<u>General Instructions</u>

A. Performing the experiments

The students should read the following guidelines before performing an experiment:

(i) Student should come prepared for the experiment to be performed

- Study all the **'Background'**, mentioned in the beginning of each experiment.
- Read the **Aim of the experiment, the apparatus required** and **theory** given in the Laboratory work Book. Read the **procedure** carefully.

(ii) **Study the experiment** very carefully, and think of the steps you will take in carrying out the work.

(iii) **Check** that you have all the required **apparatus**.

(iv) Do the work **neatly** and **methodically**, step by step.

(v) Record all the data in your Laboratory ***work*** book systematically in the Tables ***using*** PEN.

(vi) Take multiple reading to reduce **random errors**. Correct for **systematic errors**, where known.

(vii) If a mistake is made in recording a value, strike it out neatly and write the new value by its side. **Do not overwrite**. **Do not use pencil** for recording the observations.

(viii) Draw graphs, where necessary.

(ix) Put units in the final results.

(x) Compute the **percentage error**, and judge the number of significant figures to be kept in the result.

(xi) Please keep in mind that the work book, which have been provided to you, are just a basic guideline to perform the experiments. You are always encouraged to improve on your experiments beyond whatever mentioned in the manuals

Attesting the laboratory work book

The entire stress is given to the actual record made by the student in the laboratory work book that he uses in the class. **Never use pencil while taking readings in the work book. After the completion of each experiment, it should be checked and signed by the faculty present. This has to be done on the same day. Before that students should call the faculty present and show a reading. This is mandatory. Due to some reason if the student is unable to complete the experiment, he/she should still get the work book attested by the faculty present with a suitable remark. EXPERIMENTS PERFORMED DURING A PARTICULAR LAB SESSION HAS TO BE CHECKED AND WORK BOOK SIGNED BY THE FACULTY PRESENT.**

Waves

Resonance in Air Columns

What you can learn about..........

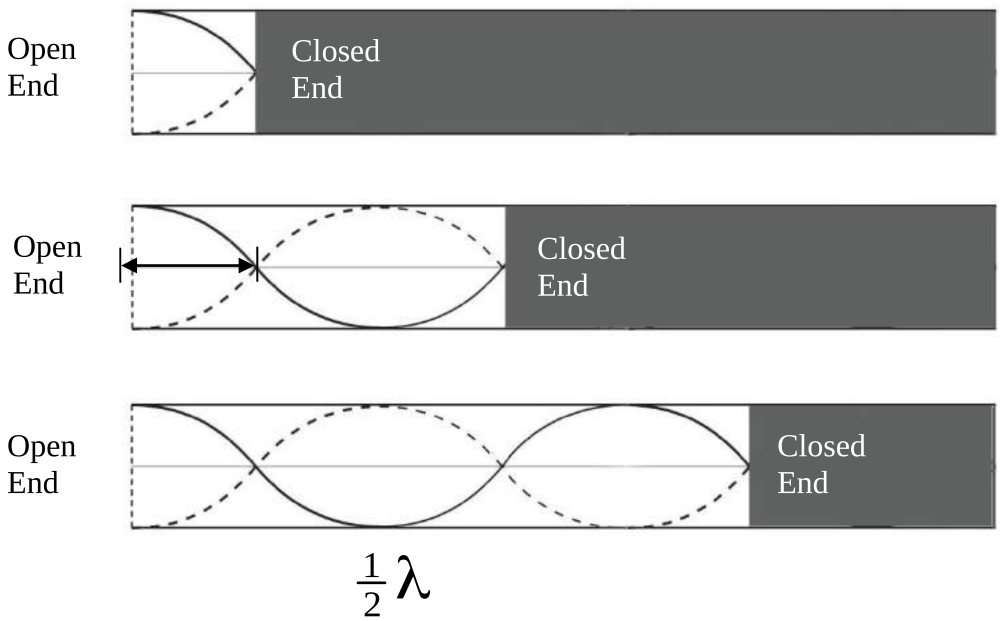
- **Open and closed organ pipes**
- **Resonance of sound waves**
- **Nodes, anti-nodes and harmonics**

Aim of the experiment:

1. To study the relationship between the wavelength and speed of sound waves using both closed and open tubes.

Equipment required:

Sine wave generator
Open speaker
Banana patch cords (set of 5)
Resonance tube



Part I: Closed Tube of Variable Length

Theory:

A resonating tube with one end open and the other end closed will always have a node at the closed end and an anti-node at the open end. A node represents an area where the velocity of the air is a minimum (zero), and an anti-node represents an area where the velocity of the air is a maximum. If the tube is resonating at a particular fixed frequency, the tube will resonate as shown below, where the curved lines represent the velocity profile of the air in the tube.

As the length of the active part of the tube is increased, the sound becomes loud at each successive node and quiet at the antinodes. Note that the distance between the nodes is ½ λ.

For all types of waves, the frequency (f) and the wavelength (λ) are related to the speed (v) of the wave as given by Equation 1.

$$v = \lambda f \tag{1}$$

Setup and Procedure:

1. **Turn on the Sine Wave Generator and turn the amplitude knob all way down (counter-clockwise).**
2. **Connect the generator to the speaker using two banana patch cords. Polarity does not matter.**
3. **Place the Resonance Tube horizontally, as shown, with the speaker near the open end. Place the speaker at a 45° angle to the end of the tube, not pointed directly into it.**

4. The inner (white) tube slides inside the blue tube to adjust the effective length of the closed tube.

5. Slide the two tubes together, so that the tube length is zero. Set the Sine Wave Generator frequency for 300 Hz and the amplitude on a reasonable level.
6. Extend the white tube, increasing the tube length. The loudness of the sound will noticeably increase as you approach resonance. Move the tube in and out to pinpoint the position that gives the loudest tone. Record this position.
7. Continue to extend the white tube and find all the positions that cause a resonance. Each of these positions represents a node in the standing wave pattern.
8. Calculate the distance between the nodes, and take the average if you have more than one value. Use this distance to calculate the wavelength of the sound wave.
9. Use the frequency of the sine wave generator to calculate the speed of the wave.
10. Set the Sine Wave Generator frequency for 400 Hz and repeat the above procedure. How does the speed of the wave compare to the previous wave at 300 Hz? Are they about the same?
11. The actual speed of the wave is the speed of sound in air, which is temperature dependent. This theoretical value can be calculated using

$$V = 331\text{ m/s} + 0.6\,T$$

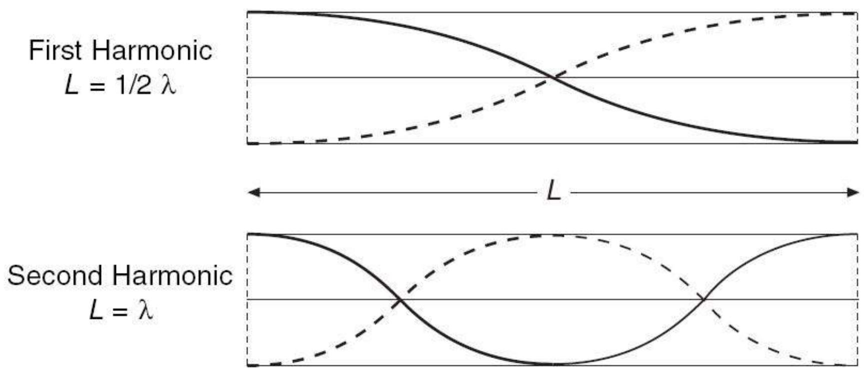
Where T is the temperature of the air in degrees Celsius. Measure the air temperature and calculate the actual speed of sound.
Compare your measured values for the speed of sound to the actual value. Calculate the percent deviation.

$$\%\,Deviation = \frac{Measured - Actual}{Actual}\,X\,1$$

Part II: Open Tube of Fixed Length

Theory:

A resonating tube with both ends open will always have an anti-node at both ends, and at least one node in between. The number of nodes is related the wavelength and the harmonic. The first harmonic (or fundamental) has one node, the second harmonic has two, etc., as shown here. For a tube of fixed length, at higher harmonics, the frequency is higher and the wavelength is shorter.



Setup and Procedure:

1. **Slide the inner tube all the way out, and separate it from the outer tube. Use only the outer blue tube with two open ends.**
2. **Set up the Sine Wave Generator and the speaker as before. Start with the frequency at 50 Hz and slowly increase it using the coarse (1.0) knob. Find the frequency of the fundamental (to the nearest 1 Hz).**
3. **Calculate the wavelength using the frequency and the actual speed of sound you calculated in Part I. How does this compare to the length of the tube?**
4. Increase the frequency of the Sine Wave Generator and determine the frequency of the second and third harmonic. How do these compare to the fundamental?

Further investigations:

1. **Return the frequency to the fundamental for the open tube, and then replace the open tube with the closed (white) tube. Is it still at resonance? Decrease the frequency until you find the fundamental resonance of the closed tube. (It may be necessary to increase the volume and /calculate the theoretical fundamental to help pinpoint the actual value.) Why is the fundamental frequency of the closed tube lower than it was for the open tube?**
2. **Increase the frequency and try to find any other harmonics. Why does a tube open at both ends play all the harmonics, but a tube with one end closed only plays the odd harmonics (1, 3, 5, etc.). What is the relationship between the tube length and the wavelength for the third harmonic of a closed tube?**

Questions:

1. In a pipe organ:
   (a) Which pipes make the low notes, the long ones or the short ones?
   (b) Which pipes sound the lower frequency, the long ones or the short ones?
   (c) Which pipes sound the longer wavelengths, the long ones or the short ones?
2. Suppose that in this experiment the temperature of the room had been lower; what effect would this have had on the distance between nodes for each reading? Explain.
3. How would an atmosphere of helium affect the pitch of an organ pipe?

CONCLUSION

Summarize the differences between an open and closed tube. Also discuss:

- How the velocity, wavelength, and frequency changed as the tube length was varied.
- How the velocity, wavelength, and frequency changed for the part of the experiment in which the tube length was constant.

b).Vibrating strings

What you can learn about......

- **Standing waves in strings**
- **Nodes and anti-nodes**
- **Resonance**

Aim of the experiment:

To study the dependence of its velocity of waves on a string as a function of string tension and length.

