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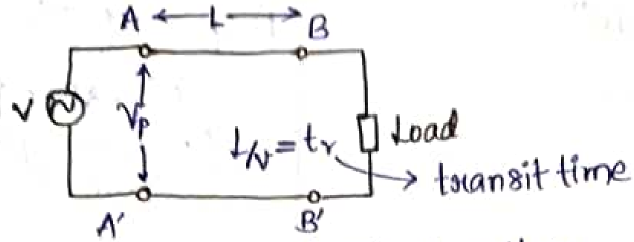
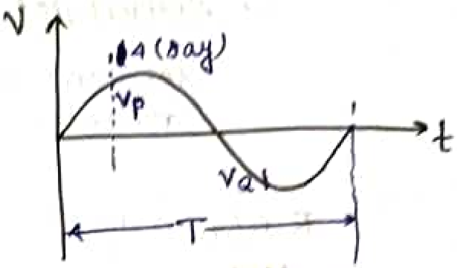
ELECTROMAGNETICS THEORY

1-08-2023

Transmission Line Theory

Transmission lines

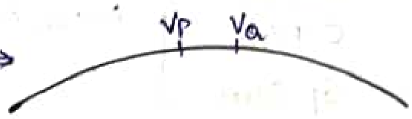




v : velocity of voltage

$t_r = \frac{L}{v}$

$t_r \ll T \Rightarrow t_r$ can be neglected. $\Rightarrow$



$\frac{L}{v} \ll \frac{1}{f}$

$\Rightarrow L \ll \frac{v}{f}$

$\Rightarrow \boxed{L \ll \lambda}$

Wavelength : Distance b/w two points of phase difference 2π.

↳ If wavelength is much larger than L, transit time can be neglected.

For $f = 50$ Hz,

$\lambda = \frac{3 \times 10^8 \text{ m/s}}{50 \text{ /s}} = 6 \times 10^6 \text{ m}$

For $f = 1$ GHz,

$\lambda = \frac{3 \times 10^{10} \text{ cm/s}}{10^9 \text{ /s}} = 30 \text{ cm}$

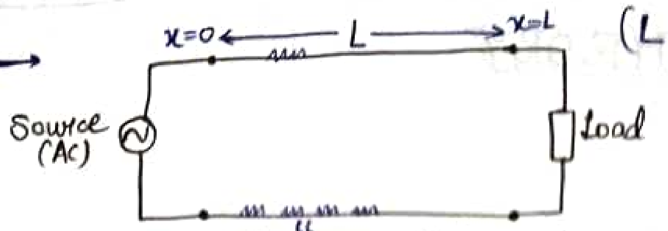
→ Antenna size should be comparable to λ (usually, $\lambda/2$).

→ More the frequency, higher the absorption of waves.

→ More the energy, higher the amplification (to cater to more antennas).

→ Receiving frequency = sending freq.

2-08-2023

→ 

$(L \ll \lambda)$

↳ "Distributed elements"
↓
resistance everywhere across the circuit.

$R = \frac{1}{\sigma}\frac{l}{A}$

Purpose of T/L
⇓
To transfer power from one place to another

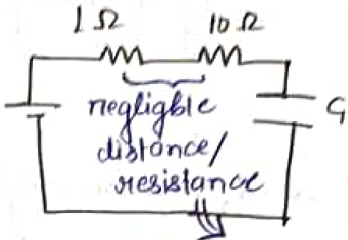
$R\ \Omega/m$ → in series

$L\ H/m$

$C\ F/m$

$G\ S/m$

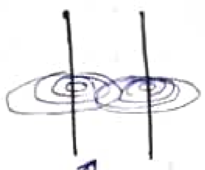
} Fundamental parameters



⇓

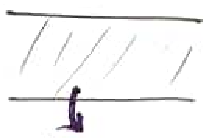
"Lumped elements"

#



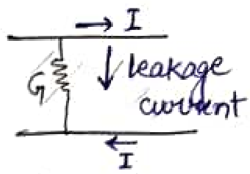
Mag. field around conductors a/c to Ampere's law
⇓
Giving inductance

#



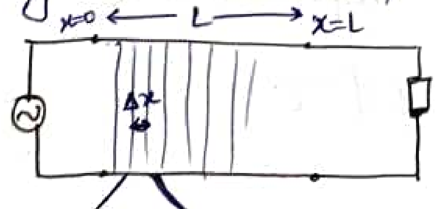
Dielectric medium b/w 2 conductors → (air, moisture, etc.)
⇓
Giving capacitance

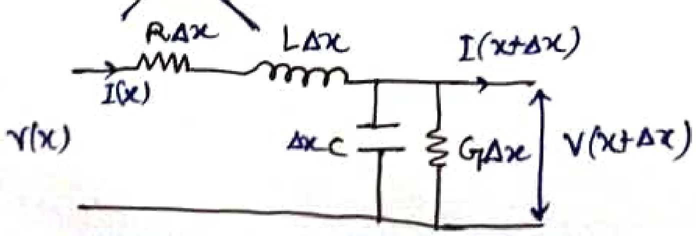
#



⇝ leakage of current in some conducting medium (if not fully insulator)
⇓
Giving G (conductance)

→ Dividing T/L into small parts:





Assume, Δx is very-very small, so we can apply KVL and KCL.

$e^{j\omega t} \rightarrow \frac{d}{dt} e^{j\omega t} = j\omega e^{j\omega t}$
signal

$$V(x+\Delta x) - V(x) = -(R+j\omega L)\Delta x\, I(x)$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{V(x+\Delta x) - V(x)}{\Delta x} = -(R+j\omega L) I(x) \Rightarrow$$

and,

$$I(x+\Delta x) - I(x) = -(G+j\omega C)\Delta x * V(x)$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{I(x+\Delta x) - I(x)}{\Delta x} = -(G+j\omega C) V(x) \Rightarrow$$

$$\boxed{\begin{aligned} \frac{dV(x)}{dx} &= -(R+j\omega L) I(x) \\ \frac{dI(x)}{dx} &= -(G+j\omega C) V(x) \end{aligned}}$$

↳ Transmission Line Equations

By Taylor series,
$V(x+\Delta x) \rightarrow$
$I(x+\Delta x) \rightarrow$

Differentiating,

$$\frac{d^2V(x)}{dx^2} = -(R+j\omega L)\frac{dI(x)}{dx}$$

$$\Rightarrow \frac{d^2V(x)}{dx^2} = +\underbrace{(R+j\omega L)(G+j\omega C)}_{\text{Const.} \rightarrow \gamma^2} V(x)$$

$$\gamma = \alpha + j\beta$$

$$\frac{d^2V(x)}{dx^2} = \gamma^2 V(x), \text{ where } \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

Solutions: $e^{\gamma x}, e^{-\gamma x}$ or $\underbrace{e^{\gamma x} + e^{-\gamma x}, e^{\gamma x} - e^{-\gamma x}}_{\text{hyperbolic fn.}}$

$c_1 e^{\gamma x} + c_2 e^{-\gamma x}$

↓
$\cosh(\gamma x)$, $\sinh(\gamma x)$
↓ ↓
$\cos(\gamma x)$ $\sin(\gamma x)$

$$\therefore \boxed{V(x) = V_1 e^{\gamma x} + V_2 e^{-\gamma x}}$$

↳ w.r.t. space

To show variation of $V(x)$ wrt. time, multiply the term by $e^{j\omega t}$.

$$\begin{aligned} V(x,t) &= (V_1 e^{\gamma x} + V_2 e^{-\gamma x}) e^{j\omega t} \\ &= V_1 e^{(\gamma x + j\omega t)} + V_2 e^{(-\gamma x + j\omega t)} \\ &= V_1 e^{\alpha x} e^{j(\omega t + \beta x)} + V_2 e^{-\alpha x} e^{j(\omega t - \beta x)} \end{aligned}$$

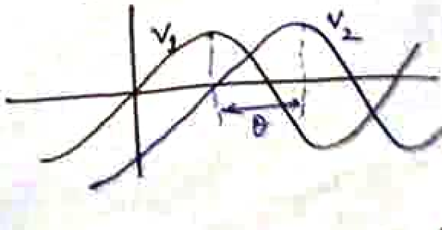
$e^{\alpha x}$: tells how the amplitude of the wave is changing (as $\alpha > 0$ & Real)
↓
defines amplitude

$e^{j(\omega t + \beta x)}$: defines phase

$e^{-\alpha x}$: amplitude

$e^{j(\omega t - \beta x)}$: phase

$e^{j\theta} \rightarrow$ here, θ = phase



→ $\gamma = \alpha + j\beta$, α, β: constants

γ: propagation constant

α: defines amplitude/attenuation
↳ attenuation constant
Neper (Np/m)

β: defines phase
↓
phase constant
(rad/m)

$\rightarrow \dfrac{d^2V(x)}{dx^2} = \gamma^2 V(x),$

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$= \alpha + j\beta$

attenuation const. (Np/m) — phase const. (rad/m)

Soln: $V(x) = V_1 e^{\gamma x} + V_2 e^{-\gamma x}$ → wrt. space

$V(x,t) = \underbrace{V_1 e^{\alpha x} e^{j(\omega t+\beta x)}}_{\text{Backward wave}} + \underbrace{V_2 e^{-\alpha x} e^{j(\omega t-\beta x)}}_{\text{moving in +ve x-dirn} \Rightarrow \text{Forward wave}}$

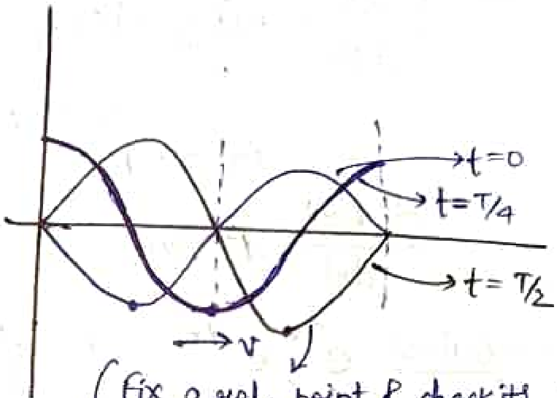
$Im\{V_2 e^{-\alpha x} e^{j(\omega t-\beta x)}\}$

$V_f = \underbrace{V_2 e^{-\alpha x}}_{=1 \text{ (say)}} \sin(\omega t - \beta x)$ ($\alpha x \to 0$)

$\approx \sin(\omega t - \beta x)$

- $\omega = \dfrac{2\pi}{T}$

$t=0 \rightarrow -\sin(\beta x)$
$t = T/4 \rightarrow \sin(\pi/2 - \beta x) = \cos(\beta x)$
$t = T/2 \rightarrow \sin(\pi - \beta x) = \sin(\beta x)$



(Fix a ref. point & check its movement to get velocity)
(has a const. phase)

→ $\omega t\uparrow - \beta x = \text{const.}$
↳ $t\uparrow$, so x should $\uparrow$ to make the term const.

#MATLAB

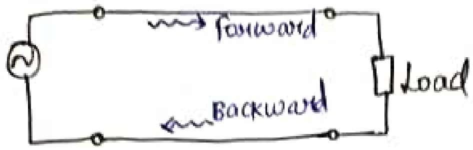
→ $\omega t + \beta x = \text{const.}$
Diff. wrt. t,

$\omega - \beta \dfrac{dx}{dt} = 0$

$\Rightarrow \boxed{v = \dfrac{\omega}{\beta}}$

→ Solution: $V(x) = V_1^- e^{\gamma x} + V_2^+ e^{-\gamma x}$

$$= \underbrace{V_1 e^{\alpha x} e^{+j\beta x}}_{\text{Backward}} + \underbrace{V_2 e^{-\alpha x} e^{-j\beta x}}_{\text{Forward}}$$



⇒ For max Power transfer, we won't want the waves to come back to the source (less wave).

$$\frac{d^2 I(x)}{dx^2} = \gamma^2 I(x) \Rightarrow I(x) = I_1 e^{\gamma x} + I_2 e^{-\gamma x}$$

↓ but it introduces 2 new constants

So, from $V(x)$ eq^n;

$$\frac{dV}{dx} = -(R+j\omega L)\, I(x)$$

$$\Rightarrow I(x) = \frac{-1}{(R+j\omega L)} \gamma \left\{ V_1 e^{\gamma x} - V_2 e^{-\gamma x} \right\}$$

$$= \sqrt{\frac{G+j\omega C}{R+j\omega L}} \left(V_2 e^{-\gamma x} - V_1 e^{\gamma x} \right)$$

$$\Rightarrow I(x) = \frac{1}{Z_0} \left(V_2 e^{-\gamma x} - V_1 e^{\gamma x} \right), \text{ where } Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

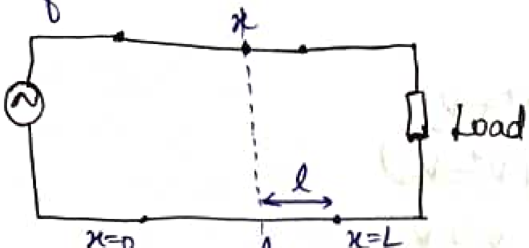
"characteristic Impedence" (So called b/c it depends only on T/L; chst. of T/L)

→ Explicitely, we don't show the variation with time.

$$I(x) = \frac{1}{Z_0} \left(V^+ e^{-\gamma x} - V^- e^{\gamma x} \right)$$

V^+, V^- ⇝ magnitudes of forward/backward waves, respectively.

→ In T/L, it is always easier to work with load as reference, instead of source.



$$\boxed{l = -x}$$

l ↘ distance from load

$$\therefore \quad V(l) = V^+ \underbrace{e^{+\gamma l}}_{\text{For.}} + V^- \underbrace{e^{-\gamma l}}_{\text{Back.}}$$

$$I(l) = \frac{1}{Z_0} \left(V^+ e^{\gamma l} - V^- e^{-\gamma l} \right)$$

→ $\frac{d^2V(x)}{dx^2} = \gamma^2 V(x)$

Solⁿ: $\underbrace{V^+e^{-\gamma x}}_{\text{forward wave}} + \underbrace{V^-e^{\gamma x}}_{\text{backward wave}}$

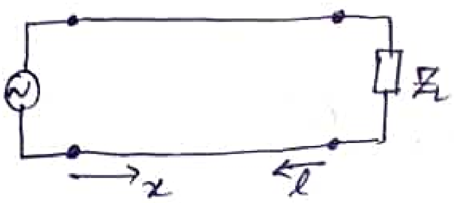
Using $\frac{dV}{dx} = -(R+j\omega L)I(x)$,

$$I(x) = \frac{1}{Z_0}\left(V^+e^{-\gamma x} - V^-e^{\gamma x}\right)$$

Z_0 → Chct. Impedence of T/L

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

As we are interested in load end,



$$V(l) = \underbrace{V^+e^{\gamma l}}_{\text{For. wave}} + \underbrace{V^-e^{-\gamma l}}_{\text{Back. wave}}$$

$$\left[\begin{aligned} e^{\gamma l}e^{j\omega t} &= e^{(\alpha+j\beta)l}e^{j\omega t} \\ &= e^{\alpha l}e^{j\underbrace{(\omega t + \beta l)}_{\text{phase}=\text{const.}}} \end{aligned}\right]$$

$$I(l) = \frac{1}{Z_0}\left(V^+e^{\gamma l} - V^-e^{-\gamma l}\right)$$

→ $\frac{\text{Forward voltage}}{\text{Forward current}} = Z_0$

$\frac{\text{Backward voltage}}{\text{Backward current}} = -Z_0$

If we move from:
Source → Load : Power dissipated $(+Z_0)$
Load → Source : Power gain $(-Z_0)$

→ Load Impedance:

$$Z_L = Z(l=0) = \frac{V^+ + V^-}{\frac{1}{Z_0}(V^+ - V^-)}$$

$$= Z_0 \frac{V^+ + V^-}{V^+ - V^-}$$

$$= Z_0 \frac{\cancel{V^+}(1 + V^-/V^+)}{\cancel{V^+}(1 - V^-/V^+)} = Z_0 \frac{1 + V^-/V^+}{1 - V^-/V^+}$$

→ Forward voltage = $V^+e^{\gamma l}$

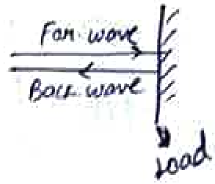
Backward voltage = $V^-e^{-\gamma l}$

$$\frac{\text{backward voltage}}{\text{forward voltage}} = \frac{V^-e^{-\gamma l}}{V^+e^{\gamma l}}$$

⇓

"Reflection coefficient" $= \frac{V^-}{V^+}e^{-2\gamma l}$

$$\boxed{\Gamma(l) = \frac{V^-}{V^+}e^{-2\gamma l}}$$



$\Gamma(l)\big|_{l=0} = \Gamma_L = \frac{V^-}{V^+}$ → "Reflection coeff." at load end (Load reflection coeff.)

$$Z_L = Z_0\frac{1+\Gamma_L}{1-\Gamma_L}$$

$$\Rightarrow \boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

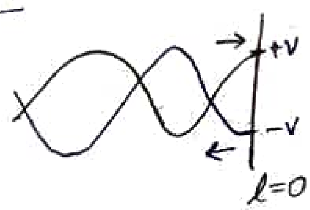
$\Gamma_L = 0$ if $Z_L = Z_0$ (Terminating the T/L with chrt. impedence)

- → no backward wave
- → ideal (max. power is given to the load)
- → "MATCHED" T/L.

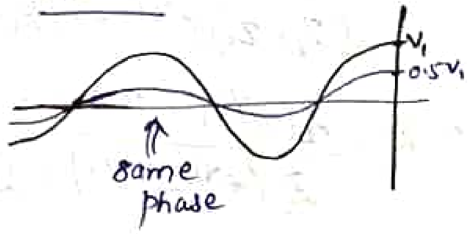
Eg. $\Gamma_L = \underbrace{-0.5}_{\text{const}} = 0.5e^{j\pi}$

↳ phase change

$\Gamma_L = -1$

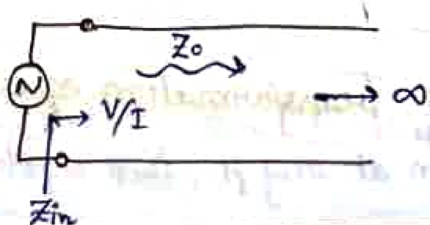


$\Gamma_L = 0.5$



Characteristic Impedence:

- ↳ Input impedence of an infinite length transmission line (Qualitative defn)
- ↳ Impedence with which T/L is terminated to get no backward wave (max. power transfer to the load). ⇝ (Mathematical defn)



* Power dissipation $= \frac{1}{2}VI^*$

9-08-2023

$$\rightarrow \Gamma(l) = \Gamma_L e^{-2\gamma l}$$

$\Gamma_L = \dfrac{Z_L - Z_0}{Z_L - Z_0} = |\Gamma_L| e^{j\theta_L}$, θ_L: phase diff. b/w incident & reflected waves.

$$V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$$

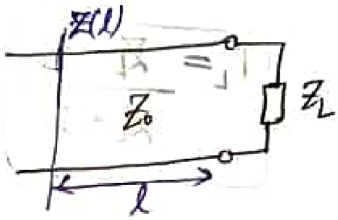
$$I(l) = \frac{1}{Z_0}\left(V^+ e^{\gamma l} - V^- e^{-\gamma l}\right)$$

$$V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$$

$$= V^+ e^{\gamma l}\left(1 + \frac{V^-}{V^+} e^{-2\gamma l}\right)$$

$$\Rightarrow V(l) = V^+ e^{\gamma l}\left(1 + \Gamma(l)\right)$$

$$I(l) = \frac{V^+ e^{\gamma l}}{Z_0}\left(1 - \Gamma(l)\right)$$



Impedance at any point along T/L,

$$Z(l) = Z_0 \frac{1 + \Gamma_L e^{-2\gamma l}}{1 - \Gamma_L e^{-2\gamma l}} \qquad \left(\because Z(l) = \frac{V(l)}{I(l)}\right)$$

$$= Z_0 \cdot \frac{(Z_L + Z_0) + (Z_L - Z_0) e^{-2\gamma l}}{(Z_L + Z_0) - (Z_L - Z_0) e^{-2\gamma l}} \cdot \frac{e^{\gamma l}}{e^{\gamma l}} \qquad \left(\because \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}\right)$$

$$= Z_0 \frac{(Z_L + Z_0) e^{\gamma l} + (Z_L - Z_0) e^{-\gamma l}}{(Z_L + Z_0) e^{\gamma l} - (Z_L - Z_0) e^{-\gamma l}}$$

$$= Z_0 \cdot \frac{Z_L(e^{\gamma l} + e^{-\gamma l}) + Z_0(e^{\gamma l} - e^{-\gamma l})}{Z_L(e^{\gamma l} - e^{-\gamma l}) + Z_0(e^{\gamma l} + e^{-\gamma l})}$$

$$Z(l) = Z_0 \frac{Z_L \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_0 \cosh(\gamma l) + Z_L \sinh(\gamma l)}$$

$$\Rightarrow \boxed{Z(l) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}}$$

$\cos(x) = \dfrac{e^{jx} + e^{-jx}}{2}$

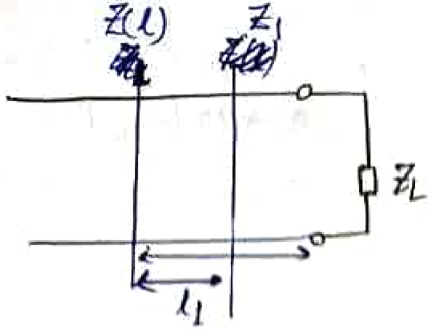
$\cosh(x) = \dfrac{e^{x} + e^{-x}}{2}$

$\sinh(x) = \dfrac{e^{x} - e^{-x}}{2}$

↳ "Impedance Transformation equation"

⇓

(If impedance is known at any pt., then impedence can be calculated at any other pt.)



$$Z(l) = Z_0 \frac{Z_1 + Z_0 \tanh(\gamma l_1)}{Z_0 + Z_1 \tanh(\gamma l_1)} \quad ; \quad Z_1 = Z_0 \frac{Z(l) - Z_0 \tanh(\gamma l_1)}{Z_0 - Z(l) \tanh(\gamma l_1)}$$

If l is measured from source, $l = -l$.

$$Z(l) = Z_0 \cdot \frac{Z_1 - Z_0 \tanh(\gamma l)}{Z_0 - Z_1 \tanh(\gamma l)} \qquad (\because \tanh(-\gamma l) = -\tanh(\gamma l))$$

Eg. $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$Z_L = 100\Omega,\ Z_0 = 50\Omega \Rightarrow \Gamma_L = 1/3$

$Z_L = 200\Omega,\ Z_0 = 100\Omega \Rightarrow \Gamma_L = 1/3$

↳ It's the ratio $\left(\dfrac{Z_L}{Z_0}\right)$ that matters.

→ $\dfrac{Z_L}{Z_0} = z_L$: "Normalised Impedance"

So, $$z(l) = \frac{z_L + z_0 \tanh(\gamma l)}{z_0 + z_L \tanh(\gamma l)}.$$

→ $V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$

γ : propagation constant.

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$= \alpha + j\beta$

α : attenuation constant (unit: Np/m)

1 Np/m: ~~Attenuation when $e^{-\alpha x}$ reduces to e^{-1} for 1V after 1m~~.

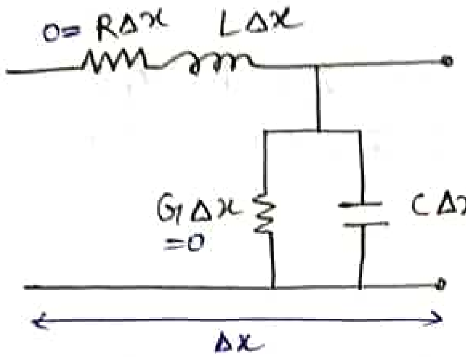
↳ Attenuation when 1 unit V reduces to e^{-1} after 1m.

→ $1\ Np/m = -8.68\ dB/m$

$20 \log e^{-1}$

$dB \rightarrow 10 \log_{10} P$

$Np \rightarrow 20 \log V \quad (\because P \propto V^2)$



→ only resistors dissipate power

→ For no power loss, all R=0

$$\gamma = \sqrt{(\cancel{R}^{0} + j\omega L)(\cancel{G}^{0} + j\omega C)}$$

$$= j\omega\sqrt{LC} \rightsquigarrow \text{Purely Imaginary } (\beta)$$

$$= \cancel{\alpha}^{0} + j\beta$$

$$\therefore V = V^+ \cancel{e^{\alpha l}}^{1} e^{j\beta l} + V^- \cancel{e^{-\alpha l}}^{1} e^{-j\beta l}$$

$$\Rightarrow \boxed{V = V^+ e^{j\beta l} + V^- e^{-j\beta l}} \Rightarrow \text{Lossless T/L}$$

- $V^+ e^{j\beta l}$

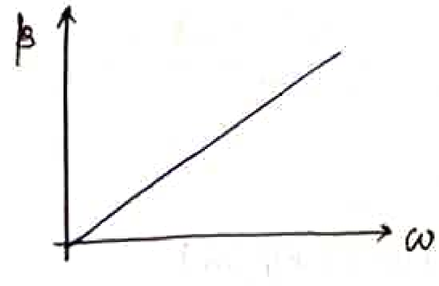
For β = real : Lossless

For $\beta = (a+jb)$: $V = V^+ e^{j(a+jb)l}$

$= V^+ e^{jal} e^{-bl}$: Lossy

→ For purely lossless T/L :

$\boxed{\alpha = 0}$; $\boxed{\beta = \omega\sqrt{LC}}$ → Linear



⇝ ω-β diagram (or ω-k diagram) (or dispersion diagram)

Input → transmitted signal : Dispersion (change in shape of wave) → not desirable

⊓ → ⊓ or ⊓ → fine ; can be amplified

10-08-2023

Lossless T/L

$\alpha = 0, \quad \beta = \omega\sqrt{LC}$

Phase velocity, $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

↳ Independent of ω.

For band (several frequency components):

$\omega_1, \omega_2, \ldots, \omega_N$

↳ all will have same phase velocity (v_p)

~~Lossless:~~

Lowloss:

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$R \ll \omega L \Rightarrow R/\omega L \ll 1$

$G \ll \omega C$

$$\therefore \gamma = \left\{ j\omega L \left(1 - j\frac{R}{\omega L}\right) j\omega C \left(1 - j\frac{G}{\omega C}\right) \right\}^{1/2}$$

$$= j\omega\sqrt{LC} \left\{ 1 - j\left(\frac{G}{\omega C} + \frac{R}{\omega L}\right) - \cancel{\left(\frac{R}{\omega L}\right)\left(\frac{G}{\omega C}\right)} \right\}^{1/2}$$

neglect

$$\approx j\omega\sqrt{LC} \left\{ 1 - j\left(\frac{G}{\omega C} + \frac{R}{\omega L}\right) \right\}^{1/2}$$

$$\approx j\omega\sqrt{LC} \left\{ 1 - j\frac{1}{2}\left(\frac{G}{\omega C} + \frac{R}{\omega L}\right) \right\} \qquad \left[\because (1+x)^n \approx 1 + nx\right]$$

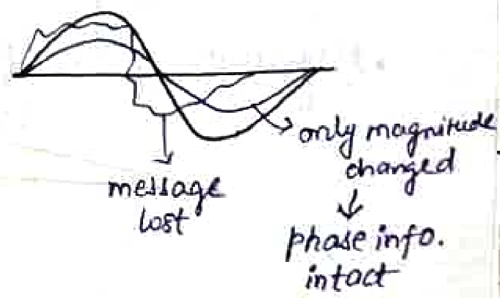
$$\approx j\omega\sqrt{LC} + \frac{\sqrt{LC}}{2}\left(\frac{G}{C} + \frac{R}{L}\right)$$

$$\therefore \boxed{\gamma \approx \underbrace{j\omega\sqrt{LC}}_{\beta} + \underbrace{\frac{1}{2}\left(G\sqrt{\frac{L}{C}} + R\sqrt{\frac{C}{L}}\right)}_{\alpha\ (\neq 0)}}$$

↳ constant

~~no change in phase~~

↳ only amplitude changes



Characteristic impedence:

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} \longrightarrow \text{Real} \Rightarrow \text{Lossless}$$

Eg. If $Z_0 = 75\,\Omega \longrightarrow$ lossless T/L

If $Z_0 = (50 + j100)\,\Omega \longrightarrow$ lossy T/L

$\rightarrow V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$

If T/L is lossless,

$$V(l) = V^+ e^{j\beta l} + V^- e^{-j\beta l}$$

$$I(l) = \frac{1}{Z_0}\left(V^+ e^{j\beta l} - V^- e^{-j\beta l}\right)$$

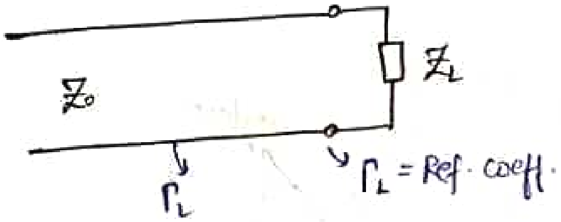
→ For lossy T/L, we only have to add the constant α.

$$\Gamma(l) = \Gamma_L e^{-j2\beta l}, \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Z_0 : Real
Z_L : can be Imaginary

$$= |\Gamma_L| e^{j\theta_L} e^{-j2\beta l}$$

$$= |\Gamma_L| e^{\underbrace{j(\theta_L - 2\beta l)}_{\text{phase term}}}$$



$$\rightarrow V(l) = V^+ e^{j\beta l} + V^- e^{-j\beta l}$$

$$= V^+ e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right)$$

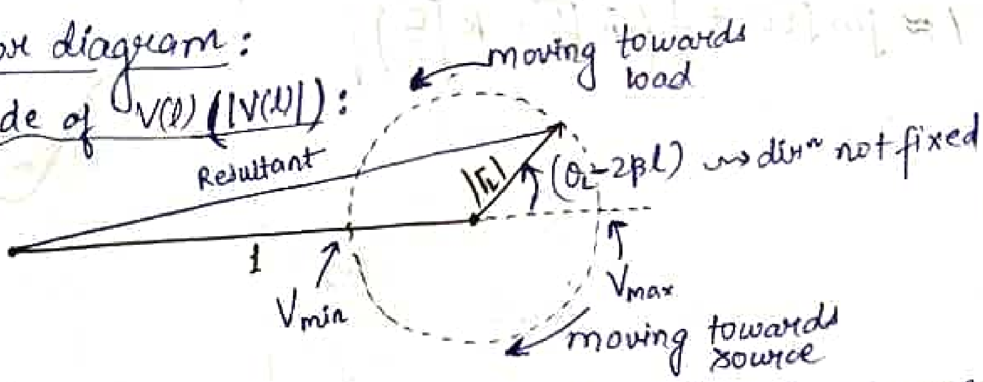
$$V(l) = V^+ e^{j\beta l}\left(1 + |\Gamma_L| e^{j(\theta_L - 2\beta l)}\right), \quad \theta_L : \text{constant}$$

$$I(l) = \frac{V^+ e^{j\beta l}}{Z_0}\left(1 - |\Gamma_L| e^{j(\theta_L - 2\beta l)}\right)$$

only l varies

Phasor diagram:

- Magnitude of $V(l)$ ($|V(l)|$):



For finding Magnitude:
$|V^+ e^{j\beta l}| = 1$
as $|e^{j\theta}| = 1$
will give some additional phase only.

↳ Voltage (magnitude) varies along the T/L sinusoidally.

↳ For V_{max}: phase $= \theta_L - 2\beta l_{max} = 0$

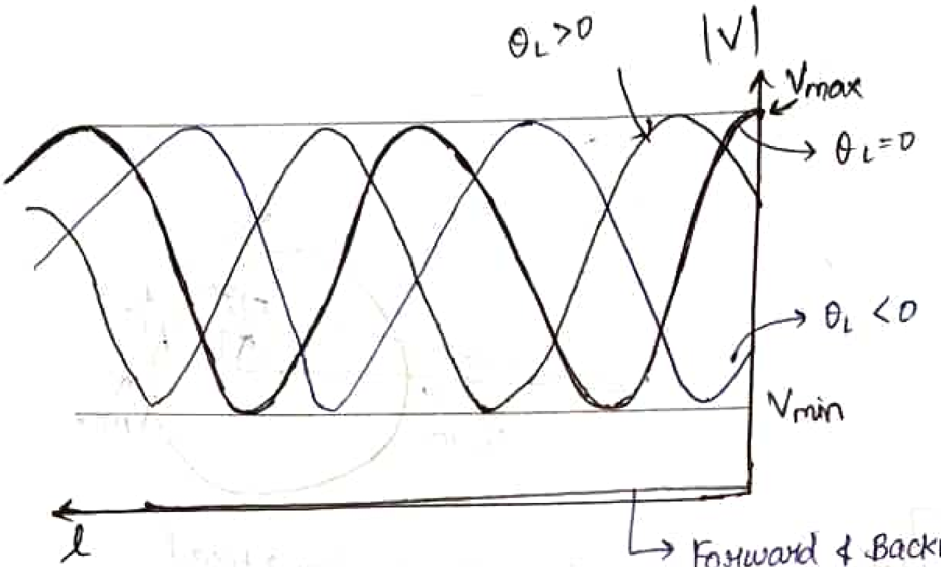
For V_{min}: phase $= \theta_L - 2\beta l_{min} = \pi$

→ Whenever there is V_{max}, there is I_{min}.
Whenever there is V_{min}, there is I_{max}.

~~→ Whenever~~

→ When moving Load → source, $l \uparrow \Rightarrow (\theta_L - 2\beta l) \downarrow \Rightarrow$ Clockwise

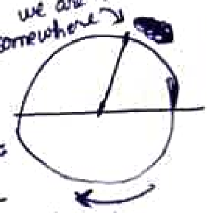
→ When moving Source → Load, $l \downarrow \Rightarrow (\theta_L - 2\beta l) \uparrow \Rightarrow$ Anticlockwise



→ Forward & Backward waves interacting
"Standing wave"
(Voltage standing wave)

• For $\theta_L = 0$, at load end,
$l = 0$
$\Rightarrow$ phase $= 0$
$\Rightarrow V_{max}$

• For $\theta_L > 0 \Rightarrow$ we are here somewhere



First we'll get V_{max}

moving towards source ($l \uparrow$)

→ $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

For real Γ_L, Z_L = real.

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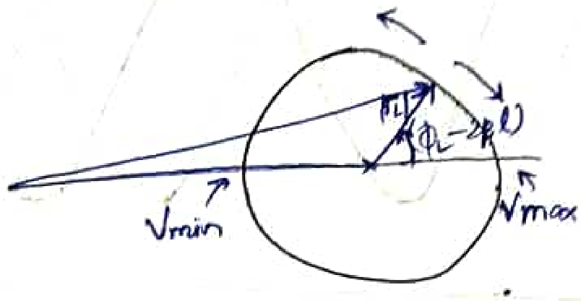
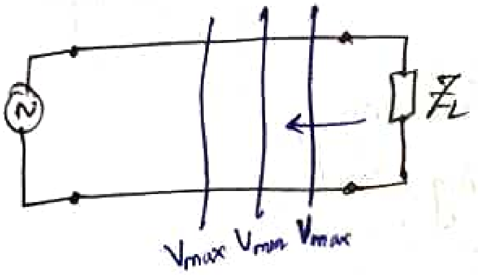
$\rightarrow V(l) = V^+ e^{j\beta l}\left(1 + |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)$

$I(l) = \frac{V^+ e^{j\beta l}}{Z_0}\left(1 - |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)$, ϕ_L : +ve, −ve or 0.

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

$= |\Gamma_L| e^{j\phi_L}$





$\Rightarrow$ V_{max}, V_{min} : value → fixed

$\rightarrow \rho = \frac{V_{max}}{V_{min}}$

(or S)

"Voltage standing Wave Ratio" (VSWR)

$\rightarrow V_{max} = |V^+|(1 + |\Gamma_L|)$

$V_{min} = |V^+|(1 - |\Gamma_L|)$

$\rho = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$

$\rightarrow -1 \le \Gamma_L \le 1 \Rightarrow 0 \le |\Gamma_L| \le 1$ } For a lossless T/L.

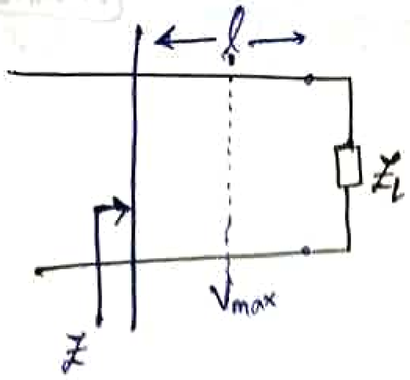
$1 \le \rho \le \infty$

$\rightarrow$ At the position of V_{max}, $I \rightarrow I_{min} \Rightarrow Z \rightarrow Z_{max}$.

$Z_{max} = \frac{V_{max}}{I_{min}} = \frac{|V^+|(1 + |\Gamma_L|)}{\frac{|V^+|}{Z_0}(1 - |\Gamma_L|)}$ ($\cancel{|Z|} \rightarrow Z$ as Z is ~ real)

$\Rightarrow \boxed{Z_{max} = Z_0 \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}} \Rightarrow \boxed{Z_{max} = Z_0 \rho} \longrightarrow$ Real

Real Real



$$Z_0 \frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}$$

$\downarrow$ complex $= R_L \pm jX_L$

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

$\cosh(x) = \frac{e^x + e^{-x}}{2}$

$\sin(jx) = j\,\frac{e^{jx} - e^{-jx}}{2j} = j\sin(jx)$

$\cos(jx) = \frac{e^{jx} + e^{jx}}{2} = \cos(x)$

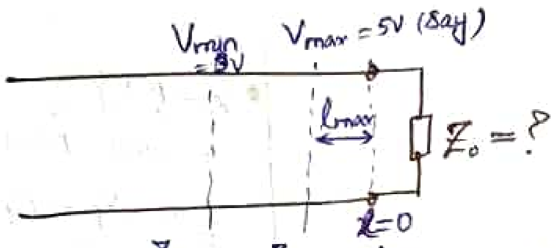
$$Z_{min} = \frac{V_{min}}{I_{max}} = \frac{|V^+|(1 - |\Gamma_L|)}{\frac{|V^+|}{Z_0}(1 + |\Gamma_L|)}$$

$$= Z_0 \frac{1 - |\Gamma_L|}{1 + |\Gamma_L|}$$

$$\Rightarrow \boxed{Z_{min} = Z_0/\rho}$$

We can't measure voltage using voltmeter in a high freq. T/L.

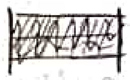
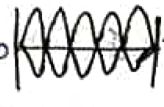
#



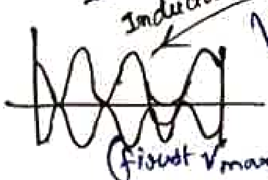
$\leftarrow$ masure V, starting at $l=0$

$\rho = \frac{V_{max}}{V_{min}} = \frac{5}{3}$

$\rightarrow \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

- $Z_L = Z_0$, $\Gamma_L = 0 \rightarrow$ MATCHED $\rightarrow$  $\rightarrow$ no reflection
- $\Gamma_L = 1$, $Z_L = \infty \rightarrow$ OPEN circuit
- $\Gamma_L = -1$, $Z_L = 0 \rightarrow$ SHORT circuit $\rightarrow$  $\rightarrow$ V_{max} doesn't move (forward or backward)
- $|\Gamma_L| = 1 \rightarrow$ Purely inductive or capacitive ~~conductive~~ $\rightarrow$ Power dissipation $= 0$.

