Graded Assignment AE222 Gasdynamics Submission Date: April 3, 2023

1) Consider that the Cp for Air is given by the NASA five coefficient polynomial as

$C_p = A_1 + A_2T + A_3T^2 + A_4T^3 + A_5T^4$, where the coefficients $A_1 \dots A_5$ are :

Temp [K]	**A1**	**A2**	**A3**	**A4**	**A5**
200–1000	1.02E+03	-2.10E-01	4.81E-04	-3.91E-08	-1.19E-10
1000–6000	8.86E+02	3.50E-01	-1.20E-04	1.88E-08	-1.11E-12

1. Study the procedure to compute normal shock properties when gas is not CPG Section 7-5 Normal Shock Waves in Imperfect Gases in Gas Dynamics Volume 1 by Zucrow and Hoffman
2. Develop a computer code (in Matlab or Python (use Matlab Live Scripts or Jupyter Notebook)) to calculate the Normal Shock Properties given inputs of M, P and T before the shock.
3. Do a comparative plot of $\frac{P_2}{P_1}, \frac{T_2}{T_1}, \frac{\rho_2}{\rho_1}, M_2, \frac{\Delta S}{R}$ vs upstream Mach number and Temperature , take $T_1 = 250K \quad and \quad 500K$. Limit Mach number such that $T_2 < 6000K$
4. Give your observations

2) Consider an ideal shock tube problem :

Take:

Driver Length = 2 m
Driven Length = 5 m

Both Driver and Driven Gases are Nitrogen at 300 K. Gas is taken to be Calorically Perfect Gas

For the given shock Mach Number Calculate:

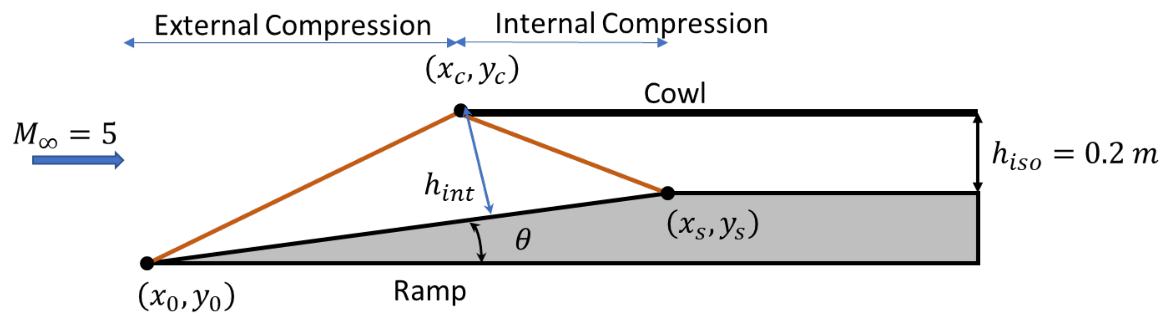
- $\frac{P_4}{P_1}$
- $\frac{P_2}{P_1}, \frac{T_2}{T_1}, U_p$
- $\frac{P_5}{P_1}, \frac{T_5}{T_1}$
- Time taken for the first shock to arrive at the end of the Driven length t_{in}
- Spatial distribution at two instants of time $0.8t_{in}$ and $1.02t_{in}$

Refer the table below for Shock Mach Number

Name	Shock Mach Number
ARAVIND C B .	1.4
SAINI JATIN RAO .	1.6
PANDYA KUSH TUSHARBHAI .	1.8
KAIN DIPENDRASINGH .	2
RAJA JANMEJAY .	2.2
P V KARTHIKEYA BHARADWAJ .	2.4
AMAL JO MAMMACHEN .	2.6
ASWATHI KRISHNA .	2.8
DEBRAJ ROY .	3
SAURABH SHARMA .	3.2
GOKUL HARISH .	3.4
S S SANJAY .	3.6
DIVYANSHU VARANDANI .	3.8
PRAVEEN S .	4
KADU RUSHIKESH PRAMOD .	4.2
PRAVEEN T .	4.4
SRISATIYA NARAYANAN B .	4.6

3) The figure shows a single ramp mixed compression intake that satisfies the shock on cowl lip and shock on ramp shoulder criteria. The design of this intake is based on a single parameter – the ramp angle θ.

- Find expressions for the coordinates (x_c, y_c) and (x_s, y_s) in terms of h_{iso}
- Find expression for $\frac{h_{int}}{h_{iso}}$ which is the area ratio for the internal compression diffuser
- Vary $5° \geq \Theta \leq 45°$, plot variations of
    - Compression ratio
    - Static Temperature ratio
    - Stagnation pressure ratio
    - Area Ratio of internal compression in the context of isentropic limit and Kantrowitz limit
    - Total length of the intake
- Give your observations

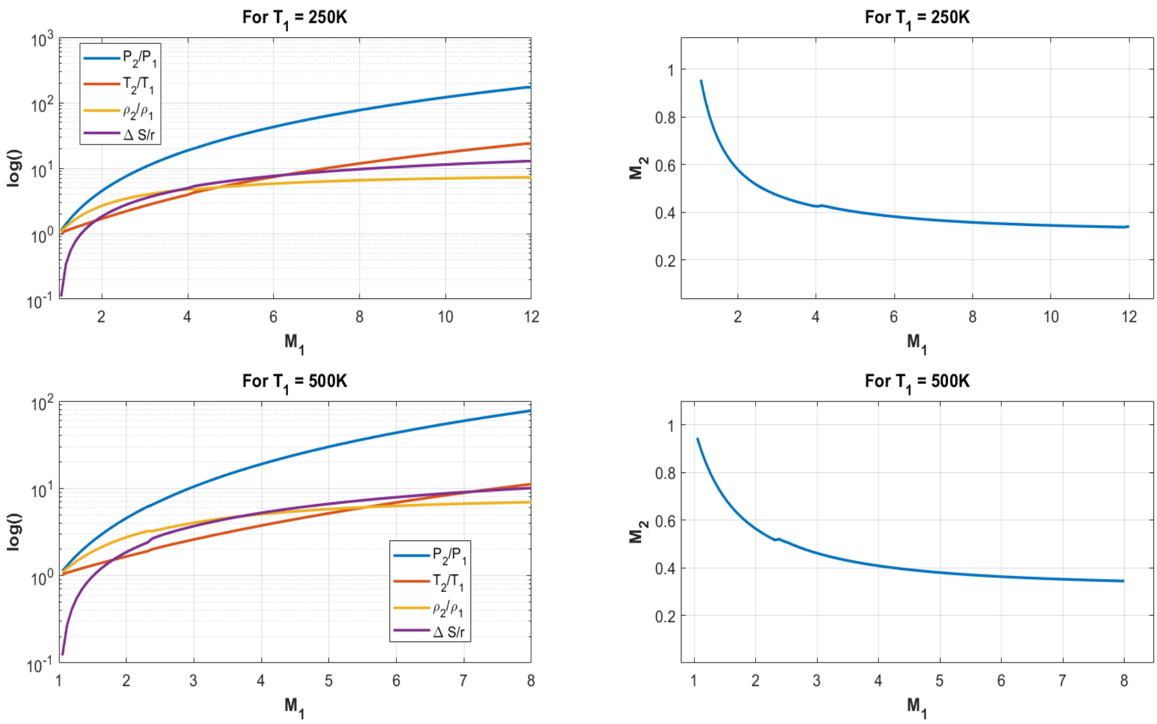


Note: Students can use Matlab live scripts or Jupyter Notebooks where code and text can be used together. The submission can be in form of Matlab Live Script /Jupyter Notebook themselves

