सञ्चारसंविधा १

COMMUNICATION SYSTEM I

30-07-2024

AV314: Communication systems I ⇒ Prerequisits:

↓

AV324: Communication Systems II

↓

Communication networks

Wireless networks

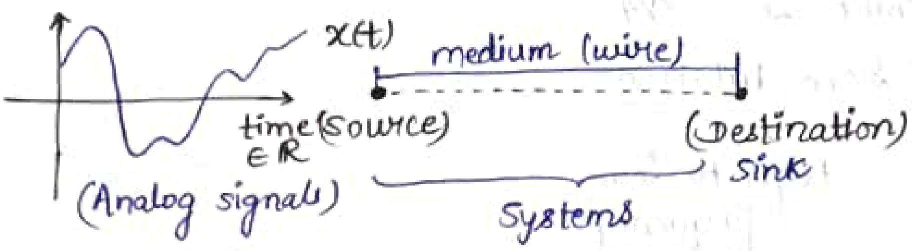
Signals and Systems

↳ Fourier series & Transform

↳ LTI

↳ Convolution

Communication Systems

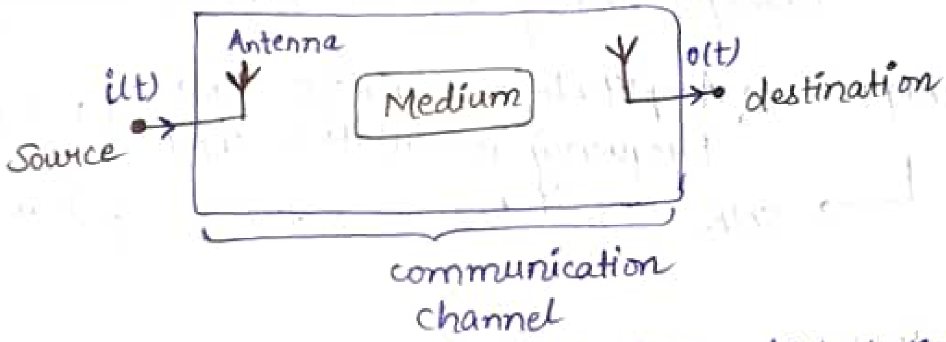


Examples:

voice

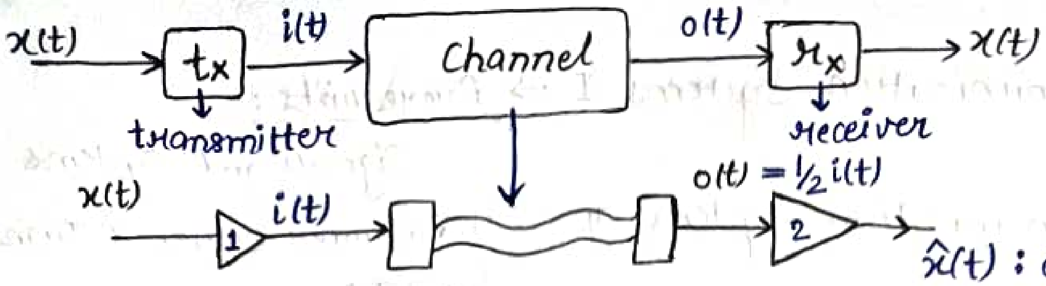
sensor data

Implicit assumption: voltage signals



↳ A system which takes a signal i(t) as i/p and gives o(t) as output, which is some function of i(t) [ideally should be same as i(t)].

Communication System Problem: To take the signal i(t) and give o(t) which is somehow related to i(t) so that i(t) can be retrieved.



$\hat{x}(t)$: estimate of $x(t)$

⇓

Design comm. system such that $\hat{x}(t)$ looks similar to $x(t)$ [Replica of $x(t)$]

Problems to Address:

Frequency : f_c

For 1 KHz signal:

$$\lambda = \frac{3\times10^8}{1\times10^3} = 300 \text{ Km}$$

Antenna size $\sim \lambda/4$

→ **Modulation & Demodulation**

- Modulation: Low to high frequency
- Demodulation: High to low frequency

→ Noise + other distortions

Aim of course: Design wireless communication system to transfer analog signals. [Analog Communication system]

↳ **Techniques:**
- Amplitude modulation (AM)
- Frequency modulation (FM)

} For Modem

- SNR (Signal to Noise Ratio) } For noises

Course Evaluation:

Internals: Class tests

Programming/ simulation assignments } MATLAB

Problems, etc.

Resources:

Textbooks:

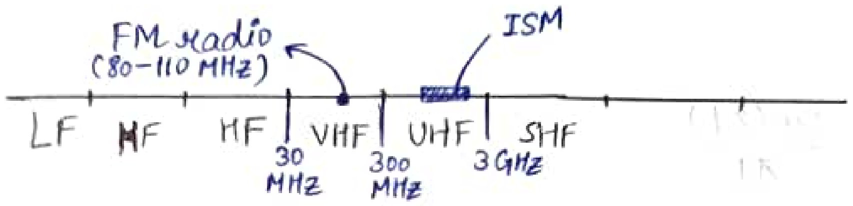
1. Upamanyu Madhav — Introduction to comm. system, 2014 (Assignments)
2. Simon Haykin — Communication Systems, 4th ed.

31-07-2024

Error $(x(t), \hat{x}(t)) = 0.01$

Given the channel, design R_x and T_x such that
error $(x(t), \hat{x}(t)) < \epsilon$

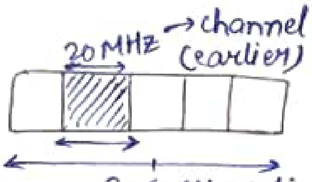
Spectrum (Wireless Spectrum)



HAM
↳ Amateur Radio

Band

WiFi :



channel : Range of frequencies in which signals can be transmitted
↳ $i(t)$ should fall within this range

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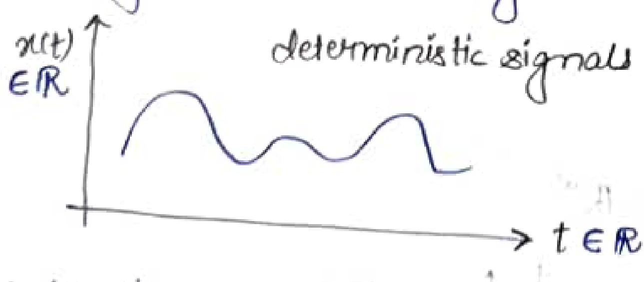
SIGNALS & SYSTEMS REVIEW

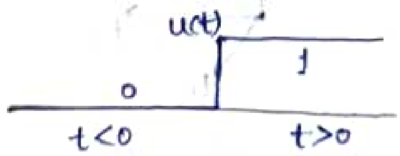
Signal: Deterministic (one value at a time) function of time.

Random signal: Value changes each time we see it.

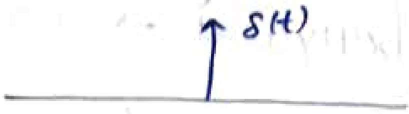


→ Continuous value
→ Continuous time

Step function:



Delta function:



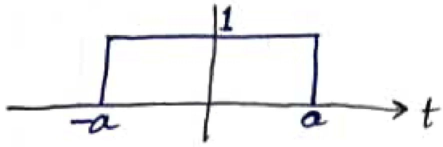
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\int_{-10}^{-5} \delta(t)\,dt = 0$$

Boxcar function / Indicator function:

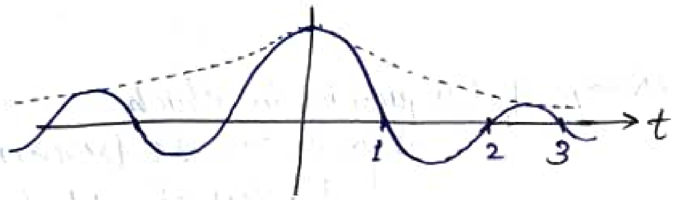
$$I_A = \begin{cases} 1, & \text{if } A \text{ is true.} \\ 0, & \text{if } A \text{ is false.} \end{cases}$$

$$I_{\{t \in [-a, a]\}}$$



Sinc function:

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



Energy of a Signal:

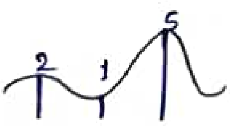
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Power of a signal:

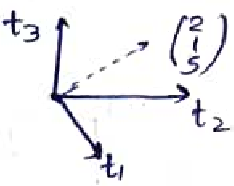
$$P_x = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

Signals as Vectors:

$$x(t) \xrightarrow{\text{sampling}} x_n \in \mathbb{R}^{\alpha}$$



$$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$



Dot product:

$x(t), y(t)$: continuous-time signals

$$\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x(t) y(t) \, dt \rightarrow \text{inner-product}$$

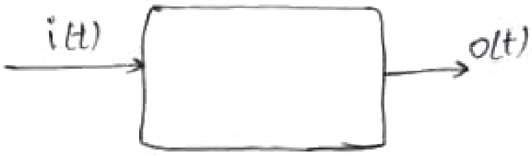
if $= 0$, then $x(t)$ and $y(t)$ are orthogonal.

→ Two signals are similar (one is replica of the other), if both lie along the same line.
↳ can be checked by using dot-product.

$$\int_{-\infty}^{\infty} x(t)\, y(t+\tau)\, dt \rightarrow \text{cross-correlation}$$
$(\tau : \text{time-lag})$

System

↳ Takes a time-signal, processes it, and produces a time-signal.



Linear: $o(t) = 5i(t)$
Non-Linear: $o(t) = 5i(t) + 5$
$= i^2(t)$

Linear System:

Homogeneity : $o(t) \leftarrow i(t)$
$\alpha o(t) \leftarrow \alpha i(t), \; \alpha \in \mathbb{R}$

Superposition : $i_1 \rightarrow o_1, \; i_2 \rightarrow o_2$
$i_1 + i_2 \rightarrow o_1 + o_2$

Time invariance:

$i(t) \longrightarrow o(t)$
$i(t-\tau) \longrightarrow o(t-\tau)$

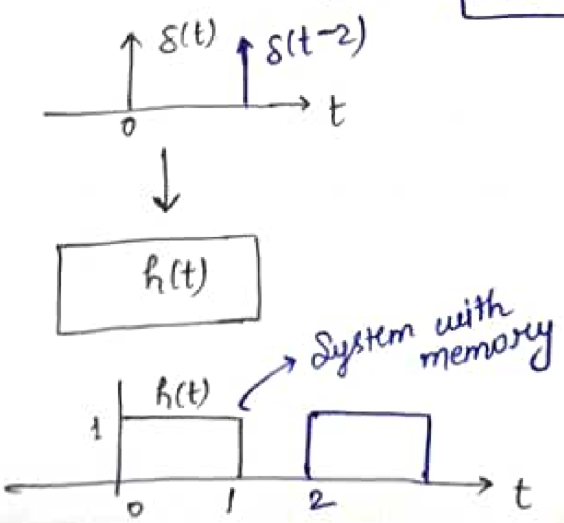
LTI systems

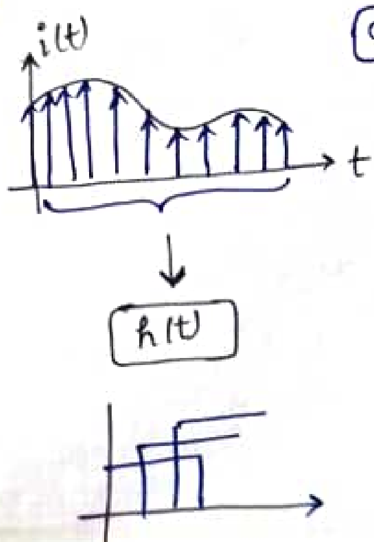
↳ Characterised by impulse response, $h(t)$.

$\delta(t) \rightarrow \boxed{\text{LTI}} \rightarrow h(t)$

02-08-2024

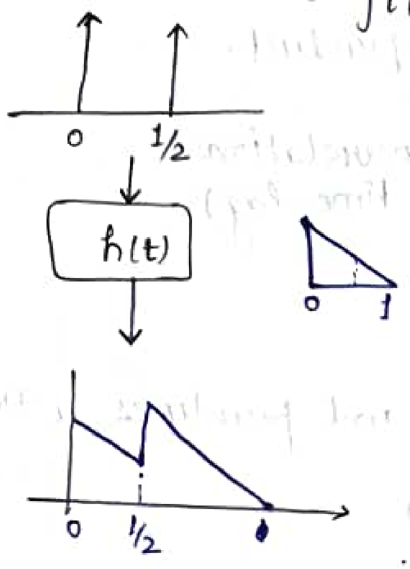
eg.



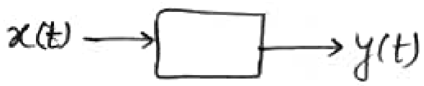


Convolution : $o(t) = \int i(\tau) \cdot h(t-\tau)\, d\tau = i(t) * h(t)$

$$= \int i(t-\tau) \cdot h(\tau)\, d\tau$$



Eg.

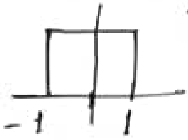


$$y(t) = \int_{-\infty}^{t} e^{u-t} x(u)\, du$$

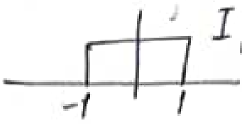
LTI ?

h(t) ?

Eg.



$I_{[-1,1]} \ast$



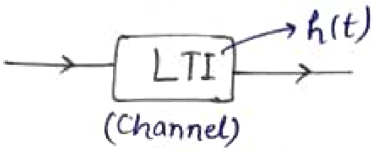
$I_{[-1,1]} = [u(t+1)-u(t-1)] \ast [u(t+1)-u(t-1)]$

$$= \int_{-\infty}^{\infty} [u(\tau+1)-u(\tau-1)][u(t-\tau+1)-u(t-\tau-1)]\,d\tau$$

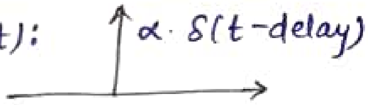
$$= \int_{-\infty}^{\infty} u(\tau+1)u(t-\tau+1)\,d\tau - \int_{-\infty}^{\infty} u(\tau+1)u(t-\tau-1)\,d\tau - \int_{-\infty}^{\infty} u(\tau-1)\,u(t-\tau+1)\,d\tau + \int_{-\infty}^{\infty} u(\tau-1)u(t-\tau-1)\,d\tau$$

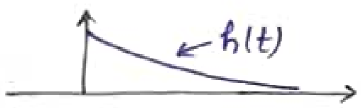
$$= \begin{cases} \left(\int_{-1}^{t+1} d\tau - \int_{-1}^{t-1} d\tau - \int_{1}^{t+1} d\tau + \int_{1}^{t-1} d\tau , \quad t>2,\ t<-2 \right. \\ \left. = (t+2) - (t) - (t) + (t-2) = 0 , \quad t>2, t<-2 \right) \\ 2-t, \quad 0 \le t \le 2 \quad \mid \quad t+2, \ -2 \le t < 0 . \end{cases}$$

LTI Models for Communication Subsystems



$h(t)$:





Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

$$e^{j2\pi f_0 t} \longrightarrow \boxed{h(t)} \longrightarrow H(f_0) e^{j2\pi f_0 t} = \int_{-\infty}^{\infty} e^{j2\pi f_0 (t-\tau)} h(\tau)\, d\tau$$

$h(t) \longleftrightarrow H(f)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

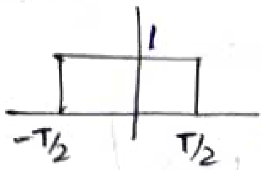
$$X(f) \qquad H(f) \qquad Y(f) = H(f)X(f)$$

FT of Some functions:

ⓐ $x(t) = \delta(t)$

$X(f) = 1$

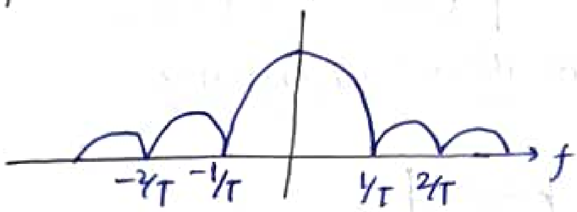
ⓑ $x(t) = I_{[-T/2, T/2]}(t)$



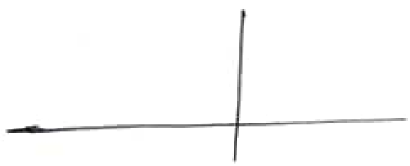
$$X(f) = T\,\mathrm{sinc}(fT) = \frac{T\sin(\pi f T)}{\pi f T} = \int_{-T/2}^{T/2} 1 \cdot e^{j2\pi f t}\,dt$$

$$= \frac{1}{2j\pi f}\left(e^{j\pi f T} - e^{-j\pi f T}\right)$$

$|X(f)|$



$\angle X(f)$



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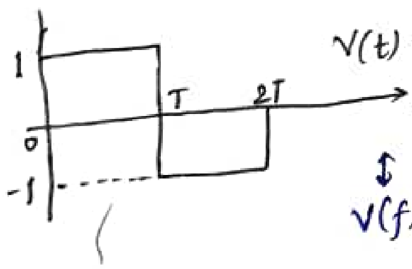


$$v(t) = I_{[0,T]} - I_{[T,2T]}.$$

$$= i(t - T/2) - i(t - 3T/2)$$

$\updownarrow$

$$V(f) = e^{-j2\pi f T/2}\,T\,\mathrm{sinc}(fT) - e^{-j2\pi f 3T/2}\,T\,\mathrm{sinc}(fT).$$

Fourier Transform Properties:

$x(t) \xleftrightarrow{F} X(f)$

- $x(t-\tau) \xleftrightarrow{F} e^{-j2\pi f \tau} X(f)$

  $y(t) \xleftrightarrow{F} Y(f)$

- $\alpha x(t) + \beta y(t) \xleftrightarrow{F} \alpha X(f) + \beta Y(f)$

- $\underbrace{e^{j2\pi f_0 T}}_{\text{complex exponential}}\, x(t) \xleftrightarrow{F} X(f - f_0)$

- Related property:

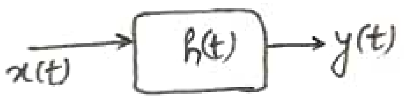
$$\cos(2\pi f_0 t)\, x(t) \xleftrightarrow{F} \tfrac{1}{2}X(f+f_0) + \tfrac{1}{2}X(f-f_0) \quad \text{[Modulation property]}$$

- $x(t) \xleftrightarrow{F} X^*(f)$

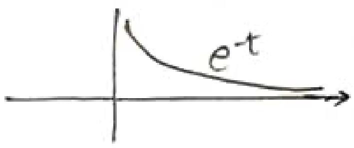
- $x(t) * y(t) \xleftrightarrow{F} X(f) \cdot Y(f)$ [Convolution property]

Proof:
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)\cdot y(t-\tau)\, d\tau \cdot e^{-j2\pi f t}\, dt$$
$$= \int_{-\infty}^{\infty} x(\tau)\left(\int_{-\infty}^{\infty} y(t-\tau) e^{-j2\pi f t}\, dt\right) d\tau$$
$$[t-\tau = u \Rightarrow dt = du]$$
$$= \int_{-\infty}^{\infty} x(\tau)\left(\int_{-\infty}^{\infty} y(u) e^{-j\omega u}\, du\right) e^{-j\omega\tau}\, d\tau$$
$$= \int_{-\infty}^{\infty}\left(x(\tau) e^{-j\omega\tau}\, d\tau\right)\left(Y(f)\right) = X(f)\cdot Y(f)$$

Eg.



$h(t) =$

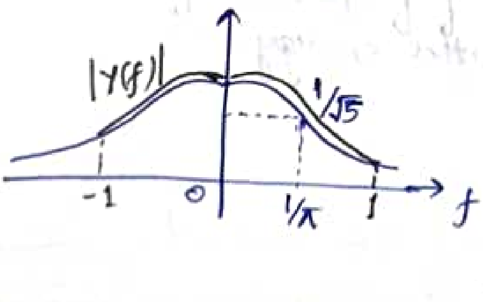


$$y(t) = \int_{-\infty}^{t} e^{u-t} x(u)\, du$$

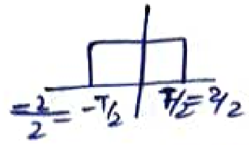
$$h(t) = \int_{-\infty}^{t} e^{u-t} \delta(u)\, du$$

$$H(f) = \int_{0}^{\infty} e^{-t} e^{-j2\pi f t}\, dt$$
$$= \frac{1}{1+j2\pi f}$$

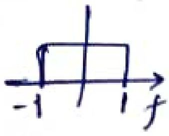
$$|H(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2}}$$



$x(t) = 2\,\text{sinc}(2t)$



$\rightarrow 2\,\text{sinc}(f \cdot 2)$
$T = 2$

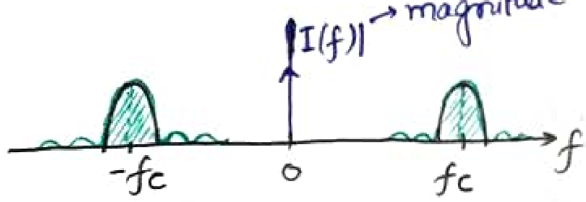


$\rightarrow \boxed{\dfrac{1}{1+j2\pi f}}$

* $X(t) \longleftrightarrow x(-f)$ [Duality property]
* $\int x(t)^2\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$

Bandwidth (BW) → Baseband (BB)
Passband (PB)

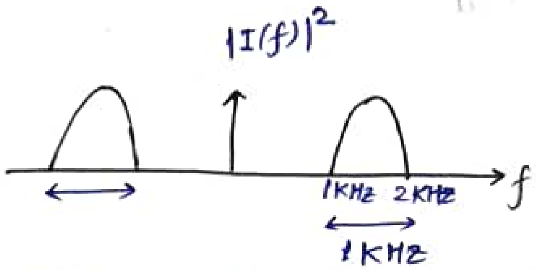
$i(t) \xrightarrow{F} I(f)$



→ double-sided spectrum
→ energy spectral density

$$\underbrace{\int_{-\infty}^{\infty} |I(f)|^2\,dt}_{\text{Energy Spectral Density}}$$

ESD :



Bandwidth (BW) : The range of frequencies for which the signal has non-negative energy.

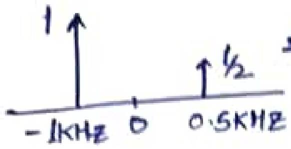
One sided B/W is 1KHz.

Two sided B/W is 2KHz.

$\rightarrow$ $i(t) \in \mathbb{R}$

$$I(-f) = I^*(f)$$

$$|I(-f)| = |I^*(f)| = |I(f)|$$

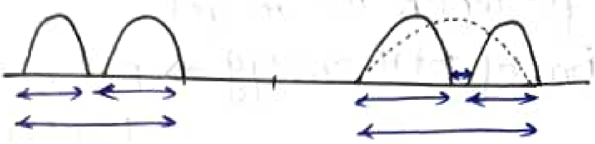


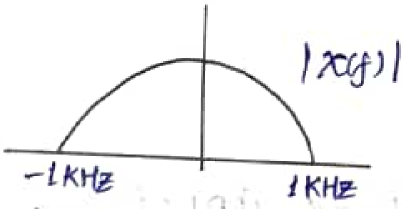
$$\Rightarrow e^{-j2\pi(1K)t} + \tfrac{1}{2} e^{+j2\pi(0.5K)t}$$

$\downarrow$

not real signal

Two-sided B/W = Total length of frequency intervals in which ESD > 0.





$\longrightarrow$ $x(t)$ (infinite)

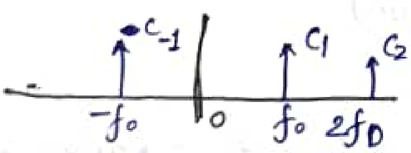
Suppose, we had $x(t) \in [-T/2, T/2]$

$$x(t) = \begin{cases} \text{sinc}(2\pi f_0 t) & \text{for } t \in [-T/2, T/2] \\ 0, & \text{otherwise.} \end{cases}$$

Digression:

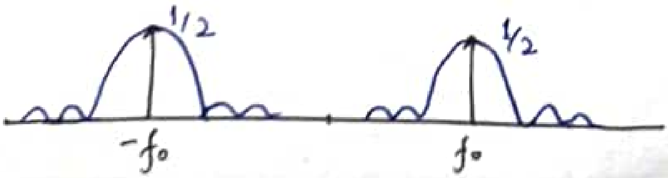
Periodic signal $p(t)$ with period $T_0 = 1/f_0$.

$$p(t) = \sum_{-\infty}^{\infty} c_k \cdot e^{j2\pi k f_0 t}$$



$$x(t) = \text{sinc}(2\pi f_0 t) \cdot I_{[-T/2, T/2]} \quad \forall t$$

$\updownarrow$ $\qquad$ $T\,\text{sinc}(fT)$



$\Rightarrow$ Infinite Bandwidth $\rightarrow$ X

Energy containment

BW:

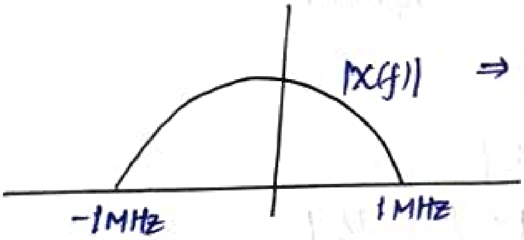
$$\alpha \int_{-\infty}^{\infty} |Y(f)|^2 df = \int_{-B/2}^{B/2} |Y(f)|^2 df \,, \quad \alpha \sim 0.9 - 0.95$$

(to get max ~~energy fraction~~ of energy)

↓

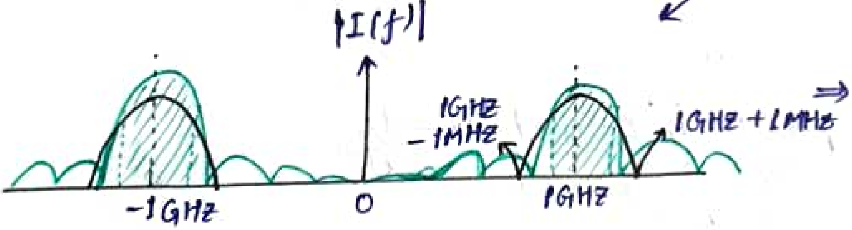
The value of B for which we can get maximum fraction of total energy ⇒ B % energy contained Bandwidth

$x(t) \longrightarrow X(f)$



⇒ Baseband (BB) : Passes through low pass filter

- Similarity: Same Bandwidth
- Differences: BB has lower freq. and can be ⊖ve.



⇒ Passband (PB): Passes through pass-band filter.

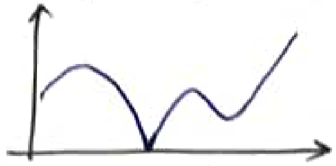
08-08-2024

$x(t)$ is real (∵ symmetric)

$$\int_{f_1}^{f_2} |X(f)|^2 df = \alpha \int_{0}^{\infty} |X(f)|^2 df$$

: Range of frequencies in which α % of energy lies.

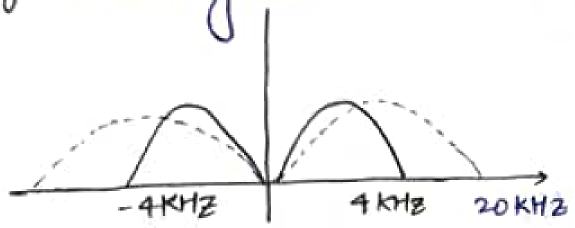
→ Message signal



$m(t)$

↳ BB signal (b/c freq. is low relative to freq. at which it is transmitted)

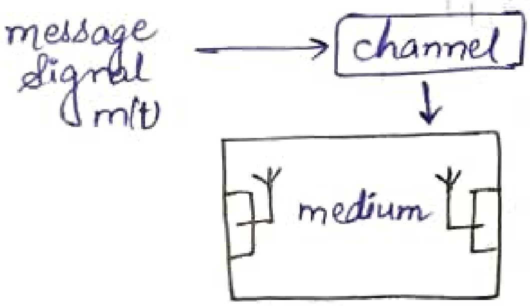
Eg. voice signals



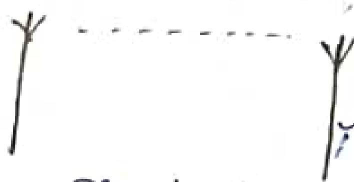
Channel

$f(i(t)) = o(t)$

Where do we get $f(\cdot)$ from?
— medium



e.g.

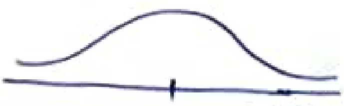


(height of antenna determines the freq. to transmit)

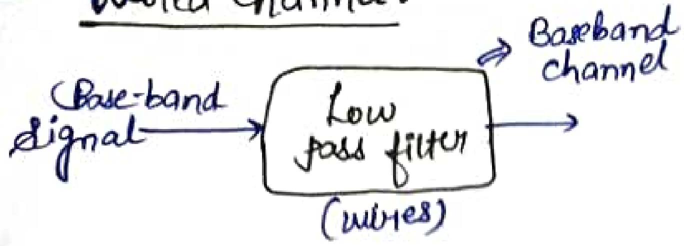
$\delta(t - t_{prop})$

⇓

→ Passband channel

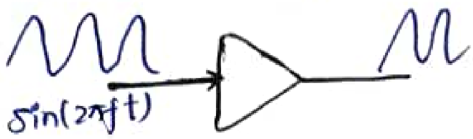


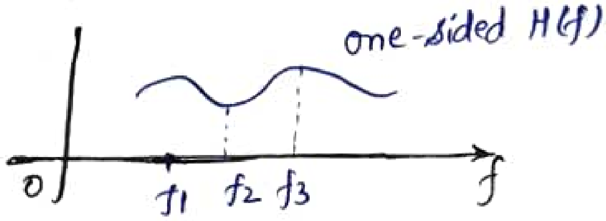
Wired Channel:



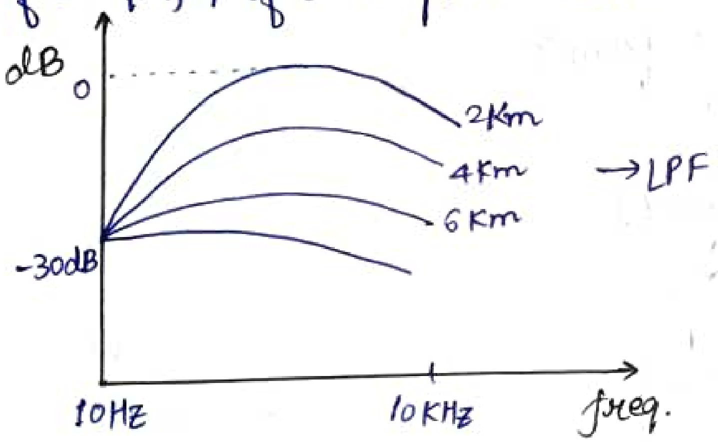
Channel as LTI filters

→ Frequency response, $H(f)$
Impulse response, $h(t)$

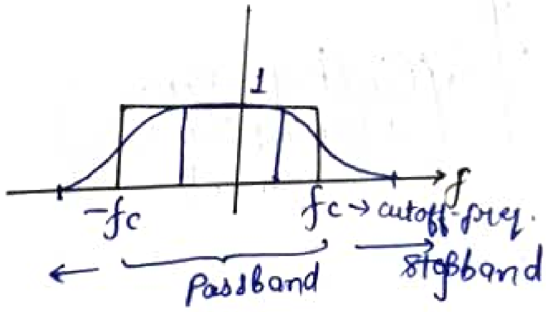




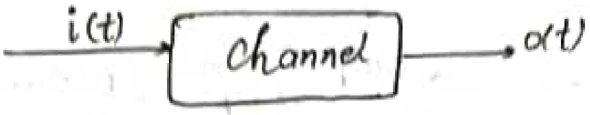
eg. of $|H(f)|$ of a telephone cable.