~~Purely~~ Inductive

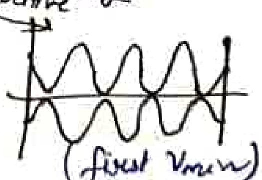


Vmin touches zero($=0$)

$$\left[\; |\Gamma_L| = \left| \frac{\pm jX - Z_0}{\pm jX + Z_0} \right| = 1 \;\right]$$

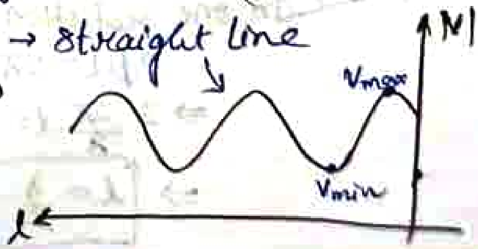
$\pm jX$: purely imaginary; Z_0: fixed

Capacitive Purely



$\longrightarrow \rho = 1 \Rightarrow V_{max} = V_{min} \rightsquigarrow |V|$ vs $l \rightarrow$ straight line

$(\Gamma_L = 0) \rightarrow Z_L = Z_0 \rightarrow$ MATCHED



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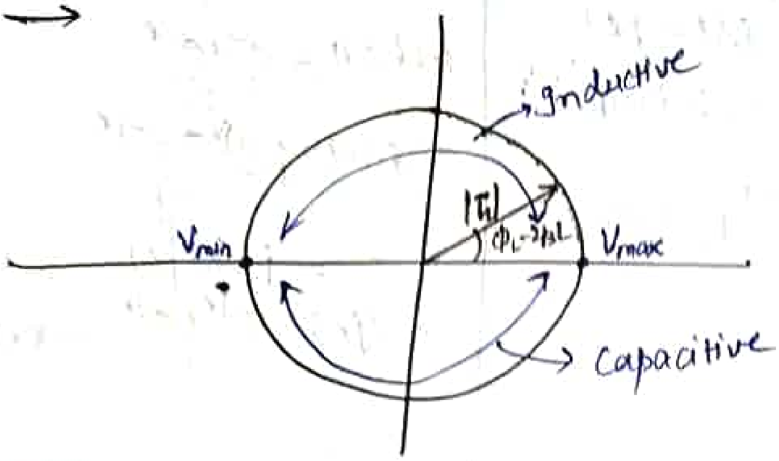
→



① $R_L > Z_0$
$\Gamma_L > 0$
↓
V_{max}

$R_L < Z_0$
$\Gamma_L < 0$
↓
V_{min}

② Inductive

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= \frac{z_L - 1}{z_L + 1}$$
$$= \frac{r + jx - 1}{r + jx + 1}$$
$$= \frac{(r - 1 + jx)(r + 1 - jx)}{(r + 1 + jx)(r + 1 - jx)}$$
$$= \frac{r^2 - 1 + x^2 + jx(\cancel{r} + 1 - \cancel{r} + 1)}{(r+1)^2 + x^2}$$

$$\Gamma_L = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} + j\frac{2x}{(r+1)^2 + x^2}$$
$$= \pm A + jB$$
$$\text{Angle} = \tan^{-1}(B/A)$$

- $Z = R + jX$
  - R ↓ Resistive
  - X ↘ Reactive

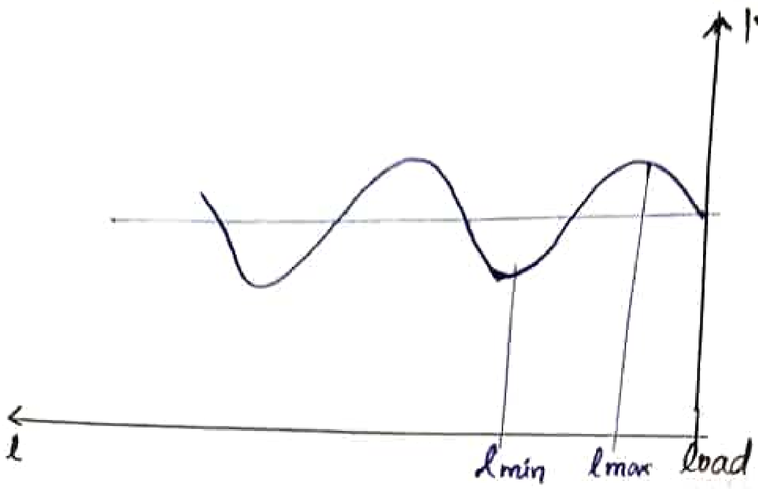
  $\frac{1}{Z} = Y_F = G + jB$ ↘ Admittance
- Passive load ↘ Resistance cannot be < 0.

→ In one rotation, angle $= 360° = 2\pi$

$$2\beta l = 2\pi$$
$$\Rightarrow 2 \cdot \frac{2\pi}{\lambda} \cdot l = 2\pi \qquad \left(\because \beta = \frac{2\pi}{\lambda}\right)$$
$$\Rightarrow \boxed{l = \frac{\lambda}{2}}$$ → distance b/w 2 consecutive V_{max}'s or V_{min}'s.
↳ fixed.

$\rightarrow V(l) = V^+ e^{j\beta l} \left(1 + |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)$, Γ_L: Reflection coeff. at load end

ϕ_L: Angle of Γ_L at load end.



Distance b/w 2 consecutive l_{min}s / l_{min}s $= \lambda/2$

Distance b/w l_{min} & l_{max} $= \lambda/4$.

Voltage Maxima

$$\phi_L - 2\beta l_{max} = \pm 2m\pi$$

$$\Rightarrow l_{max} = \frac{1}{2\beta}\{\phi_L \pm 2m\pi\}, \quad m = 0, 1, 2, \ldots$$

Voltage Minimum

$$\phi_L - 2\beta l_{min} = \pm(2m+1)\pi$$

$$\Rightarrow l_{min} = \frac{1}{2\beta}\{\phi_L \pm (2m+1)\pi\}$$

① Resistive ($R_L > Z_0$)

$Z_{max} = Z_0 S$

$Z_{min} = Z_0 / S$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{R_L - Z_0}{R_L + Z_0}$$

$$= |\Gamma_L| e^{j0^\circ}$$

$$l_{max} = \pm \frac{\cancel{2} m\pi}{\cancel{2}\beta}$$

$$= \pm \frac{\cancel{\pi} m\lambda}{2\cancel{\pi}} \quad \left(\because \beta = \frac{2\pi}{\lambda}\right)$$

$$\therefore \boxed{l_{max} = \pm m\frac{\lambda}{2}}$$

$\rightarrow$ for $m = -1$, we're going beyond the load.

② $R_L < Z_0$

$\Gamma_L = |\Gamma_L| e^{j\pi}$; $\phi_L = \pi$

$l_{max} = \frac{1}{2\beta}(\pi \pm 2m\pi)$

$= \frac{\lambda}{4\pi} \cdot \pi(1 \pm 2m)$

$\therefore \boxed{l_{max} = \frac{\lambda}{4}(1 \pm 2m)}$

$l_{max} = \frac{\lambda}{4}, \frac{3\lambda}{4}, \dots$

Voltage Min^m:

$2\beta l_{min} = \phi_L \pm (2m+1)\pi$

$\Rightarrow 2\left[\frac{2\pi}{\lambda}\right] l_{min} = \pi\{1 \pm (2m+1)\}$

$\Rightarrow l_{min} = \frac{\lambda}{4}(1 \pm (2m+1))$

$= \frac{\lambda}{4}(1 \pm 1)$

$= \frac{\lambda}{4}(2), \frac{\lambda}{2}(0)$

$= \frac{\lambda}{2}, 0$

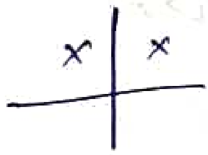
↳ First max is in b/w $\frac{\lambda}{2}$ & 0, i.e., $\frac{\lambda}{4}$.

→ Inductive:

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$, $z_L = r + jx$

$= \frac{z_L - 1}{z_L + 1}$

$= \frac{r + jx - 1}{r + jx + 1}$



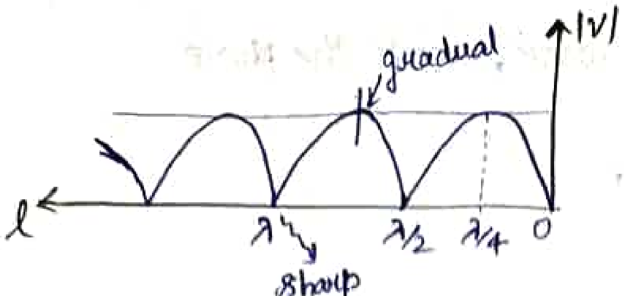
$l_{max} = \frac{\lambda}{4\pi}(\phi_L \pm 2m\pi)$

$= \frac{\lambda\phi_L}{4\pi} \pm \frac{2m\pi\lambda}{4\pi}$

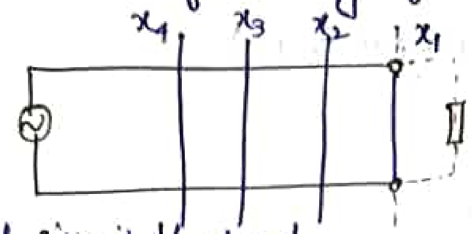
$l_{max} = \frac{\lambda\phi_L}{4\pi} \pm m\frac{\lambda}{2}$ → For $m=0$, $l_{max} \neq \frac{\lambda}{2}$

- Thus, for a given load in the T/L, we can find l_{max} & l_{min}.

→ For short-circuit load end, (voltage diff. across load = 0 V)



→ To calculate the frequency of the wave:

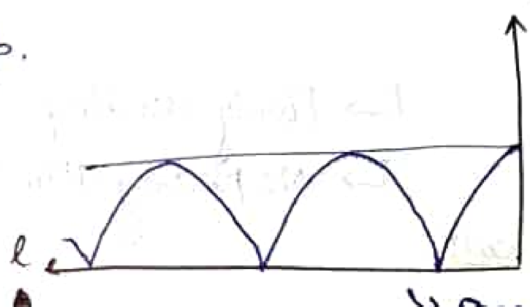


- Short-circuit the load.
- Find distances b/w minimas, say, $x_1, x_2, x_3, x_4, \dots$
- Find the average of $(x_2 - x_1)$, $(x_3 - x_2)$, $(x_4 - x_3)$, ...
- Find λ and then frequency.

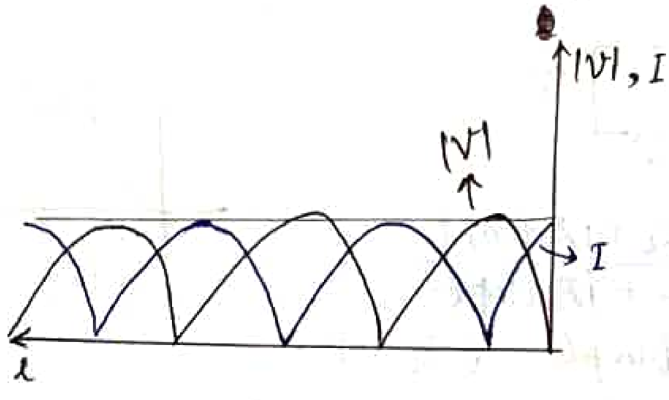
↳ It will be easier to find λ with minimas, rather than with maximas, as minimums are sharp.

$$\beta = \omega\sqrt{LC}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$



↳ Open-circuited load



→ Mathematically,

$$V(l) = V^+(e^{j\beta l} + \Gamma_L e^{-j\beta l})$$

$$= V^+(e^{j\beta l} - e^{-j\beta l}) \quad (\because \Gamma_L = -1, \text{ for short-circuit})$$

$$[\Gamma_L = 1, \text{ for open-circuit}]$$

$$= V^+ 2j \sin(\beta l)$$

$$|V(l)| = 2|V^+||\sin\beta l|$$

↝ Reason for sharp $\min^m$

→ $V(l) = 2jV^+ \sin(\beta l)$

↳ To check if it is propagating wave, check the phase.

$$v(l,t) = 2jV^+ \sin(\beta l)\, e^{j\omega t}$$
$$= 2j|V^+| \sin(\beta l)\, e^{j(\omega t + \phi)}$$

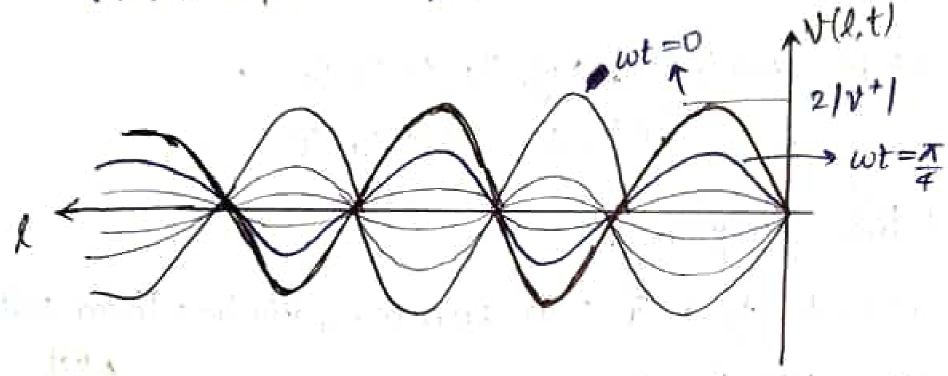
↳ has phase ϕ (say)

Let's take $\phi = 0$.

$$\Rightarrow v(l,t) = \underbrace{2j|V^+|}_{\text{Img.}} \sin(\beta l)\left(\cos(\omega t) + j\sin(\omega t)\right)$$

⇓

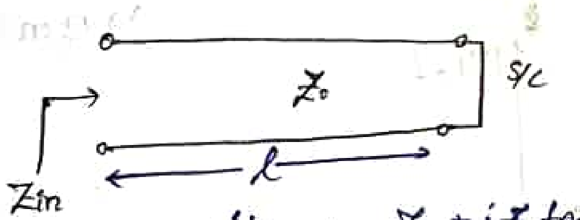
$$v(l,t) = 2|V^+| \sin(\beta l) \cos\omega t$$

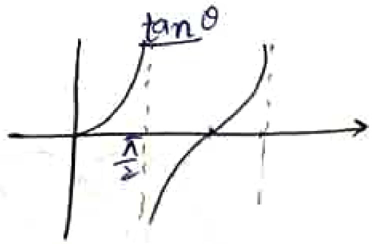


↳ Purely standing

↳ No propagation (no propagation of power)

Short Circuit





$$Z_{sc} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$
$$= jZ_0 \tan\beta l \quad (\because Z_L = 0)$$

↳ $\tan\beta l = \oplus$ve : Inductive

$\tan\beta l = \ominus$ve : Capacitive

Open Circuit

$$Z_{oc} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$
$$= -jZ_0 \cot(\beta l) \quad \left(\because Z_L = \infty \Rightarrow \frac{1}{Z_L} = 0\right)$$

→ Whenever we need inductive or capacitive circuit, we will open/short circuit the T/L.

#Via → drill in PCB to short circuit.

In PCB, open circuit is easier to work with.

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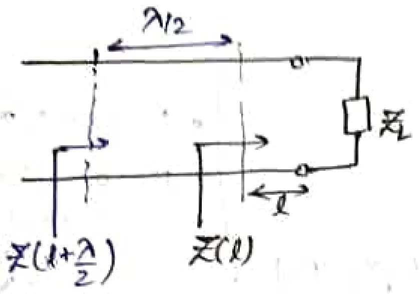
$\rightarrow Z(l) = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$z(l) = \dfrac{z_L + j\tan\beta l}{1 + jz_L \tan\beta l}$ → normalised $\left(z_L = \dfrac{Z_L}{Z_0}\right)$



$z(l+\lambda/2) = \dfrac{z_L + j\tan[\beta(l+\lambda/2)]}{1 + jz_L\tan[\beta(l+\lambda/2)]}$

$\tan(\beta l + \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2})$
$= \tan(\pi + \beta l)$
$= \tan(\beta l)$

$\Rightarrow z(l+\frac{\lambda}{2}) = \dfrac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l}$ ⇝ repeats
↳ periodic
↳ z, V, I repeat

$\Rightarrow \boxed{z(l+\frac{\lambda}{2}) = z(l)}$

Now,

$z(l+\lambda/4) = \dfrac{z_L + j\tan[\beta(l+\lambda/4)]}{1 + jz_L\tan[\beta(l+\lambda/4)]}$

$\tan(\beta l + \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4})$
$= \tan(\frac{\pi}{2} + \beta l)$
$= -\cot(\beta l)$

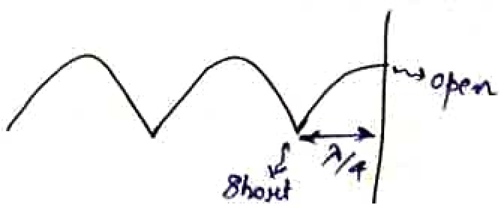
$= \dfrac{z_L - j\cot\beta l}{1 - jz_L\cot\beta l}$

$= \dfrac{z_L\tan\beta l - j}{\tan\beta l - jz_L}$

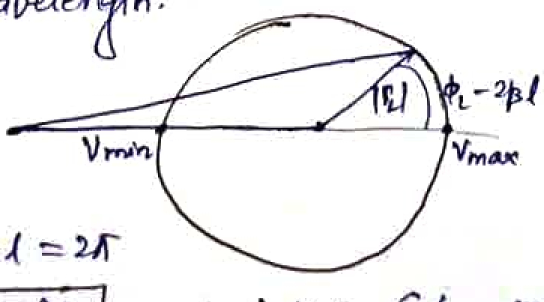
$= \dfrac{1 + jz_L\tan\beta l}{z_L + j\tan\beta l}$

l → physical length
βl → electrical length (unit: rad)

$\Rightarrow \boxed{z(l+\frac{\lambda}{4}) = \frac{1}{z(l)}}$ ⇝ If a pt. is open-circuited, moving $\frac{\lambda}{4}$ gives short-circuited



→ One complete rotation denotes half wavelength.



$2\cdot\frac{2\pi}{\lambda}\cdot l = 2\pi$

$\Rightarrow \boxed{l = \lambda/2}$ ⇝ periodicity (Vmax to Vmax)

→ **Power**

$$V(l) = V^+ e^{j\beta l} + V^- e^{-j\beta l}$$
$$= V^+ (e^{j\beta l} + \Gamma_L e^{-j\beta l})$$
$$I(l) = \frac{1}{Z_0} (V^+ e^{j\beta l} - V^- e^{-j\beta l})$$
$$= \frac{V^+}{Z_0} (e^{j\beta l} - \Gamma_L e^{-j\beta l})$$

- Power $= V * I^*$
- Power dissipating = Real part of $(V \times I^*)$

$$P(l) = \frac{1}{2} Re\{V I^*\}$$ ⇝ RMS value

$$= \frac{1}{2} Re\left\{V^+ (e^{j\beta l} + \Gamma_L e^{-j\beta l}) \frac{(V^+)^*}{Z_0} (e^{-j\beta l} - \Gamma_L^* e^{j\beta l})\right\}$$

$$= \frac{|V^+|^2}{2Z_0} Re\left\{1 - \Gamma_L^* e^{j2\beta l} + \Gamma_L e^{-j2\beta l} - |\Gamma_L|^2\right\}$$ → (not taking Z_0^* because Z_0 is real, for lossless T/L)

$$= \frac{|V^+|^2}{2Z_0} Re\left\{1 - |\Gamma_L|^2 + \underbrace{(\Gamma_L e^{-j2\beta l} - \Gamma_L^* e^{j2\beta l})}_{(Z_1 - Z_1^* = 2\,Im(Z_1))}\right\}$$

⇝ imaginary → Reactive power

$$\Rightarrow \boxed{P(l) = \frac{|V^+|^2}{2Z_0}(1 - |\Gamma_L|^2)}$$ → Independent of l

⇝ Power delivered to the load

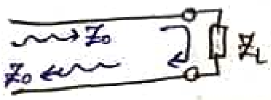
OR

Forward voltage $= V^+ e^{j\beta l}$

Reflected power $= |\Gamma_L|^2 \frac{|V^+|^2}{2Z_0}$

Power $= \frac{|V^+|^2}{2Z_0}$

[Forward wave will always see an impedence of Z_0]



\# [Reflected voltage $= \Gamma_L V^+$
Reflected power $= |\Gamma_L|^2 P^+$]

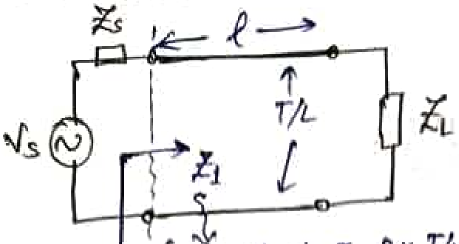
$$\therefore P(l) = \frac{|V^+|^2}{2Z_0}(1 - |\Gamma_L|^2)$$

[Backward wave will always see an impedence of Z_0 or $-Z_0$.]

Z_0: if 'reflected' is already implied

$-Z_0$: minus sign signifies 'receiving power'

V^+ calculation

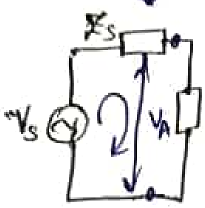


Length of T/L = L

(equivalent Z for T/L)

$$V(l) = V^+ e^{j\beta L}(1 + \Gamma_L e^{-j2\beta L}) = \frac{V_s Z_1}{Z_s + Z_1}$$

⇓



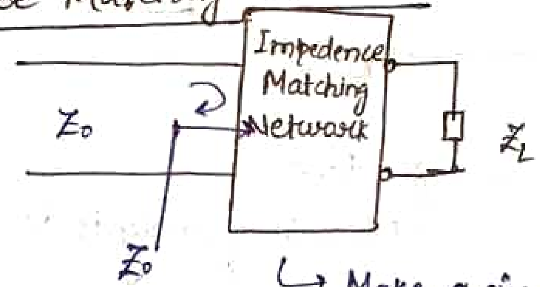
$$Z_1 = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$I = \frac{V_s}{Z_s + Z_1}$$

$$V_A = \frac{V_s Z_1}{Z_s + Z_1}$$

$$\underbrace{\underbrace{\frac{V_s}{Z_1 + Z_s}}_{\text{current}} \cdot \underbrace{Z_1}_{\text{Impedence}}}_{\text{voltage}} = V^+$$

Impedence Matching Network



↳ Make a circuit such that looking at Z_L from here gives impedence of Z_0.

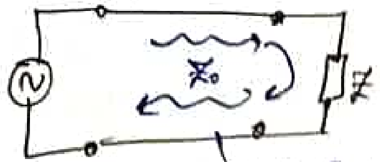
↳ Then the T/L is 'MATCHED'.

$$Z_1 \frac{Z_L + jZ_1 \tan\beta l}{Z_1 + jZ_L \tan\beta l} = Z_0$$

Variables : l, Z_1 (β is constant for a given frequency)

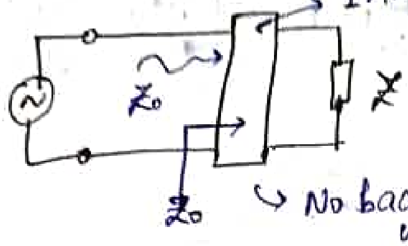
Eg of MATCHED circuit: ~~Circuit~~ Transistor Amplifier

24-08-2023

→ 

↳ ~~two~~ Backward wave may damage the source (if power is high)

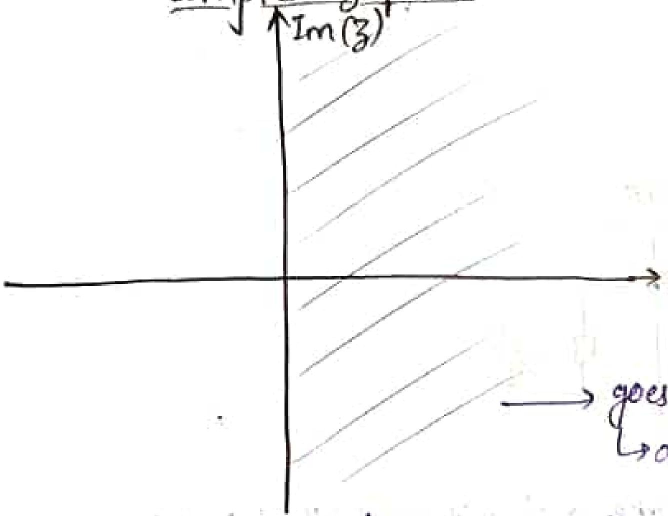
⇓



↳ No backward wave

SMITH chart

<u>Complex z-plane</u>



$$z = r \pm jx$$
↳ +, for inductive
−, for capacitive

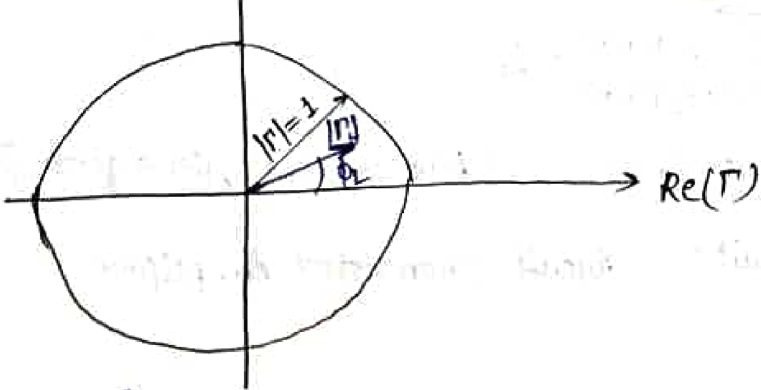
$r \geq 0$

→ goes to ∞
↳ domain of analysis is ∞.

↓ transforms $(z_L \to \Gamma_L)$

<u>Γ-plane</u>
↳ domain of analysis <u>0 to 1.</u>

$\Gamma = u + jv$



[Center : $|\Gamma_L| = 0$ → MATCHED]

→ $z = r + jx$
$\Gamma = u + jv$

$\left(\because \Gamma = \dfrac{Z - Z_0}{Z + Z_0}\right)$

$$\Gamma = \frac{z-1}{z+1}$$

$$\Rightarrow z = \frac{1+\Gamma}{1-\Gamma}$$

$$\Rightarrow r+jx = \frac{1+u+jv}{1-u-jv}$$

$$\Rightarrow r+jx = \frac{(1+u+jv)(1-u+jv)}{(1-u)^2+v^2}$$

Equating real and imaginary parts,

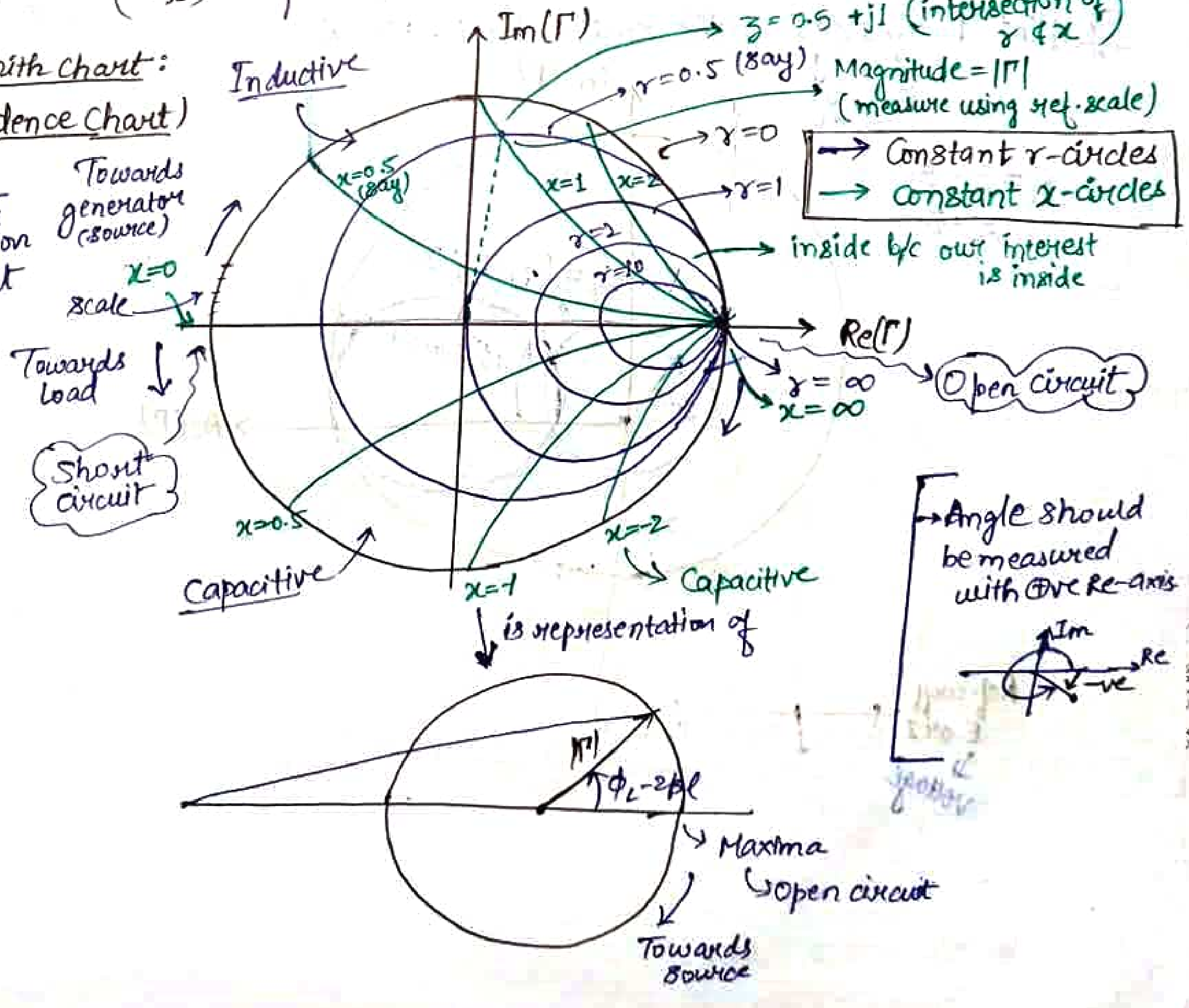
$$\left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2}$$

$$(u-1)^2 + \left(v - \frac{1}{x}\right) = \frac{1}{x^2}$$

↳ circles of

center	Radius
$\left(\frac{r}{1+r}, 0\right)$	$\frac{1}{1+r}$
$\left(1, \frac{1}{x}\right)$	$1/x$

Smith chart:
(Impedence Chart)
↓
Polar plot of reflection coefficient



 → Work with Impedance (easier)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

$G_{eq} = G_1 + G_2 + \dots + G_N$ → Easier to work with admittance.
↳ then take inverse, $\frac{1}{G_{eq}}$.

$\frac{1}{\text{Impedance}}$ = Admittance

$Z = R + jX$

$Y = G \pm jB$

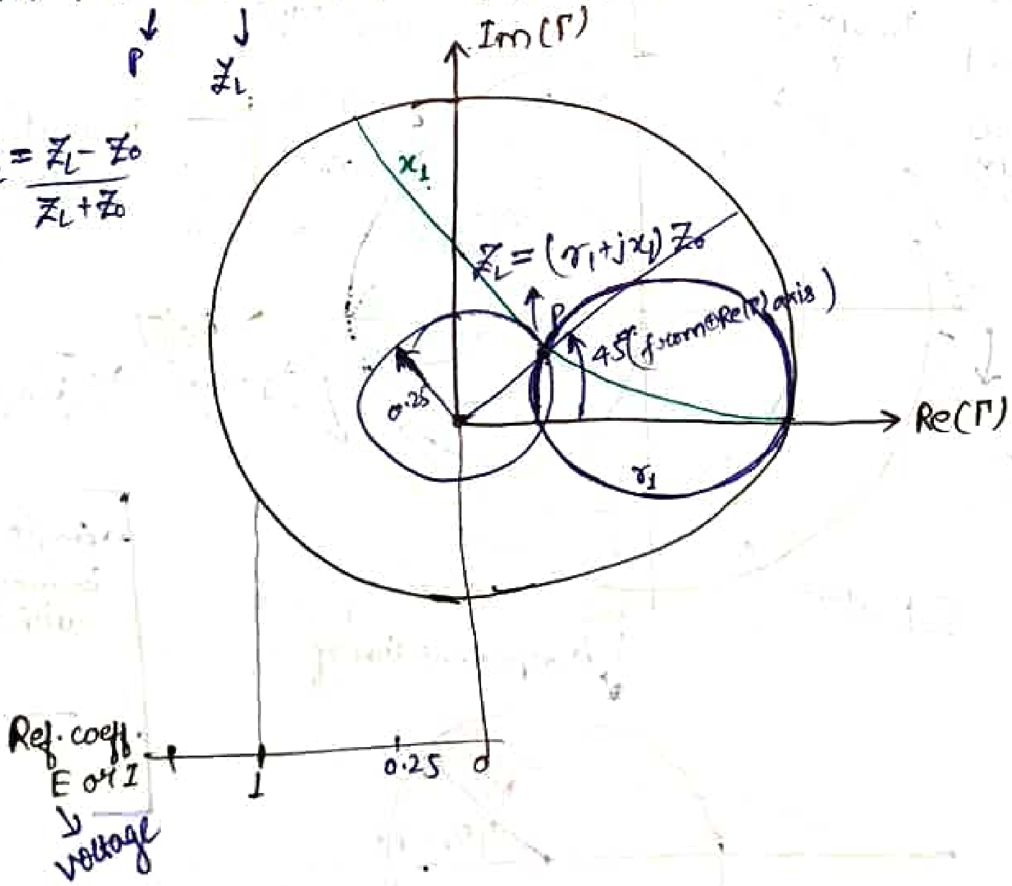
[$X = \oplus ve$ → Inductive
$B = \oplus ve$ → capacitive]

SMITH chart (Magic chart) with scale

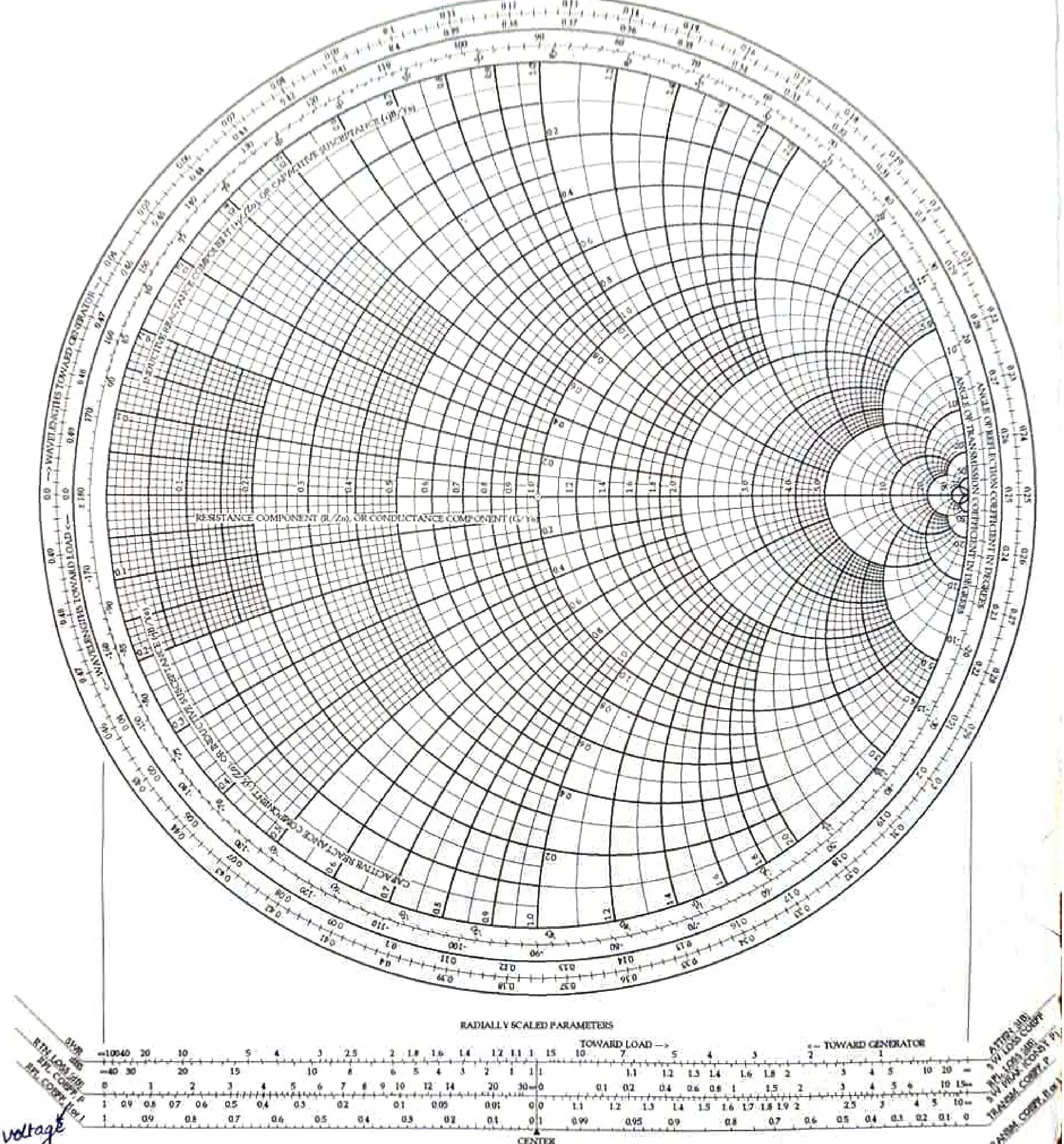
30-12-2023

Eg Plot $\Gamma = 0.25\angle 45°$ on the smith chart. Find Z_L.
(0.25 ↓ P; $45°$ ↓ Z_L)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

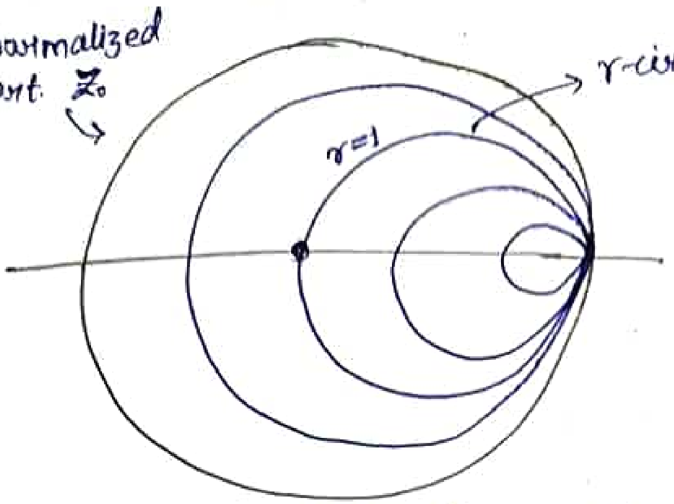


Smith Chart



ADMITTANCE CHART

→



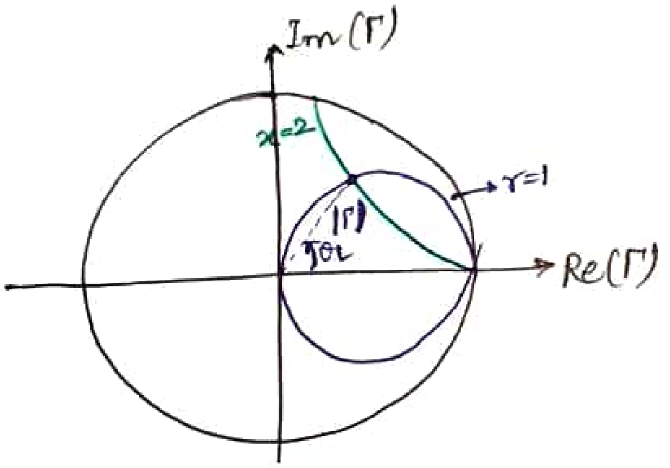
$z = r + jx$

$Z = z \times Z_0$

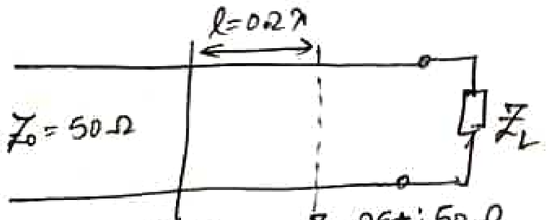
Eg. Given $Z_L = 100 + j200$. Find Γ_L. ($Z_0 = 100\,\Omega$)

Soln: Normalizing,

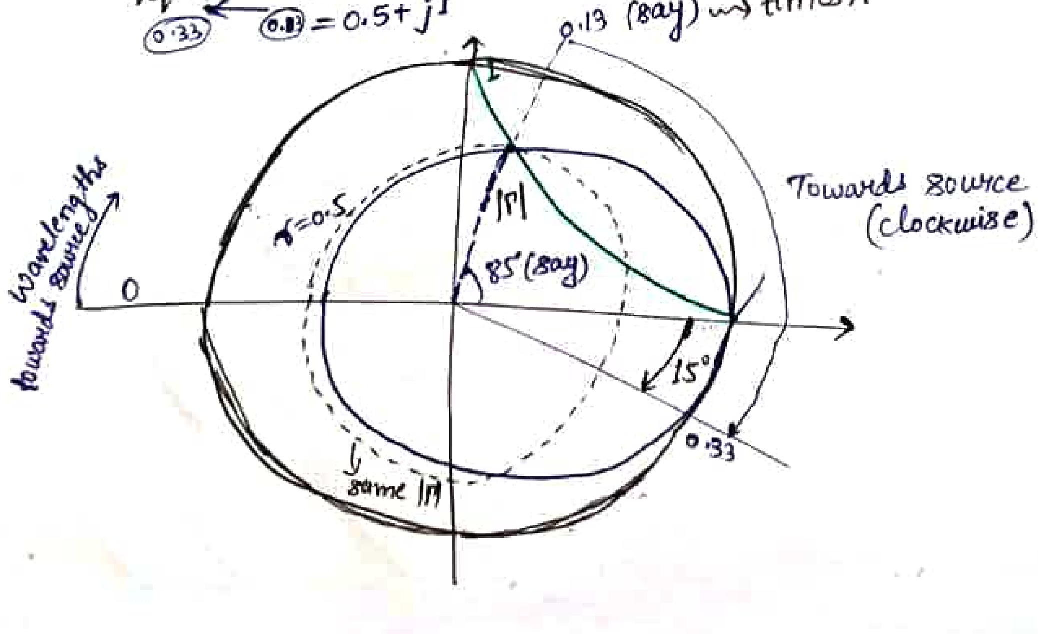
$z_L = 1 + j2$



Eg.