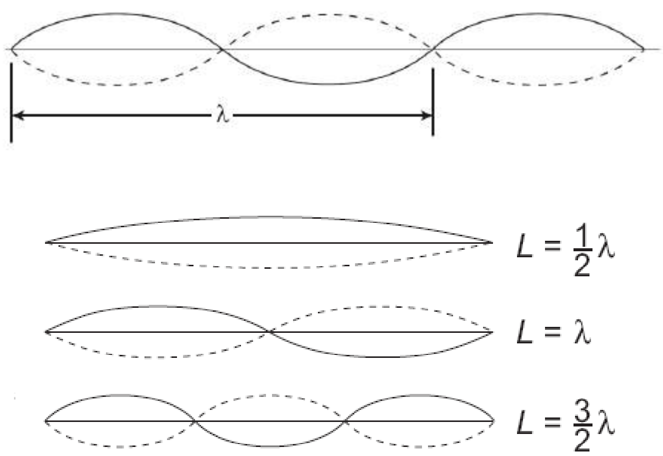
Equipment required:

String vibrator
Strings
Sine wave generator
C-clamps
Patch cords
Tape measure

Theory:

The general appearance of waves can be shown by means of standing waves in a string. This type of wave is very important because most of the vibrations of extended bodies, such as the prongs of a tuning fork or the strings of a piano, are standing waves. The purpose of this experiment is to study how the speed of the wave in a vibrating string is affected by the stretching force and the frequency of the wave.

Standing waves (stationary waves) are produced by the interference of two traveling waves, both of which have the same wavelength, speed and amplitude, but travel in opposite directions through the same medium. The necessary conditions for the production of standing waves can be met in the case of a stretched string by having waves set up by some vibrating body, reflected at the end of the string and then interfering with the oncoming waves.



Standing Waves In Strings

A stretched string has many natural modes of vibration (three examples are shown above). If the string is fixed at both ends then there must be a node at each end. It may vibrate as a single segment, in which case the length (*L*) of the string is equal to 1/2 the wavelength (λ) of the wave. It may also vibrate in two segments with a node at each end and one node in the middle; then the wavelength is equal to the length of the string. It may also vibrate with a larger integer number of segments. In every case, the length of the string equals some integer number of half wavelengths. If you drive a stretched string at an arbitrary frequency, you will probably not see any particular mode; many modes will be mixed together. But, if the tension and the string's length are correctly adjusted to the frequency of the driving vibrator, one vibrational mode will occur at a much greater amplitude than the other modes.

For any wave with wavelength λ and frequency f, the speed, v, is

$$v = \lambda f \qquad (1)$$

The speed of a wave on a string is also given by

$$v = \sqrt{\frac{F}{\mu}} \qquad (2)$$

where *F* is the tension in the string and μ is the linear density (mass/length) of the string.
L is the length of the string and *n* is the number of segments. (Note that *n* is *not* the number of nodes). Since a segment is 1/2 wavelength then

$$\lambda = \frac{2L}{n} \qquad n = 1, 2, 3 \qquad (3)$$

Setting the wave speed in Equation (1) equal to the wave speed in Equation (2) and solving for the tension gives

$$F = \mu \lambda^2 f^2 \qquad (4)$$

Substituting for the wavelength from Equation (3) yields

$$F = 4\mu L^2 f^2 / n^2 \qquad (5)$$

In this experiment, the length and the number of segments will be held constant while the frequency is varied. The tension required to achieve 2 segments will be measured for various driving frequencies. Equation (5) shows that a graph of tension versus the square of the frequency will result in a straight line with

$$dF/d(f^2) = \text{Slope} = 4\mu L^2/n^2 \qquad (6)$$

Setup and Procedure:

In this experiment, standing waves are set up in a stretched string by the vibrations of an electrically-driven clamped String Vibrator at one end of the string. The other end of the string slides over a pulley with weights suspended to keep the string tense. When the String Vibrator is turned on, standing waves form on the string for suitable values of vibrator frequency, length of the string and tension, for a given string.

1. Measure the length of the piece of string used. Measure the mass of the string and calculate the linear density, μ (mass/length). (If your balance is not precise enough to

measure that length of string, use a much longer piece of string to calculate the linear density.)

2. Clamp the String Vibrator to the table. Attach about a meter of string to the vibrator and slide the other end over a pulley to the suspender with about 150gm mass.
3. Measure the length 'L' of string between the vibrator and the top of the pulley.
4. Turn on the Sine Wave generator and turn the Amplitude knob all the way down (counter-clock wise). Connect the generator to the string vibrator using two banana patch cords. Polarity does not matter.

Part I

1. Set the Amplitude about midway. Vary the frequency and fine tune it till resonance is reached, noticed with good nodes and anti-nodes. Adjust the driving amplitude and frequency to obtain a large-amplitude wave, but also check the end of the vibrating table: the point where the string attaches should be a node. It is more important to have a good node at the blade then it is to have the largest amplitude possible. However, it is desirable to have large amplitude while keeping a good node.
2. Record the frequency. How much uncertainty is there in this value? How much can you change the frequency before you see an effect?
3. Repeat steps 1 and 2 for standing wave with two segments. The string should vibrate with a node at each end and one node in the center. Do not change the hanging mass.
4. How is the frequency of the two-segment wave related to the frequency of the one segment wave? Calculate the ratio of the frequencies. Is the ratio what you would expect?
5. With the wave vibrating in two segments, the length of the string, L, is one wavelength ($L=\lambda$). Does it look like one wavelength? Since the string vibrates up and down so fast, it is hard to see that when one side is up, the other is down. Try touching the string at an anti-node. What happens? Try touching the string at the central node. Can you hold the string at the node and not significantly affect the vibration?
6. What was the wavelength when the string was vibrating in one-segment? Use equation (1) to calculate the speed of the one-segment wave. Calculate the speed of the two-segment wave. How do these two values compare? Are they about the same? Why?
7. Calculate the tension in the string caused by the hanging mass (don't forget the mass hanger) and using your measured density of the string, calculate the speed of the wave using equation (2). How does this compare to your answer in part 6?
8. Adjust the frequency so that the string vibrates in three segments. What is the velocity now? Has it changes? Does the speed of the wave depend on the wavelength and the frequency?

Part II

1. Adjust the frequency of the Sine Wave generator so that the string vibrates in four segments. As before, adjust the driving amplitude and frequency to obtain a large amplitude wave, and clean nodes, including the node at the end of the blade. Record the frequency.

2. Add 50g to the hanging mass and repeat steps 1 and 2.
3. Repeat at intervals of 50g up to at least 250g. Record your data in a table

Note: For this part of the experiment, you will always adjust the wave vibrates in four segments

Damped Driven Harmonic Oscillator

2

What you can learn about………

- **Oscillation period**
- **Harmonic oscillations**
- **Damped oscillations**
- **Damped-Driven oscillations**

Aim of the Experiment:

(1) The resonance of a driven damped harmonic oscillator is examined by plotting the oscillation amplitude versus frequency for various amounts of damping.

(2) To calculate the period of oscillation of the driven damped harmonic oscillator.

Equipment Required:

Rotary Motion sensor
Mechanical Oscillator
Chaos Accessory
Large rod stand
120-cm long steel rods
45-cm long steed rod
Multi Clamp DC Power Supply

Extra requirements

Vernier Calipers; 20 g Hooked Mass; Data Studio (installed in the PC)

Introduction:

The oscillator consists of an aluminum disk with a pulley that has a string wrapped around it to two springs. The Rotary Motion Sensors measure the angular positions and velocities of the disk and the driver as a function of time. The damping method uses a magnet attached near the disk to induce eddy currents. These currents generate a force on the magnets that opposes the motion of the mass. The amplitude of the oscillation is plotted versus the driving frequency for different amounts of magnetic damping. Increased damping is provided by adjusting the distance between the magnet and the disk.

Theory:

The oscillating system in this experiment consists of a disk connected to two springs with nearly same spring constant. A string connecting the two springs is wrapped around the disk so that the disk oscillates back and forth. This is like a torsion pendulum. The period of a torsion pendulum is given by

$$T=2\pi\sqrt{\frac{I}{\kappa}} \qquad (1)$$

Where I is the rotational inertia of the disk and κ is the effective torsional spring constant of the springs. The rotational inertia of the disk is found by measuring the disk mass (M) and the disk radius (R). For a disk, oscillating about the perpendicular axis through its center, the rotational inertia is given by $I=\frac{1}{2}MR^2$.

When a known torque *(τ = rF, where r is the radius of the pulley and F=force (mg))* is applied to the disk, it turns through an angle θ. The torsional spring constant can be calculated as

$$\kappa=\frac{\tau}{\theta} \qquad (2)$$

When the damped oscillator is driven with a sinusoidal torque, its motion can be defined using the below differential equation.

$$I\frac{d^2\theta}{dt^2}+b\frac{d\theta}{dt}+\kappa\theta=\tau_o\cos\omega t\,,$$

Where b is a constant that describes the strength of the damping force and ω the driving frequency. The solution to this equation is

$$\Theta=\frac{\theta_0}{\sqrt{(\omega^2-\omega_0^2)^2+4\omega^2b^2}}\cos\left(\omega t-\delta\right) \quad (3)$$

where δ is the phase difference between the driving torque and the resultant motion, given by

$$\delta=\tan^{-1}\left(\frac{2\omega b}{\omega_o^2-\omega^2}\right). \qquad (4)$$

Here ω_o is the resonance frequency.

(i) As the driving frequency (ω) approaches zero, δ approaches zero as $\tan^{-1}(0)\rightarrow 0$. The resulting motion becomes in phase with the driving torque.

(ii) At resonance, $\omega = \omega_o$, results in

$$\delta=\tan^{-1}\left(\frac{2\omega_o b}{\omega_o^2-\omega_o^2}\right)=\tan^{-1}\left(\frac{\pi}{2}\right)=\infty, \text{ and}$$

$$\Theta=\frac{\theta_0}{2\omega_o b}\cos\left(\omega_o t-\frac{\pi}{2}\right) \text{(From Eq. 3)}$$

(iii) As the driving frequency (ω) goes to infinity, $\delta=\lim_{\omega\rightarrow\infty}\left[\tan^{-1}\left(\frac{2\omega b}{\omega_o^2-\omega^2}\right)\right]=\pi$. The resulting motion becomes 180° out of phase with the driving torque.

