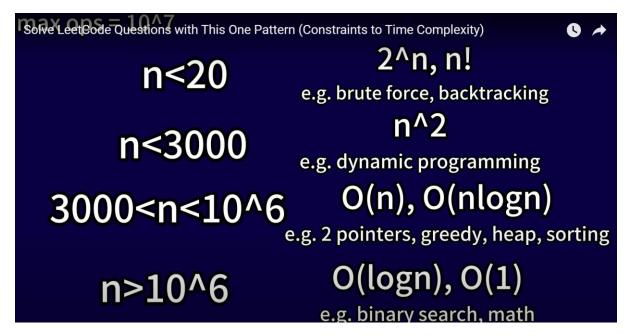
Constraints	Worst time complexity	Algorithmic solution	Examples
<i>n</i> ≤ 12	O(n!)	Recursion & Backtracking	Permutation 1n
n ≤ 25	O(2n)	Recursion , Backtracking & Bit Manipulation	All subsets of an array of size n
<i>n</i> ≤ 100	O(n4)	Dynamic Programming	4Sum
<i>n</i> ≤ 500	O(n3)	Dynamic Programming	All triangles with side length less than n
<i>n</i> ≤ 104	O(n2)	Dynamic Programming, <u>Graphs</u> , <u>Trees</u>	Bubble Sort (Slow comparison-based sorting)
<i>n</i> ≤ 106	O(n log n)	Sorting, Binary Search, Divide and Conquer	Merge Sort (Fast comparison-based sorting)
<i>n</i> ≤ 108	O(n)	Mathematical, Greedy	Min and max of element
n > 108	O(log n) or O(1)	Mathematical, Greedy	Binary Search



### **Common time complexities**

Let *n* be the main variable in the problem.

- If  $n \le 12$ , the time complexity can be O(n!).
- If  $n \le 25$ , the time complexity can be  $O(2^n)$ .
- If  $n \le 100$ , the time complexity can be  $O(n^4)$ .
- If  $n \le 500$ , the time complexity can be  $O(n^3)$ .
- If  $n \le 10^4$ , the time complexity can be  $O(n^2)$ .
- If  $n \le 10^6$ , the time complexity can be O(n log n).
- If  $n \le 10^8$ , the time complexity can be O(n).
- If  $n > 10^8$ , the time complexity can be O(log n) or O(1).

### Examples of each common time complexity

- O(n!) [Factorial time]: Permutations of 1 ... n
- $O(2^n)$  [Exponential time]: Exhaust all subsets of an array of size n
- $O(n^3)$  [Cubic time]: Exhaust all triangles with side length less than n
- O(n²) [Quadratic time]: Slow comparison-based sorting (eg. Bubble Sort, Insertion Sort, Selection Sort)
- O(n log n) [Linearithmic time]: Fast comparison-based sorting (eg. Merge Sort)
- O(n) [Linear time]: Linear Search (Finding maximum/minimum element in a 1D array),
   Counting Sort
- O(log n) [Logarithmic time]: Binary Search, finding GCD (Greatest Common Divisor) using Euclidean Algorithm

• O(1) [Constant time]: Calculation (eg. Solving linear equations in one unknown)

you get a clearer picture of how quickly each type of complexity increases with larger inputs. This is useful for understanding and comparing the efficiency of different algorithms.

log10 (n!) = 8.68 for n = 12 log10(2^n) = 7.52 for n = 25 log10(n^4) = 8.00 for n = 100 log10(n^3) = 8.09 for n = 500 log10(n^2) = 8.00 for n = 10^4 log10(n log n) = 7.29 for n = 10^6 log10(n) = 8.00 for n = 10^8

## Data Types and their ranges

Data Type	Actual Range	Approximate Range (10^)
short int	-32,768 to 32,767	±3.2 × 10^4
unsigned short int	0 to 65,535	0 to 6.5 × 10^4
int	-2,147,483,648 to 2,147,483,647	±2.1 × 10^9
unsigned int	0 to 4,294,967,295	0 to 4.3 × 10^9
long int	-2,147,483,648 to 2,147,483,647	±2.1 × 10^9
unsigned long int	0 to 4,294,967,295	0 to 4.3 × 10^9
long long int	-9,223,372,036,854,775,808 to 9,223,372,036,854,775,807	±9.2 × 10^18
unsigned long long int	0 to 18,446,744,073,709,551,615	0 to 1.8 × 10^19
float	~1.2E-38 to ~3.4E+38	~10^-38 to ~10^38
double	~2.3E-308 to ~1.7E+308	~10^-308 to ~10^308
long double	Greater than double (varies by implementation, e.g., 1.2E-4932 to 1.2E+4932)	~10^-4932 to ~10^4932 (varies)

Data Type	Actual Range	Approximate Range (10^)
char	-128 to 127 (signed) or 0 to 255 (unsigned)	±1.2 × 10^2 (signed) or 0 to 2.5 × 10^2 (unsigned)
unsigned char	0 to 255	0 to 2.5 × 10^2
signed char	-128 to 127	±1.2 × 10^2
bool	true or false	N/A
wchar_t	Implementation-defined (e.g., 0 to 4,294,967,295 for 4-byte wchar_t)	Varies

### **Types of Solutions Feasible for Upper Limits**

Given the constraints, the following table suggests feasible solutions for different input size constraints and time complexity:

Input Size Constraint	Time Complexity	Feasible Solutions
N <= 10	O(N!)	Brute force permutations, backtracking.
N <= 20	O(2^N)	Dynamic programming with bitmasks, exhaustive search.
N <= 100	O(N^3)	Floyd-Warshall for shortest paths, cubic algorithms.
N <= 1,000	O(N^2)	Dynamic programming, graph algorithms (Floyd-Warshall).
N <= 10,000	O(N log N)	Sorting algorithms, binary search, segment trees.
N <= 100,000	O(N)	Linear time algorithms, counting sort, prefix sums.
N <= 1,000,000	O(N log N)	Efficient sorting, divide and conquer, balanced trees.
N <= 10,000,000	O(N)	Linear algorithms with efficient data structures (e.g., counting sort).

