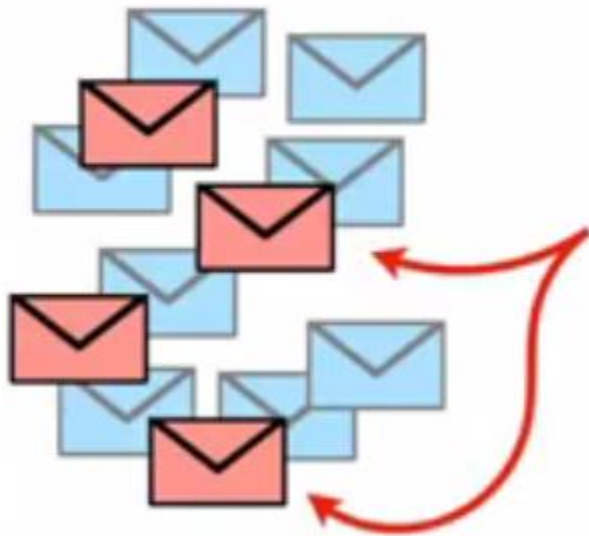


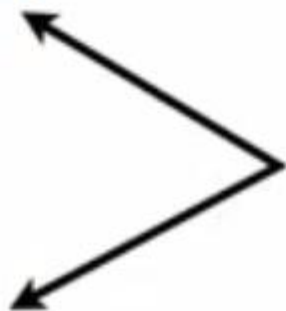
Naive Bayes



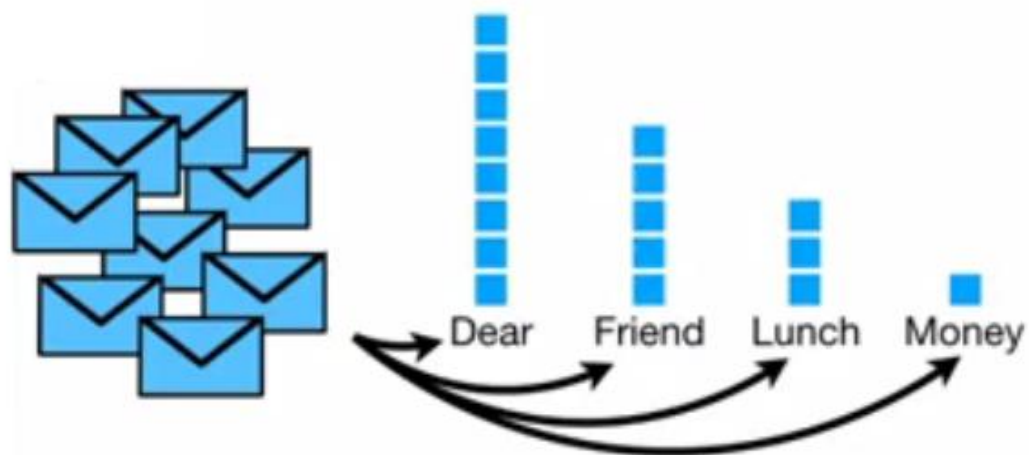
Imagine we received
normal messages from
friends and family...



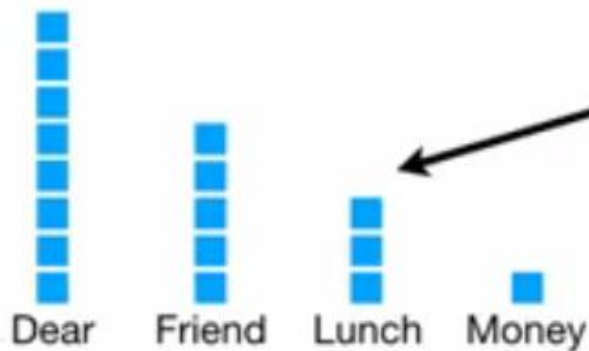
...and we also received
spam (unwanted
messages that are usually
scams or unsolicited
advertisements)...



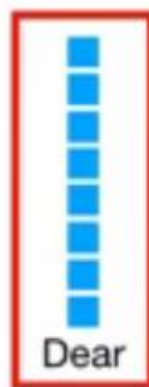
...and we wanted to filter out the **spam** messages.



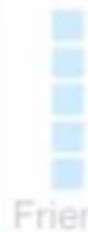
histogram of all the words that occur in the **normal messages** from friends and family.



We can use the histogram to calculate the probabilities of seeing each word, given that it was in a **normal message**.