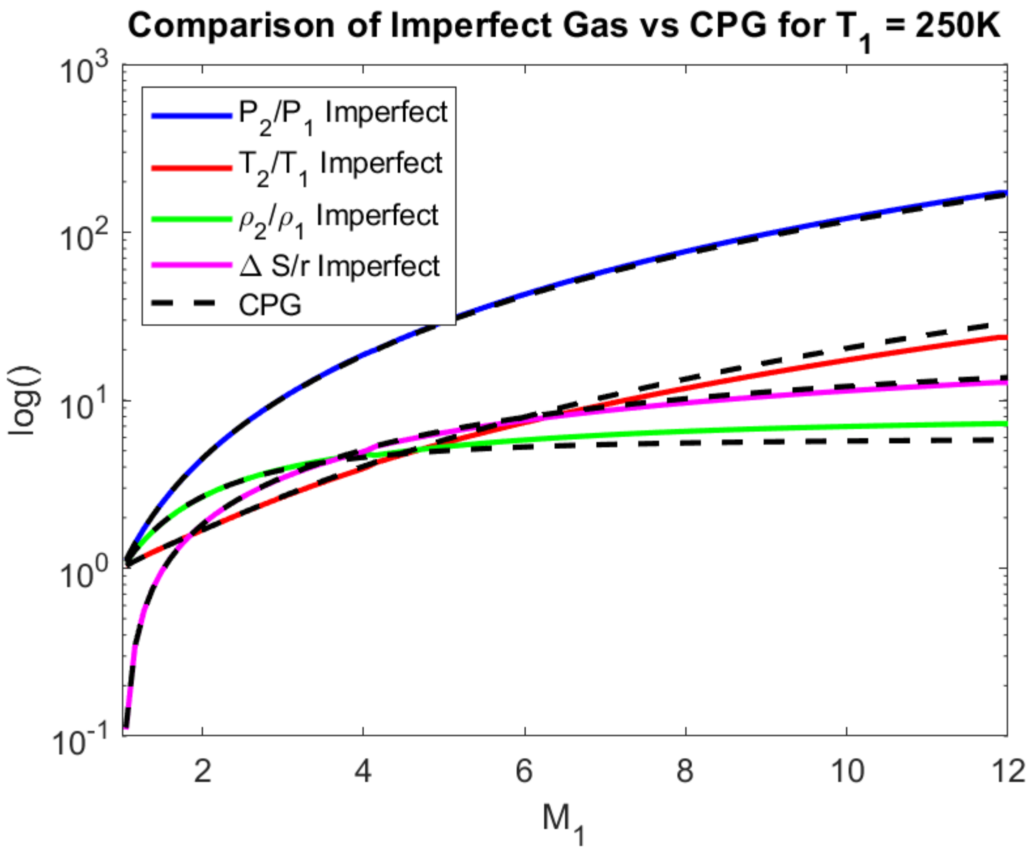
GRADED ASSIGNMENT GAS DYNAMICS – SAURABH SHARMA

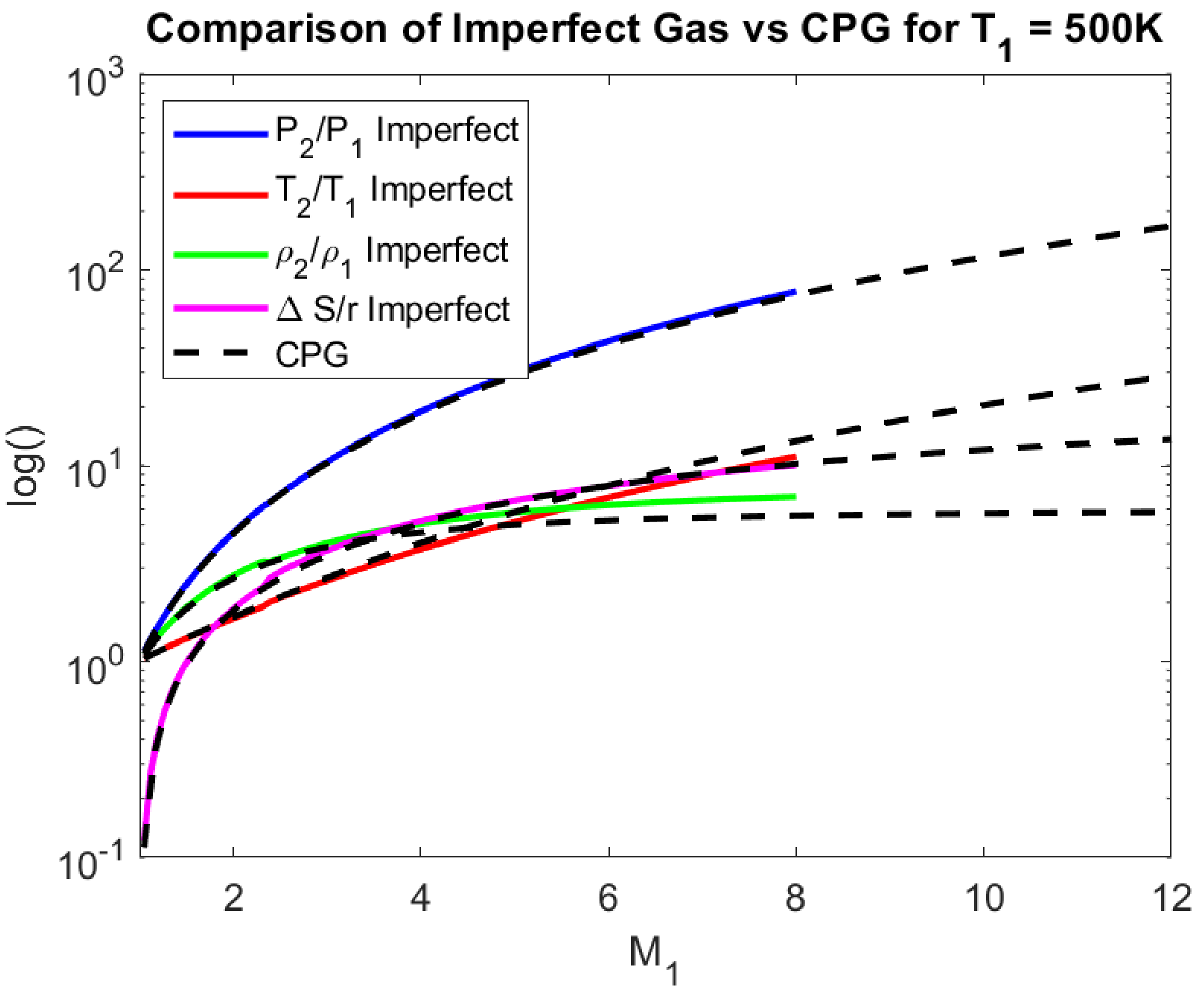
PROBLEM 1

```
After shock properties:
Mach after the shock M2 = 0.381
Temperature after the shock T2 = 1737.066 K
Pressure after the shock P2 = 563731.379 Pa
Density after the shock rho2 = 1.131 kg/m^3
```



COMPARISON PLOTS





The current observations suggest that the impact of an imperfect gas on the results of a gas dynamics analysis is not significant for lower Mach number ranges. In other words, the results obtained by considering the gas as an imperfect gas do not differ much from those obtained by assuming a constant pressure gas (CPG).

