Recurrence

$r(n, G) = 0 \quad i = 0.$

$\cancel{r(n, G)} = \max(q_i p_i + r(i-1, G - q_i w_i))$

$- \min[\text{pinei} \cdot (n_i - q_i), c_i)$

$\ldots \; q_i < q_i \min$

$\&\& \; q_i < q_i \max$

$\max(q_i p_i + r(i-1, G - q_i w_i))$

$q_i > q_i \min$

$\&\& \; q_i < q_i \max$

$i = 0$ to n (no. q iters)

Proof of Correctness

1. Feasibility:
   There exists a solution with value $\gamma(n)$
   Proof: by induction on n.

~~Optimality~~ Optimality

Let Opt(n, G) be optimal solution from items, ... items i

claim: $\gamma(n, G) \geq opt(n, G)$

proof:

By induction on i

Base $i = 0$ — no items are there $Opt(0) = \gamma(0)$.

step: consider $n > 0$

Induction Hypothesis: $\gamma(j, g)$

~~for $g < G$ $\gamma(g) \geq opt(n, G)$~~

for $g < G$ $\gamma(j, g) \geq opt(n, g)$

case i :- $q_i < q_{max}$ & $q_i < q_{imin}$

q_i is included

$\gamma(G) = \gamma(i, G)$ ~~$q_i p_i + \gamma(i-1, G - q_i w_i) -$ fixed q_i~~ $\min(q_{i}, - c_i)$

$\gamma(n, c) = \max(q_i p_i + \gamma(i-1, c-q_i w_i)$
$- \min(\text{fines} \, q_i, c_i)$

$\gamma(i-1, c-q_i w_i)$ gives the optimal solution
& $q_i p_i$ is taken such that it is max:

i.e. $Opt(i) = \{ q_i p_i \cup \overset{opt}{\gamma}(i-1, c-q_i w_i) \cup \text{fines}\, q_i, c_i)$

$opt(i) = \{ q_i p_i + opt(i-1, c-q_i w_i)$
$-(\text{fines})$

~~$opt(i) = \max($~~
$\gamma(i) = \max \{ q_i p_i + \gamma(i-1, c\, q_i w)$
$- \text{fines})$

$\geq V_i + opt(n, g)$ by I.H.
$= opt(i)$

$\therefore$ By induction, for all i
$i \in [0, \ldots n]$
$\gamma(i) \geq opt(i)$
By feasibility $\gamma(i) = opt(i)$

Runtime analysis

There are three main loop in the DP.

1. for loop goes from 1 to n
2. for loop goes from 0 to G
3. for loop goes from 0 to qi.max

$\therefore RT = O(nGq)$