Optimising a 75.6 MHz low beta Superconducting Quarter Wave Resonator Cavity

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Summer '17

Contents

1 Certification by Instructor

This is a certification that Saurabh Raj , pursuing B.Tech in *Engineering Physics* from Indian Institute of Technology, Guwahati, has successfully fulfilled the completion of project on Optimisation of 75.6 MHz low beta Quarter Wave Resonator. The objectives of the project were achieved over a time period of one month in the months of May and June 2017, where the work leading to the accomplishment of the objectives was conducted with appreciable sincerity. Also it is certified that to the best of my knowledge there has been no plagiarism in the report and all the sources from where useful information had been taken have been cited properly and in good spirit by the student.

Dr. Puneet Jain
Assistant Professor
Indian Institute of Technology , Roorkee
Date .

2 Declaration by Student

I hereby declare that all the work reflected in the project enclosed in this report are my own and there has been no plagiarism involved to the best of my knowledge. The sources from which useful details and information were taken have been duly cited. The contents of the report are my own intellectual property and usage of this report for any further studies or research would require my discretion.

Saurabh Raj
B.Tech Sophomore Undergraduate
Indian Institute of Technology , Guwahati

Date .

3 Foreword

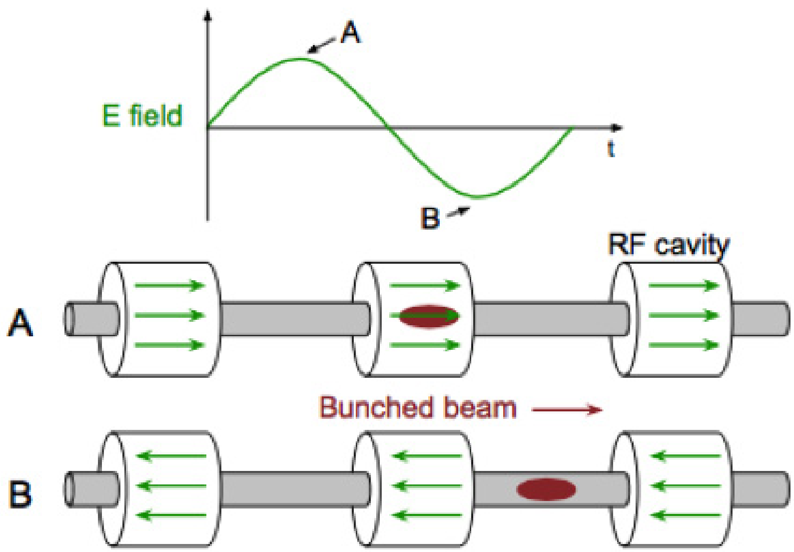
The story leading up to the actualization of this project in the summer was kind of bumpy and funny. One of the main reasons of this might be attributed to the fact that I was not yet convinced on anything whatsoever that I could feel I really would like to pursue in my further research if at all. So it was the crucial and seemingly on the edge moments of April mid weeks that I started sending out mails, approaching professors in and outside India for possible opportunities of summer academic research project.I sought projects related to Nuclear Physics, or others somewhat related to it no matter how remote.The one thing that I noticed while browsing through the faculties and their fields of interests in various institutions, was that in IIT Roorkee, there was an abrupt concentration of people working in fields of Nuclear Sciences. Finally after bumping off from a couple of professors over their on my interaction with them via emails I was approved for a project under Prof. Puneet Jain, who amusingly happens to be the only professor in the entire set of all faculties in all the IITs who work on accelerator physics.Luck also clicked in it's own regard as I was on the verge of completing my ongoing electrodynamics course that I was pursuing under my major of *Engineering Physics* which turned out to be one of the key concepts of cavity design. Hence I came to the IITR campus within 15 days of my semester getting over in my home university and started working on the mission to optimise the QWR superconducting cavity.The overall experience that I gained from IIT Roorkee helped me grow bit more and the warm atmosphere maintained by Dr. Jain throughout the timeperoid of the project is highly commendable. The expenses of the stay and extras were incurred by my parents for which I am extremely thankful. The IITR campus luckily was a piece of beauty if you see, sans the heatstrokes of the summer. Cheers.

