रेडियो-आवर्तनमणुतरङ्गसञ्चारश्च

RF AND MICROWAVE COMMUNICATION

29-07-2024

EM Spectrum

Type:	Radio	Microwave	Infrared	Visible	Ultra violet	X-Ray	Gamma Ray
wavelength (m):	10^3	10^{-2}	10^{-5}	0.5×10^{-6}	10^{-8}	10^{-10}	10^{-12}
Frequency (Hz):	10^4	10^8	10^{12}	10^{15}	10^{16}	10^{18}	10^{20}

Frequency Band:

3–30 Hz —— 10^5–10^4 km —— ELF

3–300 Hz —— 10^4–10^3 km —— SLF

3–3000 Hz —— 10^3 km – 100 km —— ULF

3–30 KHz —— 100–10 km —— VLF

3–300 KHz —— 10–1 km —— LF

300 KHz – 3 MHz —— 1 km – 100 m —— MF

3–30 MHz —— 100 – 10 m —— HF

3–300 MHz —— 10 – 1 m —— VHF

300 MHz – 3 GHz —— 1m – 10cm —— UHF

3–30 GHz —— 10 – 1 cm —— SHF

30–300 GHz —— 1cm – 1mm —— EHF

300 GHz – 3 THz —— 1mm – 0.1 mm —— THF

1–2 GHz ⟶ L-Band

2–4 GHz ⟶ S-Band

4–8 GHz ⟶ C-Band

8–12 GHz ⟶ X-Band

12–18 GHz ⟶ Ku-Band

18–27 GHz ⟶ K-Band

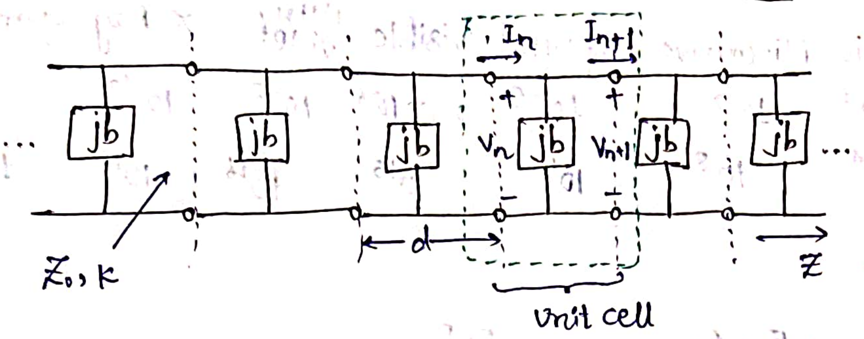
27–40 GHz ⟶ Ka-Band

40–75 GHz ⟶ V-Band

75–110 GHz ⟶ W-Band

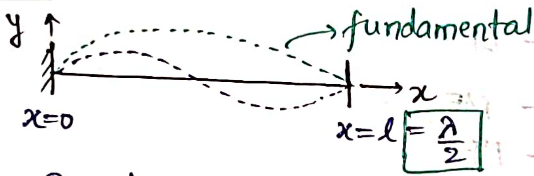
30-07-2024

Periodically Loaded Transmission Line



Rectangular Waveguide

Modes:



Boundary conditions: $y=0, \ x=0,l$

$\boxed{\dot{y}=0, \ x=0,l}$

$$f_n = \frac{n}{2l}\sqrt{\frac{T}{m}}$$

Field Distribution:

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma\vec{E} + \epsilon\frac{\partial \vec{E}}{\partial t}$$

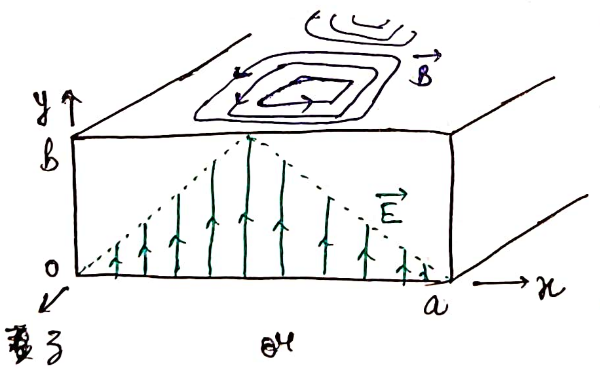
xz-plane → H-plane

yz-plane → E-plane

In z-dirn ⇒ no conduction current

⇒ Must be displacement current

⇒ $\vec{E}$ in dirn of z → not possible ⇒ TEM does not exist ⇓ TE/TM exist.

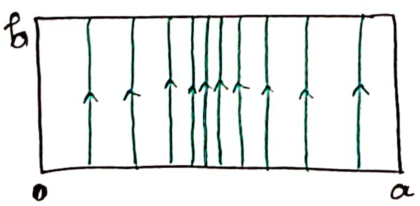


$E \sim \sin\frac{\pi}{a}x \ e^{-j\beta z}$

$H_x \sim \sin\frac{\pi x}{a}$ | $E_y \sim \sin\frac{\pi x}{a}$

$H_z \sim \cos\frac{\pi x}{a}$

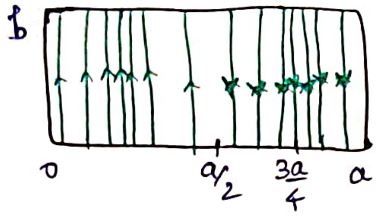
$\vec{J}_s = \hat{n}\times\vec{H}$



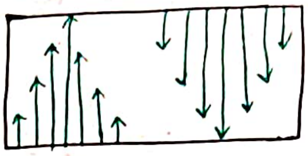
⇒ $\boxed{TE_{10}}$

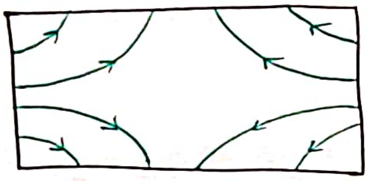
1 half cycle variation along x-axis

0 half cycle variation along y-axis

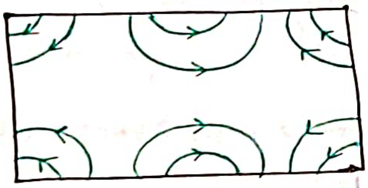


$\Rightarrow TE_{20}$ or





$\Rightarrow TE_{11}$



$\Rightarrow TE_{21}$

<u>TE_{10}:</u>

$$H_z = H_0 \cos\left(\frac{\pi}{a} x\right) e^{-j\beta z}$$

$$E_z = 0$$

$$E_x = 0$$

$$E_y = -\frac{j\omega\mu}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{\pi x}{a}\right) e^{-\gamma z}$$

$$H_x = \frac{j\beta}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

$$H_y = 0$$

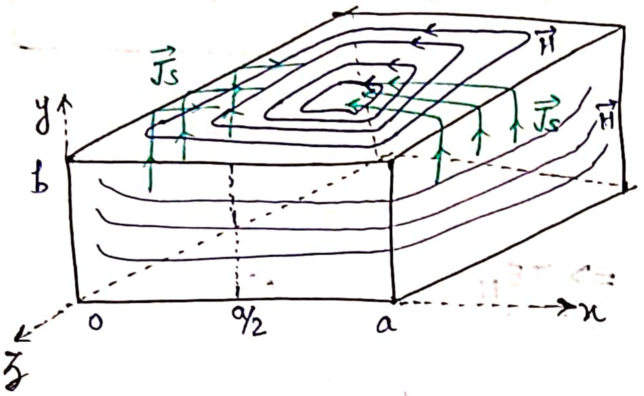
<u>Dominant Mode:</u>

$TE_{10} \rightarrow$ Electric field

$TE_{10} \rightarrow$ Magnetic field

06-08-2024

Surface Current Distribution

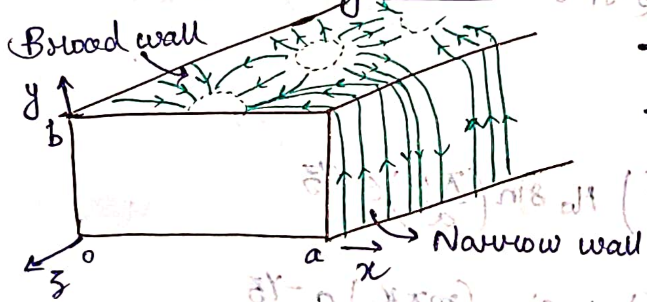


$$\vec{J_s} = \hat{n} \times \vec{H}$$

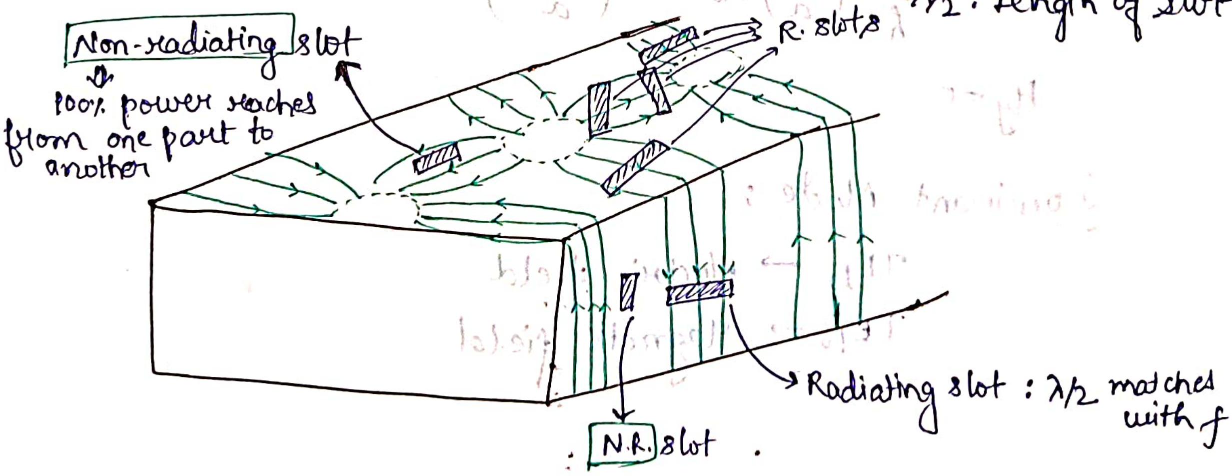
$$H_x = \frac{\beta}{K_c^2}\frac{\pi}{a} A_{mn} \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

$$H_z(x, y, z) = A_{mn} \cos\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta z)$$

Radiating and Non-Radiating slots:



→ Nulls are separated by $\lambda/2$.
→ Continuity is maintained by displacement ~~vcel~~ current.
→ Sink is created by displacement current.
$\lambda/2$: Length of slot



For TE_{10} waves:

↳ Two current components → Transverse current → cosine distribution along wide-side with zero at the center
→ Longitudinal current → Uniformly distributed along narrow side

↓ Sine distribution along wide-side with max^m at center.

→ on the narrow side of the waveguide, there is only transverse current

Rectangular Waveguides: Fields inside

↳ Assumptions:

① Waveguide is filled with lossless dielectric material.

② walls of the guide are perfect conductor.

The wave inside ~~the~~ should obey:

$$\nabla^2\vec{E} + k^2\vec{E} = 0$$
$$\nabla^2\vec{H} + k^2\vec{H} = 0$$

where, $k^2 = \omega^2\mu\epsilon$.

Results: Equations governing relationship among the fields inside the waveguide:

$$E_x = \frac{-j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial y} - \frac{j\beta}{K_c^2}\frac{\partial E_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial x} - \frac{j\beta}{K_c^2}\frac{\partial E_z}{\partial y}$$

$$H_x = \frac{j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial y} - \frac{j\beta}{K_c^2}\frac{\partial H_z}{\partial x}$$

$$H_y = -\frac{j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial x} - \frac{j\beta}{K_c^2}\frac{\partial H_z}{\partial y}$$

⇒ $E_z = 0$ and $H_z = 0$

⇓

$E_x = E_y = H_x = H_y = 0$

⇓

TEM wave can't exist in a waveguide, made of single conductor systems.

Case-I: TE Mode ($E_z = 0$, $H_z \neq 0$)

$$E_x = \frac{-j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial y} \qquad H_x = -\frac{j\beta}{K_c^2}\frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial x} \qquad H_y = \frac{-j\beta}{K_c^2}\frac{\partial H_z}{\partial y}$$

Case-II : TM Mode ($H_z = 0, E_z \neq 0$)

$$E_x = \frac{-j\beta}{K_c^2}\frac{\partial E_z}{\partial x} \qquad H_x = \frac{j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial y}$$

$$E_y = \frac{-j\beta}{K_c^2}\frac{\partial E_z}{\partial y} \qquad H_y = -\frac{j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial x}$$

Complete Solution :

TE Mode:

$$E_z = 0, \quad H_z(x,y,z) = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_x(x,y,z) = \frac{j\omega\mu}{K_c^2}\frac{n\pi}{b}A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_y(x,y,z) = -\frac{j\omega\mu}{K_c^2}\frac{m\pi}{a}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$H_x = \frac{-j\beta}{K_c^2}\frac{\partial H_z}{\partial x} = \frac{j\beta}{K_c^2}\frac{m\pi}{a}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$H_y = -\frac{j\beta}{K_c^2}\frac{\partial H_z}{\partial y} = \frac{j\beta}{K_c^2}\frac{n\pi}{b}A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

where $K_c^2 = (K^2 - \beta^2)$

$$K_c^2 = K_x^2 + K_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

TM Mode :

$$E_z(x,y,z) = B_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}, \quad H_z = 0$$

$$E_x(x,y,z) = -\frac{j\beta}{K_c^2}\frac{m\pi}{a}B_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_y(x,y,z) = -\frac{j\beta}{K_c^2}\frac{n\pi}{b}B_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$H_x(x,y,z) = \frac{j\omega\epsilon}{K_c^2}\frac{n\pi}{b}B_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$H_y(x,y,z) = -\frac{j\omega\epsilon}{K_c^2}\frac{m\pi}{a}B_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

→ TEM ($E_z = H_z = 0$) : can't propagate.

→ TE($E_z = 0$) : transverse electric : Electric lines of flux are perpendicular to the axis of the waveguide.

→ TM ($H_z = 0$) : Magnetic lines of flux are $\perp^r$ to the axis of wave-guide.

TE Modes : Cut-off Frequency

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \underbrace{\omega\sqrt{\mu\epsilon}}_{\text{phase constant of uniform plane wave in WG medium } (\beta_u)} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2}$$

For lossless case, $\gamma = j\beta$.

Cutoff : $$(\omega_c)_{mn} = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$(f_c)_{mn} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Propagation: $\omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

i.e., for $f > f_c$; $\gamma = j\beta$ and $\alpha = 0$.

In case, the wave can't propagate and attenuates very fast, such waves are called evanescent waves.

Cut-off wavelength : $$(\lambda_c)_{mn} = \frac{v_p}{(f_c)_{mn}} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$(\lambda_c)_{TE_{10}} = \frac{v_p}{(f_c)_{TE_{10}}} = 2a, \quad (\lambda_c)_{TE_{01}} = 2b.$$

Intrinsic wave impedance :

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

For TE_{10} mode, $(f_c)_{10} = \frac{v_p}{2a} \left(= \frac{1}{2a\sqrt{\mu\epsilon}}\right)$

For TE_{01} mode, $(f_c)_{01} = \frac{1}{2b\sqrt{\mu\epsilon}} = \frac{v_p}{2b}$

$(f_c)_{10} < (f_c)_{01}$ [for $a > b$]

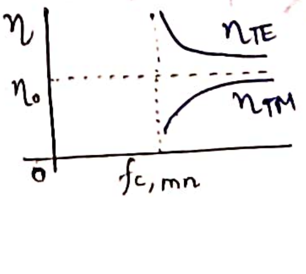
TM Modes : Cut-off Frequency

↳ Similar expressions for $(\omega_c)_{mn}$, $(f_c)_{mn}$, $(\lambda_c)_{mn}$, propagation.

Intrinsic wave impedance :

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \sqrt{\frac{\mu}{\epsilon}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \eta_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$\eta_{TE} \cdot \eta_{TM} = \eta_0^2$



Circular Waveguide

Using Maxwell's equations;

$$\vec{\nabla}\times\vec{H} = \epsilon\frac{\partial\vec{E}}{\partial t} = j\omega\epsilon\vec{E} \quad ...(1)$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} = -j\omega\mu\vec{H} \quad ...(2)$$

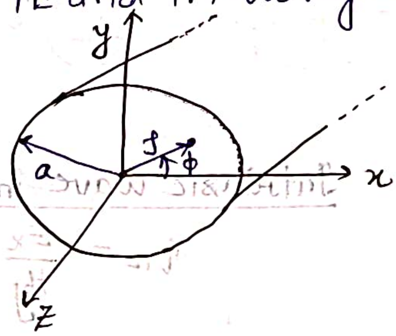
In cylindrical system,

$$\vec{\nabla}\times\vec{H} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix} = j\omega\epsilon\vec{E} \quad ...(3)$$

$$\vec{\nabla}\times\vec{E} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ E_\rho & \rho E_\phi & E_z \end{vmatrix} = -j\omega\mu\vec{H} \quad ...(4)$$

Analysis :

A hollow, round metal pipe supports TE and TM waveguide modes.



$$E_\rho = -\frac{j}{K_c^2}\left(\beta\frac{\partial E_z}{\partial\rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial\phi}\right),$$

$$E_\phi = \frac{-j}{K_c^2}\left(\frac{\beta}{\rho}\frac{\partial E_z}{\partial\phi} - \omega\mu\frac{\partial H_z}{\partial\rho}\right)$$

$$H_\rho = \frac{j}{K_c^2}\left(\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial\phi} - \beta\frac{\partial H_z}{\partial\rho}\right)$$

$$H_\phi = \frac{-j}{K_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial\rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial\phi}\right).$$

TE Modes :

$$\nabla^2 H_z + K^2 H_z = 0$$

$$H_z(\rho,\phi,z) = h_z(\rho,\phi)\, e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + K_c^2\right)h_z(\rho,\phi) = 0$$

$$h_z(\rho,\phi) = R(\rho)P(\phi)$$

$$\frac{1}{R}\frac{d^2R}{d\rho^2} + \frac{1}{\rho R}\frac{dR}{d\rho} + \frac{1}{\rho^2 P}\frac{d^2P}{d\phi^2} + K_c^2 = 0$$

$$\frac{\rho^2}{R}\frac{d^2R}{d\rho^2} + \frac{\rho}{R}\frac{dR}{d\rho} + \rho^2K_c^2 = -\frac{1}{P}\frac{d^2P}{d\phi^2}$$

The left side of this eq^n depends only on ρ, while the right side depends only on ϕ. Thus, each side must be equal to a constant.

$$-\frac{1}{P}\frac{d^2P}{d\phi^2} = K_\phi^2$$

$$\frac{d^2P}{d\phi^2} + K_\phi^2 P = 0$$

$$\Rightarrow \rho^2\frac{d^2R}{d\rho^2} + \rho\frac{dR}{d\rho} + (\rho^2K_c^2 - \cancel{K_\phi^2}^{\,n^2})R = 0 \quad \left[P(\phi) = A\sin n\phi + B\cos n\phi,\ K_\phi^2 = n^2\right]$$

↳ Bessel's differential equation

Solution:

$$R(\rho) = C\underbrace{J_n(K_c\rho)} + D\underbrace{Y_n(K_c\rho)}$$

Bessel f^n of first and second kinds.

$Y_n(K_c\rho)$ is infinite at $\rho=0 \Rightarrow D=0$

Substituting

$$h_z(\rho,\phi) = (A\sin n\phi + B\cos n\phi)J_n(K_c\rho)$$

<u>Boundary condition</u>:

$E_\phi(\rho,\phi) = 0$ at $\rho = a$ (tangential $\vec{E}=0$ at wall)

Using $E_\phi = \frac{-j}{K_c^2}\left(\frac{\beta}{\rho}\frac{\partial E_z}{\partial\phi} - \omega\mu\frac{\partial H_z}{\partial\rho}\right) = \frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial\rho}$, $(\because E_z = 0)$

$$\Rightarrow E_\phi(\rho,\phi,z) = \frac{j\omega\mu}{K_c}(A\sin n\phi + B\cos n\phi)J_n'(K_c\rho)e^{-j\beta z}$$

As $E_\phi = 0$ at $\rho = a$,

$J_n'(K_c a) = 0$ [n: no. of circumferencial variations, m: no. of radial variations]

$J_n'(p'_{nm}) = 0$, p'_{nm}: m-th root of J_n'

$$K_{cnm} = \frac{p'_{nm}}{a}$$

Propagation constant: $\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p'_{nm}}{a}\right)^2} = \sqrt{\omega^2\mu\epsilon - \left(\frac{p'_{nm}}{a}\right)^2}$

Cut-off frequency: $f_{c\,nm} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p'_{nm}}{2\pi a\sqrt{\mu\epsilon}}$, $\lambda_c = \frac{2\pi a}{p'_{nm}}$

Dominant mode: Min^m value of p'_{nm} corresponds to dominant mode.

$\hookrightarrow$ TE_{11} : dominant mode

Since $m \geq 1 \Rightarrow$ No TE_{10} mode, but TE_{01} mode ✓

Transverse field components:

$$E_\rho = \frac{-j\omega\mu n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)\, J_n(k_c\rho)\, e^{-j\beta z}$$

$$E_\phi = \frac{j\omega\mu}{k_c}(A\sin n\phi + B\cos n\phi)\, J'_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\rho = \frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)\, J'_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\phi = \frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)\, J_n(k_c\rho)\, e^{-j\beta z}$$

Wave impedance:

$$Z_{TE} = \frac{E_\rho}{H_\phi} = \frac{-E_\phi}{H_\rho} = \frac{\eta k}{\beta}$$

Dominant mode fields: (TE_{11}). [wave is excited s.t. $B=0$]

$$E_z = 0, \quad H_z = A\sin\phi\, J_1(k_c\rho)\, e^{-j\beta z}$$

$$E_\rho = \frac{-j\omega\mu}{k_c^2\rho} A\cos\phi\, J_1(k_c\rho)\, e^{-j\beta z}$$

$$E_\phi = \frac{j\omega\mu}{k_c} A\sin\phi\, J_1'(k_c\rho)\, e^{-j\beta z}$$

$$H_\rho = \frac{-j\beta}{k_c\rho} A\sin\phi\, J_1'(k_c\rho)\, e^{-j\beta z}$$

$$H_\phi = \frac{-j\beta}{k_c^2\rho} A\cos\phi\, J_1(k_c\rho)\, e^{-j\beta z}$$

TMnm Modes :

$$\left(\frac{\partial^2}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial}{\partial \rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2}+k_c^2\right)E_z=0,$$

$$E_z(\rho,\phi,z)=e_z(\rho,\phi)\,e^{-j\beta z}$$

General soln : $E_z(\rho,\phi)=(A\sin n\phi+B\cos n\phi)\,J_n(k_c\rho)$

Boundary condition :

$E_z(\rho,\phi)=0$ at $\rho=a$

$\Rightarrow$ $J_n(k_c a)=0,$

$J_n(p_{nm})=0$, p_{nm} : m^{th} root of J_n

$$k_{cnm}=\frac{p_{nm}}{a}$$

Propagation constant : $\beta_{nm}=\sqrt{k-k_c^2}=\sqrt{k^2-\left(\frac{p_{nm}}{a}\right)^2}=\sqrt{\omega^2\mu\epsilon-\left(\frac{p_{nm}}{a}\right)^2}$

cut-off frequency : $f_{cnm}=\dfrac{k_c}{2\pi\sqrt{\mu\epsilon}}=\dfrac{p_{nm}}{2\pi a\sqrt{\mu\epsilon}}$

Dominant mode : TM_{01} (but not overall dominant)
$[\because p'_{11}=1.841<p_{01}=2.405]$

Field Equations :

$$E_z=J_n\left(\frac{p_{nm}\rho}{a}\right)e^{-j\beta z}\begin{Bmatrix}\cos(n\phi)\\ \sin(n\phi)\end{Bmatrix}$$

$$H_z=0$$

$$E_\rho=\frac{-j\beta p_{nm}J_n'(p_{nm}\rho/a)}{a k_c^2}e^{-j\beta z}\begin{Bmatrix}\cos(n\phi)\\ \sin(n\phi)\end{Bmatrix}$$

$$E_\phi=\frac{-jn\beta J_n(p_{nm}\rho/a)}{r k_c^2}e^{-j\beta z}\begin{Bmatrix}-\sin(n\phi)\\ \cos(n\phi)\end{Bmatrix}$$

$H_\rho=-E_\phi/Z_e$, $H_\phi=E_\rho/Z_e$.

p_{nm} : m^{th} root of $J_n(x)$.

$Z_e=Z_0\beta/k$, $\lambda_c=2\pi a/p_{nm}$

$k_c=\dfrac{p_{nm}}{a}$, $\beta^2=k^2-k_c^2$

In TM_{01} mode;

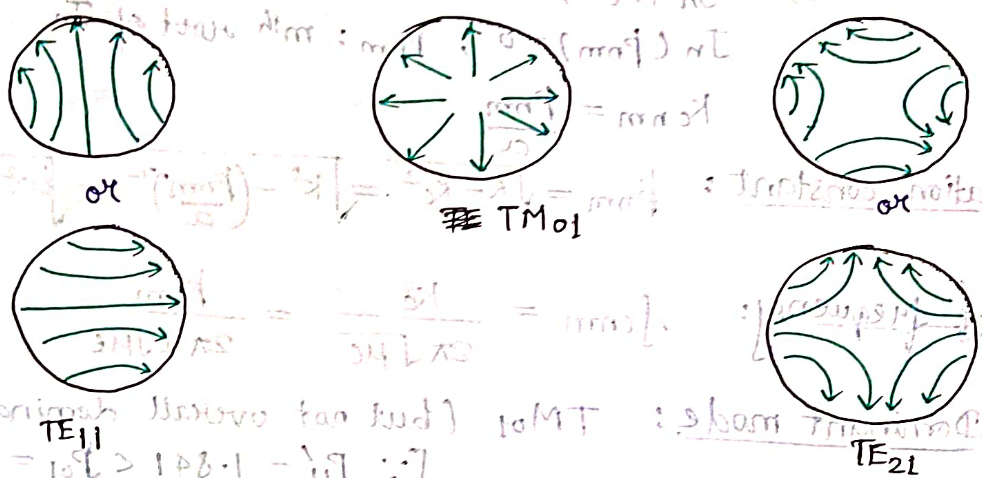
$$E_z = J_0\left(\frac{p_{01}\rho}{a}\right) e^{-j\beta z}, \quad H_z = 0$$

$$E_\rho = \frac{-j\beta p_{nm} J_0'(p_{01}\rho/a)}{a k_c^2} e^{-j\beta z}, \quad H_\rho = -\frac{E_\phi}{Z_e} = 0$$

$$E_\phi = 0, \quad H_\phi = \frac{E_\rho}{Z_e}$$

$$p_{01} = 2.405$$

Field Patterns in Circular Waveguides:



07-08-2024

Cavity Resonator

Microwave Resonators

- ↳ Applications in filters, oscillators, frequency meters, tuned amplifiers.
- ↳ Operations very similar to series and parallel RLC resonant circuits.
- ↳ Implementation using ~~micro~~ distributive elements such as microstrip line, rectangular and circular cavities, etc.

↳ Closed
- → Rectangular CR.
- → circular CR.

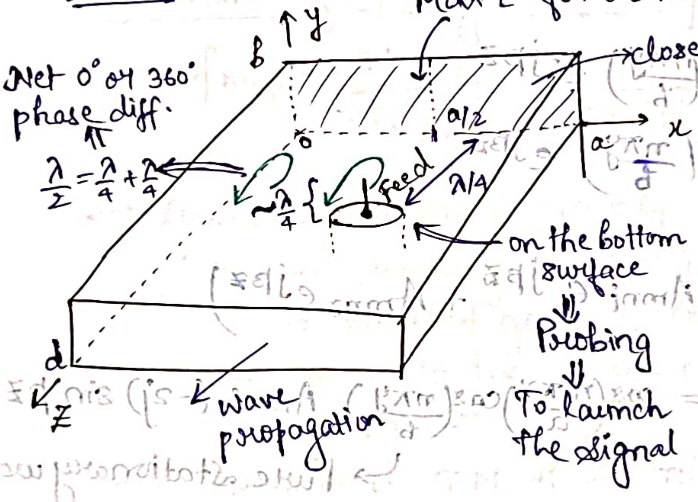
TEmn, TMmn

TE_{10}

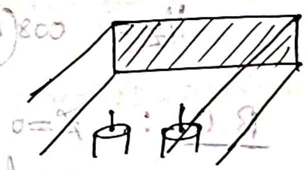
Max $\vec{E}$ for dominant TE_{10} mode

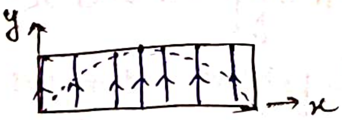
[PEC ⇒ $\Gamma = -1$ ∵ Metallic reflection]

$$\{e^{-j\beta z} + e^{j\beta z}$$



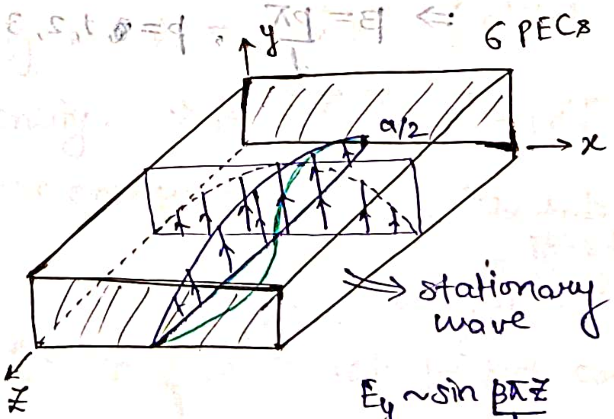
TE_{20}





$E_y \sim \sin\frac{\pi x}{a}$

6 PECs



$E_y \sim \sin\frac{p\pi z}{d}$

TEmnp, TMmnp

TE_{101} : dominant mode

TE_{102}

TE_{mnp} mode

TE mode

$$E_z = 0$$

$$H_z = A_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$E_x = \frac{-j\omega\psi}{K_c^2} \frac{\partial H_z}{\partial y}$$

$$E_y = \frac{j\omega\psi}{K_c^2} \frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-j\beta}{K_c^2} \frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\beta}{K_c^2} \frac{\partial H_z}{\partial y}$$

$$H_{zi} = A_{mni} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$H_{zr} = A_{mnr} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{j\beta z}$$

$\Downarrow$ superimpose

$$H_z = \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \left(A_{mni} e^{-j\beta z} + A_{mnr} e^{j\beta z}\right)$$

B.C. : $\left.\begin{array}{l} z=0,\ H_z=0 \\ z=d,\ H_z=0 \end{array}\right\}$ $H_z = \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) A_{mni} (-2j) \sin\beta z$

↳ Pure stationary wave

$$\sin\beta l = 0 = \sin p\pi$$

$$\Rightarrow \beta = \frac{p\pi}{d}, \quad p = 1, 2, 3, \ldots$$

(12-08-2024)

Resonance: when V and I are in phase.

Power factor : $\cos \underset{\text{phase}}{\theta} = 1$ (for resonance)

How to identify Resonance:

- → Resonance frequency
- → Quality factor

$$\text{Quality Factor } (Q) = \frac{\text{Energy stored}}{\text{Energy lost per second}}$$

↑ ⇒ Resonance ↑
⇒ Loss ↓
⇒ Bandwidth ↓

$$K_{mnp} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$: Resonance wavenumber for rectangular cavity

Resonance frequency of TE_{mnp} or TM_{mnp} mode:

$$f_{mnp} = \frac{c K_{mnp}}{2\pi\sqrt{\mu_r \epsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

→ Lowest resonant frequency: TE_{101} mode.
  ↳ corresponding to the TE_{10} dominant waveguide mode in a shorted guide of length $\lambda g/2$.
  ↳ Similar to the short-circuited $\lambda/2$ transmission line resonator.

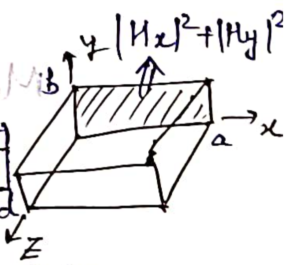
→ Dominant TM resonant mode: TM_{110} mode.

Electric energy density : $v_e = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\epsilon|\vec{E}|^2$ → energy stored in cavity.

Magnetic energy density: $v_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\mu|\vec{H}|^2$

Total ~~energy~~ electric energy $= \int_0^a\int_0^b\int_0^d \frac{\epsilon}{4}|E_y|^2 \, dx\,dy\,dz = \boxed{\frac{\epsilon abd E_0^2}{16}}$

(volume integration)



→ Power loss is b/c surface is not perfect conductor.

