EE4580 –Quasi-Optical Systems

Elliptical lens

MATLAB Instruction

27 Feb. 2020

Description

Write a Matlab program to analyze and design a planar antenna placed at the focal point of an elliptical lens made of a material with a certain ε_r. Suppose that the lens is illuminated by a planar antenna operating at 100GHz and whose far field inside an infinite medium has the following expression:

$$E^a_{far}(\theta,\phi) = \cos^n\theta\,(cos\phi\hat{\theta} - sin\phi\hat{\phi})\frac{e^{-jk_d r}}{r}$$

Where $k_d = \sqrt{\varepsilon_r}k_0$

Provide the following plots:

- The far field of the planar antenna in the infinite medium ($\varepsilon_r = 11.9$) for $n = 3$.
- Plot the equivalent aperture current distribution of an elliptical lens with $\varepsilon_r = 11.9$, $\mathrm{D} = 6\lambda_0$, and $\theta_0 = 55°$ in the main planes of the antenna.
- Plot the far field patterns radiated by the lens. Compare with the far field of an equivalent uniform aperture (airy pattern).
- Give an estimation of the directivity and gain of such lens antenna.
- Find the phase center of the lens antenna.

Feeding antenna

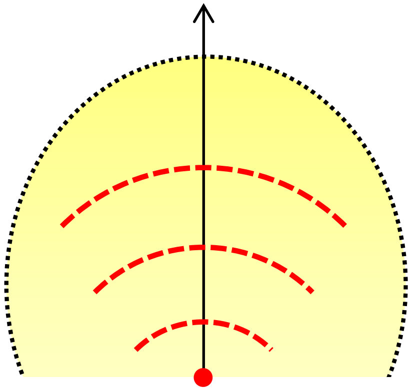
Step 1: Write a routine to evaluate the radiation of the lens feed

[Eth, Eph, Prad] = FeedLens(kd, r, th, ph, n)

Antenna far field:

$$E_{far}^{a}(\theta,\phi) = \cos^3\theta\,(\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi})\frac{e^{-jk_d r}}{r}$$

Where $k_d = \sqrt{\varepsilon_r}k_0, \varepsilon_r = 11.9$



Power radiated by the antenna (inside the lens):

$$P_{rad}^{feed} = \int_0^{2\pi}\int_0^{\pi/2} U(\theta,\phi) sin\theta d\theta d\phi$$

$$U(\theta,\phi) = \frac{\left|E_{far}^{a}((\theta,\phi)\right|^2}{2\zeta_d} r^2 \qquad \zeta_d = \zeta_0/\sqrt{\varepsilon_r}$$

Elliptical lens antenna

Step 2: Write a routine to evaluate the equivalent aperture current distribution of an elliptical lens antenna

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[Jx, Jy] = LensAperture(th, ph, r, e, er, n);
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1) Lens geometry

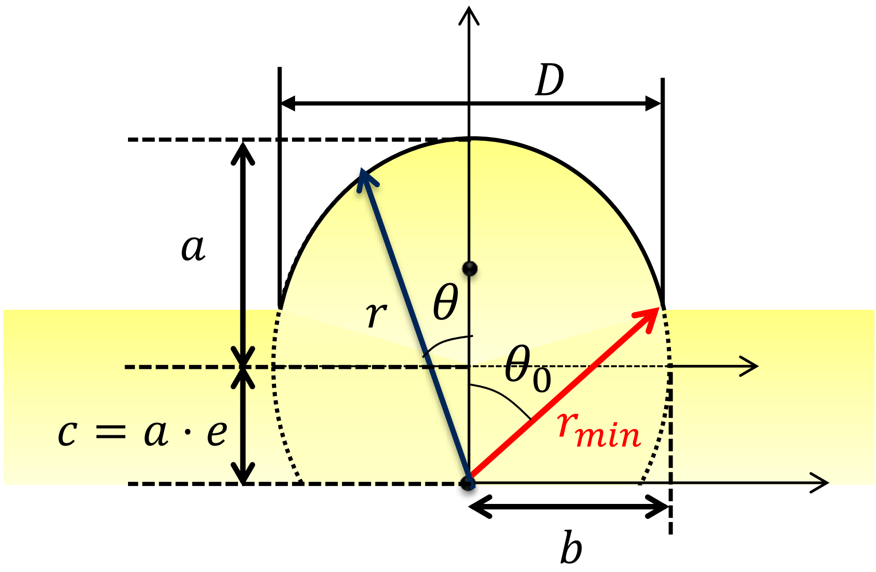
$$D = 6\lambda_0, \theta_0 = 55°, e = 1/\sqrt{\varepsilon_r}$$

$$r = a\frac{1-e^2}{1-e\cos\theta} \Rightarrow a, b, c$$

2) Surface Parametrization

$$\rho = r sin\theta \in \left[0, \frac{D}{2}\right] \qquad \phi \in [0, 2\phi]$$

$$\left(\frac{z-c}{a}\right)^2 + \left(\frac{\rho}{b}\right)^2 = 1 \Rightarrow \boxed{\theta = \text{atan}\frac{\rho}{z}}$$



