Que. No.		Sub Q. No.							Total Marks
Examiner		Marks							
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प्र. क्र. Q. No.

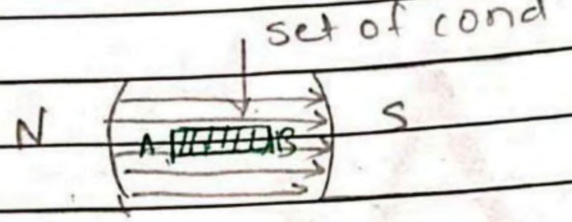
B]

AC FUNDAMENTALS.

Que. No.		Sub Q. No.								Total Marks
Examiner		Marks								
Moderator		Marks								

प्र. क्र. Q. No.

generation of 1ϕ AC



$$e = -N\frac{d\phi}{dt}$$

Anti clockwise - rotation.

velocity - ω rad/sec

	Sub Q. No.							Total Marks
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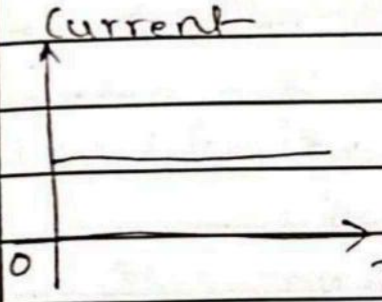
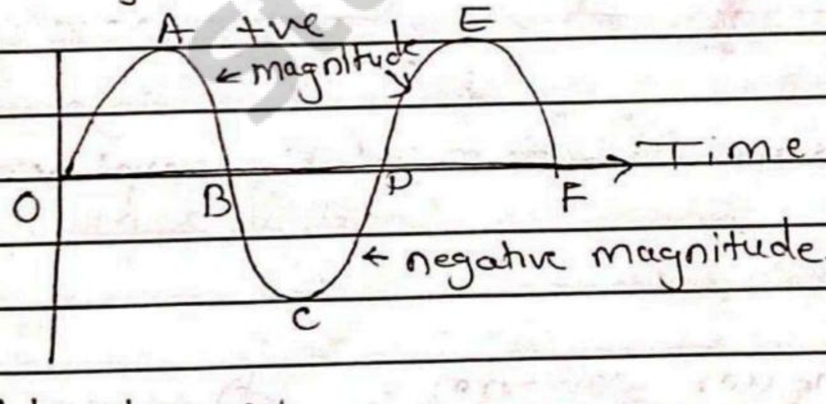
Concept of AC & DC Quantities

DC - Direct current has constant value (magnitude) with respect to time. Polarity (direction) do not change with time.



AC :

Alternating current is the current whose magnitude and direction change with the time. AC keep on alternating +ve & -ve during one cycle.



OAB - +ve half cycle

BCD - negative half cycle

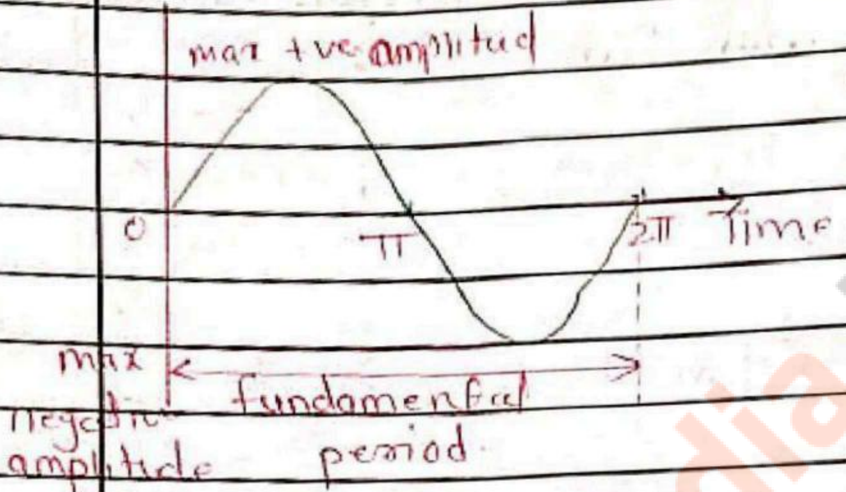
Advantages:

- Generation is easy and cheaper
- Using x'mer easily ↑ & ↓
- AC Generator have higher η than dc generator
- Negligible loss of energy during x'mission
- AC can be easily converted to DC. (rectifier)

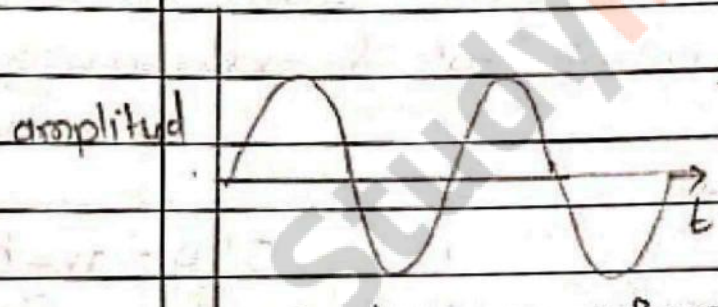
24	Que. No.		Sub Q. No.							Total Marks
	Examiner		Marks							
	Moderator		Marks							

प्र. क्र. Q. No.

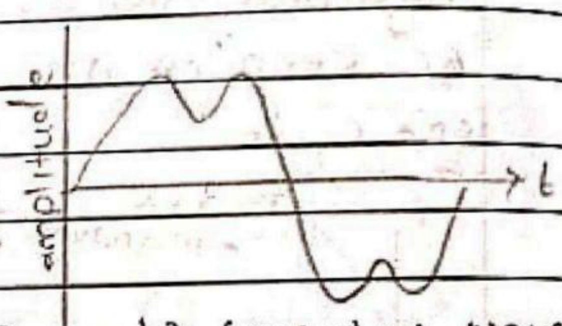
* Graphical Representation of sinusoidal voltage and current



* Diff. types of Waveform

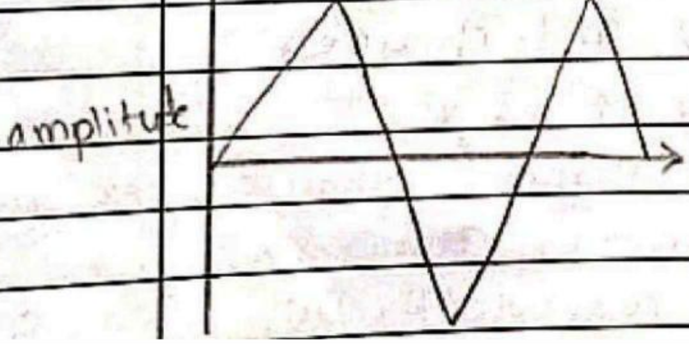


a) sine waveform

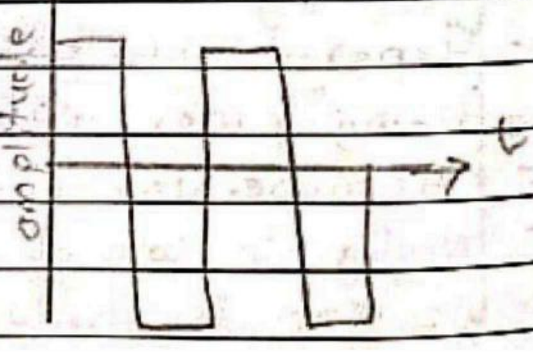


b) complex wavefo

c) Triangular waveform



d) square wavefo



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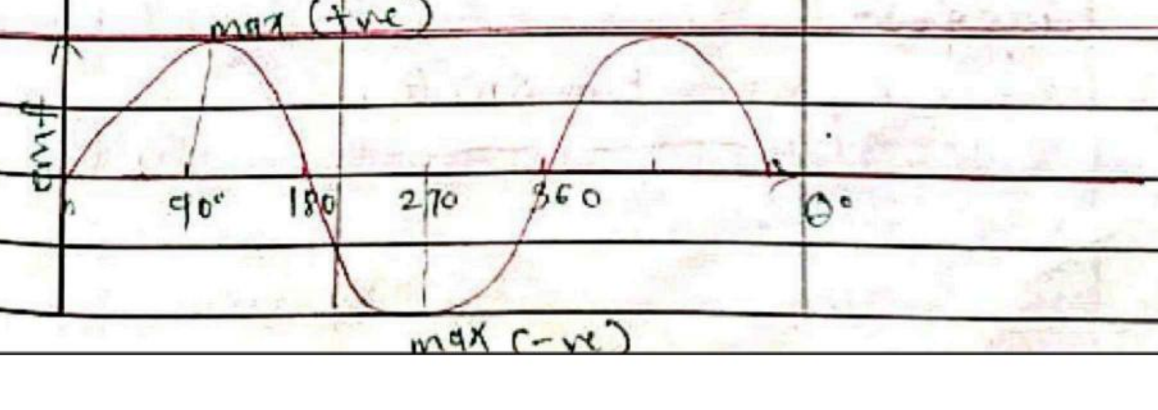
Examiner	Marks								
Moderator	Marks								

* Single phase Alternator:

* Generation of alternating voltage.

