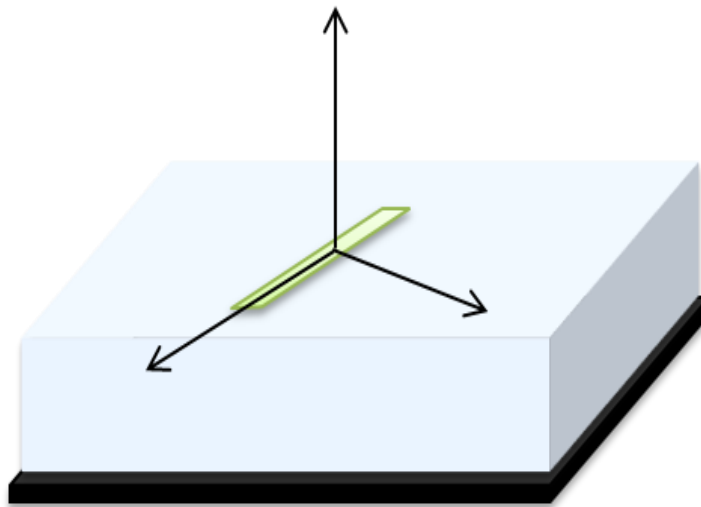


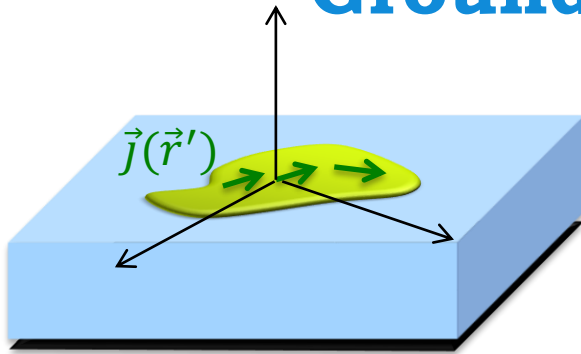
Question 1

Write a Matlab routine that finds the propagation constant of the first two surface waves (TM0 and TE1) surface waves supported by a grounded slab.

To check the program, provide a plot showing the propagation constant, normalized to k_0 , of the TM0 and TE1 surface waves for a frequency range from 1GHz to 20GHz. Consider $h=2\text{mm}$, $\epsilon_r = 10$ and $\epsilon_r = 5$.



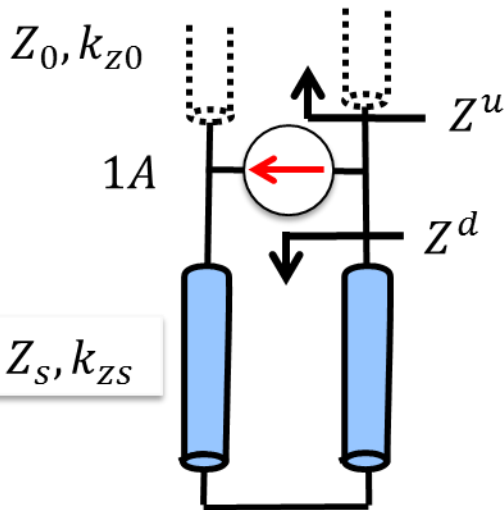
Grounded Dielectric Substrate



$$Z_{in} = \frac{Z_u Z_d}{Z_u + Z_d}$$

$$Z_u = Z_0$$

$$Z_d = jZ_s \tan(k_{zs}h)$$



$$v(z = z_s) = Z_{in}$$

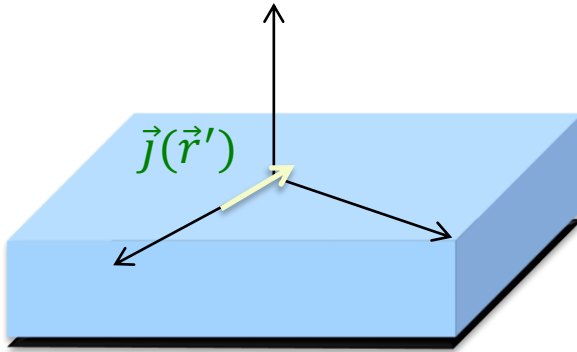
All the voltage and current solutions at any z -quote are expressed as a function of this voltage