Saurabh Raj
June 23, 2017

4 Chapter 1: Build up to a QWR

4.1 Accelerators and Resonators

A particle accelerator is a set-up which, as the name suggests, helps to accelerate charged particles over a voltage gradient. The peculiar thing about these accelerators is that, as one might presume otherwise, the acceleration is not a continuous process, but instead, happens only at certain specially designed spaces along the trajectory of the accelerating bunch of particles. The particles that need to be accelerated aren't a continuous beam of particles either. In fact, they travel as groups, all of which could be estimated to be estimated to be of almost same geometric dimensions. These groups are called bunches. The accelerating gaps that we talked about above are the spaces where we install specially designed structures solely meant to accelerate the incoming particles. They operate in a very synchronous manner in order to maintain the desired pulsating acceleration as we know from above paragraph that these bunches would arrive at these accelerating gaps only on certain points of time. This could be also be interpreted as discrete kicks to these bunches along the accelerator trajectory. These kickers are known as resonators. As we will see later, the RF resonant cavities that we tend to design in this paper are resonators operating at 75.6 MHz with $\beta_g = 0.055$.



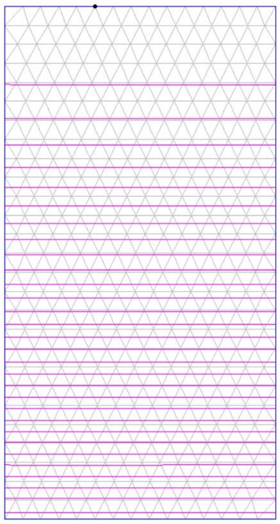
4.2 Pillbox Cavity

One of the simplest structures possible for a resonator is a pillbox cavity. It is basically a cylindrical structure in which the electromagnetic energy is trapped by feeding the cavity with EM waves via wave-guides. The resonance inside such a cavity is in TM mode. But the mode of TM that we would be interested and, in turn, would be focussing on is TM_{010} mode. In a pillbox cavity the Higher Order Modes (HOMs) generally very far apart in terms of influence over changing the quantifiable characteristics of the cavity (figures of merits). The distribution of the electric field at a certain plane perpendicular to the axis of pillbox cavity is non uniform. The higher density being near the axis, we could expect higher density of field lines utilised for the acceleration. The calculated theoretical values of two of the parameters are given below:

$$E_{pk} = 1.57 * E_{acc}$$

$$H_{pk}/E_{pk} = 1.54 * (10^{-3}) A/v$$

The field lines simulated for few given dimensions are given below. The cavity is operating at a resonant frequency of 2.4 GHz.



All calculated values below refer to the mesh geometry only.
Field normalization (NORM = 0): EZERO = 1.00000 MV/m
Frequency = 2441.26656 MHz
Particle rest mass energy = 938.272029 MeV
Beta = 0.9771826 Kinetic energy = 3479.189 MeV
Normalization factor for E0 = 1.000 MV/m = 5165.416
Transit-time factor = 0.6364632
Stored energy = 2.45655E-04 Joules
Using standard room-temperature copper.
Surface resistance = 12.89047 milliOhm
Normal-conductor resistivity = 1.72410 microOhm-cm
Operating temperature = 20.0000 C
Power dissipation = 275.4082 W
Q = 13681.8 Shunt impedance = 108.929 MOhm/m
Rs*Q = 176.365 Ohm Z*T*T = 44.126 MOhm/m
r/Q = 96.754 Ohm Wake loss parameter = 0.37103 V/pC
Average magnetic field on the outer wall = 1371.08 A/m, 1.21161 W/cm^2
Maximum H (at Z,R = 3,3.56852) = 1536.4 A/m, 1.52141 W/cm^2
Maximum E (at Z,R = 3,0.0) = 0.999932 MV/m, 0.023477 Kilp.
Ratio of peak fields Bmax/Emax = 1.9308 mT/(MV/m)
Peak-to-average ratio Emax/E0 = 0.9999

4.3 Figures of Merit

To quantify the efficiency of design of a resonant cavity we define few figures of merits. These are basically numbers that could be compared with and crossed checked with other experiments to tell us how better one's design is with respect to other available resonators, or if or not the design process is proceeding in the correct direction.

Quality Factor: The Quality Factor (*Q Factor*) tells us how efficient the cavity is. Its significance is more pronounced if one looks at the formula that is used to calculate it. It is defined as:

$$Q = \frac{\text{Energy stored any time}}{\text{Energy lost during one radian of oscillation of resonating field}}$$

Also, we know for a fact that the equation of a damped oscillation describes a wave better than the one that considers an ideal behaviour.