	Angle	Flux cutting	emf or current
i)	$\theta = 0°$	No flux cutting	zero
ii)	$\theta = 0$ to $90°$	flux cutting ↑ from 0 to maximum	emf ↑ from 0 to maximum.
iii)	$\theta = 90°$	max. flux cutting	max. +ve value
iv)	$\theta = 90°$ to $180°$	flux cutting ↓ reduced from max to 0	emf start reducing from max to 0.
v)	$\theta = 180°$	No flux cutting	zero
vi)	$\theta = 180°$ to $270°$	flux cutting in reverse direction	emf ↑ in reverse direction
vii)	$\theta = 270°$	max. flux cutting in reverse direction	max negative emf
viii)	$\theta = 270°$ to $360°$	flux cutting reduce to zero	emf reduces to zero
ix)	$\theta = 360°$	No flux cutting	zero.



26	Que. No.	Sub Q. No.							Total Marks
	Examiner	Marks							
	Moderator	Marks							

प्र. क्र. Q. No.

* Equations of Alternating Voltage and current.

1) While generating a sinusoidal vltg (emf) a condr is rotated in mag. field. Here θ is an angle thr which condr is rotated. According to faraday's law, electromagnetic induction, generated emf is given by,

$$e = B \cdot l \cdot v \sin\theta \quad \text{— (1)}$$

B = flux density of mag. field.
l = length of conductor
V = Velocity of rotation of coil.

we discussed, max emf is generated when angle θ is 90°. The max (peak) emf is denoted by Em

$$E_m = B l v \sin(90^\circ) \quad \text{— 2}$$

$\sin 90^\circ = 1$

$$E_m = B l V \quad \text{— (2)}$$

put value of (2) in (1)
we get

$$\boxed{e = E_m \sin\theta}$$

	Sub Q. No.						Total Marks
miner	Marks						
erator	Marks						

* Different forms of voltage equation.

θ is angle thr which condr is rotated. It can be expressed in terms of angular ~~(ω) as~~, frequency (ω), as

$$\theta = \omega t$$

$$e = E_m \sin\theta$$

$$e = E_m \sin\omega t \quad \text{--- (3)}$$

- If the coil angle is ϕ then eqn of voltage is

$$e = E_m \sin(\omega t + \phi)$$

This eqn in terms of max. voltage and coil angle.

angular frequency (ω) = $2\pi f$

$$\therefore e = E_m \sin(2\pi f t) \quad \text{--- (4)}$$

This is equation in term of frequency

- The frequency is reciprocal of timeperiod.

$$f = \frac{1}{T}$$

$$\therefore e = E_m \sin\left(\frac{2\pi t}{T}\right)$$

This is equation in term of period.

Examiner	Marks							Total Marks
Moderator	Marks							

5

Instantaneous value:
The value (magnitude) of alternating quantity at a particular time instant. is called as instantaneous value.

Cycle
Each repetition of one positive and one negative portion of waveform is called as cycle.

Period (T)
The time taken by alternating quantity to complete one cycle is called period of waveform.

Amplitude:
The max. value attained by alternating quantity is called as amplitude.
($-V_m$ & $+V_m$)

Frequency: (f)
The no. of cycles completed by alternating quantity per second, is called as frequency.

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

6

Que. No.		Sub Q. No.								Total Marks
Examiner		Marks								
Moderator		Marks								

प्र. क्र. Q. No.

Angular frequency / velocity (ω)

Angular frequency or angular velocity is frequency expressed in electrical radian per second

$$\omega = 2\pi f \ \text{rad/sec}$$

* Peak to peak value

The voltage measured from max. negative level ($-V_m$) to maximum positive voltage level (V_m) is called peak to peak voltage.

(It helpful to decide safe operating region of the device.

* Effective or RMS value

RMS value of ac current is equal to the d.c current flowing thr given circuit for a given time, producing same amount of heat a produced by ac current flowing thr the same ckt & for the same time

* The square root of mean value of squares of alternating quantity over one cycle.

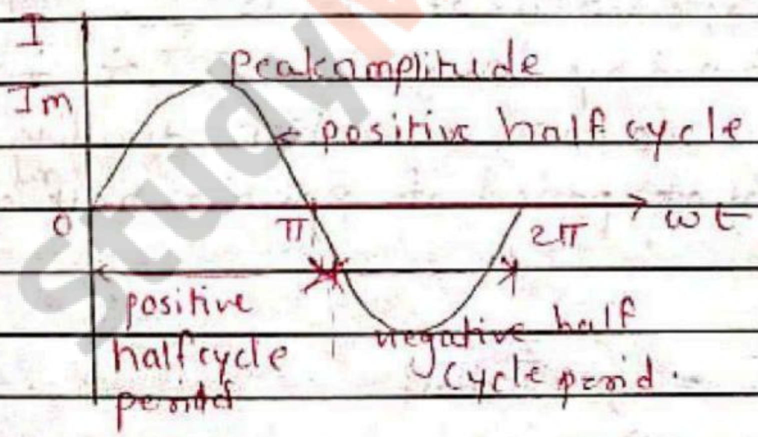
Que. No.	Sub Q. No.							Total Marks
Examiner	Marks							
Moderator	Marks							

प्र. क्र. Q. No.

* RMS value in terms of peak value (Derivation)

Step i) Draw a waveform & write math. eq^n
ii) Cal. square of current I.
iii) Cal. average value of square equation of step II.
iv) Cal. square root of average value. It gives rms value.

step i) Draw a wave form & write mathematical equation.



$$I = I_m \sin \omega t \quad \text{—} \ (1)$$

↑ peak amplitude, ω angular velocity, t ← Time

step II. Cal. the square of current
taking square of eq^n (1)

$$I^2 = I_m^2 \sin^2 \omega t$$

24. Que. No.	Sub Q. No.							Total Marks
Examiner	Marks							
Moderator	Marks							

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$$\sin^2\omega t = \frac{1-\cos 2\omega t}{2}$$ (mathematically)

$$I^2 = I_m^2\left[\frac{1-\cos 2\omega t}{2}\right]$$

$$\boxed{I^2 = \frac{I_m^2}{2}\left[1-\cos 2\omega t\right]} \quad \text{--- (2)}$$

III] Cal. average value (I^2_{av})

average value of I^2 is I^2_{av} It is obtained by integrating I^2 for +ve half cycle (0 to π) and then dividing it by period of period of positive half cycle (0 to π)

$$I^2_{av} = \frac{\int_0^{\pi} I^2 \, d\omega t}{\pi}$$

$$= \frac{\int_0^{\pi} \frac{I_m^2}{2}\left(1-\cos(2\omega t)\right] d\omega t}{\pi}$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} 1-\cos(2\omega t)\, d\omega t$$

Sub Q. No.							Total Marks
Examiner Marks							
Moderator Marks							

25

$$= \frac{I_m^2}{2\pi}\left[\int_0^{\pi} 1\, d\omega t - \int_0^{\pi} \cos 2\omega t\, d\omega t\right]$$

$$= \frac{I_m^2}{2\pi}\left[(\omega t)_0^{\pi} - \left[\frac{\sin 2\omega t}{2}\right]_0^{\pi}\right]$$

$$= \frac{I_m^2}{2\pi}\left[(\pi - 0) - \left[\frac{\sin 2\pi - \sin 0}{2}\right]\right]$$

$$= \frac{I_m^2}{2\pi}\left[[\pi - 0] - \frac{(\sin(2\pi) - \sin(0))}{2}\right]$$

$$\boxed{I_{av} = \frac{I_m^2}{2}}$$

iv) Step 4) Cal. RMS value

Rms value of current is denoted by Irms
It is obtained by taking square root of I^2

$$\therefore I_{rms} = \sqrt{\frac{I_m^2}{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I_{rms} = 0.707\, I_m}$$

26	Que. No.	Sub Q. No.							Total Marks
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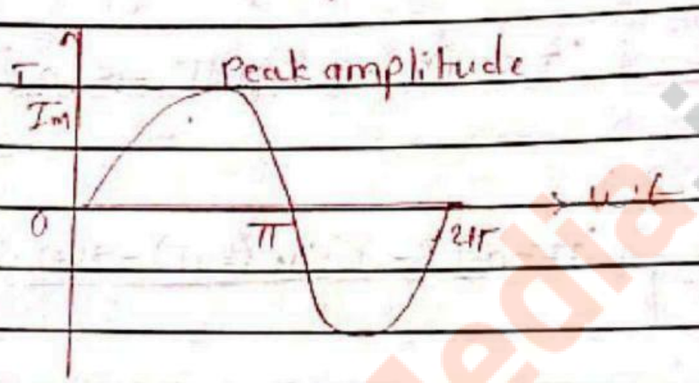
	Moderator	Marks							

प्र. क्र. Q. No.