Set up and Procedure:

1. Mount the driver on a rod base as shown in Figure 2. Slide the first Rotary Motion Sensor onto the same rod as the driver. See Figure 3 for the orientation of the Rotary Motion Sensor.

2. Rotate the driver arm until it is vertically downward. Attach a string to the driver arm and thread the string through the string guide at the top end of the driver. Wrap the string completely around the Rotary Motion Sensor large pulley. Tie one end of one of the springs to the end of this string. Tie the end of the spring close to the Rotary Motion Sensor.
3. Use two vertical rods connected by a cross rod at the top for greater stability. See Figure 3.
4. Mount the second Rotary Motion Sensor on the cross rod.

Figure 2: Driver

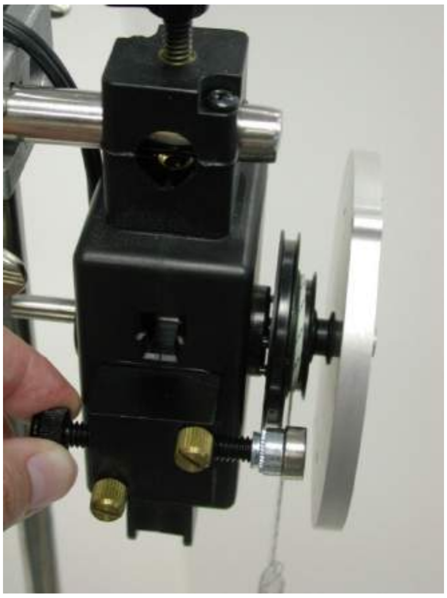


Figure 3: String and Magnet

5. Tie a short section of string (a few centimeters) to the leveling screw on the base. Tie one end of the second spring to this string.
6. Cut a string to a length of about 1.5 m. Wrap the string around the middle step of the second Rotary Motion Sensor twice. See Figure 4. Attach the disk to the Rotary Motion Sensor with the screw.
7. To complete the setup of the springs, thread each end of the string from the pulley through the ends of the springs and tie them off with about equal tension is each side: The disk should be able to rotate 180 degrees to either side without the springs hitting the Rotary Motion Sensor pulley.
8 Attach the magnetic drag accessory to the side of the Rotary Motion Sensor as shown in Figure 4. Adjust the screw that has the magnet so the

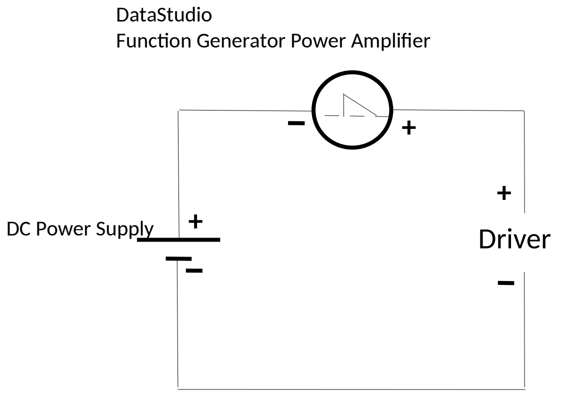
magnet is about 1.0 cm from the disk.

9. Wire the driver circuit as shown in Figure 5. In this experiment, a ramped voltage is applied to the driver using the signal generator on the 750 interface. However, since the driver motor stalls out at low voltages and it is desired to get the maximum number of data points possible, it is necessary to have an offset voltage so the minimum voltage is about 1 V. This offset voltage is supplied by the DC power supply. Plug the driver into the DC power supply and attach the digital voltmeter across the power supply.

Fig.4 Complete setup

10. Plug the disk Rotary Motion Sensor into Channels 1 and 2 on the Science Workshop 750 interface with the yellow plug in Channel 1. Plug the driver Rotary Motion Sensor into Channels 3 and 4 with the yellow plug in Channel 3. Plug the Power Amplifier into Channel A.
11. Open the Data Studio file called "Driven Harmonic".

Figure 5: Driver Wiring Diagram



procedure

1. Measure the resonant frequency. Leave the DC power supply turned off and click the signal generator off in Data Studio. Click on START, displace the disk, and let it oscillate. Click on STOP. Measure the period using the Smart cursor on the disk oscillation graph.
2. Click on START. Hang a hooked mass (20 g) on the top of one of the springs and measure the resulting angle through which the disk rotates. Click on STOP. Measure the radius of the middle step of the RMS pulley and calculate the torque caused by the weight of the 20 g mass. Calculate the torsional spring constant using Equation (2).
3. Remove the disk and measure the mass and radius of the disk. Calculate the rotational inertia of the disk.
4. Turn on the DC power supply and set the voltage on 1 V. Click on Auto on the signal generator in Data Studio. Click on START in DataStudio. Since the frequency of the signal generator ramp is set for 0.001 Hz, data collection will take 1000 seconds (18 minutes). Then click on STOP.
5. Adjust the magnetic damping screw to about 0.1 cm from the disk and repeat the data collection.

6. Adjust the magnetic damping screw to about 0.2 cm from the disk and repeat the data collection.
7. Adjust the magnetic damping screw to about 0.3 cm from the disk and repeat the data collection.

ANALYSIS

1. Using the torsional spring constant and the disk rotational inertia, calculate the theoretical period and the resonant frequency of the oscillator (ignoring friction).

$$\nu_o = \text{…………..Hz}$$

2. Examine the resonance curves for different amounts of damping. How does increasing the damping affect the shape of the curve (the width, maximum amplitude, frequency of the maximum)?
3. Is the resonant frequency for the least amount damping the same as the theoretical frequency? Calculate the percent difference.

$$100 \quad (|\nu_o - \nu_{max}|)/\nu_o = \text{…………..\%}$$

4.. Examine the graphs of the driving oscillation versus time and the disk oscillation versus time. Measure the phase difference between these oscillations at high frequency (at the beginning of the time), resonance frequency (at the time when the disk oscillation is greatest), and at low frequency (at the end of the time). Do these phase differences agree with the theory? (Compare $\frac{\Delta T}{T} \times 360^0$ from the graph with the theoretical value of $\delta . \Delta T$ is the difference in the period of oscillation between the driver and the disk.)

Earth's Magnetic Field

3

What you can learn about……

- Components of Earth's magnetic field
- Angle of inclination
- Principle of Helmholtz coils
- Magnetic flux density

Aim of the experiment:

1. Calibration of the Helmholtz coil.
2. Determination of the horizontal and vertical components of the Earth's magnetic field.

Equipments required:

Pair of Helmholtz coils
Power supply, universal
Rheostat, 100 Ohm, 1.8 A
Tesla meter
Hall Probe
Digital Multimeter
Magnetometer

Theory:

The measured magnetic flux density of the Helmholtz coil is used to derive the Earth's magnetic field. For current less coils, the magnetic needle of the magnetometer aligns itself with the horizontal component $\hat{B}_E$ (direction north/south) of the earth-magnetic field. If an additional magnetic field is $\hat{B}_H$ superimposed on this component through the Helmholtz coils, the needle of the magnetometer will be turned around by an angle α and will point in the direction of the resulting $\hat{B}_R$. Figure:1 displays the vector diagram for the magnetic flux densities involved. Fig:1(A) shows the field components for the general case of $\psi \neq 90^0$. The components drawn by a broken line represent the resulting conditions for the case when the polarity of the coil current is reversed.

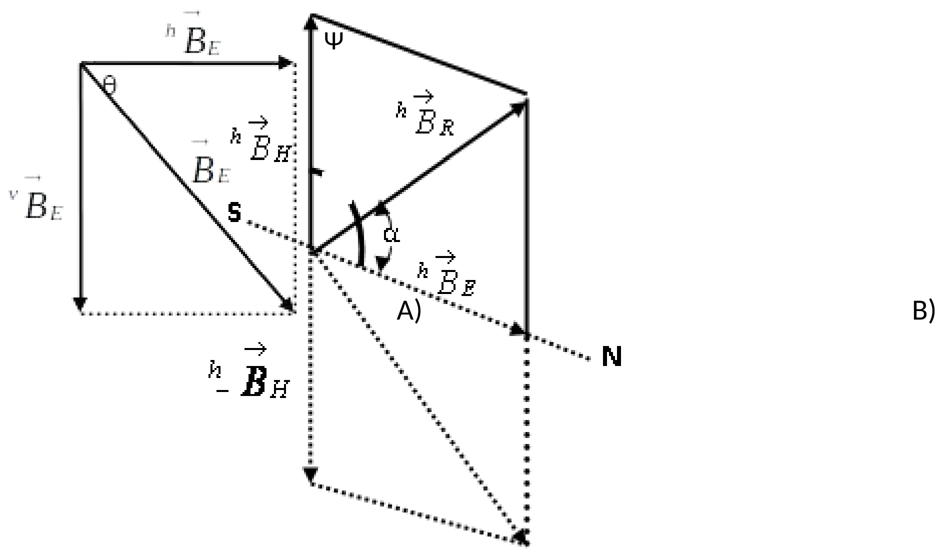


Fig:1. Vector diagram for magnetic flux densities. (A) horizontal plane (B) vertical plane

It can be then shown that

$$\frac{sin\alpha}{sin\beta} = \frac{sin\alpha}{\sin(\psi-\alpha)} = \frac{hB_H}{hB_E} (1)$$

If the coil axis is perpendicular to the north-south direction ($\psi = 90^0$) then

$$hB_E = {}^hB_H \cot\alpha \qquad (2)$$

The calibration of the magnetic flux density of the Helmholtz coil as a function of the current passing through coils yields

$$hB_H = I_H \cdot K \qquad (3)$$

From Eqns. (1) and (3) we obtain

$$\frac{\sin\alpha}{\sin\beta} hB_E = I_H \cdot K \qquad (4)$$

The horizontal component of the Earth's magnetic field is thus obtained as the slope of graph

between $\frac{\sin\alpha}{\sin\beta} \quad and \quad I_H \cdot K$ (4)

Figure 1 (B) shows the relation between the horizontal and vertical components and the angle of inclination θ which is given by

$$ {}^{v}B_{E} = {}^{h}B_{E} \tan\theta \qquad (5) $$

Hence the total flux density B_E *is* given by

$$ \left|B_{E}\right| = \sqrt{({}^{v}B_{E})^{2} + ({}^{h}B_{E})^{2}} \qquad (6) $$

Set up and Procedure:

The experimental set up is shown in Fig. 2.