However, when analyzing higher Mach number ranges, the deviation between the results obtained from an imperfect gas analysis and those obtained from a CPG analysis is found to be higher. This can be attributed to the fact that at higher Mach numbers, the post-shock temperatures also increase, resulting in a higher variation of specific heat (Cp) values.

Specific heat is a thermodynamic property that determines the amount of energy required to change the temperature of a material. In the case of gases, Cp is the specific heat at constant pressure. For an ideal gas, Cp is a constant value, which is used in CPG analyses. However, for real gases, the value of Cp varies with temperature and pressure, and this variation becomes more significant at higher temperatures.

Therefore, when analyzing high Mach number flows, it becomes crucial to consider the effects of an imperfect gas, which can better capture the Cp variation and provide more accurate results. However, for lower Mach number ranges, the CPG assumption is still valid and can be used to simplify the analysis without significantly affecting the results.

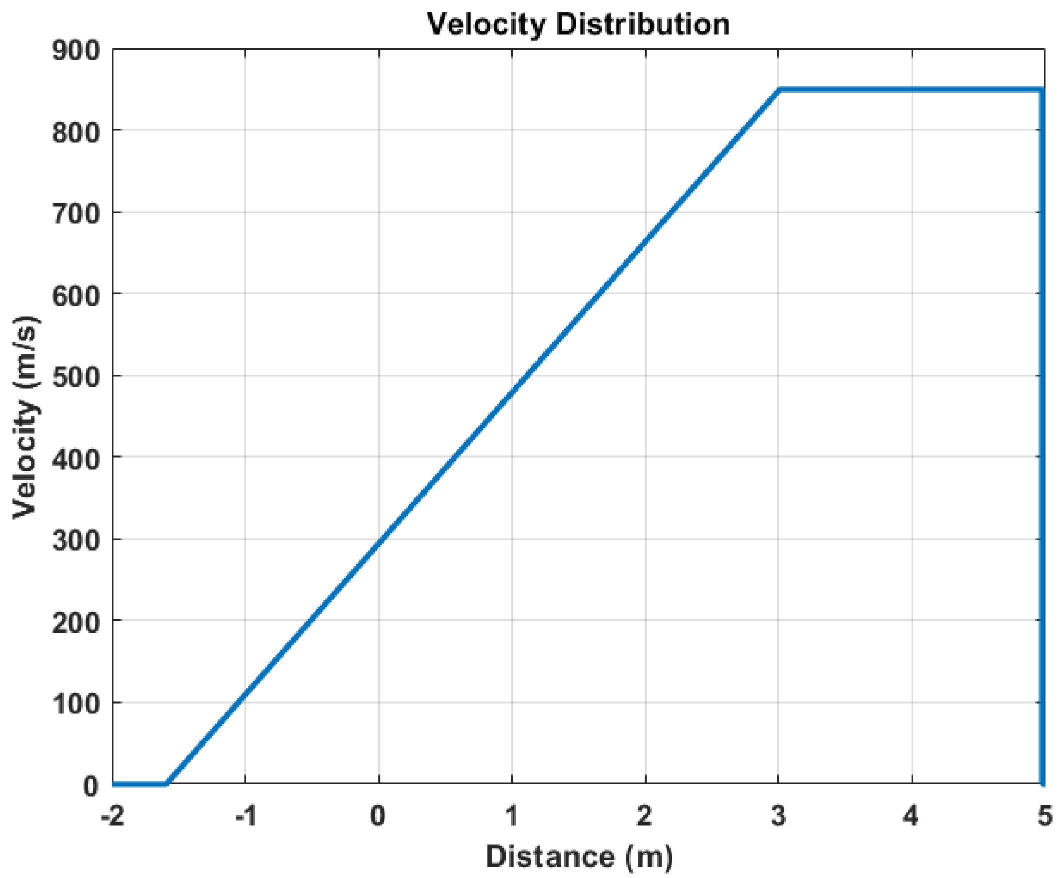
PROBLEM 2

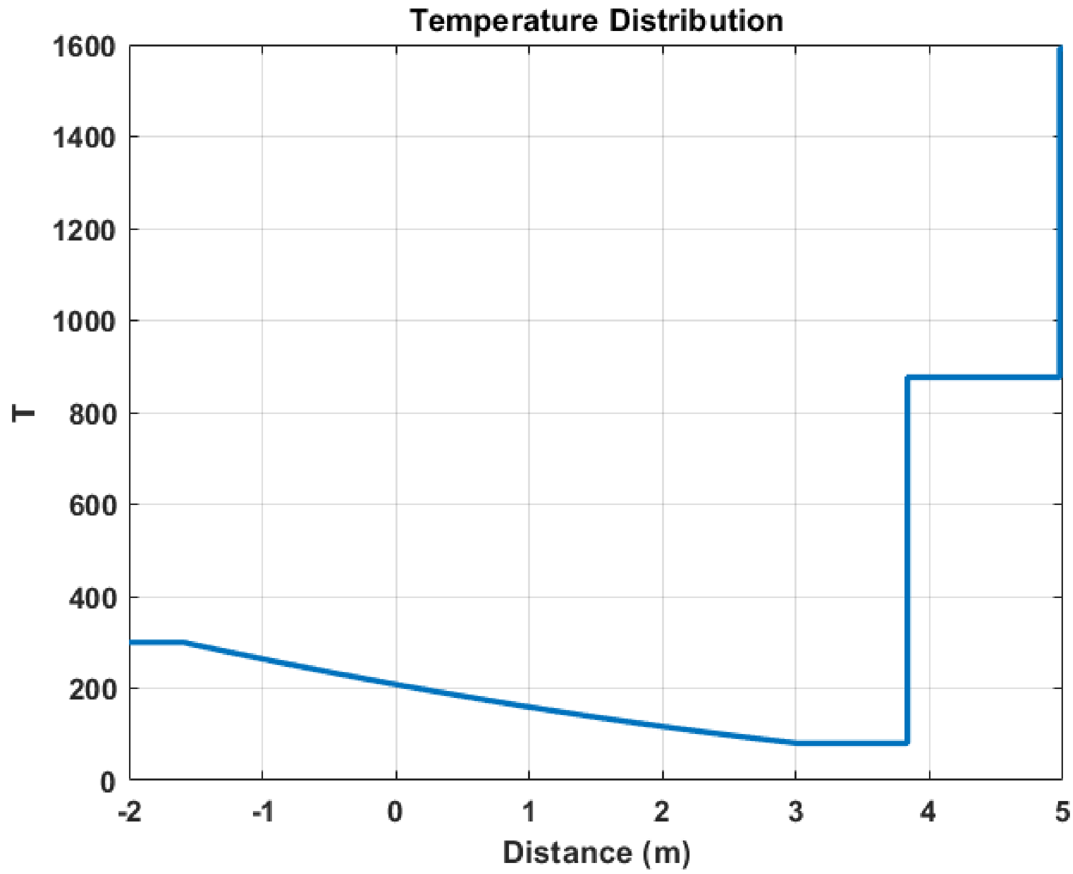
```
L4 = 2;       % Driver length [m]
L1 = 5;       % Driven length [m]
T4 = 300;     % Driver gas temperature [K]
T1 = 300;     % Driven gas temperature [K]
gamma = 1.4; % Specific heat ratio
R = 296.8;    % Gas Constant of nitrogen gas [J/kg-K]
P1= 1;        % Non Dimensionalizing pressure for the plots

%% Defining input values
M = 3.2;
```