$$\ddot{\theta} + \omega^2\theta + \gamma\dot{\theta} = 0$$

Now the solution of this equation is

$$\begin{aligned}
\theta &\sim e^{j\omega t}e^{-\gamma t} && (1)\\
U &= U_0 e^{-\gamma t} && (2)\\
P_c &= \frac{dU}{dt} = \gamma U && (3)\\
Q &= \frac{U}{\gamma U \frac{T}{2\pi}} = \frac{1}{\gamma}\frac{2\pi}{T} = \frac{\omega}{\gamma} && (4)\\
hence \quad U &= U_0 e^{-\frac{\omega}{Q}t} && (5)\\
U &\sim H^2 and && (6)\\
U &\sim E^2 && (7)\\
so \quad H &= H_0 e^{-\frac{\omega}{2Q}t} && (8)\\
also \quad E &= E_0 e^{-\frac{\omega}{2Q}t} && (9)\\
now \quad P_c &= \gamma U = \frac{\omega U}{P_c} && (10)\\
hence \quad Q &= \frac{\omega U}{P_c} && (11)
\end{aligned}$$

Now Pc depends on the resistances felt inside the cavity, which in turn is dependent on temperature. Hence for higher Q Factor we need lower Pc. This means a lower resistance, which means a lower temperature. To satisfy this cause, we operate these resonators in cryogenic environment, most commonly in the liquid Helium bath. Although the Q Factor here described is taken for an ideal case where the geometry is assumed to be absolutely flawless and perfect but in actuality, one has to keep in mind that to operate the set up, we need to make several ports and holes in the cavity creating geometric perturbations in it. This in turn loads the circuit more than expected theoretically and hence there is more loss in the cavity. The final Q_{eff} is given by

$$\frac{1}{Q_{eff}} = \frac{1}{Q_0} + \frac{1}{Q_1} + \frac{1}{Q_2} + \frac{1}{Q_3}................$$

Here the Q_1, Q_2, and so on reflect the change in Q factor due to several perturbations.

Shunt Impedance: The Shunt Impedance was required as another figure of merit for the fact that was required in order to make up for the extra losses in the cavity due to loading effect of the geometric perturbations described above. The loading leads to a decrease in Q Factor and to make up for this we hypothesize an impedance called Shunt Impedance (R_s) that we assume to be added in parallel to the LCR equivalent to any given Resonator. LCR circuit with R_s . Formula for the Shunt Impedance is given by:

$$R_s = \frac{{V_c}^2}{P_c}$$

Hpk/Eacc and Epk/Eacc: These two figures of merit vary from cavity to cavity. Although for each kind of design we can figure an approximate conservative number around which these two parameters must lie. Eg. E_{pk}/E_{acc} should be greater than 1 but should not be larger than 6-7 for a relatively efficient design.

4.4 Simulation and Data Analysis

The software that we used for the simulation of various cavity designs to reach an optimum design keeping in mind all four Figures of Merits stated above at the given frequency of 75.6 MHz was *Poisson Superfish.* This software gives a 2D plot for half structure of any symmetric cavity. The co-ordinates are specified by the user and the software creates and simulates electric and magnetic field lines inside the cavity with respect to a grid (called mesh). The generation, editing and execution of the input files, as well as extraction of data to a DBMS from output file hence generated were all managed by an external C++ script in an automated manner.

5 CHAPTER 2: Design and speculation

5.1 Parallels drawn from the Transmission Lines Theory

The Quarter Wave resonator that we need to design requires us to treat it as a transmission line that is terminated at one end by a capacitive loading and shorting($Z_l = 0$) the other end. The formula for the input impedance of a transmission line is.

$$Z_{in} = Z_0 \frac{Z_l + Z_0 tan(\gamma l)}{Z_0 + Z_l tan(\gamma l}$$

For a lossless line we can assume $\gamma = j\beta l$ where β here is the value equivalent to $\frac{2\pi}{\lambda}$

here , if we put $Z_l = 0$, and make l= $(\lambda/4)$, we see that the

$$Z_{in} = \infty$$

.

The peculiar thing about $(\lambda/4)$ is that it helps in creating a sustainable resonance inside the cavity when a standing wave is created. The current flows on the walls of the cavity and in between the open end of inner conductor and the outer conductor, there is a displacement current that keeps the circuit a closed one.

The electromagnetic energy once filled inside this transmission line in an arrangement described above would be oscillating inside the cavity in a TEM mode. The electric field in the vertical gaps on either side of the inner conductor would be flowing from inner surface of outer conductor to the inner surface of inner conductor. This field could be utilised to accelerate any charged particle in a direction perpendicular to the central co-axis.