Ideal LPF: (Brickwall response)



09-08-2024

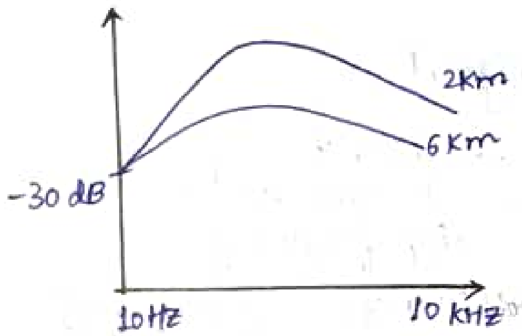


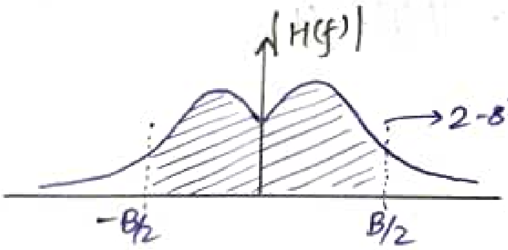
First order models for
Channels filters (LTI)

Baseband Passband

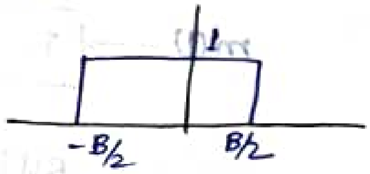
Common problem
– a baseband signal m(t)
over BB channel or
PB channel

Baseband channels



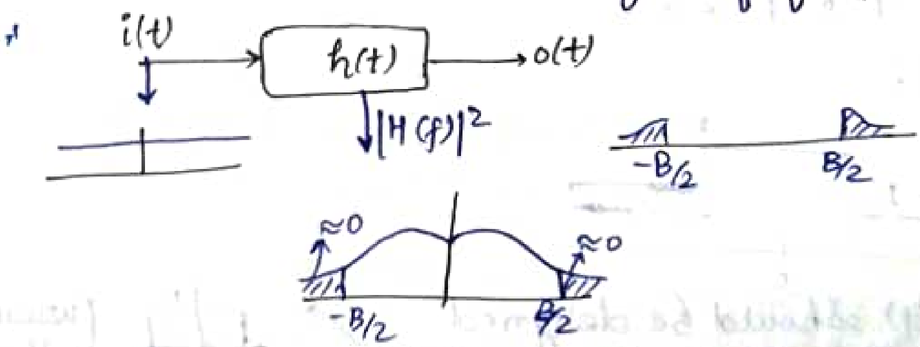


filter → Simpler model:

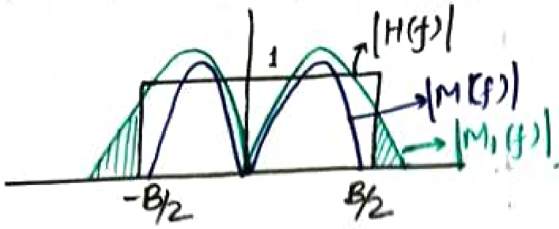


Bandwidth for a channel/System:

↳ Tells the capability of the channel (ability to transmit particular range of frequencies).



Simpler model for channel:



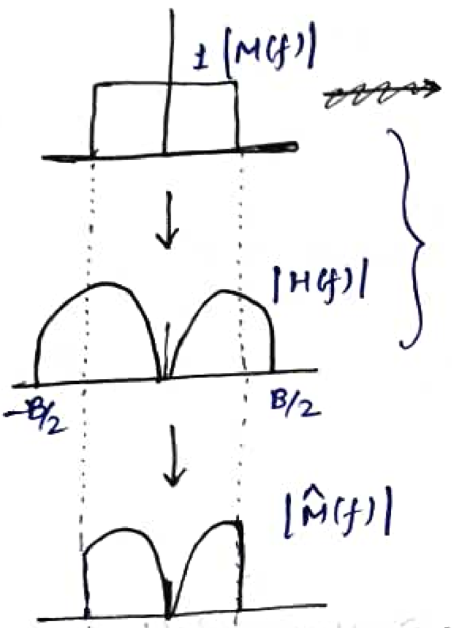
→ output same as i/p (BW matched)

→ ① change the signal

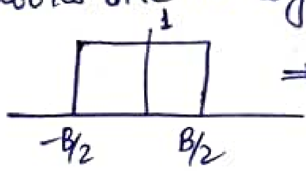
② Increase BW of channel : expensive

③ Expand the signal in time-domain to compress it in freq.-domain.

BB ⟶ BBC, if the BW of $m(t)$ $\leq$ BW of channel



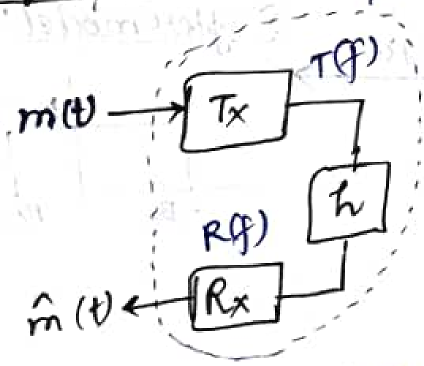
Make the system looks like the signal



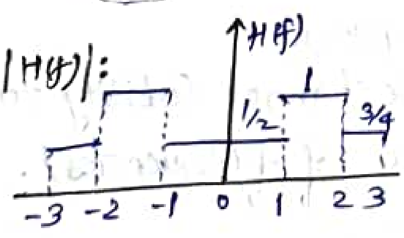
⇒ doesn't descriminate b/w any frequency

$= T(f) \cdot H(f) \cdot R(f)$ [cascading]

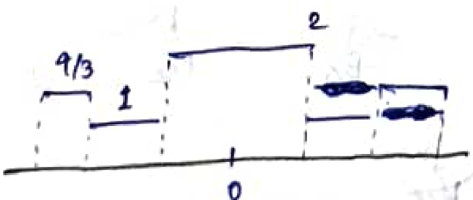
$\Rightarrow \frac{1}{H(f)} = T(f) \cdot R(f)$



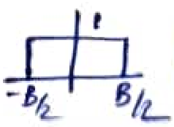
e.g. $|H(f)|$:



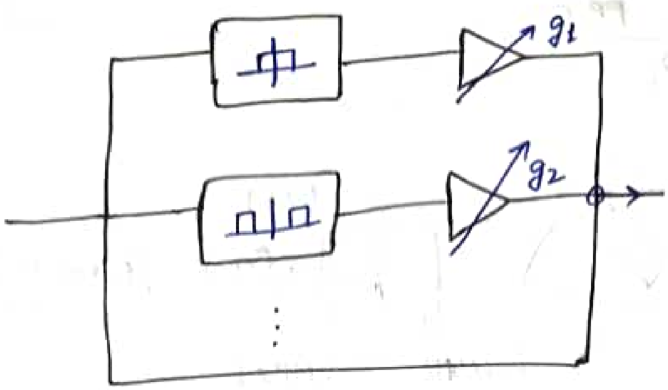
$T(f).R(f)$:



$T(f) \cdot R(f)$ should be designed so that $T(f) = R(f) \cdot H(f)$



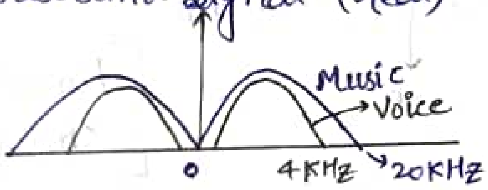
(usually at the Rx)



13-08-2024

Review

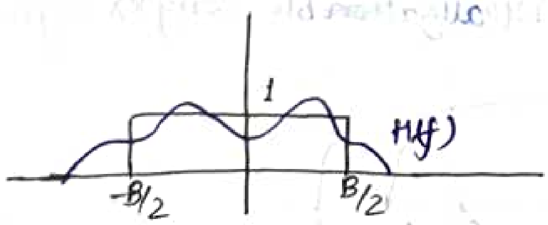
→ Message signal, $m(t)$
↳ a baseband signal (real)



→ Source has $m(t)$
↓ $T(f)$
Channel
↓ $R(f)$
sink/destination, $\hat{m}(t)$

→ Channels - think using models
Models → LTI ~~mod~~ filters
↙ Baseband (LPF)
↘ Passband (BPF)

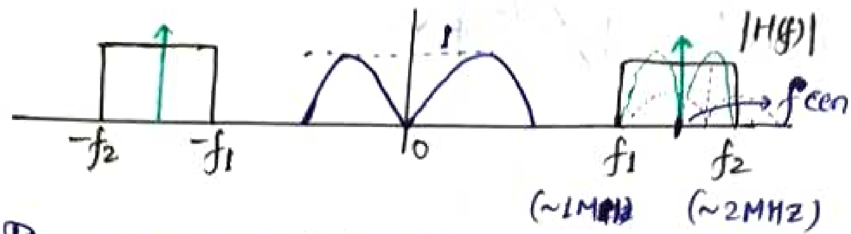
Baseband



1st check → message B/W $\leq B$
Realistic model → needs equalization

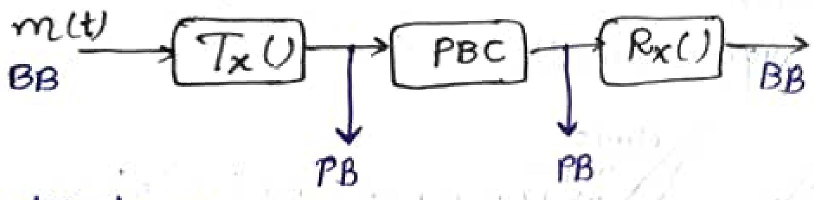
$$T(f) \cdot R(f) = \frac{1}{H(f)} \text{ (only in BW)}$$

Baseband Signals over PBC



Constraint: Phase response to be linear [adds delay in time-domain: $|H(f)|e^{-j2\pi f\tau}$]

Carrier Signal:



[Linear operation doesn't change the frequency.]

Amplitude Modulation:

A signal that can go through the channel:

$$A\cos(2\pi f t)$$

$f \in [f_1, f_2]$ → Try to keep f in the middle to maintain symmetry & not allow signal to fall outside.

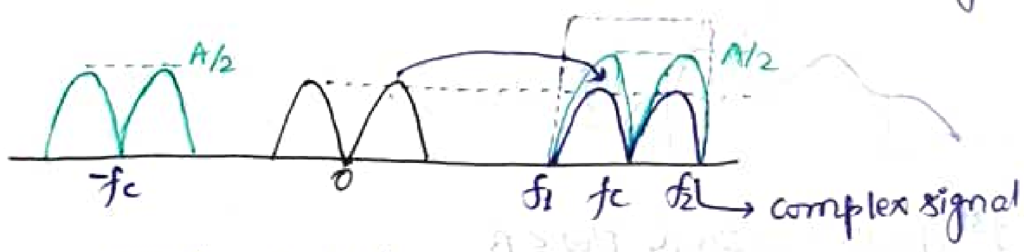
$$f_c = \frac{f_1+f_2}{2}$$

$$\underbrace{A}_{\text{carrier amplitude}}\cos(2\pi \underbrace{f_c}_{\text{carrier freq.}} t)$$

[Controls the power to be put] ⇐ carrier amplitude

Frequency translation:

$m(t)e^{+j2\pi f_c t}$ ⇝ unrealizable complex signal



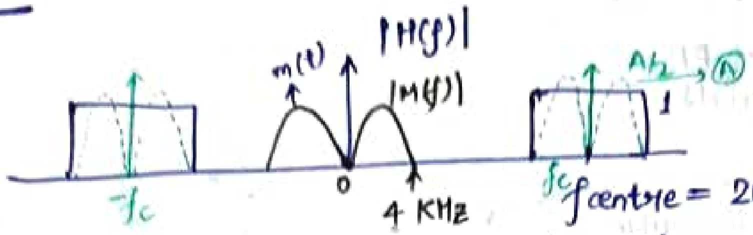
$$m(t)\,A\cos(2\pi f_c t)$$
$$= m(t)\frac{A}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$$

↓ Demodulation

multiply with cos/sin ⇒ $m(t)\,A\cos^2(2\pi f_c t) = \boxed{m(t)A}\left[\frac{1+\cos(2\pi(2f_c)t)}{2}\right]$

14-08-2024

Review

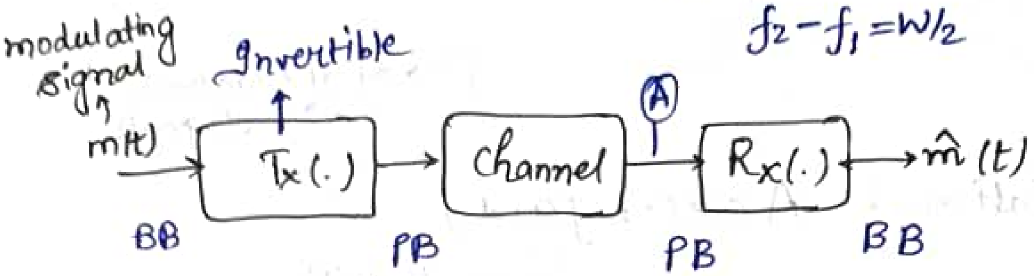


$f_{centre} = 20\,MHz$

$= 2.4\,GHz$

$f_1 \longleftrightarrow f_2$

$f_2 - f_1 = W/2$

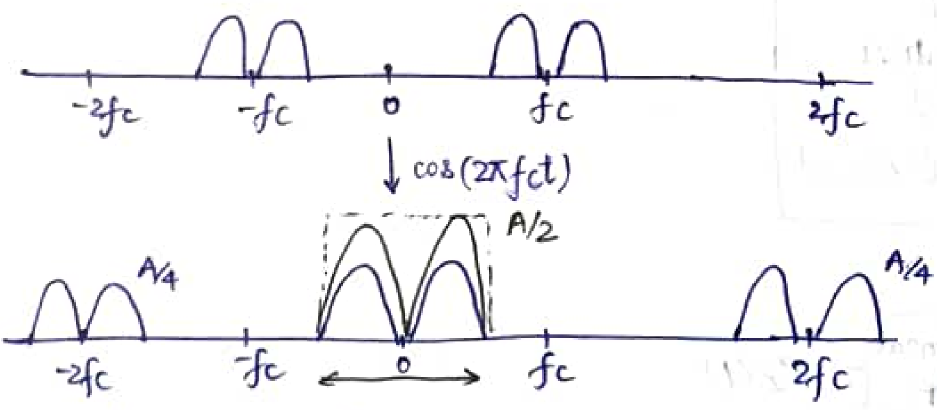


$A\cos(2\pi f_c t)$

$\underbrace{A m(t)}_{\text{Amplitude modulation}} \cdot \cos(2\pi f_c t)$ Ⓐ $\Rightarrow$ Modulated carrier/signal

$x(t) \xrightarrow{\mathcal{F}} X(t)$

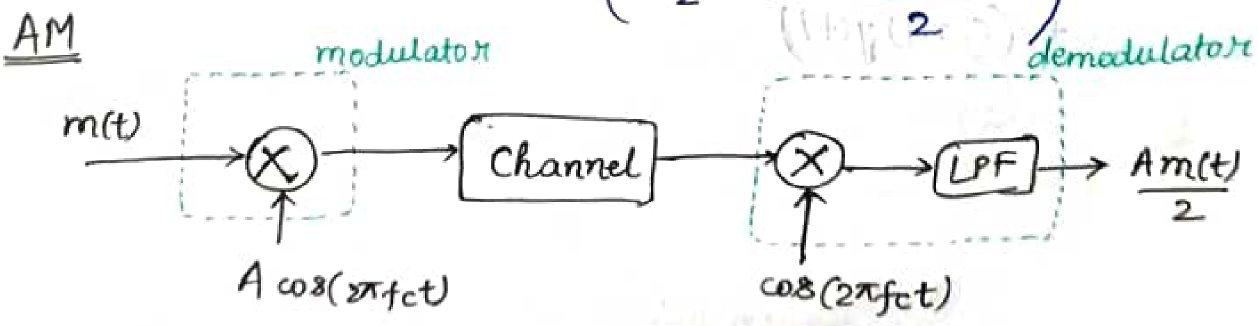
$x(t)\cos(2\pi f_c t) \longrightarrow \frac{1}{2}X(f-f_c) + \frac{1}{2}X(f+f_c)$



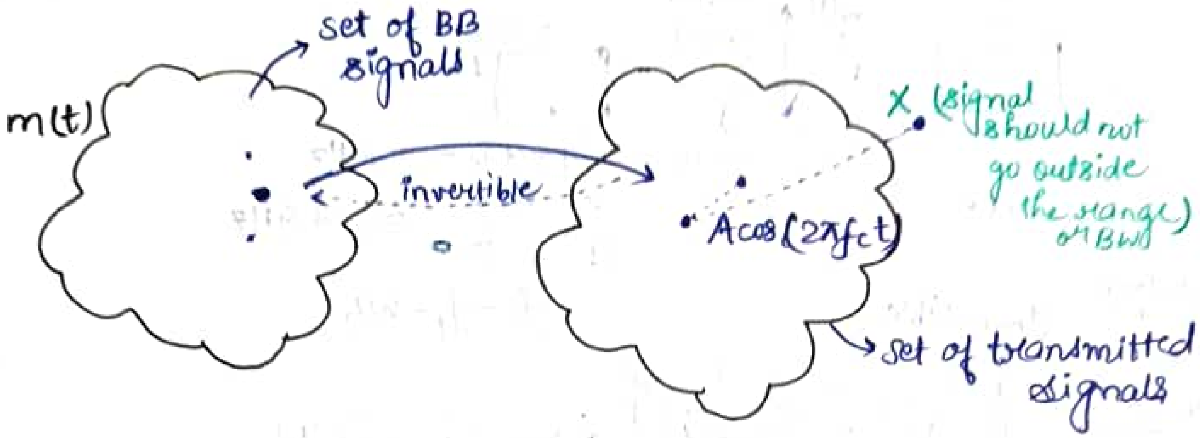
$A m(t) \cdot \cos^2(2\pi f_c t)$

$= A\, m(t)\left(\frac{1}{2} + \frac{\cos(2\pi(2f_c)t)}{2}\right)$

AM



Modulation Schemes



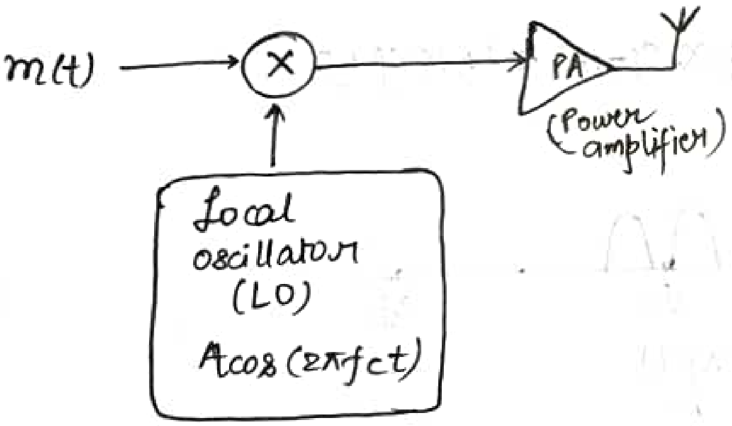
Frequency Modulation : $A \cos(2\pi f(t)\, t)$
(FM)

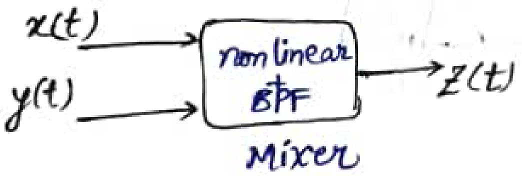
$f(t)$ ↳ acc. to $m(t)$

$$f(t) = f_c + k_f\, m(t)$$

Phase Modulation : $A \cos(2\pi f_c t + \phi(t))$
(PM)

$\phi(t)$ ↳ acc. to $m(t)$

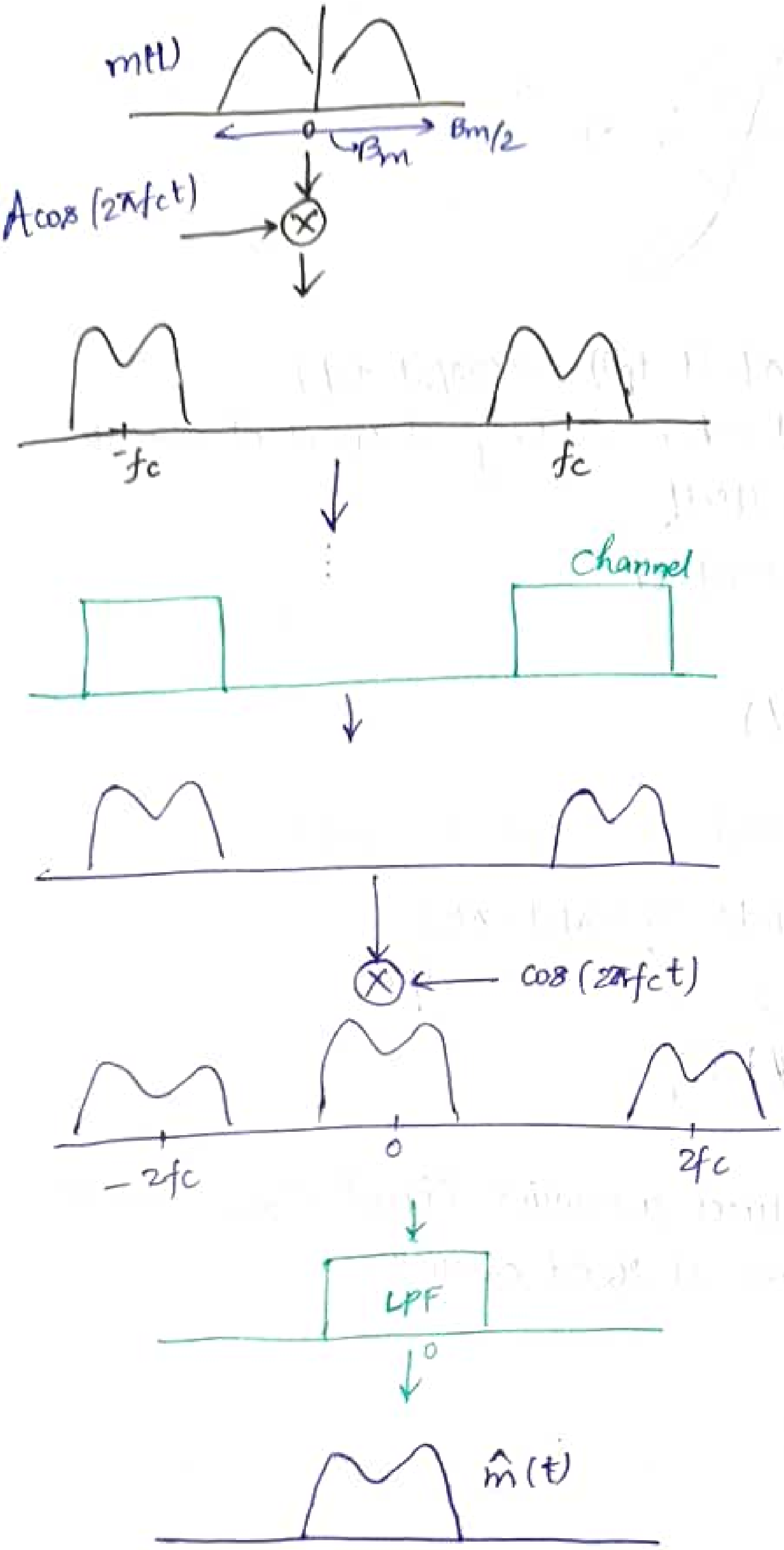


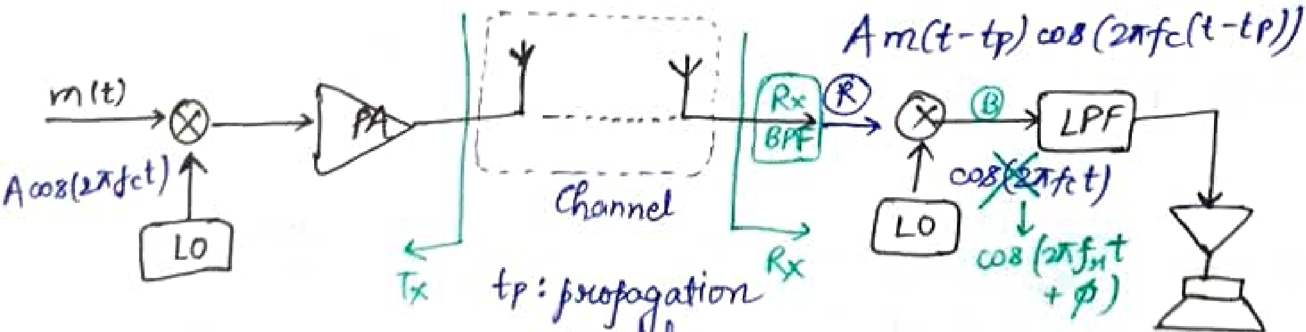


$$Z(t) = c_1 x(t) + c_2 y(t) + (c_3 x(t) y(t))$$

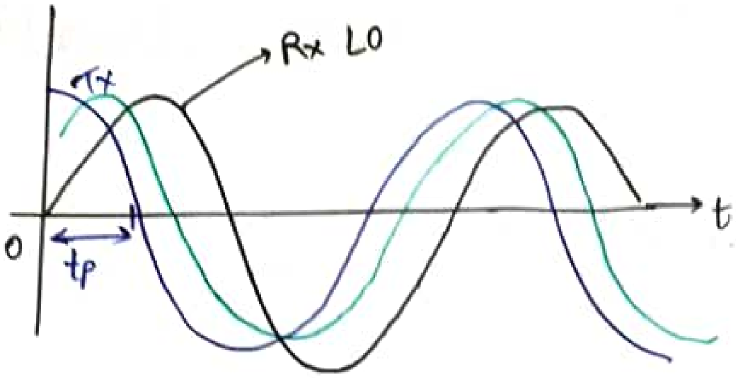
16-08-2024

Recall





t_p: propagation delay
↳ difficult to know
↳ can be done through GPS but expensive



(B) $\Rightarrow$ $A\, m(t-t_p) \cos(2\pi f_c(t-t_p)) \cdot \cos(2\pi f_r t + \phi)$

Under assumption that when looking at received signals time origin is at r_x itself.

(R) is still $A m(t) \cos(2\pi f_c t)$

LO signal at r_x:

$\cos(2\pi f_r t + \phi)$

Eg. Suppose the RX LO produces $\cos(2\pi f_c t)$.

(B) $\Rightarrow$ $A\, m(t) \cos(2\pi f_c t) \cdot \cos(2\pi f_c t + \pi/2)$

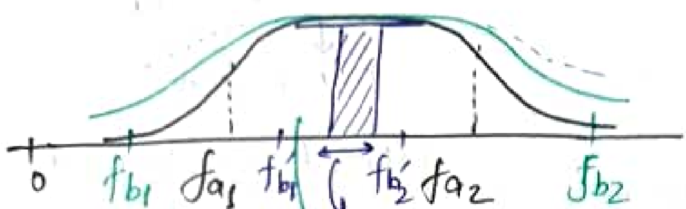
↳ No output

Suppose $\cos(2\pi f_c t + \phi)$

Assumption: Rx LO produces a carrier signal same (same frequency & phase) ~~same~~ as $r_x$ed carrier.

Phase locked loop (PLL)

BPF model for the channel



Channel (BPF) ⇒ desired range for transmission (desired signal)

OOB (out of band)

$f_{a_1} - f_{a_2}$: Range of freq. that is txed / rxed

Receiver –

just after the antenna

we need Rx BPF

– channel selection

+

amplification (LNA)

noise cutoff

Passband Communication System

20-08-2024

Recall

m(t) over PBC ← a carrier signal

↑ modulating signal

$A\cos(2\pi f_c t)$

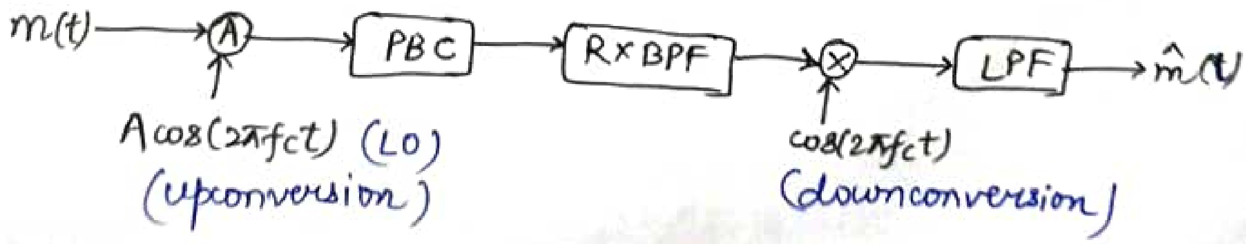
modulates

Amplitude (AM)

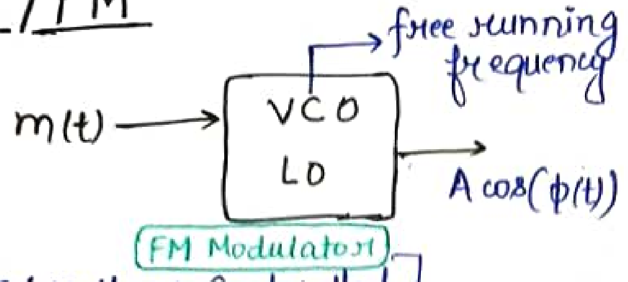
$Am(t)\cos(2\pi f_c t)$

Frequency / phase (FM) (PM)

$A\cos(2\pi f(t)\, t)$



FM / PM



[VCO: voltage Controlled Oscillator]

Ideally $A\cos(2\pi f_c t)$
instantaneous frequency.

In general,

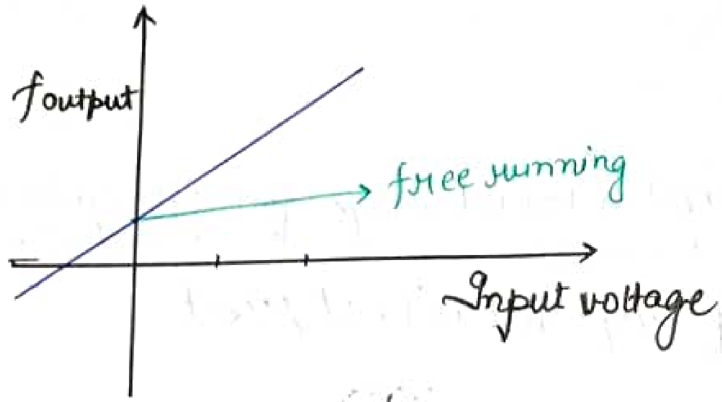
$$A\cos(\phi(t))$$

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

for a VCO

$$f(t) = f_{free} + \boxed{K_f}\, m(t)$$

↳ sensitivity parameter
↳ slope



$$\phi(t) = \int_0^t 2\pi f(u)\,du \quad , \quad \phi(0) = 0$$

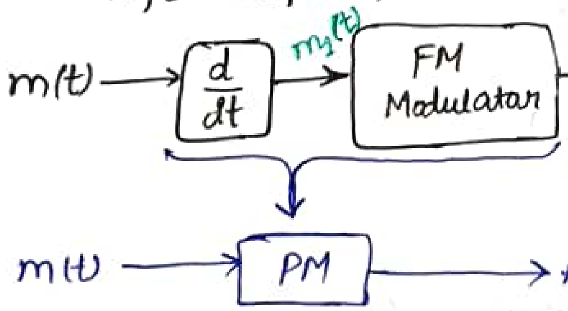
$$\phi(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du$$

So, the FM modulated signal:

$$A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$$

Phase Modulation

$$\phi(t) = 2\pi f_c t + K_p m(t)$$



$$A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t \overbrace{\frac{dm(u)}{du}}^{m_1(u)}\,du\right)$$

$$= A\cos(2\pi f_c t + \underbrace{2\pi K_f}_{K_p} m(t)).$$

$$A\cos(2\pi f_c t + K_p m(t))$$

We'll restrict to FM.

21-08-2024

Passband Communications

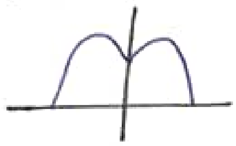
Carrier signal : $A\cos(2\pi f_c t)$

AM : $m(t) \longrightarrow \otimes \longrightarrow A\,m(t)\cos(2\pi f_c t)$, where the multiplier's second input is $A\cos(2\pi f_c t)$

FM : $m(t) \longrightarrow \boxed{\text{FM modulator}} \longrightarrow A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$

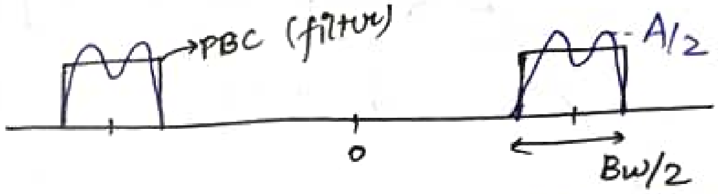
(PM is eq.)

