Program Outcomes (POs)

PO1	**Engineering knowledge**	Apply the knowledge of mathematics, science, Engineering fundamentals, and an Engineering specialization to the solution of complex Engineering problems.
PO2	**Problem analysis**	Identify, formulate, review research literature and analyze complex Engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences and Engineering sciences.
PO3	**Design / Development of Solutions**	Design solutions for complex Engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and Environmental considerations.
PO4	**Conduct Investigations of Complex Problems**	Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
PO5	**Modern Tool Usage**	Create, select, and apply appropriate techniques, resources, and modern Engineering and IT tools including prediction and modeling to complex Engineering activities with an understanding of the limitations.
PO6	**The Engineer and Society**	Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practices.
PO7	**Environment and Sustainability**	Understand the impact of the professional Engineering solutions in societal and Environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
PO8	**Ethics**	Apply ethical principles and commit to professional ethics and responsibilities and norms of Engineering practice.
PO9	**Individual and Team Work**	Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
PO10	**Communication Skills**	Communicate effectively on complex Engineering activities with the Engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
PO11	**Project Management and Finance**	Demonstrate knowledge and understanding of Engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary Environments.
PO12	**Life-long Learning**	Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological

Savitribai Phule Pune University First Year of Engineering (2024 Pattern) Course Code: ESC-104-CVL Course Name: Engineering Mechanics			
Teaching Scheme	**Credit**	**Examination Scheme**	
Theory : 2 Hours/Week **Practical : 2 Hours/Week**	**02** **01**	**CCE** **End-Semester** **Term Work**	**: 30 Marks** **: 70 Marks** **: 25 Marks**

Prerequisite Courses, if any:

- Basic Calculus, Trigonometry, Geometrical expressions, Laws of motion, Concept of mass, acceleration with Fundamental knowledge of Engineering Mathematics and Physics.

Companion Course, if any: **Laboratory Practical.**

Course Objectives:

The objectives of this course is to make students to learn basics of engineering Mechanics concepts and its application to the real-world problems, solve problems involving Forces, loads and Moments and know their applications in allied subjects.

Course Outcomes:

On completion of the course, learner will be able to:

CO1. Understand basic concept of forces, moments and couples in two-dimension force system

CO2. Apply concept of free body diagram for static equilibrium in two-dimension force system

CO3. Analyze the practical example involving friction and application of two force members

CO4. Analyze rectilinear and curvilinear motion of particle

CO5. Apply Newton's second law, work energy and impulse momentum principles for particles

Course Contents		
Unit I	**Force systems and its resultants**	**(06 Hours)**
Introduction, type of motion, fundamental concepts and principle, force system, resolution and composition of forces, resultant of concurrent force system, moment of a force, Varignon's theorem, resultant of parallel force system, couple and resultant of general force system. Introduction, centroid of basic figures, centroid of composite figure, moment of inertia of simple geometrical figure, parallel axis theorem, perpendicular axis theorem, moment of inertia of composite figure.		
Unit II	**Equilibrium**	**(06 Hours)**
Introduction, free body diagram, equilibrium of coplanar forces, equilibrium of two forces, three force principle, equilibrium of concurrent, parallel and general force system, type of load, type of support, type of beam and support reaction.		
UNIT III	**Friction and trusses**	**(06 Hours)**
Introduction, sliding and rolling friction, laws of coulomb friction, coefficient of friction, angle of repose, angle of friction, cone of friction, friction on inclined plane, ladder friction and belt friction. **Trusses:** two force and multi force member, assumption of analysis, analysis of truss, identification of zero force members, method of joint and method of section.		