Ellipsoid Parameters

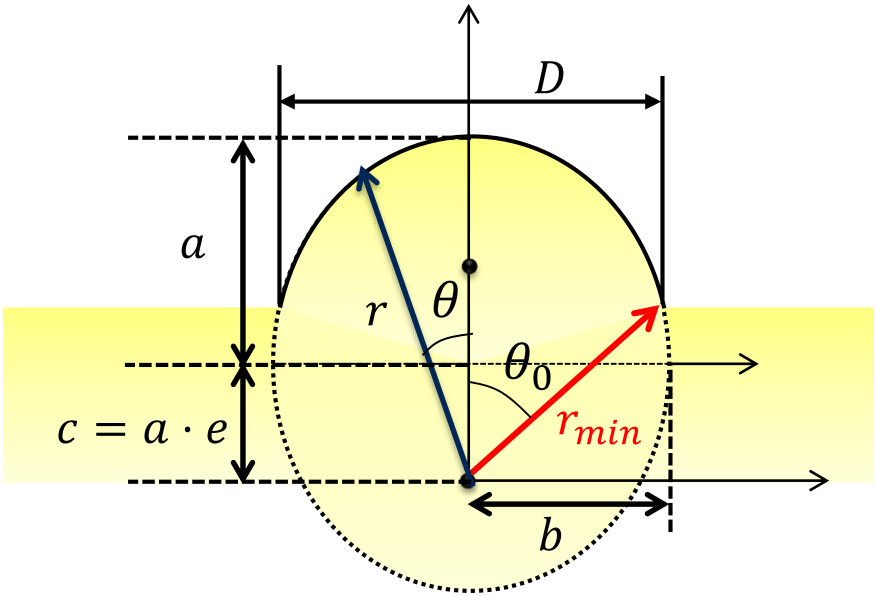
$$\sin\theta_0 = \frac{D/2}{r_{min}} \Rightarrow r_{min} = \frac{D/2}{sin\,\theta_0}$$

$$r = a\frac{1-e^2}{1-e\cos\theta} \Rightarrow r_{min} = a\frac{1-e^2}{1-e\cos\theta_0}$$

$$a = r_{min}\frac{(1-e\cos\theta_0)}{1-e^2}$$

$$c = a.e$$

$$b = \sqrt{a^2-c^2}$$



Equivalent current distribution

$$\vec{J}_s(\vec{\rho}) \approx \hat{z} \times \vec{H}_t|_S = -\frac{1}{\zeta_0}\left(\tau^{\parallel}(\theta_i)E_i^{\theta}(\vec{r})\hat{\rho} + \tau^{\perp}(\theta_i)E_i^{\phi}(\vec{r})\hat{\phi}\right)\frac{S(\vec{r})}{r}\boxed{e^{-jk_0r'}e^{-jk_dr}}$$

