July 19, 2024

Antithetic Variate Method

Goal

Apply the antithetic variate method to reduce the variance in the estimation of average sojourn times in a simulated tandem queue system.

Methodology

The approach involves manipulating the random number streams to create pairs of negatively correlated runs. This negative correlation is expected to reduce the variance of the estimated sojourn times.

Experiment Parameters

- Number of queues N: 6
- Arrival rate λ: Varied from 0.1 to 0.9 in steps of 0.1, 12 values such that $0 < \rho < 1$
- Service rate μ: Fixed at 1 for all the queues
- Number of simulations: 50 for each traffic intensity value, i.e. 50 pairs in antithetic variate calculations and 50 runs for independent variate calculation
- Total number of customers: 3000 per simulation
- Confidence interval: 95% using $z = 1.96$
- Random number generator: Expon class provided by professor which is built on Python's built-in random module
- Performance measures: Mean delay and variance of customers in the system

Using one code file for M/M/1 Tandem Queue function and related plotting function in Python. For each value of $\rho = \lambda/\mu$, perform 30 simulations to obtain reliable estimates. In each simulation, initialize the queueing system and simulate until 3000 customers have been served.

Results

The analysis of variance reduction and estimation of mean sojourn times under different simulation strategies is presented through Figures 1 and 2.

Variance of Sojourn Times

Figure 1 illustrates the comparison of sojourn time variances between antithetic variates and independent variates. The variance of sojourn times using antithetic variates is significantly lower, by approximately 60%, than that obtained using independent variates. This substantial reduction highlights the effectiveness of the antithetic variate method in reducing the variability of the estimated parameters in queue simulations.

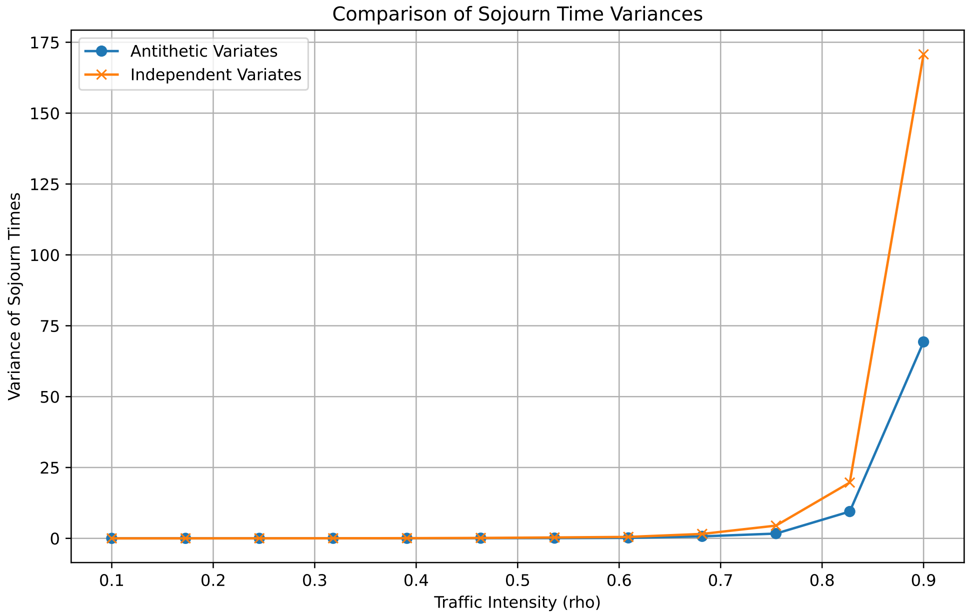

Figure 1: Comparison of sojourn time variances using antithetic and independent variates

Mean Sojourn Times

Figure 2 displays the comparison of mean sojourn times across different traffic intensities (ρ). Three curves are plotted: the analytically calculated sojourn times using the formula

$$\text{expected sojourn time} = \frac{\mu}{1-\rho}$$

the simulated results using independent variates, and those obtained with the antithetic variate adjustment. The results also include 95% confidence intervals for the simulations. The confidence intervals for the antithetic variate method are noticeably more compact, demonstrating not only the method's ability to reduce variance but also its potential to provide more precise and reliable estimates of system performance.