Dear



Friend



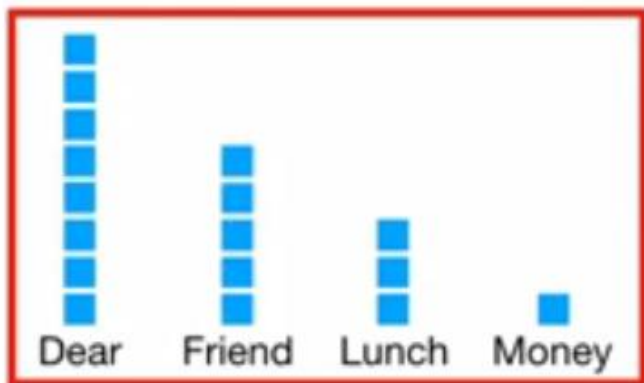
Lunch



Money

For example, the probability we see the word "**Dear**"...

$$p(\text{Dear})$$



Dear

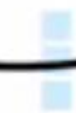
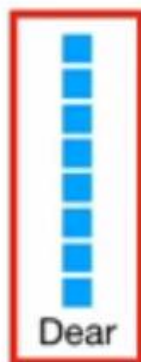
Friend

Lunch

Money

...given that we saw it in a **normal message**...

$$p(\text{Dear} \mid \text{Normal})$$



Friend

Lunch

Money

8, the total number of times **Dear** occurred in **normal messages**...

$$p(\text{Dear} | \text{Normal}) = \frac{8}{17}$$

divided by **17**, the total number of words in all of the **normal messages**.



$$p(\text{Dear} | \text{N}) = 0.47$$



Dear



Friend



Lunch



Money

$$p(\text{Dear} | \text{Normal}) = \frac{8}{17} = 0.47$$



$$p(\text{Dear} | N) = 0.47$$



Dear

$$p(\text{Friend} | N) = 0.29$$



Friend



Lunch



Money

$$p(\text{Friend} | \text{Normal}) = \frac{5}{17} = 0.29$$



$$p(\text{Dear} | N) = 0.47$$



Dear

$$p(\text{Friend} | N) = 0.29$$



Friend

$$p(\text{Lunch} | N) = 0.18$$



Lunch

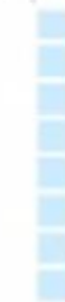


Money

$$p(\text{Lunch} | \text{Normal}) = \frac{3}{17} = 0.18$$



$$p(\text{Dear} | N) = 0.47$$



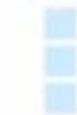
Dear

$$p(\text{Friend} | N) = 0.29$$



Friend

$$p(\text{Lunch} | N) = 0.18$$



Lunch

$$p(\text{Money} | N) = 0.06$$



Money

$$p(\text{Money} | \text{Normal}) = \frac{1}{17} = 0.06$$



$$p(\text{Dear} | N) = 0.47$$



Dear

$$p(\text{Friend} | N) = 0.29$$



Friend

$$p(\text{Lunch} | N) = 0.18$$



Lunch

$$p(\text{Money} | N)$$



Money

Now we make a **histogram** of all the words that occur in the **spam**...



Dear



Friend

Lunch



Money




$$p(\text{Dear} | N) = 0.47$$



Dear

$$p(\text{Friend} | N) = 0.29$$



Friend

$$p(\text{Lunch} | N) = 0.18$$



Lunch

$$p(\text{Money} | N) = 0$$



Money

0.29.

$$p(\text{Dear} | S) = 0.29$$



Dear



Friend



Lunch



Money

$$p(\text{Dear} | \text{Spam}) = \frac{2}{7} = 0.29$$



$$p(\text{Dear} | N) = 0.47$$



Dear

$$p(\text{Friend} | N) = 0.29$$



Friend

$$p(\text{Lunch} | N) = 0.18$$



Lunch

$$p(\text{Money} | N)$$



Money

Likewise, we calculate the probability of seeing the remaining words, given that they were in the **spam**.



$$p(\text{Dear} | S) = 0.29$$



Dear

$$p(\text{Friend} | S) = 0.14$$



Friend

$$p(\text{Lunch} | S) = 0$$



Lunch



Money

$$p(\text{Money} | S) = 0.57$$

$$p(\text{Money} | \text{Spam}) = \frac{4}{7} = 0.57$$



$$p(\text{Dear} \mid \text{N}) = 0.47$$

$$p(\text{Friend} \mid \text{N}) = 0.29$$

$$p(\text{Lunch} \mid \text{N}) = 0.18$$

$$p(\text{Money} \mid \text{N}) = 0.06$$



$$p(\text{Dear} \mid \text{S}) = 0.29$$

$$p(\text{Friend} \mid \text{S}) = 0.14$$

$$p(\text{Lunch} \mid \text{S}) = 0.00$$

$$p(\text{Money} \mid \text{S}) = 0.57$$

We Recieve a new Message

Dear Friend



$$\begin{aligned}p(\text{Dear} \mid \text{N}) &= 0.47 \\p(\text{Friend} \mid \text{N}) &= 0.29 \\p(\text{Lunch} \mid \text{N}) &= 0.18 \\p(\text{Money} \mid \text{N}) &= 0.06\end{aligned}$$

?