Fig.2

1. The Helmholtz coils, complete with mounted space-holders, are connected in series and connected with the DC-generator by the rheostat and the multimeter used as ammeter.
2. The Hall probe is fixed on the support rod with barrel base pointing inward toward the coil axis in the centre of the Helmholtz arrangement.
3. In this arrangement, the horizontal flux density ${}^{h}B_{H}$ of the pair of coils is determined as a function of the coil current I_H The slope of the graph gives the calibration factor $K = {}^{h}B_{H} / I_{H}$
4. With the help of barrel base and stand tube , the magnetometer (with a levelled graduated circle) is placed between the coils so that the centre of the graduated circle is approximately identical with the centre of the pair of coils.
5. The direction "north/south" is noted on the graduated circle for current less coils. In order to secure the direction "north/south" of the magnetic needle, the needle should be slightly turned away from its resting position several times. Possible friction resistance can be reduced by gently tapping the instrument.

6. In order to determine the horizontal component $^{h}B_{E}$ of the earth-magnetic field, the deflection angle α of the magnetic needle is measured from its resting position as a function of small coil currents. If the polarity of the coil current is reversed, the measuring series must be repeated. In determining the exact angle, the indications from both needle tips must be considered.
7. The angle φ between the direction "north/south" and the axis of the pair of coils is obtained through maximal needle deflection when the resistor is short-circuited the ammeter eliminated and the coil current set to approximately 4 A.
8. To determine the vertical component of the Earth's magnetic field, the magnetometer is placed at the centre of the current less coils. The graduated circle of the magnetometer is turned to the vertical plane so that the magnetic needle now indicates the inclination angle θ_1. Make sure that the spin axis is consistent with the direction "north/- south". The magnetometer is turned by 180° and θ_2 is measured. The average gives value of the vertical component $^{v}B_{E}$

Note

1. The zero-point of the Tesla meter has to adjusted properly before taking readings

Avoid performing this experiment very close to iron pieces or other electronic devices

Specific Charge Of The Electron-e/m

4

What you can learn about……

- Cathode rays
- Lorentz force
- Electron in crossed fields
- Electron mass
- Electron charge

Aim of the experiment:

To determine the specific charge of the electron (e/m_0) from the path of an electron beam in crossed electric and magnetic fields of variable strength.

Equipments required:

Narrow beam tube
Helmholtz coils, one pair
Power supply, 0...600 VDC
Power supply, universal
Digital multimeter
Connecting cord, l = 100 mm, red
Connecting cord, 32 A, l = 100 mm, blue
Connecting cord, 32 A, l = 750 mm, red, blue, yellow

Theory:

If an electron of mass m_0 and charge e is accelerated by a potential difference U it attains the kinetic energy:

$$eU = \frac{1}{2} m_0 v^2 \qquad (1)$$

where v is the velocity of the electron.

In a magnetic field of strength $\vec{B}$ the Lorentz force acting on an electron with velocity $\vec{v}$ is:

$$F = ev \times B$$

If the magnetic field is uniform, as it is in the Helmholtz arrangement the electron therefore follows a spiral path along the magnetic lines of force, which becomes a circle of radius r if $\vec{v}$ is perpendicular to $\vec{B}$.
Since the centrifugal force $m_0 v^2/r$ thus produced is equal to the Lorentz force, we obtain

$$v = \frac{e}{m_0} Br$$

where B is the absolute magnitude of $\vec{B}$.
From equation (1), it follows that

$$\frac{e}{m_0} = \frac{2U}{(Br)^2} \qquad (2)$$

For the Helmholtz arrangement of two coils of radius R and number of turns n the magnetic field **B** between the coils is given by

$$B = \left(\frac{4}{5}\right)^{3/2} \mu_0 n \frac{I}{R}, \qquad (3)$$

where $\mu_0 = 1.257 \times 10^{-6}$ $VsA^{-1}m^{-1}$.
Here R = 0.2m, and n=154.

Set-up and Procedure:

The experimental set up is as shown in Fig:1.

Fig:1. Experimental setup for determining the specific charge of the electron.

The electrical connection is shown in the wiring diagram in Fig:2 and Fig:3. The two coils are turned towards each other in the Helmholtz arrangement. Since the current must be the same in both coils, connection in series is preferable to connection in parallel. The maximum permissible continuous current of 5 A should not be exceeded.

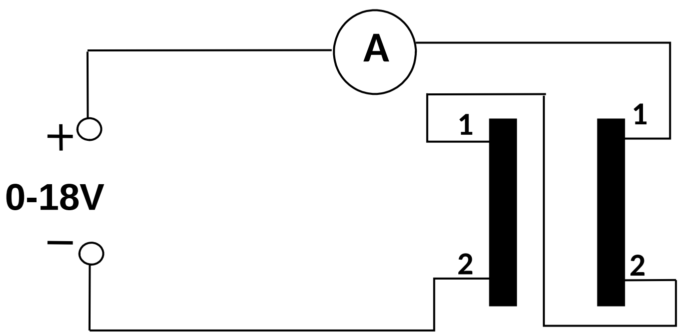


Fig:2. Wiring diagram for Helmholtz coils.

If the polarity of the magnetic field is correct, a curved luminous trajectory is visible in the darkened room. By varying the magnetic field (current) and the velocity of the electrons (Acceleration and focusing voltage) the radius of the orbit can be adjusted, such that it coincides with the radius defined by the luminous traces. When the electron beam coincides with the luminous traces, only half of the circle is observable. The radius of the circle is then 2, 3, 4 or 5cm.

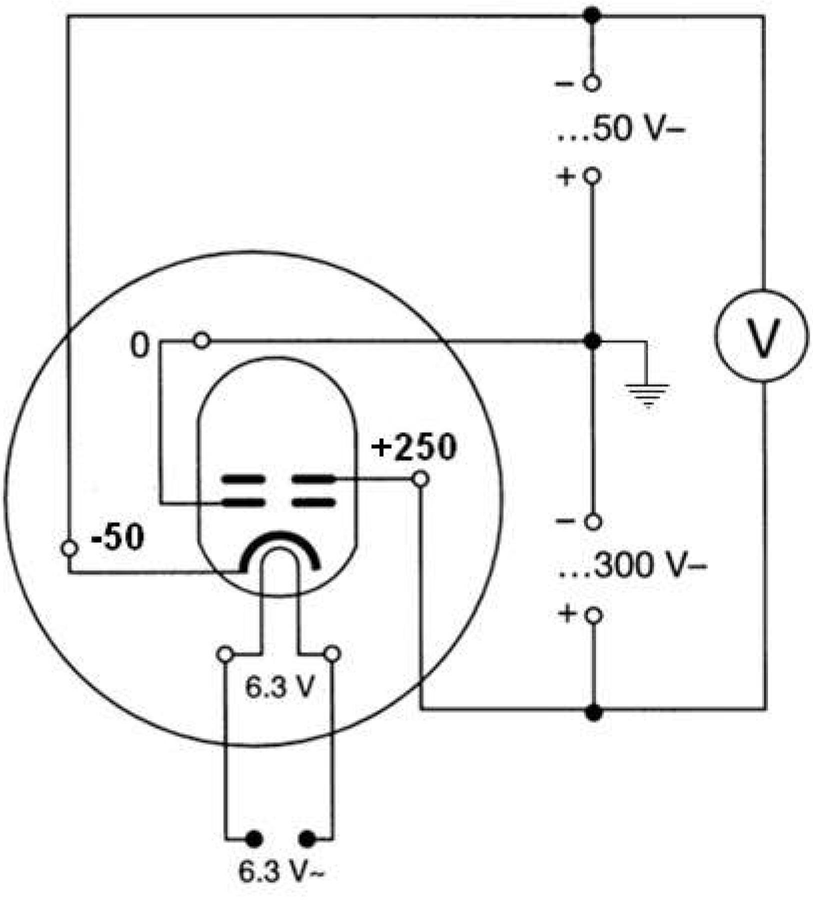


Fig:2. Wiring diagram for Narrow beam tube.

If the trace has the form of a helix this must be eliminated by rotating the narrow beam tube around its longitudinal axis.

Estimation of Celsius Equivalent of Absolute Zero

5

What you can learn about……

- Absolute zero of temperature
- Universal gas constant
- Expansion of gases

Aim of the experiment:

To estimate the value of absolute zero temperature of Celsius units.

Equipments required:

Round bottom flask
U-tube
Rubber cork L-tube
Beaker
Heater with magnetic stirrer
Thermometer
Rubber cork-double holed

Theory:

To a good approximation air at normal temperature and pressure can be treated as ideal gas. We use the expression

$$PV = nRT$$

To estimate the approximation value of absolute zero temperature in celcius units. We measure the expansion of air in a flask as function of temperature.

