INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM

End Semester Examination – May 5, 2015
CH 121- Materials Science and Metallurgy
Second Semester

Time: 3 Hours **Max. Marks: 100**

All questions carry equal marks. (10x 10 = 100)

Answer any 10 questions

Note: Read the Questions and Hints carefully before attempting to write/solve. Draw neat sketches wherever necessary.

1. (a) Suppose that liquid iron melt is undercooled until nucleation occurs, how many atoms would have to group together spontaneously for this to occur under the conditions of homogeneous nucleation and heterogeneous nucleation? Also determine the nucleation barrier (ΔG^*) for this to occur. Assume that the radius of the iron atom is 1.265 A°, Surface energy of the solid-liquid interface is 204×10^{-7} J/sq.cm and the heat of fusion (ΔH) is 1737 J/cm^3. Nucleation starts at a temperature of 1119°C. (Melting temperature of iron =1539°C). The wetting angle of the liquid iron with the mould wall is 30° and the volume of the spherical cap (V_s) for heterogeneous nucleation is given by

1812

$$V_s = \frac{\pi r^3}{3}(2 - 3.\cos\theta + \cos^3\theta)$$

$\Delta G_{Het} = \Delta G_{Hm}\left(\frac{2-3\cos\theta+\cos^3\theta}{4}\right)$

Hint: Compute the no of unit cells that can be accommodated in the sphere of critical radius. You should also know the relationship between the radius of an atom to the lattice parameter of BCC unit cell. Substitute temperatures in Kelvin only. The solid clusters assume spherical shape. (7)

(b) Assuming that the solid clusters during solidification takes up a cuboid shape with one of the side length given by 'a', calculate the critical size (a*) for homogeneous nucleation. (3)

2. (a) The presence of dislocations in a metallic material increases the strength during plastic deformation. Suppose that the metallic material does not have any dislocation and introduction of a single dislocation decreases the strength as compared to the strength attained by the material that had no dislocations. Both the statements are true. Justify with proper reasoning. (4)

$\Delta G_{Hm} = -\frac{4}{3}\pi r^3(\lambda) + \sigma \times 4\pi r^2$ $\Delta G = \Delta H - T_m \Delta S = 0$

$T_m = \frac{\Delta H}{\Delta S}$ 1.265×10^{-10} m

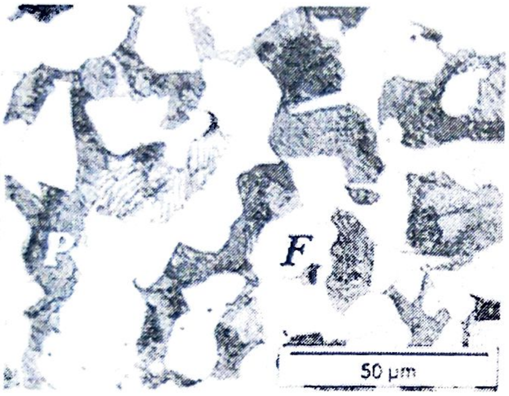
$\Delta G = \Delta H - T \times \frac{\Delta H}{T_m}$

$\Delta H\left(1 - \frac{T}{T_m}\right)$

(b) What is meant by a slip system and why is slip favourable at 45° to the axis of a cylindrical specimen when loaded under tension. Justify with proper reasoning and equations. (6)

3. (a) The room temperature microstructure of mild steel as seen under a optical microscope is shown in the below figure. Careful observation of the microstructure reveals grains of different contrast. Explain the procedure of metallography in simple words and the reason for the grains to have different contrast. (7)

Hint: 'P' and 'F' marked in the microstructure indicate grains that have different composition, thus having different contrast.



(b) Calculate the grain diameter for the ASTM grain number – 7. What is the influence of ASTM grain size number on the mechanical properties? Explain with equation (3)

4. (a) How do you make a well dispersed ceramic powder suspension in an aqueous medium for slip casting? Explain the mechanism of particle dispersion in the chosen method.

(b) Describe a process for preparation of thin alumina sheet for electronic circuit substrate application. (5+5)

5. (a) Why wet- chemical methods are superior to the solid state method for synthesis of ceramic powders? Describe one wet-chemical method for preparation of a ceramic powder.

