# Homework - 2

## AV221 - Semiconductor Devices

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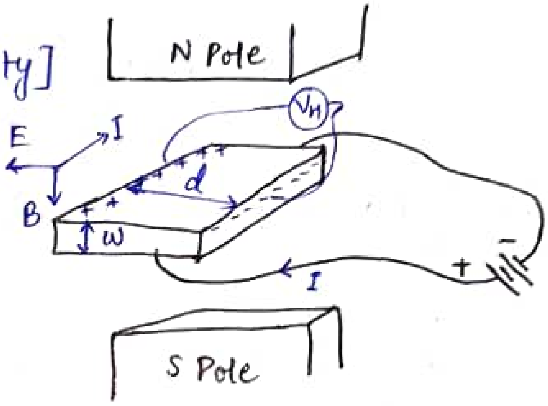
SC22B146

① What is Hall effect? How is it useful in determining the carrier concentration in a doped semiconductor?

Soln: When an electric current-carrying conductor is placed in a magnetic field, an electric field is induced in the direction perpendicular to current and magnetic field (direction is as per Fleming's left hand rule), which results into hall voltage. This effect is known as Hall effect.

Magnetic force,

$$\bar{F}_B = q(\vec{V}_d \times \vec{B}) \quad [V_d : \text{drift velocity}]$$




$$\Rightarrow F_B = q V_d B$$

Electric force,

$$F_B = qE$$

$$\Rightarrow F_B = q\left(\frac{V_H}{d}\right)$$

$$\therefore q V_d B = q \frac{V_H}{d}$$

$$\Rightarrow V_H = V_d B d$$

As current, $I_H = (ne V_d) A = (ne V_d) wd$

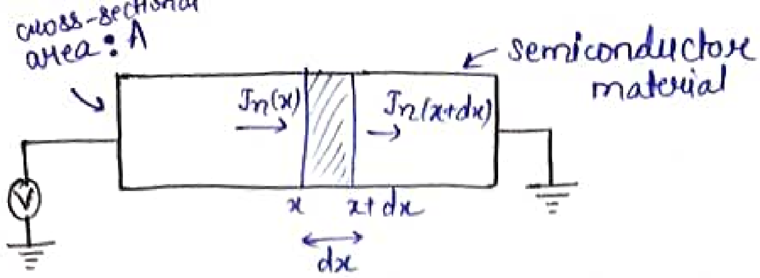
$$\therefore V_H = \left(\frac{I_H}{ne w\cancel{d}}\right) B \cancel{d}$$

$$\Rightarrow \boxed{V_H = \frac{I_H B}{ne\, d}} = \frac{B I R_H}{w}, \text{ where } R_H = \frac{1}{ne} : \text{Hall-effect coefficient.}$$

Using this relationship, carrier concentration, e or p, can be calculated.

② Derive the continuity equations for electrons and holes. Describe the significance of each term in the resulting equations.

Soln:




In the elemental material, electron concentration is increasing due to $J_n(x)$ entering and generation, and is decreasing due to $J_n(x+dx)$ current leaving and recombination.

$$\therefore \frac{dn}{dt} A dx = \left[\frac{J_n(x)}{-q}A + G_n A dx\right] - \left[\frac{J_n(x+dx)A}{-q} + R_n A dx\right]$$

$$= +\frac{A}{q} dx \left[\frac{J_n(x+dx) - J_n(x)}{dx}\right] + A dx\left[G_n - R_n\right]$$

As $\frac{J_n(x+dx) - J_n(x)}{dx}$ is the definition of $\frac{d J_n(x)}{dx}$,

$$\Rightarrow \frac{dn}{dt} A dx = \frac{A}{q} dx \left[\frac{d}{dx} J_n(x)\right] + \left[G_n - R_n\right] A dx$$

$$\Rightarrow \frac{dn}{dt} = \frac{1}{q}\left[\frac{d}{dx}(J_n(x))\right] + \left[G_n - R_n\right]$$