Soln.



31-08-2023

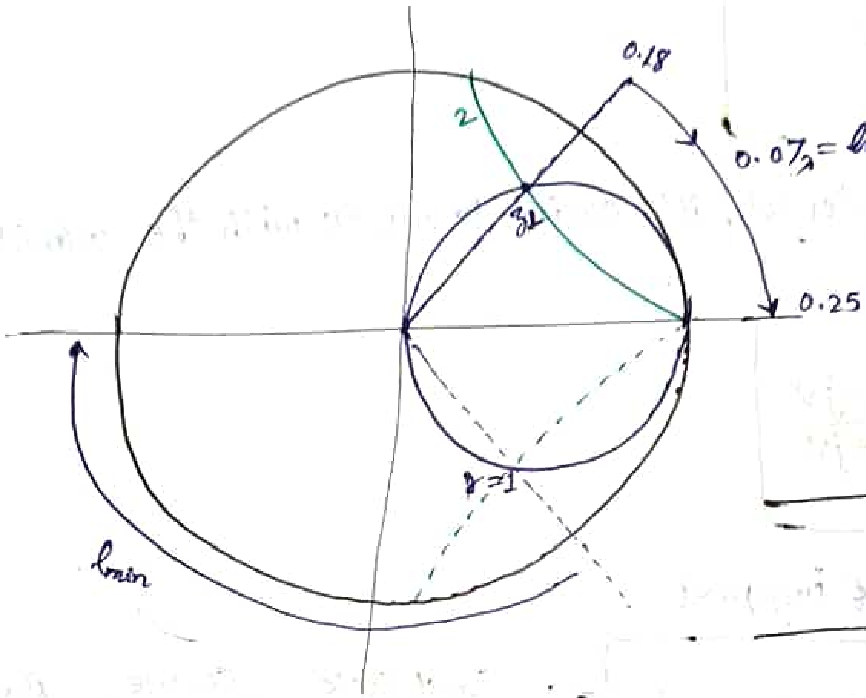
Eg. $Z_L = 100 + j200\ \Omega$

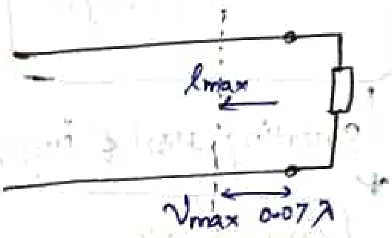
$Z_0 = 100\ \Omega$

$z_L = 1 + j2$

$l_{max} = ?$

$l_{min} = ?$

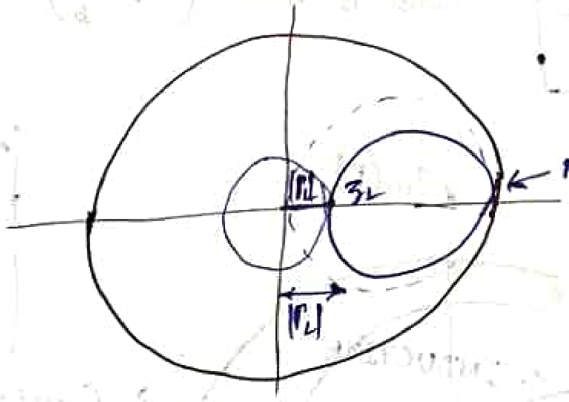




Eg. $Z_L = 100\ \Omega$

$Z_0 = 50\ \Omega$

$z_L = 2$



Smith chart ⇓ is Z-chart

Eg. $Z = 100 + j100\ \Omega$

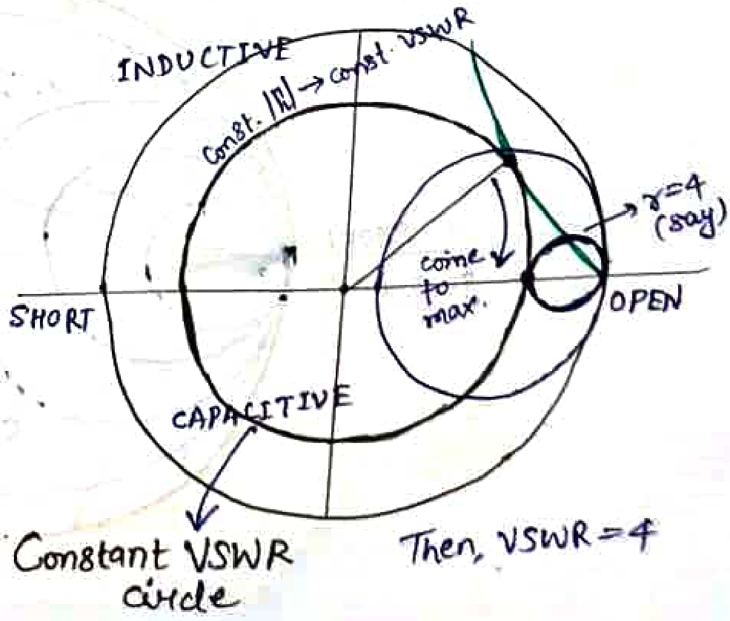
$Z_0 = 50\ \Omega$

$VSWR = ? = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$

(ρ or S)

$Z_{max} = Z_0 \rho \Rightarrow z_{max} = \rho$

$Z_{min} = Z_0 / \rho$



Then, VSWR = 4

$$\Gamma_L \leftrightarrow z_L$$

$$\Gamma_L = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma_L}{1-\Gamma_L}$$

$$r + jx = \frac{1+u+jv}{1-u-jv}$$

$\Rightarrow$

$$\left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2} \; ; \; r\text{-circles}$$

$$(u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \frac{1}{x^2} \; ; \; x\text{-circles}$$

→ For parallel impedances, it's easier to work with the admittances.

$$y_L = \frac{1-\Gamma_L}{1+\Gamma_L}$$

$\downarrow$

$$g + jb = \frac{1-u-jv}{1+u+jv}$$

↓ Equating real & img. part

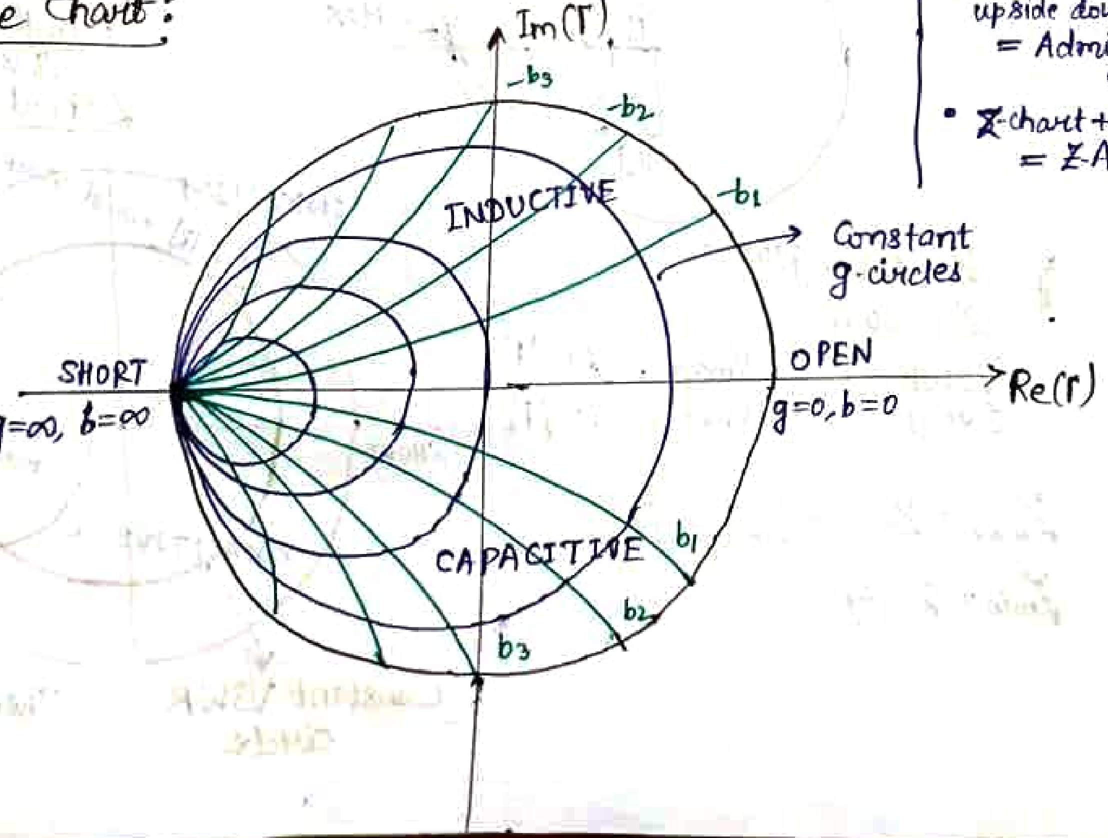
$$\left(u + \frac{g}{1+g}\right)^2 + v^2 = \frac{1}{(1+g)^2}$$

$$(u+1)^2 + \left(v + \frac{1}{b}\right)^2 = \frac{1}{b^2}$$

→

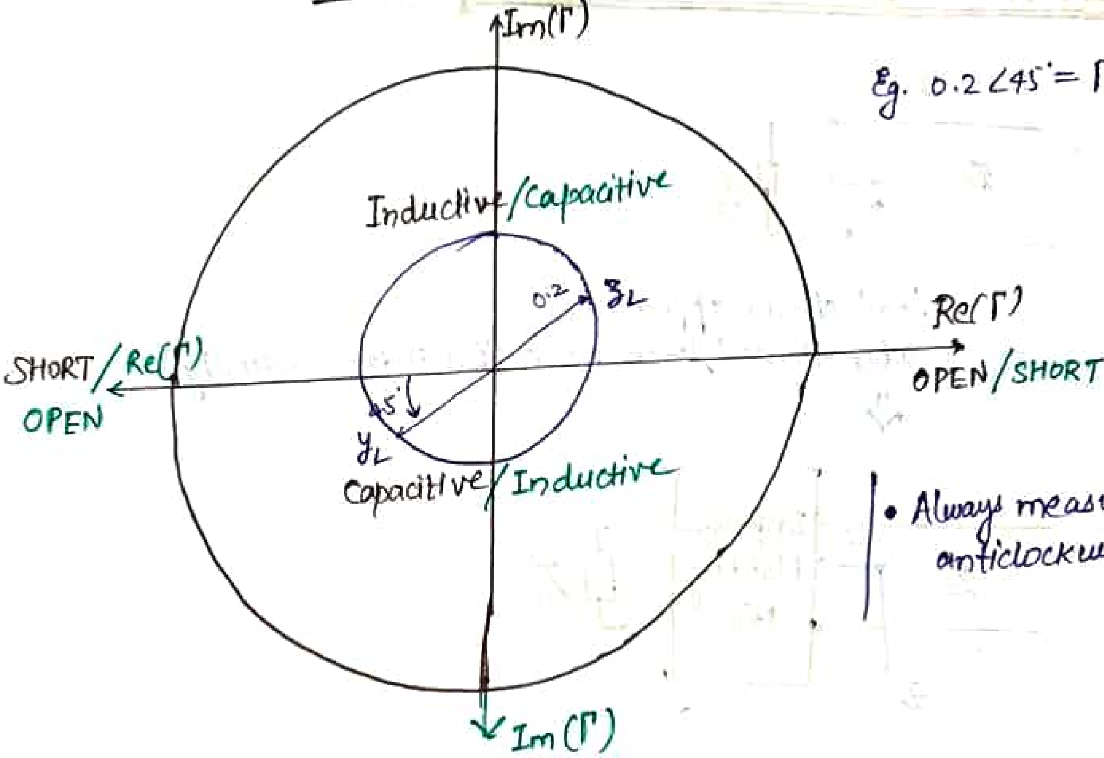
Const. circle	Centre	Radius
const. g circle	$\left(-\frac{g}{1+g}, 0\right)$	$\frac{1}{1+g}$
const. b circle	$\left(-1, -\frac{1}{b}\right)$	$\frac{1}{b}$

Admittance chart:



- Smith chart inverted upside down = Admittance chart
- Z-chart + A-chart = Z-A chart

Y-Chart



Eg. $0.2\angle 45° = \Gamma_L$

- Always measure angles anticlockwise.

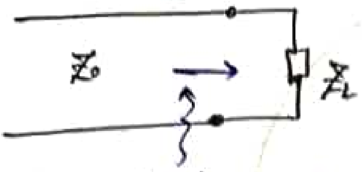
$$\Gamma_L = \frac{z_L - 1}{z_L + 1}$$

$$= \frac{1/y_L - 1}{1/y_L + 1}$$

$$= \frac{1 - y_L}{1 + y_L} = \frac{y_L - 1}{y_L + 1} e^{j\pi}$$

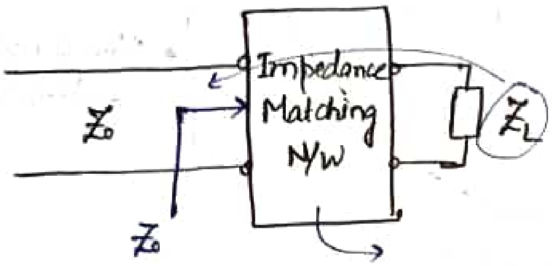
5-09-2023

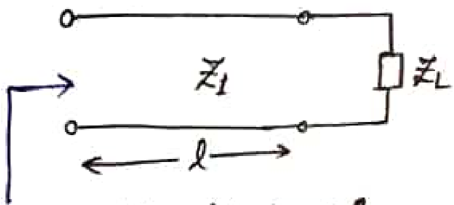
IMPEDANCE MATCHING



Load should appear 'Z_0'
when viewed from here (but it isn't actually)

$\Downarrow$





$\rightsquigarrow$ Works as Impedance Matching N/w.

$$Z_0 = Z_1 \frac{Z_L + jZ_1 \tan\beta l}{Z_1 + jZ_L \tan\beta l}$$

$\rightsquigarrow$ we introduced Z_1 and l here
↳ 2 degrees of freedom.

$$\Rightarrow Z_1 Z_0 + jZ_0 Z_L \tan\beta l = Z_1 Z_L + jZ_1^2 \tan\beta l$$

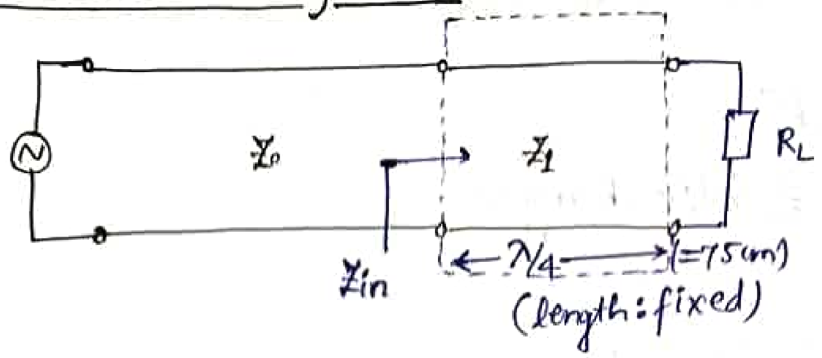
$$\Rightarrow Z_1 Z_0 + jZ_0 (R_L + jX_L)\tan\beta l = Z_1 (R_L + jX_L) + jZ_L^2 \tan\beta l.$$

Equating real and imaginary parts,

$$Z_1 = \frac{\sqrt{Z_0 R_L - R_L^2 - X_L^2}}{\sqrt{1 - R_L/Z_0}}$$

$$\tan(\beta l) = \frac{\sqrt{(1 - R_L/Z_0)(Z_0 R_L - R_L^2 - X_L^2)}}{X_L}$$

Quarter Wave Transformer



Eg. $f = 1GHz$

$\lambda = \frac{c}{f}$

$= \frac{3\times10^{10} cm/s}{10^9}$

$= 30cm$

• $R_L + jX_L$

$R_L \to Z_0$, $X_L \to 0$

\#

Z_1 Z_2 ⇝ can't be connected directly

Lossless T/L (Matched)

Quarter wave Transformer needed.

To add 2 Real

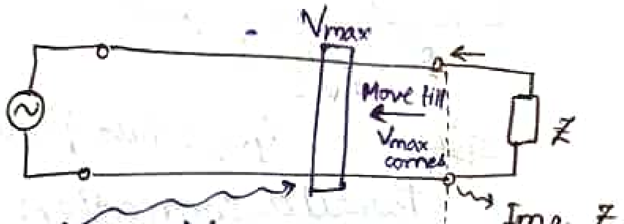
$$Z_{in} = Z_1 \frac{R_L + jX_1 \tan\beta l}{Z_1 + jR_L \tan\beta l}$$

$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$$Z_0 = \frac{Z_1^2}{R_L} \Rightarrow \boxed{Z_1 = \sqrt{Z_0 R_L}}$$

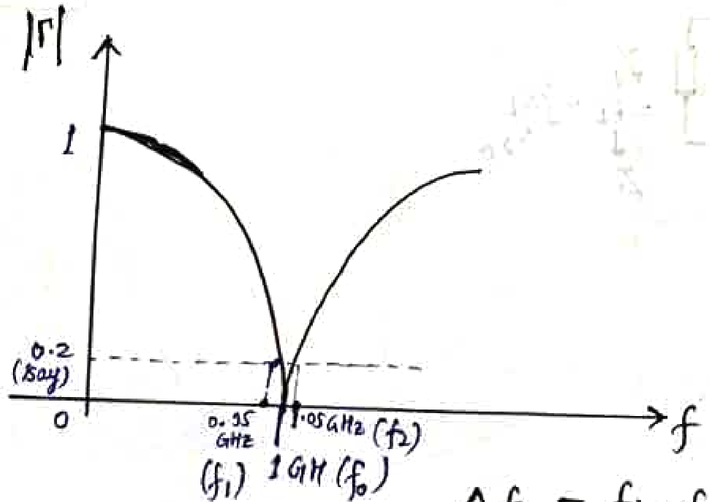
length, $l = (2m+1)\frac{\lambda}{4}$, $m = 0, 1, 2, \ldots$

\#



Insert Quarter-W.T. here

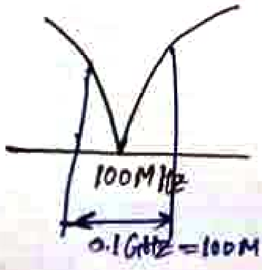
V_{max} corresponds to → $Z_{max} = Z_0/\rho$



$\Delta f = f_1 - f_2$

(Bandwidth)

$\% BW = \frac{\Delta f}{f_0} \times 100$



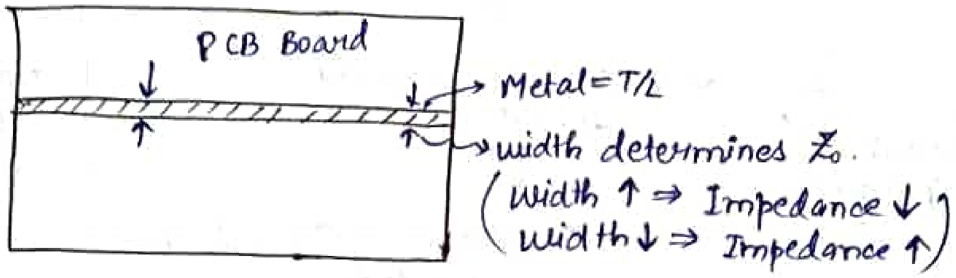
Disadvantages :

① Narrow Band

② $R_L \uparrow \longrightarrow Z_L \uparrow \longrightarrow l \uparrow$

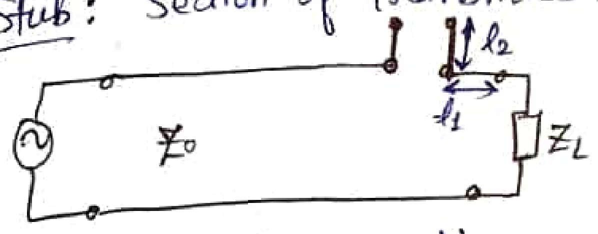
↳ If R_L changes, Z_L also changes.

#

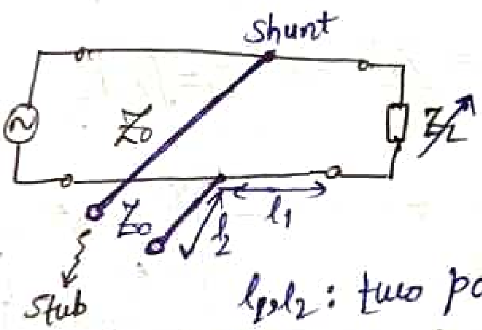


Single Stub Matching

Stub : section of transmission line.



Series connection



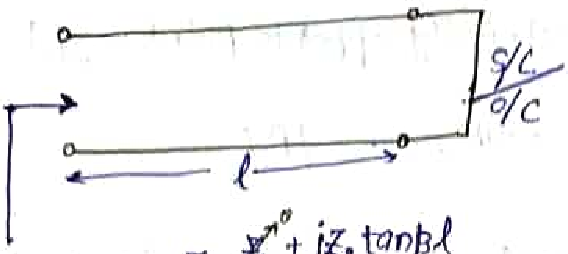
l_1, l_2 : two parameters.

Parallel connection

(Parallel/shunt stub)

7-09-2023

Input Impedance of Short / Open T/L:

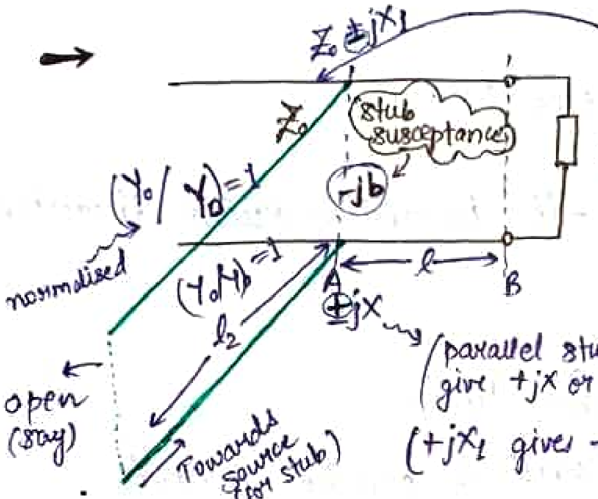


$$Z_{in,sc} = Z_0 \frac{\cancel{Z_L}^{0} + jZ_0 \tan\beta l}{Z_0 + j\cancel{Z_L}_{0} \tan\beta l}$$

$$Z_{sc} = jZ_0 \tan\beta l$$

$$Z_{oc} = -jZ_0 \cot\beta l$$

→



(parallel stub can give $+jX$ or $-jX$)

($+jX_1$ gives $-jX$)

$Z_L = R_L + jX_L \longrightarrow y_L = g_L + jb_L$

$\downarrow l$ $\downarrow$

$Z_0 \pm jX_1$ $1 \pm jb_1$

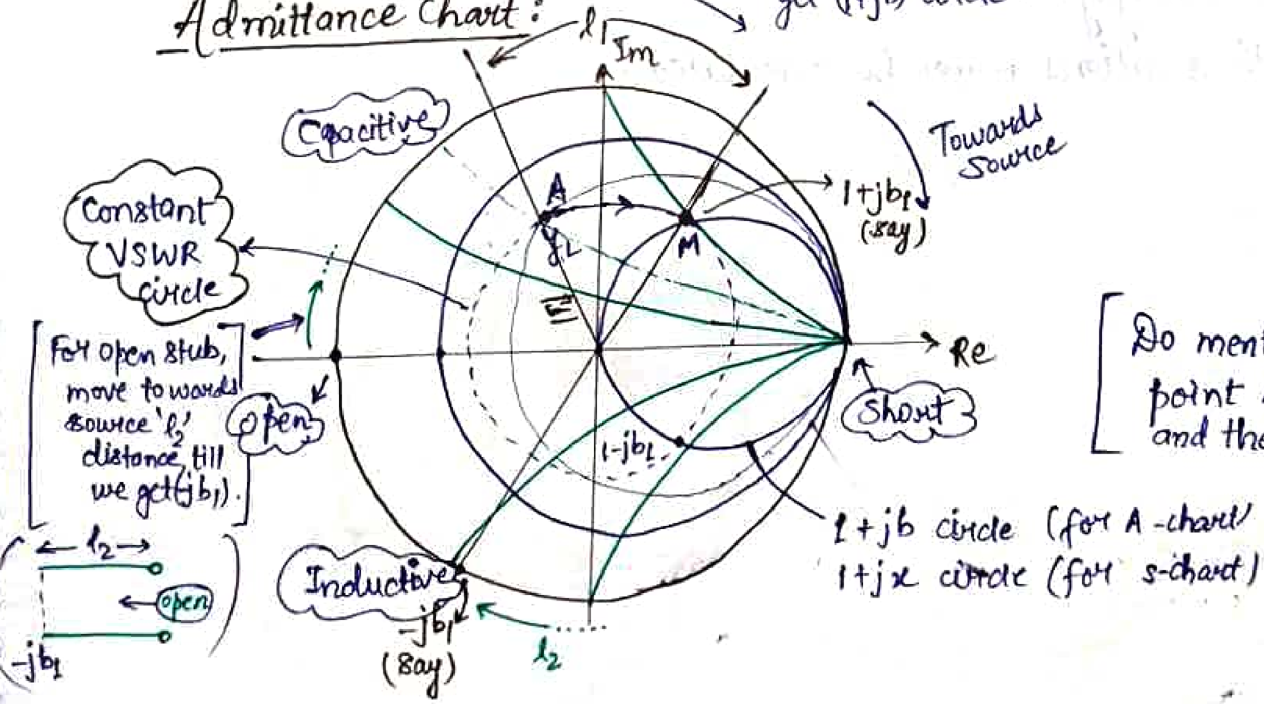
$\downarrow$

1

$\left[\begin{matrix} +jb_1 / -jb_1 \\ \downarrow \\ -jb_1 / +jb_1 \end{matrix}\right]$

Admittance Chart:

Move clockwise till we get $(1+jb)$ circle.



[Do mention what point A denotes, and the steps.]

Steps to draw:

1. Plot y_L

   (If z_L is given, plot $z_L \rightarrow$ dimmetrically opposite point $= y_L$)
   (for II stub)

2. Draw a constant VSWR circle with radius $|\Gamma_L|$

   ↓

   Move towards source (clockwise) till you get $1+jb$ circle
   $(1+jb_1) = M$

   ↓

   The amount of rotation $= 2\beta l_1$ | one full rotation ↓ $2\beta l$

   ↓

   Find out $-jb_1$

   ↓

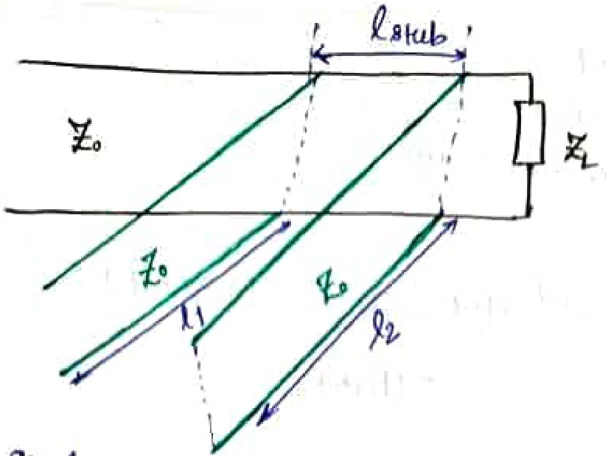
   Draw a straight line from centre to $-jb_1$.

   ↓

   Find out l_2 by rotating from $-jb_1$ in anticlockwise direction till (open) circuit condition.

→ There will be two solutions for each line:
$(1+jb_1)$ and $(1-jb_1)$, from which l_1 and l_2 can be found accordingly.
Both solutions must be calculated.

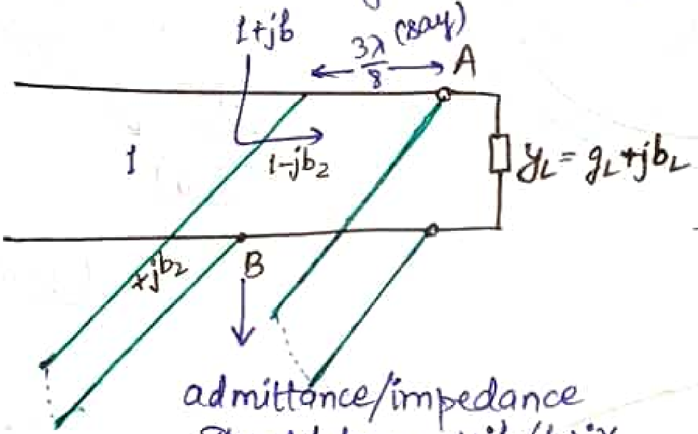
11-09-2023



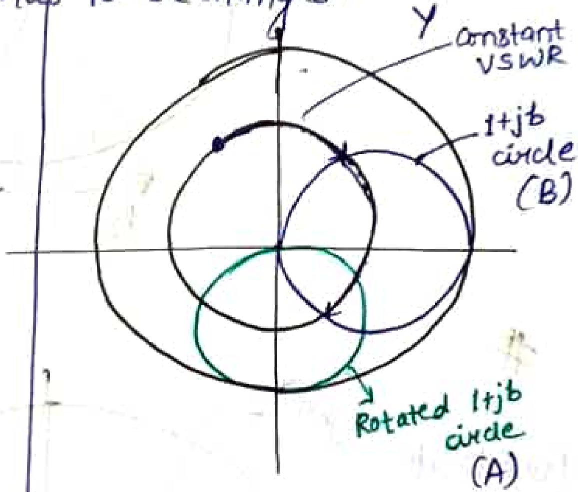
In single stub T/L, whenever load is changed, stub position needs to be adjusted

If l_{stub} the stub positions are fixed,
design parameter (goal): l_1, l_2

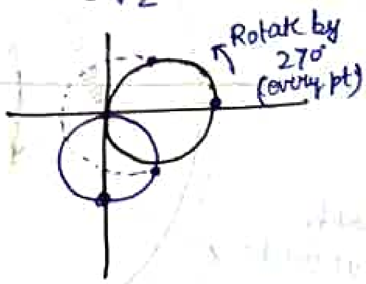
Whenever load is changed, l_1 and l_2 has to be changed.

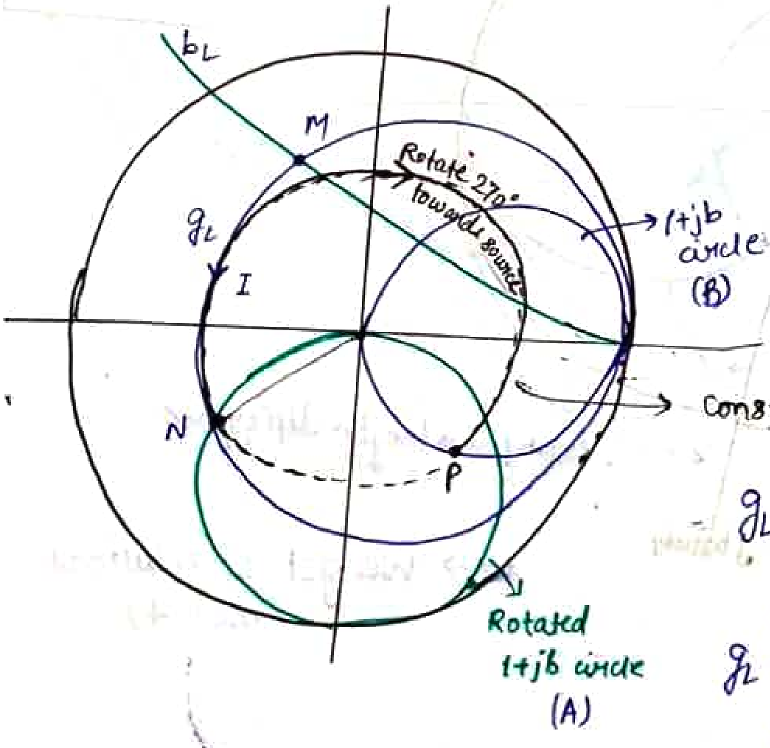


admittance/impedance should be on 1+jb/1+jx circle.



- $2\beta l = 2 \cdot \frac{2\pi}{\lambda} \cdot \frac{3\lambda}{8}$
$= 3\pi/2$
-





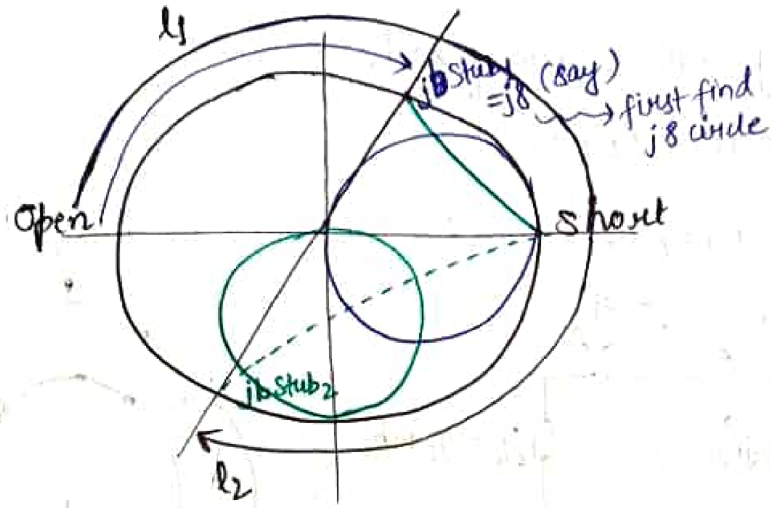
$g_L + jb_L$
↓ Img. is added
$g_L + jb_L \pm jb_1$
↓
move along g_L circle,
move clockwise or acw (doesn't matter)

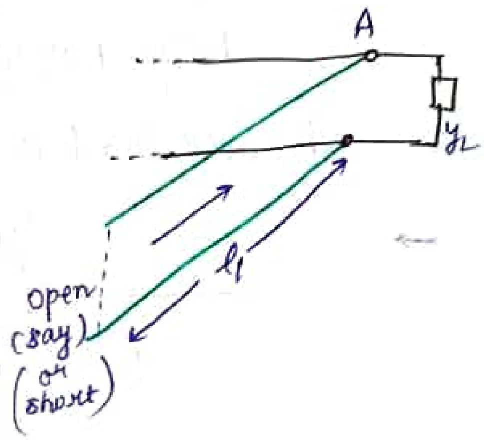
$\log(M) \rightarrow g_L + jb_L$
$\downarrow$
$N \rightarrow g_L + jb_1$

} susceptance added by stub 1
$j8 = j(b_1 - b_L)$
$= jB_{stub_1} = j8$ (say)

$P \rightarrow 1 - jb_2$

↳ The susceptance added by 2^{nd} stub $= jb_2$ (to get 1)
$= jB_{stub_2}$





12-09-2023

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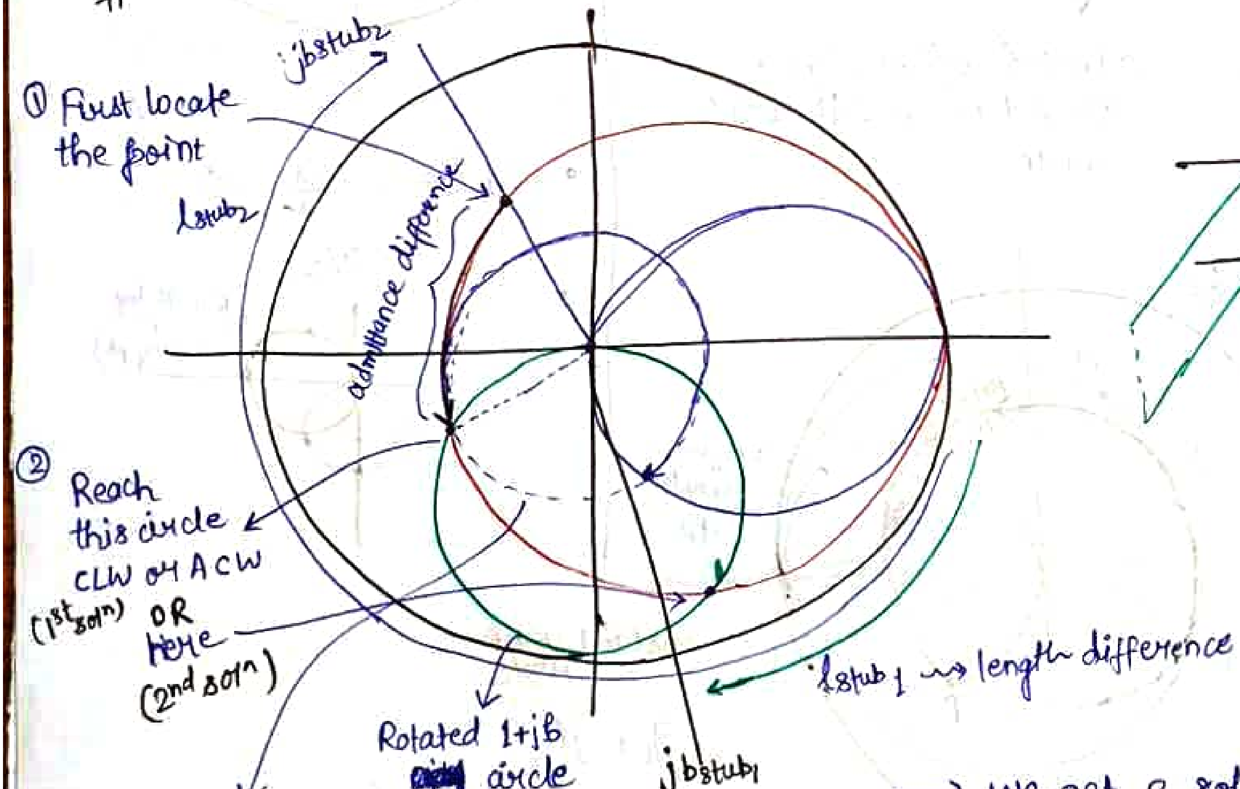
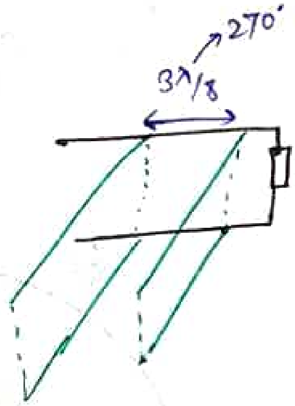
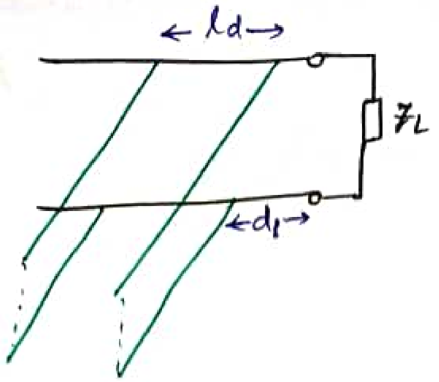
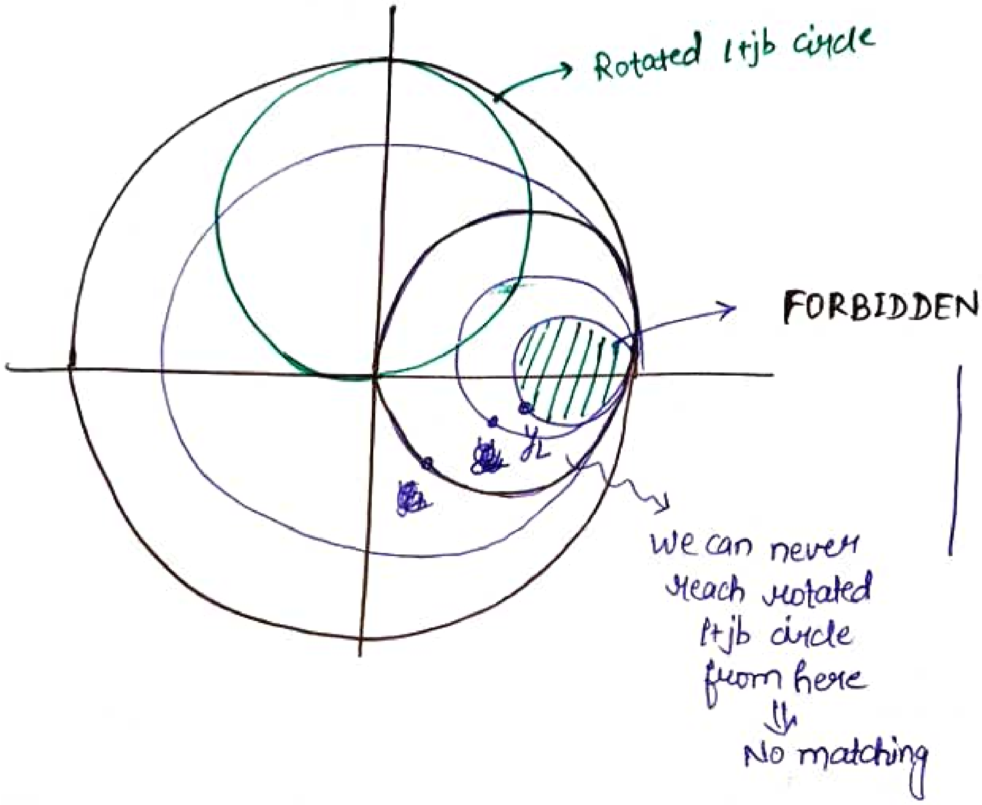




→ We get 2 solutions (not 4)

③ Locate VSWR circle using radius & Move 270°

→ 



1 GHz → $\lambda = \frac{3\times10^{10}}{10^9}$
$= 30$ cm.

→ In single stub T/L, we can always do matching.
In double stub T/L, it is possible to not get the matching.

