

# Indian Institute of Space Science and Technology

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B.Tech - 2nd Semester, 2017

MA121 - Vector Calculus and Differential Equations

Quiz-I

Date : 8th Feb, 2018

Time: 9:00am to 10:00am

( Answer all questions. Total marks 15.)

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1. (a) Show that  $W := \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 1\}$  is not a vector space over  $\mathbb{R}$  under the induced operation of the vector space  $\mathbb{R}^3$  over  $R$ . [1.5]
- (b) Show that in a vector space  $V$  over  $R$ , any element  $v$  has a unique additive inverse. Further, show that the inverse of  $v$  is  $-1 \cdot v$ . [3.5]
2. (a) Let  $f : \mathbb{R}^2 \longrightarrow R$  be given by

$$f(x, y) = \begin{cases} \frac{x^3 y}{x^4 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$$

Show that  $D_{\vec{v}}(f)|_{(0,0)} = \langle \nabla f|_{(0,0)}, \vec{v} \rangle$  for all unit vector  $\vec{v}$ . Further, show that  $f$  is not differentiable at  $(0, 0)$  [Hint: find “the limit” on moving along parabolas]. [3.5]

- (b) Let  $h(x, y) = 2e^{-x^2} + e^{-3y^2}$  denotes the height on a mountain at position  $(x, y)$ . With argument find the direction one should begin walking in order to climb the fastest from the point  $(1, 0)$ ? [1.5]
3. (a) Let  $C := \gamma(t)$ ,  $t \in [a, b]$  be a  $C^1$ -type curve. Show that  $-C$  is also of  $C^1$ -type. [2.5]
- (b) Let  $C$  be a curve given by the cartesian equation

$$x^2 = y^3, \quad y = z^2, \quad x, y, z \in \mathbb{R}, \quad x \geq 0, \quad z \geq 0.$$

Parametrize the curve with initial point  $(0, 0, 0)$ . Find the arc length function of the given curve with initial point  $(0, 0, 0)$ . [2.5]

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