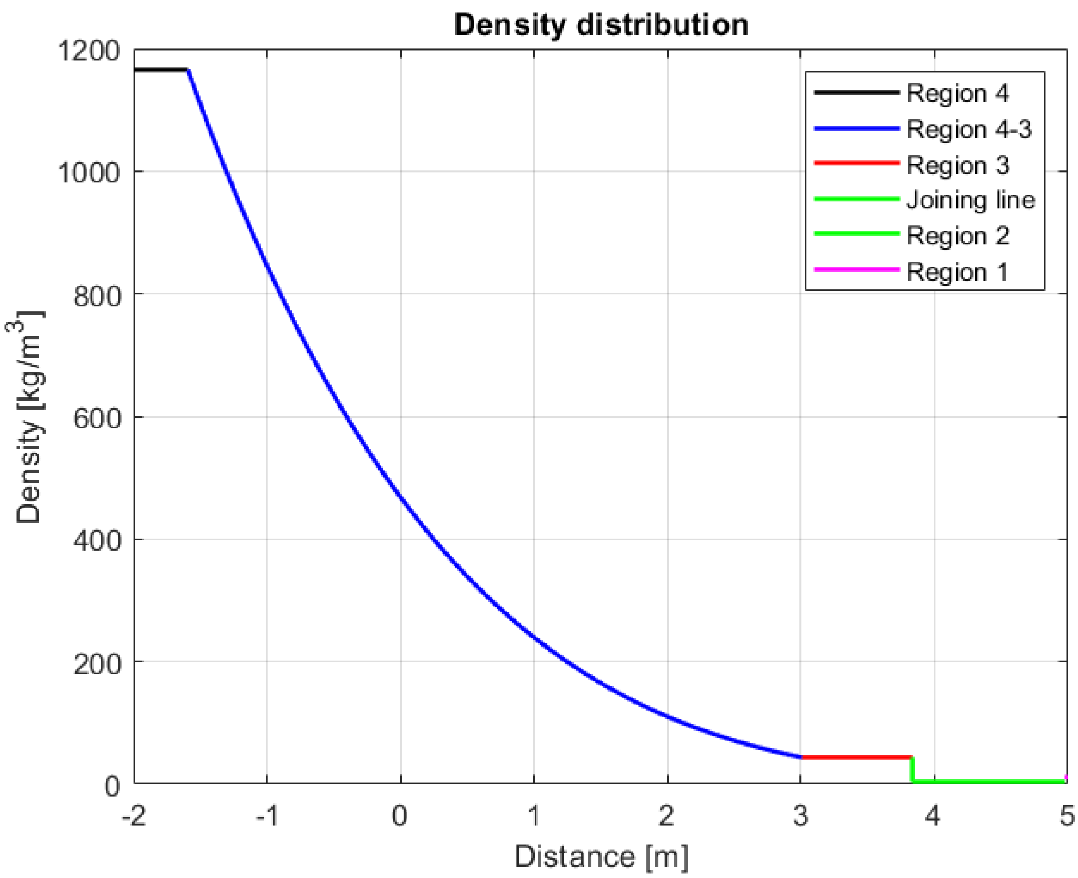
mmand Window

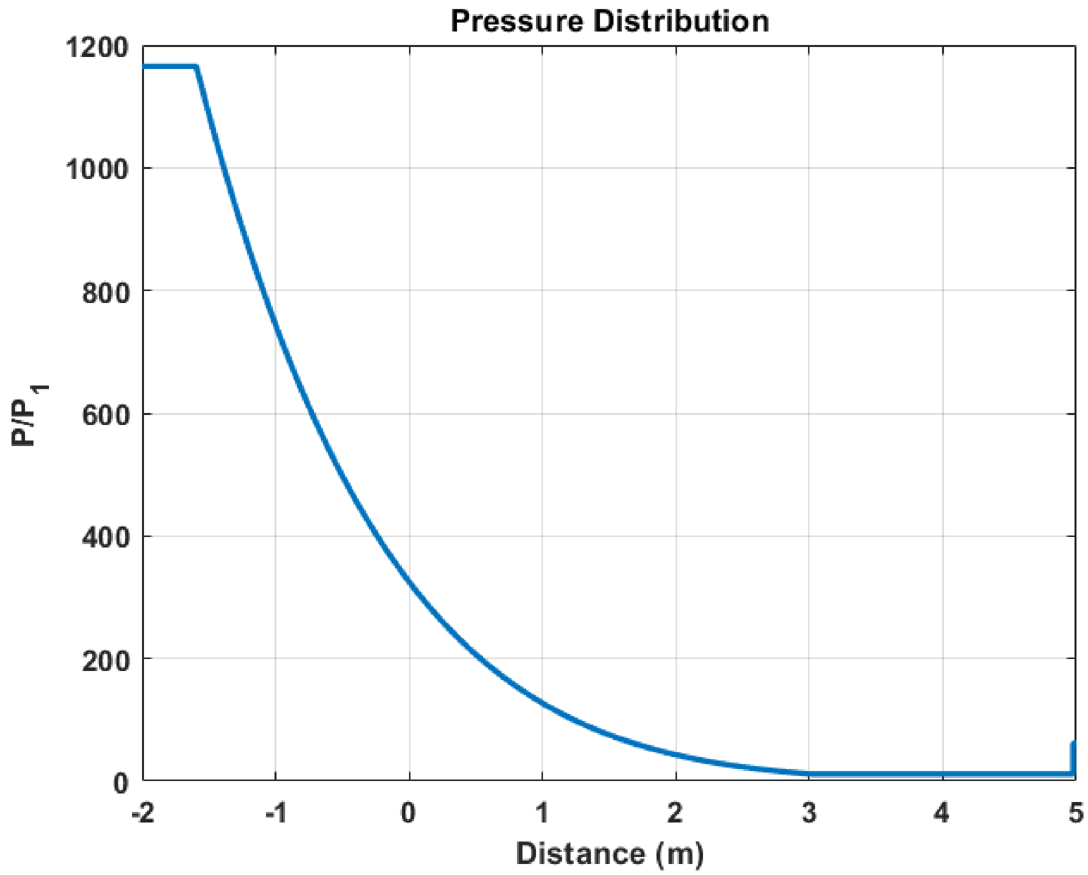
```
Pressure ratio across the shock: 11.780
Temperature ratio across the shock: 2.922
Contact surface speed: 849.566 m/s
Pressure ratio across reflected shock: 61.775
Temperature ratio across reflected shock: 5.307
Pressure ratio P4/P1: 1165.302
Time taken for the first shock to arrive at the end of the driven length: 0.004
```

