Indian Institute of Space Science and Technology

Thiruvananthapuram

MA211 - Linear Algebra

Tutorial-2

1. Find all eigenvalues and an eigenvector corresponding to each eigenvalue

   $(i)\ A = \begin{bmatrix} 4 & 0 \\ 0 & -6 \end{bmatrix}$

   $(ii)\ A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

   $(iii)\ A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

2. The product of two eigenvalues of $A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}$

   is -2. Find the third eigenvalue.

3. Verify Cayley-Hamilton theorem for

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

   and hence find A^{-1} and A^5.

4. Check whether the following are vector spaces or not.

   i. $V =$ Ker A, A is a m x n real matrix $Ker\ A = \{x \epsilon R^n / Ax = 0\}$.
   ii. $V = \{(x,\ y,\ z) \in R^3 / x + y + z = 1\}$.
   iii. $V = \{$set of entire polynomials P with real coefficients such that $p(0) = 0, p(1) = 0$ and $p(2) = 0\}$.
   iv. $V = \left\{ \begin{pmatrix} a & 1 \\ b & c \end{pmatrix} / a, b, c \in R \right\}$.
   v. $V = \{f : R \longrightarrow R / \frac{d^2 f}{dx^2} + f = 0\}$.
   vi. $V = \{3 \times 3$ real matrices with diagonal $1,\ 1,\ 1\}$.

5. Find whether the vectors are linearly independent or not.

   i. $\{(1, -3, 5), (2, 2, 4), (4, -4, 14)\}$ in R^3.
   ii. $\{-x^2, 1 + 4x^2\}$ in P_3.
   iii. $f(x) = x, g(x) = \frac{1}{x}$ in the vector space of all real valued functions from R^+ to R.
   iv. $\{1, \sin x, \sin 2x\}$ in $V = \{f : R \longrightarrow R\}$

6. Say True / False and Justify your answer

i. $\{u, v, w\}$ is linearly independent then $\{u, u+v, u+v+w\}$ also linearly independent.

ii. If $\{u, v, w\}$ is linearly independent then all its proper subsets are linearly independent.

iii. There is a set of four vectors in R^3, such that any three of which form a linearly independent set.

iv. Union of linearly independent sets again linearly independent.

7. Check whether the given set is a basis for the corresponding vector space.

   i. $\{(1, 2, 3), (3, 2, 1), (0, 0, 1)\}$ in R^3.

   ii. $\{(1, 1 + t, (1 + t)^2, (1 + t)^3)\}$ in P_3.

8. Find a basis for

   i. $V = \{\text{all } 2 \times 2 \text{ real matrices}\}$ over R.

   ii. $V = KerA$ over R, where $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 4 \end{bmatrix}$.

9. Find v such that

   i. $\{x, 1 + x^2, v\}$ is a basis for P_2.

   ii. $\{(1, 5), v\}$ is a basis for R^2.

10. Find a basis for the given subspace

    i. $W = xy$ plane in R^3

    ii. $W = \{(x, y, z)/3x + 2y + z = 0\}$ in R^3.

    iii. $W = \{\text{Polynomials } p(t) \text{ of degree } \leq 2 \text{ with real coefficients and } p(0) = 0, p(4) = 0\}$ in P_2.

11. Verify

    i. $\left\{ \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} /a + b = 0 \right\}$ is a subspace of the vector space of 2×2 real matrixes.

    ii. $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} /a, b\epsilon R \right\}$ is a subspace of the vector space of 2×2 real matrixes.

    iii. R^2 is a subspace of R^3.

    iv. Span $(\{x, x^2\})$ is a subspace of P_3.

12. Apply Gram-Schmidt process to find an orthonormal basis from the given basis

    i. $\{(1, 0, 3), (2, 2, 0), (3, 1, 2)\}$ for R^3 in this order.

    ii. $\{(1, 2, 0), (2, -1, 1), (-2, 6, 4)\}$ for R^3 in the given order.

13. Check whether the given maps are linear or not

    (a) $T : R \to R^3$ given by $T(x) = (x, x + 1, x + 2)$.

    (b) $T : R \to R^2$ given by $T(x) = (x, x^2)$.

