

# Indian Institute of Space Science And Technology

**5<sup>TH</sup> SEMESTER, 2022**  
**Campus: Valiamala**

## AVIONICS

### AVD311 - Digital Signal Processing

(Time allowed: THREE hours)

Candidate's Name: \_\_\_\_\_ ID No: \_\_\_\_\_

1. Obtain the inverse system(s) for the system whose transfer function is given by [5 Marks]

$$H(z) = \frac{1 - 0.5z^{-1}}{1 - 0.8z^{-1}}, \quad |z| > 0.8 \quad (1)$$

2. Obtain the Minimum Phase system corresponding to the Fourier magnitude Function [5 Marks]

$$H(e^{j\omega}) = \frac{17}{16} - \frac{1}{2} \cos(\omega) \quad (2)$$

3. Obtain the difference equation corresponding to following system, [2 Marks]

$$H(z) = \frac{z^{-1} - a^*}{1 - az^{-1}} \quad (3)$$

Plot the magnitude response of the system. [3 Marks]

4. Given the three sequences [6 Marks]

- (a)  $X_1(z) = 1 + 1.3z^{-1} + 0.4z^{-2}$
- (b)  $X_2(z) = 0.5 + 0.4z^{-1} + 0.8z^{-2}$
- (c)  $X_3(z) = 0.4 + 1.3z^{-1} + z^{-2}$

- (a) Find the zeros corresponding to three polynomials and plot them.
- (b) Which of them is the minimum phase and maximum phase ?
- (c) Plot the impulse responses corresponding to the  $z$ -functions shown.

5. Prove the validity of these two statements : [5 Marks]

- (a) The convolution of two minimum phase sequence (system) is minimum phase sequence. (Hint : may use a graphical method to prove it)
- (b) The sum of two minimum phase system is not necessarily a minimum phase system (Hint : Use an counter example).

**INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY**  
**THIRUVANANTHAPURAM 695 547**

**Quiz I - September 2022**

B.Tech - V Semester

**MA311 - Probability, Statistics and Numerical Methods**

Date: 12/09/2022

Time: 09.00 am - 10.00 am

Max. Marks: 15

**Answer all questions.**

1. (a) Suppose  $E$  and  $F$  are independent events. Show that  $E$  and  $F^c$  are also independent. [1.5]  
(b) A bin contains 3 different types of disposable flashlights. The probability that a type-1 flashlight will give over 100 hours of use is 0.7, with the corresponding probabilities for type-2 and type-3 flashlights being 0.4 and 0.3, respectively. Suppose that 20 percent of the flashlights in the bin are type-1, 30 percent are type-2, and 50 percent are type-3.
  - i. What is the probability that a randomly chosen flashlight will give more than 100 hours of use? [2]
  - ii. Given that a flashlight lasted over 100 hours, find type of the flashlight which is most probable (i.e., having maximum probability). [1.5]
2. (a) Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with mean 3. Find the probability that 3 or more accidents occur today. [1.5]  
(b) For a biased coin, it is known that probability of getting a head is 0.25. The coin is getting tossed until a head occurs. Find the probability that the experiment will not end before 3rd toss given that it didn't end at the 3rd toss. [1.5]  
(c) Let  $X \sim U(-1, 1)$ . Find the density of  $Y = X^2$ . [2]
3. Let  $X$  and  $Y$  denote the length of life, in years, of two components in an electronic system. If the joint density function of these variables is  $f(x, y) = e^{-(x+y)}$  where  $x > 0, y > 0$ .
  - (a) Are  $X$  and  $Y$  independent? [1.5]
  - (b) Find the conditional density function  $f_{X|Y}$  and therefore find  $P(0 < X < 1 | Y = 2)$ . [2]
  - (c) Find  $E(X + Y)$ . [1.5]

\*\*\*END\*\*\*

**Indian Institute of Space Science and Technology (IIST)**

**B. Tech, Avionics 3<sup>rd</sup> Year**

**Quiz 1 Question Paper**

**RF and Microwave Communication (AV 313), V Semester**

Marks: 15

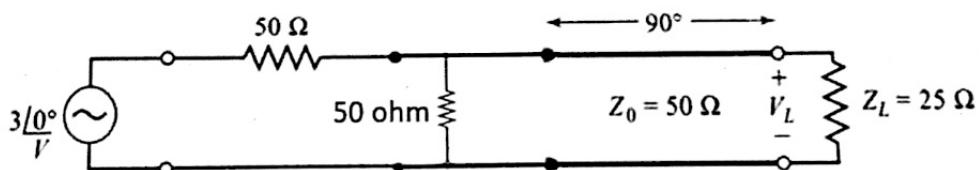
Date of Exam: September 20 '2022

Time: 1 Hr

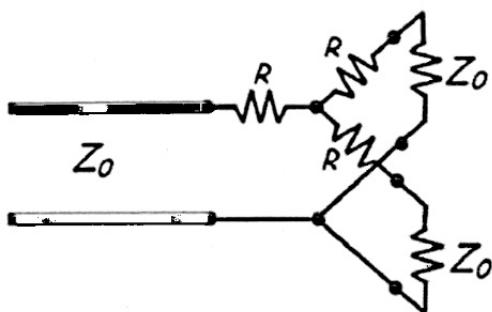
**Note:** Make suitable assumptions if necessary and mention them clearly. Smith chart problem should be done in pencil in case you want to erase it you can. In the smith chart problem write your name and roll no also mention the question number.

If it is not mentioned, the symbols and variables usually have their own meaning. It is a closed book exam; No Formula Sheet is allowed. Answer should be brief and to the point and the same questions answer should be at the same place.

1. (a) From the first principle derive the ABCD parameters of the lossless Transmission line of characteristic impedance of 75 ohm and from this obtain the ABCD matrix for the half wavelength line of the same characteristic impedance. [2]
- (b) Find out the ABCD matrix of the overall circuit given below. Also determine the voltage across the load resistor. [2]

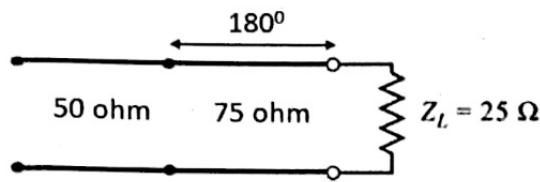


- (c) Under what condition of resistance 'R', the input of the following microwave network will be matched? Determine its scattering matrix parameters and construct the full matrix for that condition and comment on it [3]



P.T.O.

(d) Determine the input reflection coefficient of the following microwave network using the Signal Flow Graph (SFG) technique. [3]



2. (a) Design an 'L' section lumped matching circuit to transform a load  $Z_{LOAD} = 200 - j100 \text{ ohm}$  to a source impedance of  $100 \text{ ohm}$ . The matching network should have a shunt capacitor immediately facing the load impedance. Take characteristic impedance is equal to  $100 \text{ ohms}$ . Use the given ZY smith chart to solve the problem. No need to consider the Complex conjugate of the load. [3]

(b) Why the concept of equivalent voltage is required? [2]

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END

AV301 -Digital Signal Processing  
Quiz 1

Indian Institute of Space Science and Technology,  
Thiruvananthapuram - 695547

September 11, 2022

1. Suppose the  $z$ -transform of  $x[n]$  is

$$X(z) = \frac{z^{10}}{(z - \frac{1}{2})(z - \frac{3}{2})^{10}(z + \frac{3}{2})^2(z + \frac{5}{2})(z + \frac{7}{2})} \quad (1)$$

It is also known that  $x[n]$  is a stable sequence.

- Determine the region of convergence of  $X(z)$ .
- Determine  $x[n]$  at  $n = -8$ .

