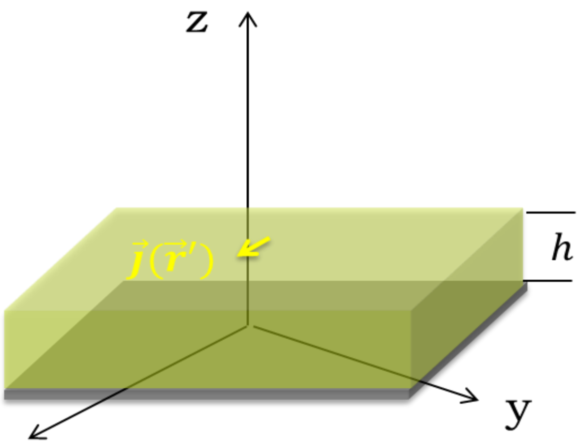
Question 1

Write a Matlab routine to calculate the spectral Green's function for the electric field given an elementary electric source placed at the top of a grounded slab of thickness h and dielectric constant e_r as shown in the figure. Consider h=$2mm$, e_r =10 and the source oriented along x.

Make a plot of the amplitude variation of the x-component of spectral field at $z = h^+$ as a function of k_x from 0 to $5k_0$ with k_y=0 at 10GHz and 20GHz.

Suggestion: check your answers using the free space SGF and the image theorem



Spectral Green's function of stratified media

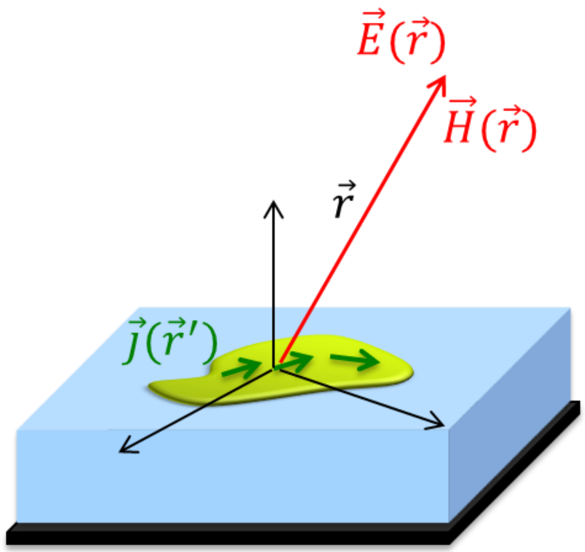
$$\vec{e}(\vec{r}) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \tilde{\tilde{G}}^{ej}(k_x, k_y, z, z') e^{-jk_x x} e^{-jk_y y} dk_x dk_y$$

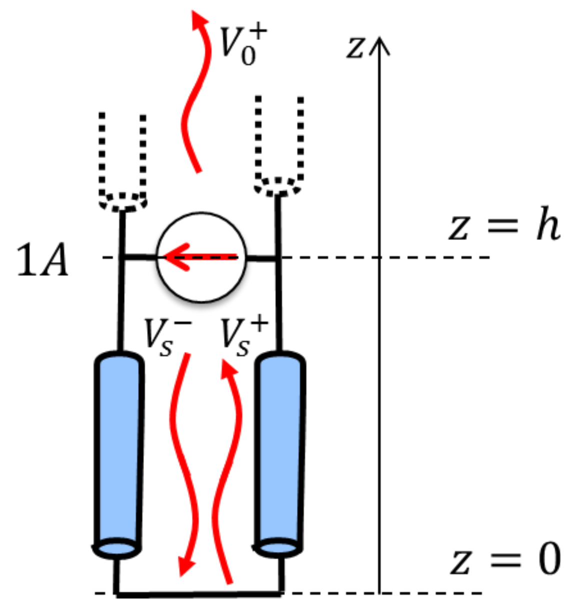
$$\tilde{\boldsymbol{G}}^{\boldsymbol{ej}} = \begin{bmatrix} -\dfrac{v_{TM} k_x^2 + v_{TE} k_y^2}{k_\rho^2} & \dfrac{(v_{TE} - v_{TM}) k_x k_y}{k_\rho^2} \\ \dfrac{(v_{TE} - v_{TM}) k_x k_y}{k_\rho^2} & -\dfrac{v_{TE} k_x^2 + v_{TM} k_y^2}{k_\rho^2} \\ \varsigma \dfrac{k_x}{k} i_{TM} & \varsigma \dfrac{k_y}{k} i_{TM} \end{bmatrix}$$

z: observation point in z (voltage/current output of the transmission line)
z': source location (generator in the transmission line)