The poles in the transversal complex plane can be found solving the following **dispersion equation**

$$D(k_\rho) = Z_u + Z_d = 0$$

for TE and TM

Grounded Dielectric Substrate



All the voltage and current solutions at any z -quote are expressed as a function of the same denominator

Voltage in the slab:

$$V_s = \frac{Z_u Z_d \sin(k_{zs} z)}{\underbrace{Z_u + Z_d}_{\text{denominator}} \sin(k_{zs} h)} = \frac{Z_u Z_d \sin(k_{zs} z)}{D(k_\rho) \sin(k_{zs} h)}$$

Current in the slab:

$$I_s = \frac{1}{Z_s} \frac{Z_u Z_d}{D(k_\rho)} \frac{j \cos(k_{zs} z)}{\sin(k_{zs} h)}$$

Voltage in the air:

$$V_0 = \frac{Z_u Z_d}{D(k_\rho)} e^{jk_{z0} h} e^{-jk_{z0} z}$$

Current in the air:

$$I_0 = \frac{1}{Z_0} \frac{Z_u Z_d}{D(k_\rho)} e^{jk_{z0} h} e^{-jk_{z0} z}$$

Solving Dispersion Equations

If we know a good guess of the solution, k_ρ^g

1) One can expand the denominator around this point using the Taylor's series:

$$D(k_\rho) \approx D(k_\rho^g) + D'(k_\rho^g)(k_\rho - k_\rho^g)$$

2) Evaluating this expansion in the actual zero, $k_{\rho 0}$:

$$D(k_{\rho 0}) \approx D(k_\rho^g) + D'(k_\rho^g)(k_{\rho 0} - k_\rho^g) = 0$$



The derivative can be done numerically

$$k_{\rho 0} = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

$$D'(k_\rho^g) \approx \frac{D(k_\rho^g + \Delta k/2) - D(k_\rho^g - \Delta k/2)}{\Delta k} \quad \Delta k = k_0/500$$

- Write a separate routine to calculate the dispersion equation:
 $[D] = \text{Den_GroundSlab}(k_0, er, h, k_{ro}, 'TE/TM')$

$$D(k_\rho) = Z_u + Z_d$$

- Write a separate routine to calculate the propagation constant:
 $[krho] = \text{findprop}(k_0, er, h, \mathbf{k}_\rho^g, 'TE/TM')$

$$k_\rho = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

$$D'(k_\rho^g) \approx \frac{D(k_\rho^g + \Delta k/2) - D(k_\rho^g - \Delta k/2)}{\Delta k}$$

$$\Delta k = k_0/500$$

Iterative Frequency Loop for solving the dispersion equation

- 1) Plot the Denominator vs k_ρ / k_0 at the highest frequency to find a good initial guess: k_ρ^g
- 2) Use this guess to find the propagation constant at the highest frequency

$$k_\rho (f_i) = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

- 3) Do a loop from the highest to the lowest frequency where you use the *normalized* propagation constant, $k_\rho^{gn} = k_\rho (f_i) / k_0(f_i)$, of the previous step as a guess to the next frequency k_ρ^0 :

$$k_\rho (f_i + 1) = k_\rho^g - \frac{D(k_\rho^g)}{D'(k_\rho^g)}$$

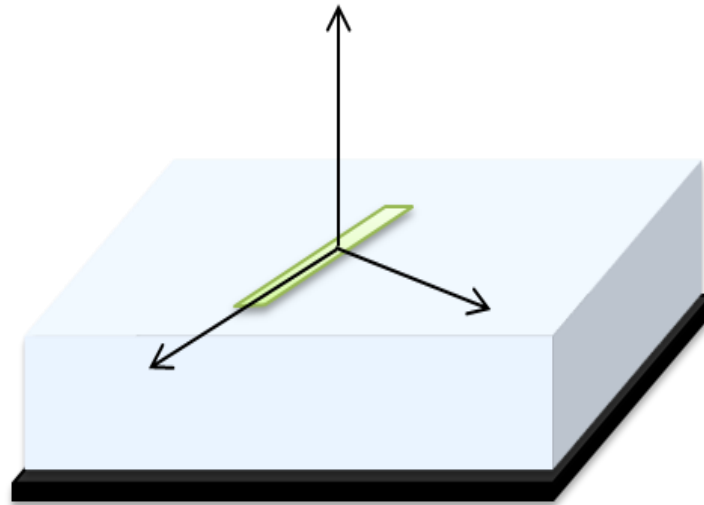
$$\text{with } k_\rho^g = k_\rho^{gn} k_0(f_i + 1)$$

