Submitted by:
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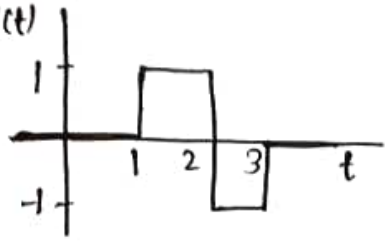
SC22B146

AV223 – Signals and System : Homework 1

① Determine an expression for the following signals. Simplify your answer.

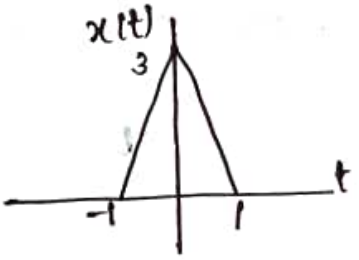
(a)



Soln: $x(t) = \begin{cases} 1, & 1<t<2 \\ -1, & 2<t<3 \\ 0, & t<1, t>3 \end{cases}$

$\Rightarrow x(t) = u(t-1) - 2u(t-2) + u(t-3)$

(b)

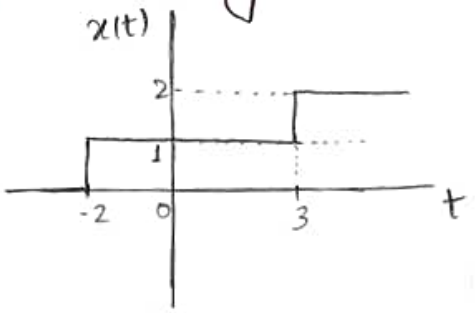


Soln: $x(t) = \begin{cases} 3(1-|t|), & |t|<1 \\ 0, & |t|\geq 1 \end{cases}$

$= 3\,tri(t)$.

② Sketch the following continuous-time signals.

