Assignment 4: Fourier Optics for Focal Plane Arrays

Part 1: Feed at the center of the focal plane of the reflector

Write a MATLAB program to analyze in reception an antenna located in the center of the focal plane of a parabolic reflector. The geometry is shown in Fig. 1(a). Suppose that the antenna can be modeled as a circular feed with a linearly x-polarized uniform amplitude distribution. The operating frequency is $100\mathrm{GHz}$. The aperture of the circular feed has a diameter of $D_f = 6\lambda_0$. Suppose that the parabola has a diameter of $D_r = 50\lambda_0$ and f-number of $f_\# = 3$. Assume that the parabolic reflector is illuminated by x-polarized plane waves with an incident angle of $\theta_{pw} = 0$ with amplitude $E_0^{PW} = 1V/m$.

1.1 Calculate the GO electric field on the equivalent FO sphere of this geometry. Plot these fields in the $\phi = 0°$ and $90°$ planes.
1.2 Evaluate the electric far field of the circular feed over the same FO sphere. Plot this field together with the GO fields
1.3 Using the analysis of antennas in reception for a generalized incidence, calculate the power received at the input terminals of the feed, assuming impedance matched condition. Provide the value of the aperture efficiency of the overall reflector system.

Assume now that incident plane waves are illuminating the reflector with the following incident angles: $\theta_{pw} = [0{:}\,6^o]$ and $\phi_{pw} = [0{:}\,360°]$.

1.4 Calculate the GO electric fields on the equivalent FO sphere of this geometry for each incident plane wave angle
1.5 Plot the power received, in dBs, by the reflector feed as function of this incident angle. This plot represents the radiation pattern of the reflector antenna. Compare this plot with the far field radiation pattern calculated using a transmission analysis approach of the same structure, as developed in MATLAB assignment 2.
1.6 Compare the aperture efficiencies, directivities and gains achieved using both Tx and Rx approaches.

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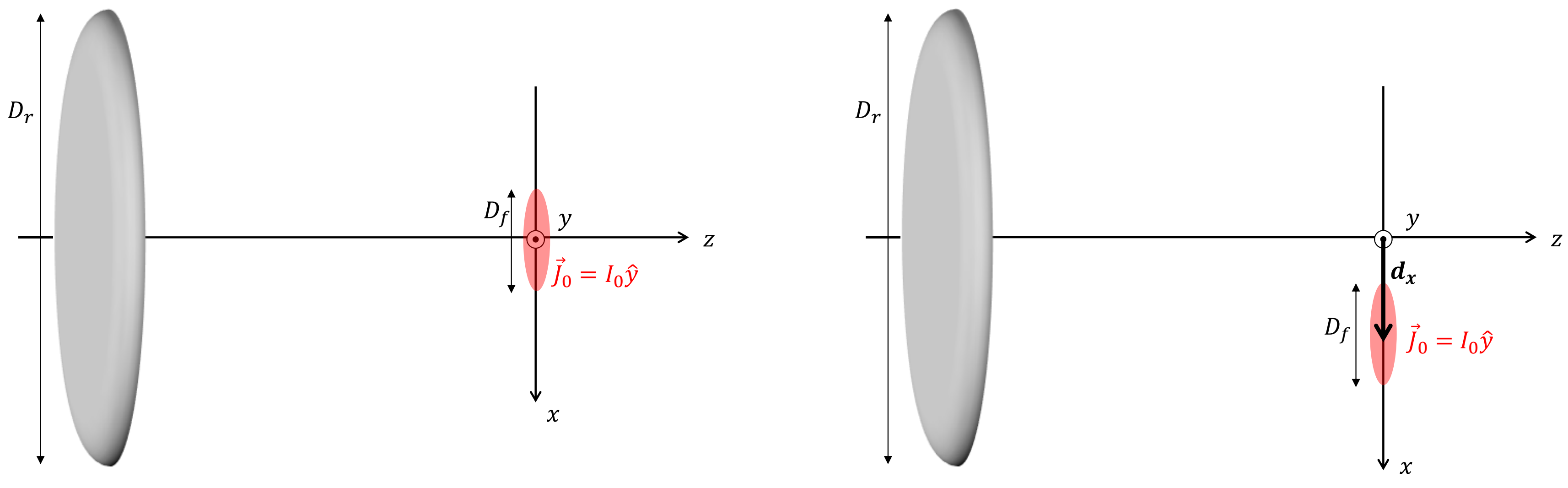
Part 2: Feed displaced in the focal plane of the parabolic reflector

Now let us consider that we displace the same circular feed in the focal plane by a certain quantity d_x as indicated in Fig. 1(b).

