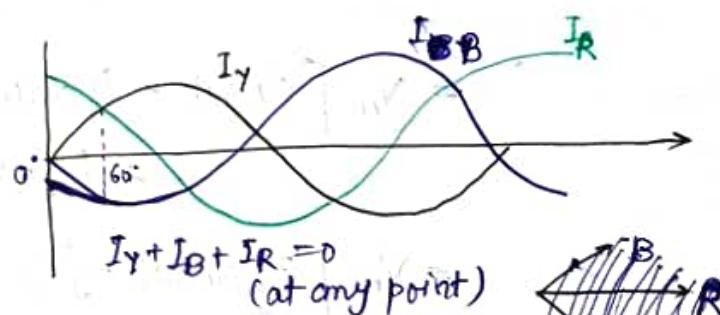
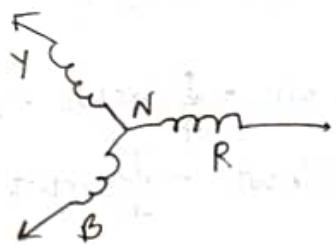
Stationary part : stator
Moving part : Rotor

↓
Produce
2 magnetic
fields
→
Cause
rotation

3-phase IM: Orientation of windings:

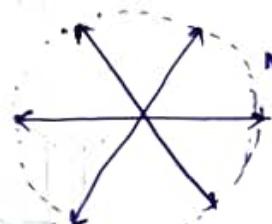
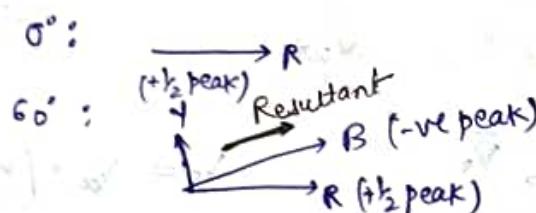


Generation of Rotating Magnetic Field:



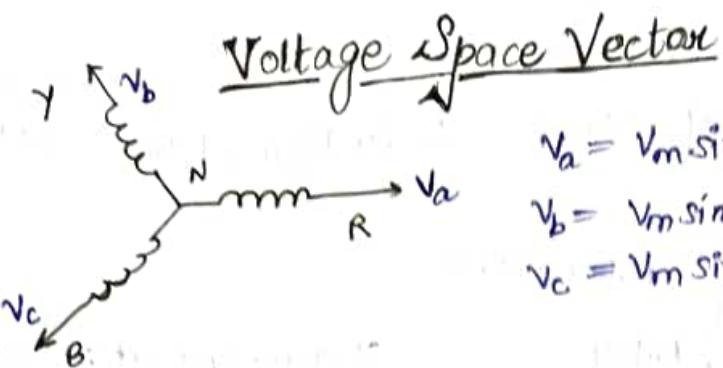
$$\text{Flux} = \frac{\text{Magneto Motive Force}}{\text{Reluctance}} = \frac{MMF}{S} = \frac{NI}{S}$$

Flux $\propto I$ \Rightarrow opposition to flux



⇒ Rotating magnetic field / flux vector

[Due to voltage vector rotating] \Rightarrow [due to current space vector rotating in space]



$$V_a = V_m \sin \omega t$$

$$V_b = V_m \sin(\omega t - 120^\circ)$$

$$V_c = V_m \sin(\omega t - 240^\circ)$$

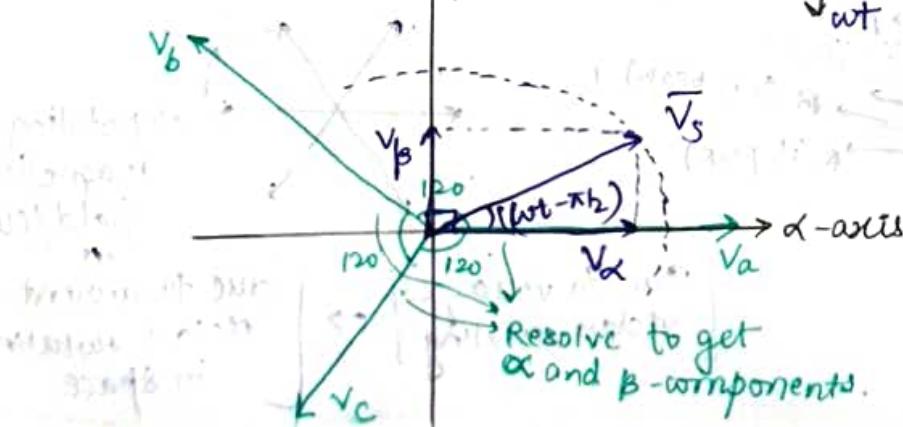
$$\begin{aligned} \bar{V}_s &= V_a e^{j0^\circ} + V_b e^{j2\pi/3} + V_c e^{j4\pi/3} \\ &= V_a + V_b [\cos 2\pi/3 + j \sin 2\pi/3] + V_c [\cos 4\pi/3 + j \sin 4\pi/3] \\ &= V_m \sin \omega t + V_m \sin(\omega t - 120^\circ) [\cos 2\pi/3 + j \sin 2\pi/3] \\ &\quad + V_m \sin(\omega t - 240^\circ) [\cos 4\pi/3 + j \sin 4\pi/3] \\ &= V_m \left[\sin \omega t + (\sin \omega t \cos 120^\circ - \cos \omega t \sin 120^\circ) (-0.5 + j\sqrt{3}/2) \right. \\ &\quad \left. + (\sin \omega t \cos 240^\circ - \cos \omega t \sin 240^\circ) (-0.5 - j\sqrt{3}/2) \right] \end{aligned}$$

(21-10-2024)

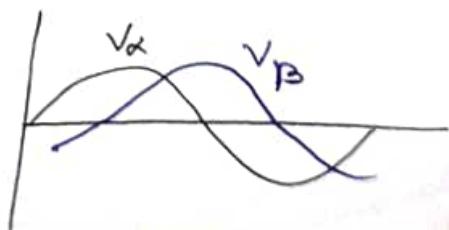
$$\begin{aligned} &= V_m \left[\sin \omega t + \left(-\frac{1}{2} \sin \omega t - \frac{\sqrt{3}}{2} \cos \omega t \right) \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right. \\ &\quad \left. + \left(-\frac{1}{2} \sin \omega t + \frac{\sqrt{3}}{2} \cos \omega t \right) \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right] \\ &= V_m \left[\sin \omega t + \frac{1}{4} \sin \omega t - j\frac{\sqrt{3}}{4} \sin \omega t + \frac{\sqrt{3}}{4} \cos \omega t - j\frac{3}{4} \cos \omega t \right. \\ &\quad \left. + \frac{1}{4} \sin \omega t + j\frac{\sqrt{3}}{4} \sin \omega t - \frac{\sqrt{3}}{4} \cos \omega t - j\frac{3}{4} \cos \omega t \right] \\ &= V_m \left[\frac{3}{2} \sin \omega t - j\frac{3}{2} \cos \omega t \right] \\ &= \frac{3}{2} V_m \left[\sin \omega t - j \cos \omega t \right] \\ &= \frac{3}{2} V_m \left[\cos(\omega t - \pi/2) + j \sin(\omega t - \pi/2) \right] \end{aligned}$$

$$\therefore \bar{V}_s = \frac{3}{2} V_m e^{j(\omega t - \pi/2)}$$

$$\begin{array}{l} \text{wt} = \pi/2 \\ \text{wt} \\ \hline a + jb \end{array}$$



$$\bar{V}_s = V_\alpha + j V_\beta$$



Two windings of spatial difference of 90° and phase difference of 90° would create a ^{rotating} voltage vector, or vice-versa.



$$V_\alpha = V_a - V_b \cos 60^\circ - V_c \cos 60^\circ = V_a - \frac{1}{2}V_b - \frac{1}{2}V_c$$

$$V_\beta = 0V_a + V_b \cos 30^\circ - V_c \cos 30^\circ = 0 + \frac{\sqrt{3}}{2}V_b - \frac{\sqrt{3}}{2}V_c$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

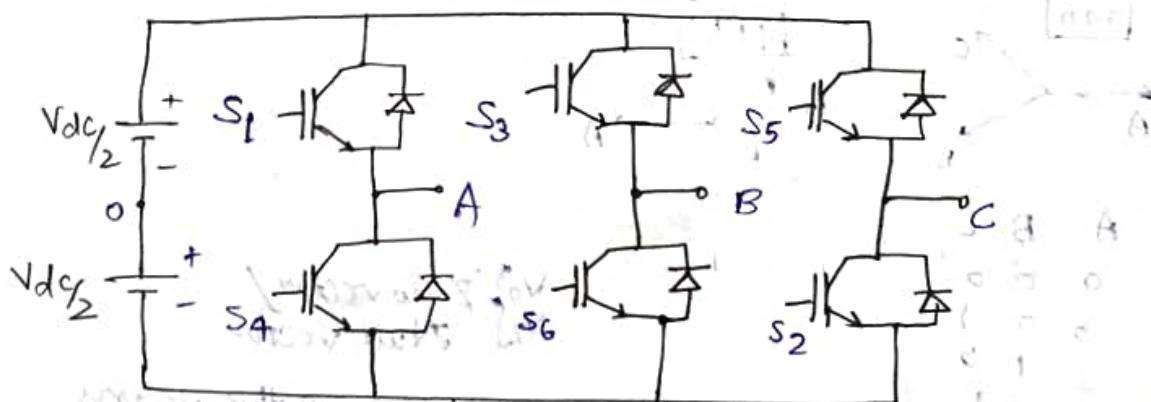
→ Clavke's Transformation

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

3 phase to 2-phase transformation

→ Three windings of spatial difference of 120° and phase difference of 120° would create a ~~rotating~~ voltage vector, or if we create a rotating voltage vector, we would get 3 sine waves of 120° of phase difference.

Space Vector PWM



$$S_1 \text{ ON} \rightarrow V_{AO} = +V_{dc}/2$$

$$S_5 \text{ ON} \rightarrow V_{CO} = +V_{dc}/2$$

$$S_1 \text{ OFF} \rightarrow V_{AO} = -V_{dc}/2$$

$$S_5 \text{ OFF} \rightarrow V_{CO} = -V_{dc}/2$$

$$S_3 \text{ ON} \rightarrow V_{BO} = +V_{dc}/2$$

3 legs of 2 states possible

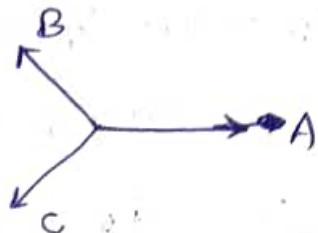
$$S_3 \text{ OFF} \rightarrow V_{BO} = -V_{dc}/2$$

Total 2^3 states = 8.

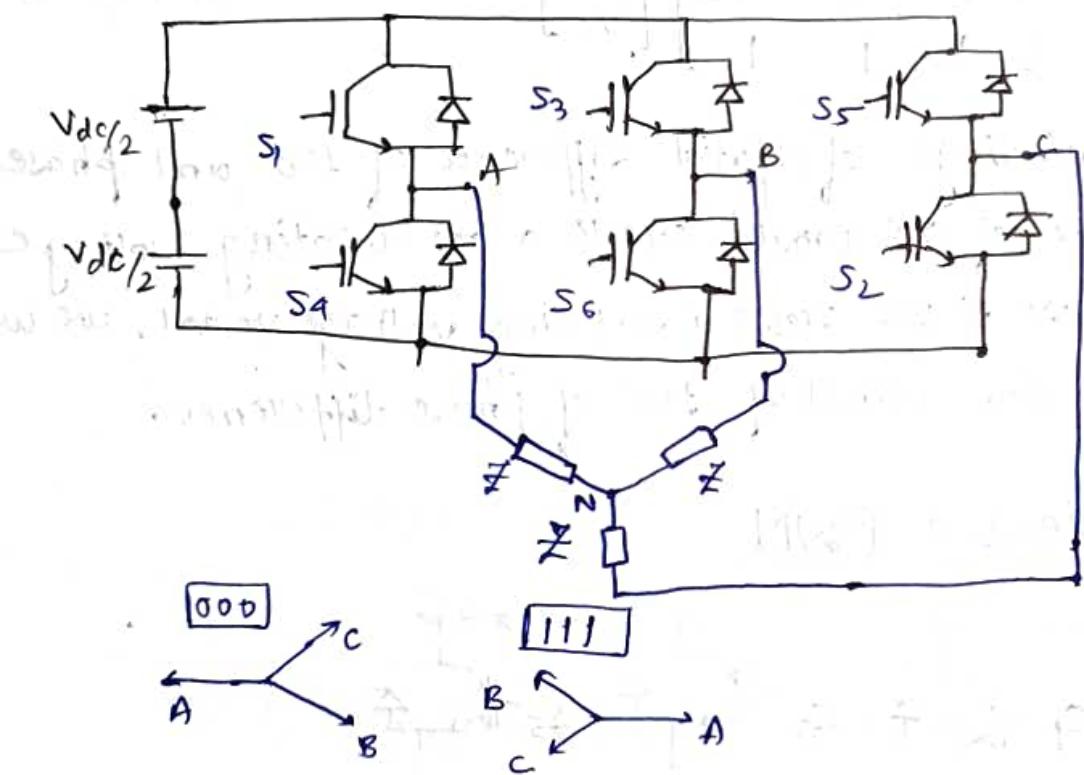
1 : ON-state of top switch in a leg of the inverter.
 0 : OFF-state "

A B C [8 switching states]

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



22-10-2024



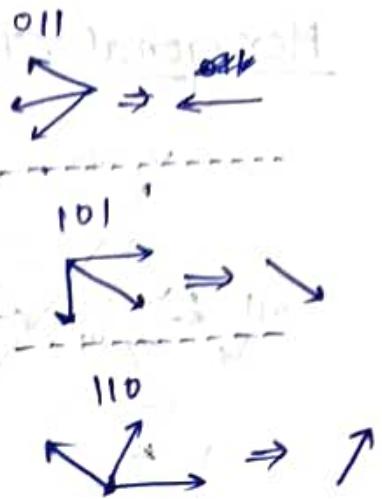
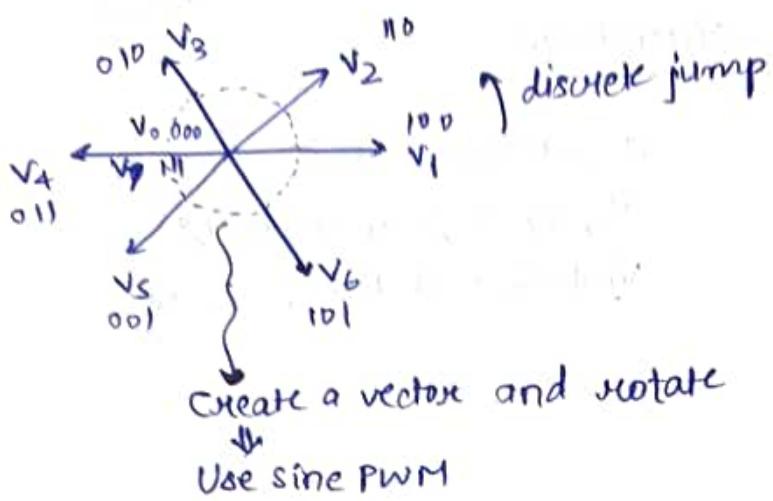
A B C

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

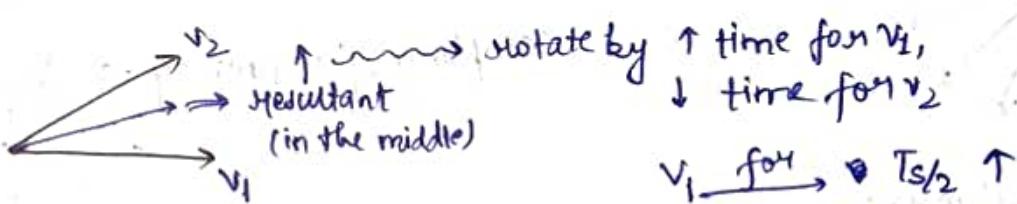
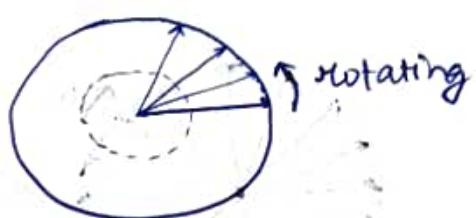
V_{00} Zero vector/
 V_1 Null vector

Active
vector

V_1 to $V_S \rightarrow$ Active vectors

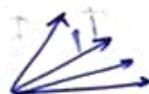


Sample the sine wave



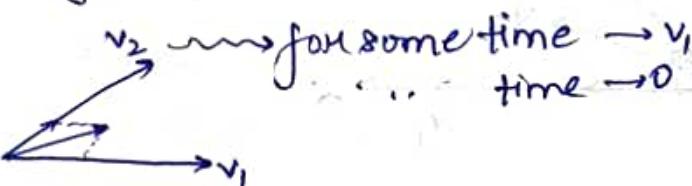
$$v_1 \xrightarrow{\text{for } T_{s/2}} \uparrow \\ v_2 \xrightarrow{\text{for } T_{s/2}} \downarrow$$

$$v_1 \rightarrow T_1' \\ v_2 \rightarrow T_2' \\ T_1' + T_2' = T_s$$



Also change the magnitude

V8e zero vector

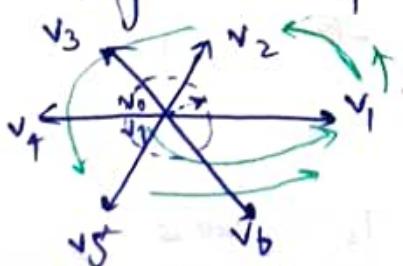


Volt - sec. Balance

$$V_b T_s = V_1 T_1 + V_2 T_2 + V_0 T_0$$

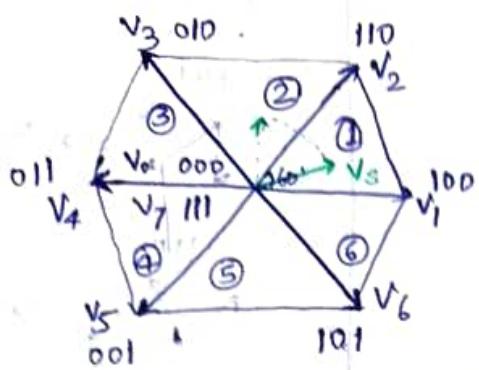
$$T_1 + T_2 + T_0 = T_s$$

Infinite sampling frequency → Smooth rotation



$$V_0 \rightarrow v_1 \rightarrow v_2 \rightarrow v_3$$

Hexagonal Space Vector Structure

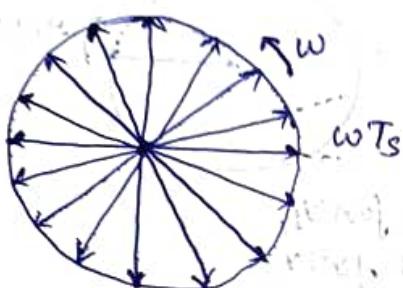


6 sectors: ① to ⑥

v_0, v_7 : Zero vectors

v_1 to v_6 : Active vectors

Reference line vector v_s



Use 3 nearest vectors to construct the vector

Min^m harmonic destruction

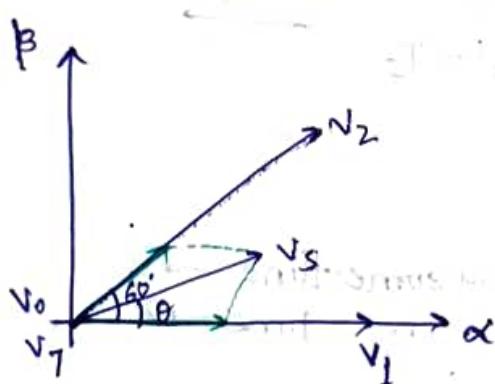
Sampling frequency \Rightarrow Smooth rotation