2. Let  $x[n]$  be a discrete time signal with  $x[n] = 0$  for  $n \leq 0$  and  $z$ -transform  $X(z)$ . Furthermore, given  $x[n]$ , let the discrete time signal  $y[n]$  is given by

$$y[n] = \begin{cases} \frac{1}{n}x[n] & n \geq 1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

- Compute  $Y(z)$  in terms of  $X(z)$ .
- Using the result of the above part, find the  $z$ -transform of

$$w[n] = \frac{1}{n + \delta[n]}u[n - 1] \quad (3)$$

3. A causal signal  $x(n)$  has poles at  $z = z_0$  and at  $z = z_0^*$ , where  $z_0^*$  corresponds to the complex conjugate of  $z_0$ . If  $y(n) = x(n) \cos(\pi n)$ , mark the location of poles and region of convergence of  $y(n)$ .
4. Given  $z_0 = 0.8e^{j\omega_0}$ , mark the following on the  $z$ -plane -  $z^*$ ,  $1/z$  and  $1/z^*$  for some assumed value of  $\omega_0$ .



Indian Institute of Space Science and Technology (IIST)  
Department of Avionics [[www.iist.ac.in](http://www.iist.ac.in)]

5<sup>th</sup> Semester B.Tech ECE Quiz-1  
AV 312: Computer Architecture and Organization, Aug-Dec 2022

Name:

Student ID:

Date: 15/09/2022

Duration: 1 hour

Max Score: 15

Make suitable assumptions, if necessary, and clearly state them. Answers should be marked with question/branch question numbers properly. Clear and legible steps are important for answers.  
COx stands for Course Objective x which can be ignored by students as these notations are used by the instructor for Outcome Based Education with Multi-track Modular Teaching in order to enhance the teaching-learning process at IIST.

1. [CO1] (1 point) Estimate the number of address lines required for an 8KByte memory chip where each location can store an 8bit word.
2. [CO1, CO2, and CO5] (2 points) Assume that you have a system design requirement where 1024byte sized memory capacity where each word is 8 bits. However, the chip available in the market is providing a capacity of 2048byte capacity with each word of 8 bits. How do you use the available chip to obtain the required storage capacity. Explain with diagram if necessary.
3. [CO1 and CO2] (3 points) Consider a 32-bit microprocessor, with a 16-bit external data bus, driven by an 8-MHz input clock. Assume that this microprocessor has a bus cycle whose minimum duration equals five input clock cycles. What is the maximum data transfer rate across the bus that this microprocessor can sustain, in bytes/s? Between the two choices of increasing the external bus width to 32 bits or to increase the clock frequency to 32MHz, which one improves the data transfer rate better? What are the pros and cons of your decision?
4. [CO2] (4 points) Assume the following performance characteristics on a cache read miss: one clock cycle to send an address to main memory and four clock cycles to access a 32-bit word from main memory and transfer it to the processor and cache.
  - a. If the cache line size is one word, what is the miss penalty (i.e., additional time required for a read in the event of a read miss)?
  - b. What is the miss penalty if the cache line size is four words and a multiple, non-burst transfer is executed?
  - c. What is the miss penalty if the cache line size is four words and a transfer is executed, with one clock cycle per word transfer?
5. [CO1 and CO2] (3 points) Assume a processor having a memory cycle time of 300 ns and an instruction processing rate of 1 million instructions per second. On average, each instruction requires one bus memory cycle for instruction fetch and one for the operand it involves.
  - a. Calculate the utilization of the bus by the processor.
  - b. Suppose the processor is equipped with an instruction cache and the associated hit ratio is 0.5. Determine the impact on bus utilization.
6. [CO1] (1 point) Among the two bus choices of asynchronous and synchronous buses which one is better? Explain why?
7. [CO5] (1 point) Briefly describe your Track-4 R&D project idea.

-----All the best-----

Page 1 of 1



5<sup>th</sup> Semester B.Tech ECE Quiz-2  
AV 312: Computer Architecture and Organization, Aug-Dec 2022

Name:

Student ID:

Date: 21/10/2022

Duration: 1 hour

Max Score: 15

Make suitable assumptions, if necessary, and clearly state them. Answers should be marked with question/branch question numbers properly. Clear and legible steps are important for answers. Course instructor will not be available during the examination. COx stands for Course Objective x which can be ignored by students as these notations are used by the instructor for Outcome Based Education with Multi-track Modular Teaching in order to enhance the teaching-learning process at IIST.

1. [CO1] (1 point) Describe how design the size of a cache in a multi-level cache system is carried out.
2. [CO1, CO2, and CO5] (5 points) Assume that you have been approached by a Billionaire named Malone Esk for designing his new smart building where all the appliances are computer controlled. He requests you to design an I/O interface for the 16-floor building with five switches to control the lighting system. Four of these switches are used to select the floor and the fifth switch is used to determine ON/OFF decision of the lights on the selected floor. Due to the sophistication of the switch, he requested to use handshaked I/O. Design the I/O system and write the program for operating it. [Hint: You can consider using 8255 programmable peripheral interfaces for the I/O design].
3. [CO1, CO2, and CO5] (5 points) Consider a computer interfaced with a Dynamic RAM chip that uses row addressing (RA) and column addressing (CA) mechanism. Each memory unit consists of four bits, utilizing four data lines of the CPU. The CPU utilized programmed I/O in combination with memory-mapped I/O to interface the RAM. The capacity of the chip is 16Mbits. The clock speed of the system is 1GHz. Buffers are available to store the address/data as needed for the system. CPU has a data bus width of 32 bits, therefore, CPU has the same word length. Memory chip utilizes only four data pins from the databus. Each addressing requires 1 clock cycle. Databus supports bulk data transfer to meet the word length of the CPU. What is the time taken for reading a word from memory? What is the maximum data transfer rate achievable?
4. [CO1 and CO2] (2 points) What is the difference between Dynamic RAM and Static RAM? Explain the operation of a Static RAM memory cell in detail.
5. [CO5] (2 points) Explain your R&D project in detail including the roles played by each member of the group.

-----All the best-----

Indian Institute of Space Science and Technology  
AV314 - Communication Systems I  
Department of Avionics

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Quiz 2 for Semester V on 26/10/2022

**Note to the student**

1. There are **4 questions** in this question paper on **2 pages**, for a total of **15 marks**.
2. Answer **all** questions. Complete credit would be given only for answers including assumptions and intermediate steps.

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**Question 1 (3 marks):** Consider a radio receiver that needs to receive a FM channel that has a one-sided bandwidth of 1MHz at 1800.5 MHz. In order to hear this channel satisfactorily, any nearby channels need to be attenuated to at least 60 dB. We want to design a superheterodyne receiver with a intermediate frequency (IF) of 300 MHz. The intermediate frequency is generated by a local oscillator that is tunable from 1900 to 2250 GHz, with any frequency divider circuits if needed (frequency division can be done only by an integer). Design a superheterodyne receiver (use a block diagram in order to present your design) to receive the channel at 1800.5 MHz. Specify the characteristics of the RF and IF filters and show how the LO signal required for mixing is generated. For the superheterodyne receiver that you have designed, what is the image frequency corresponding to the channel above?

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**Question 2 (5 marks):** Suppose  $m(t)$  is a general real valued modulating baseband signal, with energy contained in the frequency range  $[-B_m, B_m]$ . Assume that  $x(t)$  is obtained from  $m(t)$  by single side band modulation, using upper side band, using a carrier  $\cos(2\pi f_c t)$  with carrier frequency  $f_c$ , where  $f_c \gg B_m$  (i.e., the SSB signal is generated by filtering the DSB signal obtained with this carrier). The signal  $x(t)$  is transmitted over an ideal channel such that the received signal  $y(t)$  is  $x(t)$  itself. The signal  $y(t)$  is applied to an ideal FM demodulator which produces the instantaneous frequency of  $y(t)$  as output.

1. Write down an expression for  $f(t)$  in terms of  $m(t)$ .
2. Now, if suppose  $m(t)$  were a pure sinusoid then what would  $f(t)$  be?

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**Question 3 (3 marks):** Suppose  $m(t)$  is  $A_m \sin(2\pi f_m t)$ . The signal  $m(t)$  is frequency modulated using a carrier frequency of  $f_c$  and an FM modulation sensitivity of  $k_f$ . Assuming that the modulation results in a wideband FM signal, derive the exact spectrum of the FM signal.