2.1 Consider $d_x = D_f = 6\lambda_0$. Plot the pattern in reception of this displaced field and evaluate the aperture efficiency. Compare them to the feed in the center. What is the pointing direction in both cases?

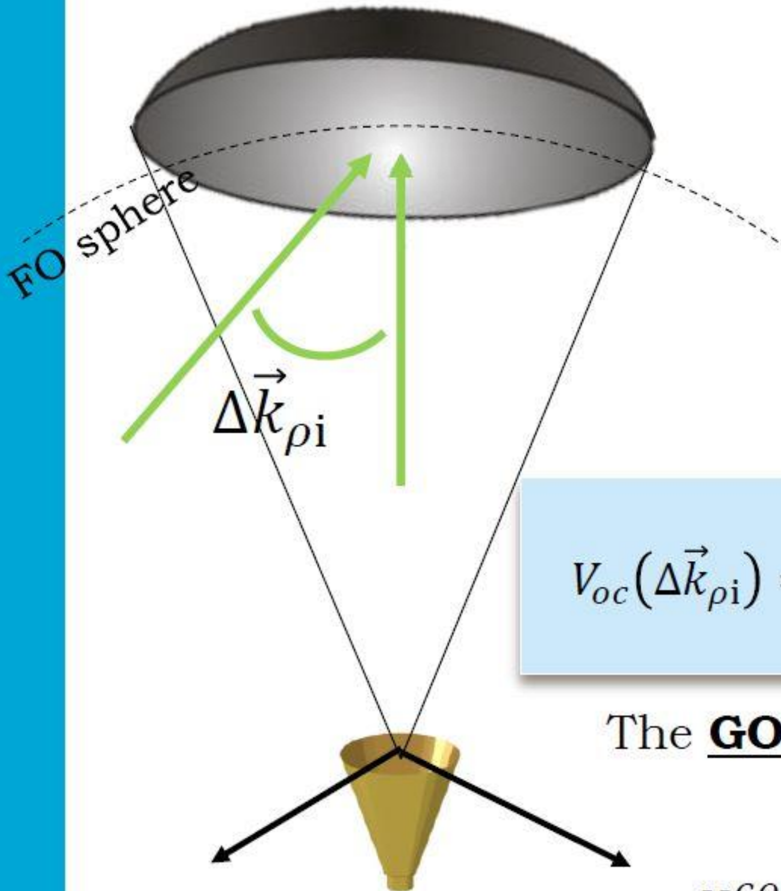
2.2 How does the aperture efficiency and pointing direction of the displaced feed changes if $d_x = D_f = 2\lambda_0 f_\#$, $d_x = D_f = \lambda_0 f_\#$ and $d_x = D_f = 0.5\lambda_0 f_\#$?



(a) (b)

Fig. 1: The scenario with feed (a) at the centered and (b) displaced in the focal plane of the reflector.

Far Field Pattern



We can evaluate now the induced voltage as a function of the plane wave incident angle on the reflector ($\Delta\vec{k}_{\rho\mathrm{i}}$)

$$V_{oc}\left(\Delta\vec{k}_{\rho\mathrm{i}}\right) = \frac{2}{\zeta_0}\int_0^{2\pi}\int_0^{\pi} \vec{V}_a^{tx}(\theta,\phi)\cdot\vec{V}_r^{GO}\left(\Delta\vec{k}_{\rho\mathrm{i}},\theta,\phi\right)\sin\theta\, d\theta d\phi$$