(a) $x(t) = u(t+2) + u(t-3)$

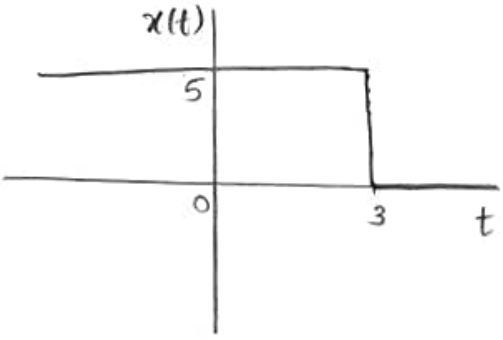


(b) $x(t) = 5u(-2t+6)$

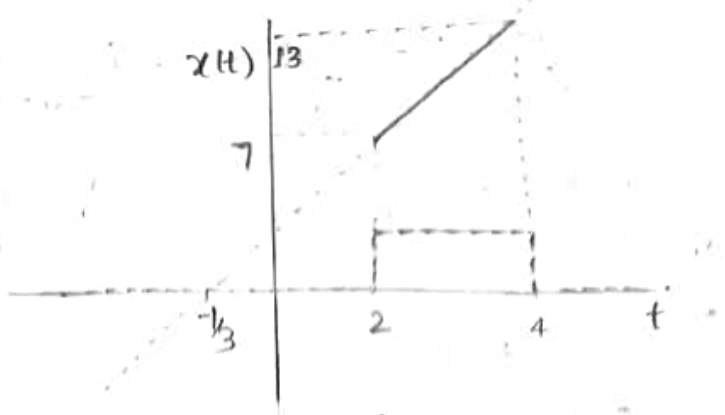
$-2t+6 \geq 0$

$\Rightarrow 2t \leq 6$

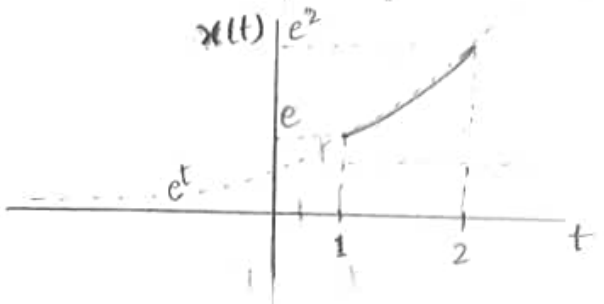
$\Rightarrow t \leq 3$



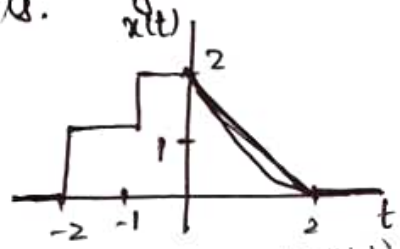
© $x(t)=\underbrace{(3t+1)}_{t\geq -\frac{1}{3}}(u(t-2)-u(t-4))$



(d) $x(t)=e^{t}(u(t-1)-u(t-2))$

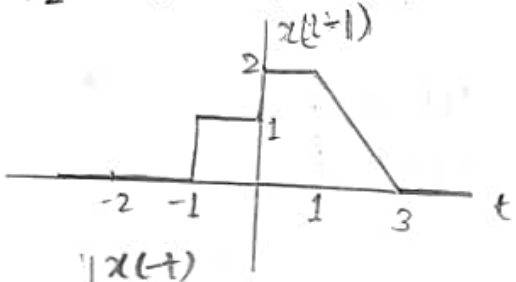


③ A continuous-time signal, $x(t)$ is shown below. Sketch each of the following signals.

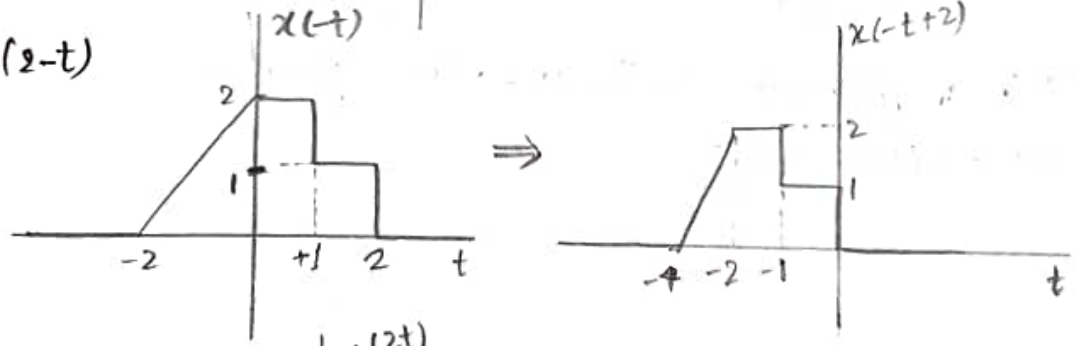


$$x(t)=\begin{cases}1, & -2<t<-1\\ 2, & -1<t<0\\ 2-t, & 0<t<2\\ 0, & x<-2,\ x>2\end{cases}$$