Derivation of Average value in term of peak value

→ 1) Draw a waveform & write math. eqⁿ.
2) Cal. average value

1)



$$I = I_m \sin\omega t$$

2) Cal. average value

It is obtained by integrating +ve half & then deviding it by +ve half cycle.

$$I_{av} = \int_0^{\pi} I \cdot d\omega t$$

$$= \frac{\int_0^{\pi} I_m \sin\omega t \, d\omega t}{\pi}$$

Q. No.	Sub Q. No.							Total Marks
Examiner	Marks							
Moderator	Marks							

$$I_{av} = \frac{I_m}{\pi}\int_0^{\pi} \sin\omega t \, d\omega t$$

$$= \frac{I_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$I_{av} = \frac{I_m}{\pi}\left[-\cos(\pi) - (-\cos(0))\right]$$

$$I_{av} = \frac{I_m}{\pi}\left[1+1\right]$$

$$I_{av} = \frac{2I_m}{\pi}$$

$$\boxed{I_{av} = 0.637 I_m}$$

* Form Factor

The ratio of rms value to average value of an alternating quantity is called form factor.

$$\boxed{K_f = \frac{\text{rms value}}{\text{average value}}}$$

For sin / cosine quantity

$$\boxed{K_f = \frac{0.707 I_m}{0.637 I_m}}$$

$$K_f = 1.11$$

4

Que. No.		Sub Q. No.							Total Marks
Examiner		Marks							
Moderator		Marks							

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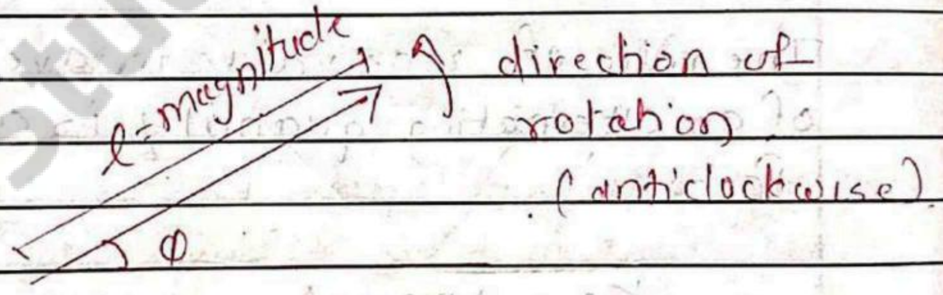
* Peak factor or Crest factor (k_p)

The ratio of max (peak) value to the rms value of an alternating quantity is called crest factor.

$$\boxed{K_p = \frac{\text{Peak value}}{\text{rms value}}}$$ Basic eq^n

$$\boxed{k_p = \frac{I_m}{0.707 I_m} = 1.414}$$ for sinusoidal quantity.

* Phasor representation of sinusoidal quantities



phasor representation is a simple method to represent alternating quantities such as sine wave, cosine wave.

$I_m = \sqrt{2}\, I_{rms}$

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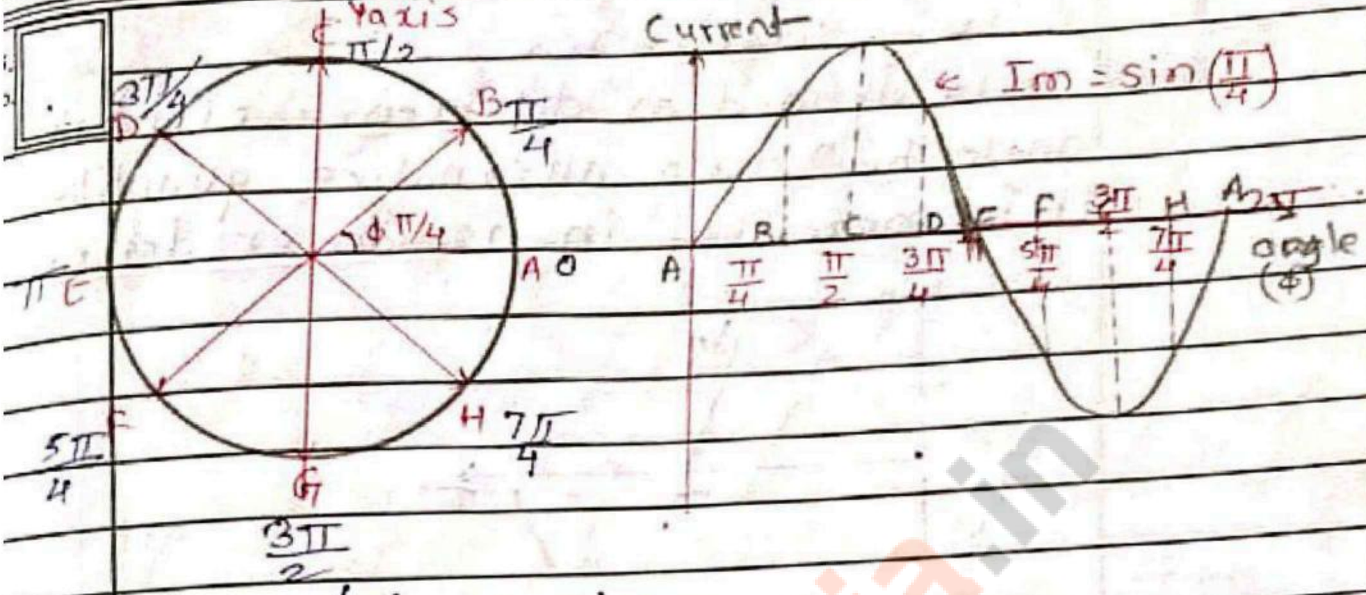
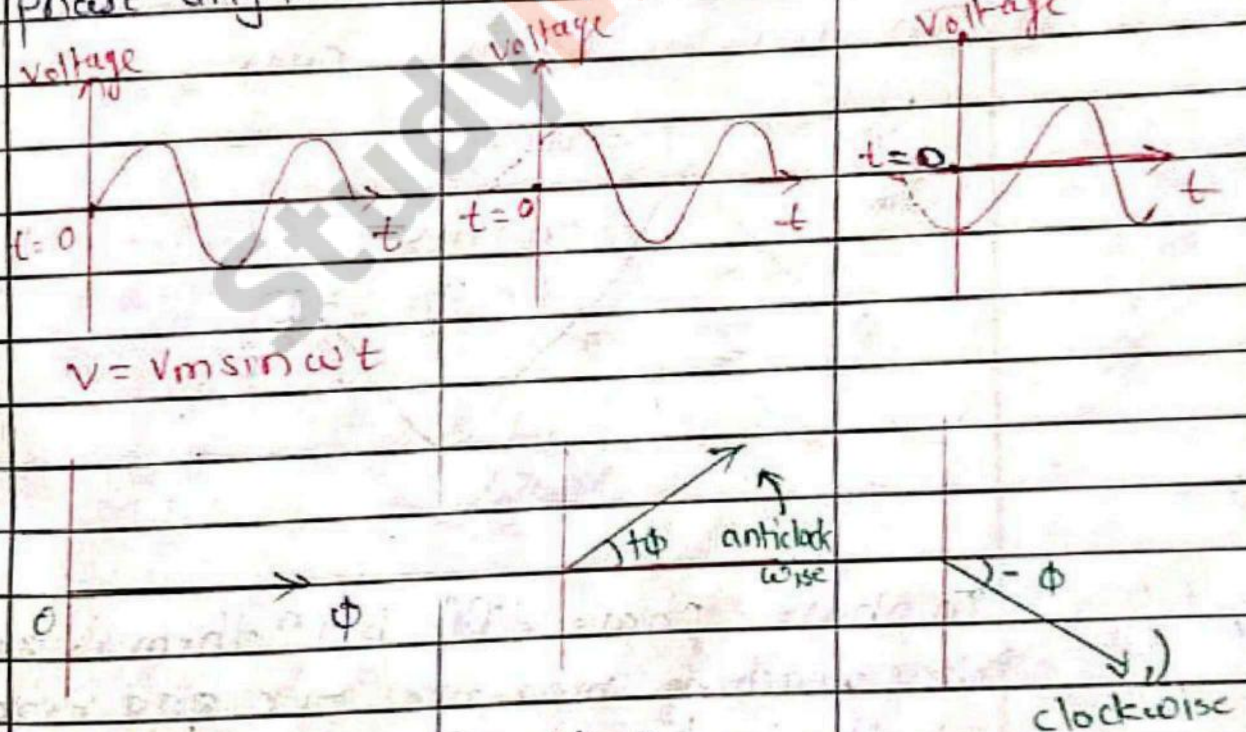


* Phase / phase angle

A position of phasor at a particular instant is expressed in term phase or phase angle.



