Homework-3
AV221 - Semiconductor Devices

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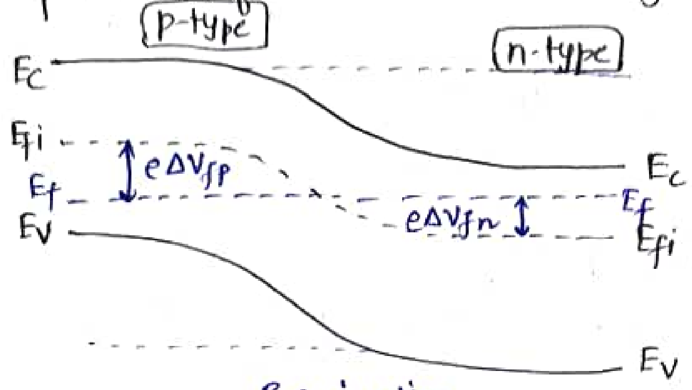
SC22B146

① Derive the expression for the built-in voltage of p-n diode.

Soln:



P-n junction

Built-in voltage, $V_{bi} = |\Delta V_{fp}| + |\Delta V_{fn}|$

Using $n_0 = n_i \exp\left(\frac{E_f - E_{fi}}{kT}\right)$

and, $e\Delta V_{fp} = E_{fi} - E_f$

$e\Delta V_{fn} = E_f - E_{fi}$

$$\Rightarrow n_0 = n_i \exp\left(\frac{e\Delta V_{fn}}{KT}\right)$$

$$\Rightarrow \Delta V_{fn} = \frac{KT}{e} \ln\left(\frac{n_0}{n_i}\right)$$

Similarly,

$$p_0 = n_i \exp\left(\frac{E_{fi} - E_f}{KT}\right)$$

$$= n_i \exp\left(\frac{e\Delta V_{fp}}{KT}\right)$$

$$\Rightarrow \Delta V_{fp} = \frac{KT}{e} \ln\left(\frac{p_0}{n_i}\right)$$

$$\therefore V_{bi} = \frac{KT}{e} \ln\left(\frac{n_0 p_0}{n_i^2}\right)$$

Take $n_0 = N_D$ & $p_0 = N_A$, $e = q$

$$\Rightarrow \boxed{V_{bi} = \frac{KT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

② Derive the expression for the depletion widths along n-sides and p-sides of p-n junction.

Soln: Poisson's eqn for one-dimensional analysis:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx},$$

$\phi(x)$: Electric ~~field~~ Potential
$E(x)$: Electric field
$\rho(x)$: volume charge density.
ϵ_s : permittivity of semiconductor

$\rho(x) = -eN_A, \quad -x_p < x < 0$

$\rho(x) = eN_D, \quad 0 < x < x_n$

Electric field for p-region.

$$E_1 = \int \frac{\rho(x)}{\epsilon_s} dx = -\frac{1}{\epsilon_s}\int eN_A dx = -\frac{eN_A x}{\epsilon_s} + C_1$$

As $E_1 = 0$ at $x = -x_p \Rightarrow C_1 = -\frac{eN_A x_p}{\epsilon_s}$

$$\Rightarrow E_1 = -\frac{eN_A}{\epsilon_s}(x + x_p), \quad -x_p \le x \le 0 \qquad \text{—(i)}$$

Similarly for n-region,

$$E_2 = \int \frac{eN_D}{\epsilon_s} dx = -\frac{eN_D}{\epsilon_s}(x_n - x), \quad 0 \le x \le x_n \qquad \text{—(ii)}$$

By (i) and (ii),

E_{max} at $x=0$

$$\Rightarrow E_{max} = -\frac{eN_A N_D}{\epsilon_s} = -\frac{eN_D x_n}{\epsilon_s}$$

$$\Rightarrow x_p N_A = x_n N_D \qquad \text{—(iii)}$$

Potential in p-region,

$$V_1(x) = -\int E(x)dx = +\int \frac{eN_A}{\epsilon_s}(x + x_p)dx = \frac{eN_A}{\epsilon_s}\left(\frac{x^2}{2} + x_p . x\right) + C_2$$

As $V_1(x) = 0$ at $x = -x_p \Rightarrow C_2 = \frac{eN_A}{2\epsilon_s} x_p^2$

$$\Rightarrow V_1(x) = \frac{eN_A}{\epsilon_s}\left(\frac{x^2}{2} + x_p x\right) + \left(\frac{eN_A}{2\epsilon_s} x_p^2\right), \quad -x_p \le x \le 0.$$

$$= \frac{eN_A}{\epsilon_s}(x + x_p)^2, \quad -x_p \le x \le 0.$$

Potential in n-region,

$$V_2(x) = \int \frac{eN_D}{\epsilon_s}(x_n - x)dx = \frac{eN_D}{\epsilon_s}\left(x_n x - \frac{x^2}{2}\right) + C_3$$

As $V_2(x) = 0$ at $x = 0 \Rightarrow V_1(x) = V_2(x) \Rightarrow C_3 = C_2$

$$\therefore V_2(x) = \frac{eN_D}{\epsilon_s}\left(x_n x - \frac{x_n^2}{2}\right) + \frac{eN_A}{2\epsilon_s} x_p^2, \quad 0 \le x \le x_n.$$

At $x = x_n$, $V_2(x) = V_{bi}$

$$\Rightarrow V_{bi} = \frac{e}{2\epsilon_s}\left(N_D x_n^2 + N_A x_p^2\right) \quad \text{(iv)}$$

