Indian Institute of Space Science and Technology (IIST)
End Semester Question Paper, November, 2022
Avionics, Electromagnetic and Wave Propagation (AV214), 3rd Semester

Marks: 100 Time: 3 Hrs

Answer all the questions. Make suitable assumptions if necessary

1. (a) A TE wave propagating in a dielectric filled rectangular waveguide of unknown permittivity has dimensions $a = 5$ cm and $b = 3$ cm. If the x-component of its electric field is given by

$$E_x = -36\cos(40\pi x)\sin(100\pi y)\sin(2.4\pi \times 10^{10}t - 52.9\pi z)\ \mathrm{V/m}$$

Find (i) mode number (ii) relative dielectric constant of the medium (iii) cut off frequency (iv) the expression of H_y. 2+3+2+3

(b) Design a rectangular air waveguide to transmit only the dominant mode in the frequency range 15-22 GHz such that the lower end of the frequency band is atleast 25% higher than the cut off frequency of the dominant mode, the cut off frequency of the next higher mode is atleast 25% higher than the center frequency of this band, and the power transmission in the dominant mode is maximized. What is the cut off frequency of the nearest mode(s) to the dominant mode? 4+3

(c) A rectangular waveguide of dimensions $a = 3$ cm and $b = 1.5$ cm has a dielectric discontinuity, as shown in Fig.1. A TE_{10} wave of frequency 6000 MHz is incident on the discontinuity from the free-space side. (i) Find the SWR in the free-space section. (ii) Find the length and the permittivity of a quarter-wave section required to achieve a match between the two media. Assume $\mu = \mu_o$ for the quarter-wave section. 4+4

2. (a) The electric field of a uniform plane wave propagating in perfect dielectric medium having $\mu = \mu_0$ is given by

$$\overline{E} = 10\cos\left(2\pi \times 10^7 t - 0.2\pi x\right)\hat{z}$$

Find (i) the frequency (ii) the wavelength (iii)the phase velocity (iv)the relative permittivity of the medium (v) the associated magnetic field. 1+1+1+2+2

(b) For a uniform plane wave having $\overline{E} = E_x(z,t)\hat{x}$ and $\overline{H} = H_y(z,t)\hat{y}$ and propagating in the +z direction in a perfect dielectric medium, the time variation of E_x in a constant z-plane and the distance variation of H_y for a fixed time are observed to be periodic as shown in Fig.2 for two complete cycle. Find the relative permittivity and permeability of the medium. 6

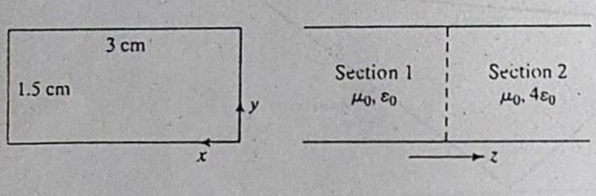

Figure 1: Question no 1(c)

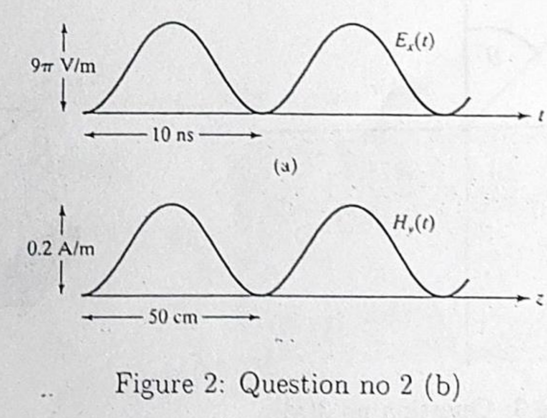

Figure 2: Question no 2 (b)

(c) A common material in dielectric radomes for aeronautical applications is fiberglass. For L-band (1-2 GHz), fiberglass has a typical relative dielectric constant of approximately $\varepsilon_r = 4.6$. (i) Assuming a flat plane radome, determine the minimum thickness of fiberglass that causes no reflections at the center of the L-band. (ii)Using the thickness found in (i), find the percentage of incident power which transmits to other side of the radome at each end of L-band frequency (i.e 1 GHz and 2 GHz). 3+5

(d) Consider the following complex phasor expression of a time harmonic electric field in free space

$$\overline{E} = (3\hat{x} + 4\hat{y} + j5\hat{z})e^{-j(8x-6y)\pi} \text{ mV/m}$$

(i) Is this a uniform plane wave? Justify your answer. What is the frequency? (ii) What is the direction of propagation and the state of polarization (specify type and sense)? (iii) Find the associated magnetic field. 1+1+2

