Homework-1

AV221 - Semiconductor Devices

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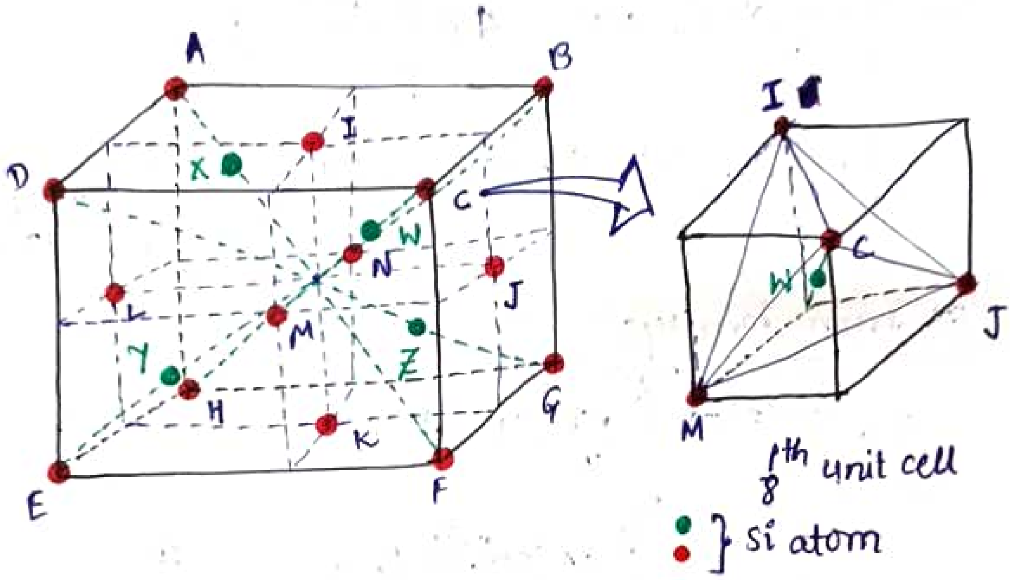
SC22B146

① Draw the crystal structure of Silicon and GaAs.

Soln: Si has diamond structure in which the silicon atoms form FCC as well as the alternate tetrahedral voids.



GaAs has zinc blend crystal structure similar to diamond structure in which As ~~form~~ forms FCC and Ga takes alternate tetrahedral voids

● → As
● → Ga

② Calculate the number of atoms present in 1 cm³ of silicon.

Soln: Density of Si material = 2.33 g/cm³

⇒ Weight of 1cm³ Si = 2.33 g

⇒ Moles of Si = $\frac{2.33}{28.08}$ [Molar mass of Si = 28.08 g/mol.]

$= 0.083$

⇒ No. of Si atoms $= 0.083 \times 6.023 \times 10^{23}$ [Avogadro's no, $N_A = 6.023 \times 10^{23}$]

$\approx 5 \times 10^{22}$

∴ There are $\sim 5 \times 10^{22}$ Si atoms in 1 cm³ of Si.

(3) Using the density of states and the Fermi probability function, derive the expression for the carrier density of holes in the valence band and electrons in the conduction band in terms of Fermi integrals. Invoking Boltzmann approximation, recalculate the holes and electron densities. Assume that the semiconductor is moderately doped.

Soln: Density of state, $g(E) = \frac{4\pi (2m^*)^{3/2}}{h^3}\sqrt{E}$

Fermi probability function, $f(E) = \frac{1}{1+e^{\left[\frac{E-E_f}{KT}\right]}}$

Electron density:

$$n_0 = \int_{E_c}^{\infty} n(E)\,dE = \int_{E_c}^{\infty} g_{CB}(E)\,f(E)\,dE$$

$$= \int_{E_c}^{\infty} \frac{4\pi (2m_e^*)^{3/2}}{h^3}\sqrt{E-E_c}\cdot\frac{dE}{1+e^{\left(\frac{E-E_f}{KT}\right)}}$$

$$= K'\int_{E_c}^{\infty} \frac{\sqrt{E-E_c}}{1+e^{\left(\frac{E-E_f+E_c-E_c}{KT}\right)}}\,dE \quad , \text{ where } K' = \frac{4\pi (2m_e^*)^{3/2}}{h^3}$$