## **Special Constraints**

- **Definition:** Unique conditions specific to the problem, like distinct values or specific properties of data (e.g., sorted array, connected graph).
- Impact: Adds complexity and requires consideration of additional factors in the solution.

# **Special Constraints and Their Solutions**

### 1. Distinct Elements

o **Example:** All array elements are distinct.

 Feasible Solutions: Sorting algorithms, unique combinations, or permutations can be leveraged effectively.

# 2. Graph Connectivity

- o **Example:** The graph is connected.
- Feasible Solutions: DFS, BFS, and union-find algorithms for efficient traversal and connectivity checks.

## 3. Sorted Input

- **Example:** The input array is already sorted.
- Feasible Solutions: Binary search, two-pointer techniques, and efficient merge operations.

a comprehensive table that suggests feasible solutions based on different input size constraints and their corresponding time complexities:

### **Detailed Feasible Solutions for Different Input Sizes**

Input Size Constraint	Time Complexity	Space Complexity	Feasible Solutions
N <= 10	O(N!)	O(N)	Brute force permutations, backtracking, exhaustive search.
N <= 20	O(2^N)	O(2^N)	Dynamic programming with bitmasks, exhaustive search, subset generation.
N <= 100	O(N^3)	O(N^2)	Floyd-Warshall for shortest paths, cubic algorithms, dynamic programming with three nested loops.
N <= 10^3	O(N^2)	O(N^2)	Dynamic programming (e.g., longest common subsequence), graph algorithms (e.g., Floyd-Warshall, matrix multiplication).
N <= 10^4	O(N log N)	O(N)	Efficient sorting algorithms (e.g., merge sort, quicksort), binary search, line sweeping algorithms, segment trees.
N <= 10^5	O(N)	O(N)	Linear time algorithms (e.g., counting sort, radix sort, prefix sums, two-pointer techniques, linear scan).
N <= 10^6	O(N log N)	O(N)	Balanced tree algorithms, divide and conquer strategies, efficient data structures (e.g., Fenwick trees, AVL trees).

Input Size Constraint	Time Complexity	Space Complexity	Feasible Solutions
N <= 10^7	O(N)	O(N)	Linear algorithms with efficient implementation (e.g., counting sort, radix sort, linear scan for specific problems).
N <= 10^8	O(N log log N)	O(N)	Sieve of Eratosthenes for prime number generation, certain amortized linear algorithms.
N <= 10^9	O(N)	O(N)	Specific linear algorithms, data streaming algorithms, counting frequencies.
N <= 10^{18}	O(log N)	O(1)	Binary search on ranges, logarithmic algorithms, number theory algorithms (e.g., modular arithmetic, logarithmic exponentiation).

## Small Input Size (N <= 10)

- O(N!)
  - Brute force solutions.
  - o Permutations and combinations.
  - Backtracking algorithms.

# Moderate Input Size (N <= 20)

- O(2^N)
  - Subset generation.
  - o Dynamic programming with bitmasks.
  - o Recursive algorithms with memoization.

# Larger Input Size (N <= 100)

- O(N^3)
  - o Floyd-Warshall algorithm for all pairs shortest paths.
  - Matrix multiplication.
  - $\circ\quad$  Dynamic programming for problems like the traveling salesman problem with small N.

# Significant Input Size (N <= 1,000)

#### O(N^2)

- o Dynamic programming for problems like the longest common subsequence.
- o Graph algorithms like Floyd-Warshall.
- o Matrix chain multiplication.

### Large Input Size (N <= 10,000)

# • O(N log N)

- o Efficient sorting algorithms like merge sort and quicksort.
- o Binary search algorithms.
- o Data structures like segment trees for range queries.

### Very Large Input Size (N <= 100,000)

### • O(N)

- o Linear algorithms like counting sort, radix sort.
- o Prefix sums for range sum queries.
- o Two-pointer techniques for problems involving subarrays or subsequences.

### Extremely Large Input Size (N <= 1,000,000)

### • O(N log N)

- o Balanced tree algorithms like AVL trees.
- o Efficient divide and conquer strategies.
- o Data structures like Fenwick trees for range queries.

#### Ultra Large Input Size (N <= 10,000,000)

#### O(N)

- o Highly optimized linear algorithms.
- Counting sort for specific ranges.
- o Linear scan algorithms for specific types of problems.

### Massive Input Size (N <= 100,000,000)

#### • O(N log log N)

- Sieve of Eratosthenes for prime number generation.
- o Amortized linear time algorithms for specific problem domains.

# Giga Input Size (N <= 1,000,000,000)

#### O(N)

Data streaming algorithms.

- Counting frequencies in large data sets.
- Highly specialized linear algorithms.

# N <= 10^{18}

- Time Complexity: O(log N)
- Space Complexity: O(1)
  - o Binary search on ranges, logarithmic algorithms like modular exponentiation, and number theory algorithms.

#note

Standard CPU operation 10^8 ops/s