(b) What is the crystal structure of SiC? How SiC ceramic components are prepared by reaction bonding process? (5+5)

6. (a) The yield strength of titanium with grain size of 17×10^{-6} m is found to be 448.175MPa. The same material at a grain size of 0.8×10^{-6} m has yield strength of 565MPa. Determine the strength of titanium with a grain size of 0.2×10^{-6} m.

(b) Explain injection moulding of ceramics? What are the limitations of injection moulding ? (5+5)

ZrO_2

7. (a) What is the crystal structure of cubic zirconia? How fully stabilized zirconia function as an oxygen sensor?

c = c

(b) What are nanomaterials? What is the reason for the exceptionally high reactivity of nanomaterial (5+5)

8. (a) Write down the mechanism of formation of low density polyethylene starting from the monomer.

crystalline isotactic / syndiotactic Ziegler Natta catalyst

(b) How can you obtain highly polypropylene $\left[\!\!\begin{array}{c}CH_3\\ |\end{array}\!\!\right]_n$ for fiber manufacture ? (5+5)

9. (a) You need to prepare polystyrene with polydispersity index (PDI) ~ 1.001. $\frac{M_w}{M_n} \simeq 1.001$

Explain the reaction and mechanism

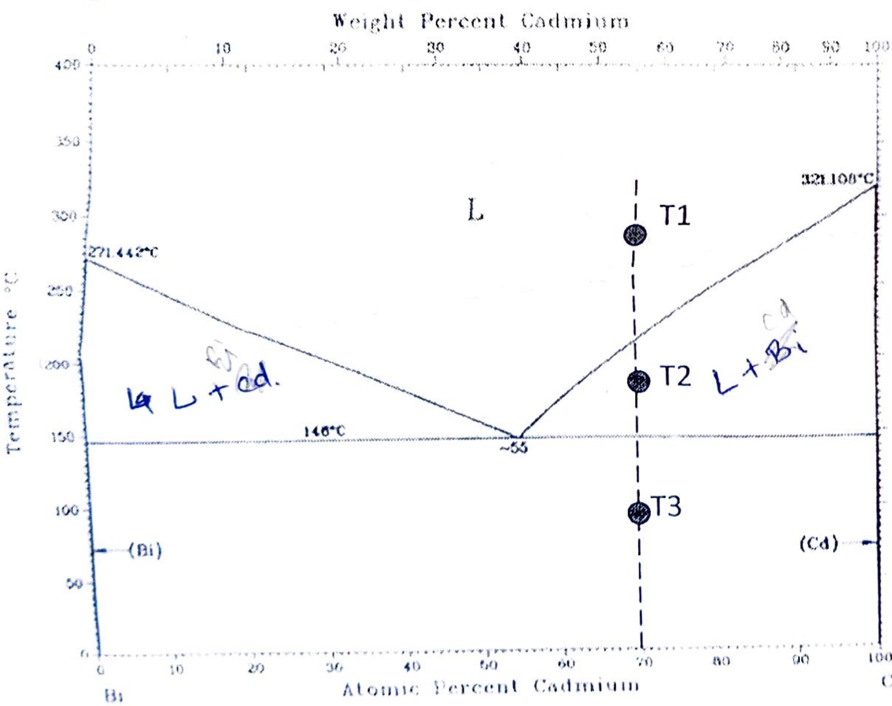
(b) Explain the polymerization technique by which you can obtain spherical polymer particles with 0.2- 1 mm diameter. monomer prefer to go in hydrophobic part (5+5)

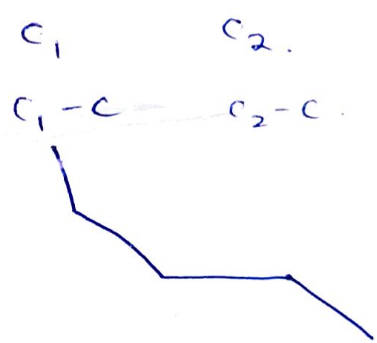
10. Condensation polymerization of a diacid and a diamine was carried out with slight excess (molar) of the diamine.

(a) Derive an expression for Xn (degree of polymerization) in terms of 'p' and 'r'.
p = extent of polymerization defined in terms of the functional groups present in minority.
r = N_A/N_B (N_A and N_B are initial number of acid and amine functional groups respectively

(b) Calculate the feed ratio of adipic acid($HOOC(CH_2)_4COOH$) and hexamethylene diamine ($H_2N(CH_2)_6NH_2$) that should be employed to obtain a polyamide of approximately 15,000 molecular weight at extent of polymerization 0.995. *(Hint: You may use the equation derived in 10.a)* (6+4)

11.