Constant phase term can be neglected

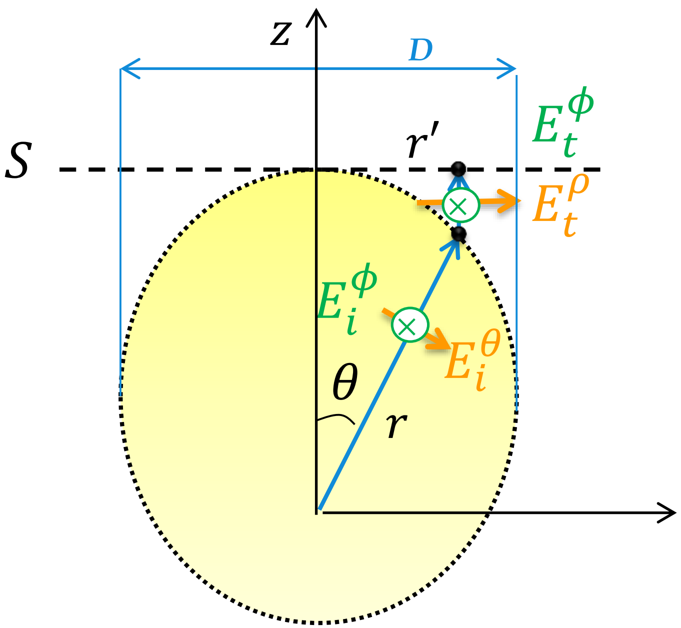
3) Equivalent current

$$E_{far}^{a}(\theta,\phi) = \cos^3\theta\,(\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi})\frac{e^{-jk_dr}}{r}$$

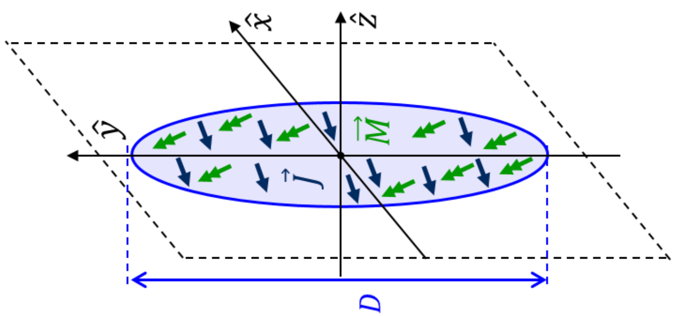
$$\hat{\rho} = \hat{x}cos\phi + \hat{y}sin\phi \qquad \hat{\phi} = -\hat{x}sin\phi + \hat{y}cos\phi$$

$$\Rightarrow \vec{E}_i(\vec{r}) = \left(E_i^{\theta}(\theta,\phi)\hat{\theta} + E_i^{\phi}(\theta,\phi)\hat{\phi}\right)\frac{e^{-jk_dr}}{r}$$

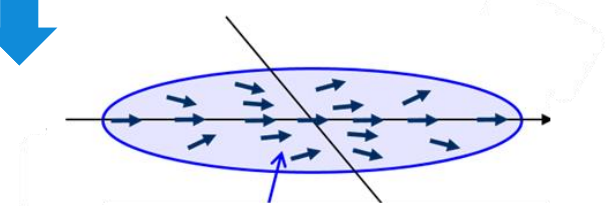
$$\vec{J}_s(\vec{\rho}) \approx -\frac{2}{\zeta_0}\left(\hat{x}\left(\tau^{\parallel}(\theta_i)E_i^{\theta}(\theta,\phi)cos\phi - \tau^{\perp}(\theta_i)E_i^{\phi}(\theta,\phi)sin\phi\right) + \hat{y}\left(\tau^{\parallel}(\theta_i)E_i^{\theta}(\theta,\phi)sin\phi + \tau^{\perp}(\theta_i)E_i^{\phi}(\theta,\phi)cos\phi\right)\right)\frac{S(\vec{r})}{r}$$



Both $\vec{J}_s(\vec{\rho})$ and $\vec{M}_s(\vec{\rho})$



$2\,\vec{J}_s(\vec{\rho})$



Equivalent current distribution

$$\vec{J}_s(\vec{\rho}) \approx -\frac{2}{\zeta_0}\left(\hat{x}\left(\tau^{\parallel}(\theta_i)E_i^{\theta}(\theta,\phi)cos\phi - \tau^{\perp}(\theta_i)E_i^{\phi}(\theta,\phi)sin\phi\right) + \hat{y}\left(\tau^{\parallel}(\theta_i)E_i^{\theta}(\theta,\phi)sin\phi + \tau^{\perp}(\theta_i)E_i^{\phi}(\theta,\phi)cos\phi\right)\right)\frac{S(\vec{r})}{r}$$

4) Transmission coefficients

$$\tau^{\perp} = 1 + \Gamma^{\perp} = \frac{2\zeta_0 cos\theta_i}{\zeta_0 cos\theta_i + \zeta_d cos\theta_t}$$