Implementation of the square root: $k_{z0} = -j\sqrt{-(k_0^2 - k_\rho^2)}$

Transmission line Solution





$v_{TE/TM}(k_\rho, z, z')$
$i_{TE/TM}(k_\rho, z, z')$

$$Z_i^{TM} = \frac{\zeta_i k_{zi}}{k_i}$$

$$Z_i^{TE} = \frac{\zeta_i k_i}{k_{zi}}$$

In the air:

$$V_0 = V_0^+ e^{-jk_{z0}z}, I_0 = \frac{V_0^+}{Z_0} e^{-jk_{z0}z}$$

In the substrate:

$$V_s = V_s^+ e^{-jk_{zs}z} + V_s^- e^{jk_{zs}z}, I_s = \frac{V_s^+}{Z_s} e^{-jk_{zs}z} - \frac{V_s^-}{Z_s} e^{jk_{zs}z}$$

Transmission line Solution

Voltage solution in the slab:

$$V_s(z) = \frac{Z_u Z_d}{Z_u + Z_d} \frac{sin(k_{zs} z)}{sin(k_{zs} h)}$$

Current solution in the slab:

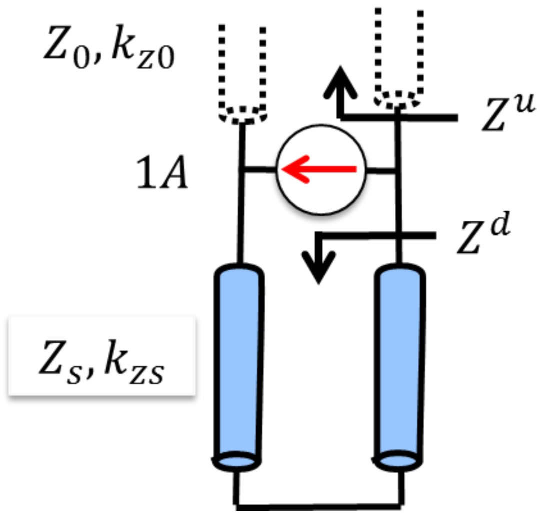
$$I_s(z) = \frac{V_s^+}{Z_s}\left(e^{-jk_{zs}z} + e^{jk_{zs}z}\right) = \frac{1}{Z_s} \frac{Z_u Z_d}{Z_u + Z_d} \frac{j\cos(k_{zs} z)}{sin(k_{zs} h)}$$

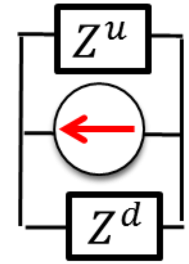
Voltage solution in the air:

$$V_0(z) = \frac{Z_u Z_d}{Z_u + Z_d} e^{jk_{z0}h} e^{-jk_{z0}z}$$

Current solution in the air:

$$I_0(z) = \frac{1}{Z_0} \frac{Z_u Z_d}{Z_u + Z_d} e^{jk_{z0}h} e^{-jk_{z0}z}$$





$$Z_u = Z_0$$

$$Z_d = jZ_s \tan(k_{zs} h)$$

Routines

- Solution of the equivalent transmission line:

$$[v_{TM}, v_{TE}, i_{TM}, i_{TE}] = trxline\_GroundSlab(k0, er, h, kro, z)$$

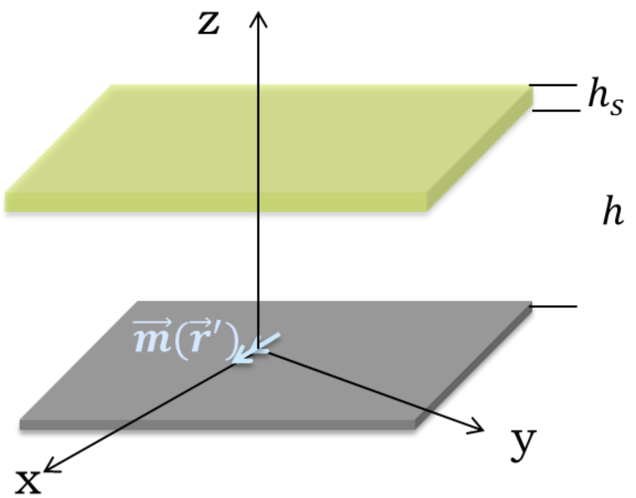
- Dyadic SGF :

$$[Gxx, Gyx, Gzx] = SpectralGFej(k0, er, kx, ky, v_{TM}, v_{TE}, i_{TM}, i_{TE})$$

Question 2

Write a Matlab routine to calculate the spectral Green's function for the electric field given by an elementary x-oriented magnetic source radiating at z=0 with the presence of a ground plane and a dielectric layer of thickness hs located at a distance of h from the ground plane, as shown in the figure.
Consider h=15mm, hs =2.1mm, ε_r=12.