For the given phase diagram, draw the microstructures at temperatures T1, T2 & T3 and the cooling curve for the alloy 70at% Cd. There is a mandate from the industry to use alloys that has a minimum of 80% of eutectic mixture for a particular application. Your help is sought to arrive at the alloy composition range. Mention the solution with the detailed steps of determining the same and assumptions if any. (10)

INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

End Semester Examination - May 2015

B. Tech - II Semester

MA121 - Vector Calculus and Differential Equations

Date: 13/05/2015 **Time: 1.30 pm - 4.30 pm** **Max. Marks: 100**

SECTION A (Attempt all 10 questions - 10x5= 50 marks.)

1. Is it true that if each f_n and the point wise limit function $f(x) = \lim_{n\to\infty} f_n(x)$ are continuous, then f_n converges to f uniformly. Justify your answer with an example. [5]

2. Show that the series $f(x) = \sum_{n=0}^{\infty} e^{-nx}\cos nx$ converges uniformly on $0 < x < \infty$. Also check whether the following term by term derivative of the series can be justified: [5]
$$f'(x) = -\sum_{n=0}^{\infty} ne^{-nx}(\cos nx + \sin nx).$$

3. Define directional derivative of a function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ at a point P_0 along a vector $\vec{v}$. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by $f(x,y) = \sqrt{x^2+y^2}$. Let $\vec{v}$ be a **unit vector** in $\mathbb{R}^2$; does $D_{\vec{v}}(f)|_{(0,0)}$ exist? Is f differentiable at $(0,0)$? [3+2].

4. Define arc length function of a smooth curve $C : \vec{\gamma}(t),\ t \in [a,b]$ with initial point $\gamma(a)$. Let C be a curve given by $C : x^2+y^2 = 1,\ y = x\tan z$ with initial point $(1,0,0)$. Give a parametrization to the curve. Find the "arc length function" of the curve. [2+3]

5. Let $\vec{\gamma} : [a,b] \longrightarrow \mathbb{R}^3$ be a non-constant smooth curve. Show that $\vec{\gamma}'(t) \neq \vec{0}$ for all $t \in [a,b]$. Express $-\vec{\gamma}$ in terms of $\vec{\gamma}$. Is $-\vec{\gamma}$ a smooth curve? Justify your answer. [3+1+1]

6. Using Green's theorem find the area of the region bounded by $x^2+y^2 = 4,\ y \geq 0$; $x - y - 2 = 0$; and $x + y + 2 = 0$ on the XY-plane. [5]

7. Does Picard theorem guarantee that there exist an interval on which the following initial value problems have a unique solution? If yes, find it and also justify your answer. [5]

   (a) $\frac{dy}{dx} = y^{\frac{1}{2}},\quad y(0) = 0.$

   (b) $\frac{dy}{dx} = x^2|y|,\quad y(0) = \frac{1}{2}.$

8. Find the general solution of $y'' - f(x)y' + [f(x) - 1]y = 0$, where f is a continuous function on $\mathbb{R}$. [5]

9. Using method of undetermined coefficients, find particular solution of the following differential equations. [5]

   (a) $y'' + y = \sin x$.

   (b) $y''' + 2y'' - y' = 3x^2 - 2x + 1$.

10. Find the first three terms of the Legendre series of the following functions. [5]

(a) $f(x) = \begin{cases} 0, & -1 \le x < 0, \\ x, & 0 \le x \le 1. \end{cases}$

(b) $f(x) = e^x$.

SECTION B (Attempt any 5 questions - 5x10= 50 marks.)

11. Let $f_n(x) = n^c x(1-x^2)^n$ for x real and $n \ge 1$.

(a) Prove that $\{f_n\}$ converges pointwise on $[0,1]$ for every real c. [2]

(b) Find the values of c for which the convergence is uniform on $[0,1]$. [4]

(c) Also determine the values of c for which the following is true [2]

$$\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 \lim_{n\to\infty} f_n(x)dx.$$

(d) Suppose $\{g_n\}$ is a sequence of continuous functions satisfying the integral identity given in question (c). Does it imply the uniform convergence of $\{g_n\}$? [2]