$m(t) \xrightarrow{\mathcal{F}} M(f)$



PBC :

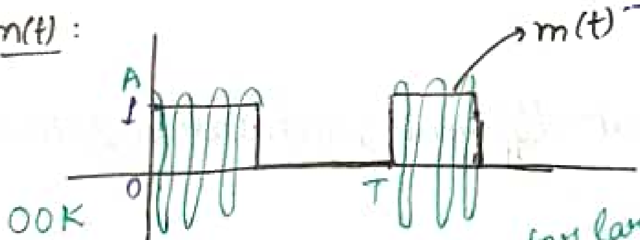
$A\,m(t)\cos(2\pi f_c t)$ $\xrightarrow{\mathcal{F}}$



$A\cos(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du)$

AM :

m(t) :

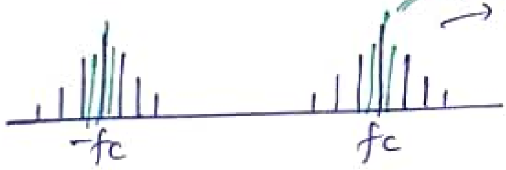


m(t) → periodic [Boxcar]

$B_m < Bw/2$

for larger T (more congested)

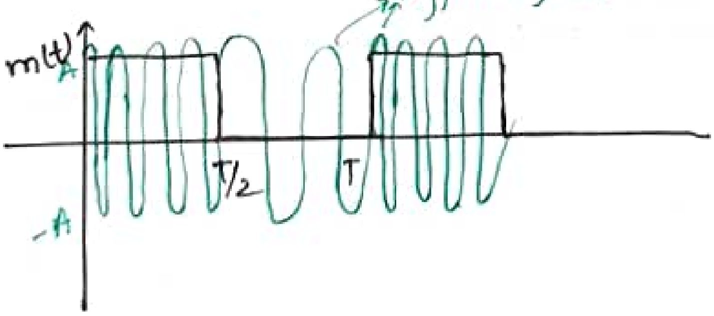
→ sampled sinc



Amp(m(t)) ↑↑
⇒ f_0 ↑↑

FM :

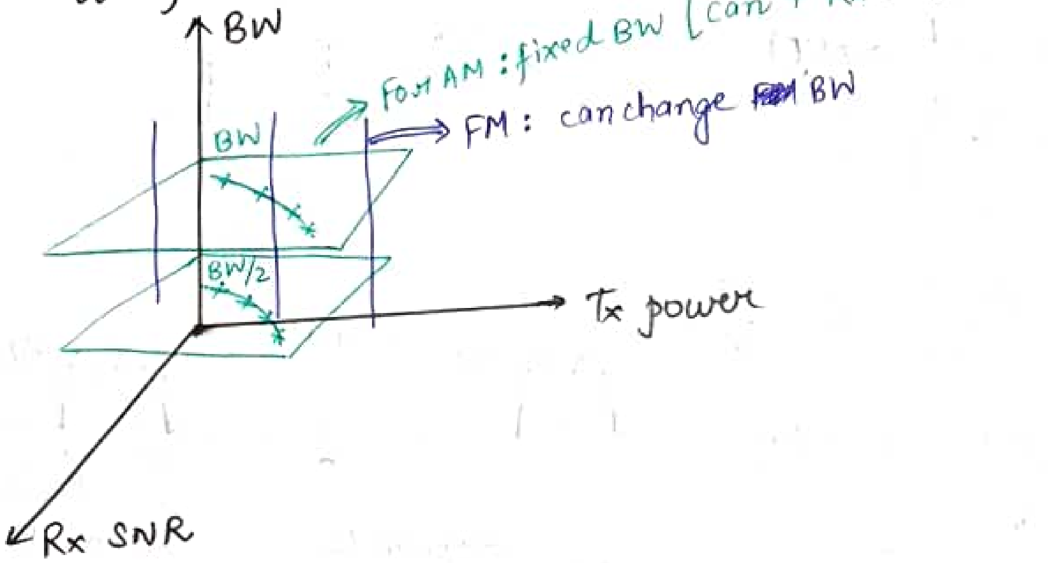
different freq.



Performance metrices:

- Bandwidth
- Power (Transmit) ↑ ⇒ Battery drain ↑
- Rx SNR
- complexity

Tradeoff space:



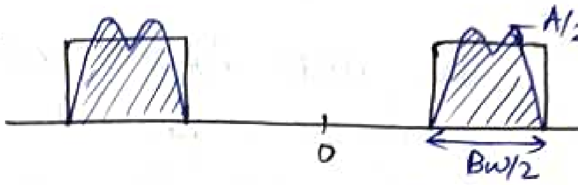
Motivate why we look at different modulation schemes

Amplitude Modulation Schemes

- ↳ Double sideband suppressed carrier (DSB, DSBSC)
- ↳ Plain AM / Large carrier AM (LCAM)
- ↳ Single sideband SSB
- ↳ Vestigial sideband (VSB) } X

$m(t) \longrightarrow \otimes \longrightarrow A\,m(t)\cos(2\pi f_c t) \rightarrow$

↑ $A\cos(2\pi f_c t)$



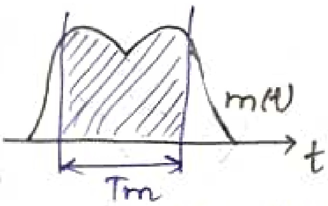
$B_m < B_W/2$

DSB spectrum:

$$\frac{A}{2}M(f-f_c) + \frac{A}{2}M(f+f_c)$$

Transmit ~~power~~ energy: $\frac{A^2}{4} \times 2 \times E_m$

$$= \frac{A^2}{2}E_m$$



$P_m = \frac{E_m}{T_m}$

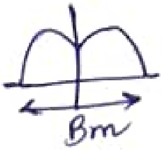
↳ Time duration during which most of the energy is transmitted

$\boxed{\frac{A^2}{2}P_m} \Rightarrow$ Transmitted power

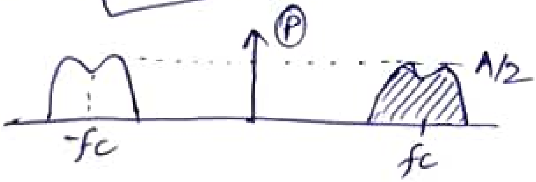
22-08-2024

$m(t) \longrightarrow \otimes \longrightarrow$ (P) Modulation Upconversion

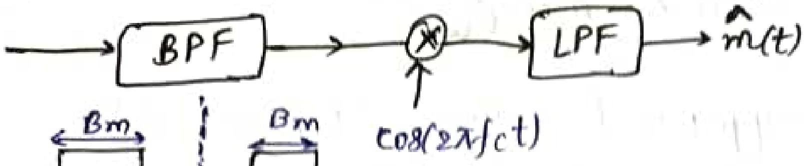
↑ $A\cos(2\pi f_c t)$

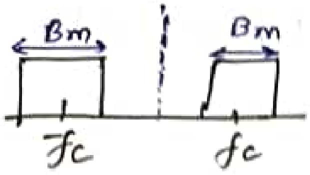


Tx B/W : $\boxed{2B_m}$



DSB Rx Demodulation:





Eg. with multiple channels

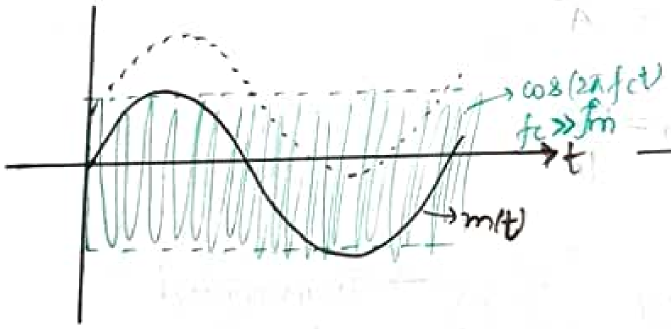
Eg. $m(t) = A_m \sin(2\pi f_m t)$

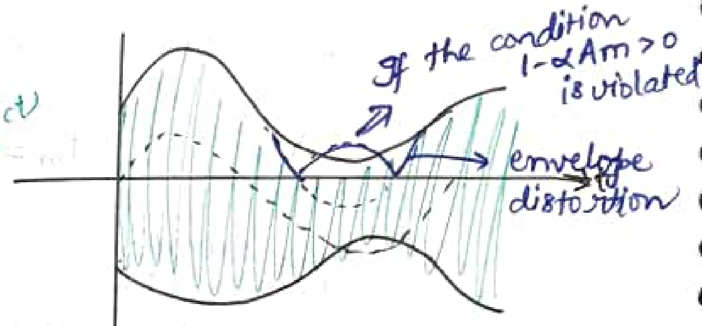
Draw magnitude spectrum of the DSB modulated signal.
Draw the time domain modulated signal.

Soln: $m(t) = A_m \sin(2\pi f_m t)$

$A(1 + \alpha A_m \sin(2\pi f_m t)) \cdot \cos(2\pi f_c t)$

↑ Modulated signal

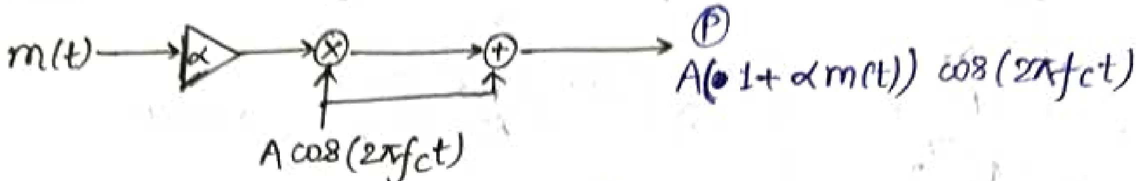




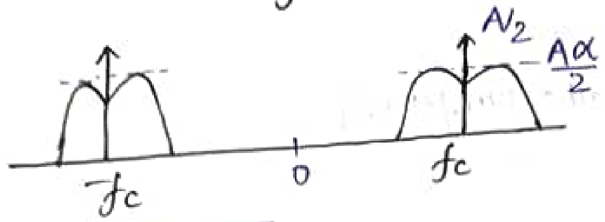
$1 - \alpha A_m > 0$ ⇐ condition for m(t) not touching t-axis

In LCAM, ~~we~~ ensure that envelope distortion does not happen.

Plain AM or Large Carrier AM (LCAM)



Ⓟ



Tx BW : $\boxed{2B_m}$

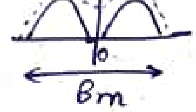
Tx power: $\int_{T_{eff}} A^2 (1+\alpha m(t))^2 \cos^2(2\pi f_c t)\, dt$

$$= \int_{T_{eff}} \frac{A^2}{2}\left(1+\alpha^2 m(t)^2 + 2\alpha m(t)\right)\left(1+\cos(2\pi \cdot 2f_c\, t)\right) dt$$

$$\approx \int_{T_{eff}} \frac{A^2}{2}\left(1+\alpha^2 m(t)^2 + 2\alpha m(t)\right) dt$$

$$\approx \int_{T_{eff}} \left(\frac{A^2}{2} + \frac{\alpha^2 A^2 m(t)^2}{2}\right) dt$$

[For signal with no dc-component $\Rightarrow \int m(t)\, dt = 0$ (mean=0)]

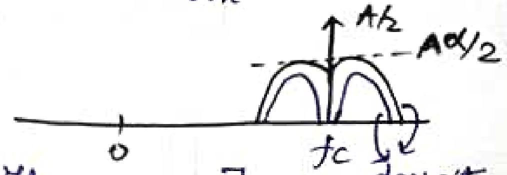


$$= \frac{A^2}{2} T_{eff} + \frac{\alpha^2 A^2}{2} E_m$$

Divide by Teff,

$$\boxed{\text{Tx Power} = \frac{A^2}{2} + \frac{\alpha^2 A^2}{2} P_m}$$

[Carrier power + scaled DSB power]

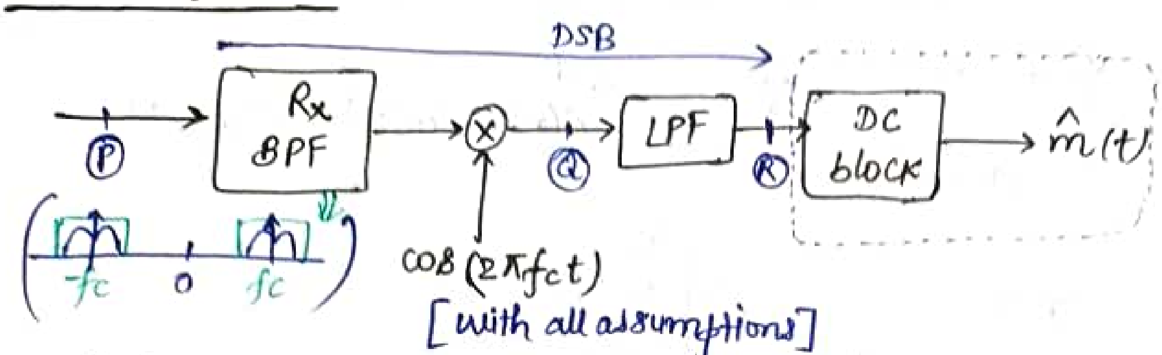


doesn't share freq.
⇓
Orthogonal

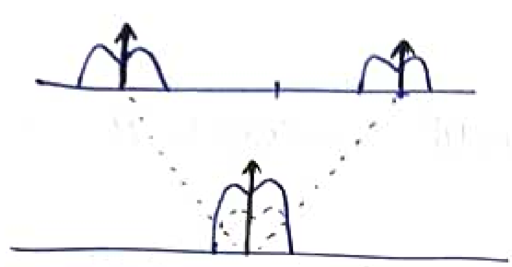
⇓
orthogonal
(doesn't share time)

23-08-2024

LCAM Receiver



For DSB:

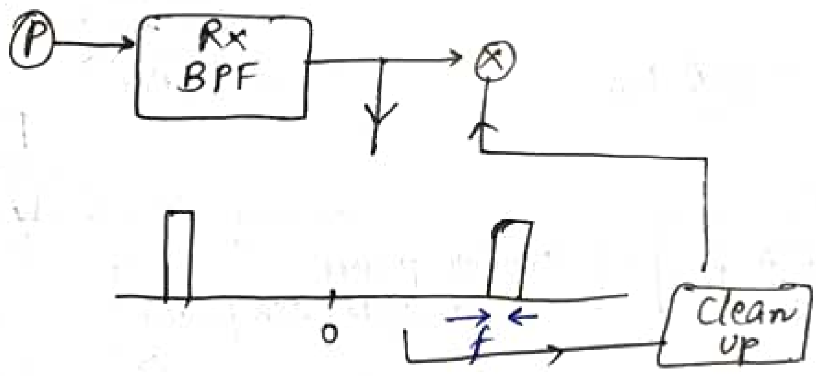


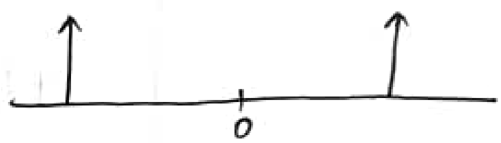
$P \Rightarrow A(1+\alpha m(t)) \cos(2\pi f_c t)$

$Q \Rightarrow A(1+\alpha m(t)) \cos^2(2\pi f_c t)$

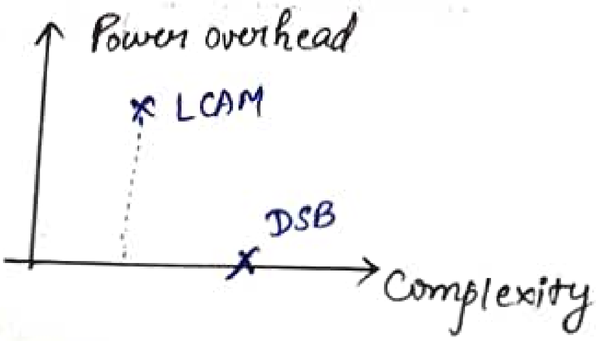
$R \Rightarrow \frac{A}{2} + \frac{A\alpha}{2} m(t)$

$\hat{m}(t) = \frac{\alpha A}{2} m(t)$.





$\cos(2\pi f_c(t - t_{propagation}))$



→ Pilot signals [in-band] ⇝ helps in demodulating using an envelope detector.

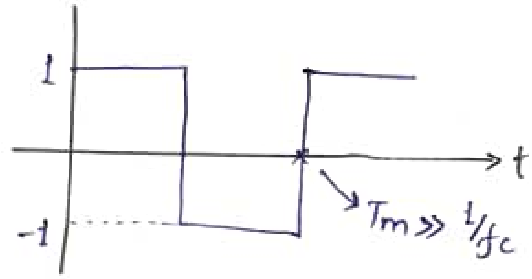
→ Out of band schemes
↳ Needs more BW

→ go out of band in the carrier
⇓
Then, frequency division

27-08-2024

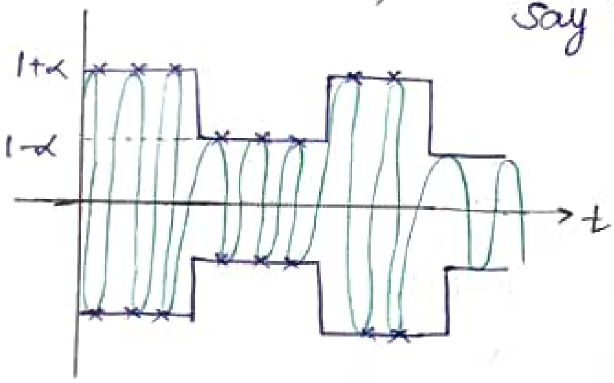
LCAM signal in time-domain :

Eg. Consider $m(t)$ to be

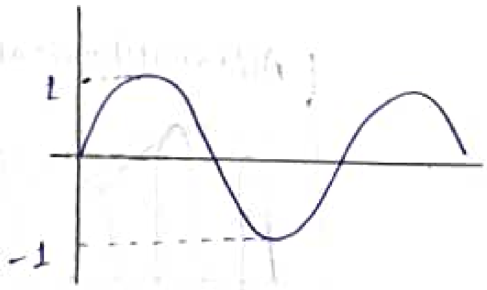


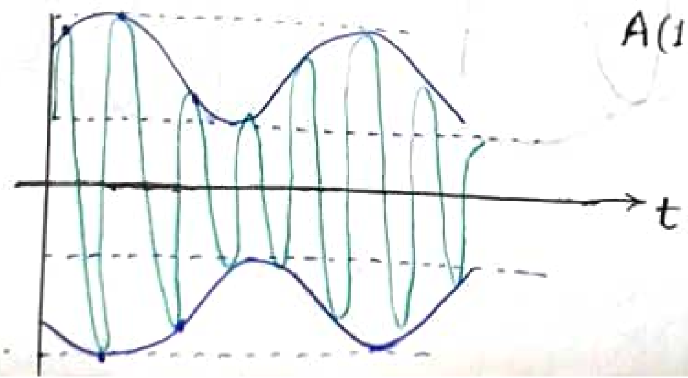
$A(1+\alpha m(t))\cos(2\pi f_c t)$

Say $\alpha < 1$



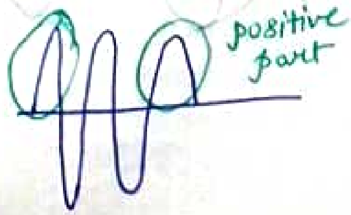
Eg. Consider $m(t)$ to be $\cancel{A_m}^{1} \sin(2\pi f_m t)$.



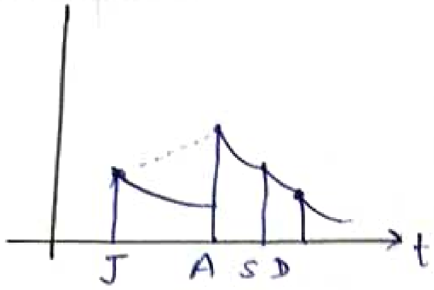


$A(1+\alpha m(t))\cos(2\pi f_c t)$

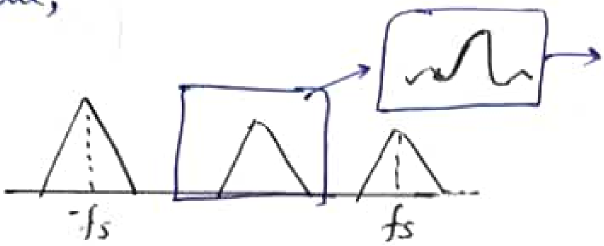
$\alpha < 1$



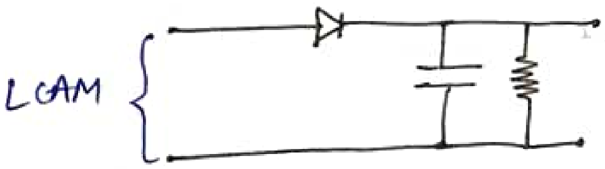
Interpolation:

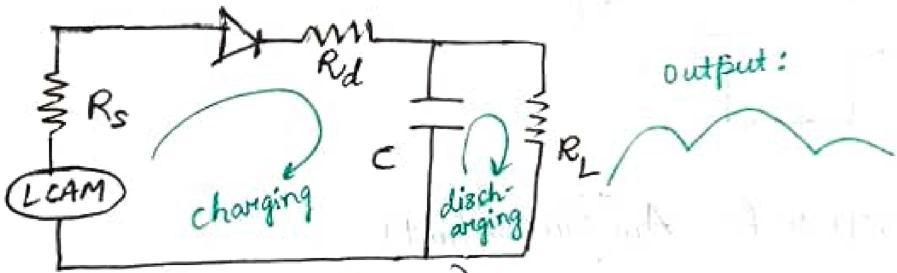


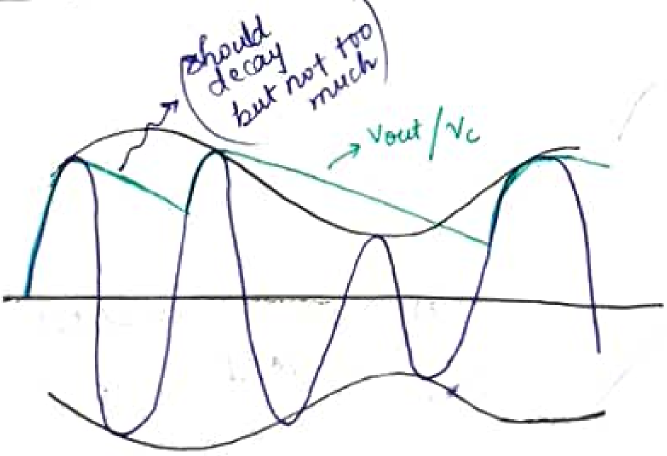
Recall,



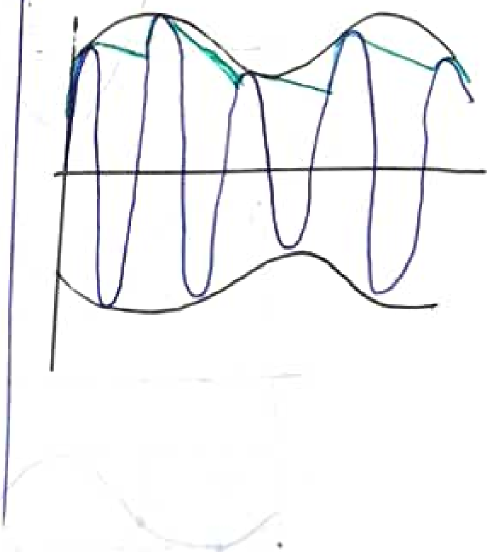
Envelope detector:







$A(1+\kappa m(t))\cos(2\pi f_c t)$:



$\rightarrow$ BPF $\rightarrow$ ED $\rightarrow$ Dc Block

} LCAM receiver with ED $\rightarrow$ coherent receiver
↓
less complex
↓
envelope based receiver

Constraints :

$$\underbrace{C(R_s + R_d)}_{\text{charging time constant}} \ll 1/f_c$$

$$\frac{1}{f_c} \ll \underbrace{CR_L}_{\text{disch. time constant}} \ll \underbrace{\frac{1}{B_m}}_{\text{time taken for modulating signal to change}}$$

$B_m \rightarrow$ (B/W of m(t))

Receiver :

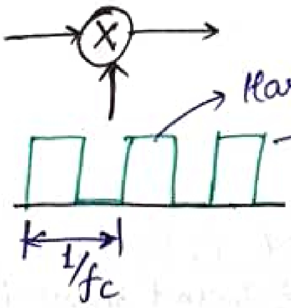
$\rightarrow$ BPF $\rightarrow \otimes \rightarrow$ LPF

$\uparrow$

LO | $\cos(2\pi f_c t)$

↳ coherent Receivers : Generate ~~of~~ frequency matched signals

A way to understand E.D :



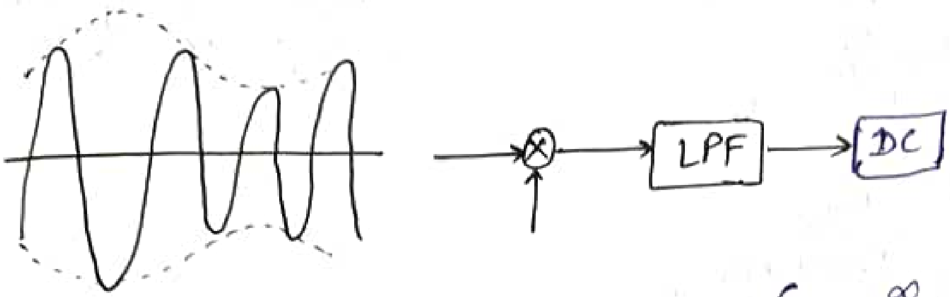
Harmonic

can be thought of diode generating it

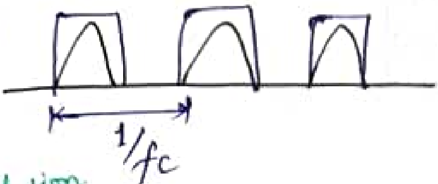
(28-08-2024)

Coherent (DSB)
Non-coherent (ED)
} Demodulation schemes

$R_x LO$

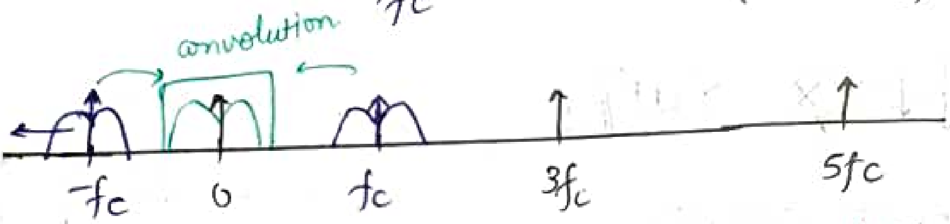


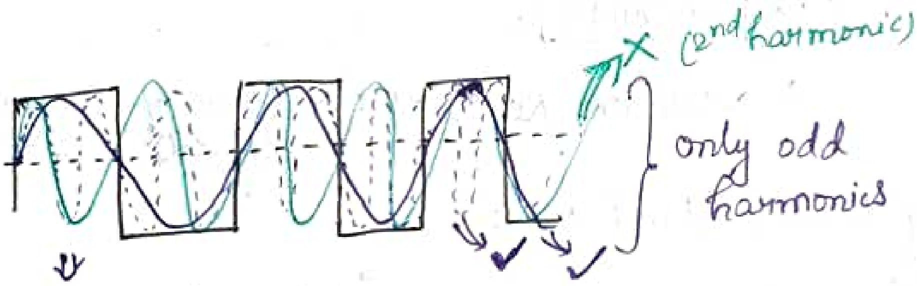
Plain AM (LCAM)

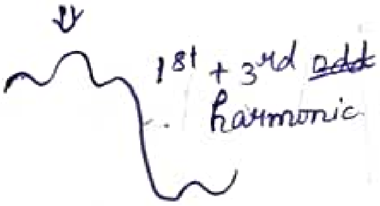


$$= \left(a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_c t + \phi_n)\right)$$

$$A(1 + \alpha m(t)) \cos(2\pi f_c t)$$



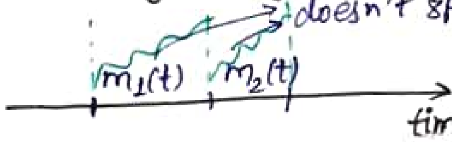




Multiplexing

Time division multiplexing (TDM):

doesn't share time ⇒ Time orthogonal

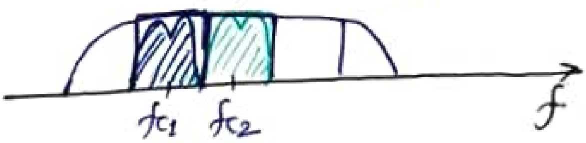


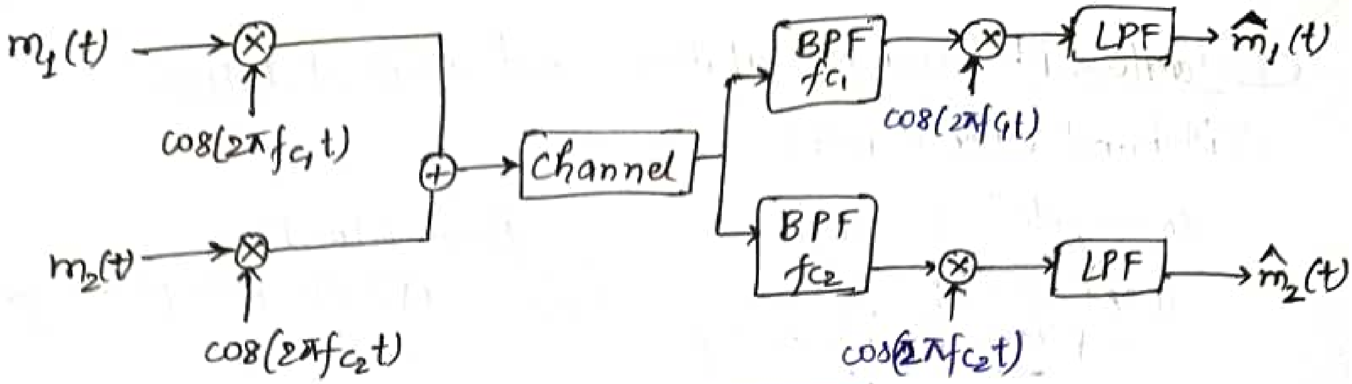
⇒ One BW, one channel

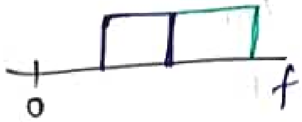
Frequency division multiplexing (FDM):

orthogonal

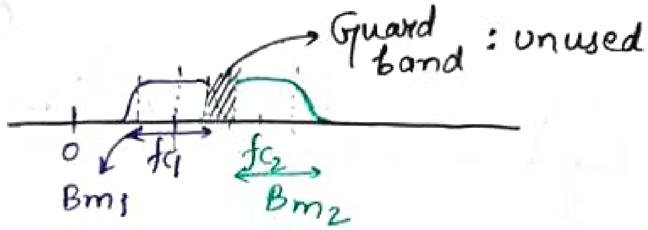
If two signals are orthogonal, they can be multiplexed.



