

Quiz II - March 2023

B.Tech - IV Semester

MA221 - Integral Transforms, PDE and Calculus of Variations

Date: 27/03/2023

Time: 09.00 am - 10.00 am

Max. Marks: 15

Answer all questions.

1. Does the following PDE has a unique solution? If yes, then find it and if not, justify your answer.

(i)  $(y - u) \frac{\partial u}{\partial x} + (u - x) \frac{\partial u}{\partial y} = x - y$

with the initial data  $u = 0$  on  $y = -x$ .

[3]

(ii)  $(y + u) \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = x - y, y > 0$

with the initial data  $u = 1 + x$  on  $y = 1$ .

[3]

2. Find the general solution of the following PDE

$$(D' - 5)^{10} u = e^{2x+5y}, \text{ where } D' \equiv \frac{\partial}{\partial y}.$$

[2]

3. Find the general solution of the following PDE

$$x^2 \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + xyu = 1.$$

[3]

4. Reduce the following PDE to a canonical form

$$\frac{\partial^2 u}{\partial x^2} + (1 + y^2)^2 \frac{\partial^2 u}{\partial y^2} - 2y(1 + y^2) \frac{\partial u}{\partial y} = 0.$$

[2]

5. Check the compatibility of the following PDEs. If they are compatible, then find their common solutions

$$xp - yq = x \text{ and } x^2 p + q = xz,$$

where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$ .

[2]

\*\*\*END\*\*\*