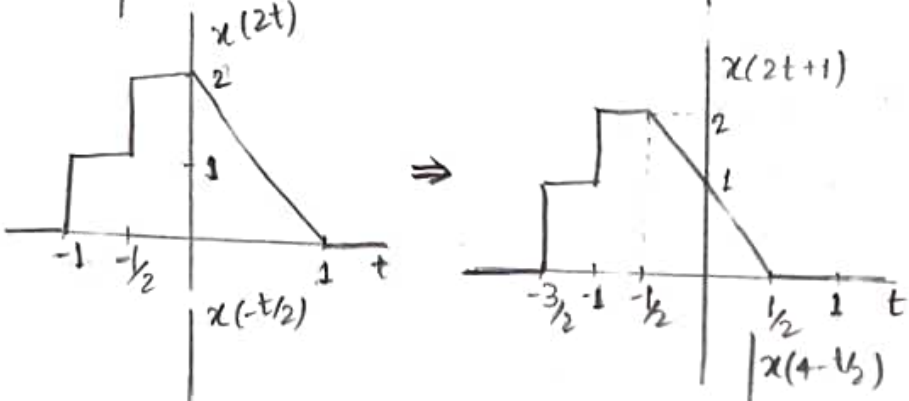
(a) $y(t)=x(t-1)$



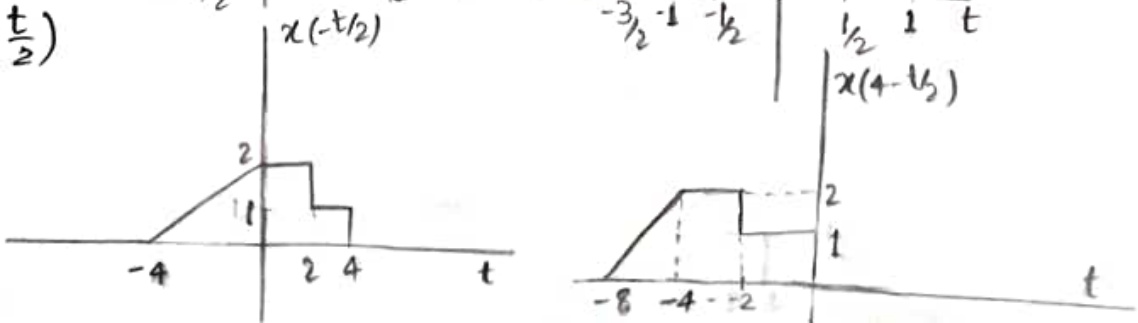
(b) $y(t)=x(2-t)$



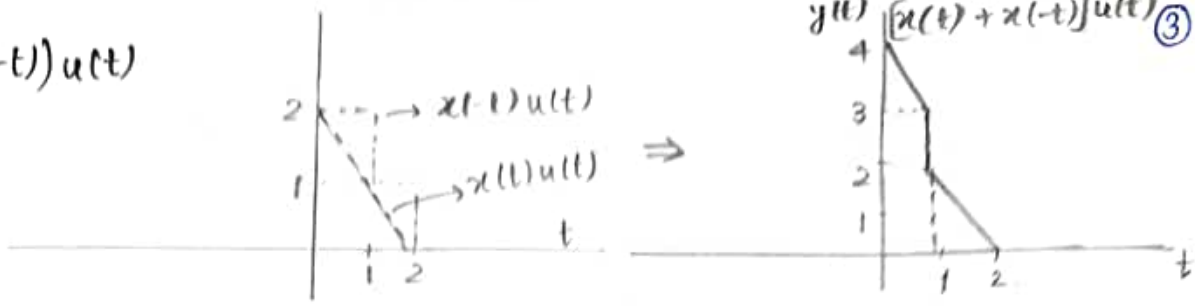
© $y(t)=x(2t+1)$



(d) $y(t)=x\left(4-\frac{t}{2}\right)$

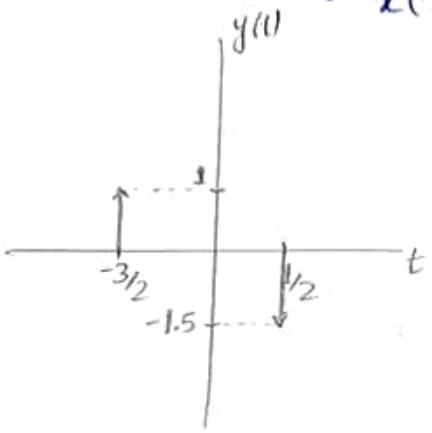


(e) $y(t) = [x(t) + x(-t)]u(t)$



(f) $y(t) = x(t)\left[\delta(t+3/2) - \delta(t-1/2)\right] = x(t)\,\delta(t+3/2) - x(t)\,\delta(t-1/2)$

$= x(-3/2) - x(+1/2)$



(4) Determine whether or not the following continuous-time signals are periodic. If the signal is periodic, determine what the fundamental frequency is.