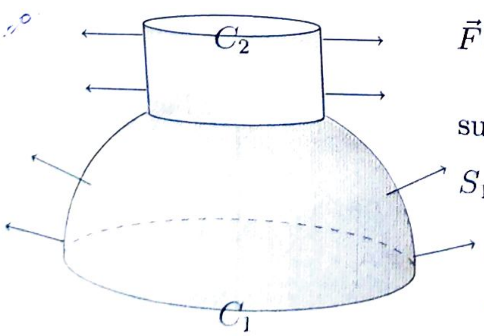
12. (a) Check whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ converges uniformly on $\mathbb{R}$. [3]

(b) Prove the following with appropriate justification: $\displaystyle\int_0^1 \sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2} dx = \frac{1}{2}$. [7]

13. (a) Let $\vec{F}(x,y,z) = (ye^z + z\cos x,\ xe^z + 1,\ xye^z + \sin x + 1)$ be a vector field and C a curve defined by $C = C_1 * C_2 * C_3$ where C_1 is the straight line segment $(0,0,0) \longrightarrow (\pi, 0, -\pi)$; $C_2 : \gamma(t) = (\pi+t,\ (\pi+t)\sin(\pi+t),\ (\pi+t)\cos(\pi+t))$ where $t \in [0,\pi]$; and C_3 is the straight line segment $(2\pi, 0, 2\pi) \longrightarrow (1,0,0)$. Evaluate $\displaystyle\int_C \vec{F}$, if exists. Is the integral path independent? Justify your answer. [3+2]

(b) Suppose that at time $t=0$ a particle is ejected from the surface $x^2+y^2-z^2 = -1$ at the point $(1,1,\sqrt{3})$ along the normal to the surface, which is directed toward the xy plane with a speed of 10 units per second. When and where does it cross the xy plane? [5]

14. (a) Verify multiply connected version of Stoke's theorem for the vector field



$\vec{F}(x,y,z) = \left(\dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2}, e^z\right)$ over the surface $S = S_1 \sqcup S_2$ where

$S_1 : x^2+y^2+z^2 = 5,\ 0 \le z \le 2$ and

$S_2 : x^2+y^2 = 1,\ 2 \le z \le 4$ with outward orientation (see corresponding figure). [5]

(b) Let $\vec{F}$ be a smooth vector field with domain $\mathbb{R}^2 - \{(0,0)\}$ having Curl zero. Let C_1 and C_2 be any two positively oriented loops on XY-plane around $(0,0)$ such that C_1 and C_2 do not intersect each other (e.g. concentric circles). Using **Green's theorem on simply-connected domains** show that the line integrals of $\vec{F}$ along C_1 and along C_2 are the same. [5]

2

15. Find the general solution of the following differential equation

$$x^2y'' + (x^2 - 3x)y' + 3y = 0.$$

[10]

16. Consider the following second order differential equation

$$y'' + P(x)y' + Q(x)y = R(x), x \in I = (-1, 1). \tag{1}$$

Then prove or disprove the following for arbitrary real constants c_1 and c_2:

(a) If $R(x) = 0 \quad \forall x \in I$, y_1 and y_2 are any two solutions of (1), then $c_1y_1 + c_2y_2$ is the general solution of (1). [3]

(b) If $R(x) \neq 0$ for some $x \in I$, y_1 and y_2 are any two solutions (1), then $c_1y_1 + c_2y_2$ is the general solution of (1). [3]

(c) Let $R(x) = 0 \quad \forall x \in I$, then there exist P and Q such that x^2 and $x|x|$ are solutions of (1). Justify your answer. [4]

END

PH.121: Physics II **09-05-2015**

Max. Marks: 100

PART A: Attempt ALL questions from 1 – 10; 5 mark each

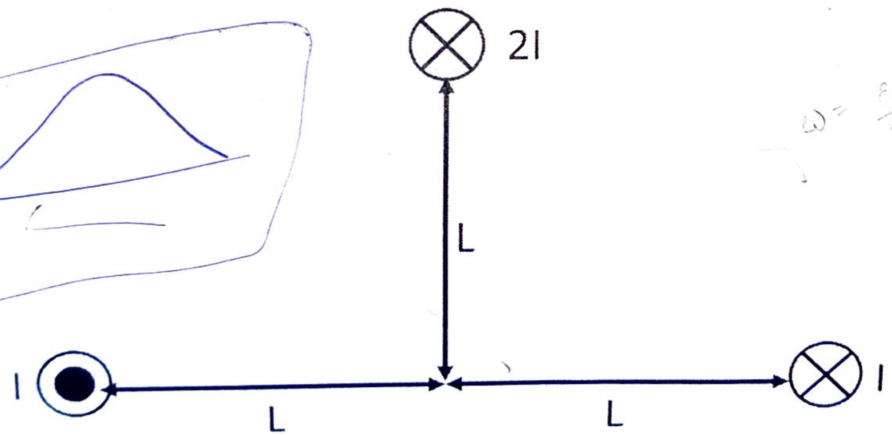
1. Show the validity of stokes theorem for $\boldsymbol{E} = \hat{\imath}xy - \hat{\jmath}(x^2 + 2y^2)$. Use a triangular loop in **x,y** plane with points **(0,0)**, **(2,0)** and **(2,2)**.

