INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

Quiz II - March 2014

B. Tech - II Semester

MA121 - Vector Calculus and Differential Equations

Date: 12/03/2014 **Time: 9.00 am - 10.00 am** **Max. Marks: 15**

Attempt all questions

1. Check whether the sequence $f_n(x) = \dfrac{n \ln x}{x^n},\ x \in [1, \infty)$ converges uniformly on the given interval? (2)

2. Let $\{f_n(x)\}$ be a sequence of continuous functions defined on a finite interval $[a, b]$. Suppose that $\{f_n(x)\}$ converges uniformly on $[a, b]$. Then show that

$$\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b \lim_{n\to\infty} f_n(x)dx.$$

   State whether the converse of this statement is true or false. Justify your answer with an example. (4)

3. Show that the limit of the derivatives need not be equal to the derivative of the limit if a sequence of differentiable functions converges uniformly but their derivatives converges only pointwise by taking the sequence $f_n(x) = \dfrac{x}{1 + nx^2},\ x \in R$. (3)

4. Evaluate $\displaystyle\sum_{n=0}^{\infty} \int_0^{\frac{1}{2}} \frac{x^n(1 - x^2)}{\sqrt{1 + x}} dx$ in the simplest form with appropriate justification. (4)

5. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$ converges uniformly on [0,1]. (2)

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