a) zero phase

b) positive phase

c) Negative phase

6

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Examiner		Marks							
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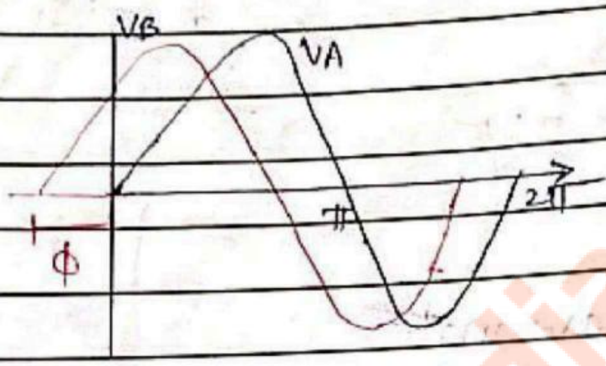
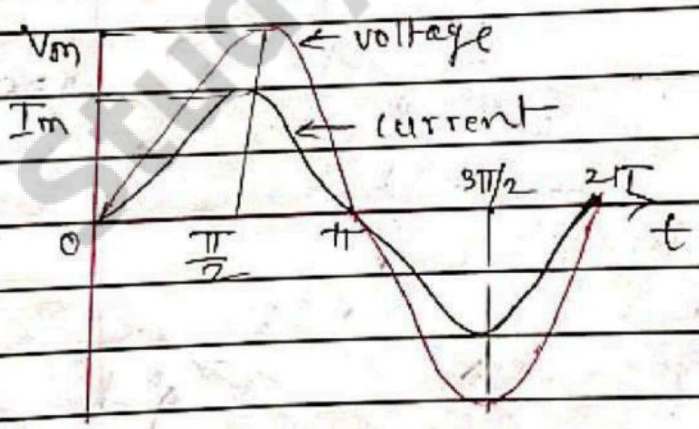
Phase difference

It is defined as difference of phase angle betⁿ two alternating quantities. It is measured in radian or degree.



* Type of phase difference

i) In phase or zero phase



In phase - phase diff. betⁿ them is zero. They reaching max +ve, -ve and zero value at the same time.

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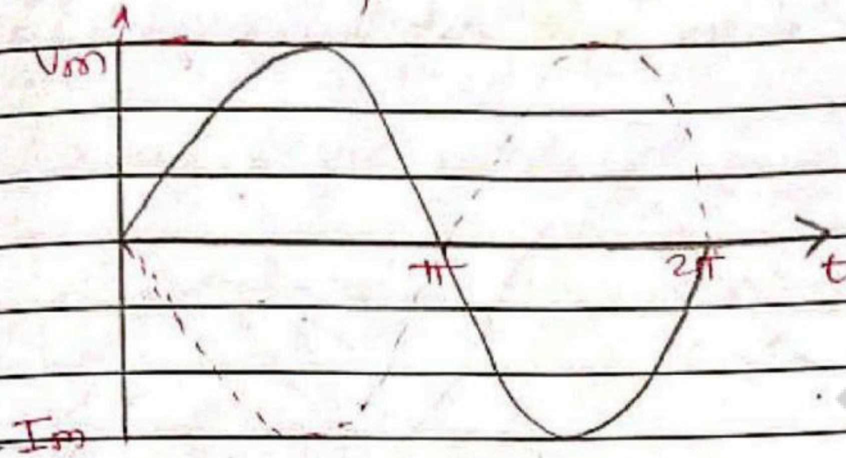
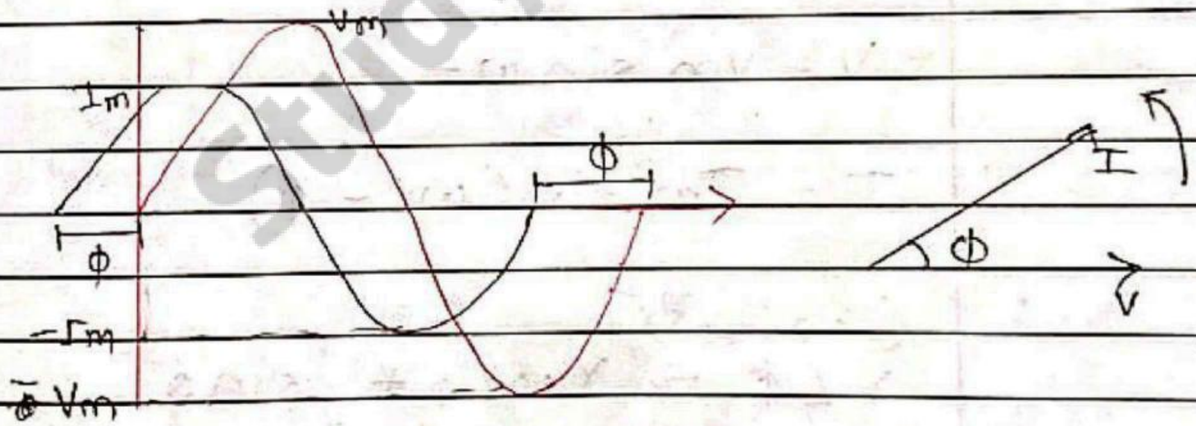
Moderator	Marks							

2) Out of phase



- They are said to out of phase when phase diff. is 180°

3) leading phase



- Current waveform reaches its max. min & zero value prior to voltage waveform So these two having leading phase diff.

$V = V_m \sin \omega t$

$I = I_m \sin(\omega t + \phi)$

I lead to v by angle ϕ

24

Que. No.		Sub Q. No.							Total Marks
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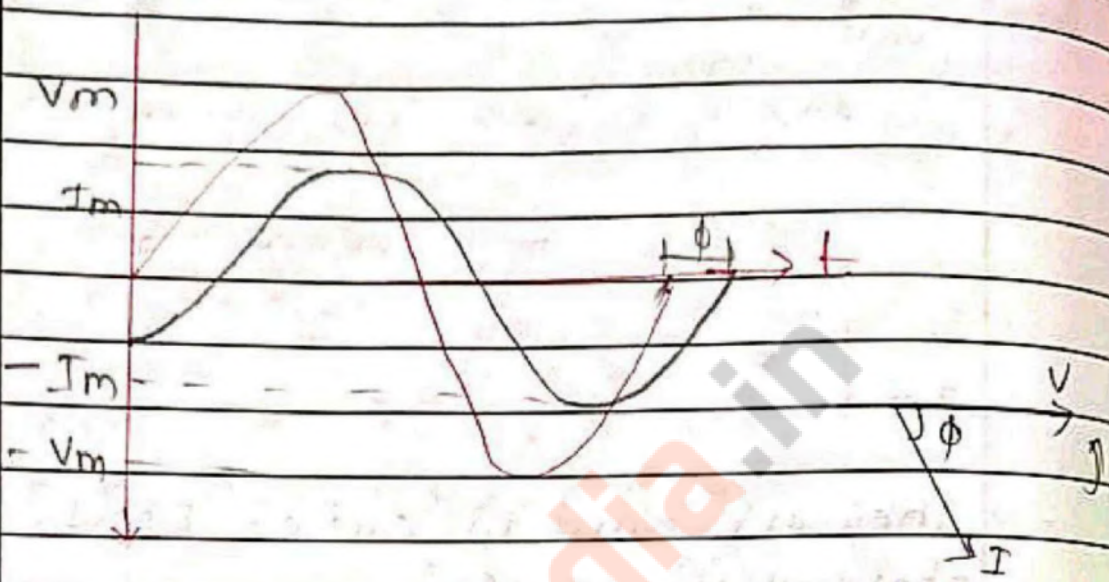
4) Lagging phase



- Here current waveform reaches it max +ve, -ve & zero value later than voltage waveform.

$V = V_m \sin \omega t$

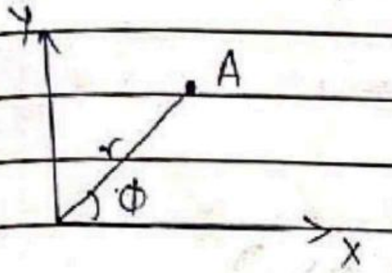
$I = I_m \sin(\omega t - \phi)$

Conversion
$r\angle\phi \rightarrow r\cos\phi + j\sin\phi$
polar $\rightarrow$ rectangular

$x + jy \Rightarrow r\angle\phi = \sqrt{x^2+y^2}\ \angle \tan^{-1}\frac{y}{x}$

* Rectangular & Polar representation of phasor :- Basically phasor contain magnitude & angle. Two methods to represent phasor.

1] Polar Representation.

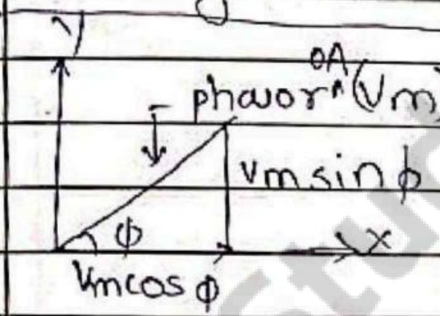


$v = V_m \sin(\omega t + \phi)$

$V_m = r.$

$v = r \angle \phi$

2) Rectangular Representation.



