Indian Institute of Space Science and Technology

Thiruvananthapuram

MA211 - Linear Algebra

Tutorial-I

1. Reduce the following into row-reduced echelon form

(i) $\begin{bmatrix} 0 & 3 & 4 & 1 \\ 3 & 1 & 2 & 2 \\ 1 & 5 & 2 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 2 & 4 & 1 & 4 \\ 1 & 1 & 3 & 2 & 1 \\ 3 & 2 & 5 & 1 & 4 \\ 1 & 0 & 3 & 1 & 2 \end{bmatrix}$

2. Find the rank of

(i) $\begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 0 & 2 & 1 \\ 1 & 0 & 4 & 5 \\ 0 & 1 & 2 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 3 & 4 \\ 3 & 0 & 4 \\ 2 & 3 & 1 \\ 0 & 3 & 5 \end{bmatrix}$

3. Using Gauss-Jordan elimination method find solutions of

(a)
$$\begin{array}{rcrcrcrcl} x & + & y & + & 2z & + & 3w & = & 13 \\ x & - & 2y & + & z & + & w & = & 8 \\ 3x & + & y & + & z & - & w & = & 1 \end{array}$$

(b)
$$\begin{array}{rcrcrcl} x & + & 2y & - & 4z & = & 3 \\ x & - & 2y & + & 3z & = & -1 \\ 2x & + & 3y & - & z & = & 5 \end{array}$$

(c)
$$\begin{array}{rcrcrcl} 2x & - & y & + & z & = & 3 \\ x & - & 3y & + & z & = & 3 \\ -5x & - & 2z & = & -5 & & \end{array}$$

(d)
$$\begin{array}{rcrcrcl} 2x_1 & + & 3x_2 & + & x_3 & = & 0 \\ x_1 & + & 3x_2 & + & x_3 & = & 4 \\ 6x_1 & + & 9x_2 & + & 4x_3 & = & 8 \end{array}$$

(e)
$$\begin{array}{rcrcrcrcl} 3x_1 & + & 6x_3 & = & 6 & & & & \\ x_1 & + & x_2 & + & 5x_3 & + & x_4 & = & 9 \\ 2x_2 & + & 4x_3 & + & 2x_4 & = & 10 & & \end{array}$$

4. Test consistency and solve the system

(a)
$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 14 \\ x + 4y + 7z &= 30 \end{aligned}$$

(b)

$$\begin{aligned} 2x + 6y &= -11 \\ 6x + 20y + 6z + 3 &= 0 \\ 6y - 18z &= -1 \end{aligned}$$

5. Find values of $'a'$ for which the linear system

$$\begin{aligned} x + y - z &= 2 \\ x + 2y + z &= 3 \\ x + y + (a^2 - 5)z &= a \end{aligned}$$

has (i) no solution (ii) a unique solution (iii) infinitely many solutions.

6. Consider the system of equations

$$\begin{aligned} x - 3y + 2z &= \alpha \\ 2x - 4y - 3z &= \beta \\ x + 3y - 7z &= \gamma \\ 2x - 3y - 5z &= \delta \end{aligned}$$

Find the relation between $\alpha,\ \beta,\ \gamma$ and δ so that the system may have a solution and also find that solution.

7. Find the values of a for which the system

$$\begin{aligned} x + 2y + 3z &= ax \\ 3x + y + 2z &= ay \\ 2x + 3y + z &= az \end{aligned}$$

has non-trivial solution.

8. Find the values of a and b for which the system

$$\begin{aligned} 3x + 2y + z &= 6 \\ 3x + 4y + 3z &= a \\ 6x + 10y + bz &= a \end{aligned}$$

has (i) unique solution, (ii) no solution, (iii)infinitely many solutions.

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Assignment-I

Submit the answers of questions 3(c), 3(e), 4(a), 7, 8 on or before 17-10-2023.