Distributions for second time instant:

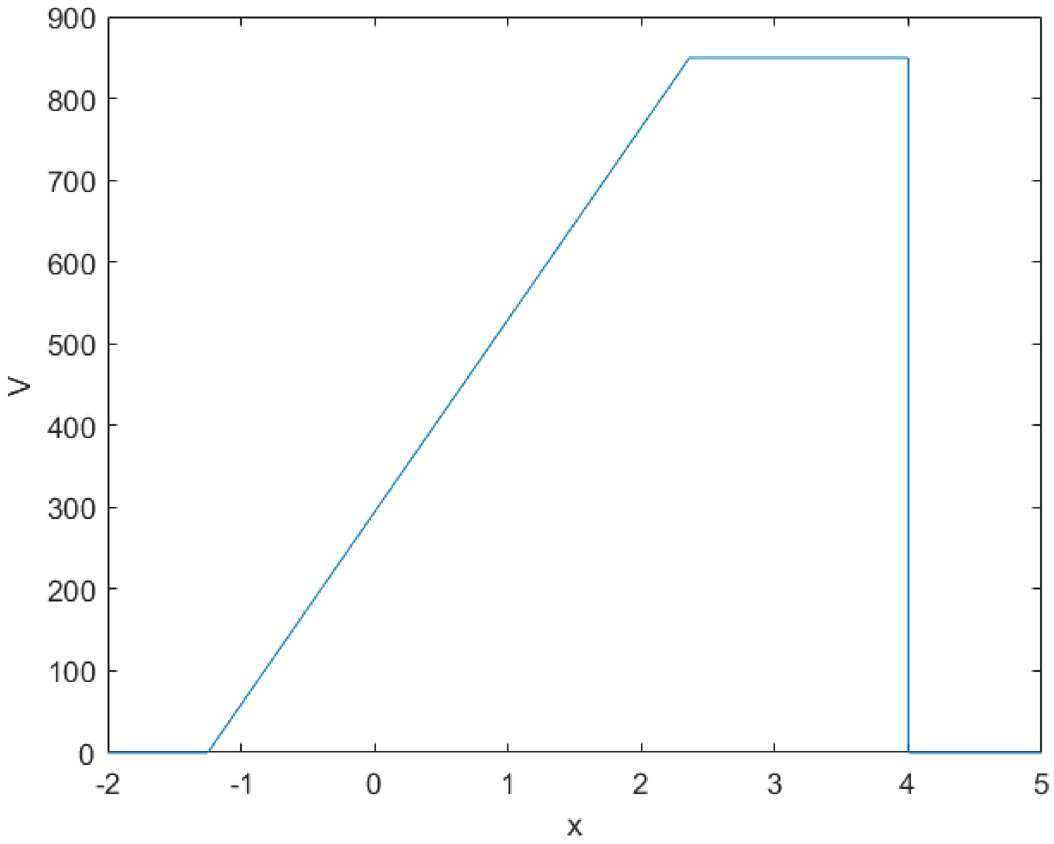


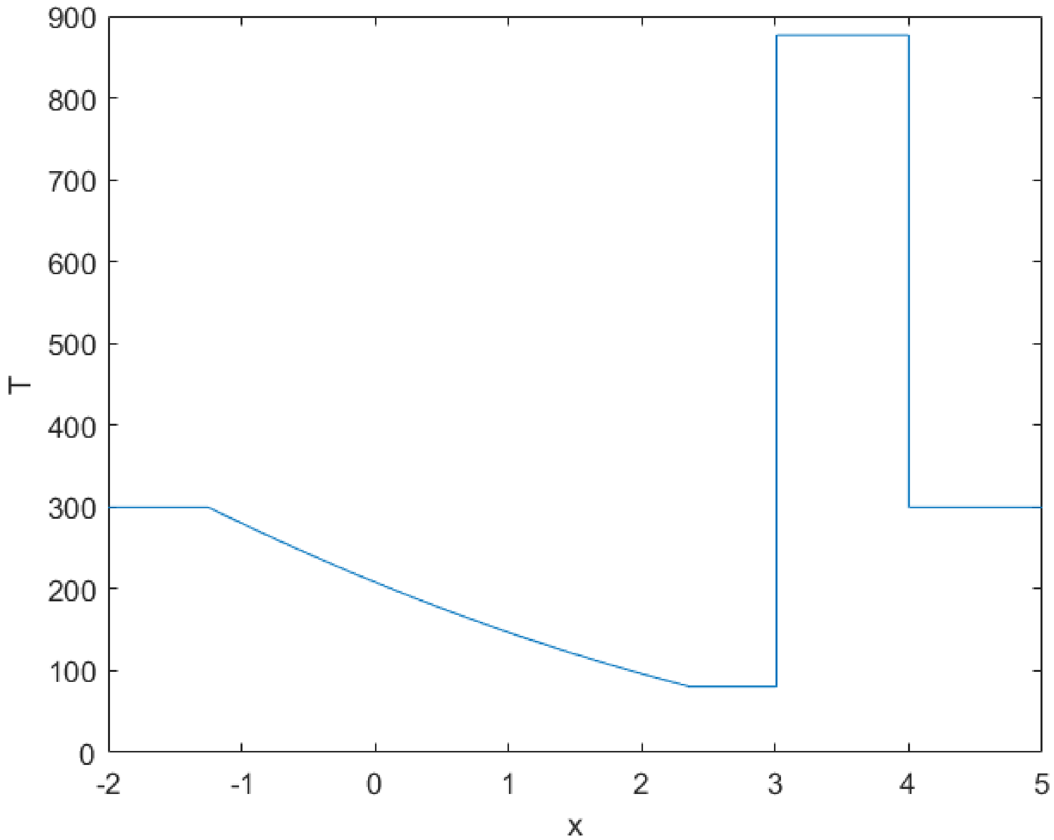

Temperature Distribution
T
1600
1400
1200
1000
800
600
400
200
0
-2
-1
0
1
2
3
4
5
Distance (m)