Set up and Procedure:

1. Make the connections a per the instruction.
2. Turn on the heater and set it to 50^0C
3. Carefully note the change in the volume in the U tube as a function of temperature.
4. Note the values at every 1^0C changes (Say 31^0C, 32^0C, 33^0C……)
5. Then change the heater setting to 60^0C

Inductance of Solenoids

6

What you can learn about……

- Law of inductance
- Lenz's law
- self-inductance
- solenoids
- transformer
- oscillatory circuit
- resonance
- damped oscillation
- logarithmic decrement
- Q factor

Aim of the experiment:

To connect coils of different dimensions (length, radius, number of turns) with a known capacitance C to form an oscillatory circuit. From the measurements of the natural frequencies,
to calculate the inductances of the coils and determine the relationships between
1. Inductance and number of turns
2. Inductance and length
3. Inductance and radius

Equipments Required:

Cobra3 Basic Unit
Power supply, 12 V
RS 232 data cable
Cobra3 Universal writer software
Cobra3 Function generator module
Induction coil, 300 turns, dia. 40 mm
Induction coil, 300 turns, dia. 32 mm
Induction coil, 300 turns, dia. 25 mm
Induction coil, 200 turns, dia. 40 mm
Induction coil, 100 turns, dia. 40 mm
Induction coil, 150 turns, dia. 25 mm
Induction coil, 75 turns, dia. 25 mm
Coil, 1200 turns
PEK capacitor /case 1/ 470 nF/250 V
Connection box
Connecting cord, 250 mm, red
Connecting cord, 250 mm, blue
Connecting cord, 500 mm, red
Connecting cord, 500 mm, blue

PC, Windows® 95 or higher

Theory:

If a current of strength I flows through a cylindrical coil (solenoid) of length *l*, cross sectional area A = π · r^2, and number of turns N, a magnetic field is set up in the coil. When *l* >> r
the magnetic field is uniform and the field strength H is easy to calculate:

$$H = I.\frac{N}{l} \qquad (1)$$

The magnetic flux through the coil is given by

$$\Phi = \mu_0 \cdot \mu \cdot H \cdot A \qquad (2)$$

Where μ_0 is the magnetic field constant and μ the absolute permeability of the surrounding medium.
When this flux changes, it induces a voltage between the ends of the coil,

$$\begin{aligned} U_{ind} &= -N.\frac{d\phi}{dt} \\ &= -N.\mu_0.\mu.A.\frac{N}{l}.\frac{dI}{dt} \\ &= -L.\frac{dI}{dt} \end{aligned} \qquad (3)$$

Where

$$L = \mu_0.\mu.\pi.\frac{N^2.r^2}{l} \qquad (4)$$

Where μ_0 is the coefficient of self-induction (inductance) of the coil.
Inductivity Equation (4) for the inductance applies only to very long coils *l* >> r, with a uniform magnetic field in accordance with (1).
In practice, the inductance of coils with *l* > r can be calculated with greater accuracy by an approximation formula

$$L = 2.1\times10^{-6}.N^2.r.\left(\frac{r}{l}\right)^{3/4}$$

$$\text{for } 0 < \frac{r}{l} < 1 \qquad (5)$$

In the experiment, the inductance of various coils is calculated from the natural frequency of an oscillating circuit.

$$\omega_0 = \frac{1}{\sqrt{LC_{tot}}} \qquad (6)$$

C_{tot} is the sum of the capacitance of the known capacitor and the input capacitance C_i of the Cobra3 input.
The internal resistance R_i of the Cobra3 input exercises a damping effect on the oscillatory circuit and causes a negligible shift (approx. 1%) in the resonance frequency.
The inductance is therefore represented by

$$L = \frac{1}{4\pi^2 f_0^2 C_{tot}} \quad (7)$$

Where $C_{tot} = C + C_i$ and $f_0 = \frac{\omega_0}{2\pi}$

Set-up and procedure:

Set up the experiment as shown in Fig: 1 + 2. A square wave voltage of low frequency ($f \approx$ 500 Hz) is applied to the excitation coil L. The sudden change in the magnetic field induces a voltage in coil L1 and creates a free damped oscillation in the L1C oscillatory circuit, the frequency f_o of which is measured with the Cobra3 interface.
Coils of different lengths *l*, diameters 2r and number of turns N are available (Tab.1). The diameters and lengths are measured with the vernier caliper and the measuring tape, and the number of turns are given.

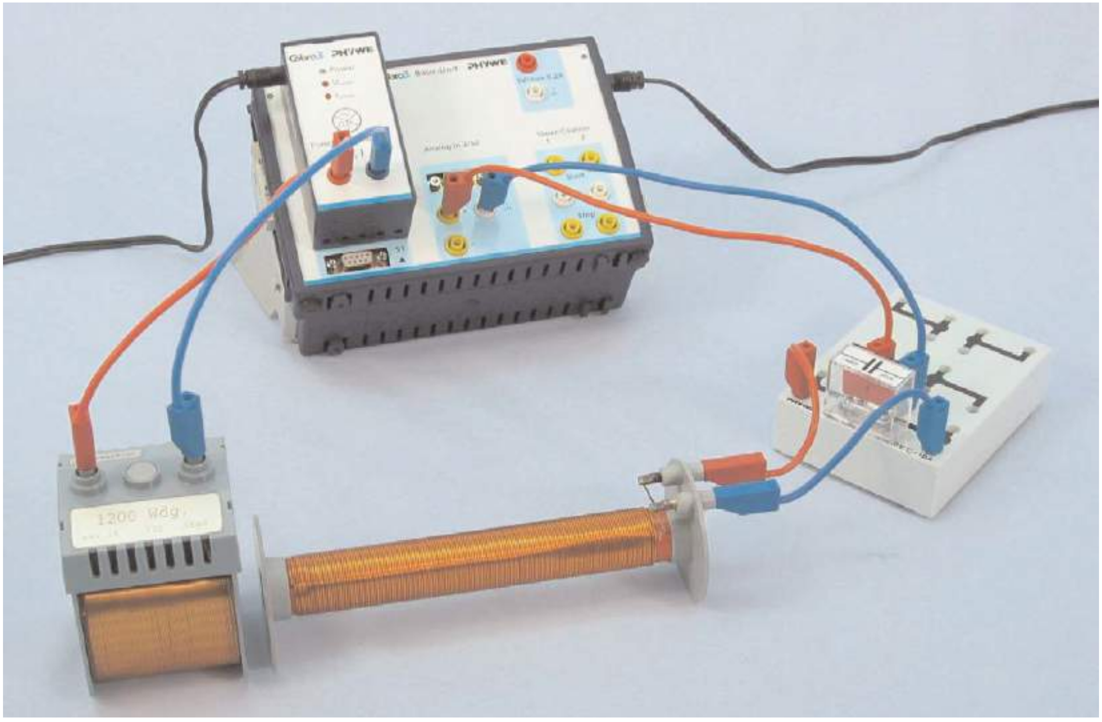


Fig:1. Experimental set-up.

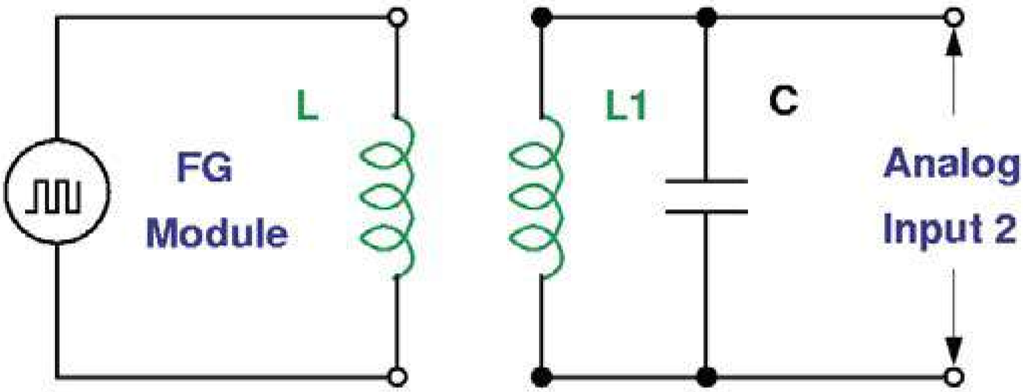


Fig:2. Set-up for inductance measurement

As a difference in length also means a difference in the number of turns, the relationship between inductance and number of turns found in Task 1 must also be used to solve Task 2.

The distance between L1 and L should be as small as possible so that the effect of the excitation coil on the resonant frequency can be disregarded. There should be no iron components in the immediate vicinity of the coils.

Connect the Cobra3 Basic Unit to the computer port COM1, COM2 or to USB port (for USB computer port use USB to RS232 Converter 14602.10). Start the measure program and select Cobra3 Universal Writer Gauge. Begin the measurement using the parameters given in Fig: 3.
For the measurement of the oscillation period the “Survey Function” of the Measure Software is used.

Fig:4 shows the rectangular signal and the damped oscillation behind each peak. Determine the frequency f_0 of this damped oscillation,

$$f_0 = \frac{1}{T}$$

Where T is the oscillation period.

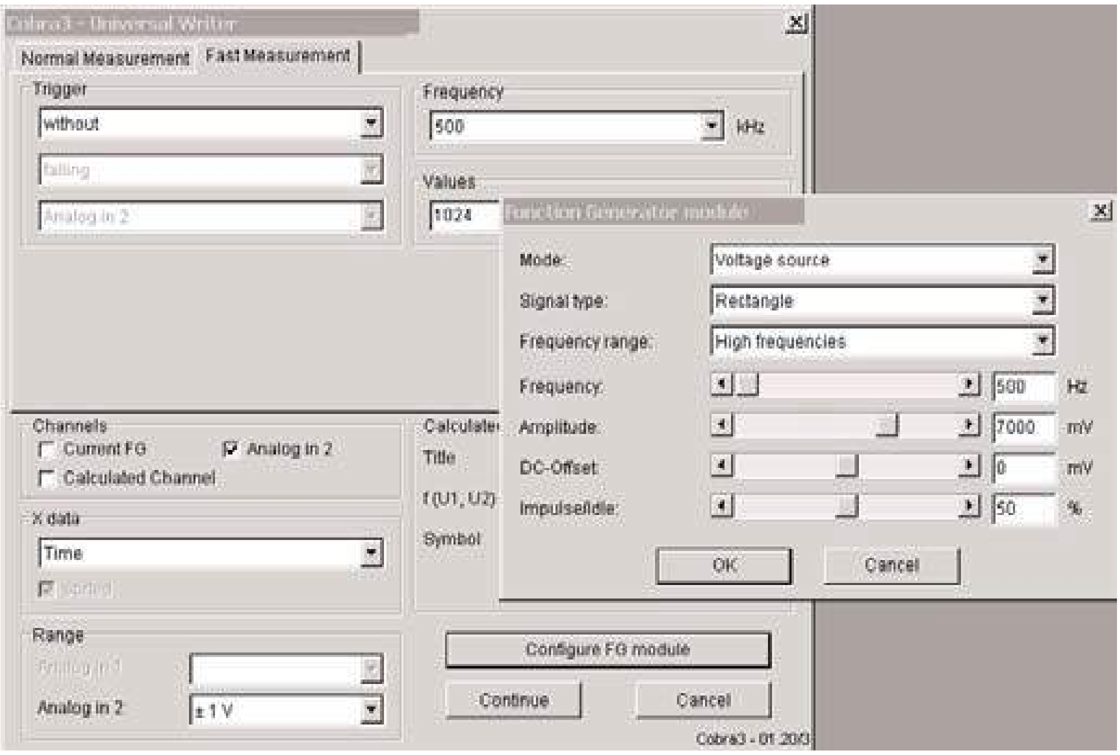


Fig:3. Measuring parameters.

