RF ASSIGNMENT

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SC22B146

(2) For 3-section coupled-line coupler,

$$C(\theta) \quad S_{31}(\theta) = \left|\frac{V_3^-}{V_1^+}\right| = 2\sin\theta\left[C_1\cos 2\theta + \frac{C_2}{2}\right], \quad (N=3)$$

$$= 2C_1\sin\theta\cos2\theta + C_2\sin\theta$$

$$= C_1(\sin3\theta - \sin\theta) + C_2\sin\theta$$

$$= C_1\sin3\theta + (C_2 - C_1)\sin\theta$$

So, $|C(\theta)|_{\theta=\pi/2} = |S_{31}(\theta)|$

Maximally flat condition :

$$\left(\frac{d^m|C(\theta)|}{d\theta^m}\right)\Bigg|_{\theta=\pi/2} = 0, \quad m = M-1 = \left(\frac{N+1}{2}\right) - 1 = 1$$

$$\Rightarrow \frac{dC(\theta)}{d\theta}\Bigg|_{\theta=\pi/2} = 0$$

$$\Rightarrow \left(3C_1\cos3\theta + \cancel{(C_2-C_1)\cos\theta}\right)\Bigg|_{\theta=\pi/2} = 0$$

Now, $\dfrac{d^2C(\theta)}{d\theta^2}\Bigg|_{\theta=\pi/2} = 0$

$$\Rightarrow \left(-9C_1\sin3\theta - (C_2 - C_1)\sin\theta\right)\Big|_{\theta=\pi/2} = 0$$

$$\Rightarrow 9C_1 - C_2 + C_1 = 0$$

$$\Rightarrow 10C_1 - C_2 = 0 \Rightarrow 10C_1 = C_2$$

For 30 dB -coupler,

$$-20\log_{10} C = 30 \Rightarrow C = 10^{-3/2}$$

Also, $C(\theta)|_{\theta=\pi/2} = -2C_1 + C_2$

$$\therefore -2C_1 + C_2 = 10^{-3/2}$$

$$\Rightarrow 8C_1 = 10^{-3/2}$$

$$\Rightarrow C_1 = 1/8 \times 10^{-3/2} = 0.00395$$

$$\Rightarrow C_2 = 10/8 \times 10^{-3/2} = 5/4 \times 10^{-3/2}$$

$$= 0.03953$$

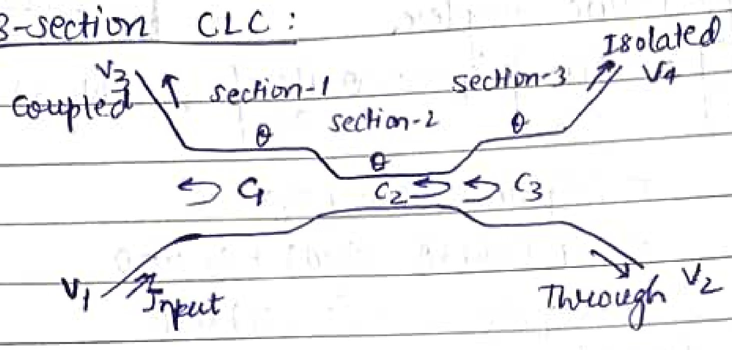
$$\therefore Z_{oe_1} = Z_0\sqrt{\frac{1+C_1}{1-C_1}} = 50\sqrt{\frac{1+0.00395}{1-0.00395}} = 50.198\,\Omega = Z_{oe_3}$$

$$Z_{oo_1} = Z_0\sqrt{\frac{1-C_1}{1+C_1}} = 50\sqrt{\frac{1-0.003953}{1+0.003953}} = 49.8027\,\Omega$$