UNIT IV	Kinematics of particle	(06 Hours)
Introduction, basic concept, rectilinear motion: motion with uniform acceleration, gravitational acceleration and variable acceleration, curvilinear motion: rectangular components, motion of projectile, normal and tangential components.		
UNIT V	**Kinetics of particle**	**(06 Hours)**
Introduction, Newton's second law of motion, equation of motion, Newton's law of gravitation, application of Newton's second laws to rectilinear and curvilinear motion, conservative and non-conservative forces, work energy principle, conservation of energy, impulse momentum principle and impact		

Learning Resources

Text Books:

1. Engineering Mechanics, Ferdinand Singer, 3rd edition, Harper and Row
2. Engineering Mechanics (Statics and Dynamics) by Hibbeler R. C., Pearson Education

Reference Books:

1. Engineering Mechanics, S Timoshanko and Young, Tata McGraw Hill Education Pvt. Ltd. New Delhi.
2. Vector Mechanics for Engineers – Statics, Beer and Johnston, Tata McGraw Hill
3. Vector Mechanics for Engineers – Dynamics, Beer and Johnston, Tata McGraw Hill.
4. Engineering Mechanics - Statics and Dynamics, Meriam J. L. and Kraige L.G., John Wiley and Sons

The CO-PO mapping table

PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	1	2	-									
CO2	1	2	-									
CO3	1	1	3									
CO4	1	1	3									
CO5	1	2	3									

S.I. SYSTEM

Fundamental units of S.I system

Sr. No.	Physical quantities	Unit	symbol
1	Length	Metre	m
2	Mass	Kilogram	Kg
3	Time	Second	S
4	Temperature	Kelvin	K

Supplementary units of S.I. system

Sr. No.	Physical quantities	Unit	symbol
1	Plane angle	Radian	Rad

Sr. No.	Physical quantities	Unit	symbol
1	Force	Newton	N
2	Work	Joule	J, N.m
3	Power	Watt	W
4	Energy	Joule	J, N.m
5	Area	Square metre	m^2
6	Volume	Cubic metre	m^3
7	Pressure	Pascal	Pa
8	Velocity/speed	metre per second	m/s
9	Acceleration	metre/second^2	m/s^2
10	Angular velocity	radian/second	rad/s
11	Angular acceleration	radian/second^2	rad/s^2
12	Momentum	kilogram metre/second	Kg.m/s
13	Torque	Newton metre	N.m
14	Density	Kilogram/metre^3	Kg/m^3
15	Couple	Newton.metre	N.m
16	Moment	Newton.metre	N.m

S.I. Prefixes

Multiplication factor	Prefix	Symble
10^{12}	Tera	T
10^9	Giga	G
10^6	Mega	M
10^3	kilo	k
10^2	hecto	h
10^1	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p

UNIT CONVERSION

1 m = 100 cm = 1000 mm	1 Mpa = 1 N/mm^2
1 km = 1000 m	1 Gpa = 10^3 N/mm^2
1 cm^2 = 100 mm^2	1 Pascal = 1 N/m^2
1 m^2 = 10^6 mm^2	1 degree = $\frac{\pi}{180}$ radians
1 kgf = 9.81 N = 10 N	
1 kN = 10^3 N	

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PuneEngineers Channel

https://t.me/PuneEngineers

SPPU Engineering Group

https://t.me/SPPUBTECH

UNIT I FORCE SYSTEMS AND ITS RESULTANTS (06 HOURS)

Introduction, type of motion, fundamental concepts and principle, force system, resolution and composition of forces, resultant of concurrent force system, moment of a force, Varignon's theorem, resultant of parallel force system, couple and resultant of general force system. Introduction, centroid of basic figures, centroid of composite figure, moment of inertia of simple geometrical figure, parallel axis theorem, perpendicular axis theorem, moment of inertia of composite figure.

INTRODUCTION:

The subject of Engineering Mechanics is that branch of Applied Science, which deals with the laws and principles of Mechanics, along with their applications to engineering problems. As a matter of fact, knowledge of Engineering Mechanics is very essential for an engineer in planning, designing and construction of his various types of structures and machines. In order to take up his job more skillfully, an engineer must pursue the study of Engineering Mechanics in a most systematic and scientific manner. Applications of Engineering Mechanics are found in analysis of forces in the components of roof truss, bridge truss, machine parts, parts of heat engines, rocket engineering, aircraft design etc.

