AV 323 - Communication Systems II
Assignment II

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SC22B146

1. A computer puts out binary data at the rate of 56 kilobits per second. The computer output is transmitted using a baseband binary PAM system that is designed to have a raised-cosine pulse spectrum. Determine the transmission bandwidth required for each of the following roll-off factors: (a) $\alpha = 0.25$, (b) $\alpha = 0.5$, (c) $\alpha = 0.75$, and (d) $\alpha = 1.0$.

Soln: Raised-cosine filter frequency response,

$$H(f) = \begin{cases} 1 & , |f| \le \frac{1-\alpha}{2T} \\ \frac{1}{2}\left[1+\cos\left(\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right)\right] & , \frac{1-\alpha}{2T} < |f| \le \frac{1+\alpha}{2T} \\ 0 & , |f| > \frac{\alpha+1}{2T} \end{cases}$$

$\therefore$ Total transmission BW

$$= \frac{1+\alpha}{2T} = \frac{(1+\alpha)}{2} R_b \quad , \alpha: \text{roll-off factor.}$$

(a) $\alpha = 0.25 \Rightarrow BW = \frac{56k}{2}(1+0.25) = 28 \times 1.25k = 35$ KHZ

(b) $\alpha = 0.5 \Rightarrow BW = 28 \cdot (1+0.\cancel{75}) = 28 \times 1.\cancel{75}k = 42$ KHZ

(c) $\alpha = 0.75 \Rightarrow BW = 28\ (1.75)k = 49$ KHZ.

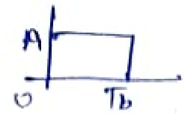
(d) $\alpha = 1.0 \Rightarrow BW = 28(2.0)k = 56$ KHZ.

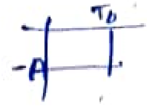
2. A digital source ~~bits~~ puts out a bit sequence $(B_1, B_2, B_3, \dots)$ which is assumed to be an IID random process with $B_K \in \{0,1\}$ and $P_r\{B_K = 1\} = \frac{1}{2}$. We will consider a baseband comm. system in which the above bits are transmitted using a bipolar NRZ line code with amplitude of A and signalling time of T_b. Suppose be the signal transmitted into the baseband channel. Write down an expression for $x(t)$. Due to ISI in the baseband comm. channel the received signal is

$$Y(t) = x(t) + 0.5\, x(t - T_b).$$

Assume that the receiver has timing synchronization and samples the received signal from the baseband channel at the middle of each bit interval T_b in order to obtain the samples $y[n]$. The $y[n]$ samples are fed into a threshold decoder with a threshold value of 0 in order to decode the bits. Derive the probability of bit error for the above receiver.

Soln: $p(t)$: NRZ pulse (Bipolar)

1: 

0: 

Transmitted signal,

$$x(t) = \sum_i A_i \cdot p(t - iT_b)$$

Received signal,

$$y(t) = x(t) + 0.5\,x(t - T_b)$$

Sampling at middle of each bit interval,

$$y(n) = y(nT_b + T_b/2)$$

$$= x(nT_b + T_b/2) + 0.5\,x(nT_b - T_b/2).$$

As only the pulse from current and previous bit contribute,

$$x(nT_b + T_b/2) = A_n$$

$$x(nT_b - T_b/2) = A_{n-1}$$

$$\therefore\; y(n) = A_n + 0.5\,A_{n-1}$$

Thresholding: Case-(i) $A_n = A$

(a) $A_{n-1} = A \Rightarrow y(n) = A + 0.5A = 1.5A > 0 \rightarrow$ correct

(b) $A_{n-1} = -A \Rightarrow y(n) = A - 0.5A = 0.5A > 0 \rightarrow$ correct

Case-(ii) $A_n = -A$

(a) $A_{n-1} = A \Rightarrow y(n) = -A + 0.5A = -0.5A < 0 \rightarrow$ correct

(b) $A_{n-1} = -A \Rightarrow y(n) = -A - 0.5A = -1.5A < 0 \rightarrow$ correct.

$\therefore$ No error without noise.

