EE4580 – Quasi-Optical Systems

Reflector Antennas

MATLAB Instruction

20 Feb. 2020

Question:

Write a MATLAB routine to analyze a parabolic reflector. Suppose that the reflector is illuminated by a circular feed with a linearly y-polarized uniform amplitude distribution. The operating frequency is $60\ \mathrm{GHz}$. The aperture of the circular feed has a diameter of 3λ. Suppose that the parabola has a focal length $f = 100\ \mathrm{cm}$.

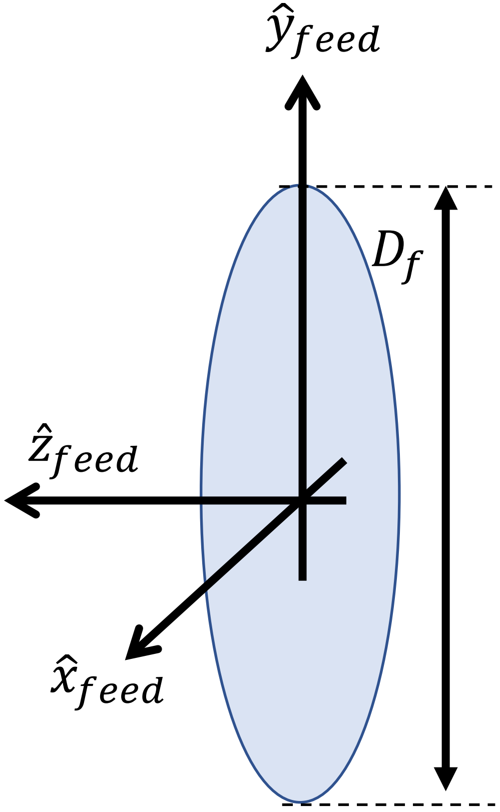
Provide the following plots:

- The far field pattern of the feed of the reflector.
- Plot the equivalent aperture current distribution for a reflector with $f/D = 0.5$.
- Plot the far field patterns radiated by the parabolic reflector for $f/D = 5$. Compare with the far field of a uniform aperture.
- Plot the taper efficiency, the spillover efficiency, and the aperture efficiency vs the diameter D of the reflector for $0.5 < f/D < 5$.
- Plot the maximum possible directivity (i.e. uniform aperture distribution), the directivity and the gain vs the diameter D of the reflector for $0.5 < f/D < 5$.

1. Far field Pattern of the Feed

Feed:

- Circular feed
- Constant amplitude
- y-polarized
- $f = 60$ GHz
- $D_f = 3\lambda$