As $J_n(x) = qn\mu_n E + D_n q \frac{dn}{dx}$,

$$\Rightarrow \frac{dn}{dt} = \frac{1}{q}\frac{d}{dx}\left[qn\mu_n E + q D_n \frac{dn}{dx}\right] + \left[G_n - R_n\right]$$

$$= \left[\mu_n \frac{d}{dx}(nE) + D_n \frac{d^2 n}{dx^2}\right] + \left[G_n - R_n\right]$$

$$= \mu_n n \frac{dE}{dx} + \mu_n E \frac{dn}{dx} + D_n \frac{d^2 n}{dx^2} + G_n - R_n$$

$$\Rightarrow \frac{dn_p}{dt} = \mu_n n_p \frac{dE}{dx} + \mu_n E \frac{dn_p}{dx} + D_{n_p} \frac{d^2 n_p}{dx^2} + G_n - \left[\frac{n_p - n_{p_0}}{\tau_n}\right]$$

[For p-type material]

Similarly,
For holes in n-type material,

$$\frac{dP_n}{dt} = -\mu_p P_n \frac{dE}{dx} - \mu_p E \frac{dP_n}{dx} + D_{P_n} \frac{d^2 P_n}{dx^2} + G_p - \left[\frac{P_n - P_{n_0}}{\tau_p}\right]$$

③ Derive Einstein's relationship.

Soln: For non-uniformly doped semiconductor under equilibrium, $J_n = 0$ for both electrons and holes individually.

$$\therefore \; ne\mu_n E + D_n e \frac{dn}{dx} = 0 \qquad -(i)$$

and, $n = n_i \exp\left[\frac{E_f - E_{fi}}{KT}\right]$

$$\Rightarrow \frac{dn}{dx} = n \cdot \frac{1}{KT}\left(\frac{dE_f}{dx} - \frac{dE_{fi}}{dx}\right)$$

As $E_f$ is constant at thermal equilibrium.

$$\Rightarrow \frac{dn}{dx} = \frac{n}{KT}\left(-\frac{dE_{fi}}{dx}\right)$$

Also, $\frac{dE_{fi}}{dx} = eE_{fi}$.

$$\Rightarrow \frac{dn}{dx} = -\frac{ne}{KT} E_{fi} \qquad -(ii)$$

From (i) and (ii),

$$\frac{\cancel{n}\mu_n E}{D_n} - \frac{\cancel{n}e E_{fi}}{KT} = 0$$

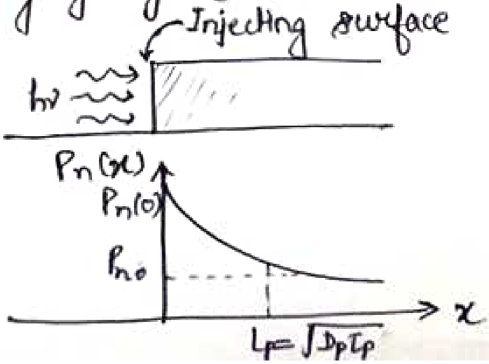
$$\Rightarrow \mu_n - \frac{D_n e}{KT} = 0$$

$$\Rightarrow \boxed{\frac{D_n}{\mu_n} = \frac{KT}{e}}.$$

Similarly, for holes,

$$\boxed{\frac{D_p}{\mu_p} = \frac{KT}{e}}.$$

④ Setup and solve the continuity equations for an n-type semiconductor where excess carriers are injected from one side as a result of light illumination. Assume that the light penetration is negligibly small.



Soln: Continuity eqn:

$$\frac{dP_n}{dt} = -\mu_p\left[P_n\frac{dE}{dx} + E\frac{dP_n}{dx}\right] + D_p\frac{d^2P_n}{dx^2} + G_p - \left(\frac{P_n - P_{no}}{\tau_p}\right)$$

where, $\frac{dP_n}{dt} = 0$, at steady state

$E = 0$, as no electric field is applied.

$G_n = 0$ for $x > 0$ (generation is only happing at $x = 0$).

$$\therefore D_p\frac{d^2P_n}{dx^2} = \frac{P_n - P_{no}}{\tau_p}$$

Take $P_n - P_{no} = y$

$\Rightarrow dP_n = dy$

$$\therefore \frac{d^2y}{dx^2} = \frac{y}{D_p\tau_p}$$

$$\Rightarrow y = k_1 e^{-\frac{x}{\sqrt{D_p\tau_p}}} + k_2$$

Apply boundary conditions:

At $x = 0$, $P_n = P_n(0)$

At $x \to \infty$, $P_n = P_{no}$.