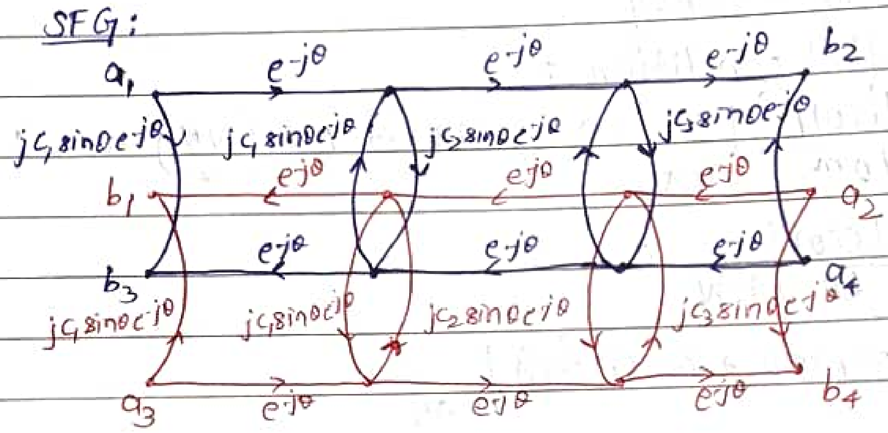
$$Z_{oe_2} = Z_0\sqrt{\frac{1+C_2}{1-C_2}} = 50\sqrt{\frac{1+0.03953}{1-0.03953}} = 52.0172\,\Omega$$

$$Z_{oo_2} = Z_0\sqrt{\frac{1-C_2}{1+C_2}} = 50\sqrt{\frac{1-0.03953}{1+0.03953}} = 48.0611\,\Omega$$

3-section CLC :



SFG :



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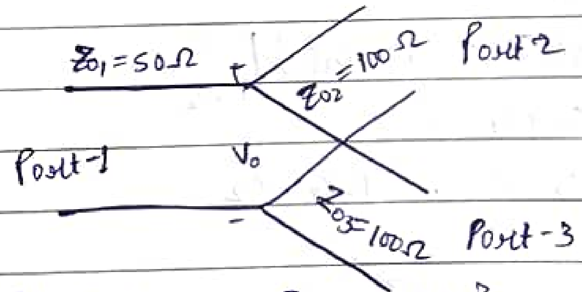


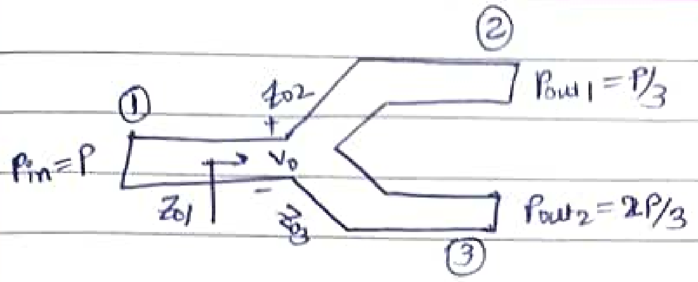
At port-1, $P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0}$

$$P_2 = \frac{1}{2}\frac{V_0^2}{2Z_0} = \frac{P_{in}}{2}$$

$$P_3 = \frac{1}{2}\frac{V_0^2}{2Z_0} = \frac{P_{in}}{2}$$

$\therefore$ Power division ratio $= P_2 : P_3 = 1 : 1$.

(b)



$$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_{01}}$$

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_{02}} = \frac{P_{in}}{3} = \frac{1}{6}\frac{V_0^2}{Z_{01}} \Rightarrow Z_{02} = 3Z_{01}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_{03}} = \frac{2P_{in}}{3} = \frac{1}{3}\frac{V_0^2}{Z_{02}} \Rightarrow Z_{03} = \frac{3}{2}Z_{01}$$

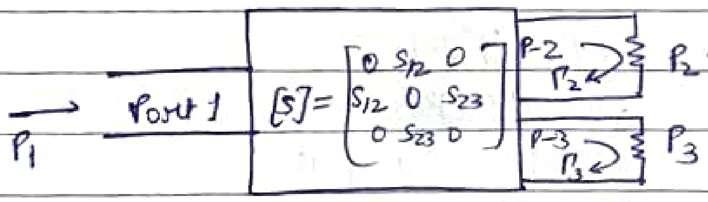
For $Z_{01} = 50\Omega = Z_0$,

$$Z_{02} = 3Z_0 = 150\Omega$$

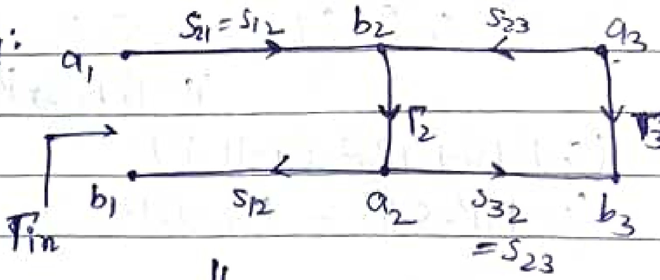
$$Z_{03} = \tfrac{3}{2}Z_0 = 75\Omega$$

Also, $Z_{in} = Z_{02} \parallel Z_{03} = \frac{150\times 75}{225} = 50\Omega$.

