Surprise Quiz Solution

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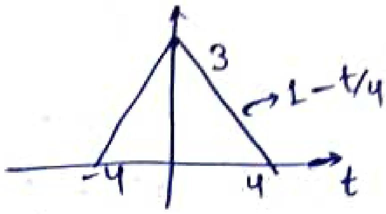
SC22B146

① Find the signal energy of $x(t) = 3\,\text{tri}(t/4)$.

Soln: Signal energy,

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$= \int_{-\infty}^{\infty} |3(\text{tri}\,t/4)|^2\,dt$$



$$= 9 \times 2 \int_0^4 (1 - t/4)^2\,dt$$

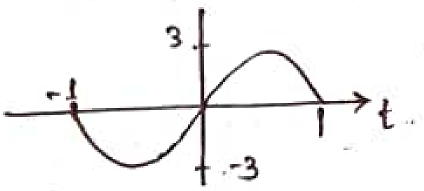
$$= 18 \int_0^4 \left(1 + \frac{t^2}{16} - \frac{t}{2}\right) dt$$

$$= 18 \left[t + \frac{t^3}{48} - \frac{t^2}{4}\right]_0^4$$

$$= 18 \left[\cancel{4} + \frac{\cancel{64}^{4}}{\cancel{48}_{3}} - \frac{\cancel{16}}{\cancel{4}}\right]$$

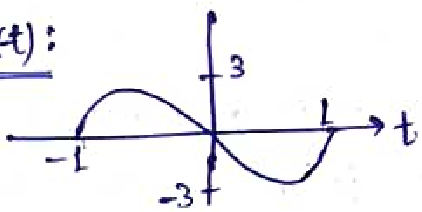
$$= 24.$$

② $g(t)$:

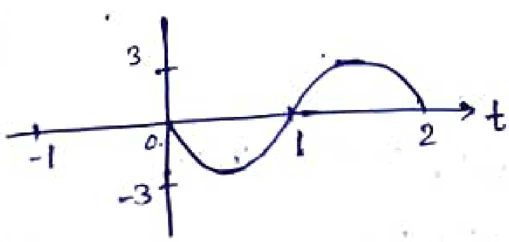


Plot $g(-t)$, $g(t-1)$, $g(2t)$.

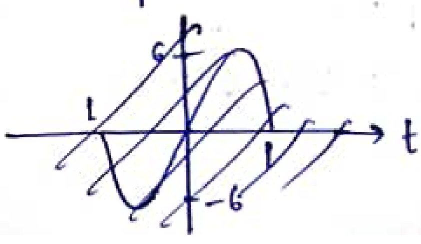
Soln: $g(-t)$:

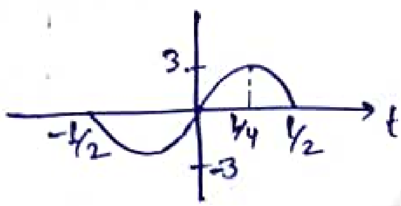


$g(t-1)$:



$g(2t)$:





③ Find the even and odd parts of these functions.

ⓐ $g(t) = 2t^2 - 3t + 6$

Soln: $g_{even}(t) = \frac{g(t) + g(-t)}{2}$

$$= \frac{(2t^2 - \cancel{3t} + 6) + (2t^2 + \cancel{3t} + 6)}{2}$$

$$= 2t^2 + 6$$

$g_{odd}(t) = \frac{g(t) - g(-t)}{2}$

$$= [(\cancel{2t^2} - 3t + \cancel{6}) - (\cancel{2t^2} + 3t + \cancel{6})]/2$$

$$= -3t$$

ⓑ $20\cos(40\pi t - \pi/4) = h(t)$

Soln: $h_{even} = \frac{20\cos(40\pi t - \pi/4) + 20\cos(40\pi t + \pi/4)}{2}$

$$= 10\left[\cos(40\pi t)\cos(\pi/4) + \cancel{\sin(40\pi t)\sin(\pi/4)} + \cos(40\pi t)\cdot\frac{1}{\sqrt{2}} - \sin(40\pi t)\cdot\frac{1}{\sqrt{2}}\right]$$

$$= \frac{20}{\sqrt{2}}\cos(40\pi t) = 10\sqrt{2}\cos(40\pi t)$$

$h_{odd} = \frac{20\cos(40\pi t - \pi/4) - 20\cos(40\pi t + \pi/4)}{2}$

$$= 10\left[\frac{\cancel{\cos 40\pi t}}{\sqrt{2}} + \frac{\sin 40\pi t}{\sqrt{2}} - \frac{\cancel{\cos 40\pi t}}{\sqrt{2}} + \frac{\sin(40\pi t)}{\sqrt{2}}\right]$$

$$= \frac{20}{\sqrt{2}}\sin 40\pi t = 10\sqrt{2}\sin(40\pi t).$$

④ Show that a system with excitation $x(t)$ and response $y(t)$ described by $y(t) = x(t-5) - x(3-t)$ is linear, non-causal and non-invertible.

Soln: Linearity: For i/p x_1, o/p : y_1

$$y_1(t) = x_1(t-5) - x_1(3-t)$$

For i/p x_2, o/p : y_2

$$y_2(t) = x_2(t-5) - x_2(3-t)$$

For i/p x_1+x_2, o/p: y_3

$$y_3(t) = (x_1+x_2)(t-5) - (x_1+x_2)(3-t)$$

$$= [x_1(t-5) + x_1(3-t)] + [x_2(t-5) + x_2(3-t)]$$

$$= y_1(t) + y_2(t)$$

$\therefore$ System is ~~linear~~ additive.

Now,

For i/p $= \alpha x$, o/p: y_4

$$y_4(t) = \alpha x(t-5) - \alpha x(3-t)$$

$$= \alpha y.$$

$\therefore$ System is homogeneous.

Hence, the system is linear.

<u>Causality:</u> $y(t) = x(t-5) - x(3-t)$

At $t=0$, $y(0) = x(-5) - x(3)$.

Output at $t=0$ requires input at $t=3$.

Hence, the system is non-causal.

<u>Invertibility:</u> Give i/p $= y$, o/p $\overset{?}{=} x$.

$$y(t-5) - y(3-t) = Y(t)$$

$$t \to t+5 \Rightarrow y(t) - y(3-(t+5)) = Y(t+5)$$

$$\Rightarrow y(t) - y(-2-t) = Y(t+5)$$

Thus, we cannot get x as output and hence the system is non-invertible.

(5) Find the impulse response ~~of~~ $h(t)$ of

$$\frac{dy(t)}{dt} + ay(t) = x(t).$$

Soln: $(D+a)y(t) = x(t)$, where $D \equiv \frac{d}{dt}$. For $x=0 \Rightarrow y(t) = c_1 e^{-at}$.

As $b_0 = 0 \Rightarrow h(t) =$ other characteristic mode terms at $t \geq 0$.

$$\lambda + a = 0 \Rightarrow \lambda = -a$$

$$\therefore h(t) = (c_1 e^{-at})\, u(t).$$

Here, order, $N = 1$.

So, $h(0) = 1 = c_1$

$$\therefore h(t) = e^{-at} u(t).$$