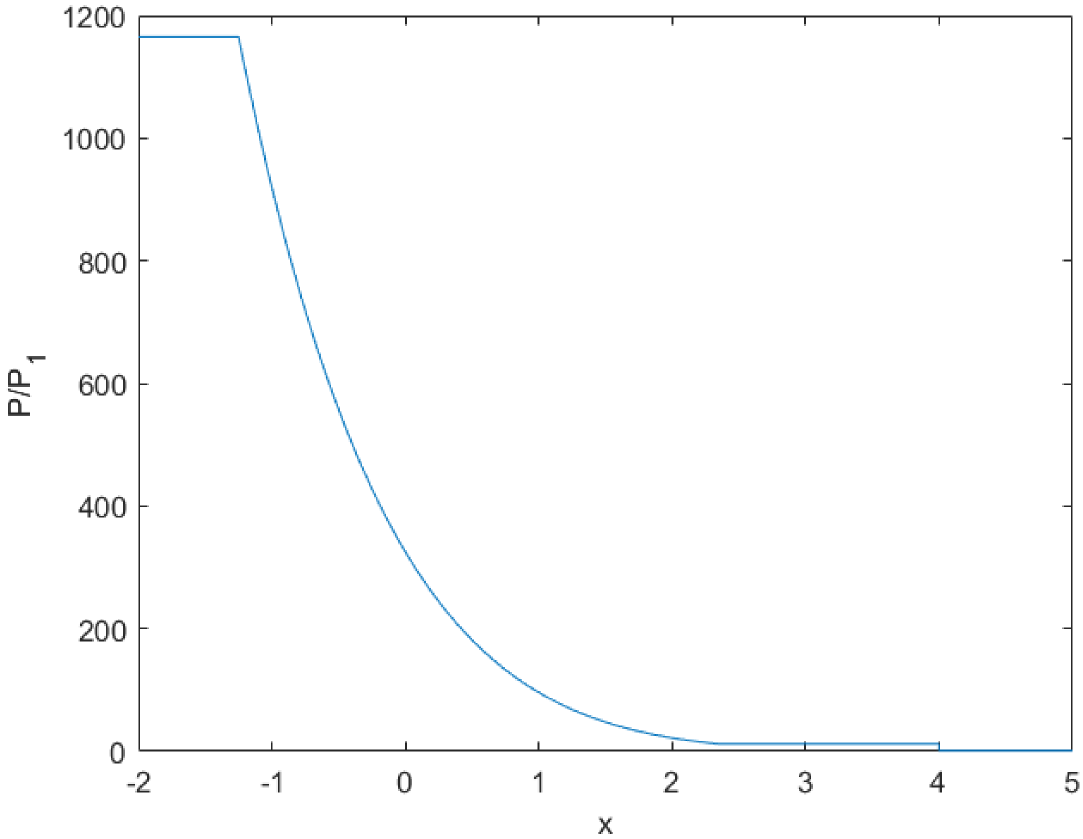

Density distribution
Density [kg/m^3]
1200
1000
800
600
400
200
0
-2
-1
0
1
2
3
4
5
Distance [m]
Region 4
Region 4-3
Region 3
Joining line
Region 2
Region 1

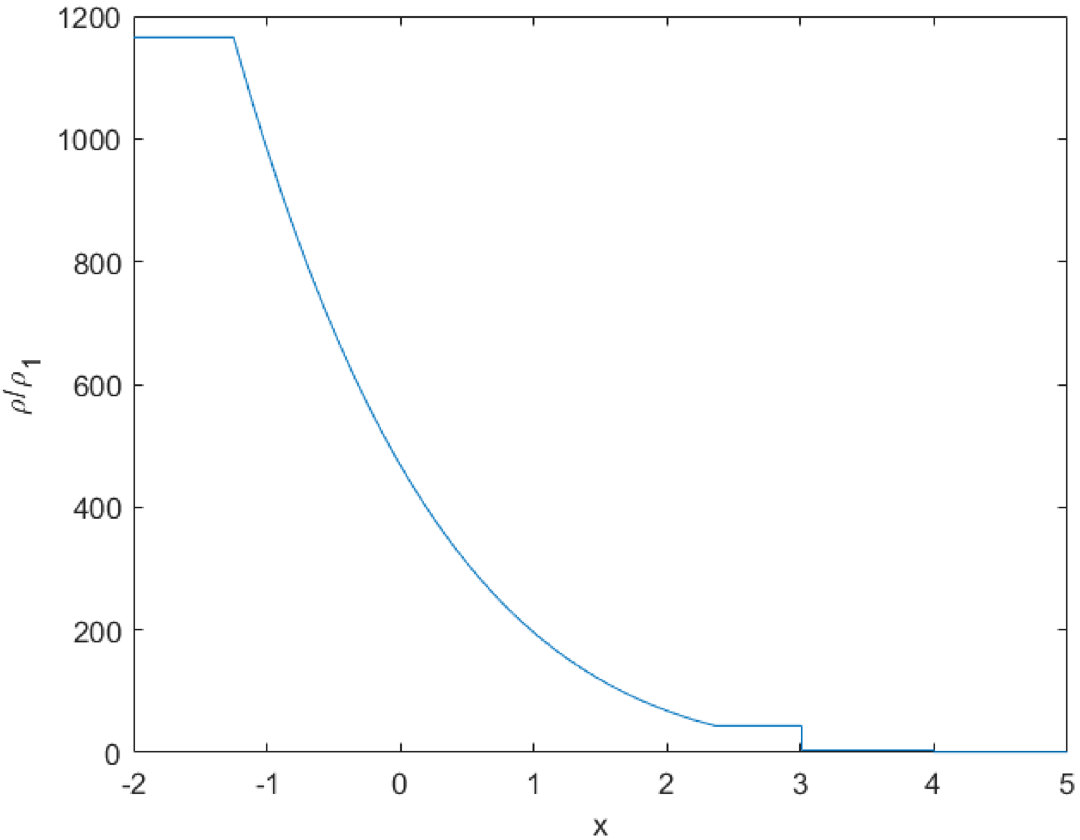

Pressure Distribution
1200
1000
800
600
400
200
0
P/P1
-2
-1
0
1
2
3
4
5
Distance (m)

Distribution for first-time instant:





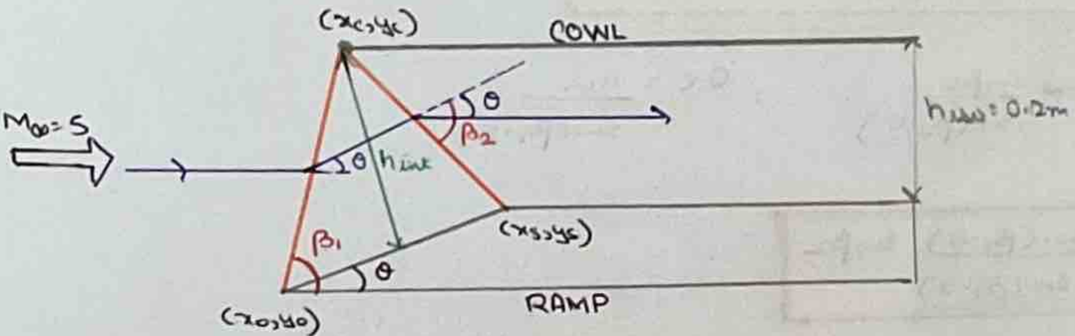

1200
1000
800
600
400
200
0
P/P1
-2
-1
0
1
2
3
4
5
x


1200
1000
800
600
400
200
0
ρ/ρ1
-2
-1
0
1
2
3
4
5
x

SOLUTION 3

- SAURABH SHARMA
(21498)