?

And we want to
decide if is a **normal**
message or **spam**.



$$\begin{aligned}p(\text{Dear} \mid \text{S}) &= 0.29 \\p(\text{Friend} \mid \text{S}) &= 0.14 \\p(\text{Lunch} \mid \text{S}) &= 0.00 \\p(\text{Money} \mid \text{S}) &= 0.57\end{aligned}$$



$p(\text{Dear} | N) = 0.47$
 $p(\text{Friend} | N) = 0.29$
 $p(\text{Lunch} | N) = 0.18$
 $p(\text{Money} | N) = 0.06$

Dear Friend

$$p(N) = \frac{8}{8 + 4} = 0.67$$



$p(\text{Dear} | S) = 0.29$
 $p(\text{Friend} | S) = 0.14$
 $p(\text{Lunch} | S) = 0.00$
 $p(\text{Money} | S) = 0.57$

since **8** of
the **12** messages are
normal messages, our
initial guess will be **0.67**.

Dear Friend



$$p(\text{N}) = 0.67$$

$$\begin{aligned}p(\text{Dear} \mid \text{N}) &= 0.47 \\p(\text{Friend} \mid \text{N}) &= 0.29 \\p(\text{Lunch} \mid \text{N}) &= 0.18 \\p(\text{Money} \mid \text{N}) &= 0.06\end{aligned}$$

$$p(\text{N}) = \frac{8}{8 + 4} = 0.67$$



$$\begin{aligned}p(\text{Dear} \mid \text{S}) &= 0.29 \\p(\text{Friend} \mid \text{S}) &= 0.14 \\p(\text{Lunch} \mid \text{S}) &= 0.00 \\p(\text{Money} \mid \text{S}) &= 0.57\end{aligned}$$



$$p(\text{N}) = 0.67$$

$$p(\text{Dear} \mid \text{N}) = 0.47$$

$$p(\text{Friend} \mid \text{N}) = 0.29$$

$$p(\text{Lunch} \mid \text{N}) = 0.18$$

$$p(\text{Money} \mid \text{N}) = 0.06$$

Now we multiply that initial guess by probability that the word **Dear** occurs in a **normal message**...

...and the probability that the word **Friend** occurs in a **normal message**.

$$p(\text{N}) \times p(\text{Dear} \mid \text{N}) \times p(\text{Friend} \mid \text{N})$$



$$p(\text{Dear} \mid \text{S}) = 0.29$$

$$p(\text{Friend} \mid \text{S}) = 0.14$$

$$p(\text{Lunch} \mid \text{S}) = 0.00$$

$$p(\text{Money} \mid \text{S}) = 0.57$$



$$p(\text{N}) = 0.67$$

$$\begin{aligned}p(\text{Dear} \mid \text{N}) &= 0.47 \\p(\text{Friend} \mid \text{N}) &= 0.29 \\p(\text{Lunch} \mid \text{N}) &= 0.18 \\p(\text{Money} \mid \text{N}) &= 0.06\end{aligned}$$

Now we multiply that initial guess by probability that the word **Dear** occurs in a **normal message**...

...and the probability that the word **Friend** occurs in a **normal message**.

$$p(\text{N}) \times p(\text{Dear} \mid \text{N}) \times p(\text{Friend} \mid \text{N})$$

$$0.67 \times 0.47 \times 0.29$$



$$\begin{aligned}p(\text{Dear} \mid \text{S}) &= 0.29 \\p(\text{Friend} \mid \text{S}) &= 0.14 \\p(\text{Lunch} \mid \text{S}) &= 0.00 \\p(\text{Money} \mid \text{S}) &= 0.57\end{aligned}$$



$$p(N) = 0.67$$

$$p(\text{Dear} | N) = 0.47$$

$$p(\text{Friend} | N) = 0.29$$

$$p(\text{Lunch} | N) = 0.18$$

$$p(\text{Money} | N) = 0.06$$



$$p(\text{Dear} | S) = 0.29$$

$$p(\text{Friend} | S) = 0.14$$

$$p(\text{Lunch} | S) = 0.00$$

$$p(\text{Money} | S) = 0.57$$

Now we multiply that initial guess by probability that the word **Dear** occurs in a **normal message**...