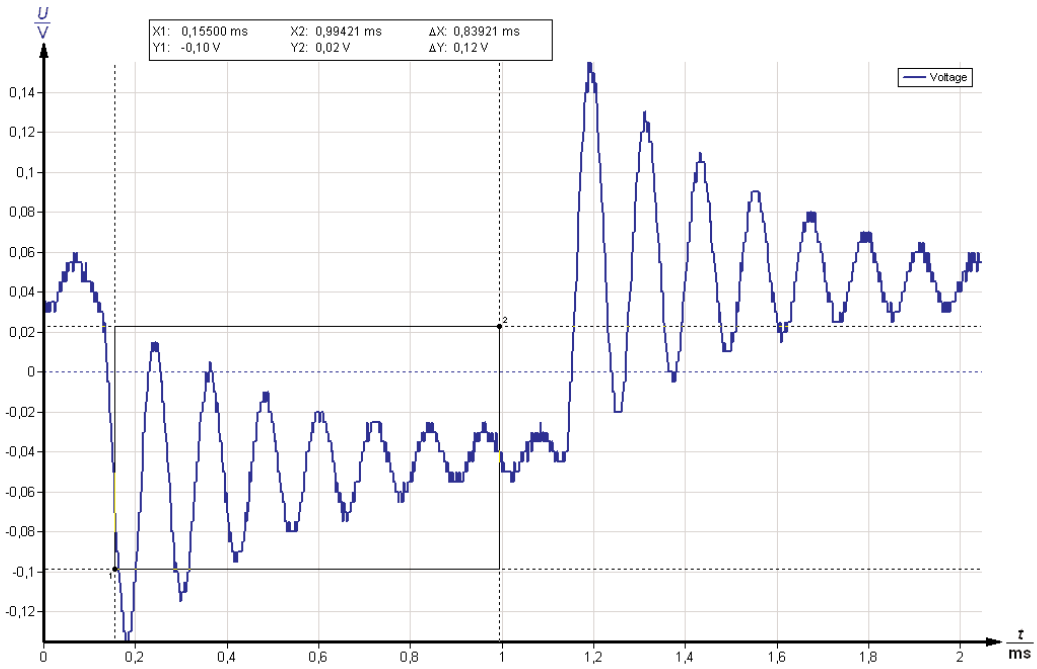


Fig:4. Measurement of the oscillation period with the “Survey Function

7. Ratio of specific heats

What you can learn about:

- **Heat capacities of gases**
- **Specific heat capacity**
- **Oscillations**

Aim of the experiment:

1. To determine the ratio of specific heat capacities of gases

Equipment Required:

Heat Engine/Gas Law Apparatus
Large Rod Stand
45 cm Long Steel Rod
Low Pressure Sensor
ScienceWorkshop 500 or 750 Interface
DataStudio Software

Theory:

A cylinder is filled with air and a Pressure Sensor is attached. The piston is plucked by hand and allowed to oscillate. The oscillating pressure is recorded as a function of time and the period is determined. The ratio of specific heat capacities is calculated using the period of oscillation, according to Ruchhardt's method.

In Ruchhardt's Method, a cylinder of gas is compressed adiabatically by plucking the piston. The piston will then oscillate about the equilibrium position. Gamma, the ratio of specific heat, can be determined by measuring the period of oscillation.

If the piston is displaced downwards a distance x, there will be a restoring force which forces the piston back toward the equilibrium position.

Just like a mass on a spring, the piston will oscillate. The piston acts as the mass and the air acts as the spring.

Figure 1: The piston is plucked by hand

The period of oscillation of a mass on a spring (or for the piston and air) is

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (1)$$

To determine the spring constant, **k,** for air, calculate the force when the piston is displaced a distance x. When the piston is displaced downward a distance x, the volume decreases by a very small amount compared to the total volume: dV = xA where A is the cross-sectional area of the piston.

The resulting force on the piston is given by F = (dP)A where dP is the small change in pressure. To find a relationship between dP and dV, we assume that if the oscillations are small and rapid, no heat is gained or lost by the gas. Thus the process is adiabatic and

$$PV^{\gamma} \qquad (2)$$

Where, $\gamma = \frac{C_P}{C_V} =$ Ratio of Molar Specific Heats (3)

For a diatomic gas, C_V = 5/2 R and C_P = 7/2 R, so γ = 7/5.

Taking a derivative of Equation (2) gives

$$P\gamma V^{\gamma-1} dV + V^{\gamma} dP = 0 \qquad (4)$$

Solving for dP, $dP = -\frac{P\gamma V^{\gamma-1}}{V^{\gamma}} dV$ (5)

Since dV = xA, $$dP = -\frac{\gamma PxA}{V}$$ (6)

Plugging into F = (dP)A gives

$$F = -\left(\frac{\gamma PA^2}{V}\right)x \qquad (7)$$

Comparing this to F = -kx shows that

$$k = \left(\frac{\gamma PA^2}{V}\right) \qquad (8)$$

Substituting into the period equation for k gives $$T = 2\pi\sqrt{\frac{mV}{\gamma PA^2}}$$ (9)

Solving for the volume gives $$V = \frac{\gamma A^2 PT^2}{4\pi^2 m}$$. The total volume is A(h+h_o), where h is the height measured on the labeled scale and h_o is the unknown height below zero on the label. Substituting in for the volume and solving for the height of the piston, h, gives

$$h = \left(\frac{\gamma AP}{4\pi^2 m}\right)T^2 - h_o \qquad (10)$$

Thus, if the piston height is plotted versus the square of the period, the resulting graph will be a straight line with $\text{slope} = \left(\frac{\gamma AP}{4\pi^2 m}\right)$ and y-intercept h_o.
Therefore the ratio of specific heats is given by

$$\gamma = \frac{4\pi^2 m(slope)}{AP} \qquad (11)$$

where m = mass of piston, A = cross-sectional area of piston, P = atmospheric pressure, and the slope is from the graph of h vs. T^2.

Setup and Procedure:

1. Find the mass (m) of the piston (given on the apparatus label) and the cross-sectional area (A)of the piston (the piston diameter is given on the apparatus label).
2. Click on Start in the DataStudio program.
3. Using the tip of your finger, pluck the top of the piston. Click Stop on the computer.
4. Using the Smart Cursor, determine the period of the oscillation from the pressure versus time graph. Expand the area of the graph that shows the oscillation. Measure the period by measuring the time for several peaks and dividing by the number of peaks. Type this period and the corresponding piston height into the table in DataStudio.

5. Lower the piston to 8 cm and repeat the procedure. Then continue to lower the piston in steps of 1 cm, repeating the procedure at each piston position down to 1 cm.
6. Unless a barometer is available, assume the atmospheric pressure is 1.01 x 10^5 Pa.
7. Using the slope of the resulting graph of h vs. T^2, calculate γ for air and compare to the ideal value.
8. If another gas is available, determine γ for that gas. NOTE: Another gas, such as Helium, can be introduced into the cylinder by moving the piston to its lowest position, attaching a rubber balloon filled with Helium to the unused port and opening the hose clamp and letting the Helium from the balloon flow into the cylinder, pushing the piston up to the top. Then the hose clamp is closed with the piston at 9 cm. Never attach a high pressure hose directly to the apparatus.

Appendix C
Least Square Fitting

1. Uses for a Least Squares Fit: Linear Dependence

What's the reason for wanting to do a Least Squares Fit? Why bother with finding a best fit line to a set of data in the first place? Well, a graph is used to show whether there is a relation between the dependent variable (the y-axis) and the independent variable (the x-axis). Usually, one looks for a linear relation, that is to say whether the data points fall roughly on a line or not. A linear relation has the form $y = a + bx$, which is useful for showing direct relationships such as $F=ma$ and $V=IR$. A graph of the force of gravity vs. mass would yield a line with a slope equal to the acceleration due to gravity. A graph of voltage vs. current would give a value for resistance. This is good stuff!

What about equations which are non-linear? How could calculating a best fit line using the Least Squares Fitting method help with that? Here are two examples of equations that may appear non-linear but can be made linear.

The first example involves the magnetic force on electrons and the circular motion the electrons undergo in a uniform magnetic field. The equation is $eB = \frac{m}{r}\sqrt{\frac{2eV}{m}}$, which doesn't seem like it could be graphed easily. However, one wants to graph the charge vs. the mass of the electron, as was done in 1897 by Sir J. J. Thompson. Therefore, that messy equation can be rearranged as follows:

$$\frac{eBr}{m} = \sqrt{\frac{2eV}{m}} \longrightarrow \frac{e^2B^2r^2}{m^2} = \frac{2eV}{m} \longrightarrow \frac{eB^2r^2}{m} = 2V \longrightarrow e = \frac{2V}{B^2r^2}m$$

This involves simple algebra, however, one had to know ahead of time what variables one wanted to graph. In order to graph charge vs. mass, the charge had to be alone on the left-hand side, and the mass had to be on the right-hand side, to the first power only, and with a prefactor of known variables (V, B, and r). This gives a linear graph of the charge vs. the mass with a slope of $\frac{2V}{B^2r^2}$.