(a) $x(t) = 5\sin\left(4t - \frac{\pi}{6}\right)$

Soln: Sinusoidal functions are ~~continue~~ periodic.

$\therefore x(t)$ is periodic with period

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}.$$

(b) $x(t) = e^{\cos(t)}$

Soln: $x(t) = e^{\cos t}$

As $\cos(t)$ repeats itself with period 2π,

$x(t) = e^{\cos t}$ should also repeat with period 2π.

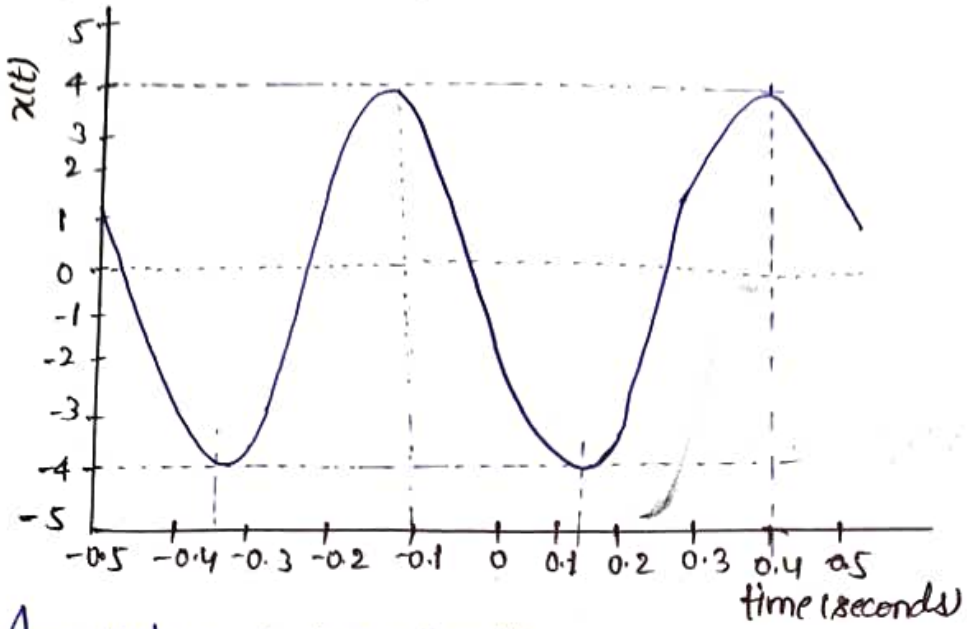
$$x(t+2\pi) = e^{\cos(t+2\pi)} = e^{\cos(t)} = x(t).$$

$\therefore x(t)$ is periodic with period 2π.

(c) $x(t) = t\,e^{\cos(t)}$

Soln: As t is non-periodic and $e^{\cos(t)}$ is periodic, the product $te^{\cos(t)}$ will also be non-periodic.

⑤ For the following waveform, determine the amplitude, period, frequency, time shift, and phase delay. Write an expression for the waveform.



Soln: Amplitude = 4 - (0) = 4 units.

Period = 0.1 - (-0.4) = 0.5 s.

Frequency = $\frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi$ rad/s = 2 Hz.

Time shift = 0.1 s.

Phase delay: $\phi = 0.4\pi$ rad.

Expression: $\omega = 2\pi f = 4\pi$ rad/s.