1. Far field Pattern of the Feed

Recap

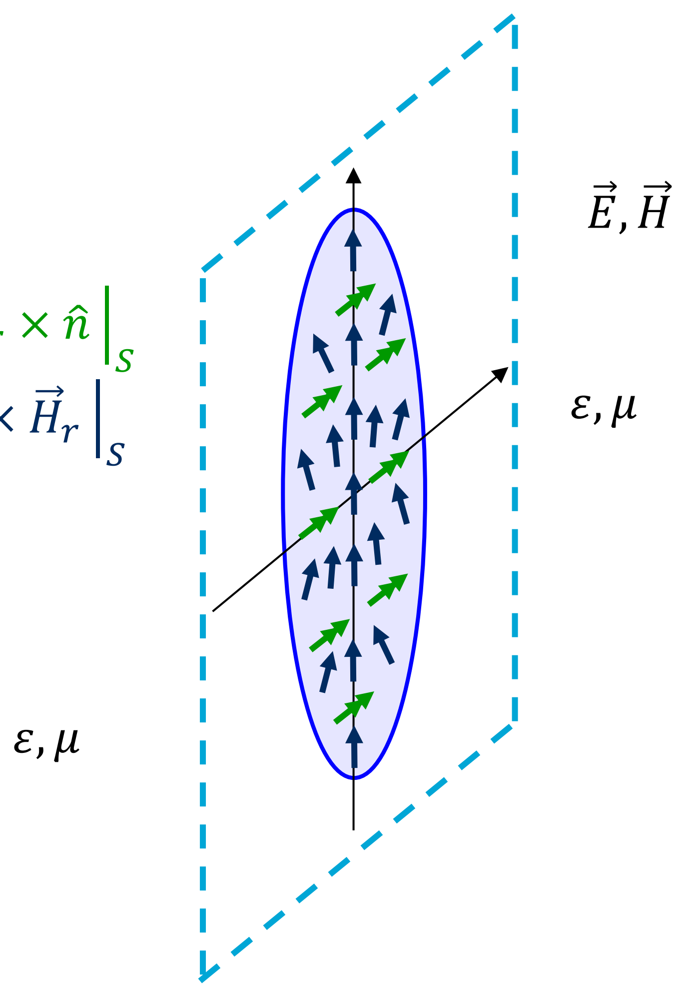
Equivalent Aperture Integral Approach

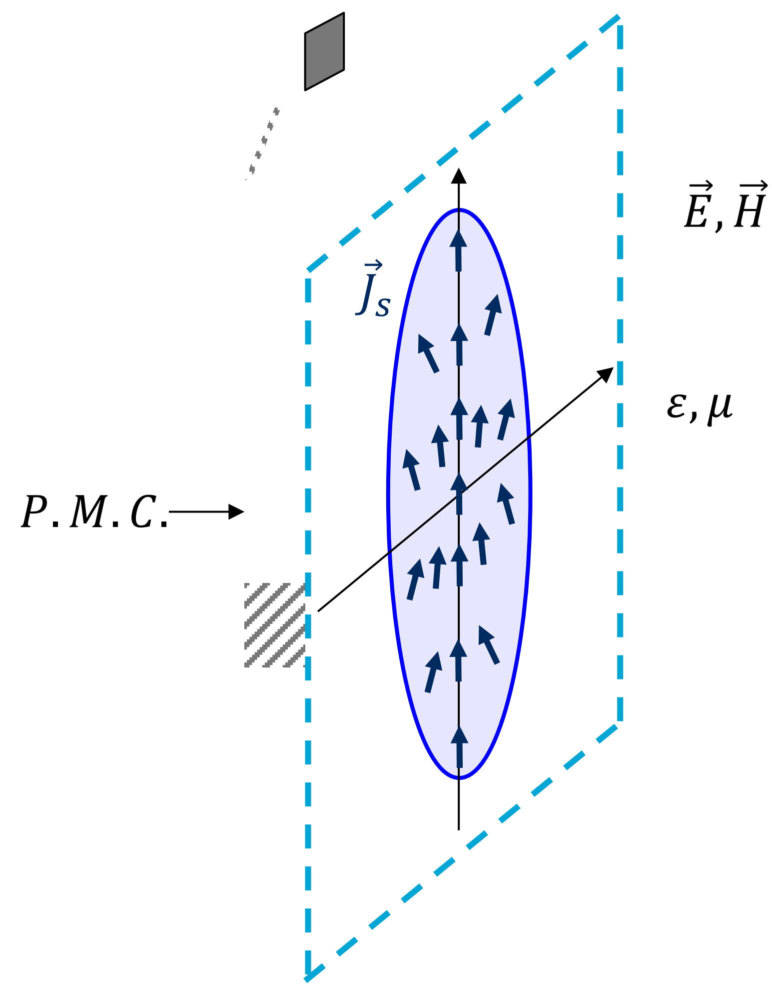
Two choices

Love's formulation | *Schelkunoff's formulation*