The <u>**GO field**</u> can be evaluated easily for a slightly off-broadside case:

$$V_r^{GO}\left(\vec{r},\overrightarrow{\Delta k}_{\rho i}\right)$$
$$\approx V_r^{GO}\left(\vec{r},\overrightarrow{\Delta k}_{\rho i}=0\right)\ e^{-j\vec{k}_\rho\cdot\overrightarrow{\Delta\rho}_{fp}}\ e^{-j\vec{k}_\rho\cdot\overrightarrow{\Delta\rho}_{fp}\delta_n(\theta)}$$

Step 1: Write a routine to evaluate the GO voltage

```
[Vgoth,Vgoph]=GOField(th,ph,thi,phi,F,D)
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$$\vec{e}_r^{GO}(\vec{r},\overrightarrow{\Delta k}_{\rho i}=0) = -\frac{2}{1+\cos\theta} E_0^{pw}\left((\hat{y}\cdot\hat{\rho})\hat{\theta} + (\hat{y}\cdot\hat{\phi})\hat{\phi}\right) \quad \theta < \theta_0 \qquad \tan\left(\frac{\theta_0}{2}\right) = \frac{D}{4F}$$

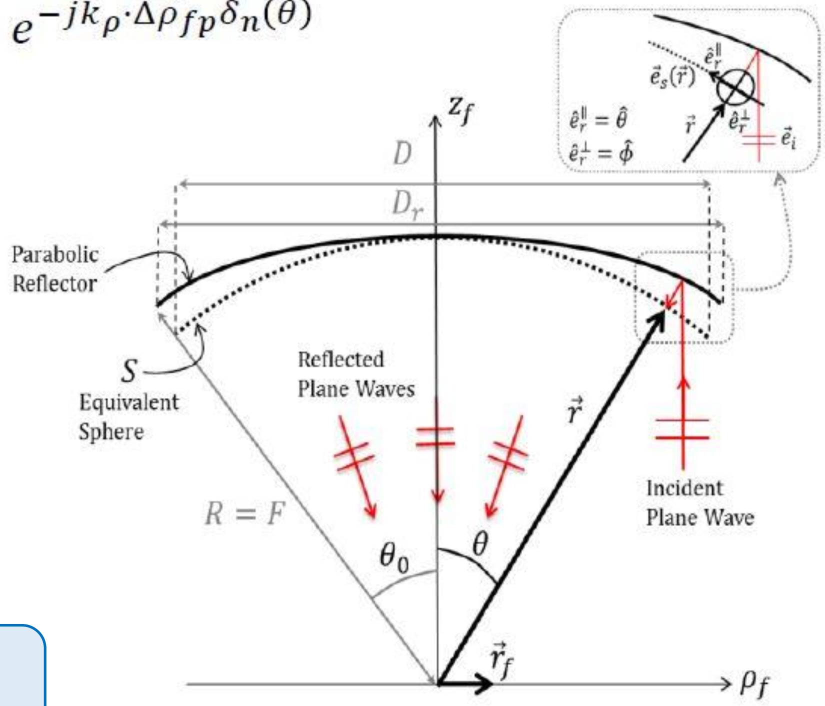
$$\vec{e}_r^{GO}(\vec{r},\overrightarrow{\Delta k}_{\rho i}) \approx \vec{e}_r^{GO}(\vec{r},\overrightarrow{\Delta k}_{\rho i}=0)\ e^{-j\vec{k}_\rho\cdot\overrightarrow{\Delta\rho}_{fp}}\ e^{-j\vec{k}_\rho\cdot\overrightarrow{\Delta\rho}_{fp}\delta_n(\theta)}$$

$$\frac{1-\cos\theta}{1+\cos\theta} = \delta_n(\theta)$$

$$\overrightarrow{\Delta\rho}_{fp} = F\overrightarrow{\Delta k}_{\rho i}/k_0$$

$$\overrightarrow{\Delta k}_{\rho i} = k_0 \sin\Delta\theta_i\,(\cos\phi_i\,\hat{x} + \sin\phi_i\,\hat{y})$$

$$\boxed{\vec{e}_r^{GO}(\vec{r},\overrightarrow{\Delta k}_{\rho i}=0) = \vec{V}_r^{GO}(\vec{r},\overrightarrow{\Delta k}_{\rho i}=0)\frac{e^{jkR}}{R}}$$



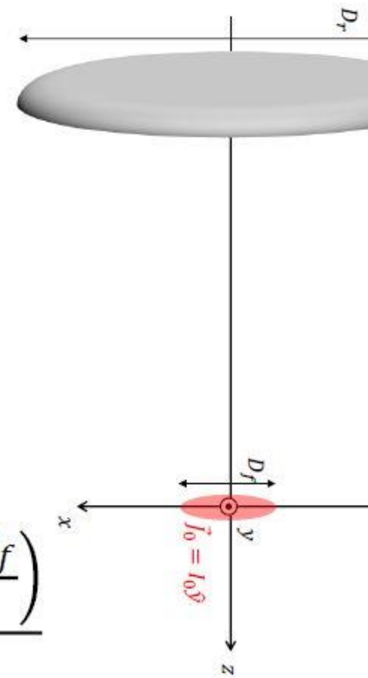
Step 2: Write a routine to evaluate the antenna in tx far field

