Indian Institute of Space Science and Technology

Modern Signal Processing (AVD611)

Tutorial 3

1. A linear Shift invariant system is given by $H(z) = \frac{1-0.5z^{-1}}{1-0.75z^{-1}}$, this is excited by zero mean exponentially correlated noise $x(n)$ with an autocorrelation sequence $r_x(k) = \left(\frac{1}{3}\right)^{|k|}$. Let $y(n)$ be the output. Find the output power spectrum $P_y(z)$, autocorrelation $r_y(k)$, crosscorrelation $r_{xy}(k)$,

2. Find the power spectrum of the wss random process if the autocorrelation sequence is given by (i) $r_x(k) = \delta(k) + 2(0.5)^{|k|}$ (ii)$r_x(k) = \begin{cases} 10 - |k|\,; & |k| < 10 \\ 0 & otherwise \end{cases}$

3. Find the autocorrelation sequence if the power spectrum is given by $p_x(e^{j\omega}) = \frac{-2z^2+5z-2}{3z^2+10z+3}$.

4. Consider we have a zero mean process $x(n)$ with autocorrelation $r_x(k) = 10\left(\frac{1}{2}\right)^{|k|} + 3\left(\frac{1}{2}\right)^{|k-1|} + 3\left(\frac{1}{2}\right)^{|k+1|}$. Find a stable causal filter which when excited by $x(n)$ will produce zero mean unit variance white noise.

5. If $x(n)$ is a MA(2) process that is generated by the difference equation $x(n) = 4v(n) - 2v(n-1) + v(n-2)$ where $v(n)$ is zero mean unit variance white noise, find the system function of the two step predictor and evaluate the mean square error.

6. A signal $x(n)$ is observed in a noisy environment, $y(n) = x(n) + 0.8x(n-1) + v(n)$ where $v(n)$ is white noise with variance $\sigma_v^2 = 1$ uncorrelated with $x(n)$. Where $x(n)$ is wss AR(1) with autocorrelation values $r_x = [4, 2, 1, 0.5]^T$.
(a) Find the noncausal IIR Wiener filter, H(z) that produces the minimum MSE of $x(n)$
(b) Find the causal IIR Wiener filter, H(z) that produces the minimum MSE of $x(n)$

7. An AR(2) process is defined by the difference equation $x(n) = x(n-1) - 0.6x(n-2) + w(n)$, where $w(n)$ is the white noise process with variance σ_w^2. Use the Yule walker equation to solve for the values of autocorrelation $r_x(0)$, $r_x(1)$ and $r_x(2)$.

8. consider a signal $x(n) = s(n) + w(n)$, where $s(n)$ is an AR(1) process the satisfies the difference equation $s(n) = 0.4s(n-1) + w(n)$, where $v(n)$ is a white noise sequence with variance $\sigma_v^2 = 0.49$ and $w(n)$ is a white noise sequence with variance $\sigma_w^2 = 1$. The process $v(n)$ and $w(n)$ are uncorrelated.
(a)Determine the autocorrelation sequence $r_s(m)$ and $r_x(m)$.
(b)Design a Wiener filter of length M=2 to estimate $s(n)$.
(c)Determine the MMSE for M=2.