$$\vec{M}_s \approx \vec{E}_r \times \hat{n}\Big|_S$$

$$\vec{J}_s \approx \hat{n} \times \vec{H}_r\Big|_S$$





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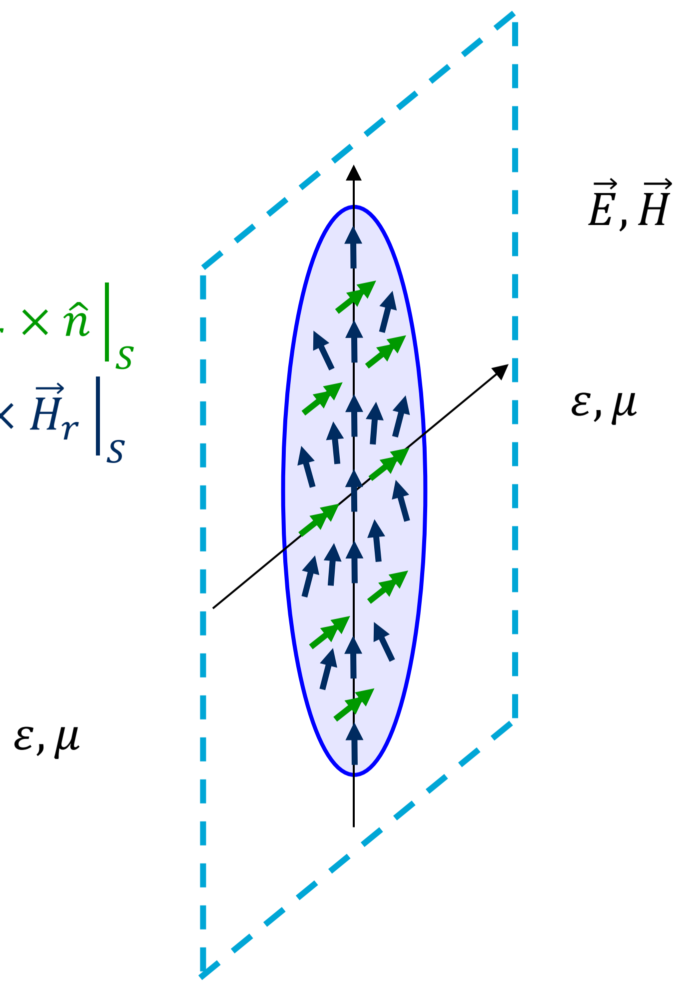
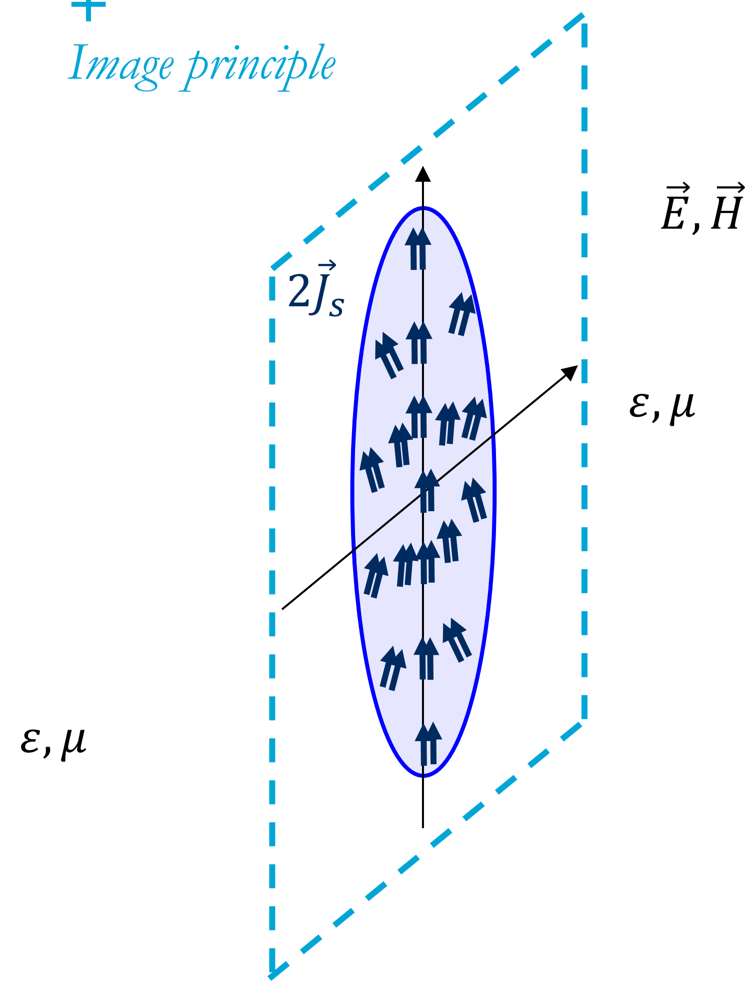