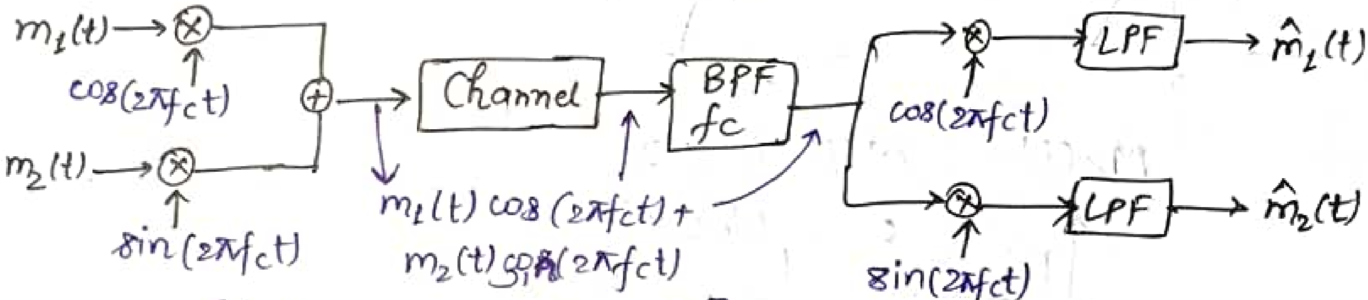




CDMA
OFDMA



Quadrature Carrier Multiplexing (QCM):



$$\left[\begin{array}{l} \frac{1}{2}\left(M_1(f-f_c)+M_1(f+f_c)\right)+ \\ \frac{1}{2j}\left(M_2(f-f_c)-M_2(f+f_c)\right) \end{array}\right]$$

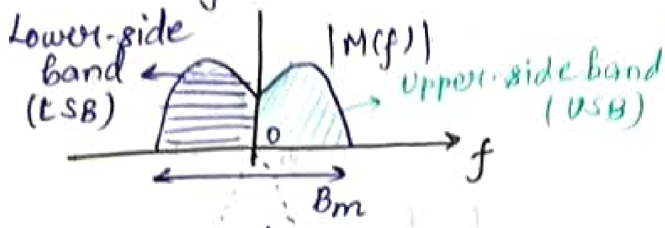
$$m_1(t)\cos^2(2\pi f_c t) + m_2(t)\cos(2\pi f_c t)\sin(2\pi f_c t)$$

$$= m_1(t)\left[\frac{1+\cos(2\pi 2 f_c t)}{2}\right] + m_2(t)\,\frac{\sin(2\pi 2 f_c t)}{2}$$

29-08-2024

Single sideband Modulation and Demodulation

Sideband signal $m(t)$

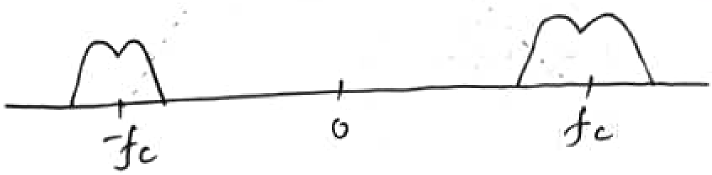


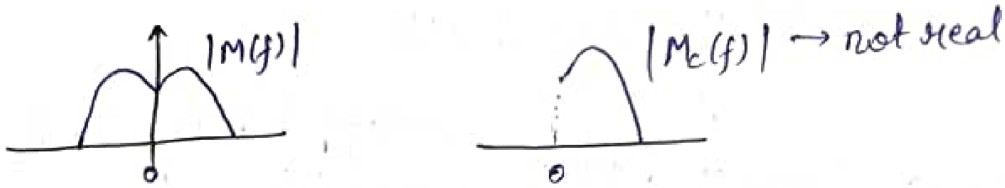
Real valued:

$M(-f) = M^*(f) \Rightarrow$ Symmetric

$\downarrow$ DSB

$A/2\,(M(f-f_c) + M(f+f_c))$





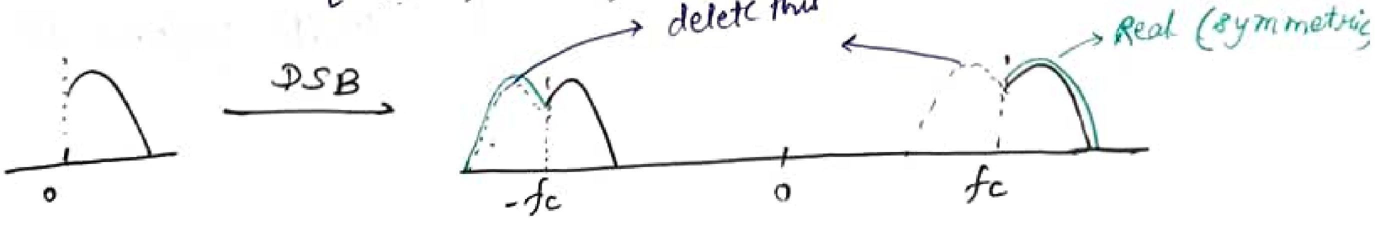
$$M_c(f) = \begin{cases} M(f) & \text{if } f \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

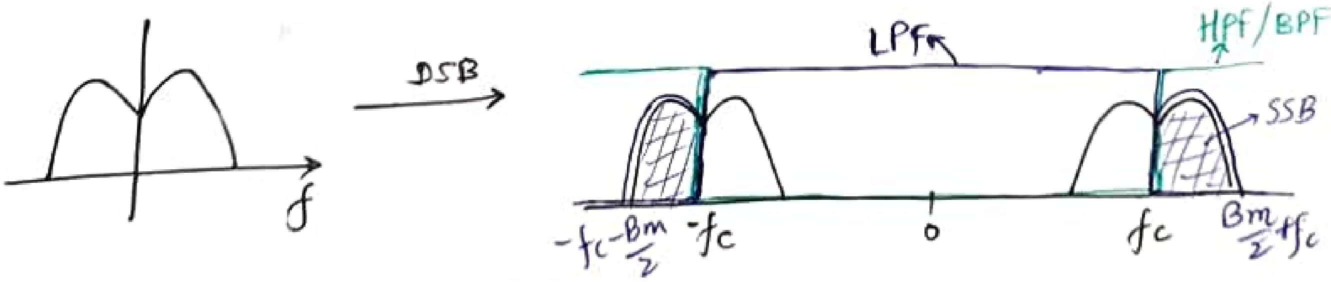
$\updownarrow F$

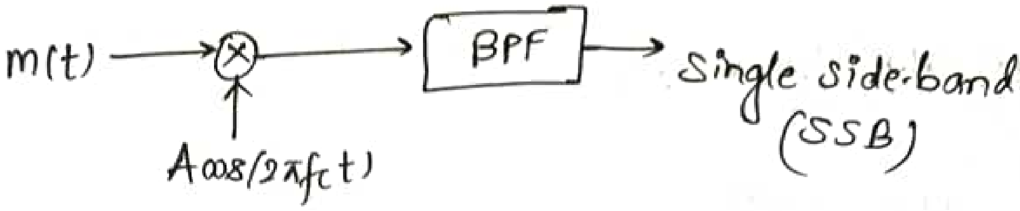
$m_c(t)$

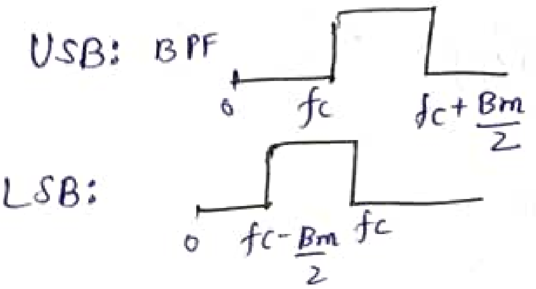
$m_c(t) \overset{\equiv}{\longleftrightarrow} m(t)$

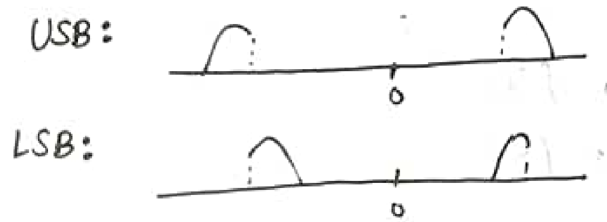
$$M(f) = \begin{cases} M_c(f) & \text{for } f \geq 0 \\ M_c^*(-f) & , f < 0 \end{cases}$$





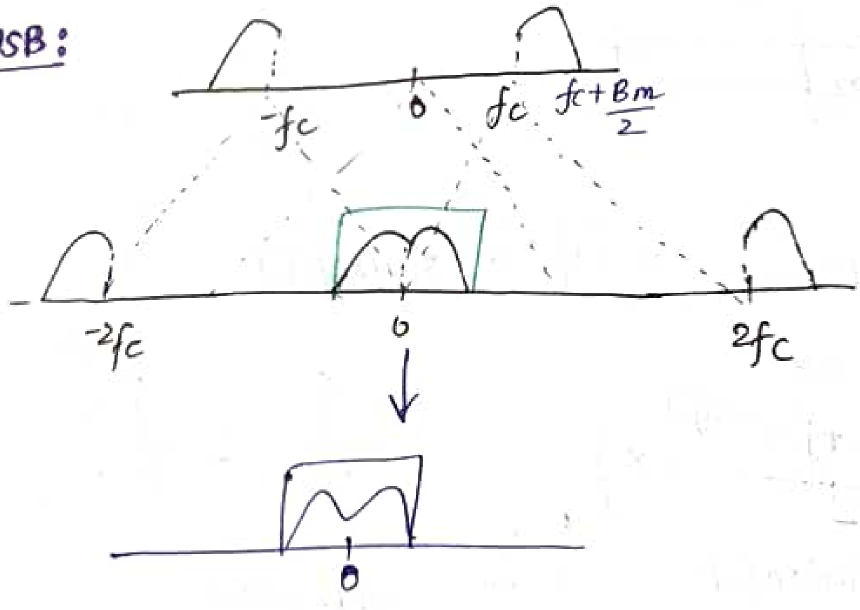






⇒ Tx power ?

for USB:



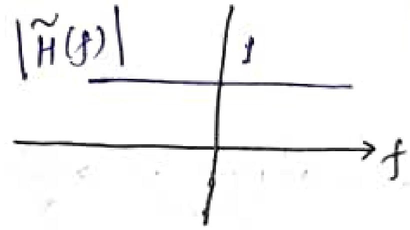
Hilbert Transform Filter

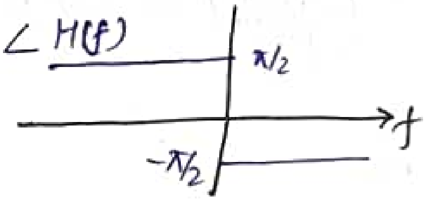
$$x(t) \longrightarrow \boxed{\tilde{h}(t)} \longrightarrow y(t)$$

$$\begin{array}{ccc} x(t) & \tilde{h}(t) & y(t) \\ \downarrow & \downarrow & \downarrow \\ X(f) & \tilde{H}(f) & Y(f) = X(f)\cdot\tilde{H}(f) \end{array}$$

$$\tilde{H}(f) = -j\,sgn(f) = \begin{cases} -j, & f \geq 0 \\ +j, & f < 0 \end{cases}$$

$$= \begin{cases} e^{-j\pi/2}, & f \geq 0 \\ e^{j\pi/2}, & f < 0 \end{cases}$$

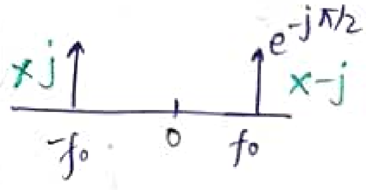




Eg. $\cos(2\pi f_0 t) \longrightarrow \boxed{\tilde{H}(f)} \longrightarrow \sin(2\pi f_0 t)$

$$\frac{(-j)\,e^{j2\pi f_0 t} + (j)\,e^{-2\pi f_0 t}}{2} \times \frac{j}{j}$$

$$= \sin(2\pi f_0 t)$$

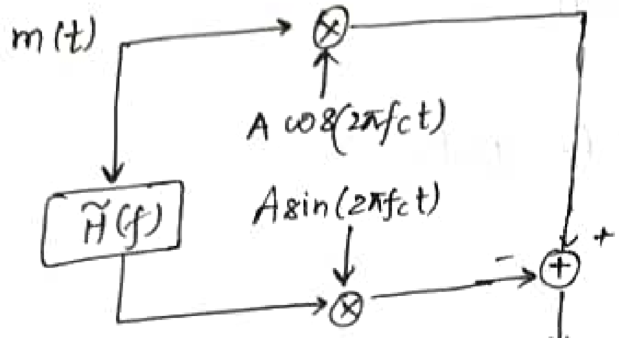


Eg. $\sum a_n \cos(2\pi f_n t) + \sum b_n \sin(2\pi f_n t) \longrightarrow \boxed{\tilde{H}(f)} \longrightarrow \sum a_n \sin(2\pi f_n t) + \sum b_n \cos(2\pi f_n t)$

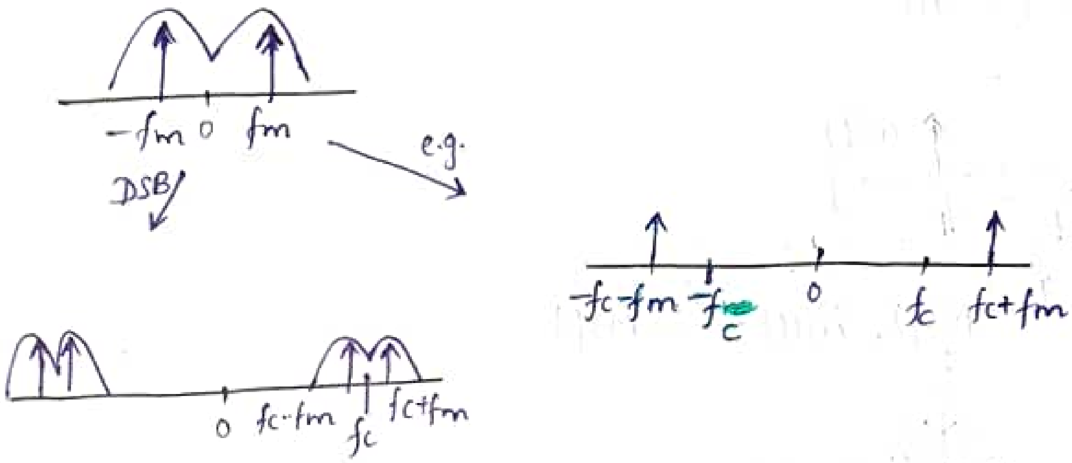
LTI filter

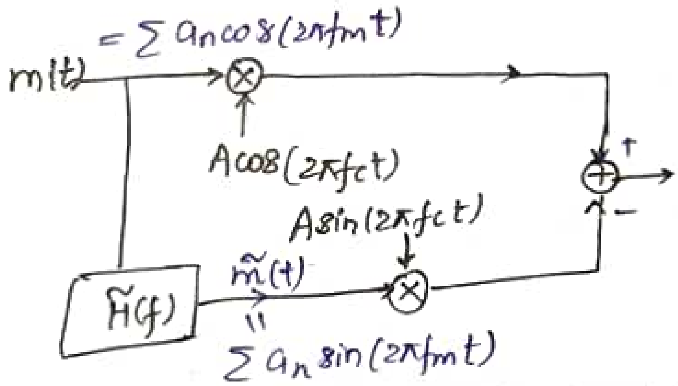
wideband phase shifter

Eg $m(t) = A_m \cos(2\pi f_m t)$



$A A_m \cos(2\pi(f_c+f_m)t)$ } upper-sideband of DSB (USB)

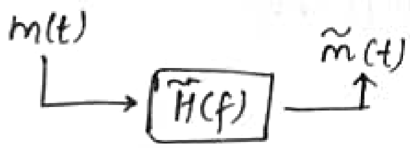




$USB = \sum a_n \cos(2\pi(f_c+f_m)t)$

: Phase Descrimination Method

1. Frequency descrimination method (using BPF)
2. Phase desc. method
3. Weaver's method (Assignment)

Eg: 

$m(t), \tilde{m}(t)$: real

Spectrum of $m(t) \pm j\tilde{m}(t)$

03-09-2024

→ $m(t)$ (real) $\longrightarrow \boxed{-j\,\text{sgn}(f)} \longrightarrow \tilde{m}(t)$ (real)

$m(t) * \tilde{h}(t)$

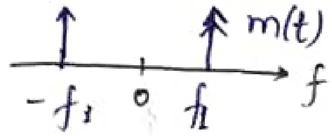
$\tilde{H}(-f) = \tilde{H}^*(f) \Rightarrow h(t)$ should be real.

$m(t) \pm j\tilde{m}(t) \longrightarrow F$

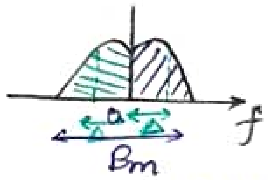
Eg. If $m(t) = \cos(2\pi f_1 t)$

$\cos(2\pi f_1 t) + j\sin(2\pi f_1 t)$

$= e^{j2\pi f_1 t}$



$m(t) \xleftrightarrow{F} M(f)$, $\tilde{m}(t) \xrightarrow{F} \tilde{M}(f)$



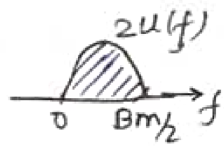
$M(f) = U(f) + L(f)$

$\tilde{M}(f) = -jU(f) + jL(f)$

Now, $m(t) \pm j\tilde{m}(t) \xrightarrow{F} M(f) + j\tilde{M}(f)$ =

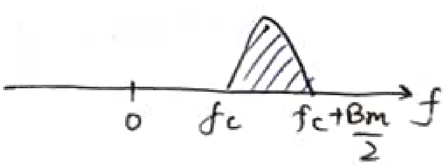
$= U(f) + \cancel{L(f)} + U(f) - \cancel{L(f)}$

$= 2U(f)$



Suppose we take

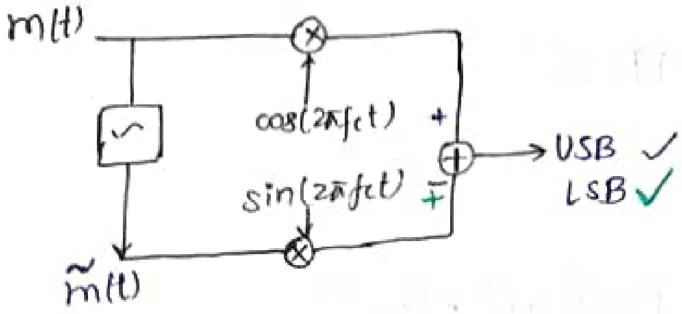
$(m(t) + j\tilde{m}(t)) \cdot e^{j2\pi f_c t} \xrightarrow{F}$



Suppose [Take real part of signal]

$Re\{(m(t) \mp j\tilde{m}(t)) \cdot e^{j2\pi f_c t}\} \cdot \cos(\;) + \sin(\cdot)$

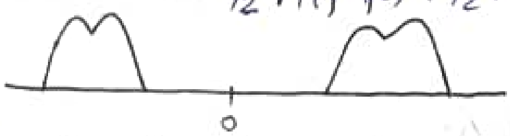
$= m(t)\cos(2\pi f_c t) \pm \tilde{m}(t)\sin(2\pi f_c t)$



$$F\left\{ m(t)\cos(2\pi f_c t) - \tilde{m}(t)\sin(2\pi f_c t) \right\}$$

$$= F\left\{ m(t)\cos(2\pi f_c t) \right\} - F\left\{ \tilde{m}(t)\sin(2\pi f_c t) \right\} \quad \Big\} \; U(f-f_c) + L(f+f_c)$$

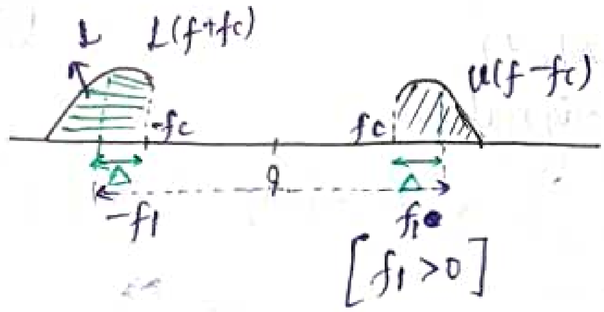
$$\tfrac{1}{2} M(f-f_c) + \tfrac{1}{2} M(f+f_c)$$



$$\tfrac{1}{2} U(f-f_c) + \cancel{\tfrac{1}{2} L(f-f_c)} + \cancel{\tfrac{1}{2} U(f+f_c)} + \tfrac{1}{2} L(f+f_c)$$

$$\frac{-\cancel{j}U(f-f_c) + jL(f-f_c)}{2\cancel{j}} - \frac{-jU(f+f_c) + jL(f+f_c)}{2\cancel{j}}$$

$$= \frac{-U(f-f_c)}{2} + \cancel{\frac{L(f-f_c)}{2}} + \cancel{\frac{U(f+f_c)}{2}} - \frac{L(f+f_c)}{2}$$



Frequency Modulation

Review on FM (PM ≡ FM)

In FM, how do we get the passband signal?

Instantaneous frequency of the signal,

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t) \quad \text{for } A\cos(\phi(t))$$
$$\text{or } A\cos(2\pi f_c t + \phi(t))$$

$$f(t) = K_f \cdot m(t)$$

K_f has units Hz/V

$\phi(0) = 0$

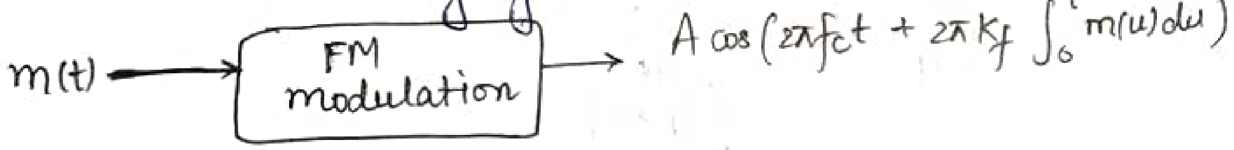
$$\phi(t) = \cancel{2\pi f}\; 2\pi K_f \int_0^t m(u)\,du$$

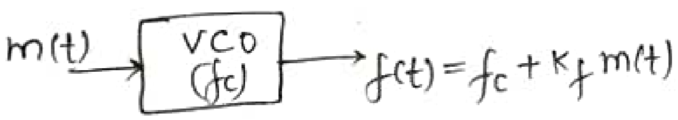
FM signal:

$$A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$$

05-09-2024

$$m(t) = A_m \cos(2\pi f_m t)$$
↳ modulating signal





→ FM allows change in BW.

Single tone → single frequency

Suppose $m(t) = A_m \cos(2\pi f_m t)$,

then FM signal : $A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$

$$= A\cos\left(2\pi f_c t + \cancel{2\pi} K_f A_m \frac{\sin(2\pi f_m t)}{\cancel{2\pi} f_m}\right)$$

$$= A\cos\left(2\pi f_c t + \frac{K_f A_m}{f_m}\sin(2\pi f_m t)\right)$$

$K_f A_m$: equal to max^m frequency deviation.

f_m : equal to BW

$$\boxed{\beta = \frac{k_f A_m}{f_m}}$$: modulation index

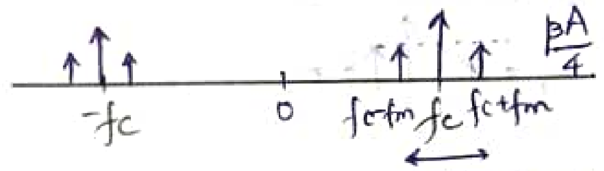
FM signal = $A\cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

$= A\cos(2\pi f_c t)\cdot\cos(\beta\sin(2\pi f_m t)) - A\sin(2\pi f_c t)\cdot\sin(\beta\sin(2\pi f_m t))$

$\beta \ll 1$: regime leads to narrow band FM signals.

$\cos\theta \approx 1$, $\sin\theta \approx \theta$

$= A\cos(2\pi f_c t) - A\sin(2\pi f_c t)(\beta\sin(2\pi f_m t))$



$2f_m$: one-sided B/W

$$\boxed{\text{Power} = \frac{A^2}{2}}$$

(since β is very small, so we can ignore power count. from $\beta A/2$)

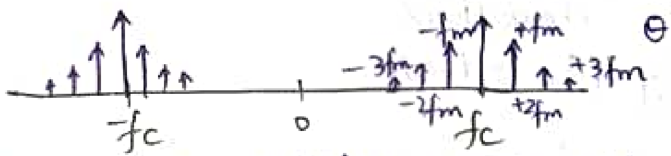
Suppose β is not small,
then $\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}, \dots$, Here $\theta = \beta\sin(2\pi f_m t)$

$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \dots$

$\theta^2 = \beta^2 \sin^2(2\pi f_m t)$

$= \beta^2 \frac{(1-\cos(4\pi f_m t))}{2}$

$\theta^3 = \frac{\beta^3}{2}[\sin(2\pi f_m t + (.) f_m + (.) 3f_m$



Theoretically, FM has infinite BW.

Effective BW: (for any β)

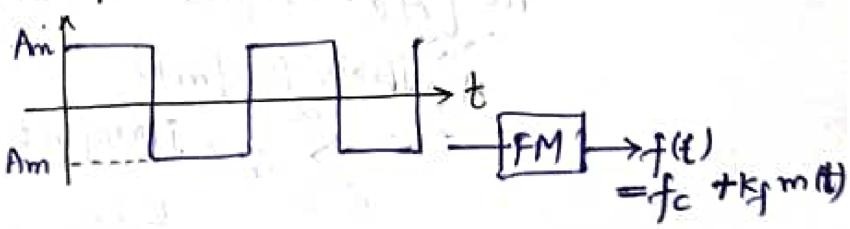
Onesided-BW $= 2(f_m + \Delta f)$ ← $k_f + A_m$

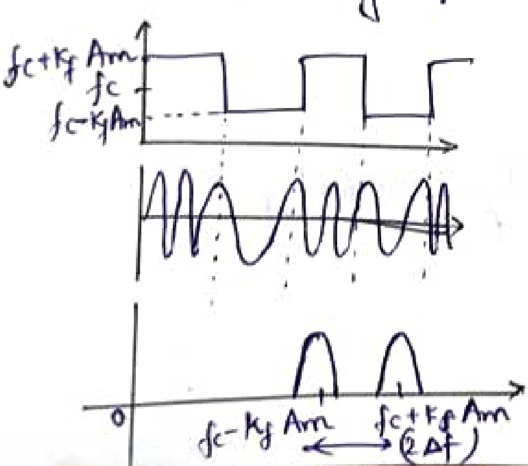
↳ Carson's rule for $m(t) = A_m\cos(2\pi f_m t)$

→ For smaller β, $\Delta f = k_f A_m$ will be very small,

so, $BW = 2f_m$ ($\beta \ll 1$ case)

→ For larger β, $m(t)$:





06-09-2024

FM Signal Analysis

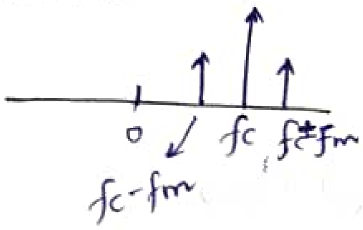
$$A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$$

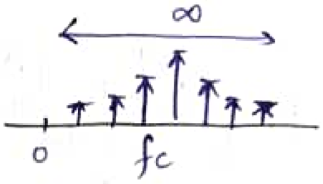
What is the $F(\cdot)$ of this?

$$\boxed{m(t) = A_m \cos(2\pi f_m t)}$$

(NBFM) $\beta \ll 1$ — Narrow Band Freq. Modulation

$\beta \not\ll 1$ (WBFM) → wideband F.M. $\Rightarrow$ B/W $\approx \infty$





Carson's Rule:

FM B/W is [one-sided]

$$2\underbrace{\Delta f}_{K_f A_m} + 2f_m$$

$K_f A_m$ (max. frequency deviation)

Motivation for why this is true.

F.M. signal for $m(t) = A_m\cos(2\pi f_m t)$

$$\boxed{A\cos\left(2\pi f_c t + \beta\sin(2\pi f_m t)\right)}$$ → not periodic in general

$\left.\begin{matrix} f_c \cdot T_p, \\ f_m \cdot T_p \end{matrix}\right\} \in \mathbb{Z}_+ \Rightarrow$ Periodic

$$\text{Re}\left\{A e^{j2\pi f_c t} e^{j\beta\sin(2\pi f_m t)}\right\}$$

$$A e^{j\beta\sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

$$c_n = A f_m \int_{-1/2f_m}^{1/2f_m} e^{j\beta\sin(2\pi f_m t)} \cdot e^{-j2\pi n f_m t}\,dt$$

Take $u = 2\pi f_m t$

$$C_n = \frac{A}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin(u) - nu)}\,du$$

$$\left[\ \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin u - nu)}\,du \right.$$

$\Downarrow$

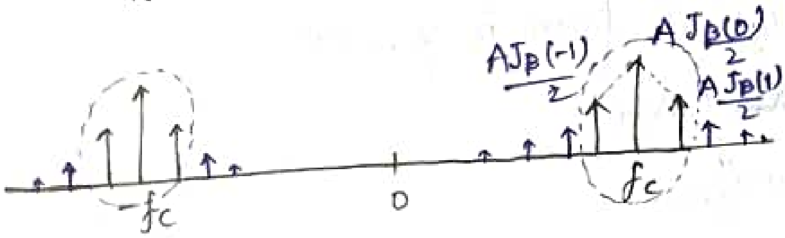
Bessel f^n: $J_\beta(n) \longrightarrow n^{th}$ order, 1^{st} kind $\left.\right]$

$$\therefore\ C_n = A\,J_\beta(n)$$

$$\sum_{n=-\infty}^{\infty} A\,J_\beta(n)\,e^{j2\pi n f_m t}$$

$$(*) = Re\left\{\sum_{n=-\infty}^{\infty} A\,J_\beta(n)\,\underbrace{e^{j2\pi n f_m t}\cdot e^{j2\pi f_c t}}_{e^{j2\pi(f_c + f_m\cdot n)t}}\right\}$$

$$= \sum_{n=-\infty}^{\infty} A\,J_\beta(n)\cos(2\pi(f_c + nf_m)t)$$



$\underline{\beta \ll 1:}$

$J_\beta(0) = 1$

$J_\beta(1) = \beta/2$

$J_\beta(n) \approx 0$

$$\sum_{n=-\infty}^{\infty} J_\beta^{\,2}(n) = 1 \Rightarrow \text{Power} = \sum_{n=-\infty}^{\infty} \frac{A^2 J_\beta^{\,2}(n)}{2}$$

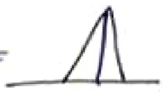
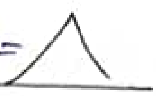
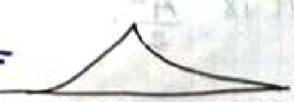
$$\therefore \text{FM power} \longrightarrow \boxed{A^2/2}$$

$$A\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(u)\,du\right) \rightarrow i(t)$$

$$= A\cos(2\pi f_c t + \beta i(t))$$

$$\cos(2\pi f_c t) - \cos(\beta i(t))$$

$$-\sin(2\pi f_c t) - \sin(\beta i(t))$$

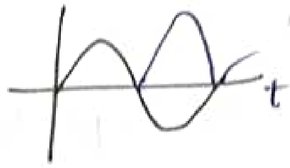
$i(t) =$  $i^2(t) =$  $i^3(t) =$ 

For a general signal $m(t)$,

$$2\Delta f + 2(B_m/2)$$

$\downarrow$

$$K_f \, \frac{\max|m(t)|}{t}$$



10-09-2024

FM - transmitter

(modulation schemes)

Recall

FM signal: $A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$

Spectrum

$\hookrightarrow$ no analytical form in general

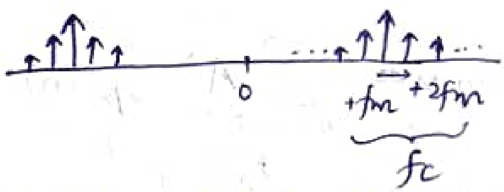
$m(t) = A_m \cos(2\pi f_m t)$

$$\boxed{\beta = K_f A_m / f_m}$$

$\beta \ll 1$: NBFM — like DSB + carrier

$\beta \not\ll 1$: WBFM

$$\sum A J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$



Carson's Rule: $\underline{2\Delta f} + 2f_m$

$\downarrow$

$K_f A_m$: max^m freq. deviation

General $m(t) \longrightarrow B_m$ (2 sided)

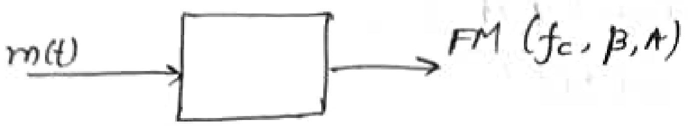
$$\beta = \Delta f / (B_m/2)$$

$$2\Delta f + B_m$$

Power is $\frac{A^2}{2}$.

FM modulators → Direct methods
f_c, β, A → Indirect methods

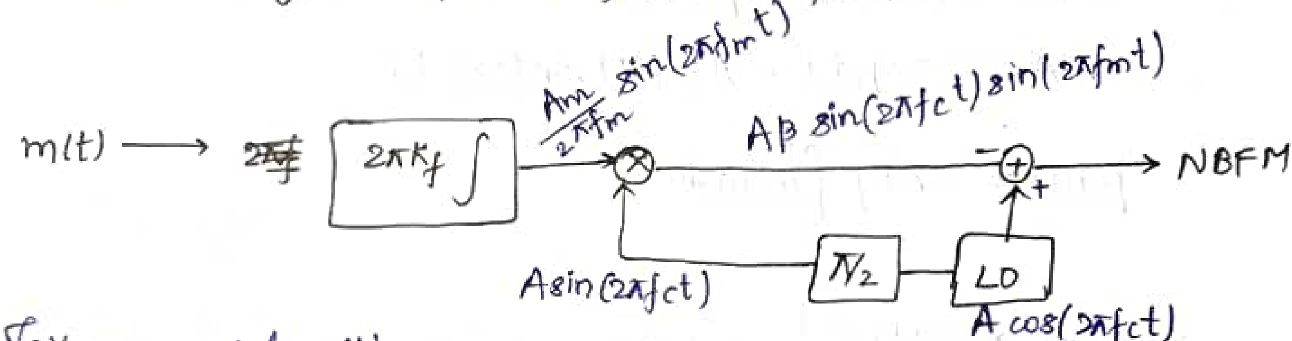
Indirect methods



How can you generate a NBFM signal?

Eg. For $A_m \cos(2\pi f_m t)$ as $m(t)$

$$A\cos(2\pi f_c t) - A\beta \sin(2\pi f_c t)\sin(2\pi f_m t)$$



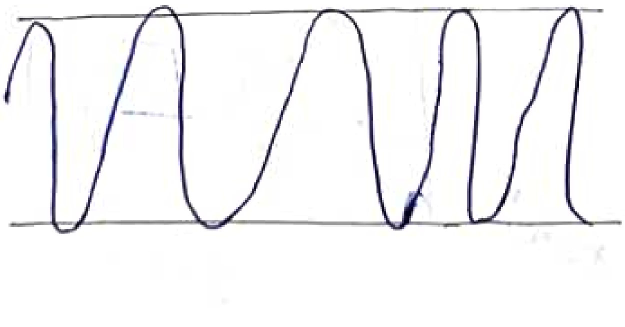
For a general m(t),

$$A\cos\left(2\pi f_c t + \underbrace{2\pi k_f \int_0^t m(u)\,du}_{\to \frac{A_{max}}{\pi B_m}}\right)$$

$$A\cos\left(2\pi f_c t + \overset{\beta}{\frac{k_f A_{max}}{(B_m/2)}} i(t)\right)$$

$$A\left(\cos(2\pi f_c t)\cdot\underbrace{\cos(\beta i(t))}_{1} - \sin(2\pi f_c t)\cdot\underbrace{\sin(\beta i(t))}_{\beta i(t)}\right) \quad \text{if } \beta \ll 1$$

FM signals have constant envelope.