3 sine waves in 3 phases

Volt-sec Balance:

$$v_1 T_1 + v_2 T_2 + v_3 T_3 = v_s T_s$$

$$T_1 + T_2 + T_3 = T_s$$



[v_2 has to be applied for a longer time]

Apply Volt-sec Balance along α -axis

$$v_1 T_1 + v_2 \cos 60^\circ T_2 = v_s \cos \theta T_s \quad \text{--- (1)}$$

Volt-sec balance along β -axis:

$$0 + v_2 \sin 60^\circ T_2 = v_s \sin \theta T_s \quad \text{--- (2)}$$

$$\Rightarrow T_2 = T_s \cdot \frac{v_s \sin \theta}{\sin 60^\circ} \quad \text{--- (3)}$$

$$\text{--- (3), (1)} \Rightarrow v_1 T_1 + v_2 \cos 60^\circ \cdot \frac{v_s \sin \theta}{\sin 60^\circ} T_s = v_s \cos \theta T_s$$

$$\Rightarrow V_1 T_1 = T_s V_s \cos \theta - \frac{V_s \cos 60^\circ \sin \theta T_s}{\sin 60^\circ}$$

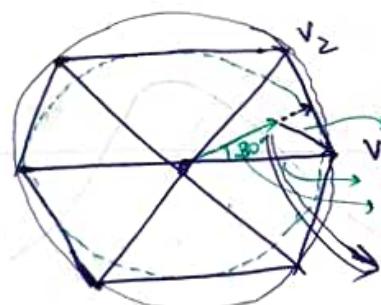
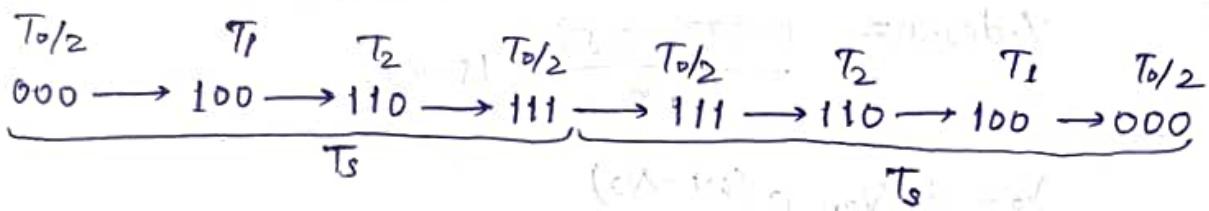
$$= T_s V_s \left[\frac{\sin 60^\circ \cos \theta - \frac{1}{2} \cos 60^\circ \sin \theta}{\sin 60^\circ} \right]$$

$$\Rightarrow T_1 = \frac{T_s V_s \sin(60^\circ - \theta)}{V_s \sin 60^\circ}$$

$$T_1 = T_s \cdot \frac{V_s \sin(60^\circ - \theta)}{V_{dc} \sin 60^\circ}$$

$$T_2 = T_s \cdot \frac{V_s \sin \theta}{V_{dc} \sin 60^\circ}$$

$$T_0 = T_s - (T_1 + T_2)$$



Maximum magnitude realizable is given by the radius of the circle inscribed inside the hexagon.

$$\begin{aligned} \text{Maxim } V_s &= V_{dc} \cos 30^\circ \\ &= V_{dc} \cdot \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{bmatrix} V_d \\ V_p \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$V_d = V_{an} - \frac{1}{2} V_{bn} - \frac{1}{2} V_{cn}$$

$$= V_{an} - \frac{1}{2} (V_{bn} + V_{cn}) = V_{an} - \frac{1}{2} (-V_{an}) = \frac{3}{2} V_{an}$$

$$\begin{cases} V_{an} + V_{bn} + V_{cn} = 0 \\ (V_{bn} + V_{cn}) = -V_{an} \end{cases}$$

$$V_d = \frac{3}{2} V_{an}$$

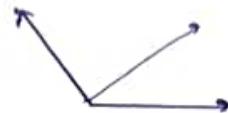
$$\Rightarrow V_{an} = \frac{2}{3} V_d$$

Max. m:

$$V_{A0(\max)} = \frac{\text{Max. mag.}}{V_d/2}$$

$$V_{AB} = V_{A0} - V_{B0} = \sqrt{3} V_{dc}/2$$

$$V_{A0} = \frac{V_{AB}}{\sqrt{3}}$$



$$V_{an(\max)} = \frac{\sqrt{3} V_{dc}/2}{\sqrt{3}} = V_{dc}/2$$

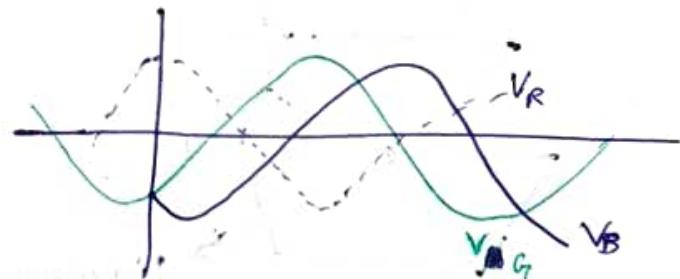
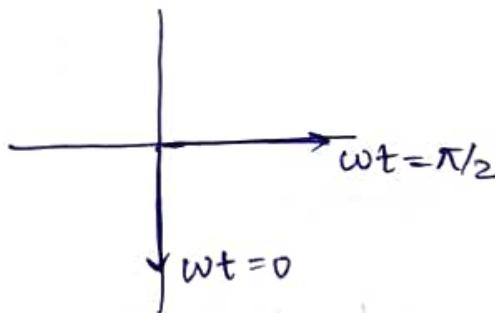
$$= 0.5 V_{dc}$$

$$\text{Max. mag. of } V_{an} = \frac{2}{3} \cdot V_{dc} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3} V_{dc}}{3}$$

$$= 0.577 V_{dc}$$

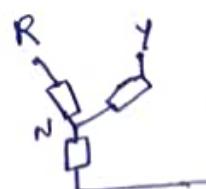
$$\% \text{ deviation} = \frac{(0.577 - 0.5)}{0.5} = 15.4\%$$

$$V_o = \frac{3}{2} V_{an} e^{(wt - \pi/2)}$$

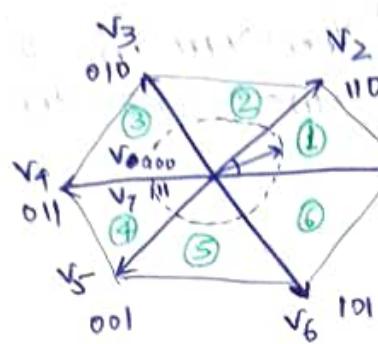


$$V_R > V_Y > V_B : \text{sector-1}$$

$$V_Y > V_R > V_B : \text{sector-2}$$

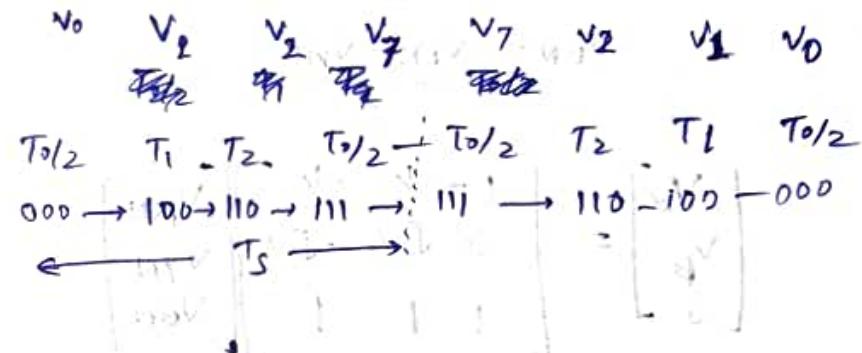
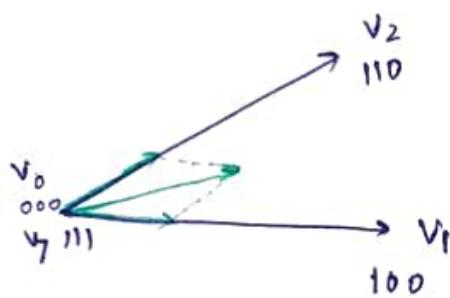


\Leftarrow Conventional SVPWM



Dwell time : T_1, T_2, T_0 ; $T_S = T_0 + T_1 + T_2$

$$T_0 = T_S - (T_1 + T_2)$$



Sector ① : $V_R > V_Y > V_B$

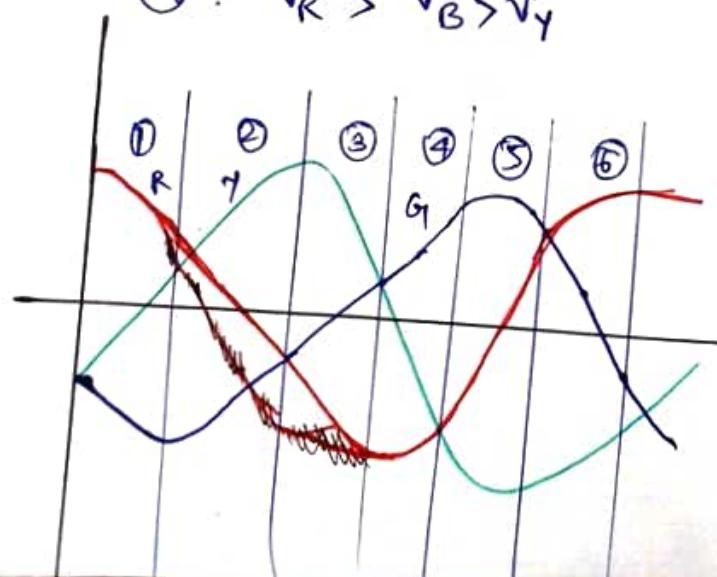
② : $V_Y > V_R > V_B$

③ : $V_Y > V_B > V_R$

④ : $V_B > V_Y > V_R$

⑤ : $V_B > V_R > V_Y$

⑥ : $V_R > V_B > V_Y$



Phase voltages: V_{RN} , V_{YN} , V_{BN}

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} - V_{YN}$$

$$V_{YB} = V_{YN} - V_{BN}$$

$$V_{BR} = V_{BN} - V_{RN}$$

$$\begin{bmatrix} V_{RN} + V_{YN} + V_{BN} = 0 \\ V_{YN} + V_{BN} = -V_{RN} \end{bmatrix}$$

$$\begin{aligned} V_{RY} - V_{BR} &= V_{RN} - V_{YN} - (V_{BN} - V_{RN}) \\ &= 2V_{RN} - (V_{YN} + V_{BN}) \\ &= 2V_{RN} - (-V_{RN}) = 3V_{RN} \end{aligned}$$

$$V_{RN} = \frac{V_{RY} - V_{BR}}{3}$$

$$V_{YN} = \frac{V_{YB} - V_{RY}}{3}$$

$$V_{BN} = \frac{V_{BR} - V_{YB}}{3}$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_{RN} \\ V_{YN} \\ V_{BN} \end{bmatrix}$$

$$\begin{aligned} V_\alpha &= V_{RN} - \frac{1}{2}(V_{YN} + V_{BN}) \\ &= \frac{1}{2}V_{RN} \end{aligned}$$

$$V_\beta = \frac{\sqrt{3}}{2}(V_{YN} - V_{BN})$$

$$\therefore \bar{V}_s = V_\alpha + jV_\beta$$

Switching state	Pole voltages, V_{RD}, V_{YD}, V_{BD}	Line voltages	Phase voltages	V_A, V_B	\bar{V}_S
000	$V_{RD} = -V_{dc}/2$ $V_{YD} = -V_{dc}/2$ $V_{BD} = -V_{dc}/2$	$V_{RY} = 0$ $V_{YB} = 0$ $V_{BR} = 0$	$V_{RN} = 0$ $V_{YN} = 0$ $V_{BN} = 0$	$V_A = 0$ $V_B = 0$	0.60
001	$V_{RD} = -V_{dc}/2$ $V_{YD} = -V_{dc}/2$ $V_{BD} = +V_{dc}/2$	$V_{RY} = 0$ $V_{YB} = -V_{dc}$ $V_{BR} = +V_{dc}$	$V_{RN} = -V_{dc}/3$ $V_{YN} = -V_{dc}/3$ $V_{BN} = 2V_{dc}/3$	$V_A = -\frac{V_{dc}}{2}$ $V_B = \frac{\sqrt{3}V_{dc}}{2}$	$V_{dc} < 240^\circ$
110	$V_{RD} = V_{dc}/2$ $V_{YD} = V_{dc}/2$ $V_{BD} = -V_{dc}/2$	$V_{RY} = 0$ $V_{YB} = V_{dc}$ $V_{BR} = -V_{dc}$	$V_{RN} = V_{dc}/3$ $V_{YN} = V_{dc}/3$ $V_{BN} = -2V_{dc}/3$	$V_A = \frac{V_{dc}}{2}$ $V_B = \frac{\sqrt{3}V_{dc}}{2}$	

Während der 001 und 110 Zustand kann nicht
zwei Phasen gleichzeitig aufgeladen werden

zwei Phasen gleichzeitig aufgeladen

Während der 001 Zustand kann nur eine Phase aufgeladen werden
 $V_R = V_{dc}$
 $V_Y = 0$
 $V_B = 0$

Während der 110 Zustand kann nur eine Phase aufgeladen werden
 $V_R = 0$
 $V_Y = 0$
 $V_B = V_{dc}$

Stromfluss durch die Spulen ist

gleichzeitig

• eingeschaltet als + 1/3 an der

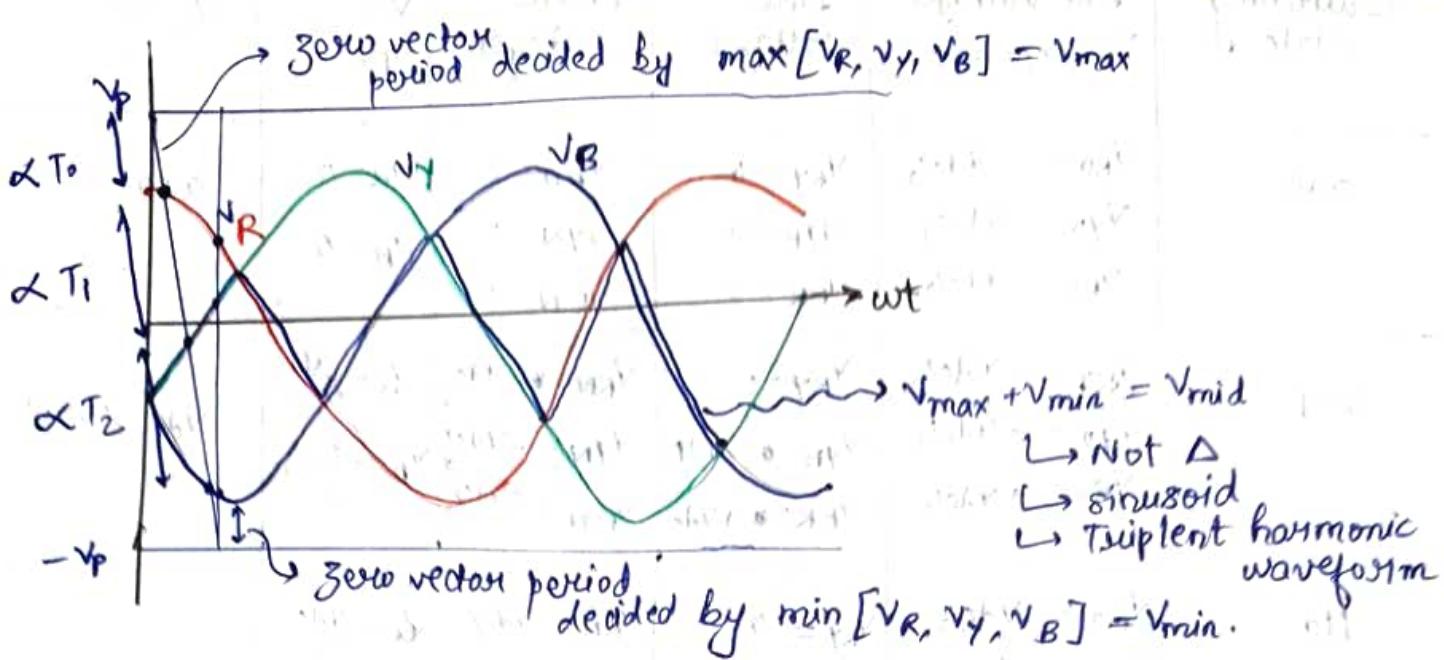
$$(V_R + V_{RN}) + (V_{RN} + V)$$

$$V + V_{RN} = \frac{2}{3}V + V$$

$$V = \frac{2}{3}V_{dc} + \frac{2}{3}V_{dc}$$

$$V = \frac{4}{3}V_{dc} + \frac{2}{3}V_{dc} + \frac{2}{3}V_{dc} + \frac{2}{3}V_{dc}$$

$$\left(\frac{4}{3}V_{dc} + \frac{2}{3}V_{dc} \right) = 2V_{dc}$$



000 - 100 - 110 - 111

111 - 110 - 100 - 000

→ Zero vectors are not divided equally in sinusoid PWM.