3. (a) A uniform plane wave propagating in air has an electric field given by

$$\overline{E}_i(x, y) = E_0(0.5\hat{x} + 0.5\sqrt{3}\hat{y} - e^{j\pi/2}\hat{z})e^{-j2\sqrt{3}\pi x + j2\pi y}$$

where E_0 is a real constant. The wave is incident on the planar interface (located at $y = 0$) of a dielectric with $\mu_r = 1$ and $\varepsilon_r = 3$, as shown in Fig.3. (a) Find out the wave frequency and the angle of incidence. (b) What is the polarization (including sense of rotation if any) of the incident wave? (c) Write the complete expression of reflected electric field. (d) What is the polarization of the reflected wave. 2+2+3+2

(b) Consider a ground to air communication system as shown in Fig. 4. The receiver antenna is on an aircraft over a huge lake circling at a horizontal distance of 8 km from the transmitter antenna. The transmitter antenna is located right at the shore mounted on top of a 50 m tower above the lake water level overlooking the lake and the transmits a parallel polarized (with respect to plane of incidence) signal. The transmitter operates in VHF band. The pilot of the aircraft experiences a noise (unwanted signal) in his receiver due to the direct wave and the reflected wave and needs to adjust his altitude to minimize this interference. Assuming the lake to be flat and lossless with $\varepsilon_r = 79$, calculate the critical height of the aircraft in order to achieve clear transmission between the transmitter and receiver. 5

(c) An experiment is designed to measure the refractive index of a prism using the principle of total internal reflection. In this experiment, a plane polarized collimated monochromatic beam

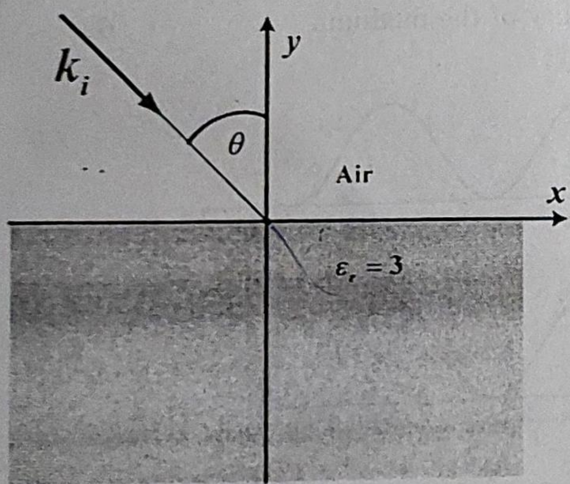

Figure 3: Question no 3(a)

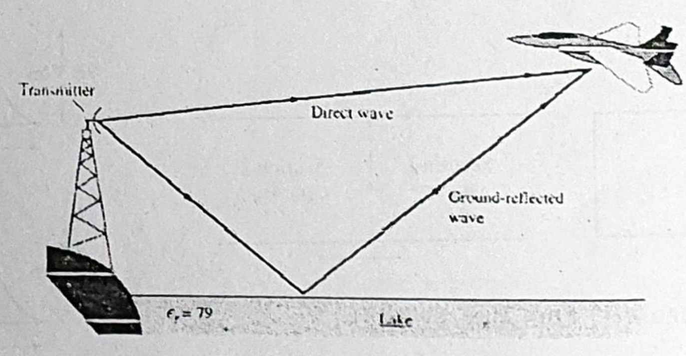

Figure 4: Question no 3(b)

of light is obliquely incident from one side of the prism at an incidence angle of θ_i as shown in Fig.5. The incidence angle θ_i of the beam is adjusted to a critical value ψ_c such that the incidence angle on the other side of the prism is equal to the critical angle of incidence (θ_c). Thus, by measuring the refracting angle A of the prism and the critical angle ψ_c, the refractive index n_p of the prism can be calculated. Show that

$$n_p^2 = 1 + \left(\frac{\sin\psi_c + \cos A}{\sin A}\right)^2$$

6

(d) A plane wave is incident at 45° upon a perfectly conducting screen located at $x = 0$. The plane wave consists of two components as follows:

$$\overline{E}_{i\perp} = \hat{y}E_0 e^{jk(x-z)/\sqrt{2}}$$

$$\overline{E}_{i\parallel} = (\hat{x} + \hat{z})\frac{jE_0}{\sqrt{2}} e^{jk(x-z)/\sqrt{2}}$$

(i) Write down the parallel and perpendicular components of the reflected electric field and show that the tangential component of electric field satisfy the boundary condition on the conducting screen. (ii) What are the polarizations of incident and reflected waves? 3+2