11-09-2024

FM Transmitter / Signal Generation $\langle$ direct, indirect

FM signal : Tx power, BW, Band channel
(A) (β) (f_c)

$\beta \ll 1$: Narrowband FM
$\beta \not\ll 1$: wideband FM

NBFM: $m(t)$

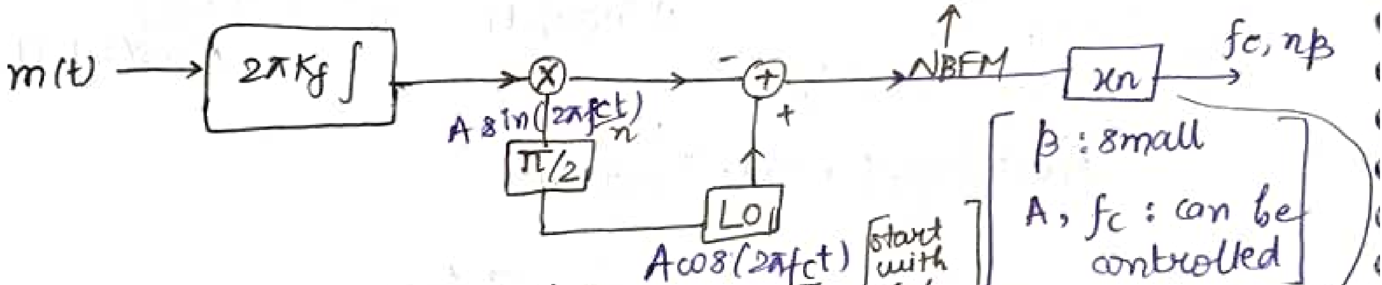
$$A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$$

$$= A\cos\left(2\pi f_c t + \beta i(t)\right)$$

$$= A\cos(2\pi f_c t) - A\beta i(t)\sin(2\pi f_c t)$$

$|i(t)| < \beta$

$$\beta i(t) = 2\pi K_f \int_0^t m(u)\,du$$



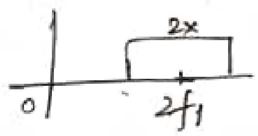
[start with fc/n]

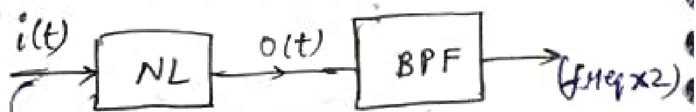
[β : small
A, f_c : can be controlled]

Indirect method (Edwin Armstrong Method) : (to compensate for increased f_c)

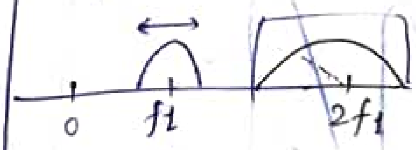
$$\underbrace{A\cos\left(2\pi f_c t + \frac{\Delta f}{B_m/2}\, i(t)\right)}_{\phi(t)}$$

Filter X
Any kind of a non-linear device





$$o(t) = a_0 + a_1 i(t) + a_2 i(t)^2 + a_3 i(t)^3 + \dots$$



one-sided spectra

$\rightarrow \boxed{\times 2} \dots \boxed{\times 2} \rightarrow \times 2^n$ (n)

$\rightarrow \boxed{\times 3} \dots \boxed{\times 3} \rightarrow \times 3^n$

Example of an indirect FM generation:

$m(t) = \frac{B_m}{2} = 10\,KHz$

↓

NBFM ← $200\,KHz = f_c$

$\Delta f = 25\,Hz \Rightarrow \beta = 2.5m\,(\ll 1)$

↓

$\times 64$ ⟶ FM signal center freq. $= 12.8\,MHz$

$\beta = 0.16 \left(= 64 \times \frac{25}{m}\right)$

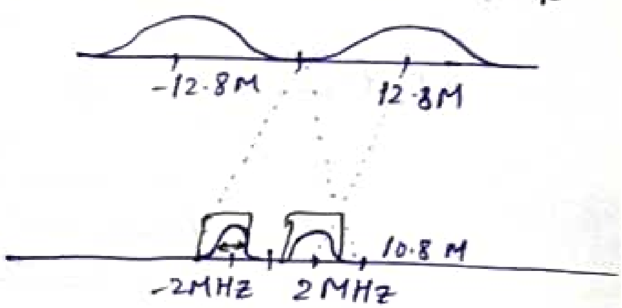
↓

$\otimes$ ← LO $(10.8\,MHz)$

(Mixing) for freq. translation

WBFM:

$BW \approx 2 \times \Delta f + 2\left(\frac{B_m}{2}\right)$
$= 3200 + 20K$



↓

BPF @ 2MHz

↓

$\times 48$

↓

FM signal $f_c = 2\,MHz \times 48$
$= 96\,MHz$

β is now 7.6 $(= 0.16 \times 48)$

$\Delta f = \beta \times \frac{B_m}{2} \approx 80\,KHz$

12-09-2024

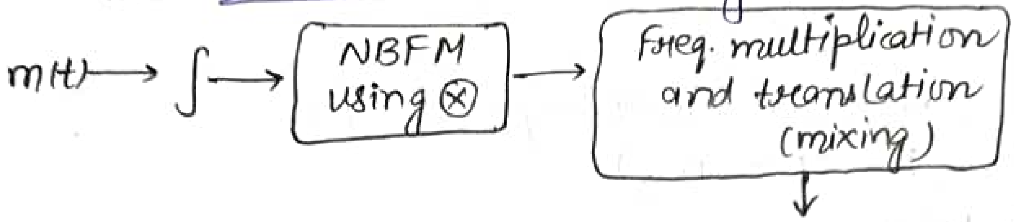
FM generation

need Tx power — A

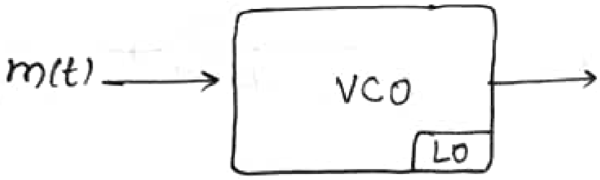
BW — β

Band/Channel — f_c

Indirect - method (Armstrong's):



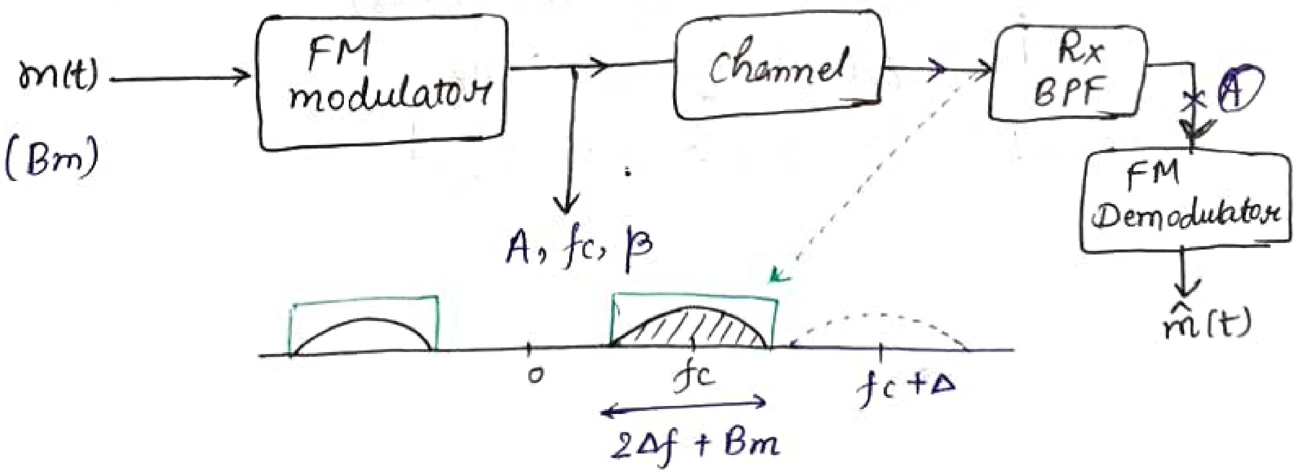
Direct method:



$$A\cos(\underbrace{2\pi f_f\, t + \psi(t)}_{\phi(t)})$$

f_f : free running frequency of the vco $(m(t)=0)$

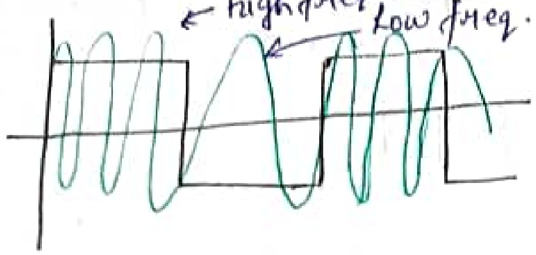
$$\frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_f + \underbrace{\frac{1}{2\pi}\frac{d}{dt}\psi(t)}_{K_f m(t)}$$



FM-demodulation:

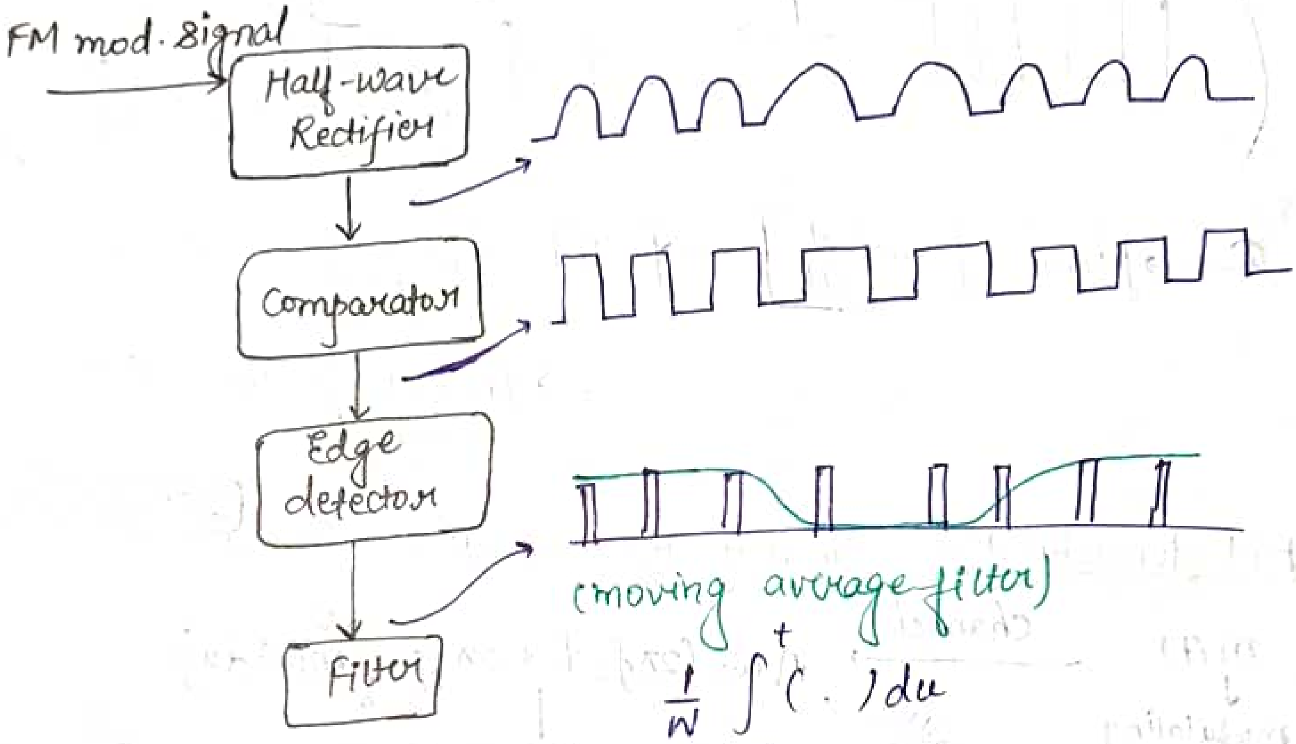
$$A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$$

Eg: $m(t) \rightarrow$ square wave



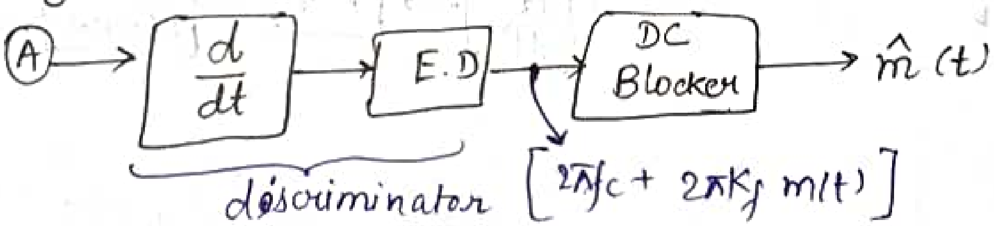
Get the square wave from sinusoids.

(Pulse/Peak) Counting method for FM demodulation:



$$\frac{1}{W}\int_{t-W}^{t}(\,.\,)\,du$$

Using differentiator:

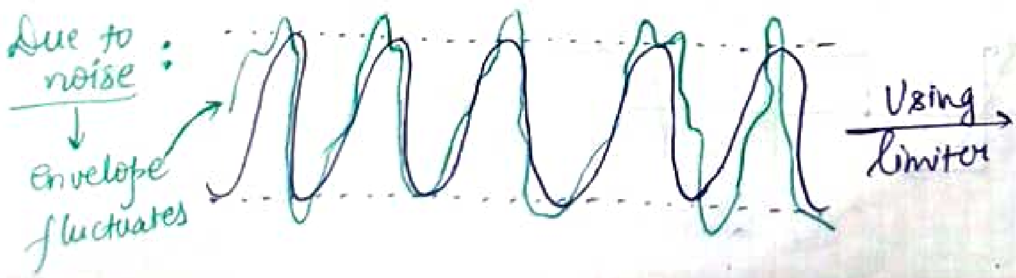


$\left[2\pi f_c + 2\pi K_f\, m(t)\right]$

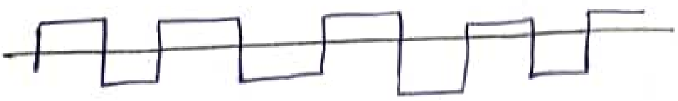
$$\frac{d}{dt}\left(A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)\right)$$

constant envelope

$$= A\sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right) \times \left(\left[2\pi f_c + 2\pi K_f\, m(t)\right]\right)$$

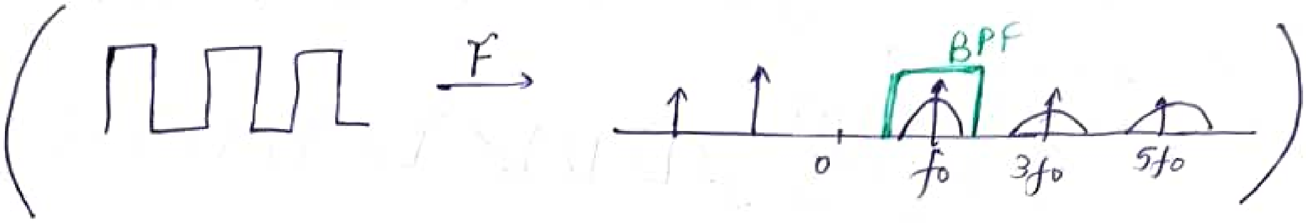


↓ (Using limiter)
↳ clips/removes the noise ⇒ Produce the signal which has zero crossing
⇓
Extracts the signal which has constant envelope



↓

[BPF] → 



Ⓐ → [Limiter] → $\left[\frac{d}{dt}\right]$ → [E.D] → [DC blocker] → $\hat{m}(t)$

⇒ Limiter-discriminator method

$\boxed{20\text{-}09\text{-}2024}$

FM demodulation : Limiter-discrimination method:

$m(t)$ $\xrightarrow{\text{channel}}$ $A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$

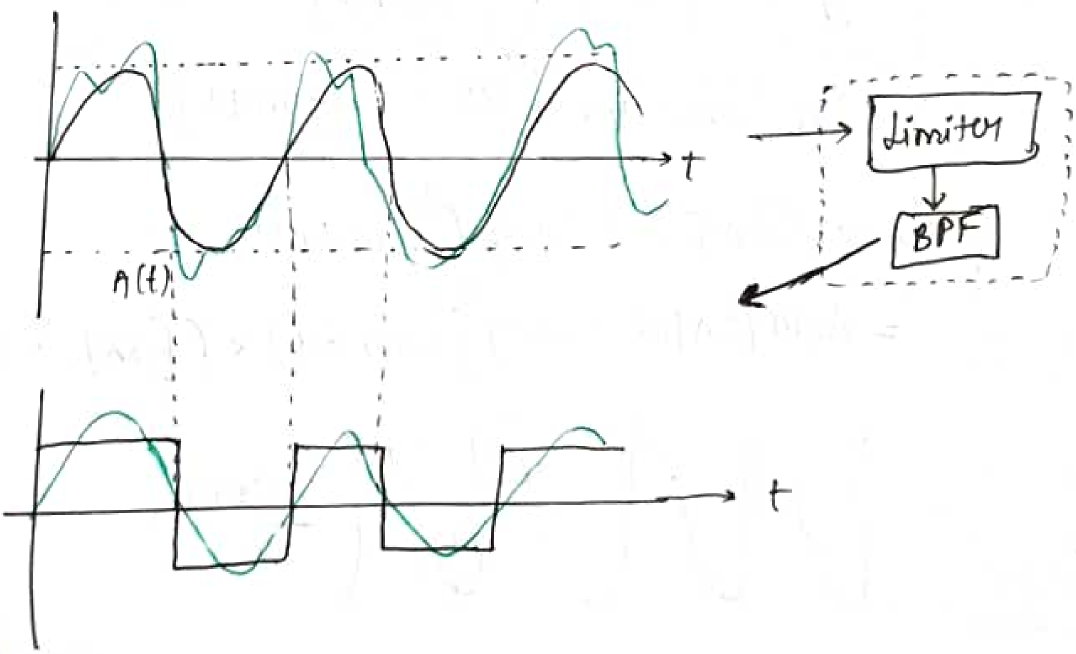
↓ modulating signal

$\underbrace{\left(2\pi f_c + 2\pi K_f\, m(t)\right)}_{\text{envelope}} A \sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$

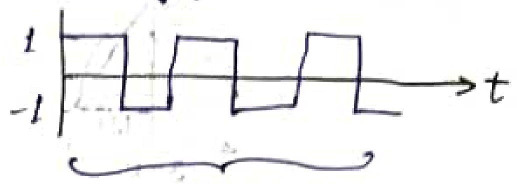
ideally constant envelope

↓

$\left[\frac{d}{dt}\right]$ → [E.D.] → [DC Blocker] → $\hat{m}(t)$

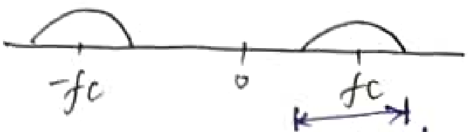


$i(t) = A\cos\left(2\pi f_c t + 2\pi K_f \int m(u)\,du\right) \longrightarrow \boxed{sgn(\cdot)} \longrightarrow o$

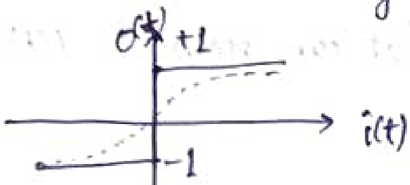


$\underbrace{\qquad\qquad}_{\text{Spectrum?}}$

F



given by Carson's rule

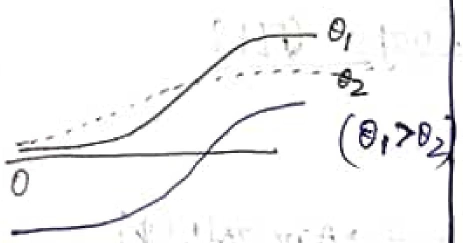


Non-linear : (Mapping)

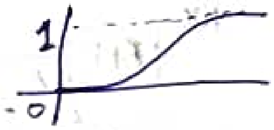
$o(t) = a_0 + a_1 i(t) + a_2 i(t)^2 + a_3 i(t)^3$

Approximation :

$$\left(\frac{1}{1+e^{-\theta i}} - \frac{1}{2}\right)2$$



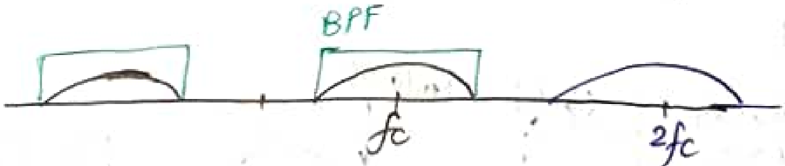
$\frac{1}{1+e^{-\theta i}}$



⇓

sigmoid function

(Activation f^n)



$\boxed{01\text{-}09\text{-}2024}$

FM : Phase Locked Loop Demodulation

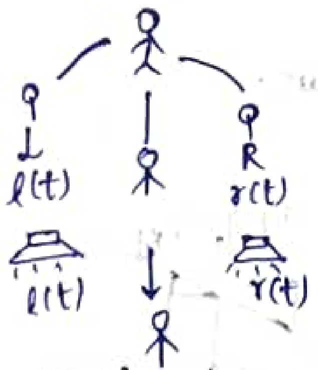
$m(t) \rightarrow \boxed{\text{modulation}} \rightarrow \boxed{\text{Channel}}$

↓

voice signal (baseband)

Reading Assignment :

Broadcast FM → Upamanyu Madhav book



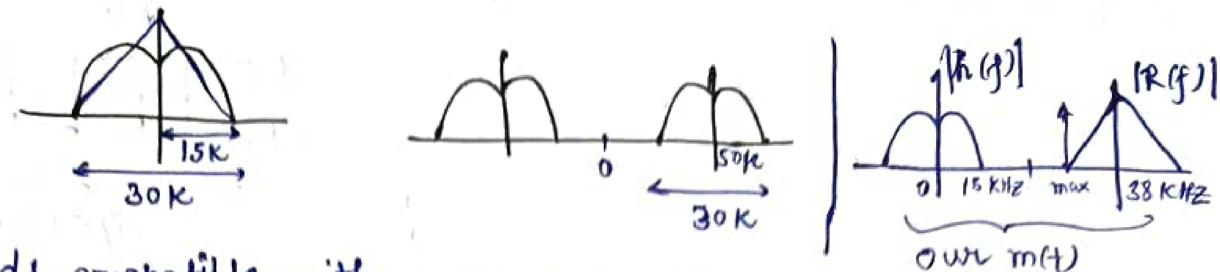
Stereo signals :

$\underbrace{l(t),\ r(t)}_{15\text{ kHz}}$: two baseband signals

(1 sided BW)

→ Modulator needs a baseband signal.

An idea:



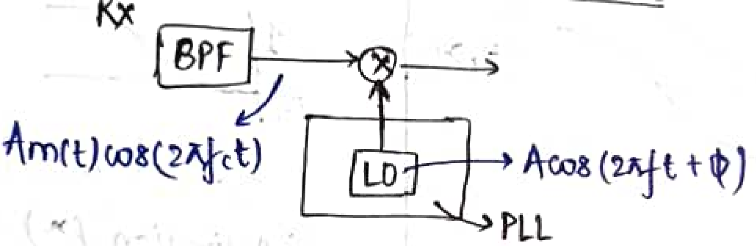
→ Backwards compatible with a mono receiver.

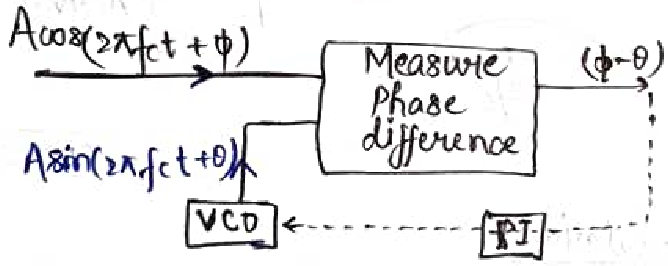
In the actual stereo-FM system

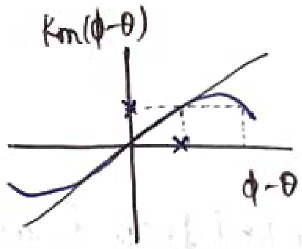
$$l(t) + r(t) = u(t)$$
$$l(t) - r(t) = v(t)$$

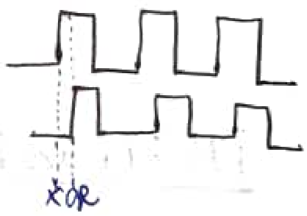
Eg. Draw and discuss a block diagram that can recover $l(t)$ and $r(t)$ from the received FM signals.

Phase Locked Loops (PLL)









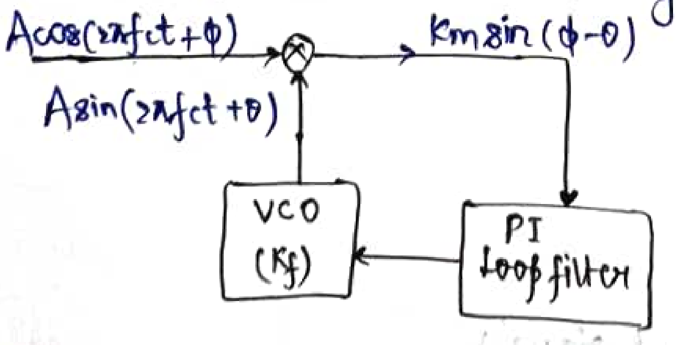
Using mixer/multiplier,

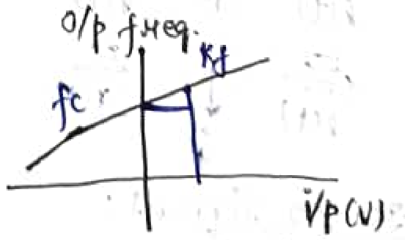
$$A\cos(2\pi f_c t + \phi)\, A\sin(2\pi f_c t + \theta)$$

$$A^2 \sin(\phi - \theta) + A^2 \sin(4\pi f_c t + \phi + \theta)$$

↳ Not linear function of $(\phi - \theta)$

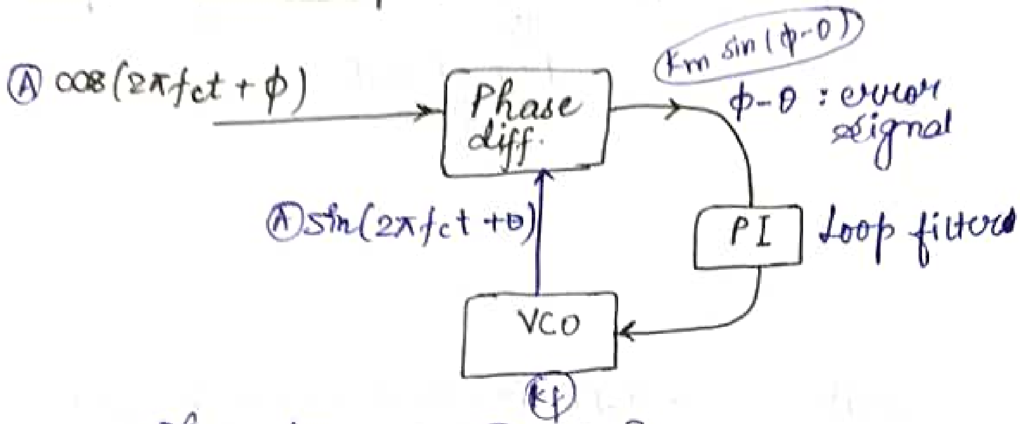
for $2\phi \neq \theta$ values, we get some value.





09-10-2024

Phaselocked Loops



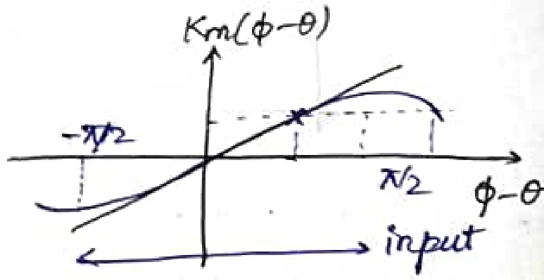
Phase diff. $\longrightarrow \otimes \longrightarrow$?

$$\cos(2\pi f_c t + \phi) \cdot \sin(2\pi f_c t + \theta)$$
$$= \boxed{\sin(\phi - \theta)} + \sin(2\pi(2f_c t) + \phi + \theta)$$

Ideal phase differentiator: memoryless

$\cos(2\pi f_c t + \phi) \longrightarrow \boxed{\otimes} \longrightarrow \phi - \theta$

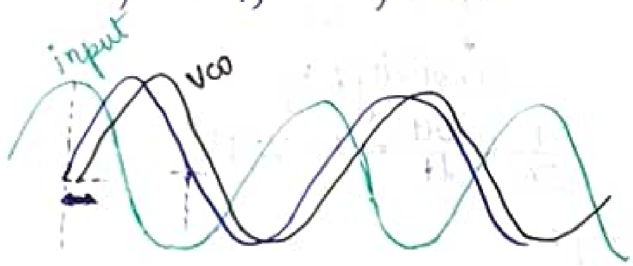
$\uparrow \sin(2\pi f_c t + \theta)$

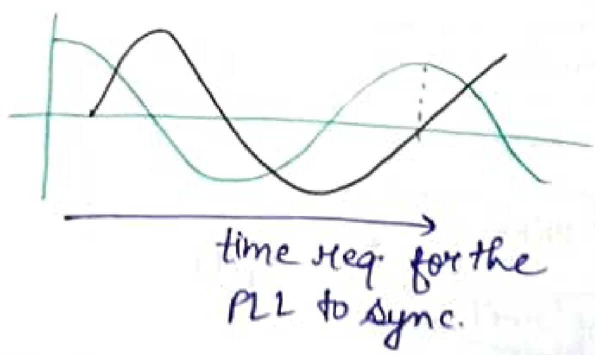


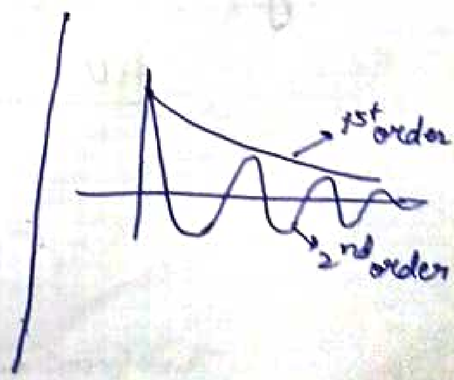
VCO

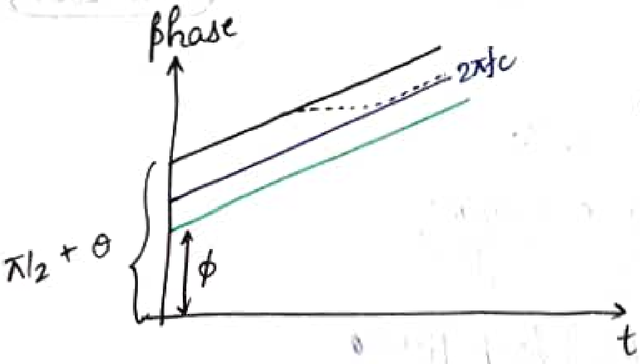
$v_i(t) \longrightarrow \boxed{VCO} \longrightarrow \sin(2\pi f(t) \cdot t)$

$$f(t) = f_f + K_f \cdot V_i(t)$$





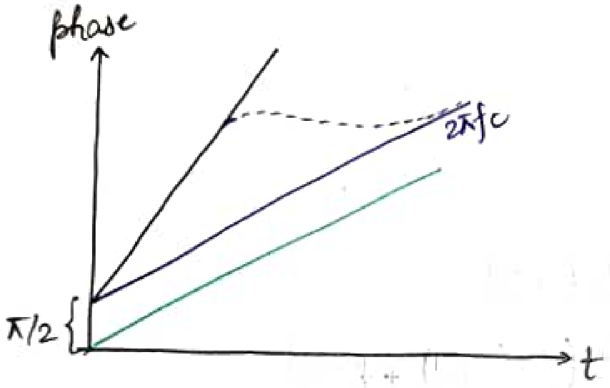




→ PLL is able to track change in phase

⇓

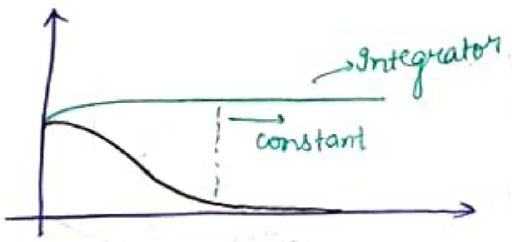
Proportional filter

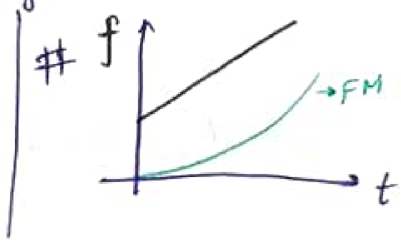


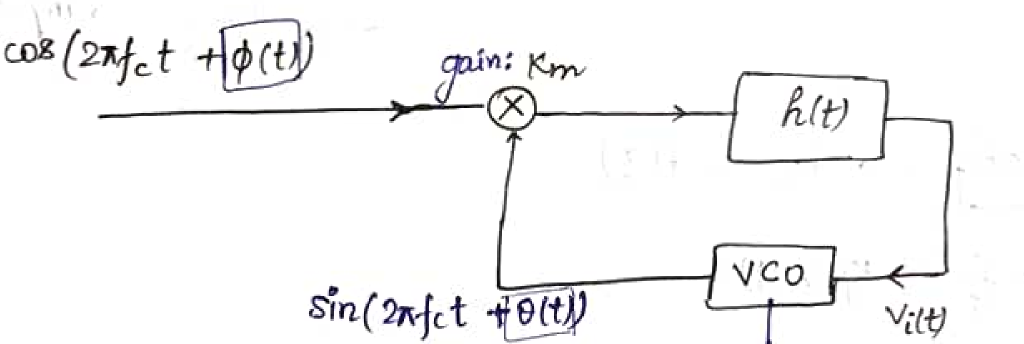
→ PLL is able to track change in frequency