---

**Question 4 (2 marks each):**

1. Consider a discrete time random process which is an infinite collection of random variables  $X_1, X_2, \dots, X_n, \dots$ . Suppose we define  $X_n = n + Z$  where  $Z$  is a Gaussian random variable with mean 0 and variance 1. Prove or disprove the statement “The random process  $X_1, X_2, \dots, X_n, \dots$  is IID”.
  2. Assume that there are two coins given to Alice. Alice proceeds to toss the coins, one with her left and the other with her right hand. The results of the coin tosses are as follows (the result is denoted as  $(l, r)$  where  $l$  is the result of the left coin toss and  $r$  is the result of the right coin toss. H denotes heads and T denotes tails):  $(H, H), (H, T), (T, H), (T, T), (T, \bar{H}), (\bar{T}, H), (\bar{T}, \bar{H}), (H, \bar{H}), (\bar{H}, H), (\bar{H}, \bar{H})$ . Are the two tosses done independently of one another? Please show all steps.
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Best of luck!

**INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY**  
**5<sup>th</sup> Sem. B.Tech Electronics and communication Engineering**

**Environmental Science and Engineering (CH311)**

**Quiz H, October 27, 2022**

**Time: 1 hour**

**Maximum marks: 20**

**(Select most suitable answer from the bracket, each question carries 1/2 marks)**

1. The most harmful particulate matter is of the size -----  
[(a)  $>10 \mu\text{m}$  (b) 10 to 5  $\mu\text{m}$  (c) 10 to 100  $\mu\text{m}$  (d)  $<2.5 \mu\text{m}$ ]
2. The concentration of current CO<sub>2</sub> concentration in atmosphere is close to -----  
[(a) 500 ppm, (b) 400 ppm, (c) 300 ppm, (d) 450 ppm]
3. ----- is a micronutrient in the soil  
[(a) Li (b) P (c) Ar (d) B]
4. Green chemistry aims to  
[(a) Reduce waste (b) Reduce cost (c) Reduce reaction time (d) Avoid catalyst]
5. ----- is a greenhouse gas  
[(a) N<sub>2</sub> (b) H<sub>2</sub>O (c) He (d) H<sub>2</sub>]
6. Improper disposal of CFL lamp leads to  
[(a) Pb pollution (b) Cr pollution (c) As pollution (d) Hg pollution]
7. Which of the plastic given below is not recyclable  
[(a) Bakelite (b) PVC (c) Polyethylene (d) Polypropylene]
8. The most suitable alternative for chlorofluorocarbon as a blowing agent for production of polymer foams is  
[(a) Supercritical water (b) Supercritical ethanol (c) Supercritical CO<sub>2</sub> (d) N<sub>2</sub>O<sub>4</sub>]

**(Answer all questions briefly. Each question carries 2 marks)**

7. Explain the principle and application of electrostatic precipitator
8. What you mean by thermal NO<sub>x</sub> and fuel NO<sub>x</sub> ?
9. Explain how the organic volatiles enhances the ozone formation in the atmosphere
10. Suggest an alternative coal combustion technology to minimize SO<sub>x</sub>
11. What is greenhouse effect?

**(Write short note on the following. Each question carries 3 marks)**

12. Green Chemistry

13. Energy from waste

**Indian Institute of Space Science and Technology (IIST)**  
**B. Tech, Avionics 3<sup>rd</sup> Year**  
**Quiz 2 Question Paper**  
**RF and Microwave Communication (AV 313), V Semester**

Marks: 15

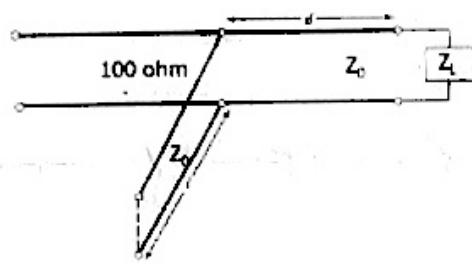
Date of Exam: October 28 ' 2022

Time: 1 Hr

**Note:** Make suitable assumptions if necessary and mention them clearly. Smith chart problem should be done in pencil in case you want to erase it you can. In the smith chart problem write your name and roll no also mention the question number.

If it is not mentioned, the symbols and variables usually have their own meaning. It is a closed book exam; No Formula Sheet is allowed. Answer should be brief and to the point and the same questions answer should be at the same place.

1. (a) A load impedance of  $90-j25$  ( $Z_L$ ) is to be matched to 100 ohms using a shorted stub of length  $l$  as shown by the dotted line in the figure below. Determine the length  $d$ , and  $l$  using the provided Z smith chart. Take  $Z_0 = 50$  ohm. [2]



Load  
to a

- (b) Design a three-section binomial multi-section transformer to match a 100-ohm line to a 50-ohm line. Calculate the best return loss that can be obtained between 2 and 3 GHz (considering 2.5 GHz as resonating frequency) to attain a fractional bandwidth (FBW) of 70 percent. Also draw the signal flow graph of the above multi-section transformer considering the theory of small reflection. [2] + [1] = [3]

- (c) What is Bode Fano criteria. Consider a fixed type of load to define the same. [1]

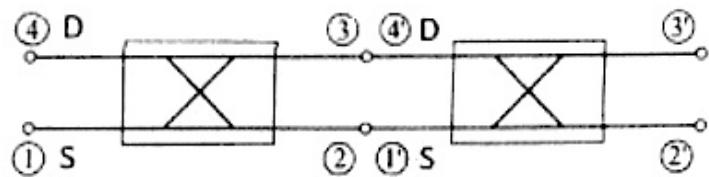
2. (a) Obtain the scattering parameters of the 3 dB branch line coupler (Quadrature Hybrid) for equal power division using Even and Odd Mode analysis. [3]  
 You may use the following conversion formulae if required.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{A+B/Z-CZ-D}{A+B/Z+CZ+D} & \frac{2(AD-BC)}{A+B/Z+CZ+D} \\ \frac{2}{A+B/Z+CZ+D} & \frac{-A+B/Z-CZ+D}{A+B/Z+CZ+D} \end{bmatrix}$$

- (b) Find the coupling factor (C) and the directivity (D), return loss (RL) and isolation (I) of the directional coupler whose scattering matrix is given below [4]

$$\{S\} = \begin{bmatrix} 0.1\angle 40^\circ & 0.944\angle 90^\circ & 0.178\angle 180^\circ & 0.0056\angle 90^\circ \\ 0.944\angle 90^\circ & 0.1\angle 40^\circ & 0.0056\angle 90^\circ & 0.178\angle 180^\circ \\ 0.178\angle 180^\circ & 0.0056\angle 90^\circ & 0.1\angle 40^\circ & 0.944\angle 90^\circ \\ 0.0056\angle 90^\circ & 0.178\angle 180^\circ & 0.944\angle 90^\circ & 0.1\angle 40^\circ \end{bmatrix}$$

- (c) Two identical rat race 3 dB couplers are connected as shown below. Find the resulting phase and amplitudes at port 2' and 3' (at final output stage of the second coupler) relative to port 1 (of the first coupler). (Assume the incoming signal at port 1 is of magnitude 1 and phase of 0 degree and port 4 at the input is matched. S stands for Sum Port and D Stands for Difference Port in the figure) [2]



END



भारतीय अंतरिक्ष विज्ञान एवं प्रौद्योगिकी संस्थान, तिरुवनंतपुरम्,  
केरल

बीटेक तीसरा वर्ष /B. Tech - 3<sup>rd</sup> year

क्विज-2 /Quiz-2

MA 311 - प्रायिकता, सांख्यिकी एवं आंकिक विधियों

MA 311 - Probability, Statistics and Numerical Methods

निर्धारित समय /Time: 1 घण्टे /hr      दिनांक /Date: 19-10-2022      अधिकतम अंक /Max mark: 15