18-09-2023

ELECTROMAGNETIC WAVES

$\bar{E}$: Electric field or electric field intensity
$\bar{H}$: Magnetic field strength
$\bar{D}$: Displacement vector
$\bar{B}$: Magnetic flux density

Maxwell's Equations :

① $\vec{\nabla}.\vec{D} = \rho_v$; ρ_v : electric charge density (Gauss law)
↳ Differential form

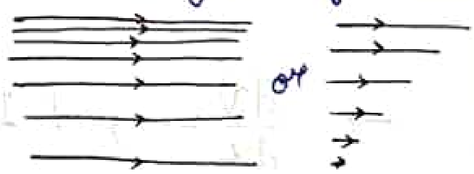
$$\iiint \vec{\nabla}.\vec{D}\, dv = \iiint \rho_v\, dv$$ → Integral form

↓

$$\oiint \vec{D}.d\vec{s} = Q$$

$$d\vec{s} = \hat{n}\, ds$$

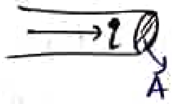
- Divergence ⇝ Sink or source
  - ↳ 0, if whatever is coming in = whatever is going out
  - ↳ sink → IN > OUT
  - ↳ source → OUT > IN
- Curl → Rotation involved
  - ↳ 0, for uniform field
  - ↳ dirⁿ along axis of rotation



② $\bar{\nabla}.\bar{B} = 0$ (Gauss law for magnetic field)

③ $\bar{\nabla} \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$,

$\bar{J}$: current density (A/m²)
(volume c.d.)



$J = \frac{i}{A}$

④ $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ (Faraday's law)

↳ Something is missing in these equations (but not wrong)
for static charges

↳ 2 more equations.

Continuity equation : ~~$\nabla\times$~~ $\bar{\nabla}.\bar{J} = -\frac{\partial \rho}{\partial t}$

$\bar{\nabla}.(\bar{\nabla}\times\bar{H}) = \bar{\nabla}\bar{J} = 0$
0

Electromagnetic field
↓
$\vec{E}$ & $\vec{B}$ coexist
↓
Power can be transferred

$\overline{\nabla}\times\overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$

$\therefore \overline{\nabla}\cdot(\cancel{\overline{\nabla}\times\overline{H}}) = \overline{\nabla}\cdot\overline{J} + \overline{\nabla}\cdot\frac{\partial \overline{D}}{\partial t} = 0$

$\Rightarrow \overline{\nabla}\cdot\overline{J} = -\frac{\partial}{\partial t}(\overline{\nabla}\cdot\overline{D})$ ($\because \overline{\nabla}$ is space derivative)

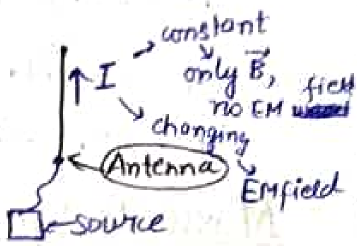
$= -\frac{\partial \rho}{\partial t}$

$\overline{J} = \sigma\overline{E}$: conduction current density

$\frac{\partial \overline{D}}{\partial t}$: Displacement current density

$\overline{J}, \rho_v$: source of $\vec{E}$ & $\vec{B}$

#



Source free: $\rho_v = 0$, $J = 0$

$\overline{\nabla}\cdot\overline{D} = 0$

$\overline{\nabla}\cdot\overline{B} = 0$

$\overline{\nabla}\times\overline{H} = \frac{\partial \overline{D}}{\partial t}$ ($\overline{B}$ is coupled with $\overline{E}$) $= \frac{\partial}{\partial t}(\epsilon\overline{E}) = \epsilon\frac{\partial \overline{E}}{\partial t} = j\omega\epsilon\overline{E}$

$\overline{\nabla}\times\overline{E} = -\frac{\partial \overline{B}}{\partial t}$ ($\overline{E}$ is coupled with $\overline{B}$) $= -\frac{\partial}{\partial t}(\mu\overline{H}) = -\mu\frac{\partial \overline{H}}{\partial t} = -j\omega\mu\overline{H}$

where ϵ : permittivity
μ : permeability
$\mu_0 = 4\pi\times10^{-7}$ H/m

(for not-changing medium)

Time-domain — Frequency-domain

$\rightarrow \frac{dV}{dx} = -j\omega L I$

$\frac{dI}{dx} = -j\omega C V$

coupled $\Rightarrow$ cannot be solved directly.

$\Downarrow$ (same for $\overline{E}$ & $\overline{H}$)

$\Downarrow$ Take double-derivative

$\Downarrow$

$\frac{d^2V}{dx^2} = \gamma^2 V$, $\gamma = \sqrt{(\cancel{R}+j\omega L)(\cancel{G}+j\omega C)}$

$= j\omega\sqrt{LC}$

(R, G = 0 for no-loss)

$\rightarrow \overline{\nabla}\times\overline{H} = j\omega\epsilon\overline{E}$

$\Rightarrow \overline{\nabla}\times(\overline{\nabla}\times\overline{H}) = (\overline{\nabla}\times j\omega\epsilon\overline{E})$ [Take curl]

$\Rightarrow \overline{\nabla}(\cancel{\overline{\nabla}\cdot\overline{H}}) - \nabla^2 H = j\omega\epsilon\,\overline{\nabla}\times\overline{E}$ [for isotropic]

ϵ comes out of $\overline{\nabla}$

$\Rightarrow -\nabla^2\overline{H} = j\omega\epsilon(-j\omega\mu\overline{H})$

$\Rightarrow -\nabla^2 H = \omega^2\mu\epsilon\overline{H}$

$$\Rightarrow \boxed{\nabla^2 \bar{H} + \omega^2 \mu \epsilon \bar{H} = 0}$$

Similarly,

$$\boxed{\nabla^2 E + \omega^2 \mu \epsilon \bar{E} = 0}$$

} for source free, isotropic, linear, lossless
"Wave Equation" (vector)

↳ If $\bar{H}$ is known, $\bar{E}$ can be calculated.
If $\bar{E}$ is known, $\bar{H}$ can be calculated.
⇓
One information is sufficient

19-09-2023

$\frac{dV}{dx} = -(\cancel{R}^{0} + j\omega L)I$

$$\frac{dI}{dx} = -(\cancel{G}^{0} + j\omega C)V$$

$$\Rightarrow \frac{d^2V}{dx^2} = \gamma^2 V \quad \Big\| \quad \frac{d^2I}{dx^2} = \gamma^2 V \longrightarrow V(x) = V^+ e^{-\gamma x} + V^- e^{\gamma x}$$

$$\gamma = \sqrt{(\cancel{R}^{0} + j\omega L)(\cancel{G}^{0} + j\omega C)} = j\omega\sqrt{LC}$$

$$\gamma = \overset{0}{\alpha} + j\beta = j\omega\sqrt{LC}; \quad \beta = \omega\sqrt{LC}$$

$$\therefore \frac{d^2V}{dx^2} - \gamma^2 V = 0$$

$$\Rightarrow \boxed{\frac{d^2V}{dx^2} + \omega^2 LC V = 0}$$

↳ comparing with $\nabla^2 \bar{H} + \omega^2 \mu \epsilon \bar{H} = 0$; $\nabla^2 E + \omega^2 \mu \epsilon \bar{E} = 0$,

$$\mu \xrightarrow[\text{to}]{\text{corresponds}} L$$

$$\epsilon \xrightarrow[\text{to}]{\text{corresponds}} C$$

↳ open air acts like transmission line
↳ Wireless

$$\rightarrow \bar{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$\rightarrow \boxed{\bar{\nabla}^2 \bar{E} + \beta^2 \bar{E} = 0}$

① $\bar{E}$ is uniform $\Rightarrow \bar{\nabla}^2 E = 0 \Rightarrow \bar{E} = 0$

$\Rightarrow$ Completely uniform $\bar{E}$ is not possible.

② $\bar{E}$ is varying $\rightarrow$ $\vec{E}$ is uniform in a plane perpendicular to the field vector (y z-plane) $\rightarrow$ ✗

$\bar{E} = \hat{x} E_x(z)(\alpha y)$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

$\Downarrow$

Not a possible solution (as $\bar{H} = 0$)

$\bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ E_x & 0 & 0 \end{vmatrix} = -j\omega\mu\bar{H}$$

$\Rightarrow \bar{H} = 0$

$\rightarrow$ $\vec{E}$ is uniform in a plane containing the field (xy plane (or xz plane)) $\rightarrow$ ✓

$$\frac{\partial}{\partial x} = 0 = \frac{\partial}{\partial y}$$

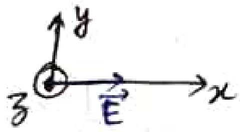
$\Downarrow$

Possible solution

$\rightarrow$ $\vec{E} \perp$ to propagation dirn.

$\rightarrow$ $\vec{E}$ & $\vec{H}$ are orthogonal

$\rightarrow$ $\perp$ to each other & $\perp$ to dirn of propagation (or transverse to dirn of propagation) $\rightarrow \perp^{r}$



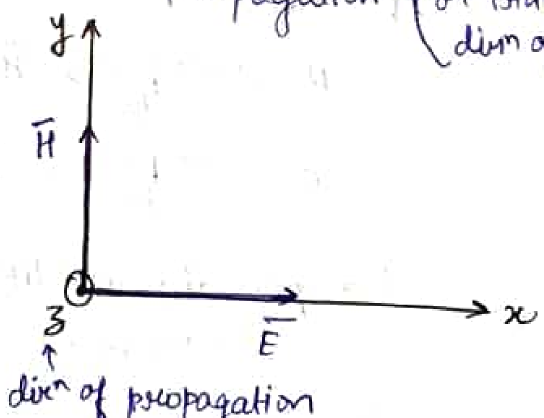
$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x(z) & 0 & 0 \end{vmatrix} = -j\omega\mu\bar{H}$$

$\Rightarrow \hat{y} \frac{\partial E_x}{\partial z} = -j\omega\mu\bar{H}$

For xz-plane: $\vec{E} \rightarrow$ uniform

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \frac{\partial}{\partial y} & 0 \\ E_x & 0 & 0 \end{vmatrix} = -j\omega\mu\bar{H}$$

$\Rightarrow \hat{z}\left(-\frac{\partial E_x(x)}{\partial y}\right) = -j\omega\mu\bar{H}$



$\rightarrow$ No component of $\bar{E}$ or $\bar{H}$ in the dirn of propagation

If direction of propagation: $\hat{z}$,

$E_z = H_z = 0$

$\rightarrow$ "Transverse Electromagnetic Waves" (TEM)

TE waves ⇝ $\bar{E}$ may have component in the dirn of propagation.

TM waves ⇝ $\bar{H}$ have component in the dirn of propagation
($\bar{H}$ is transverse to dirn of propagation)

→ $\nabla^2 \bar{E} + \beta^2 \bar{E} = 0$

For $\bar{E} = \hat{x} E_x(z) \Rightarrow \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$ (∵ $\bar{E}$ is uniform in the ~~dirn~~ plane ⊥ to dirn of propagation)

$$\Rightarrow \boxed{\frac{\partial^2 E_x(z)}{\partial z^2} + \beta^2 E_x(z) = 0}$$

$$\Rightarrow \boxed{E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}}\, e^{j\omega t}$$

Forward wave — Backward wave

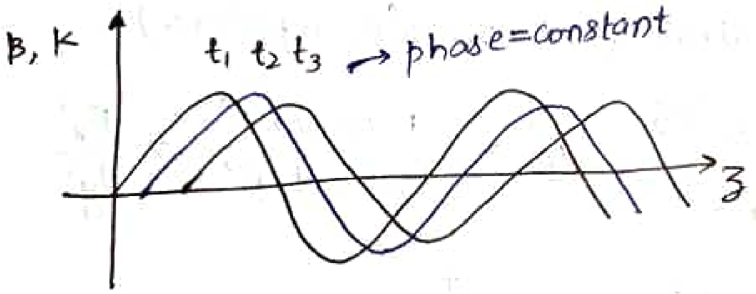
$e^{j(\omega t - \beta z)}$

phase $= \omega t\uparrow - \beta z\uparrow =$ constant

ω: fixed

β: constant for fixed ω and constant medium

$\beta \leftrightarrow k$
β → in EM; k → in optics; → same → phase constant or wave number



→ $\omega t - \beta z = \text{constant}$

$\Rightarrow \omega - \beta \frac{dz}{dt} = 0$

$\Rightarrow \frac{dz}{dt} = \frac{\omega}{\beta}$

$\Rightarrow \boxed{v_p = \frac{\omega}{\beta}} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} = \frac{1}{\sqrt{LC}} = c$

$\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m

$\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m $= \frac{10^{-9}}{36\pi}$ F/m

Generally, in medium
$\mu = \mu_0$ ⇝ non-magnetic
ϵ ⇝ changes

↳ In free space, EM waves travel with the velocity of light.

$\epsilon = \epsilon_0 \epsilon_r$

$\mu = \mu_0 \mu_r$

Generally, when we say a field changes, we talk about changing of its phase.

20-09-2023

As $\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$,

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -j\omega\mu\bar{H}$$

$$\Rightarrow \hat{y}\frac{\partial}{\partial z}E_x = -j\omega\mu\bar{H}$$

$$\Rightarrow \hat{y}\left\{\frac{(+j\beta)E_x^+e^{-j\beta z} - j\beta E_x^- e^{j\beta z}}{j\omega\mu}\right\} = +j\omega\mu\bar{H} \left[\because E_x(z) = E_x^+e^{-j\beta z} + E_x^- e^{j\beta z}\right]$$

$$\Rightarrow H_y = \frac{\beta}{\omega\mu}E_x^+e^{-j\beta z} - \frac{\beta}{\omega\mu}E_x^-e^{j\beta z}$$

$$= \frac{1}{\sqrt{\mu/\epsilon}}\left(E_x^+e^{-j\beta z} - E_x^-e^{j\beta z}\right)$$

$$\therefore H_y = \frac{1}{\eta}\left(E_x^+e^{-j\beta z} - E_x^-e^{j\beta z}\right)$$

$\frac{[H]}{[E]} = \frac{A/m}{V/m} \rightarrow$ unit of Impedance $\left(\frac{1}{\sqrt{\mu/\epsilon}}\right)$

η: Intrinsic impedance of the medium (= characteristic impedance).

~~Forward $\vec{E}$ / Backward $\vec{E}$~~

$$\frac{E_x^+e^{-j\beta z}}{\frac{1}{\eta}E_x^+e^{-j\beta z}} = \eta = \frac{\text{Forward } \vec{E}}{\text{Forward } \vec{H}} = \frac{E_x^+}{H_y^+}$$

$$\frac{E^-e^{j\beta z}}{-\frac{1}{\eta}E^-e^{+j\beta z}} = -\eta = \frac{E_x^-}{H_y^-}$$

$\rightarrow$ $\bar{E} = \hat{x}E_x(z)$

$\bar{H} = \hat{y}H_y(z)$

$\bar{E}\times\bar{H}$ $\leftarrow$ vector will be the dirn of propagation

If $E = \hat{y}E_y(z)$,

$$E_y = \underbrace{E^+e^{-j\beta z}}_{E_y^+} + \underbrace{E^-e^{j\beta z}}_{E_y^-}$$

$$\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$$

$$\Rightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = -j\omega\mu\bar{H}$$

$$\Rightarrow \hat{x}\left(-\frac{\partial E_y}{\partial z}\right) = -j\omega\mu\vec{H}$$

$$\Rightarrow \quad H_x = -\frac{1}{j\omega\mu}\left\{j\beta E^+ e^{-j\beta z} - j\beta E^- e^{j\beta z}\right\}$$

$$= \underbrace{-\frac{1}{\sqrt{\mu/\epsilon}} E^+ e^{-j\beta z}}_{H_x^+} + \underbrace{\frac{1}{\sqrt{\mu/\epsilon}} E^- e^{j\beta z}}_{H_x^-}$$

$$\therefore \frac{E_y^+}{H_x^+} = -\eta \; ; \quad \frac{E_y^-}{H_x^-} = \eta$$

→ If z ← dirn of propagation.

$$\frac{E_x^+}{H_y^+} = \eta \qquad (\text{As } \hat{x}\times\hat{y} = \hat{z} \Rightarrow +\eta)$$

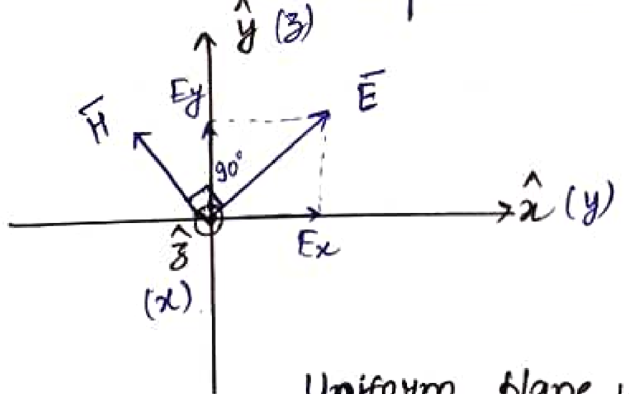
$$\frac{E_y^+}{H_x^+} = -\eta \qquad (\text{As } \hat{y}\times\hat{x} = -\hat{z} \Rightarrow -\eta)$$

→ If x : dirn of propagation,

$$\frac{E_z^+}{H_y^+} = -\eta$$

$$\frac{E_y^+}{H_z^+} = \eta$$

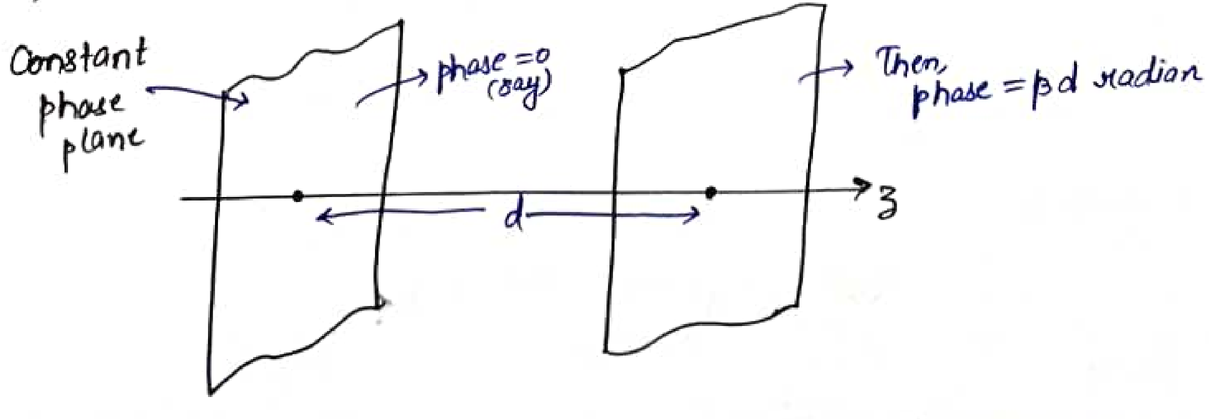
→ $\vec{E}$ and $\vec{H}$ are in the same phase (but are orthogonal).



⇝ $\vec{E}$ & $\vec{H}$ can have x & y-components and not z-component (for wave propagating in $\hat{z}$) as they are transverse

Uniform plane wave

→

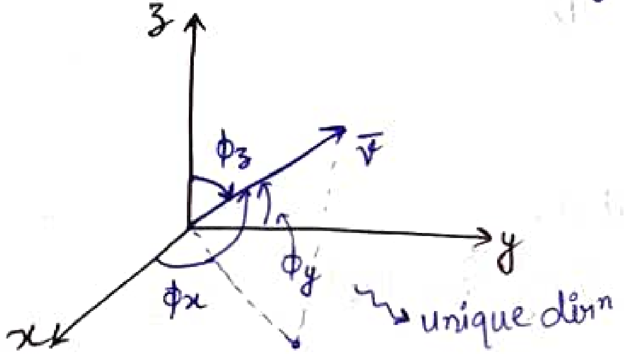


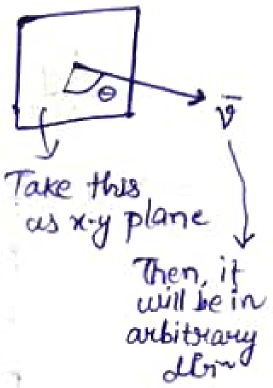
→ $\bar{E} = (\hat{x} E_x + \hat{y} E_y) e^{-j\beta x} \rightarrow \times$ (only forward wave shown)

$\bar{E} = (\hat{x} E_x + \hat{y} E_y) e^{-j\beta z} \rightarrow \checkmark$

$\bar{E} = (\hat{x} E_y - \hat{y} E_y) e^{-j\beta z} \rightarrow \checkmark$

→ **Direction cosines:** Express fields in arbitrary directions.





$v_x = v \cos\phi_x$

$v_y = v \cos\phi_y$

$v_z = v \cos\phi_z$

$\rightarrow$

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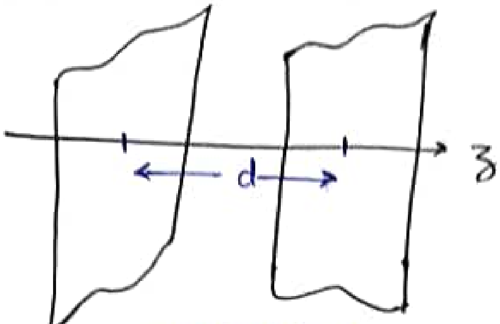
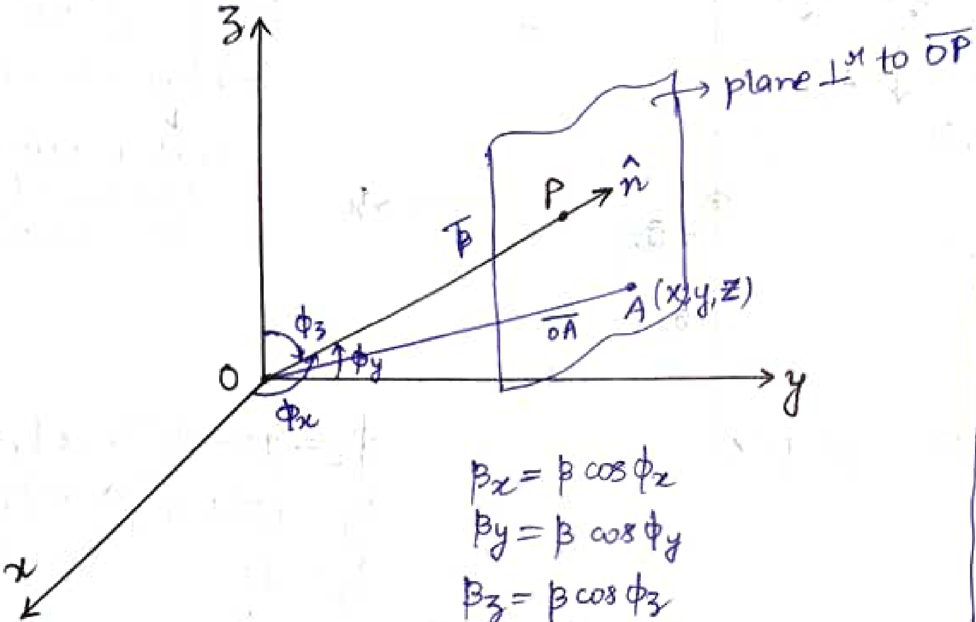
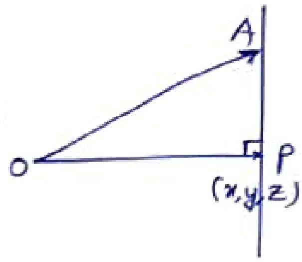


TEM/VPW

μ, ϵ, σ (conductivity) — characterize the medium





$\beta_x = \beta \cos\phi_x$

$\beta_y = \beta \cos\phi_y$

$\beta_z = \beta \cos\phi_z$

OP is the normal component of $\overline{OA}$.

$\overline{OP} = x\hat{x} + y\hat{y} + z\hat{z}$

$\rightarrow \overline{OA}\cdot\hat{n}$

$\hat{n}$: normal unit vector w.r.t. constant phase plane

$\overline{OA} = \bar{r}$

$|\overline{OP}| = \overline{OA}\cdot\hat{n} = \hat{n}\cdot\bar{r} = \text{constant}$

β rad/m $\rightarrow$ phase constant

$\bar{\beta} = \hat{n}\beta$

Phase at the constant phase plane

$= \beta\,|\overline{OP}|$ $\rightarrow$ distance $\times$ phase constant

$= \beta\,\hat{n}\cdot\bar{r}$

wave $\rightarrow$ $\bar{E} = \bar{E}_0\, e^{-j\beta\hat{n}\cdot\bar{r}}$

$\bar{E}_0 \rightarrow \hat{x}E_{0x} + \hat{y}E_{0y} + \hat{z}E_{0z}$

$\bar{E}_x(z) = E_{x_0}\, e^{-j\beta z}\,\hat{x}$ ($\beta z \rightarrow$ phase)

$\bar{E}(z) = \bar{E}_{x_0}\, e^{-j\beta z}$

$\bar{E}_{x_0} \rightarrow (E_{x_0}\hat{x})$

$\bar{E} = \bar{E}_0\, e^{-j\bar{\beta}\cdot\bar{r}}$

$= \bar{E}_0\, e^{-j(\hat{x}\beta_x + \hat{y}\beta_y + \hat{z}\beta_z)\cdot(x\hat{x} + \hat{y}y + \hat{z}z)}$

$\therefore \bar{E} = \bar{E}_0\, e^{-j(x\beta_x + y\beta_y + z\beta_z)}$

$= \bar{E}_0\, e^{-j\beta(x\cos\phi_x + y\cos\phi_y + z\cos\phi_z)}$

β: phase constant/wave no.

$\bar{\beta}$: propagation vector

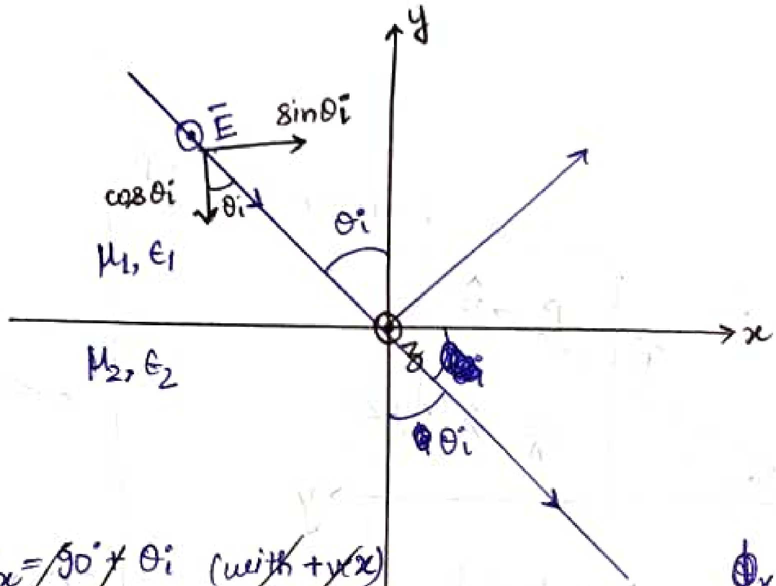
$\bar{\beta} = \beta\hat{n}$

Special Case:

$\cos\phi_x = \cos\phi_y = 0 \Rightarrow \beta_x = \beta_y = 0.$

$\therefore \quad \bar{E} = \bar{E}_0 \, e^{-j\beta_z z}$

→



Interface plane : xz

Incident plane : xy

↓

Incident, reflected & transmitted waves lie in this plane

~~$\phi_x = 90° + \theta_i$ (with +y(x))~~

~~$\phi_y = \phi_i$~~

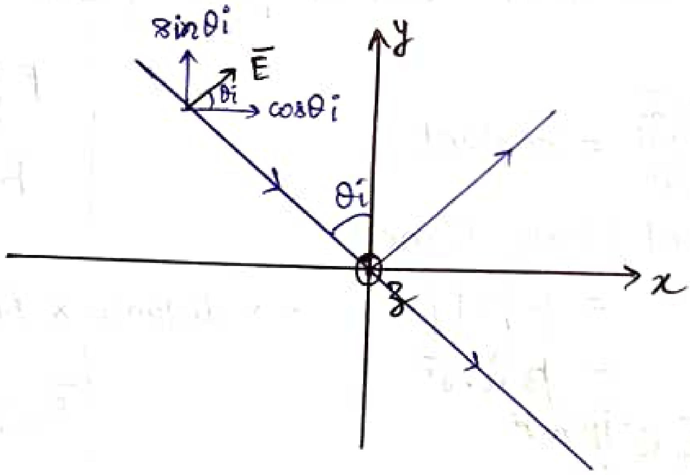
~~$\phi_z = 90°$~~

$$\left.\begin{aligned} \phi_x &= -(90° - \theta_i) \\ \phi_y &= 180° + \theta_i \\ \phi_z &= 90° \end{aligned}\right\} \begin{aligned} &\Rightarrow \cos\phi_x = \sin\theta_i \\ &\Rightarrow \cos\phi_y = -\cos\theta_i \\ &\Rightarrow \cos\phi_z = 0 \end{aligned}$$

⇓

Away from origin & with +ve axes.

$$\therefore \quad \bar{E} = \hat{z} E_0 \, e^{-j\beta(x\sin\theta_i - y\cos\theta_i)}$$



$$\therefore \bar{E} = E_0\left(\hat{x}\cos\theta_i + \hat{y}\sin\theta_i\right) \underbrace{e^{-j\beta(x\sin\theta_i - y\cos\theta_i)}}_{\text{constant}}$$

27-09-2023

$\rightarrow$ If $\bar{E} = \bar{E}_0 e^{-j\bar{\beta}\cdot\bar{r}}$

$= \bar{E}_0 e^{-j\beta(x\cos\phi_x + y\cos\phi_y + z\cos\phi_z)} = \overbrace{(\hat{x}E_{x_0} + \hat{y}E_{y_0} + \hat{z}E_{z_0})}^{\bar{E}_0} e^{-j(x\beta_x + y\beta_y + z\beta_z)}$

<u>To find $\bar{H}$.</u>

$\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$

$\Rightarrow -j\omega\mu\bar{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$

$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -j\beta_x & -j\beta_y & -j\beta_z \\ E_x & E_y & E_z \end{vmatrix}$

$\Rightarrow \omega\mu\bar{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \beta_x & \beta_y & \beta_z \\ E_x & E_y & E_z \end{vmatrix}$

$E_x = E_{x0} e^{-j(x\beta_x + y\beta_y + z\beta_z)}$

$E_y = E_{y0} e^{-j(x\beta_x + y\beta_y + z\beta_z)}$

$E_z = E_{z0} e^{-j(x\beta_x + y\beta_y + z\beta_z)}$

$\hookrightarrow \frac{\partial}{\partial x}E_x = \underbrace{-j\beta_x}\, E_{x0} e^{-j(x\beta_x + y\beta_y + z\beta_z)}$

\# $\hat{x}\left\{\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right\} \leftarrow \begin{matrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_y & E_z \end{matrix}$

$= \hat{x}\left\{-j\beta_y E_{z0} e^{-j(\ )} - j\beta_z E_{y0} e^{-j(\ )}\right\}$

$\Rightarrow \omega\mu\bar{H} = \bar{\beta}\times\bar{E}$

$\Rightarrow \omega\mu\bar{H} = \hat{n}\beta\times\bar{E}$ $\quad (\because \bar{\beta} = \hat{n}\beta)$

$\Rightarrow \bar{H} = \frac{\beta}{\omega\mu}\hat{n}\times\bar{E}$

$= \frac{\omega\sqrt{\mu\epsilon}}{\omega\mu}\hat{n}\times\bar{E}$

$\therefore \boxed{\bar{H} = \frac{1}{\eta}\hat{n}\times\bar{E}}$ $\quad \left(\frac{E}{H} = \eta\right)$

$\Rightarrow \hat{n}\times\bar{H} = \frac{1}{\eta}\hat{n}\times\hat{n}\times\bar{E}$

$\Rightarrow \eta(\hat{n}\times\bar{H}) = \hat{n}(\cancel{\hat{n}\cdot\bar{E}}^{\,0}) - \bar{E}(\hat{n}\cdot\hat{n})$ $\quad [\because (\bar{E}, \bar{H}) \perp$ dirn of propagation $(\hat{n})]$

$\Rightarrow \boxed{\bar{E} = -\eta(\hat{n}\times\bar{H})}$

Eg. $\bar{E} = \hat{x} E_x e^{-j\beta z}$

$$\bar{H} = \frac{1}{\eta} \hat{n} \times \bar{E}$$
$$= \frac{1}{\eta} \hat{z} \times \hat{x} (E_x e^{-j\beta z})$$
$$= \hat{y} \frac{1}{\eta} E_x e^{-j\beta z}$$

Eg. $\bar{E} = (\hat{x} E_x + \hat{y} E_y) e^{-j\beta z}$

$$\bar{H} = \frac{1}{\eta} \hat{z} \times (\hat{x} E_x + \hat{y} E_y) e^{-j\beta z}$$
$$= \frac{1}{\eta} \cancel{E_x e^{-j\beta z}} \{\hat{y} E_x - \hat{x} E_y\} e^{-j\beta z}$$
$$= \hat{y} \underbrace{\frac{E_x}{\eta} e^{-j\beta z}}_{H_y} - \hat{x} \underbrace{\frac{E_y}{\eta} e^{-j\beta z}}_{H_x}$$

Dirn of propagation $\rightarrow$ +ve z

E_y H_x

$y \times x = -z$

$\therefore \frac{E_y}{H_x} = -\beta$

$\rightarrow$ $\bar{E} = \hat{x} E_x e^{-j\beta z}$

$$\bar{E}(z,t) = \hat{x} E_x e^{-j\beta z} e^{j\omega t}$$
$$= \hat{x} E_x e^{j(\omega t - \beta z)}$$
$$= \hat{x} E_x \sin(\omega t - \beta \cancel{z}^{0})$$ (looking at z=0; for simplicity)
$$= \hat{x} E_x \sin \omega t$$

POLARISATION

Possibilities:

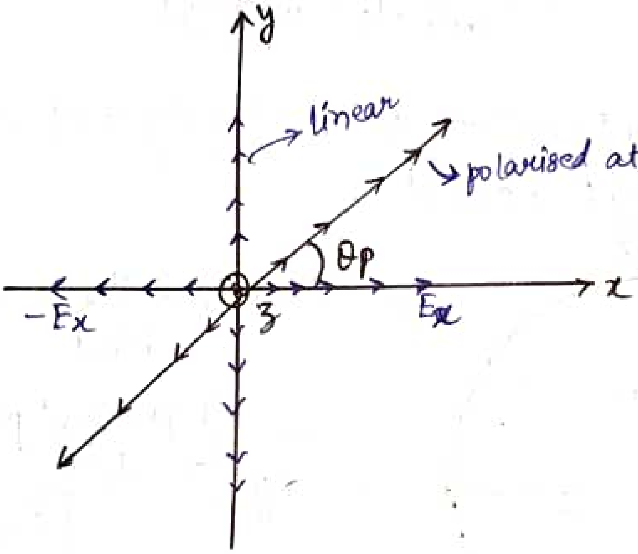
① $\bar{E}(z,t) = \hat{x} E_x e^{-j\beta z} e^{j\omega t}$

$= \hat{x} E_x e^{j(\omega t - \beta z)}$

$= \hat{x} E_x \sin(\omega t - \beta z)$ (looking at $z=0$; for simplicity) → at a particular location

$= \hat{x} E_x \sin \omega t$

② $\bar{E}(z,t) = \hat{y} E_y \sin \omega t$



③ $\bar{E} = (\hat{x} E_x + \hat{y} E_y) \sin \omega t$

$= \hat{x} E_x \sin \omega t + \hat{y} E_y \sin \omega t$

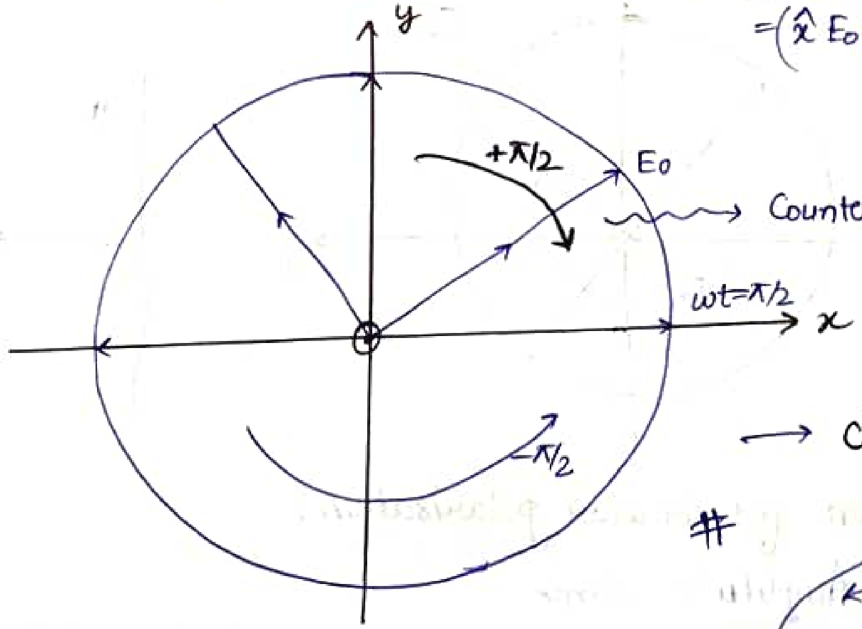
④ $\bar{E} = \{\hat{x} E_0 \sin(\omega t) + \hat{y} E_0 \sin(\omega t + \pi/2)\} e^{-j\beta z}$

$= (\hat{x} E_0 \sin \omega t + \hat{y} E_0 \cos \omega t)$

($z=0$; y-component : leading, x-comp. : lagging)

⑤ $\bar{E} = (\hat{x} E_0 \sin \omega t + \hat{y} E_0 \sin(\omega t - \pi/2))$

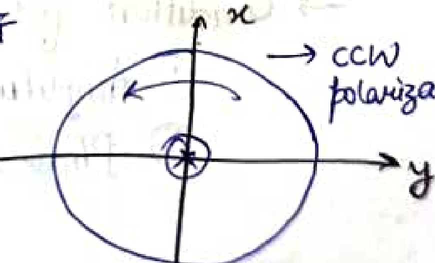
$= (\hat{x} E_0 \sin \omega t - \hat{y} E_0 \cos \omega t)$



→ Counter-clockwise (CCW) polarization → Always look from the back

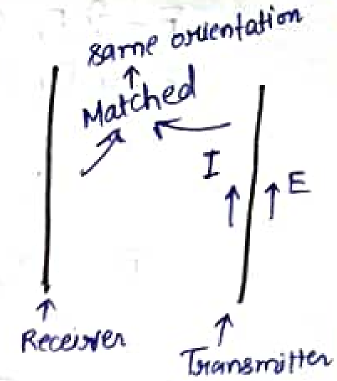
→ Circular Polarisation → CW, CCW

#



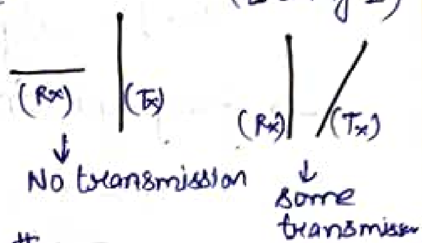
→ CCW polarization

#

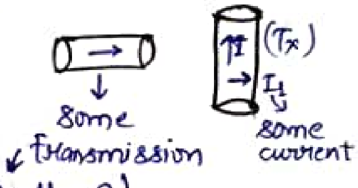


$\bar{J} = \sigma \bar{E}$ ($\bar{E}$ along I)

#



#



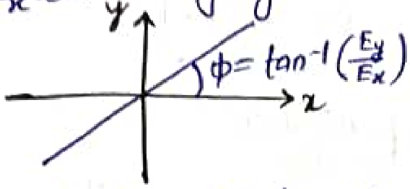
$\bar{E} = \hat{x} E_x \cos \omega t$, $\bar{E} = \hat{y} E_y \cos \omega t$ } linearly polarised (linear)

3-10-2023

→ $\bar{E} = \hat{x} E_x \cos\omega t$
$\bar{E} = \hat{y} E_y \cos\omega t$ } linearly polarised (linear)

phase diff = 0
angle = 90°

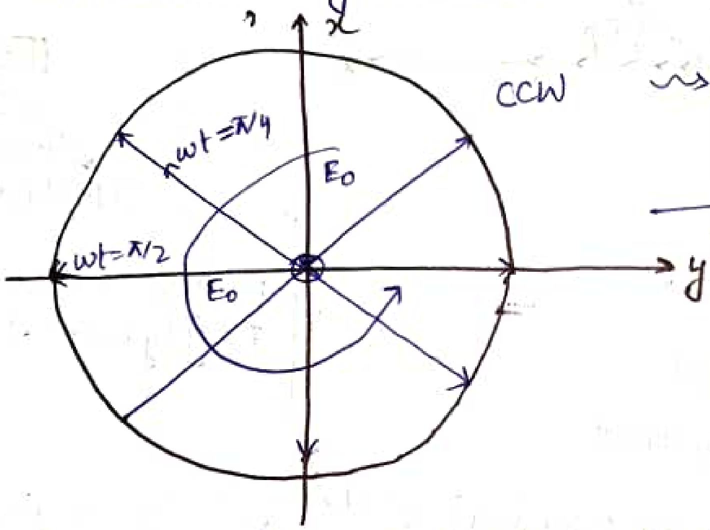
$\bar{E} = \hat{x} E_x \cos\omega t + \hat{y} E_y \cos\omega t$ } linear



↳ Linearly polarised: phase difference = 0.
(amplitude = anything)

→ propagation → +ve z

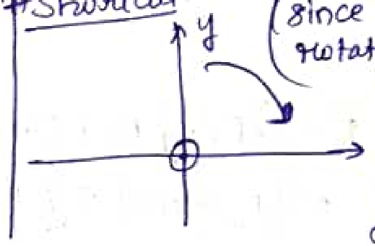
$\bar{E} = \hat{x} E_0 \cos\omega t + \hat{y} E_0 \cos(\omega t + \pi/2)$ ⇝ leading component: y
$= \hat{x} E_0 \cos\omega t - \hat{y} E_0 \sin(\omega t)$



CCW ⇝ Look the wave from behind

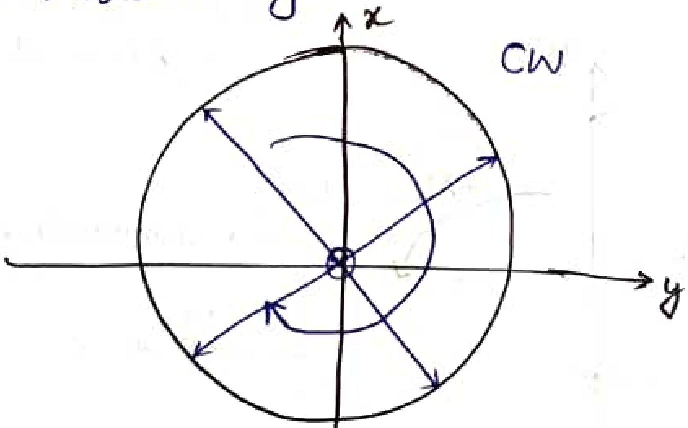
⟶ Left hand polarised

#Shortcut



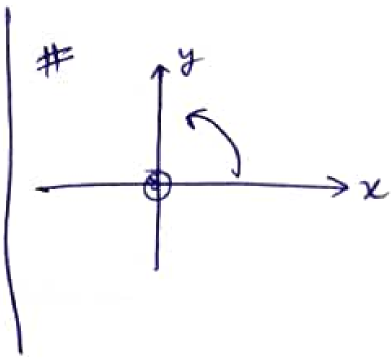
(since y is leading, rotate from y to x) ⇓ gives dirⁿ of polarisation

$\bar{E} = \hat{x} E_0 \cos\omega t + \hat{y} E_0 \cos(\omega t - \pi/2)$ ⇝ leading x
$= \hat{x} E_0 \cos\omega t + \hat{y} \sin\omega t$



CW

#



→ Condition for circular polarisation:

(i) Amplitude same.