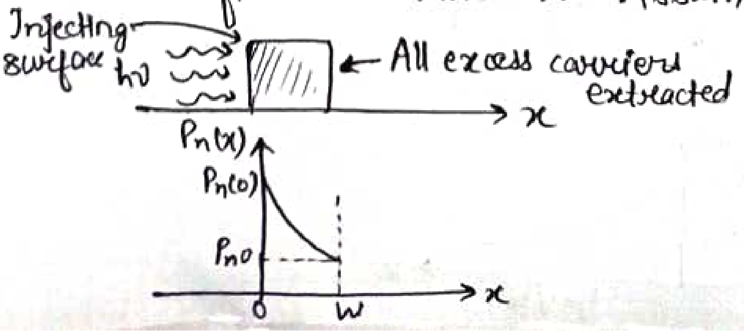
$$\Rightarrow k_2 = P_n - P_{no} = P_{no} - P_{no} = 0$$

$$k_1 = P_n - P_{no} = P_n(0) - P_{no}$$

$$\therefore P_n - P_{no} = [P_n(0) - P_{no}]\, e^{-\frac{x}{\sqrt{D_p\tau_p}}}$$

$$\Rightarrow \boxed{P_n = P_{no} + [P_n(0) - P_{no}]\, e^{-\frac{x}{L_p}}} \quad [\text{As } L_p = \sqrt{D_p\tau_p}]$$

⑤ Set up and solve the continuity equations for an n-type semiconductor where excess carriers are injected from one side as a result of light illumination, and the injected excess carriers are completely extracted after a distance W. Assume that the light penetration is negligibly small.



Soln: Using continuity equations and $\frac{dP_n}{dt}=0$, $E=0$, $G_n=0$.

$$D_p \frac{d^2 P_n}{dx^2} = \frac{P_n - P_{n0}}{\tau_p}$$

$$\Rightarrow P_n - P_{n0} = K_1 e^{-\frac{x}{L_P}} + K_2$$

Apply boundary conditions,

At $x=0$, $P_n = P_n(0)$

At $x=W$, $P_n = P_{n0}$.

$x=0 \Rightarrow \quad K_1 + K_2 = P_n(0) - P_{n0}$

$x=W \Rightarrow \quad P_{n0} - P_{n0} = 0 = K_1 e^{-\frac{W}{L_P}} + K_2$

$$\Rightarrow K_2 = -K_1 e^{-W/L_P}$$

$$\Rightarrow K_1 - K_1 e^{-W/L_P} = P_n(0) - P_{n0}$$

$$\Rightarrow K_1 = \frac{P_n(0) - P_{n0}}{1 - e^{-W/L_P}}$$

So, $K_2 = \left(\frac{P_n(0) - P_{n0}}{e^{-W/L_P} - 1}\right) e^{-W/L_P}$

$$\therefore \boxed{P_n = P_{n0} + \frac{P_n(0) - P_{n0}}{1 - e^{-W/L_P}}\left[e^{-x/L_P} - e^{-W/L_P}\right]}.$$

6. For a n-type Si sample with a doping concentration $N_D = 10^{16}$ per cm³; the sample is illuminated with a laser beam so that EHPs are generated at a rate of $10^{18}$ per cm³ per second. calculate the majority and minority carrier concⁿ at steady state. Assume that the majority carrier life time is $\tau_p = 10^{-6}$s. Using these values, plot the excess carrier concⁿ ($e^-$ and hole) once laser is turned off. Also calculate the Fermi-level positions. Take $n_i = 1.5\times10^{10}\ cm^{-3}$.

Soln: Given that

$N_D = n_0 = 10^{16}/cm^3$, $G_L = 10^{18}/cm^3/s$, $\tau_p = 10^{-6}s$, $n_i = 1.5\times10^{10}/cm^3$.