$x(t) = 4\cos(4\pi t + \phi)$

At $t = -0.1$ s,

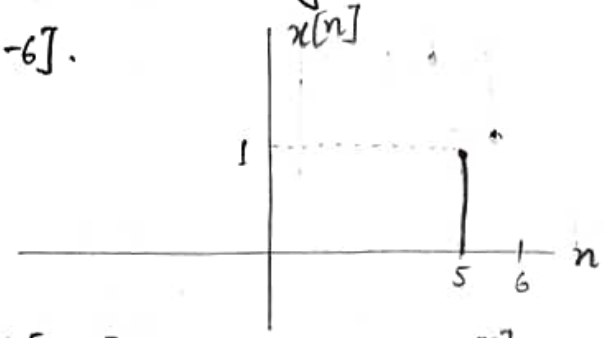
$4 = 4\cos(4\pi(-0.1) + \phi)$

$\Rightarrow \cos(-0.4\pi + \phi) = 1 = \cos(0)$

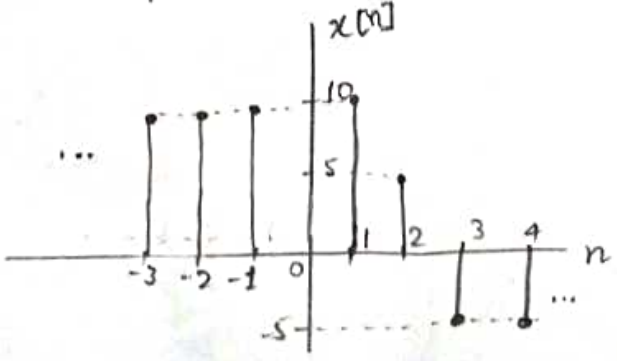
$\Rightarrow \phi = 0.4\pi$ rad.

⑥ Sketch the following discrete-time signals.

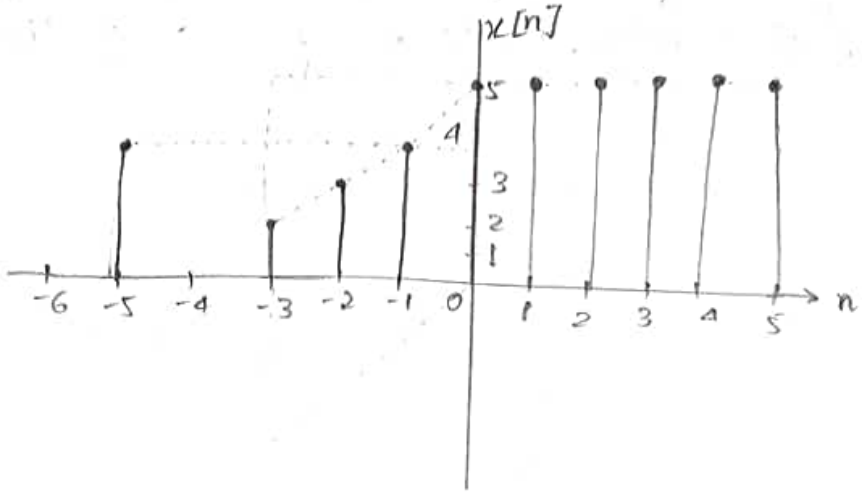
ⓐ $x[n] = u[n-5] - u[n-6]$.



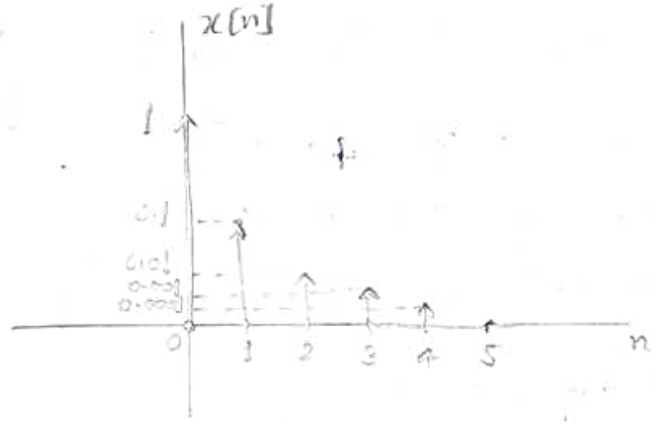
ⓑ $x[n] = 10\,u[\underbrace{-n+2}_{n\le 2}] - 5[\underbrace{n-2}_{n\ge 2}]$



