AVD862 - Digital Image Processing
Assignment - 1

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(1) $L=8$, $MN=4096$

r_K	n_K	c_K	$S_K=(L-1)\frac{c_K}{MN}$	
0	790	790	1.35	$\rightarrow$ 1
1	~~850~~ 1023	1813	3.09	$\rightarrow$ 3
2	850	2663	4.55	$\rightarrow$ 5
3	656	3319	5.67	$\rightarrow$ 6
4	329	3648	6.23	$\rightarrow$ 6
5	245	3893	6.65	$\rightarrow$ 7
6	122	4015	6.86	$\rightarrow$ 7
7	81	4096	7.00	$\rightarrow$ 7

Resulting histogram:

0 : 0
1 : 790 (from r_0)
2 : 0
3 : 1023 (from r_1)
4 : 0
5 : 850 (from r_2)
6 : 985 (from r_3, r_4 : 656 + 329)
7 : 448 (from r_5, r_6, r_7 : 245+122+81)

Total = 4096

Mapping:
$0 \rightarrow 1$
$1 \rightarrow 3$
$2 \rightarrow 5$
$3 \rightarrow 6$
$4 \rightarrow 6$
$5 \rightarrow 7$
$6 \rightarrow 7$
$7 \rightarrow 7$

(2)

z_K	n_K	Target count ($\times$4096)
0	0	$\rightarrow$ 0
1	0	$\rightarrow$ 0
2	0	$\rightarrow$ 0
3	0.15	$\rightarrow$ 614.4
4	0.20	$\rightarrow$ 819.2
5	0.30	$\rightarrow$ 1228.8
6	0.20	$\rightarrow$ 819.2
7	0.15	$\rightarrow$ 614.4

$P_z(K)$	c_K	$P_c(K)=\frac{c_K}{MN}$
0	790	0.193
0	1813	0.443
0	2663	0.650
0.15	3319	0.810
0.35	3648	0.891
0.65	3893	0.951
0.85	4015	0.981
1.00	4096	1.000

For each r_k, find smaller z such that $P_z(k) \geq P_c(k)$:
(Mapping)

$r_0 \rightarrow z_4$
$r_1 \rightarrow r_5$
$r_2 \rightarrow z_6$
$r_3 \rightarrow z_6$
$r_4 \rightarrow z_7$
$r_5 \rightarrow z_7$
$r_6 \rightarrow z_7$
$r_7 \rightarrow z_7$

③ An ideal low pass filter is a filter that passes all frequencies components, $|w|$, less than a cutoff, $|w_c$, with gain 1, and completely rejects (gain 0) frequencies above w_c.

In 1D, $H(w) = \begin{cases} 1, & |w| \leq w_c \\ 0, & |w| > w_c. \end{cases}$

No, in general, ideal LPF is theoretical with perfect cutoff. Its impulse response in the spatial domain is an infinite sinc function (non-causal).

Impulse response:

$$h(n) = \frac{w_c}{\pi}, \quad n = 0$$
$$= \frac{\sin(w_c n)}{\pi n}, \quad n \neq 0.$$

④ Properties of convolution:

ⓐ Linearity: $a(f * g) + b(f * h) = f * (ag + bh)$
ⓑ Commutative: $f * g = g * f$.
ⓒ Associative: $f * (g * f) = (f * g) * h$
ⓓ Distributive: $f * (g + h) = (f * g) + (f * h)$
ⓔ shift invariance: If $f'_{(x)} = f(x - x_0)$, then $f' * g = (f * g)(x - x_0)$
ⓕ Convolution theorem:
Convolution in spatial domain is multiplication in freq. domain and vice-versa.

$$\begin{bmatrix} 0 & 1/6 & 0 \\ 1/6 & 1/3 & 1/6 \\ 0 & 1/6 & 0 \end{bmatrix} * \begin{bmatrix} 5 & 15 & 20 & 45 & 50 \\ 10 & 35 & 40 & 60 & 70 \\ 20 & 75 & 100 & 85 & 75 \\ 30 & 40 & 85 & 135 & 80 \\ 40 & 10 & 45 & 50 & 65 \end{bmatrix}$$

Applying the mask on shaded/dotted pixels:

$$\begin{bmatrix} 5 & 15 & 20 & 45 & 50 \\ 10 & 35.83 & 50.83 & 60.83 & 70 \\ 20 & 57.5 & 81.67 & 90 & 75 \\ 30 & 46.67 & 81.67 & 95 & 80 \\ 40 & 10 & 45 & 50 & 65 \end{bmatrix} \approx \begin{bmatrix} 5 & 15 & 20 & 45 & 50 \\ 10 & 36 & 51 & 61 & 70 \\ 20 & 58 & 82 & 90 & 75 \\ 30 & 47 & 82 & 95 & 80 \\ 40 & 10 & 45 & 50 & 65 \end{bmatrix}$$

5.

$$\begin{bmatrix} 5 & 15 & 20 & 45 & 50 \\ 10 & 35 & 45 & 60 & 70 \\ 20 & 75 & 100 & 85 & 75 \\ 30 & 40 & 85 & 135 & 80 \\ 40 & 10 & 45 & 50 & 65 \end{bmatrix}$$

$$\Big\downarrow \text{ sort and take median}$$

$$\begin{bmatrix} 5 & 15 & 20 & 45 & 50 \\ 10 & 20 & 45 & 60 & 70 \\ 20 & 40 & 75 & 80 & 75 \\ 30 & 40 & 75 & 80 & 80 \\ 40 & 10 & 45 & 50 & 65 \end{bmatrix}$$

(5) Laplacian is a second-order derivative operator that measures the sum of second-partial derivatives.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

In image processing, it highlights regions of rapid intensity change, useful for edge detection.

Types of Laplacian operator:

(i) 4-Neighbour Laplacian:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(ii) 8-Neighbour Laplacian:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(iii) Laplacian of Gaussian (LoG): Gaussian smoothening followed by Laplacian to detect edges.

Eg. Mexican Hat kernel:

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 \\ -1 & -2 & 16 & -2 & -1 \\ 0 & -1 & -2 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$