At steady-state,

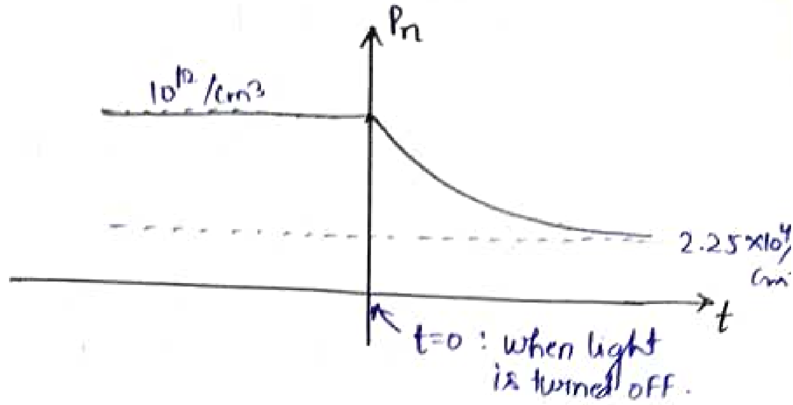
$n_0 = 10^{16}/cm^3$.

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.25\times10^{20}}{10^{16}} = 2.25\times10^4/cm^3.$$

When light is ON,

$$p_n = p_{n_0} + G_L \tau_p$$
$$= 2.25\times10^4 + 10^{18}\times10^{-16}$$
$$\approx 10^{12}/cm^3.$$

$$n_n = n_0 + G_L \tau_p$$
$$= 10^{16} + 10^{12}$$
$$\approx 10^{16}/cm^3.$$




Now,

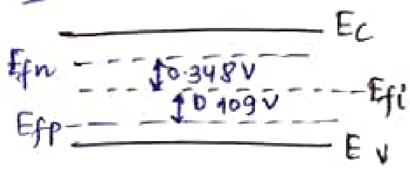
Using $n_n = n_i \exp\left(\frac{E_{fn} - E_{fi}}{kT}\right)$

$$\Rightarrow E_{fn} - E_{fi} = kT \ln\left(\frac{n_n}{n_i}\right) = 26\times10^{-3} \ln\left(\frac{10^{16}}{1.5\times10^{10}}\right)$$
$$= \boxed{0.348\ V}$$

and, $E_{fi} - E_{fp} = kT \ln\left(\frac{p_n}{n_i}\right) = 26\times10^{-3} \ln\left(\frac{10^{12}}{1.5\times10^{10}}\right)$

$$= \boxed{0.109\ V}$$

$\Rightarrow$




⑦ A sample of intrinsic Si of volume 1cm³ is illuminated by a light pulse of power 1mW for 10 μs from a Helium-Neon laser emitting at 632.8nm. Assume that the photons are absorbed uniformly throughout the Si and that each photon generates 1 EHP. By how much does the conductivity of the Si sample change after laser irradiation. Assume no carrier recombination.

Soln: Given that $V = 1\,cm^3$, $P = 1\,mW$, $t = 10\,\mu s$, $\lambda = 632.8\,nm$.

So, total energy $= Pt = 10^{-3}\times10^{-5} = 10^{-8}\,J$.

Energy of one photon $= \frac{hc}{\lambda} = \frac{6.625\times10^{-34}\times3\times10^{8}}{632.8\times10^{-9}} = 3.14\times10^{-19}\,J$

$\Rightarrow$ No. of photons $= \frac{10^{-8}}{3.14\times10^{-19}} = 3.18\times10^{10}$ (N)

Original conductivity,

$$\sigma_0 = e(n_0\mu_n + p_0\mu_p) = en_i(\mu_n + \mu_p)$$

New $\sigma_N =$ ~~$e(N\mu_n + N\mu_p)$~~ $e(n_i + N)(\mu_n + \mu_p)$

% change in conductivity $= \frac{\sigma_N - \sigma_0}{\sigma_0}\times100$

$$= \frac{e(n_i + N) - n_i)(\mu_n + \mu_p)}{n_i e(\mu_n + \mu_p)}\times100 = \frac{N}{n_i}\times100$$

$$= \frac{3.18\times10^{10}}{1.5\times10^{10}}\times100 = \boxed{212\ \%}.$$