- **DIVISIONS OF ENGINEERING MECHANICS**

The subject of Engineering Mechanics may be divided into the following two main groups:

1. Statics, and 2. Dynamics.

1. STATICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

2. DYNAMICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the following two branches :

a) Kinetics, and 2. Kinematics.

a) KINETICS

It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

b) KINEMATICS

It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

❖ FUNDAMENTAL UNITS

Every quantity is measured in terms of some internationally accepted units, called fundamental units.

All the physical quantities in Engineering Mechanics are expressed in terms of three fundamental quantities, *i.e.*

1. Length 2. Mass and 3. Time

❖ DERIVED UNITS

Sometimes, the units are also expressed in other units (which are derived from fundamental units) known as derived units e.g. units of area, velocity, acceleration, pressure etc.

Systems of units: Generally we use four systems of units.

- ❖ **Centimeter-gram-second (C.G.S) system**: In this system the units of fundamental quantities i.e. mass, length and time are expressed in gram, centimeter and second respectively.
- ❖ **Metre-kilogram-second (M.K.S) system**: In this system the units of fundamental quantities i.e. mass, length and time are expressed in kilogram, meter and second respectively.
- ❖ **Foot-pound-second (F.P.S) system**: In this system the units of fundamental quantities i.e. mass, length and time are expressed in pound, foot and second respectively.
- ❖ **International systems of units (S.I system**): This system consists of seven fundamental quantities. In this system the units of fundamental quantities i.e. mass, length and time are expressed in kilogram, meter and second respectively.

In this study material we shall use only the S.I. system of units.

FUNDAMENTAL S.I UNITS

QUANTITIES	FUNDAMENTAL UNIT	SYMBOL
Length	Meter	m
Mass	Kilogram	Kg
Time	Second	S
Electric current	Ampere	A
Luminous intensity	Candela	Cd
Thermodynamic temperature	Kelvin	K

SOME S.I DERIVED UNITS

QUANTITIES	DERIVED UNIT	SYMBOL
Force	Newton	N
Moment	Newton-meter	Nm
Work done	Joule	J
Power	Watt	W
Velocity	Meter per second	m/s
Pressure	Pascal or Newton per square meter	Pa or N/m^2

❖ **SCALAR QUANTITIES**

A quantity which is specified by its magnitude is called as *Scalar quantity*. Example: mass, length, area, volume, time etc.

❖ **VECTOR QUANTITIES**

A quantity which is specified by its magnitude and direction is called as *Vector quantity*. Example: force, weight, velocity, displacement, acceleration, moment

- ❖ **MASS :** It is the total quantity of matter contained in a body. The S.I unit of mass is kilogram (kg).
- ❖ **Weight :** It is the force with which a body is attracted towards the centre of earth. The S.I unit of weight is Newton (N).
- ❖ **Rigid body :** A body which doesn't deform under the action of forces is known as a rigid body . In actual practice, there is no body which can be said to be rigid in true sense of terms.
- ❖ **Deformable body :** A body which deforms under the action of forces is known as a deformable body.
- ❖ **Elastic body :** A body which can deform under the action of external forces and come back to its original shape and size after the removal of forces is known as *elastic body*.
- ❖ **Force:** It is an external agency either push or pull which changes or tends to change the state of rest or of uniform motion of a body, upon which it acts.