1. Derive the estimate

$$n \geq \frac{\log(b_0 - a_0) - \log(\epsilon)}{\log(2)} - 1$$

involving  $b_0 - a_0$  and  $\epsilon$  where  $[a_0, b_0]$  is the initial interval and  $n$  is the number of steps that must be taken in the bisection method to guarantee that  $|\alpha - x_n| \leq \epsilon$  where  $\alpha$  is the root of the equation  $f(x) = 0$ . (5 marks)

2. If  $A > 0$ , then  $\alpha = \sqrt{A}$  is a root of either equation

$$x^2 - A = 0, \quad \frac{A}{x^2} - 1 = 0$$

Explain why Newton's method applied to first equation converges for arbitrary starting value  $x_0 > 0$ , whereas the same method applied to the second equation produces positive iterates  $x_n$  converging to  $\alpha$  only if  $x_0$  is in some interval  $0 < x_0 < b$ . Determine  $b$ . (5 marks)

3. Consider the following approximations

$$(a) x'(t_k) \approx \frac{x(t_{k+1}) - x(t_{k-1})}{2h}$$

$$(b) x'(t_{k+1}) \approx \frac{3x(t_{k+1}) - 4x(t_k) + x(t_{k-1})}{2h}$$

Derive the numerical schemes for the above approximations for the problem  $x'(t) = f(t, x(t))$ . What is the truncation error? Is it a consistent method, if so then what is the order? (5 marks)

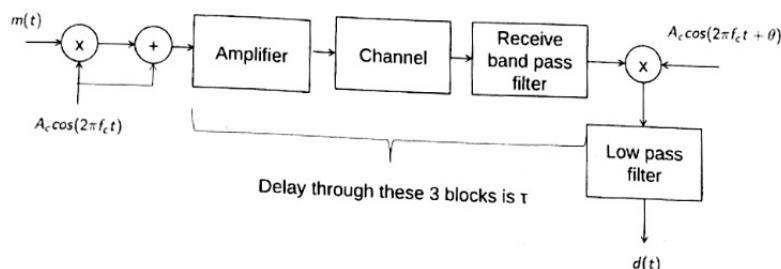
\*\*\*\*\*End of questions\*\*\*\*\*

**Indian Institute of Space Science and Technology**  
**AV314 - Communication Systems I**  
**Department of Avionics**

**Quiz 1 for Semester V on 16/09/2022**

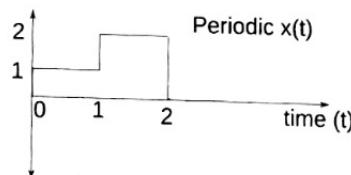
**Note to the student:** There are **6** questions in this question paper on **2** pages, for a total of **15 marks**. Answer **all** questions.

**Question 1 (2 marks):** Consider the block diagram of an AM scheme shown below. Derive the time domain expression for  $d(t)$ . The signal  $m(t)$  is baseband. State all assumptions that you are making in your derivations. You can assume that the amplifier, the channel, and the receive bandpass filter (with amplifier) are all ideal and have enough bandwidth to accommodate their respective input signals. Please note that no credit will be given without a statement of your assumptions.



**Question 2 (5 marks):**

1. Consider a periodic signal  $x(t)$  which has a period of 2 seconds. The signal is defined over one period as shown below. Find out the Fourier series representation of  $x(t)$ . (2 marks)



2. Suppose  $x(t)$  is the following aperiodic signal. Find out the Fourier transform of  $x(t)$ . (3 marks, full credit would be given for the answers that use standard functions such as step, δ, sin, cos, real-exponential, sinc etc.)

$$x(t) = \begin{cases} t, & \text{for } t \in [0, 1], \\ 2-t, & \text{for } t \in [1, 2], \\ 0, & \text{otherwise.} \end{cases}$$

**Question 3 (2 marks):** Using a signal flow diagram/block diagram, explain how you would transmit and receive a baseband voice signal, with a two-sided bandwidth of 10kHz over a passband channel of two-sided bandwidth 10 kHz, centered at 1 GHz. What modulation and demodulation scheme would you use? Clearly indicate these in your block diagram. Also draw the Fourier magnitude spectrum of the signal that you obtain at the output of the modulator. (Hint: Would DSB or SSB be appropriate in this case?)

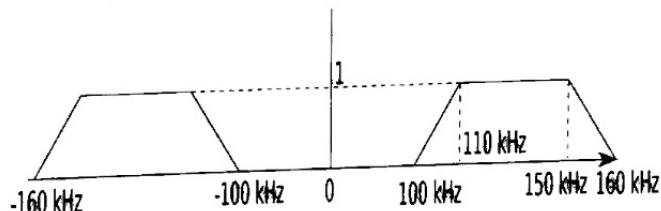
**Question 4 (2 marks):** Let  $m(t) = 2\cos(2\pi 500t) + 5\cos(2\pi 1000t)$  be a baseband audio signal. This signal is transmitted over a baseband channel with spectrum  $H(f)$  as follows

$$H(f) = \begin{cases} 1, & \text{for } |f| \in [0, 800], \\ 0.3, & \text{for } |f| \in (800, 1200], \\ 0.5, & \text{for } |f| \in (1200, 1500], \\ 0, & \text{otherwise.} \end{cases}$$

Describe what transmitter and receiver functions you would use (i.e. the  $t(\cdot)$  and  $r(\cdot)$  functions) so that you would be able to recover  $m(t)$  at the destination. (All frequencies are in Hertz.) Comment on how you will realize the  $t(\cdot)$  and  $r(\cdot)$  functions.

**Question 5 (2 marks):** Suppose  $m_1(t)$  and  $m_2(t)$  are two baseband signals, with Fourier spectra  $M_1(f)$  and  $M_2(f)$  and with two sided bandwidths of 10kHz and 20kHz respectively.

- Using a block diagram describe a system that can simultaneously transmit these two signals over a channel modelled as an LTI filter with the following  $|H(f)|$ . You can assume that the phase response is linear in the passband. Draw the signal spectra of the signals which are obtained at each point in your block diagram.



- Using a block diagram describe a system that can receive the above signal from the output of the channel  $H(f)$  and extract out *good replicas* of  $m_1(t)$  and  $m_2(t)$  from the received signal.

**Question 6 (2 marks)** Consider a pilot based DSBSC system which is used to transmit a baseband signal  $m(t)$  with two sided bandwidth  $B_m$  modulated by a carrier signal  $A\cos(2\pi f_c t)$ . The pilot signal is  $A_p\cos(8\pi f_c t)$ . Assume that the channel is ideal, centered at  $f_c$  and has a bandwidth large enough to accommodate the DSBSC signal as well as the pilot. Describe a receiver for this system that can demodulate  $m(t)$  using the carrier. (Hint: Recall that a pilot signal is an extra signal sent along with the information carrying signal to aid demodulation.)



Indian Institute of Space Science and Technology (IIST)

Department of Avionics [[www.iist.ac.in](http://www.iist.ac.in)]

5<sup>th</sup> Semester B.Tech ECE Finals

AV 312: Computer Architecture and Organization, Aug-Dec 2022

Name:

Student ID:

Date: 02/12/2022

Duration: 3 hours

Max Score: 50

Answers should be marked with question/branch question numbers properly. Clear and legible steps are important for answers. COx stands for Course Objective x which can be ignored by students as these notations are used by the instructor for Outcome Based Education with Multi-track Modular Teaching in order to enhance the teaching-learning process at IIST.

Make suitable assumptions, if necessary, and clearly state them.

1. [CO1, CO2, and CO5] (20 points) The billionaire Malone Esk strikes again. He signed a new deal with you for designing his new Mars Colony housing module. Despite your initial busy schedule, finally, you gave in to his friendly push to take this responsibility. His only reason for signing you for this design mission was your passion and deep interest for computer architecture and organization.