Loss due to surface current :

$$P_{loss}/area = \frac{1}{2}|\vec{J_s}|^2 R_s$$

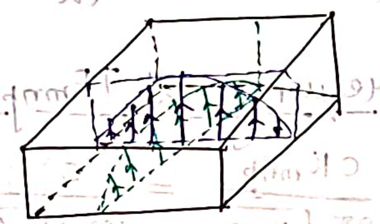
$$= \frac{1}{2}|H_{tan.}|^2 R_s \quad (\because \vec{J_s} = \hat{n}\times\vec{H})$$

$$\left[\underbrace{R_s}_{\text{decides power loss}} = \sqrt{\frac{\omega\mu}{2\sigma}} \downarrow\downarrow \Rightarrow \text{Power loss} \downarrow\downarrow\right]$$

$$\Rightarrow P_{loss} = \iint_S \frac{1}{2}|H_{tan.}|^2 R_s \, ds$$

Unloaded /Standalone Q : considering nothing before & after.

TE101 Mode :



$E_z = 0$

$E_x = 0$

$$E_y = E_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{p\pi z}{d}\right)$$

$$H_x = \frac{-jE_0}{Z_{TE}}\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{p\pi z}{d}\right) \quad [p=1]$$

$$H_z = \frac{j\pi E_0}{k\eta a}\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{p\pi z}{d}\right)$$

$H_y = 0$

Stored Energy :

Electric stored energy :

$$v_e = \frac{\epsilon}{4}\int_v |E_y|^2 dv = \frac{\epsilon abd}{16} E_0^2$$

Magnetic stored energy :

$$v_m = \frac{\mu}{4}\int_v \left(|H_x|^2 + |H_z|^2\right) dv$$

$$= \frac{\mu abd}{16} E_0^2\left(\frac{1}{Z_{TE}^2} + \frac{\pi^2}{k^2\eta^2 a^2}\right)$$

$$P_c = \frac{R_s}{2}\left[2\int_{y=0}^{b}\int_{x=0}^{a}|H_x(z=0)|^2\,dx\,dy + 2\int_{z=0}^{d}\int_{y=0}^{b}|H_z(x=0)|^2\,dy\,dz + 2\int_{z=0}^{d}\int_{x=0}^{a}\left(|H_x(y=0)|^2 + |H_z(y=0)|^2\right)dx\,dz\right]$$

$$= \frac{R_s E_0^2 \lambda^2}{8\eta^2}\left(\frac{l^2ab}{d^2} + \frac{bd}{a^2} + \frac{l^2 a}{2d} + \frac{d}{2a}\right).$$

$$Q_c = \frac{2\omega W_e}{P_c}$$

$$= \frac{k^3 abd\eta}{4\pi^2 R_s}\,\frac{1}{\left[\left(\frac{l^2ab}{d^2}\right) + \left(\frac{bd}{d^2}\right) + \left(\frac{l^2a}{2d}\right) + \left(\frac{d}{2a}\right)\right]}$$

$$= \frac{(kad)^3 b\eta}{2\pi^2 R_s\left[2l^2a^3b + 2bd^3 + l^2a^3d + ad^3\right]}$$

13-08-2024

Dielectric filled Cavity

Non-reciprocal material: Propagation constant is not same in positive and negative z (dirn of prop)

$\Rightarrow$ Attenuation constant is different.

↳ Eg. ferrites.

→ Extra loss due to the loss of the dielectric.

→ Total loss = Conduction loss + Dielectric loss.

→ Quality factor is degraded.

Air-filled:

$$Q = \frac{\omega_0 (V_e + V_m)}{P_c} = \frac{2\omega_0 V_e}{P_c}$$

Dielectric-filled:

$$Q = \frac{\omega_0 (V_e + V_m)}{P_c + P_d} = \frac{2\omega_0 V_e}{P_c + P_d}$$

$$\frac{1}{Q} = \frac{P_c + P_d}{2\omega_0 V_e} = \frac{1}{\frac{2\omega_0 V_e}{P_c}} + \frac{1}{\frac{2\omega_0 V_e}{P_d}}$$

$$\Rightarrow \boxed{\frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_d}}$$

Dielectric field waveguide:

$$\vec{\nabla} \times \vec{H} = \vec{J_c} + \vec{J_d}$$
$$= \sigma\vec{E} + j\omega\epsilon\vec{E}$$
$$= \sigma\vec{E} + j\omega(\epsilon' - j\epsilon'')\vec{E}$$
$$= (\sigma + \omega\epsilon'')\vec{E} + j\omega\epsilon'\vec{E}$$

$$\boxed{\tan\delta = \frac{\sigma + \omega\epsilon''}{\omega\epsilon'}}$$: Loss tangent

$$\boxed{\tan\delta = \frac{\epsilon''}{\epsilon'}}$$

Effective conductivity for dielectric having only dielectric loss:

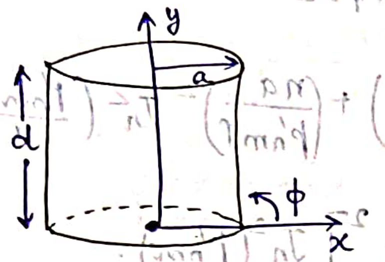
$$\sigma = \omega\epsilon''$$

Power dissipated in the dielectric:

$$P_d = \frac{1}{2}\int_V \vec{J}\cdot\vec{E}^* \, dv = \frac{\omega\epsilon''}{2}\int_V |\vec{E}|^2 dv = \frac{abd\,\omega\epsilon'' |E_0|^2}{8}$$

$$Q_d = \frac{2\omega_0 V_e}{P_d} = \frac{2\omega_0 \left(\frac{\epsilon' abd}{16} E_0^2\right)}{\frac{\omega\epsilon'' abd}{8} E_0^2} = \frac{\epsilon'}{\epsilon''} = \frac{1}{\tan\delta}$$

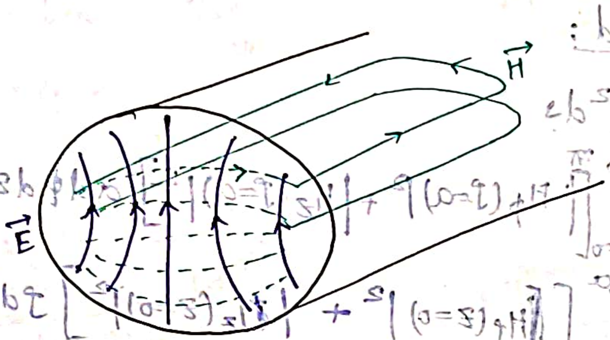
Circular Cavity Resonator



$$\beta_{nm} = \sqrt{k^2 - \left(\frac{P_{nm}}{a}\right)^2}$$

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}}\sqrt{\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$



Degenerate modes → modes of same frequency

$TE_{11} \rightarrow$ dominant mode

TE_{nml} mode:

$$E_z = 0$$

$$E_\rho = \frac{jk\eta a^2 n H_0}{(P'_{nm})^2 \rho} J_n\left(\frac{P'_{nm}\rho}{a}\right) \sin n\phi \sin\frac{l\pi z}{d}$$

$$E_\phi = \frac{jk\eta a H_0}{P'_{nm}} J'_n\left(\frac{P'_{nm}\rho}{a}\right) \cos n\phi \sin\frac{l\pi z}{d}$$

$$H_z = H_0 J_n\left(\frac{P'_{nm}\rho}{a}\right) \cos n\phi \sin\frac{l\pi z}{d}$$

$$H_\rho = \frac{\beta a H_0}{P'_{nm}} J'_n\left(\frac{P'_{nm}\rho}{a}\right) \cos n\phi \cos\frac{l\pi z}{d}$$

$$H_\phi = \frac{-\beta a^2 n H_0}{(P'_{nm})^2 \rho} J_n\left(\frac{P'_{nm}\rho}{a}\right) \sin n\phi \cos\frac{l\pi z}{d}.$$

Energy stored:

$$W = 2W_e = \frac{\epsilon}{2}\int_{z=0}^{d}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a}\left(|E_\rho|^2+|E_\phi|^2\right)\rho\, d\rho\, d\phi\, dz$$

$$= \frac{\epsilon k^2\eta^2 a^2 \pi d H_0^2}{4(p'_{nm})^2}\int_{\rho=0}^{a}\left[J_n'^2\left(\frac{p'_{nm}\rho}{a}\right)+\left(\frac{na}{p'_{nm}\rho}\right)^2 J_n^2\left(\frac{p'_{nm}\rho}{a}\right)\right]\rho\, d\rho$$

$$= \frac{\epsilon k^2\eta^2 a^4 H_0^2 \pi d}{8(p'_{nm})^2}\left[1-\left(\frac{n}{p'_{nm}}\right)^2\right]J_n^2(p'_{nm}).$$

Power Loss per second:

$$P_c = \frac{R_s}{2}\int_S |\vec{H}_{tan.}|^2 ds$$

$$= \frac{R_s}{2}\left\{\int_{z=0}^{d}\int_{\phi=0}^{2\pi}\left[|H_\phi(\rho=a)|^2+|H_z(\rho=a)|^2\right]a\, d\phi\, dz + 2\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a}\left[|H_\rho(z=0)|^2+|H_z(z=0)|^2\right]\rho\, d\rho\, d\phi\right\}$$

$$= \frac{R_s}{2}\pi H_0^2 J_n^2(p'_{nm})\left\{\frac{da}{2}\left[1+\left(\frac{\beta a n}{(p'_{nm})^2}\right)^2\right] + \left(\frac{\beta a^2}{p'_{nm}}\right)^2\left(1-\frac{n^2}{(p'_{nm})^2}\right)\right\}$$

$$Q_c = \frac{\omega_0 W}{P_c} = \frac{(ka)^3\eta a d}{4(p'_{nm})^2 R_s}\frac{1-\left(\frac{n}{p'_{nm}}\right)^2}{\left\{\frac{ad}{2}\left[1+\left(\frac{\beta a n}{p'_{nm}}\right)^2\right]+\left(\frac{\beta a^2}{p'_{nm}}\right)^2\left(1-\frac{n^2}{(p'_{nm})^2}\right)\right\}}$$

14-08-2024

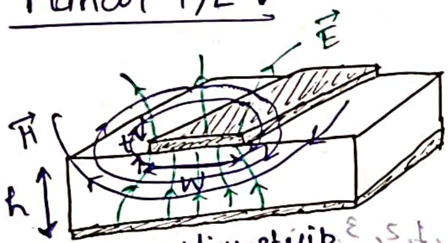
Transmission Line Resonator

Types of Transmission Line

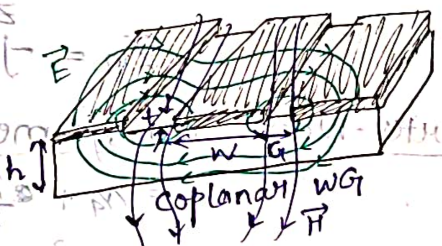
- Balanced and Unbalanced
- TEM and non-TEM
- Planar and non-planar
- Homogeneous and ~~non-ho~~ Inhomogeneous

Coaxial line | Two-wire line | Planar line | Wire above conducting plane | Microstrip line

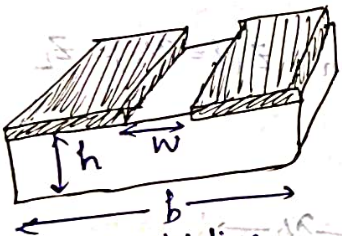
Planar T/L :



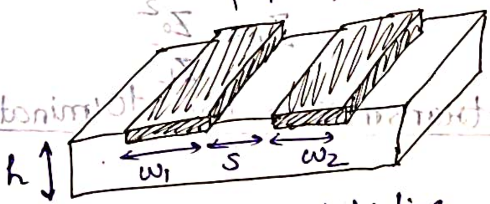
Microstrip



coplanar WG



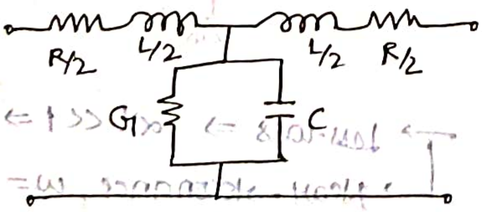
Slot line



coplanar stripline

Transmission Line

Equivalent circuit :



$$Z_{in} = \frac{V_S}{I_S} = Z_0\left[\frac{Z_L + Z_0 \tanh \gamma z}{Z_0 + Z_L \tan \gamma z}\right]$$

$$= Z_0 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$

$$= Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

For lossless T/L

Matched Line: $Z_L = Z_0$ $\quad \left(\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \right)$

$\Gamma(l) = 0 \Rightarrow$ No reflection

Short-circuited Line: $Z_L = 0, \quad \Gamma = -1$

$$V(z) = -2jV_0^+ \sin\beta z,$$

$$I(z) = 2\frac{V_0^+}{Z_0}\cos\beta z$$

$$Z_{in} = jZ_0 \tan\beta l$$

Open-circuited Line: $Z_L = \infty, \quad \Gamma = 1$

$$V(z) = 2V_0^+ \cos\beta z$$

$$I(z) = \frac{-2jV_0^+}{Z_0}\sin\beta z$$

$$Z_{in} = -jZ_0 \cot\beta l$$

Quarter-wave Transformer:

$$l = \lambda/4 + \frac{\lambda}{2}n, \quad n = 0, 1, 2, 3, \ldots$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$\lambda/2$-transmission line terminated in Z_L: $Z_{in} = Z(l = \lambda/2) = Z_L$

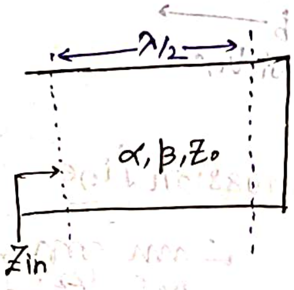
T/L Resonators

↳ All realistic resonators have a finite $Q = f_0/BW$.

→ Non-zero BW ⇒ Resonator is lossy

→ T/L Resonator have $\alpha \neq 0$.

→ Lossy T/L.



Short-circuited $\lambda/2$ T/L:

↳ ⓐ $\tanh\alpha l \approx \alpha l$

ⓑ $\Delta\omega$ is small.

→ Low-loss ⇒ $\alpha l \ll 1 \Rightarrow \tan\alpha l \approx \alpha l$.

↳ Near-resonance, $\omega = \omega_0 + \underbrace{\Delta\omega}_{\text{small}}$.

$$\beta l = \frac{\omega l}{v_p} = \frac{(\omega_0 + \Delta\omega)l}{\frac{\omega_0\lambda}{2\pi}} = \frac{2\pi l}{\lambda}\left(1 + \frac{\Delta\omega}{\omega_0}\right) = \pi\left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

$$\Rightarrow \tan\beta l = \tan\left(\pi + \frac{\Delta\omega\pi}{\omega_0}\right) = \tan\frac{\Delta\omega\pi}{\omega_0}$$

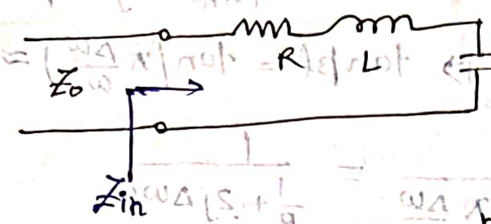
$$Z_{in} = Z_0 \tan(\alpha + i\beta)l = Z_0 \frac{\tan\alpha l + j\tan\beta l}{1 + j\tan\alpha l \tan\beta l}$$

$$\approx Z_0 \frac{\alpha l + j\frac{\Delta\omega}{\omega_0}\pi}{1 + i\alpha l\frac{\Delta\omega}{\omega_0}\pi} \approx Z_0\left(\alpha l + j\frac{\Delta\omega}{\omega_0}\pi\right)$$

For series resonance , $Z_{in} \rightarrow R + 2jL\Delta\omega$.

Comparing, $R = Z_0 \alpha l$, $L = \frac{\pi}{2}\frac{Z_0}{\omega_0}$, $C = \frac{1}{\omega_0^2 L}$

Eq. circuit :



Quality Factor : $Q = \frac{\omega_0 L}{R} = \frac{\pi}{2\alpha l} = \frac{\beta}{2\alpha}$. $\uparrow \Rightarrow \alpha \downarrow$

Short-circuited $\lambda/4$ T/L :

$$Z_{in} = Z_0 \tanh \gamma l$$
$$= Z_0 \tanh(\alpha + j\beta)l = Z_0 \frac{\tanh \alpha l + j\tan \beta l}{1 + j\tan\beta l \cdot \tanh \alpha l}$$
$$= Z_0 \frac{1 - j\cot\beta l \cdot \tanh \alpha l}{\tanh \alpha l - j\cot \beta l}$$

$$\beta l = \frac{\omega l}{v_p} = \frac{(\omega_0 + \Delta\omega)l}{\frac{\omega_0}{2\pi}\lambda} = \frac{2\pi l}{\lambda}\left(1 + \frac{\Delta\omega}{\omega_0}\right) = \frac{\pi}{2}\left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

$$\Rightarrow \cot\beta l = \cot\left(\frac{\pi}{2} + \frac{\Delta\omega\pi}{2\omega_0}\right) = -\tan\frac{\pi}{2}\frac{\Delta\omega}{\omega_0} = -\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}$$

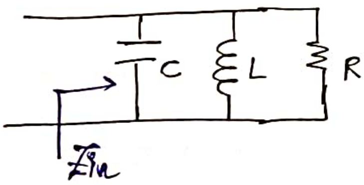
$$Z_{in} = Z_0 \frac{1 + j\alpha l \pi \Delta\omega/2\omega_0}{\alpha l + j\pi\Delta\omega/2\omega_0} \approx \frac{Z_0}{\alpha l + j\pi\Delta\omega/2\omega_0}.$$

For parallel resonance,

$$Z_{in} = \frac{1}{1/R + 2j\Delta\omega C}$$

Comparing, $C = \frac{\pi}{4\omega_0 Z_0}$, $R = \frac{Z_0}{\alpha l}$, $L = \frac{1}{\omega_0^2 C}$

Eq. ckt. :



Quality factor: $Q = \omega_0 RC = \frac{\pi}{4\alpha l} = \frac{\beta}{2\alpha}$, $l = \frac{\pi}{2\beta}$

Open-circuited $\lambda/2$ T/L :

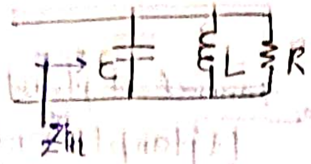
$$Z_{in} = Z_0 \coth(\alpha l + j\beta l) = Z_0 \frac{1 + j\tanh\alpha l \cdot \tan\beta l}{\tanh\alpha l + j\tan\beta l}$$

$$\beta l = \pi\left(1 + \frac{\Delta\omega}{\omega_0}\right) \Rightarrow \tan\beta l = \tan\left(\pi\frac{\Delta\omega}{\omega_0}\right) \approx \pi\frac{\Delta\omega}{\omega_0}$$

$$Z_{in} \approx \frac{Z_0}{\alpha l + j\pi\frac{\Delta\omega}{\omega_0}} = \frac{1}{\frac{1}{R} + 2j\Delta\omega C}$$

Comparing, $C = \frac{\pi}{2\omega_0 Z_0}$, $R = \frac{Z_0}{\alpha l}$, $L = \frac{1}{\omega_0^2 C}$

Eq. ckt :

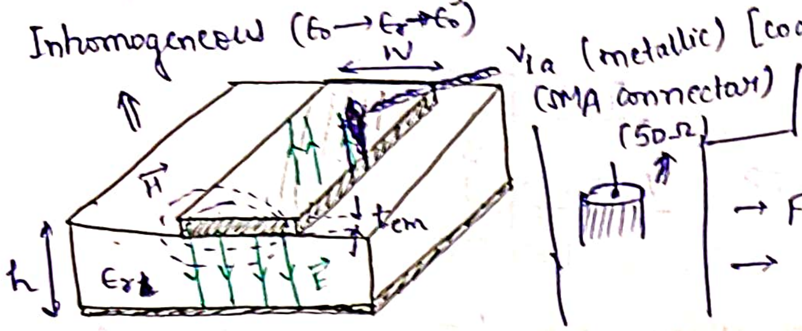


Quality factor : $Q = \omega_0 RC = \frac{\beta}{2\alpha}$, $l = \frac{\pi}{\beta}$ at resonance.

20-08-2024

Half-Wave Microstrip Resonator

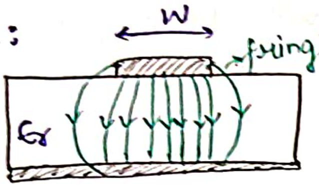
Inhomogeneous ($\epsilon_0 \rightarrow \epsilon_r \rightarrow \epsilon_0$)



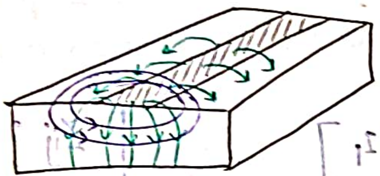
$t = 35\,\mu m$
$h = 1.575\,mm$
$\epsilon_r = 2.2 - 10.2$

→ Field distribution is like TE mode.
→ Quasi TEM mode

Cross-section:



→ Dirn of fringing field reverses after every $\lambda/2$ distance.



⟹

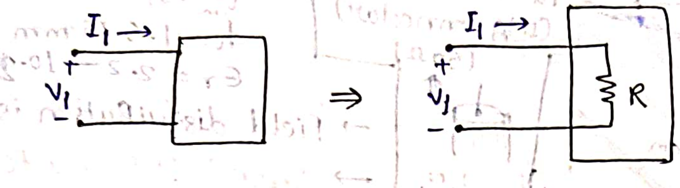
Effective ϵ_r for microstrip

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left(1 + \frac{12h}{W}\right)^{-1/2}$$

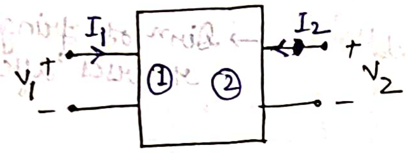
27-08-2024

Multiport Networks

One-Port Network:



Two-Port Networks:



Z-parameters:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Z_{ij} : transimpedance

$$\Rightarrow [V] = [Z][I]$$

~~Transin~~
Impedance matrix

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}, \quad I_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}, \quad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

In general, $Z_{ij} = \frac{V_i}{I_j}\Big|_{I_k=0,\, k\neq j}$ ← open-circuit parameters

↳ convenient for series connected networks.

Admittance (Y) Parameters: convenient for parallel connected networks.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow [I] = [Y][V]$$

$Y_{ij} = \frac{I_i}{V_j}\Big|_{V_k=0,\, k\neq j}$ ← short-circuit parameters

Relation b/w [Z] and [Y] matrices:

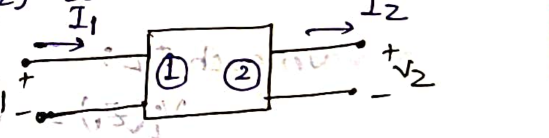
$$[V] = [Z][I]$$
$$[I] = [Y][V]$$
$$\Rightarrow [V] = [Z][Y][V]$$
$$[Z][Y] = [U] = \text{identity matrix}$$
$$\therefore [Y] = [Z]^{-1}$$

Reciprocal N/W: Does not contain non-reciprocal devices or materials (e.g., ferrites or active devices).

$$\Rightarrow Z_{ij} = Z_{ji} \quad (Y_{ij} = Y_{ji})$$
$$\Rightarrow [Z] \text{ and } [Y] \text{ are symmetric} \quad (\because \text{Inverse of symmetric matrix is symmetric})$$

→ Reciprocal materials have reciprocal permittivity and permeability tensors. (S) $\epsilon = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$, ($\epsilon_{xy}$: apply $\vec{E}$ in y-dirn & calculate polarization in x-dirn.)

ABCD Parameters



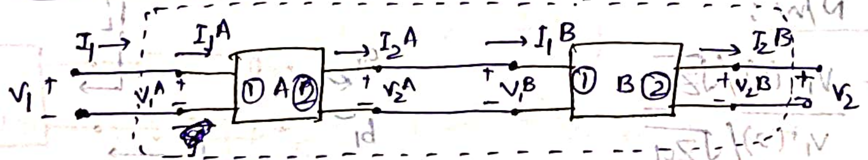
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2' \end{bmatrix}$$

$$I_2' = -I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2'=0}, \quad C = \left.\frac{I_1}{V_2}\right|_{I_2'=0}$$

$$B = \left.\frac{V_1}{I_2'}\right|_{V_2=0}, \quad D = \left.\frac{I_1}{I_2'}\right|_{V_2=0}$$

Cascaded N/W:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1A \\ I_1A \end{bmatrix} = [ABCD^A] \begin{bmatrix} V_2A \\ I_2A' \end{bmatrix}$$

$$= [ABCD^A] \begin{bmatrix} V_1B \\ I_1B \end{bmatrix}$$

$$= \underbrace{[ABCD^A][ABCD^B]} \begin{bmatrix} V_2B \\ I_2B' \end{bmatrix}$$

$$= [ABCD^{AB}] \begin{bmatrix} V_2 \\ I_2' \end{bmatrix}$$

Scattering (S) Parameters:

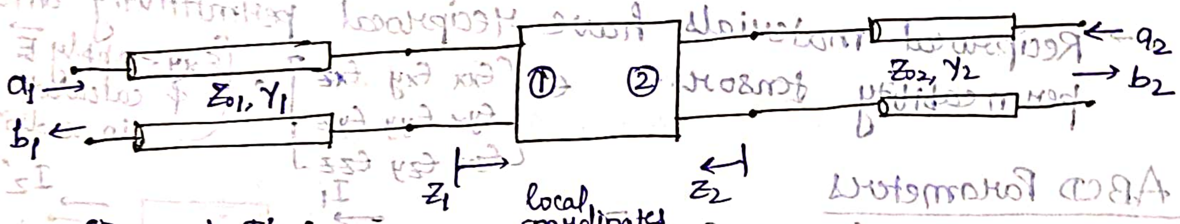
↳ At high frequencies, Z, Y, h and ABCD parameters are difficult (if not impossible) to measure.

↳ Required open and short-circuit conditions are difficult to achieve.

↳ ~~Requi~~ V and I are not uniquely defined.

↳ Even if defined, V & I are difficult to measure (particularly I).

→ S-parameters: Best representation for multi-port n/w at high frequency.



On each T/L:

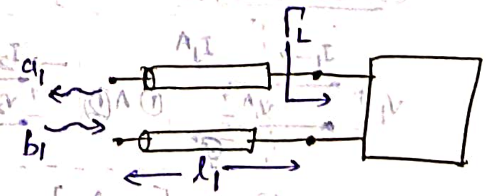
$$V_i(z_i) = V_{i0}^{+} e^{-Y_i z_i} + V_{i0}^{-} e^{+Y_i z_i} = V_i^{+}(z_i) + V_i^{-}(z_i)$$

$$I_i(z_i) = \frac{V_i^{+}(z_i)}{Z_{0i}} - \frac{V_i^{-}(z_i)}{Z_{0i}}, \quad i = 1, 2$$

Incoming wave f^n : $a_i(z_i) = V_i^{+}(z_i) / \sqrt{Z_{0i}}$

Outgoing wave f^n : $b_i(z_i) = V_i^{-}(z_i) / \sqrt{Z_{0i}}$

For One-Port N/W:



$$\Gamma_L = \frac{V_1^{-}(0)/\sqrt{Z_{01}}}{V_1^{+}(0)/\sqrt{Z_{01}}} = \frac{b_1(0)}{a_1(0)} = S_{11} \Rightarrow b_1(0) = \Gamma_L a_1(0) = S_{11} a_1(0)$$

For Two-Port N/W:

$$b_1(0) = S_{11} a_1(0) + S_{12} a_2(0)$$

$$b_2(0) = S_{21} a_1(0) + S_{22} a_2(0)$$

$$\Rightarrow \begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}}_{\text{scattering matrix}} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix} \Rightarrow [b] = [S][a]$$

$S_{11} = \left. \frac{b_1(0)}{a_1(0)} \right|_{a_2=0}$ ← o/p is matched ← I/p reflection coeff. with o/p matched

$S_{12} = \left. \frac{b_1(0)}{a_2(0)} \right|_{a_1=0}$ ← I/p is matched ← Reverse transmission coeff. with i/p matched

$S_{21} = \left. \frac{b_2(0)}{a_1(0)} \right|_{a_2=0}$ ← Forward tx coeff. with o/p matched

$S_{22} = \left. \frac{b_2(0)}{a_2(0)} \right|_{a_1=0}$ ← o/p reflection coeff. with i/p matched.

In general, $S_{ij} = \left. \frac{b_i(0)}{a_j(0)} \right|_{a_K=0,\ K\neq j}$ ← all ports except j are semi-infinite (or matched).

S_{21}: How much signal is attenuated at port 2 when launched at port 1.

→ For reciprocal n/w, S-matrix is symmetric

$$\Rightarrow S_{ij} = S_{ji}, \quad i \neq j.$$

→ If all lines entering the n/w have same char. impedance,

$$S_{ij} = \left. \frac{V_i^-(0)}{V_j^+(0)} \right|_{V_K^+=0,\ K\neq j}$$

09-09-2024

Properties of S-matrix:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Unitary property: $S_{11}S_{11}^* + S_{21}S_{21}^* + S_{31}S_{31}^* = 1$

$$\Rightarrow |S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1,$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = 1$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1.$$

→ $\sum_{K=1}^{n} S_{Ki} \cdot S_{Ki}^* = 1$ (lossless microwave)

Zero-property: $S_{11}S_{12}^* + S_{21}S_{22}^* + S_{31} \cdot S_{32}^* = 0$

$$S_{11}S_{13}^* + S_{21}S_{23}^* + S_{31} \cdot S_{33}^* = 0$$

$$S_{12}S_{13}^* + S_{22}S_{23}^* + S_{32}S_{33}^* = 0$$

→ $\sum_{K=1}^{n} S_{Ks} \cdot S_{Kr}^* = 0, \quad r \neq s$

→ The column vectors and the row vectors form an orthogonal set.

Eg. Matched terminated, lossless, reciprocal.

Unitary property: $|S_{21}|^2 + |S_{31}|^2 = 1$

$|S_{12}|^2 + |S_{32}|^2 = 1$

$|S_{13}|^2 + |S_{23}|^2 = 1$

Zero property: $S_{31} \cdot S_{32}^* = 0$

$S_{21} \cdot S_{23}^* = 0$

$S_{12} \cdot S_{13}^* = 0$

} Atleast 2 are zeros.

⇒ Solution is not consistent.

⇒ A 3-port network cannot be matched-terminated, lossless and reciprocal simultaneously.

E-plane: Plane constituted by $\vec{E}$ and dirn of propagation.

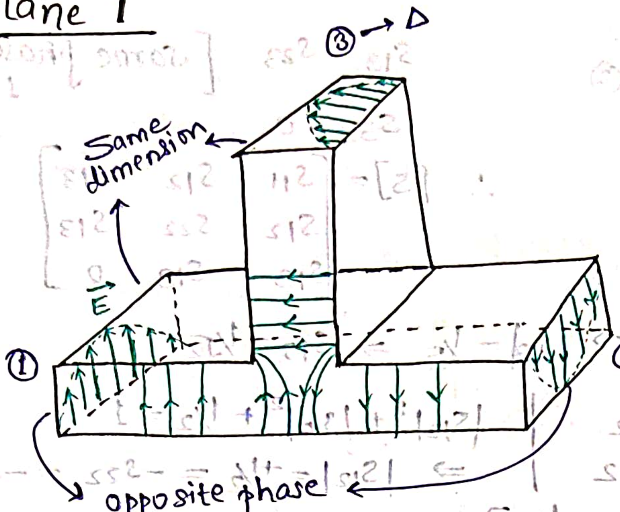
03-09-2024

Microwave Power Divider

E-plane T

SIW: Substrate Integrated waveguide

→ Non-reciprocal (by default)
↳ No reciprocal material



→ A 3-port network cannot be matched terminated, reciprocal and lossless simultaneously.

→ If wave is launched from port-3, it is divided equally b/w ① and ②, but in opposite phase.

→ If launched at P_1 and P_2, it gets subtracted at P_3 (that's why port-3 is called Δ arm).