(4)



SFG:



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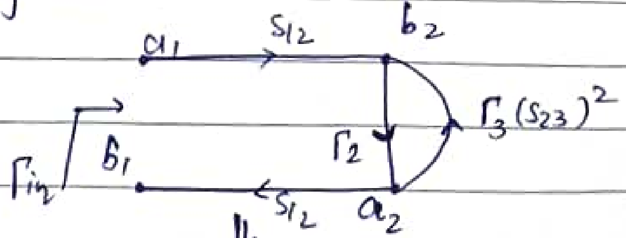
By Series Rule:



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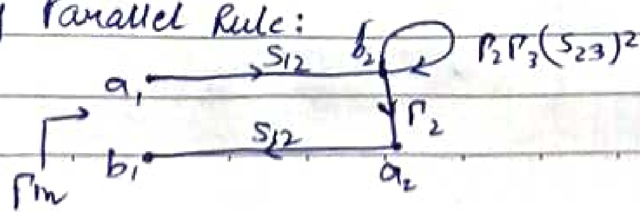
By Parallel Rule:



$$b_2 = a_1 \cdot \frac{S_{12}}{1 - \Gamma_2 \Gamma_3 (S_{23})^2}$$

$$b_1 = a_1 \frac{S_{12}^2 \Gamma_2}{1 - \Gamma_2 \Gamma_3 S_{23}^2}$$

$$b_3 = b_2 \Gamma_2 S_{23} = a_1 \frac{S_{12} S_{23} \Gamma_2}{1 - \Gamma_2 \Gamma_3 S_{23}^2} \quad ; \quad \Gamma_{in} = \frac{S_{12}^2 \Gamma_2}{1 - \Gamma_2 \Gamma_3 (S_{23})^2}$$

$$\therefore \frac{P_2}{P_1} = \frac{b_2^2 - a_2^2}{a_1^2 - b_1^2} = \frac{b_2^2 (1 - |\Gamma_2|^2)}{a_1^2 (1 - |\Gamma_{in}|^2)}$$

$$= \left(\frac{b_2}{a_1}\right)^2 \left[\frac{(1 - |\Gamma_2|^2)}{1 - \frac{|S_{12}|^2 |\Gamma_2|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2}}\right]$$

$$= \frac{|S_{12}|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2} \cdot \frac{(1 - |\Gamma_2|^2) |1 - \Gamma_2 \Gamma_3 S_{23}^2|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 - |S_{12}^2 \Gamma_2|^2}$$

$$= \frac{|S_{21}|^2 (1 - |\Gamma_2|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 - |S_{12}^2 \Gamma_2|^2}$$

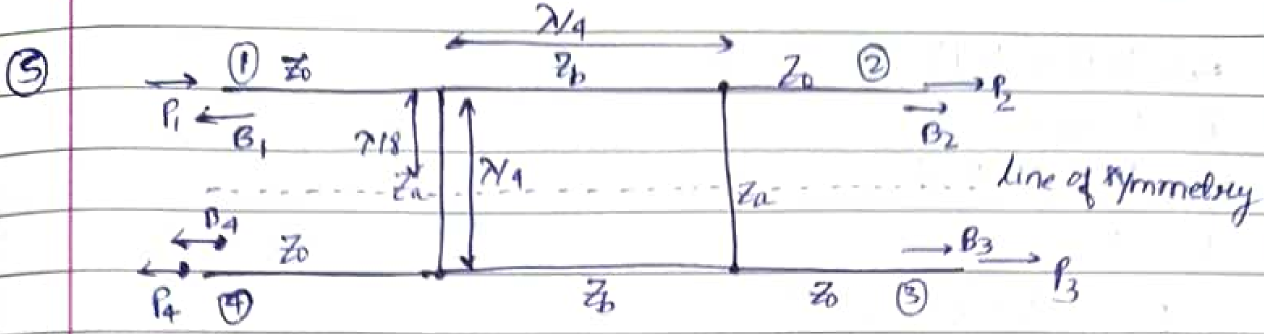
and,

$$\frac{P_3}{P_1} = \frac{b_3^3 - a_3^2}{a_1^2 - b_1^2} = \frac{b_3^2 (1 - |\Gamma_3|^2)}{a_1^2 (1 - |\Gamma_{in}|^2)}$$