Including AWG Noise:

$$n(t) \sim N(0, \sigma^2) = N(0, N_0/2)$$

$$y(n) = A_n + 0.5A_{n-1} + \cancel{n(n)}\, N$$

For $A_n = A$, $A_{n-1} = A$,

$$y(n) = A + 0.5A + n(n) = 1.5A + \cancel{n(n)}\, N$$

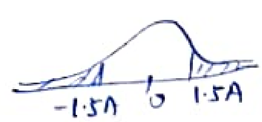
$$P_r(\text{error}) = P_r\{1.5A + N < 0\}$$

$$= P_r\{N < -1.5A\}$$

$$= P_r\{N > 1.5A\}$$

$$= P_r\left\{\frac{N}{\sqrt{N_0/2}} > \frac{1.5A}{\sqrt{N_0/2}}\right\}$$

$$= Q\left(\frac{1.5A}{\sqrt{N_0/2}}\right)$$



For $A_n = A,\ A_{n-1} = -A$,

$$y(n) = A - 0.5A + N = 0.5A + N.$$

$$P_r\{\text{error}\} = P_r\{0.5A + N < 0\}$$

$$= P_r\left\{\frac{N}{\sqrt{N_0/2}} < -\frac{0.5A}{\sqrt{N_0/2}}\right\} = P_r\left\{\frac{N}{\sqrt{N_0/2}} > \frac{0.5A}{\sqrt{N_0/2}}\right\}$$

$$= Q\left(\frac{0.5A}{\sqrt{N_0/2}}\right)$$

For $A_n = -A,\ A_{n-1} = A$,

$$y(n) = -A + A(0.5) + N = -0.5A + N$$

$$P_r\{\text{error}\} = P_r\{-0.5A + N > 0\}$$

$$= P_r\left\{\frac{N}{\sqrt{N_0/2}} > +\frac{0.5A}{\sqrt{N_0/2}}\right\}$$

$$= Q\left(\frac{0.5A}{\sqrt{N_0/2}}\right)$$

For $A_n = -A,\ A_{n-1} = -A$,

$$y(n) = -A - 0.5A + N = -1.5A + N$$

$$P_r\{\text{error}\} = P_r\{-1.5A + N > 0\}.$$

$$= P_r\left\{\frac{N}{\sqrt{N_0/2}} > \frac{1.5A}{\sqrt{N_0/2}}\right\}$$

$$= Q\left(\frac{1.5A}{\sqrt{N_0/2}}\right).$$

$\therefore$ Total error probability

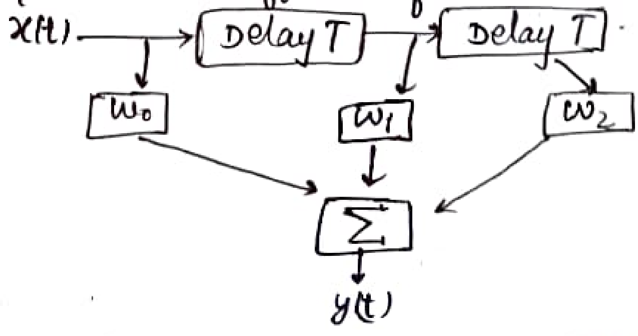
$$= \frac{1}{4}Q\left(\frac{1.5A}{\sqrt{N_0/2}}\right)\times 2 + \frac{1}{4}Q\left(\frac{0.5A}{\sqrt{N_0/2}}\right)\times 2$$

$$= \frac{1}{2}Q\left(\frac{1.5A}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{0.5A}{\sqrt{N_0/2}}\right).$$

③ consider a channel, the output of which in response to a signal $s(t)$ is defined by

$$x(t) = a_1 s(t - t_1) + a_2 s(t - t_2),$$

in the absence of noise. This models a channel with multipath distortion. Suppose we use the following tapped delay line equalizer to equalize the effect of the channel.



(i) Obtain the transfer function of the channel.

(ii) Obtain the parameters of the tapped delay line equalizer in terms of a_0, a_1, t_1 and t_2 which can be used for approximate zero forcing equalization of the channel.

Sol^n: (i) Channel:

$$x(t) = a_1 \delta(t-t_1) + a_2 \delta(t-t_2)$$

$$\Rightarrow X(f) = a_1 S(f) e^{-j2\pi f t_1} + a_2 S(f) e^{-j2\pi f t_2}$$

$$\Rightarrow H(f) = \frac{X(f)}{S(f)} = a_1 e^{-j2\pi f t_1} + a_2 e^{-j2\pi f t_2} \rightarrow \text{T.F. of the channel}$$

(ii) Equalizer:

$$y(t) = w_0 x(t) + w_1 x(t-T) + w_2 x(t-2T)$$

$$= w_0 a_1 \delta(t-t_1) + w_0 a_2 \delta(t-t_2) + w_1 a_1 \delta(t-t_1-T) + w_1 a_2 \delta(t-t_2-T) + w_2 a_1 \delta(t-t_1-2T) + w_2 a_2 \delta(t-t_2-2T)$$