Launching wave from port-3:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

[$S_{23} = -S_{13}$, $S_{33} = 0$ (Assume P_3 to be matched terminated from where signal is launched)]

By Unity property: $|S_{13}|^2 + |S_{13}|^2 = 1$

$\Rightarrow |S_{13}| = 1/\sqrt{2} \Rightarrow S_{13} = \frac{1}{\sqrt{2}}$ (or $-\frac{1}{\sqrt{2}}, j\frac{1}{\sqrt{2}}, -j\frac{1}{\sqrt{2}}$)

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & 1/\sqrt{2} \\ S_{12} & S_{22} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

$S_{12} = S_{22}$, $S_{11} = S_{12}$ (By Zero property)

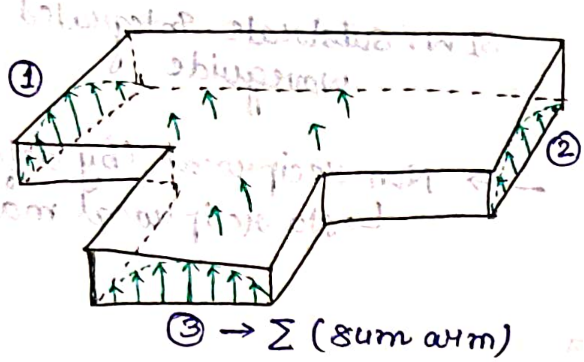
By unity property: $|S_{12}|^2 + |S_{12}|^2 + 1/2 = 1$

$\Rightarrow |S_{12}|^2 = 1/4$

$\Rightarrow |S_{12}| = 1/2$.

$$\therefore [S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

H-plane T



$S_{13} = S_{23}$ [same phase at port 1 and 2

$S_{33} = 0$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

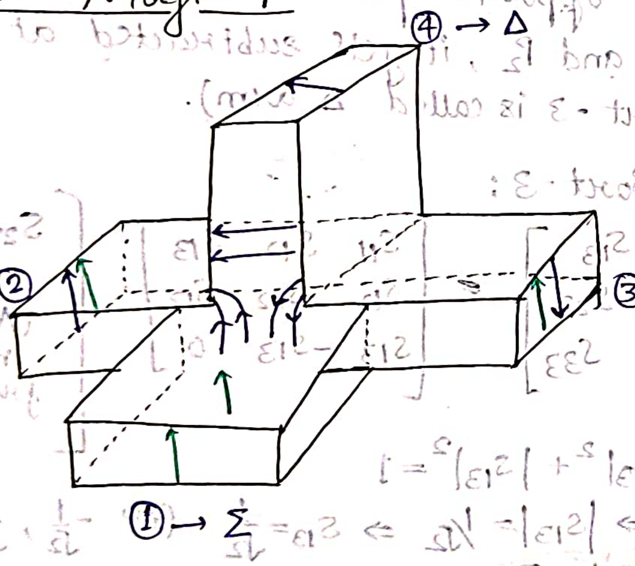
Unity p/ty: $2|S_{13}|^2 = 1 \Rightarrow |S_{13}| = 1/\sqrt{2} \Rightarrow S_{13} = 1/\sqrt{2}$

$S_{12} = -S_{22}$ | $|S_{12}|^2 + |S_{12}|^2 + 1/2 = 1$

$S_{11} = -S_{12}$ | $\Rightarrow |S_{12}| = +1/2 = -S_{22} = -S_{11}$

$$\therefore [S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Hybrid-T / Magic-T



→ Launching wave at ①, no power reaches ④, and vice-versa.

$S_{34} = -S_{24}$

$S_{31} = S_{21}$

$S_{41} = S_{14} = 0 \rightarrow$ Orthogonal

$|S_{11}| = |S_{44}| = 0$

$$\therefore [S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{12} & S_{23} & S_{33} & -S_{24} \\ 0 & S_{24} & -S_{24} & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & +1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$S_{24} = -1/\sqrt{2}$, $S_{12} = 1/\sqrt{2}$

$|S_{13}|^2 + 1/2 = 1 \Rightarrow S_{13} = 1/\sqrt{2}$

$|S_{22}|^2 + |S_{23}|^2 = 0 \Rightarrow S_{22} = S_{23} = 0 = S_{33}$.

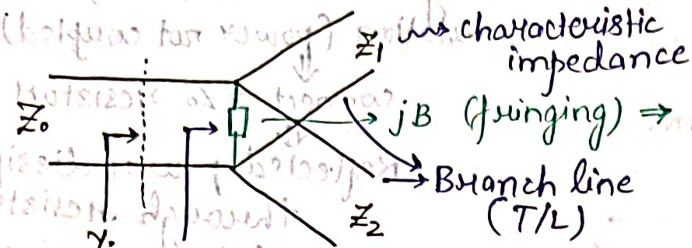
01-09-2021

→ In circular waveguide, TE_{21} mode in T-n/w is used to ensure locked condition for transmitter and receiver in satellite comms.

→ Rx and Tx antenna beam should be overlapping.

T-Junction Power Divider

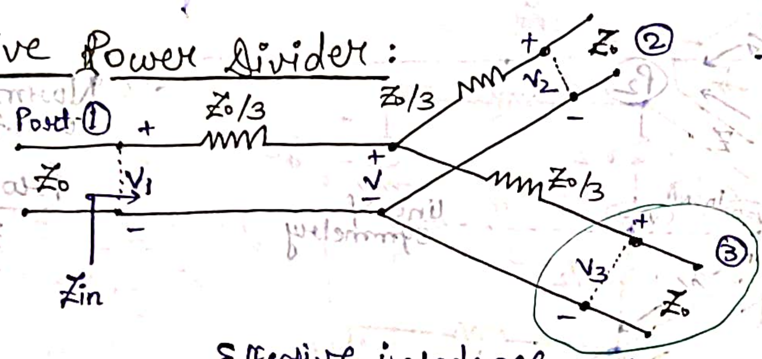
- → Resistive Power divider
- → Wilkinson Power divider



⇒ [Ideally = 0] → can cause radiation / localised heating

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} \quad \left(\text{ideally} = \frac{1}{Z_1} + \frac{1}{Z_2}\right) \quad \left[\text{Expected } Y_{in} = \frac{1}{Z_0}\right]$$

Resistive Power Divider:



→ Matched-terminated at all ports.

→ Not full power at o/p; some gets reflected/attenuated (compromise for matched-termination)

Effective impedance (at each port) : $\left(Z_0 + \frac{Z_0}{3}\right) = \frac{4Z_0}{3}$

$$Z_{in} = \left(\frac{4Z_0}{3}\right) \parallel \left(\frac{4Z_0}{3}\right) + \frac{Z_0}{3}$$

$$= \frac{2Z_0}{3} + \frac{Z_0}{3} = Z_0$$

$S_{11} = S_{22} = S_{33} = 0$ (All ports are m.t.)

$$V = \frac{2Z_0/3}{\frac{2Z_0}{3} + \frac{Z_0}{3}} V_1 = \frac{2}{3} V_1$$

$$V_2 = \frac{Z_0}{Z_0 + \frac{Z_0}{3}} V = \frac{3}{4} V = \frac{3}{4} \times \frac{2}{3} V_1 = \frac{1}{2} V_1 \Rightarrow \text{Power: } 25\% \text{ of } P_1.$$

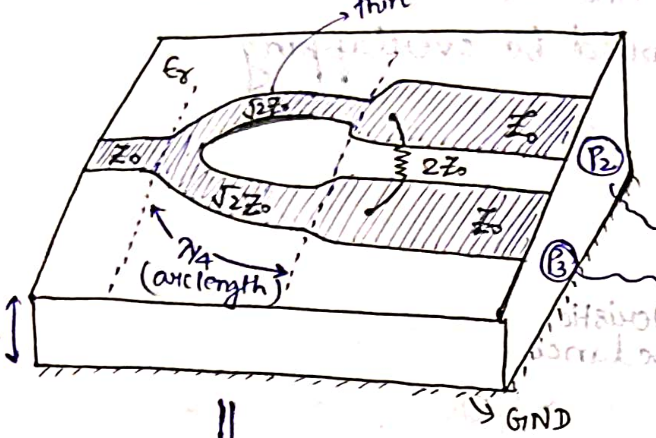
$S_{21} = V_2/V_1 = 1/2$ | $P_{in} = \frac{V_1^2}{V_0}$ | $\boxed{P_2 = P_3 = \frac{1}{4} P_{in}}$ ⇝ problem!

$S_{31} = 1/2$

$$\therefore [S] = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

(Properties not applicable for lossy)

Wilkinson Power Divider :



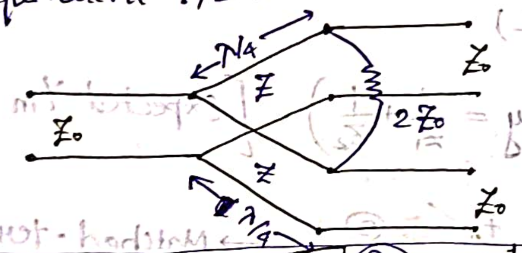
$Z_0 = 50\Omega$

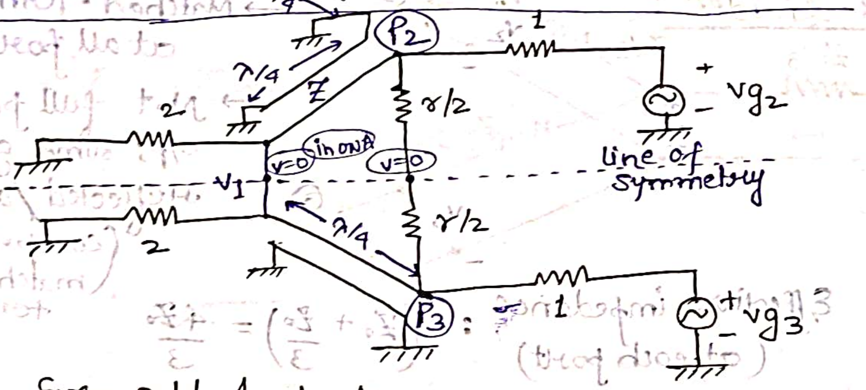
→ same $\sqrt{2}Z_0$ impedance for the arc ensures equal power goes to ② and ③ ⇒ Equal split

may be mismatched ; violating isolation (power not coupled) b/w 2 ports
⇓
connect $2Z_0$ resistor
⇓
Reflected power dissipates through resistor (ensuring proper isolation)

⇓

Equivalent T/L :





→ Normalised impedances w.r.t. Z_0.

Prove that :
$Z = \sqrt{2}$
$r = 2$

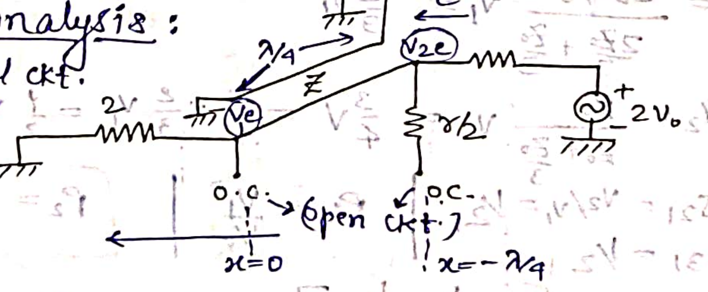
Even-odd Analysis :

① Even-mode : $V_{g2} = V_{g3} = 2V_0$ (or V_0)

② Odd-mode : $V_{g2} = -V_{g3} = 2V_0$

$V_{g2} = 4V_0$
$V_{g3} = 0$

Even-mode analysis :
↳ symmetrical ckt.



$$Z_{in}^e = \frac{Z^2}{2} = 1 \quad \text{(make} = 1 \text{, for matching)}$$

$$\boxed{Z = \sqrt{2}}$$

$\boxed{V_2^e = V_0}$, $V_1^e = ?$

$$V(x) = V^+\left[e^{-j\beta x} + \Gamma e^{j\beta x}\right]$$

$$V(x=-\lambda/4) = V_2^e = V_0 = V^+\left[\underbrace{e^{j\beta\lambda/4}}_{j} + \Gamma \underbrace{e^{-j\beta\lambda/4}}_{-j}\right]$$

$$= jV^+(1-\Gamma)$$

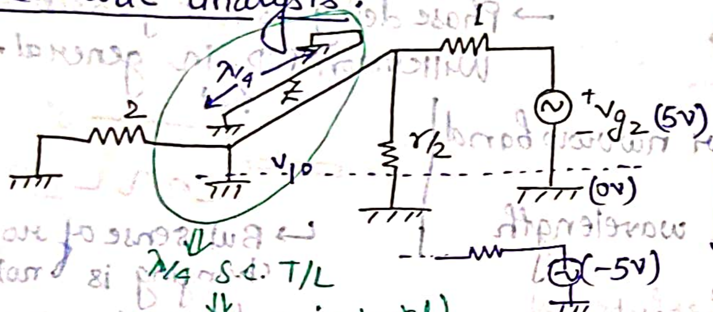
$$\Rightarrow \boxed{V^+ = \frac{-jV_0}{1-\Gamma}}$$

$\# \; e^{j\beta\lambda/4} = e^{\left(j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right)} = e^{j\pi/2} = j$

$$V(x=0) = V_1^e = V^+(1+\Gamma) = -jV_0\left(\frac{1+\Gamma}{1-\Gamma}\right) = \boxed{-jV_0\sqrt{2} = V_1^e}$$

$$\left(\frac{\Gamma}{1} = \frac{2-\sqrt{2}}{2+\sqrt{2}} \Rightarrow \frac{1+\Gamma}{1-\Gamma} = \frac{4}{2\times\sqrt{2}} = \sqrt{2}\right)$$

Odd-mode analysis:



← -90° phase delay at o/p port $(-j)$

$\Uparrow$ ∴

50% power goes into ② & ③ separately

$\lambda/4$ S.C. T/L
$\Downarrow$
OC $(-jZ_0 \tan\beta l)$

$r/2 = 1 \Rightarrow \boxed{r=2}$

$\boxed{V_2^o = V_0 ; V_1^o = 0}$

$$\frac{V_1}{V_2} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{-jV_0\sqrt{2}}{2V_0} = \boxed{-j/\sqrt{2}} \Rightarrow S_{12} = S_{21}$$

EMA:

$\boxed{V_2^e = V_0, Z = \sqrt{2} \quad V_1^e = -jV_0\sqrt{2}}$

OMA:

$\boxed{r = 2 \quad V_2^o = V_0, V_1^o = 0}$

09-09-2024

Quadrature Hybrid (90° Hybrid)

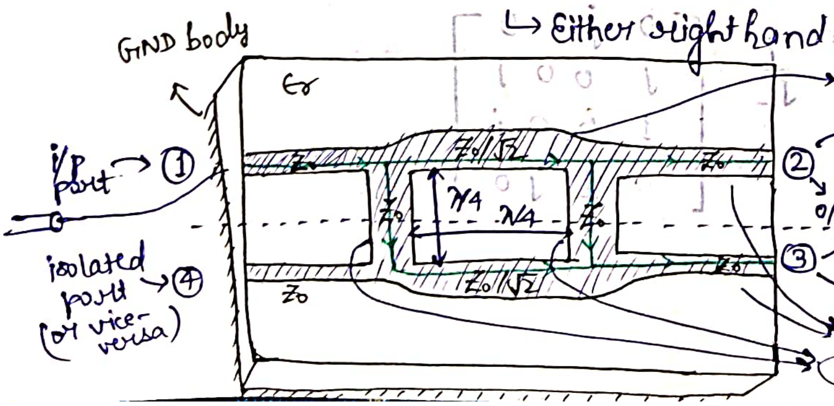
(Branchline Coupler):

↳ 1 i/p, 2 o/p, 1 isolated port [4-port Microwave Passive P.D.]

↳ o/p ports maintain 90° phase difference (same magnitude)

↳ Produces Circular Polarisation

↳ Either right hand (CW) or left hand (ACW) polarise



→ less impedance line ⇒ Wide line

→ Through port (if wave launched from ①)

o/p ports

--- symmetric wrt this line

→ Coupled port

Out-of-phase paths [λ/4]

In-phase paths ⇒ Adds up

$\Downarrow$ path diff. $= \lambda/2 \Rightarrow 180^\circ$
$\Downarrow$
cancels
$\Downarrow$
④ : isolated

$[\phi = \beta\lambda = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}]$

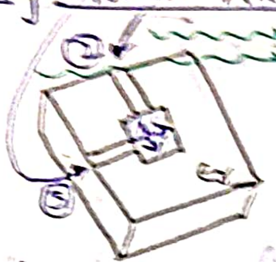
③ gets delayed by 90° wrt ②

$\lambda \to 360^\circ$

120   120

② and ③ gets same power $\Rightarrow 1/\sqrt{2}$
$\Downarrow$
$S_{21} = -j/\sqrt{2}$
$S_{31} = -1/\sqrt{2}$

Microstrip Patch Antenna:



same magnitude & 90° phase diff $\Rightarrow$ circularly polarised

→ Phase delay is not possible in Wilkinson P.D. in general.

But

0°

∠90°

↳ But sense of rotation changing is not possible

↳ By changing ① with ④.

Limitation: only for narrow band
$\Downarrow$
for $\lambda/4$ wavelength

→ Fractional BW $= \frac{\text{Absolute BW}}{\text{Central BW}}$

→ Harmonic Response $\Rightarrow \lambda/4$ line can work as $3\lambda/4$ line.

↳ May or may not be good depending on the application.

S-parameters:

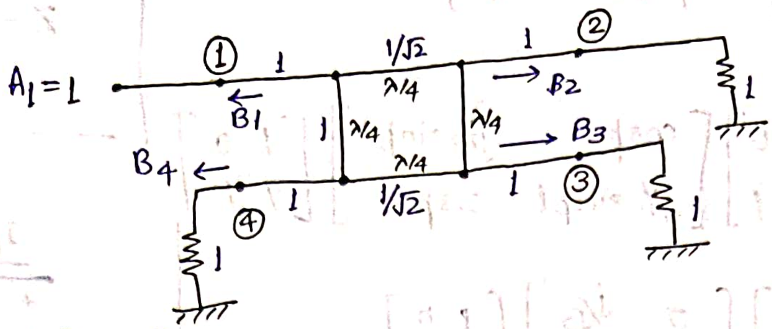
If excited from ①:

P_1 ⇓	P_2 ⇓	P_3 ⇓	P_4 ⇓
$S_{11} = 0$	$S_{12} = \frac{1}{\sqrt{2}}\angle -90^\circ$	$S_{13} = \frac{1}{\sqrt{2}}\angle -180^\circ$	$S_{14} = 0$
$S_{21} = \frac{1}{\sqrt{2}}\angle -90^\circ$	$S_{22} = 0$	$S_{23} = 0$	$S_{24} = \frac{1}{\sqrt{2}}\angle -180^\circ$
$S_{31} = \frac{1}{\sqrt{2}}\angle -180^\circ$	$S_{32} = 0$	$S_{33} = 0$	$S_{34} = \frac{1}{\sqrt{2}}\angle -90^\circ$
$S_{41} = 0$	$S_{42} = \frac{1}{\sqrt{2}}\angle 180^\circ$	$S_{43} = \frac{1}{\sqrt{2}}\angle -90^\circ$	$S_{44} = 0$

$$\therefore \text{S-matrix, } [S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

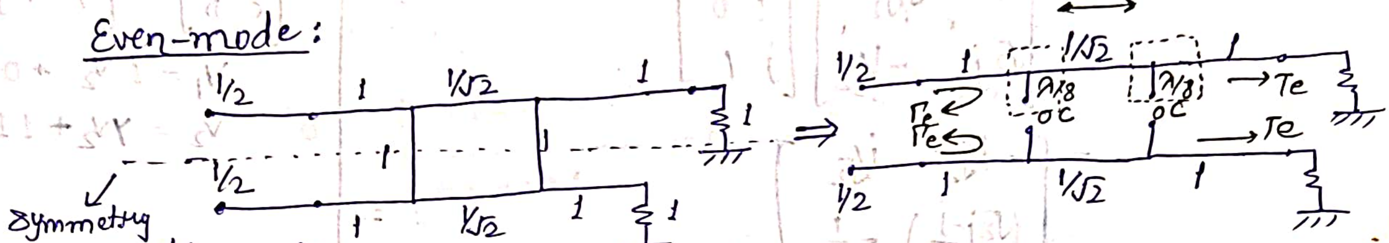
(10-09-2024)

90° Hybrid/Branchline Coupler : Analysis



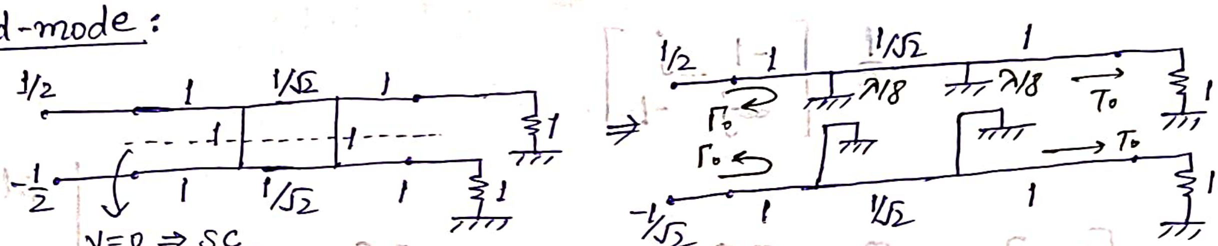
$$\begin{matrix}1\\0\end{matrix}\Bigg\} \begin{pmatrix}1/2\\1/2\end{pmatrix} + \begin{pmatrix}1/2\\-1/2\end{pmatrix}$$

Even-mode :



Symmetry (no current passing through this) ⇒ oc

Odd-mode :



V=0 ⇒ SC

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

$$B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

$$B_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

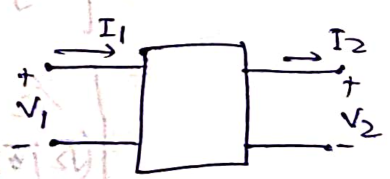
By ABCD parameter,

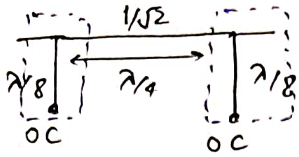
$$\boxed{\Gamma = \frac{A + B - (C+D)}{A+B+C+D}}$$

$$\boxed{T = \frac{2}{A+B+C+D}}$$

ABCD parameters :

$$\begin{bmatrix}V_1\\I_1\end{bmatrix} = \begin{bmatrix}A & B\\C & D\end{bmatrix}\begin{bmatrix}V_2\\I_2\end{bmatrix}$$





[Lossless OC T/L]

$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$

$Z \rightarrow jZ_0 \cot\beta l$

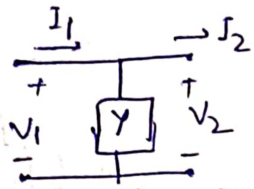
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ jY_0 \sin\beta l & \cos\beta l \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ \sqrt{2}j & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j/\sqrt{2} \\ \sqrt{2}j & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/\sqrt{2} & j/\sqrt{2} \\ \left(\sqrt{2}j - \frac{j}{\sqrt{2}}\right) & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$V_1 = 1 V_2 + 0 I_2$

$V_2 = Y V_2 + 1 I_2$

$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\lambda}{4}$

$Z = jZ_0 \tan\beta l$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ jY_0 \sin\beta l & \cos\beta l \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} \cancel{\cos\beta l}\ 0 & j/\sqrt{2} \\ \cancel{jY_0\sin\beta}\ \sqrt{2}j & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j/\sqrt{2} \\ \sqrt{2}j & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & j/\sqrt{2} \\ \left(\sqrt{2}j - \frac{1}{\sqrt{2}}j\right) & 1/\sqrt{2} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$\Gamma_e = 0$

$\Gamma_o = 0$

<u>Even</u>: $A = \frac{-1}{\sqrt{2}},\ B = j/\sqrt{2},\ C = j/\sqrt{2},\ D = -1/\sqrt{2}$

$$\Gamma_e = \frac{A+B-(C+D)}{A+B+C+D} = \frac{-1 + j - (j/\sqrt{2} - 1/\sqrt{2})\sqrt{2}}{-1+j+j-1} = 0$$

$$T_e = \frac{2}{A+B+C+D} = \frac{2}{(-1+j+j-1)1/\sqrt{2}} = \frac{\sqrt{2}}{\cancel{2}}\frac{1}{(j-1)} = \frac{-1}{\sqrt{2}}(1+j)$$

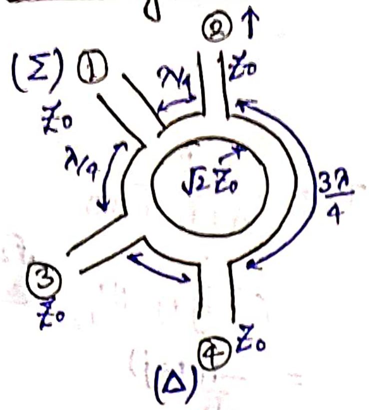
<u>Odd</u>: $A = 1/\sqrt{2},\ B = j/\sqrt{2}$

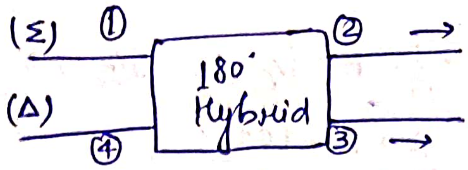
$C = j/\sqrt{2},\ D = 1/\sqrt{2}$

$$\Gamma_o = \frac{1+j-(j+1)}{1+j+j+1} = 0$$

$$T_o = \frac{2}{\frac{1}{\sqrt{2}}(1+j+j+1)} = \frac{\sqrt{2}}{\cancel{2}}\frac{1}{(j+1)} = \frac{1}{\sqrt{2}}(1-j)$$

180° Hybrid

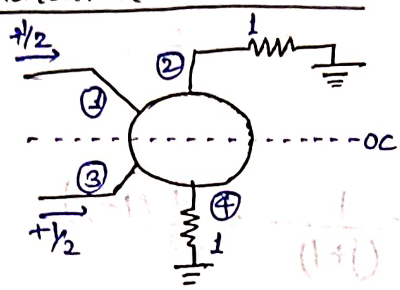


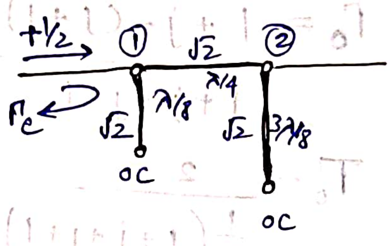


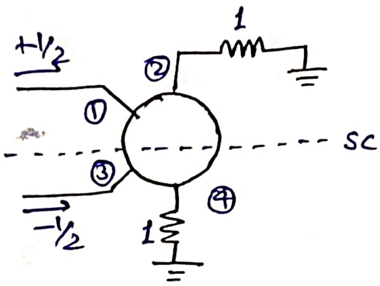
$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

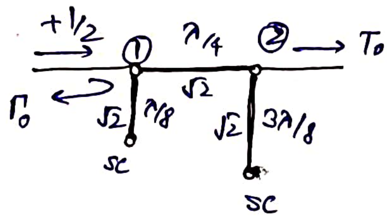
Even and Odd Mode Analysis:

Excitation at Port 1:









$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$, $\quad B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$

$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o$, $\quad B_4 = \frac{1}{2}T_e - \frac{1}{2}T_o$

Even:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

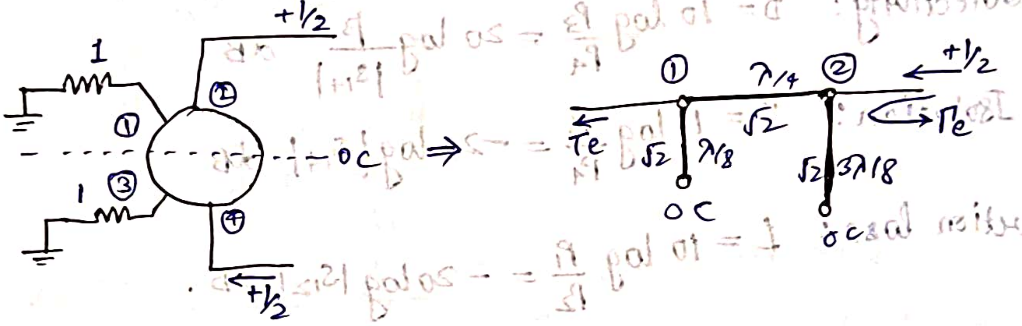
odd:

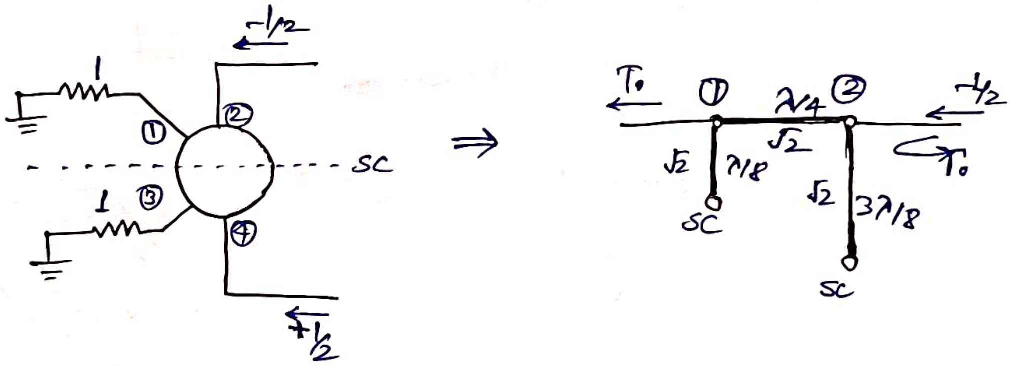
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

$$\left.\begin{aligned} \Gamma_e &= \frac{-j}{\sqrt{2}} \\ T_e &= \frac{-j}{\sqrt{2}} \\ \Gamma_o &= \frac{j}{\sqrt{2}} \\ T_o &= \frac{-j}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \begin{aligned} B_1 &= 0 \\ B_2 &= \frac{-j}{\sqrt{2}} \\ B_3 &= \frac{-j}{\sqrt{2}} \\ B_4 &= 0 \end{aligned}$$

Excitation at port-4 :

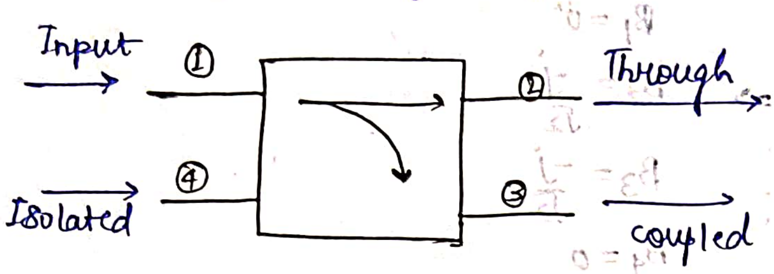




$$\left.\begin{aligned} B_1 &= \tfrac{1}{2}T_e - \tfrac{1}{2}T_o \\ B_2 &= \tfrac{1}{2}\Gamma_e - \tfrac{1}{2}\Gamma_o \\ B_3 &= \tfrac{1}{2}T_e + \tfrac{1}{2}T_o \\ B_4 &= \tfrac{1}{2}\Gamma_e + \tfrac{1}{2}\Gamma_o \end{aligned}\right\} \Rightarrow \begin{aligned} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_e &= \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix} \\ \begin{bmatrix} A & B \\ C & D \end{bmatrix}_o &= \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix} \end{aligned}$$

$$\Rightarrow \left.\begin{aligned} \Gamma_e &= \frac{j}{\sqrt{2}} \\ T_e &= \frac{-j}{\sqrt{2}} \\ \Gamma_o &= \frac{-j}{\sqrt{2}} \\ T_o &= \frac{-j}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \begin{aligned} B_1 &= 0 \\ B_2 &= j/\sqrt{2} \\ B_3 &= -j/\sqrt{2} \\ B_4 &= 0. \end{aligned}$$

Directional Couplers: Figure of Merits



Coupling: $C = 10 \log \frac{P_1}{P_3} = -20 \log \beta \ dB$

Directivity: $D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} \ dB$

Isolation: $I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| \ dB$

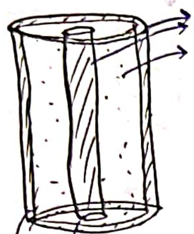
Insertion loss: $L = 10 \log \frac{P_1}{P_2} = -20 \log |S_{12}| \ dB$.

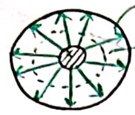
17-09-2024

→ As frequency ↑, metal starts loosing metallic property
↳ e^- don't carry electrons.