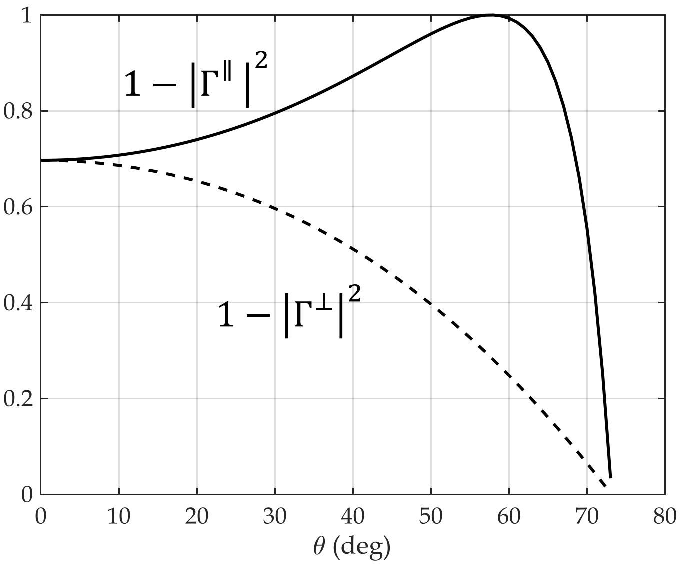
$$\tau^{\parallel} = (1 + \Gamma^{\parallel})\frac{cos\theta_i}{cos\theta_t} = \frac{2\zeta_0 cos\theta_i}{\zeta_0 cos\theta_t + \zeta_d cos\theta_i}$$

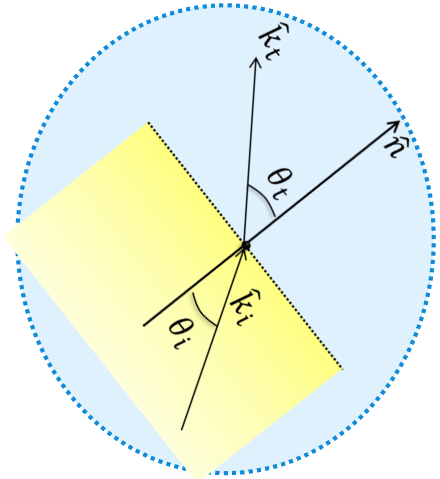
5) Spreading factor

$$S(\vec{r}) = \sqrt{\frac{cos\theta_{\boldsymbol{t}}}{cos\theta_{\boldsymbol{i}}}\frac{ecos\theta - 1}{e - cos\theta}} = 1$$

Relate θ to θ_i:

$$\begin{cases} cos\theta_i = \hat{k}_i \cdot \hat{n} = \dfrac{1 - ecos\theta}{\sqrt{1 + e^2 - 2ecos\theta}} \\ cos\theta_{\boldsymbol{t}} = \hat{k}_t \cdot \hat{n} = \dfrac{\cos\theta - e}{\sqrt{1 + e^2 - 2ecos\theta}} \end{cases}$$





$$\hat{n} = \frac{(\cos\theta - e)\hat{z} + \sin\theta\,\hat{\rho}}{\sqrt{1 + e^2 - 2ecos\theta}}$$

Far field of the lens antenna

Step 3: Far field radiation of the lens antenna

$$k_{xf} = k_0 \sin\theta_f \cos\phi_f$$

$$k_{yf} = k_0 \sin\theta_f \sin\phi_f$$

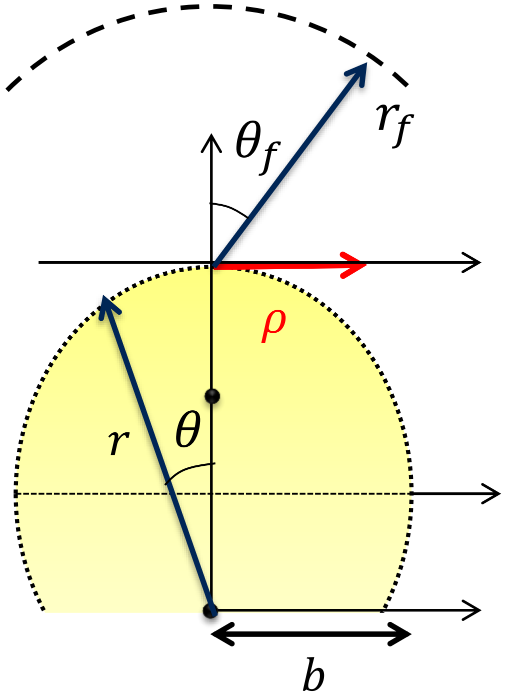
$$k_{zf} = k_0 \cos\theta_f$$

Calculate FT of currents:

$$J_s^{x/y}(k_{xf}, k_{yf}) = \int_0^a \int_0^{2\pi} J_s^{x/y}(\rho,\phi) e^{jk_{xf}\rho\cos(\phi)} e^{jk_{yf}\rho\sin(\phi)} \rho d\rho d\phi$$

