Indian Institute of Space Science and Technology

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MA 221 - PDE, Calculus of variations and Complex Analysis

Tutorial-2

1. Determine the region in which the following PDE is hyperbolic, parabolic or elliptic. Also, find the canonical form on the respective region.

   (i) $xu_{xx} + u_{yy} = y^2$

   (ii) $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x$

   (iii) $u_{xx} - \sqrt{y}u_{xy} + \frac{x}{4}u_{yy} + 2xu_x - 3yu_y + 2u = \exp(x^2 - 2y), \quad y \geq 0.$

   (iv) $u_{xx} - \sqrt{y}u_{xy} + xu_{yy} = \cos(x^2 - 2y), \quad y \geq 0$

2. Obtain the general solution of the following PDE

   (i) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} + xyu_x + y^2u_y = 0$

   (ii) $u_{xx} + u_{xy} - 2u_{yy} - 3u_x - 6u_y = (2x - y)$

   (iii) $yu_x + 3yu_{xy} + 3u_x = 0, y \neq 0.$

3. Transform the following equation to the form $u_{\xi\eta} = cu$, c is a constant.

   (i) $u_{xx} - u_{yy} + 3u_x - 2u_y + u = 0$

   (ii) $3u_{xx} + 7u_{xy} + 2u_{yy} + uy + u = 0$

4. Transform the equation $u_{xx} + yu_{yy} + \sin(x + y) = 0$ into the canonical form. Hence, find the general solution.

5. Solve the following initial boundary value problem.

   (i)

$$\begin{aligned} u_{tt} &= c^2u_{xx}, \quad 0 < x < 1, \quad t > 0 \\ u(x,0) &= x(1-x), \quad 0 \leq x \leq 1 \\ u_t(x,0) &= 0, \quad 0 \leq x \leq 1 \\ u(0,t) &= u(1,t) = 0, \quad t > 0 \end{aligned}$$

   (ii)

$$\begin{aligned} u_{tt} &= c^2u_{xx}, \quad 0 < x < \pi, \quad t > 0 \\ u(x,0) &= 0, \quad u_t(x,0) = 8\sin^2 x, \quad 0 \leq x \leq \pi \\ u(0,t) &= u(\pi,t) = 0, \quad t > 0 \end{aligned}$$

(iii)

$$\begin{aligned} u_{tt} &= u_{xx} + x^2 \\ u(x,0) &= x, \quad u_t(x,0) = 0, \quad 0 \le x \le 1 \\ u(0,t) &= 0, \quad u(1,t) = 0, \quad t > 0 \end{aligned}$$

(iv)

$$\begin{aligned} u_t &= 4u_{xx}, \quad 0 < x < 1, \quad t > 0. \\ u(x,0) &= x^2(1-x), \quad 0 \le x \le 1 \\ u(0,t) &= 0, \quad u(1,t) = 0 \end{aligned}$$

(v)

$$\begin{aligned} u_t - ku_{xx} &= Ae^{-ax}, \quad 0 < x < \pi \\ u(x,0) &= \sin x, \quad 0 \le x \le \pi \\ u(0,t) &= u(\pi,t) = 0, \quad t \ge 0 \end{aligned}$$

(vi)

$$\begin{aligned} \nabla^2 u &= 0, \quad 0 \le r \le 10, \quad 0 \le \theta \le \pi. \\ u(10,\theta) &= \frac{400}{\pi}(\pi\theta - \theta^2) \\ u(r,0) &= u(r,\pi) = 0, \quad u(0,\theta) \text{ is bounded }. \end{aligned}$$

6. Solve the following PDE

(i) $(D - D' - 1)(D - D' - 2)u = 0$

(ii) $(D^2 + DD' + D + D' + 1)u = 0$

(iii) $(D^2 + 3DD' + 2D'^2)u = x + y$

(iv) $(D^2 - 3DD' + 2D'^2)u = l^{2x+3y} + \sin(x+y)$

(v) $(x^2D^2 - 2xyDD' + y^2D'2 - xD + 3yD')u = 8(\frac{x}{y})$

(vi) $(D^2 + DD' - 6D'^2)u = y\cos x$

(vii) $(D^2 + 2DD' + D'^2 - 2D - 2D')u = 0$

(viii) $(2D^2 - D'^2 + D)u = 0.$

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