Question 2

Write a Matlab routine to calculate the electric field contribution, in cylindrical coordinates, associated to the TM₀ surface wave excited by an elementary dipole on top of a grounded slab.

Provide the following plots ($h=2\text{mm}$, $\epsilon_r=10$, $f=10\text{GHz}$):

- Real and imaginary part variation of the electric field as a function of the radial distance.
- Amplitude variation of the electric field as a function of z .
- Amplitude variation of the electric field as a function of ϕ .



TM surface wave

Expressions for an *electric current oriented along x*

$$E_\rho(\rho, z) \approx \text{Res}[v_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

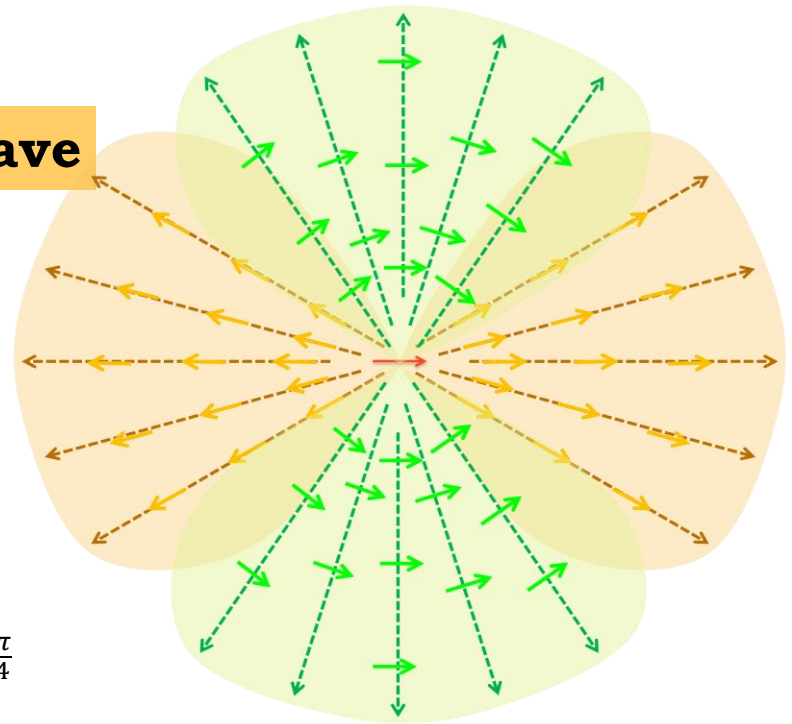
$$E_z(\rho, z) \approx -\frac{\zeta k_{\rho i}}{k} \text{Res}[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$H_\phi(\rho, z) \approx \text{Res}[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

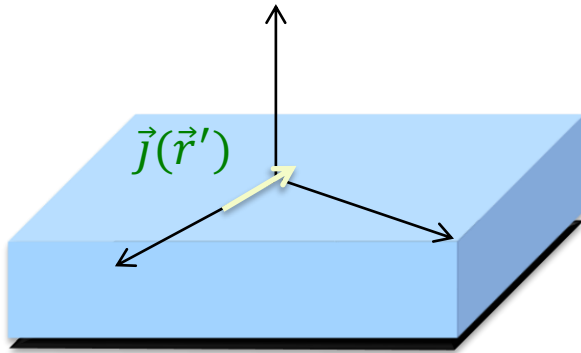
$$C = j \sqrt{\frac{k_{\rho i}}{2\pi}} e^{j\frac{\pi}{4}}$$