Calculate far field:

$$\vec{E}_{lens}^{far}(\vec{r}_f) = jk_{zf}\tilde{G}_{fs}^{ej}(k_{xf}, k_{yf})\vec{J}_s(k_{xf}, k_{yf})\frac{e^{-jk_0 r_f}}{2\pi r_f}$$



Coordinate system for radiation outside the lens: $[r_f, \theta_f, \phi_f]$

Dir. & gain of the lens antenna

Step 4: directivity and gain of the lens antenna

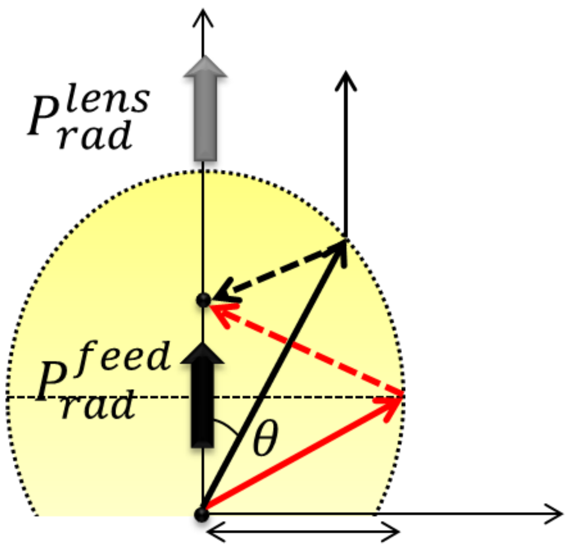
Calculate directivity $D(\theta_f, \phi_f) = 4\pi U_{lens}/P_{rad}^{lens}$

$$U_{lens}(\theta_f, \phi_f) = \frac{\left|\vec{E}_{lens}^{ff}(\theta_f, \phi_f)\right|^2}{2\zeta_0} r_f^2$$

$$P_{rad}^{lens} = \int_0^{2\pi} \int_0^{\pi/2} U_{lens} sin\theta_f d\theta_f d\phi_f$$

Radiation efficiency of the lens eff.: $\eta_{rad} = \frac{P_{rad}^{lens}}{P_{rad}^{feed}}$

Gain: $G(\theta_f, \phi_f) = D(\theta_f, \phi_f)\eta_{rad}$

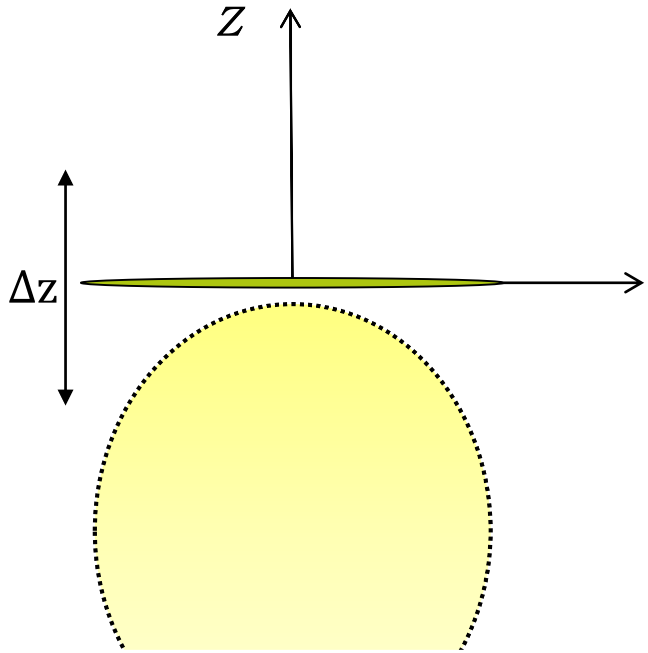


Phase center

Step 5: Phase center evaluation

Change in the z-origin of the reference system causes a phase shift in the far field:

$$\vec{E}_{lens}^{ff}(\Delta z) = \vec{E}_{lens}^{ff}(\Delta z = 0)e^{jk_0 cos\theta_f \Delta z}$$



To evaluate the phase center:

- Plot the phase in the two main planes as a function of Δz
- Do a parametric analysis for $\Delta z \in [-3\lambda_0, 3\lambda_0]$
- The phase center will be the Δz where the variation of the phase in the main lobe is the lowest