Make a plot of the amplitude variation of the y-component of spectral field at $z = h + h_s^+$ as a function of ky from 0 to k0 with kx=0 for the following frequencies: 9GHz, 9.5GHz, 10GHz, 10.5GHz and 11GHz



Routines

- Solution of the equivalent transmission line:

$$[v_{TM}, v_{TE}, i_{TM}, i_{TE}] = trxline\_Superstrate(k0, er, h, kro, z)$$

- Dyadic SGF :

$$[Gxx, Gyx, Gzx] = SpectralGFem(k0, er, kx, ky, v_{TM}, v_{TE}, i_{TM}, i_{TE})$$

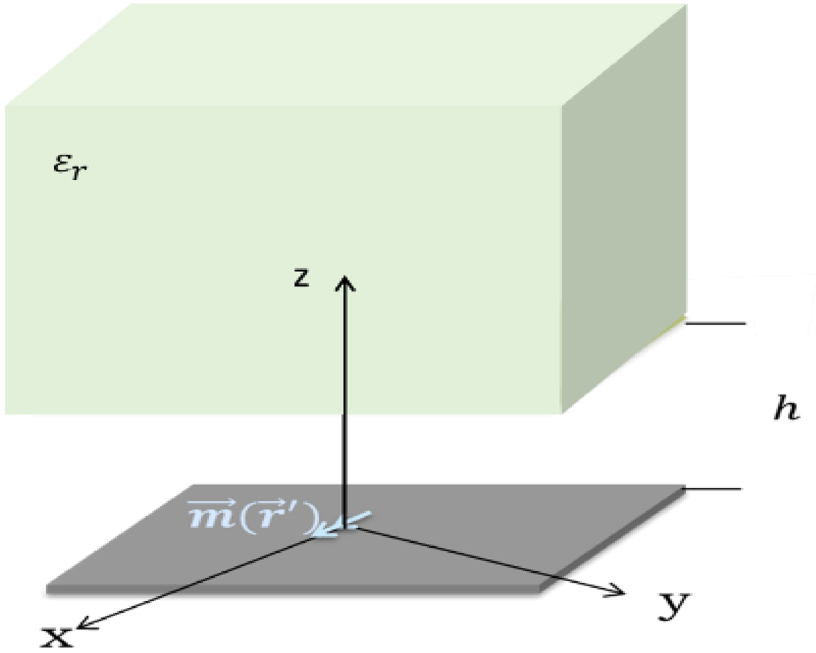
$$\underline{\underline{\mathbf{G}}}^{em}(k_x, k_y, z, z') = \tag{A.8}$$

$$\begin{bmatrix} \frac{k_x k_y}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z') - {}^mV_{TE}(k_\rho, z, z')\right) & \frac{-1}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z')k_x^2 + {}^mV_{TE}(k_\rho, z, z')k_y^2\right) \\ \frac{1}{k_\rho^2}\left({}^mV_{TM}(k_\rho, z, z')k_y^2 + {}^mV_{TE}(k_\rho, z, z')k_x^2\right) & \frac{k_x k_y}{k_\rho^2}\left({}^mV_{TE}(k_\rho, z, z') - {}^mV_{TM}(k_\rho, z, z')\right) \\ -\frac{k_y}{k_{zi}} Z_{TMi}\ {}^mI_{TM}(k_\rho, z, z') & \frac{k_x}{k_{zi}} Z_{TMi}\ {}^mI_{TM}(k_\rho, z, z') \end{bmatrix}$$

Question 2

Write a Matlab routine to calculate the spectral Green's function for the electric field given by an elementary x-oriented magnetic source at z=0 radiating into an infinite medium with a permittivity of er in the presence of a ground plane and an air layer of thickness h, as shown in the figure.
Consider h=5mm and a frequency of 30GHz.

Make a plot of the amplitude variation of the y-component of spectral field at $z = h^+$ as a function of kx from 0 to 2k0 with ky=0 for the following values of the permittivity er=2.5, 4 and 12.



Routines

- Solution of the equivalent transmission line:

$$[v_{TM}, v_{TE}, i_{TM}, i_{TE}] = trxline\_Superstrate(k0, er, h, kro, z)$$

- Dyadic SGF :

$$[Gxx, Gyx, Gzx] = SpectralGFem(k0, er, kx, ky, v_{TM}, v_{TE}, i_{TM}, i_{TE})$$