Sampling at mid of bit-time T,

$$y(mT+T/2) = w_0 a_1 \delta(mT+\tfrac{T}{2}-t_1) + w_0 a_2 \delta(mT+T/2-t_2) + w_1 a_1 \delta(mT+T/2-T-t_1) + a_2 w_1 \delta(mT-\tfrac{T}{2}-t_2) + w_2 a_1 \delta((m-1)T-\tfrac{T}{2}-t_1) + w_2 a_2 \delta((m-1)T-\tfrac{T}{2}-t_2),$$

$$m = 0, 1, 2, \ldots$$

To zero-force ISI,

$$g(T/2) = 1; \quad g(mT+\tfrac{T}{2}) = 0, \; m \neq 0.$$

$m=0$: $w_0 a_1 \delta(\tfrac{T}{2}-t_1) + w_0 a_2 \delta(\tfrac{T}{2}-t_2) + w_1 a_1 \delta(-\tfrac{T}{2}-t_1) + a_2 w_1 \delta(-\tfrac{T}{2}-t_2) + a_1 w_2 \delta(-\tfrac{3T}{2}-t_1) + w_2 a_2 \delta(-\tfrac{3T}{2}-t_2) = 1$

$m=1$: $w_0 a_1 \delta(\tfrac{3T}{2}-t_1) + w_0 a_2 \delta(\tfrac{3T}{2}-t_2) + a_1 w_1 \delta(\tfrac{T}{2}-t_1) + a_2 w_1 (\tfrac{T}{2}-t_2) + w_2 a_1 \delta(-\tfrac{T}{2}-t_1) + w_2 a_2 \delta(-\tfrac{T}{2}-t_2) = 0.$

$m=2$: $w_0 a_1 \delta(\tfrac{5T}{2}-t_1) + w_0 a_2 \delta(\tfrac{5T}{2}-t_2) + w_1 a_1 \delta(\tfrac{3T}{2}-t_1) + a_2 w_1 \delta(\tfrac{3T}{2}-t_2) + w_2 a_1 \delta(\tfrac{T}{2}-t_1) + w_2 a_2 \delta(\tfrac{T}{2}-t_2) = 0$

Assuming $\frac{T}{2} > t_1, t_2$ and to maintain causality,

$$\Rightarrow w_0\left[a_1 \delta\left(\tfrac{T}{2}-t_1\right) + a_2 \delta\left(\tfrac{T}{2}-t_2\right)\right] = 1,$$

$$w_0\left[a_1 \delta\left(\tfrac{3T}{2}-t_1\right) + a_2 \delta\left(\tfrac{3T}{2}-t_2\right)\right] + w_1\left[a_1 \delta\left(\tfrac{T}{2}-t_1\right) + a_2 \delta\left(\tfrac{T}{2}-t_2\right)\right] = 0$$

$$w_0\left[a_1 \delta\left(\tfrac{5T}{2}-t_1\right) + a_2 \delta\left(\tfrac{5T}{2}-t_2\right)\right] + w_1\left[a_1 \delta\left(\tfrac{3T}{2}-t_1\right) + a_2 \delta\left(\tfrac{3T}{2}-t_2\right)\right] + w_2\left[a_1 \delta\left(\tfrac{T}{2}-t_1\right) + a_2 \delta\left(\tfrac{T}{2}-t_2\right)\right] = 0$$

$$\Rightarrow w_0 = \frac{1}{a_1 \delta\left(\tfrac{T}{2}-t_1\right) + a_2 \delta\left(\tfrac{T}{2}-t_2\right)},$$

$$w_1 = -\frac{\left[a_1 \delta\left(\tfrac{3T}{2}-t_1\right) + a_2 \delta\left(\tfrac{3T}{2}-t_2\right)\right]}{\left[a_1 \delta\left(\tfrac{T}{2}-t_1\right) + a_2 \delta\left(\tfrac{T}{2}-t_2\right)\right]^2},$$

$$w_2 = \frac{-\left[a_1 \delta\left(\tfrac{5T}{2}-t_1\right) + a_2 \delta\left(\tfrac{5T}{2}-t_2\right)\right]\left[a_1 \delta\left(\tfrac{T}{2}-t_1\right) + a_2 \delta\left(\tfrac{T}{2}-t_2\right)\right] + \left[a_1 \delta\left(\tfrac{3T}{2}-t_1\right) + a_2 \delta\left(\tfrac{3T}{2}-t_2\right)\right]^2}{\left[a_1 \delta\left(\tfrac{T}{2}-t_1\right) + a_2 \delta\left(\tfrac{T}{2}-t_2\right)\right]^3}.$$