⇓

Integrator

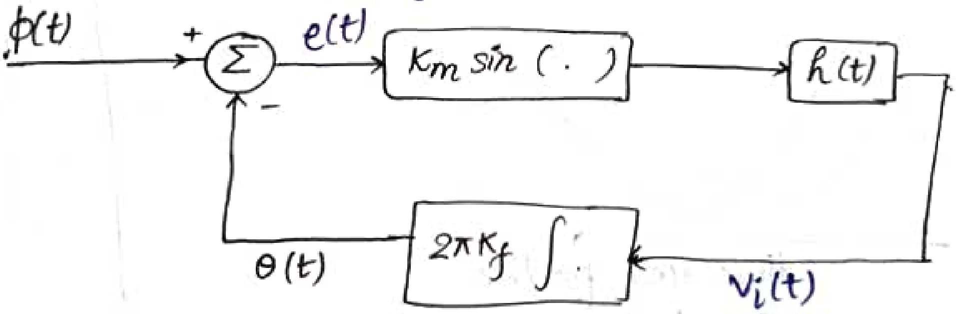






$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = K_f \cdot V_i(t)$$

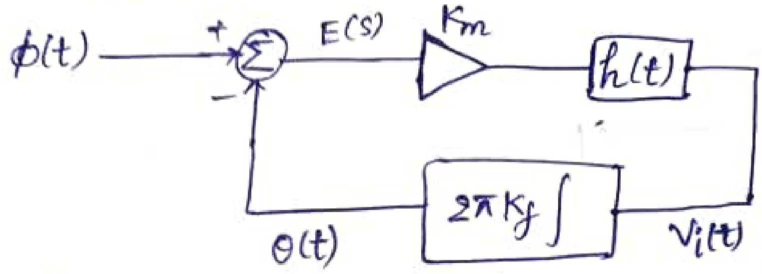
Signals which are interesting:



$$e(t) = \phi(t) - \theta(t)$$
$$= \phi(t) - 2\pi K_f \int v_i(t)\,dt$$
$$= \phi(t) - 2\pi K_f \int_0^t \left(\int_0^u h(\tau).\,K_m \sin(e(u-\tau)).\,d\tau \right).\,du$$
$$\Rightarrow \frac{de(t)}{dt} = \frac{d\phi(t)}{dt} - 2\pi K_f \int_0^t \left(h(\tau).\,K_m \boxed{\sin(e(t-\tau))}\right) d\tau$$

Small error analysis:

$e(t) \approx 0.$



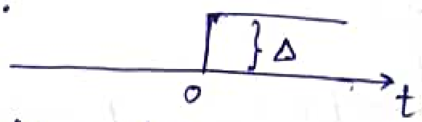
Laplace domain : $\Phi(s)$, $\Theta(s)$, $H(s)$.

$$E(S) = \Phi(s) - \Theta(s)$$
$$= \Phi(s) - \frac{2\pi K_f K_m}{S}.\,H(S)\,E(S)$$
$$\Rightarrow E(S) = \frac{s\Phi(s)}{S + \underbrace{\boxed{2\pi K_f K_m}}_{K} H(S)}$$
$$= \frac{s\Phi(s)}{S + K H(S)}$$

Case: $H(S) = 1.$

$$E(S) = \frac{S\,\Phi(S)}{S+K}$$

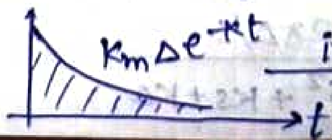
(i) $\phi(t)$:



$$\Phi(s) = \Delta/s.$$
$$\therefore E(S) = \frac{\Delta}{S+K}$$

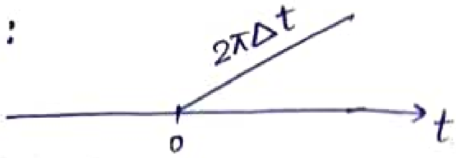
e(t) :

$V_i(t)$:



$\xrightarrow{\text{integrate}}$ constant

(ii) $\phi(t)$:



$$E(S) = \frac{s(2\pi\Delta/s^2)}{s+k}$$

$$= \frac{\cancel{2\pi\Delta}\; 2\pi\Delta}{s(s+k)}$$

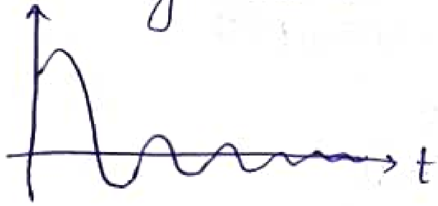
$$\Rightarrow e(t) = \frac{2\pi\Delta}{k}\left(1 - e^{-kt}\right)$$

Case : $H(s) = 1 + a/s$

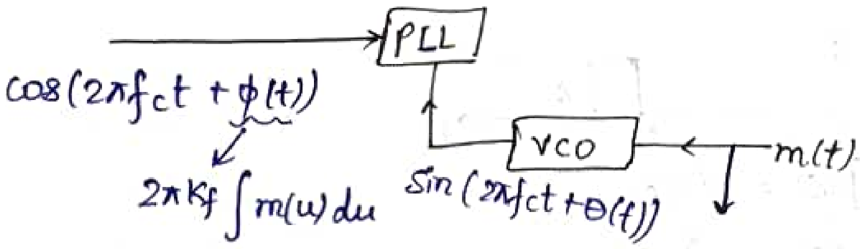
$$E(s) = \frac{s^2 \cdot 2\pi\Delta/s^2}{s^2 + ks + ka}$$

$$= \frac{2\pi\Delta}{s^2 + ks + ka}$$

$e(t)$: Diminishing sinusoid

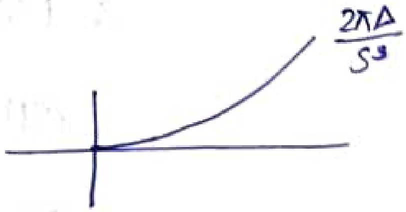


FM demodulation



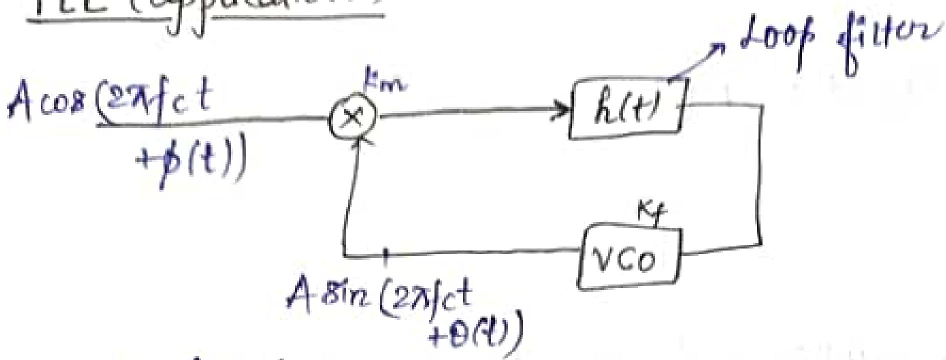
$$\phi(t) - \theta(t) = e$$

Suppose $\phi(s) = \dfrac{2\pi\Delta}{s^3}$

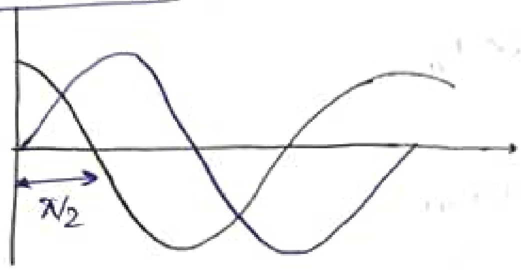


$$E(s) = \frac{2\pi\Delta/s}{s^2 + ks + ka}$$

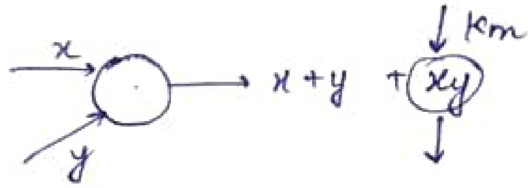
PLL (applications)



Under 'lock'



#



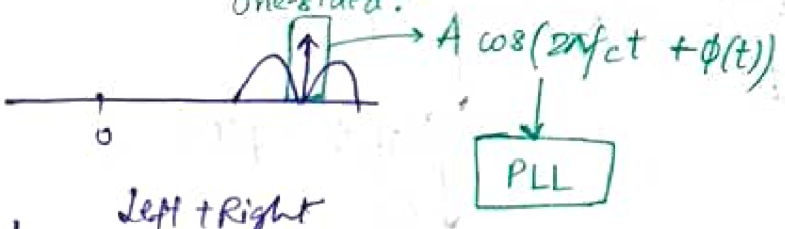
DSB demodulation :

$A\, m(t) \cdot \cos(2\pi f_c t + \phi(t))$

Error signal :

$A\, m(t) \sin(\phi(t) - \theta(t))$

$A\,(1 + \alpha m(t)) \cdot \cos(2\pi f_c t + \phi(t))$



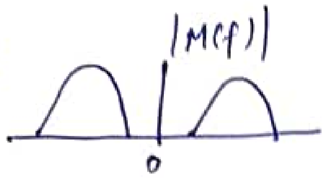
Can use PLLs if we have a pilot signal easily !

$A^2 m(t)^2 \cos^2(2\pi f_c t + \phi(t))$

@ $2f_c$

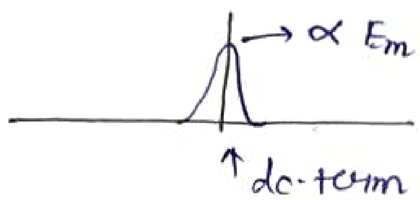
$A^2 m(t)^2 \cos(2\pi(2f_c)t + 2\phi(t))$.

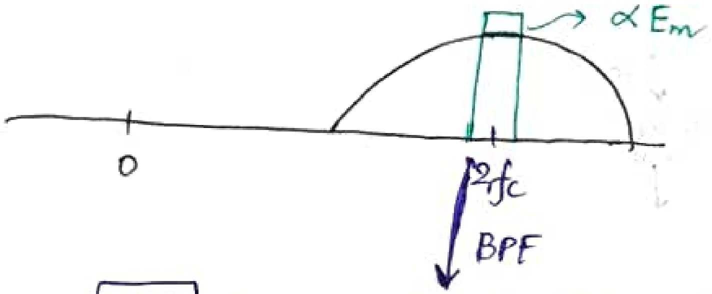
$m(t)$:



↳ no dc-term $m(t)$ is a baseband voice-signal.

$m^2(t) \xrightarrow{\mathcal{F}}$

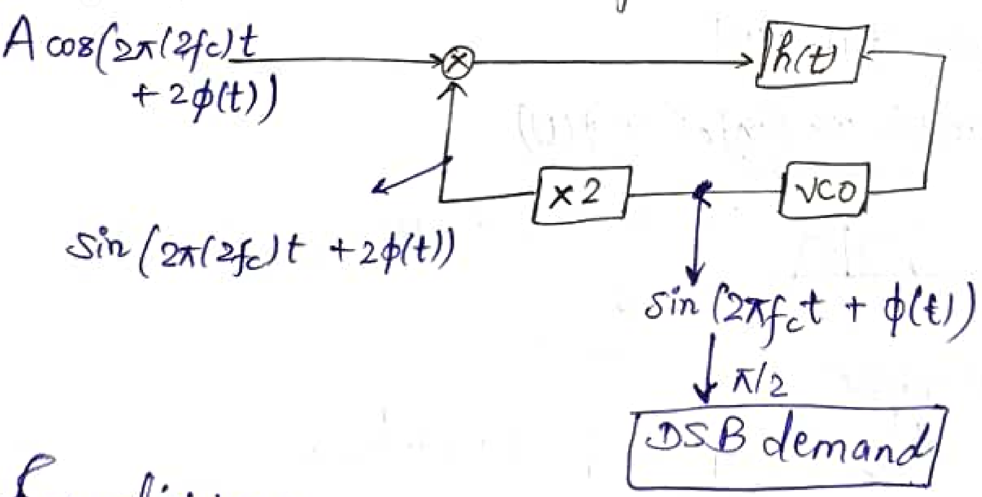




$\boxed{PLL} \leftarrow K\cdot\cos(2\pi(2f_c)t + 2\phi(t))$

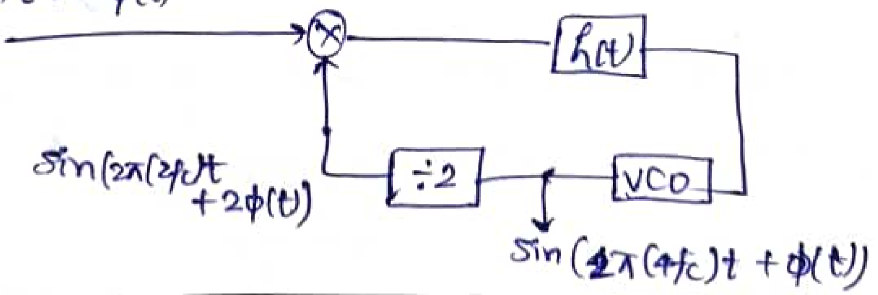
↓

Frequency division and multiplication:



Freq. division:

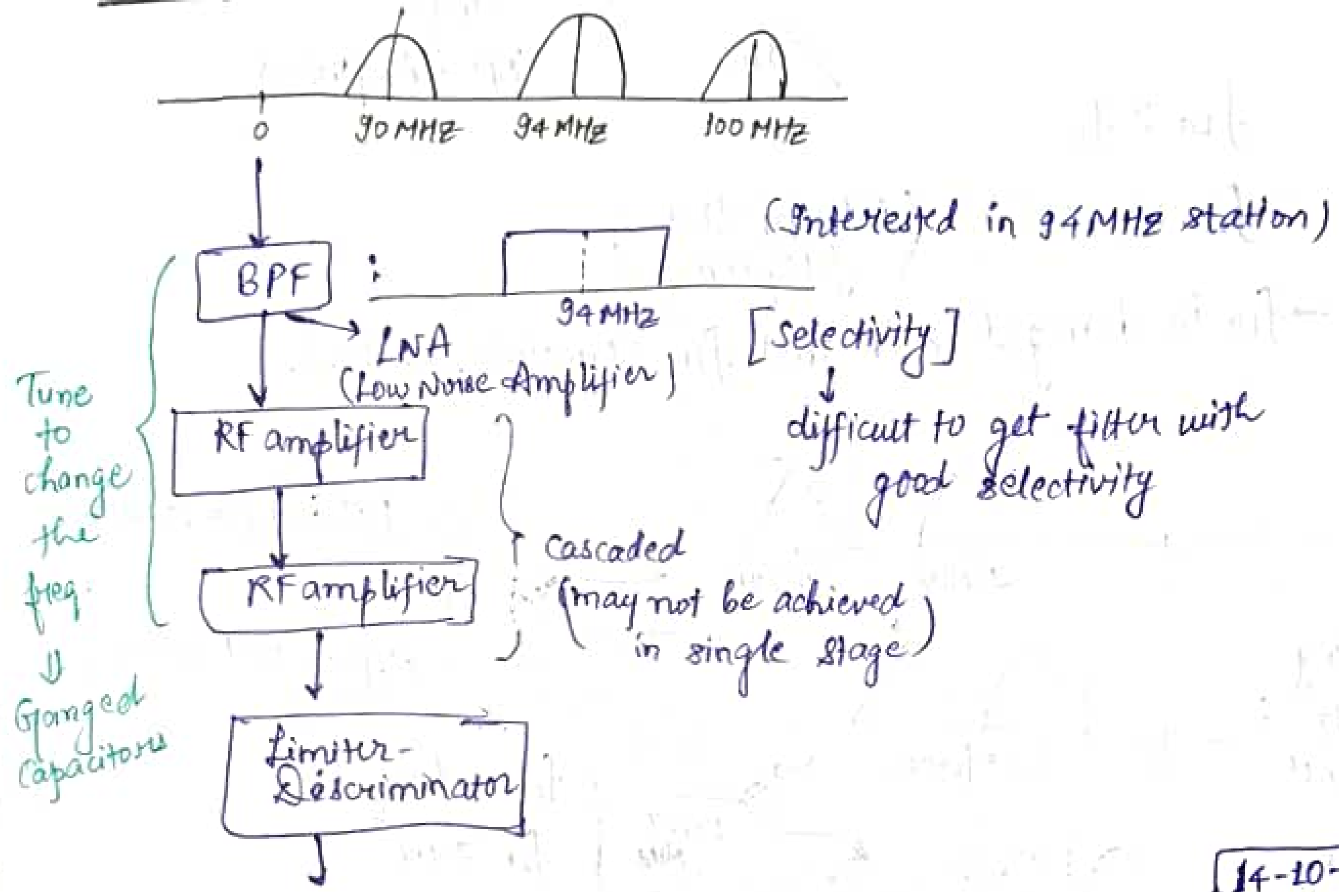
$A\cos(2\pi(2f_c)t + 2\phi(t))$



Receiver Architectures

→ Tuned RF (TRF) (old)
→ Superheterodyne (old)
→ Zero IF architecture

TRF



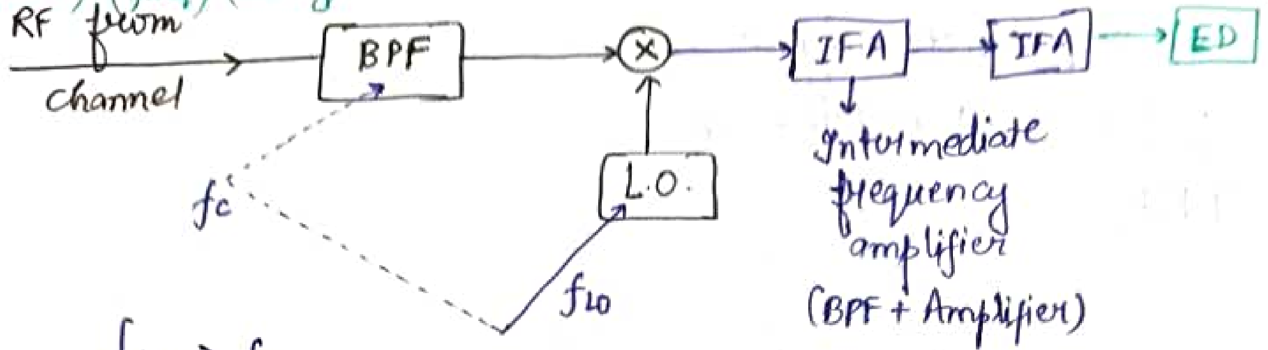
14-10-2024

TRF
↳ For fixed channel
↳ If tuning, joint/ganged tuning of all stages together.
↳ Rx selectivity

→ LNA → BPF amp. → BPF amp. → ED →

Superheterodyne Architectures

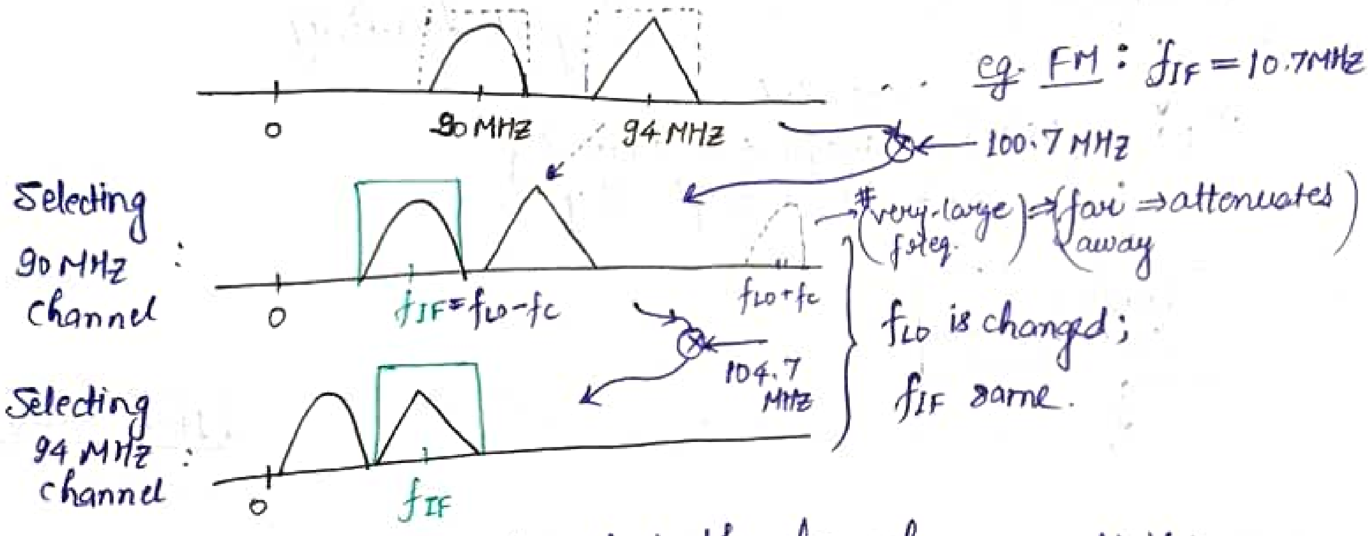
(b/c $f_{LO} > f_c$) (2 diff freq) (mixing)



$f_{LO} > f_c$

$f_{LO} - f_c = f_{IF}$ (intermediate frequency)

→ f_{LO} is changed such that f_{IF} remains constant.



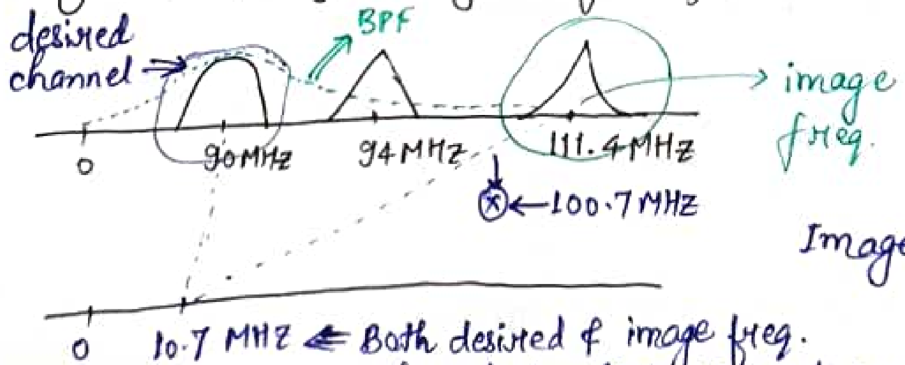
→ IFA or filter to select the channel.

→ working at a lower ~~center~~ fixed intermediate center frequency (f_{IF}).

→ Compared to IRF, it is easier to make selective filters.

→ works even without BPF; required to cutoff noise.

Image frequency/Image signal problem : Without BPF



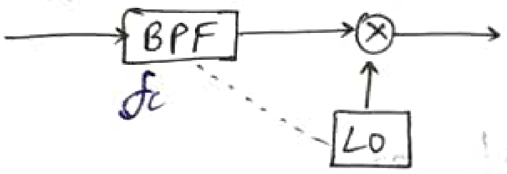
$[f_{IF} = 10.7 \text{ MHz}]$

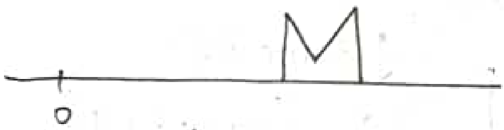
Image freq. $= f_{LO} + f_{IF}$

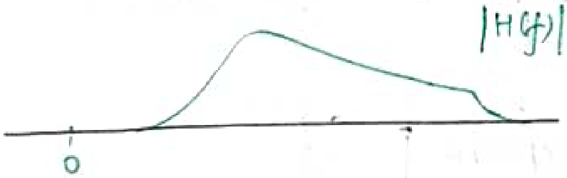
10.7 MHz ⇐ Both desired & image freq. shifts to ~~imediate~~ intermediate freq.

BPF (frontend) : Removes the image signal
↳ Need not be very selective.
↳ Easier to design than TRF, where BPF is very selective.

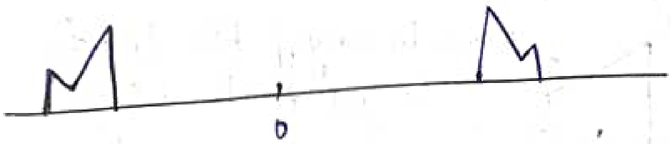
Zero IF : Homodyne (tuning of same frequency)
↳ $f_{IF} = 0 \Rightarrow f_{LO} = f_c \Rightarrow$ DSB Receiver (but tunable)

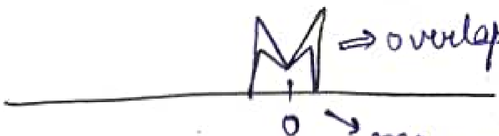






$|H(f)|$ → channel [We're not sure what kind of channel we'll get]





⇒ overlaps

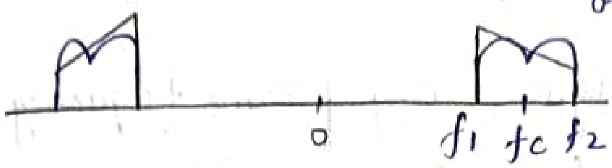
↘ may not be same as the original signal (M).

15-10-2024

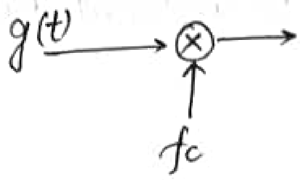
Zero IF: Complex Baseband representation

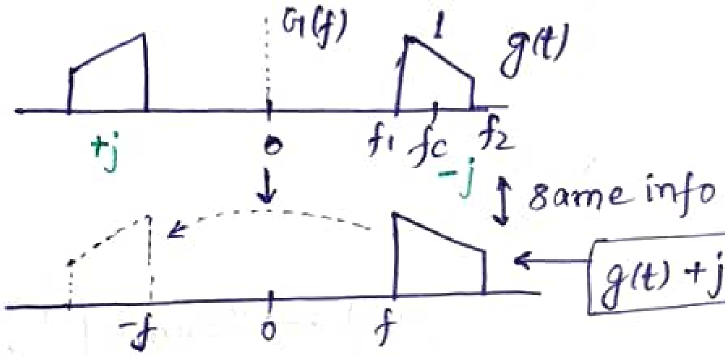
A general passband signal (real!):

$G(-f) = G^*(f)$

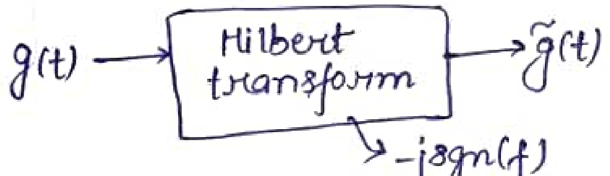


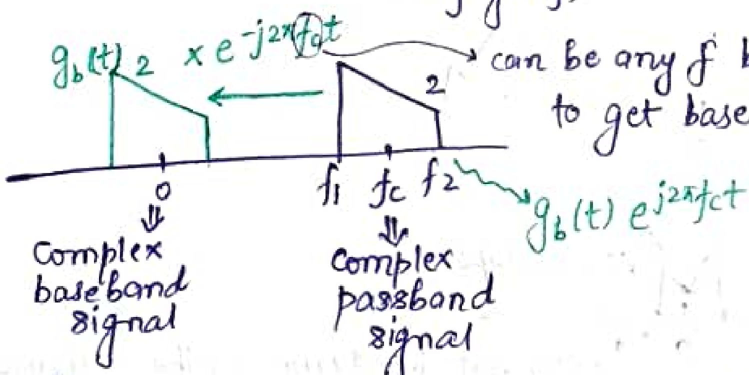
$f_c = \frac{f_1+f_2}{2}$





$\boxed{g(t) + j\tilde{g}(t)}$





can be any f b/w f_1 & f_2 to get baseband signal

⇓

Complex baseband representation of a passband signal.

$$g_b(t)\, e^{j2\pi f_c t}$$

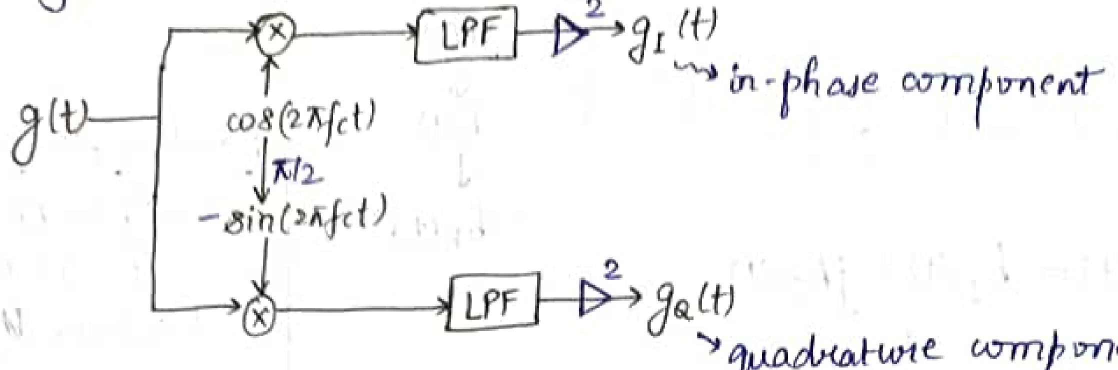
$$\mathrm{Re}\{\underbrace{g_b(t)}_{g_I(t) + j g_Q(t)}\ \underbrace{e^{j2\pi f_c t}}_{\cos(2\pi f_c t) + j\sin(2\pi f_c t)}\} = g(t)$$

$g_I(t)$, $g_Q(t)$ → baseband signals

$$= g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t) = g(t)$$

↳ In-phase-quadrature (IQ) representation of passband signal.

To get $g_I(t)$, $g_Q(t)$ from $g(t)$, use multiplexing.



→ Converting a passband signal to a baseband signal, makes it easy to do signal processing with it by converting it to digital signal and making use of software.

$$\{g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)\} \times \frac{e(t)}{e(t)}$$

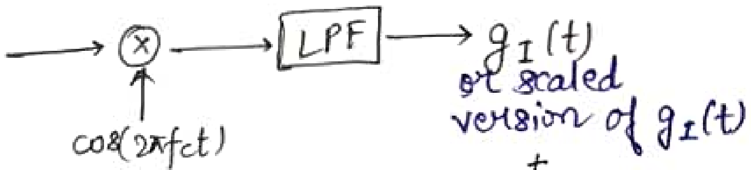
$$e(t) = \sqrt{g_I^2(t) + g_Q^2(t)}$$

$$\alpha(t) = \tan^{-1}\left(\frac{g_Q(t)}{g_I(t)}\right)$$

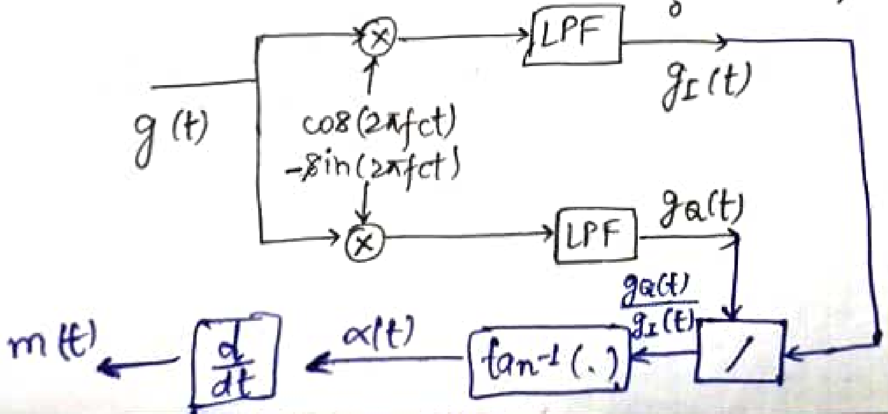
$$= \underbrace{e(t)}_{\text{envelope}} \cdot \cos(2\pi f_c t + \underbrace{\alpha(t)}_{\text{phase}}) = g(t)$$: Envelope-phase representation of a passband signal.

