

Assignment Solutions: - Time and Space Complexity 2

- 1) Calculate the time complexity for the following code snippet.

```
for(int i = 0; i < n; i++) {  
    for(int j = 0; j * j < n; j++) {  
        cout << "PhysicsWallah ";  
    }  
}
```

$O(n * \sqrt{n})$

- 2) Calculate the time complexity for the following code snippet.

```
int c = 0;  
for(int i = 0; i < n; i++) {  
    for(int j = 1; j < n; j *= 2) {  
        c++;  
    }  
}
```

$O(n \log n)$ as the first loop 'i' will be iterated n times and the inner loop will only traverse $\log n$ times so in total the overall time complexity becomes $O(n \log n)$.

- 3) Calculate the time complexity for the following code snippet.

```
int c = 0;  
for(int i = 0; i < n; i++) {  
    for(int j = 1; j * j < n; j *= 2) {  
        c++;  
    }  
}
```

Let us analyze how many times the inner loop will iterate. Let us see the values of j for that.

$J = 1, 2, 4, \dots 2^k$

So $2^k * 2^k < n$
So $2^{(k+1)} < n$
So Time complexity becomes $\log N$.

4) Calculate the time complexity for the following code snippet.

```
int c = 0;
for(int i = n; i > 0; i /= 2) {
    for(int j = 0; j < i; j++) {
        c++;
    }
}
```

Here the inner loop will be traversed 'i' times so let us see the values of 'i' here.
Values of 'i' will be $n, n/2, n/4, n/8$ and so on
So the total number of iterations in the above nested loop will be $n + n/2 + n/4 + n/8 + \dots$
Which sums to $2n$
So time complexity becomes $O(2n) \sim O(n)$

5) Calculate the time complexity for the following code snippet.

```
int c = 0;
for(int i = 1; i < n; i*=2) {
    for(int j = n; j > i; j--) {
        c++;
    }
}
```

Lets us calculate the number of iterations in the above nested loop here, we get
Values of 'i' will be $1, 2, 4, 8, 2^k$
So the total number of iterations will be $(n-1) + (n-2) + (n-4) + \dots + (n-2^k)$
This sum becomes $n*k - (1+2+4+ \dots + 2^k)$
Which becomes $n*k - (2^{(k+1)})$
Here k is number of terms which is $O(\log N)$
Hence the overall time complexity becomes $n \log n - n$
 $\sim O(n \log n)$