$v(t) = x + jy$

Here

$x = V_m \cos\phi$, $y = V_m \sin\phi$

$V_t = V_m \cos\phi + j V_m \sin\phi$

e.g. $i = \underbrace{5}_{I_m (r)} \sin(\underbrace{100\pi t}_{\omega t} + \underbrace{30^\circ}_{\phi})$

polar representation $= r \angle \phi = 5 \angle 30^\circ$

Rectangular representation

$= r\cos\phi + j\, r\sin\phi$

$= 5\cos 30^\circ + j\, 5\sin 30^\circ$

$= 4.33 + j2.5$

26	Que. No.		Sub Q. No.							Total Marks
	Examiner		Marks							
	Moderator		Marks							

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* Addition & Subtraction of phasor

- alway is rectangular form.

$A = x_1 + jy_1$
$B = x_2 + jy_2$

$A + B = (x_1 + x_2) + j(y_1 + y_2)$
$A - B = (x_1 - x_2) + j(y_1 - y_2)$

* Multiplication & Division of phasor

- alway in polar form.

$A_1 = r_1 \angle \phi_1$
$B = r_2 \angle \phi_2$

$A \times B = r_1 \times r_2 \angle (\phi_1 + \phi_2)$

$\frac{A}{B} = \frac{r_1}{r_2} \angle (\phi_1 - \phi_2)$

	Sub Q. No.							Total Marks
er	Marks							
or	Marks							

Cal.

mode - 2 complx

polar to Rect

Rect to polar.

fX 991 ES

AC Fundamental (Numericals)

1) A 50 Hz sinusoidal voltage has rms value of 200V. At t = 0, the instantaneous value is +ve and half of its max. value. Write down the expression for voltage and sketch the waveform.

→ Given,

$F = 50 Hz.$

$V_{rms} = 200V.$

$V_m = \sqrt{2} V_{rms} = \sqrt{2} \times 200 = 282.84 V.$

Expression for instantaneous voltage

$$V(t) = V_m \sin(\omega t + \phi) \quad \text{— (1)}$$

To cal. ϕ, we will consider vltg at $t=0$.

At $t = 0$

$$V(t) = \frac{V_m}{2} = \frac{282.84}{2} = 141.42 V.$$

put this value in eq (1) at $t=0$.

$$141.42 = 282.84 \sin(0 + \phi)$$

$$\sin\phi = \frac{141.42}{282.84} = 0.50$$

$$\phi = \sin^{-1}(0.5) = \frac{\pi}{6} \ (30°) \text{ (from table)}$$

26	Que. No.		Sub Q. No.							Marks
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$\omega = 2\pi f = 2\pi \times 50 = 100\pi$

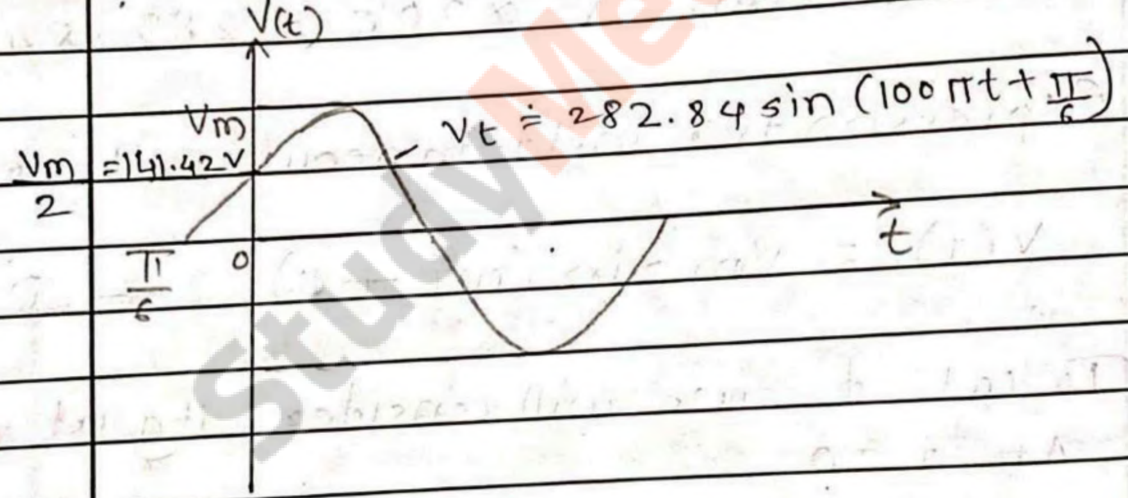
put values of V_m, ω & ϕ in eq^n (1)

$V(t) = 282.84 \sin\left(100\pi t + \frac{\pi}{6}\right)$

$V(t) = 282.84 \sin\left(100\pi t + 30^\circ\right)$

This is expression of voltage

II] sketch the waveform.



The waveform is sinusoidal

At $t = 0$, amplitude is $\frac{Vm}{2} = 141.42V$ &

the phase shift is positive $\frac{\pi}{6}$ rad (30°

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Examiner	Marks							
Moderator	Marks							

2. The RMS value of 50 Hz sinusoidally varying current is 20A. when t=0, its value is 10A. obtain equation of current. Find the value of current when t = 0.002 sec. Also sketch the waveform.

→ Given

$f = 50Hz$

$I_{rms} = 20A$. 30A

At $t = 0$, $I_{rms} = 10A$. 15A

i] Equation of current

$$i(t) = I_m \sin(\omega t + \phi)$$

Here,

$I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 20 = 28.28$ A.

and $\omega = 2\pi f = 2\pi \times 50 = 100\pi$

$$\boxed{i(t) = 28.28 \sin(100\pi t + \phi)} \quad \text{— (1)}$$

ii] Calculate ϕ.

At = $t = 0$, $i_{rms} = 10A$, $i(t) = \sqrt{2} \times I_{rms}$

$= \sqrt{2} \times 10 = 14.14$ A

$t = 0$.

$10\ \cancel{14.14} = 28.28 \sin(0 + \phi)$

$\sin\phi = \dfrac{10\ \cancel{14.14}}{28.28} = 0.5$

4	Que. No.		Sub. Q. No.								Total Marks
	Examiner		Marks								
	Moderator		Marks								

प्र. क्र. Q. No.

$\phi = \sin^{-1} 0.5 = \frac{\pi}{6}$ (in radian mode)

$\phi = 30°$.

3) cal. i at $(t) = 0.02$ sec.

$30 \times \sqrt{2}$

$i(t) = 28.28 \sin(100\pi t + 30°)$

$i(t) = 28.28 \sin(100 \times \pi \times 0.02 + 30°)$

$i_t = 28.28 \sin(2\pi + 30°)$

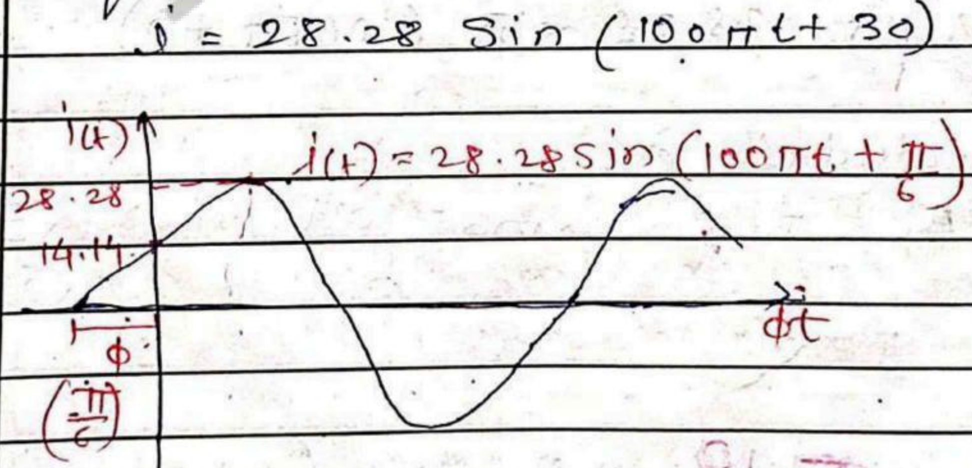
$i(t) = 28.28 [(\underbrace{\sin 2\pi}_{0}) + (\underbrace{\sin 30}_{0.5})]$

$i(t) = 28.28 (0.5)$

$i(t) = 14.14$ A.

iv) equation of current

$i = 28.28 \sin(100\pi t + 30)$



Que. No.	Sub Q. No.							Total Marks
Examiner	Marks							
Moderator	Marks							

3. A sinusoidally varying a.c voltage is given by $v = 141.4 \sin(100\pi t)$ volts. find (i) Rms. (ii) average value. (iii) freq.

