**INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY**
**THIRUVANANTHAPURAM 695 547**

## Summer Supplementary Examination - June 2016

**B. Tech - II Semester**

**MA121 - Vector Calculus and Differential Equations**

**Date: 29/06/2016** **Time: 9.30 am - 12.30 pm** **Max. Marks: 100**

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### SECTION A ( Answer all 10 questions - 10x5= 50 marks.)

1. Solve $y(y^2 - 2x^2)dx + x(2y^2 - x^2)dy = 0$.
2. Solve $2xyy' = y^2 - 2x^3,\ y(1) = 2$.
3. A body is heated to $110°C$ and placed in air at $10°C$. After 1 hour its temperature is $60°C$. How much additional time is required for it to cool to $30°C$ ?
4. Solve $\dfrac{d^2y}{dx^2} - y = 2x^4 - 3x + 1$.
5. Check the uniform convergence of $\dfrac{x^k}{1 + x^k}$ for $|x| > 1$.
6. Show that $\displaystyle\sum_{1}^{\infty} \frac{x}{(n + x^2)^2}$ converges uniformly for $x \in \mathbb{R}$.
7. Define directional derivative of a function $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ at a point $P_0$ along a vector $\vec{v}$. Check whether the directional derivative of $f(x, y, z) = e^{xy} + z$ at the point $P_0 = (1, 1, 1)$ along the vector $\vec{v} = (0, 4, 3)$ exists, and if exists find the value of the directional derivative. [2+1+2]
8. State Green's theorem over simply-connected regions. Let $D$ be a circular region on $xy$-plane given by $D = \left\{ (x, y) \middle| \ x^2 + y^2 \leq 4 \right\}$. With explanation find the area of the region $D$ using Green's theorem . [1+4]
9. Define arc length function of a continuously differentiable curve $\gamma : [a, b] \longrightarrow \mathbb{R}^3$. Find arc length function of the curve $C : y = 4x^2,\ z = 5,\ x \geq 0$ with initial point $(0, 0, 5)$. [2+3]
10. Let $\vec{\gamma} : [a, b] \longrightarrow \mathbb{R}^3$ be a non-constant smooth curve. Show that $\vec{\gamma}\,'(t) = \vec{0}$ for all $t \in [a, b]$ is not possible. Express $-\vec{\gamma}$ in terms of $\vec{\gamma}$. Is $-\vec{\gamma}$ a smooth curve? Justify your answer. [2+2+1]

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### SECTION B ( Answer any 5 questions - 5x10= 50 marks.)

11. Find the general solution of

$$8x^2y'' + 10xy' - (1 + x)y = 0.$$

12. (a) Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

    (b) Show that $J_1''(x) = -J_1(x) + \frac{1}{x} J_2(x)$.

13. (a) Show that $\int_0^{\pi/2} (\sum_{k=1}^{\infty} \frac{\cos(kx)}{k^2}) dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3}$.

(b) Check whether $f_n(x) = 1 - \frac{x^n}{n}$ converges on $[0, 1]$ and check the continuity of the limit function.

14. Let $f_n(x) = x - \frac{x^n}{n}$; $x \in [0, 1]$. Shows that sequence $\{f_n\}$ converges uniformly but the sequence $\{f_n'\}$ of its derivatives is not uniformly convergent.

15. (a) State Stoke's theorem. Verify Stoke's theorem for $\vec{F}(x, y, z) = (x, y, 0)$ over the surface $S : x^2 + y^2 + z^2 = 1, \ z \geq 0$ [5]

(b) Let $\vec{F}(x, y) = \left( \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$. Find the domain of $\vec{F}$. Calculate $Curl(\vec{F})$. Let $C : x^2 + y^2 = 1$. Find $\int_C \vec{F}$. Is the field $\vec{F}$ conservative? Justify your answer.

[0.5+1+ 2 + 1.5]

16. (a) Let $\vec{F}(x, y) = (e^y + ye^x, e^x + xe^y)$ for all $(x, y) \in \mathbb{R}^2$. Suppose $C = C_1 * C_2$ be a curve in $\mathbb{R}^2$ given by $C_1 : \frac{x^2}{4} + \frac{y^2}{9} = 1, \ y \geq 0$ oriented positively and $C_2$ is the straight line segment from $(-2, 0)$ to $(-4, 2)$. Find $\int_C \vec{F}$. Is the vector field $\vec{F}$ conservative? Justify your answer. [3+2]

(b) Let a surface $S$ is given by $\vec{r}(u, v) = (2\cos\ u, \ 2\sin\ u, \ v)$ where $u \in [0, 2\pi]$ and $v \in [-1, 1]$. Represent the surface $S$ pictorially. Find unit normal vector at any point $P$ on the surface $S$. Using surface integral find the surface area of $S$. [1+1+3]

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