Indian Institute of Space Science and Technology

Thiruvananthapuram

MA 211 - Integral Transforms

Instructor: Dr. Kaushik Mukherjee — Tutorial-2

---

1. (a) Show that $f(x) = x^2$ is of exponential order for any $\sigma > 0$, where n is a fixed positive integer.
   (b) Without using definition, show that $f(x) = (x+1)^2$ possesses the Laplace transform $\widehat{f}(s)$ for $s > 0$. Also, compute $\widehat{f}(s)$.

2. Let K be a postive real number.
   (a) Show that $f(x) = e^{Kx}$ is of exponential order for any $\sigma \geq \sigma_c = K$.
   (b) Without using definition, show that $f(x) = \sinh Kx = \dfrac{e^{Kx} - e^{-Kx}}{2}$, possesses the Laplace transform $\widehat{f}(s)$ for $s > K$. Also, compute $\widehat{f}(s)$.

3. (a) Show that $f(x) = \cos x$ is of exponential order for any $\sigma \geq \sigma_c = 0$.
   (b) Without using definition, show that $f(x) = (x+1)\cos(x+1)$ possesses the Laplace transform $\widehat{f}(s)$ for $s > 0$. Also, compute $\widehat{f}(s)$.

4. Show that if $f : [0, \infty) \to \mathbb{R}$ is piecewise continuous on $[0, A]$, for every real number $A > 0$ and is of exponential order with $\sigma > \sigma_c$, then show that $\lim_{s\to\infty} \widehat{f}(s) = 0$.

5. Consider the function $f(x) = \begin{cases} x, & \text{for} \quad 0 \leq x \leq a, \\ 10, & \text{for} \quad x > a. \end{cases}$

   Verify whether f is piece-wise continuous on every interval $[0, A]$, for $A > 0$ and is of exponential order for any $\sigma \geq \sigma_c = 0$. Compute $\widehat{f}(s)$ if it exits for $s > 0$.

6. Consider the function $f(x) = \begin{cases} 0, & \text{for} \quad 0 \leq x \leq a, \\ \frac{1}{x-a}, & \text{for} \quad x > a. \end{cases}$

   (a) Show that f is not piece-wise continuous on every interval $[0, A]$, for $A > 0$ but is of exponential order for any $\sigma \geq \sigma_c = 0$.
   (b) Does the above observation assure existence of the Laplace transform $\widehat{f}(s)$ for $s > 0$?

7. Consider the function $f(x) = \begin{cases} 0, & \text{for} \quad x = 0, \\ \frac{1}{\sqrt{x}}, & \text{for} \quad x > 0. \end{cases}$

   (a) Show that f is not piece-wise continuous on every interval $[0, A]$, for $A > 0$ but is of exponential order for any $\sigma \geq \sigma_c = 0$.
   (b) Verify whether f possesses the Laplace transform $\widehat{f}(s)$ for $s > 0$, using definition.
   (c) Does the above observation contradict the existence of $\widehat{f}(s)$ according the theorem of existence of the Laplace transform ?

8. Consider the function $f(x) = \begin{cases} 0, & \text{for} \quad x = 0, \\ \frac{1-\cos(x)}{x}, & \text{for} \quad x > 0. \end{cases}$

Verify whether f is piece-wise continuous on every interval $[0, A]$, for $A > 0$ and is of exponential order for any $\sigma \geq \sigma_c = 0$. Compute $\widehat{f}(s)$ if it exits for $s > 0$.