→ Given

$$V = V_m \sin \omega t$$

$$V = \underbrace{141.4}_{V_m} \sin \underbrace{100\pi}_{\omega} t$$

1) Rms value. V_{rms}.

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 99.98\ V$$

2) Cal. of avg. value.

$$V_{av} = 0.637\ V_m$$

$$= 0.637 \times 141.4$$

$$V_{av} = 90.0718\ V$$

3) freq.

$$\omega = 2\pi f$$

$$100\pi = 2\pi f$$

$$f = \frac{100\pi}{2\pi}$$

$$f = 50\ Hz$$

6

Que. No.		Sub Q. No.							Total Marks
Examiner		Marks							
Moderator		Marks							

1) An alternating current varying sinusoidally with a freq. 50 Hz has a RMS value of 10 A. Write down eq^n inst value of this current quantity and find its value for t = 0.0015 sc & t = 0.0075 after passing th^r zero the increasing negatively.

→ Given

$f = 50 Hz$, $I_{rms} = 10 A$.

$t_1 = 0.0015$ ~~m~~sec

$t_2 = 0.0075$ ~~m~~sec.

$i = I_m \sin \omega t$

$I_m = \sqrt{2} I_{rms} = \sqrt{2} \times 10 = 14.14 A$.

Equation for instantaneous value of I

$i(t) = I_m \sin \omega t$

$= 14.14 \sin (2\pi f) \cdot t$

$i(t) = 14.14 \sin (2 \times \pi \times 50) t$.

$i(t) = 14.14 \sin (100\pi) t$ — (1)

1) $i(t) = ?$ when $t = 0.0015$ ms

$i(t) = 14.14 \sin (100\pi) \times 0.0015$

$= \cancel{6.41 A \quad (0.641) \quad 6.41}$

$= 6.41 A$

Sub Q. No.							Total Marks	23
Marks								
Marks								

when $t = 0.0075$

$i(t) = I_m \sin \omega t$

$= 14.14 \sin (100\pi \times 0.0075)$

$i(t) = \cancel{-0.8584} - 10\ A.$

24	Que. No.		Sub Q. No.							Total Marks
	Examiner		Marks							
	Moderator		Marks							

प्र. क्र. Q. No. 2.

A sinusoidally varying alternating voltage of 100v (RMS value) with 50Hz freq. is applied to a ckt find.

a) mathematically eqⁿ of voltage
b) Time period.
c) The inst. value of voltage at $t = 1.667\,ms$
d) The time when inst. voltage is 100V
e) V_{avg} . f) V_m.

→ Given.

$V_{rms} = 100V$.

$f = 50Hz$, $\omega = 2\pi f$, $\omega = 100\pi$

a) equation of voltage.

$V_m = V_{rms} \times \sqrt{2}$

$= 100 \times \sqrt{2}$

$V_m = 141.42V$.

$V(t) = V_m \sin \omega t$

$= 141.42 \sin \cancel{\omega t} (100\pi) t$.

b) Time period $(T) = \frac{1}{f} = \frac{1}{50} = 0.02\,sec$.

c) $V(t)$ when $t = 1.667\,ms$

$V(t) = 141.42 \sin(\omega) \times 1.667 \times 10^{-3}$

$V(t) = 141.42 \sin(100\pi) \times 1.667 \times 10^{-3}$

Sub Q. No.							
Examiner Marks							Total Marks
Moderator Marks							

$V(t) = 70.72\ V$

d) $t = ?$ when $V(t) = 100V$.

$V(t) = 141.42 \sin(100\pi)t$

$100 = 141.42 \sin(100\pi) \cdot t$

$\sin(100\pi t) = \frac{100}{141.42}$

$\sin 100\pi t = 0.707$

$t = \frac{\sin^{-1} 0.707}{100\pi}$

$t = \cancel{0.143 \text{ sec}}$

2.5×10^{-3} sec

e) $V_{avg} = ?$

$V_{avg} = 0.637\ V_m$

$V_{av} = 0.637 \times 141.42$

$V_{av} = 90.08\ V.$

f) $V_m = 141.42V$.

III] # Single phase AC Circuits.

- R, L, C Circuit Elements:

AC Circuit

AC Fundamentals - Pure R
- Pure L
- Pure C.

AC series circuit - R+L
R+C
R+L+C

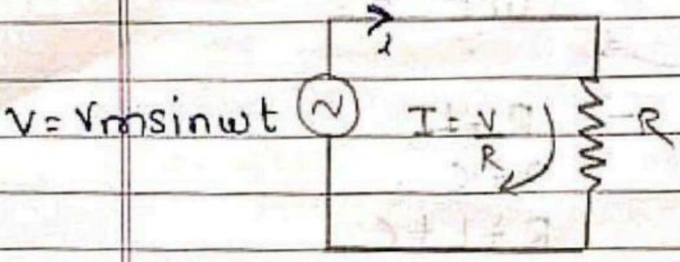
1. Basic three elements are used in AC circuits.
1) Resistor (R) (2) Inductor (L) (3) Capacitor (C)
We will discuss AC response of following circuit.
1. Purely resistive AC circuit
2. Purely inductive AC circuit
3. purely capacitive AC circuit.

In this cases, we will assume the equation of the applied voltage
$V = V_m \sin \omega t$ & determine
current equation in the following steps:
1. Draw a circuit diagram
2. Establish equation of current 'i'
3. Draw waveform of v & i
4. Draw phasor diagram of v & i
5. Determine 'opposition' to the current flow.
6. Determine phase angle (p.f angle) between v & i

7. Determine <u>power factor</u> of circuit
8. Determine <u>power consumed</u> by the circuit

* Purely Resistive AC Circuit

1) Circuit diagram.



2) Current Equation

applied voltage, $V = V_m \sin\omega t$ — ①

The basic equation of current.
ohm's law, $V = IR$

$$I = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$

$$\therefore I = \left(\frac{V_m}{R}\right) \sin\omega t \quad \text{—— ②}$$

skip. eqⁿ ② is instantaneous equation of current for purely resistive AC circuit. The std. eqⁿ of current is

$$I = I_m \sin(\omega t + \phi) \quad \text{—— ③}$$

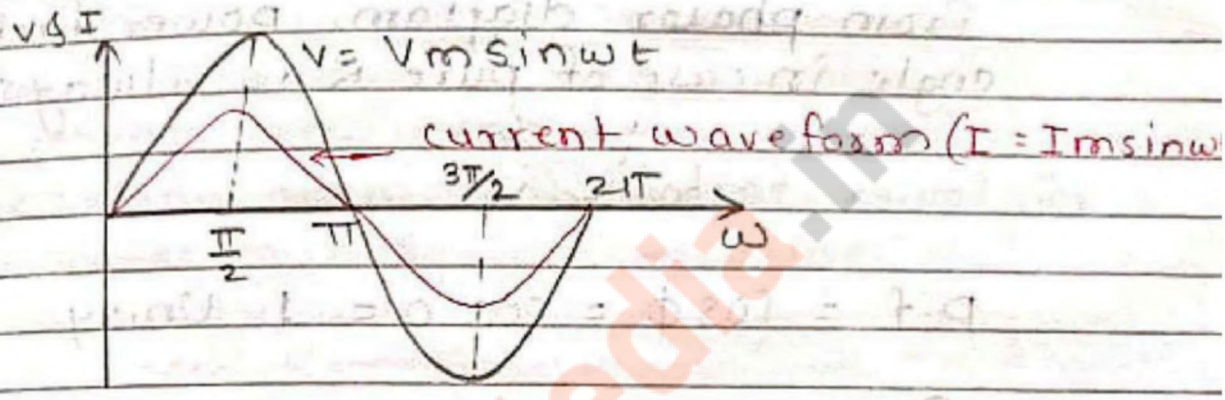
where,
I_m = peak (max) value of current
ϕ = phase difference

Comparing eqⁿ ② & ③, we get

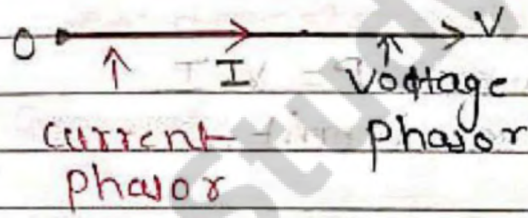
$I_m = \frac{V_m}{R}$ & $\phi = 0$

$$\boxed{I = I_m \sin \omega t}$$

3) Voltage & current waveform.



4) phasor Diagram.



$\phi = 0$, I & V waveform in phase.

5. Impedance:

1. Rectangular Form: $Z = R + jX$
   R = Resistance
   jX = Reactance
2. Polar Form: $Z = |Z| \angle \phi$

where, $|Z|$ = magnitude of Z
ϕ = phase difference.

vi) opposition to current flow

$$I_m = \frac{V_m}{R}$$

vii) power factor angle $(\phi) = 0$
From phasor diagram, power factor angle in case of pure R is always zero

viii) Power factor:

$$p.f = \cos\phi = \cos 0 = 1 \text{ Unity}$$

ix) Power.