His Mars colony consists of small housing modules each is an enclosure with access control and ambience control. You are required to do a computer-controlled ambience maintenance for his housing module. The requirements are listed as follows.

- a. Temperature to be maintained between 20-25 degree Celsius. If temperature exceeds, AC needs to be turned ON to reduce the temperature. If temperature is lower than the acceptable range, then a heater is to be turned ON.
- b. Humidity to be maintained at a healthy level of 30-50%. If humidity is lower than the acceptable range, then a small water spray device needs to be turned ON to improve the humidity. If humidity is higher than the range, then a device called Esk-Humidity-Remover needs to be turned on to bring the humidity from ambience. Esk-Humidity-Remover is a magical device invented by Esk himself to create water out of thin air. A side-benefit of his device is to reduce humidity in the ambience.
- c. The lights are to be controlled using switches. Three LED lights are to be controlled using three switches. Another two switches are used for intensity levels. There are four intensity levels possible for the LED lighting system.
- d. Access to the module is controlled through a switch system where switch pad that generates a binary single digit code is used for access. Code number 9 is used for the default access. The door is actuated by a liver mechanism which is controlled by an electrical relay.

Detailed design diagram, interface circuitry, interface chips, and program to control and manage the Mars colony module are essential for him.

2. [CO5] (3 points) Briefly explain the major results obtained in your Track-4 R&D project.

3. [CO1, CO2] (2 points) Consider a processor similar to 8085 which needs to interface with a memory of size 64KB. Due to the lower price of the 16K memory chips which has four-bit words in each cell, you preferred to use that chip. Design the interface circuitry necessary to achieve and operate the 64KB memory with the processor.
4. [CO1] (5 points) Compare and Contrast RAID0-RAID6 configurations. Explain with details.
5. [CO1, CO2] (5 points) A computer consists of a processor and an I/O device D connected to main memory M via a shared bus with a data bus width of one word. The processor can execute a maximum of  $10^6$  instructions per second. An average instruction requires five machine cycles, three of which use the memory bus. A memory read or write operation uses one machine cycle. Suppose that the processor is continuously executing "background" programs that require 95% of its instruction execution rate but not any I/O instructions. Assume that one processor cycle equals one bus cycle. Now suppose the I/O device is to be used to transfer very large blocks of data between M and D.
- If programmed I/O is used and each one-word I/O transfer requires the processor to execute two instructions, estimate the maximum I/O data-transfer rate, in words per second, possible through D.
  - Estimate the same rate if DMA is used.
6. [CO1, CO2] (5 points) Consider a magnetic disk drive with 8 surfaces, 512 tracks per surface, and 64 sectors per track. Sector size is 1 KB. The average seek time is 8 ms, the track-to-track access time is 1.5 ms, and the drive rotates at 3600 rpm. Successive tracks in a cylinder can be read without head movement.
- What is the disk capacity?
  - What is the average access time? Assume this file is stored in successive sectors and tracks of successive cylinders, starting at sector 0, track 0, of cylinder i.
  - Estimate the time required to transfer a 5-MB file.
  - What is the burst transfer rate?
7. [CO1, CO2, CO3] (5 points) Assume an 8088 is executing a program in which the probability of a program jump is 0.1. For simplicity, assume that all instructions are 2 bytes long.
- What fraction of instruction fetch bus cycles is wasted?
  - Repeat if the instruction queue is 8 bytes long.
8. [CO1, CO2, CO3] (5 points) Explain the following:
- Speedup factor.
  - Methods of increasing the capacity in optical storage disks.
  - Serpentine reading.
  - NAND Flash Vs NOR Flash memory.

All the best

Indian Institute of Space Science and Technology  
AV314 - Communication Systems I  
Department of Avionics

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**Final Exam [50 marks] — 5th Semester, B.Tech (ECE) — 05/12/2022 [3 hrs]**

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**Note to the student**

1. There are **10 questions** in this question paper on **4 pages**, for a total of **50 marks**.
2. Answer all questions.
3. Credit will be given only for answers which are written legibly, clearly, and with complete steps. Clearly state any assumptions that you are making.
4. **Your answer to a single question should be on contiguous pages of your answer sheet and should not be interleaved with other answers.**
5. The notation  $\mathbb{I}_A(x)$  is used for the indicator function (or boxcar function).

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**Question 1 (5 marks):** Let  $m(t)$  be the signal  $\cos(2\pi f_m t)$ . Assume that  $x_1(t)$  is obtained by the SSB-modulation of  $m(t)$  using the carrier  $\cos(2\pi f_{ct})$ . Also assume that  $f_m$  is very small compared to  $f_c$ . In order to try and de-modulate the SSB signal using envelope detection, we transmit the signal  $x(t) = x_1(t) + a \cos(2\pi f_{ct} t)$ . Suppose we assume that  $x(t)$  is received as it is without any distortion or noise addition. Obtain the output of an ideal envelope detector (which acts like an ideal rectifier followed by an ideal filter) when  $x(t)$  is applied to it. Is it possible to recover  $m(t)$  back from  $x(t)$  using this ideal envelope detector? Are there any conditions that  $a$  needs to satisfy for this to happen? (Hint: can ideal envelope detection be thought of as implicit coherent demodulation?)

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**Question 2 (5 marks):**

1. For a general baseband signal  $m(t)$  write down the transmit SSB signal for upper sideband modulation (assuming a carrier frequency of  $f_c$ )
2. Assume that this SSB signal is received without any distortion or delay at a SSB receiver
3. Assume that this SSB receiver uses a locally generated carrier signal for demodulation.
4. Assume that the local carrier signal generated while demodulating the SSB signal has a frequency error (offset) of  $\Delta$  (assumed to be non-zero) with respect to the carrier frequency  $f_c$  used to generate the SSB signal. Write down the time domain representation of this local carrier signal clearly showing the frequency error.

- 
5. Write down what the demodulated signal is when the SSB signal consists of the upper sideband only. Comment on the effect of the frequency error on the demodulated signal.
- 

**Question 3 (5 marks):**

1. Draw the complete block diagram of a coherent DSBSC demodulator with carrier synchronization, clearly stating the functionality of each of the blocks.
  2. In your block diagram show how noise is modelled as adding on the received signal.
  3. Derive the signal to noise ratio at the output of a DSBSC receiver assuming that the noise at the receiver input is zero mean additive white Gaussian noise with a power spectral density of  $N_0/2$ . State all necessary assumptions.
- 

**Question 4 (4 marks):** Suppose  $m(t)$  is a baseband signal of two sided bandwidth  $B_M$ . We are given a channel which takes a passband time domain signal  $i(t)$  as input and produces a signal  $o(t)$  as output. The passband signal  $i(t)$  needs to be centered at  $f_c$  and can have a one-sided bandwidth of  $100B_M$ . But the channel is non-linear and the input output relationship is given as  $o(t) = i(t) + 0.75(i(t))^2 + 0.25(i(t))^3$ . Suppose you are given the option to transmit  $m(t)$  through the channel using amplitude modulation or frequency modulation. Justify mathematically, which one of the above schemes you would prefer. (Hint: think about the distortions that are produced by the non-linear channel on an amplitude modulated or frequency modulated  $i(t)$ .)

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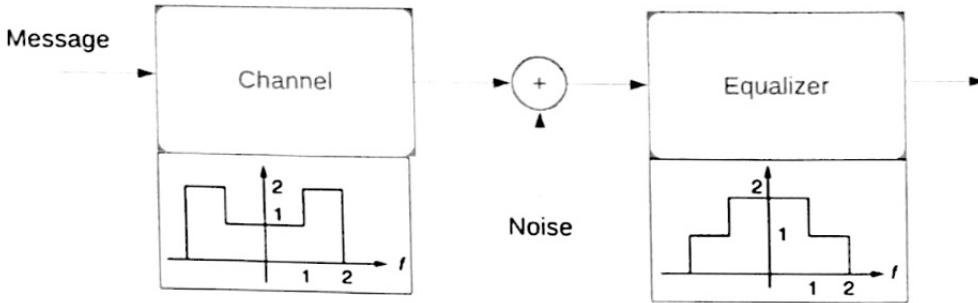
**Question 5 (5 marks):** Consider a FM receiver system which uses a limiter-discriminator for demodulation of a baseband signal  $m(t)$ . For such systems, explain why pre-emphasis filtering needs to be done on  $m(t)$  before frequency modulation and de-emphasis filtering after demodulation. Please note that your explanation should include a discussion of the noise signal in the FM receiver.