$k_{\rho i}$: *pole location in the spectrum*

TM Wave



Residues



$$E_{\rho}(\rho, z) \approx \text{Res}[v_{TM}(k_{\rho}, z, z')]\big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$E_z(\rho, z) \approx -\frac{\zeta k_{\rho i}}{k} \text{Res}[i_{TM}(k_{\rho}, z, z')]\big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$H_{\phi}(\rho, z) \approx \text{Res}[i_{TM}(k_{\rho}, z, z')]\big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

Fields inside the slab:

$$\text{Res}[v_{TE/TM}(k_{\rho}, z, z')]\big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{\sin(k_{zs} z)}{\sin(k_{zs} h)}$$

$$\text{Res}[i_{TE/TM}(k_{\rho}, z, z')]\big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_s} \frac{j \cos(k_{zs} z)}{\sin(k_{zs} h)}$$

Fields inside the air:

$$\text{Res}[v_{TE/TM}(k_{\rho}, z, z')]\big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} e^{jk_{z0} h} e^{-jk_{z0} z}$$

$$\text{Res}[i_{TE/TM}(k_{\rho}, z, z')]\big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_0} e^{jk_{z0} h} e^{-jk_{z0} z}$$

$$D'(k_{\rho}^g) \approx \frac{D(k_{\rho}^g + \Delta k/2) - D(k_{\rho}^g - \Delta k/2)}{\Delta k}$$

- Write a separate routine to calculate the residues:

$$[VtmR, ItmR] = Residue_GroundSlab(k0, er, h, ksw, z)$$

Fields inside the slab:

$$Res[v_{TE/TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{\sin(k_{zs} z)}{\sin(k_{zs} h)}$$

$$Res[i_{TE/TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_s} \frac{j \cos(k_{zs} z)}{\sin(k_{zs} h)}$$

Fields inside the air:

$$Res[v_{TE/TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} e^{jk_{z0} h} e^{-jk_{z0} z}$$

$$Res[i_{TE/TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} = \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_0} e^{jk_{z0} h} e^{-jk_{z0} z}$$

$$k_{\rho i} = ksw$$

- Write a separate routine to calculate the field components:

$$[Erho, Ez, Hphi] = TMSwFields(k0, ksw, er, VtmR, ItmR, \rho, \phi, z)$$

$$E_\rho(\rho, z) \approx Res[v_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$E_z(\rho, z) \approx -\frac{\zeta k_{\rho i}}{k} Res[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$H_\phi(\rho, z) \approx Res[i_{TM}(k_\rho, z, z')] \Big|_{k_{\rho i}} J_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

- FT of the current distribution (PWS):
 $Jx = FTcurrent(k0, kx, ky, l, w)$

$$k_x = k_{sw} \cos \phi$$

$$k_y = k_{sw} \sin \phi$$

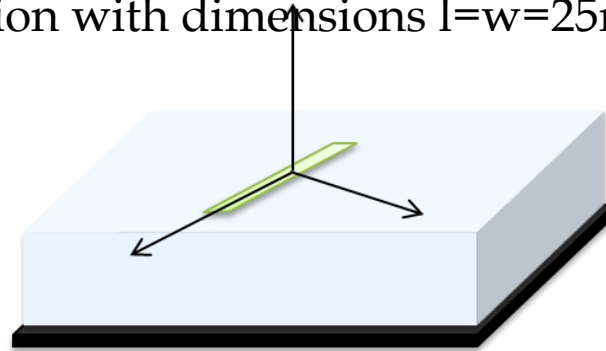
$$C = j \sqrt{\frac{k_{\rho i}}{2\pi}} e^{j\frac{\pi}{4}}$$