2 conductors ⇒ TEM mode

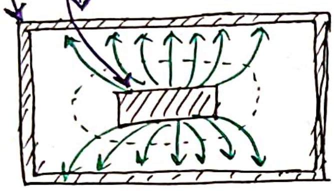
Coaxial cable :





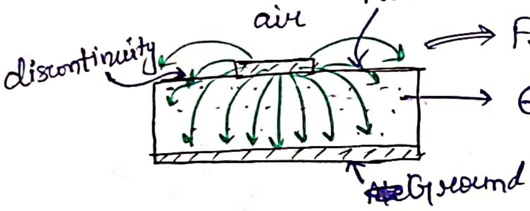
$$\hat{k} = \hat{E} \times \hat{H}$$

⇓ Flattening



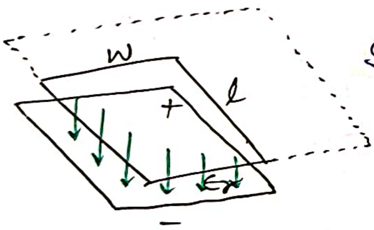
(Strip line)
→ TEM

⇓ splitting



→ Fringing effect ⇒ Due to high ϵ_r below

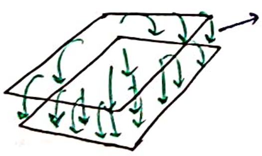
→ $\epsilon_r = 9.9$ ↑ ⇒ More field lines below ⇒ less fringing
⇒ cost ↑↑
⇒ Bad for antenna
(it wants to radiate)



Infinite parallel plates :

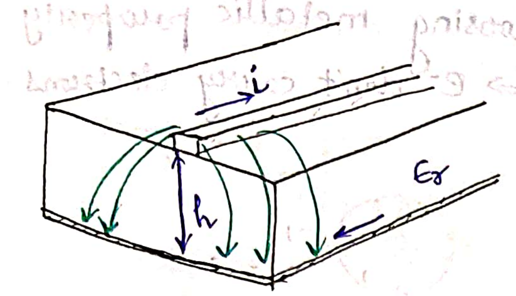
$$C = \frac{\epsilon_r \epsilon_0 A}{d}$$

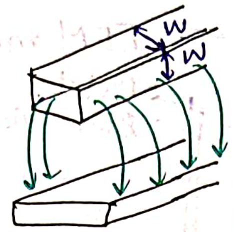
⇓ Infinite parallel plates :



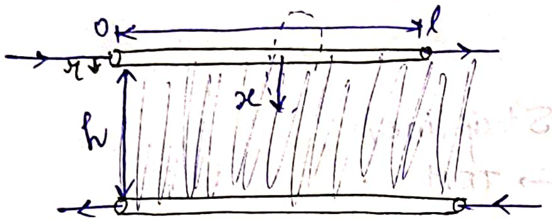
→ fringing → depends on : ϵ_r, d, perimeter (P), thickness

$$C = \frac{\epsilon_0 \epsilon_r A}{d} + \frac{\epsilon_0 \epsilon_r}{d} \frac{Pt}{(K)}$$





$\Downarrow$



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow B(2\pi x) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0}{2\pi x} I$$

$$\phi = \int \vec{B}\cdot d\vec{s}$$

$$= \int_r^h \frac{\mu_0 I}{2\pi x}\, dx$$

$$= \frac{\mu_0}{2\pi} \ln \frac{h}{r}$$

$$\frac{d\phi}{dt} = \frac{\mu_0}{2\pi} \ln\frac{h}{r}\left(\frac{di}{dt}\right)$$

$$V_L = -L\frac{di}{dt}$$

$$\therefore \boxed{L = \frac{\mu_0}{2\pi} \ln\frac{h}{r}}$$

Capacitance, $C = \left(\frac{\epsilon_0 \epsilon_r W l}{h}\right) + \left(\frac{\epsilon_0 \epsilon_r}{h} \cdot \frac{(l+W)W}{K}\right)$

for $l >> W$,

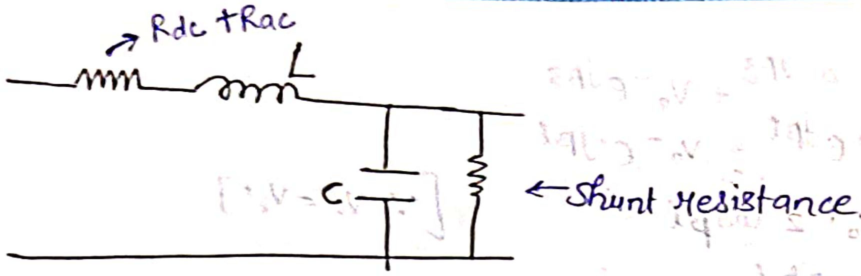
$$\boxed{C = \frac{\epsilon_0 \epsilon_r l W}{h}\left[1 + \frac{1}{K}\right]}$$

$W\downarrow$ or $r\downarrow \Rightarrow L\uparrow$, $C\downarrow$

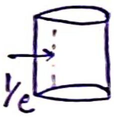
$h\uparrow \Rightarrow L\uparrow$, $C\downarrow$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



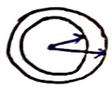
$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$$



$\frac{1}{3} \approx 1/e$

$\downarrow\uparrow\ \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$: Skin depth

$\Downarrow$ Hollow conductor

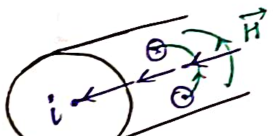


$$R = \frac{\rho l}{\pi(r_1^2 - r_2^2)}$$

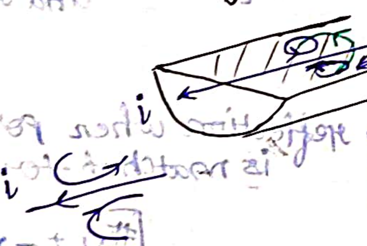
$f\uparrow \Rightarrow R\uparrow$

$\sqrt{f} \propto R$

→



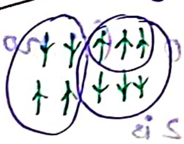
$\Downarrow$ split



← Eddy current

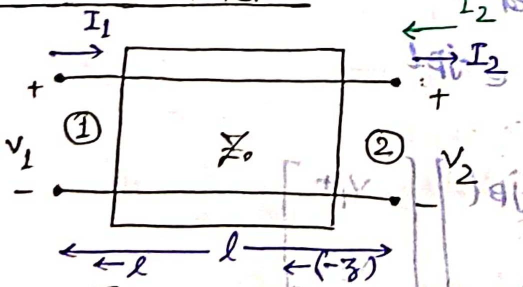
$\frac{d\phi}{dt} = E \Rightarrow$ Back EMF

$\Downarrow$



= Net current decreases

ABCD Parameter:



(Lossless T/L)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ ⇒ Port-2: open ckt.

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$V(-l) = V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l}$$

$$V(-l) = V_0^+ \, 2\cos\beta l \qquad [\because V_0^- = V_0^+]$$

$$\Rightarrow V_1 = 2V_0^+ \cos\beta l$$

$$V(0) = V_2 = 2V_0^+$$

S-parameter:

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

[∵ system is passive and reciprocal]

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+ = 0} = 0$$

(∵ no reflection when port-2 is matched-terminated)

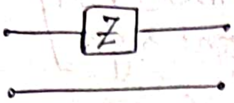
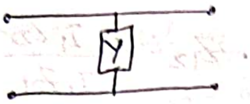
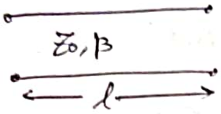
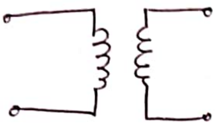
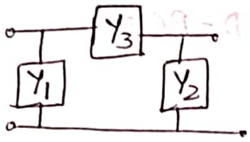
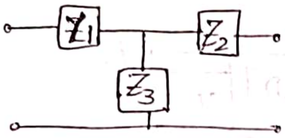
⇒ Port-2 is matched-terminated

[# $V_2^+ = V_2$]

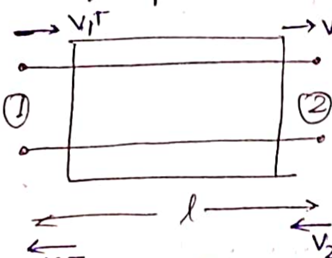
$$S_{12} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+ = 0} = e^{-j\beta l}$$

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{+j\beta l} & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

ABCD Parameters of Some Useful Two-Port Circuits

Circuit	ABCD Parameters
① 	$A=1 \quad B=Z$ $C=0 \quad D=1$
② 	$A=1 \quad B=0$ $C=Y \quad D=1$
③ 	$\begin{bmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & \cos\beta l \end{bmatrix}$
④ 	$\begin{bmatrix} N & 0 \\ 0 & 1/N \end{bmatrix}$
⑤ 	$\begin{bmatrix} 1+\frac{Y_2}{Y_3} & \frac{1}{Y_3} \\ Y_1+Y_2+\frac{Y_1Y_2}{Y_3} & 1+\frac{Y_1}{Y_3} \end{bmatrix}$
⑥ 	$\begin{bmatrix} 1+\frac{Z_1}{Z_3} & Z_1+Z_2+\frac{Z_1Z_2}{Z_3} \\ \frac{1}{Z_3} & 1+\frac{Z_2}{Z_3} \end{bmatrix}$

Eg. Find out the [S] parameter of the lossless T/L.



From defn, $\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$

$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = 0$ [so port ② reflection has to be stopped] → terminate with Z_0

↳ No reflection at the i/p.

$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = e^{-j\beta l}$ [there is a ⊕ve phase shift]

Wave going from port ① to ②, while ② being Z_0 terminated; no reflection.

$\therefore [S] = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$ → depends upon the length & frequency.

$|S_{21}| = |S_{12}|$

Conversion from Z to ABCD

$$V_1 = I_1 Z_{11} - I_2 Z_{12}$$
$$V_2 = I_1 Z_{21} - I_2 Z_{22}$$

(Sign change since [ABCD] has different sign)

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = \frac{I_1 Z_{11}}{I_1 Z_{21}} = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{V_1}{I_2}\Big|_{V_2=0} = \frac{I_1 Z_{11} - I_2 Z_{12}}{I_2}\Big|_{V_2=0} = Z_{11}\frac{I_1}{I_2}\Big|_{V_2=0} - Z_{12} = Z_{11}\frac{I_1 Z_{22}}{I_1 Z_{21}} - Z_{12}$$

$$= \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{I_1}{I_1 Z_{21}} = \frac{1}{Z_{21}}$$

$$D = \frac{I_1}{I_2}\Big|_{V_2=0} = \frac{I_2 Z_{22}/Z_{21}}{I_2} = \frac{Z_{22}}{Z_{21}}.$$

For reciprocal N/w, $Z_{12} = Z_{21}$ and $AD - BC = 1$.

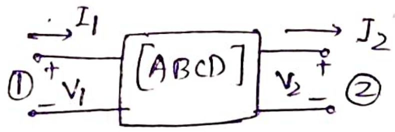
Conversion from S to ABCD

$$S_{11} \rightarrow \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}$$

$$S_{12} \rightarrow \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} \rightarrow \frac{2}{A + B/Z_0 + CZ_0 + D}$$

$$S_{22} \rightarrow \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Proof: $V_1 = AV_2 - BI_2$ ...①

$e^{-j\beta (l)}$ $(=0)$ $\leftarrow (V_1^+ + V_1^-) = A(\cancel{V_2^+} + V_2^-) - B\dfrac{(\cancel{V_2^+} - V_2^-)}{Z_0}$

↳ [measured at phase ref. plan]

$I_1 = CV_2 - DI_2$ ...②

$$\frac{V_1^+ - V_1^-}{Z_0} = \cancel{CV_2^+} + CV_2^- - \left[\frac{\cancel{DV_2^+} - DV_2^-}{Z_0}\right]$$

To calculate $S_{11} = \dfrac{V_1^-}{V_1^+}\Big|_{V_2^+=0}$.

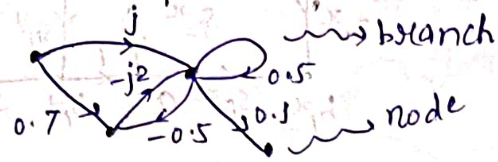
$\Rightarrow V_1^+ + V_1^- = AV_2^- + \dfrac{BV_2^-}{Z_0} \longrightarrow (V_1^+ + V_1^-) = \dfrac{(AZ_0 + B)}{Z_0} V_2$.

$V_1^+ - V_1^- = CZ_0 V_2^- + DV_2^- \longrightarrow (V_1^+ - V_1^-) = (CZ_0 + D)V_2^-$

$$\frac{V_1^-}{V_1^+} = \frac{V_1^+ + V_1^- - V_1^+ + V_1^-}{V_1^+ + V_1 + V_1^+ - V_1^-} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}$$

$$\therefore S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}$$

Signal Flow Graph (SFG)



→ Each port of a device is represented by two nodes — 'a' and 'b'.

a : normalized amplitude of wave incident on that port evaluated at the plane of that port.

b : normalized amplitude of the wave exiting that port.

$$a_n = \frac{V_n^+ (z_n = z_{nP})}{\sqrt{Z_{0n}}}, \quad b_n = \frac{V_n^- (z_n = z_{nP})}{\sqrt{Z_{0n}}}$$

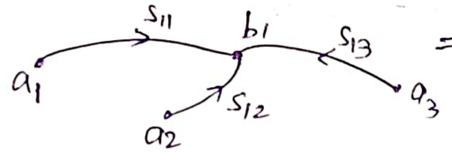
Total voltage at a port : $V_n (z_n = z_{nP}) = (a_n + b_n)\sqrt{Z_{0n}}$

→ The value of the branch connecting two nodes is simply the value of the scattering parameter relating two voltage values.

$$a_n = \frac{V_n^+ (z_n = z_{nP})}{\sqrt{Z_{0n}}} \xrightarrow{S_{mn}} b_m = \frac{V_m^- (z_m = z_{mP})}{\sqrt{Z_{0m}}}$$

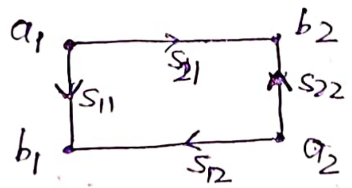
→ Signal flow graph is the graphical representation of the equation : $\boxed{b_m = a_n S_{mn}}$

→

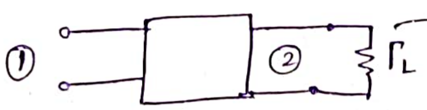


$\Rightarrow b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3$

→ Two-port device with $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$:



→

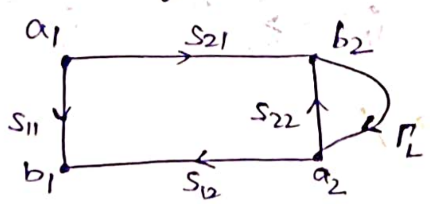


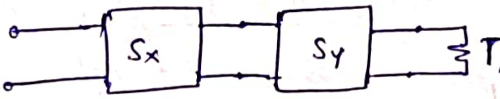
→ terminated with some load Γ_L.

$$V_2^+ (z_2 = z_{2P}) = \Gamma_L V_2^- (z_2 = z_{2P})$$

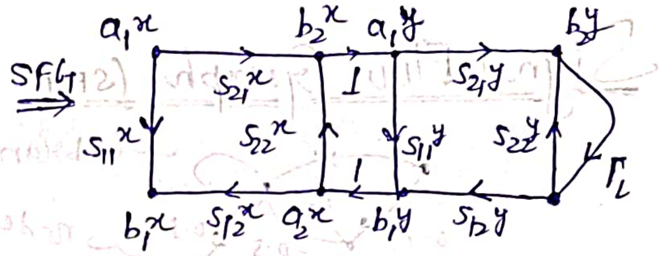
$$\Rightarrow a_2 = \Gamma_L b_2$$

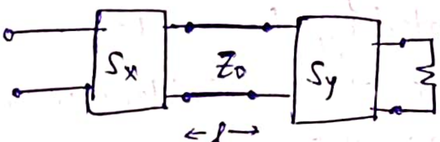
⇓



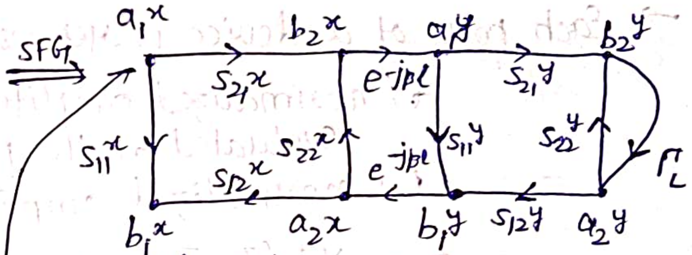
→ 

$a_1^y = b_2^x$
$b_1^y = a_2^x$

SFG ⟹ 

→ 

$S = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$

SFG ⟹ 

No incoming branch
⇒ Independent node
⇒ $\boxed{\text{Source}}$

Represents complex amplitude of the wave incident on the one-port n/w.
↳ If zero, no power is incident on the n/w
↳ Rest of the nodes (wave amp.) will be zero.

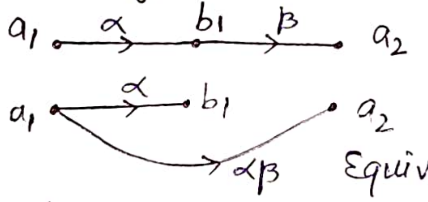
Reduction Rules in SFG:

① Series Rule:

Complex eqⁿs:

$$\left.\begin{aligned} b_1 &= \alpha a_1 \\ a_2 &= \beta b_1 \end{aligned}\right\} \Rightarrow a_2 = \beta b_1 = \beta(\alpha a_1) = \alpha\beta a_1$$

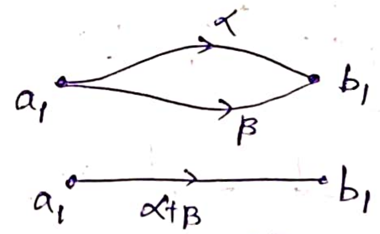
⇓ SFG



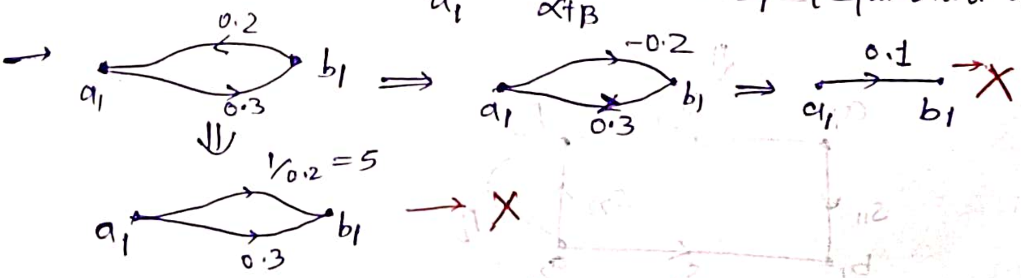
Equivalent SFGs

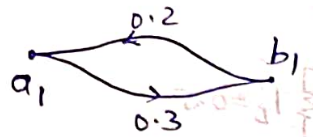
② Parallel Rule:

$b_1 = \alpha a_1 + \beta a_1$
$b_1 = (\alpha + \beta) a_1$

⇒ 

[Equivalent SFG]

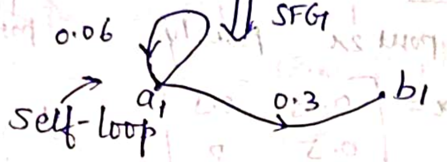
→ 



$\Rightarrow$ $b_1 = 0.3a_1$, $a_1 = 0.2b_1$

$\Rightarrow$ $a_1 = 0.06a_1$, $b_1 = 0.3a_1$

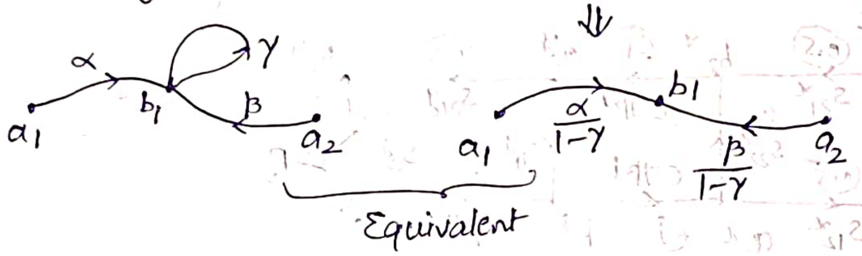
$\Downarrow$ SFG



[Self-loop node will always have atleast one other incoming branch.]

③ Self-Loop Rule :

$$b_1 = \alpha a_1 + \beta a_2 + \gamma b_1 \Rightarrow b_1 = \frac{\alpha}{1-\gamma} a_1 + \frac{\beta}{1-\gamma} a_2$$

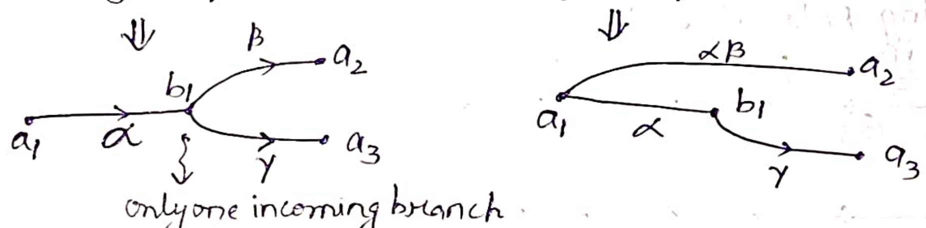


④ Splitting Rule :

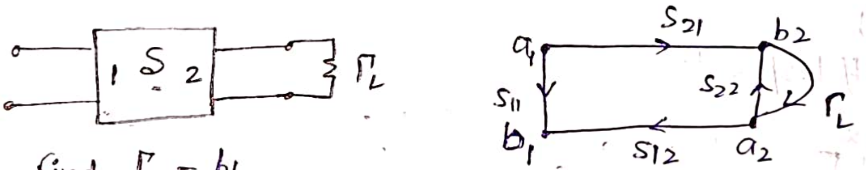
$b_1 = \alpha a_1$ $\qquad\Longrightarrow\qquad$ $b_1 = \alpha a_1$

$a_2 = \beta b_1$ $\qquad\qquad\quad$ $a_2 = \alpha\beta a_1$

$a_3 = \gamma b_1$ $\qquad\qquad\quad$ $a_3 = \gamma b_1$

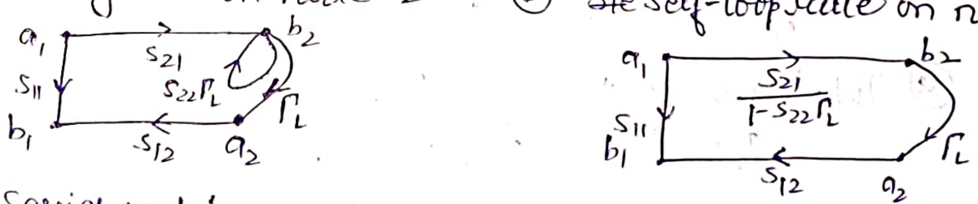


Eg.

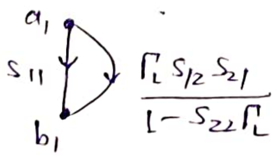


Find $\Gamma_1 = \frac{b_1}{a_1} = S_{11}$ (iff $\Gamma_L = 0$)

Soln: ① Splitting Rule on node a_2: ② Self-loop rule on node b_2:



③ Series rule:

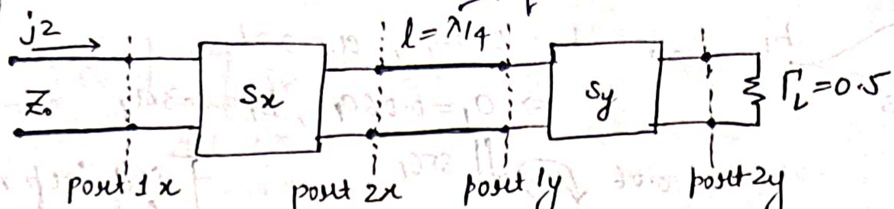


④ Parallel Rule :

$$a_1 \downarrow \left(S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1-S_{22}\Gamma_L}\right) \Leftarrow \frac{b_1}{a_1} .$$

b_1

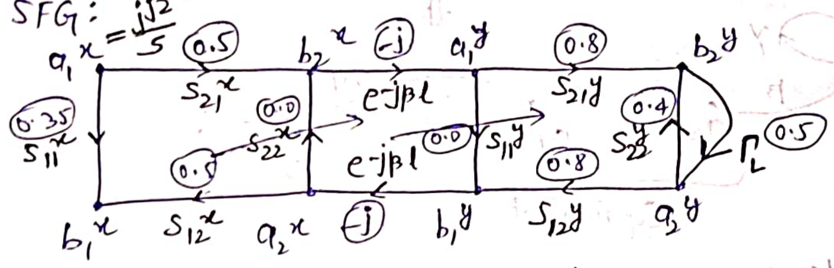
Eg. (Input)



Given: $Z_0 = 50\,\Omega$, $S_x = \begin{bmatrix} 0.35 & 0.5 \\ 0.5 & 0 \end{bmatrix}$, $S_y = \begin{bmatrix} 0 & 0.8 \\ 0.8 & 0.4 \end{bmatrix}$

Draw the complete signal flow graph of this circuit, and then reduce the graph to determine: (a) the total current through load Γ_L (b) the power delivered to (i.e., absorbed by) port 1x.

Soln: SFG: $a_1^x = \frac{j\sqrt{2}}{5}$



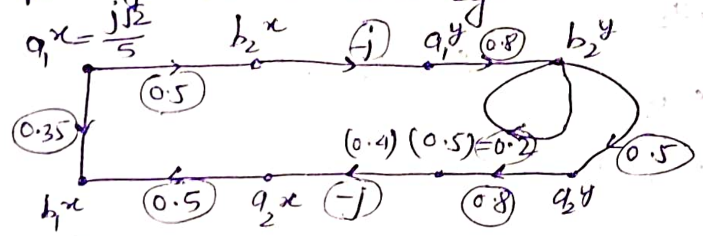
⇒ Remove zero valued branches

Wave incident on port 1 of device Sx:

$$a^x = \frac{V_{1x}^+ (Z_{1x} = Z_{1xP})}{\sqrt{Z_0}} = \frac{j2}{\sqrt{50}} = \frac{j\sqrt{2}}{5}$$

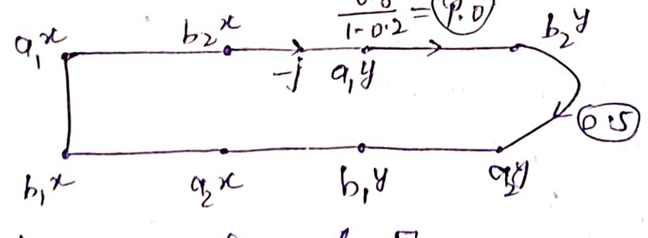
⇓

Splitting Rule at node a_{2y}.



⇓

Self-loop rule at node b_{2y}:

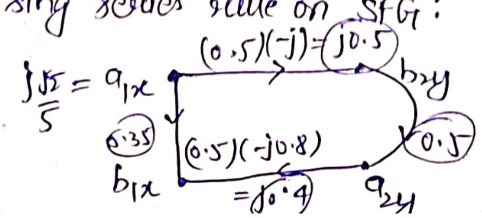


(a) Total current through Γ_L,

$$I_L = -I(z_{2y} = z_{2yP}) = -\frac{V_{2y}^+ (z_{2y} = z_{2yP}) - V_{2y}^- (z_{2y} = z_{2yP})}{Z_0}$$

$$\Rightarrow I_L = -\frac{a_{2y} - b_{2y}}{\sqrt{Z_0}} = \frac{b_{2y} - a_{2y}}{\sqrt{50}}$$ ← we need to determine the value of nodes a_{2y} & b_{2y}.

Using series rule on SFG:



$\Rightarrow b_{2y} = -j0.5\, a_1^x = -j0.5 \times \frac{j\sqrt{2}}{5} = 0.1\sqrt{2}$

$a_{2y} = 0.5 \times b_{2y} = 0.05\sqrt{2}$

$\therefore I_L = \dfrac{b_{2y} - a_{2y}}{\sqrt{50}} = \dfrac{(0.1-0.05)\sqrt{2}}{\sqrt{50}} = \dfrac{0.05}{5} = 10\,mA.$

(b) Power delivered to port 1x,

$$P_{abs.} = P^+ - P^- = \frac{|V_{1x}^+ (z_{1x} = z_{1xP})|^2}{2Z_0} - \frac{|V_{1x}^- (z_{1x} = z_{1xP})|^2}{2Z_0}$$

$$\Rightarrow \quad P_{abs.} = \frac{|a_{1x}|^2 - |b_{1x}|^2}{2} \leftarrow \text{Requires knowledge of nodes } a_{1x} \text{ \& } b_{1x}.$$

By series rule on SFG:

$a_{1x} = j\sqrt{2}/5$

0.35 $(-j0.5)(0.5)(-j0.4) = -0.1$

b_{1x}

$\Downarrow$

Parallel Rule:

$a_{1x} = j\sqrt{2}/5$

$0.35 - 0.1 = 0.25$

b_{1x}

$\therefore b_{1x} = 0.25\,a_{1x} = (0.25)\left(\dfrac{j\sqrt{2}}{5}\right) = j0.05\sqrt{2}$

$\therefore P_{abs.} = \dfrac{|j\sqrt{2}/5|^2 - |j0.05\sqrt{2}|^2}{2} = \dfrac{0.08 - 0.005}{2} = 37.5\,mW.$

Impedance Transformation

Wideband Matching Network → A matching n/w that provides an adequate match over a wide range of frequencies.

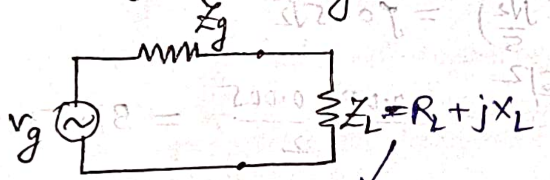
Tradeoff for a Matching N/W :

1. Bandwidth
2. Complexity
3. Implementation
4. Adjustability.

L Networks

↳ Consists of a single capacitor and a single inductor.
↳ Drawbacks : ① Narrow-band
② C & L are difficult to make at microwave frequencies.

Matching N/W Analysis :



absorbed power → delivered by the source

$$P_L = \frac{1}{2}\{V_L I_L^*\} = \frac{1}{2}\mathrm{Re}\left\{\left(V_g \frac{Z_L}{Z_g+Z_L}\right)\left(\frac{V_g}{Z_g+Z_L}\right)^*\right\}$$

$$= \frac{1}{2}|V_g|^2 \frac{\mathrm{Re}\{Z_L\}}{|Z_g+Z_L|^2} = \frac{1}{2}|V_g|^2 \frac{R_L}{|Z_g+Z_L|^2}$$

Max when $Z_L = Z_g^*$

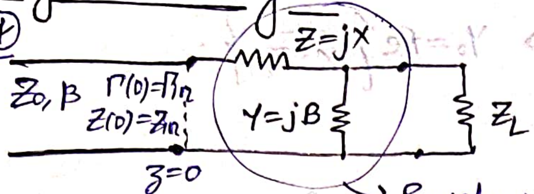
$$P_L^{max} = P_{avl.} = \frac{1}{2}|V_g|^2 \frac{R_g}{|Z_g+Z_g^*|^2} = \frac{1}{2}|V_g|^2 \frac{R_g}{|2R_g|^2} = \frac{|V_g|^2}{8R_g}$$

available power from the source → depends on source parameters only (V_g, R_g)

L-Matching N/W Analysis

~~Circuit A~~

Matching N/W (A) :



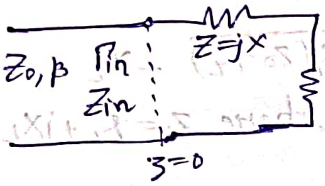
→ Purely reactive → capacitor/inductor

For $\Gamma_{in}=0 \Rightarrow Z_{in}=Z_0$

$\Rightarrow Re\{Z_{in}\}=Z_0\ ,\ Im\{Z_{in}\}=0$ } → aids to find unknowns (X, B).

Part-①: Selecting $Y=jB$ → Combine $\underbrace{Y \ \& \ Z_L}$ in parallel.

$Y_1 = Y+\frac{1}{Z_L} = jB+Y_L$



$$Z_1=\frac{1}{Y_1}=\frac{1}{jB+Y_L}=\frac{Z_L}{1+jBZ_L}=Z_0+jX_1$$

jX_1 → Reactance part of matching ckt.

For perfect match: $Re\{Z_{in}\}=Re\{Z_1\}$

$$\Rightarrow Z_0 = Re\left\{\frac{Z_L}{Z_L+jBZ_L}\right\}$$

Part-②: Selecting $Z=jX$

↳ Remove img. part of Z_1 by setting $Z=jX$

$$X=-X_1=-Im\left\{\frac{Z_L}{Z_L+jBZ_L}\right\}$$

$$Z_{in}=Z+Z_1$$

$$= -jX_1+Z_0+jX_1=Z_0 \longleftrightarrow \text{Perfect Match.}$$

$$\therefore B=\frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2+X_L^2-Z_0R_L}}{R_L^2+X_L^2} \longleftrightarrow \text{Two solutions for } B \text{ (\& thus } X)$$

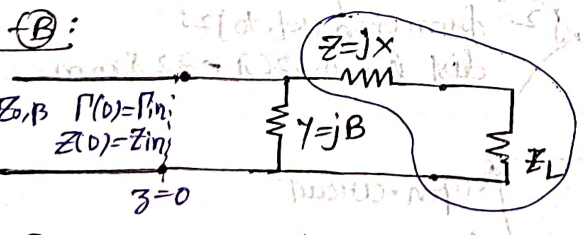
$$X=\frac{1}{B}+\frac{X_LZ_0}{R_L}-\frac{Z_0}{BR_L}\ ,\ \text{where } Z_L=R_L+jX_L$$

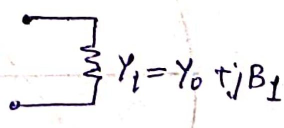
→ For jB to be purely img (reactive), B must be real

$\Rightarrow \boxed{R_L > Z_0}$

⇒ Normalized load z_L' lies inside $r=1$ circle.