$$= \frac{|S_{12} S_{23} \Gamma_2|^2 \quad (1 - |\Gamma_3|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 \left(1 - \frac{|S_{12}^2 \Gamma_2|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2}\right)}$$

$$= \frac{|S_{12}|^2 |S_{23}|^2 |\Gamma_2|^2 (1 - |\Gamma_3|^2)}{(|1 - \Gamma_2 \Gamma_3 S_{23}^2|)^2 - |S_{12}^2 \Gamma_2|^2}$$

(5)

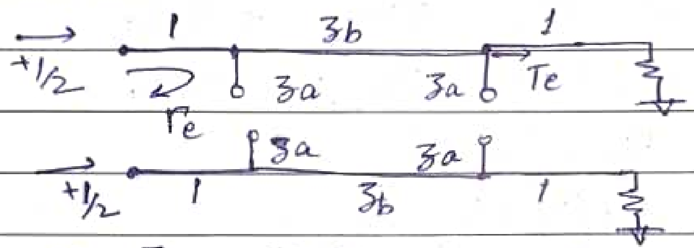


Normalised impedances: $Z_0 : 1$

$Z_a : z_a$

$Z_b = z_b$

Even-mode :



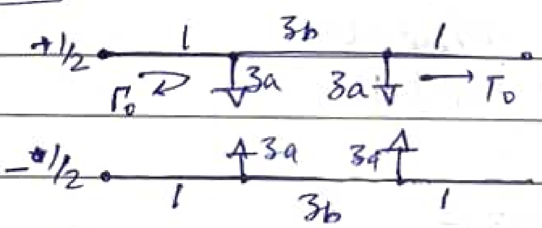
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j/z_a & 1 \end{bmatrix} \begin{bmatrix} \cos\beta l & j z_b \sin\beta l \\ j/z_b \sin\beta l & \cos\beta l \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/z_b & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ j/z_a & 1 \end{bmatrix} \begin{bmatrix} 0 & j z_b \\ j/z_b & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/z_a & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & j z_b \\ j/z_b & -z_b/z_a \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/z_a & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -z_b/z_a & j z_b \\ j/z_b - \frac{j z_b}{z_a^2} & -z_b/z_a \end{bmatrix}$$

Odd-mode :



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -j/z_a & 1 \end{bmatrix} \begin{bmatrix} 0 & j z_b \\ j/z_b & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/z_a & 1 \end{bmatrix}$$

$$= \begin{bmatrix} z_b/z_a & j z_b \\ j/z_b - \frac{j z_b}{z_a^2} & z_b/z_a \end{bmatrix}$$

$$B_1 = \tfrac{1}{2}\Gamma_e + \tfrac{1}{2}\Gamma_o$$

$$B_2 = \tfrac{1}{2}T_e + \tfrac{1}{2}T_o$$

$$B_3 = \tfrac{1}{2}\Gamma_e - \tfrac{1}{2}\Gamma_o$$

$$B_4 = \tfrac{1}{2}T_e - \tfrac{1}{2}T_o$$

$$\Gamma = \frac{A+B-(C+D)}{A+B+C+D}$$

$$T = \frac{2}{A+B+C+D}$$

Now,

$$\Gamma_e = \frac{-\cancel{z_b}/z_a + jz_b - j/z_b + jz_b/z_a^2 + z_b/\cancel{z_a}}{-z_b/z_a + jz_b + j/z_b - jz_b/z_a^2 - z_b/z_a}$$

$$= \frac{j(z_b - 1/z_b + z_b/z_a^2)}{-2z_b/z_a + j\left(z_b + 1/z_b - z_b/z_a^2\right)}$$

$$\Gamma_o = \frac{+\cancel{z_b}/z_a + jz_b - j/z_b + jz_b/z_a^2 - z_b/\cancel{z_a}}{z_b/z_a + jz_b + jz_b - jz_b/z_a^2 + z_b/z_a}$$