Schelkunoff's formulation
+
Image principle



Far field of the Circular Feed

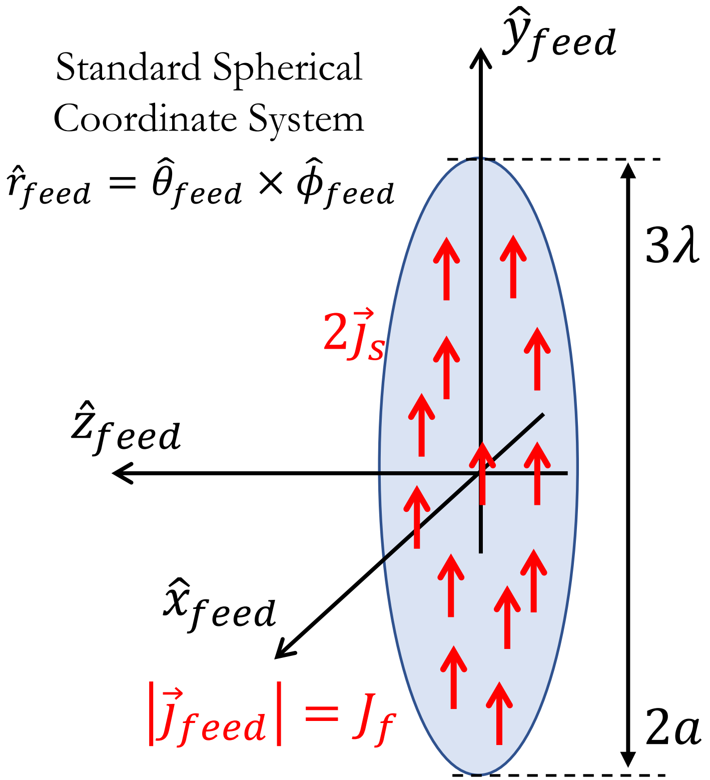
Far field of a given current:

Spatial representation:

$$\vec{E}^{ff}_{feed}(\theta_{feed},\phi_{feed}) = -j\omega\mu(\tilde{I}-\hat{r}\hat{r})\vec{J}_{FT}\frac{e^{-jkr}}{4\pi r}$$

Spectral representation:

$$\vec{E}^{ff}_{feed}(\theta_{feed},\phi_{feed}) = jk_{zs}\tilde{G}^{fc}(k_{xs},k_{ys},z,z')\vec{J}_{FT}\frac{e^{-jkr}}{2\pi r}$$



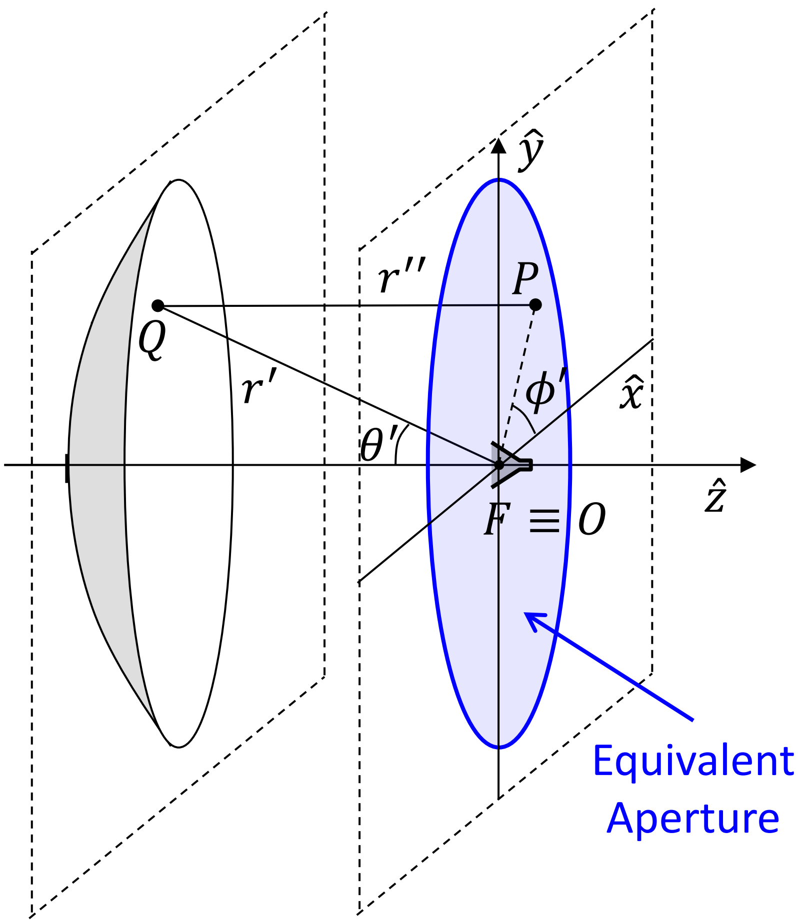
$$\vec{E}^{ff}_{feed}(\theta_{feed},\phi_{feed}) = -2jk\zeta J_f J_{FT}\frac{e^{-jkr_{feed}}}{2\pi r_{feed}}[\cos\theta_{feed}\sin\phi_{feed}\hat{\theta}_{feed}+\cos\phi_{feed}\hat{\phi}_{feed}]$$

