# Exploration II

1) A satellite of mass 1000 kg is circulating the earth in and orbit with speed of 10000 m/s at the closest approach (perigee) of 1000 km. The eccentricity of the orbit is 0.1. Find the distance of farthest approach (apogee), speed at apogee, angular momentum of the system and using Kepler’s third law calculate the time period of the orbit.

2) A two dimensional harmonic oscillator is a good example of central force. Write to the total energy equation for such an example. Plot Total energy, centrifugal barrier term, U(r) and effective potential $U_{eff}(r)$ as a function of r. Identify the conditions on total energy for different types of orbits.

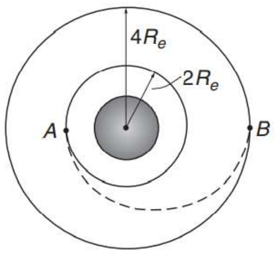
3) Prove that for a pair of particles interacting with a central force, the total angular momentum of the system is conserved and the trajectories of the particle will always stay in a plane.

4) Consider a system of two particles interacting by gravitational force. Write the potential of the interaction as well as the effective potential for the system. Show that the effective potential will have one and only one minima for finite value of r (assuming nonzero angular momentum).

5) A particle of mass 50 g moves under an attractive central force of magnitude $4r^3$ dynes. The angular momentum is equal to 1000 g· cm2/s. (a) Find the effective potential energy. (b) Indicate on a sketch of the effective potential the total energy for circular motion. (c) The radius of the particle’s orbit varies between $r_0$ and $2r_0$. Find $r_0$.

6) For what values of n are circular orbits stable with the potential energy $U(r) = -A/r^n$, where $A > 0$?

7) (Advanced problem. Not for quiz)




10.12 *Hohmann transfer orbit*

A space vehicle is in circular orbit about the Earth. The mass of the vehicle is 3000 kg and the radius of the orbit is $2R_e = 12\,800$ km. It is desired to transfer the vehicle to a circular orbit of radius $4R_e$.

(*a*) What is the minimum energy expenditure required for the transfer?

(*b*) An efficient way to accomplish the transfer is to use a semi-elliptical orbit (known as a Hohmann transfer orbit), as shown. What velocity changes are required at the points of intersection, *A* and *B*?

8) (Advanced problem. Not for quiz)

If the velocity or momentum vector of a particle is translated so as to start from the center of force, then the head of the vectors traces out the particle's hodograph, a locus curve of considerable antiquity in the history of mechanics but one that has had something of a revival in connection with the dynamics of space vehicles. By taking the cross product of L with the Laplace-Runge-Lenz vector A, show that the hodograph for elliptical Kepler motion is, in terms of the momentum, a circle of radius mk/l with origin on the y axis a distance A/l displaced from the center of force.