DERIVATIONS

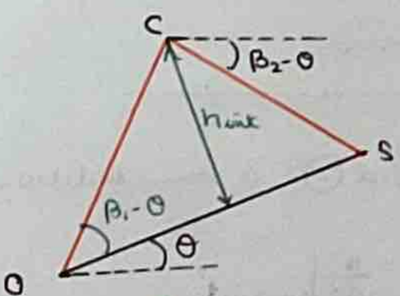


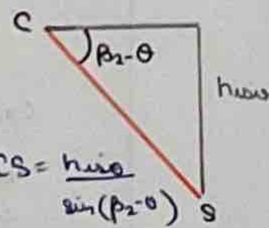
We know that,

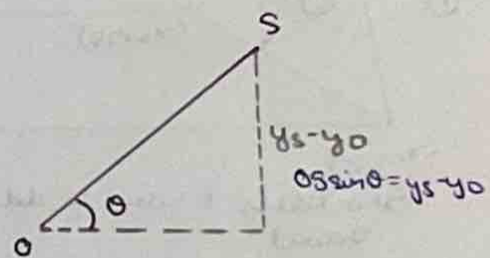
$\beta_1 = f(M_\infty, \theta)$, $\beta_2 = f(M_2, \theta)$

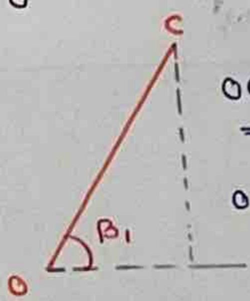
where, $M_2 = f(M_\infty, \beta_1) = f(M_\infty, \theta)$

$\Rightarrow \beta_2 = f(M_\infty, \theta)$



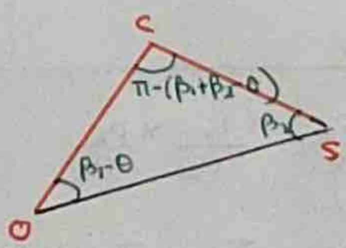






$OC \sin\beta_1 = y_c - y_0$
$= h_{iso} + y_s - y_0$

$OC \cos\beta_1 = x_c - x_0$



$$\frac{OC}{\sin\beta_2} = \frac{CS}{\sin(\beta_1 - \theta)} = \frac{OS}{\sin(\pi - \beta_1 - \beta_2 + \theta)}$$

Also, $x_c + CS\cos(\beta_2 - \theta) = x_s$

$OC \sin(\beta_1 - \theta) = h_{int}$

This gives : $OS = \sin(\beta_1 + \beta_2 - \theta) \dfrac{h_{iso}}{\sin(\beta_2 - \theta)}$

$OC = \sin\beta_2 \dfrac{h_{iso}}{\sin(\beta_2 - \theta)}$

After solving above relations :

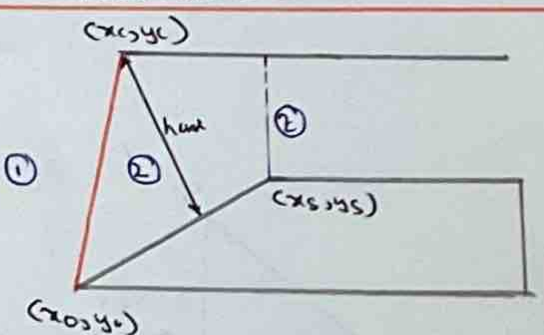
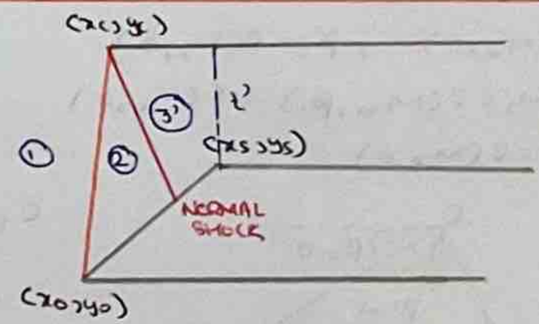
$$\boxed{\begin{aligned} x_c &= x_0 + h_{iso} \frac{\cos\beta_1 \sin\beta_2}{\sin(\beta_2 - \theta)} \\ y_c &= y_0 + h_{iso}\left[\frac{\sin\theta \sin(\beta_1 + \beta_2 - \theta)}{\sin(\beta_2 - \theta)} + 1\right] \end{aligned}} \Rightarrow (x_c, y_c)$$

$$x_s = x_0 + h_{iso}\left[\frac{\cos\beta_1 \sin\beta_2}{\sin(\beta_2-\theta)} + \cot(\beta_2-\theta)\right] \Longrightarrow (x_s, y_s)$$

$$y_s = y_0 + h_{iso}\left[\frac{\sin\theta \sin(\beta_1+\beta_2-\theta)}{\sin(\beta_2-\theta)}\right]$$

We have, $OC = h_{iso}\dfrac{\sin\beta_2}{\sin(\beta_2-\theta)}$, $OC = \dfrac{h_{in}}{\sin(\beta_1-\theta)}$

$$\therefore \boxed{\frac{h_{in}}{h_{iso}} = \frac{\sin(\beta_1-\theta)\sin\beta_2}{\sin(\beta_2-\theta)}}$$

ISENTROPIC LIMIT	KANTROWITZ LIMIT
	
Shockless & sonic condition at throat	Normal shock at 2 & sonic condition at throat
$\dfrac{A_2}{A_t} = \dfrac{A}{A^*}\Big\rvert_{M=M_2}$	$\dfrac{A_2}{A_t'} = \dfrac{A_3}{A_t'} = \dfrac{A}{A^*}\Big\rvert_{M=M_3}$