Question 3

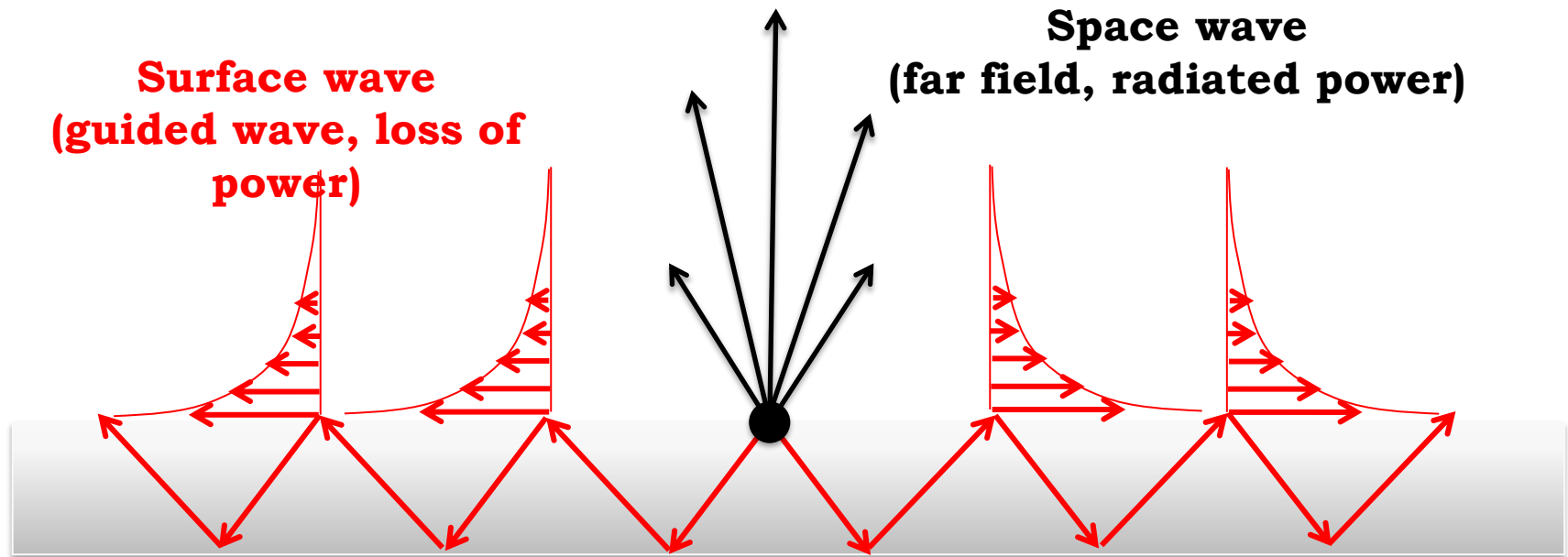
Write a Matlab routine to calculate the power radiated into the TM₀ surface wave of a dipole antenna printed on top of a grounded slab.

Provide the following plots ($h=2\text{mm}$, $\epsilon_r=10$, $f=1\text{-}20\text{GHz}$):

- The far field power and surface wave power radiated by an elementary dipole ($J_x=1$) normalized to the power radiated by the same dipole in free space
- The TM₀ surface wave efficiency for:
 - an elementary dipole,
 - a dipole with $l=5.3\text{mm}$ and $w=0.5\text{mm}$ (assume a PWS distribution along the dipole and uniform across), and
 - a *uniform* current distribution with dimensions $l=w=25\text{mm}$.



Antenna SW Efficiency



We are going to characterize the loss of efficiency into SW:

1. Find the SW propagation constant (dispersion equation)
2. Calculate the SW fields (Residue)
3. Power into SW vs radiated power