...and the probability that the word **Friend** occurs in a **normal message**.

$$p(N) \times p(\text{Dear} | N) \times p(\text{Friend} | N)$$

$$0.67 \times 0.47 \times 0.29 = 0.09 \quad p(N | \text{Dear Friend})$$

In a simple way, we can think of **0.09** as the score that **Dear Friend** gets if it is a **Normal Message**.

$$p(\text{N} \mid \text{Dear Friend}) \propto 0.09$$

Dear Friend



$$p(\text{N}) = 0.67$$

$$\begin{aligned} p(\text{Dear} \mid \text{N}) &= 0.47 \\ p(\text{Friend} \mid \text{N}) &= 0.29 \\ p(\text{Lunch} \mid \text{N}) &= 0.18 \\ p(\text{Money} \mid \text{N}) &= 0.06 \end{aligned}$$



$$\begin{aligned} p(\text{Dear} \mid \text{S}) &= 0.29 \\ p(\text{Friend} \mid \text{S}) &= 0.14 \\ p(\text{Lunch} \mid \text{S}) &= 0.00 \\ p(\text{Money} \mid \text{S}) &= 0.57 \end{aligned}$$

$$p(\text{S}) = \frac{4}{4 + 8} = 0.33$$

$$p(\text{N} \mid \text{Dear Friend}) \propto 0.09$$

Dear Friend



$$p(\text{N}) = 0.67$$

$$\begin{aligned} p(\text{Dear} \mid \text{N}) &= 0.47 \\ p(\text{Friend} \mid \text{N}) &= 0.29 \\ p(\text{Lunch} \mid \text{N}) &= 0.18 \\ p(\text{Money} \mid \text{N}) &= 0.06 \end{aligned}$$



$$p(\text{S}) = 0.33$$

$$\begin{aligned} p(\text{Dear} \mid \text{S}) &= 0.29 \\ p(\text{Friend} \mid \text{S}) &= 0.14 \\ p(\text{Lunch} \mid \text{S}) &= 0.50 \\ p(\text{Money} \mid \text{S}) &= 0.57 \end{aligned}$$

$$p(\text{S}) \times p(\text{Dear} \mid \text{S}) \times p(\text{Friend} \mid \text{S})$$

$$p(\text{N} \mid \text{Dear Friend}) \propto 0.09$$

Dear Friend



$$p(\text{N}) = 0.67$$

$$\begin{aligned} p(\text{Dear} \mid \text{N}) &= 0.47 \\ p(\text{Friend} \mid \text{N}) &= 0.29 \\ p(\text{Lunch} \mid \text{N}) &= 0.18 \\ p(\text{Money} \mid \text{N}) &= 0.06 \end{aligned}$$

$$0.33 \times 0.29 \times 0.14 = 0.01 \propto p(\text{S} \mid \text{Dear Friend})$$



$$p(\text{S}) = 0.33$$

$$\begin{aligned} p(\text{Dear} \mid \text{S}) &= 0.29 \\ p(\text{Friend} \mid \text{S}) &= 0.14 \\ p(\text{Lunch} \mid \text{S}) &= 0.00 \\ p(\text{Money} \mid \text{S}) &= 0.57 \end{aligned}$$

$$p(N | \text{Dear Friend}) \propto 0.09$$

Dear Friend

score we got

for **Normal Message**, **0.09**

...is greater than the score we
got for **Spam**, **0.01**...

$$0.33 \times 0.29 \times 0.14 = 0.01 \propto p(S | \text{Dear Friend})$$

Dear Friend



...we will decide that **Dear
Friend** is a **Normal Message**.



$$p(N) = 0.67$$

$$p(\text{Dear} | N) = 0.47$$

$$p(\text{Friend} | N) = 0.29$$

$$p(\text{Lunch} | N) = 0.18$$

$$p(\text{Money} | N) = 0.06$$



$$p(S) = 0.33$$

$$p(\text{Dear} | S) = 0.29$$

$$p(\text{Friend} | S) = 0.14$$

$$p(\text{Lunch} | S) = 0.00$$

$$p(\text{Money} | S) = 0.57$$



$$p(\text{N}) = 0.67$$

$$\begin{aligned}p(\text{Dear} \mid \text{N}) &= 0.47 \\p(\text{Friend} \mid \text{N}) &= 0.29 \\p(\text{Lunch} \mid \text{N}) &= 0.18 \\p(\text{Money} \mid \text{N}) &= 0.06\end{aligned}$$



$$p(\text{S}) = 0.33$$

$$\begin{aligned}p(\text{Dear} \mid \text{S}) &= 0.29 \\p(\text{Friend} \mid \text{S}) &= 0.14 \\p(\text{Lunch} \mid \text{S}) &= 0.00 \\p(\text{Money} \mid \text{S}) &= 0.57\end{aligned}$$

Dear Friend



we decided that **Dear Friend**
as a **normal message**
because **0.09** > **0.01**.

$$p(\text{N}) \times p(\text{Dear} \mid \text{N}) \times p(\text{Friend} \mid \text{N}) = 0.09$$

$$p(\text{S}) \times p(\text{Dear} \mid \text{S}) \times p(\text{Friend} \mid \text{S}) = 0.01$$