Matching N/w (B):





For $\Gamma_{in}=0 \Rightarrow Y_{in}=Y_0$

$$\left[Y_{in}=jB+\frac{1}{jX+Z_L}\right]$$

$\Rightarrow Re\{Y_{in}\}=Y_0\ ,\ Im\{Y_{in}\}=0.$

Set $Z = jX$ such that

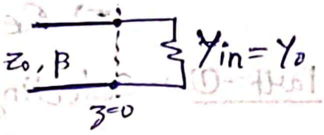
$$Y_1 = \frac{1}{Z + Z_L} = \frac{1}{jX + Z_L} \Rightarrow Y_0 = Re\left\{\frac{1}{jX + Z_L}\right\}.$$

For perfect match,

$$B = -Im\{Y_1\} = -Im\left\{\frac{1}{jX + Z_L}\right\}$$

so that

$$Y_{in} = Y + Y_1 = -jB_1 + (Y_0 + jB_1) = Y_0$$



Solving X & B,

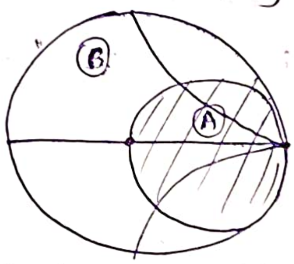
$$B = \pm \frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}, \quad X = \pm \sqrt{R_L(Z_0 - R_L)} - X_L$$

where $Z_L = R_L + jX_L$.

↓ Two sol^ns. for B (& thus X)

→ For jB & jX to be purely img. (reactive), B & X must be real. ⇒ $\boxed{R_L < Z_0}$

⇒ Normalized load z_L' lies outside $r=1$ circle.



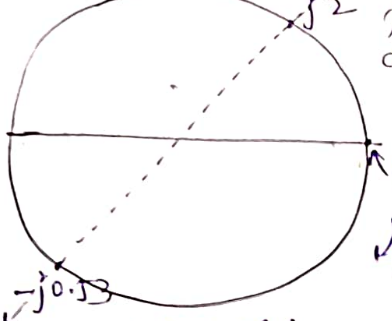
→ Validity Regions for N/w Ⓐ & Ⓑ.

Eg. For an open-circuited 50-Ω T/L operated at 3 GHz and with phase velocity of 77% of speed of light, find the line lengths to create a 2pF capacitor and 5.3nH inductor. Use smith chart for solving this problem.

Sol^n: At 3GHz, $X_C = \frac{1}{j\omega C} = -j26.5\,\Omega \xrightarrow{\text{normalised}} z_C = \frac{X_C}{Z_0} = -j0.53$

$X_L = \frac{1}{j\omega L} = -j100\,\Omega \xrightarrow{\text{normalized}} z_L = \frac{X_L}{Z_0} = -j2$

wavelength, $\lambda = \frac{v_p}{f} = 77\,mm$.



from open-ckt. to j2:
dist. $l_2 = 0.426\lambda = 32.8\,mm$

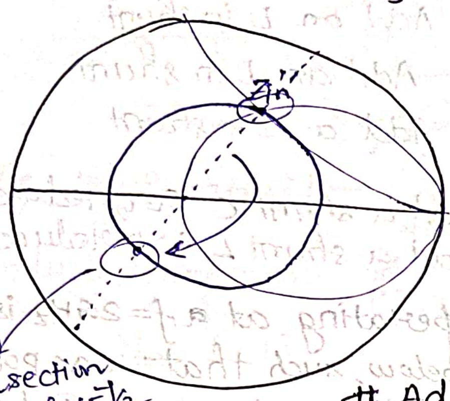
from open-ckt. to -0.53:
dist. $l_1 = 0.172\lambda = 13.24\,mm$.

Eg. Convert the following norm. i/p impedance Z_{in}' into norm. i/p admittance using Smith chart.

$$Z_{in}' = 1+j1 = \sqrt{2}\, e^{j(\pi/4)}$$

[First approach
↳ direct inversion]

Soln:



intersection of $r=\frac{1}{2}$ & $x=\frac{1}{2}$ circle.

Steps:

① Mark normalized impedance.

② Identify phase angle and magnitude of associated reflection coeff.

③ Rotate the ref. coeff. by $\angle 180°$.

④ Identify x-and r-circle of the rotated ref. coeff.

Admittance Transformation:

$$Z_{in}(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)}, \quad y_{in}(z) = \frac{1-\Gamma(z)}{1+\Gamma(z)} = \frac{1+e^{-j\pi}\Gamma(z)}{1-e^{-j\pi}\Gamma(z)}$$

(Equivalent to a 180° rotation of the reflection coeff. in complex Γ-plane)

Admittance Smith Chart (Y-Smith Chart)

↳ 180° rotated Smith Chart

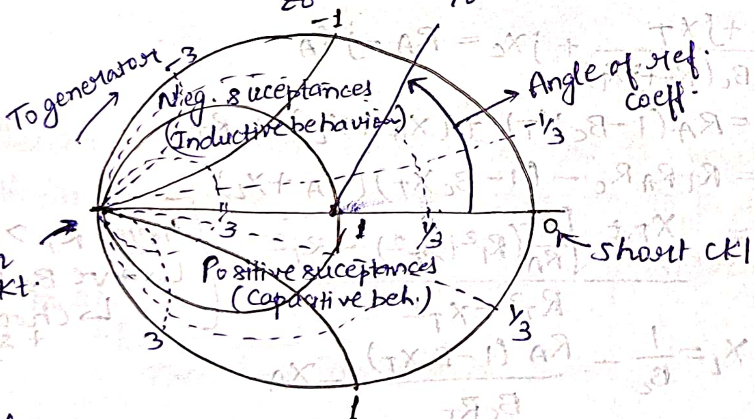
→ Preserves:

① Dirn in which angle of ref. coeff. is measured.

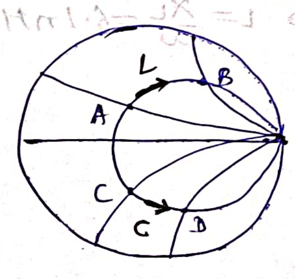
② Dirn of rotation (either towards or away from generator).

$$r = \frac{R}{Z_0} \Rightarrow g = \frac{G}{Y_0} = Z_0 G$$

$$x = \frac{X}{Z_0} \Rightarrow b = \frac{B}{Y_0} = Z_0 B$$



Adding Reactive Element in a Circuit using Z-Smith chart:



Constant R-circle → L or C in series

(CW) A → B : Add an L in series

(CCW) B → A : Add a C in series

(CCW) C → D : Add a C in series

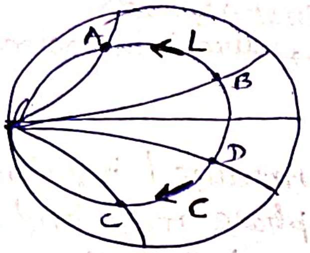
(CW) D → C : Add an L in series.

In Z-plane,

CW → add a series L (or reduce series C)

CCW → add a series C (or reduce series L)

Y-smith chart:



(CW) $A \to B$: Add a C in shunt

(CCW) $B \to A$: Add an L in shunt.

(CCW) $C \to D$: Add an L in shunt

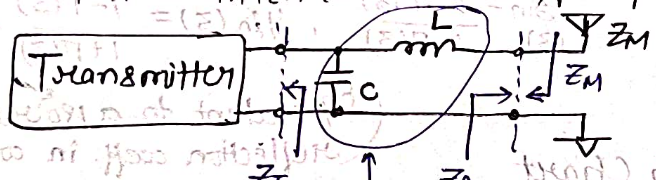
(CW) $D \to C$: Add a C in shunt.

In Y-plane,

CW → add a shunt C (or reduce shunt L)

CCW → add a shunt L (or reduce shunt C)

Eg: The o/p imp. of a transmitter operating at a $f = 2GHz$ is $Z_T = (150 + j75)\Omega$. An L-Type M/N, shown below, such that max power is delivered to the antenna whose i/p impedance is $Z_A = (75 + j15)\Omega$.



Soln: For max. power transfer, $Z_M = Z_A^* = (75 - j15)\Omega$.

$$Z_M = \frac{1}{Z_T^{-1} + jB_c} + jX_L = Z_A^* \quad \leftarrow B_c = \omega C, \quad \leftarrow X_L = \omega L$$

Take $Z_T = R_T + jX_T$, $Z_A = R_A + jX_A$

$$\Rightarrow \frac{R_T + jX_T}{1 + jB_c(R_T + jX_T)} + jX_c = R_A - jX_A$$

$$\Rightarrow R_T = R_A(1 - B_c X_T) + (X_A + X_L) B_c R_T,$$

$$X_T = R_T R_A R_c - (1 - B_c X_T)(X_A + X_L)$$

$$\Rightarrow B_c = \frac{X_T \pm \sqrt{\frac{R_T}{R_A}(R_T^2 + X_T^2) - R_T^2}}{R_T^2 + X_T^2}$$

Here, $R_T > R_A$
↳ +ve B_c (capacitor)
↳ choose + sign

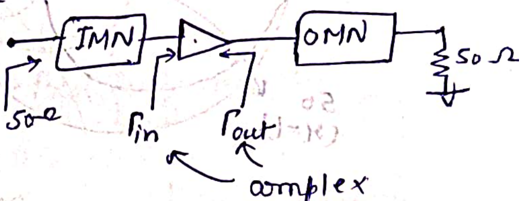
$$\Rightarrow X_L = \frac{1}{B_c} - \frac{R_A(1 - B_c X_T)}{B_c R_T} - X_A$$

$\therefore B_c = 9.3\,mS \Rightarrow C = \frac{B_c}{\omega} = 0.73\,pF$

$X_L = 76.9\,\Omega \Rightarrow L = \frac{X_L}{\omega} = 6.1\,nH.$

Eg. A load $Z_L = 10 + j10\,\Omega$ is to be matched to a $50\text{-}\Omega$ line. Design two matching n/w and specify the values of L and C at a freq. of 500 MHz.

- Load → Source
- Complex conjugate not considered.
- Most amplifier matching i/p & o/p like this.



Take Char. Impedance = 50Ω.

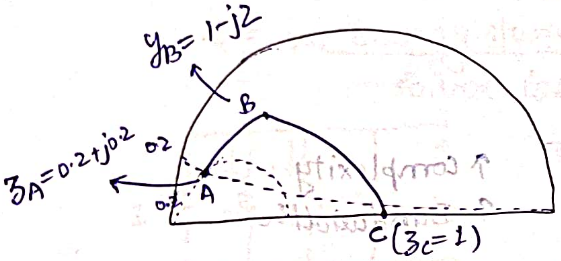
A ($z_L = \frac{10+j10}{50} = 0.2 + j0.2$) ⟶ B along const.-resistance circle (along unit const-conductance circle)

Inductor impedance, $z_L = j0.4 - j0.2$
$= j0.2$

$y_B = 1 - j2$

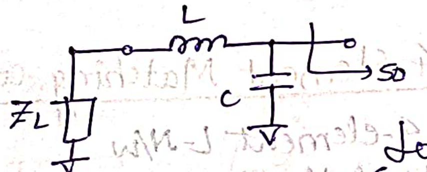
B ⟶ C (origin) along const-conductance circle.

Capacitor admittance, $y_C = 0 - (-j2) = j2$
or, $z_C = \frac{1}{j2} = -j0.5$ $\Bigg\} \Rightarrow y_{IN} = z_{IN} = 1$ or $Z_{IN} = 50\,\Omega$



$$L = \frac{10}{2\pi(500 \times 10^6)} = 3.18\,nH$$

$$C = \frac{1}{25(2\pi)500 \times 10^6} = 12.74\,pF$$



$Z_A = Z_L = [10 + j10]\,\Omega$

Low Pass Matching
(High Pass: other soln)

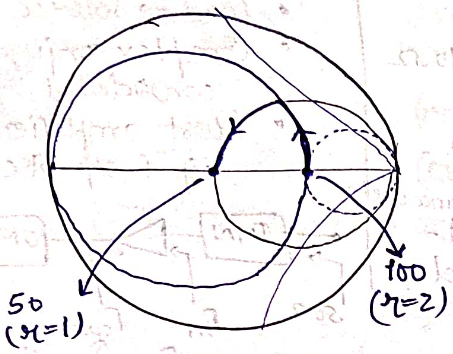
Eg. Design an L-section matching n/w to match a series RC load with an impedance $Z_L = 200 - j100\,\Omega$ to a $100\,\Omega$ line at a f. of 500 MHz.

[Solve in ZY Smith Chart Only]

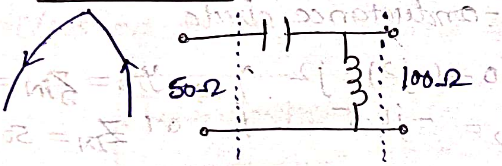
Take Char. Imp. = 100Ω

(23-10-2024)

Possible 3 Element Matching Configurations

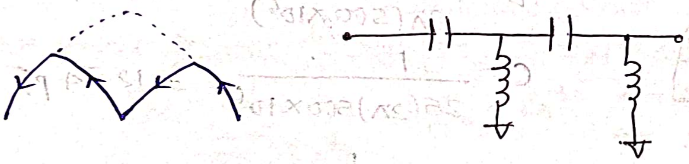


2-Elements:



(Series C, Shunt L)

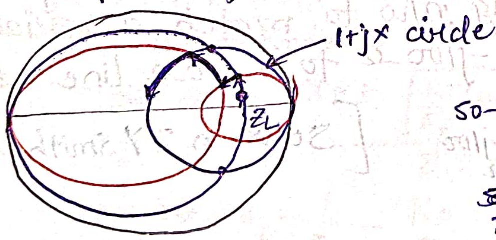
3-Elements:

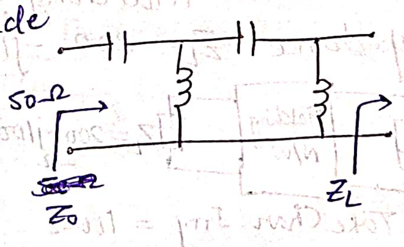


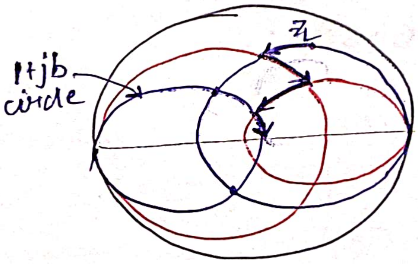
↑ complexity
↑ Bandwidth

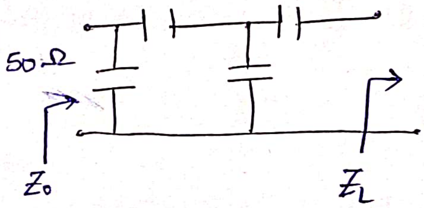
Possible 4 Element Matching configurations

Possible 4-element L-N/w
shorter paths for a wider operational B/W.

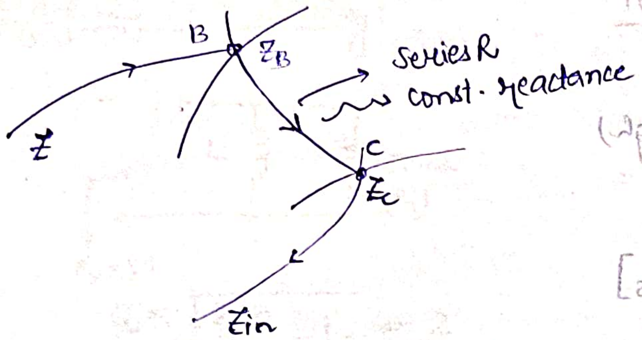


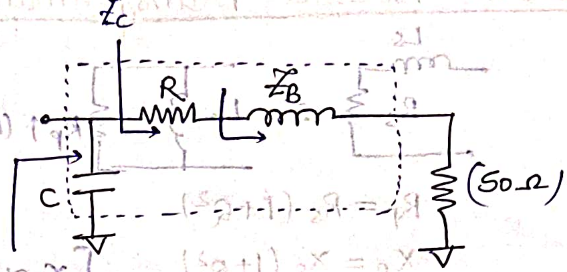






Reverse Problem:





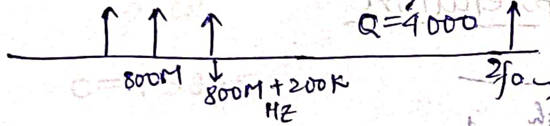
↳ lossy
↳ Poor performance

About Q

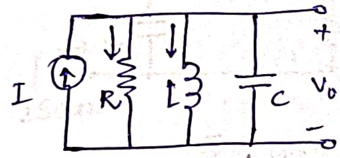
800 MHz signal freq. (center) → 200 KHz channel spacing

$$(\omega_2 - \omega_1) = \frac{\omega_0}{Q}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$



$$Q = \omega_0 \frac{\text{Energy stored}}{\text{Power lost}}$$

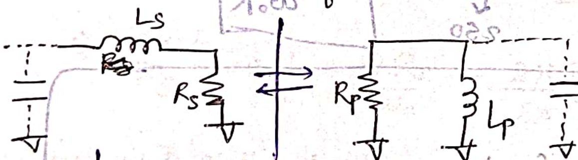


$$Q = \frac{\omega_0 \left(\frac{1}{2} C V^2\right)}{\frac{1}{2} \frac{V_0^2}{R}} = \omega_0 CR = \frac{CR}{\sqrt{LC}} = \sqrt{\frac{C}{L}} R = \frac{\sqrt{LC}}{L} R = \frac{R}{\omega_0 L}$$

Series Parallel Transformation

@ ω_0

$$\uparrow Q = \frac{\omega_0 L_s}{R_s \downarrow}$$



when $R_p \to \infty$, inductor is high quality (pure) in parallel

$$\omega_0 = \frac{1}{\sqrt{L_p C_p}}$$

$$\uparrow Q = \frac{R_p \uparrow}{\omega_0 L_p}$$

$$Z_{in,p} = \frac{1}{\frac{1}{R_p} + \frac{1}{j\omega_0 L_p}} = \frac{j\omega_0 L_p R_p}{R_p + j\omega_0 L_p}$$

$$= \frac{j\omega_0 L_p R_p (R_p - j\omega_0 L_p)}{(R_p + j\omega_0 L_p)(R_p - j\omega_0 L_p)}$$

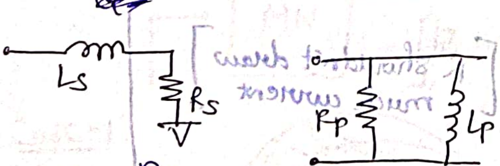
$$= \frac{j\omega_0 L_p R_p (R_p - j\omega_0 L_p)}{(R_p^2 + \omega_0^2 L_p^2)}$$

$$= \frac{\omega_0^2 L_p^2 R_p + j\omega_0 L_p R_p^2}{(R_p^2 + \omega_0^2 L_p^2)}$$

$$= \frac{R_p}{1+Q^2} + \frac{j\omega_0 L_p Q^2}{1+Q^2}$$

$$1 + Q^2 = \frac{\omega_0^2 L_p^2 + R_p^2}{\omega_0^2 L_p^2}$$

$$\Rightarrow \frac{1}{(1+Q^2)} = \frac{\omega_0^2 L_p^2}{\omega_0^2 L_p^2 + R_p^2}$$



$$R_s = \frac{R_p}{1+Q^2}$$

$$L_s = \frac{L_p Q^2}{1+Q^2}$$

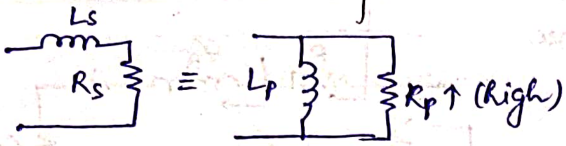
$$Q' = \frac{\omega_0 L_s}{R_s} = \frac{(\omega_0 L_p Q^2)(1+Q^2)}{(1+Q^2) R_p} = \left(\frac{\omega_0 L_p}{R_p}\right) Q^2 = Q$$

$\frac{\omega_0 L_p}{R_p} \to \frac{1}{Q}$

$$X_p = X_s \frac{1+Q^2}{Q^2}$$

(28-10-2024)

Series Parallel Transformation



$$R_p = R_s(1+Q^2)$$

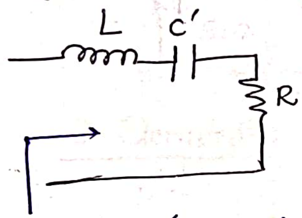
$$X_p = \frac{X_s(1+Q^2)}{Q^2} \qquad [X_p \approx X_s]$$

① @ ω_0

② Q = constant.

Lumped L Matching - Q interpretation

Eg.

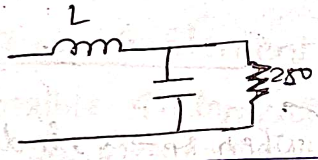


$$Q = \sqrt{\frac{R_{High}}{R_{Low}} - 1}$$

① L ✓

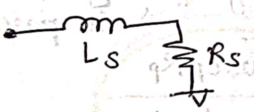
② C ?

$$\overset{\checkmark}{Q} = \frac{\overset{\checkmark}{\omega_0} L \Rightarrow \checkmark}{\underset{\checkmark}{R(=50)}} \qquad \omega_0 = \frac{1}{\sqrt{LC'}} \rightarrow \times$$



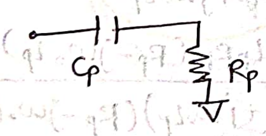
$$Q = \omega_0 \underset{250}{\underset{\downarrow}{R}} C \Rightarrow \boxed{C = \frac{Q}{\omega_0 R}}$$

①



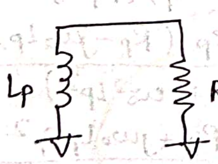
$$Q = \frac{\omega_0 L_s}{R_s \downarrow} \uparrow$$

②



$$Q = \frac{1}{\omega_0 C_p R_p}$$

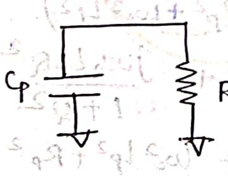
③



$$Q = \frac{R_p}{\omega_0 L_p}$$

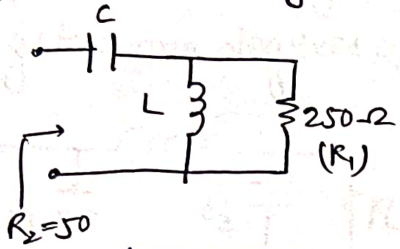
[R shouldn't draw much current]

④



$$Q = \omega_0 C_p R_p$$

Highpass L-matching:



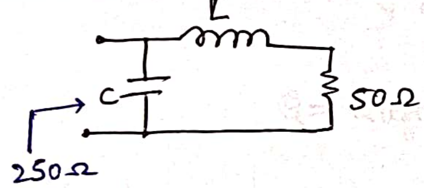
$$Q=\sqrt{\frac{250}{50}-1} = 2$$

$$Q= \frac{R_1}{\omega_o L} \Rightarrow L = 19.89\ nH$$

$$Q= \frac{1}{\omega_o C R_2}$$

$$C = 1.59\ pF$$

Upward xform example (Low-pass)



$\omega_o = 1\ GHz$

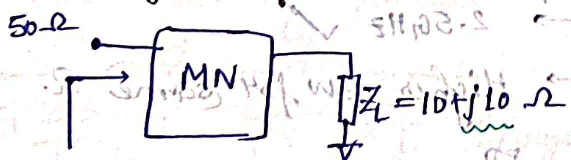
$$Q = \sqrt{\frac{250}{50}-1} = 2$$

$$Q = \omega_o C R_p \Rightarrow C = \frac{Q}{\omega_o R_p} = \frac{2}{2\pi(250)}\ nF$$

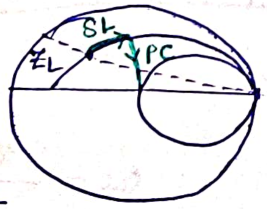
$$= 1.27\ pF$$

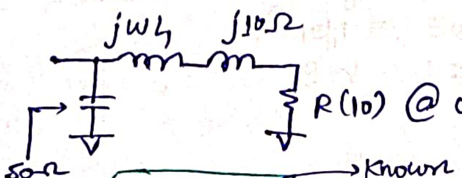
$$Q = \frac{\omega_o L}{R_s} \Rightarrow L = \frac{Q R_s}{\omega_o} = \frac{2\times 50}{2\pi(1)}\ nH = 15.9\ nH$$

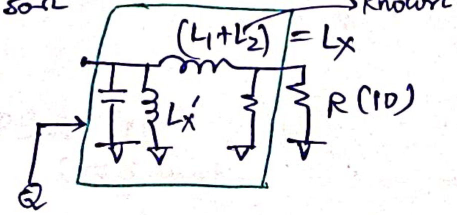
Eg. [Using Q- Method] A load $Z_{load} = 10 + j10\ \Omega$ is to be matched to a 50-Ω line. Design two matching n/ws and specify the values of L and C at a freq. of 500 MHz.



Case-1:

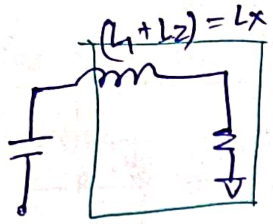






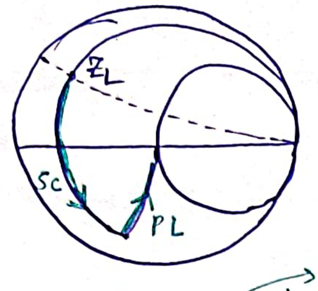
$$Q = \sqrt{(5-1)} = 2$$

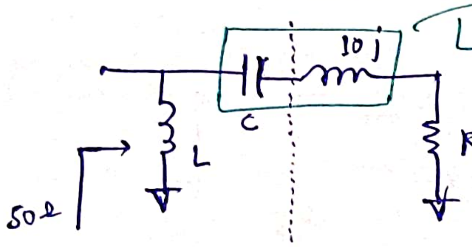
$$Q = \frac{\cancel{\quad}}{\omega_o C R} \Rightarrow C = \frac{Q}{Q\,\omega_o R}$$



$$Q = \frac{\omega_0 L_X}{10}$$

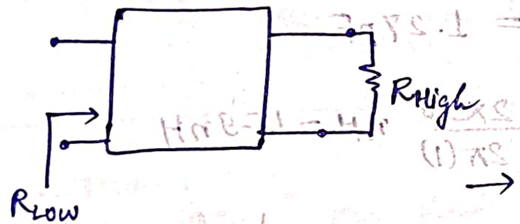
Case-2:





→ should not resonate out
↳ Choose C such that the effective impedance is that of a capacitor.

→



$$Q = \sqrt{\frac{R_{high}}{R_{low}} - 1}$$

→ Bandwidth should be decided by the design specification, not by impedances (elements in the circuit).

$f_{op.} = 800\,MHz\ [f_c] \xrightarrow{\text{carrier freq.}}$ 2.5GHz ✓

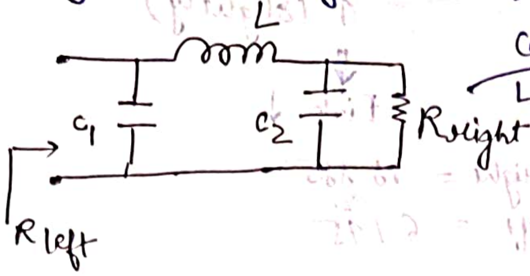
$BW = 10\,MHz \longrightarrow$ Higher BW for same Q. → Higher freq

$Q = 80 \longrightarrow 80$

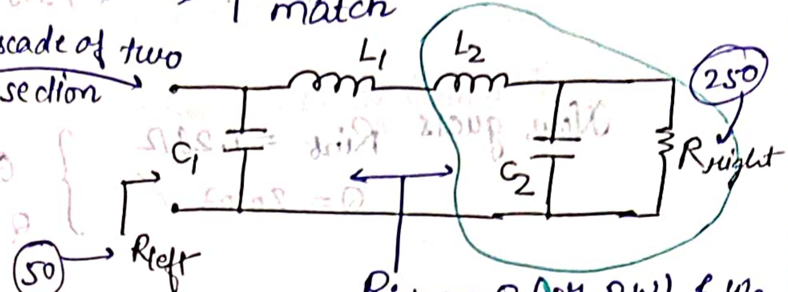
29-10-2021

Lumped L Matching - Q interpretation

To get more degrees of freedom → π match / T match



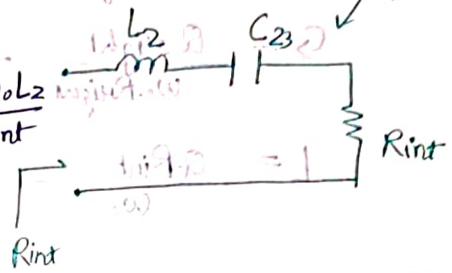
cascade of two L-section



R_{int} : can be arbitrarily chosen → freedom

R_{int} on Q (or BW) & ω_0

[$R_{int} < 50$]



$$Q_{right} = \sqrt{\left(\frac{R_{right}}{R_{int}} - 1\right)} = \frac{\omega_0 L_2}{R_{int}}$$

$$Q_{int} = \sqrt{\frac{R_{left}}{R_{int}} - 1} = \frac{\omega_0 L_1}{R_{int}}$$

$$Q = Q_{left} + Q_{right} = \frac{\omega_0}{R_{int}}(L_1 + L_2)$$

$$= \sqrt{\frac{R_{right}}{R_{int}} - 1} + \sqrt{\frac{R_{left}}{R_{int}} - 1}$$

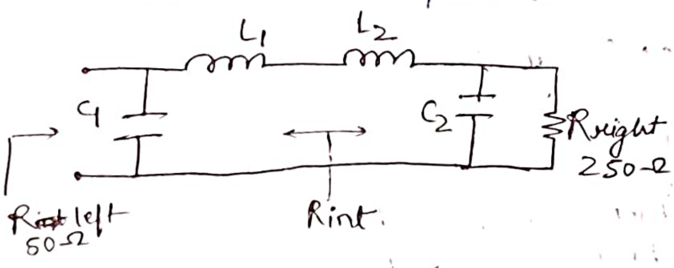
$$\Rightarrow \quad C_1 = \frac{Q_{left}}{\omega_0 R_{left}}, \quad C_2 = \frac{Q_{right}}{\omega_0 R_{right}}$$

$$R_{int} = \left(\frac{\sqrt{R_{left}} + \sqrt{R_{right}}}{Q}\right)^2 \quad \text{[Starting point]}$$

π match property : Inductor parasities can be absorbed in C_1 & C_2.