The equation of the radial electric field inside an ideal transmission line is given by

$$E_\rho = \frac{E_0 a}{\rho} sin(\omega t) cos(\frac{p\pi z}{2L})$$

Just for the sake of simplicity if we put

$$cos(\frac{p\pi z}{2L}) = 1$$

Then

$$E_\rho = \frac{E_0 a}{\rho} sin(\omega \frac{\rho}{\beta c})$$

Hence if we fix a certain energy difference between the incoming and the outgoing particle of a given charge, then we can calculate the geometric beta for such a system.

$$\Delta E = 2 \int_b^a \frac{E_0 a}{\rho} sin(\omega \frac{\rho}{\beta c}) d\rho$$

In this paper we have tried to accelerate a proton incoming at 1 MeV and want it escaping the cavity at energy of 2 MeV. By applying this condition in the energy equation above we get value of radius of inner conductor = 0.1 m. Now, again we need to satisfy one more condition that restricts our choices of outer radius of the coax.

$$a + b = \frac{3\beta\lambda}{2}$$

This restriction occurs because we want an accelerating field all along the accelerating gap. For this condition, the time difference between the time when the particle enters the first accelerating gap and the second accelerating gap should be

$$\frac{3\beta_g T}{2}$$

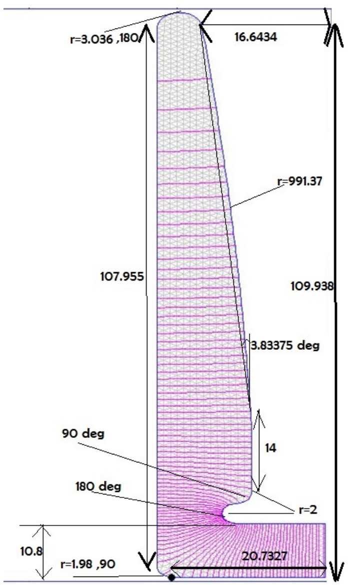
. That gives us

$$b = \frac{3\beta\lambda}{2} - a \approx 0.2271m$$

Now we have at least a starting point where to start checking for the match in the resonant frequency that we desire by varying the width of the capacitive termination at the bottom, or the tapering of the gap between the inner and outer conductor.

5.2 Design One

The following design gives us an impressive set of results for the figure of merits. The dimensions are being mention herewith.All lengths are in centimetre:



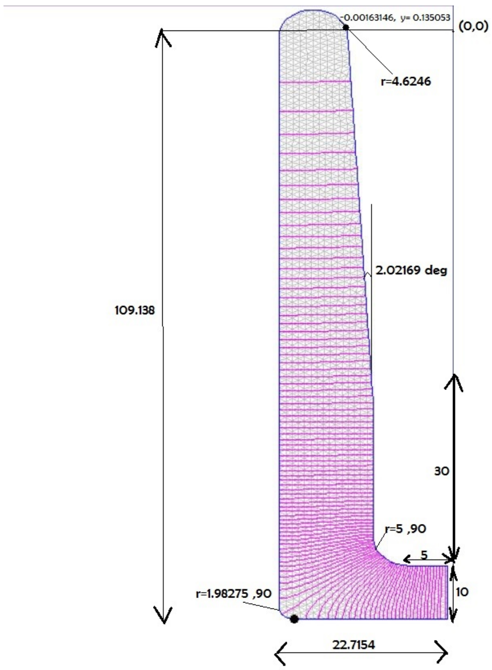
All calculated values below refer to the mesh geometry only.
Field normalization (NORM = 4): ENORM = 1.00000 MV/m
for the integration path from point Z1,R1 = 0.86138 m, 0.10000 m
to ending point Z2,R2 = 0.86138 m, 0.22715 m
Frequency = 75.67506 MHz
Particle rest mass energy = 938.272029 MeV
Beta = 0.0545253 Kinetic energy = 1.398 MeV
Normalization factor for E0 = 1.299 MV/m = 1128.926
Transit-time factor = 0.6358696
Stored energy = 0.3732565 Joules
Superconductor surface resistance = 10.0569 nanoOhm
Operating temperature = 2.0000 K
Power dissipation = 69.8330 mW
Q = 2.5414E+09 Shunt impedance = 2.6093E+06 MOhm/m
Rs*Q = 25.559 Ohm Z*T*T = 1.0550E+06 MOhm/m
r/Q = 44.835 Ohm Wake loss parameter = 0.00533 V/pC
Average magnetic field on the outer wall = 1627.11 A/m, 1.33127 uW/cm^2
Maximum H (at Z,R = 0.299492,0.133776) = 3720.03 A/m, 6.9587 uW/cm^2
Maximum E (at Z,R = 0.981896,0.13701) = 3.4077 MV/m, 0.331914 Kilp.
Ratio of peak fields Bmax/Emax = 1.3718 mT/(MV/m)
Peak-to-average ratio Emax/E0 = 2.6235

Frequency	Beta	Q	r/Q	Rs	Bmax/Emax	Emax/E0
75.67506 MHz	0.0545253	2.54E+09	44.835 Ohm	2.6093E+06 MOhm/m	1.3718 mT/(MV/m)	2.6235

For this design the origin has been selected on the top right corner in horizontal line with the point with height 16.634 cm. Hence the drift tube is planned to be located on the horizontal line which is 86.138 cm below the origin. All the calculations hence is done along that line itself. As a precautionary measure to minimise the chances of large multipacting we have chosen corners which are smooth and no abrupt pointed edge is inculcated in the design. The calculations are being done in acryogenic environment at temperature of 2K.