(ii) Phase difference = 90°.

Notations in Antenna theory:

① Right hand (RHCP) = CW

② Left hand (LHCP) = CCW

↳ Point thumb in the direction of propagation, if it is the left hand curling in the dirn of polarisation, it is left hand polarised, otherwise right hand polarised.

→ Propagation → +ve z.

$$\bar{E} = \hat{x} E_0 e^{-j\beta z} e^{j\omega t} + \hat{y} E_0 e^{-j\beta z} e^{j(\omega t + \pi/2)}$$ ⇝ leading y

$$= (\hat{x} + \underbrace{j}_{+ve \Rightarrow \text{leading } y}\hat{y}) E_0 e^{-j\beta z} e^{j\omega t}$$ ⇝ compact form

$$\bar{E} = \hat{x} E_0 e^{j\omega t} e^{-j\beta z} + \hat{y} E_0 e^{j(\omega t - \pi/2)} e^{-j\beta z}$$

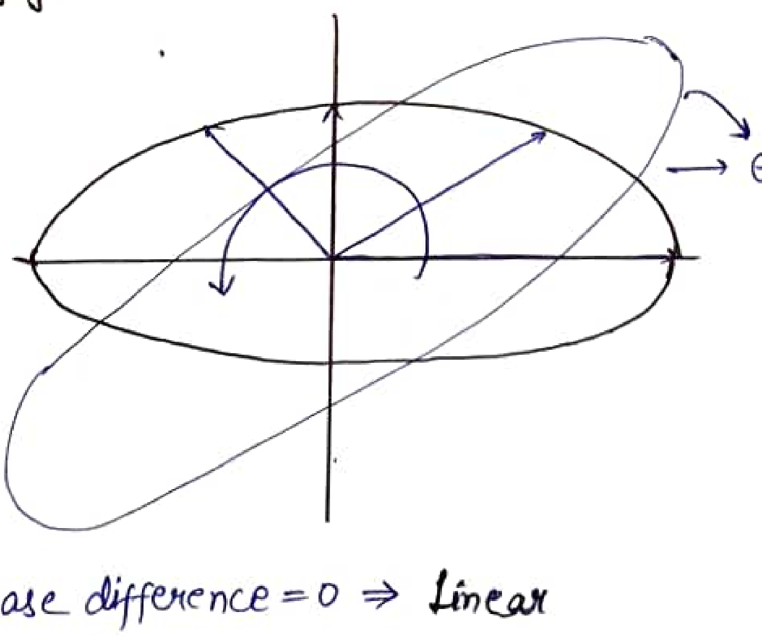
$$= (\hat{x} \underbrace{-j}_{-ve \Rightarrow \text{lagging } y \Rightarrow \text{leading } x}\hat{y}) E_0 e^{j\omega t} e^{-j\beta z}$$

$\bar{E} = (\hat{x}) E_0 e^{j(\omega t - \beta z)}$ ⇝ linearly polarised

$= (\hat{x} + j\hat{y}) E_0 e^{j(\omega t - \beta z)}$ ⇝ circularly polarised

$= (j\hat{x} + j\hat{y}) E_0 e^{j(\omega t - \beta z)}$ ⇝ linear

→ Elliptically polarised:



→ elliptically polarised

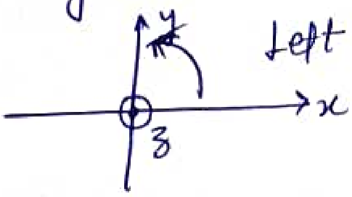
↳ If major axis = minor axis
⇓
circularly polarised

① If the phase difference = 0 ⇒ Linear

② If the phase difference = $\pm\frac{\pi}{2}$, and magnitudes are same ⇒ Circular

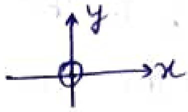
→ $\bar{E} = (j\hat{x} + \hat{y}) E_0 e^{j(\omega t + \beta z)}$ ⇝ wave propagation : $-z$

Left hand (LHCP)



→ $\bar{E} = (-j\hat{x} + \hat{y}) E_0 e^{j(\omega t + \beta z)}$ → RHCP

$e^{-j\pi/2}\hat{x} - e^{j\pi}\hat{y}$



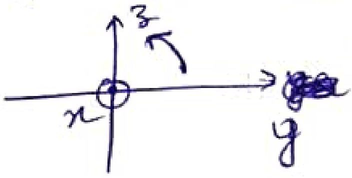
→ $\bar{E} = (-j\hat{x} - \hat{y}) E_0 e^{j(\omega t + \beta z)}$ ⇝ leading x

↳ LHCP

→ $\bar{E} = (j\hat{z} - \hat{y}) E_0 e^{j(\omega t + \beta x)}$ ⇝ propagation: $-x$

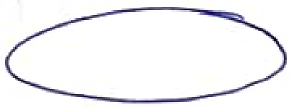
$e^{j\pi/2}\hat{z} + e^{j\pi}\hat{y}$ ⇝ leading y

→ LHCP



$\vec{E} = \hat{x} E_1 \cos\omega t + \hat{y} E_2 \sin\omega t$

$|\bar{E}| = \sqrt{(E_1 \cos\omega t)^2 + (E_2 \sin\omega t)^2}$



04-10-2023

$\rightarrow$ $E_x = E_1 \cos(\omega t) \Rightarrow \cos\omega t = \frac{E_x}{E_1} \Rightarrow \sin\omega t = \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}$

$E_y = E_2 \cos(\omega t + \delta)$

$$\Rightarrow \cos(\omega t + \delta) = \frac{E_y}{E_2}$$

$$\Rightarrow \cos\omega t \cos\delta - \sin\omega t \sin\delta = \frac{E_y}{E_2}$$

$$\Rightarrow \frac{E_x}{E_1}\cos\delta - \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}\sin\delta = \frac{E_y}{E_2}$$

$$\Rightarrow \left(\frac{E_x}{E_1}\cos\delta - \frac{E_y}{E_2}\right) = \sqrt{1-\left(E_x/E_1\right)^2}\sin\delta$$

$$\Rightarrow \left(\frac{E_x}{E_1}\right)^2\cos^2\delta + \left(\frac{E_y}{E_2}\right)^2 - \frac{2E_xE_y}{E_1E_2}\cos\delta = \left(1-\left(\frac{E_x}{E_1}\right)^2\right)\sin^2\delta$$

$$\Rightarrow \boxed{\left(\frac{E_x}{E_1}\right)^2 + \left(\frac{E_y}{E_2}\right)^2 - \frac{2E_xE_y}{E_1E_2}\cos\delta = \sin^2\delta}$$

Conditions:

① $\delta = 0$ (no phase difference)

$$\left(\frac{E_x}{E_1} - \frac{E_y}{E_2}\right)^2 = 0 \Rightarrow \frac{E_x}{E_1} = \frac{E_y}{E_2}$$

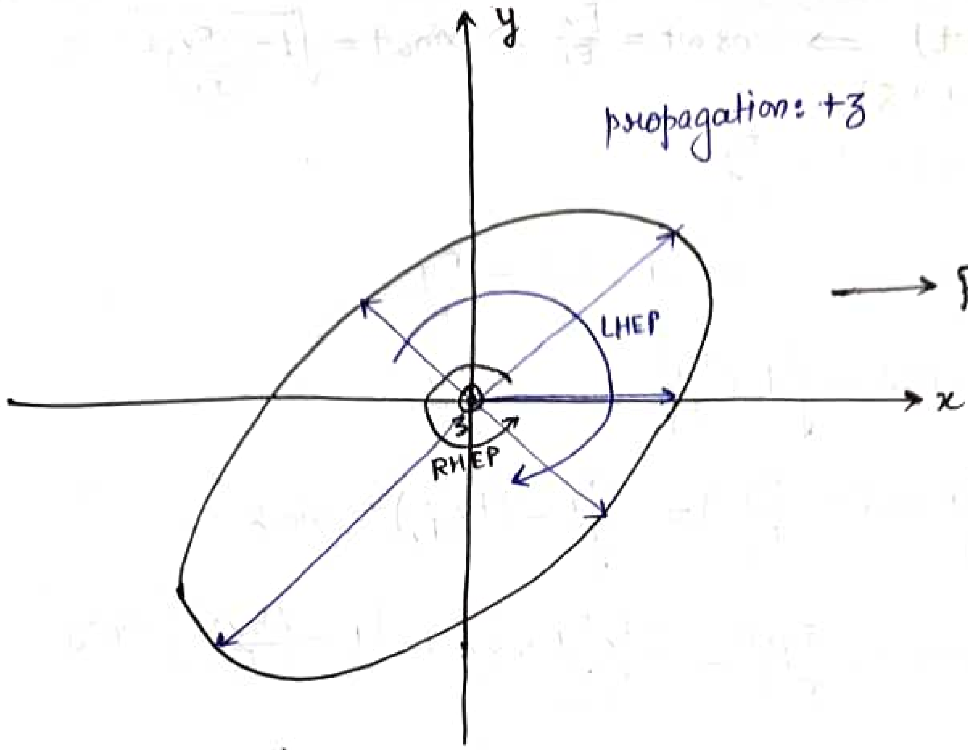
$$\Rightarrow \boxed{E_y = \frac{E_2}{E_1}E_x}$$

② $\delta = \pm \pi/2$, $E_1 = E_2 = E_0$.

$$\boxed{\left(\frac{E_x}{E_0}\right)^2 + \left(\frac{E_y}{E_0}\right)^2 = 1} \rightarrow \text{Circularly polarised}$$

③ $E_1 \neq E_2$

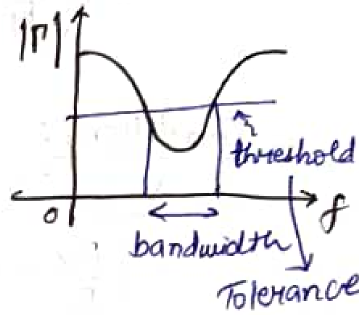
$$\boxed{\left(\frac{E_x}{E_1}\right)^2 + \left(\frac{E_y}{E_2}\right)^2 = 1} \rightarrow \text{Elliptically polarised}$$



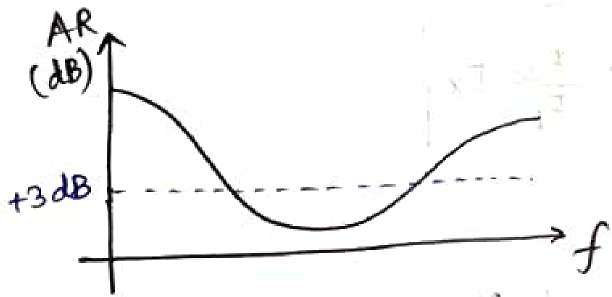
Polarisation
→ RHEP
→ LHEP

→ Polarization ellipse

Axial Ratio (AR) $= \dfrac{\text{major axis}}{\text{minor axis}}$

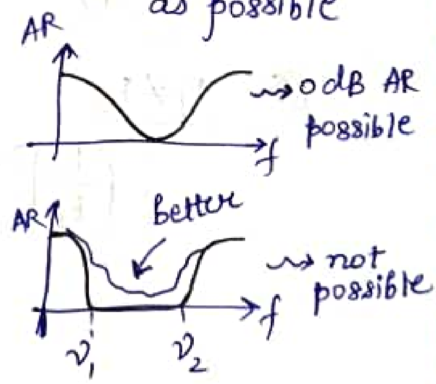


AR = 1
$\Rightarrow 10 \log(AR) = 0$ → Circular Polarisation

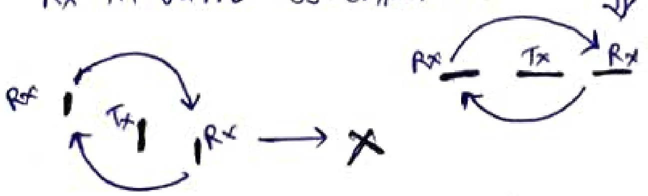


[AR: 0 to 3 dB
↳ can be considered circularly polarised]
↳ Design goal: as low as possible

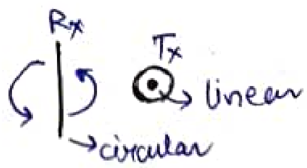
Generally, we talk about
$1 < |AR| < \infty$
Circularly polarised — Linearly polarised



Copolar measurement : Measuring Tx $\vec{E}$ with Rx in same orientation.



→ ✗ → ✓



⇓
constant $\vec{E}$ transmitted
(but not 100% transmission)

9-10-2023

Lossless, Sourcefree

$\alpha=0, \ \beta=\omega\sqrt{\mu\epsilon}$

$\bar{\nabla}\cdot\bar{E}=0$
$\bar{\nabla}\cdot\bar{B}=0$ } No free charge (source free)

$\bar{\nabla}\times\bar{E}=-j\omega\mu\bar{H}$

| $\bar{J}=\sigma\bar{E}$

$\bar{\nabla}\times\bar{H}=\bar{J}+j\omega\epsilon\bar{E}$, J: current density ($A/m^2$) ↳ b/c of conductivity of medium

$=\sigma\bar{E}+j\omega\epsilon\bar{E}$

$=\sigma\bar{E}+j\omega\epsilon_0\epsilon_r\bar{E}$

| $\underset{\text{conduction}}{\bar{J}}, \ \underset{\text{displacement}}{j\omega\epsilon\bar{E}}$

$=j\omega\epsilon_0\left(\epsilon_r+\frac{\sigma}{j\omega\epsilon_0}\right)\bar{E}$

$=j\omega\epsilon_0\left(\epsilon_r-j\frac{\sigma}{\omega\epsilon_0}\right)\bar{E}=j\omega\overbrace{\left(\epsilon_0\epsilon_r-j\frac{\sigma}{\omega}\right)}^{\epsilon_c}$, ϵ_c: complex permittivity

$=j\omega\epsilon\,\underline{\epsilon_{rc}}\,\bar{E}$ → For conducting medium

ϵ_{rc}: complex ϵ_r

$\epsilon_{rc}=\epsilon_r-j\frac{\sigma}{\omega\epsilon_0}$

→ ① complex
→ ② Frequency dependent

$\bar{\nabla}\times\bar{H}=j\omega\epsilon\bar{E}$
$=j\omega\epsilon_0\underline{\epsilon_r}\bar{E}$
→ For non-conducting ($\sigma=0$)
⇓
Dielectric

$\epsilon_{rc}=\epsilon_r'-j\epsilon''$

real, 'minus' has to be there, imaginary

→ $\bar{\nabla}\times\bar{E}=-j\omega\mu\bar{H}$

$\Rightarrow \bar{\nabla}\times\bar{\nabla}\times\bar{E}=-j\omega\mu\,\bar{\nabla}\times\bar{H}$

[For isotropic; ϵ,μ same in all 3-D directions]

$\epsilon_{rc}=10^5-j$ ⇝ good dielectric

$\epsilon_{rc}=1-j10^{0.5}$ ⇝ good conducting medium

$\Rightarrow \nabla(\cancel{\bar{\nabla}\cdot\bar{E}})^{0}-\bar{\nabla}^2\bar{E}=-j\omega\mu(\sigma\bar{E}+j\omega\epsilon\bar{E})$

$\Rightarrow -\bar{\nabla}^2\bar{E}=-j\omega\mu(\sigma+j\omega\epsilon)\bar{E}$

$\Rightarrow \bar{\nabla}^2\bar{E}-\underbrace{j\omega\mu(\sigma+j\omega\epsilon)}_{\gamma^2}\bar{E}=0$

$\Rightarrow \boxed{\bar{\nabla}^2\bar{E}-\gamma^2\bar{E}=0}$

$\nabla^2 V-\gamma^2 V=0$ ($R=0$)

$\Rightarrow \nabla^2 V-(\cancel{R}^{0}+j\omega L)(G+j\omega C)V=0$

$\Rightarrow \nabla^2 V-j\omega L(G+j\omega C)V=0$

$L \leftrightarrow \mu$
$C \leftrightarrow \epsilon$
$G \leftrightarrow \sigma$

Lossless: $\bar{\nabla}^2\bar{E}+\beta^2 E=0$

Lossy: $\bar{\nabla}^2\bar{E}-\gamma^2\bar{E}=0$

γ^2 ↑ $j\omega\mu(\sigma+j\omega\epsilon)$

Lossless: $\frac{d^2E_x}{dz^2} + \beta^2 E_x = 0$; Auxilliary Eq^n : $m^2+\beta^2=0$

Sol^n $\Rightarrow E_x = E^+ e^{-j\beta z} + E^- e^{j\beta z}$

Lossy: $\frac{d^2E_x}{dz^2} - \gamma^2 E_x = 0$

Sol^n $\Rightarrow \bar{E}_x = E^+ e^{-\gamma z} + E^- e^{\gamma z}$

$= E^+ \underbrace{e^{-\alpha z}} e^{-j\beta z} + E^- \underbrace{e^{\alpha z}} e^{j\beta z}$

denotes attenuation

Propagation constant

$\gamma = \sqrt{j\omega\mu(\sigma+j\omega\epsilon)} = \frac{j\omega\sqrt{\mu\epsilon}}{\beta}$

$= \alpha + j\beta$ → phase

Attenuation

$j\omega\epsilon_0\left(\epsilon_r - j\frac{\sigma}{\omega\epsilon_0}\right)$

$\epsilon_r \gg \frac{\sigma}{\omega\epsilon_0}$ → good dielectric ($\neq$ ideal dielectric) (pure)

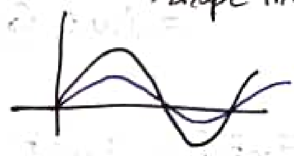
$\epsilon_r \ll \frac{\sigma}{\omega\epsilon_0}$ → good conductor

Lossless : $\alpha=0$, $\beta=\omega\sqrt{LC}$

Low loss : $\alpha\neq 0$, $\beta=\omega\sqrt{LC}$

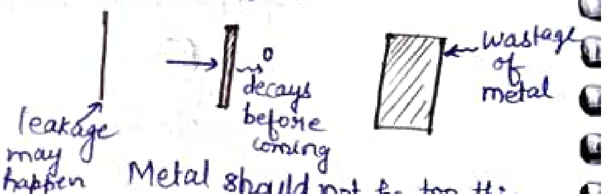
$\neq f(\omega)$

→ shape intact



\# Closed metal

→ EM shield



Metal should not be too thin or too thick.

10-10-2023

→ $\nabla^2\bar{E} - \gamma^2\bar{E} = 0$

$\bar{E} = \hat{x}E_x^+ e^{-\gamma z} + \hat{x}E_x^- e^{\gamma z}$

$= \hat{x}E_x^+ e^{-\gamma z} e^{-j\beta z} + \hat{x}E_x^- e^{\alpha z} e^{+j\beta z}$

$\gamma = \sqrt{j\omega\mu(\sigma+j\omega\epsilon)}$

Conduction current $= \sigma E$

Displacement current $= j\omega\epsilon E$

$\frac{\sigma}{\omega\epsilon} \gg 1$: Good conductor

$\frac{\sigma}{\omega\epsilon} \ll 1$: Good dielectric

Lossless ($\sigma/\omega\epsilon \ll 1$)

$$\gamma = \{j\omega\mu(\sigma + j\omega\epsilon)\}^{1/2}$$

$$= j\omega\sqrt{\mu\epsilon}\left(1 - j\frac{\sigma}{\omega\epsilon}\right)^{1/2}$$

$$(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 + \ldots$$

$$= j\omega\sqrt{\mu\epsilon}\left(1 - j\frac{\sigma}{2\omega\epsilon} + \underbrace{\left(\frac{1}{8}\right)\left(\frac{\sigma}{\omega\epsilon}\right)^2 - \ldots}_{\text{neglected}}\right)$$

$$\approx j\omega\sqrt{\mu\epsilon}\left(1 - j\frac{\sigma}{2\omega\epsilon}\right)$$

$$\alpha = \omega\sqrt{\mu\epsilon}\,\frac{\sigma}{2\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$\beta = \omega\sqrt{\mu\epsilon}$ → for lossless medium (same as in T/L)

↳ Non-dispersive medium

→ $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$\Rightarrow \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

Squaring both side and equating real & imaginary parts,

$$\left.\begin{aligned} \alpha &= \omega\sqrt{\frac{\mu\epsilon}{2}}\left\{\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right\}^{1/2} \\ \beta &= \omega\sqrt{\frac{\mu\epsilon}{2}}\left\{\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right\}^{1/2} \end{aligned}\right\} \text{(General case)}$$

↳ Different frequency waves will have different attenuation and different wave velocities.

↳ Dispersion, in conducting medium.

→ $E_x = E_x^+ e^{-\gamma z} + E_x^- e^{\gamma z}$

$\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H}$

$$\Rightarrow \vec{H} = -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cancel{\frac{\partial}{\partial x}}^{0} & \cancel{\frac{\partial}{\partial y}}^{0} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix}$$

$$= -\hat{y}\frac{1}{j\omega\mu}\frac{\partial E_x}{\partial z}$$

$$= \hat{y}\frac{1}{j\omega\mu}\left(\gamma E_x e^{-\gamma z} - \gamma E_x^- e^{\gamma z}\right)$$

$$= \hat{y}\frac{\gamma}{j\omega\mu}\left(E_x^+ e^{-\gamma z} - E_x^- e^{\gamma z}\right)$$

$$E_x = E_x^+ e^{-\gamma z} + E_x^- e^{\gamma z}$$

$$\therefore H_y = \frac{1}{\left(\frac{j\omega\mu}{\gamma}\right)}\left(E_x^+ e^{-\gamma z} - E_x^- e^{\gamma z}\right)$$

$$= \frac{1}{\eta}\left(E_x^+ e^{-\gamma z} - E_x^- e^{\gamma z}\right)$$

$$v = v^+ e^{-\gamma z} + v^- e^{-\gamma z}$$

$$I = \frac{1}{Z_0}\left(v^+ e^{-\gamma z} - v^- e^{\gamma z}\right)$$

Impedance, $\eta = \frac{j\omega\mu}{\gamma}$

$$= \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

↳ E & H are not in the same phase (for lossy medium)

Lossless:

$\frac{E}{H} = \eta = \sqrt{\frac{\mu}{\epsilon}}$ (real)

$E = \eta H \Rightarrow$ E & H are in the same phase

$\uparrow$ real

Good conductor:

$\sigma \gg \omega\epsilon$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\approx \sqrt{j\omega\mu\sigma}$$

$$= \sqrt{\omega\mu\sigma}\, e^{j\pi/2}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}\,(1+j)$$

$$e^{j\pi/2} = \cos(\pi/2) + j\sin(\pi/2) = j$$

For non-dispersive medium, ω & β should be linear.

$$\beta = \alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$

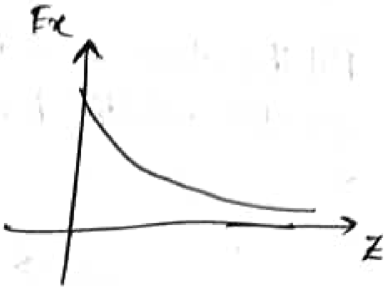
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\approx \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\pi/4}}$$

$E_x^+ e^{-\gamma z}$
$= E_x^+ e^{-\alpha z} e^{j\beta z}$

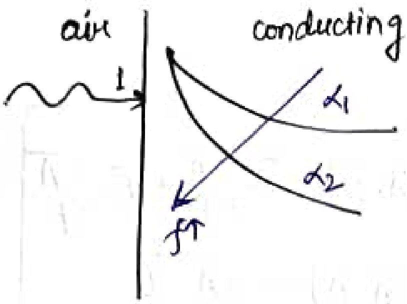
$$e^{-\alpha z} \xrightarrow{\text{reduces to}} e^{-1}$$



at the distance $z = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$

skin depth (δ)

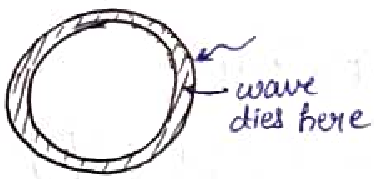
$\propto \frac{1}{\sqrt{f}} \Rightarrow f\uparrow \Rightarrow$ wave decays faster



→ At high frequency, wave is confined to the skin of the conductor.

"Skin effect"

Instead of having the entire conductor, we can have ring/pipe.



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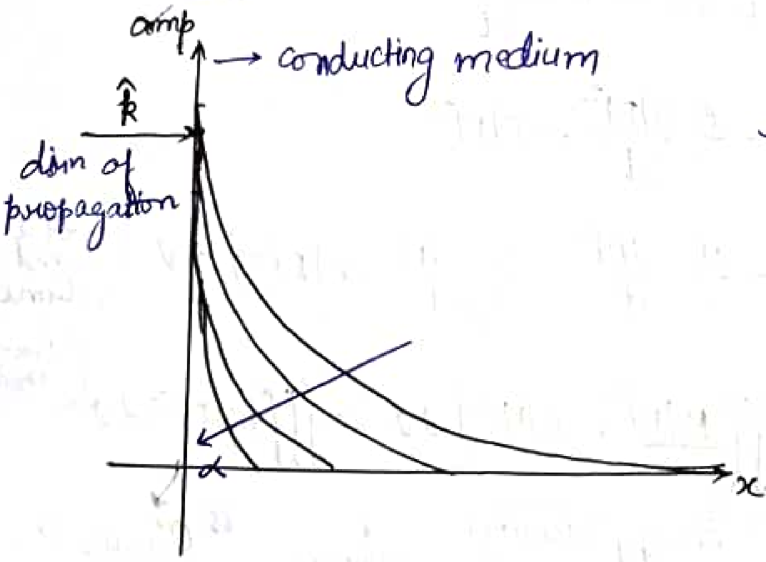
→ $E \sim e^{-\alpha x}$
$= e^{-\sqrt{\pi f \mu \sigma}\, x}$



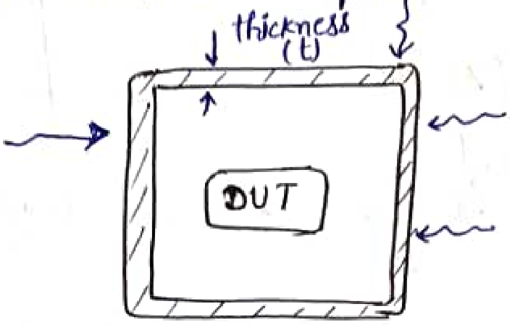
Skin effect:

$$x = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}} = \delta = \text{skin depth}$$

Measuring the radiation from a conductor

Problem: There are many other signals generating other fields.

Put the device under test (DUT) in a chamber with much greater width than skin depth.



$$\frac{\partial}{\partial t}(\bar{A}\cdot\bar{B}) = \bar{A}\cdot\frac{\partial \bar{B}}{\partial t} + \bar{B}\cdot\frac{\partial \bar{A}}{\partial t}$$

$$\frac{\partial}{\partial t}(\bar{A}\cdot\bar{A}) = 2\bar{A}\cdot\frac{\partial \bar{A}}{\partial t}$$

$$\frac{\partial}{\partial t}(|A|^2) = 2\bar{A}\cdot\frac{\partial \bar{A}}{\partial t}$$

POWER

Maxwell Eq^n: $\bar{\nabla}\times\bar{H} = \bar{J} + \epsilon\frac{\partial \bar{E}}{\partial t}$

$$\bar{\nabla}\times\bar{E} = -\mu\frac{\partial \bar{H}}{\partial t}$$

vector identity: $\bar{\nabla}\cdot(\bar{E}\times\bar{H}) = \bar{H}\cdot(\bar{\nabla}\times\bar{E}) - \bar{E}(\bar{\nabla}\times\bar{H})$

$$= \bar{H}\cdot\left(-\mu\frac{\partial \bar{H}}{\partial t}\right) - \bar{E}\cdot\left(\bar{J} + \epsilon\frac{\partial \bar{E}}{\partial t}\right)$$

$$= -\mu\bar{H}\frac{\partial \bar{H}}{\partial t} - \bar{E}\cdot\bar{J} - \epsilon\bar{E}\cdot\frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \bar{\nabla}\cdot(\bar{E}\times\bar{H}) = -\mu\bar{H}\frac{\partial \bar{H}}{\partial t} - \bar{E}\cdot\sigma\bar{E} - \epsilon\bar{E}\cdot\frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \bar{\nabla}\cdot(\bar{E}\times\bar{H}) = -\frac{\mu}{2}\frac{\partial |H|^2}{\partial t} - \frac{\epsilon}{2}\frac{\partial |E|^2}{\partial t} - \sigma|E|^2$$

$$\Rightarrow \iiint_v \bar{\nabla}\cdot(\bar{E}\times\bar{H})\,dv = \iiint_v \left(-\frac{\mu}{2}\frac{\partial |H|^2}{\partial t} - \frac{\epsilon}{2}\frac{\partial |E|^2}{\partial t} - \sigma|E|^2\right) dv$$ (over a closed volume)

(Divergence Theorem)

$$\Rightarrow \underbrace{\oint\!\!\oint (\bar{E}\times\bar{H})\cdot d\bar{s}}_{\text{Unit: Watt}} = \underbrace{-\frac{\partial}{\partial t}\iiint \overbrace{\left(\underbrace{\frac{\mu|H|^2}{2}}_{\text{Magnetic energy density}} + \underbrace{\frac{\epsilon|E|^2}{2}}_{\text{Electric energy density}}\right)}^{\text{energy (density)}} dv}_{\text{Decrease in power}} - \underbrace{\iiint \sigma|E|^2 dv}_{\text{"Ohmic" loss (due to resistance)}}$$

↳ Gives net outward flow of power from a given volume.

"Poynting Theorem"

→ $\underbrace{\oiint (\bar{E}\times\bar{H})\cdot d\bar{s}}_{\text{Total power going out}} \rightarrow$ watt

→ $\bar{P} = \bar{E}\times\bar{H} \rightarrow W/m^2$

↳ Poynting vector (Surface power density)

→ Power flows $\perp^{r}$ to $\bar{E}$ & $\bar{H}$.

⇒ Power flows in the direction of propagation of wave.

→ Power will flow only if there is electromagnetic wave (changing E and $\bar{H}$ with time).

$$\bar{E}(x,y,z) = \hat{e}\,\bar{E}_0(x,y,z)\, e^{j\omega t} e^{j\phi_e}$$

($\hat{e}$: any arbitrary dirn
ϕ_e : initial E phase)

$$\Rightarrow \bar{E}(x,y,z) = \hat{e}\, E_0(x,y,z)\cos(\omega t+\phi_e)$$

$$\bar{H}(x,y,z) = \hat{h}\, H_0(x,y,z)\, e^{j\omega t} e^{j\phi_h}$$

$$\Rightarrow \bar{H}(x,y,z) = \hat{h}\, H_0(x,y,z)\cos(\omega t+\phi_h)$$

$$\bar{P} = \bar{E}\times\bar{H} = E_0H_0\cos(\omega t+\phi_e)\cos(\omega t+\phi_h)\,(\hat{e}\times\hat{h})$$

$$= \frac{E_0H_0}{2}\left[\cos(2\omega t+\phi_e+\phi_h)+\cos(\phi_e-\phi_h)\right](\hat{e}\times\hat{h})$$

↳ Instantaneous power (density)
↓
not much useful

<u>Average over time period</u>:

$$P_{av.} = \frac{1}{T}\int_0^T \frac{E_0H_0}{2}\left[\cancelto{0}{\cos(2\omega t+\phi_e+\phi_h)} + \underbrace{\cos(\phi_e-\phi_h)}_{\substack{\text{constant} \\ \text{(not a } f^n \text{ of time)}}}\right](\hat{e}\times\hat{h})\,dt$$

$$= \frac{E_0H_0}{2}\cos(\phi_e-\phi_h)\,(\hat{e}\times\hat{h})$$

$$=\frac{1}{2}Re\left\{\underbrace{E_0 e^{j\phi_e} e^{j\omega t}\hat{e}}_{\bar{E}} \times \underbrace{H_0 e^{-j\phi_h} e^{-j\omega t}\hat{h}}_{\bar{H}^*}\right\}$$

$Re\{e^{j(\phi_e-\phi_h)}\} = \cos(\phi_e-\phi_h)$

$$\therefore \boxed{\bar{P}_{av} = \frac{1}{2} Re(\bar{E}\times\bar{H}^*)}.$$

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Lossless: $\bar{E} = \hat{x} E_0 e^{-j\beta z}$

$\bar{H} = \frac{1}{\eta}\hat{z}\times\bar{E}$, $\hat{z}$: dirn of propagation.

$= \frac{1}{\eta}\hat{y} E_0 e^{-j\beta z}$

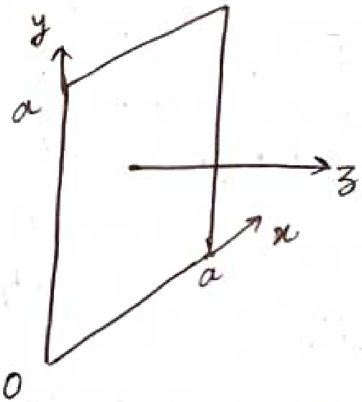
$\bar{P}_{av} = \frac{1}{2}Re\{\bar{E}\times\bar{H}^*\}$

$= \frac{1}{2}Re\left\{E_0 \frac{E_0^*}{\eta}\right\}\hat{z}$

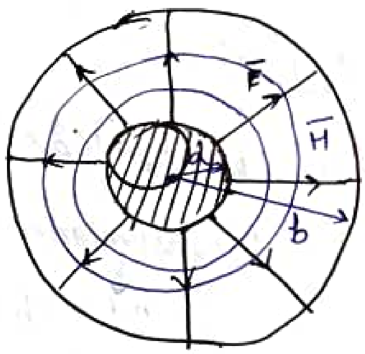
$= \frac{|E_0|^2}{2\eta}\hat{z}$ $\left[\because \eta^*_{lossless} = \sqrt{\frac{\mu}{\epsilon}}\right.$

$\bar{P}_{total} = \int_0^a\int_0^b P_{av}\, d\hat{z}\, dx\, dy\, \hat{z}$

$= \iint P_{av}\, dx\, dy$



Eg



$\bar{E} = (\text{given})\,\hat{\rho}$

Co-ordinate system
Cartesian : $x\ y\ z$
Spherical : $r\ \theta\ \phi$
Cylindrical : $\rho\ \phi\ z$

Find the power through the cross section.

Soln: $\bar{H} = (\hat{\phi})$

Elemental: $\rho\, d\rho\, d\phi$

$\bar{P}_{av} = f(\rho,\phi)\hat{z}$

$\int_a^b\int_0^{2\pi} f(\rho,\phi)\, \rho\, d\rho\, d\phi$

Lossy:

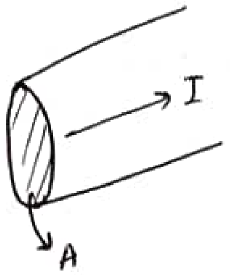
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\overline{P}_{av} = \frac{1}{2} Re\{\overline{E} \times \overline{H}^*\}$$

$$= \frac{|E_0|^2}{2} Re\left\{\frac{1}{\eta^*}\right\}$$

$$\eta_{\text{conducting medium}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$$

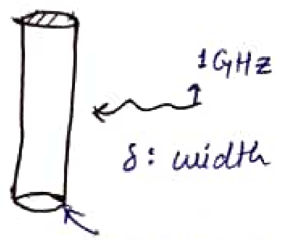
→



→ For a high frequency signal, current remains on the surface.

