Appendix A. Appendix

A.1 Details on $\mathcal{EXSPN}$

The steps involved in converting an arbitrary SPN into a CSI-tree is illustrated in figure 5.

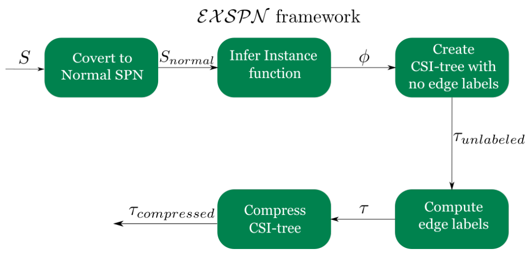

Figure 5: Flowchart illustrating the steps involved in converting an arbitrary SPN into a more interpretable CSI-tree.

A.2 Proofs

Algorithm 4: RetrieveSPN

input : CSI-tree $\tau = (G_{csi}, \psi_{csi}, \chi, \zeta)$
output: $G_{normal}, \psi_{normal}$

initialize: $G_{normal} = G_{csi}, \psi_{normal} = \psi_{csi}$
$st = [G_{normal}.root]$
while *st is not empty* **do**
  $N_{current} = st.pop()$
  if $N_{current}$ *has not been visited and is not the root node* **then**
    if $|\chi(N_{current})| = |\psi_{csi}(N_{current})|$ **then**
      Replace $N_{current}$ with N_p
      Append $|\chi(N_{current})|$ leaf nodes
    if $|\chi(N_{current})| \neq |\psi_{csi}(N_{current})|$ **then**
      for k *in* $\chi(N_{current})$ **do**
        if $|k| = 1$ **then**
          Append N_l s.t. $\psi(N_l) = k$
        if $|k| \neq 1$ **then**
          Append N_s s.t. $\psi(N_s) = k$ **for** c *in* $ch(N_{current})$ **do**
            Append N_p to N_s if $\psi_{csi}(c) = k$
          end
      end
  Add $ch(N_{current})$ to st
end
return $G_{normal}, \psi_{normal}$

Theorem 2 *The CSI-tree $\tau = (G_{csi}, \psi_{csi}, \chi, \zeta)$, inferred from an SPN, $S_{normal} = (G_{normal}, \psi_{normal}, w_{normal}, \theta_{normal})$, using $\mathcal{EXSPN}$ can infer G'_{normal}, and ψ'_{normal} which encodes the same CSIs as S_{normal}.*

Table 4: Dataset details.

Dataset	Type	$\|\mathbf{X}\|$	Train	Test
Synthetic	Continuous	4	22,500	7,500
Mushroom	Categorical	23	4,233	1,411
Plants	Binary	70	17,411	5,804
NLTCS	Binary	16	16,180	5,394
MSNBC	Binary	17	291,325	97,109
Abalone	Mixed	9	3,132	1,045
Adult	Mixed	15	33,916	11,306
Wine quality	Continuous	12	4,872	1,625
Car	Categorical	7	1,296	432
Yeast	Mixed	9	1,113	371
nuMoM2b	Categorical	8	8,832	388

Table 5: Hyperparameters(HP)

Component	HP	Value
SPN	rows	GMM
	mis	1% of $\|Train\|$
		5% of $\|Train\|$
		1% of $\|Train\|$
DT	max_depth	2
	mid	0.1
	class_weight	balanced
$\mathcal{EXSPN}$	min_precision	0.7
	min_recall	0.7
	n_instances	$5 \times mis$
		mis (Synthetic)

Proof Algorithm 4 that retrieves G_{normal} and ψ_{normal} associated with S_{normal}, given τ provides the proof. It performs DFS over G_{normal} and identifies two main cases of $N_{current}$: **1.** A node where the length of the partition function $\chi(N_{current})$ is equal to length of the scope function $N_{current}$[**line 6**] **2.** A node where that's not the case[**line 10**]. In the first case, it replaces $N_{current}$ with a product node N_p such that $\psi(N_p) = \psi_{csi}(N_{current})$ and adds $|\chi(N_{current})|$ number of leaf nodes. In the second case, it iterates over the subsets in $\chi(N_{current})$, adds a leaf node if that subset k is a singleton set and adds an intermediate sum node N_s for all other children associated with a subset k[**lines 10-17**]. Then, it returns computational graph G and scope function ψ. Although this algorithm retrieves only G_{normal} and ψ_{normal} which define the structure of the SPN, the parameters of the SPN w_{normal} and θ can also be retrieved by modifying $\mathcal{EXSPN}$ to produce a CSI-tree that stores these parameters in the leaf nodes of the CSI-tree alongside the scope ψ_{csi} and the partition χ. This modification is trivial and does not improve the interpretability. Hence, we do not consider it. ■

A.3 Details on Datasets and Hyperparameters

Tables 4 and 5 present statistics of the datasets and the hyperparameters used, respectively.

We generated the synthetic dataset by sampling 10,000 instances each from the following 3 multivariate Gaussians.

$$\mathcal{N}\left[\begin{pmatrix}2\\2\\2\\2\end{pmatrix}, 0.01\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\right],$$

$$\mathcal{N}\left[\begin{pmatrix}-8\\4\\4\\4\end{pmatrix}, 0.01\begin{pmatrix}1&0&0&0\\0&1&1&1\\0&1&1&1\\0&1&1&1\end{pmatrix}\right],$$

$$\mathcal{N}\left[\begin{pmatrix}8\\8\\8\\8\end{pmatrix}, 0.01\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&1\\0&0&1&1\end{pmatrix}\right]$$

A.4 Additional Experimental Results

Table 6 presents the mean log-likelihood over the test set for the SPNs trained on each of the datasets.