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**Question 6 (8 marks):** Consider a baseband communication system shown in the figure below.

The message signal itself is modelled as a random process with a power spectral density  $2(1 - |f|/2)\mathbb{I}_{[-2,2]}(f)$ . The message signal is passed through a channel with magnitude response as shown (below the channel block). Receiver noise is modelled as a bandlimited white noise with power spectral density  $0.25\mathbb{I}_{[-3,3]}(f)$ . The equalizer is used to remove the distortion caused due to the channel. The magnitude response of the equalizer is also shown in the figure (below the equalizer block). Find

1. The signal power at the channel input
2. The signal power at the channel output



3. The SNR at the equalizer input
4. The SNR at the equalizer output

**Question 7 (5 marks):** A continuous time random process  $X(t), t \in (-\infty, \infty)$  is defined as  $X(t) = A \cos(2\pi f_c t + \Theta)$  where  $A, f_c$  are constants and  $\Theta$  is a discrete random variable, uniformly distributed in  $\{0, \pi/4, \pi/2, 3\pi/4, \pi\}$ . Is  $X(t)$  wide sense stationary? Justify mathematically.

**Question 8 (5 marks):** A stationary Gaussian process ( $X(t)$ ) has zero mean and power spectral density  $S_{xx}(f) = \mathbb{I}_{[-B, B]}$  where  $\mathbb{I}$  is the boxcar function. Determine the joint probability density function of the random variables  $X(t_1)$  and  $X(t_2)$  where  $t_1 < t_2$  are two arbitrary time instants.

**Question 9 (3 marks):** Suppose the autocorrelation function of a wide sense stationary random process is given as  $R_x(\tau) = (1 - |\tau|)\mathbb{I}_{-1,1}(\tau)$ . Prove whether or not this is a valid autocorrelation function.

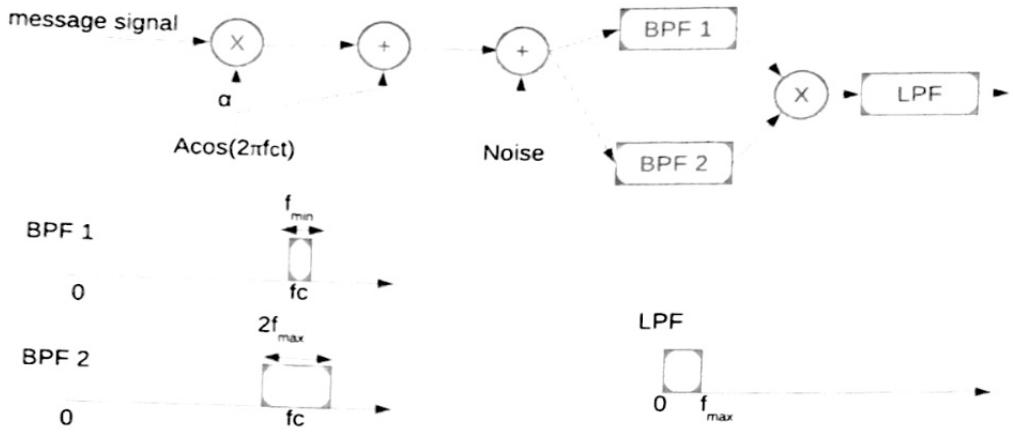
**Question 10 (5 marks):** Consider a plain-AM carrier-assisted modulator and demodulator scheme as shown in the figure below.

It is assumed that the baseband message signal has an energy spectrum which is bounded in frequency and lies in the range  $[-f_{max}, f_{max}]$ . Furthermore, it is also assumed that the baseband message signal has zero energy in the range  $[-f_{min}, f_{min}]$ , where  $0 < f_{min} < f_{max}$ . The carrier frequency is assumed to be  $f_c$ , carrier amplitude is  $A$ .

(a) Assuming a modulation sensitivity of  $\alpha$ , sketch the energy spectrum of the transmitted AM signal.

At the receiver, the signal is assumed to be corrupted with zero mean additive white Gaussian noise with a power spectral density of  $N_0/2$ .

At the receiver, since the carrier signal can be bandpass filtered, we recover the carrier using a sharp bandpass filter of magnitude 1, centered at  $f_c$  and with one-sided bandwidth  $f_{min}$ . The plain AM signal is bandpass filtered using a bandpass filter of magnitude 1, centered at



$f_c$ , and with a one-sided bandwidth of  $2f_{max}$ . (These filters are denoted as BPF 1 and BPF 2 in the figure. Their one sided magnitude responses are also shown).

The recovered carrier signal (corrupted by noise) is used to coherently multiply (mix with) the plain AM signal as shown. Then, the mixed signal is low pass filtered using a LPF of one-sided bandwidth  $f_{max}$  and magnitude 1 in the passband (the magnitude response is shown in the figure).

(b) Write down the desired-signal component and the noise component in the signal after the low pass filter. Your answer should be expressed using baseband signals. Here you can assume that  $A$  is much larger compared to the noise signal or message signal amplitudes.

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Best of luck!

**Indian Institute of Space Science and Technology (IIST)**  
**B. Tech, Avionics 3<sup>rd</sup> Year**  
**End-Sem Question Paper**  
**RF and Microwave Communication (AV 313), V Semester**

Full Marks: 50

Date of Exam: December 09' 2022

Time: 3 Hr

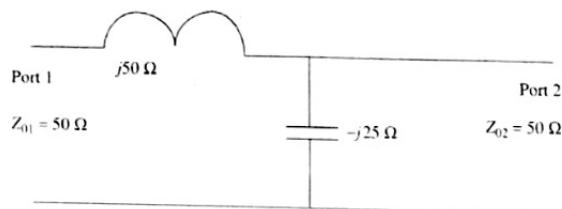
**Note:** Make suitable assumptions if necessary and mention them clearly. Smith chart problem should be done in pencil in case you want to erase it you can. In the smith chart problem write your name and roll no also mention the question number.

If it is not mentioned, the symbols and variables usually have their own meaning. It is a closed book Exam; No Formula Sheet is allowed. Answer should be brief and to the point. Answer of a particular question no must be written at the same place.

1. (a) A two-port network is known to have the following scattering matrix

$$[S] = \begin{bmatrix} 0.15\angle 0^\circ & 0.85\angle -45^\circ \\ 0.85\angle 45^\circ & 0.2\angle 0^\circ \end{bmatrix}$$

- i) Determine whether the network is lossless or reciprocal?
  - ii) If port 2 is terminated with open load, what is the return loss seen at port 1 in dB?  
[2] + [2] = [4]
- (b) Find the scattering parameters of the two-port network shown in Figure below. [3]



- (c) Prove the following relation between Z and S parameter starting from the basic definition. [3]

$$[Z] = ([U] + [S]) ([U] - [S])^{-1}$$

2. (a) A load  $Z_L = (10 + j10)$  ohm is to be matched to a source impedance 50-ohm using a 'L' type lumped matching network and the frequency of operation is 500 MHz You have to match the load impedance to the source impedance in such a way so that a capacitor faces next to the source impedance of 50 ohm. Solve the problem in the given ZY smith chart. [4]

- (b) Design a single-section quarter-wave matching transformer to match a 350-ohm load to a 100-ohm line. [2] + [2] = [4]