1. Instantaneous Power (P)

∵ Instantaneous power $P = V.I$
For purely resistive circuit,

$$V = V_m \sin \omega t \quad \& \quad I = I_m \sin \omega t$$

$$\therefore P = V_m \sin \omega t \cdot I_m \sin \omega t$$

$$P = V_m I_m \sin^2 \omega t$$

$$\therefore \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$P = V_m I_m \left[\frac{1 - \cos 2\omega t}{2}\right]$$

$$\therefore\ P = \frac{V_m I_m}{2}\left[1-\cos 2\omega t\right]$$

$$\therefore\ P = \underbrace{\frac{V_m I_m}{2}}_{\text{constant term}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t)}_{\text{fluctuating term}}$$

i) the first term does not contain any sine or cosine component. so it is constant term.

ii) The second term contains $\cos(2\omega t)$ so it will change with respect to time. so it is called fluctuating term.

ii) Average Power

- The avg. power of sine wave / cosine wave for one complete cycle is zero.

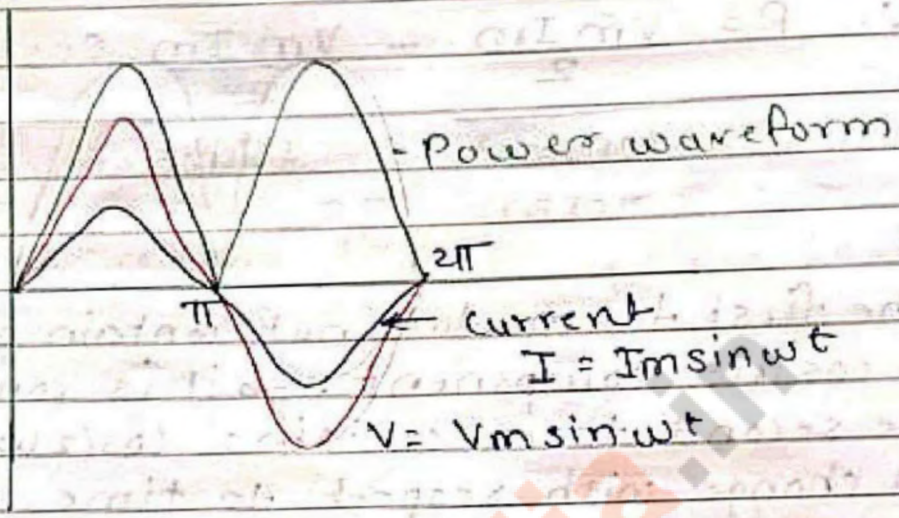
$$P_{av} = \frac{V_m \cdot I_m}{2}$$

$$2 = \sqrt{2} \times \sqrt{2}. \quad \text{(we can write)}$$

$$P_{av} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

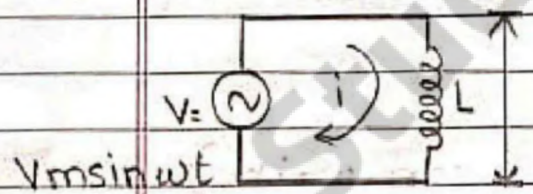
$$P_{av} = V_{rms} \cdot I_{rms} \text{ watts.}$$

Waveform for Power



* Purely Inductive Circuit:

1) Circuit diagram.



$e = -L \frac{di}{dt}$ (self induced emf)

2) Current equation.

Supply voltage (V) is equal and opposite to self induced emf.

$$V = -e = -\left(-L\frac{di}{dt}\right) = L\frac{di}{dt}$$

$\therefore V = V_m \sin\omega t$

$$\therefore \; V_m \sin \omega t = L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_m \sin \omega t}{L}$$

equation of current can be obtained by integrating di

$$i = \int di = \int \frac{V_m}{L} \sin \omega t \, dt$$

$$I = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$= \frac{V_m}{L} \left[-\frac{\cos \omega t}{\omega} \right] = -\frac{V_m}{\omega L} (\cos \omega t)$$

Use this, $\cos \omega t = \sin \left(\frac{\pi}{2} - \omega t \right)$

$$i = -\frac{V_m}{\omega L} \left[\sin \left(\frac{\pi}{2} - \omega t \right) \right]$$

But,

$$\sin \left(\frac{\pi}{2} - \omega t \right) = \sin - \left(\omega t - \frac{\pi}{2} \right) = -\sin \left(\omega t - \frac{\pi}{2} \right)$$

Using identity $\sin(-\theta) = -\sin\theta$.

$$= -\sin \left(\omega t - \frac{\pi}{2} \right)$$

$$i = -\frac{V_m}{\omega L} \left[-\sin \left(\omega t - \frac{\pi}{2} \right) \right]$$

$$\boxed{i = \frac{V_m}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)}$$

The term $\frac{V_m}{\omega L}$ is called a peak value I_m

$$I_m = \frac{V_m}{\omega L}$$

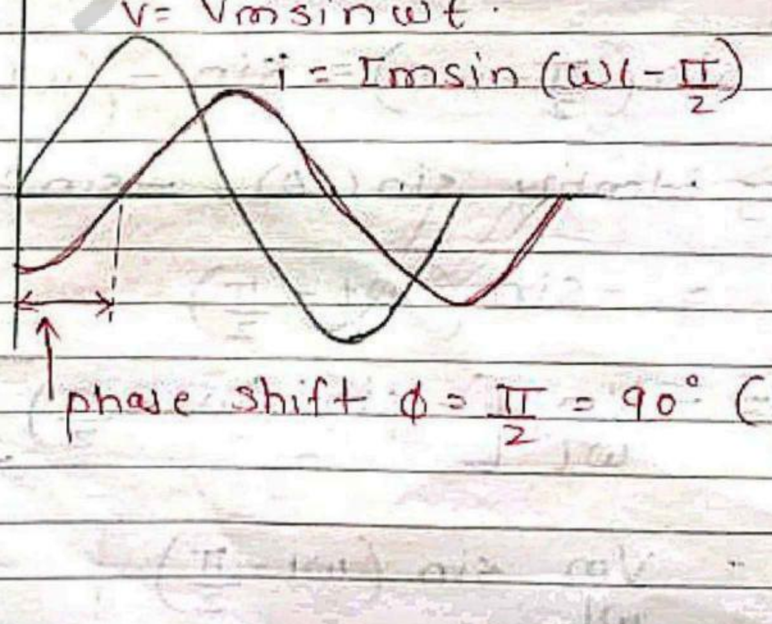
$\omega L = X_L = 2\pi f L$ is called reactance of inductance.

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

ωt = lagging phase, $\frac{\pi}{2}$ = phase shift

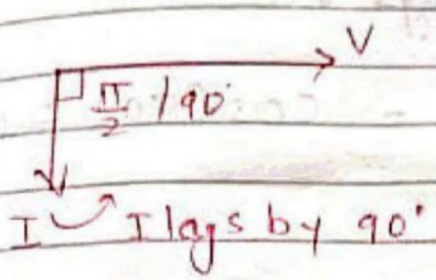
- The term $-\frac{\pi}{2}$ indicates that, current in purely inductive ckt lags by $-\frac{\pi}{2}$ or $-90°$.

iii) Voltage & Current Waveform.



phase shift $\phi = \frac{\pi}{2} = 90°$ (lagging).

iv) phasor diagram.



I & V are rms valu

v) Impedance:

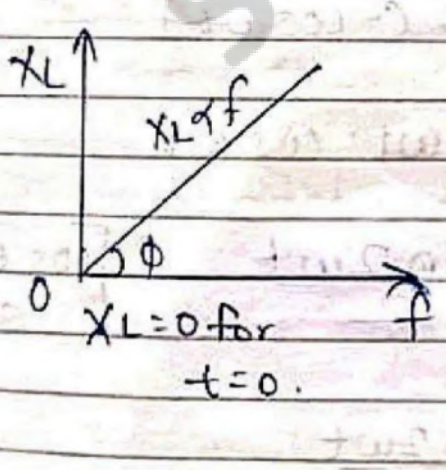
1) Rectangular form: $Z = R + jX_L$
2) polar form $Z = |Z| \angle \frac{\pi}{2}$ Ω

vi) Opposition to current flow:

$X_L = \omega L = 2\pi f L = \frac{2\pi}{T} L$: & X_L is inductive reactance. Unit Ω.

for DC $f = 0$.

$X_L = 0$. hence pure L acts as short ckt.



vii) Power factor angle (ϕ)
$-90°$ as I lags V.

viii) Power factor = $\cos(90) = 0$ lagging.

ix) Power

Instantaneous Power

$P = v \times i$

$v = V_m \sin\omega t$, $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

$P = V_m \sin\omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

$\sin\left(\omega t - \frac{\pi}{2}\right)$ $\therefore$ $\sin\left(\theta - \frac{\pi}{2}\right) = -\cos\theta$

$P = V_m \sin\omega t \cdot I_m \cdot (-\cos\omega t)$

$P = V_m I_m \sin\omega t\,(-\cos\omega t)$

$P = -V_m I_m \sin\omega t \cdot \cos\omega t$.