Table 7 presents the minimum support and minimum confidence parameters used for the apriori algorithm and the mean confidence of the association rules over the test set.

Tables 8, 9 and 10 present all the CSIs extracted from the Earthquake, Cancer and Asia Bayesian Networks, and fraction of datapoints that match with the ground truth CSIs.

Table 6: The mean log-likelihood over the test set (LL) for the SPNs trained on each of the datasets.

Dataset	LL
Synthetic	2.83
Mushroom	-8.98
Plants	-14.03
NLTCS	-6.3
MSNBC	-6.68
Abalone	18.99
Adult	-5.52
Wine quality	-3.55
Car	-7.92
Yeast	46.28
nuMoM2b	-6.92

Table 7: The Minimum Support (MS) and Minimum Confidence (MC) parameters used for the apriori algorithm and the mean confidence of the association rules over the test set (TC).

Dataset	TC	MS	MC
Synthetic	0.94	0.5	0.7
Mushroom	0.91	0.5	0.7
Plants	0.84	0.15	0.7
NLTCS	0.83	0.25	0.7
MSNBC	0.75	0.01	0.7
Abalone	0.87	0.25	0.7
Adult	0.85	0.5	0.7
Wine quality	0.86	0.5	0.7
Car	0.93	0.1	0.7
Yeast	0.9	0.5	0.7
nuMoM2b	0.87	0.5	0.7

Table 8: The CSIs extracted from data samples from the Earthquake Bayesian Network and the fraction of datapoints that matched with the ground truth CSIs

Antecedent	Consequent	Matched
(Alarm = 1)	(Earthquake, MaryCalls)	1.0
(Alarm = 1)	(Earthquake, JohnCalls)	1.0
(Alarm = 1)	(JohnCalls, MaryCalls)	1.0
(Alarm = 1)	(Burglary, Earthquake)	0.0
(Alarm = 1)	(Burglary, MaryCalls)	1.0
(Alarm = 1)	(Burglary, JohnCalls)	1.0
¬(Alarm = 1)	(Earthquake, MaryCalls)	1.0
¬(Alarm = 1)	(Earthquake, JohnCalls)	1.0
¬(Alarm = 1)	(JohnCalls, MaryCalls)	1.0
¬(Alarm = 1)	(Burglary, Earthquake)	0.0
¬(Alarm = 1)	(Burglary, MaryCalls)	1.0
¬(Alarm = 1)	(Burglary, JohnCalls)	1.0

Table 9: The CSIs extracted from data samples from the Cancer Bayesian Network and the fraction of datapoints that matched with the ground truth CSIs

Antecedent	Consequent	Matched
(Cancer = 1)	(Pollution, Smoker)	0.0
(Cancer = 1)	(Pollution, Xray)	1.0
(Cancer = 1)	(Dyspnoea, Smoker)	1.0
(Cancer = 1)	(Dyspnoea, Xray)	1.0
(Cancer = 1)	(Smoker, Xray)	1.0
(Cancer = 1)	(Dyspnoea, Pollution)	1.0
¬(Cancer = 1)	(Pollution, Smoker)	0.0
¬(Cancer = 1)	(Pollution, Xray)	1.0
¬(Cancer = 1)	(Dyspnoea, Smoker)	1.0
¬(Cancer = 1)	(Dyspnoea, Xray)	1.0
¬(Cancer = 1)	(Smoker, Xray)	1.0
¬(Cancer = 1)	(Dyspnoea, Pollution)	1.0

Table 10: The CSIs extracted from data samples from the Asia Bayesian Network and the fraction of datapoints that matched with the ground truth CSIs

Antecedent	Consequent	Matched
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(bronc, smoke)	0.0
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(bronc, xray)	1.0
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(bronc, lung)	1.0
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(dysp, smoke)	1.0
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(dysp, xray)	1.0
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(dysp, lung)	1.0
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(dysp, tub)	1.0
((either = 1)∧¬(asia = 1))∨¬(either = 1)	(bronc, tub)	1.0
(((either = 1)∧¬(asia = 1))∨¬(either = 1))∧¬(lung = 1)	(smoke, xray)	1.0
(((either = 1)∧¬(asia = 1))∨¬(either = 1))∧¬(lung = 1)	(tub, xray)	1.0
(((either = 1)∧¬(asia = 1))∨¬(either = 1))∧¬(lung = 1)	(smoke, tub)	1.0
(either = 1)∧(asia = 1)	(lung, tub)	0.0
(either = 1)∧(asia = 1)	(bronc, lung)	1.0
(either = 1)∧(asia = 1)	(dysp, lung)	1.0
(either = 1)∧(asia = 1)	(dysp, xray)	1.0
(either = 1)∧(asia = 1)	(tub, xray)	1.0
(either = 1)∧(asia = 1)	(smoke, tub)	1.0
(either = 1)∧(asia = 1)	(dysp, tub)	1.0
(either = 1)∧(asia = 1)	(bronc, tub)	1.0
(either = 1)∧(asia = 1)	(smoke, xray)	0.56
(either = 1)∧(asia = 1)	(bronc, xray)	1.0
(either = 1)∧(asia = 1)	(lung, smoke)	1.0
(either = 1)∧(asia = 1)	(lung, xray)	1.0
((either = 1)∧(asia = 1))∧(bronc = 1)	(dysp, smoke)	1.0
((either = 1)∧(asia = 1))∧¬(bronc = 1)	(dysp, smoke)	1.0