- (i) Determine the percent bandwidth of this transformer, for  $\text{SWR} \leq 2$ ? If the design frequency is 4 GHz.
- (ii) Sketch the layout of a microstrip circuit using the following given formulae, including dimensions, to implement this matching transformer. Assume the substrate is 0.159 cm thick (d), with a relative permittivity of 2.2, with copper conductors 0.035 mm thick. [Try with  $W/d < 2$  first, W is the microstrip line width,  $Z_0$  is characteristic impedance of the line]

$$\begin{aligned}\varepsilon_r &= \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left(1 + \frac{12d}{W}\right)^{-\frac{1}{2}} \\ A &= \frac{Z_0}{60} \sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r + 1}{\varepsilon_r + 2} \left(0.23 + \frac{0.11}{\varepsilon_r}\right) \\ B &= \frac{377\pi}{2Z_0\sqrt{\varepsilon_r}} \\ \frac{W}{d} &= \begin{cases} \frac{8e^A}{e^{2A} - 2} & \text{for } W/d < 2 \\ \frac{2}{\pi} \left[ B - 1 - \ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r} \left\{ \ln(B - 1) + 0.39 - \frac{0.61}{\varepsilon_r} \right\} \right] & W/d \geq 2 \end{cases}\end{aligned}$$

- (c) Using the theory of small reflection and drawing suitable Signal Flow Graph (SFG) prove that the approximate solution of the input reflection coefficient of the quarter wave transformer is 0. [2]

3. (a) Obtain the scattering matrix of the 3dB Transmission line based lossless divider. [3]  
 (b) Characterize the 3dB 180 Degree coupler for its difference port only using even and odd mode analysis. Obtain the suitable scattering parameters. Why port 4 is called difference port? [you may use the following relation if required] [4]

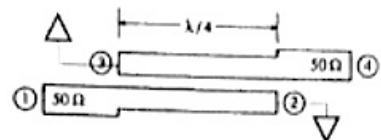
$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{A+B/Z-CZ-D}{A+B/Z+CZ+D} & \frac{2(AD-BC)}{A+B/Z+CZ+D} \\ \frac{2}{A+B/Z+CZ+D} & \frac{-A+B/Z-CZ+D}{A+B/Z+CZ+D} \end{bmatrix}$$

- (c) Explain what is the odd and even mode capacitance of a coupled line. [3]  
 (d) (i) Draw the SFG of a three-section coupled line coupler and from it obtain the coupling coefficient polynomial for the same.  
 (ii) Design a three-section 20 dB coupled line coupler with a binomial (maximally flat) response, a system impedance of 50 ohm, and a centre frequency of 3 GHz using the developed equations. [3] + [2] = [5]

4. (a) Design a low-pass, third-order, maximally flat filter using only series stubs. The cutoff frequency is 6 GHz and the impedance is 50. Take pi equivalent circuit to realize the same.  $g_1 = 1, g_2 = 2, g_3 = 1$  [5]

(b) Design a stepped-impedance low-pass filter having a cutoff frequency of 3 GHz and a fifth-order 0.5 dB equal-ripple response. Assume  $R_0 = 50 \text{ ohm}$ ,  $Z_{\text{low}} = 15 \text{ ohm}$ , and  $Z_{\text{high}} = 120 \text{ ohm}$ . Find the required electrical lengths of the five sections. You can assume suitable configuration (for practical realization) Low pass lumped prototype with  $g_1 = 1.70$ ,  $g_2 = 1.229$ ,  $g_3 = 2.54$ ,  $g_4 = 1.22$ ,  $g_5 = 1.70$ ,  $g_6 = 1$  [5]

(c)



If port 2 and 3 are terminated at short circuit for the above coupled line coupler, determine power exiting from all the port. The coupling between the coupler is  $c$  at the resonating wavelength. Assume port 1 is input port, Port 2 as through port, Port 3 as coupled port and Port 4 as isolated port. [5]

END

# Indian Institute of Space Science And Technology

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**5<sup>TH</sup> SEMESTER, 2022**

**Campus: Valiamala**

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## AVIONICS

### AVD311 - Digital Signal Processing

(Time allowed: THREE hours)

Candidate's Name: Pratik Prakash Aker ID No: ScroB137

- 1. [Inverse System]** The system function of a communication channel is given by

$$H(z) = (1 - 0.9e^{j0.4\pi}z^{-1})(1 - 0.9e^{-j0.4\pi}z^{-1})(1 - 1.5e^{j0.6\pi}z^{-1})(1 - 1.5e^{-j0.6\pi}z^{-1})$$

- (a) Determine a causal, stable  $H_c(z)$  (that compensates for the channel induced distortion) such that  $H(z)H_c(z) = 1$ . [3 Marks]
- (b) Sketch the pole-zero plot of  $H(z)$ ,  $H_c(z)$  and other systems involved in this process. [3 Marks]

- 2. [The Magnitude Function]** Determine the minimum phase system whose magnitude squared function is  
[3+3 = 6 Marks]

$$1. |H(e^{j\omega})|^2 = \frac{\frac{5}{4} - \cos(\omega)}{\frac{10}{9} - \frac{2}{3}\cos(\omega)} \quad 2. |H(e^{j\omega})|^2 = \frac{2(1 - a^2)}{(1 + a^2) - 2a\cos(\omega)}$$

- 3. [Inverse System]** The frequency response of a stable LTI system is known to be real and even. Is the inverse system stable. Reason it out. [3 Marks]

- 4. [Minimum Phase Sequences]** Let  $x(n)$  be a real valued minimum phase sequence. Modify  $x(n)$  to obtain another real valued minimum phase sequence  $y(n)$  such that  $y(0) = x(0)$  and  $y(n) = |x(n)|$ . [5 Marks]

- 5.** Let  $h(n)$  be a real filter with linear (nonzero) or nonlinear phase response. What will be phase of resultant signal for the two cases listed below. [3 + 3 = 6 Marks]

Problem 1:  $g(n) = h(n) * x(n)$   
 $f(n) = h(n) * g(-n)$   
 $y(n) = f(-n)$

Problem 1:  $g(n) = h(n) * x(n)$   
 $f(n) = h(n) * x(-n)$   
 $y(n) = g(n) + f(-n)$

CONTINUED

6. [DTFT, DFT, Zero padding] Given that  $x = [1 \ 1 \ 1 \ 1 \ 1]$ .

- (a) Obtain the Discrete Time Fourier Transform of the sequence  $x$ . [2 marks]
- (b) Obtain the 5-pt Discrete Fourier Transform of the sequence  $x$ . [2 marks]

Create  $y = [1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]$ . Now,

- (a) Obtain the Discrete Time Fourier Transform of the sequence  $x$ . [2 marks]
- (b) Obtain the 5-pt Discrete Fourier Transform of the sequence  $x$ . [2 marks]
- (a) What relationship do you observe between  $X(e^{j\omega})$  and  $Y(e^{j\omega})$ . [2 marks]
- (b) What relationship do you observe between  $X(k)$  and  $Y(k)$ . [2 marks]

7. In a motion picture, as the car races faster and faster, why do the wheels seem to go backwards, stop and go forwards ? Give correct technical explanation from signal processing point of view. [3 marks]

8. A signal  $x(n)$  consists of two frequency components  $\omega_0$  and  $\omega_1$ , where  $|\omega_1 - \omega_0| \ll \omega_0$  or  $\omega_1$ . To resolve these components in the Fourier spectra, what should be the length of the signal record,  $N$ . [4 marks]  
*Assumption* : Rectangle time window is used for observing the signal.