2. A pair of circular coils has the same axis and radius **'a'**. They are separated by a distance **'2d'**. Find the value of **'d'** for which the magnetic field in the axis mid-way will be most uniform. Consider that both the coils are carrying same amount of current in same direction.

3. Calculate the force required to hold two hemispheres of radius **R** each with charge **Q/2** together.

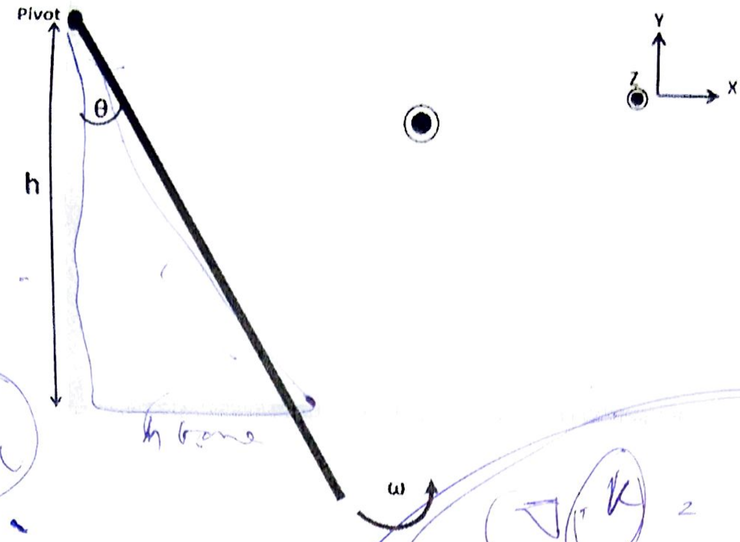
4. The magnetic field of a toroid coil is given by $\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0 NI}{2\pi s}\hat{\boldsymbol{\phi}}$. Find the vector potential **A**.

5. Three long wires carrying current **I, I** and **2I** are located as shown below. The direction of currents carried by these wire is also shown. Determine the force experienced by the top wire (current **2I**) due to the fields of the two bottom conductors.



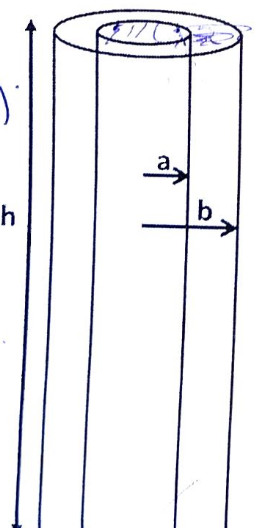
6. A long resistive bar with resistance **'R'** is attached to a conducting right angle frame at one end by a pivot a distance **'h'** as shown in the figure. The free end of the bar is resting on the frame making a triangular circuit. Assume that the frame has negligible resistance. A constant magnetic field **'B'** is applied perpendicular as shown. The bar starts at **t = 0** along the **y** axis and is rotated counterclockwise about the pivot with an angular velocity **ω**, such that the pivot angle is **θ=ωt**. Assume that the bar never loses contact with the horizontal side of the frame. **(a)** Find the magnetic flux penetrating

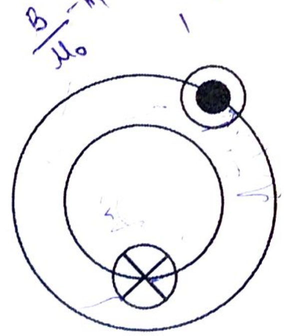
the circuit? **(b)** Find the induced emf and the current. Indicate the direction of flow of the current.