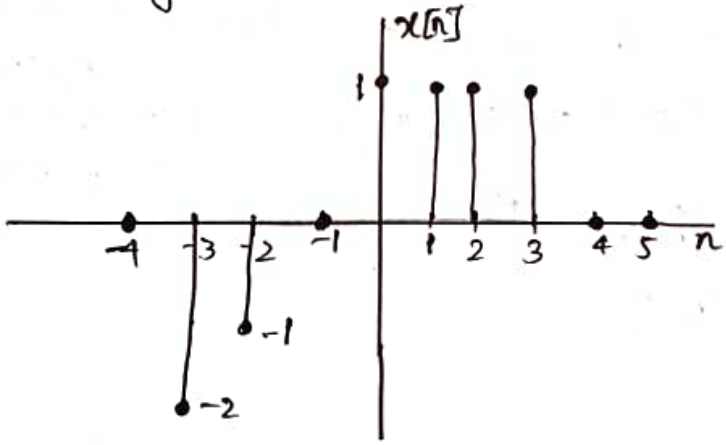
© $x[n] = 4\delta[n+5] + (n+5)u[n+3] - nu[n]$



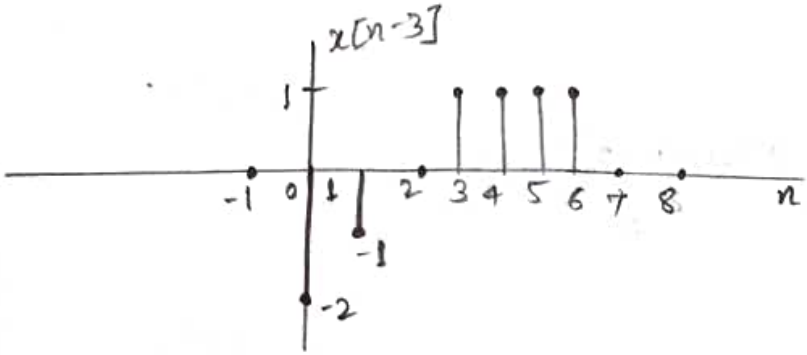
ⓓ $x[n] = (0.1)^n (u[n] - u[n-5])$



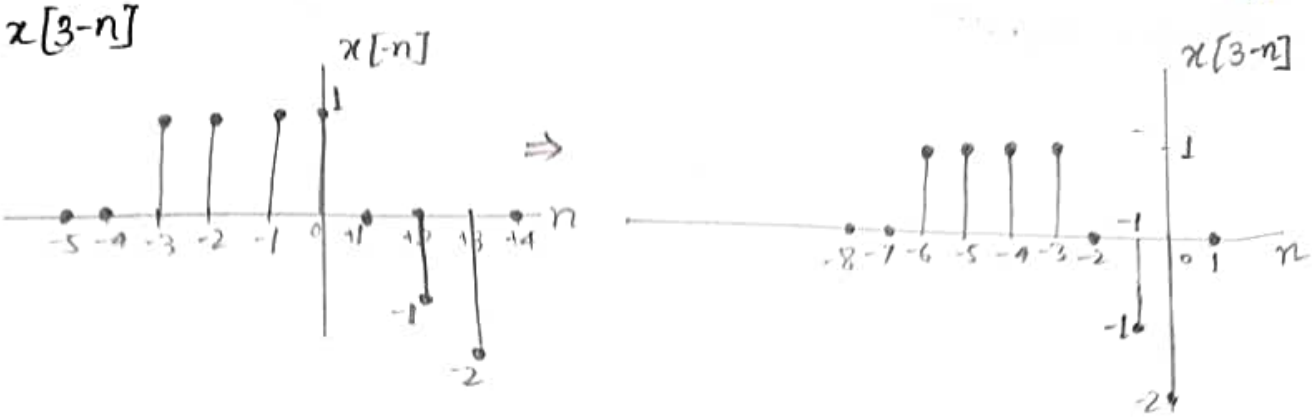
⑦ A discrete-time signal, $x[n]$, is shown below. Sketch each of the following signals.