```
[Vath,Vaph,Prad]=FeedField(th,ph,Df)
```

$$\vec{J}_0 = circ(\rho, a)\hat{y}$$

$$\vec{E}_{far} = \underbrace{Airy(\theta)\left(\cos\phi\,\hat{\phi} + \cos\theta\,\sin\phi\,\hat{\theta}\right)}_{\vec{V}_a^{tx}}\frac{e^{-jk_0R}}{R}$$

$$Airy(\theta) = A_f\,\frac{J_1\left(\frac{k_0\sin\theta\,D_f}{2}\right)}{\frac{k_0\sin\theta\,D_f}{2}}$$



Step 3: Write a routine to evaluate the received power

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[Prx,eta]=RxPower (th,ph,Vgoth,Vgoph,Vath,Vaph,Prad)
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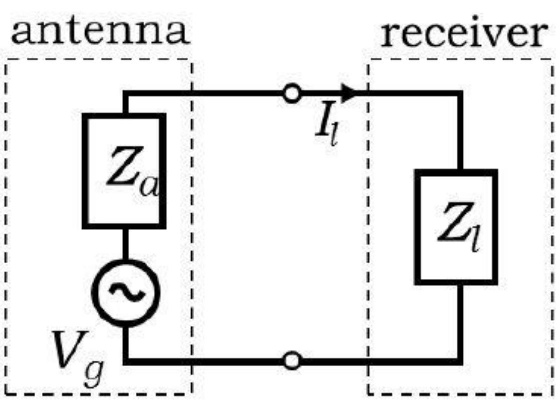

Current into the load

$I_l = V_g/(Z_a + Z_l)$

$$V_g = \frac{2}{\zeta_0}\int_0^{2\pi}\int_0^{\pi} \vec{V}_a^{tx}(\theta,\phi)\cdot\vec{V}_r^{GO}(\theta,\phi)\sin\theta\, d\theta d\phi$$

$$P_{rad} = \frac{1}{2\zeta_0}\int_0^{2\pi}\int_0^{\pi} \left|\vec{V}_a^{tx}(\theta,\phi)\right|^2 \sin\theta\, d\theta d\phi$$

$$P_{rx} = \frac{|V_g|^2}{8R_a} = \frac{|V_g|^2}{16P_{rad}}$$

$$\eta_{ap} = \frac{P_{rx}}{\frac{|E_0^{pw}|^2}{2\zeta_0}A_r}$$

Step 4: Modified the antenna far fied for off-focus

[Vath,Vaph,Prad]=FeedField(th,ph,Df,dx)

$$\vec{J}_{dx}(x,y) = \vec{J}_0(x - dx, y)$$

$$\vec{E}^{dx}_{far}(\vec{r}) = \vec{E}^{0}_{far}(\vec{r})e^{jk_x d_x}$$

$$k_x = k_0 \sin\theta \cos\phi$$