7. The electric field free space is given by $\boldsymbol{E} = \mathbf{10}\,\mathbf{sin(10^8}\boldsymbol{t} - \boldsymbol{kz})\hat{\boldsymbol{x}}$ volts/m. Given $\sigma = 0$, $\mu = \mu_o$, $\varepsilon = \varepsilon_o$. Calculate **H** and the propagation constant **k** using Maxwell's equations.

8. Consider a coaxial cable with inner radius **'a'** and outer radius **'b'** and length **'h'**, held apart with vacuum in between. The cross sections showing the direction of current flow **(I)** is shown. Calculate the inductance per unit length of the transmission line.





Cross section

9. Using Maxwell's equations show that the tangential component of the electric field intensity $\boldsymbol{E}$ is continuous even at the surface of discontinuity if one of the surfaces is a conductor.

10. In free space the electric field intensity is given by $\boldsymbol{E} = \boldsymbol{E_o}\sin(\omega t - kz)\hat{x}$. Verify whether it satisfies wave equation

PART B: Attempt any FIVE questions from 11 – 17; 10 mark each

shell

E.B.

11. Two concentric conducting spheres of inner and outer radii **a** and **b**, respectively, carry charges **±Q**. The empty space between the spheres is half-filled by a hemispherical shell of dielectric (of dielectric constant **k=** $\varepsilon_1/\varepsilon_o$), as shown in the figure. Find the electric field everywhere between the spheres

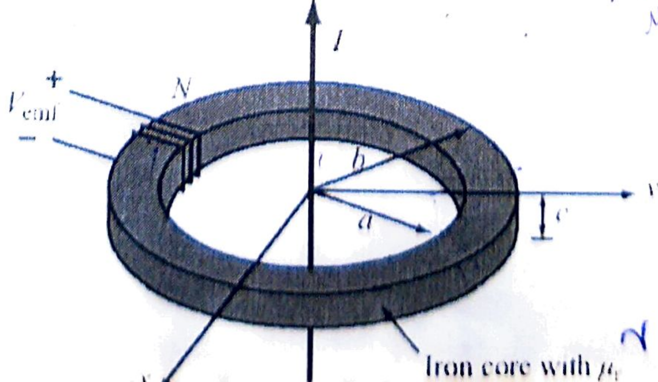
12. Suppose we have some charges and currents which produces electric and magnetic fields, **E** and **B** respectively at time **'t'**. In the next instant of time 'dt', if the charges move a bit, find the rate of work done on the charges $\frac{dW}{dt}$. Using the result, show that $\frac{\partial}{\partial t}(u_{mech} + u_{em}) = -\nabla.\boldsymbol{S}$, where **S** is the Poynting's vector.

13. An air spaced transmission line consists of two parallel cylindrical conductors of **2mm** diameter each with their centres **10mm** apart. (a) Calculate maximum potential difference one can apply before the air between them breaks down (at **3 MV/m**). (b). What is the capacitance between them?

$\omega = \frac{1}{2}\varepsilon_o E^2 = \frac{1}{2}CV^2$ $\frac{A\varepsilon_o}{d}$

14. The transformer shown in figure consists of a long wire coincident with the z-axis carrying a current **I** = $\mathbf{I_o}$ **cos(ωt)**, coupling magnetic energy to a toroidal coil situated in the **x–y** plane and centred at the origin. The toroidal core uses iron material with relative permeability μ_r, around which 100 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the figure. Derive expression for induced EMF (V_{emf}). Calculate its value at frequency $f = 60\ Hz$, μ_r = **4000**, **a = 5 cm**, **b = 6 cm**, **c = 2 cm**, and $\mathbf{I_o}$ **= 50 A**.

$S = \frac{1}{\mu_0}(E\times B)$ $\frac{\mu_0 I}{2\pi r}$

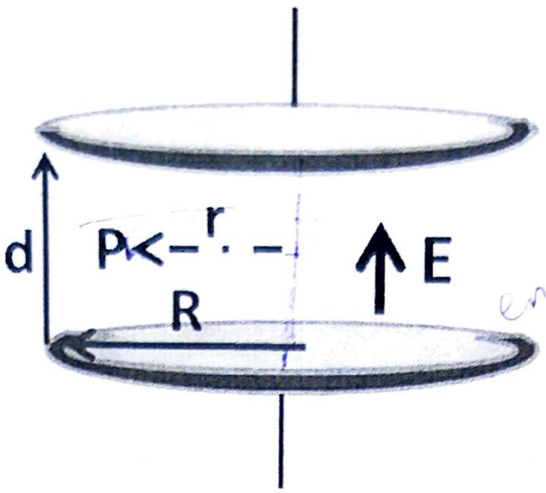


$\frac{\sigma}{2\varepsilon_0}$

$V = \frac{\sigma}{\varepsilon_0}(d-b)$

15. Prove $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ using Ampere's law and continuity equation. Verify it experimentally using charging of a capacitor for non-steady currents.