Adding some common voltage (offset) to all the waveforms

↳ Modified reference waveforms:

$$V_R^* = V_R + V_{CM} \rightarrow \text{common mode voltage}$$

$$V_Y^* = V_Y + V_{CM}$$

$$V_B^* = V_B + V_{CM}$$

$$V_{\max}^* = V_{\max} + V_{CM}$$

$$V_{\min}^* = V_{\min} + V_{CM}$$

↳ V_{RD}, V_{YD}, V_{BD} (pole voltages) → affected

line voltages } No effect
phase voltages }

↳ No effect on the performance.

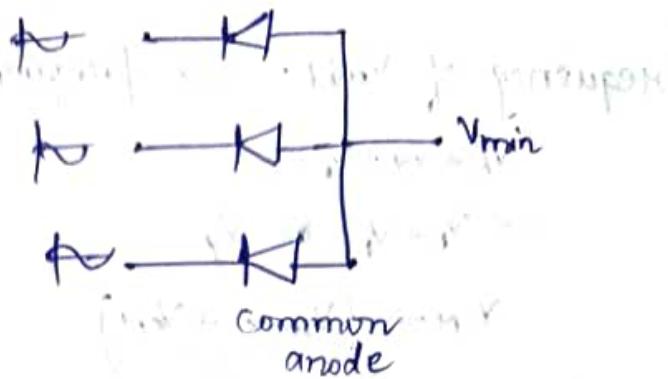
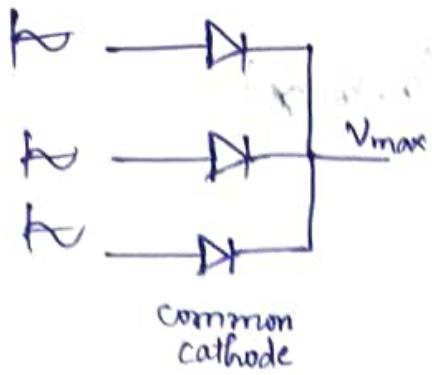
$$V_p - V_{\max}^* = V_{\min}^* - (-V_p)$$

$$\Rightarrow V_p - V_{\max}^* = V_{\min}^* + V_p$$

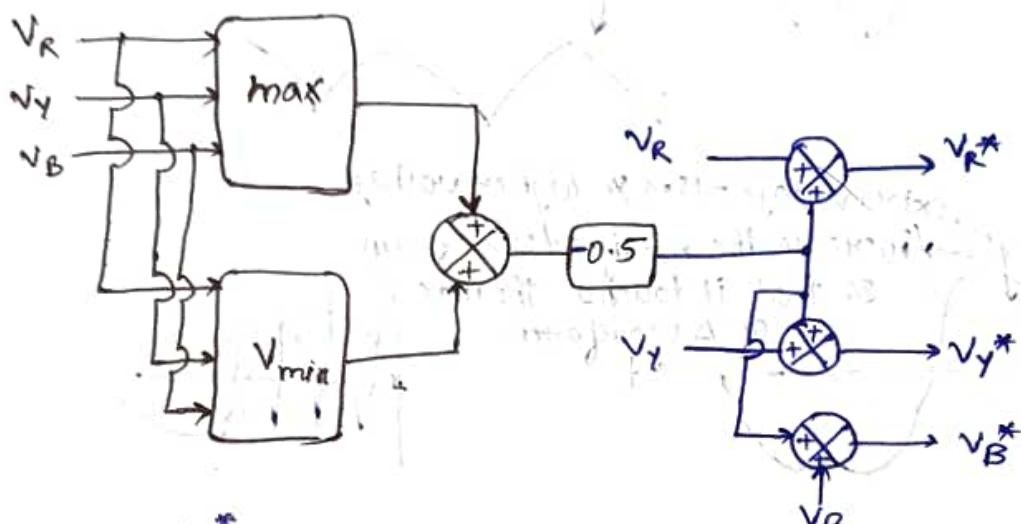
$$\Rightarrow V_{\max}^* + V_{\min}^* = 0$$

$$\Rightarrow V_{\max} + V_{CM} + V_{\min} + V_{CM} = 0$$

$$\Rightarrow V_{CM} = -\left(\frac{V_{\max} + V_{\min}}{2}\right)$$



01-11-2024



$$V_R^* = V_R + V_{CM}$$

$$V_y^* = V_y + V_{CM}$$

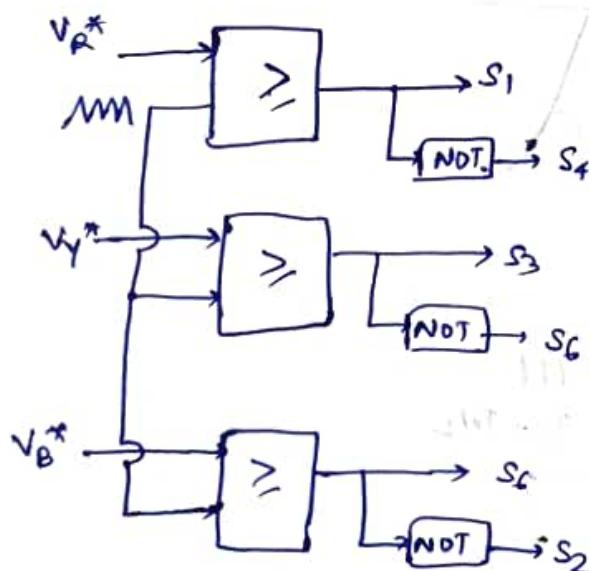
$$V_B^* = V_B + V_{CM}$$

$$V_{CM} = - \left(\frac{V_{max} + V_{min}}{2} \right)$$

$$V_R = V_m \sin \omega t$$

$$V_y = V_m \sin(\omega t - 120^\circ)$$

$$V_B = V_m \sin(\omega t - 240^\circ)$$



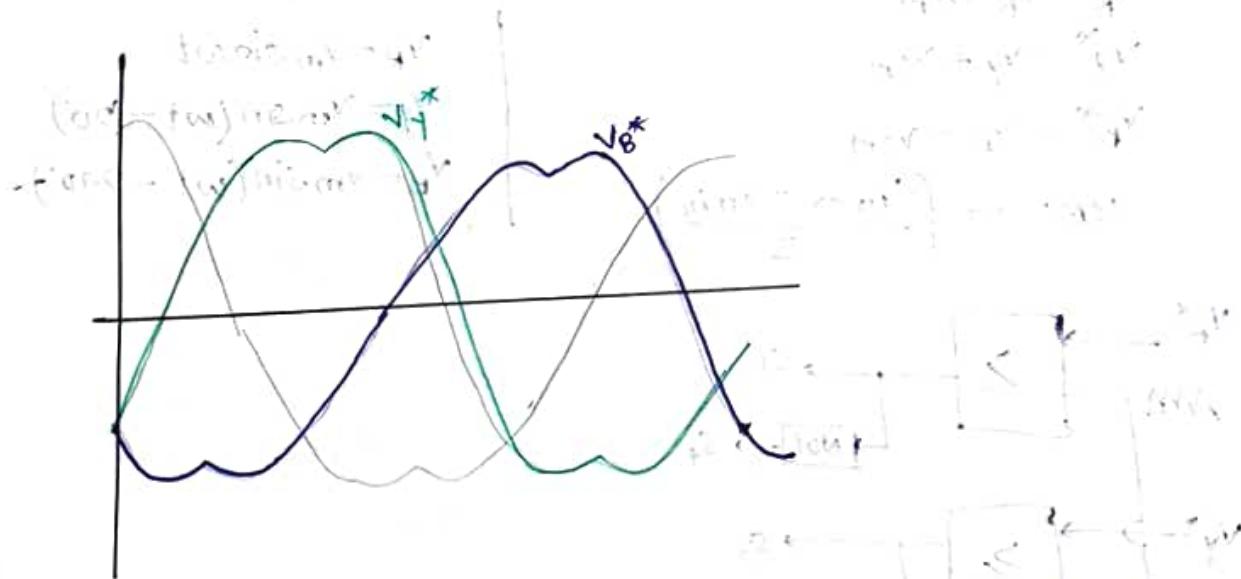
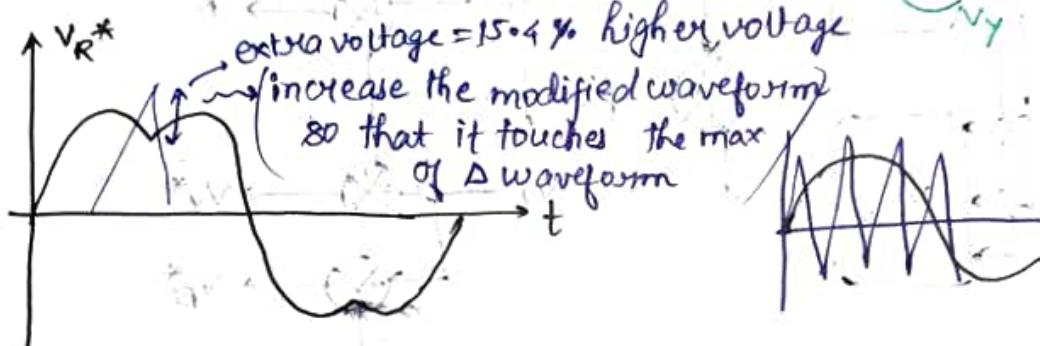
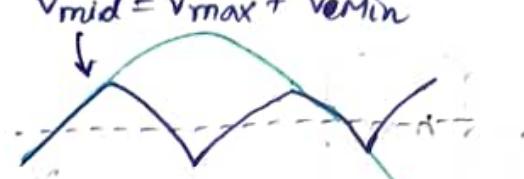
Frequency of V_{mid} = 3 x frequency of (V_R, V_B, V_Y) .

$$V_R + V_Y + V_B = 0$$

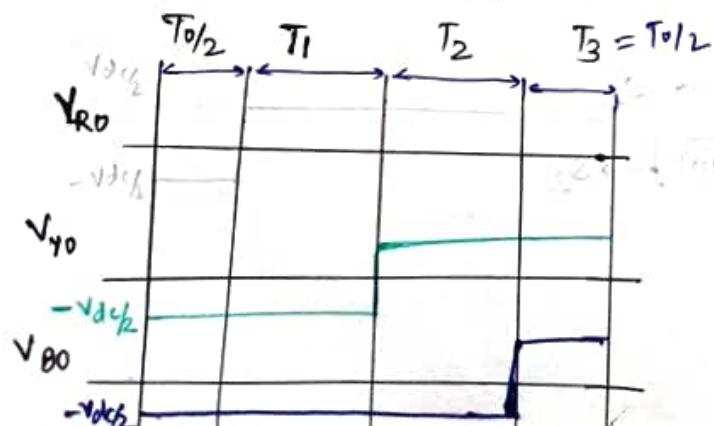
$$\Rightarrow V_R + V_B = -V_Y$$

$$V_{CM} = -\left(\frac{V_{max} + V_{min}}{2}\right)$$

$$= -\left(\frac{V_{mid}}{2}\right) \quad V_{mid} = V_{max} + V_{min}$$



000 — 100 — 110 — 111

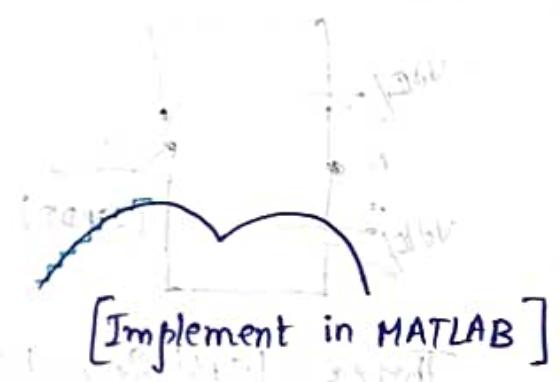


$$T_{0/2} + T_1 + T_2 + T_{0/2} = T_S$$

$$\begin{aligned} V_{RO} &= \frac{V_{dc}/2}{T_S} \left[-T_{0/2} + T_1 + T_2 + T_{0/2} \right] \\ &= \frac{V_{dc}}{2} \frac{(T_1 + T_2)}{T_S} \end{aligned}$$

$$\begin{aligned} V_{YO} &= \frac{V_{dc}/2}{T_S} \left[-T_{0/2} - T_1 + T_2 + T_{0/2} \right] \\ &= \frac{V_{dc}/2}{T_S} \left[-T_1 + T_2 \right] \end{aligned}$$

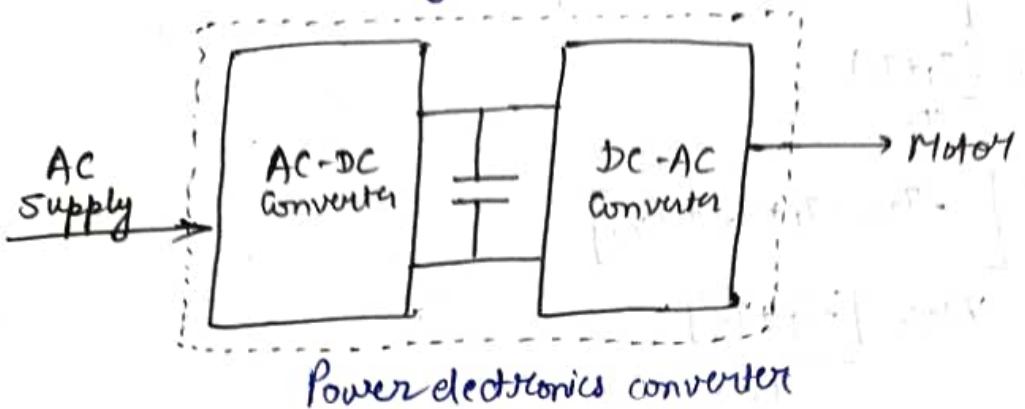
$$\begin{aligned} V_{BO} (\text{avg.}) &= \frac{V_{dc}/2}{T_S} \left[-\frac{T_0}{2} - T_1 - T_2 + T_{0/2} \right] \\ &= \frac{V_{dc}/2}{T_S} \left[-T_1 - T_2 \right] \\ &= -\frac{V_{dc}/2}{T_S} \left[T_1 + T_2 \right] \\ &= -V_{RO} \end{aligned}$$



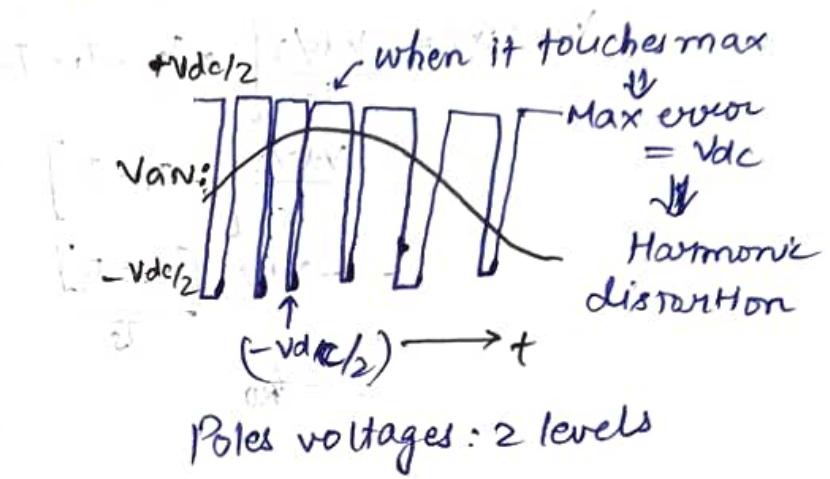
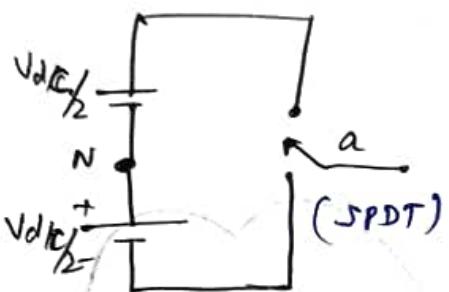
Opションで反並列二極管を導入する

Multilevel Inverters and PWM Schemes

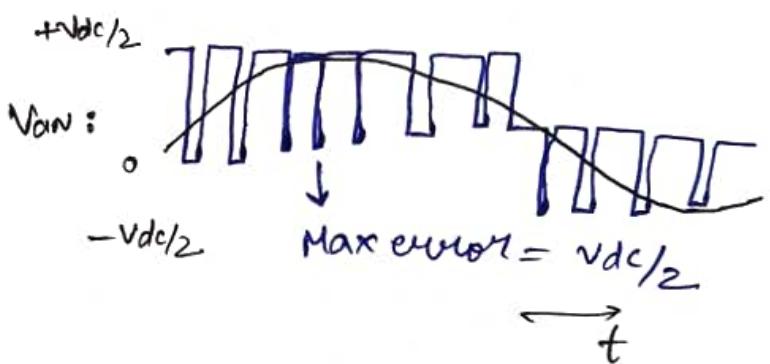
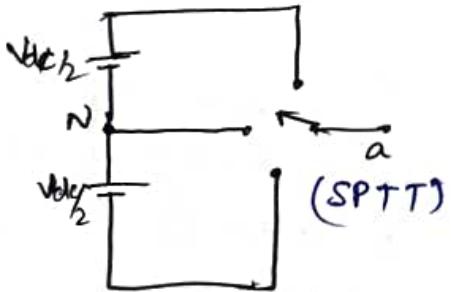
[Multilevel Voltage Source Inverters]



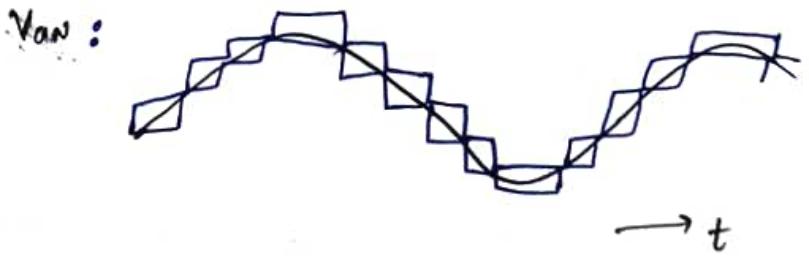
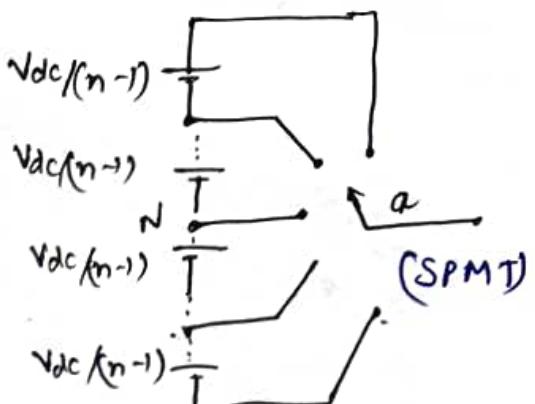
Two-level Inverter - SPWM



Three-level Inverter



Multilevel Inverter



↳ Quality of o/p voltage ↑
↳ Reduced harmonic distortion.