$$= \frac{j(z_b - 1/z_b + z_b/z_a^2)}{2z_b/z_a + j(z_b + 1/z_b - z_b/z_a^2)}$$

$B_1 = \tfrac{1}{2}(T_o + \Gamma_e) = 0$ [Reflection at port 1 = 0]

$$\Rightarrow \frac{1}{2}j(z_b - 1/z_b + z_b/z_a^2)\left(\frac{2z_b/z_a + j(z_b + 1/z_b - z_b/z_a^2) + \frac{2z_b}{z_a} - j}{\left(\frac{2z_b}{z_a}\right)^2 + \left(z_b + 1/z_b - z_b/z_a^2\right)^2}\right) = 0$$

$$\Rightarrow z_b - 1/z_b + z_b/z_a^2 = 0$$

$$\Rightarrow z_b^2 - 1 + z_b^2/z_a^2 = 0$$

$$\Rightarrow z_b^2(1 + 1/z_a^2) = 1$$

$$\Rightarrow z_b = \frac{z_a}{\sqrt{1+z_a^2}}$$

$$\Rightarrow z_a = \frac{z_b}{\sqrt{1-z_b^2}}$$

$$\therefore T_e = \frac{2}{-z_b/z_a + jz_b + j/z_b - jz_b/z_a^2 - z_b/z_a} = \frac{2}{-z_b/z_a + j(z_b + 1/z_b - z_b/z_a^2)}$$

$$= \frac{1}{-z_b/z_a + jz_b}$$

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$$T_o = \frac{2}{z_b/z_a + jz_b + j/z_b - jz_b/z_a^2 + z_b/z_a} = \frac{2}{2z_b/z_a + j\left(z_b + 1/z_b - \frac{z_b}{z_a^2}\right)}$$

$$= \frac{1}{z_b/z_a + jz_b}$$

$$B_2 = \frac{1}{2}(T_e + T_o) = \frac{1}{2}\left(\frac{1}{-\frac{z_b}{z_a} + jz_b} + \frac{1}{\frac{z_b}{z_a} + jz_b}\right)$$

$$= \frac{-j}{z_b(1 + 1/z_a^2)}$$

$$B_3 = \frac{1}{2}(T_e - T_o) = \frac{1}{2}\left[\frac{1}{-\frac{z_b}{z_a} + jz_b} - \frac{1}{\frac{z_b}{z_a} + jz_b}\right]$$

$$= \frac{-1/z_a}{z_b(1 + 1/z_a^2)}$$

$$P_2/P_3 = \alpha$$

$$\Rightarrow P_2 = \alpha P_3$$

$$\Rightarrow |B_2|^2 = \alpha |B_3|^2$$

$$\Rightarrow 1 = \alpha/z_a^2$$

$$\Rightarrow z_a = \sqrt{\alpha} \Rightarrow Z_a = Z_0\sqrt{\alpha}$$

$$z_b = \frac{z_a}{\sqrt{1 - z_a^2}} = \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}} \Rightarrow Z_b = \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}} Z_0$$

Now,

$$B_4 = \frac{1}{2}(\Gamma_e - \Gamma_o)$$

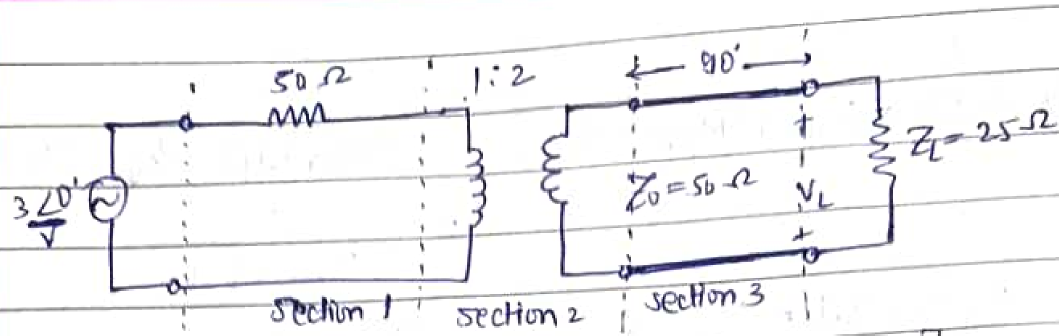
$$= j/2\left(b - 1/b + b/a^2\right)(\cdot) = \gamma$$