$$J_{FT} = Airy(ka\sin\theta_{feed}) = 2\pi a^2\frac{J_1(ka\sin\theta_{feed})}{ka\sin\theta_{feed}}$$

$$\vec{H}^{ff}_{feed}(\theta_{feed},\phi_{feed}) = \frac{1}{\zeta}\hat{r}_{feed}\times\vec{E}^{ff}_{feed}(\theta_{feed},\phi_{feed})$$

2. Current on Equivalent Aperture

Recap



Parabola Definition: $|P - Q| + |Q - F| = 2f$

(r', θ') parametrization:

$$r' = \frac{2f}{1 + \cos\theta'} = \frac{f}{\cos^2\left(\frac{\theta'}{2}\right)} = f\left[1 + \tan^2\left(\frac{\theta'}{2}\right)\right]$$

(ρ', z) parametrization: $z = -f + \frac{(\rho')^2}{4f}$

2. Current on Equivalent Aperture

Recap

Useful formulas:

$$\vec{E}_a(\rho',\phi') \approx \vec{E}_r(\rho',\phi') = \sqrt{2\zeta U_{feed}\left(2\tan^{-1}\left(\frac{\rho'}{2f}\right),\phi'\right)}\,\frac{4f}{4f^2+(\rho')^2}\,e^{-j2kf}\,\hat{e}_r$$

$$U_{feed}(\theta',\phi') = \frac{1}{2\zeta}\left|\vec{f}_{pattern}(\theta',\phi')\right|^2$$

$$\left|\vec{f}_{pattern}(\theta',\phi')\right|^2 = \left|f_{pattern,\theta'}(\theta',\phi')\hat{\theta}' + f_{pattern,\phi'}(\theta',\phi')\hat{\phi}'\right|^2 = \left|E^{FF}_{feed,\theta'}(\theta',\phi')\hat{\theta}' + E^{FF}_{feed,\phi'}(\theta',\phi')\hat{\phi}'\right|^2 (r')^2$$
$$= \left|\vec{E}^{ff}_{feed}(\theta',\phi')\right|^2 (\boldsymbol{r}')^{\mathbf{2}}$$

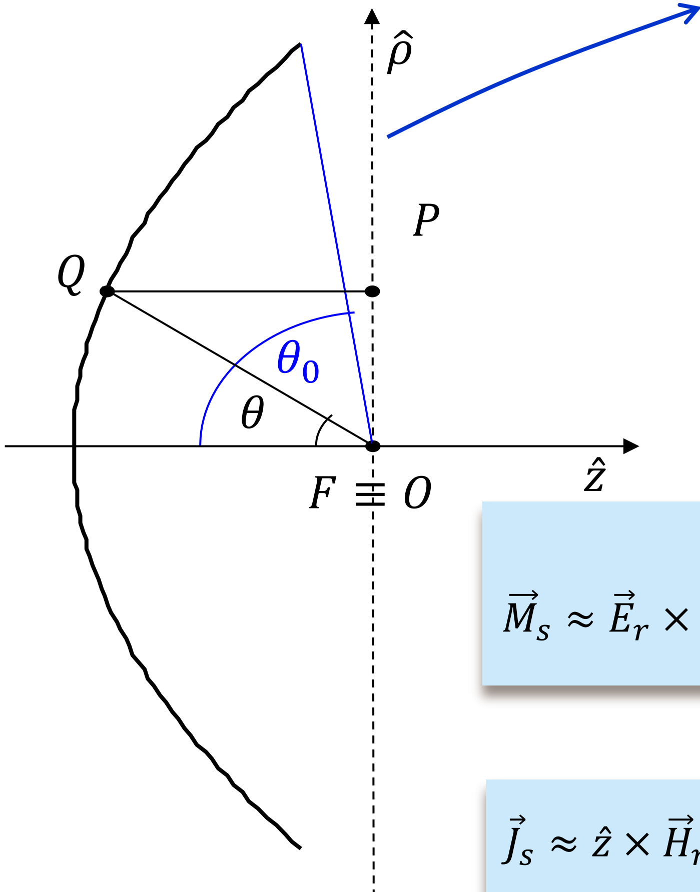
$$\bar{R} = -\hat{\rho}'\hat{\theta}' - \hat{\phi}'\hat{\phi}'$$