Multilevel Power Converters - Advantages:

- Enhanced quality of output waveforms with reduced harmonic distortions.
- Nearly sinusoidal output voltage at higher no. of levels - hence expensive and bulky filters can be avoided, in many applications.
- High voltage waveform can be generated using switching devices of low voltage ratings without employing series connections.
- Low voltage stress on semiconductor devices.
- Transformer for stepping up voltages can be avoided in many applications.

Multilevel Inverters - Advantages:

- Enhanced quality of the output voltage at low switching frequencies. Hence, switching loss reduces.
- Reduction in torque pulsation in drives.
- Reduction in dv/dt .
- Reduced EMI problems.
- Reduction in CM voltage and associated problems.
- facilitates elimination of CM voltages, etc.

Multilevel Inverters - Disadvantages:

(As the no. of levels increases)

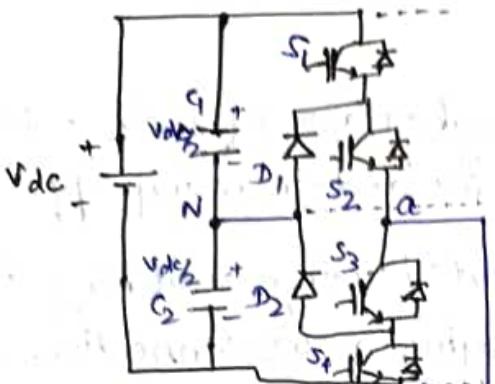
- Complexity of the power and control circuits increases.
- Large no. of components increases costs.
- Reliability decreases due to the presence of large no. of devices - each device is a potential failure point.

Conventional Multilevel Inverters Topologies :

- Neutral Point Clamped (NPC) topology
- Flying capacitor (FC) topology
- Cascaded H-bridge (CHB) topology

3-Phase, 3-Level NPC Inverter

[Diode clamped MLI]



[One Leg]

$C_1, C_2 \rightarrow$ split DC link voltage into two.
N \rightarrow Neutral point

Diodes clamp the switch voltages to $V_{dc}/2$.

$S_1, S_2, S_3, S_4 \rightarrow$ current bidirectional 2-quadrant sc switcher.

$S_1, S_3 \}$ operate in complementary manner

Pole voltage V_{an}

$$\begin{array}{c} +V_{dc}/2 \\ \text{---} \\ -V_{dc}/2 \end{array}$$

Switch States				Pole voltage V_{an}
S_1	S_2	S_3	S_4	$+V_{dc}/2$
1	1	0	0	
0	1	1	0	0
0	0	1	1	$-V_{dc}/2$

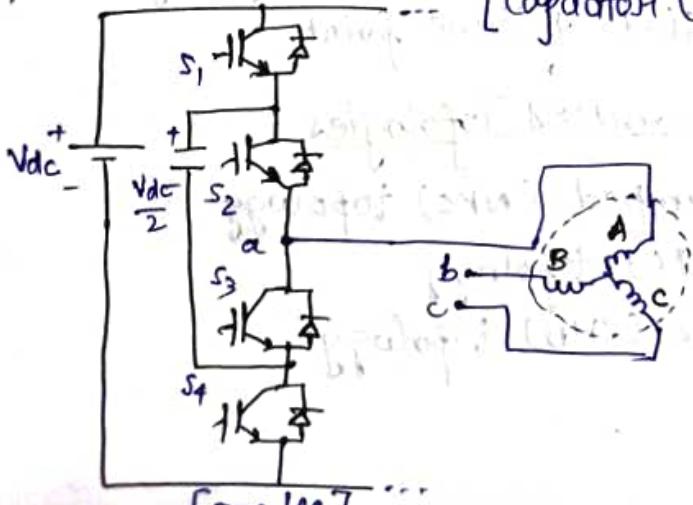
NPC Inverter - Disadvantages: fluctuations in capacitor voltage depending on the load current drawn from DC link.

↳ Requires large no. of clamping diodes as the no. of levels increases.

↳ N-level NPC inverter requires $3(N-1)(N-2)$ clamping diodes of voltage rating $(\frac{V_{dc}}{N-1})$.

Flying Capacitor Multilevel Inverter (FCMLI)

[Capacitor Clamped MLI]



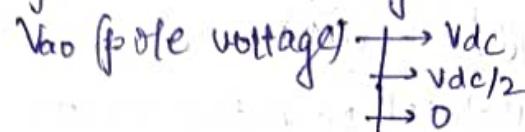
→ floating voltages across the capacitors are used for generating different voltage level.

→ 3-Level, 3-Phase FCI.

[One Leg]

→ Capacitor voltages have to be maintained at the required level under all conditions of operation. This can be achieved by making use of the redundancy in switching states that exists for generating different voltage levels.

$s_1, s_2, s_3, s_4 \rightarrow$ current bi-directional 2-quadrant SC switched capacitor C is charged to a voltage of $V_{dc}/2$



Switch States				Pole Voltage
s_1	s_2	s_3	s_4	V_{ao}
1	1	0	0	V_{dc}
0	1	0	1	$V_{dc}/2$
1	0	1	0	$V_{dc}/2$
0	0	1	1	0

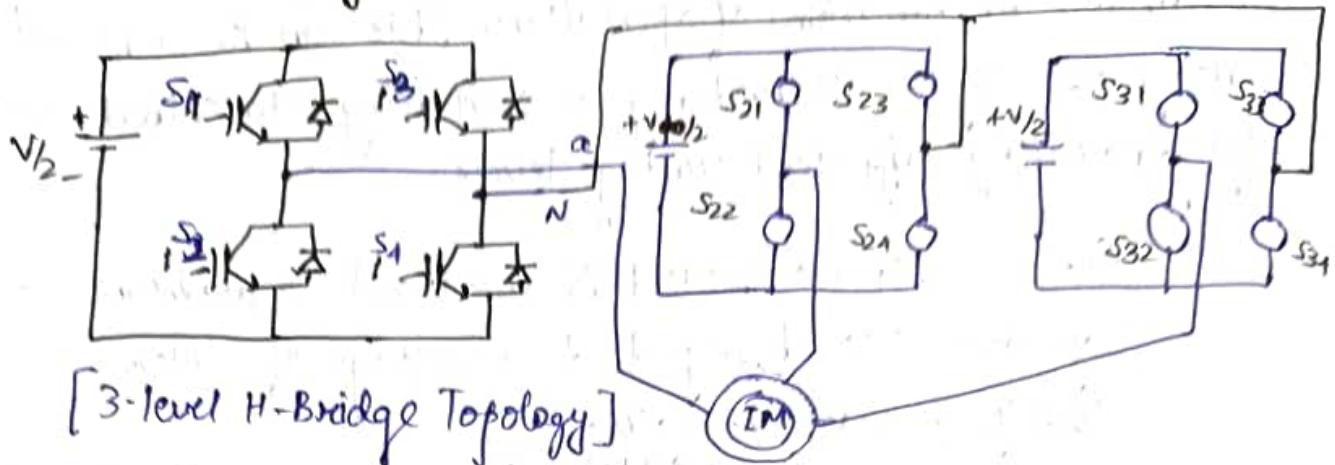
3-Level FCMLI - Balancing of capacitor voltages:

- Capacitor-voltage is not affected while generating V_{dc} and 0.
- There are 2 methods for generating $V_{dc}/2$ - called redundancy.
- for a given dim of the load current, these two methods produce opposite effects (charging / discharging) on the capacitor.
- The method of generation of $V_{dc}/2$ is selected based on the voltage level of the capacitor & the dim of load current so as to keep the capacitor voltage within a specified band ($V_{dc}/2 \pm \Delta V$).

FCMLI - Features:

- ↳ Requires large no. of capacitors to achieve higher no. of voltage levels.
- ↳ A 3-phase N-level flying capacitor based MLI with a DC link voltage of V requires $\frac{3(N-1)(N-2)}{2}$ capacitors of voltage rating $(\frac{V}{N-1})$.

Cascaded H-Bridge (CHB) Inverter



- H-bridge cells fed from isolated DC voltage sources are connected in series to realize MLI structure. The no. of H-bridge cells per phase required depends on the no. of voltage levels.
- Disadvantage: Requirement of large no. of isolated DC voltage sources.

$S_1, S_2 \}$ operate in complementary manner.

Pole voltage : V_{AN}

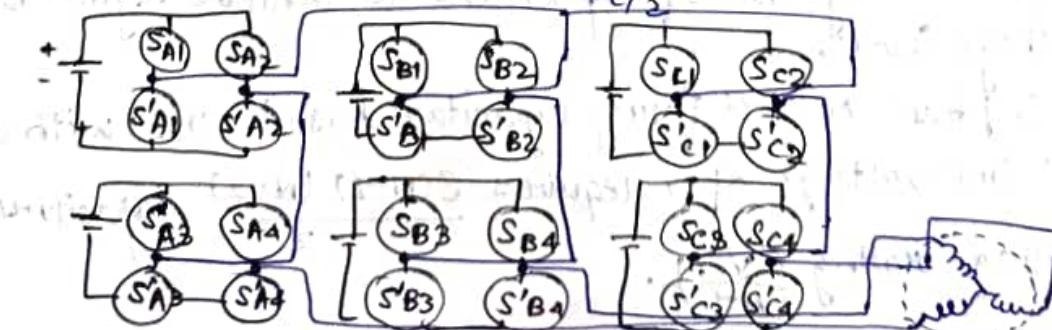
$$\begin{cases} \rightarrow +V_{dc}/2 \\ \rightarrow -V_{dc}/2 \\ \rightarrow 0 \end{cases}$$

Switch states				Pole voltage V_{AN}
S_1	S_2	S_3	S_4	
1	0	0	1	$+V_{dc}/2$
0	1	1	0	$-V_{dc}/2$
1	1	0	0	0
0	0	1	1	0

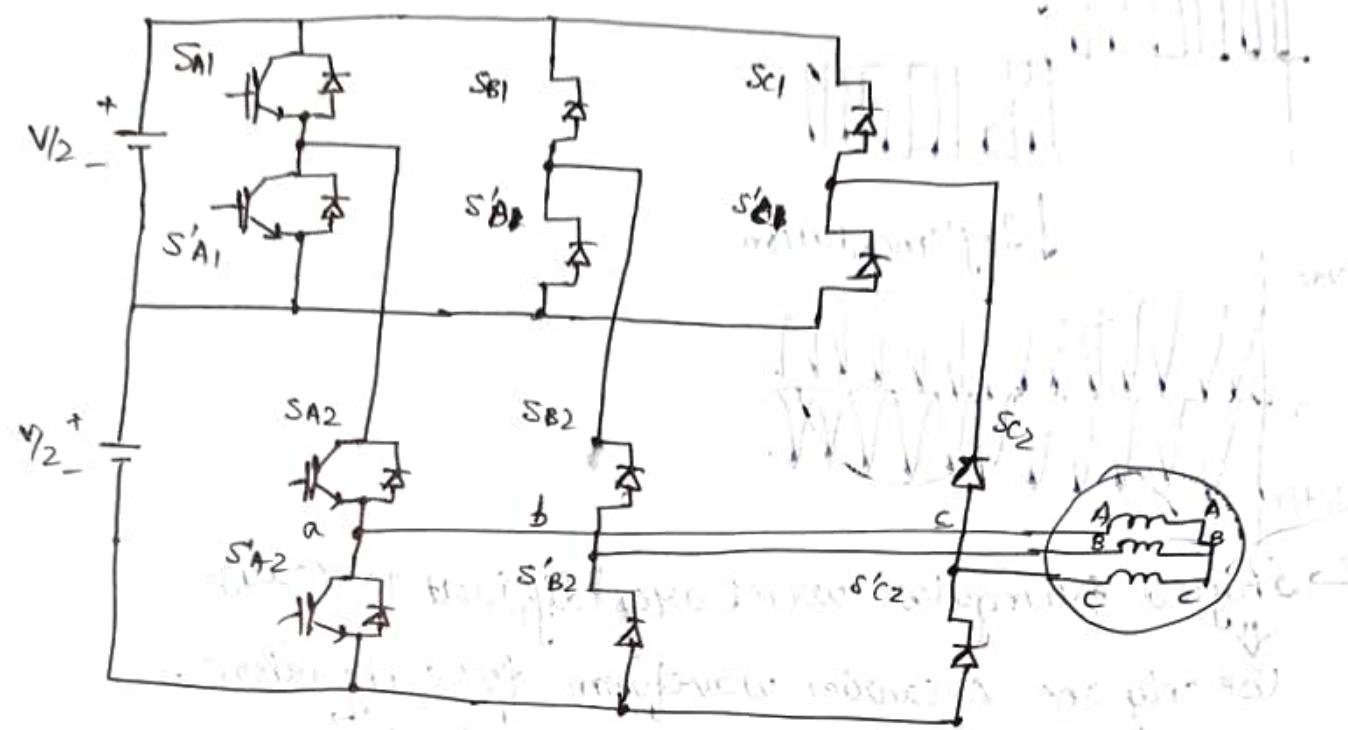
5-level, 3-phase, Cascaded H-Bridge Inverter :

Pole voltage levels :

$$\left\{ \begin{array}{l} +V_{dc}/2 \\ +V_{dc}/4 \\ 0 \\ -V_{dc}/4 \\ -V_{dc}/2 \end{array} \right.$$



Multilevel Inverter by cascading two-level Inverters



Switch states

S_{A1}	S'_{A1}	S_{A2}	S'_{A2}
----------	-----------	----------	-----------

1	0	1	0
---	---	---	---

0	1	1	0
---	---	---	---

0	0	0	1
---	---	---	---

Pole voltage

V_{A0}

V

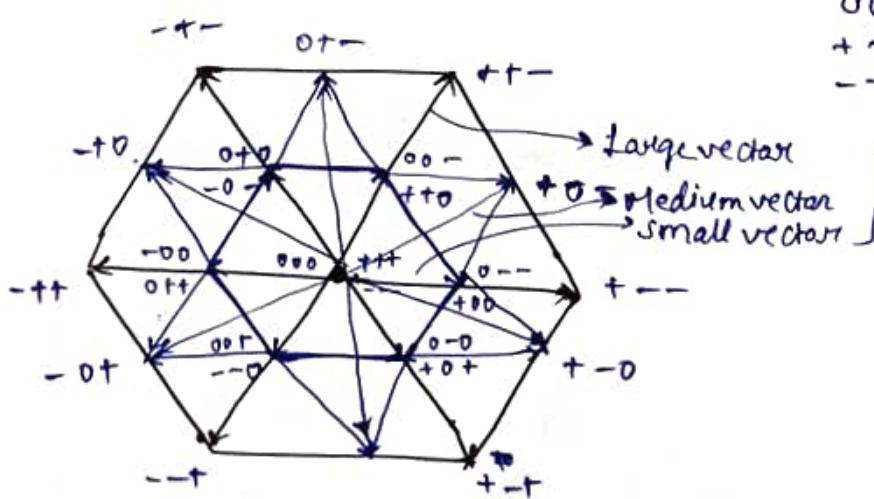
$V/2$

0

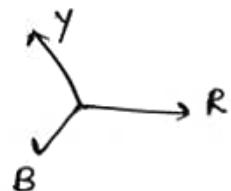
3 states levels: $+V_2 \rightarrow +$
 $0 \rightarrow 0$
 $-V_2 \rightarrow -$

} $3^3 = 27$ states

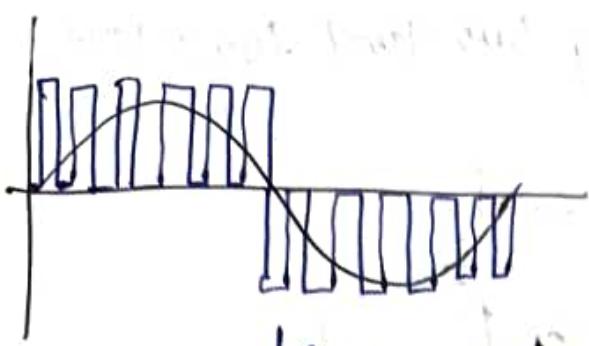
$\hookrightarrow 21$ active vectors



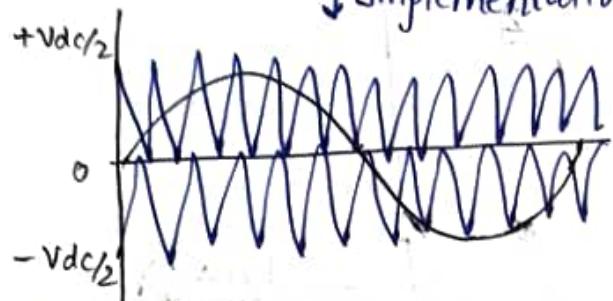
$\begin{matrix} 000 \\ +++ \\ --- \end{matrix}$ } Zero vectors
 $\rightarrow 18$ space vector locations



[RYB]



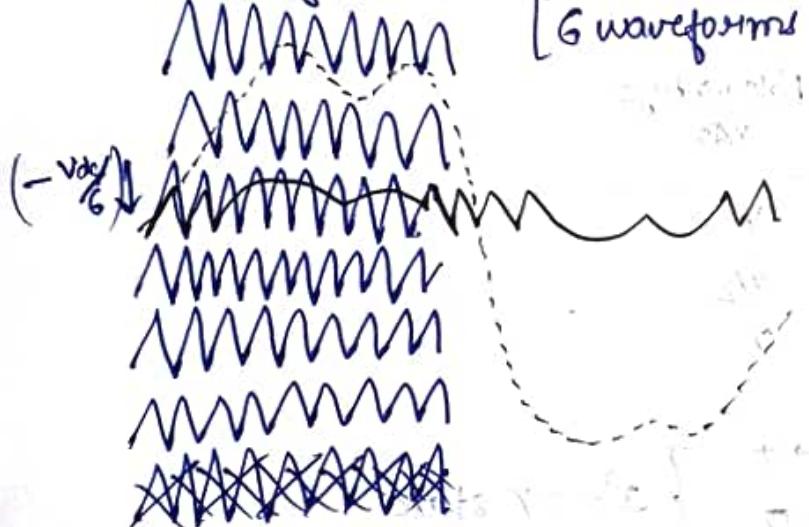
↓ Implementation



↳ Shifted triangular waves are difficult to realize.