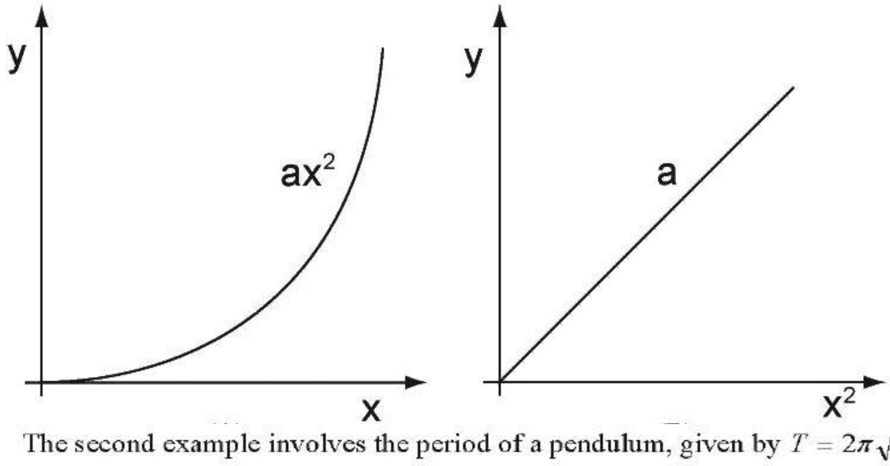


Figure 1: Linearization of an equation. (A) shows a graph of y vs. x of the quadratic function ax^2. (B) shows a graph of y vs. x^2, a linear relation with slope a.

The second example involves the period of a pendulum, given by $T = 2\pi\sqrt{l/g}$. In lab one measures the period and the length of the pendulum. What to do? Squaring the equation gives $T^2 = \frac{4\pi^2}{g}l$. This yields a linear graph of the square of the period vs. the length of the pendulum with a slope of $\frac{4\pi^2}{g}$.

2. Methods of Finding the Best Fit Line: Estimating, Using Excel, and Calculating Analytically

A graph can be used to estimate the best fit line as opposed to calculating the best fit line from the data points. The basic idea is to draw a line through the data with as many data points above it as below it within error. For full instructions refer to the **Making Graphs** section in the lab manual. Estimating the best fit line is a good idea when there are fewer than 5 data points. Picking where the line should go is a skill that improves with practice. The downside of this is that the best fit line and the uncertainties really are just estimates based on "eye-balling" the graph.

A step up from getting a best fit line by hand is having Excel graph and fit a **trendline** (mentioned in the **Graphing with Excel** document). Excel uses the Least Squares Fit method to calculate the best fit line. Using Excel is a good idea for data sets larger than 5 points, for the program takes care of the whole process; but, while an Excel graph will give the equation of the best fit line, it won't give the uncertainty in the slope. Also, it is always dangerous to use a result that is not fully understood.

Calculating the best fit line by using Least Squares Fit method is good for data sets with more than 5 points and gives better results for more data points. The analytic method is good for when error is small. The method will give numbers for slope, intercept, the uncertainties in the slope and intercept, as well as the correlation coefficient (an indication of how good the data fit the line). This document goes through the derivation and the method of doing a least squares fit. At the end, there are instructions for using Excel's LINEST function, which calculates all of the numbers of the fit for you. Why bother reading this whole document when you could just skip to the end? It's important to know where your numbers come from rather than just accepting them from a program.

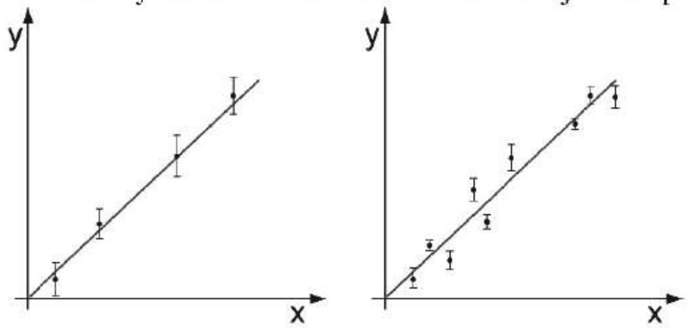


Figure 2: Comparison of (A) data where it's easy to estimate the best fit line and (B) data where it's best to use a Least Squares Fit.

The key to calculating a Least Squares Fit is a well-organized data sheet (tired of hearing that?). Equations will be used to calculate the values for the slope, the intercept, and the uncertainty in those values. Throughout the document, the equations will refer to the independent variable as just x and the dependent variable as just y. When applying Least Squares Fitting to a data set, keep in mind which variable is the *independent* one (the variable one changes in lab) and which one is the *dependent* one (the variable one checks in lab to see what effects changes in the independent variable produce).

3. How to Calculate a Least Squares Fit

Consider the distances from each point to the best fit line, as shown in **Figure 3**, also called the deviations. If a line is a really good fit, those deviations will be as small as possible. The Least Squares Fitting method attempts to minimize the square of the deviations. (Squaring the deviations makes the math far more manageable, where working with the absolute values of the distances would lead to discontinuous derivatives.) The sum of all of the squares of the deviations is called the residual, χ^2 (χ is pronounced "ky" to rhyme with "guy"), given by equation (1), where y_i are the data points and y_{true} is the y-value from the best fit line.

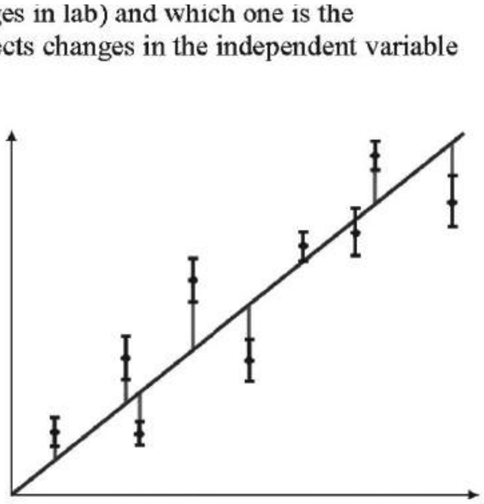


Figure 3: Example of deviations from the best fit line (blue).

$$\chi^2 \equiv \sum_{i=1}^{N} (y_i - y_{true})^2 \qquad (1)$$

If the data follows a linear relation, then y_{true} can be expressed in the general form $y_{true} = mx + b$, making the residual

$$\chi^2 = \sum_{i=1}^{N} (y_i - (mx_i + b))^2 = \sum_{i=1}^{N} (y_i - mx_i - b)^2 \qquad (2)$$

Note that this formula still uses the general form of the equation for the line: b and m have not been specified! To find the line that best fits the data, the residual should be as small as possible. Variables b and m must be chosen so that they minimize χ^2. This is where the method gets its name: making the sum of the squares of the deviations the least it can be. Any statistical book will give the details of this minimization; they are also available on the website under **Useful Documents and Websites**. The end result is that the values of b and m are given by the following equations:

$$b = \frac{\left(\sum_{i=1}^{N} (x_i^2)\right)\left(\sum_{i=1}^{N} (y_i)\right) - \left(\sum_{i=1}^{N} (x_i)\right)\left(\sum_{i=1}^{N} (x_i y_i)\right)}{N\left(\sum_{i=1}^{N} (x_i^2)\right) - \left(\sum_{i=1}^{N} (x_i)\right)^2} \qquad (3)$$

$$m = \frac{N\left(\sum_{i=1}^{N} (x_i y_i)\right) - \left(\sum_{i=1}^{N} (x_i)\right)\left(\sum_{i=1}^{N} (y_i)\right)}{N\left(\sum_{i=1}^{N} (x_i^2)\right) - \left(\sum_{i=1}^{N} (x_i)\right)^2} \qquad (4)$$

4. Uncertainty in the Dependent Variable, Slope, and Intercept

The formula for the standard deviation is $\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} (x_i - \bar{x})^2}$. That formula applies to finding the standard deviation of a number of measurements of the same "true" value, which are randomly distributed around that "true" value (assuming that systematic errors have been reduced). An example of that would be finding the distance to a block sitting on the table: various measurements of the distance might be slightly off the "true" distance to the block, but that "true" distance is the same for all the measurements because the block is sitting still. The "true" value for the distance is approximated by the average of the distance measurements.

Consider the measurement of the distance to a glider as it moves along a track. Time is the independent variable (x) and the distance to the glider is the dependent variable (y). In this example these data are separate measurements in themselves, each one randomly distributed around the "true" value for the distance at a particular instant. If lots of measurements of the distance to the glider were taken *at one instant* by lots of rangers instead of just one, those measurements would be randomly distributed around a "true" value for the distance to the glider *at one instant in time.* The "true" value for the distance to the glider at that instant could be approximated by the average value of the measurements from all those rangers --- but that's just one instant!

The mean of the distance ($\bar{y}$) is a good approximation to the "true" value of the distance *if the glider is not moving*. If the glider was given a push and is moving along a track, measurements of distance versus time would follow a linear pattern. As seen in **Section 3**, the best fit line ($y_{true} = mx + b$) is a good approximation to the "true" value of the distance at any time *if the glider is moving*.

The standard deviation of the dependent variable, σ_y, is calculated a little differently than σ_x. When the dependent variable is changing with the independent variable in a linear fashion, the standard deviation of the dependent variable can be found by the following:

$$\sigma_y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(y_i - y_{true})^2} = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(y_i - mx - b)^2} \qquad (5)$$

In the formula for the standard deviation, σ_x, the factor before the sum is $\frac{1}{N-1}$, but in equation (5) for σ_y it is $\frac{1}{N-2}$. Why the difference? Consider a data set with only two points (i.e. $N = 2$). Two points define a line, so (of course!) the line through those two points is a perfect fit. For a data set consisting of less than three points, the uncertainty σ_y is undefined.

The uncertainty in b, the intercept, and m, the slope, can be found by propagation of the uncertainty in the dependent variable, σ_y, in the formula for b and m. In doing this, it's assumed that most of the error comes from the dependent variable. This propagation may be found in any statistics book and is also available on the website under **Useful Documents and Websites**.