Zero IF Receivers

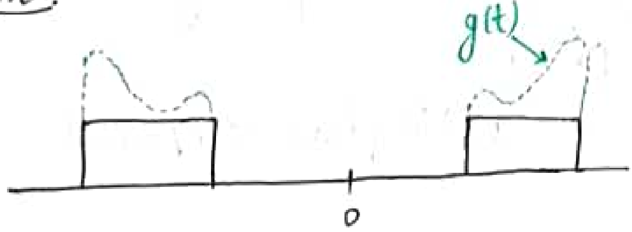
① $g(t) = A m(t) \cos(2\pi f_c t)$



② $g(t) = A\cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(u)\,du\right)$



System :



$$H(f) \downarrow h(t) \downarrow h_b(t)$$

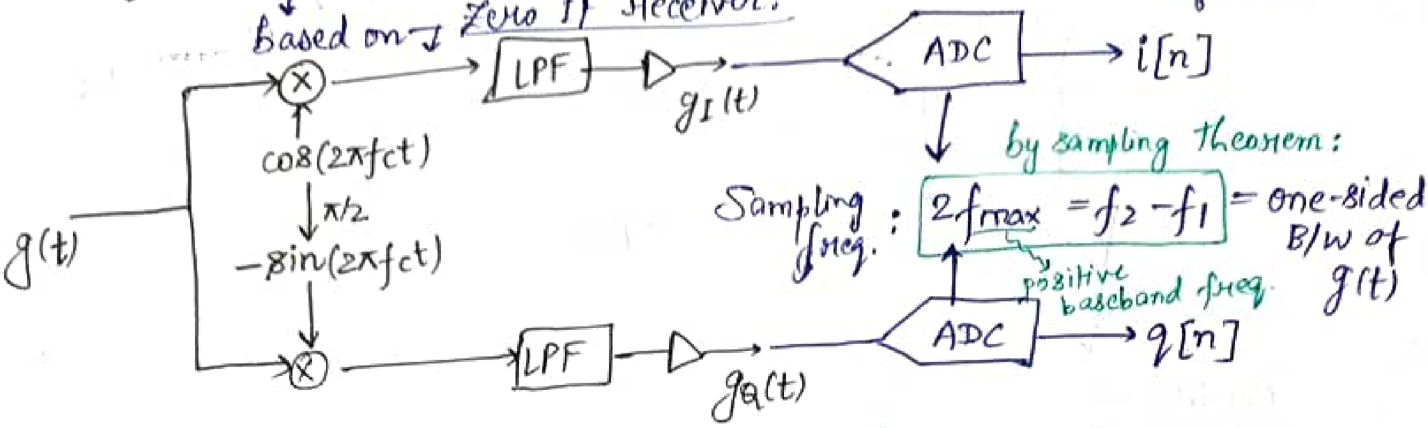
$$h_b(t) = h_I(t) + j h_Q(t)$$

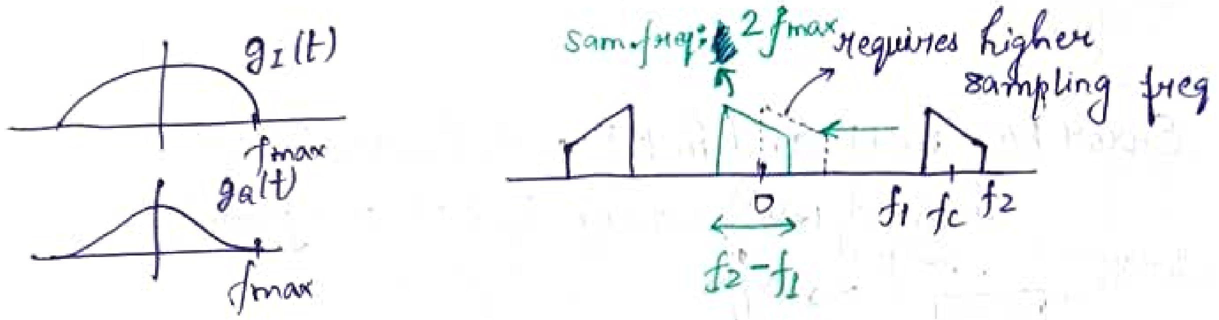
$$g_b(t) \longrightarrow \boxed{h_b(t)} \longrightarrow$$

Reading Assignment:
complex BB rep. of system
↳ Source: Madhav or Simon Haykins books

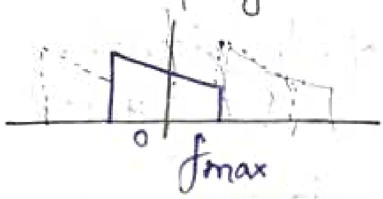
Software defined Radios (Receiver) : Implemented in microprocessor or software.

↓ Based on → Zero IF receiver:



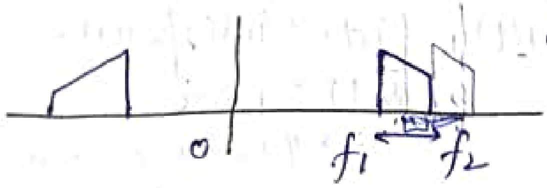


Baseband Sampling theorem:



sampling freq. = $2f_{max}$

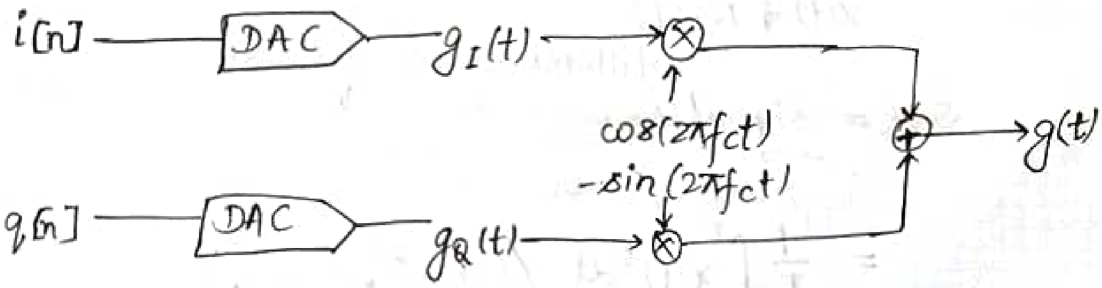
Passband Sampling theorem:



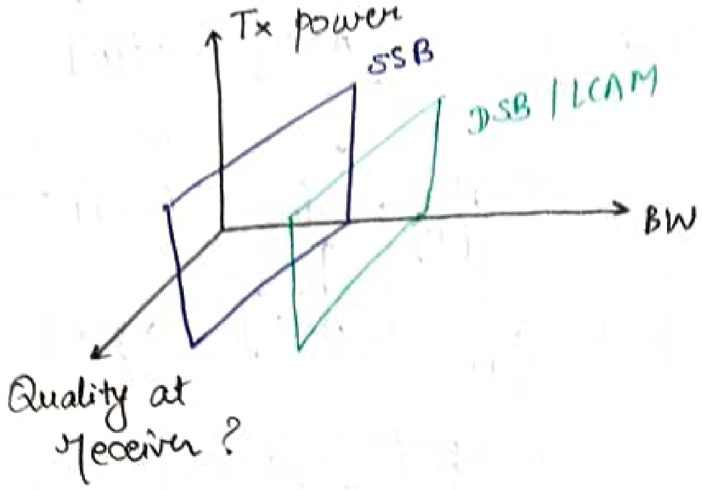
Sampling freq. = $f_2 - f_1$

Software defined Radios (Transmitters):

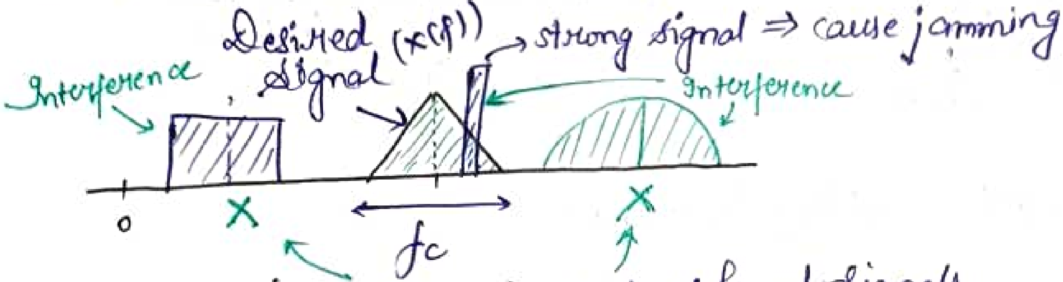
↳ Reverse the receiver



Recall Tradeoffs :



Error b/w $m(t)$ and $\hat{m}(t)$: $e(m(t), \hat{m}(t))$.



→ Selectivity : Removing out of band signals.
→ In-band interference cannot be filtered.

Noise: Unpredictable random signal

Signal-to-Noise Ratio (SNR): At receiver [Mechanics of SNR calc.]

$x(t) + i(t) + N(t)$, $i(t)$: interference signals
$N(t)$: noise
→ Jamming cannot be removed here.

↓ from channel

R_x [BPF]

↓

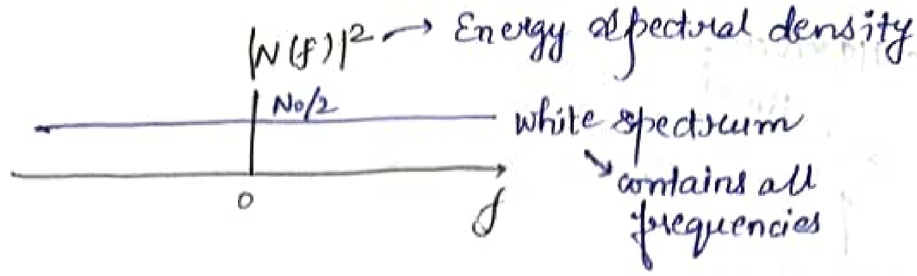
$x(t) + N_f(t)$ ← filtered version of noise

$$SNR = \frac{\text{Signal power}}{\text{Noise power}}$$

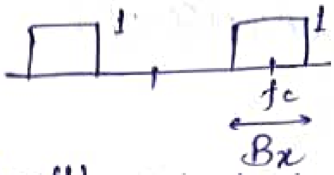
$$= \frac{1}{T}\int_0^T x(t)^2 dt \Big/ \underbrace{\left(\frac{1}{T}\int_0^T N_f^2(t)\,dt\right)}_{\text{avg.}}$$

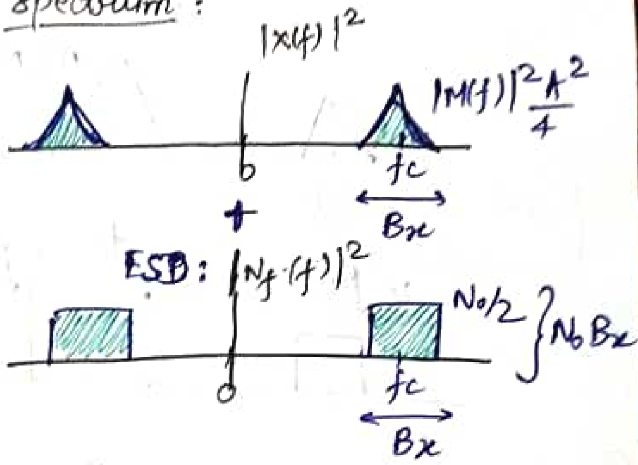
← unpredictable

Believe



$x(t)+N(t) \longrightarrow$ BPF $\longrightarrow$ O/p spectrum :





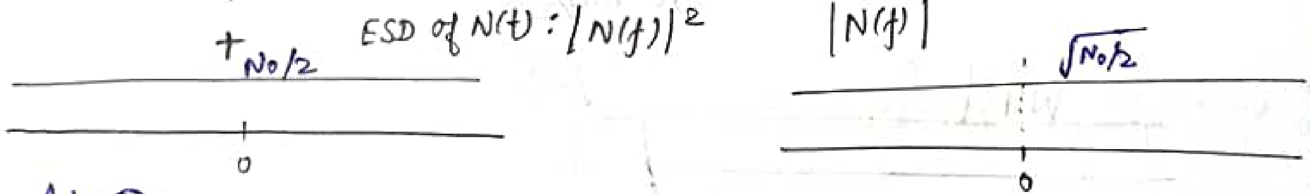
e.g. $x(t) = A m(t) \cos(2\pi f_c t)$

SNR just after BPF: $\left(\dfrac{\frac{A^2 P_m}{2}}{N_0 B_x}\right)$

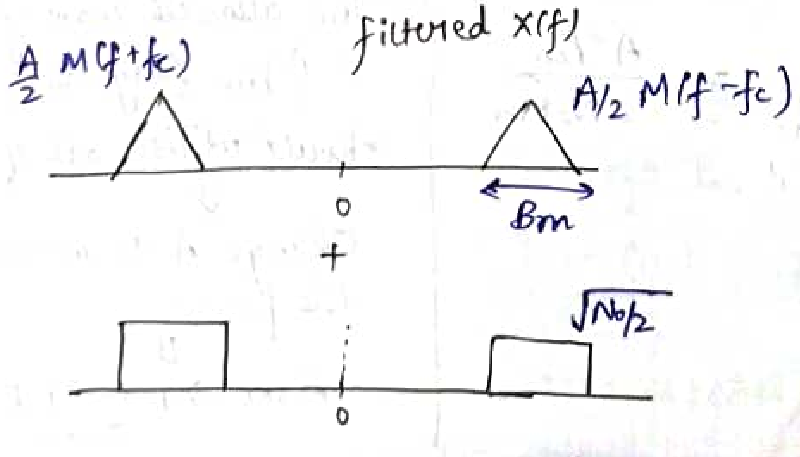
16-10-2024

from channel $x(t)+N(t) \longrightarrow$ R_x BPF $\longrightarrow$ (A)

R_x BPF → Noise rejection

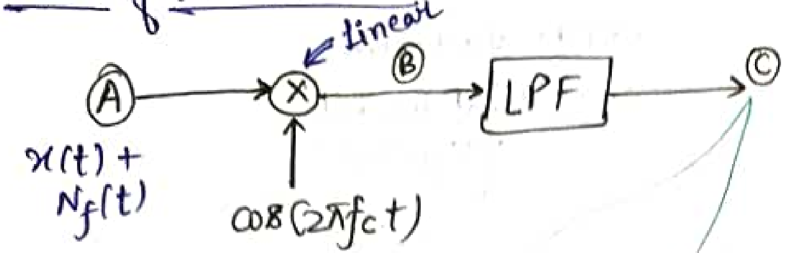


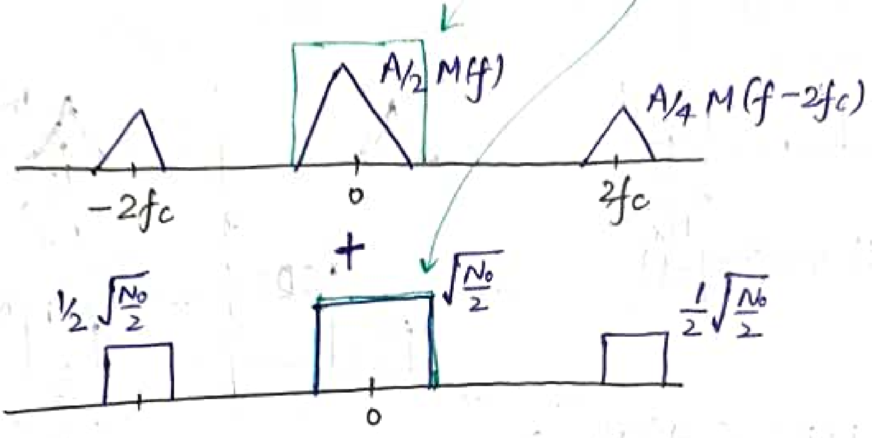
At (A):



SNR after BPF: $\dfrac{A^2 P_m}{2 N_0 B_m}$ | SNR before BPF ≈ 0 [Huge improvement in SNR]

Rest of the Receiver:

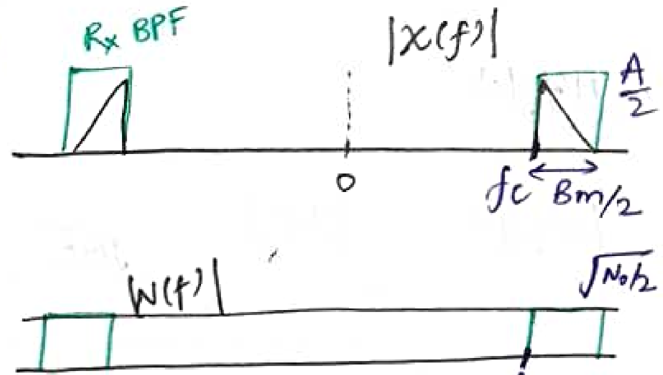




$$\text{Final SNR} = \frac{\frac{A^2}{4} \cdot P_m}{\frac{N_0}{2} \cdot B_m}$$

For SSB

$x(t)$:



$\xrightarrow{\text{to make SSB power = DSB power}} A'$

$$\frac{A^2}{4} P_m = \frac{A'^2}{2} P_m$$

At (A), after the Rx BPF:

$$SNR = \frac{\frac{A^2}{4} P_m}{\frac{N_0}{2} \cdot \frac{2B_m}{2}} = \frac{A^2 P_m}{2 N_0 B_m}$$

If the allowed maxm power is $\frac{A^2}{2} P_m$ (say 1W), we should utilise all of it

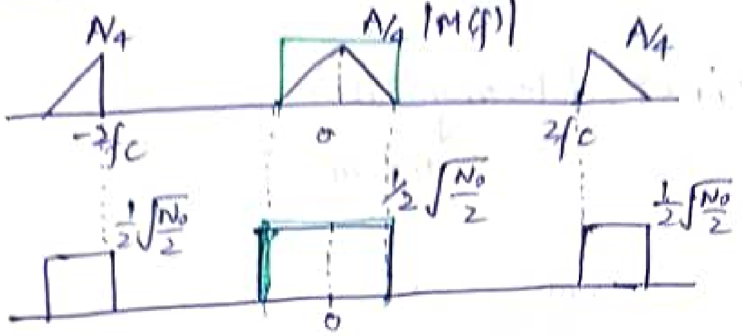
$\Downarrow$

Change A to increase the power

$\Downarrow$

Make DSB & SSB power same.

At Ⓑ:

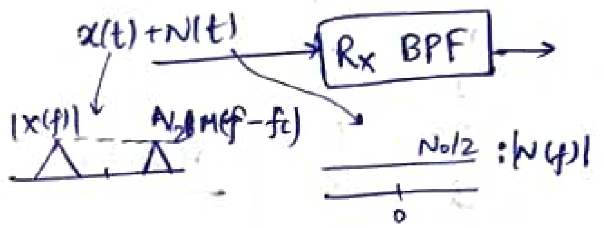


$$SNR = \frac{\frac{A^2}{16} P_m}{\frac{N_0}{8} B_m} = \frac{A^2 P_m}{2 N_0 B_m}$$

SAURABH KUMAR
SC22B146

Exercise: ① Compare SSB, DSB SNR if Tx powers are same.
② SNR for LCAM, assuming Tx power is same as DSB.
[Submit by 23rd Oct.]

Sol^n: ① DSB Tx power $= \frac{A^2}{2} P_m$



SSB Tx power $= \frac{A'^2}{4} P_m$

$$\because \frac{A^2}{2} P_m = \frac{A'^2}{4} P_m \Rightarrow A'^2 = 2A^2 \Rightarrow A' = \sqrt{2} A$$

SNR of DSB $= \dfrac{\frac{A^2}{2} P_m}{\frac{N_0}{2}(B_m \times 2)} = \dfrac{A^2 P_m}{2 N_0 B_m}$ [BW = $B_m \times 2$]

SNR of SSB $= \dfrac{\frac{A'^2}{4} P_m}{\frac{N_0}{2}(B_m)} = \dfrac{2A^2 P_m}{2 B_m N_0} = \dfrac{A^2 P_m}{N_0 B_m}$ [BW = B_m]

$$\therefore \boxed{SNR(SSB) = 2\, SNR(DSB)}$$

② DSB Tx power $= \frac{A^2 P_m}{2}$

LCAM Tx power $= \frac{A'^2}{2} + \frac{\alpha A'^2 P_m}{2} = \frac{A'^2}{2}(1 + \alpha P_m)$

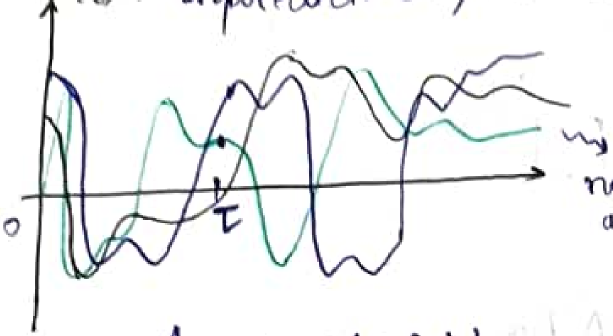
SNR of LCAM $= \dfrac{\frac{A'^2}{2}(1+\alpha P_m)}{\frac{N_0}{2} \times (2B_m)} = \dfrac{A^2 P_m}{2 N_0 B_m} = SNR(DSB)$

Also, Tx P(DSB) = Tx P(LCAM) $\Rightarrow \frac{A^2 P_m}{2} = \frac{A'^2}{2}(1 + \alpha P_m)$

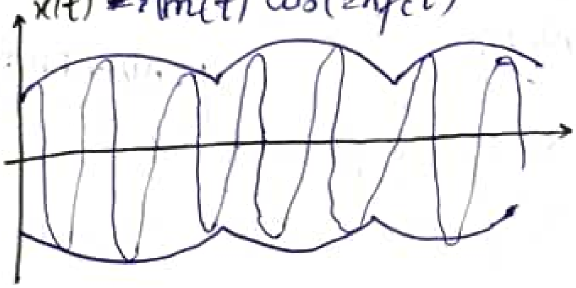
$$\therefore \boxed{SNR(LCAM) = SNR(DSB)}$$

Modelling Noise

$N(t)$: Unpredictable/not deterministic → random signals



$x(t) = Am(t)\cos(2\pi f_c t)$

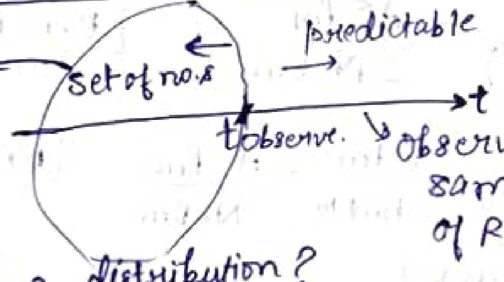


Observation of values at 1 min (τ):

n	
1	0.1
2	10
3	0.5
⋮	⋮

random → how do we model such phenomena?

Random Variable:



support of the R.V. [Possible values]

observation/measurement/sample of R.V.

RV: support with distribution

and a distribution?

↳ tells how many times a particular no. would appear on observing multiple times at t_{obs}.

Distribution of a R.V.

X $\quad P_X(x)$ = fraction of observations with value 'x'.
(dist.)

e.g. 0 1 0 0 1 0 0 0

$P_X(1) = 1/4$

$P_X(0) = 1 - P_X(1)$

$\sum P_X(x) = 1$

$x \in S \to$ support

e.g. Rolling a dice

1 2 4 6 1 2 3 4

5 1 1 2 1 2 1 4

$S = \{1, 2, \dots, 6\}$

$p_X(6) = 1/16$

$p_X(5) = 1/16$

$p_X(4) = 3/16$

$\vdots$

II Fitting a r.v. model

e.g. 10 persons roll a 10 die and their sum is calculated.

$\{10, \dots\dots, 60\}$

Fit a "reasonable" r.v. here.

Suppose Binomial, then n is max value.

Binomial r.v. (n, p)

Support: $\{0, \dots, n\}$

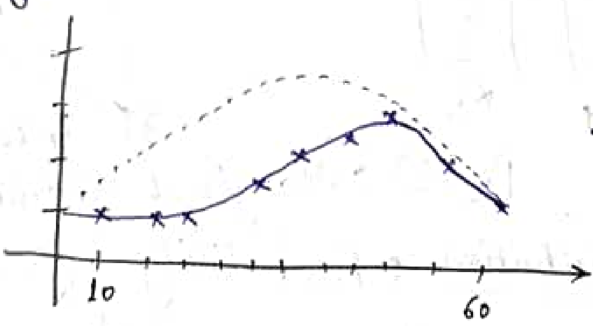
$p_X(x) = \binom{n}{x} \cdot p^x (1-p)^{n-x}$

np = avg. = expectation

p = average/n

Tradeoff: we may not get accurate ~~value~~ fit.

Histogram:



→ we may not get a good fit.

⇒ Advantage:

we would require less parameters

No need to calculate for all the values.

S is discrete.

RV : discrete R.V.

$p_X(x)$ = PMF.

Values at τ:

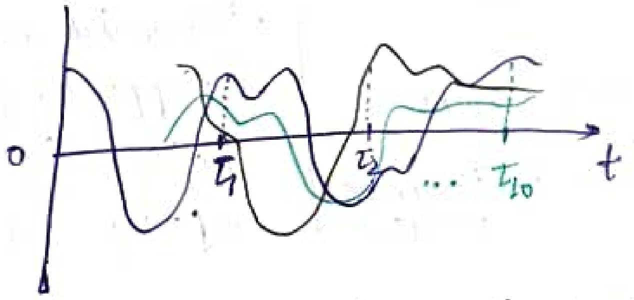
n	—
1	0.1
2	10
3	0.5
⋮	⋮

Here, values are continuous.

$S \subseteq \mathbb{R}$ [Continuous R.V.]

$f_X(x)$: PDF

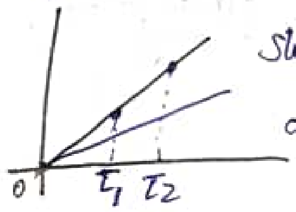
= fraction of time samples $\in [x, x+dx]$



Observe values at 2 times τ_1 and τ_2.

n	X_1	X_2
1	0.1	0.5
2	1	3
3	3	2

e.g. Signals : random slopes

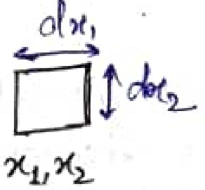


$P_{X_1}(x_1)$ $P_{X_2}(x_2)$: Marginal distributions

Marginalization

Joint distribution : Probability of seeing a pair of values.

↳ $P_{X_1 X_2}(x_1, x_2)$



$f_{X_1 X_2}(x_1, x_2)$: PDF

$$\boxed{P_{X_1}(x_1) = \sum_{x_2} P_{X_1 X_2}(x_1, x_2)}$$

fraction of seeing x_1

sum of probabilities of observing x_1 & some x_2 over all x_2.

Suppose 10 times

$P_{X_1X_2\ldots X_{10}}(x_1, \ldots, x_{10})$

n					
1	0.1	0.01	0.2	...	1.2
2					
⋮					

Standard models

17-10-2024

Random variables → vectors − Process

DRV X : $P_X(x)$

CRV X : $f_X(x)$

Multiple r.v.s : $X_1 X_2 \rightarrow \begin{bmatrix} P_{X_1,X_2}(x_1, x_2) \\ f_{X_1X_2}(x_1, x_2) \end{bmatrix}$

Standard distributions:

fit data → R.V. models

For 10 times

1 : 0.1, 0.2, 0.9, ..., 9

2 : 1, 0.1, 1, ... 10

↓

Model :

$X_1, X_2, \ldots, X_{10}$ → 10^{10} parameters

$10(M)$ $10(M)$ $10(M)$ (M^{10})

$P_{X_1X_2\ldots X_{10}}(x_1, x_2, \ldots, x_{10})$

Standard models : 10 length

Random variables are independent.

$$P_{X_1\ldots X_{10}}(x_1, \ldots, x_{10}) = P_{X_1}(x_1) \ldots P_{X_{10}}(x_{10})$$

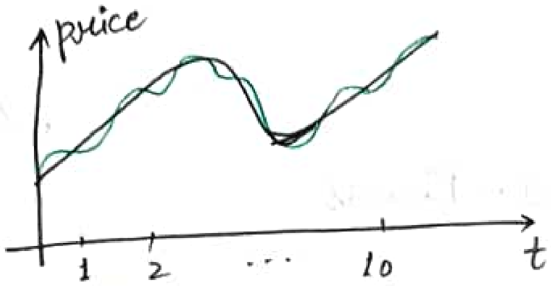
$10(M) + \ldots + 10(M) \rightarrow 10\times10\ (10M)$ parameters.

Independent and identically distributed models: (IID)

↳ Independent R.V.s
↳ Each of the distribution is same.

$$P_{X_1X_2\cdots X_{10}}(x_1,\ldots,x_{10}) = P_X(x_1)\cdots P_X(x_{10}) \rightarrow \text{only M parameters}$$

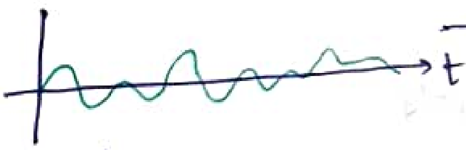
Eg.



Deterministic → ~~trend~~ 'Trend'
+
noise

$p_1 \ p_2 \ \ldots \ p_{10}$

detrended (Removing deterministic part)



↓
model using indep. & identically distributed.

Random Process

↳ An ordered collection of random variables with index

t = time.

$t \in \mathbb{R}$ → cont. time R.Ps
or $t \in \mathbb{Z}$ → disc. time R.P.s
} also be finite.

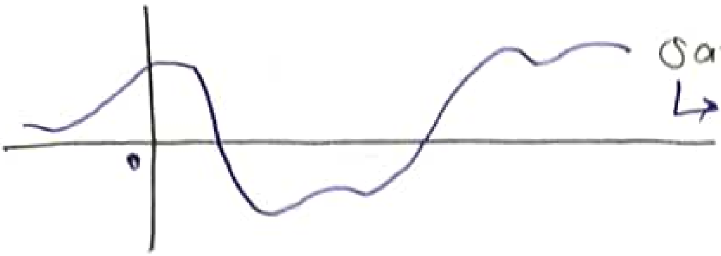
$X(t), t \in \mathbb{R} \longrightarrow$ CTRP
$X_n, n \in \mathbb{Z} \longrightarrow$ DTRP

Model for Noise Signal

$(N(t), \forall t \in \mathbb{R})$ ⇝ uncountable collection of R.V.s

↘ noise → R.V.

↳ one R.V. at each time.



Sample function
↳ Realization of R.P.

Problem:

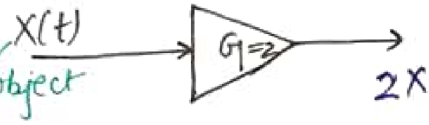
D.T.R.P.

$(x(t),\ t \in \mathbb{Z})$

$X(t)$ is IID. ⇝ PDF in $[-1,1]$

$f_X(x) = \text{Unif.}\,[-1,1]$

ⓐ

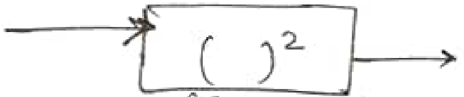


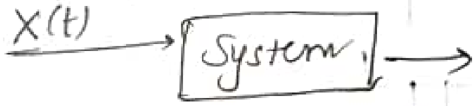
math. object

Describe model

⇝ has joint dist. ⇝ R.P.

has joint dist. ⓑ



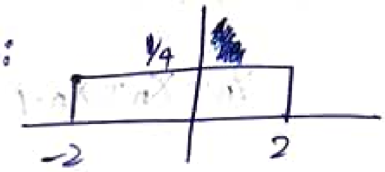


18-10-2024

Soln: ⓐ Joint dist. of $2X(t) = Y(t)$,

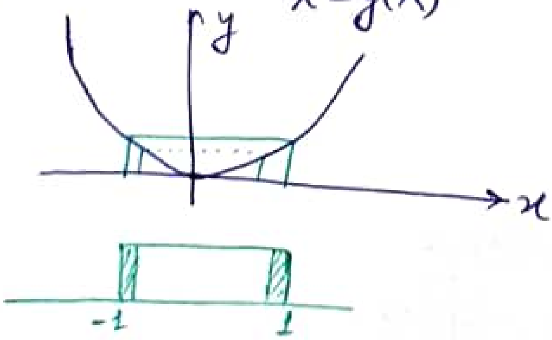
$$f_{Y(1)|Y(2)}(y_1, y_2) = f_{Y(1)}(y_1)$$

$f_{Y(1)}(y)$:

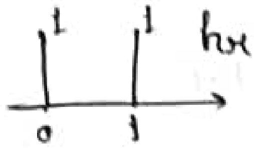


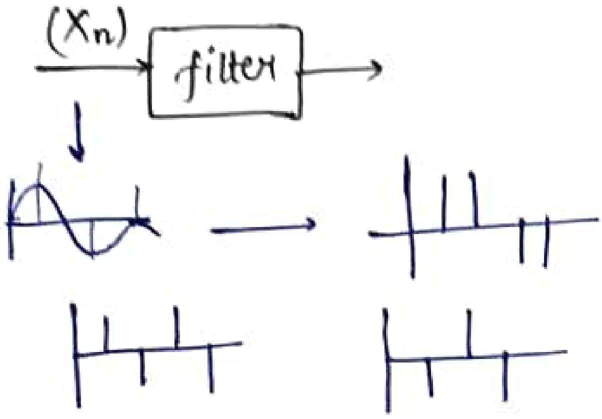
$$f_Y(y).dy = P_r\{Y \in [y, y+dy]\}$$
$$= P_r\{X \in [y/2,\ y/2 + dy/2]\}$$
$$= f_X(y/2).\,dy/2$$

ⓑ $X = g(X)$



Eg.

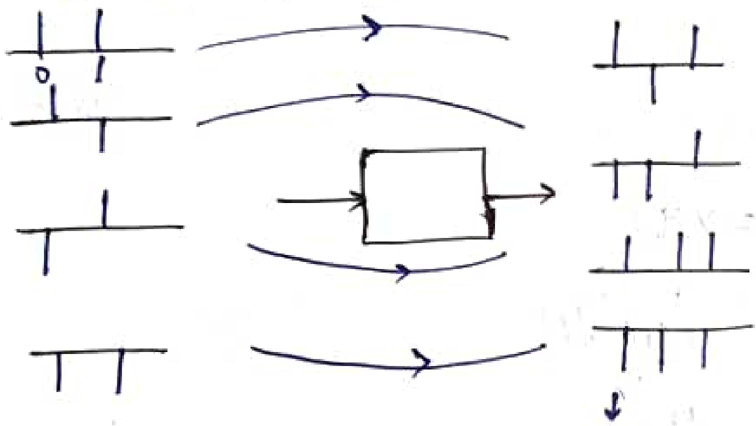




$$Y_n = X_n + X_{n-1}$$

Suppose input is IID.

$$\begin{array}{cc} Y_{10} & Y_{11} \\ \| & \| \\ X_{10} + X_9 & X_{11} + X_{10} \end{array}$$



$$Y_n = X_n + X_{n-1}$$

→ $X + Y$: DRVs

$X \perp Y$ $\quad P_X(x), P_Y(y)$

$$P_r\{X + Y = Z\} = \sum P_X(x) P_r\{Y = Z - x \mid X = x\}$$

$X(t)$ or (X_n) : Joint distribution

Eg. $P_{X_1 X_2 X_3}(x_1, x_2, x_3)$

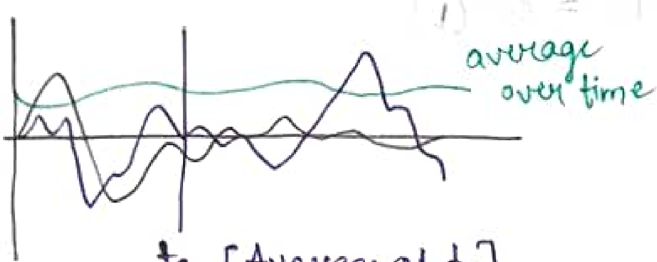
Structured models
— Independent
— IID

For our applications,

$$x(t) \rightarrow \boxed{} \rightarrow y(t)$$

$y(t)$ also has a joint distribution.
— usually difficult to characterise.