    (c) $T : P_3 \to P_4$ by $T(p(x)) = p(x) + x^4$.

(d) $T : P_4 \to P_4$ by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_2x + a_3x^2$.

(e) $T : R^2 \to R^3$ by $T(x, y) = (x + 3, 2y, x + y)$

(f) T : { real sequences } $\to R$ by $T($ a sequence $) = n^{th} term + (n-1)^{th} term$.

14. Find range (T), ker (T) and their dimensions

    (a) $T : R^3 \to R^3$ by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2x)$

    (b) $T : P_3 \to P_3$ by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_0 - a_3x^2$.

    (c) $T : R^4 \to R^3$ by $T(x, y, z, u) = (x - y + z + u, x + 2z - u, x + y + 3z - 3u)$.

    (d) $T : R^4 \to R^3$ by $T(x, y, z, u) = (x+2y+3z+2u, 2x+4y+7z+5u, x+2y+6z+5u)$.

15. Find the matrix of T

    (a) $T : R^2 \to R^2$ by $T(x, y) = (2x - 7y, 4x + 3y)$ with respect to the same ordered basis $B = \{(1, 3), (2, 5)\}$ for domain and codomain.

    (b) $T : R^3 \to R^4$ by $T(x, y, z) = (x + y, y + z, x + z, x + y + z)$ with respect to the ordered basis $B_1 = \{(1, 0, 1),\ (1, 1, 0),\ (0, 1, 1)\}$ for R^3 and $B_2 = \{(0, 1, 1, 1),\ (1, 0, 1, 1),\ (1, 1, 0, 1),\ (1, 1, 1, 0)\}$ for R^4.

    (c) $T : P_3 \to P_2$ by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1x + a_2x^2$ with respect to the ordered basis $B_1 = \{1, x, x^2, x^3\}$ for P_3 and $B_2 = \{5, 5 + x, 5 + x^2\}$ for P_2.

16. Find $T(1, 3, 5)$, where $T : R^3 \to R^2$ has the matrix representation $[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ with respect to the ordered basis $B_1 = \{(1, 0, 1),\ (1, 1, 0),\ (0, 1, 1)\}$ for R^3 and $B_2 = \{(2, 3), (5, 3)\}$ for R^2.

17. Find $T(2, 7)$, where $T : R^2 \to R^4$ has the matrix representation $[T] = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$ with respect to the ordered basis $B_1 = \{(1, 1),\ (2,\ 3)\}$ for R^2 and $B_2 = \{(1, 0, 0, 0),\ (0, 1, 0, 0),\ (0, 0, 1, 0),\ (0, 0, 0,$ for R^4.

18. Let $[T] = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ be the matrix representation of $T : R^3 \to R^3$ with respect to the ordered basis $B = \{(1, 0, 0),\ (0, 1, 1),\ (0, 0, 1)\}$ for both domain and codomain. Find T.

19. Let $T : R^2 \to R^4$ has the matrix representation $[T] = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$ with respect to the ordered basis $B_1 = \{(1, 1),\ (2,\ 3)\}$ for R^2 and $B_2 = \{(1, 0, 0, 0),\ (0, 1, 0, 0),\ (0, 0, 1, 0),\ (0, 0, 0, 1)\}$ for R^4.
    Find T.

20. Let $[T] = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$ be the matrix representation of $T : R^3 \to R$ with respect to the ordered basis $B_1 = \{(1, 0, 0),\ (0, 1, 1),\ (0, 0, 2)\}$ for R^3 and $B_2 = \{8\}$ for R. Find T.

21. Let $[T] = \begin{bmatrix} 8 & 11 \\ -6 & -11 \end{bmatrix}$ be the matrix representation of $T : R^2 \to R^2$ with respect to the ordered basis $B_1 = \{(1, -2),\ (2, 5)\}$ for R^2 and $B_2 = \{(1, -2),\ (2, 5)\}$ for R^2. Find T.

Assignment-II

Submit the answers of questions 2, 4(ii), 4(vi), 5(ii), 7(ii), 8(ii), 11(ii), 12(i), 13(c), 14(c), 15(c), 19 on or before 22-11-2023.