16. A parallel-plate capacitor consists of two circular plates, each with radius R, separated by a distance d. The electric field E between the plates is uniform and directed upwards



energy $\frac{1}{2}\epsilon_0 E^2$

$\frac{\epsilon_0 E^2}{2}$

$\frac{1}{2} CV^2$ $\frac{1}{2}$

(a) What is the total energy stored in the electric field of the capacitor? Assume that the electric field is uniform between the plates and zero outside of the plates.

(b) Suppose that the electric field is increasing with time ($dE/dt > 0$). The point P is located between the plates at radius $r < R$ (see figure). Derive an expression for the magnitude of the magnetic field B at point P and indicate its direction there on the sketch.

(c) What is the Poynting vector at point P? Give both direction and magnitude.

(d) Using the Poynting vector, determine the total electromagnetic energy flowing into or out of the capacitor per unit time across $r = R$. What is its direction (in or out ?). Write down an equation relating this quantity to the electric energy contained in the capacitor.

$\int \frac{B^2}{2\mu_0} +$

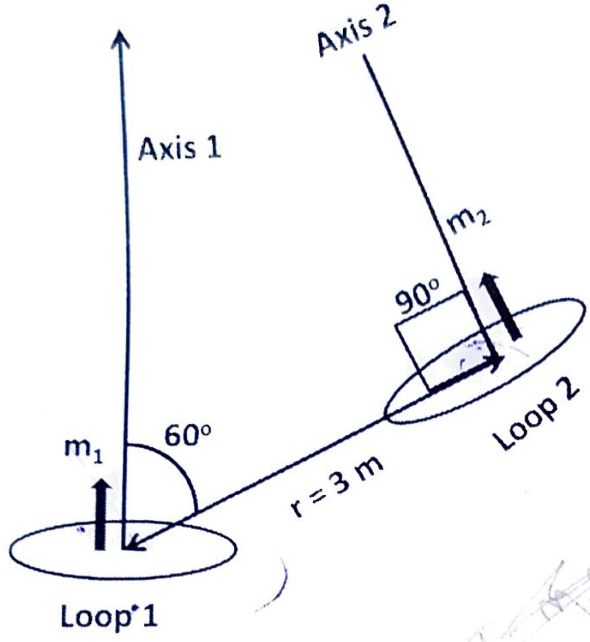
17. Two circular loops carries a current of **5 A** and has **20** turns each. The area of each loop is **5 cm^2**. They are arranged as shown below. Find the torque on **loop 2**. Assume that the field experienced by the loops due to each other is homogeneous throughout.

$B = \frac{\mu_0 I}{2R}$

$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

$B = \frac{\mu_0}{2R}$

$J_d = \epsilon_0 \frac{\partial E}{\partial t}$



Axis 1
Axis 2
m_2
90°
m_1
60°
r = 3 m
Loop 2
Loop 1

Indian Institute of Space Science and Technology
Department of Aerospace Engineering
AE 141: Engineering Graphics: Jan-April 2015
End Semester Examination
SET 2 Part- A (Manual drawing)

Max time: 2 hours Max Marks: 40

1. ABCD is a rhombus of diagonals AC, 100mm and BD, 70mm. Its corner C is on HP and the plan of the rhombus is a square. Draw its projections and find its inclination with HP.

(6 Marks)

2. A pentagonal pyramid 30 mm base edge and 60 mm height is resting on one of its corner on HP with two of its triangular faces equally inclined to HP and the apex in VP. The slant edge containing the corner on which it rest is at 60^0 with HP. Draw its projections.

(12 Marks)

3. A hexagonal prism, base edge 30 mm and height 60 mm is resting on its base on HP with one base edge parallel to VP. A pentagonal hole, side 25 mm is drilled through this prism in such a way that one face of the hole is vertical, parallel to and is in line with the axis of the prism. Draw the development of the lateral surface of the prism, if the axis of the hole is perpendicular to VP and is at a height of 30 mm above the base of the prism.

(11 Marks)

4. Draw the isometric view of the solid given below. **(11 Marks)**