Current density $= \frac{I}{A}$ amp/m² (assumption: I is uniform)

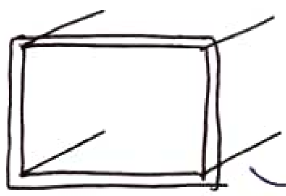
→



Surface current density,

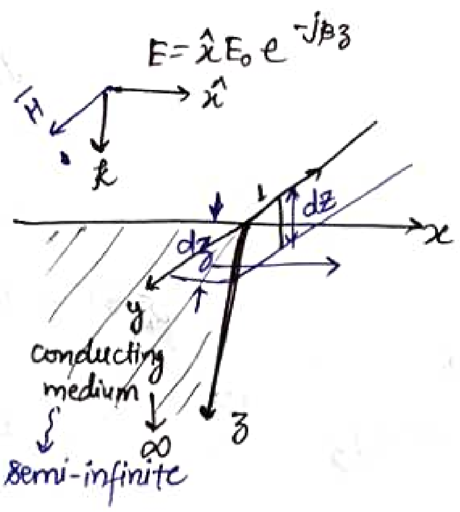
$$J_s = \frac{I}{\text{width}}$$

→



→ Rectangular Waveguide (eg., pipe)

→



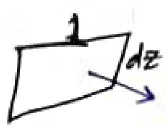
Induced current density, → volume c.d. (A/m²)

$$\underset{(A/m^2)}{\overline{J}} = \sigma \overline{E}$$

$$= \sigma E_0 e^{-\alpha z} e^{-j\beta z} \hat{x}$$

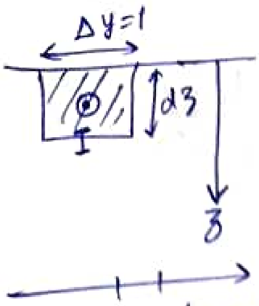
$$= \sigma E_0 e^{-\gamma z} \hat{x}$$

$$\text{Current} = \hat{x} \sigma E_0 e^{-\gamma z} dz$$



$(\because \text{Area} = dz \times 1 = dz)$

$$\text{Total Current/unit width} = \hat{x}\int_0^\infty \sigma E_0 e^{-\gamma z}dz$$



$$\Rightarrow \bar{J}_s = \hat{x}\,\frac{\sigma E_0}{-\gamma}e^{-\gamma z}\Big|_0^\infty$$

$$\therefore \boxed{\bar{J}_s = \hat{x}\,\frac{\sigma E_0}{\gamma}} \text{ A/m}$$

(confined to the surface)

$\gamma = \alpha + j\beta$

$e^{-\gamma z} = e^{-\alpha z}e^{-j\beta z}$

$\downarrow$

0 (because of $e^{-\alpha z}$ term only) as $z \to \infty$

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$$\eta_{conducting}\ (\eta_c) = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} \qquad (\because \sigma \gg \omega\epsilon)$$

$$= \sqrt{\frac{j\omega\mu\sigma}{\sigma^2}}$$

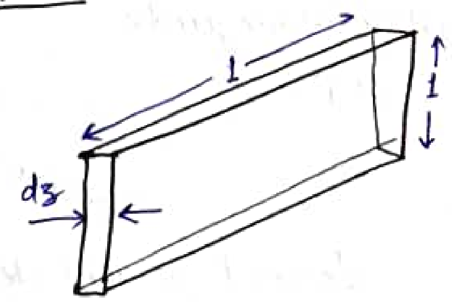
$$\gamma = \sqrt{j\omega\mu(\sigma + \cancel{j\omega\epsilon})} \approx \sqrt{j\omega\mu\sigma}$$

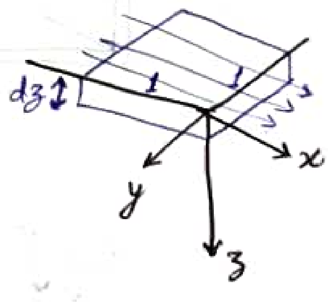
$$\therefore \boxed{\eta_c = \frac{\gamma}{\sigma}}$$

$$|E_0| = \eta_c |H_0|$$

$$|H_0| = \frac{|E_0|}{\eta} = \frac{\sigma|E_0|}{\gamma} = |J_s|$$

Power loss





$$dR = \frac{1}{\sigma}\frac{l}{A} = \frac{1}{\sigma}\frac{1}{dz}$$

$$\text{Ohmic loss} = \frac{1}{2}|I(z)|^2 dR$$

$$= \frac{1}{2}\sigma|E_0|^2 e^{-2\alpha z}dz$$

$$|e^{-\gamma z}|^2 = e^{-\alpha z}\cancel{e^{-j\beta z}} \cdot e^{-\alpha z}\cancel{e^{j\beta z}} = e^{-2\alpha z}$$

Power loss per unit area [for 1×1 area]

$$= \frac{1}{2}\sigma |E_0|^2 \int_0^{\alpha} e^{-2\alpha z} dz$$

$$= \frac{1}{2}\sigma |E_0|^2 \left. \frac{e^{-2\alpha z}}{-2\alpha} \right|_0^{\alpha}$$

$$= \frac{1}{2}\sigma |E_0|^2 \frac{1}{2\alpha}$$

$$= \frac{1}{2}\cancel{\sigma} \frac{|\gamma|^2 |J_s|^2}{\sigma \cancel{\sigma}} \frac{1}{2\alpha}$$

$$= \frac{1}{2} \frac{\omega\mu\sigma |J_s|^2}{\sigma} \frac{1}{\sqrt{2}\sqrt{\omega\mu\sigma}}$$

$$= \frac{1}{2} |J_s|^2 \frac{\sqrt{\omega\mu}}{\sqrt{2\sigma}}$$

$$= \frac{1}{2} |J_s|^2 \underbrace{\sqrt{\frac{\omega\mu}{2\sigma}}}$$

$$= \frac{1}{2} |J_s|^2 R_s$$

For linear : $\frac{1}{2} I^2 R$

For surface : $\frac{1}{2} |J_s|^2 R_s$

surface resistance

$$\left[\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\frac{\omega\mu\sigma}{2}}(1+j) \right]$$

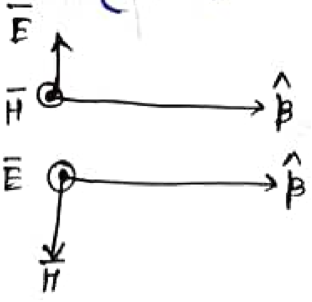
$$\left[\eta_c = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} e^{j\pi/4} = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = R_s + jX_s \right]$$

Interface:

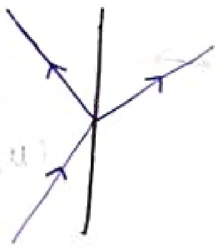
Medium 1	Medium 2
$(\sigma_1, \mu_1, \epsilon_1)$	$(\sigma_2, \mu_2, \epsilon_2)$

Assume $\sigma = 0$ purely dielectric (lossless)

(μ_1, ϵ_2) $\quad$ (μ_2, ϵ_2)



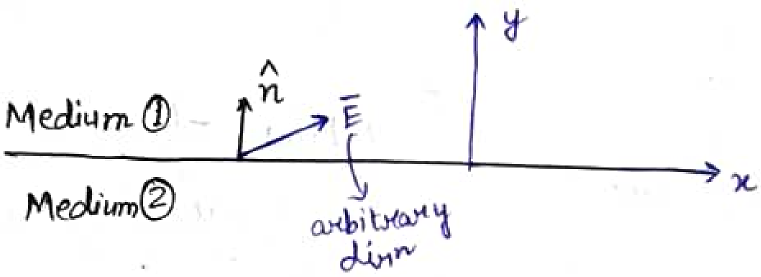
Wave will be reflected as well as transmitted



Boundary conditions

for ① $\bar{E}$ → $\perp$ component
→ $\parallel$ component (tangential component

for ② $\bar{H}$ → $\perp$
→ $\parallel$



Interface plane : xz

$\hat{n} \times \bar{E}$: tangential

$\hat{n} \cdot \bar{E}$: normal

1.) $\hat{n} \times (\bar{E}_2 - \bar{E}_1) = 0$: Tangential component of $\bar{E}$ is continuous across the interface

2.) $\hat{n} \times (\bar{H}_2 - \bar{H}_1) = 0$ → (if the interface or any of the medium is conducting)

$= \bar{J}_s$, otherwise

3.) $\hat{n} \cdot (\bar{D}_2 - \bar{D}_1) = 0$

4.) $\hat{n} \cdot (\bar{B}_2 - \bar{B}_1) = 0$:

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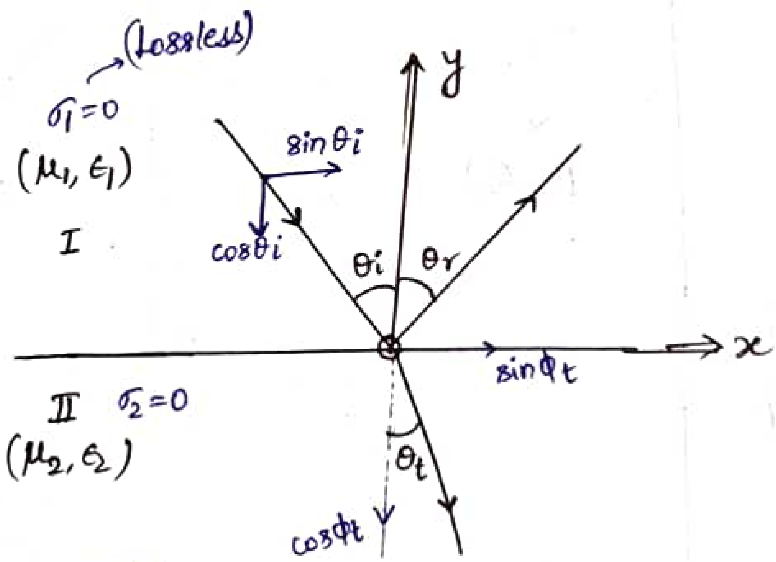
→



xy : plane of incidence

xz : plane of interface

$\bar{E} e^{-j\bar{\beta}\bar{r}}$

$= \bar{E} e^{-j(x\beta_x + y\beta_y + z\beta_z)}$

$= E e^{-j(x\beta\cos\phi_x + y\beta\cos\phi_y + z\beta\cos\phi_z)}$

$$\bar{E}_i = \underbrace{\bar{E}_{io}}_{\text{constant vector}} e^{-j\beta_1(x\sin\theta_i - y\cos\theta_i)}$$

$$\bar{E}_r = \bar{E}_{ro}\, e^{-j\beta_1(x\sin\theta_r + y\cos\theta_r)}$$

$$\bar{E}_t = \bar{E}_{to}\, e^{-j\beta_2(x\sin\theta_t - y\cos\theta_t)}$$

$\bar{E} = \hat{x}E_o e^{-j\beta z}$
$= \bar{E}_o \underbrace{e^{-j\beta z}}_{\text{only this will change}}$
$\rightsquigarrow z$

Applying boundary condition on $\bar{E}$,

$$(\bar{E}_{io})_{tan.} e^{-j\beta_1 x\sin\theta_i} + (\bar{E}_{ro})_{tan.} e^{-j\beta_1 x\sin\theta_r} = (\bar{E}_{to})_{tan.} e^{-j\beta_2 \sin\theta_t} \quad [\because \underbrace{y=0}_{\text{at } xz \text{ plane}}]$$

$$\underbrace{\beta_1\sin\theta_i = \beta_1\sin\theta_r}_{} = \beta_2\sin\theta_t \quad \text{[Phase terms must be same]}$$

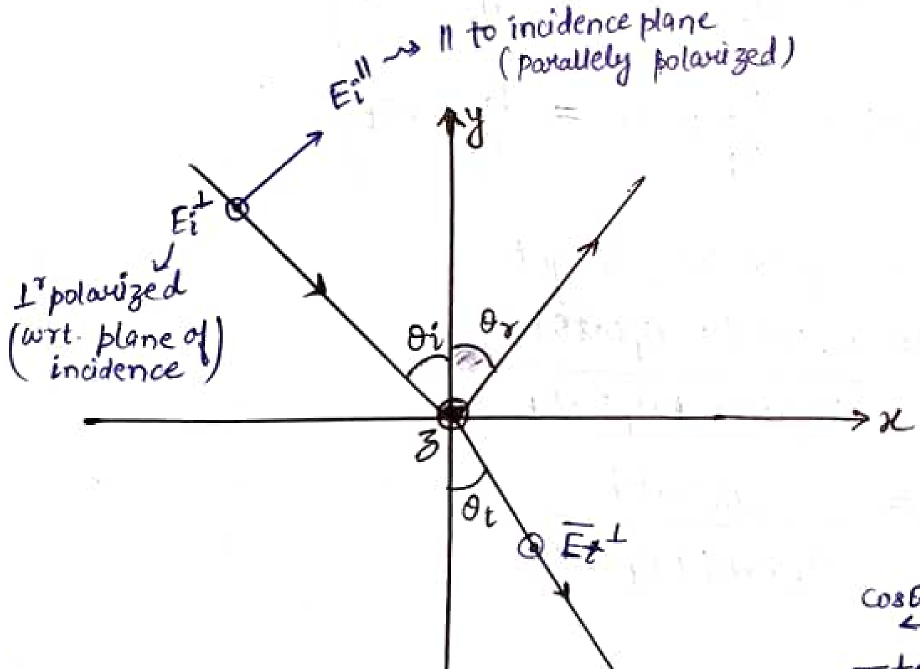
$$\Rightarrow \boxed{\theta_i = \theta_r}$$

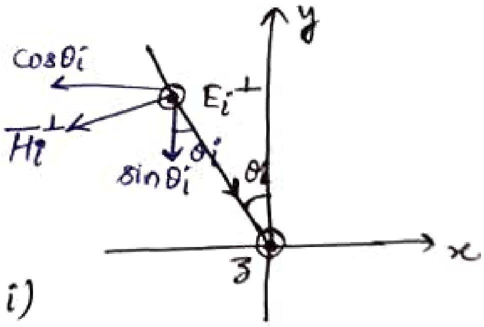
and, $\beta_1\sin\theta_i = \beta_2\sin\theta_t \Rightarrow \omega\sqrt{\mu_1\epsilon_1}\sin\theta_i = \omega\sqrt{\mu_2\epsilon_2}\sin\theta_t$

$$\Rightarrow \frac{\omega}{c}\sqrt{\epsilon_{r_1}}\sin\theta_i = \frac{\omega}{c}\sqrt{\epsilon_{r_2}}\sin\theta_t$$

$$\Rightarrow \boxed{n_1\sin\theta_i = n_2\sin\theta_t}$$

→



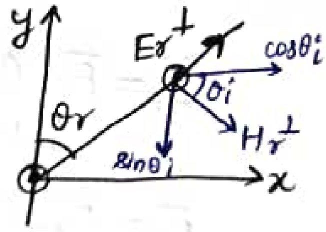


Perpendicular Polarized Incident wave:

$$\bar{E}_i^{\perp} = \hat{z}\underbrace{E_{io}}_{\text{scalar constant}} e^{-j\beta_1(x\sin\theta_i - y\cos\theta_i)}$$

$$\bar{H}_i^{\perp} = \frac{E_{io}}{\eta_1}(-\hat{x}\cos\theta_i - \hat{y}\sin\theta_i)\, e^{-j\beta_1(x\sin\theta_i - y\cos\theta_i)}$$

[$\because$ Phases must be same for $\bar{E}_i^{\perp}$ & $\bar{H}_i^{\perp}$
$\because$ $E_i^{\perp}/H_i^{\perp} = 1/\eta$]



Reflected field:

$$\bar{E}_r^{\perp} = \hat{z}\, E_{r0}\, e^{-j\beta_1(x\sin\theta_i + y\cos\theta_i)}$$

$$\bar{H}_r^{\perp} = \frac{E_r}{\eta_1}(\hat{x}\cos\theta_i - \hat{y}\sin\theta_i)\, e^{-j\beta_1(x\sin\theta_i + y\cos\theta_i)}$$

Transmitted wave field:

$$\bar{E}_t^{\perp} = \hat{z}\, E_{t0}\, e^{-j\beta_2(x\sin\theta_t - y\cos\theta_t)}$$

$$\bar{H}_r = \frac{E_{t0}}{\eta_2}(-\hat{x}\overset{\cos}{\cancel{\sin}}\theta_t - \hat{y}\sin\theta_t)\, e^{-j\beta_2(x\sin\theta_t - y\cos\theta_t)}$$

Boundary condition on $(E)_{tan}$:

$$\boxed{E_{i0} + E_{r0} = E_{t0}} \Rightarrow \boxed{1 + \Gamma_{\perp} = T_{\perp}}$$

Boundary condition on $(\bar{H})_{tan}$:

$$\boxed{-\frac{E_{i0}}{\eta_1}\cos\theta_i + \frac{E_{r0}}{\eta_1}\cos\theta_i = -\frac{E_{t0}}{\eta_2}\cos\theta_t}$$

$$\Rightarrow \boxed{\frac{1}{\eta_1}\cos\theta_i - \frac{\Gamma_{\perp}}{\eta_1}\cos\theta_i = \frac{T_{\perp}}{\eta_2}\cos\theta_t}$$

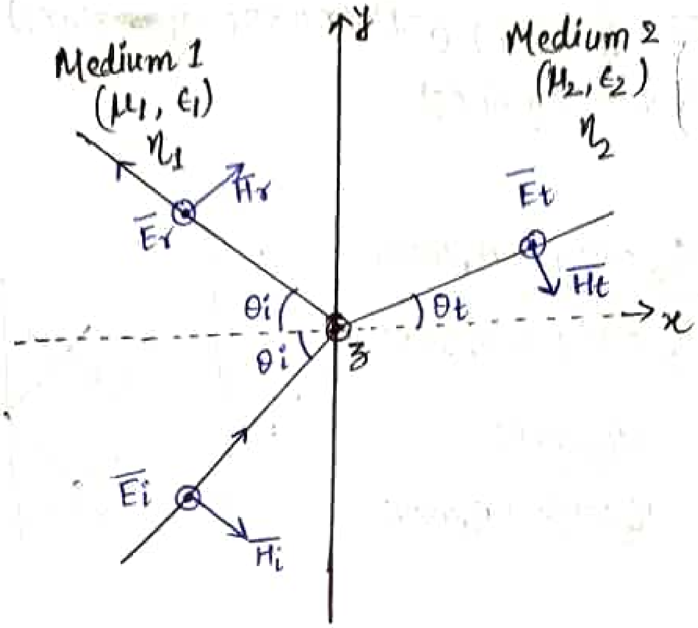
Solving these equations, we get

$$\Gamma_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$T_{\perp} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

31-10-2023

Perpendicular polarised

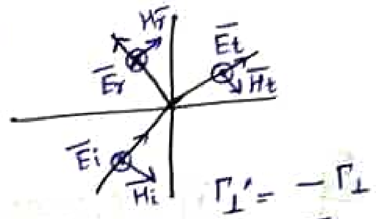


$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$ → negative if $\bar{E}_r \equiv \otimes$

↳ $\frac{\text{Reflected } \bar{E}}{\text{Incident } \bar{E}}$

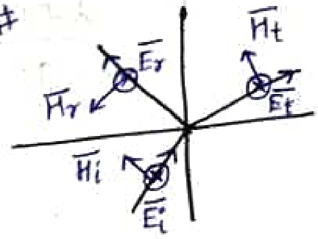
$$T_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

#



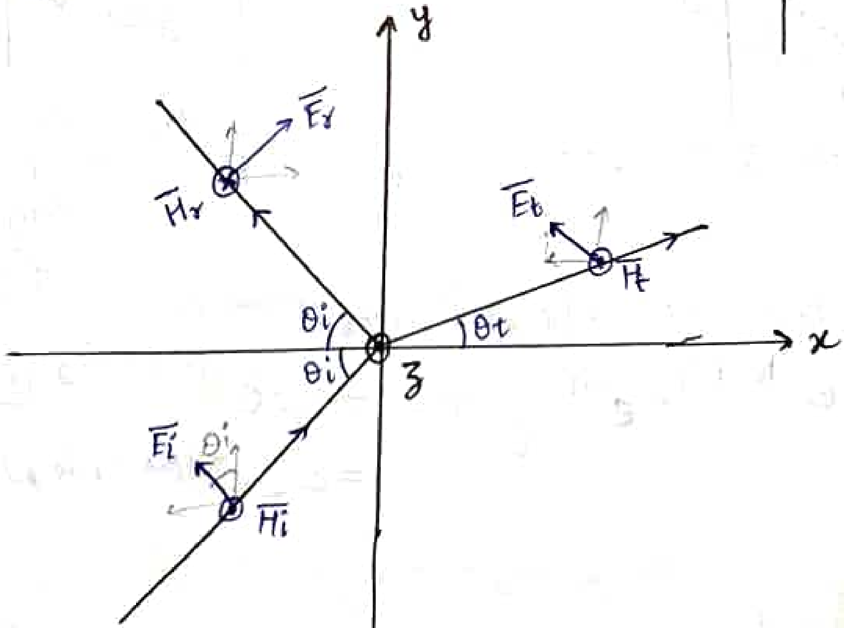
$\Gamma_\perp' = -\Gamma_\perp$
$T_\perp' = T_\perp$

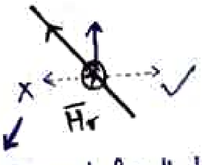
#



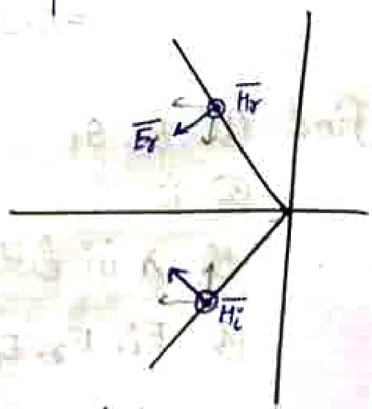
$\Gamma_\perp' = \Gamma_\perp$
$T_\perp' = T_\perp$

Parallel polarised





$\bar{E}_r$ cannot be ∥ to $\hat{\beta}$



$\bar{E} = -\frac{1}{\eta} \hat{\beta} \times \bar{H}$

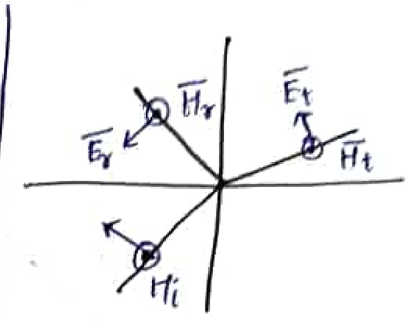
Incident waves:

$$\bar{E_i} = \eta H_0 (-\hat{x} \sin\theta_i + \hat{y} \cos\theta_i)\, e^{-j\beta_1(x\cos\theta_i + y \sin\theta_i)}$$

$$\bar{H_i} = \hat{z} H_0\, e^{-j\beta_1(x\cos\theta_i + y\sin\theta_i)}$$

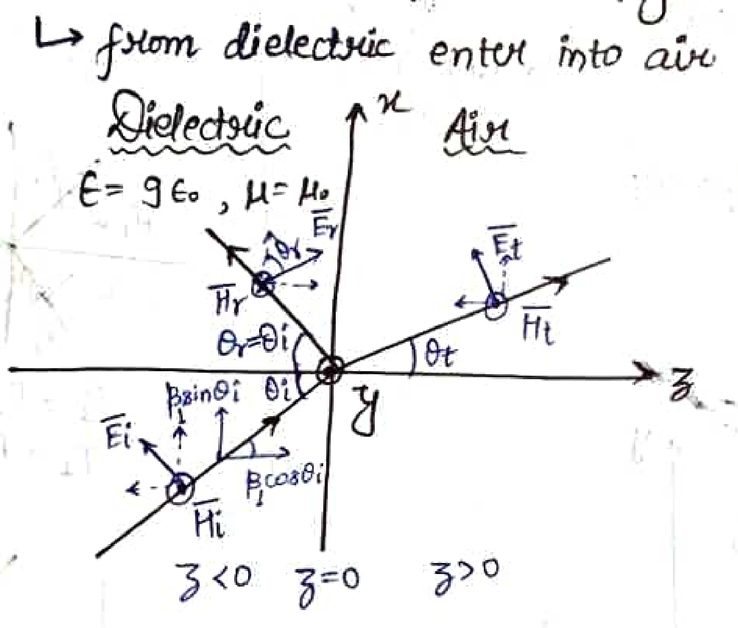
$$\frac{(E_r^{\parallel})_{tan.}}{(E_i^{\parallel})_{tan.}} = \Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

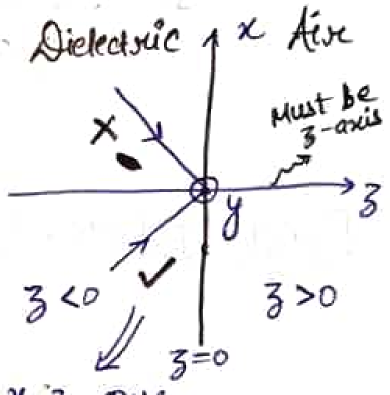
$$T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$



Eg. $\bar{H} = 0.2 \cos(10^9 t - Kx - K\sqrt{8}\, z)\hat{y}$ A/m

↳ from dielectric enter into air at the interface at $z=0$.





$$\bar{H} = 0.2 \cos(10^9 t - Kx - K\sqrt{8}\, z)\,\hat{y} \text{ A/m}$$

$$= 0.2\, e^{-jK(x+\sqrt{8}z)}\, e^{\omega t}\,\hat{y} \text{ A/m} = 0.2\, e^{-j\beta(x+\sqrt{8}z)}\, e^{\omega t}\,\hat{y} \text{ A/m}$$

$$= 0.2\, e^{-j(\beta x + \beta\sqrt{8}z)}$$

find ① θ_i, θ_t

② k

③ λ in Both medium

④ E_i, E_r, E_t.

1-11-2023

$e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}$

$\downarrow$

$\beta_1 \sin\theta_i = \beta$

$\beta_1 \cos\theta_i = \beta\sqrt{8}$

$\Rightarrow \tan\theta_i = \frac{1}{\sqrt{8}}$

$\Rightarrow \theta_i = 19.5^\circ = \theta_r$

$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$

$\Rightarrow \theta_t = \sin^{-1} 10$

$= 90^\circ$

$\beta_1 = \omega\sqrt{\mu_1 \epsilon_1} = \frac{10^8}{c}\sqrt{9} = 10$ rad/m.

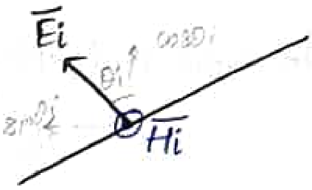
As $\beta_1 \sin\theta_i = \beta$

$\Rightarrow \beta = 10 \sin(19.5^\circ)$

$= 3.33$.

<u>Incident</u>

$\bar{E_i} = \eta_1 (0.2) (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i) e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}$



$= 25.1 (\hat{x}\, 0.94 - \hat{z}\, 0.33) e^{-j(3.33x + 9.43z)}$

In medium ①,

$\eta_1 = \sqrt{\frac{\mu}{\epsilon}}$

$= \sqrt{\frac{\mu_0}{\epsilon_0 9}}$

$= \frac{373}{3}$

In medium ②,

$\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_0}}$

$= 373$

$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

$= -1$

$T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{2\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

$= 6$

$\bar{E_r} = 25.1\, \Gamma_{\parallel} (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r) e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)}$

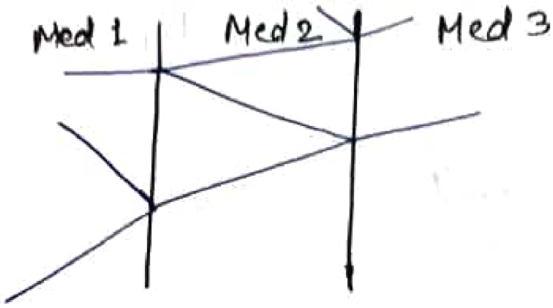
$= 25.1 (-1) (\hat{x}\, 0.94 + \hat{z}\, 0.33) e^{-j(x\, 3.33 - 9.43z)}$ propagation along +ve x & −ve z.

→ In lossless medium,

Transmitted power + Reflected power = Incident power.

→ If incident angle > critical angle,

⇒ fully reflection



↳ If we restrict wave in a medium

↳ waveguide

→ For non-conducting medium (lossless),

$\eta_1, \eta_2, \ldots$ → real quantities

⇒ Incident and transmitted waves are in the same phase.

For conducting medium (lossly),

$\eta_1, \eta_2, \ldots$ → complex quantities

⇒ Incident and transmitted waves are out of phase.

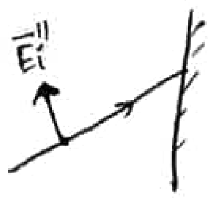
→ If incident wave is linearly polarised, (lossless)

$\bar{E}_{\parallel}$ and $\bar{E}_{\perp}$ are $\perp^r$ and phase difference $= 0$.

If the wave is RHCP,

$\bar{E}_i^{\parallel}$ & $\bar{E}_i^{\perp}$ are $\perp^r$, phase difference $= \frac{\pi}{2}$

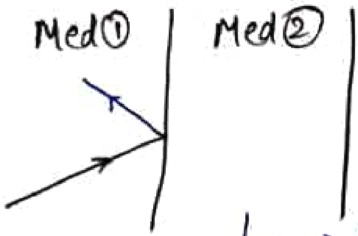
$1e^{j\pi/2}$ $\quad$ 1 $\quad$ & $\bar{E}_i^{\parallel}$ is leading.



↳ Phase difference : Remains same

RHEP/LHEP : Magnitudes : change (as $\underbrace{\Gamma_{\parallel}, \Gamma_{\perp}, T_{\parallel}, T_{\perp}}_{\text{Real}}$ change)
(of $\bar{E}_i^{\parallel}$ & $\bar{E}_i^{\perp}$)

→ Med① | Med②



↳ Perfect electric conductor (PEC) ($\sigma = \infty$)

↳ No transmitted wave

$E_i + E_r = 0$

$\Gamma_E = -1$

$\Rightarrow \Gamma_{\parallel} = -1, \ \Gamma_{\perp} = -1$

$\Downarrow$

$\Gamma_H = 1$ for $\overline{H}$

Electric field gets inverted after reflection

$\overline{H}$ remains same after reflection.

Perfect Magnetic Conductor (PMC) : $\Gamma_E = 1, \ \Gamma_H = -1$

$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$

$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2} \sin\theta_i$

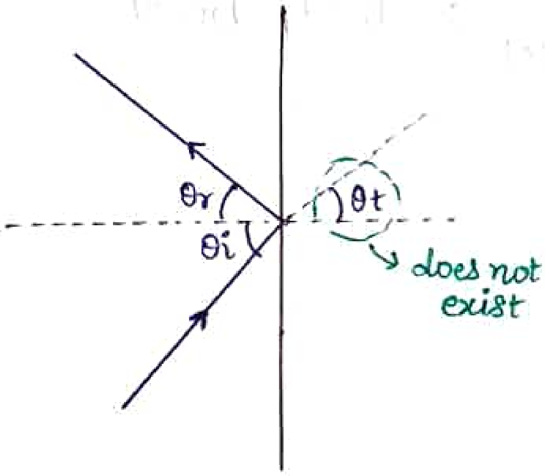
$\Rightarrow \cos\theta_t = \pm\sqrt{1 - \left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i}$

$\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i \geq 1$

$\frac{\beta_1}{\beta_2} \sin\theta_i \geq 1$

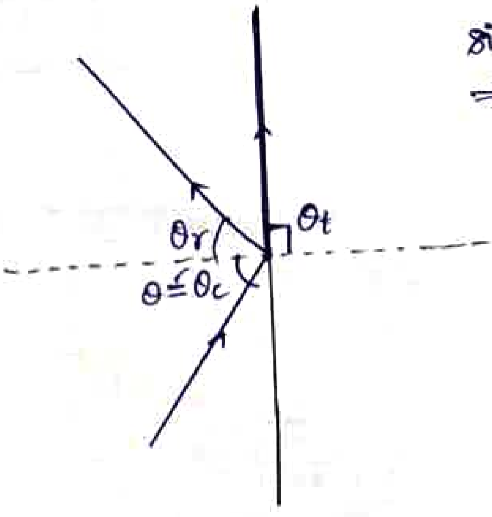
$\theta_t \rightarrow$ complex quantity

$\Rightarrow$ no θ_t exists $\Rightarrow$ no transmission



$\frac{\beta_1}{\beta_2} \sin\theta_c = 1$

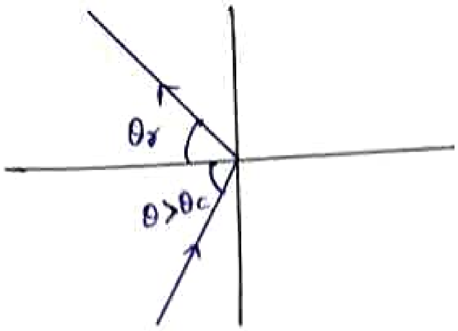
$\Rightarrow \boxed{\sin\theta_c = \frac{\beta_2}{\beta_1}}$: Critical angle (θ_c)



$\sin\theta \leq 1$

$\Rightarrow \beta_2 \leq \beta_1$

$\rightarrow$ Denser medium to rarer medium



Total Internal Reflection

$\theta_i > \theta_c$

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\Gamma_{\parallel} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$\cos\theta_t = j\sqrt{\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i - 1}$$

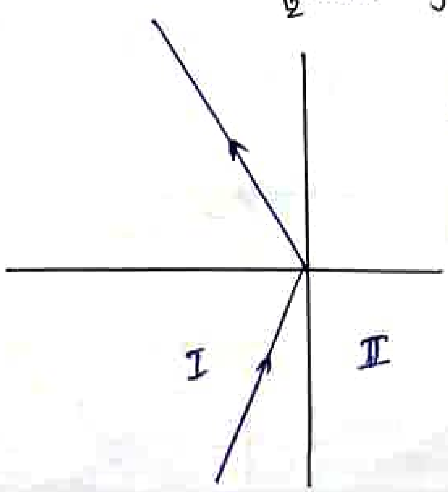
$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - j\eta_1\sqrt{\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i - 1}}{\eta_2 \cos\theta_i + j\eta_1\sqrt{\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i - 1}}$$

$$|\Gamma_\perp| = |\Gamma_\parallel| = 1$$

Reflected power = Incident power

$$T_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$= \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + j\eta_1\sqrt{\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i - 1}}$$

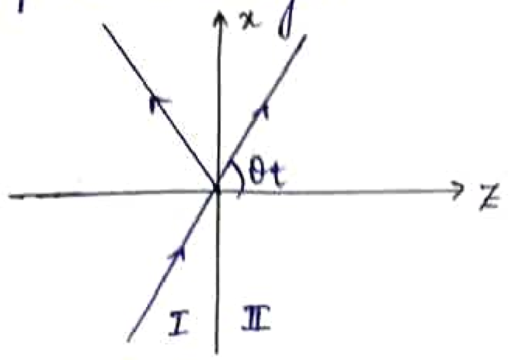


In medium II, field is not completely zero.

$$\cos\theta_t = \pm j\sqrt{\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i - 1}$$

Transmitted Field

Only phase is changed



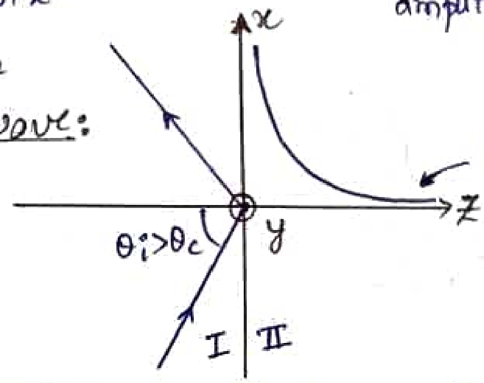
$$\bar{E}_t = \bar{E}_{to}\, e^{-j\beta_2 (x \sin\theta_t + z\cos\theta_t)}$$

$$= \bar{E}_{to}\, e^{-j\beta_2 \left\{ x \frac{\beta_1}{\beta_2}\sin\theta_i \pm z j\sqrt{\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i - 1} \right\}}$$

$$= \bar{E}_{to}\, \underbrace{e^{-jx\beta_1 \sin\theta_i}}_{e^{-j\beta_x x}}\; \underbrace{e^{\pm \beta_2 z\sqrt{\left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_i - 1}}}_{\text{amplitude attenuation in z-dirn}}$$

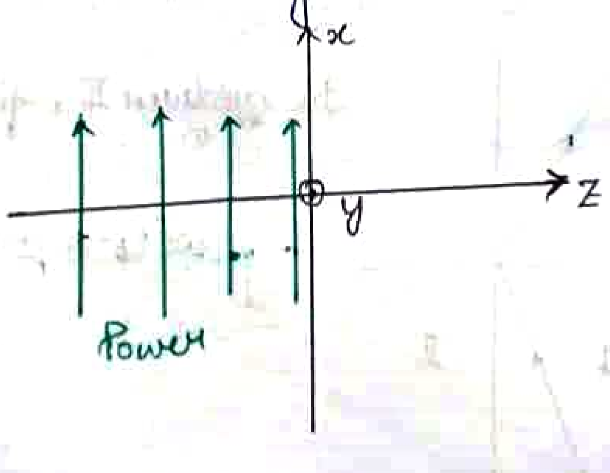
wave in x-dirn

Surface wave:



wave is attenuated quickly in this side

- In medium II, the transmitted wave is not there so → -ve sign.
- Power flow direction is on surface not towards z.
- In TIR, field is confined more towards interface of medium II.
- But power exists everywhere in medium I.



$$\rightarrow \Gamma_{\parallel} = \frac{-\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Brewster's Angle (θ_B): (or Polarisation angle)

$\underline{\theta_B^{\parallel}}$

$$\theta_i = \theta_B^{\parallel} \Rightarrow \Gamma_{\parallel} = 0$$

$$\Rightarrow \eta_2 \cos\theta_t - \eta_1 \cos\theta_B^{\parallel} = 0$$

As $\Rightarrow$ $\beta_1 \sin\theta_B^{\parallel} = \beta_2 \sin\theta_t$ (Snell's law)

$$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2} \sin\theta_B^{\parallel}$$

$$\Rightarrow \cos\theta_t = \sqrt{1 - (\beta_1/\beta_2)^2 \sin^2\theta_B^{\parallel}}$$

$$\Rightarrow \sqrt{1 - \left(\frac{\beta_1}{\beta_2}\right)^2 \sin^2\theta_B^{\parallel}} = \frac{\eta_1}{\eta_2}\cos\theta_B^{\parallel}$$

$$\Rightarrow 1 - \left(\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\right)\sin^2\theta_B^{\parallel} = \frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}\cos^2\theta_B^{\parallel}$$

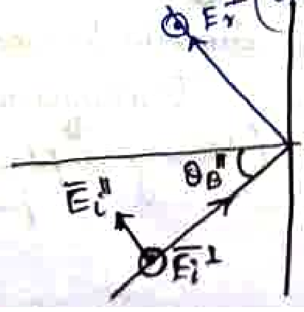
$$\Rightarrow 1 + \tan^2\theta_B^{\parallel} - \left(\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\right)\tan^2\theta_B^{\parallel} = \frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}$$

$$\Rightarrow \tan^2\theta_B^{\parallel}\left\{1 - \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\right\} = \frac{\mu_1\epsilon_2 - \mu_2\epsilon_1}{\mu_2\epsilon_1}$$

$$\Rightarrow \tan^2\theta_B^{\parallel}\,\frac{\mu_2\epsilon_2 - \mu_1\epsilon_1}{\cancel{\mu_2}\epsilon_2} = \frac{\mu_1\epsilon_2 - \mu_2\epsilon_1}{\cancel{\mu_2}\epsilon_1}$$

$$\Rightarrow \boxed{\tan\theta_B^{\parallel} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left\{\frac{\mu_1\epsilon_2 - \mu_2\epsilon_1}{\mu_2\epsilon_2 - \mu_1\epsilon_1}\right\}^{1/2}} \rightarrow \text{Depends on medium parameters}$$

(only $E_r^{\perp}$ component)



$\hookrightarrow$ Application: communication b/w two aircraft (aeroplanes).
$\downarrow$
To cancel out interference.
$\downarrow$
By adjusting height of the planes.

$$\tan\theta_B^{\perp} = \sqrt{\frac{\mu_2}{\mu_1}} \left\{ \frac{\mu_2\epsilon_1 - \mu_1\epsilon_2}{\mu_2\epsilon_2 - \mu_1\epsilon_1} \right\}^{1/2}$$

For non-magnetic materials, $(\mu = \mu_0)$

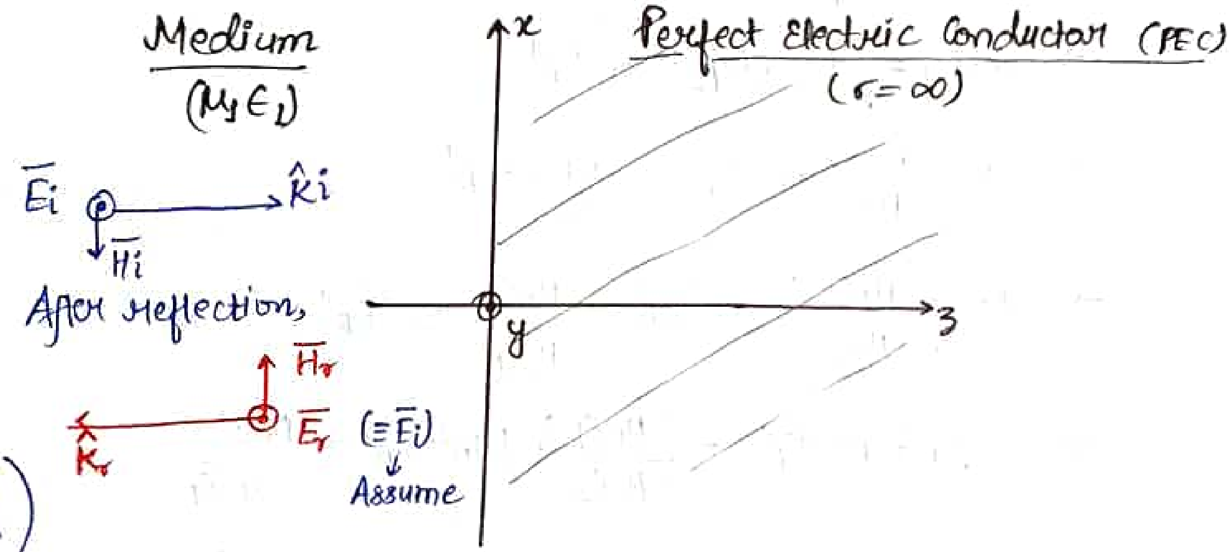
$$\boxed{\tan\theta_B^{\parallel} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

$$\cancel{\tan\theta_B^{\perp}} = \left\{ \frac{\epsilon_1 - \epsilon_2}{\epsilon_2 - \epsilon_1} \right\}^{1/2} \rightarrow \text{Complex}$$

$\downarrow$
X

$\hookrightarrow$ No Brewster angle for $\perp$ polarization

Normal incidence:



$\left(\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}\right)$

Incident field

$$\bar{E}_i = \hat{y}\, E_{i0}\, e^{-j\beta_1 z}$$

$$\bar{H}_i = -\hat{x}\, \frac{E_{i0}}{\eta_1}\, e^{-j\beta_1 z}$$

Reflected field

$$\bar{E}_r = \hat{y}\, E_{r0}\, e^{j\beta_1 z}$$

$$\bar{H}_r = \hat{x}\, \frac{E_{r0}}{\eta_1}\, e^{j\beta_1 z}$$

Using boundary conditions,

$\Rightarrow E_{i0} + E_{r0} = 0$

$\Rightarrow \boxed{\Gamma = -1} \rightarrow$ For pure electric conductor.