Let $\frac{E-E_c}{KT} = u$, and $\frac{E_f-E_c}{KT} = u_f$

$\Rightarrow dE = KT\,du$

| when $E = E_c \Rightarrow u = 0$
$E = \infty \Rightarrow u = \infty$

$$\therefore n_0 = K'\int_{0}^{\infty} \frac{\sqrt{u}\cdot\sqrt{KT}\cdot KT\,du}{1+e^{(u-u_f)}}$$

$$= 4\pi\left(\frac{2KTm_e^*}{h^2}\right)^{3/2}\int_0^{\infty}\frac{u^{1/2}\,du}{1+e^{(u-u_f)}}$$

$$\Rightarrow \boxed{n_0 = N_c \underbrace{\int_0^{\infty}\frac{u^{1/2}\,du}{1+e^{(u-u_f)}}}_{\text{Fermi's half integral}}}$$, where $N_c = 4\pi\left(\frac{2KTm_e^*}{h^2}\right)^{3/2}$ is the effective density of states of CB.

For $E \gg E_f$ or $(E - E_f) \gg KT$,

$$f(E) = \frac{1}{1+e^{(E-E_f)/KT}} \approx \frac{1}{e^{\frac{E-E_f}{KT}}} = e^{-\left(\frac{E-E_f}{KT}\right)}$$

[Boltzmann approximation]

$$\therefore n_0 = \int_{E_c}^{\infty} g_{CB}(E)\, f(E)\, dE$$

$$= K' \int_{E_c}^{\infty} e^{-\left(\frac{E-E_f+E_c-E_c}{KT}\right)} \sqrt{E-E_c}\; dE$$

Let $u = \frac{E-E_c}{KT}$

$\Rightarrow du = \frac{dE}{KT}$

$$\therefore n_0 = K' e^{-\frac{(E_c-E_f)}{KT}} \int_0^{\infty} u^{1/2} \sqrt{KT}\, e^{-u} \cdot KT\, du$$

$$= K' (KT)^{3/2} e^{-\left(\frac{E_c-E_f}{KT}\right)} \underbrace{\int_0^{\infty} e^{-u} u^{1/2}\, du}_{\text{gamma function } = \sqrt{\pi}/2}$$

$$= \cancel{4}^{2}\pi \frac{(2m_e^*)^{3/2}}{h^3} (KT)^{3/2} e^{-\left(\frac{E_c-E_f}{KT}\right)} \cdot \frac{\sqrt{\pi}}{\cancel{2}}$$

$$\Rightarrow \boxed{n_0 = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} e^{-\frac{(E_c-E_f)}{KT}}}$$

<u>Hole density:</u>

$$P_0 = \int_{-\infty}^{E_v} p(E)\, dE = \int_{-\infty}^{E_v} (1-f(E))\, g_{VB}\, dE$$

$$= \int_{-\infty}^{E_v} \frac{e^{\left(\frac{E-E_f}{KT}\right)}}{1+e^{\left(\frac{E-E_f}{KT}\right)}} \cdot 4\pi \frac{(2m_h^*)^{3/2}}{h^3} \sqrt{E_v - E}\; dE$$

$$= K'' \int_{-\infty}^{E_v} \frac{e^{\left(\frac{E-E_f+E_v-E_v}{KT}\right)}}{1+e^{\left(\frac{E-E_f+E_v-E_v}{KT}\right)}} \sqrt{E_v - E}\; dE, \text{ where}$$

$$K'' = 4\pi \frac{(2m_h^*)^{3/2}}{h^3}$$

Let $\frac{E_v - E}{KT} = u'$

$\Rightarrow dE = -KT\, du'$

when $E = -\infty$, $u = \infty$

$E = E_v$, $u = 0$

and, $u_f' = \frac{E_v - E_f}{KT}$

$$\therefore p_0 = K'' \int_{\infty}^{0} \frac{e^{(u_f' - u')} + 1 - 1}{1 + e^{(u_f' - u')}} \sqrt{KT}\sqrt{u'}.(-KT)\, du'$$

$$= 4\pi \left(\frac{2KT\, m_h^*}{h^2}\right)^{3/2} \int_0^{\infty} \left(1 - \frac{1}{e^{(u_f' - u')}}\right) u'^{1/2}\, du'$$