Conclusion

The results obtained from employing the antithetic variate method in simulating tandem queues demonstrate a significant reduction in the variance of the estimated sojourn times compared to independent variate simulations. A key benefit of this reduction is the consequent narrowing of the 95% confidence intervals around the mean sojourn times. Narrower confidence intervals imply a higher degree of precision in our estimates, which in turn leads to increased confidence in the reliability and accuracy of the simulation results.

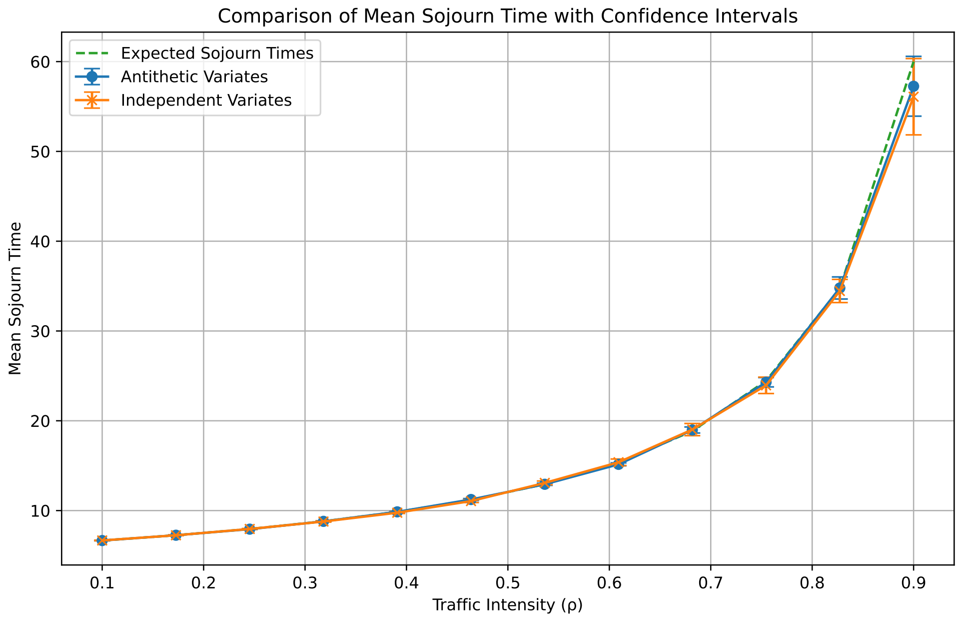

Figure 2: Comparison of mean sojourn times with confidence intervals against traffic intensity (ρ)

Control Variate Method

Goal

To investigate the application of the control variate method to similarly reduce the variance in estimating sojourn times.

Methodology

The control variate method involves using a known variable that is correlated with the output of interest to adjust estimations and reduce variance. In this simulation, the service time has been chosen as the control variate because of its direct correlation with the sojourn time, the primary output of interest. Adjustments to the estimators are systematically made based on the correlation between the sojourn times and the service times. This method capitalizes on the known properties of the service time to stabilize the estimation of sojourn times, effectively reducing the variability of the simulation results.

Experiment Parameters

- Number of queues N: 8
- Arrival rate λ: Varied from 0.1 to 0.9 in steps of 0.1, 10 values such that $0 < \rho < 1$
- Service rate μ: Fixed at 1 for all the queues
- Number of simulations: 50 for each traffic intensity (ρ) value
- Total number of customers: 3000 per simulation
- Confidence interval: 95% using $z = 1.96$

- Random number generator: Expon class provided by professor which is built on Python's built-in random module
- Performance measures: Mean delay and variance of customers in the system

Results

The evaluation of the control variate method on our simulation's accuracy and precision is illustrated through the analyses presented in Figure 3. This figure contains two crucial plots that demonstrate the method's effectiveness under different traffic intensities (ρ).

