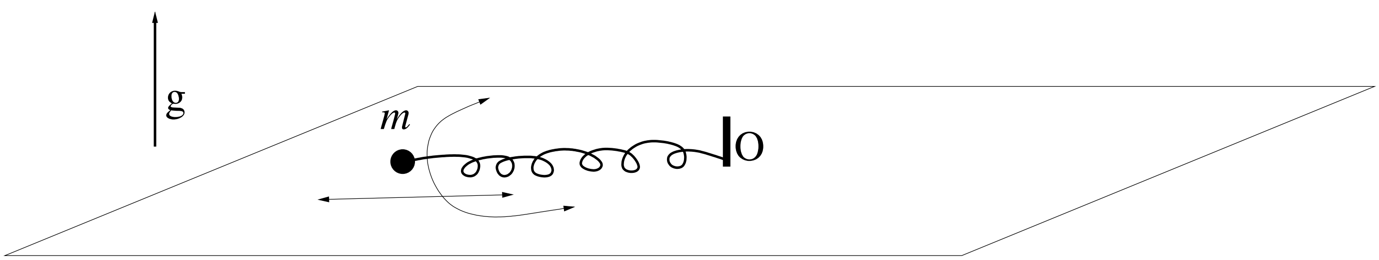
INDIAN INSTITUTE OF SPACE SCIENCE & TECHNOLOGY

B. Tech(I Year) **Physics - I (PH111)** **Quiz 1**

15 Dec' 2022 **Duration:1 Hrs** **Full Marks: 30**

Answer ALL questions (All questions carry equal marks)

1. A pendulum of mass m (assumed to be a point mass) connected to the origin by a massless spring of spring constant k and free length r_0, is otherwise free to move on a frictionless horizontal plane (perpendicular to gravity, see figure). The spring is allowed to stretch or contract only along its length, but not bend. Write the equation of motion for this spring pendulum, and identify two constants of motion.



Answer Key:

$$m(\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + m(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} = -k(r - r_0)\hat{\mathbf{r}}$$

$$m(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} = 0 \Rightarrow mr^2\dot{\theta} = l \text{ is one constant.}$$

From the $\hat{\mathbf{r}}$ part, $E = \frac{1}{2}(m\dot{r}^2 + \frac{l^2}{mr^2} + k(r - r_0)^2)$ is the other other constant.

2. A point mass m is free to slide inside a tube without friction (see figure). From time $t = 0$, the tube is rotated with a constant angular velocity ω about the point O. If the point mass is initially at rest, at a distance r_0 from the point O: a) find its radial distance as a function of time, b) Find the magnitude and direction of the force acting on the mass as a function of r.

Answer key: Any force experienced is only along the $\hat{\theta}$ direction. Hence $\ddot{r} - r\dot{\theta}^2 = 0$. But $\dot{\theta} = \omega$, and $\ddot{\theta} = 0$:

Solution: a) $r = r_0 \cosh(\omega t)$, b) $\mathbf{F} = 2\dot{r}\dot{\theta}\hat{\theta} = \omega^2 r_0 \sinh(\omega t)\hat{\theta} = \omega^2(r^2 - r_0^2)^{1/2}\hat{\theta}$

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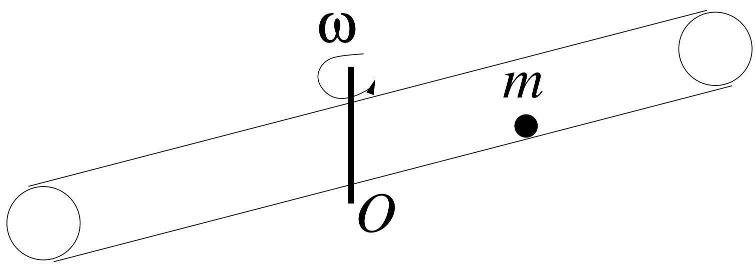
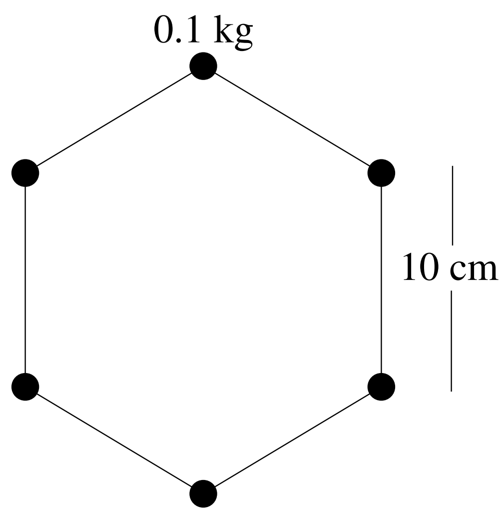


3. Six equal point masses of 0.1 kg each are fixed at the vertices of a regular hexagon in a plane with sides of length 10 cm, using thin rigid rods of negligible mass. Evaluate the moment of inertia about an axis a) passing through the center of the hexagon perpendicular to the plane, b) axis passing through one of the masses and perpendicular to the plane, c) Axis passing through two adjacent masses, d) axis passing through two masses separated by one mass in between.



Answer key: a) $6ml^2$, b) $12ml^2$, c) $\frac{15}{2}ml^2$, d) $\frac{9}{2}ml^2$.

4. i) A 2.00 kg mass oscillates with an initial amplitude of 3.00 cm. The force constant of the spring is 400 N/m. Find (a) the period, and (b) the total initial energy. (c) Now a mild damping is applied and the energy decreases by 1.00 percent per period. Find the linear damping coefficient b (in kg/s).

   ii) Show that the ratio of the amplitudes for two successive oscillations is constant when mild damping is present.

   Answer key:

i) Recall $x(t) = A\cos(\omega_0 t + \phi)$, and energy $E_0 = \frac{1}{2}kA^2$. From given initial conditions, $\phi = 0$.

a) Period $T = 2\pi\sqrt{\frac{m}{k}} = \frac{\sqrt{2}}{10} \cdot \pi$.

b) Energy $E_0 = 0.18$ J.

c) With damping, $x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t) \sim Ae^{-\frac{b}{2m}t}\cos(\omega_0 t)$ (for weak damping), and $E = \frac{1}{2}kA^2 e^{-\frac{b}{m}t}$

From data given $\frac{E_1}{E_0} = e^{-\frac{b}{m}T} = 0.99 \Rightarrow b = -\frac{m}{T}\ln(0.99) = \frac{-1}{10\sqrt{2}\cdot\pi}\ln(0.99)$.

ii) From expressions given in c) above, $\frac{x((n+1)T)}{x(nT)} = e^{-\frac{bT}{2m}}$.