$\Downarrow$

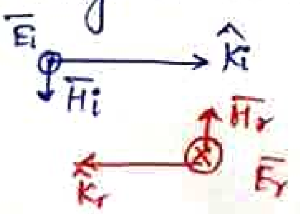
Similar to short circuit transmission line

$\Downarrow$

(Keeping a plane in the path of wave)

same char. impedance as intrinsic impedance $\uparrow$ of medium

$\hookrightarrow$ E must change its orientation.



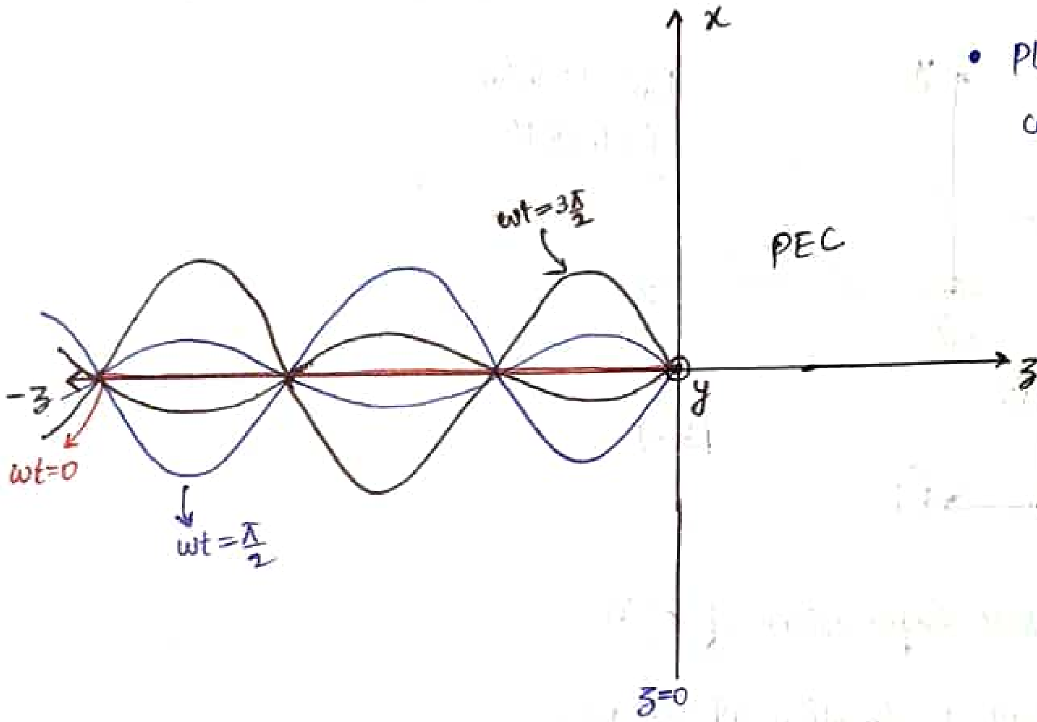
Total electric field in medium ①,

$$\bar{E}_{tot.} = \bar{E}_i + \bar{E}_r$$
$$= \hat{y} E_{i0} e^{-j\beta_1 z} - \hat{y} E_{i0} e^{j\beta_1 z}$$
$$= \hat{y} E_{i0} \left(e^{-j\beta_1 z} - e^{j\beta_1 z}\right)$$

$$\therefore \bar{E}_{tot} = -2j\,\hat{y}\,E_{i0} \sin(\beta_1 z)\, e^{j\omega t} \xrightarrow[\text{Real part}]{\text{Taking}} \left[\bar{E}_{tot\,Real} = 2E_{i0} \sin(\beta_1 z) \sin(\omega t)\right]$$

$$|\bar{E}_{tot}| = 2E_{i0} \sin(\beta_1 z)$$

no ($e^{j\beta z}$) term



- Placing a perfect electric conductor in path of wave

  ↓ similar to

  Short-circuit TL

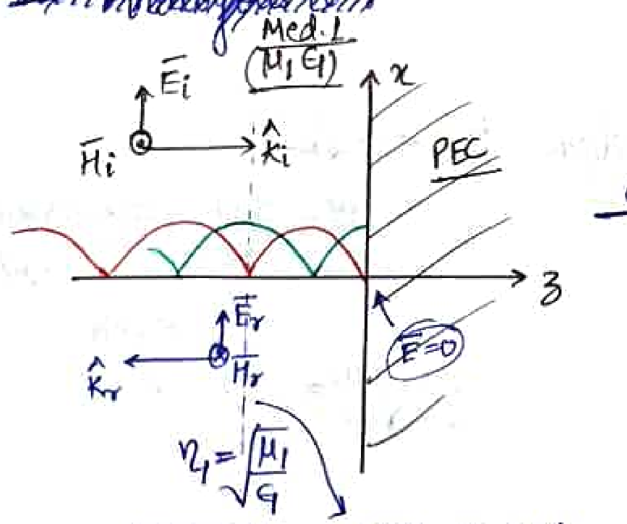
  ⇒ Standing wave

  ⇓

  No movement of power

$\boxed{13\text{-}11\text{-}2023}$

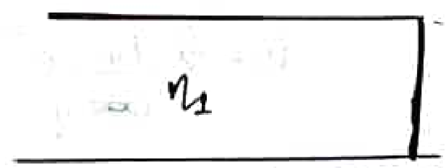
Tangent component of $\bar{E}$ on a perfect conductor is always zero.



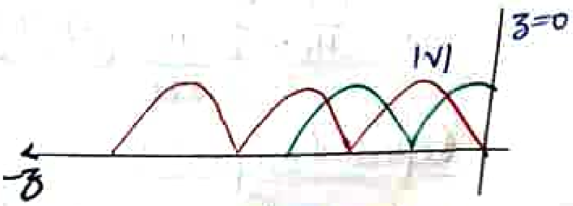
$\boxed{\Gamma = -1}$

shifting plate here doesn't change anything

analogous to

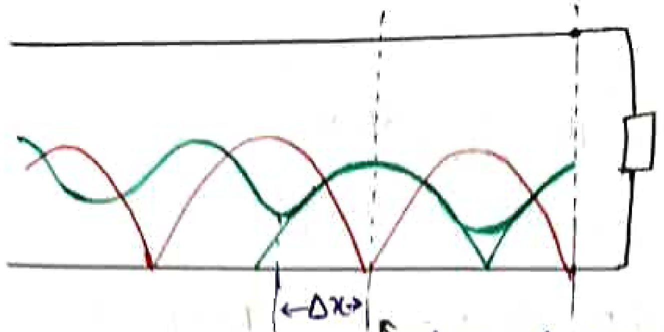


Transmission line



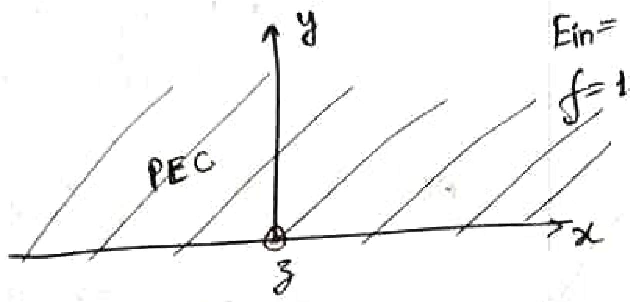
VSW pattern

↳ Have same char. impedance as the intrinsic impedance of medium



shifting load by $\frac{\lambda}{2}$ doesn't change anything to the left of it.

Eg



$E_{in} = 10$ V/m

$f = 1.5$ GHz

$\bar{E}_r \quad \bar{H}_r \quad \hat{K}_i \quad \hat{K}_r \quad \bar{E}_i \quad \hat{H}_i$

$\Gamma = -1$

find: ① Phasor expression of $\bar{E}, \bar{H}$

② Nearest location where $\bar{E} = 0$.

③ Nearest location where $\bar{H} = 0$

Soln: ①

Incident:

$$\bar{E}_i = \hat{y}\, E_{in}\, e^{-j\beta_{air} y}, \quad \text{where } \beta_{air} = \omega\sqrt{\mu_0 \epsilon_0}$$

$$= \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{\cancel{2}\pi \times 1.5\times 10^{9}}{3\times 10^{8}}$$

$$= 10\pi$$

$$\bar{H}_i = \hat{x}\, \frac{E_{in}}{\eta}\, e^{-j\beta_{air} y}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega$$

$$\Rightarrow \bar{E}_i = \hat{y}\, 10\, e^{-j10\pi y}$$

$$\bar{H}_i = \hat{x}\, \frac{10}{377}\, e^{-j10\pi y}$$

Reflected signal:

$$\bar{E}_r = -\hat{z}\, \cancel{E}\, 10\, e^{j10\pi y}$$

$$\bar{H}_r = \hat{x}\, \frac{10}{377}\, e^{j10\pi y}$$

Total electric field,

$$\bar{E}_{total} = 2j\,\hat{z}\,10 \left\{ \frac{e^{-j10\pi y} - e^{j10\pi y}}{2j} \right\} e^{j\omega t}$$

$$= -\hat{z}\,20j\,\sin(10\pi y)\,e^{j\omega t}$$

Phasor expression : Take either real or imaginary part.

Real part :

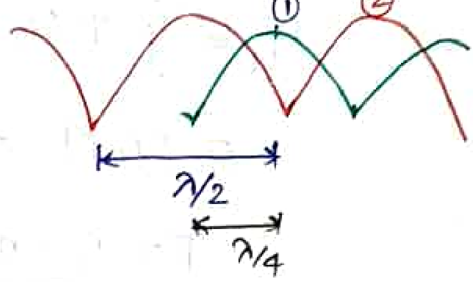
$$\bar{E}_{tot} = \hat{z}\,20\sin(10\pi y)\sin(\omega t).$$

(2) $\bar{E} = 0 \Rightarrow \sin(10\pi y) = 0$

$\Rightarrow 10\pi y = \pm n\pi,\ n = 0, 1, 2, \ldots$

$\Rightarrow y = \pm 0.1$

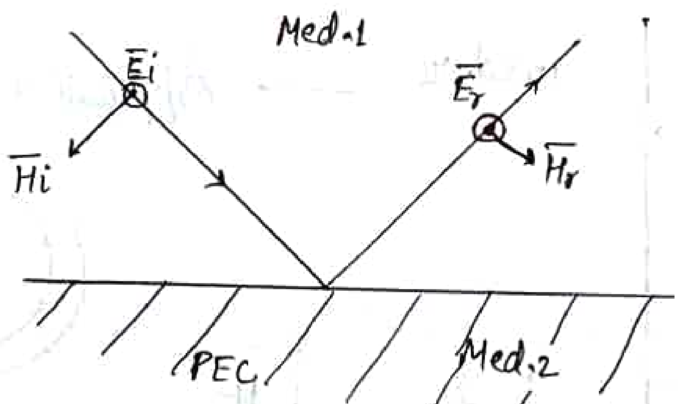
$\Rightarrow y = -0.1$ ✓ $(y \neq 0.1)$ No wave here.



Phase diff. b/w (1) & (2) $= \pi/2$.

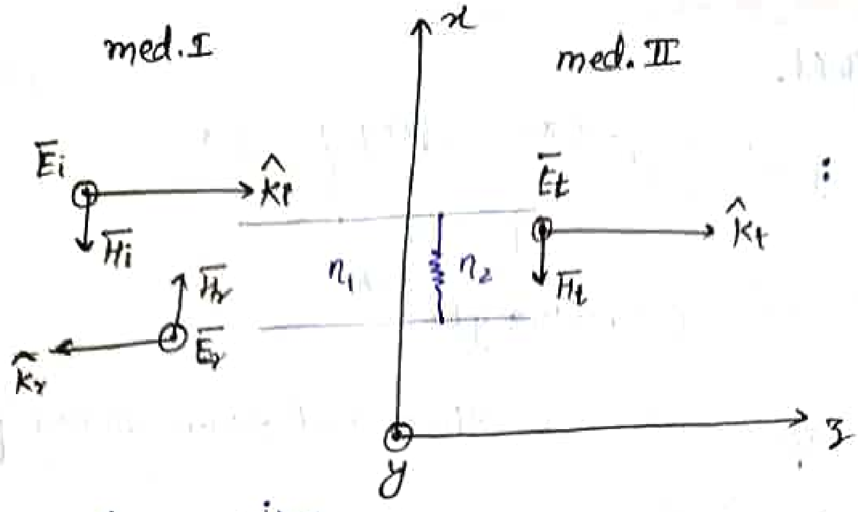
$\left[\ \beta\frac{\lambda}{4} = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}\right.$

Eg



Find the reflection coefficient.

Sol

→ 

: Normal Incidence ↓ analogous to Transmission line

$$\bar{E}_i = \hat{y} E_{io} e^{-j\beta_1 z}$$

$$\bar{E}_r = \hat{y}\, \Gamma E_{io} e^{+j\beta_1 z}$$

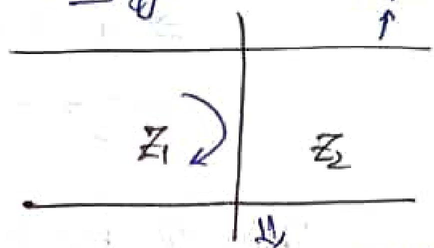
$$\bar{E}_t = \hat{y}\, T E_{io} e^{-j\beta_2 z}$$

$$E_{io}(1+\Gamma) = T E_{io}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$T = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Analogy:



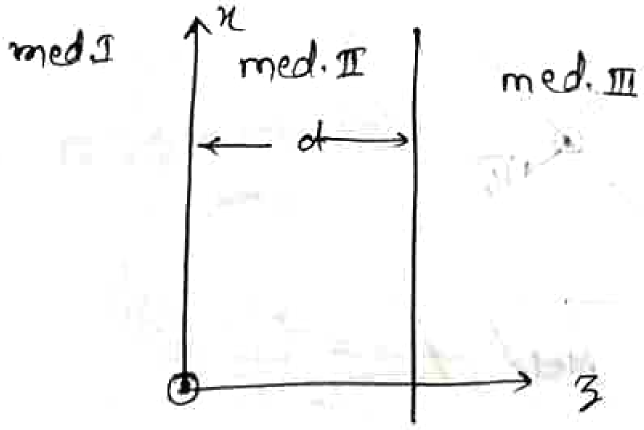
⇓

2 Mismatched T/Ls

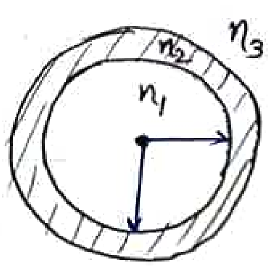
⇓

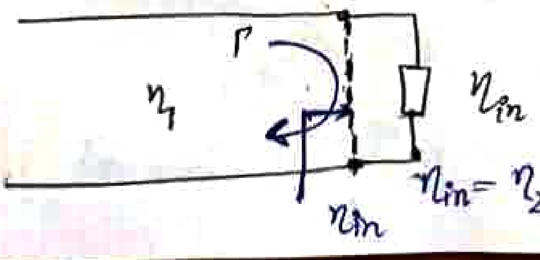
Reflection

$$\text{Ref. coeff.} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

→ 

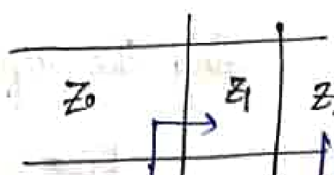
→ Application in RADOM





$$\eta_{in} = \eta_2 \frac{\eta_3 + j\eta_2 \tan\beta d}{\eta_2 + j\eta_3 \tan\beta d}$$

#

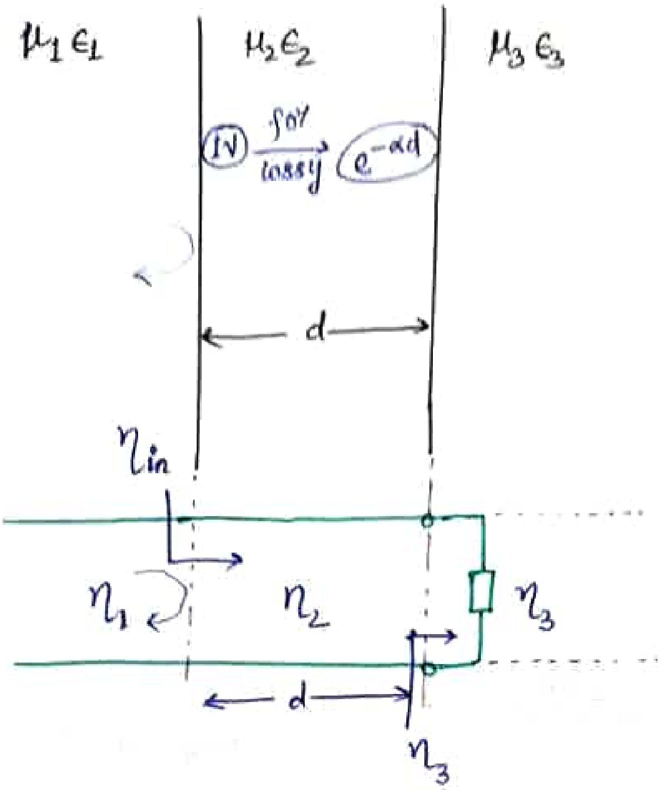


Imp. Transformation:

$$Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$\Gamma = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1}$$

14-11-2023



→ Amplitudes will remain same for the lossless media
↳ will change for lossy media

$$\eta_{in} = \eta_2 \frac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d}$$

$$= \eta_2 \frac{\eta_3 \cos\beta_2 d + j\eta_2 \sin\beta_2 d}{\eta_2 \cos\beta_2 d + j\eta_3 \sin\beta_2 d}$$

$$\Gamma = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1}$$

For no reflection: $\Gamma = 0 \Rightarrow \eta_{in} = \eta_1$

$$\Rightarrow \eta_2 \frac{\eta_3 \cos\beta_2 d + j\eta_2 \sin\beta_2 d}{\eta_2 \cos\beta_2 d + j\eta_3 \sin\beta_2 d} = \eta_1$$

$$\Rightarrow \eta_2\eta_3 \cos\beta_2 d + j\eta_2^2 \sin\beta_2 d = \eta_1\eta_2 \cos\beta_2 d + j\eta_1\eta_3 \sin\beta_2 d$$

Real: $\eta_3 \cos\beta_2 d = \eta_1 \cos\beta_2 d \Rightarrow \boxed{(\eta_3 - \eta_1)\cos\beta_2 d = 0}$

Imaginary: $\eta_2^2 \sin\beta_2 d = \eta_1\eta_3 \sin\beta_2 d \Rightarrow \boxed{(\eta_2^2 - \eta_1\eta_3)\sin\beta_2 d = 0}$

① $\eta_3 = \eta_1$

② $\cos \beta_2 d = 0$

$\Rightarrow \beta_2 d = (2n+1)\frac{\pi}{2}$

$\Rightarrow \frac{2\pi}{\lambda_2} d = (2n+1)\frac{\pi}{2}$

$\Rightarrow \boxed{d = (2n+1)\frac{\lambda_2}{4}}$

If $\eta_1 \neq \eta_3$

$\eta_2^2 - \eta_1\eta_3 = 0 \Rightarrow \boxed{\eta_2 = \sqrt{\eta_1\eta_3}} \rightarrow$ Quarter wave Transformer

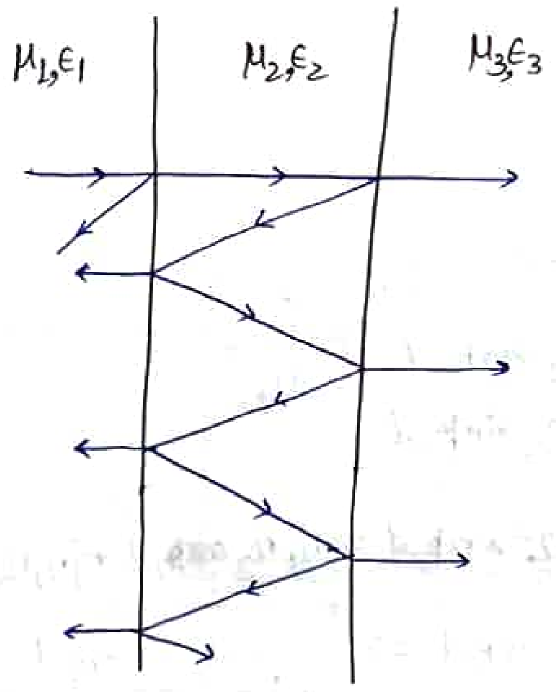
$\sin \beta_2 d = 0$

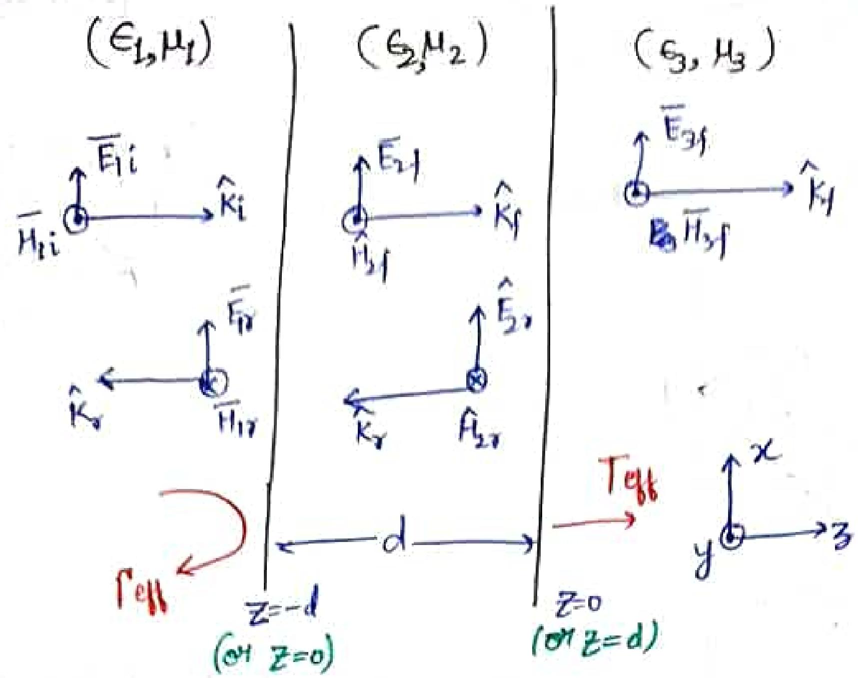
$\Rightarrow \beta_2 d = n\pi$

$\Rightarrow \frac{2\pi}{\lambda_2} d = n\pi$

$\Rightarrow \boxed{d = n\frac{\lambda_2}{2}}$

→





$$\bar{E}_1 = \bar{E}_{1i} + \bar{E}_{1r} = \hat{x} E_{i0} \left\{ e^{-j\beta_1 \overbrace{(z+d)}^{z}} + \Gamma_{eff}\, e^{+j\beta_1 \underbrace{(z+d)}_{z}} \right\}$$

Phase diff = 0 for $z=-d$

contribution from all the fields

$$\bar{H}_1 = \hat{y} \frac{E_{i0}}{\eta_1} \left\{ e^{-j\beta_1 (z+d)} - \Gamma_{eff}\, e^{j\beta_1 (z+d)} \right\}$$

$z < -d$

$$\bar{E}_2 = \hat{x} E_{20} \left\{ e^{-j\beta_2 z} + \Gamma_{23}\, e^{j\beta_2 z} \right\}$$

$$\bar{H}_2 = \hat{y} \frac{E_{20}}{\eta_2} \left\{ e^{-j\beta_2 z} - \Gamma_{23}\, e^{j\beta_2 z} \right\}$$

$-d < z < 0$

$$\bar{E}_3 = \hat{x} E_{10} T_{eff}\, e^{-j\beta_3 z}$$

$$\bar{H}_3 = \hat{y} \frac{E_{10}}{\eta_3} T_{eff}\, e^{-j\beta_3 z}$$

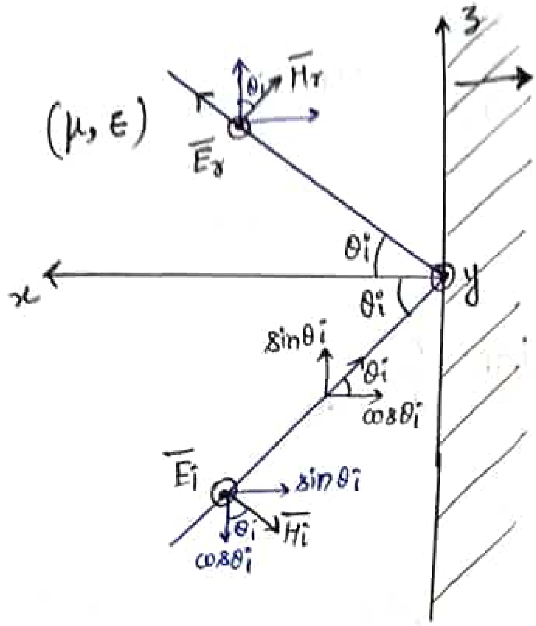
\# Book
↳ D.K. CHENG

By boundary conditions:

$\bar{E}$: $E_{i0} (1 + \Gamma_{eff}) = E_{20} (e^{j\beta_2 d} + \Gamma_{23} e^{-j\beta_2 d})$ → Equating E's tangential component at 1-2 interface (Put $z=-d$)

$\bar{H}$: $\frac{E_{i0}}{\eta_1} (1 - \Gamma_{eff}) = \frac{E_{20}}{\eta_2} (e^{j\beta_2 d} - \Gamma_{23} e^{-j\beta_2 d})$

Reflection from PEC



Perpendicularly polarized: E is $\perp^r$ to plane of incidence

Given: Perpendicular Polarization

Incident:

$$\overline{E}_i = \hat{y}\, E_{io}\, e^{-j\beta(-x\cos\theta_i + z\sin\theta_i)}$$

$$\overline{H}_i = \frac{E_{io}}{\eta}(-\hat{x}\sin\theta_i - \hat{z}\cos\theta_i)\, e^{-j\beta(-x\cos\theta_i + z\sin\theta_i)}$$

Reflected:

$$\overline{E}_r = \hat{y}\, \cancel{\Gamma}^{-1} E_{io}\, e^{-j\beta(x\cos\theta_i + z\sin\theta_i)}$$

$$\overline{H}_r = -(-\hat{x}\sin\theta_i + \hat{z}\cos\theta_i)\frac{E_{io}}{\eta}\, e^{-j\beta(x\cos\theta_i + z\sin\theta_i)}$$

$E_{io} + \Gamma E_{io} = 0 \quad (x=0) \rightarrow$ Boundary condition

$\Rightarrow \boxed{\Gamma = -1}$

$$\therefore \overline{E}_{tot} = \hat{y}\, E_{io}\left\{e^{-j\beta(-x\cos\theta_i + z\sin\theta_i)} - e^{-j\beta(x\cos\theta_i + z\sin\theta_i)}\right\}$$

$$= \hat{y}\, E_{io}\, e^{-j\beta z\sin\theta_i}\, 2j\left\{\frac{e^{j\beta x\cos\theta_i} - e^{-j\beta x\cos\theta_i}}{2j}\right\}$$

$$\Rightarrow \boxed{\overline{E}_{total} = j\hat{y}\, 2E_{io}\, e^{-j\beta z\sin\theta_i}\sin(\beta x\cos\theta_i)}\; e^{j\omega t}$$

and,
$$\overline{H}_{tot} = \hat{x}\frac{E_{io}}{\eta}\left(-\sin\theta_i\, e^{-j\beta(-x\cos\theta_i + z\sin\theta_i)} + \sin\theta_i\, e^{-j\beta(x\cos\theta_i + z\sin\theta_i)}\right)$$
$$+\hat{z}\frac{E_{io}}{\eta}\left(-\cos\theta_i\, e^{-j\beta(-x\cos\theta_i + z\sin\theta_i)} - \cos\theta_i\, e^{-j\beta(x\cos\theta_i + z\sin\theta_i)}\right)$$

$$= \hat{x}\,\frac{E_{io}}{\eta}\sin\theta_i\, e^{-j\beta z\sin\theta_i}\left\{-e^{j\beta x\cos\theta_i} + e^{-j\beta x\cos\theta_i}\right\}$$

$$-\hat{z}\,\frac{E_{io}}{\eta}\cos\theta_i\, e^{-j\beta z\sin\theta_i}\left\{e^{j\beta x\cos\theta_i} + e^{-j\beta x\cos\theta_i}\right\}$$

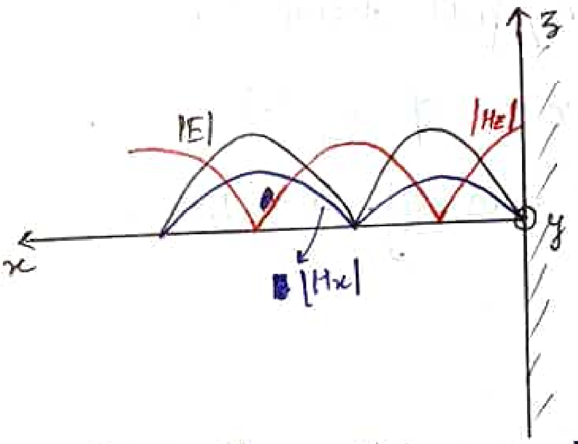
$$\Rightarrow \boxed{\begin{aligned}\bar{H}_{total} = &-2j\,\hat{x}\,\frac{E_{io}}{\eta}\sin\theta_i\,\underbrace{e^{-j\beta z\sin\theta_i}}_{prop.}\,\underbrace{\sin(\beta x\cos\theta_i)}_{standing}\\ &-2\,\hat{z}\,\frac{E_{io}}{\eta}\cos\theta_i\,\underbrace{e^{-j\beta z\sin\theta_i}}_{prop}\,\underbrace{\cos(\beta x\cos\theta_i)}_{standing}\end{aligned}}$$

E $\downarrow$ E_y H $\to$ $-H_x$ H_z

$\Downarrow$

- Field is propagating in $+z$ direction ($e^{-j\beta z\sin\theta_i}$ term)
- Field is standing in x direction ($\sin/\cos(\beta x\cos\theta_i)$ term)

- $e^{-j\beta z\sin\theta_i} \rightarrow$ propagating (in $+z$)
- $\sin(\beta x\cos\theta_i)$, $\cos(\beta x\cos\theta_i)$ $\rightarrow$ standing in x



- H_x & E $\rightarrow$ phase difference $= \pi$
  $\hookrightarrow$ Power ✓
- H_z & E $\rightarrow$ phase difference $= \pi/2$
  $\hookrightarrow$ No power

E & $\bar{H}$ have $\frac{\pi}{2}$ phase diff.
$\Downarrow$
Power $= 0$

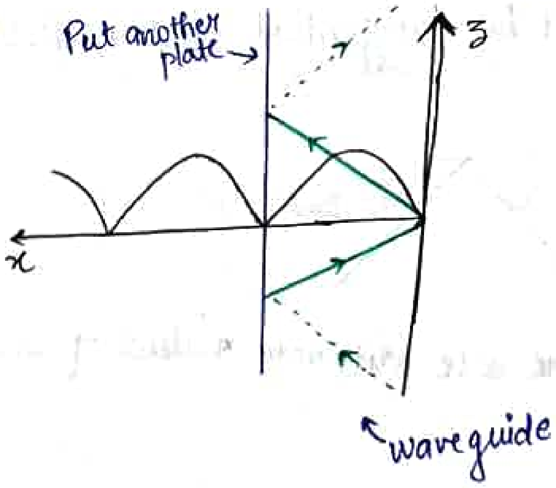
$\frac{1}{T}\int_0^T \frac{V^2}{R}\sin^2\omega t\,(d\omega t)$

$V\sin(\omega t)\,V\sin(\omega t+\pi)$

$\hookrightarrow -\frac{V^2}{R}\int \sin^2(\omega t)\,d\omega t$

$V\sin\omega t.\,\frac{V}{R}\sin(\omega t+\frac{\pi}{2})$

$\hookrightarrow P = 0$

- 

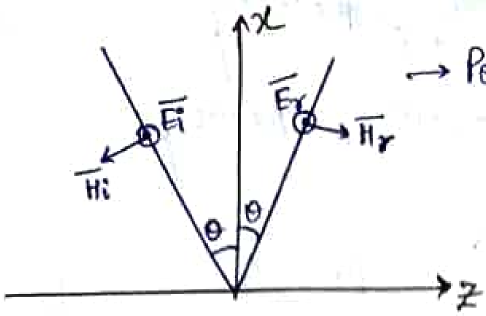
$\beta x\cos\theta_i = \pm n\pi$

TEM ✗

TE ✓ (TE^z) $\hookrightarrow E_z = 0$

TM^z $\hookrightarrow H_z = 0$

16-11-2023



→ Perpendicular polarization

Electric field minima :

$$\sin(\beta x \cos\theta) = 0 \Rightarrow \beta x \cos\theta = m\pi$$

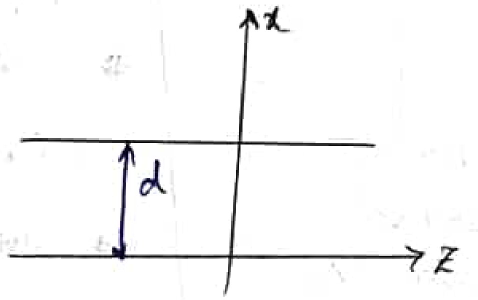
$$\Rightarrow x = \frac{m\pi}{\beta\cos\theta} = \frac{m\lambda\pi}{2\pi\cos\theta}$$

$$\Rightarrow \boxed{x = m\frac{\lambda}{2\cos\theta}}$$

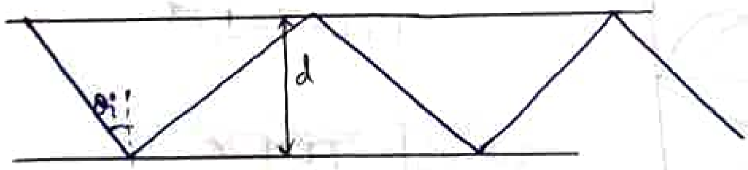
We can place conducting plate at some other minima, which will not affect wave pattern / field distribution.

Angle of incidence ⇄ Spacing (d)

$$\boxed{\cos\theta = m\frac{\lambda}{2d}}, \quad m = 0, 1, 2, \ldots$$



θ can't be $\frac{\pi}{2}$ as there can't be tangential electric field at interface.



Since, m is integer, there are discrete values of angle of incidence.

① Angle of incidence is discrete

② $|\cos\theta| \leq 1$.

$m\frac{\lambda}{2d}$, for some values of m, $\frac{m\lambda}{2d}$ will be >1, but $|\cos\theta| \le 1$.

Hence, there are limited no. of positions / ~~allote~~ allowed ~~s~~pace.

(3) $\cos\theta = \frac{m\lambda}{2d}$

$\boxed{d < \frac{\lambda}{2}}$ → even for $m=1$

→ not allowed $|\cos\theta| > 1$

- Separation between plates must be $\geq \frac{\lambda}{2}$

$\therefore \boxed{d_{min} = \frac{\lambda}{2}}$.

If $\lambda\uparrow \Rightarrow m$ values will decrease and vice versa.

$\cos\theta\Big|_{m=1} = \frac{\lambda\uparrow}{2d}$

At some λ, $\frac{\lambda}{2d} > 1$, hence there is some cutoff frequency.

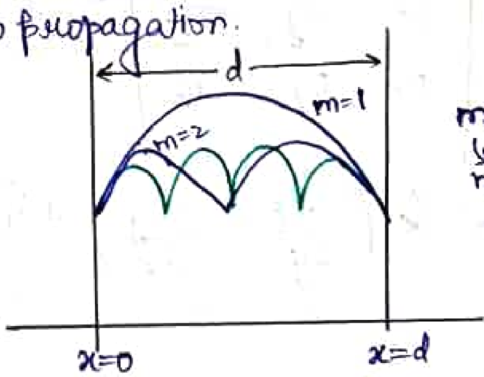
Cutoff frequency : Min^m frequency at which wave will propagate.
Cut off wavelength: Max^m wavelength at which wave will propagate.

$\bar{E} = \hat{y}\, 2j\, E_{i0} \sin[\beta x \cos\theta]\, e^{-j\beta z \sin\theta}$

$\bar{E} = \hat{y}\, 2j\, E_{i0} \sin\left[\beta x \frac{m\lambda}{2d}\right] e^{-j\beta z\sqrt{1-\left(\frac{m\lambda}{2d}\right)^2}}$

$= \hat{y}\, 2j\, E_{i0} \sin\left[\frac{m\pi x}{d}\right] e^{-j\beta z\sqrt{1-\left(\frac{m\lambda}{2d}\right)^2}}$

whenever $\frac{m\lambda}{2d} > 1$, there is no propagation.

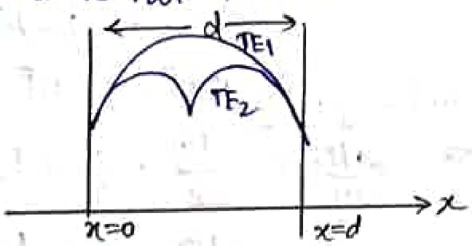


m : MODE → no. of half cycles

$m=0$ → not possible

$\bar{E}=0 \Rightarrow \bar{H}=0$

This is not TEM but $TE^z \Rightarrow E_z = 0$



$1-\left(\frac{m\lambda}{2d}\right)^2 \geq 0 \Rightarrow \frac{m\lambda_c}{2d} = 1$

$\Rightarrow \boxed{\lambda_c = \frac{2d}{m}}$ → cutoff wavelength

$\Rightarrow \boxed{f_c = \frac{c}{\lambda_c} = \frac{cm}{2d}}$ → cutoff frequency

For parallel polarization,

E_x, E_z, H_y — TM_m

TM_0 → possible

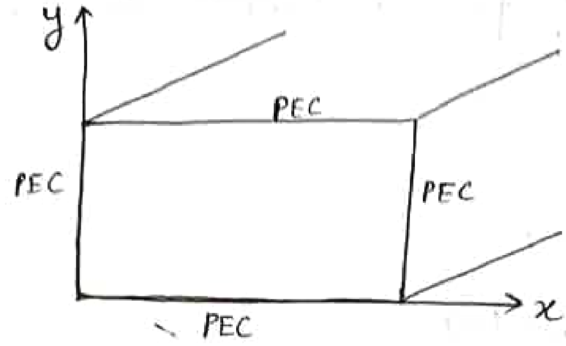
m can be 0

cutoff freq. = 0

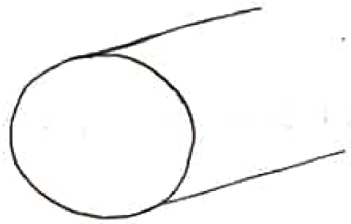
It doesn't have cutoff frequency

18-11-2023

Rectangular Wave Guide



Circular wave Guide (cylindrical)



→ TE^z ↓ $E_z = 0$ $(H_z \neq 0)$

TM^z ↓ $H_z = 0$ $(E_z \neq 0)$

TEM^z ↓ $E_z, H_z = 0$

$$\bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H}$$

$$\Rightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\bar{H}$$

$$\Rightarrow \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -j\omega\mu\bar{H}$$

$$\Rightarrow \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

Similarly, by $\bar{\nabla} \times \bar{H} = j\omega\mu\bar{E}$

$$\Rightarrow \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

We want to express E_x, E_y, H_x, H_y in terms of E_z / H_z.