$$\alpha = 1/3$$

$$Z_a = Z_0\sqrt{1/3} = 28.86\,\Omega$$

$$Z_b = Z_0\frac{\sqrt{1/3}}{\sqrt{4/3}} = 25\,\Omega$$

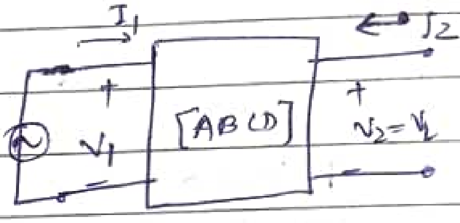
(6)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & j50 \\ j/50 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/25 & 1 \end{bmatrix}$$

load

$$= \begin{bmatrix} 3j & 25j \\ j/25 & 0 \end{bmatrix}$$



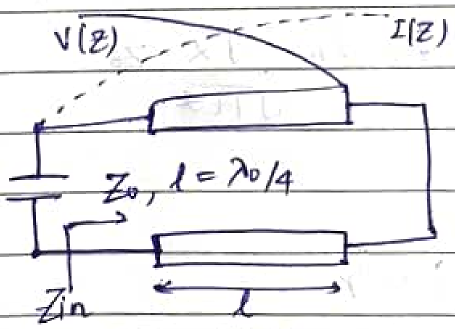
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_L \\ I_2 \end{bmatrix}$$

$V_1 = V_L A + B I_2$

$\Rightarrow V_1 = V_L A \quad (\because I_2 = 0)$

$\Rightarrow V_L = \dfrac{V_1}{A} = \dfrac{3}{3j} = 1\angle -90^\circ$

(7)



$f = 6\text{ GHz}$

$Z_{in} = Z_0 \tanh(\alpha + j\beta)l$

$= Z_0 \dfrac{\tan\alpha l + j\tan\beta l}{1 + j\tan\alpha l \tan\beta l}$

$= Z_0 \dfrac{1 - j\tan\alpha l \cot\beta l}{-j\cot\beta l + \tan\alpha l}$

$l = \lambda/4, \quad \omega = \omega_0 + \Delta\omega$

$\Rightarrow \beta l = \dfrac{\omega_0 l}{v_p} + \dfrac{\Delta\omega l}{v_p} = \dfrac{\pi}{2} + \dfrac{\pi\Delta\omega}{2\omega_0}$

$\Rightarrow \cot\beta l = \cot\left(\dfrac{\pi}{2} + \dfrac{\pi\Delta\omega}{2\omega_0}\right) = -\tan\left(\dfrac{\pi\Delta\omega}{2\omega_0}\right) \simeq -\pi\dfrac{\Delta\omega}{2\omega_0}$

$$\therefore Z_{in} = Z_0 \frac{1 + j\alpha l \cancel{\pi \Delta\omega/2\omega_0}^{\,0}}{\alpha l - j\pi\Delta\omega/2\omega_0} \quad [\text{for } \alpha l \text{ to be small}]$$

$$= \frac{1}{\frac{\alpha l}{Z_0} - j\frac{\Delta\omega\pi}{Z_0 2\omega_0}}$$

Comparing with parallel RLC circuit,

$$Z_{in} = \frac{1}{1/R + 2j\Delta\omega C}$$

$$\Rightarrow R = \frac{Z_0}{\alpha l}, \quad C = \frac{\pi}{4\omega_0 Z_0} \quad \left[\because L = \frac{1}{\omega_0^2 C}\right]$$