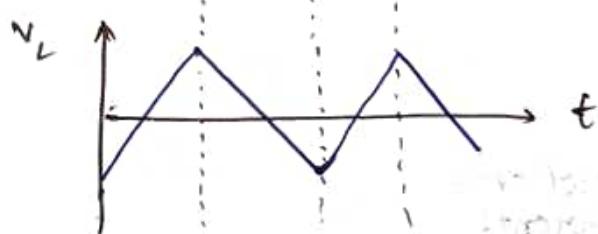
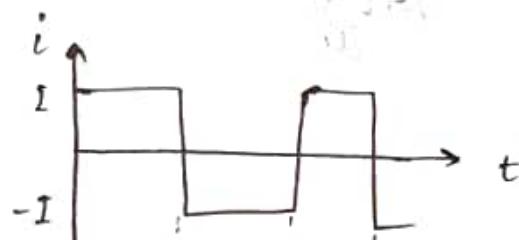
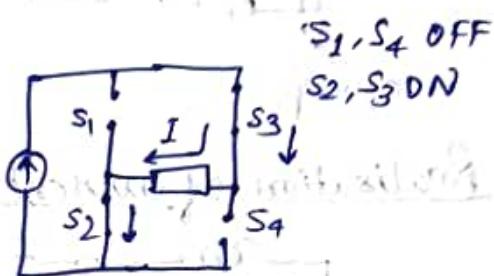
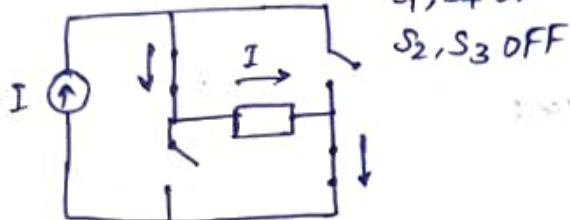
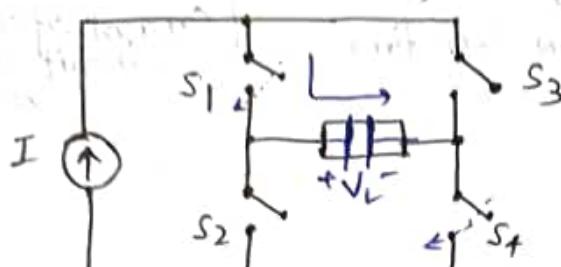
Use only one carrier waveform for comparison.

[6 waveforms for 7 levels]



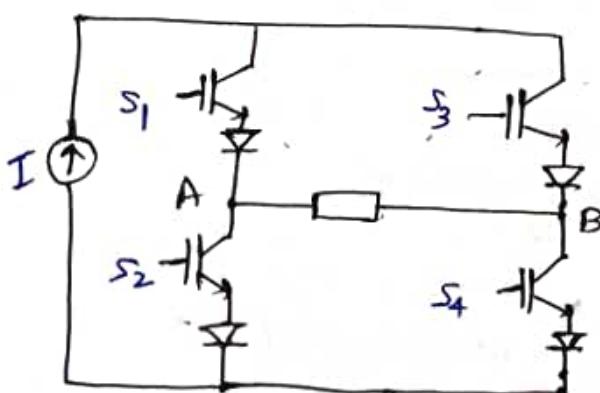
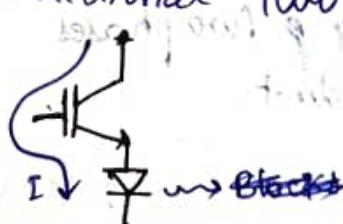
→ Whenever Hef. voltage waveform traverses from one carrier region to the next, it is shifted and scaled to reside within one triangle.

Current Source Inverter (CSI)



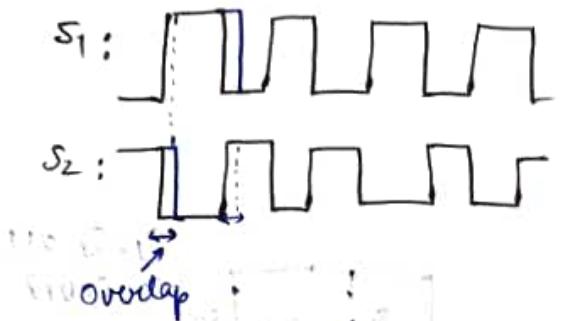
[Capacitor circuit]

→ Switch conducts current only in forward direction & voltage switches.
↳ voltage bidirectional two quadrant switch.

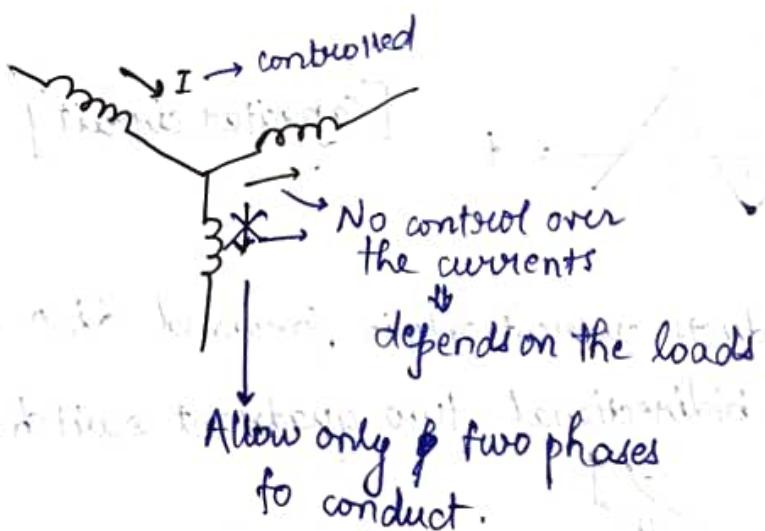
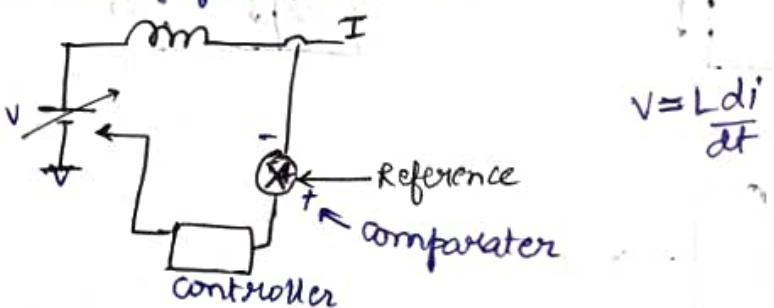


→ Controlled current
↳ No failure of components due to current.

Turn ON $S_1, S_4 \Rightarrow$ Turn OFF $S_1, S_4 \Rightarrow$ Any delay would
 OFF S_2, S_3 ON S_2, S_3 cause damage
 simultaneously (blast).
 But finite T_{ON}, T_{OFF} time [cannot open
 circuit current source]

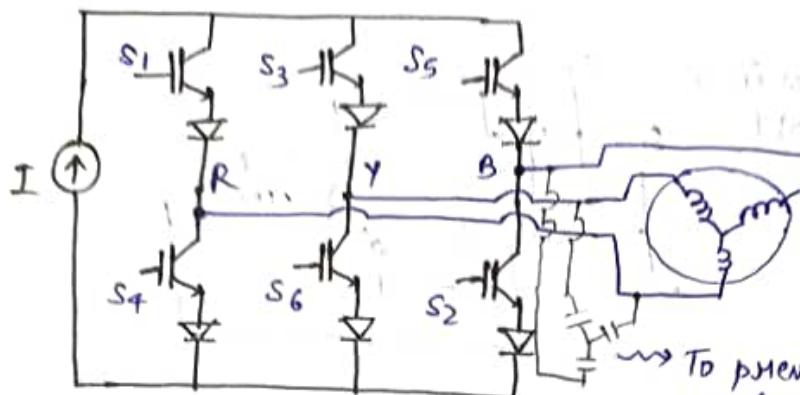


Realisation of current source:



Maximum utilization
 depending on
 of sub-diodes in
 common

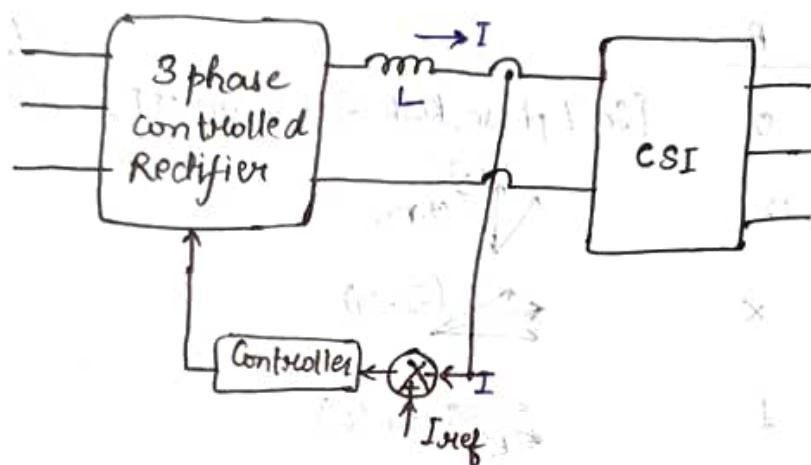
Three Phase Current Source Inverters



Two phase conduction mode

→ Only two phases can be allowed to conduct.

To prevent voltage spike: $v = L \frac{di}{dt}$ when current is turned ON/OFF.

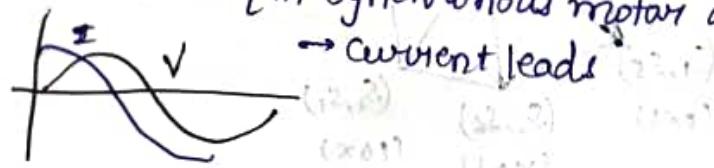


SCR: silicon Controlled Rectifier

↓

→ blocks voltage in both diode
→ allows current in one diode.
→ can be turned ON
→ cannot be turned OFF; → NO PWM
To turn OFF, current has to fall below certain level [decided by load].

Load commutation [in synchronous motor drives]



Zero State: Current does not flow through the load.
OR, By passing current away from the load.

Zero vector {
 (S₁, S₄) - ON
 (S₃, S₆) - ON
 (S₅, S₂) - ON
 }

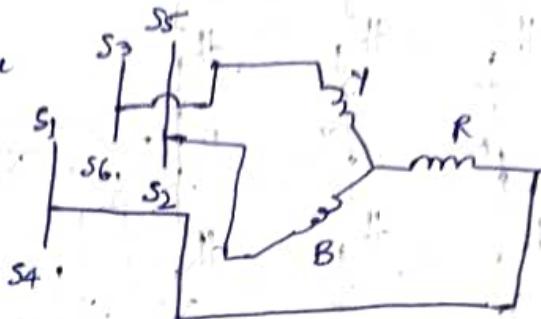
→ In voltage source,
turn OFF the source.

States:

1 → ON

0 → OFF

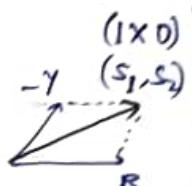
X → Both switches in a leg are OFF



Switching states:

ON states	R	Y	B
S_1, S_2 ON	1	X	0
S_1, S_3	X	1	0
S_3, S_4	0	1	X
S_4, S_5	0	X	1
S_5, S_6	X	0	1
$S_6 - S_1$	1	0	X

[In Y phase, both S_3 & S_6 are OFF]



(S_1, S_3)
(X#0)

(S_3, S_4)

(S_4, S_5)

(S_5, S_6)

(S_6, S_1)

$(X10)$

(S_3, S_2)

(1x0)

(01X)

(S_1, S_5)

$(0x1)$

(S_5, S_6)

$(x01)$

(S_6, S_1)

$(10X)$

[Total 16 states]

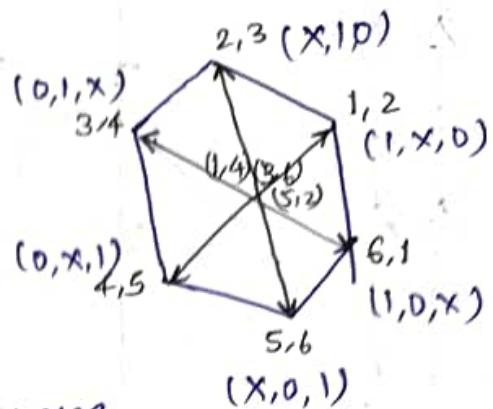
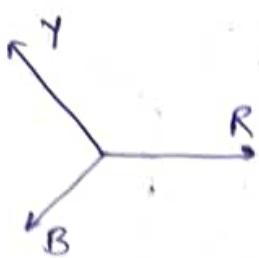
[Switching states for each phase with respect to neutral point]



- Total no. of present w/wf for one terminal = 8 states w/wf
- Total no. of states from two phases of YB = 16
- Number of states = 16
- Number of states = 16
- Number of states = 16

Active: (1,2), (2,3), (3,4), (4,5), (5,6)

Zero: (1,4), (3,6), (5,2)



⇒ phase shift in current source inverter in comparison to voltage source inverter.

Applying Ampere-second balance:

$$I_x T_s = I_1 T_1 + I_2 T_2 + I_0 T_0$$

[I_x is the current we want to generate.]

→ Voltage Source Inverter: Buck Inverter

→ Current Source Inverter: Boost Inverter.

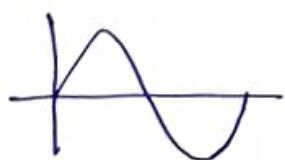
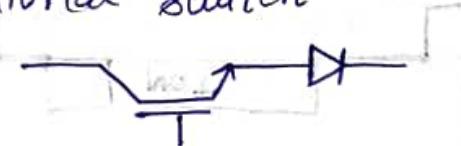


SCR (Silicon Controlled Rectifier)

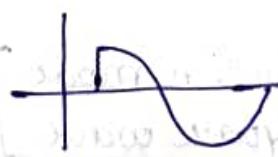
For making it to conduct we have to add gate drive current.

→ Voltage bidirectional switch

→ Equivalent to



Normal diode

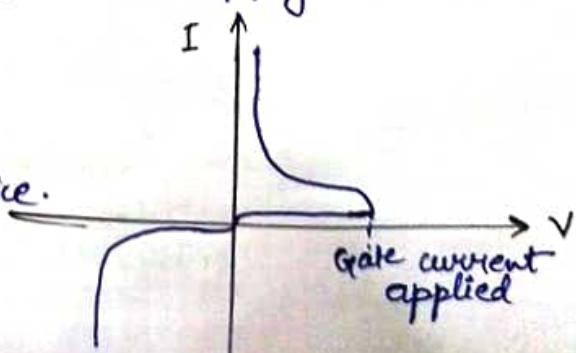


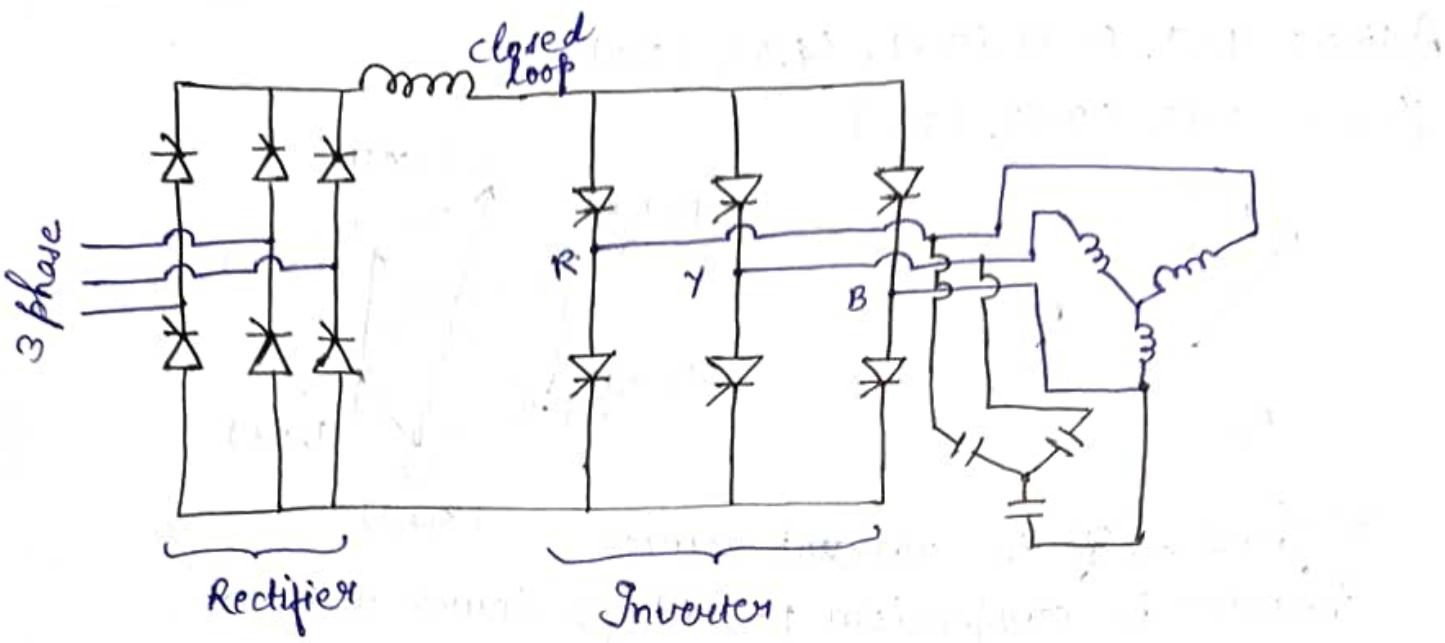
SCR diode

[we can apply controlled voltage]

→ To switch off, reduce the current

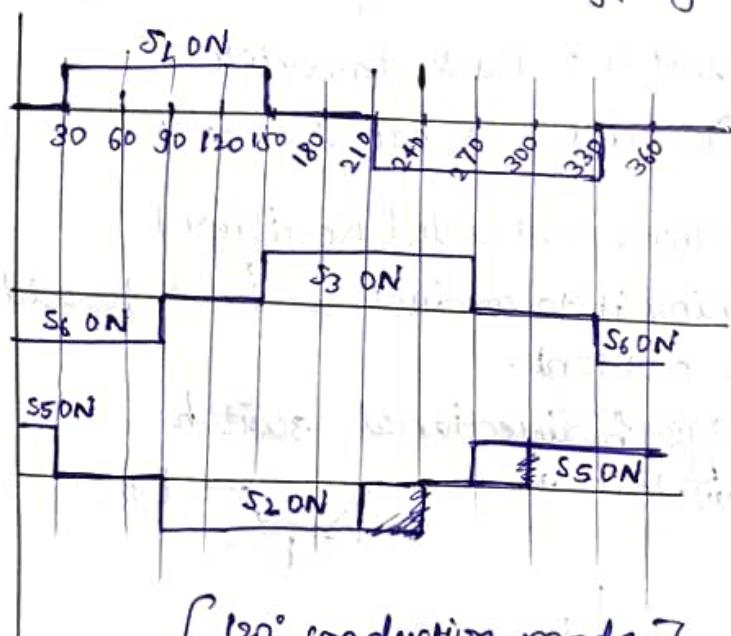
↪ Semi-controlled device.