$\sin\omega t \cdot \cos\omega t = \frac{\sin 2\omega t}{2}$ $\left[\cos\theta \cdot \sin\theta = \frac{\sin 2\theta}{2}\right]$

$P = -V_m I_m \frac{\sin 2\omega t}{2}$

$$\boxed{P = -\frac{V_m I_m}{2} \sin(2\omega t).}$$

average Power.

$$P_{av} = \int_0^{2\pi} P$$

$$P_{av} = 0.$$

* Purely Capacitive Circuit

1) circuit diagram:

$v = V_m \sin \omega t$

2) Current equation

1. Voltage applied to purely capacitive circuit is,

$$v = V_m \sin \omega t.$$

The charge q on the plates of capacitor is related to value of capacitor (C)

$$q = Cv = C V_m \sin \omega t.$$

2. Current is defined as the rate of change of charge,

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[C V_m \sin \omega t\right]$$

$$i = C V_m \frac{d}{dt} \sin \omega t$$

$$i = C V_m \omega \cos \omega t.$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \cos \omega t.$$

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

the term $\frac{V_m}{\frac{1}{\omega C}}$ is denoted by I_m.

Here,

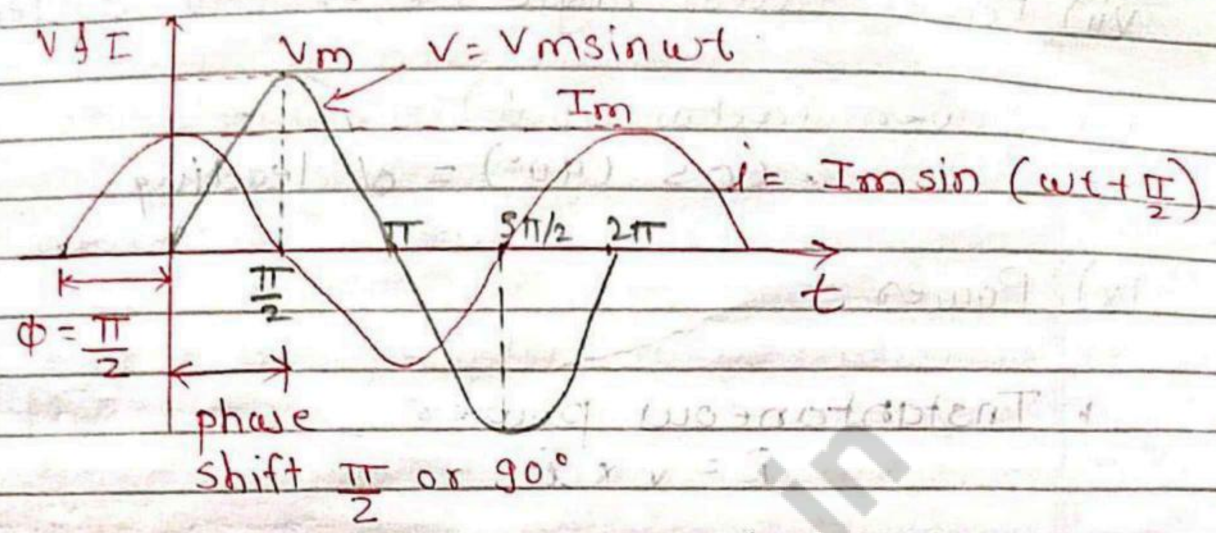
$\frac{1}{\omega C} = X_C$ ... reactance of capacitance

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

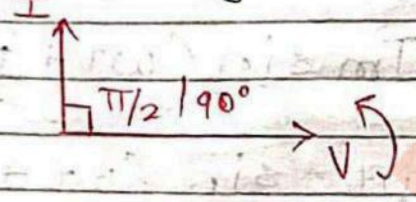
I_m = peak current
ωt = leading phase
$\frac{\pi}{2}$ = phase shift.

The term $+\frac{\pi}{2}$ indicates that current leads voltage by $\frac{\pi}{2}$ or 90°.

3) Voltage & Current waveform.



4) phasor diagram



v) Impedence

1) Rectangular $Z = R + jX$

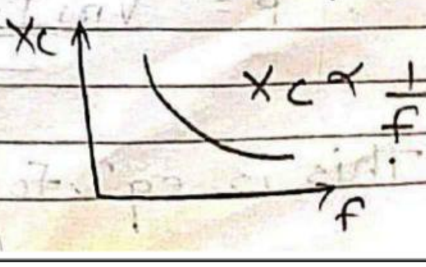
Polar $Z = X_C \angle -\frac{\pi}{2}\ \Omega$

vi) opposition to current flow

- $\frac{1}{\omega C} = \frac{1}{2\pi f C}$, X_C called capacitive reactan

$\therefore X_C = \infty$, pure C acts as open ckt.

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$



vii) Power factor Angle $(\phi) \rightarrow +90^\circ$ as leads V

viii) power factor (p.f)

$\cos(90^\circ) = 0$ leading.

ix) Power

- Instantaneous power

$$P = v \times i$$

$$v = V_m \sin \omega t, \quad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore P = V_m \sin \omega t \cdot I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore P = V_m I_m \sin \omega t \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta}$$

$$P = V_m I_m \sin \omega t \cdot \cos \omega t$$

$$\sin\theta \cdot \cos\theta = \frac{\sin 2\theta}{2}$$

$$P = V_m I_m \frac{\sin 2\omega t}{2}$$

$$\therefore \boxed{P = \frac{V_m I_m}{2} \sin(2\omega t)}$$

This is eq^n for instantaneous power.

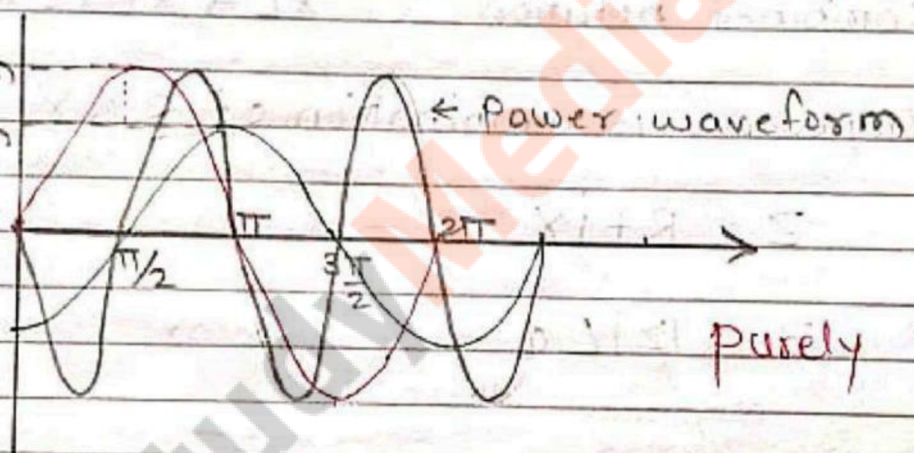
(2ω) means double the original freq. (ω)

· Average power:

$$P_{av} = \int_0^{2\pi} P$$

$$P_{av} = 0.$$

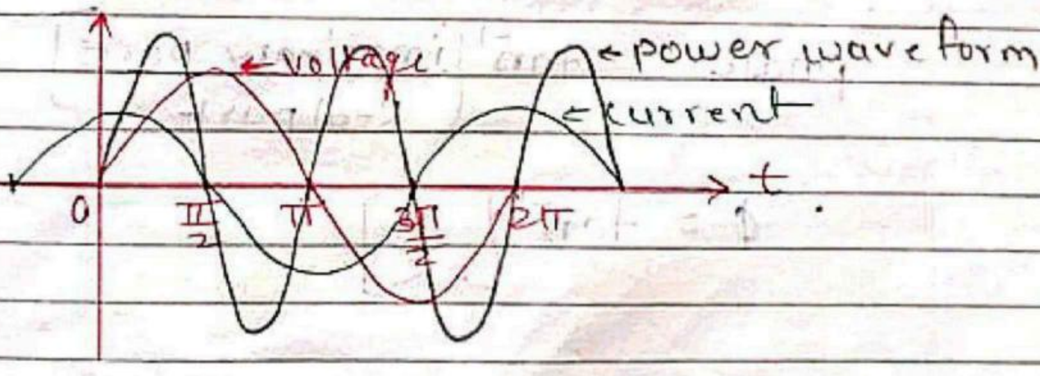
* Waveform for Power (purely inductive)



purely inductive

for inst power equal +ve & -ve cycle, result
$P_{av} = 0$.

* Waveform for power (capacitive)



$P_{av} = 0$.