For $E >> E_f$ or $(E - E_f) >> KT$,

$$f(E) \approx e^{-\frac{(E - E_f)}{KT}} \quad \text{[Boltzmann approximation]}$$

$$\therefore p_0 = \int_{-\infty}^{E_v} (1 - f(E))\, g_{VB}(E)\, dE$$

$$= K'' \int_{-\infty}^{E_v} \left[1 - e^{-\frac{(E - E_f + E_v - E_v)}{KT}}\right] \sqrt{E_v - E}\, dE$$

Let $u' = \frac{E_v - E}{KT}$

$\Rightarrow dE = -KT\, du'$

$$\therefore p_0 = K'' \int_{\infty}^{0} \left[1 - e^{u'}\left(e^{\frac{E_f - E_v}{KT}}\right)\right] \sqrt{u'}.\sqrt{KT}.(-KT\, du')$$

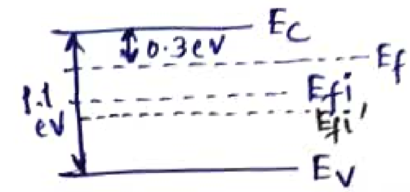
$$= K'' (KT)^{3/2} \int_0^{\infty} \left(1 - e^{u'} e^{\frac{E_f - E_v}{KT}}\right) u'^{1/2}\, du'$$

$$\Rightarrow \boxed{p_0 = 2\left[\frac{2\pi m_h^* KT}{h^2}\right]^{3/2} e^{-\left(\frac{E_f - E_v}{KT}\right)}}$$

(4) For an n-type Si sample, the Fermi energy is assumed to be 0.3 eV below the conduction band edge. Calculate the electron and hole concentrations in the sample at 300K. For Silicon, assume that the Bandgap is 1.1 eV, $n_i = 1.5\times10^{10}/cm^{-3}$. You can assume that the intrinsic Fermi level is at the mid-bandgap. Recalculate the values by using the exact relation for the position of intrinsic Fermi level, and compare the results. [Use $m_e = 0.98\,m_0$, $m_h = 0.49\,m_0$.]

Soln: Given that $E_g = 1.1\,eV$

$$\Rightarrow E_c - E_{fi} = 0.55\,eV$$



$$\therefore E_f - E_{fi} = 0.55 - 0.3\,eV$$
$$= 0.25\,eV$$

Using $n_0 = n_i \exp\left(\frac{E_f - E_{fi}}{KT}\right)$

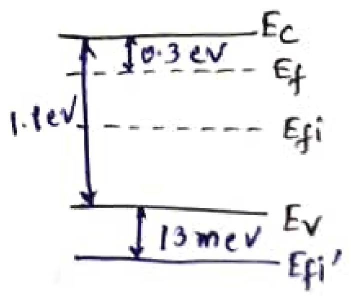
$$= 1.5\times10^{10} \exp\left(\frac{0.25\,eV}{25\,meV}\right) \qquad [KT \approx 25\,meV \text{ at } 300K]$$
$$= 1.5\times10^{10} \times 22{,}026.47$$
$$= 3.3\times10^{14}\,/cm^3.$$

and, $p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{3.3\times10^{14}} = 6.8\times10^{5}/cm^3.$

Exact value of E_{fi}:

$$E_{fi}' = \left(\frac{E_c + E_v}{2}\right) + \frac{3}{4}KT \ln\left[\frac{m_h}{m_e}\right]$$

$$\Rightarrow E_{fi}' - E_v = \frac{3}{4}(25\,meV) \ln\left[\frac{0.49}{0.98}\right]$$
$$\approx -13\,meV$$



$$\therefore E_f - E_{fi} = 0.55\,eV + 13\times10^{-3}\,eV$$
$$= 0.563\,eV$$

Now, $n_0 = 1.5\times10^{10} \exp\left(\frac{0.563}{25m}\right)$

$$= 9.04\times10^{19}\,/cm^3$$

and, $p_0 = \frac{(1.5\times10^{10})^2}{9.04\times10^{19}} = 2.49\,/cm^3.$