$$\eta_{sw} = \frac{P_{rad}}{P_{rad} + P_{sw}}$$

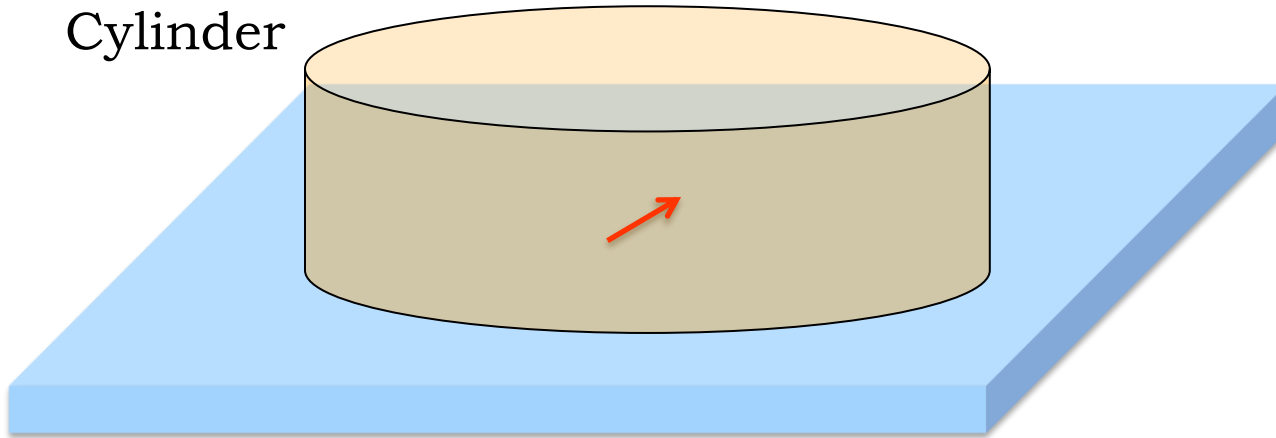
SW Power

We need to characterize the power launched into surface waves

By integrating the Poynting vector around a closed surface surrounding the antenna

$$P_{sw} = \frac{1}{2} \sum_i \iint_s \operatorname{Re}[\vec{E}_{sw_i} \times \vec{H}_{sw_i}^*] d\vec{s}$$

Cylinder



Since the fields at the top and bottom of the cylinder are zero

$$P_{sw}^{TM} = \frac{1}{2} \int_0^{2\pi} \int_0^{\infty} \operatorname{Re}[-E_z H_{\phi}^*] \rho_s dz d\phi$$

TM SW Power

$$P_{SW}^{TM} = \frac{1}{2} \int_0^{2\pi} \int_0^{\infty} \text{Re}[-E_z H_{\phi}^*] \rho_s dz d\phi$$



$$E_z(\rho, z) \approx -\frac{\zeta k_{\rho i}}{k} \text{Res}[i_{TM}(k_{\rho}, z, z')] \Big|_{k_{\rho i}} C_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$H_{\phi}(\rho, z) \approx \text{Res}[i_{TM}(k_{\rho}, z, z')] \Big|_{k_{\rho i}} C_x(k_{\rho i}, \phi) \frac{C \cos \phi e^{-jk_{\rho i} \rho}}{\sqrt{\rho}}$$

$$C = j \sqrt{\frac{k_{\rho i}}{2\pi}} e^{j\frac{\pi}{4}}$$

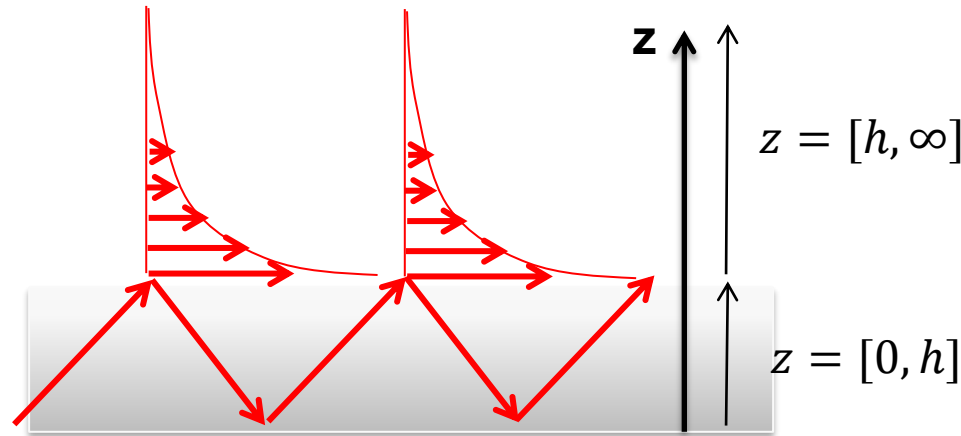
$$P_{SW}^{TM} = \frac{1}{2} \frac{k_{\rho i}^2}{2\pi} \int_0^{\infty} \frac{\zeta}{k} \left| \text{Res}[i_{TM}(k_{\rho}, z, z')] \Big|_{k_{\rho i}} \right|^2 dz \int_0^{2\pi} |C_x(k_{\rho i}, \phi)|^2 \cos^2 \phi d\phi$$