Small R_{int} → comparable to routing resistance → trouble

Eg



π match example :

$\omega_0 = 2.5$ GHz

$R_{right} = 250\,\Omega$

$R_{left} = 50\,\Omega$

BW = 125 MHz

$$Q = \frac{2.5\text{ GHz}}{125\text{ MHz}} = 20$$

Initial guess: $R_{int} = \left(\frac{\sqrt{250} + \sqrt{50}}{20}\right)^2 = 1.309\,\Omega$ ↓↓ (very low)

Crosscheck: $Q = Q_{right} + Q_{left}$

$$= \sqrt{\frac{250}{1.309} - 1} + \sqrt{\frac{50}{1.309} - 1} = 19.89 < 20$$

↑ (slightly)

⇓

R_{int} ↓

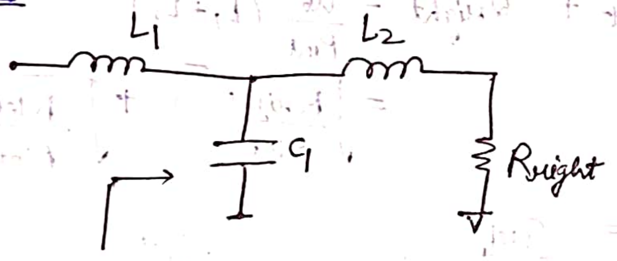
Now, guess $R_{int} = 1.29\,\Omega$, $Q = 20.03$ ✓ $\Big\}$ $Q_{right} = 13.885$, $Q_{left} = 6.145$

$$C_1 = \frac{Q_{left}}{\omega_0 \cdot R_{left}} = \frac{6.145}{2\pi(2.5G)\,50} = 7.83\ pF$$

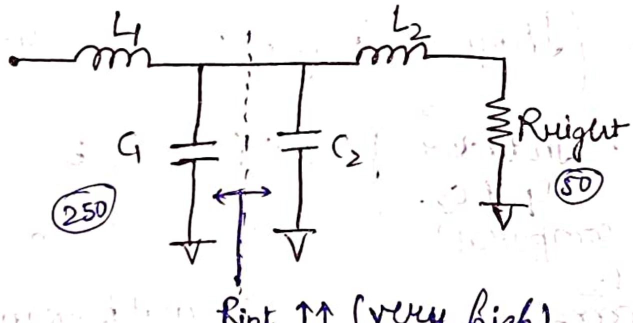
$$C_2 = \frac{Q_{right}}{\omega_0 \cdot R_{right}} = \frac{13.885}{2\pi(2.5G)\,250} = 3.53\ pF$$

$$L = \frac{Q \cdot R_{int}}{\omega_0} = \frac{20\,(1.29)}{2\pi(2.5G)} = 1.64\ nH$$

T-Type



⇓

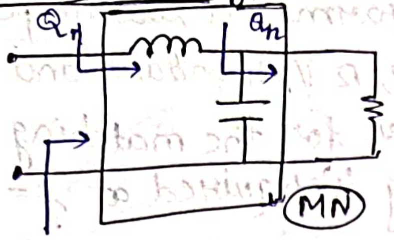


R_{int} ↑↑ (very high)

$$Q = Q_{right} + Q_{left}$$

$$= \sqrt{\frac{R_{int}}{R_{right}} - 1} + \sqrt{\frac{R_{int}}{R_{left}} - 1}$$

Nodal Quality Factor:



Q_L : Loaded Quality factor → overall Q.F. of both sides

$$= \left(\frac{\omega_0}{\Delta\omega}\right) = \frac{f_0}{BW}$$

Nodal Quality factor : Q_n

$$Q_n = \left(\frac{Q_L}{2}\right).$$

$$\Gamma = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1}$$

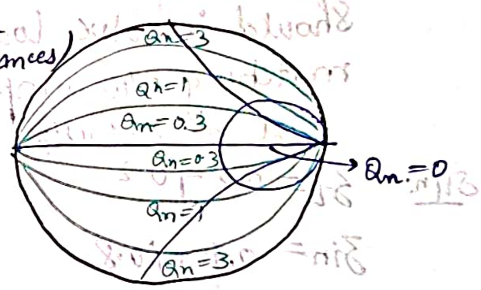
$$\bar{Z}_L = \frac{1+\Gamma}{1-\Gamma} = \frac{(1+\Gamma_r + j\Gamma_i)(1-\Gamma_r + j\Gamma_i)}{(1-\Gamma_r - j\Gamma_i)(1-\Gamma_r + j\Gamma_i)}$$

$$r + jx = \text{real}\{\bar{Z}_L\} + \text{img}\{\bar{Z}_L\}$$

$$\Rightarrow \bar{Z} = r + jx = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} + j\frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

[Smith chart expression]

$$\therefore Q_n = \frac{|x|}{r} = \frac{2|\Gamma_i|}{1-\Gamma_r^2 - \Gamma_i^2} = \frac{|b|}{g} \text{ (for admittances)}$$



$$\Rightarrow 1-\Gamma_r^2 - \Gamma_i^2 = \frac{2|\Gamma_i|}{Q_n}$$

$$\Gamma_r^2 + \Gamma_i^2 + \frac{2\Gamma_i}{Q_n} + \frac{1}{Q_n^2} = 1 + \frac{1}{Q_n^2}$$

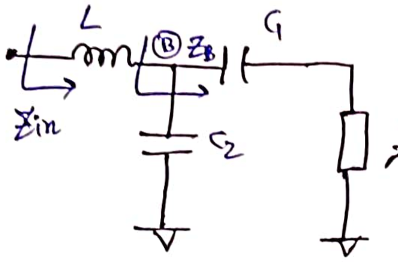
$$\Gamma_r^2 + \left(\Gamma_i \pm \frac{1}{Q_n}\right)^2 = 1 + \frac{1}{Q_n^2}$$

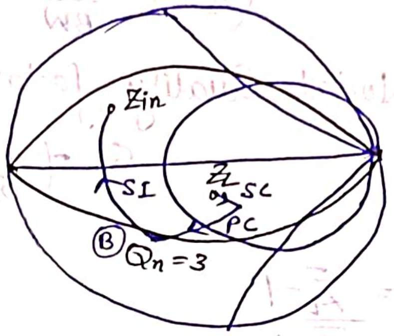
	center	Radius
$Q_n = 0$:	$(0, \infty)$,	(∞)
$Q_n = 1$:	$(0, 1)$,	(1.414)
$Q_n = \infty$:	$(0, 0)$	, (1)

Eg. Design a T-type n/w that transforms a load impedance of $Z_L = (60 - j30)\,\Omega$ into $Z_{in} = (10 + j20)\,\Omega$ i/p impedance and that has a max. Q_n of 3. Compute the values for the matching n/w components, assuming that matching is required at $f = 1$ GHz.

Soln: $z_L = (60 - j30)/50 = 1.2 - j0.6$

$z_{in} = (10 + j20)/50 = 0.2 + j0.4$



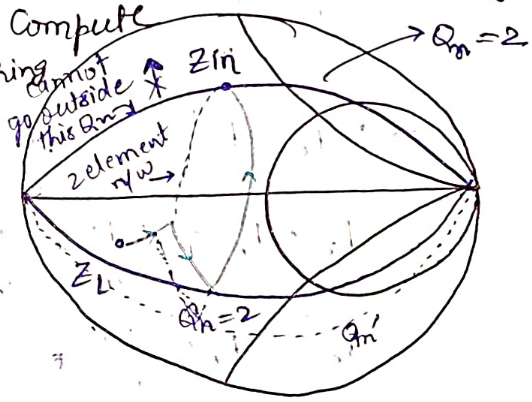


$\Rightarrow$ $C_1 = 8.72\,pF$

$C_2 = 3.53\,pF$

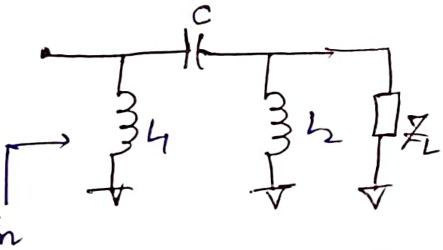
$L = 7.85\,nH$

Eg. For a broadband amplifier, it is required to develop a Pi-type matching n/w that transforms a load impedance $Z_L = (10 - j10)\,\Omega$ into an i/p impedance $Z_0 = (20 + j40)\,\Omega$. The design should involve lowest possible Q_n. Compute matching n/w comp. values for matching at $f = 2.4$ GHz.



Soln: $z_L = 0.2 - j0.2$

$z_{in} = 0.4 + j0.8$



$\Rightarrow$ $C = 1.65\,pF$

$L_1 = 1.66\,nH$

$L_2 = 1.31\,nH$

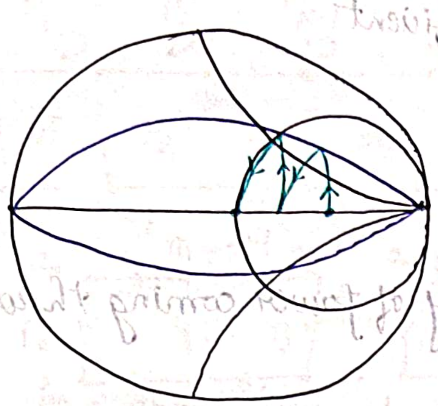
$Q = 0.8, 1.8$, etc. are not possible

Higher Q $\Rightarrow$ lower B/W

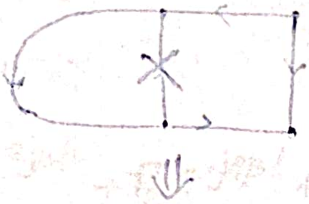
$\rightarrow$ Q cannot be lower than the values at Zin and Z_L.

30-10-2024

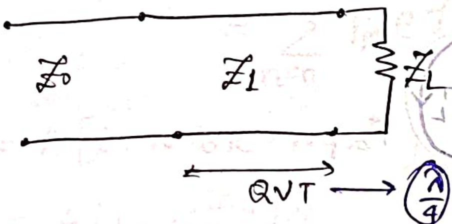
Real-to-Real:



$\Rightarrow$ complexity $\uparrow\uparrow$

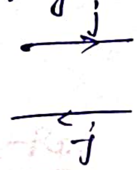


Quarter wave Transformer



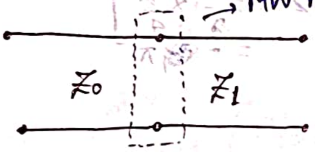
$\longrightarrow$ Narrow band system

$$\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \Rightarrow$$



$$Z_1 = \sqrt{Z_0 Z_L} \Rightarrow \frac{Z_1^2}{Z_L} = Z_0$$

MW n/w $\longrightarrow$ lossless



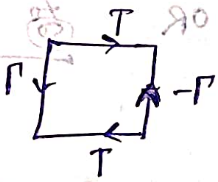
$$S_{11} = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \Gamma$$

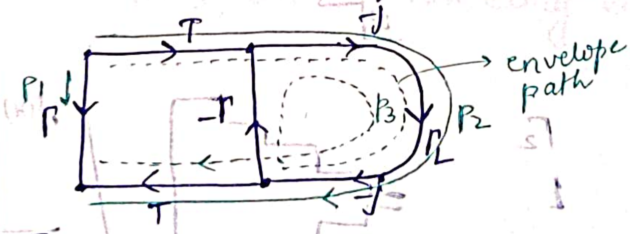
$$\boxed{\Gamma^2 + T^2 = 1}$$

$$\Rightarrow T^2 = 1 - \Gamma^2$$

$$[S] = \begin{bmatrix} \Gamma & T \\ T & -\Gamma \end{bmatrix}$$

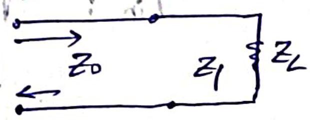
Signal flow graph:





envelope path

P_1, P_2 : forward path



$P_1 = \Gamma$

$P_2 =$

$P_3 =$

$$\Gamma_1 = \frac{Z_1 - \frac{Z_1^2}{Z_L}}{Z_1 + \frac{Z_1^2}{Z_L}} = \frac{Z_L - Z_1}{Z_L + Z_1} = Z_L$$

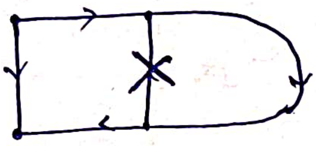
$P = P_1 + P_2 + P_3 + P_4 + \ldots$ $\longrightarrow$ insignificant $\Rightarrow$ Remove

$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$ $\Downarrow \Rightarrow$ Marginal reflection coefficient

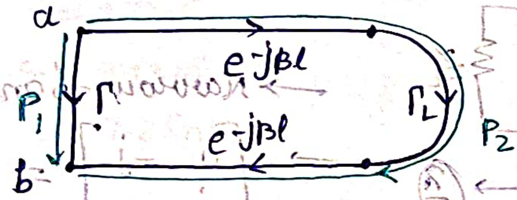
$Z_1^2 = Z_0 Z_L$

$$\boxed{\frac{Z_1^2}{Z_L} = Z_0}$$

Remove $P_3, P_4, \ldots$ (Majority of power coming through P_1 & P_2).



$\Downarrow$



$\frac{b}{a} = (\Gamma - T^2\Gamma_L)$ $\Leftarrow$ $[\because \Gamma = \Gamma_L]$

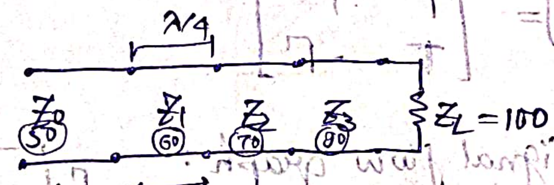
$\Gamma(1 + e^{-j2\beta l})$ $\leftarrow$ $\left[-2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = -\pi\right]$

$\Gamma(1 + e^{-j\pi})$

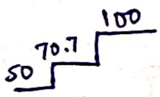
$$\boxed{\Gamma_{in} = 0}$$

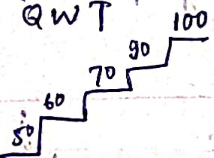
\# QWT

OR



Which is better?

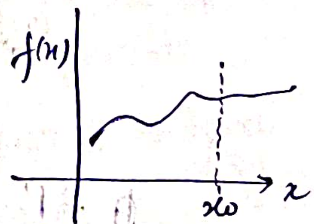




Γ_{in} :	Γ_0	Γ_1	Γ_2
	1	2	1

$\downarrow$

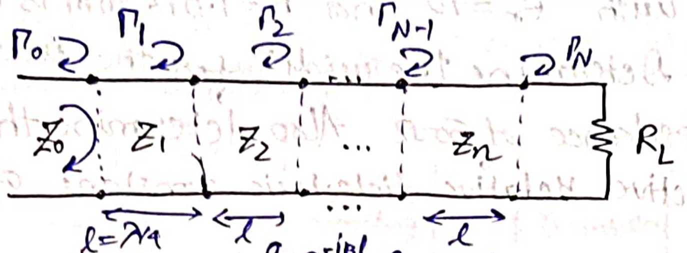
Broadband response

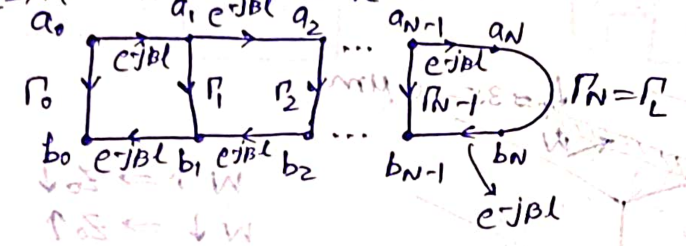


f(x) is flat at x_0 if all derivates are zero.

04-11-2024

Broadband Multi-section Transformer





$$\frac{b_0}{a_0} = \Gamma_{in}(\beta) \Rightarrow \frac{b_0}{a_0} = \Gamma_0 + \Gamma_1 e^{-2j\beta l} + \Gamma_2 e^{-j4\beta l} + \dots + \Gamma_N e^{-j2N\beta l}$$

$$= \sum_{n=0}^{N} \Gamma_n e^{-j2n\beta l}$$

$$\Gamma(\theta) = A\left[1 + N C_1 x + N C_2 x^2\right]$$

$$\hookrightarrow \Gamma(\theta) = A\left[1 + e^{-j\theta 2}\right]^N$$

$$\Gamma(0) = A\,2^N = \frac{R_L - R_0}{R_L + R_0}$$

$$\theta = \beta \underset{\lambda/4}{l} = \pi/2$$

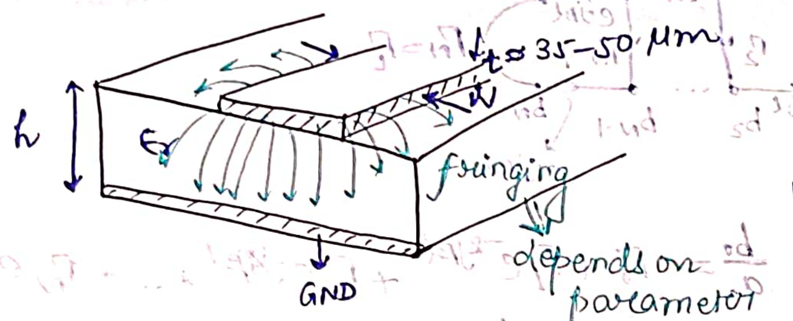
$$\Rightarrow \frac{d^{N-1}}{d\theta^{N-1}}\left[\Gamma(\theta)\right]\Big|_{\theta=\pi/2} = 0$$

$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

→ At $\omega = 0$ [DC], all the cascading vanishes ⇒ only Z_0, R_L remain.

Eg. A microstrip material with $\epsilon_r = 10$ and $h = 1.016$ mm is used to build a narrow T/L. Determine the width for the μstrip T/L to have a char. impedance of 50 Ω. Also determine the wavelength and the effective relative dielectric constant of the μstrip line.



$W \uparrow \Rightarrow Z_0 \downarrow$

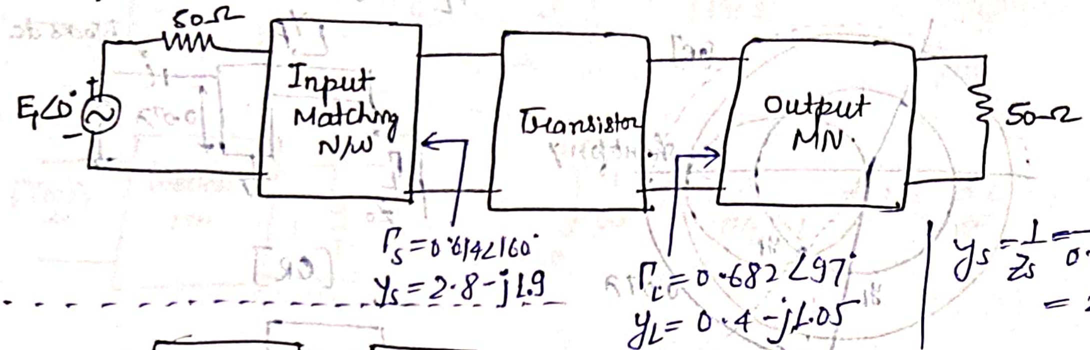
$W \downarrow \Rightarrow Z_0 \uparrow$

$Z_0 = \sqrt{\frac{L}{C}}$, $Z \uparrow \Rightarrow C \downarrow$

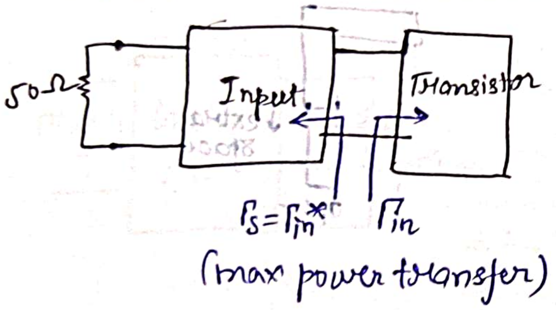
$Z \approx [10-15]\,\Omega$

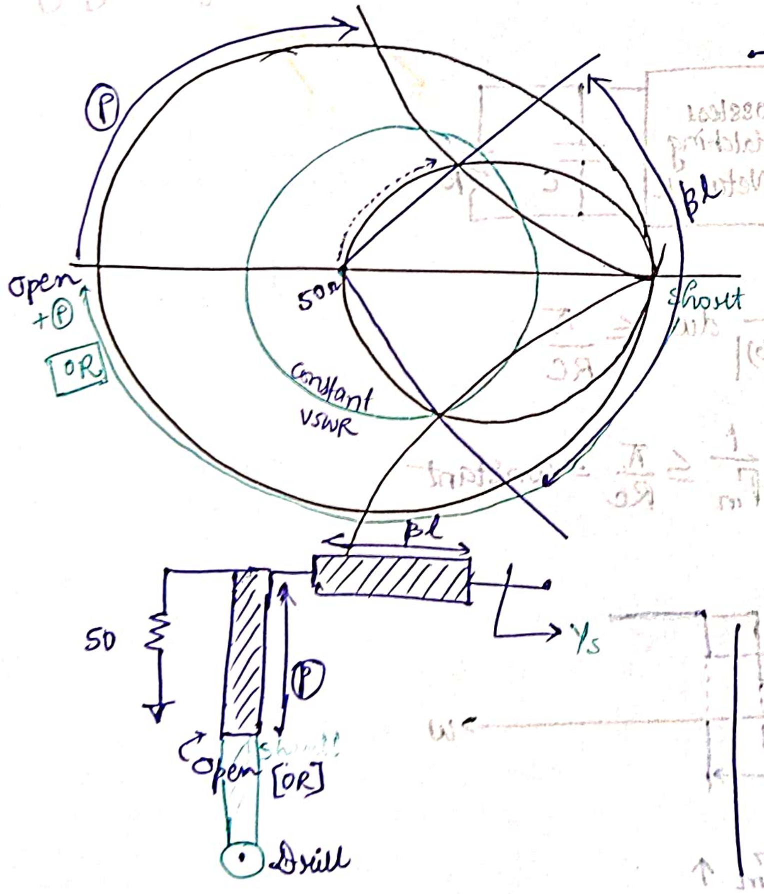
$W \approx [3-3.5]$ mm

Eg. Design 2 μstrip matching n/w for amplifier whose reflection coeff. for a good ~~system~~ match in a 50Ω system are $\Gamma_s = 0.614\angle 160°$ and $\Gamma_L = 0.682\angle 97°$.



$y_s = \frac{1}{Z_s} = \frac{1}{0.245 + j1.65}$
$= 2.8 - j1.9$





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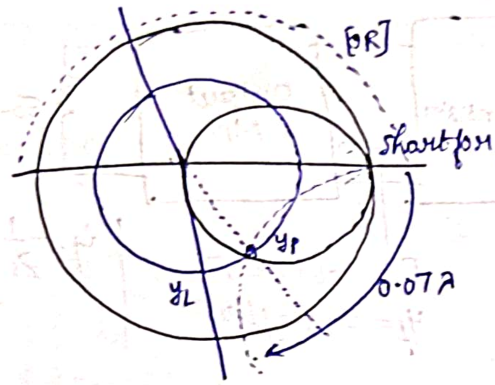
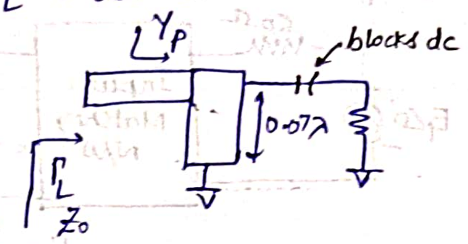
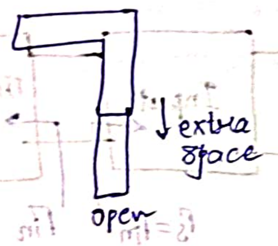
Soln:

$\Gamma_L = 0.682 \angle 97^\circ$

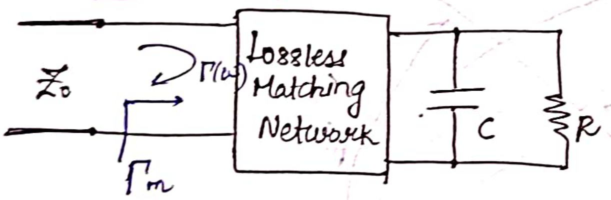




[OR]

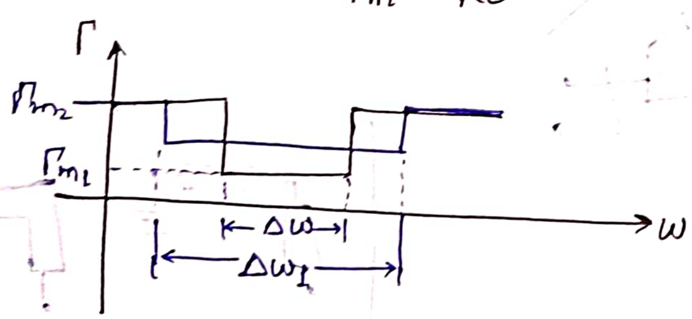


Bode-Fano Criterion → Determines the B/W for any general load.

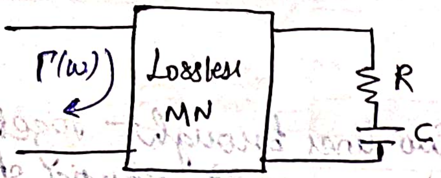


$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|} \, d\omega \le \frac{\pi}{RC}$$

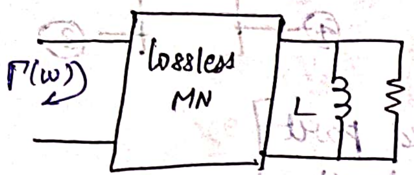
$$\Rightarrow \Delta\omega \ln \frac{1}{\Gamma_m} \le \frac{\pi}{RC} = \text{constant}$$



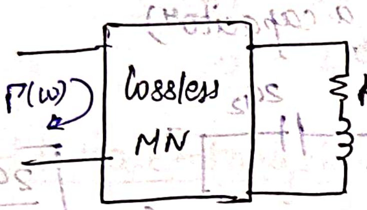
$\Delta\omega \uparrow \Rightarrow \Gamma_m \uparrow$



$$\int_0^\infty \frac{1}{\omega^2} \ln \frac{1}{|\Gamma(\omega)|}\, d\omega < \pi RC \;(\text{time})$$

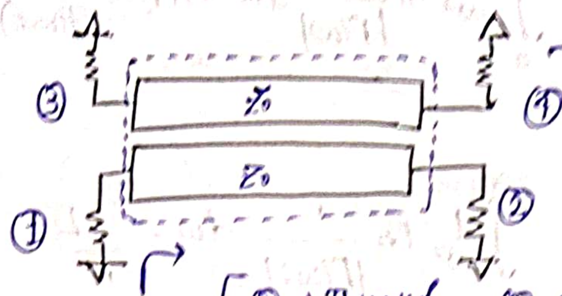


$$\int_0^\infty \frac{1}{\omega^2} \ln \frac{1}{|\Gamma(\omega)|}\, d\omega < \frac{\pi L}{R}$$

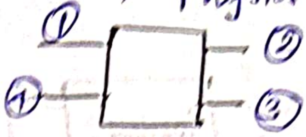


$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|}\, d\omega < \frac{\pi R}{L}$$

Coupled line coupler



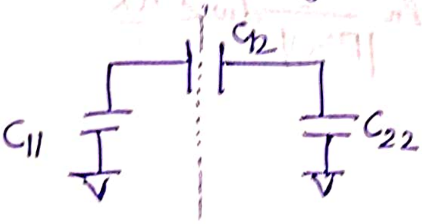
→ Two lines brought together
⇒ Physical changes



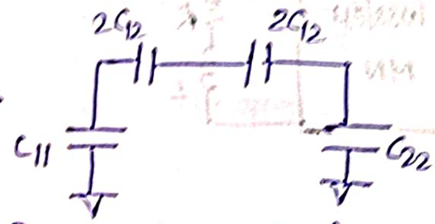
[② : Through   ④ : Isolated port
  ③ : Coupled   ① : Input port]

⇓

Capacitively coupled (connected by a capacitor)

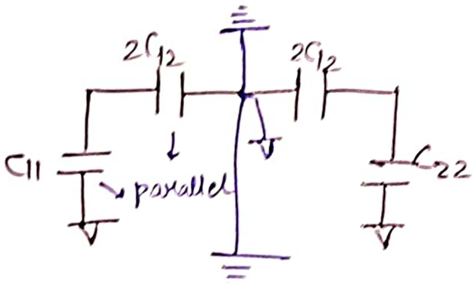


⇒

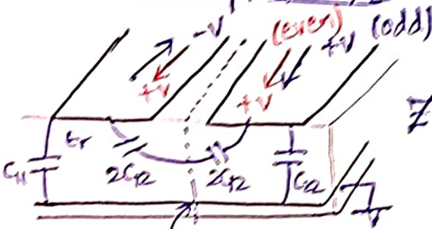


$$\left[\frac{2C_{12}\times 2C_{12}}{4C_{12}} = C_{12}\right]$$

Equ. capacitance n/w for a 3-wire coupled T/L

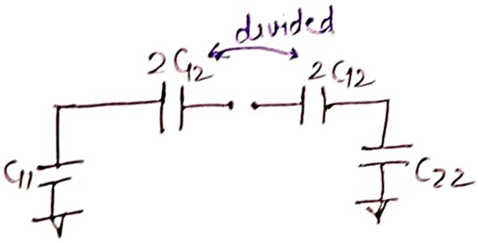


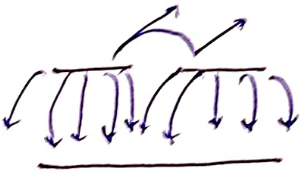
$C_0 = C_{11} + 2C_{12}$



plane of coupler symmetry

$$Z_0^o = \sqrt{\frac{L}{C_o}} = \frac{\sqrt{LC_o}}{C_o} = \frac{1}{v_{po}C_o}$$



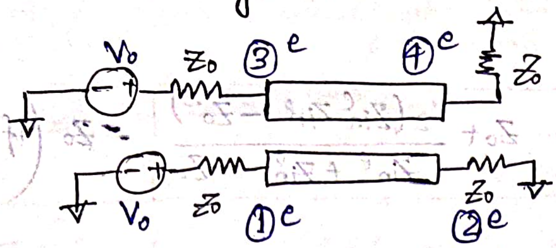


$$Z_0^e = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{v_{pe}C_e}$$

$\underbrace{C_e = C_{11} = C_{22}}$
[For two typical identical T/Ls]

06-11-2024

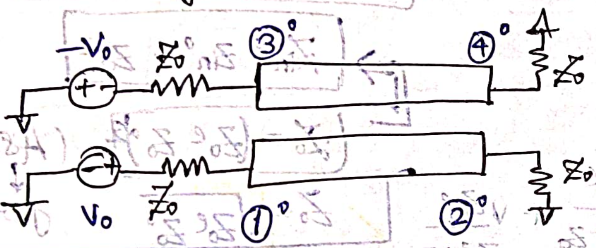
Even Mode Analysis:



$$\left.\begin{aligned} V_1^e &= V_3^e \\ V_2^e &= V_4^e \end{aligned}\right\} \text{[By symmetry]}$$

$$\left.\begin{aligned} I_1^e &= I_3^e \\ I_2^e &= I_4^e \end{aligned}\right\}$$

Odd Mode Analysis:

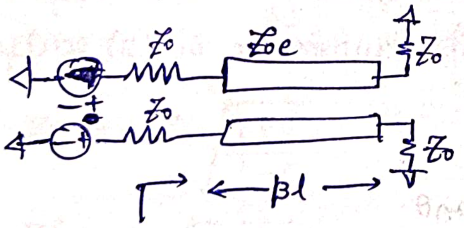


$$\left.\begin{aligned} V_1^o &= -V_3^o \\ V_2 &= -V_4^o \end{aligned}\right\} \qquad \left.\begin{aligned} I_1^o &= -I_3^o \\ I_2^o &= -I_4^o \end{aligned}\right\}$$

$$V_1 = V_1^o + V_1^e; \qquad V_2 = V_2^e + V_2^o;$$

$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o \qquad V_4 = V_2^e - V_2^o$$

#



$$\text{Input impedance} = Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$Z_L \rightarrow Z_0$

$Z_0 \rightarrow Z_{oe,o}$

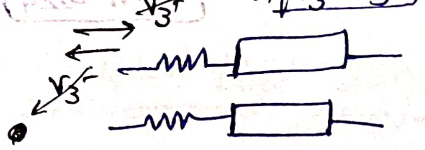
$$V_1^{e,o} = V\frac{Z_{in}^{eo}}{Z_{in}^{eo} + Z_0}$$

$$I_1^{eo} = \frac{V}{Z_{in}^{eo} + Z_0}$$

$$Z_{in}^e = Z_{oe}\frac{Z_0 + jZ_{oe}\tan\beta l}{Z_{oe} + jZ_0 \tan\beta l}$$

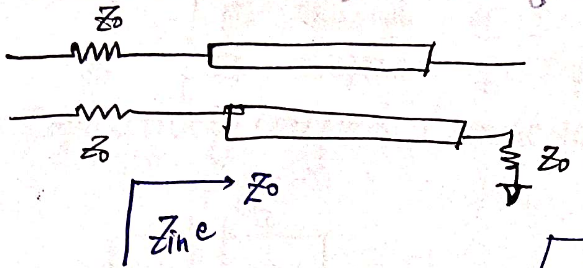
$$Z_{in}^o = Z_{oo}\frac{Z_0 + jZ_{oo}\tan\beta l}{Z_{oo} + jZ_0 \tan\beta l}.$$

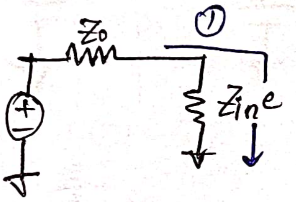
$V_3^+ \to 0 \Rightarrow \boxed{V_3 = V_3^-}$



$$\boxed{Z_{in}^e = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = Z_0 + \frac{2(Z_{in}^e Z_{in}^o - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2Z_0}} = Z_0 \left(\text{if } Z_0^2 (= Z_{in}^e Z_{in}^o) = Z_0^e Z_0^o\right)$$

$$\boxed{V_1^e = V \times \frac{Z_{in}^e}{Z_{in}^e + Z_0}} \; ; \quad V_1^o = V \times \frac{Z_{in}^o}{Z_{in}^o + Z_0}$$





$$\boxed{Z_{in}^e Z_{in}^o - Z_0^2 = 0}$$

$$\boxed{Z_{in}^e Z_{in}^o = Z_0^2}$$

$$\boxed{Z_0 = (Z_0^e Z_0^o)^{1/2}} \text{ (Assumption) for matching}$$

$$Z_0 = \sqrt{Z_0^e Z_0^o}$$

Derivation:

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = \frac{\frac{V Z_{in}^e}{Z_{in}^e + Z_0} + \frac{V Z_{in}^o}{Z_{in}^o + Z_0}}{\frac{V}{Z_{in}^e + Z_0} + \frac{V}{Z_{in}^o + Z_0}}$$

$$= \frac{Z_{in}^e (Z_{in}^o + Z_0) + Z_{in}^o (Z_{in}^e + Z_0)}{Z_{in}^e + Z_{in}^o + 2Z_0} = \frac{Z_0 (Z_{in}^e + Z_{in}^o + 2Z_0) + 2 Z_{in}^o Z_{in}^e - 2Z_0^2}{Z_{in}^e + Z_{in}^o + 2Z_0}$$

$$= Z_0 + \frac{2(Z_{in}^e Z_{in}^o - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2Z_0}$$

If $Z_0 = \sqrt{Z_0^e Z_0^o}$

$$\Rightarrow Z_{in}^e = Z_0^e \frac{\sqrt{Z_0^o} + j\sqrt{Z_0^e}\tan\theta}{\sqrt{Z_0^e} + j\sqrt{Z_0^o}\tan\theta}$$

$$Z_{in}^o = Z_0^o \frac{\sqrt{Z_0^e} + j\sqrt{Z_0^o}\tan\theta}{\sqrt{Z_0^o} + j\sqrt{Z_0^e}\tan\theta}$$

$$V_1^e = V\frac{Z_{in}^e}{Z_{in}^e + Z_0} = V\frac{Z_0^e\,\dfrac{Z_0 + jZ_0^e\tan\theta}{Z_0^e + jZ_0\tan\theta}}{Z_0^e\,\dfrac{Z_0 + jZ_0^e\tan\theta}{Z_0^e + jZ_0\tan\theta} + Z_0}$$

$$= V\frac{Z_0^e Z_0 + jZ_{0e}^2\tan\theta}{2Z_0^e Z_0 + jZ_0^{e2}\tan\theta + jZ_0^2\tan\theta} \times \frac{1/Z_0^e}{1/Z_0^e} = V\frac{Z_0 + jZ_{0e}\tan\theta}{2Z_0 + jZ_{0e}\tan\theta + jZ_0^o\tan\theta}$$

$$\boxed{V_3 = V_1^e - V_1^o = V\frac{j(Z_0^e - Z_0^o)\tan\theta}{2Z_0 + j(Z_0^e + Z_0^o)\tan\theta}}$$

; V_1^o can be simply found out by interchanging $Z_0^e \rightarrow Z_0^o$

first 'o' is not changing.

denominator is also same after changing

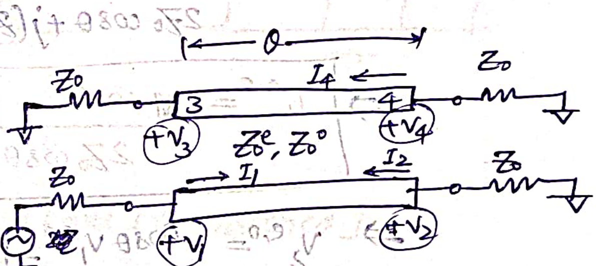
Let $C = \dfrac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$: Coupling coefficient

$$\Rightarrow V_3 = V\frac{\left[j\dfrac{Z_{0e} - Z_0^o}{Z_{0e} + Z_0^o}\tan\theta\right]}{\left[\dfrac{2Z_0}{Z_0^e + Z_0^o} + j\tan\theta\right]} = V\left(\frac{jC\tan\theta}{\dfrac{2Z_0}{Z_0^e + Z_0^o} + j\tan\theta}\right).$$

$$\because \sqrt{1 - C^2} = \sqrt{1 - \left(\frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}\right)^2} = \frac{\sqrt{(Z_0^e + Z_0^o)^2 - (Z_0^e - Z_0^o)^2}}{Z_0^e + Z_0^o}$$

$$= \frac{2\sqrt{Z_0^e Z_0^o}}{Z_0^e + Z_0^o} = \frac{2Z_0}{Z_0^e + Z_0^o}$$

$$\Rightarrow \boxed{V_3 = V\frac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta}}$$



Three Important Design Equations:

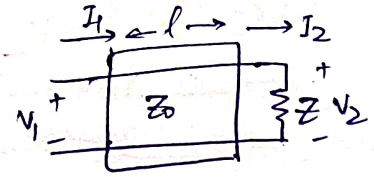
Coupling (C) is maximum for $l = \lambda/4$.