Synchronous Motor: can be controlled to be leading or lagging power.

Non-synchronous Motor: Lagging current.

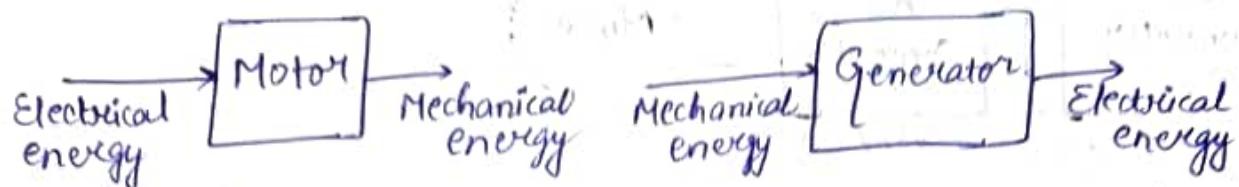


[120° conduction mode]
[Quasi-square wave]

[approximate switching performance test]

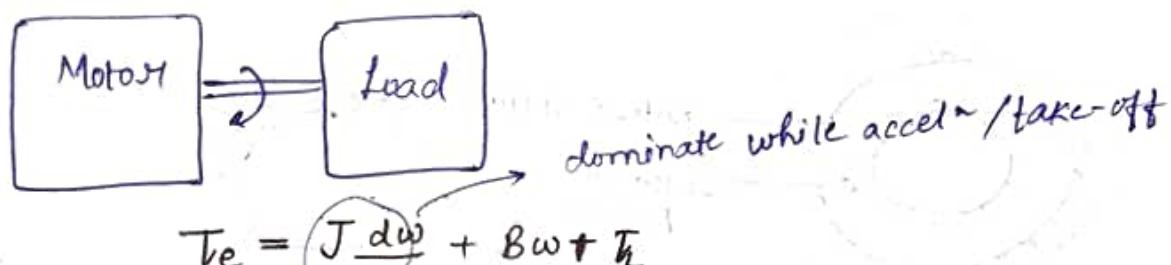
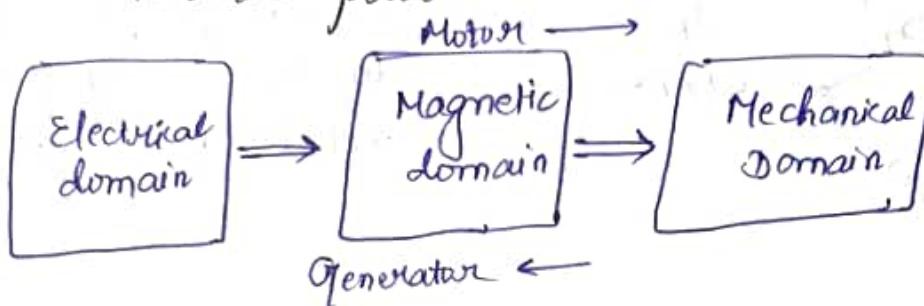
Switching
frequency

Control of Electric Motors



To control:
 Torque (T) \rightarrow N.m
 Speed (ω) \rightarrow rad/sec.

$$\text{Mechanical power} = \omega \cdot T$$



$$T_e = J \frac{d\omega}{dt} + B\omega + T_L$$

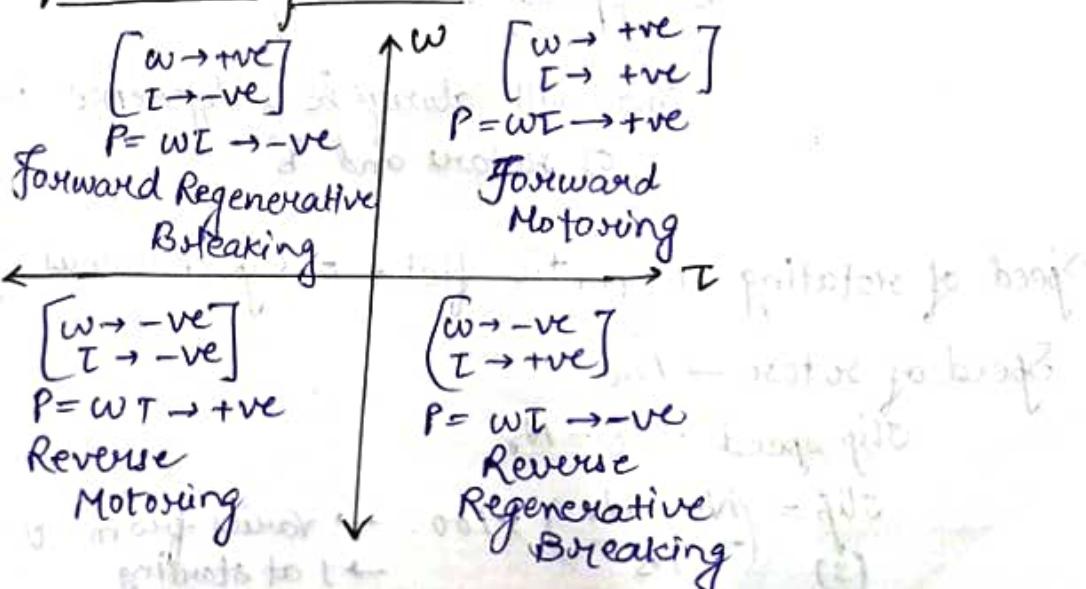
T_L : load torque

ω : velocity (rad/sec)

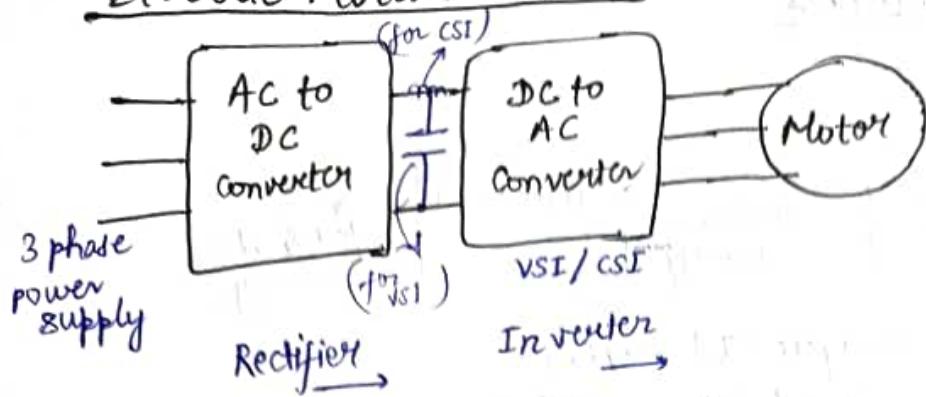
J : MI

B : coeff. of friction.

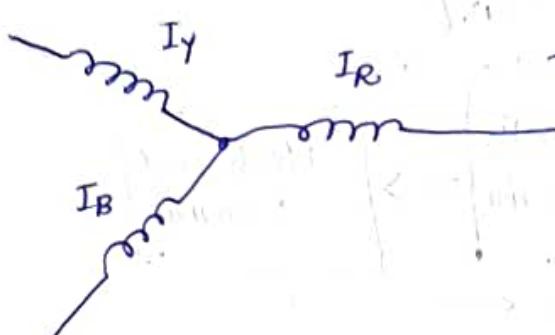
Four quadrant operation:



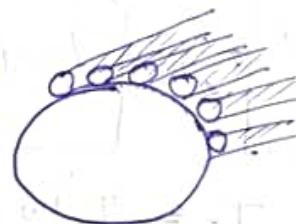
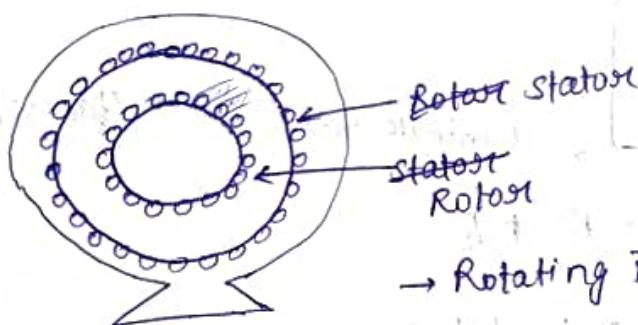
Electric Motor Drive



18-11-2024



→ 3 windings of 120° spatial difference carrying currents with 120° phase difference.



→ Rotating \vec{B} with constant magnitude.

→ Rotor conductors oppose the magnetic field by rotating themselves, along with rotating \vec{B} and try to achieve its speed.

\vec{B} appears stationary w.r.t. rotors.

There will always be a difference in speed of rotors and \vec{B} .

Speed of rotating magnetic field = Synchronous speed (N_s)

Speed of rotor $\rightarrow N_r$

$$\text{Slip speed} = N_s - N_r$$

$$\text{Slip} = \left(\frac{N_s - N_r}{N_s} \right) \times 100 \quad \begin{aligned} &\rightarrow \text{varied from } 0 \text{ to } 1. \\ &\rightarrow 1 \text{ at starting} \\ &\rightarrow 0 \text{ when } N_s = N_r. \end{aligned}$$

↪ $s > 1$ when $N_s \neq N_r$ rotates in opposite direction
↪ Regenerative braking -
(due to torque in
opposite dirn)

↪ $s < 0 \rightarrow$ not possible in motor.
↪ in generator

→ Back EMF

↪ Asynchronous Motor: Induction motor

↪ works on the speed less than synchronous speed.
↪ Based on electromagnetic induction.

→ Frequency / voltage induced in rotor depends on the slip
(relative speed).

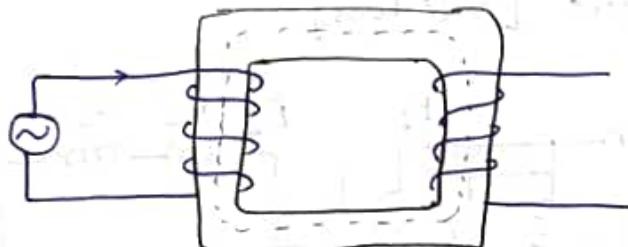
$f_s \rightarrow$ stator frequency

$f_r \rightarrow$ rotor frequency

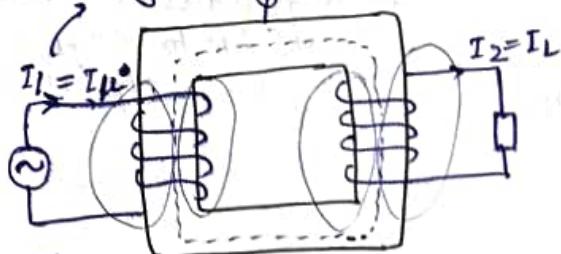
$$f_r = s f_s$$

↪ Analysis of torque, speed using motor modelling in
steady or transient state.

#



$$\text{flux} = \frac{\text{mmf}}{\text{Reluctance}} = NI$$



$$I_1 = I_M + I_{2'}$$

$$\text{Magnetising current} = \frac{Nd\phi}{dt}$$

$$f_r = 5f_s, \quad s = \frac{N_s - N_r}{N_s}$$

(slip)

f_s : frequency of the voltage applied to the stator;

$$\phi = \frac{\text{MMF}}{\text{Reluctance}} = \frac{NI}{s} \quad | f_s: \text{frequency of induced voltage in stator}$$

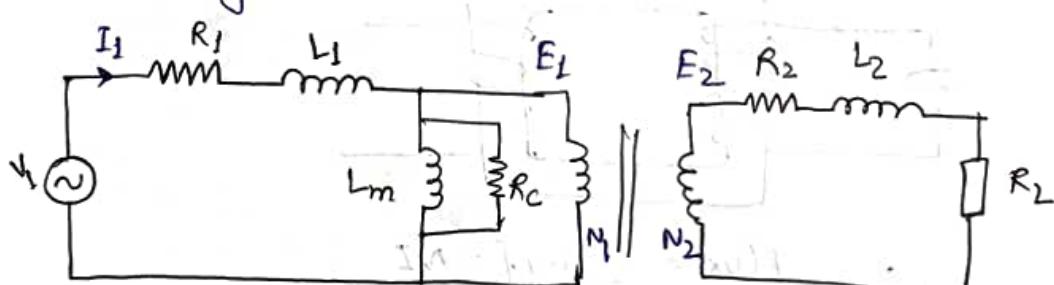
high for air,
so low leakage

→ Resistance of coil also contributes to the loss.

Losses (magnetic)

→ Hysteresis

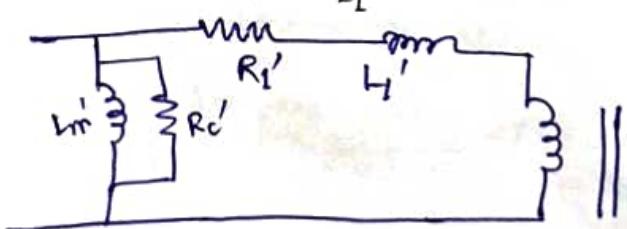
→ Eddy current [core loss]



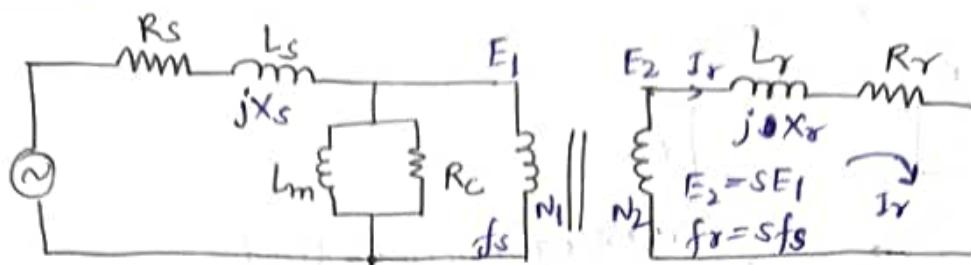
$$I_1^2 R_1 = I_2^2 R_1' \quad (R_1' \text{ after transforming})$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \Rightarrow \frac{E_1}{N_1} = \frac{E_2}{N_2}$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$



In motor



$$I_r = \frac{SE_1}{R_r + j\omega X_r} = \frac{E_1}{R_r + j\omega X_r}$$

$$X_s = 2\pi f_s L_s$$

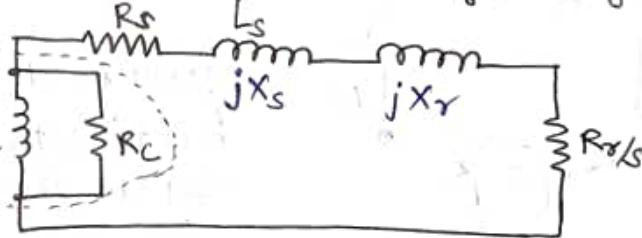
$$X_r = 2\pi f_s L_r$$

If $R_r = R_r/s$, then can say $E_2 = E_1$, $sX_r = X_r$.

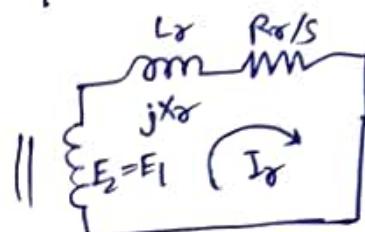
$$\text{Mechanical energy} = I_r^2 \frac{R_r}{s}$$

Equivalent circuit (after transforming):

Shifting its position won't affect much as R_s & jX_s are very small



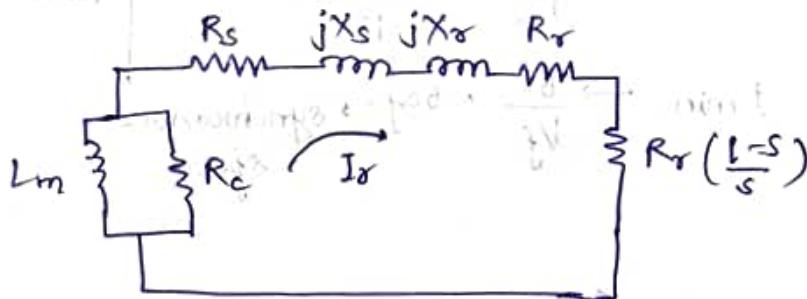
$$R_r/s = R_r + R_r \left(\frac{1-s}{s} \right)$$



freq. dependence gone

$$\text{Power dissipated at motor} = I_r^2 \frac{R_r}{s}$$

$$\begin{aligned} \text{Loss in secondary} \\ = \text{copper loss} + \text{Mechanical loss} \end{aligned}$$



$$P_{\text{mechanical phase}} = I_r^2 \frac{R_r(1-s)}{s}$$

$$\begin{aligned} \text{For 3-phase, } P_{\text{mech.}} &= 3I_r^2 \frac{R_r(1-s)}{s} \\ &= \omega_{\text{mech.}} \times T \end{aligned}$$

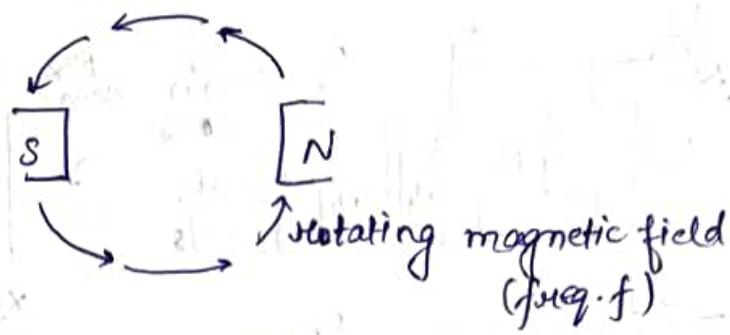
$$\begin{aligned} \omega_{\text{mech.}} &= \omega_s - s\omega_s \\ &= \omega_s(1-s) \end{aligned}$$

$$\text{Torque, } T = 3I_r^2 R_r \frac{(1-s)}{s \cdot \omega_{\text{mech.}}}$$

$$= 3I_r^2 R_r \frac{(1-s)}{s} \cdot \frac{1}{\omega_s(1-s)}$$

$$= \frac{3I_r^2 (R_r/s)}{\omega_s}$$

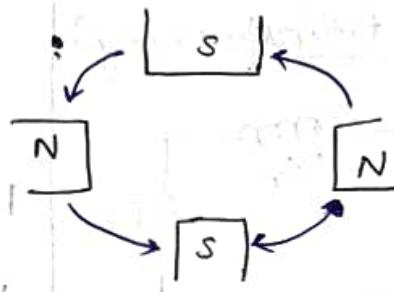
$$\rightarrow \omega_{\text{mech.}} = \omega_s, \omega_s = \omega_s(1-s)$$



(Synchronous speed) No. of rpm = 60f.