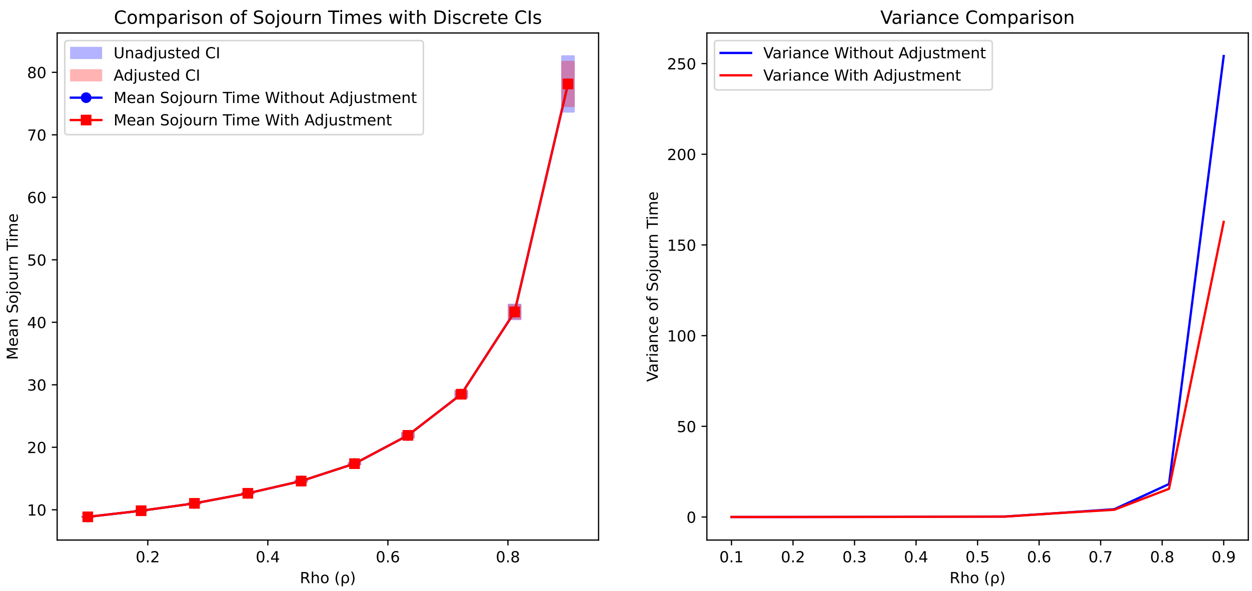

Figure 3: Comparative analysis of mean sojourn times and variance reduction using control variates against traffic intensity (ρ)

Mean Sojourn Times Comparison

The first plot in Figure 3 compares the mean sojourn times against traffic intensity for two different cases: one using the control variate method and one without it. Remarkably, the plots of mean delays for both studies lie exactly on top of each other, which confirms that the mean estimates are consistent and accurate, affirming that the control variate method does not alter the expected value of the output but stabilizes its estimation.

Variance Reduction Analysis

The second plot in Figure 3 displays the variance of sojourn times as a function of traffic intensity for both the controlled and uncontrolled variate cases. This visualization clearly shows that the variance for the control variate method is consistently lower than that for the simulations without the control variate, particularly at higher traffic intensities. The reduction in variance is quantified at approximately 40%, which underscores the significant enhancement in precision provided by incorporating the control variate.

Conclusion

This analysis demonstrates that while the control variate method maintains the integrity of mean estimates, it significantly tightens the confidence intervals and reduces the variance of the estimates. Such enhancements are particularly beneficial in high-traffic scenarios where precision in system performance predictions is crucial for effective planning and operational management.