The uncertainties are given by the following equations:

$$\sigma_b = \sqrt{\frac{\sigma_y^2\left(\sum_{i=1}^{N}(x_i^2)\right)}{N\left(\sum_{i=1}^{N}(x_i^2)\right) - \left(\sum_{i=1}^{N}(x_i)\right)^2}} \qquad (6)$$

$$\sigma_m = \sqrt{\frac{N\sigma_y^2}{N\left(\sum_{i=1}^{N}(x_i^2)\right) - \left(\sum_{i=1}^{N}(x_i)\right)^2}} \qquad (7)$$

5. Calculating the r^2 Value

Thanks to the efforts of everyone from Mr. Babbage to Mr. Gates, it's no longer necessary to watch the best years of you life slip past while you hand-process large data sets. In fact, at the push of a button we can even see how well any TWO data sets relate to each other by linear regression. But as you might suspect, the quality of all this cyber-math must be reported with something more objective than the words "good" or "bad". So when your linear regression program spits out a slope and a y-intercept, it probably also gives you a number labeled either "r" or "correlation coefficient". What is that number?

The first thing you need to know about it is that it ONLY tells you how well the two data sets match a STRAIGHT LINE. It does not tell you how well the data matches any other mathematical function.

The second important thing to know is that the correlation coefficient has a range from 0 to 1. If every matched pair of data values is a point that fits exactly on a single line, r will equal 1; a perfect fit. If the points are close to the line but not perfect, r will be less than 1 by a small amount. Finally, if the points are all over the place, and not even close to favoring the chosen line, r will equal 0. This means there is no LINEAR relation between the two data sets.

A third thing to know is that we've been using the coefficient's nickname. Its full name is the "Pearson product-moment correlation coefficient". Knowing this may get you past a tough question on Jeopardy some day.

Now let's go back to that second thing. How do you get a calculation to be 1 when the points match the line and 0 when they miss?

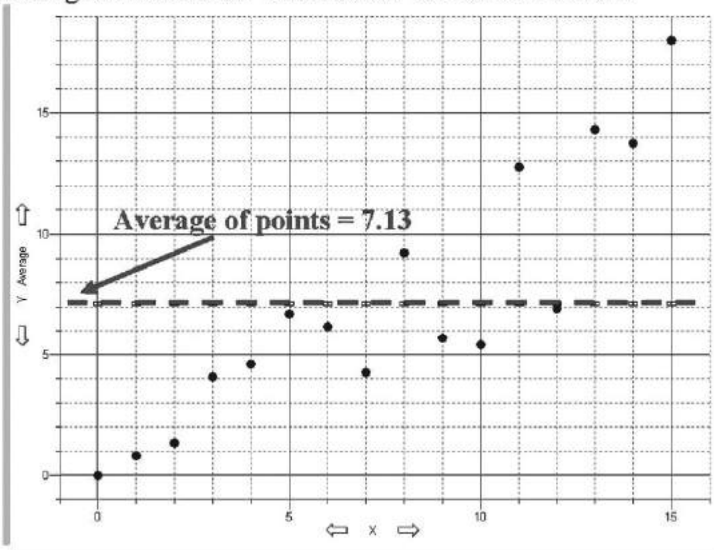


Figure 4: Average of dependent variable values.

Start by calculating the average of all the "y" values $(\bar{y})$. On a graph, this "y" average will be a horizontal line that runs through all the data points around the mid-height point, as shown by the dotted line in **Figure 4**. Why start with the average? The horizontal average line is no better than any other guess as a model for the line that fits the data. It gives a "worst case scenario" to use as a basis of comparison. To see how good (or bad) this fit is, you could take the difference between each actual y value (y_i) and the average $(\bar{y})$, and add them up...

Ooops! That's always going to be zero by definition. The average is the value that's right in the middle of a data set. Half the values are larger and half are smaller, so the sum of positive differences will exactly match the negative differences. To remedy this, after finding each difference, square it (to make all values positive) and then add them all up (since we are only using this number for comparison, we'll just remember to square all other values that will be compared to it):

$$\sum (y_i - \bar{y})^2 \qquad (8)$$

This sum of the square of the differences is a good way of determining how good the fit is. A small value for this sum indicates all the numbers are all pretty close to the average. A large value indicates the numbers are widely scattered. This sum will be used as a comparison for any additional attempts to find a line that fits the data.

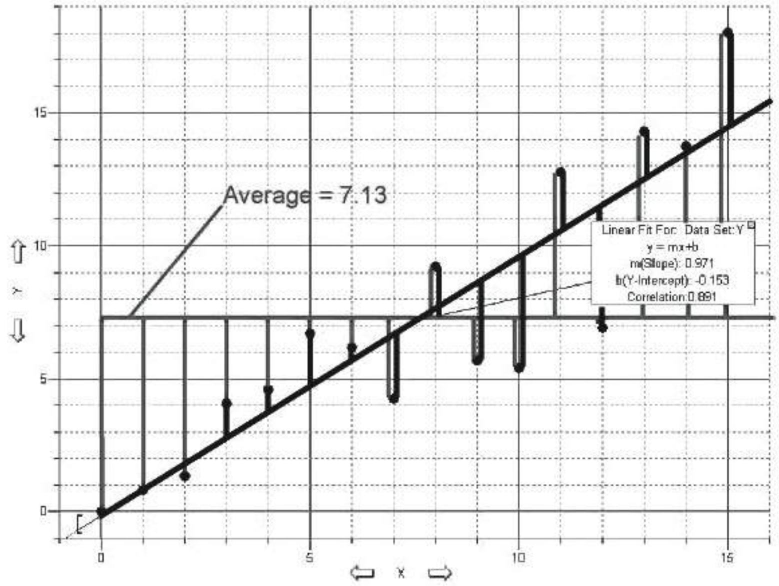


Figure 5: Straight line fit (black) to data with the equation of the fit y = 0.971x - 0.153 and correlation coefficient 0.891. Average of y-values (red) is 7.13. Note the smaller deviations for the best fit line.

You then calculate a "y_{calc}" value for each x value using the equation of the line you wish to try to fit to the data:

$$y_{calc} = mx + b$$

To see how well this line matches the data, a similar sum of squared differences is calculated.

$$\sum (y_i - y_{calc})^2 \qquad (9)$$

It might seem that a simple comparison with equation (8) might be the way to evaluate how good this fit is, but we can do much better. Consider subtracting equation (9) from equation (8) to get equation (10).

$$\sum (y_i - \bar{y})^2 - \sum (y_i - y_{calc})^2 \qquad (10)$$

If the chosen line $y_{calc} = mx + b$ fits no better than the horizontal average line of the numbers, then equation (9) will be just as large as equation (8). If that is the case, the result of (10) will be zero.

$$\sum (y_i - \bar{y})^2 - \sum (y_i - y_{calc})^2 = 0 \qquad (11)$$

If the chosen line $y_{calc} = mx + b$ is an exact fit, then each calculated value (y_{calc}) is exactly equal tc the corresponding data value (y_i), and equation (9) will be zero. If that is the case, the result of equation (10) will be equation (8).

$$\sum (y_i - \bar{y})^2 - 0 = \sum (y_i - \bar{y})^2 \qquad (12)$$

So equation (10) is equal to zero for a line that completely misses the data and equal to equation (8) for an exact match to the data. If we divide equation (10) by equation (8), then it will equal zero for line that completely misses and equal 1 for an exact match. This process of dividing equation (10) b equation (8) is called normalization. When normalized, the correlation coefficient, r^2, is adjusted to have a range from 0 to 1.

$$r^2 = \frac{\sum (y_i - \bar{y})^2 - \sum (y_i - y_{calc})^2}{\sum (y_i - \bar{y})^2} \qquad (13)$$

You may see other formulae for r that involve standard deviations. They are alternate methods to calculate the same number. In the past, calculators had only standard deviation buttons so these othe formulae may have been easier to use. Today it is common for the correlation coefficient to be built into the software, so the difficulties of calculation are no longer an issue. We have developed the equation using averages because it is much easier to see why it takes the form it does.

Figure 6A is almost a perfect fit, for the points fall very close to the best fit line. **Figure 6B** is a good fit, for the points fall fairly close to the best fit line. **Figure 6C** is a bad fit, for the points do not fall close to the best fit line (and if you look closely you'll note that they seem to fall in a

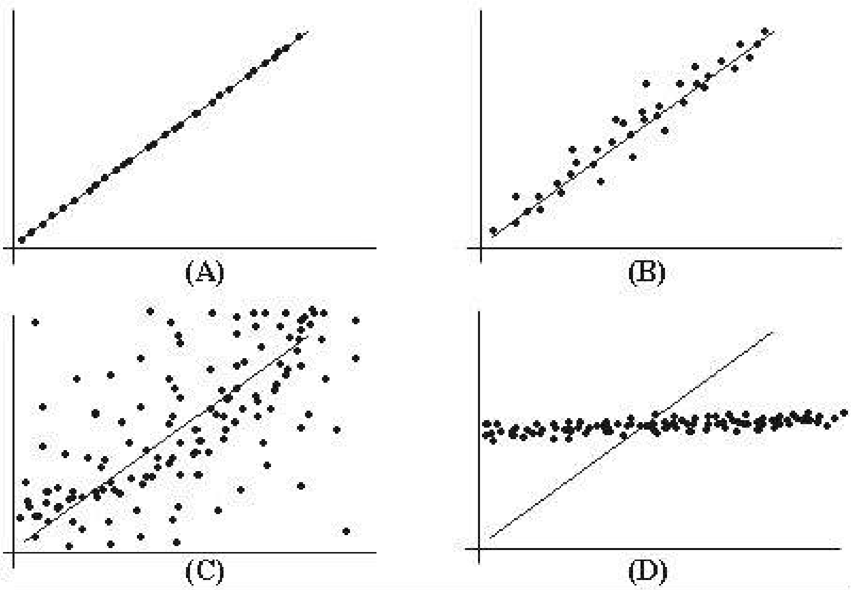


Figure 6: Graphs with different r^2 values. (A) is a perfect fit. (B) is a good fit. (C) is a bad fit, not linearly related. (D) is a bad fit, zero relationship between the variables.

parabola and not a line!). **Figure 6D** is a bad fit for the points do not fall along the chosen best fit line. The data in **Figure 6D** falls on a horizontal line, which indicates that the variables are unrelated, for changes in the independent variable produce no changes in the dependent variable.