S.I Unit: N, KN

❖ **Effects of force :**

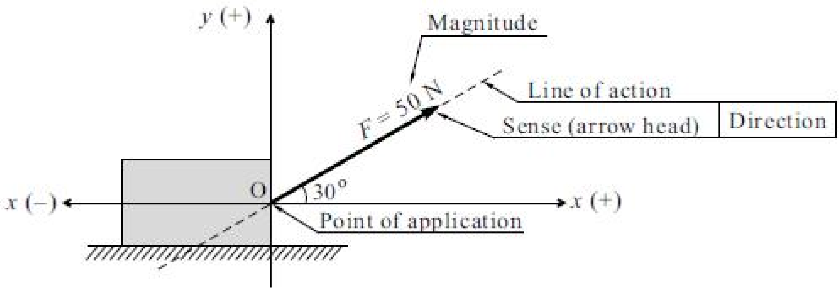
i) It may change the state of a body i.e. the state of rest or state of motion.
ii) It may accelerate or retard the motion of a body.
iii) It may turn or rotate the body on which it acts.
iv) It may deform the body on which it acts.

❖ **Characteristics of force.**

i) Magnitude: The quantity of force e.g. 10 N, 100 kN etc.
ii) Direction: It is the line along which the force acts. It is also called as line of action of the force.
iii) Point of application: The point at which the force acts on the body.
iv) Sense or Nature:

➢ Pull: If the arrow head is pointed away from the point of application, the nature of the force is pull.

➢ Push: If the arrow head is pointed towards the point of application, the nature of the force is push



❖ **Force System:**

Force System is a collection or pattern or group of various forces acting on a rigid body. It is of two types and its classification depends upon number of forces acting in planes.

❖ Classification of force system

The force system is classified into two categories
(a) Coplanar Force System (b) Non-Coplanar Force System

(a) Co-planar force system

Coplanar force systemis the one where a number of forces work in a single plane or common plane. It is further divided into:

(i) Concurrent coplanar force system: If a number of forces work through a common point in a common plane, then the force system is called concurrent coplanar force system.

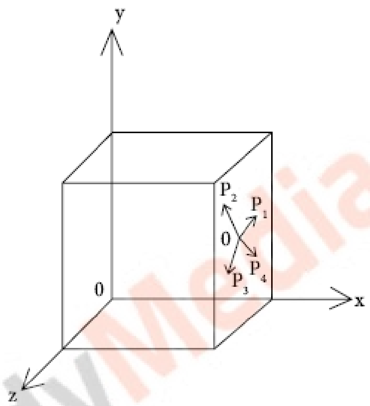


Fig. Concurrent coplanar force system

ii) Collinear coplanar force system: If a number of forces have single line of action in a common plane, then the force system is called collinear coplanar force system.

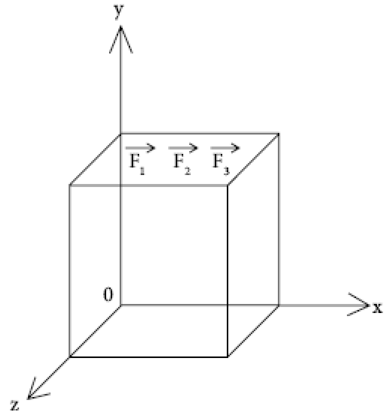


Fig. Collinear coplanar force system

(iii) Parallel coplanar force system: If a number of forces have parallel line of action in a common plane, then the force system is called parallel coplanar force system.

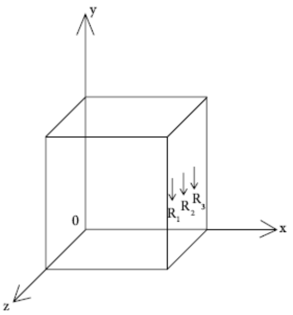


Fig. Parallel coplanar force system

(iv) Non-parallel coplanar force system: If a number of forces do not have parallel line of action in a common plane, then the force system is known as non-parallel coplanar force system.

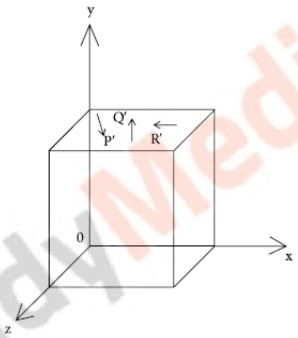


Fig. Non parallel coplanar force system

(b) **Non-coplanar force system**:

It is the one where a number of forces work in different planes.