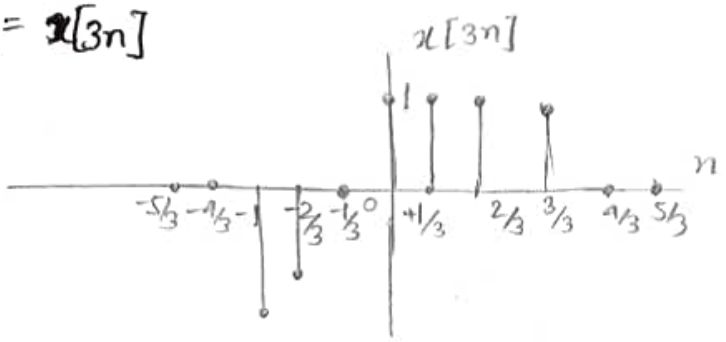
ⓐ $y[n] = x[n-3]$



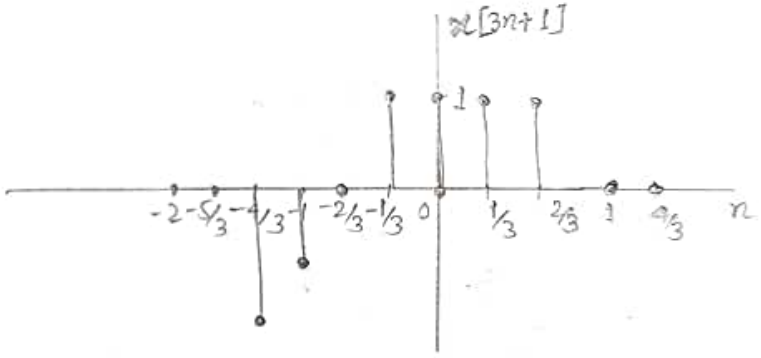
(b) $y[n] = x[3-n]$



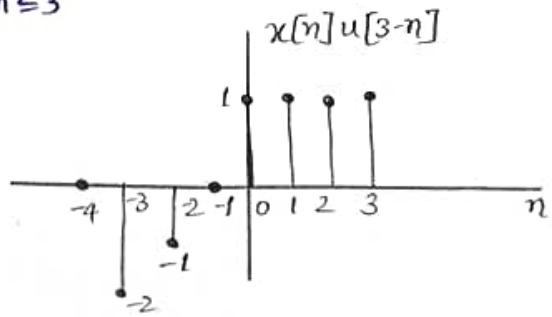
(c) $y[n] = x[3n]$



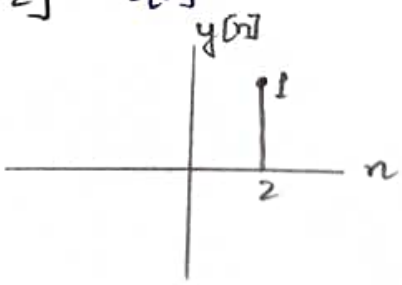
(d) $y[n] = x[3n+1]$



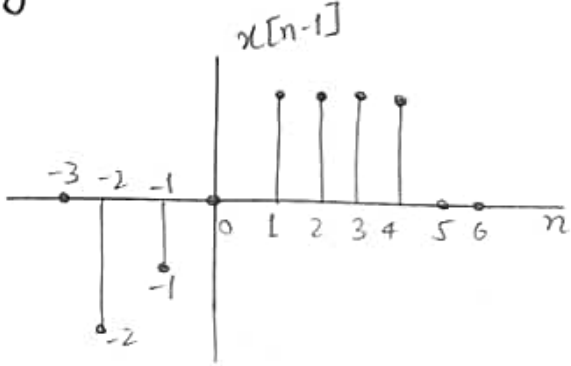
(e) $y[n] = x[n]\,u[3-n]$, $n \le 3$



(f) $y[n] = x[n-2]\,\delta[n-2] = x[2]$



Q $y[n] = x[(n-1)^2]$



⇒