$E\,Y(t) = \mu(t)$



t_0 [Average of t_0]

Law of large numbers:

X is R.V. (P_X: pmf)

$x_1, x_2, \dots, x_n$

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$\bar{x} \approx EX$

$\quad = \sum P_X(x)$

$E(Y^2(t))$: Expected power fn of time

Variance, $\sigma_Y^2(t) = E(Y^2(t)) - (E\,Y(t))^2$

2nd moment of RV

$$(X(t)) \rightarrow \boxed{h(t)} \rightarrow (Y(t))$$

$\mu_X(t)$ $\quad$ $\mu_Y(t)$

$$Y(t) = \int_0^\infty h(\tau)\, x(t-\tau)\, d\tau$$

$$E\,Y(t) = \int \left(\int_0^\infty h(\tau)\, x(t-\tau)\, d\tau \right) f_Y(y)\, dy$$

$$= \int_0^\infty \left(\int h(\tau)\, x(t-\tau)\, f_Y(y)\, dy \right) d\tau$$

$$= \int_0^\infty h(\tau)\, \mu_X(t-\tau)\, d\tau$$

$\mu_X(t) = \mu_X$ (constant)

$$\mu_Y = \mu_X \underbrace{\int_0^\infty h(\tau)\, d\tau}_{H(0)}$$

25-10-2024

$$(X(t)) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Distribution is difficult

$$\boxed{\mu_Y(t)} = \int h(\tau) . \boxed{\mu_X(t-\tau)} d\tau$$

$$\mu_Y = H(0)\,\mu_X$$

Other such functions: Summary Statistics
↳ gives the information about the complete distribution using a single value.

$\mathbb{E}X$, $\mathbb{E}X^2$, $Var(X)$

$\mathbb{E}X = \mu_X(t)$, $Var(X(t)) = \sigma_X^2(t)$

To find $\mathbb{E}(Y(t))^2$

$\mathbb{E}[Y(t_1) . Y(t_2)]$

digression

X and Y are RVs.

$\mathbb{E}XY = \iint f_{XY}(x,y) . xy \, dx dy$: correlation of RVs

Covariance : $cov(X,Y) = \mathbb{E}XY - \mathbb{E}X . \mathbb{E}Y$

$$= \mathbb{E}[\underbrace{(X - \mathbb{E}X)}_{X_c} . \underbrace{(Y - \mathbb{E}Y)}_{Y_c}]$$

mean=0 center R.V.

$Y(t_1) = \int h(u) . x(t_1 - u) . du$

$Y(t_2) = \int h(u) . x(t_2 - v) . dv$

$$\mathbb{E}[Y(t_1) . Y(t_2)] = \mathbb{E}\left[\int h(u) . x(t_1 - u) . du \int h(v) . x(t_2 - v) \, dv\right]$$

$$= \mathbb{E}\left[\iint h(u) . h(v) . X(t_1 - u) . x(t_2 - v) \, du \, dv\right]$$

$$= \iint h(u) . h(v) . \mathbb{E}[X(t_1 - u) . x(t_2 - v)] . du . dv$$

Wide Sense Stationary (WSS) : Weakly stationary

↳ $(X(t))$ ↳ weak form of strictly stationary

conditions:
- $\mu_X(t) = \mu_X$
- $\boxed{\mathbb{E}\, X(t_1) \cdot X_2(t_2)} = R_X(\boxed{t_1 - t_2})$

Autocorrelation f^n of t_1, t_2

Strictly Stationary R.P

↳ $(X(t))$

$$f_{X(t_1) X(t_2) \ldots X(t_m)}(x_1, x_2, \ldots, x_m) = f_{X(t_1+\tau) \cdot X(t_2+\tau) \ldots X(t_m+\tau)}(x_1, x_2, \ldots, x_m)$$

$$\forall\, m, (t_1, \ldots, t_m), \tau$$

$$f_X(x) = f_{X+\tau}(x)$$

$$\Rightarrow \int f_X(x)\, x\, dx = \int f_{X+\tau}(x)\, x\, dx$$

Example of same mean but different dist.: Suppose we get diff. gaussian dist. at every time $\Rightarrow$ Mean $\rightarrow$ same
Variance $\rightarrow$ different.

30-10-2024

WSS Random Process

$(X(t))$

① $E\,X(t) = \mu_x$ [summary statistics] = constant.

② $E\,X(t_1) \cdot X(t_2) = R_x(|t_1 - t_2|)$

$$\underset{WSS}{\xrightarrow{(X(t))}} \boxed{h(t)} \longrightarrow \underset{(WSS)}{(Y(t))}$$

$$\mu_y(t) = \int \underbrace{\mu_x(\tau)}_{\mu_x \text{ for WSS}} \cdot h(t-\tau)\,d\tau$$

$$\boxed{\mu_y = \mu_x \cdot H(0)} \longleftarrow \int h(\tau) e^{-j2\pi(0)t}\,d\tau = \int h(\tau)\,d\tau$$

$E\,Y(t_1)Y(t_2)$

$$Y(t_1) = \int h(u) \cdot X(t_1 - t_2) \cdot du$$

$$Y(t_2) = \int h(v) \cdot X(t_2 - v)\,dv$$

$$E\,Y(t_1) \cdot Y(t_2) = \iint h(u) \cdot h(v) \cdot \underbrace{E\left[X(t_1 - u) \cdot X(t_2 - v)\right]}_{R_x(t_1 - u - t_2 + v)} du\,dv$$

$$R_x(\boxed{t_1 - t_2} - (u - v))$$

If $(X(t))$ is WSS $\longrightarrow$ function of $(t_1 - t_2)$

$$R_y(t_1 - t_2)$$

$\Rightarrow$ $(Y(t))$ is also WSS.

$$E\,Y(t_1) \cdot Y(t_2) = \iint h(u) \cdot h(v) \cdot R_x(t_1 - t_2 - (u - v)) \cdot du\,dv$$

[Take $u - v = w \Rightarrow u = v + w$.]

$$= \iint h(v + w) \cdot h(v) \cdot R_x(t_1 - t_2 - w) \cdot dw \cdot dv$$

$$= \int R_x(t_1 - t_2 - w) \cdot \underbrace{\left(\int h(v+w)\, h(v) \cdot dv\right)}_{g(w) \to \text{autocorrelation}} dw$$

$$= \int R_x(t_1 - t_2 - w)\, g(w)\, dw$$

$$= R_x * g(t_1 - t_2)$$

$$R_X \xrightarrow{} \boxed{g(\,)} \xrightarrow{} R_Y$$
$$\downarrow FT \qquad\qquad \downarrow FT$$
$$S_X(f) \qquad\qquad S_Y(f)$$

$$S_Y(f) = S_X(f) \cdot G(f)$$

$$g(\tau) = \int h(u)\, h(u+\tau)\, du$$

$$G(f)$$

\# $h(t) * h(-t)$

$\downarrow FT$

$$H(f)\, H^*(f) = |H(f)|^2$$

$$\boxed{S_Y(f) = S_X(f) \cdot |H(f)|^2} \rightarrow \text{PSD: Power Spectral Density}$$

01-11-2024

\# WSS Random Process : $(X(t))$

- $\mathbb{E}\, X(t) = \mu_X(t) = \mu_X$
- $\mathbb{E}\, X(t_1) X(t_2) = R_X(t_1 - t_2)$
  $= R_X(\tau)$

$(X(t))$ is WSS.

$$\xrightarrow{} \boxed{h(t)} \xrightarrow{} (Y(t))\text{ is also WSS.}$$

μ_X
$R_X(\tau)$

$\mu_Y = \mu_X H(0)$

$$R_Y(\tau) = \int g(u) \cdot R_X(\tau - u)\, du$$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) \cdot e^{-j2\pi f t} \cdot d\tau$$

$$S_Y(f) = \int_{-\infty}^{\infty} R_Y(\tau)\, e^{-j2\pi f t} \cdot d\tau$$

$$\boxed{\begin{aligned} \underset{\text{PSD}}{\underset{\downarrow}{S_Y(f)}} &= G(f) \cdot S_X(f) \\ &= |H(f)|^2\, S_X(f) \end{aligned}}$$

\#

1Ω

$(X(t))$

$$\boxed{\mathbb{E}(X^2(t)) = R_X(0)} \quad (\text{for } t_1 = t_2)$$

$\downarrow$
power

Take inverse fourier transform,

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f)\, e^{j2\pi f\tau}\, df$$

$$\Rightarrow \boxed{R_X(0) = \int_{-\infty}^{\infty} S_X(f)\, df}$$

↳ Power spectral density

[Reading Assignment : PSD of bandpass]

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$\boxed{|X(f)|^2\, |H(f)|^2 = |Y(f)|^2}$$

Eg. WSS with mean $=0$ $\rightarrow$ IID

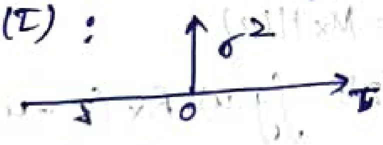
↳ $(X(t))$: $X(t) \sim N(0, \sigma^2)$

or $\sim$ $\boxed{\begin{matrix} -1 & 1 & \\ 1/2 & 1/2 & \omega_p \end{matrix}}$

$\mu_X = 0$

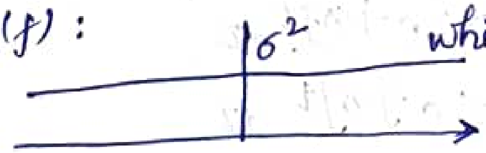
$$\mathbb{E}\, X(t_1).X(t_2) = \begin{cases} \sigma^2, & t_1 = t_2 \\ 0, & t_1 \neq t_2 \end{cases}$$

$R_X(\tau)$:



$\Rightarrow S_X(f) = \sigma^2$

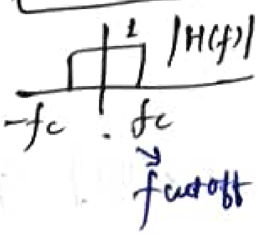
$S_X(f)$:

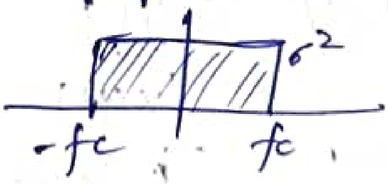


white spectrum

$$\text{Process} \longrightarrow \boxed{\text{LPF}} \longrightarrow$$

Output PSD :





Gaussian Processes

$(X(t))$:

Sequence of RVs : $(X(t_1)\ X(t_2)\ \ldots\ X(t_m))$

take values $\to x_1 \to N(x_1, \mu, \sigma^2)$, x_2 '', x_m

$X \sim N(\mu, \sigma^2)$ (gaussian dist.)

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

05-11-2011

Gaussian Random Process : Noise Model

$(X(t))$: Random process model joint distribution.

Gaussian:

$X(t) \sim N(\mu, \sigma^2)$

$M_X(t) = M_X$

$\mathbb{E}X(t_1)X(t_2) = R_X(t_1 - t_2)$

Eg. IID, $X(t) \sim N(\mu, \sigma^2)$

Joint distribution of any collection of RVs from this process is multidimensional Gaussian distribution.

Collection of RVs: (m times) $t_1, t_2, \ldots, t_m$.

Multidim. Gaussian dist. : $f_{X(t_1)X(t_2)\ldots X(t_m)}$

Eg. $f_{X(t_1)X(t_2)}(x_1, x_2)$

If $m=1$, $MN(x) = N(x, \mu, \sigma^2)$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $m=2$, $MN(\bar{x}) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Eg. If indep. $N(x_1, \mu_1, \sigma_1^2)$

$x_1 \perp x_2$ $\quad N(x_2, \underbrace{\mu_2}_{\vec{\mu}}, \sigma_2^2)$

$$f_{x_1 x_2}(\underbrace{x_1, x_2}_{\vec{x}}) = \frac{1}{\sqrt{2\pi |\Sigma|}} e^{-\left(\frac{1}{2}(\vec{x}-\vec{\mu})^T (\Sigma^{-1}(\vec{x}-\vec{\mu}))\right)}$$

$$\Sigma : \text{conv. matrix} = \begin{pmatrix} \sigma_1^2 & \text{conv}(x_1, x_2) \\ \text{conv}(x_1, x_2) & \sigma_2^2 \end{pmatrix}$$

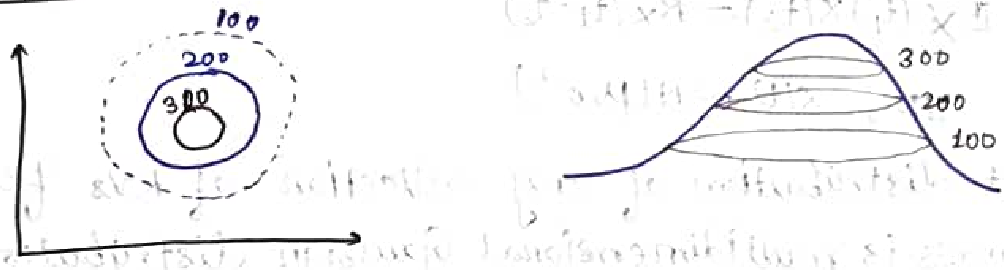
$$\text{conv.}(x, y) = E\,XY - EX \cdot EY$$

$$X \perp Y \Rightarrow \text{conv}(X, Y) = 0$$

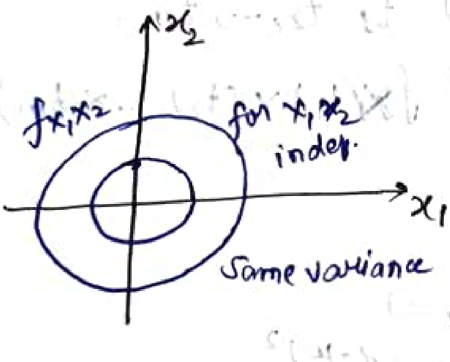
$$\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}\right)^T \begin{pmatrix} \frac{1}{\sigma_1^2} & 0 \\ 0 & \frac{1}{\sigma_2^2} \end{pmatrix} \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}\right)$$

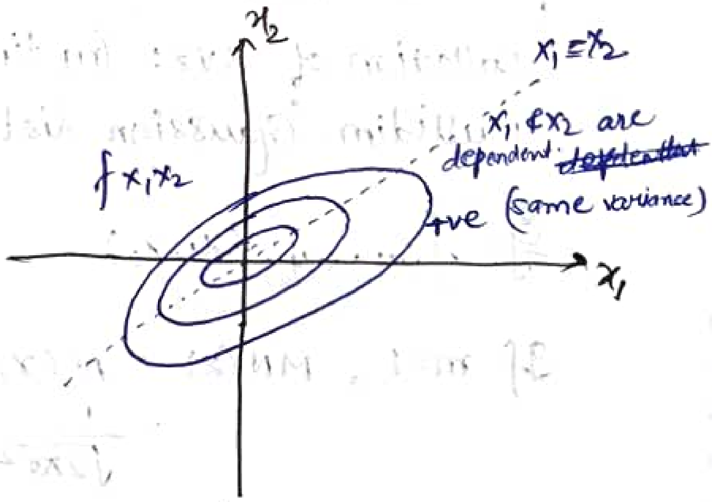
HW: MN( ) $\longrightarrow$ N.N

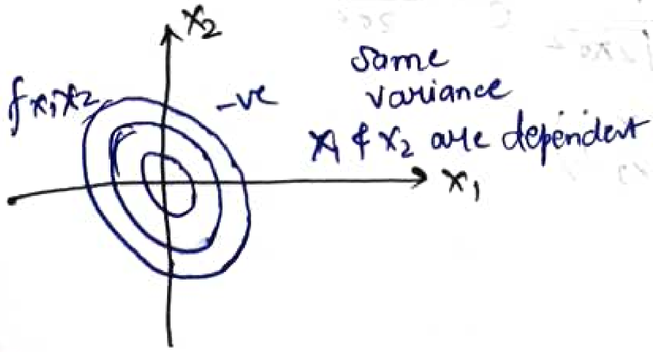
Contour Plane

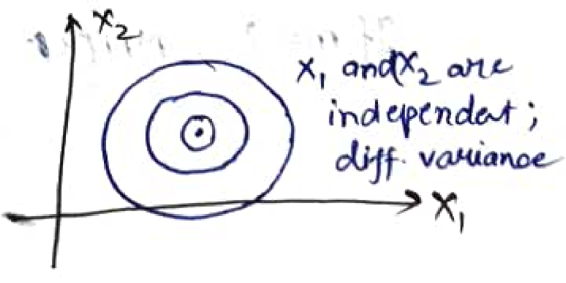


Eg.









If $m>1$,

$$f_{X(t_1)X(t_2)\ldots X(t_m)}(x_1, x_2, \ldots, x_m) = \frac{1}{\sqrt{2\pi|\Sigma|}} e^{-\frac{1}{2}\left[(\vec{x}-\vec{\mu})^T \Sigma^{-1} (\vec{x}-\vec{\mu})\right]}$$

$$\vec{\mu} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_m \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_1^2 & & \\ & conv(x_i;x_j) & \\ & & \sigma_m^2 \end{pmatrix}$$

↑ mean vector — $m \times m$ convar. matrix

Another definition for GP:

Any linear combination of $(X(t_1), \ldots, X(t_m))$ $\forall m$ $\forall (t_1, \ldots, t_m)$ is gaussian

linear combination : $\sum_{i=1}^{m} a_i X(t_i)$

Eg. $X(t) = \overset{N(0,\sigma^2)}{N} \cos(2\pi f_c t)$

Gaussian $N(0, \cos^2(2\pi f_c t)\sigma^2)$

$(X(t))$

$a_1 X(t_1) + \ldots + a_m X(t_m)$

$N(\sum a_i \cos(2\pi f_c t_i)) \sim N(0, ?) \Rightarrow$ GRP

Eg. Find $f_{X(t_1)X(t_2)}(x_1, x_2)$ distribution.

$$f_{X(t_1)X(t_2)}(x_1, x_2) = \frac{1}{\sqrt{2\pi|\Sigma|}} e^{-(\vec{x}-\vec{\mu})^T \Sigma^{-1} (\vec{x}-\vec{\mu})}$$

$$\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma^2 \cos^2(2\pi f_c t_1) & \sigma^2 \cos(2\pi f_c t_1)\cos(2\pi f_c t_2) \\ \sigma^2 \cos(2\pi f_c t_1)\cos(2\pi f_c t_2) & \sigma^2 \cos^2(2\pi f_c t_2) \end{pmatrix}$$

$conv(x(t_1), (x(t_2)))$

$= E\, X(t_1) X(t_2)$

$= E\, N \cos(2\pi f_c t_1) \cdot N \cos(2\pi f_c t_2)$

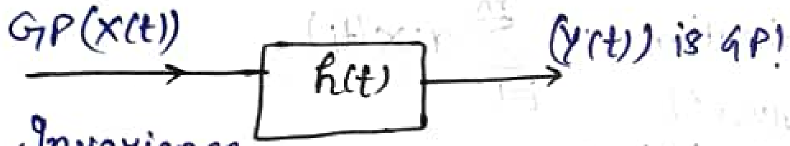
07-11-2024

Correction

$$f_{X(t_1)\dots X(t_m)}(x_1,\dots,x_m) = \frac{1}{\sqrt{(2\pi)^m|\Sigma|}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu})}$$

Recall:

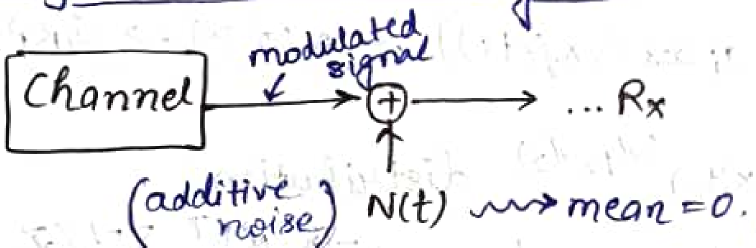
- Gaussian RPs - $(X(t))$
  - any collection of RVs
    $(X(t_1), X(t_2), \dots, X(t_m))$
    $\forall m \ \forall (t_1,\dots,t_m) \sim \mu N(\vec{\mu}, \Sigma)$
  - any linear combination
    $\forall \alpha_i \ \ \sum \alpha_i X(t_i) \sim N(\cdot)$

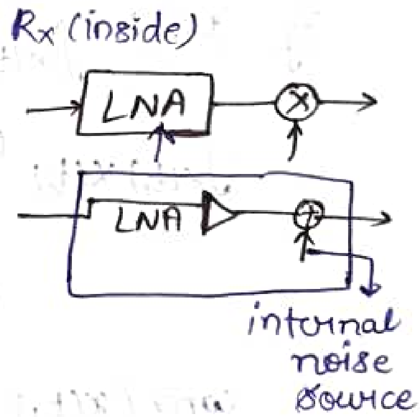


Invariance property

→ Noise models are GP models.

In a passband comm. system





IID GP $\left\{ \begin{array}{l} \text{Gaussian RP} \\ \text{White RP} \end{array} \right\}$

White RP ⇓ $(R_N(\tau) = \delta(\tau))$

Constant Spectral density ⇝ delta in time domain.

What is the covariance?

$$\text{cov}(X(t_1), X(t_2))$$
$$= \mathbb{E}X(t_1)X(t_2)$$
$$= 0 \text{ if } t_1 \neq t_2.$$

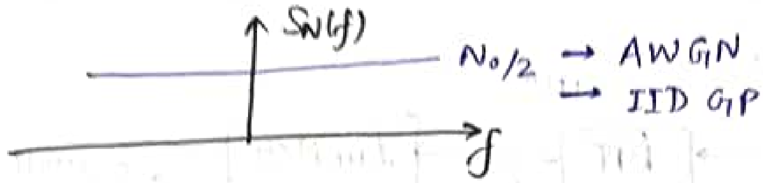
cov = 0 ⇒ Independence

Independence $\not\Rightarrow$ cov = 0

⇓

cov = 0 for Gaussian RP

→ Noise is an additive white G.P.



<u>Recall</u>: deterministic signal

$$\underbrace{\int N(t)\, e^{-j2\pi f t}\, dt}_{\text{More correct}}$$

N(t) is a RP.

— AWGN

— PSD.

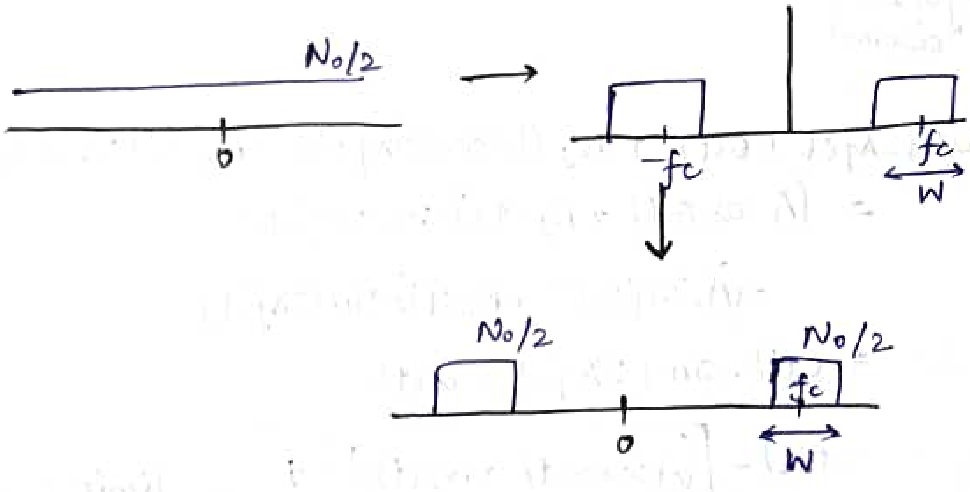
<u>H/w assignment</u>:

Revisit the SNR analysis.

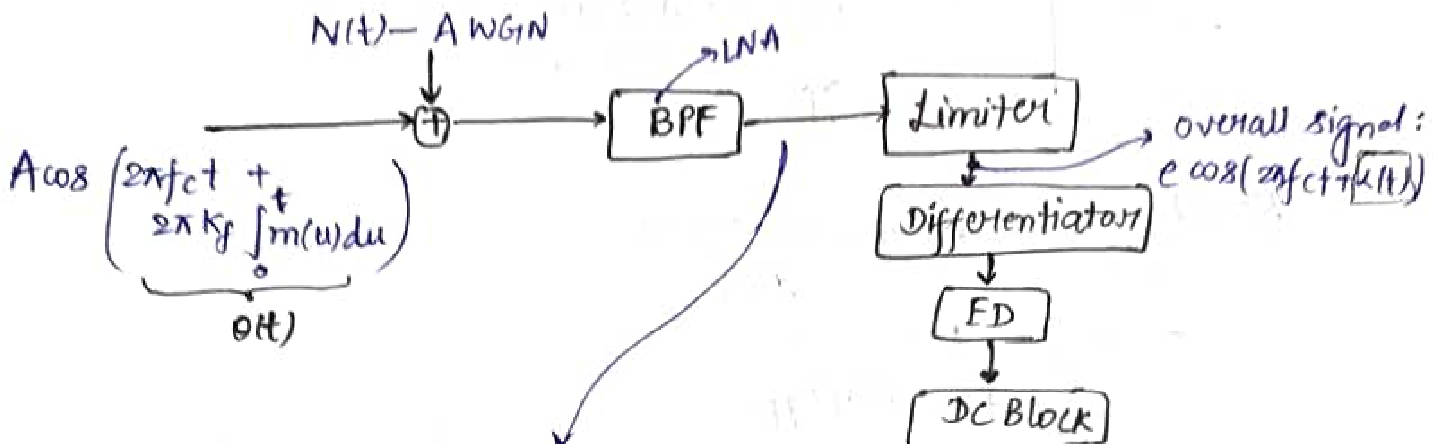
$A m(t) \cos(2\pi f_c t) + \boxed{N(t)}$

$$\boxed{S_{N_B}(f) = S_N(f) \cdot |H(f)|^2}$$

<u>Eg.</u>



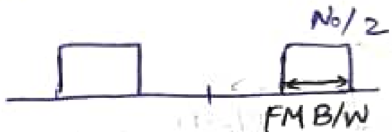
Back to SNR Analysis — SNR-FM



$A\cos(2\pi f_c t + \theta(t))$: Signal

$N(t) \rightarrow$ BPF $\rightarrow N_B(t)$: Noise

- also GP
- Mean = 0
- PSD :



↳ Not white

↳ coloured Noise

↳ Autocorrelation ⇝ (sinc . cos) (Inverse FT)

$$N_B(t) = N_I(t)\cos(2\pi f_c t) - N_Q(t)\sin(2\pi f_c t)$$

↓ like passband channel

$$A\cos(2\pi f_c t + \theta(t)) + N_I(t)\cos(2\pi f_c t) - N_Q(t)\sin(2\pi f_c t)$$

$$= (A\cos\theta(t) + N_I(t))\cos(2\pi f_c t) - (A\sin(\theta(t)) + N_Q(t))\sin(2\pi f_c t)$$

$$= e(t)\,.\sin(2\pi f_c t + \alpha(t))$$

$$e(t) = \sqrt{(A\cos\theta(t) + N_I(t))^2 + (A\sin\theta(t) + N_Q(t))^2}$$

⇝ limiter removes the variation in $e(t)$ but doesn't do anything to $\alpha(t)$.

$$\alpha(t) = \tan^{-1}\left(\frac{A\sin\theta(t) + N_Q(t)}{A\cos\theta(t) + N_I(t)}\right)$$

$$A\cos(2\pi f_c t + \theta(t)) \xrightarrow{\;+\,N(t)\;} \boxed{BPF} \xrightarrow{N_B(t)} \boxed{\text{Limiter}} \to \boxed{\text{Diff.}} \to \boxed{FD} \to \boxed{\text{DC Block}} \to$$

$\theta(t) = 2\pi K_f \int m(u)\,du$

BPF output: $e(t)\cos(2\pi f_c t + \alpha(t))$

Limiter output: $e\cos(2\pi f_c t + \alpha(t))$

Diff. output: $e\sin(2\pi f_c t + \alpha(t)).\boxed{(2\pi f_c + \alpha'(t))}$

SNR analysis is for $\alpha'(t)$

$$\alpha(t) = \tan^{-1}\left(\frac{A\sin(\theta(t)) + N_Q(t)}{A\cos(\theta(t)) + N_I(t)}\right)$$

$$N_B(t) = N_I(t)\cos(2\pi f_c t) - N_Q(t)\sin(2\pi f_c t).$$

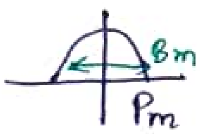
For Narrow-band FM

$\left.\begin{array}{l} \text{\& } A \gg N_I(t) \quad \forall t \end{array}\right\}$ Assumptions

$$\alpha(t) = \tan^{-1}\left(\theta(t) + \frac{N_Q(t)}{A}\right) \qquad \left[\begin{array}{l} \because \sin\theta(t) \approx \theta(t) \\ \cos\theta(t) \approx 1 \\ A + N_I(t) \approx A \end{array}\right]$$

$$= \theta(t) + \frac{N_Q(t)}{A}$$

$$\alpha'(t) = \underbrace{\theta'(t)}_{\substack{2\pi K_f m(t) \\ \text{(desired)}}} + \underbrace{\frac{N_Q'(t)}{A}}_{\text{Noise}}$$

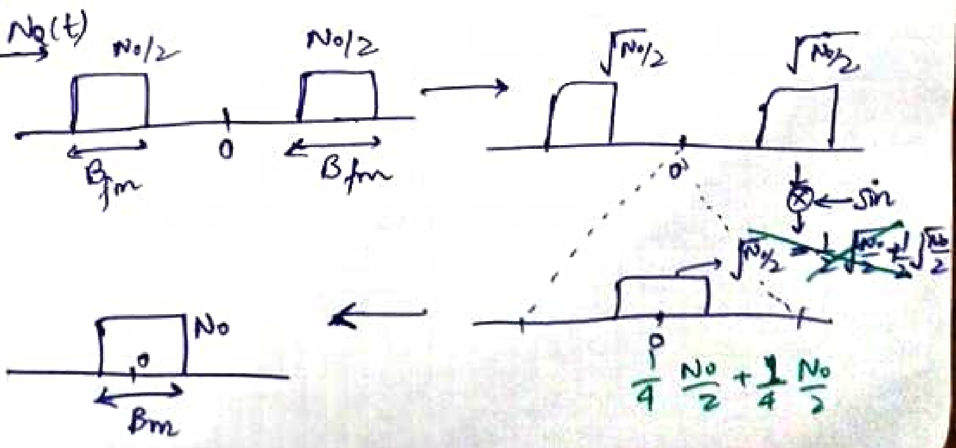


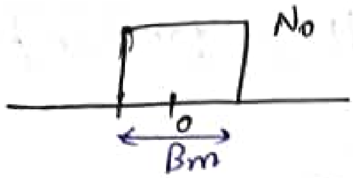
$(4\pi^2 K_f^2 P_m)$

$$N_B(t) \to \otimes \to \boxed{LPF} \xrightarrow{} N_Q(t)$$

$2 \to$ gain

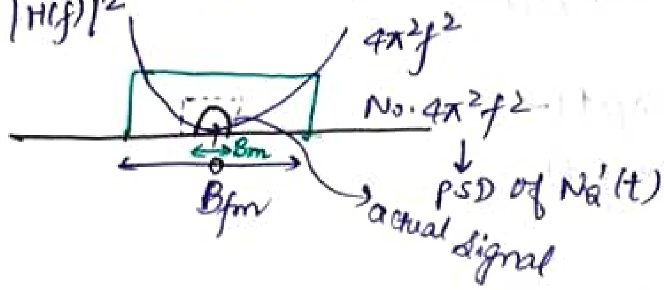
$-\sin(2\pi f_c t)$





$$N_Q(t) \longrightarrow \boxed{d/dt} \longrightarrow N_Q'(t)$$

$$H(f) = j\,2\pi f$$



$$\text{Noise power} = \frac{2}{A^2}\int_0^{Bm/2} 4\pi^2 N_o f^2\, df$$

$$= \frac{4\pi^2 N_o Bm^3}{12A^2}$$

$$SNR = \left(\cancel{4\pi^2} K_f^2 P_m\right)\left(\frac{12A^2}{\cancel{4\pi^2} N_o Bm^3}\right)$$

$$= \frac{12\, A^2 P_m}{N_o}\left(\frac{K_f}{Bm}\right)^2\left(\frac{1}{Bm}\right)$$