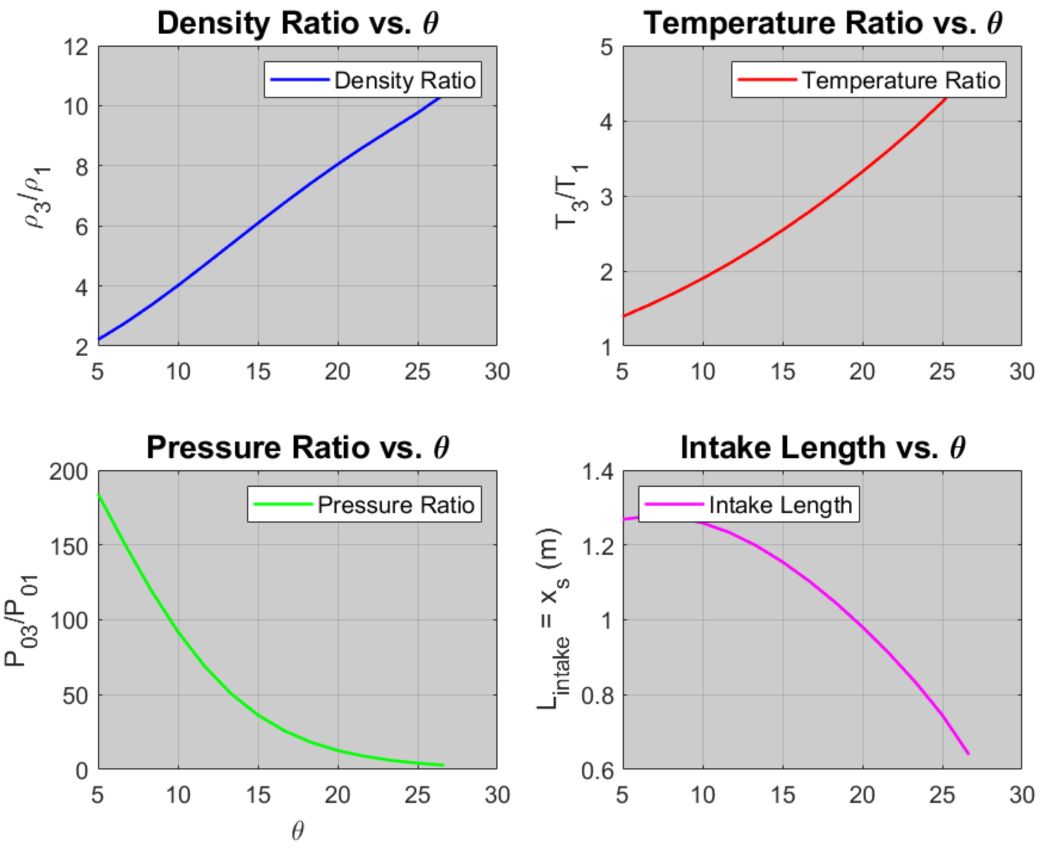
(•) Compression ratio $= \dfrac{P_3}{P_1} = \dfrac{P_3}{P_2} \times \dfrac{P_2}{P_1}$

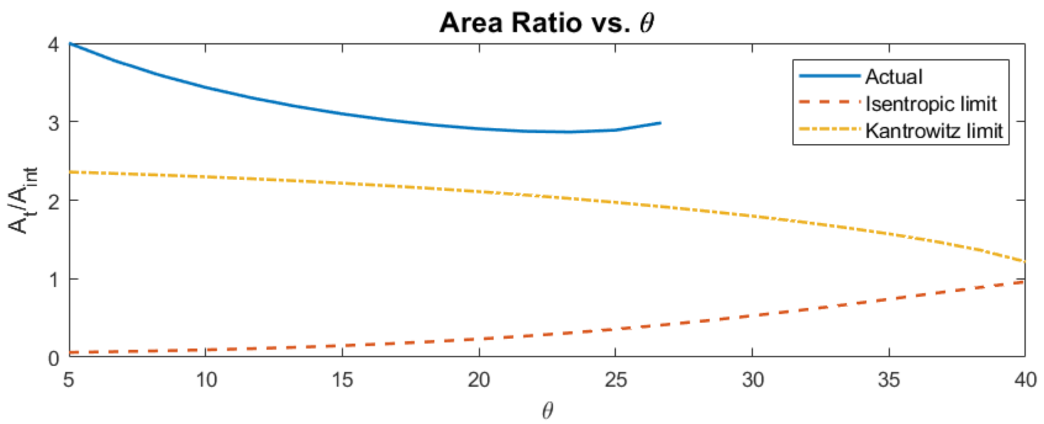
(•) Static temperature ratio $= \dfrac{T_3}{T_1} = \dfrac{T_3}{T_2} \times \dfrac{T_2}{T_1}$

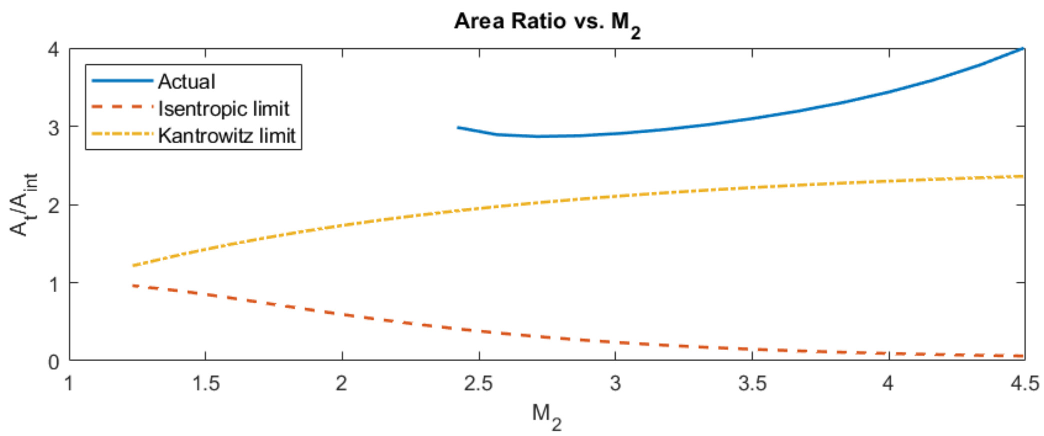
(•) Stagnation pressure ratio $= \dfrac{P_{03}}{P_{01}} = \dfrac{P_{03}}{P_3}\dfrac{P_3}{P_2}\dfrac{P_2}{P_1}\dfrac{P_1}{P_{01}}$

(•) Total length of intake $= \boxed{x_s = h_{iso}\left[\dfrac{\cos\beta_1 \sin\beta_2}{\sin(\beta_2-\theta)} + \cot(\beta_2-\theta)\right]}$

PROBLEM 3


Density Ratio vs. θ
Density Ratio
ρ3/ρ1
Temperature Ratio vs. θ
Temperature Ratio
T3/T1
Pressure Ratio vs. θ
Pressure Ratio
P03/P01
θ
Intake Length vs. θ
Intake Length
Lintake = xs (m)


Area Ratio vs. θ
Actual
Isentropic limit
Kantrowitz limit
At/Aint
θ


Area Ratio vs. M2
Actual
Isentropic limit
Kantrowitz limit
At/Aint
M2

Based on our observations, we noticed that as theta increases, the compression and static temperature ratios also increase. However, the stagnation pressure ratio and intake length decrease as theta increases.

We also found that the proposed design for the intake is self-starting, as the actual curve in the area ratio plots is above the Kantrowitz limit. However, this design, which includes two consecutive oblique shock waves (shock on cowl lip and shock on-ramp shoulder criteria), only exists within a specific range of theta.

Interestingly, we observed that the 'Actual' condition curves for all plots terminate around theta = 27 degrees. This is because, as theta increases beyond this point, the M2 value also increases, and the 2nd shock is determined by both the M2 and theta values. When theta exceeds approximately 27 degrees, the 2nd oblique shock wave detaches, and the solution no longer exists.