As $j\omega\epsilon E_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$

$$\Rightarrow E_x = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)$$

For wave propagating in z-dirⁿ;

$\therefore E_z(x,y,z) = E_z(x,y)\, e^{-\gamma z}$

$H_z(x,y,z) = H_z(x,y)\, e^{-\gamma z}$

variation in z: $e^{-j\beta z}$, $e^{-\gamma z}$
$\downarrow$ for lossless, $\downarrow$ for lossy

$$\Rightarrow \frac{\partial}{\partial z} E_z(x,y,z) = \frac{\partial}{\partial z} E_z(x,y)\, e^{-\gamma z}$$

$$= -\gamma\, E_z(x,y)\, e^{-\gamma z}$$

$$\frac{\partial}{\partial z} H_{x|y|z} = \frac{\partial}{\partial z}\left[H_{x|y|z}(x,y)\, e^{-\gamma z}\right]$$

$$= -\gamma\, H_{x|y|z}(x,y)\, e^{-\gamma z}$$

$$\Rightarrow E_x = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right)$$

$$\Rightarrow E_x = \frac{1}{j\omega\epsilon}\left\{\frac{\partial H_z}{\partial y} + \gamma\left(-\frac{1}{j\omega\mu}\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right]\right)\right\}$$

$$\Rightarrow E_x = \frac{1}{j\omega\epsilon}\left\{\frac{\partial H_z}{\partial y} - \frac{\gamma}{j\omega\mu}\left(-\gamma E_x - \frac{\partial E_z}{\partial x}\right)\right\}$$

$$\Rightarrow E_x = \frac{1}{j\omega\epsilon}\frac{\partial H_z}{\partial y} + \frac{\gamma^2 E_x}{(j\omega\epsilon)(j\omega\mu)} + \frac{\gamma}{j\omega\epsilon . j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x = \frac{1}{j\omega\epsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma^2 E_x}{\omega^2\epsilon\mu} - \frac{\gamma}{\omega^2\mu\epsilon}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x\left(1 + \frac{\gamma^2}{\omega^2\mu\epsilon}\right) = \frac{1}{j\omega\epsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma}{\omega^2\mu\epsilon}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x\left(\frac{\gamma^2 + \omega^2\mu\epsilon}{\omega^2\mu\epsilon}\right) = \frac{1}{j\omega\epsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma}{\omega^2\mu\epsilon}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x},$$

where $\boxed{\gamma^2 + \omega^2\mu\epsilon = h^2}$

$$\therefore E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$\boxed{\begin{matrix} x \to y \\ y \to -x \end{matrix}}$

$H \to E$
$E \to H$
$\mu \to -\epsilon$
$\epsilon \to -\mu$

Similarly,

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}.$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x}.$$

$E_x \sim \frac{\partial E_z}{\partial x}$ (same), $E_x \sim \frac{\partial H_z}{\partial y}$ (cross)

$E_y \sim \frac{\partial E_z}{\partial y}$, $E_y \sim \frac{\partial H_z}{\partial x}$

→ TEM wave is not possible inside a rectangular waveform.

TM^z : $H_z = 0 \quad (E_z \neq 0)$

$$\nabla^2 E_z + \omega^2\mu\epsilon E_z = 0$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2\mu\epsilon E_z = 0$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z + \omega^2\mu\epsilon E_z = 0$$

$$\left[\because \frac{\partial E_z}{\partial z} = -\gamma E_z \Rightarrow \frac{\partial^2 E_z}{\partial z^2} = \gamma^2 E_z\right]$$

$$\Rightarrow \boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0}, \text{ where } h^2 = \gamma^2 + \omega^2\mu\epsilon$$

$$\left.\begin{matrix}\bar\nabla \times \bar\nabla \times E = -j\omega\mu\bar H \\ \bar\nabla \times \bar H = j\omega\epsilon\bar E\end{matrix}\right\}$$

$$\Rightarrow \bar\nabla \times \bar\nabla \times \bar E = -j\omega\mu(j\omega\epsilon\bar E)$$

$$\Rightarrow \bar\nabla(\bar\nabla\cdot E) - \bar\nabla^2 E = +\omega^2\mu\epsilon\bar E$$

$$\Rightarrow \bar\nabla^2 \bar E + \omega^2\mu\epsilon\bar E = 0$$

#

$$\Rightarrow \nabla^2 E_z + \omega^2\mu\epsilon E_z = 0$$
$$\nabla^2 E_y + \omega^2\mu\epsilon E_y = 0$$
$$\nabla^2 E_x + \omega^2\mu\epsilon E_x = 0$$

For lossless medium,

$e^{-\gamma_z z} = e^{-j\beta_z z}$, β_z : effective β in z-dirn.

$$\gamma^2 + \omega^2\mu\epsilon = h^2$$

$$\Rightarrow (j\beta_z)^2 + \omega^2\mu\epsilon = h^2 \Rightarrow -\beta_z^2 + \omega^2\mu\epsilon = h^2$$

Method of separation of variables:

$$E_z(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$\Rightarrow YZ\frac{d^2X}{dx^2} + XZ\frac{d^2Y}{dy^2} + XY\frac{d^2Z}{dz^2} + \omega^2\mu\epsilon XYZ = 0$$

$$\Rightarrow \underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{\beta_x^2} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{\beta_y^2} + \underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{\beta_z^2} + \omega^2\mu\epsilon = 0$$

individually constant

$$\frac{1}{X}\frac{d^2X}{dx^2} = \beta_x^2 \Rightarrow \frac{d^2X}{dx^2} = X\beta_x^2$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = \beta_y^2 \Rightarrow \frac{d^2Y}{dy^2} = Y\beta_y^2$$

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = \beta_z^2 \Rightarrow \frac{d^2Z}{dz^2} = Z\beta_z^2$$

$\frac{d^2X}{dx^2} - \beta_x^2 X = 0 \rightarrow$ Auxiliary roots : $\pm\beta_x$

↳ Roots: $e^{\beta_x x}$, $e^{-\beta_x x}$, $\underbrace{\cosh(\beta_x x),\ \sinh(\beta_x x)}_{\text{combinations of } e^{\beta_x x},\, e^{-\beta_x x}}$

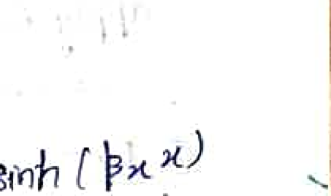
$\therefore$ Solution: $c_1 e^{\beta_x x} + c_2 e^{-\beta_x x}$

or $\underbrace{c_1\cosh(\beta_x x) + c_2\sinh(\beta_x x)}$

$\cosh(x) = \frac{e^x + e^{-x}}{2}$

None of these functions say that our wave is ~~stationary~~ standing (as all are exp. f^n)

↳ Between two plates:



$\therefore$ Our assumption is wrong.

Take $\frac{1}{X}\frac{d^2X}{dx^2} = -\beta_x^2$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -\beta_y^2$$

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = -\beta_z^2$$

$$\Rightarrow \frac{d^2X}{dx^2} + \beta_x^2 X = 0 \rightarrow (e^{j\beta_x x}, e^{-j\beta_x x}, \cos(\beta_x x), \text{sine}(\beta_x x))$$

physically not correct (mathematically correct)

Solution: $C_1 e^{j\beta_x x} + C_2 e^{-j\beta_x x}$ ⇝ Sine/cosine functions indicate standing wave.

or $X = C_1 \cos(\beta_x x) + C_2 \sin(\beta_x x)$

$Y = C_3 \cos(\beta_y y) + C_4 \sin(\beta_y y)$

$Z = C_5 e^{-j\beta_z z} + \cancel{C_6 e^{+j\beta_z z}}$ → no reflected wave

$$\therefore -(\beta_x^2 + \beta_y^2 + \beta_z^2) + \omega^2 \mu \epsilon = 0$$

As $E_z = XYZ$

$$= (C_1 \cos(\beta_x x) + C_2 \sin(\beta_x x)) \times (C_3 \cos(\beta_y y) + C_4 \sin(\beta_y y)) \cdot e^{-j\beta_z z}.$$

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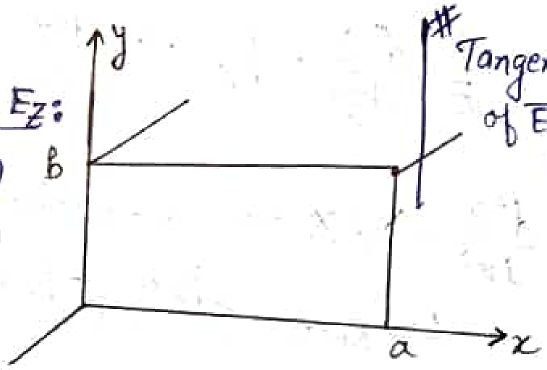
To find constants C_1, C_2, C_3, C_4,

<u>apply boundary conditions on E_z:</u>

(as E_x, E_y are still unknown)

① $E_z = 0$ at $x = 0, a$

② $E_z = 0$ at $y = 0, b$



Tangential component of $\vec{E}$ is always zero on PEC

$$E_z|_{x=0} = 0 \Rightarrow C_1 (C_3 \cos(\beta_y y) + C_4 \sin(\beta_y y)) e^{-j\beta_z z} = 0$$

$\Rightarrow C_1 = 0$ or $C_3 \cos(\beta_y y) + C_4 \sin(\beta_y y) = 0$ ✗

(as this will always be zero then, irrespective of x)

$\Rightarrow C_1 = 0$

$$E_z|_{y=0} = 0 \Rightarrow C_2 \sin(\beta_x x)(C_3 + 0) e^{-j\beta_z z} = 0$$

$\Rightarrow C_3 = 0$

$$\therefore E_z(x, y, z) = C \sin(\beta_x x) \cos(\beta_y y) e^{-j\beta_z z}$$

$E_z|_{x=a} = 0 \Rightarrow 0 = C' \sin(\beta_x a) \sin(\beta_y y) e^{-j\beta_z z}$

$\Rightarrow \beta_x a = m\pi, \; m = 0, 1, 2, \ldots$ [as $c \neq 0$, $\sin(\beta_y y) \neq 0$ as field $=0$ irrespective of x]

$\Rightarrow \beta_x = \frac{m\pi}{a}$

$E_z|_{y=b} = 0 \Rightarrow 0 = c \sin\left(\frac{m\pi}{a} x\right) \sin(\beta_y b) e^{-j\beta_z z}$

$\Rightarrow \beta_y b = n\pi, \; n = 0, 1, 2, \ldots$

$\Rightarrow \beta_y = \frac{n\pi}{a}$

$$\therefore \boxed{E_z(x, y, z) = C_{mn} \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta_z z}} \; : TM_{mn}$$

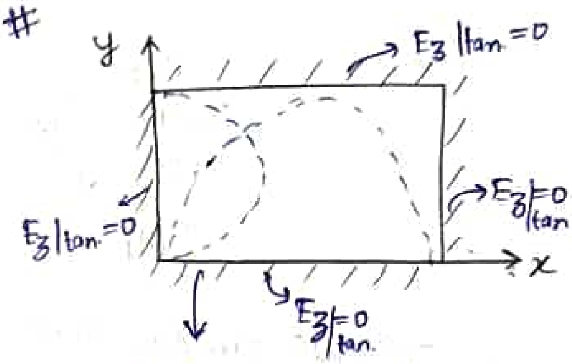
$$E_x = -\frac{\gamma}{\gamma^2 + \omega^2\mu\epsilon} \frac{\partial E_z}{\partial x}$$

$$E_y = -\frac{\gamma}{\gamma^2 + \omega^2\mu\epsilon} \frac{\partial E_z}{\partial y}$$

$$H_x = +\frac{j\omega\epsilon}{\gamma^2 + \omega^2\mu\epsilon} \frac{\partial E_z}{\partial y}$$

$$H_y = -\frac{j\omega\epsilon}{\gamma^2 + \omega^2\mu\epsilon} \frac{\partial E_z}{\partial x}.$$

#



Only sine function can give such E_z. (not cosine or anything else)

As $\frac{1}{X} \frac{d^2X}{dx^2} + \frac{1}{Y} \frac{d^2Y}{dy^2} + \frac{1}{Z} \frac{d^2Z}{dz^2} + \omega^2\mu\epsilon = 0$

$\Rightarrow -\beta_x^2 - \beta_y^2 - \beta_z^2 + \omega^2\mu\epsilon = 0$

$\Rightarrow -\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 - \beta_z^2 + \omega^2\mu\epsilon = 0$

$\Rightarrow \beta_z = \sqrt{\omega^2\mu\epsilon - \left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right\}}$

If $\omega^2\mu\epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\Rightarrow \beta_z \rightarrow$ imaginary

$\Rightarrow e^{-j\beta_z z} = e^{(\text{real})}$: "Evanescent mode"

↳ No propagation

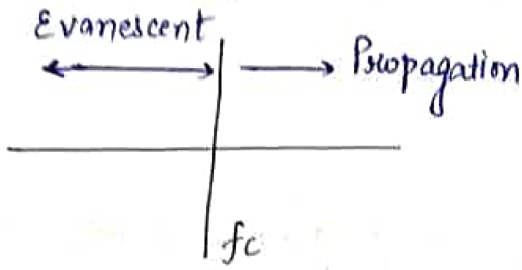
If $\omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\Rightarrow \beta_z =$ real ↳ Propagation

If $\beta_z = 0$

$\Rightarrow \omega_c^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\Rightarrow \boxed{f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right\}^{1/2}}$ → for TM as well as TE



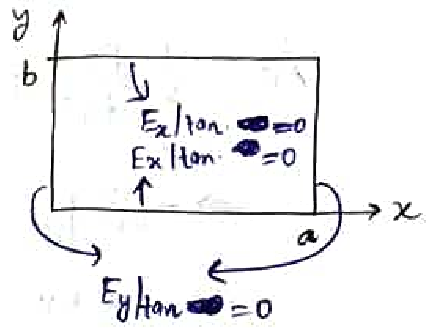
$\therefore$ TM_{mn}

$\left.\begin{array}{l} TM_{00} \\ TM_{01} \\ TM_{10} \end{array}\right\} \times$

TM_{11} ✓

→ we can substitute γ by $(j\beta)$.

TE : $E_z = 0$, $H_z \neq 0$



$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial^2 z^2} + \omega^2 \mu\epsilon H_z = 0$$

$$\Rightarrow H_z = (C_1 \cos\beta_x x + C_2 \sin\beta_x x) \times (C_3 \cos\beta_y y + C_4 \sin\beta_y y)\, e^{-j\beta_z z}$$

Boundary conditions:

$E_x = 0$ at $y = 0, b$

$E_y = 0$ at $x = 0, a$

TE_{00} → × (as $H_z = C_{mn} e^{-j\beta_z z}$ $\Rightarrow E_x = E_y = \dots = 0$ → ×)

TE_{01} ✓ $\Rightarrow E_y \neq 0 \Rightarrow$ E & H coexist ✓

TE_{10} ✓

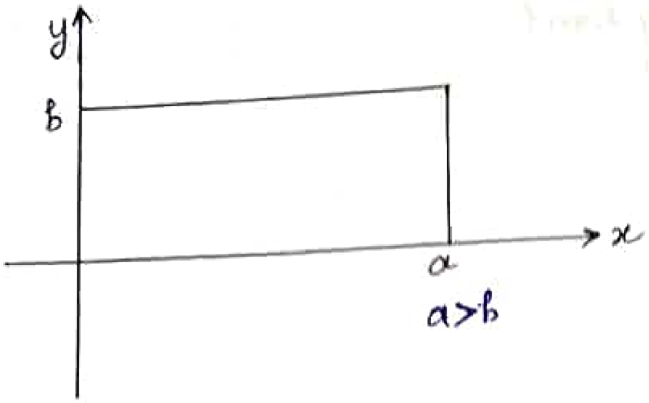
$E_x \sim \frac{\partial H_z}{\partial y}$

$\sim (C_1 \cos\beta_x x + C_2 \sin\beta_x x)(-C_3 \beta_y \sin\beta_y y + C_4 \beta_y \cos\beta_y y)\, e^{-j\beta_z z}$

$\Rightarrow C_1 = 0,\ C_2 = 0$

Similarly, $\boxed{H_z = C_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_z z}}$

21-11-2023



$$TE_{mn}: \ H_z = A_{mn} \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_z z}$$

$$TM_{mn}: \ E_z = A_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_z z}$$

$$(f_c)_{mn} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \rightsquigarrow \text{cutoff frequency}$$

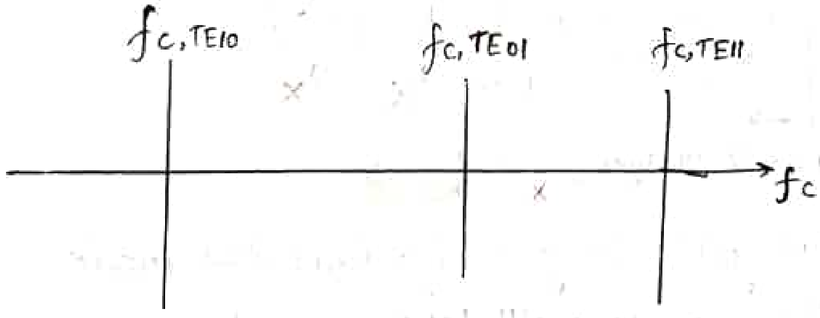
$$\beta_z = \sqrt{\omega^2\mu\epsilon - \left\{\underbrace{\left(\frac{m\pi}{a}\right)^2}_{\beta_x^2} + \underbrace{\left(\frac{n\pi}{b}\right)^2}_{\beta_y^2}\right\}}$$

$$TM_{11}: \ (f_c)^{TM_{11}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$

$$TE_{10}: \ (f_c)^{TE_{10}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left(\frac{\pi}{a}\right)$$

$$= \frac{1}{2a\sqrt{\mu\epsilon}}$$

Mode
↓
particular field distribution



$f_{c,TE_{10}}$: Min^m frequency that can pass through the waveguide

$f_{c,TE_{10}}$: Min^m frequency
- ↳ "Fundamental mode": Mode having lowest cutoff frequency.
- ↳ Frequencies more than this pass through the waveguide.

"Degenerate mode" ⇝ Mode in which two have same cutoff frequency but different field distribution.

1 GHz → 40 GHz : Frequency band

L → 1–2 GHz
S → 2–4 GHz
C → 4–8 GHz
X → 8–12 GHz
Kn → 12–18 GHz
K → 18–26 GHz
Ka → 26–40 GHz.

Eg. $a = 22.86\,mm$, $b = 10.16\,mm$ } x-band

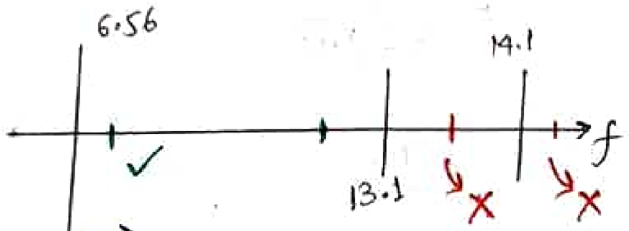
Lowest frequency:

$$(f_c)_{10}^{TE} = \frac{1}{2a\sqrt{\mu\epsilon}}$$
$$= 6.56\ GHz$$

↳ waveguide will not work for frequency above this.

$(f_c)_{01} = 14.1\ GHz$

$(f_c)_{20} = 13.1\ GHz$

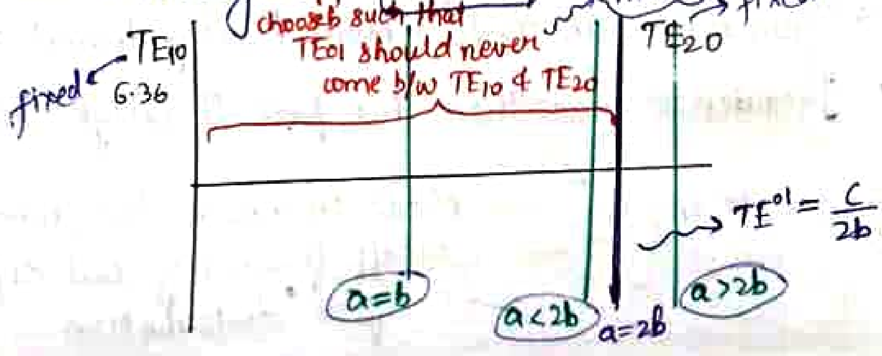


Take 25% margin (C X X ✓) Band

↳ we use waveguide only in fundamental mode.

↳ As higher mode we'll take, more frequencies will be there.

Standard waveguide: $\boxed{a \approx 2b}$. (to have single mode of operation)

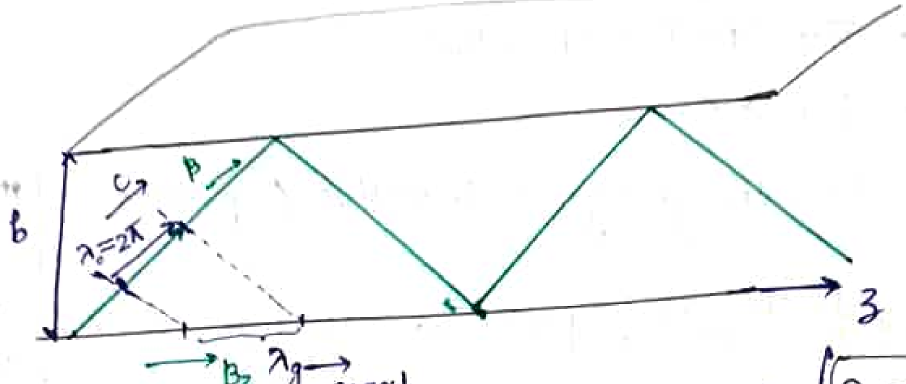


→★ a is taken slightly higher than 2b.

Cutoff wavelength:

$$\lambda_c = \frac{c}{f_c} = \frac{c}{c/2a}$$

$$\Rightarrow \lambda_c = 2a$$



$\beta = \omega\sqrt{\mu\epsilon}$

$$\boxed{\lambda_0 = \frac{2\pi}{\omega\sqrt{\mu_0\epsilon_0}}}$$

~~speed~~ phase velocity $> c$ → not actual velocity

↓

fast wave

Guided wavelength $(\lambda_g) > \lambda_0$

$$\beta_z = \sqrt{\omega^2\mu\epsilon - \left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right\}}$$

$$\beta_z = \sqrt{\omega^2\mu\epsilon - \omega_c^2\mu\epsilon} \quad (\text{for } \beta_z = 0)$$

where $\boxed{\omega_c^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$: cutoff frequency (ω_c)

$$\beta_z = \sqrt{\omega^2\mu\epsilon - \omega_c^2\mu\epsilon}$$

$$\therefore \boxed{\beta_z = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$= \beta\sqrt{1 - (f_c/f)^2}$$

$$\lambda_g = \frac{2\pi}{\beta_z} = \frac{2\pi}{\beta\sqrt{1-(f_c/f)^2}}$$

$$= \frac{\lambda_0}{\sqrt{1-(f_c/f)^2}}$$

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$(\mu, \epsilon) \rightarrow \eta = \sqrt{\frac{\mu}{\epsilon}}$

$$\text{TE}: \left(\frac{E_x}{H_y}\right)_{TE} = Z_{TE} = \frac{\omega\mu}{\beta_z} = \frac{\omega\mu}{\beta\sqrt{1-(f_c/f)^2}} = \frac{\omega\mu}{\omega\sqrt{\mu\epsilon}\sqrt{1-(f_c/f)^2}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

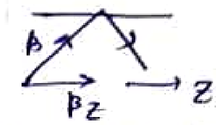
$$Z_{TM} = \frac{E_x}{H_y} = \frac{\beta_z}{\omega\epsilon} = \eta\sqrt{1-(f_c/f)^2}$$

$f > f_c \Rightarrow Z_{TE}$: Real $\rightarrow$ Resistive

$f = f_c \Rightarrow Z_{TE} \rightarrow \infty$

$f < f_c \Rightarrow Z_{TE}$: Imaginary $\rightarrow$ Inductive /capacitive.

#



$-(\beta_x^2+\beta_y^2+\beta_z^2)+\beta^2=0$

$$\beta_z = \begin{cases} +\beta\sqrt{1-(f_c/f)^2}, & f > f_c \rightsquigarrow \text{For propagation in } +z \quad e^{-j\beta_z z} \\ 0, & f = f_c \\ -j\beta\sqrt{(f_c/f)^2-1}, & f < f_c \rightsquigarrow \text{For propagation in } +z \text{ dir}^n \end{cases}$$

$$Z_{TE} = \begin{cases} \frac{\eta}{\sqrt{1-(f_c/f)^2}}, & f > f_c \\ \infty, & f = f_c \\ j\frac{\eta}{\sqrt{(f_c/f)^2-1}}, & f < f_c \end{cases}$$

$f \gg f_c \Rightarrow Z_{TE} \rightarrow \eta$

TE$_{10}$ $(a > b)$

$E_x = 0$

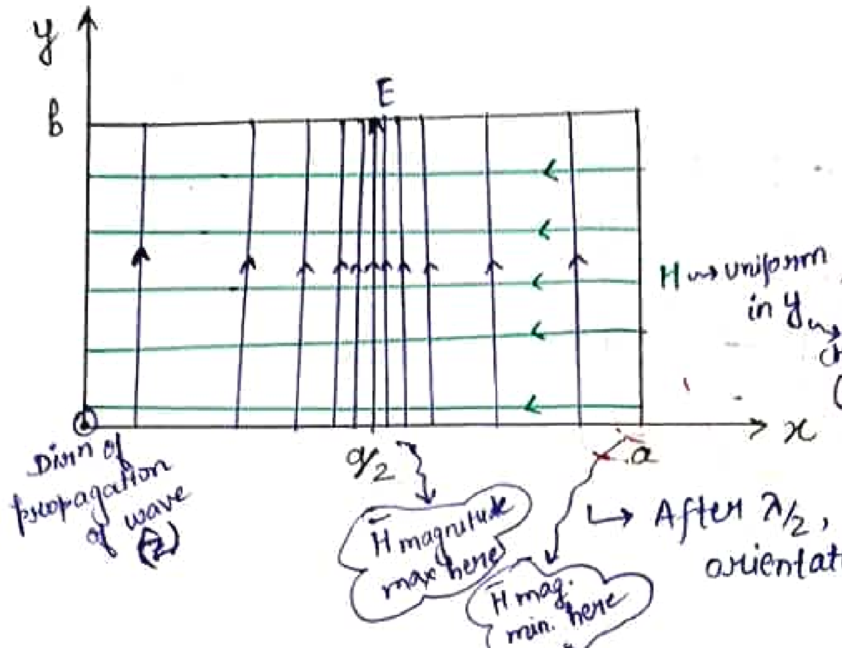
$E_y = -\frac{j\omega\mu\pi}{h^2 a} A_{10} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_z z}$

$E_z = 0$

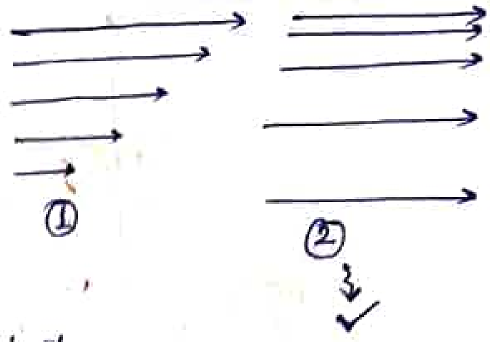
$H_x = \frac{j\beta_z \pi}{h^2 a} A_{10} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_z z}$

$H_y = 0$

$H_z = A_{10} \cos\left(\frac{\pi x}{a}\right) e^{-j\beta_z z}$

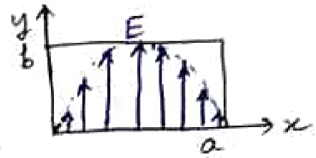


Field Representation:

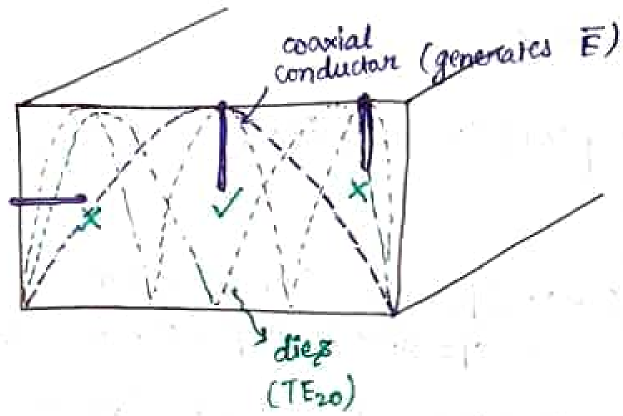


↳ After $\lambda/2$, both $\bar{E}$ & $\bar{H}$ change orientation.

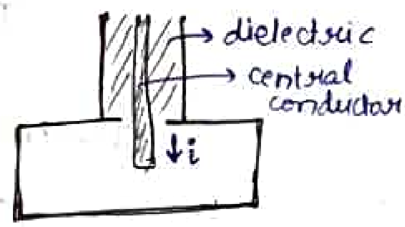
In ①st representation,



Waveguide to Coaxial Adapter:



(generates $\bar{E}$) → should be ∥ to $\bar{E}$.
↳ should be in the centre to get maxm $\bar{E}$ at $a/2$. (power)
↳ Erect or inverted



23-11-2023

TE₁₀

$$E_y = -\frac{j\omega\mu\pi}{h^2 a} A_{10} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi}{\lambda_g} z\right)$$

$$H_x = \frac{\beta_z \pi}{h^2 a} A_{10} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi z}{\lambda_g}\right)$$

$$H_z = A_{10} \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{2\pi z}{\lambda_g}\right)$$

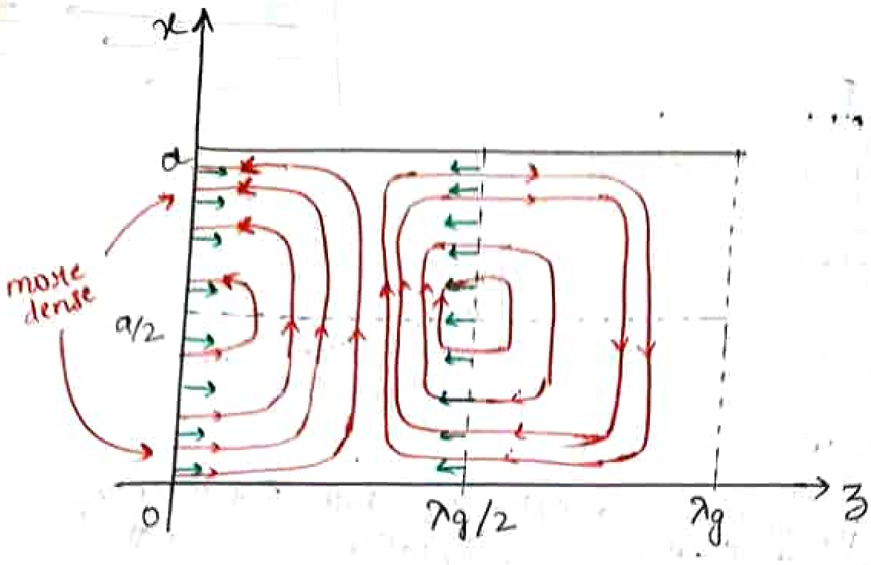
(only real parts)

At $z=0$,

$H_x = 0$, $H_z \neq 0$ ⇝ along z

At $z = \lambda_g$,

$H_x = 0$, H_z = opposite dirn



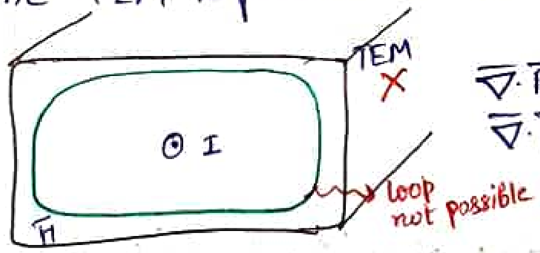
At $z=0$, $\frac{x}{a} > a/2$,

$$H_z = A_{10} \underbrace{\cos(\pi x/a)}_{\ominus ve}$$

→ In rectangular plates (waveguide),

TE ✓
TM ✓
TEM ✗ ⇝ not possible (cannot sustain)
↳ possible in ∥ plates, and coaxial lines.

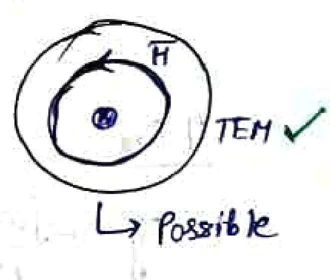
Assume TEM is possible in rectangular waveguide.



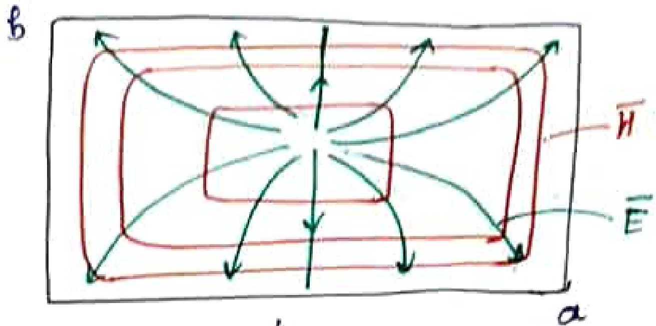
$\overline{\nabla}\cdot\overline{B}=0$
$\overline{\nabla}\cdot\overline{E}=0$

Then, $\overline{B}$ is circular, then there must be a current source at the centre, but the conductor is hollow (no source at centre).
Hence, this assumption is wrong.

#



TM_{11}



↳ Possible due to displacement current ✓

$TE_{10}, TE_{01}, TE_{11}, TM_{11}$

Other modes are repeatation of these.

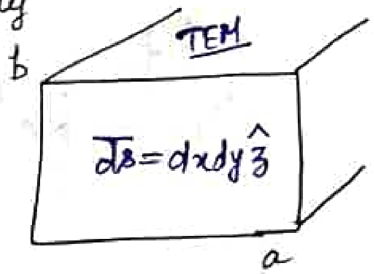
Power density:

$$\bar{P}_{av} = \frac{1}{2}\,\mathrm{Re}\left\{\bar{E}\times\bar{H}^*\right\}$$

$$= \frac{1}{2}\,\mathrm{Re}\left\{(\hat{x}E_x+\hat{y}E_y+\hat{z}E_z)\times(\hat{x}H_x+\hat{y}H_y+\hat{z}H_z)^*\right\}$$

$$= \frac{1}{2}\,\mathrm{Re}\left\{\hat{z}E_xH_y^* - \hat{y}E_xH_z^* - \hat{z}E_yH_x^* + \hat{x}E_yH_z^* + \hat{y}E_zH_x^* - \hat{x}E_zH_y^*\right\}$$

$$\bar{P}_{tot} = \iint_S \bar{P}_{av}\cdot\hat{z}\,ds = \frac{1}{2}\,\mathrm{Re}\int_0^a\!\!\int_0^b (E_xH_y^* - E_yH_x^*)\,dx\,dy$$



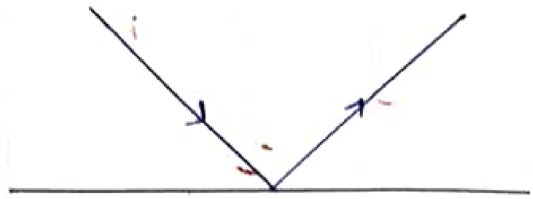
$$\bar{P}_{tot}^{TE} = |A_{mn}|^2\frac{\omega\mu\beta_z}{2h^4}\int_0^a\!\!\int_0^b\left(\frac{n\pi}{b}\right)^2\cos^2\left(\frac{m\pi x}{a}\right)\sin^2\left(\frac{n\pi y}{b}\right)+\left(\frac{m\pi}{a}\right)^2\sin^2\left(\frac{m\pi x}{a}\right)\cos^2\left(\frac{n\pi y}{b}\right)\,dx\,dy$$

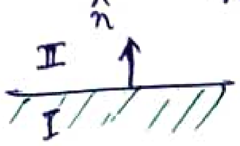
$$\frac{1}{2}\int_0^a \sin^2\left(\frac{m\pi x}{a}\right)dx = \frac{1}{2}\int_0^a\left(1-\cos\left(\frac{2m\pi x}{a}\right)\right)dx = \begin{cases} a/2, & m\neq 0\\ 0, & m=0\end{cases}$$

$$\frac{1}{2}\int_0^a \cos\left(\frac{m\pi x}{a}\right)dx = \begin{cases} a/2, & m\neq 0\\ a, & m=0\end{cases}$$

$$\epsilon_q = \begin{cases} 1, & q=0\\ 2, & q\neq 0\end{cases}$$

$$P_{tot.}^{TE} = |A_{mn}|^2 \frac{\omega\mu\beta}{2h^4}\left\{\left(\frac{n\pi}{b}\right)^2 \frac{a}{\epsilon_{om}}\,\frac{b}{2\epsilon_{on}} + \left(\frac{m\pi}{a}\right)^2 \frac{a}{2\epsilon_{om}}\,\frac{b}{\epsilon_{om}}\right\}$$

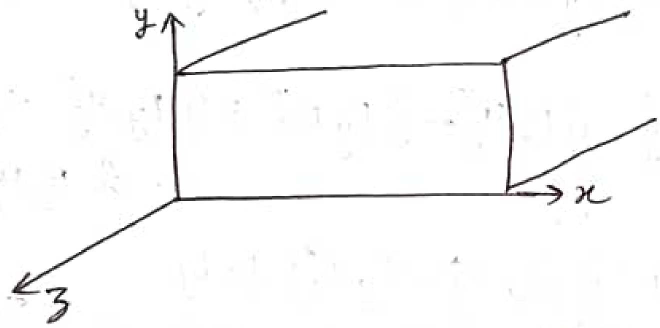




$$\hat{n}\times(\bar{H}_2-\bar{H}_1)=\bar{J}_s$$

$$\hat{n}\times\bar{H}_2=\bar{J}_s \quad \text{(if I is conductor, } H_1=0\text{)}$$

Surface current:

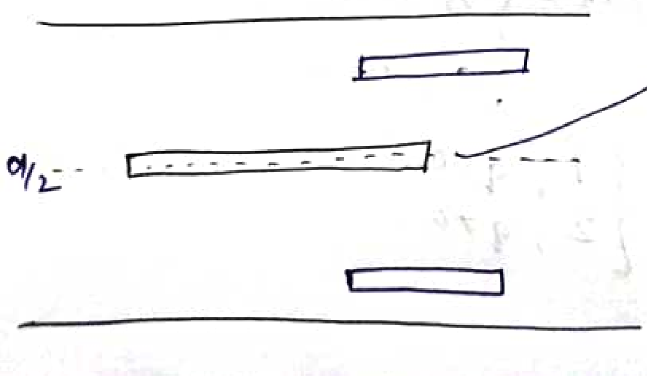


$$\bar{J}_s^{\,top} = -\hat{y}\times(\hat{x}H_x+\hat{z}H_z)$$
$$=\hat{z}H_x-\hat{x}H_z$$
$$=\hat{x}J_{sx}+\hat{z}J_{sz}$$

$$(J_s)_z = -A_{mn}\cos\left(\frac{\pi x}{a}\right)e^{-j\beta_z z}$$

$$(J_s)_x = \frac{j\beta_z\pi}{h^2 a}\cos\left(\frac{\pi x}{a}\right)e^{-j\beta_z z}$$

Slotted Waveguide Section:



Cut a slot and probe it inside waveguide without affecting its operation (To find field Max^m/Min^m)