The integral in z and phi are not coupled
They can be performed independently for
faster computation

$$P_{SW}^{TM} = \frac{1}{2} \frac{k_{\rho i}^2}{2\pi} I_z I_{\phi}$$

Z-integration

$$I_z = \int_0^{\infty} \frac{\zeta}{k} \left| \text{Res}[i_{TM}(k_\rho, z, z')] \right|_{k_{\rho i}}^2 dz = \frac{\zeta_0}{k_0} \int_0^h \frac{1}{\epsilon_r} \left| \text{Res}[i_{TM}(k_\rho, z, z')] \right|_{k_{\rho i}}^2 dz + \frac{\zeta_0}{k_0} \int_h^{\infty} \left| \text{Res}[i_{TM}(k_\rho, z, z')] \right|_{k_{\rho i}}^2 dz$$



The z -integration domain is divided in two regions associated to the different dielectrics

$$\text{Res}[i_{TM}(k_\rho, z, z')] = \begin{cases} \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_s} \frac{j \cos(k_{zs} z)}{\sin(k_{zs} h)} \\ \frac{Z_u Z_d}{D'(k_{\rho i})} \frac{1}{Z_0} e^{jk_{z0} h} e^{-jk_{z0} z} \end{cases}$$

$$k_{z0} = \sqrt{k_0^2 - k_{\rho i}^2} = -j \sqrt{k_{\rho i}^2 - k_0^2} = -j \alpha_{z0}$$

It is possible doing the z -integration in analytical form

$$\int_0^h \cos^2(k_{zs} z) dz = \frac{h}{2} (1 + \text{sinc}(2k_{zs} h))$$

$$\int_h^{\infty} e^{-2\alpha_{z0}(z-h)} dz = \frac{1}{2\sqrt{k_{\rho i}^2 - k_0^2}}$$

phi-integration: numerical

$$I_\phi = \int_0^{2\pi} |J_x(k_{\rho i}, \phi)|^2 \cos^2 \phi d\phi$$

The integral in phi can be done numerically for any current distribution

However if we consider a short dipole ($J_x(k_{\rho i}, \phi) = I_0 l$)
 $I_\phi = \pi$

- Write a separate routine to calculate the power radiated into the surface wave for an elementary dipole:

$$[P_{sw}] = P_{swTMelem}(k_0, er, h, k_{sw})$$

$$P_{sw}^{TM} \Big|_{\delta} = \frac{1}{2} \frac{k_{\rho i}^2}{2\pi} I_z I_{\phi}^{\delta}$$

$$I_{\phi}^{\delta} = \pi$$

$$I_z = \int_0^{\infty} \frac{\zeta}{k} \left| \text{Res}[i_{TM}(k_{\rho}, z, z')] \Big|_{k_{\rho i}} \right|^2 dz \quad \text{Numeric or analytic}$$

- Write a separate routine to calculate the FT of a uniform current distribution

$$\tilde{J}_x(k_x, k_y) = L \text{sinc}\left(\frac{k_x L}{2}\right) \text{sinc}\left(\frac{k_y W}{2}\right)$$

- Write a separate routine to calculate the phi integral for the specific current distribution:

$$[I_{\phi}] = \text{PhiInt_PWS}(k_0, k_{sw}, l, w)$$

$$[I_{\phi}] = \text{PhiInt_Uniform}(k_0, k_{sw}, l, w)$$

$$I_{\phi} = \int_0^{2\pi} |J_x(k_{\rho i}, \phi)|^2 \cos^2 \phi d\phi$$

The power can be evaluated using the power of the elementary current ($P_{sw}^{TM} \Big|_{\delta}$) via

$$P_{sw}^{TM} = P_{sw}^{TM} \Big|_{\delta} \frac{I_{\phi}}{I_{\phi}^{\delta}}$$