4. (a) A lossless transmission line is operating with $Z_0 = 40\Omega$, $f = 20$ MHz and $\beta = 7.5\pi$ rad/m. With a short circuit replacing the load, a minimum is found at a point on the line marked by a small spot. With the load installed, it is found that $s = 1.5$ and a voltage minimum is located 1 m toward the source from the spot. (i) Find the load impedance Z_L (ii) What is the value of load impedance which will produce the same VSWR with $|V_{max}|$ at the marked spot? **(Do not use Smith Chart)** 3+2

(b) For a lossless transmission line, the load reflection coefficient is expressed as, $\Gamma_L = 0.38\angle 138°$ and the wavelength along the line is is 60 mm. At a distance $l = 20.8$ mm from the load, the real part of complex line admittance appears to be approximately equal to the characteristic admittance of the line $Y_0 = 10$ mS. Design the admittance matching shunt short circuited stub that is to be connected at this location. **(Do not use Smith Chart)** 4

(c) A transmission line and load are given as shown in Fig. 6. It is required to calculate the length of the stubs so that the load is matched to the line. Use Smith chart to find out the length of the stubs. 12

(d) Draw and explain the voltage standing wave pattern along the length of transmission line having normalized load admittance $y_L = 0.3 - j1.6$. 4

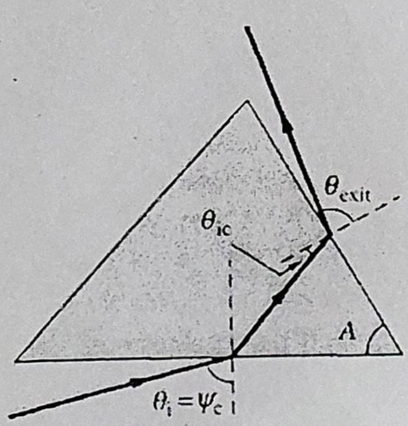

Figure 5: Question no 3(c)

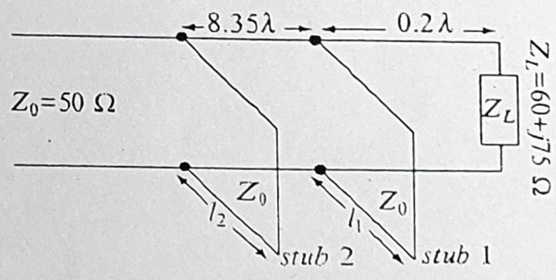

Figure 6: Question no 4

Universal constants: $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Reflection and transmission coefficient for oblique incidence:

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad \Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$T_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad T_\parallel = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Waveguide field equations:

Quantity	TE_{mn} Mode	TM_{mn} Mode
E_z	0	$B \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) e^{-j\beta z}$
H_z	$A \cos(\frac{m\pi x}{a}) \cos(\frac{n\pi y}{b}) e^{-j\beta z}$	0
E_x	$\frac{j\omega\mu n\pi}{k_c^2 b} A \cos(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) e^{-j\beta z}$	$\frac{-j\beta m\pi}{k_c^2 a} B \cos(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) e^{-j\beta z}$
E_y	$\frac{-j\omega\mu m\pi}{k_c^2 a} A \sin(\frac{m\pi x}{a}) \cos(\frac{n\pi y}{b}) e^{-j\beta z}$	$\frac{-j\beta n\pi}{k_c^2 b} B \sin(\frac{m\pi x}{a}) \cos(\frac{n\pi y}{b}) e^{-j\beta z}$
H_x	$\frac{j\beta m\pi}{k_c^2 a} A \sin(\frac{m\pi x}{a}) \cos(\frac{n\pi y}{b}) e^{-j\beta z}$	$\frac{j\omega\varepsilon n\pi}{k_c^2 b} B \sin(\frac{m\pi x}{a}) \cos(\frac{n\pi y}{b}) e^{-j\beta z}$
H_y	$\frac{j\beta n\pi}{k_c^2 b} A \cos(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) e^{-j\beta z}$	$\frac{-j\omega\varepsilon m\pi}{k_c^2 a} B \cos(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) e^{-j\beta z}$
k_c^2	$\sqrt{(m\pi/a)^2 + (n\pi/b)^2}$	$\sqrt{(m\pi/a)^2 + (n\pi/b)^2}$
k	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$
β	$\sqrt{k^2 - k_c^2}$	$\sqrt{k^2 - k_c^2}$

Propagation Constant:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1 + (\frac{\sigma}{\omega\varepsilon})^2} - 1\right]^{\frac{1}{2}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1 + (\frac{\sigma}{\omega\varepsilon})^2} + 1\right]^{\frac{1}{2}}$$