9. Why does a dot placed on a rotating disc smear into an arc as the speed of rotation increase. Explain the observation by considering the eye-brain system as a LTI system. [5 marks]
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# Indian Institute of Space Science and Technology

End Semester Examination

## B.Tech 5th Semester

### MA 311 - Probability, Statistics and Numerical Analysis

Date : 26th Nov, 2002

Time: 9.30 am to 12.30 pm

Max. Marks: 100

#### SECTION A (Each question carry 5 marks. Maximum 50 marks)

- Let  $(S, P)$  be a probability space and  $A, B \in S$ . If  $P(A) > 0$ , show that  $P(A \cap B | A) \geq P(A \cap B | A \cup B)$ .
- Let  $X \sim g(p)$ . Show that  $P(X \geq n+k | X \geq n) = P(X \geq k)$ .
- Let  $X \sim p(\lambda)$ . Show that for any  $n \in \mathbb{N}$ , we have  $E(X^n) = \lambda E((1+X)^{n-1})$ .
- Let  $X$  and  $Y$  be independent random variables such that  $X + Y \sim N(\mu_1, \sigma_1^2)$  and  $Y \sim N(\mu_2, \sigma_2^2)$ . Does  $X$  follow normal distribution?
- State Chebyshev's theorem. Derive Weak Law of Large numbers from Chebyshev's theorem.
- Let  $X, Y, Z, W \sim N(\mu, \sigma^2)$  and are independent. Show that

$$\frac{\sqrt{3}(W - \mu)}{\sqrt{X^2 + Y^2 + Z^2 + 3\mu^2 - 2\mu(X + Y + Z)}} \sim t(3).$$

- With explanation give examples of an unbiased and a biased estimator of population mean such that variance of both of them tends to zero as the sample size tends to infinity.
- Give an example of a positive sequence  $\{\epsilon_n\}$  converging to zero in such a way that  $\lim_{n \rightarrow \infty} \frac{\epsilon_{n+1}}{\epsilon_n^p} = 0$  for some  $p > 1$ , but not converging with any order  $p' > p$ .
- Consider the iteration  $x_{n+1} = x_n^3$ . Give a detailed discussion on the convergence of the sequence  $\{x_n\}$  based on the initial value  $x_0$ .
- Derive the following numerical schemes used for solving the first order ODE,  $x' = f(t, x)$  with  $x(t_0) = x_0$ .
  - $x_{k+1} = x_k + h f\left(t_k + \frac{h}{2}, x_k + \frac{h}{2}f(t_k, x_k)\right)$
  - $x_{k+1} = x_k + \frac{h}{2} [f(t_k, x_k) + f(t_k + h, x_k + h f(t_k, x_k))]$

Check whether the proposed schemes are consistent? Find the order of the truncation error.

#### SECTION B (Maximum marks 50)

- (a) A large industrial firm uses three local motels to provide overnight accommodations for its clients. From past experience it is known that 20% of the clients are assigned rooms at the Ramada Inn, 50% at the Sheraton, and 30% at the Lakeview Motor Lodge. The plumbing is faulty in 5% of the rooms at the Ramada Inn, in 4% of the rooms at the Sheraton, and in 8% of the rooms at the Lakeview Motor Lodge. With the proper use of the Theorem of Total probability and the Bayes Theorem,

- i. find the probability that a client will be assigned a room with faulty plumbing. [2.5]  
ii. find the probability that a person with a room having faulty plumbing was assigned accommodations at the Lakeview Motor Lodge. [2.5]
- (b) The joint density of  $X$  and  $Y$  is given by  $f(x, y) = \frac{12}{5}x(2 - x - y)$  when  $x, y \in (0, 1)$  and  $f(x, y) = 0$  otherwise.
- Find  $f_Y$  and  $f_{X|Y}$ . [2.5]
  - Find  $P(X \geq 0.5 | Y = 0.5)$ . [1]
  - Are  $X$  and  $Y$  independent? [1.5]
2. (a) What do we mean by Poisson approximation to Binomial? What is the corresponding thumb rule? Five percent (5%) of Christmas tree light bulbs manufactured by a company are defective. The company's Quality Control Manager is quite concerned and therefore randomly samples 100 bulbs coming off of the assembly line. What is the probability that the sample contains at most five defective bulbs? [2 + 1 + 3]
- (b) In an exam, the mean score of the students is 70 and the variance is 16. What is the probability that a student's score will lie within the 54 and 86? With proper explanation find a bound on
- the probability that a student's score will exceed 86. [2]
  - the probability that a student's score will lie within 2 standard deviations on either side of the mean score? [2]
3. (a) Civil engineers believe that  $W$ , the amount of weight (in units of 1000 pounds) that a certain span of a bridge can withstand without structural damage resulting, is normally distributed with mean 400 and standard deviation 40. Suppose that the weight (again, in units of 1000 pounds) of a car is a random variable with mean 3 and standard deviation 0.3. With proper explanation find the approximate number of cars that would have to be on the bridge span for the probability of structural damage to exceed 0.1? Given that  $\Phi(-1.28) = 0.1003$  and  $\Phi(-1.29) = 0.0985$ . [4]
- (b) The mgf of  $X$  is given by  $M_X(t) = e^{2e^t-2}$  and of  $Y$  is given by  $M_Y(t) = (\frac{3}{4}e^t + \frac{1}{4})^{10}$ . If  $X$  and  $Y$  are independent, find  $P(X + Y = 2)$ . [3]
- (c) Show that  $\bar{X}$  is a sufficient estimator for  $\lambda$  where the population is following  $p(\lambda)$ . [3]
4. (a) Suppose that the lifetime of Badger brand light bulbs is modeled by an exponential distribution ( $\frac{1}{\lambda}e^{-x/\lambda}$ ) with unknown parameter  $\lambda$ . We test 10 bulbs and find they have lifetimes of 2, 3, 1, 3.5, 4, 2.5, 1.5, 3, 4.5, 2 years, respectively. Give an ML estimate for  $\lambda$ . [4]
- (b) Suppose from place A a signal of constant value  $\mu$  is being sent and the signal is being received at place B added with a random noise following normal distribution with mean 0. Suppose from A the signal has been sent 9 times and at B the recorded values are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5.
- Give an estimate of  $\mu$ .
  - With explanation find a 95% confidence interval for  $\mu$ .
  - If and variance of the population is 9, how many times the value of the signal should be recorded at B to be 90% confident that the error in estimation will not be more than 0.2?

Given that  $\Phi(-1.64) = 0.0505$ ,  $\Phi(-1.65) = 0.0495$ ,  $\Phi(-1.96) = 0.025$ ,  $\Phi(-2.57) = 0.0051$  and  $\Phi(-2.58) = 0.0049$ . [1+3+2]

5. Consider the iteration  $x_{n+1} = \Phi(x_n)$ ,  $\Phi(x) = \sqrt{2+x}$ . [10]
- For what values of  $x_0$  the iterates  $x_n$  remain on the same side of root  $\alpha = 2$  as  $x_0$  and converge monotonically to  $\alpha$ .
  - Does the iteration converges globally? What is the order of convergence?
  - If  $0 < x_0 < 2$ , how many iteration steps are required to obtain  $\alpha$  with an error less than  $10^{-10}$ ?
6. (a) Consider the function  $f(x) = \ln^2 x - x - 1$ ,  $x > 0$ . Let  $\alpha$  be the root of the equation  $f(x) = 0$ . What is the largest positive  $b \leq 1$  such that Newton's Method, started with  $x_0 = b$ , converges to  $\alpha$ ?
- (b) Consider the equation  $x \tan x = 1$ . Discuss the real roots of this equation like how many roots, approximate location and symmetry properties. Use appropriate graphs. Using bisection method, determine how many bisections would be required to find the smallest positive root to within an error of  $\frac{1}{2} \times 10^{-8}$ . Is your answer valid for all roots. [10]

\*\*\*END\*\*\*

Note: The Poisson and t - distribution tables are on the back side of this page.

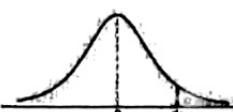


Table A.4 Critical Values of the t-Distribution

<i>v</i>	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228

Table A.2 Poisson Probability Sums  $\sum_{z=r}^{\infty} p(z; \mu)$

<i>r</i>	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6268	0.5321	0.4405
5	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8		1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9			1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10				0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11				1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12					1.0000	0.9999	0.9997	0.9992	0.9980
13						1.0000	0.9999	0.9997	0.9993
14							1.0000	0.9999	0.9998
15								1.0000	0.9999
16									1.0000