INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

Backlog Examination - May 2015

B. Tech - II Semester

MA121 - Vector Calculus and Differential Equations

Date: 06/05/2015 **Time: 9.30 am - 12.30 pm** **Max. Marks: 100**

SECTION A (Attempt all 10 questions - 10x5= 50 marks.)

1. Let a surface S be given by $\vec{r}(u,v) = (2\cos u, 2\sin u, v)$ where $u \in [0, 2\pi]$ and $v \in [-1, 1]$. Represent the surface S pictorially. Using surface integral find the surface area of S.

2. Let D be a elliptical region on xy-plane given by $D = \left\{ (x,y) \middle| \, \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \right\}$. Using Green's theorem find the area of the region D.

3. Define directional derivative. From the first principle find the directional derivative of the function $f(x,y,z) = xe^y + \sin z$ at the point $(1,1,0)$ along the vector $\vec{v} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

4. Find unit normal vector $\vec{n}$ to the surface S given by $x^2 + y^2 - z^2 + 1 = 0$ at the point $(1, 1, \sqrt{3})$. Find the parametric equation of the straight line L passing through the point $(1, 1, \sqrt{3})$ and parallel to the vector $\vec{n}$. Also find the point at which the straight line L intersect xy-plane.

5. Discuss the pointwise and uniform convergence of the sequence $f_n(x) = x^n(1-x),\ x \in [0,1]$.

6. Check whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{xe^{-nx}}{n^2}$ converges uniformly on $(0, +\infty)$.

7. Solve $\dfrac{dy}{dx} + y = xy^3$.

8. Solve $(x - 2y + 1)dy + (4x - 3y - 6)dy = 0$.

9. Prove or disprove the following:
   If y_1 and y_2 are two linearly independent solutions of $\dfrac{d^2y}{dx^2} + P(x)\dfrac{dy}{dx} + Q(x)y = 0$, then $c_1y_1 + c_2y_2$ is the general solution of this differential equation.

10. Using variation of parameter method, find the general solution of $\dfrac{d^2y}{dx^2} + y = \tan x$.

SECTION B (Attempt any 5 questions - 5x10= 50 marks.)

11. (a) Using Stoke's theorem find the surface integral $\int_S \vec{F}.d\vec{S}$ where $\vec{F} = (x^2, 0, 3y^2)$ and S is the upper hemisphere "$x^2 + y^2 + z^2 = 1,\ z \geq 0$ ". [5]

    (b) Verify divergence theorem for the vector field $\vec{F} = (x, 1, z)$ over the volume V : $x^2 + y^2 + z^2 \leq 1$. [5]

12. (a) Find the arc length function for the circle $\mathcal{C} := \{(x, y)|\ x^2 + y^2 = r^2\}$ where $r \in \mathbb{R}$ with initial point $(r, 0)$. Using this arc length function find the length of the curve formed by traversing 3 times the circle $\mathcal{C}$. [5]

(b) Let $\vec{F} = \left(\dfrac{x}{x^2 + y^2}, \dfrac{y}{x^2 + y^2}, e^z\right)$. Calculate $Curl(F)$. Find the domain of $\vec{F}$. Is $\vec{F}$ a conservative vector field? Evaluate $\int_C \vec{F}.d\vec{r}$ where $C := \{(cos\, t,\ sin\, t, t)|\ t \in [0, \pi]\}$. [5]

13. Write down the conditions on a sequence $\{f_n\}$, under which the following holds: $\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b \lim_{n\to\infty} f_n(x)dx$. Hence evaluate the following with appropriate justification: [10]

$$\lim_{n\to\infty} \int_1^2 \left(\frac{x^2+1}{8}\right)^n \sin nx\ dx.$$

14. State the Weierstrass test for uniform convergence of a series $\sum_{n=1}^{\infty} f_n(x)$ on an interval I.

Check whether the function $f(x) = \sum_{n=1}^{\infty} \dfrac{\cos^n x}{n^3}$ is differentiable on $(-\infty, +\infty)$. Justify your answer. [10]

15. Find the general solution of the following differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1)y = 0$$

[10]

16. (a) Discuss existence and uniqueness of solution of the following differential equation.

$$\frac{dy}{dx} = x^2 + y^2,\ y(0) = 0.$$

[4]

(b) Find the eigen (characteristic) values and eigen (characteristic) functions of the following differential equation.

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) = 0,\ y(\pi) = 0.$$

[6]

END