5.3 Design Two

The following design gives us an impressive set of results for the figure of merits. The dimensions are being mention herewith.All lengths are in centimetre:



All calculated values below refer to the mesh geometry only.
Field normalization (NORM = 4): ENORM = 1.00000 MV/m
for the integration path from point Z1,R1 = 1.04138 m, 0.10000 m
to ending point Z2,R2 = 1.04138 m, 0.22715 m
Frequency = 75.55636 MHz
Particle rest mass energy = 938.272029 MeV
Beta = 0.0504073 Kinetic energy = 1.194 MeV
Normalization factor for E0 = 4.457 MV/m = 2682.953
Transit-time factor = 0.6371657
Stored energy = 3.4624895 Joules
Superconductor surface resistance = 10.0567 nanoOhm
Operating temperature = 2.0000 K
Power dissipation = 586.2098 mW
Q = 2.8041E+09 Shunt impedance = 3.3888E+06 MOhm/m
Rs*Q = 28.200 Ohm Z*T*T = 1.3758E+06 MOhm/m
r/Q = 49.066 Ohm Wake loss parameter = 0.00582 V/pC
Average magnetic field on the outer wall = 4374.94 A/m, 9.62433 uW/cm^2
Maximum H (at Z,R = 0.125257,0.128635) = 12106.5 A/m, 73.6999 uW/cm^2
Maximum E (at Z,R = 0.977929,0.084116) = 8.33999 MV/m, 0.812776 Kilp.
Ratio of peak fields Bmax/Emax = 1.8242 mT/(MV/m)
Peak-to-average ratio Emax/E0 = 1.8712

Frequency	Beta	Q	r/Q	Rs	Bmax/Emax	Emax/E0
75.55636 MHz	0.0504073	2.8041E+09	49.066 Ohm	3.3888E+06 MOhm/m	1.8242 mT/(MV/m)	1.8712

For this design the origin has been indicated. Hence the drift tube is planned to be located on the horizontal line which is 104.138 cm below the origin. All the calculations hence is done along that line itself. As a precautionary measure to minimise the chances of large multipacting we have chosen corners which are smooth and no abrupt pointed edge is inculcated in the design. The calculations are being done in acryogenic environment at temperature of 2K.

6 Conclusion

From the experience gained from the conceptualisation and simulation of the above two designs we notice that the value of resonant frequency is extremely sensitive with the angle of tapering of the distance between inner and outer conductor as one goes along the axis away from the drift tube. Another interesting finding that was found was that the value of geometric beta calculated by the Superfish is very sensitive to the value of the capacitive gap that we give at the bottom of the resonating structure.

The superconducting QWR cavity hence designed were for proton and that too for low beta ion acceleration cases in a cryogenic environment. This paper describes its various characteristics and other fundamental properties.

7 Acknowledgment

The findings of this paper and corresponding research for the same was carried out in the able and encouraging environment of Indian Institute of Technology, Roorkee for which I am grateful to this magnificent institution. Secondly I would like to extend my hearties gratitude to Prof. Puneet Jain of the Department of Physics, IIT Roorkee, for recognising my interest in the field and giving me the ever so important opportunity to work on the same in the summer of 2017. The encouragement and deep insight provided by Dr. Jain paved the way for a fulfilling and successful understanding of the subject and I would be forever grateful for the same. Lastly, but by all means not the least, I would like to thank my parents who always gave me the cushion of support, both emotionally and financially to carry on with this project and keep my focus intact. Gratitude.

8 References

- Quarter wave resonators for beta $\sim$ 1 Accelerators,Ilan Ben-Zvi
- RF Theory and Design ,Jeremiah Holzbauer,USPAS - Grand Rapids
- Los Alamos National Library, `http://laacg.lanl.gov/laacg/services/serv_codes.phtml`
- Erk Jensen, Cavity Basics, Superfish
- `http://www.lnl.infn.it/~newweb/index.php/en/home-3/100-accelerators`