(A) $\leftarrow Z_0^e = Z_0\sqrt{\dfrac{1+C}{1-C}}$

(B) $\leftarrow Z_0^o = Z_0\sqrt{\dfrac{1-C}{1+C}}$

$Z_0 = \sqrt{Z_0^e Z_0^o}$

#



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \rightarrow \text{To calculate } V_2$$

This matrix to be inverted.

$$\begin{bmatrix} V_1^{eo} \\ I_1^{eo} \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{oe,o}\sin\theta \\ jY_{oe,o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^{eo} \\ I_2^{eo} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_2^{eo} \\ I_2^{eo} \end{bmatrix} = \begin{bmatrix} \cos\theta & -jZ_0^{eo}\sin\theta \\ -jY_0^{eo}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_1^{eo} \\ I_1^{eo} \end{bmatrix} \quad \text{[Matrix Inversion]}$$

$$V_1^{eo} = V\frac{Z_0 + jZ_0^{eo}\tan\theta}{2Z_0 + j(Z_0^e + Z_0^o)\tan\theta} = V\frac{Z_0\cos\theta + jZ_{oe,o}\sin\theta}{2Z_0\cos\theta + j(Z_0^e + Z_0^o)\sin\theta}$$

$$I_1^e = \frac{V}{Z_{in}^e + Z_0} = \frac{V}{\left(Z_0^e\dfrac{Z_0 + jZ_0^e\tan\theta}{Z_0^e + jZ_0\tan\theta}\right) + Z_0} = V\frac{Z_0^e + jZ_0\tan\theta}{Z_0^e(Z_0 + jZ_0^e\tan\theta) + Z_0(Z_0^e + jZ_0\tan\theta)}$$

$$= V\frac{Z_0^e + jZ_0\tan\theta}{2Z_0^eZ_0 + j(Z_0^{e2} + Z_0^2)\tan\theta} \times \frac{1/Z_0^e}{1/Z_0^e} = V\frac{1 + j\sqrt{Z_0^o/Z_0^e}\tan\theta}{2Z_0 + j(Z_0^e + Z_0^o)\tan\theta}$$

$$= V\frac{\cos\theta + j\sqrt{Z_0^o/Z_0^e}\sin\theta}{2Z_0\cos\theta + j(Z_0^e + Z_0^o)\sin\theta}$$

$$\Rightarrow \boxed{I_1^{e,o} = V\frac{\cos\theta + j\sqrt{Z_0^{o,e}/Z_{oe,o}}\sin\theta}{2Z_0\cos\theta + j(Z_{oe} + Z_{oo})\sin\theta}}$$

$$\Rightarrow V_2^{e,o} = \cos\theta\, V_1^{eo} - jZ_{oe,o}\sin\theta\, I_1^{e,o}$$

$$= V\frac{(Z_0\cos\theta + jZ_{oe,o}\sin\theta)\cos\theta}{2Z_0\cos\theta + j(Z_{oe} + Z_{oo})\sin\theta} - V\frac{jZ_{oe,o}\sin\theta(\cos\theta + j\sqrt{Z_{oo,e}/Z_{oe,o}}\sin\theta)}{2Z_0\cos\theta + j(Z_{oe} + Z_{oo})\sin\theta}$$

$$= V\frac{Z_0\cos^2\theta + jZ_{oe,o}\sin\theta\cos\theta - jZ_{oe,o}\sin\theta\cos\theta + \sqrt{Z_{oe}Z_{oo}}\sin^2\theta}{2Z_0\cos\theta + j(Z_{oe} + Z_{oo})\sin\theta}$$

$$\Rightarrow \boxed{V_2^{e,o} = V\frac{Z_0}{2Z_0\cos\theta + j(Z_{oe} + Z_{oo})\sin\theta}}$$

For $\theta = \pi/2$, coupler is $\lambda/4$ long,
$V_3/V_0 = C$,
$V_2/V_0 = -j\sqrt{1 - C^2}$

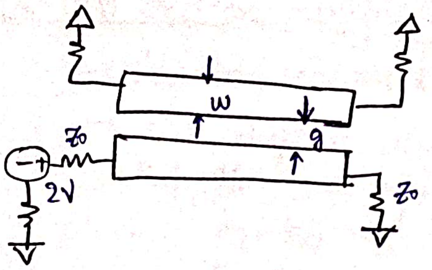
For $\theta \ll \pi/2$ (very low) $\Rightarrow$ $P_3 = 0$, $P_2 \rightarrow$ full Power

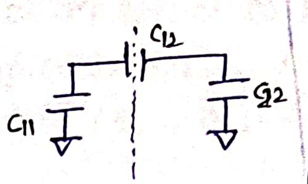
For $\theta = \pi/2$, Power at 3 $\rightarrow$ maximum $\Rightarrow$ Coupler operate in this regim

$$V_4 = V_4^e + V_4^o = V_2^e - V_2^o = \boxed{0}$$

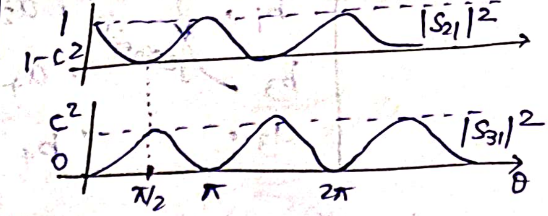
$$\boxed{V_2 = V_2^e + V_2^o = V_0\frac{\sqrt{1 - C^2}}{\sqrt{1 - C^2}\cos\theta + j\sin\theta}}\ ;\quad \boxed{V_3 = V_0\frac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta}}.$$

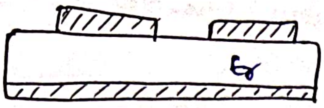
08-11-2024

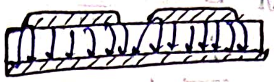


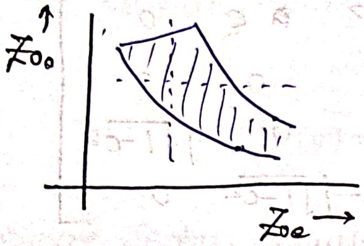


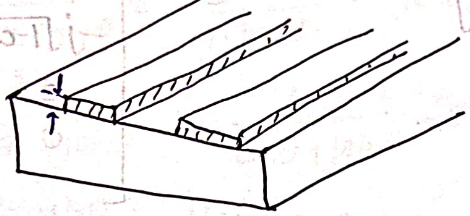
Freq. response at P-2 & 3:

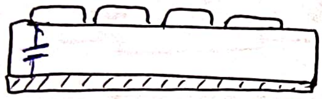


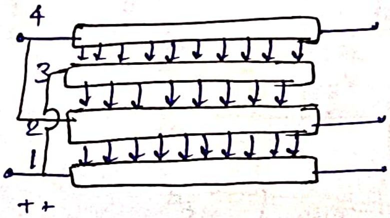


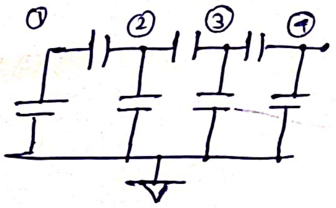


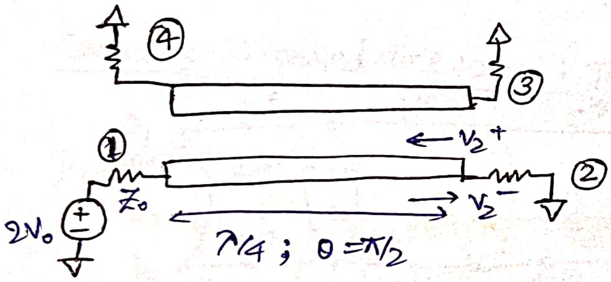












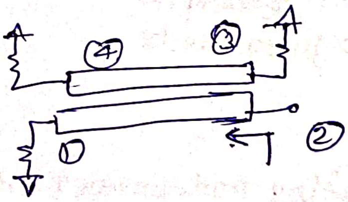
$S_{11} = 0$ [Port 1 is matched]

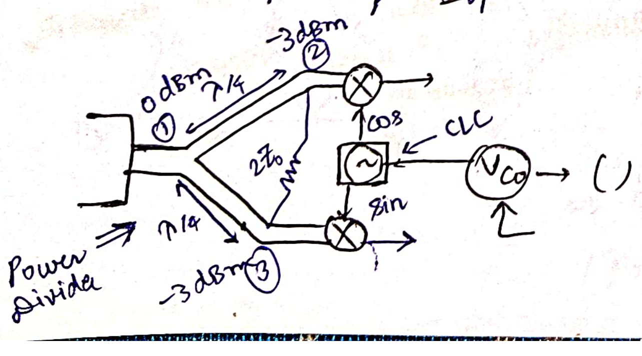
$S_{31} = c$

$S_{41} = 0$

$S_{21} = \frac{V_2^-}{V_1^+} = -j\sqrt{1-c^2}$

$$V_4 = V_4^e + V_4^o$$
$$= V_2^e - V_2^o$$
$$\frac{V_2}{V_1} = \frac{\cancel{V_2^+} + V_2^-}{V_1^+ + \cancel{V_1^-}}$$



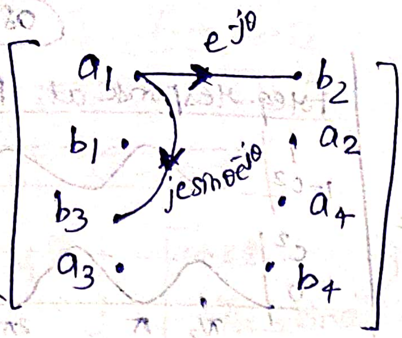


$S_{21} = \frac{-j}{\sqrt{2}}$

$S_{31} = \frac{-j}{\sqrt{2}}$

$S_{23} = \frac{-1}{2}$

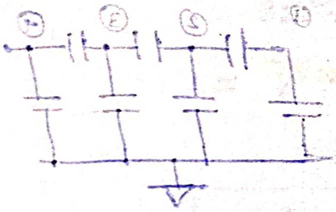
$$\begin{bmatrix} a_1 & b_2 \\ b_1 & a_2 \\ b_3 & a_4 \\ a_3 & b_4 \end{bmatrix}$$



$$S_{21} = -j\sqrt{1-c^2} = \frac{b_2}{a_1}$$

$$S_{31} = c = \frac{b_3}{a_1}.$$

$\therefore$ scattering matrix: $S = \begin{bmatrix} 0 & -j\sqrt{1-c^2} & c & 0 \\ -j\sqrt{1-c^2} & 0 & 0 & c \\ c & 0 & 0 & -j\sqrt{1-c^2} \\ 0 & c & -j\sqrt{1-c^2} & 0 \end{bmatrix}$



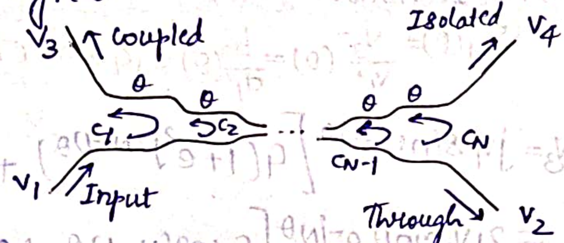
Multi Section Coupled Line Coupler

↳ Adding coupled lines in series to increase coupler bandwidth.

→ The couplers are typically designed s.t. they are symmetric, i.e.,

$C_1 = C_N$
$C_2 = C_{N-1}$
$C_3 = C_{N-2}$, where

∴ N is odd
↘ b/c phase char. are usually better.

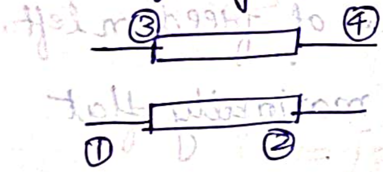


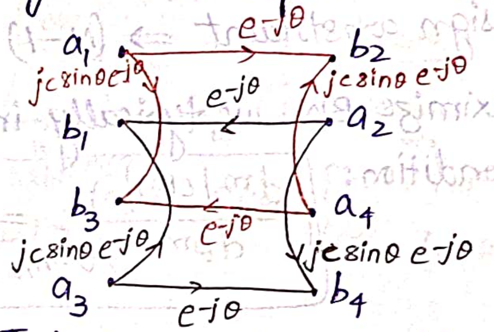
→ Coupling: $S_{31}(\theta) = \dfrac{jc\tan(\theta)}{\sqrt{1-c^2}+\tan\theta} \approx \dfrac{jc\tan\theta}{1+j\tan\theta} = jc\sin\theta \cdot e^{-j\theta}$

$$S_{21}(\theta) = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\cos\theta + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin\theta} = e^{-j\theta}, \quad \theta = \beta l = \omega T$$

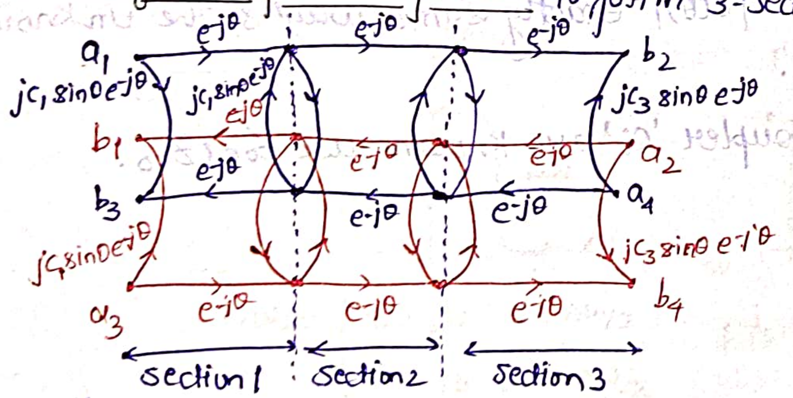
$T = l/v_p$.

Single Section Coupled Line Coupler: Use the approx. to construct signal flow graph of a single-section coupler:





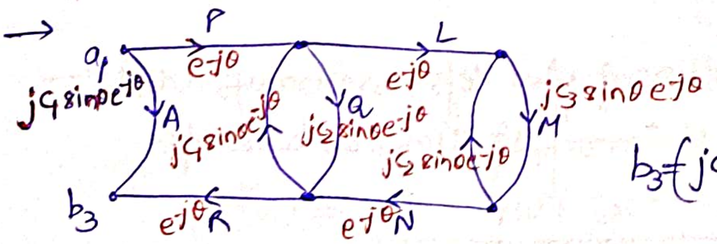
Cascading 3 coupled line pairs: To form 3-section coupled line coupler.



→ Decouples into 2 separate graphs (blue & red graphs)

→ These 2 graphs are essentially identical and emphasize the symmetric structure of coupled-line coupler.

→



Here, propagation paths: $\boxed{A}$ $\boxed{PQR}$ $\boxed{PLMNR}$

$$b_3 = \left(jc_1\sin\theta e^{-j\theta} + e^{-j\theta} jc_2\sin\theta e^{-j\theta} e^{-j\theta} + e^{-j2\theta} jc_3\sin\theta e^{-j\theta} e^{-j2\theta}\right) a_1$$

$$\Rightarrow b_3 = \left(jc_1\sin\theta e^{-j\theta} + jc_2 e^{-j3\theta} + jc_3\sin\theta e^{-j5\theta}\right) a_1$$

[Forwarded path considered]

By approximations:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1 \sin\theta\, e^{-j\theta} + jc_2 \sin\theta\, e^{-j3\theta} + jc_3 e^{-j\theta} \sin\theta$$

For N-sections:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1 \sin\theta\, e^{-j\theta} + jc_2 \sin\theta\, e^{-j3\theta} + \dots + jc_N \sin\theta\, e^{-j(2N-1)\theta}$$

(N: odd)

$$V_3 = jV_1 \sin\theta\, e^{-j\theta}\left[c_1\left(1+e^{-2j(N-1)\theta}\right) + c_2\left(e^{-j2\theta}+e^{-2j(N-2)\theta}\right) + \dots + c_M e^{-j(N-1)\theta}\right]$$

$$= 2jV_1 \sin\theta\, e^{-jN\theta}\left[c_1 \cos(N-1)\theta + c_2 \cos(N-3)\theta + \dots + \tfrac{1}{2} c_M\right], \text{ where } M = \frac{N+1}{2}.$$

At $\theta = \pi/2 \Rightarrow \sin(\pi/2) = 1$; [Symmetric Coupler]

$$\boxed{|C(\theta)|_{\theta=\pi/2} = |S_{31}(\theta)| = c_1\, 2\cos\left\{(N-1)\frac{\pi}{2}\right\} + c_2\, 2\cos\left\{(N-3)\frac{\pi}{2}\right\} + c_3\, 2\cos\left\{(N-5)\frac{\pi}{2}\right\} + \dots + c_M.}$$

...first design eqn.

→ $|C(\theta)|_{\theta=\pi/2}$ is set to the value necessary to achieve the desired coupling value.

↳ 1 design constraint ⇒ $(M-1)$ degrees of freedom left.

→ To maximize BW, we typically impose maximally flat condition:

$$\boxed{\left(\frac{d^m |C(\theta)|}{d\theta^m}\right)_{\theta=\pi/2} = 0}, \quad m = 1, 2, \dots, M-1$$

→ Here, we synthesise $|C(\theta)|$ exactly same way as broadband QWT matching so the unknown are $[c_1, c_2, \dots, c_N]$.

Once individual coupler 'c_i' are known, use Z_{oe} & Z_{oo}.

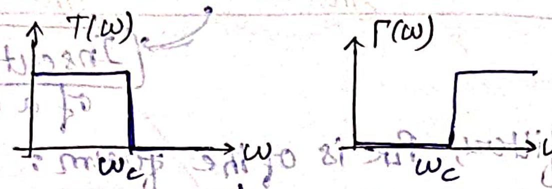
Microwave Filter (Insertion Loss Method)

→ A lossless filter can be described in terms of its power transmission coefficient, $T(\omega)$ or its power reflection coefficient, $\Gamma(\omega)$, as the two values are completely dependent.

① Passband : $T(\omega)=1$, $\Gamma(\omega)=0$

② Stopband : $T(\omega)=0$, $\Gamma(\omega)=1$.

Ideal low-pass filter :



+ Linear phase = Perfect MW Filter ⇒ Impossible to build !

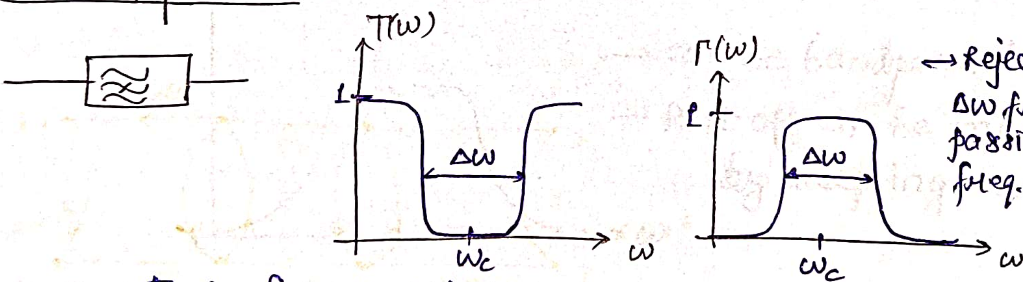
③ Band-Pass Filter :

→ Passes signals within Δω freq. BW, while rejecting at all freq outside.

$$T(\omega)=\begin{cases}\approx 1, & |\omega-\omega_0|<\frac{\Delta\omega}{2}\\ \approx 0, & |\omega-\omega_0|>\frac{\Delta\omega}{2}\end{cases}$$

$$\Gamma(\omega)=\begin{cases}\approx 0, & |\omega-\omega_0|<\frac{\Delta\omega}{2}\\ \approx 1, & |\omega-\omega_0|>\frac{\Delta\omega}{2}.\end{cases}$$

④ Band-Stop Filter :



→ Rejects signals within Δω freq. BW, while passing signals at all freq. outside this BW.

$$T(\omega)=\begin{cases}\approx 0 & |\omega-\omega_0|<\Delta\omega/2\\ \approx 1, & |\omega-\omega_0|>\Delta\omega/2\end{cases}$$

$$\Gamma(\omega)=\begin{cases}\approx 1, & |\omega-\omega_0|<\Delta\omega/2\\ \approx 0, & |\omega-\omega_0|>\Delta\omega/2.\end{cases}$$

→ Filter functions for only possible (i.e., realizable) filters expressed as finite polynomials :

$$T(\omega)=\frac{a_0+a_1\omega+a_2\omega^2+\dots}{b_0+b_1\omega+b_2\omega^2+\dots+b_N\omega^{2N}}$$

← The order N of the (denominator) polynomial is likewise the order of the filter.

→ Power loss ratio, $\boxed{P_{LR} = \frac{P_1^+}{P_2^-} = \frac{1}{1-\Gamma(\omega)}}$

$P_{LR} \to \infty$ when $\Gamma(\omega)=1$

$P_{LR} = 1$ when $\Gamma(\omega)=0$

For lossless filter, $\boxed{P_{LR} = \frac{1}{T(\omega)}}$

$P_{LR}(dB)$: $\boxed{P_{LR}(dB) = 10 \log_{10} P_{LR} = -10 \log_{10} T(\omega)}$

$\boxed{\text{Insertion loss}}$ of a lossless filter

→ For realizable filters, P_{LR} is of the form:

$\cancel{P_{LR}(\omega) = \frac{1+M(\omega^2)}{K+N}}$ $\boxed{P_{LR}(\omega) = 1 + \frac{M(\omega^2)}{N(\omega^2)}}$ → polynomial with term $\omega^2, \omega^4, \omega^6$, etc

Specifying polynomials ⇒ specifying frequency behaviour of a realizable filter.

(12-11-2024)

Microwave filter (Insertion Loss Method)

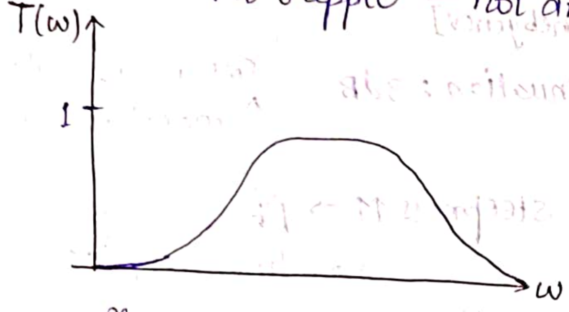
① <u>Butterworth</u>

↳ Maximally flat filters

↳ Primary characteristics :

ⓐ Gradual roll-off

ⓑ No ripple — not anywhere!!



→ The Butterworth low-pass filters have a power loss ratio equal to :

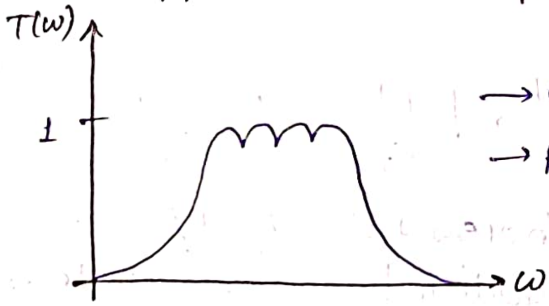
$$\boxed{P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}$$

where ω_c is the low-pass cutoff frequency, and N specifies the order of the filter.

② <u>Chebychev</u> : Equal-ripple filters

↳ Steep roll-off (but not as steep as elliptical)

↳ Passband ripple (but not stop-band ripple).



→ can be bandpass, low-pass, etc.

→ Roll-off can be made steeper by accepting more ripple

Chebychev's low-pass filters have power ratio :

$$\boxed{P_{LR}(\omega) = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)}$$, k: specifies passband ripple

$T_N(x)$: chebychev polynomial of order N.

ω_c : low-pass cutoff frequency.

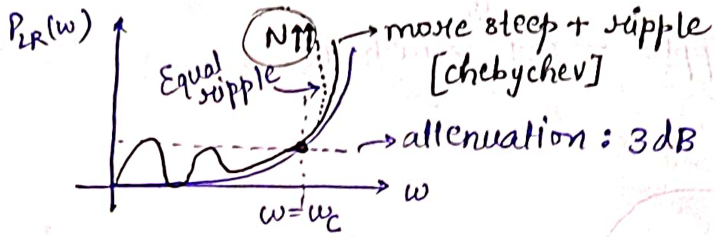
Comparison (Butterworth /Binomial)

For a low-pass filter,

$$\boxed{P_{LR} = 1 + K^2\left(\frac{\omega}{\omega_c}\right)^{2N}} \approx K^2\left(\frac{\omega}{\omega_c}\right)^{2N}, \quad \omega >> \omega_c \quad [\text{falloff rate}]$$

Insertion loss ↑ @ rate 20N dB/decade.

At edge of band, $P_{LR} = 1 + K^2$, $\omega \approx \omega_c$
$= 2$, for $K = 1$



For $\omega > \omega_c$, attenuation ↑ monotonically with f.

steepness ↑↑ ⇒ Phase response non-linearity ↑↑

→ Higher-order filters phase response are non-linear [$N \approx 10$].

Comparison (Chebychev)

Equal Ripple: Chebychev polynomial used to specify the insertion loss.

$$\boxed{P_{LR} = 1 + K^2 T_N^2\left(\frac{\omega}{\omega_c}\right)} \Rightarrow \text{Sharper cutoff.}$$

$$T_N(x) = \cos(N\cos^{-1}x), \quad x = \frac{\omega}{\omega_c}$$

$$= \cosh(N\cosh^{-1}x) \quad \text{for } \omega > \omega_c.$$

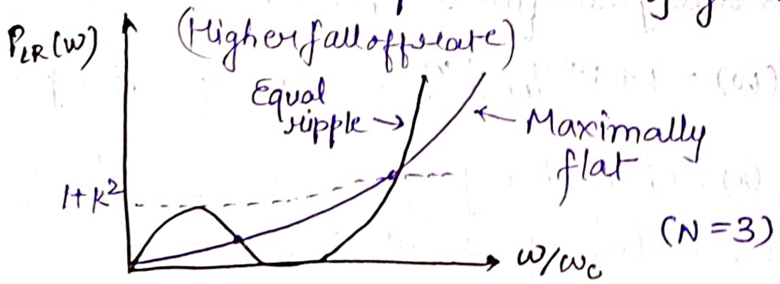
$$P_{LR} = 1 + K^2$$

$$P_{LR} \approx \frac{K^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N}, \text{ for large } x \quad \left[T_N(x) \approx \tfrac{1}{2}(2x)^N\right]$$

$(\omega >> \omega_c)$ [K^2 determines passband ripple level]

↳ increases @ 20N dB/decade.

↳ insertion loss is $(2^{2N})/4$ greater than binomial response at any given freq. $\omega >> \omega_c$.

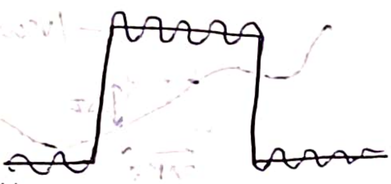


Elliptical Method

↳ Exhibit very-steep roll-off, i.e., transition from pass-band to stop-band is very rapid.

↳ Exhibit ripple in the passband, i.e., the value of T will vary slightly within the passband.

↳ They exhibit ripple in the stopband, i.e., the value of T will vary slightly within the stop band.



→ Roll-off can be made steeper by accepting more ripple.

Elliptical filters

↳ closest to ideal (steepest roll-off)

Phase Response: Butterworth $\angle S_{21}(\omega)$ → close to linear

Chebychev $\angle S_{21}(\omega)$ → Not very linear

Elliptical $\angle S_{21}(\omega)$ → Non-linear

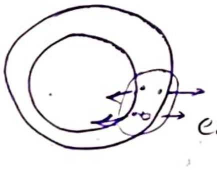
→ As filter roll-off improves, the phase response gets worse. ⇒ No "best" filter

$\boxed{\tau(\omega) = -\frac{d}{d\omega}\angle S_{21}(\omega)}$ → Phase delay ⇒ Important to consider when designing/specifying/selecting a MW filter

↳ otherwise, output signal could be distorted

⇓

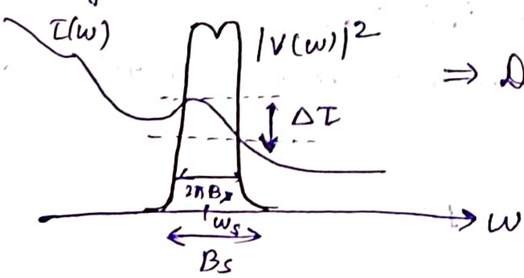
By signal dispersion.



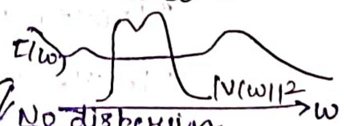
$v_p v_g = c^2$

Filter Dispersion

↳ Phase delay changes by a precipitous value $\Delta\tau$ across signal bandwidth B_s:



⇒ ~~Also~~ Dispersion will occur



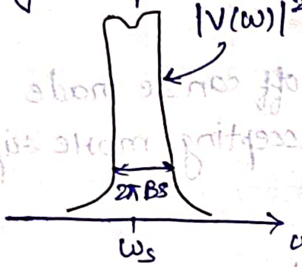
⇓ No dispersion

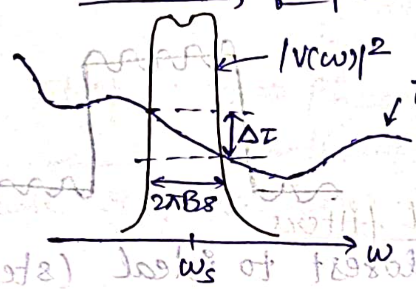
(As signal B/W is ~ constant — each freq. component will be delayed by same amount)

Signal dispersion : If Different frequencies that comprise a signal propagate at different velocities through a MW filter (i.e, each signal frequency has a different delay τ), the output signal will be distorted.

