Indian Institute of Space Science and Technology

Thiruvananthapuram

MA 211 - Fourier Series and Integral

Instructor: Dr. Kaushik Mukherjee

---

1. (a) Find the Fourier series for the square wave function $f : [-\pi, \pi] \to \mathbb{R}$ by

$$f(x) = \begin{cases} -1, & \text{for} \quad -\pi \leq x \leq 0 \\ 1, & \text{for} \quad 0 < x \leq \pi. \end{cases}$$

   (b) Hence, deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$.

2. (a) Find the Fourier series for the periodic function $f : \mathbb{R} \to \mathbb{R}$ with period 2π defined on $[-\pi, \pi)$ by

$$f(x) = \begin{cases} -\pi, & \text{for} \quad -\pi \leq x < 0 \\ x, & \text{for} \quad 0 \leq x < \pi. \end{cases}$$

   (b) Hence, deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

3. Consider the function $f : [0, \pi] \to \mathbb{R}$ defined by $f(x) = |\cos x|$. Find (a) Fourier series for f, (b) Fourier sine series for f, (c) Fourier cosine series for f.

   Sketch the graph of the functions represented by each of these series on $[-3\pi, 3\pi]$ (Justify your answer).

4. (a) Find the Fourier integral of the function $f : \mathbb{R} \to \mathbb{R}$, defined by

$$f(x) = \begin{cases} |\sin(x)|, & \text{for} \quad |x| \leq \pi, \\ 0, & \text{otherwise}, \end{cases}$$

   (b) Using (a), show that

$$\int_0^\infty \frac{\cos \lambda\pi + 1}{1 - \lambda^2} \cos \frac{\pi\lambda}{2} \, d\lambda = \frac{\pi}{2}.$$

5. Establish the following identity using the Fourier integral

$$\int_0^\infty \frac{\sin \lambda}{\lambda} \cos \lambda x \, d\lambda = \begin{cases} \frac{\pi}{2}, & \text{for} \quad |x| < 1, \\ 0, & \text{for} \quad |x| > 1. \end{cases}$$