Using (iii) & (iv),

$$V_{bi} = \frac{e}{2\epsilon_s}\left(N_D \frac{x_p^2 N_A^2}{N_D^2} + N_A x_p^2\right)$$

$$\Rightarrow x_p^2 = \frac{2\epsilon_s V_{bi}}{e}\left(\frac{1}{\frac{N_A^2}{N_D} + N_A}\right) = \frac{2\epsilon_s V_{bi}}{e}\frac{N_D}{N_A}\left[\frac{1}{N_A + N_D}\right]$$

$$\Rightarrow \boxed{x_p = \sqrt{\frac{2\epsilon_s V_{bi}}{e} \cdot \frac{N_D}{N_A}\left(\frac{1}{N_A + N_D}\right)}}.$$

Similarly,

$$\boxed{x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{e} \cdot \frac{N_A}{N_D}\left(\frac{1}{N_A + N_D}\right)}}.$$

3 ~~Setting~~ Stating all the assumptions, derive the shockley diode equation.

Soln: By Boltzmann's expression,

$$V_{bi} = \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$\Rightarrow \frac{n_i^2}{N_A} = N_D \exp\left(-\frac{eV_{bi}}{KT}\right)$$

Assuming complete ionization, $n_{no} \approx N_D$, $N_A \approx p_{po}$.

$$\Rightarrow \frac{n_i^2}{p_{po}} = n_{no}\exp\left(-\frac{eV_{bi}}{KT}\right)$$

$$\Rightarrow n_{po} = n_{no}\exp\left[-\frac{eV_{bi}}{KT}\right]$$

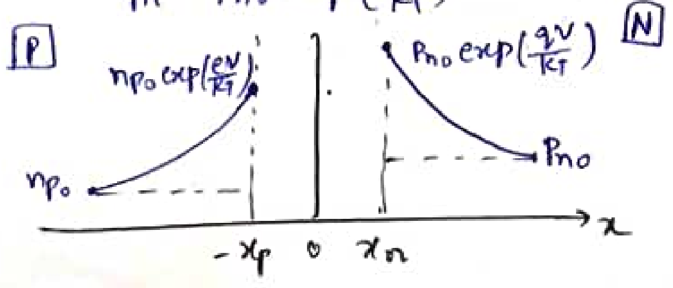
Applying forward voltage V, $V_{bi} \to V_{bi} - V$,

$$n_p = n_{no}\exp\left[-\frac{e(V_{bi}-V)}{KT}\right] = \underbrace{n_{no}\exp\left(-\frac{eV_{bi}}{KT}\right)}_{n_{po}}\exp\left(\frac{eV}{KT}\right)$$

$\Rightarrow n_p = n_{po}\exp\left(\frac{eV}{KT}\right)$, $n_{no} \approx$ constant for low-level injection.

Similarly for holes,

$$p_n = p_{no}\exp\left(\frac{eV}{KT}\right)$$



Using continuity eqn,

$$\cancel{\frac{dp_n}{dt}} = -p_n \mu \cancel{\frac{dE}{dx}} - \cancel{\mu_p E \frac{dp_n}{dx}} + D_p \frac{d^2 p_n}{dx^2} + \cancel{G_p} - \frac{p_n - p_{no}}{\tau_p}$$

At steady state $\frac{dp_n}{dt} = 0$

$E = 0$ for no electric field applied.

$G_n = 0$ for no generation.

$$\Rightarrow \frac{d^2 p_n(x)}{dx^2} = \frac{p_n - p_{no}}{D_p \tau_p}$$

$$\Rightarrow p_n - p_{no} = K_1 \exp\left(\frac{-x}{\sqrt{\tau_p D_p}}\right) + K_2$$

Applying boundary conditions,

At $x = x_n$, $p_n(x_n) = p_{no} \exp\left(\frac{qV}{KT}\right)$

At $x \to \infty$, $p_n(\infty) = p_{no}$.

$$\Rightarrow K_2 = 0$$

$x = x_n \Rightarrow$ $$p_{no} \exp\left(\frac{qV}{KT}\right) - p_{no} = K_1 \exp\left(\frac{-x}{L_p}\right)$$

$$\Rightarrow K_1 = p_{no}\left[\exp\left(\frac{qV}{KT}\right) - 1\right]\left[\exp\left(\frac{x_n}{L_p}\right)\right]$$

$$\therefore p_n(x) = p_{no} + p_{no}\left[\exp\left(\frac{qV}{KT}\right) - 1\right] \exp\left[\frac{x_n - x}{L_p}\right].$$

Now,

$$J_p = -qD_p \frac{dp_n(x)}{dx}$$

$$= -qD_p\, p_{no}\left[\exp\left(\frac{qV}{KT}\right) - 1\right] \exp\left(\frac{x_n - x}{L_p}\right)\left(-\frac{1}{L_p}\right)\Bigg|_{x = x_n}$$

$$= -q\frac{D_p}{L_p} \cdot p_{no}\left[\exp\left(\frac{qV}{KT}\right) - 1\right].$$

Similarly,

$$J_n = -q\frac{D_n}{L_n} \cdot n_{po}\left[\exp\left(\frac{qV}{KT}\right) - 1\right].$$

$$\therefore J_{total} = I = J_p + J_n$$

$$= \left[\exp\left\{\frac{qV}{KT}\right\} - 1\right]\underbrace{\left[qA\left(\frac{D_p \cdot p_{no}}{L_p} + \frac{D_n n_{po}}{L_n}\right)\right]}_{I_0}$$

$$\therefore \boxed{I = I_0\left[\exp\left(\frac{qV}{KT}\right) - 1\right]}.$$