Mechanical speed Vs. Electrical speed

$$\omega_{\text{mech.}} = \frac{\omega_{\text{electrical}}}{P/2}$$



four part, mechanical it has to rotate by 180° to rotate electrical field by 360° .

$$(\text{Syn. speed}) N_s = \frac{60f}{P/2} = \frac{120f}{P}$$

RPM \rightarrow Rev. per minute

$$1 \text{ min.} \rightarrow \frac{60}{1f} = 60 \text{ f} \rightarrow \text{synchronous speed}$$

$$2\omega_2 - 2\omega_1 = 4\pi m \omega$$

$$(2-1)2\omega_1 =$$

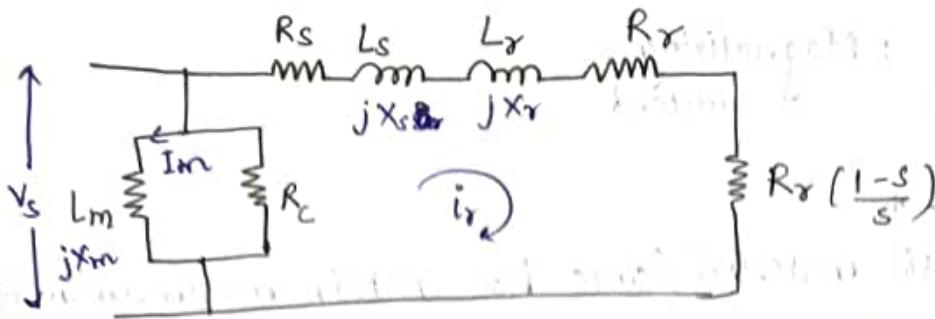
$$\frac{(2-1)}{2} \times 2\omega_1 = 1 \times 2\omega_1 = 2\omega_1$$

$$T \propto \omega$$

$$\frac{(2-1)}{2} \times 2\omega_1 = T \text{ (suppose)}$$

$$\frac{(2-1)}{2} \times 2\omega_1 =$$

$$(2-1), 2\omega_1 =$$



$$[X_m = 2\pi f L_m]$$

$$X_s = 2\pi f L_s$$

$$X_r = 2\pi f L_r$$

$$T = \frac{3 I_s^2 R_s / s}{\omega_s}$$

$$I_s = \frac{V_s}{\sqrt{(R_s + R_r/s)^2 + (X_s + X_r)^2}}, \quad T = \frac{3 V_s^2 R_s}{s \omega_s \left[(R_s + R_r/s)^2 + (X_s + X_r)^2 \right]}$$

$$I_s = \frac{V_s}{(R_s + R_r/s) + j(X_s + X_r)}$$

→ Torque is very sensitive to voltage.

→ If s is very small, i.e., very small slip.

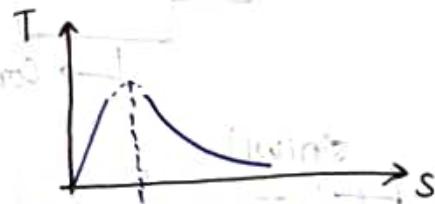
$$T \propto \frac{V_s^2 (R_r/s)}{\omega_s (R_r/s)^2} = \frac{V_s^2}{\omega_s (R_r/s)} = \frac{s V_s^2}{\omega_s R_r}$$

$$\Rightarrow T \propto s$$

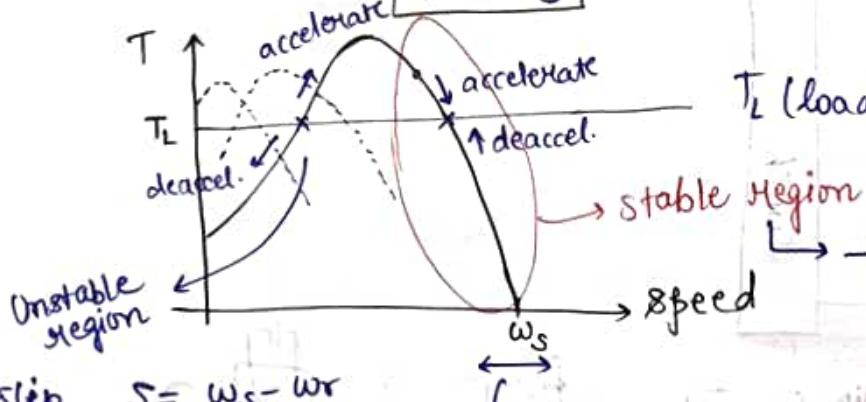
For large values of s ,

$$T \propto \frac{V_s^2 - R_r}{s \omega_s}$$

$$\Rightarrow T \propto 1/s$$



At some point, torque will be maximum.



T_L (load torque)

stable Region

↓ -ve slope

↓ Maximum
slip

$$\text{slip, } s = \frac{\omega_s - \omega_r}{\omega_s}$$

this range of speed is

only possible here

→ We can change freq. to change ω_s to get wider range of speed variations.

$$|I_m| = \frac{V_s}{X_m} = \frac{V_s}{2\pi f L_m} : \text{Magnetising current}$$

$f \downarrow, |I_m| \uparrow$
 $\hookrightarrow \text{flux } \uparrow$

→ Every magnetic material / core has certain maximum limit.

$$T \propto \phi^2 \omega_{\text{slip}}$$

→ we need to keep $|I_m|$ constant.

→ $\frac{V_s}{f}$ should remain constant (V_s should be such.)

Constant V/f Ratio control

$$\frac{V}{\omega} \rightarrow \frac{V}{\text{rad/sec.}} \rightarrow \text{volt-sec.}$$

$$V = \frac{d\Phi}{dt}$$

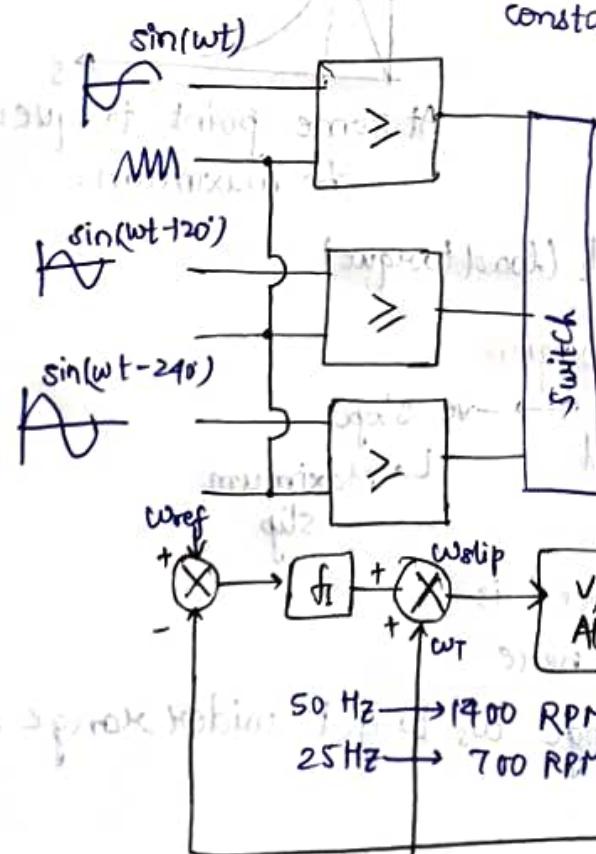
$$\rightarrow \frac{V}{\omega} = \text{constant} \Rightarrow \text{flux} = \text{constant}$$

→ At rated flux, machine can deliver rated torque only when load requires it. \Rightarrow Deliver Rated torque.

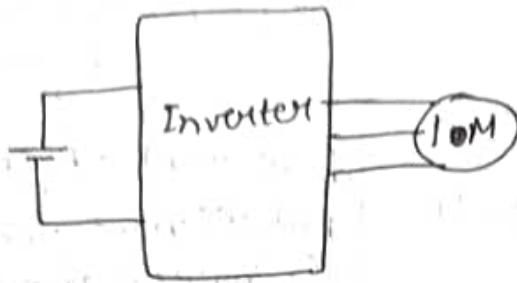


Control it to have

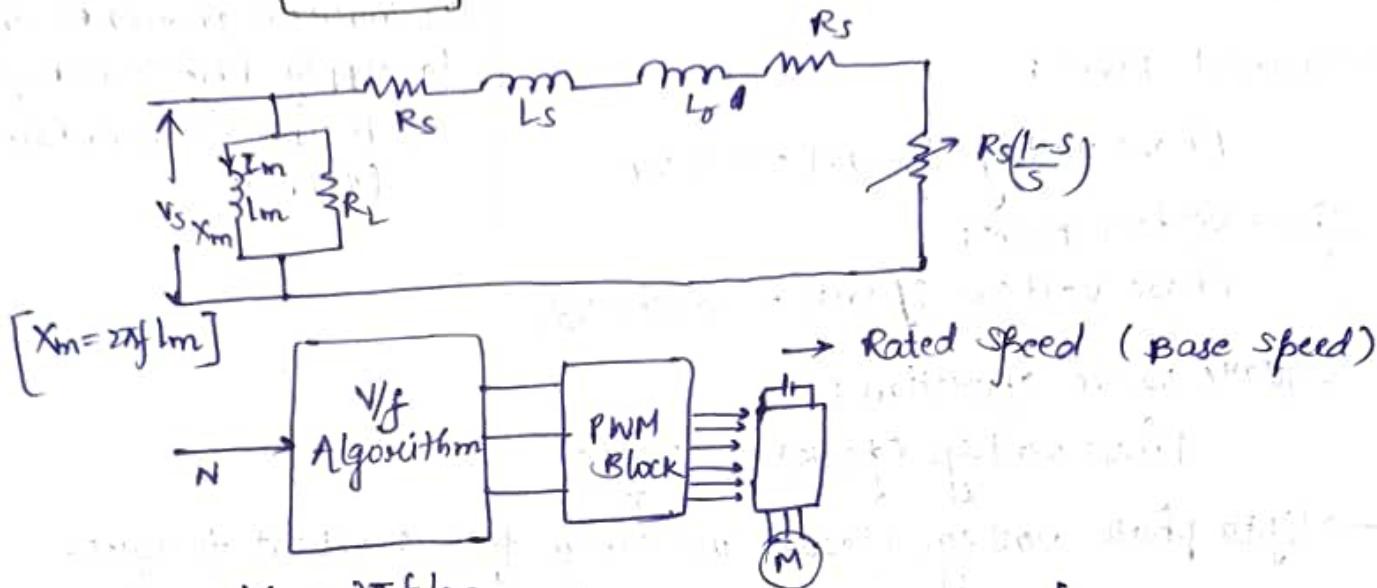
$$\text{constant } \frac{V}{f}$$



Constant V/f Ratio Control



$$N_s = \frac{120f}{P}$$



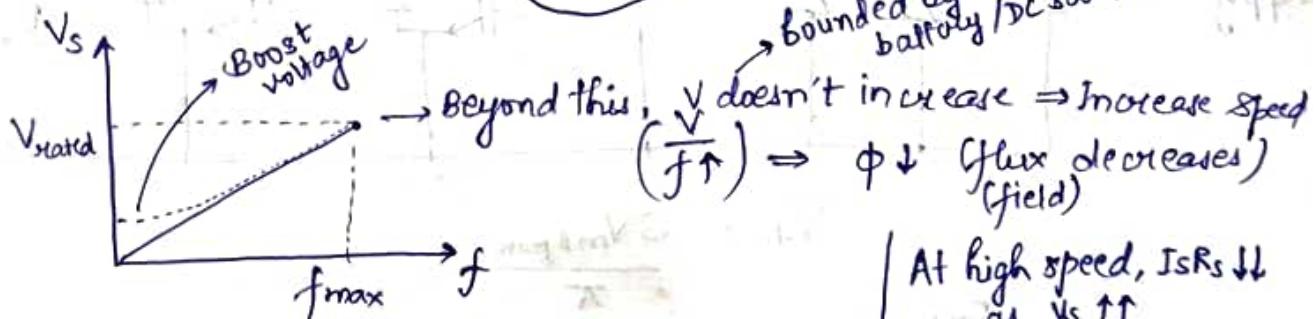
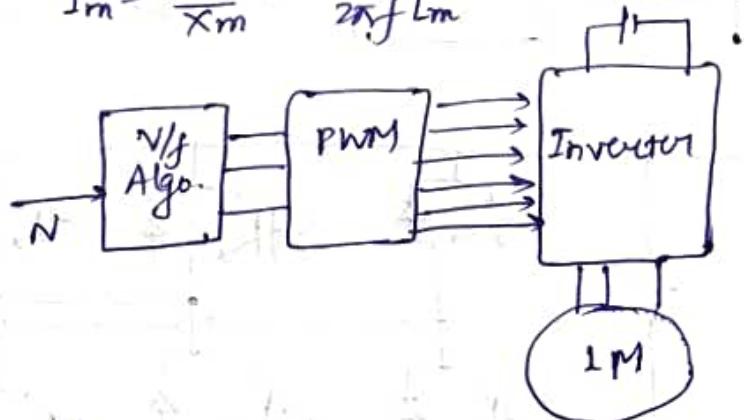
$415V, 3ph, 50Hz, 1440 RPM$

→ Rated speed (base speed)

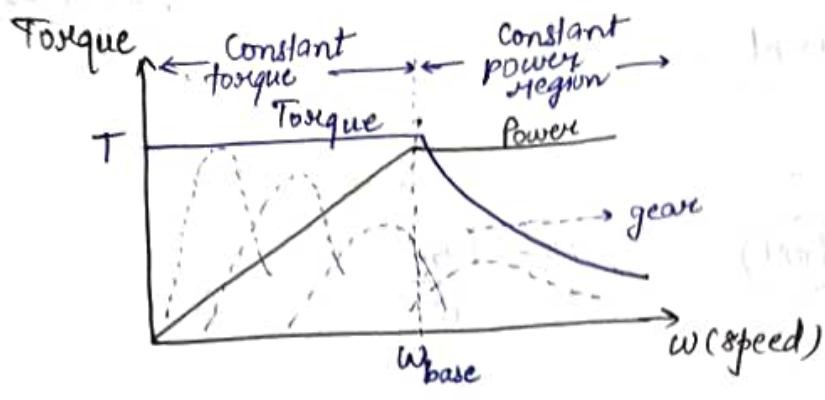
 $\phi = L_m I_m$

$$I_m = \frac{V_s}{X_m} = \frac{V_s}{2\pi f L_m}$$

$$\frac{415}{2}, 25Hz \rightarrow 720 RPM$$



→ Machine does not get actual voltage at the low speed
↳ Give boost voltage



$$\omega \times T = \text{Power} = \text{constant}$$

beyond ω_{base}

If we want a higher speed but still want constant vectors the inverter should be able to supply higher voltage to be able to maintain (V/f) ratio.

Sinusoid PWM:

$$\text{Phase voltage (peak)} = 0.5 V_{\text{dc}}$$

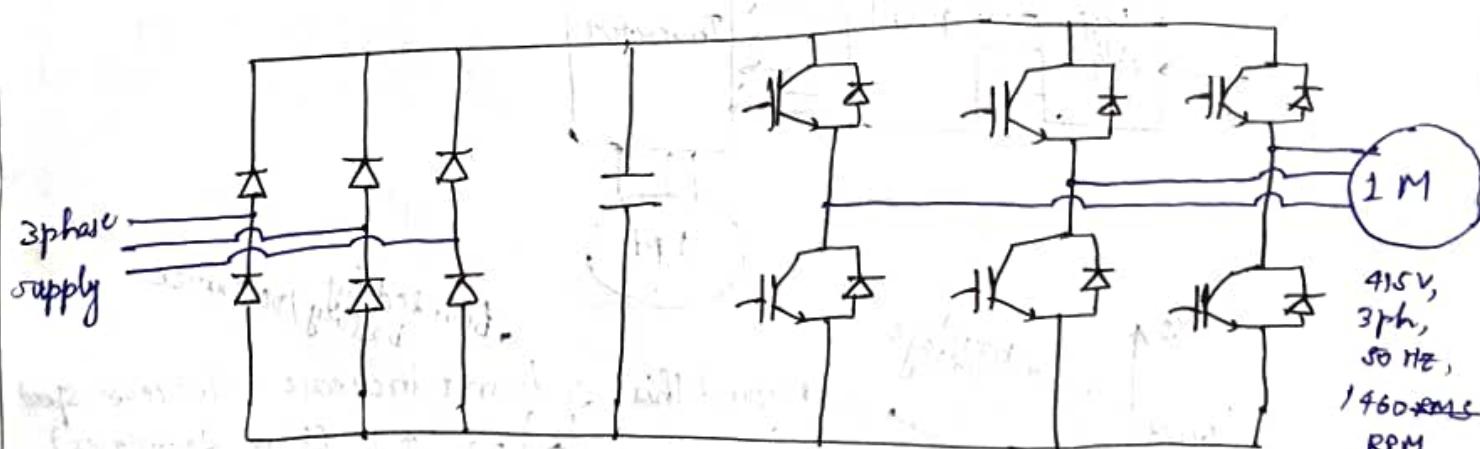
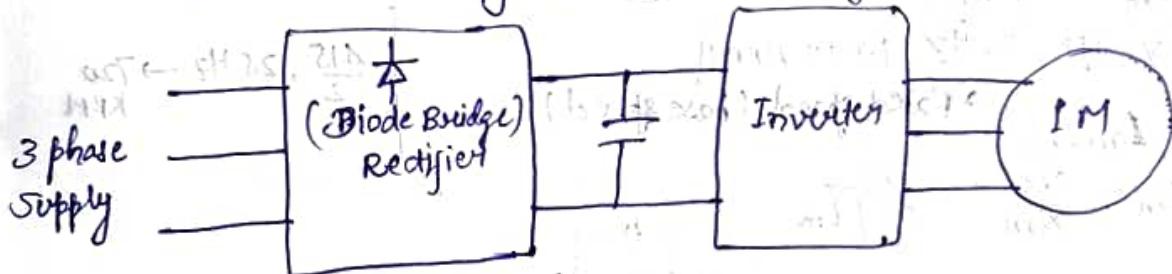
Space-vector PWM:

$$\text{Phase voltage (peak)} = 0.577 V_{\text{dc}}$$

Square wave operation:

$$\text{Phase voltage (peak)} = \frac{2 V_{\text{dc}}}{\pi}$$

→ Up to phase voltage (peak), we can go for constant torque.



$$V_{\text{dc}} = \frac{3 V_{\text{m peak}}}{\pi}$$

For 415V,

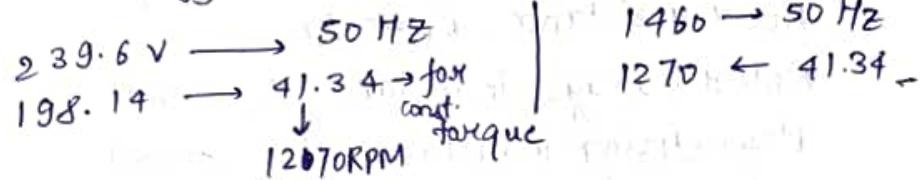
$$V_{\text{dc}} = \frac{3 \times 415 \times \sqrt{3}}{\pi} = 460.4 \text{ V.}$$

SPWM:

$$V_p(\text{rms}) = \frac{V_{dc}}{2\sqrt{2}} = \frac{560.4}{2\sqrt{2}} = 198.14$$

Phase voltage required to run the motor at rated speed

$$= \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$



SVPWM:

$$V_p(\text{rms}) = \frac{0.577 V_{dc}}{\sqrt{2}} = 228.48$$

$$228.48 \rightarrow \frac{50}{239.6} \times 228.64 = 17.67 \text{ Hz} \rightarrow 1372 \text{ rpm}$$

Square Wave Operation : (6-steps)

$$\frac{2 V_{dc}}{\sqrt{2}\pi} = \frac{2 \times 560.4}{\sqrt{2}\pi} = 252.05$$

Inverter in
overmodulation region.