$$\hat{e}_r = \bar{R}\cdot\hat{e}_i$$

$$\vec{H}_a(\rho',\phi') \approx \vec{H}_r(\rho',\phi') = \frac{1}{\zeta}\hat{r}''\times\vec{E}_r(\rho',\phi') = \frac{1}{\zeta}\hat{z}\times\vec{E}_r(\rho',\phi')$$

2. Current on Equivalent Aperture

Recap



$$\vec{E}_r(P) = \tilde{R}(Q) \cdot \vec{E}_{feed}(Q)\, e^{-jkr''}$$

$$\vec{E}_{feed} = \left(E_{feed}^{\theta}\hat{\theta} + E_{feed}^{\phi}\hat{\phi}\right)\frac{e^{-jkr}}{r}$$

Phase: $|P - Q| + |Q - F| = 2f$

Amplitude: $$\frac{1}{|Q-F|} = \frac{1}{f\left[1 + \left(\frac{\rho'}{2f}\right)^2\right]} = \frac{\cos^2\left(\frac{\theta'}{2}\right)}{f}$$

$$\vec{M}_s \approx \vec{E}_r \times \hat{z}\Big|_S = \left[\left(-E_{feed}^{\theta}\hat{\rho} - E_{feed}^{\phi}\hat{\phi}\right) e^{-jk2f}\,\frac{\cos^2\left(\frac{\theta'}{2}\right)}{f}\right] \times \hat{z}$$

Cartesian coordinates

$$\vec{J}_s \approx \hat{z} \times \vec{H}_r\Big|_S = \frac{1}{\zeta}\hat{z} \times (\hat{z} \times \vec{E}_r)\Big|_S$$

3. Far field Patterns of the Reflector

Recap

$$\vec{E}_{ref}^{ff}\left(\theta_{ff},\phi_{ff}\right) = jk_{zs}\tilde{G}^{fc}\left(k_{xs},k_{ys},z,z'\right)\vec{J}_{FT}\left(k_{xs},k_{ys}\right)\frac{e^{-jkr}}{2\pi r}$$

$$\vec{J}_{FT}\left(k_x,k_y\right) = \iint_S \vec{J}(\vec{r}\,')e^{jk\hat{r}\cdot\vec{r}\,'}dS'$$

4. Performance of the Antenna Reflector

Recap

Useful formulas:

$$\eta_t = \frac{A_{eff}}{A} = \frac{1}{A}\frac{\left|\iint_A \vec{E}_a(\rho',\phi')\,dA\right|^2}{\iint_A |\vec{E}_a(\rho',\phi')|^2 dA}$$

$$\eta_s = \frac{\int_0^{2\pi}\int_0^{\theta_0} U_{feed}(\theta',\phi')\sin\theta'\,d\theta' d\phi'}{\int_0^{2\pi}\int_0^{\pi} U_{feed}(\theta',\phi')\sin\theta'\,d\theta' d\phi'}$$

Only in Cartesian Coordinates

$$\theta_0 = 2\,\mathrm{acot}(4f_{\#}^{r})$$

$$\left|\iint_A \vec{E}_a(\rho',\phi')\,dA\right|^2 =$$

$$\left|\hat{x}\iint_A E_{ax}(\rho',\phi')\,dA + \hat{y}\iint_A E_{ay}(\rho',\phi')\,dA + \hat{z}\iint_A E_{az}(\rho',\phi')\,dA\right|^2$$

$$\eta_{ap} = \eta_s\eta_t$$

$$D_M = \frac{4\pi}{\lambda^2}A$$

$$D = D_M\eta_t$$

$$G = D_M\eta_{ap} = \frac{4\pi}{\lambda^2}\eta_s\eta_t A$$