Constant phase delay $\Rightarrow$ No distortion (not a strict requirement)
$\Downarrow$
Difficult to build

Eg. Signal Spectrum: BW = B_s Hz. Phase delay of a filter:





↳ Signal would be distorted, but only b/c $\tau(\omega)$ changes significantly ~~ace~~ across B_s, if it passed through filter.

→ ~~$\Delta\tau$~~ For no (or insignificant) dispersion:
(Subjective) Rule of thumb:

$$\boxed{f_s \Delta\tau \le 0.1 \quad \text{or,} \quad \omega_s \Delta\tau \le \frac{\pi}{5}}$$

$[\omega_s = 2\pi f_s]$

For precise zero distortion $\Rightarrow \Delta\tau = 0$

→ For any type of filter, we can improve roll-off (i.e., increase stop-band attenuation) by increasing the filter order N, but has deleterious effects:

- makes phase response $\angle S_{21}(\omega)$ worse (i.e, more nonlinear).
- increases filter cost, weight and size.
- increases filter insertion ~~loast~~ Loss.
- makes filter performance more sensitive to temperature, aging, etc.

→ For practical, keep N<10.

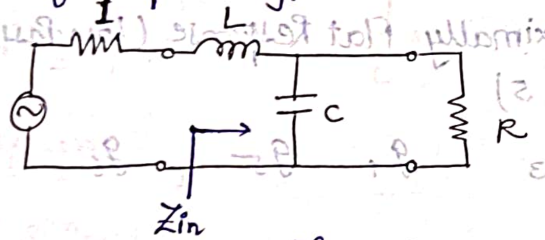
MW Filter (Design steps)

Process of filter design by insertion loss method:

Filter specifications → Low-pass prototype design → Scaling and conversion → Implementation

Insertion Loss Method:

Low-pass filter prototype, $N=2$:



Input impedance, $Z_{in} = j\omega L + \dfrac{R(1-j\omega RC)}{1+\omega^2R^2C^2}$

As $\Gamma = \dfrac{Z_{in}-1}{Z_{in}+1}$,

Power loss ratio:

$$P_{LR} = \frac{1}{1-|\Gamma|^2} = \frac{1}{1-\left[\frac{(Z_{in}-1)}{(Z_{in}+1)}\right]\left[\frac{Z_{in}^*-1}{Z_{in}^*+1}\right]} = \frac{|Z_{in}+1|^2}{2(Z_{in}+Z_{in}^*)}$$

$$Z_{in}+Z_{in}^* = \frac{2R}{1+\omega^2R^2C^2}$$

$$|Z_{in}+1|^2 = \left(\frac{R}{1+\omega^2R^2C^2}+1\right)^2 + \left(\omega L - \frac{\omega CR^2}{1+\omega^2R^2C^2}\right)^2$$

$$\Rightarrow P_{LR} = \frac{1+\omega^2R^2C^2}{4R}\left[\left(\frac{R}{1+\omega^2R^2C^2}+1\right)^2 + \left(\omega L - \frac{\omega CR^2}{1+\omega^2R^2C^2}\right)^2\right]$$

$$= \frac{1}{4R}\left(R^2+2R+1+R^2\omega^2C^2+\omega^2L^2+\omega^4L^2C^2R^2-2\omega^2LCR^2\right)$$

$$= 1+\frac{1}{4R}\left[(1-R)^2+(R^2C^2+L^2-2LCR^2)\omega^2+L^2C^2R^2\omega^4\right] \quad \dots (A)$$

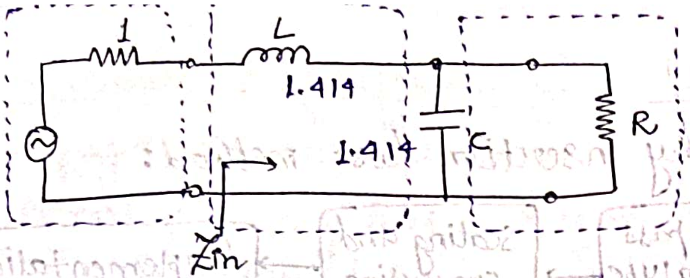
$\Rightarrow$ $\boxed{P_{LR} = 1+\omega^4}$ ... (B) [Butterworth filter Prototype Function

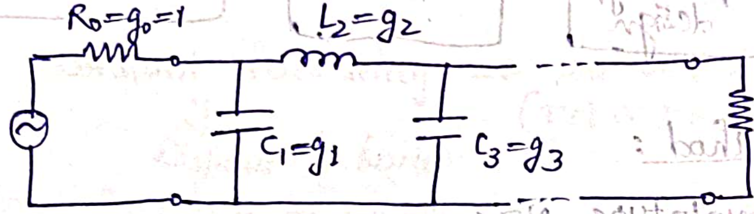
Comparing (A) and (B)

$\Rightarrow R=1,$

$C^2+L^2-2LC = (C-L)^2 = 0 \Rightarrow L=C,$

$\frac{1}{4}L^2C^2 = \frac{1}{4}L^4 = 1 \Rightarrow L=C=\sqrt{2}$





→ Element Values for Maximally Flat Response (Low-Pass Filter)
($g_0=1$, $\omega_c=1$, $N=1$ to 5)

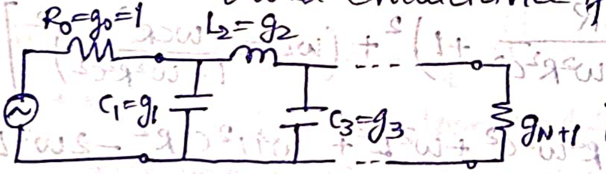
N	g_1	g_2	g_3	g_4	g_5	g_6
1	2.0000	1.0000				
2	1.4142	1.4142	1.0000			
3	1.0000	2.0000	1.0000	1.0000		
4	0.7654	1.8478	1.8478	1.7654	1.0000	
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000

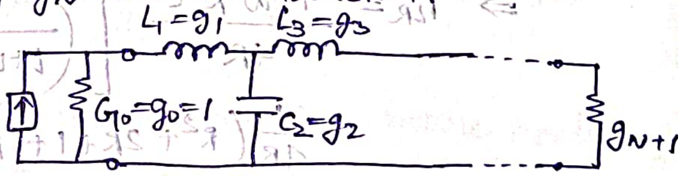
→ Between series and shunt connections, g_K definition:

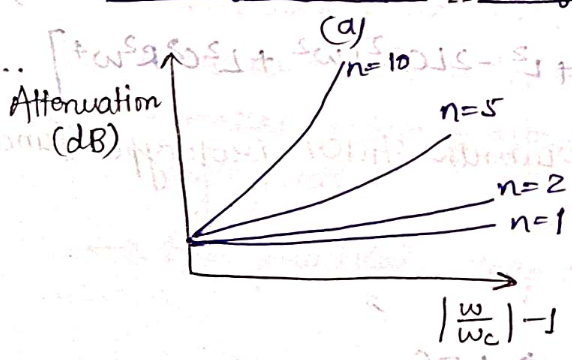
$g_0 = \begin{cases} \text{generator resistance [a]} \\ \text{generator conductance [b]} \end{cases}$

$\underset{(K=1 \text{ to } N)}{g_K} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$

$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is shunt capacitor} \\ \text{load conductance if } g_N \text{ is series inductor} \end{cases}$







Maximally Flat Response (Alternative ~~Respons~~ Approach)

T.F. for Butterworth Response:

$$\boxed{|T(s)|^2 = \frac{1}{1+\omega^{2n}}}$$

Poles of Butterworth filter:

$1+(\omega)^{2n}=0$

$\Rightarrow (\omega)^{2n}=-1$

$\Rightarrow \left(\frac{s}{j}\right)^{2n}=-1$ $\quad\left[s=j\omega \Rightarrow \omega=\frac{s}{j}\right]$

$\Rightarrow (-1)^n(s^2)^n=-1$

$\Rightarrow (-1)^n s^{2n}=-1$

$$S_K = \begin{cases} e^{j\left(\frac{2K-1}{2n}\right)\pi}, & n: \text{even} \\ e^{j\left(\frac{K\pi}{n}\right)}, & n: \text{odd} \end{cases} \quad [K=1,2,3,\ldots,2n]$$

$n=3$

$K=6$: $e^{\frac{jK\pi}{n}}$

$= e^{\frac{j\pi}{3}}, e^{\frac{j2\pi}{3}}, e^{\frac{j3\pi}{3}}, e^{\frac{j4\pi}{3}}, e^{\frac{j5\pi}{3}}, e^{\frac{j6\pi}{3}}.$

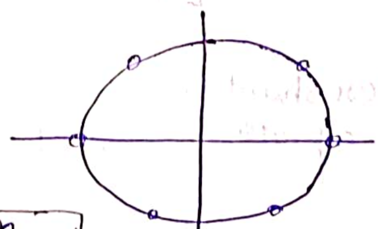
Maximally Flat (Butterworth) Response

Derivation of driving point impedance function:

$|\Gamma(s)|^2 = 1-\frac{1}{1+\omega^{2n}}$

$= \frac{\omega^{2n}}{1+\omega^{2n}}$

$= \frac{(s/j)^{2n}}{1+(s/j)^{2n}} = \boxed{\frac{(-s^2)^n}{1+(-s^2)^n}}$



$$\Rightarrow \boxed{|\Gamma(s)|^2 = \frac{-s^{2n}}{\prod_{K=1}^{n}(s-s_K)\prod_{K=n+1}^{2n}(s-s_K)}}$$

$$\Rightarrow |\Gamma(s)|^2 = \frac{-s^n \cdot s^n}{\left(\prod_{K=n+1}^{2n}(s-s_K)\right)\left(\prod_{K=1}^{n}(s-s_K)\right)}$$

$\Gamma(-s) \qquad \Gamma(+s)$

$$= \Gamma(s) \cdot \Gamma(-s)$$

Poles for n=3 B.F. :

$s_1 = -1$

$s_2 = -0.5 + j0.8660$

$s_3 = -0.5 - j0.8660$

$$\Gamma_1(s) = \frac{s^n}{\prod_{K=1}^{n}(s-s_K)} = \frac{s^3}{s^3+2s^2+2s+1}$$

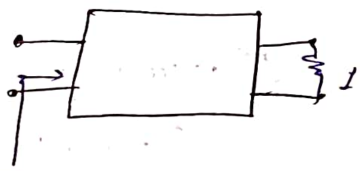
Reflection coeff. assuming the filter is terminated by 1Ω impedance at both ends,

$$Z_1(s) = \frac{1+\Gamma_1(s)}{1-\Gamma_1(s)} = \frac{2s^3+2s^2+2s+1}{2s^2+2s+1}$$

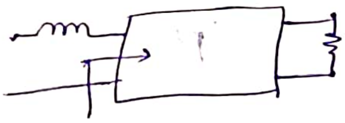
$$= s + \frac{s+1}{(2s^2+2s+1)}$$

$1.(s) \rightarrow L=1$

$Z_1'(s)$



$Z_1(s)$



$Z_1'(s)$

For shunt capacitor : $\dfrac{2s^2+2s+1}{s+1}$

Chebychev's case:

BW:

CH: $P_{LR}(\omega) = 1 + K^2 T_N^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$

$= 1 + K^2 T_N^2 (\omega)^{2N}$

$1 + K^2 (\cos(N \cos^{-1} x))$

$= 1 + K^2 (2x^2 - 1)^2$

For $\omega = 0 \longrightarrow \sim 1 + K^2$

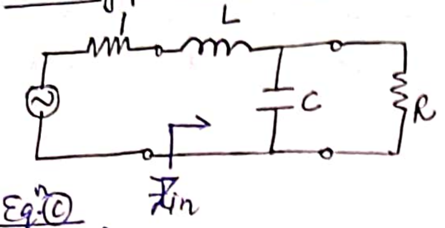
$\cos(N \cos^{-1} x)$

$= \cos(3 \cos^{-1} x)$ [N=3]

$\equiv 1 + K^2 (\underbrace{3x^3 - 3x}_{\text{does not contribute}})$

~ 1

Equal-Ripple Low Pass Filter



$T_N(x) = \cos(N \cos^{-1} x)$

Eq. (C)

$$1 + K^2(4\omega^4 - 4\omega^2 + 1) = 1 + \frac{1}{4R}\left[(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4\right]$$

→ Eq.ⁿ (1)

At $\omega = 0$, $K^2 = \frac{(1-R)^2}{4R}$

$\Rightarrow R = 1 + 2K^2 \pm 2K\sqrt{1+K^2}$ (N: Even)

$T_2(x) = (2x^2 - 1)$
$T_3(x) = (4x^3 - 3x)$

$\Rightarrow 4K^2 = \frac{1}{4R} L^2C^2R^2$

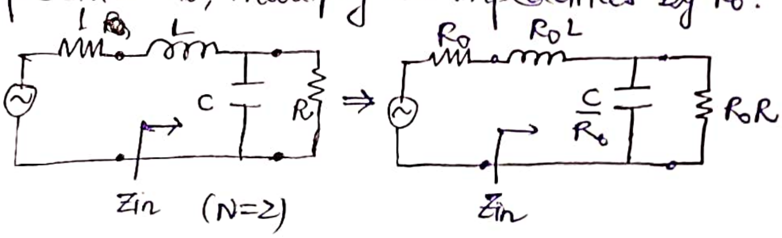
$-4K^2 = \frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2)$

↳ Normalised having source impedance $R_0 = 1$ and cutoff freq. $\omega_c = 1$ rad/sec.

Only Impedance Scaling:

↳ For source impedance R_0, multiply all impedances by R_0.

$L' = R_0 L$
$C' = \frac{C}{R_0}$
$R_s' = R_0$
$R_L' = R_0 R_L$

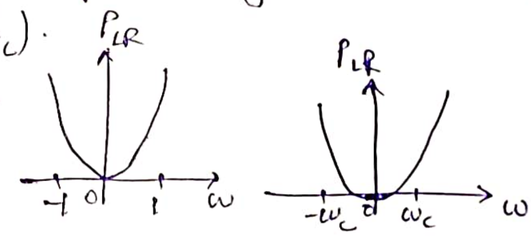


Only Frequency Scaling:

↳ To change cutoff-freq. of LP prototype from unity to ω_c, scale freq. dependence of the filter by $(1/\omega_c)$.

$\omega \rightarrow \frac{\omega}{\omega_c}$ substitution

Net power loss ratio, $P_{LR}'(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right)$

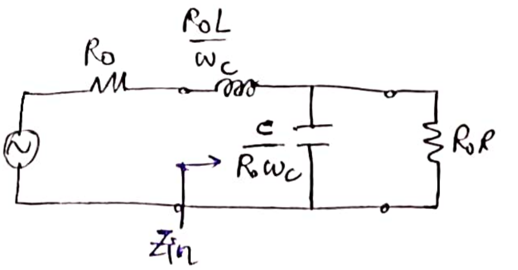


$$\left.\begin{aligned} jX_K &= j\frac{\omega}{\omega_c}L_K = j\omega L_K' \\ jB_K &= j\frac{\omega}{\omega_c}C_K = j\omega C_K' \end{aligned}\right\} \Rightarrow \begin{aligned} L_K' &= L_K/\omega_c \\ C_K' &= C_K/\omega_c \end{aligned}$$

Both Frequency & Imp. Scaling of LPF:

$L_K' = \frac{R_0 L_K}{\omega_c}$

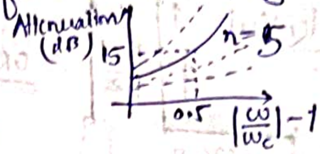
$C_K' = \frac{C_K}{R_0 \omega_c}$



Eg. Design a maximally flat low-pass filter with a cutoff-freq. of 2GHz, impedance of 50Ω and atleast 15dB insertion loss at 3GHz.

Soln: Find required order of filter to satisfy insertion loss of at 3GHz.

$$\left|\frac{\omega}{\omega_c}\right| - 1 = 0.5$$



$\Rightarrow n = 5$

From table for $N=5$,

$g_0 = 1,\ g_1 = 0.6180,\ g_2 = 0.6180,\ g_3 = 2,\ g_4 = 1.6180,\ g_5 = 0.6180,\ g_6 = 1.$

$$\left[L_k' = \frac{R_0 L_K}{\omega_c},\ C_k' = \frac{C_K}{R_0 \omega_c}\right]$$

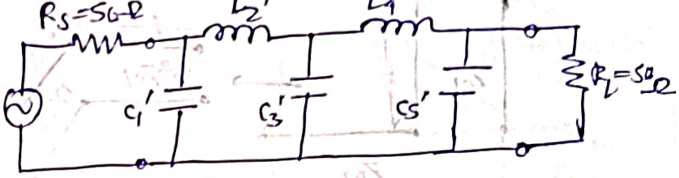
$$C_1' = \frac{0.6180 \rightarrow g_1}{50 \times (2\pi \times 2 \times 10^9)} = 0.984\ pF$$

$L_2' = 6.438\ nH$

$C_3' = 3.183\ pF$

$L_4' = 6.438\ nH$

$C_5' = 0.984\ pF$



Filter Transformation

Richard's Transformation:

$$\Omega = \tan\beta l = \tan\left(\frac{\omega l}{v_p}\right)$$

↳ maps ω to Ω-plane, which repeats with period of $\frac{\omega l}{v_p} = 2\pi$.

Reactance: $jX_L = j\Omega L = jL\tan\beta l$.

Susceptance: $jB_c = j\Omega C = jC\tan\beta l$

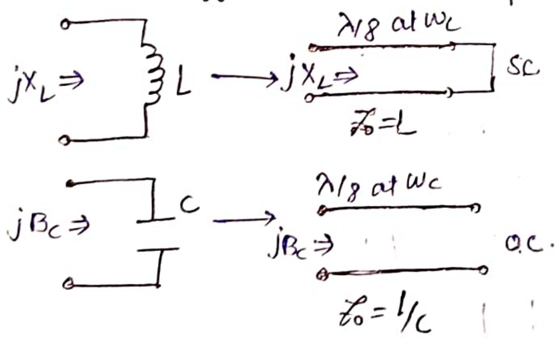
⇓

Inductor ⟶ short-circuited stub of length βl and char. imp. (L).

Capacitor ⟶ open-circuited stub of length βl and char. imp. $\left(\frac{1}{C}\right)$.

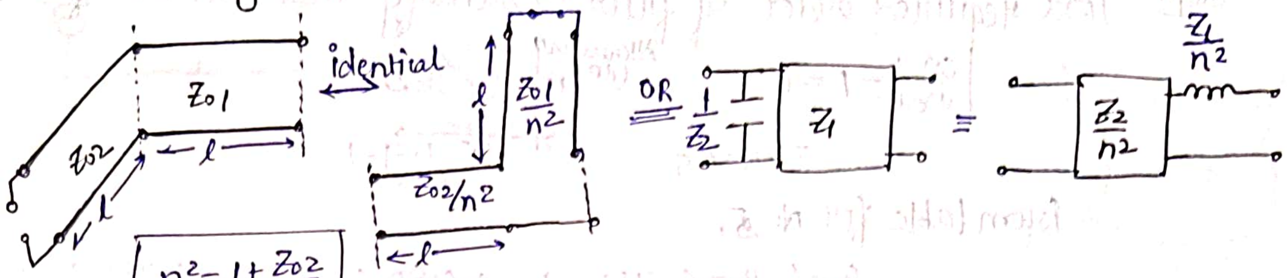
[Assume unity filter impedance]

Cutoff at $\Omega = 1 = \tan\beta l$.



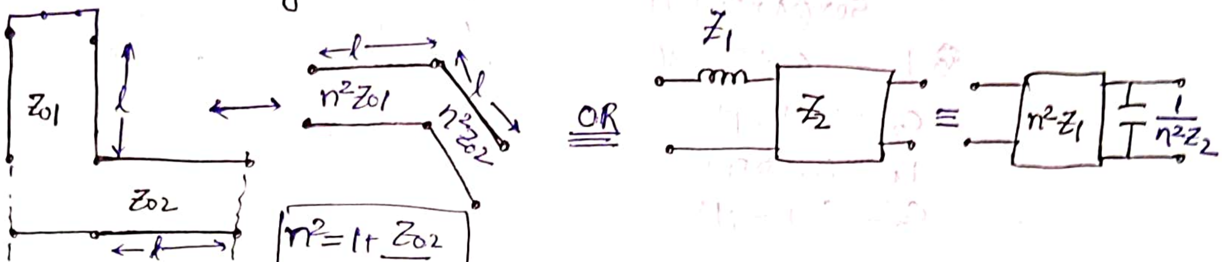
Kuroda's Identity → To change impractical char. imp. into more realizable ones.

① First Identity:



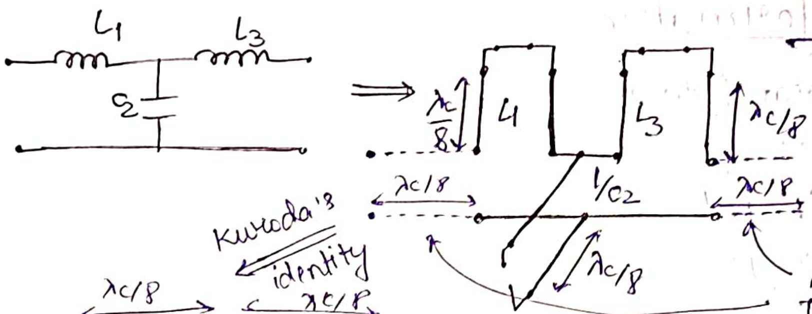
$$\boxed{n^2 = 1 + \frac{Z_{02}}{Z_{01}}}$$

② Second Identity:

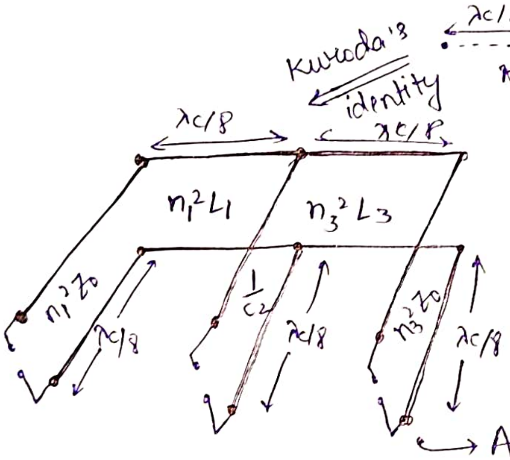


$$\boxed{n^2 = 1 + \frac{Z_{02}}{Z_{01}}}$$

→



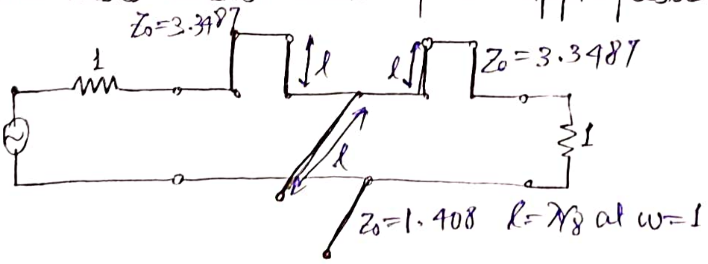
→ Problems:
① Infinitely close stubs.
② Series stubs are difficult to construct.
⇓
Add a short length of T/L (Z_0, $l = \lambda_c/8$) at beginning & end of filter
Cause phase shift but ref. & trans. f^{n}s remain same.



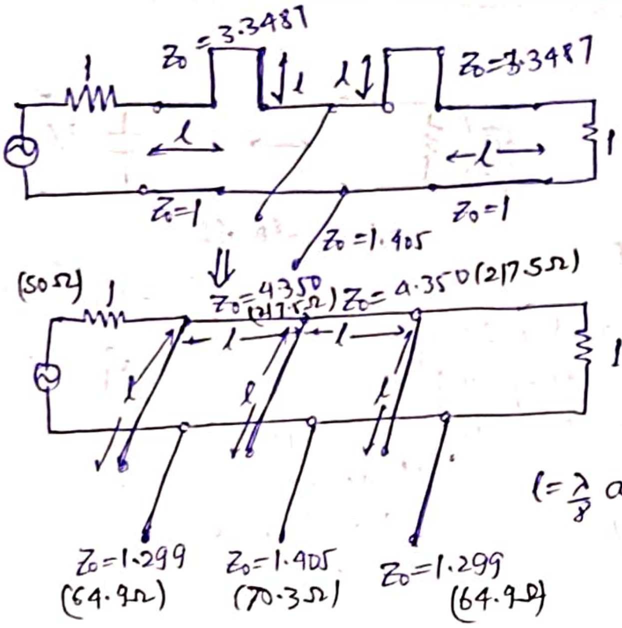
$$\boxed{n^2 = 1 + \frac{Z_{02}}{Z_{01}}}$$

→ All shunt stubs.

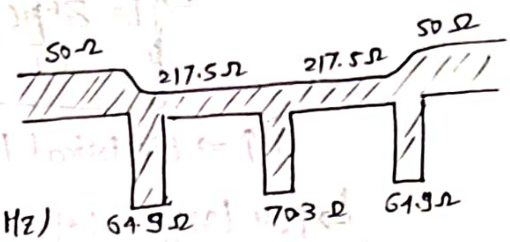
Eg. Design a low-pass filter for fabrication using microstrip lines. The specifications include a cutoff freq. of 4 GHz, an impedance of 50 Ω, and a third-order 3dB equal ripple passband response.



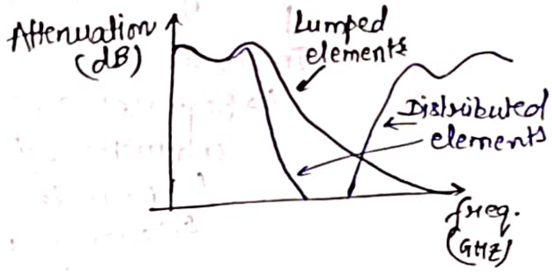
$g_1 = 3.3487 = L_1$
$g_2 = 0.7117 = C_2$
$g_3 = 3.3487 = L_3$
$g_4 = 1.0000 = R_L$.



$$n^2 = 1 + \frac{Z_2}{Z_1} = 1 + \frac{1}{3.3487} = 1.299$$

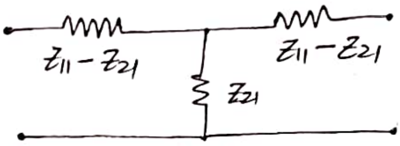


$l = \frac{\lambda}{8}$ at $\omega = 1$ (4 GHz)

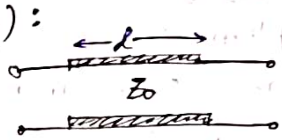


Stepped Impedance Low Pass Filter

→ Any symmetric two-port n/w of impedance matrix, $Z = \begin{bmatrix} Z_{11} & Z_{21} \\ Z_{21} & Z_{11} \end{bmatrix}$ can be modeled as T-circuit: [OR vice versa]



→ For length l T/L (a symmetric 2-port n/w):



$Z_{11} = Z_{22} = -jZ_0 \cot\beta l$

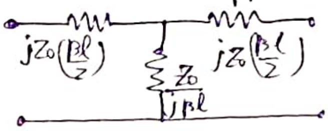
$Z_{12} = Z_{21} = -jZ_0 \operatorname{cosec}\beta l$

$\Rightarrow Z_{11} - Z_{12} = jZ_0 \tan\left(\frac{\beta l}{2}\right) \approx jZ_0\left(\frac{\beta l}{2}\right)$ (for small βl)

↳ $\beta l \ll 1$ → electrically short

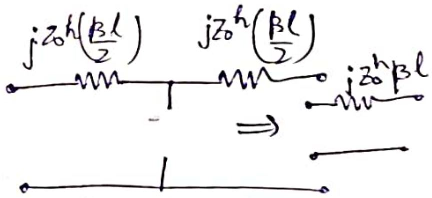
$Z_{12} = Z_{21} = -jZ_0 \operatorname{cosec}\beta l \approx \frac{Z_0}{j\beta l}$

⇓ Modeled as T. ckt (approx.)



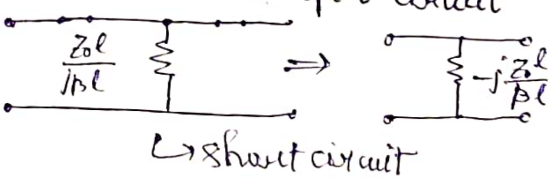
For large char. impedance of T/L → Z_0^h

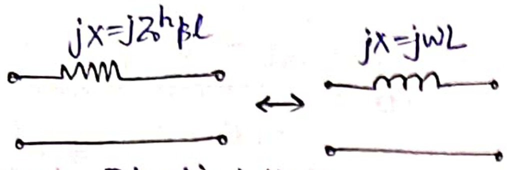
$\frac{Z_0^h}{j\beta l} \approx \infty$ for $\beta l \ll 1$ & $Z_0^h \gg Z_0$ ⇒



For low char. impedance → Z_0^l

$Z_0^l \ll Z_0$ & $\beta l \ll 1$ ⇒ $Z_{11} - Z_{12} \approx jZ_0^l\left(\frac{\beta l}{2}\right) \approx 0$





Identical iff

$jZ_0^h\beta l = j\omega L \Rightarrow \omega L = Z_0^h \beta l$

$\Rightarrow \boxed{L = \frac{Z_0^h \beta l}{\omega} = \frac{Z_0^h l}{v_p}}$ → Not a f^n of freq.(ω).
→ Not exact (but approx.)
↳ True for low freq.
↳ Valid upto ω_c (cutoff freq.)
↳ For small electrical length $< \pi/4$ radians

$f \uparrow \Rightarrow$ Electrical length $\uparrow$
↳ For large cutoff-frequency
↓
Increase Z_0^h
↳ Requires a very narrow conductor width(w)
↳ Bounded by manufacturing tolerances power handling capability and/or line loss.

Identical iff

$-j\frac{Z_0^l}{\beta l} = \frac{-j}{\omega C} \Rightarrow \frac{\beta l}{Z_0^l} = \omega C.$

$\Rightarrow \boxed{C = \frac{\beta l}{\omega Z_0^l} = \frac{l}{v_p Z_0^l}}$

$\therefore \boxed{\beta_c l = \frac{\omega L}{Z_0^h} < \frac{\pi}{4}}$

↳ For very large ω_c 1/or c
↳ Requires very small Z_0^l
↳ Requires making conductor width w^l very large.

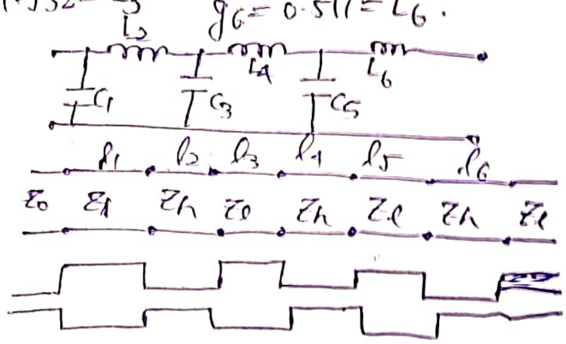
Eg. Design a stepped-impedance low-pass filter having a maximally flat response and a cutoff freq. of 2.5 GHz. It is desired to have more than 20dB insertion loss at 4GHz. The filter impedance is 50. The highest practical line impedance is 120 and the lowest is 20. Consider the effect of losses when this filter is implemented with a microstrip substrate having $d = 0.158$ cm, $\epsilon_r = 4.2$, $\tan\delta = 0.02$, and copper conductors of 0.5 mil thickness.

Sol^n: $\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{4.0}{2.5} - 1 = 0.6 \Rightarrow n = 6 \Rightarrow g_1 = 0.517 = C_1$ $\quad g_4 = 1.932 = L_4$

$g_2 = 1.414 = L_2$ $\quad g_5 = 1.414 = C_5$

$g_3 = 1.932 = C_3$ $\quad g_6 = 0.517 = L_6$.

Section	$Z_i = Z_l$ or Z_h (Ω)	βl_i (deg)	w_i (mm)	l_i (mm)
1	20	11.8	11.3	2.05
2	120	33.8	0.428	6.63
3	20	44.3	11.3	7.69
4	120	46.1	0.428	9.04
5	20	32.4	11.3	5.63
6	120	12.3	0.428	2.41



$\beta l = \frac{L R_0}{Z_h}$ (inductor)

$\beta l = \frac{C Z_l}{R_0}$ (capacitor)

[length evaluated at cutoff freq.]