(Transistor on & off)

freq

min. switching frequency for starting M

$$f_{min} = \frac{V_{dc}}{R_{load}} = \frac{560.4}{0.2 \times 10^3} = 2.8 \text{ Hz} = (60/10) \text{ s}^{-1}$$

$$f_{max} = \frac{252.05}{0.2 \times 10^3} = 1.26 \text{ Hz}$$

$$f_{avg} = \frac{252.05 + 2.8}{2} = 127.425 \text{ Hz}$$

6-step SVPWM (3/6 = T, duty cycle)

freq

$$\frac{1}{2} \sin \left(\frac{\pi}{6} \right) = \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{9.25}{2} = 1.26$$

(6-1) duty

$$\left(\frac{1}{2} \right)^2 = \frac{1}{4} \times \frac{9.25}{2} = 1.1875$$

[duty cycle]

$$\left(\frac{1}{2} \right)^2 \left[\frac{200.22 + 2.8}{(1.1875) + (1.26)} \right] = 0.95$$

Q1 A 3 phase $\frac{400}{\sqrt{3}}$ RMS, 50Hz, 4-pole-start connected squirrel cage induction motor is driven by a 3 phase two level voltage source inverter (VSI) operated with sinusoidal PWM. The VSI is fed from a 600 V DC source. The motor is controlled by maintaining constant V/f ratio. The equivalent circuit parameters are as given below.

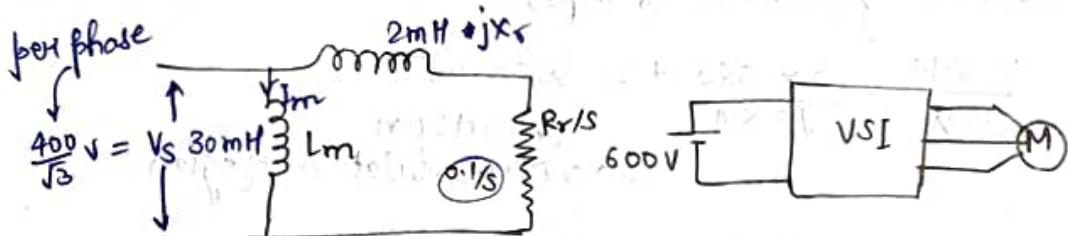
Rotor resistance, $R_r = 0.1\Omega$

Rotor leakage inductance, $L_r = 2 \text{ mH}$

Magnetizing inductance, $L_m = 30 \text{ mH}$

The other parameters can be neglected.

If the motor has to run at 700 RPM and produces a torque of 100 Nm. Determine the amplitude modulation index at which the inverter has to be inverted.



(per phase equivalent circuit)

Magnitude of magnetizing current, $I_m = \frac{V_s}{\omega_s L_m}$

$$\omega_s (\text{electrical}) = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s}$$

$$V_s = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$\frac{V_s}{\omega_s} = \frac{230.94}{100\pi} = 0.7351$$

$$\text{Torque output, } T = 3 \left(\frac{P}{2} \right) \frac{I_r^2 R_r (1-s)}{\omega r}$$

$$= 3 \times \frac{P}{2} \frac{I_r^2 R_r (1-s)}{s} = \frac{3 \left(\frac{P}{2} \right) I_r^2 R_r}{\omega_s (1-s)} \frac{s}{s}$$

$$= 3 \left(\frac{P}{2} \right) \left[\frac{V_s^2}{(R_r/s)^2 + (X_r)^2} \right] \left(\frac{R_r}{s} \right)$$

$$= \frac{3P/2}{s^2 \omega_s^2} \left[\frac{V_s^2 + s^2 \omega_s^2}{(R_r^2/s^2) + (\omega_s L_r)^2} \right] \left(\frac{R_r}{s} \right) \quad [X_r = \omega_s L_r]$$

$$= 3\left(\frac{P}{2}\right) \frac{V_s^2 R_r s_{ws}}{w_s^2 (R_r^2 + (s_{ws} l_r)^2)} \quad [s_{ws} = w_s - w_r = w_{slip}]$$

$$\approx 3\left(\frac{P}{2}\right) \frac{\left(\frac{V_s}{w_s}\right)^2 R_r \cdot w_{slip}}{R_r^2 + (w_{slip} l_r)^2}$$

For small values of s , $(w_{slip} l_r)^2$ can be neglected.

$$\Rightarrow T \approx 3\left(\frac{P}{2}\right) \left(\frac{V_s}{w_s}\right)^2 \left(\frac{R_r w_{slip}}{R_r^2}\right)$$

$$\Rightarrow \boxed{T \approx 3\left(\frac{P}{2}\right) \left(\frac{V_s}{w_s}\right)^2 \left(\frac{w_{slip}}{R_r}\right)}$$

$$T = 100 \text{ Nm}, \frac{V_s}{w_s} = 0.7357$$

$$\Rightarrow 100 = 3\left(\frac{P}{2}\right) \times (0.7357)^2 \left(\frac{w_{slip}}{0.1}\right)$$

$$\Rightarrow w_{slip} = 3.0843 \text{ rad/sec. (Electrical)}$$

Synchronous speed = $w_s = w_r + w_{slip}$

$$\begin{aligned} \text{Rotational speed} &= w_{s, \text{mech.}} = 700 \text{ RPM} = 700 \times \frac{2\pi}{360} = 23.3\pi \text{ rad/sec.} \\ &= 73.3 \text{ rad/sec.} \end{aligned}$$

$$\text{Rotor electrical speed, } w_{re} = w_{rm} \times \frac{P}{2} = 73.3 \times \left(\frac{4}{2}\right) = 146.61 \text{ rad/sec.}$$

$$\begin{aligned} \text{Synchronous speed, } w_{se} &= w_{re} + w_{slip} \\ &= 146.61 + 3.0843 \\ &= 149.692 \text{ rad/sec.} \end{aligned}$$

$$\text{Stator fundamental frequency} = \frac{149.692}{2\pi} = 23.82 \text{ Hz}$$

$$\text{Stator voltage to be applied, } = \left(\frac{V_s}{w_s}\right) \times 149.692 = 110.04 \text{ V}$$

For sinusoid PWM, $\uparrow \text{RMS value}$

$$\text{peak value of fundamental phase voltage} = m \frac{V_{dc}}{2}$$

$$V_{dc} = 600 \text{ V}$$

$$V_{s, \text{peak}} = 110.04 \sqrt{2} = 155.618 \text{ V}$$

$$\Rightarrow 155.618 = m \times \frac{600}{2}$$

$$\Rightarrow m = 0.5187$$

\nwarrow modulation index.

Q) The equivalent circuit parameter (for phase values) of a 3-phase, 440 V, 50 Hz, 4-pole, 1440 RPM star connected induction motor driven by a 3-phase voltage source inverter (VSI) is given below.

$$R_s = 0.2 \Omega, R_r = 0.4 \Omega$$

Stator leakage reactance, $X_s = 2 \Omega$

Rotor leakage reactance, $X_r = 2 \Omega$

Magnetising reactance, $X_m = 40 \Omega$

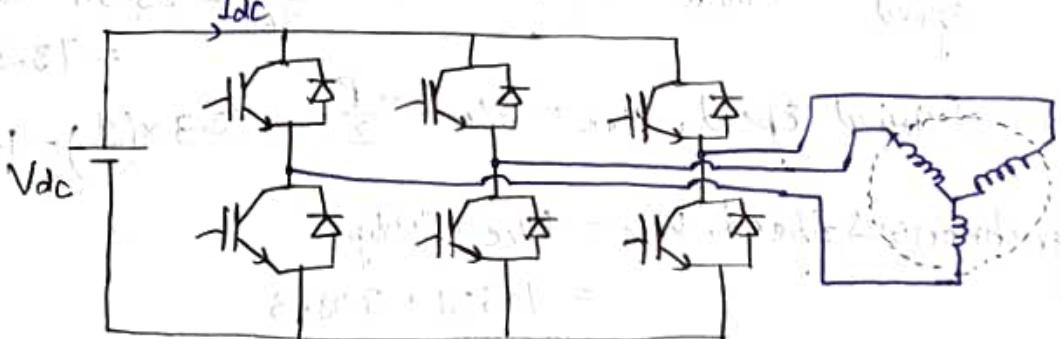
(All reactances at 50 Hz)

Resistance representing core loss = 200Ω .

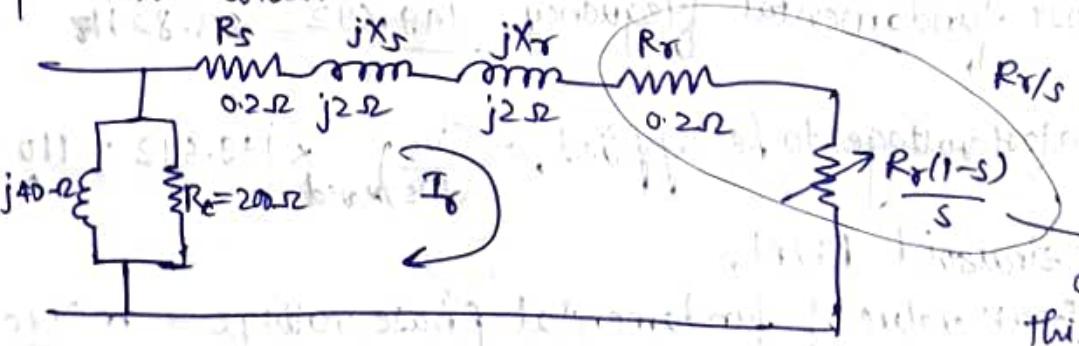
If the mechanical loss is ~~100 W~~, evaluated the following at stated speed :

- ① shaft torque of motor.
- ② efficiency of the motor.
- ③ DC voltage requirements of the inverter if it is operated in square mode.

Soln:



Equivalent circuit:



$$\text{Synchronous speed} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ RPM}$$

$$\text{Slip} = \frac{1500 - 1440}{1500} = 0.04$$

Power developed in this resistor is mechanical power

$$\text{Rotor current, } I_r = \frac{440 / \sqrt{3}}{\sqrt{\left(\frac{R_r}{S} + R_s\right)^2 + (X_s + X_r)^2}} = \frac{254}{\sqrt{10^2 + 4^2}} = 23.183 \text{ A.}$$

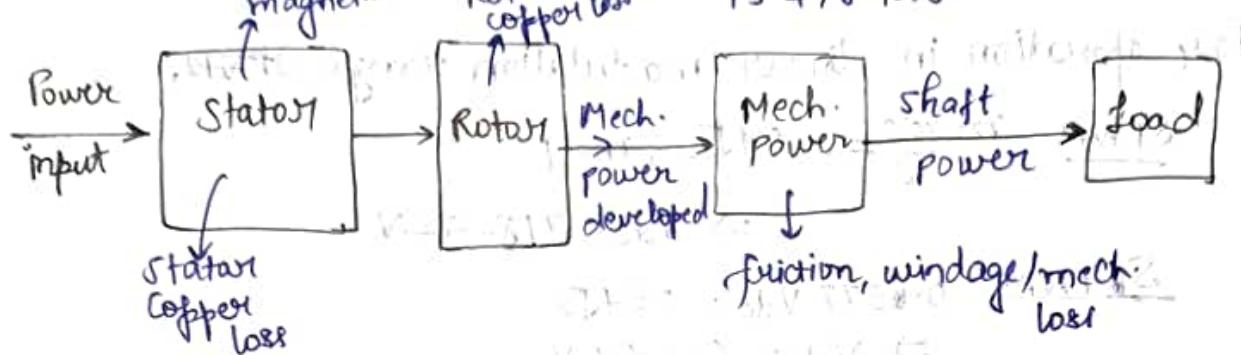
$$\text{Mechanical power developed} = 3 I_r^2 \times R_r \frac{(1-s)}{s}$$

↓
for all
3-phases

$$= 3 (23.183)^2 \frac{0.4 (1-0.04)}{0.01}$$

$$= 15478.6 \text{ W}$$

$$= 15.478 \text{ kW}$$



$$\text{Shaft power} = 15478.6 - 100 = 15378.6 \text{ W}$$

$$\uparrow \text{Mechanical loss}$$

$$\text{Shaft Torque} = \frac{\text{Shaft power}}{\text{Mechanical speed}} = \frac{15378.6}{(?)}$$

$$\text{Mechanical speed} = 1440 \text{ RPM}$$

$$= 1440 \times \frac{2\pi}{60}$$

$$= 48\pi \text{ rad/sec.}$$

$$\therefore \text{Shaft torque} = \frac{15378.6}{48\pi} = 101.982 \text{ N-m.}$$

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output Power}}{\text{Output Power} + \text{Losses}}$$

$$= \frac{\text{Output power}}{(\text{O/P power} + \text{mech. losses} + \text{rotor copper loss})}$$

total 3phases

$$\text{core loss} = \frac{3 \times 254^2}{200} = 967.74 \text{ W}$$

(O/P power + mech. losses + rotor copper loss) + core loss + stator copper loss

$\rightarrow P$ Total active power

$$\text{Total copper loss} = 3 I_s^2 (R_r + R_s) = 3 (23.123)^2 (0.2 + 0.4) \text{ W}$$

$$= 967.413 \text{ W}$$

$$\text{Efficiency} = \frac{15378.6}{15378.6 + 967.74 + 967.413 + 100} = 0.8831 = 88.31\%$$

Square wave mode operation:

$$\text{peak fundamental voltage} = \frac{V_{dc}/2}{\pi} = 254\sqrt{2}$$

$$\Rightarrow V_{dc} = 564.25 \text{ V}$$

For operation in linear modulation range ~~range~~,

$$\text{SPWM } m \frac{V_{dc}}{2} = 254\sqrt{2}, m=1$$

$$\Rightarrow V_{dc} = 718.42 \text{ V}$$

$$\text{SPWM } 0.0577 V_{dc} = 254\sqrt{2}$$

$$\Rightarrow V_{dc} = 622.54 \text{ V}$$

Apparent power, $S = \sqrt{P^2 + Q^2}$

$$\text{Active power, } P = \sqrt{3} VI \cos \phi$$

$$\text{Reactive power, } Q = \sqrt{3} I^2 \sin \phi$$

$$Q = \left(\frac{V_s^2}{X_m} + I_s^2 (X_L + X_R) \right) \times 3$$

$$\text{Power factor} = \frac{P}{S}$$

$$1 \text{ HP} = 746 \text{ W}$$

→ Inverter rating is based on apparent power (VA rating).

→ Reactive power is not supplied by DC source.

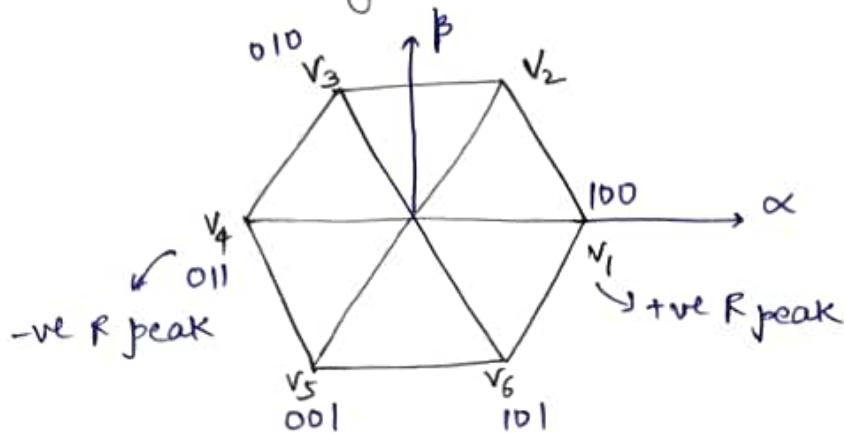
→ Inverter switching takes care of reactive power.
(circulating inside circuit)

→ $V_{dc} \times I_{dc}$ → Total active power.

→ Active power includes all losses also.

→ Depending on active-reactive current, switch rating can be determined.

Q) What is the magnitude of reference space vector?



So When reference vector coincides with α -axis, v_2 is the magnitude of reference vector.

$$V_x = \frac{3}{2} V_a.$$

#	111	011	001	000
	v_7	v_4	v_5	v_0
	$T_{0/2}$		$T_2 = \frac{V_{\text{ref}} \cdot \sin \theta}{V_{DC} \cdot \sin 60^\circ}$	$T_{0/2}$
	$T_1 = \left(\frac{V_{\text{ref}} \cdot \sin(60^\circ - \theta)}{V_{DC} \cdot \sin(60^\circ)} \right) T_S$			

$$T_0 = T_S - (T_1 + T_2), \quad T_S = \frac{1}{f_S}.$$