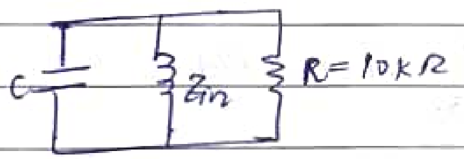
(a) $Z_{in} = jZ_0 \tan\beta l$

$= jZ_0$

$$Z_0 = \frac{1}{\omega_0 C} \Rightarrow C = \frac{1}{\omega_0 Z_0}$$

$$= \frac{1}{(2\pi)(6\times10^9)(50)} = 0.53\ pF.$$

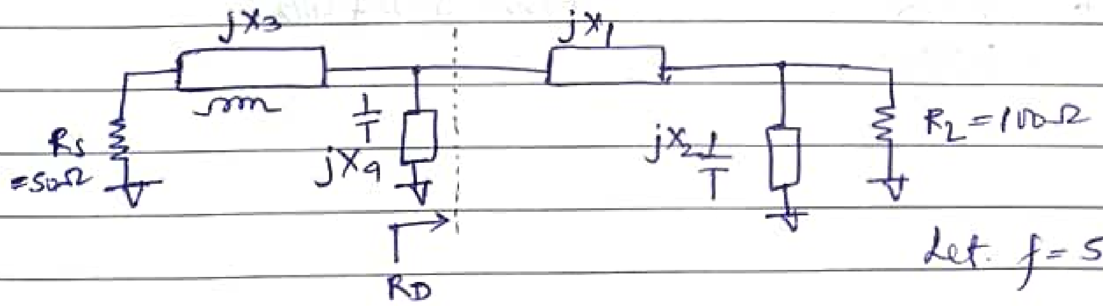
(b)



$$Q = \omega_0 RC = 2\pi(6\times10^9)\times10^4\times0.53\times10^{-12}$$

$$\simeq 200.$$

(8)



Let $f = 5GHz$.

(i) $RD = 60\Omega$

$$Q_{Right} = \sqrt{\frac{100}{60} - 1} = 0.816$$

$$Q_{left} = \sqrt{\frac{60}{50} - 1} = 0.447$$

$Q_R = X_2 R_L$

$$\Rightarrow X_2 = \frac{0.816}{100} = 0.00816\ \Omega$$

$Q_R = \frac{X_1}{R_D} \Rightarrow X_1 = 0.816 \times 60$
$= 48.96\,\Omega$.

$Q_L = \frac{X_3}{R_S} \Rightarrow X_3 = 0.447 \times 50$
$= 22.35\,\Omega$

$Q_L = X_4 R_D \Rightarrow X_4 = \frac{0.447}{60} = 0.00745\,\Omega$

$Q = Q_L + Q_R$
$= 1.263$

$B/W = 3.95$ GHz.

(ii) $R_D = 70.7\,\Omega$

$Q_R = \sqrt{\frac{100}{70.7} - 1} = 0.643$

$Q_R = X_2 R_L \Rightarrow X_2 = 0.00643\,\Omega$.

$Q_R = X_1 / R_D \Rightarrow X_1 = 45.46\,\Omega$.

$Q = Q_R + Q_L = 1.286\,\Omega$

$Q_L = \sqrt{\frac{70.7}{50} - 1} = 0.643$

$Q_L = X_3 / R_S \Rightarrow X_3 = 32.15\,\Omega$

$Q_L = X_4 R_D \Rightarrow X_4 = 0.0091\,\Omega$

$B/W = 3.88$ GHz.

(iii) $R_D = 80\,\Omega$

$Q_R = \sqrt{\frac{100}{80} - 1} = 0.5$

$X_2 = 0.005\,\Omega$

$X_1 = 40\,\Omega$

$Q_L = \sqrt{\frac{80}{50} - 1} = 0.774$

$X_S = 38.7\,\Omega$

$X_4 = 0.009675\,\Omega$

$Q = Q_L + Q_R$
$= 1.274$

$B/W = 3.924$ GHz.

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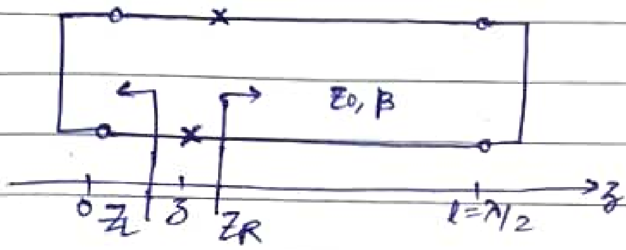

For lossless and short-circuited T/L,

$$Z = jZ_0 \tan\beta l$$

$$\Rightarrow Z_R = jZ_0 \tan\beta z$$

$$Z_L = jZ_0 \tan\beta(\lambda/2 - z)$$

$$= jZ_0 \tan(\pi - \beta z)$$

$$= -jZ_0 \tan\beta z.$$

$$\Rightarrow \boxed{Z_L = Z_R^*}$$