It is further divided into:

(i) Concurrent non-coplanar force system: If a number of forces work through a common point in different planes, then the force system is called concurrent noncoplanar force system.

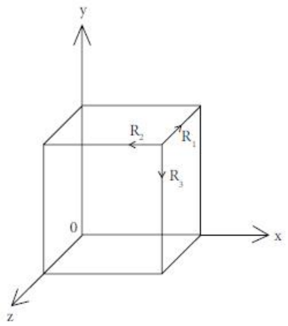


Fig. Concurrent non-coplanar force system

(ii) Non-parallel non-coplanar force system: When a number of forces are having different line of action in three different planes, then the force system is called nonparallel non-coplanar force system.

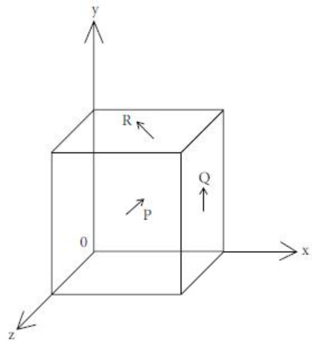
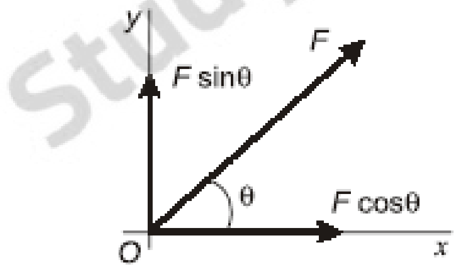


Fig. Non-parallel non-coplanar force system

(iii) Parallel non-coplanar force system: When a number of forces are having parallel line of action in two different planes, then the force system is known as parallel noncoplanar force system is force system can exist in two, two-dimensional planes only.
For example: the compressive forces in four legs of a table represent this force system

- **Resolution of a force**: The way of representing a single force into number of forces without changing the effect of the force on the body is called as resolution of a force.



$$F_x = F\cos\theta,\ F_y = F\sin\theta$$

- **Composition of force**: The process of finding out the resultant force of a given force system is called as composition of forces.

LAWS OF MECHANICS :

The following basic laws and principles are considered to be the foundation of mechanics:

(i) Newton's first and second laws of motion
(ii) Newton's third law
(iii) The gravitational law of attraction
(iv) The parallelogram law
(v) The Principle of Transmissibility of forces.

- **Newton's First Law :** It states that every body continues to be in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it.
- **Newton's Second Law :**It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it.

Thus according to this law:

Force $\propto$ rate of change of momentum. But momentum = mass $\times$ velocity

As mass do not change,

Force $\propto$ mass $\times$ rate of change of velocity

i.e., Force $\propto$ mass $\times$ acceleration

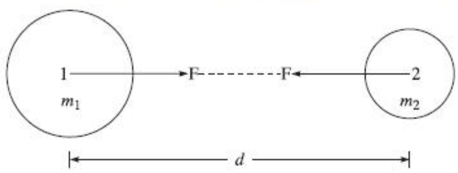
$$F \propto m \times a \qquad \text{...(1.3)}$$

- **Newton's Third Law :** It states that *for every action there is an equal and opposite reaction.*

Newton's Law of Gravitation : Every body attracts the other body. The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them. According to this law the force of attraction between the bodies of mass m_1 and m_2 at distance '*d*' as shown in Fig. is where *G* is the constant of proportionality and is known as constant of gravitation

$$F = G\frac{m_1 m_2}{d^2} \qquad \text{...(1.4)}$$

where *G* is the constant of proportionality and is known as constant of gravitation.



❖ PARALLELOGRAM LAW OF FORCES :

It states, "*If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.*"

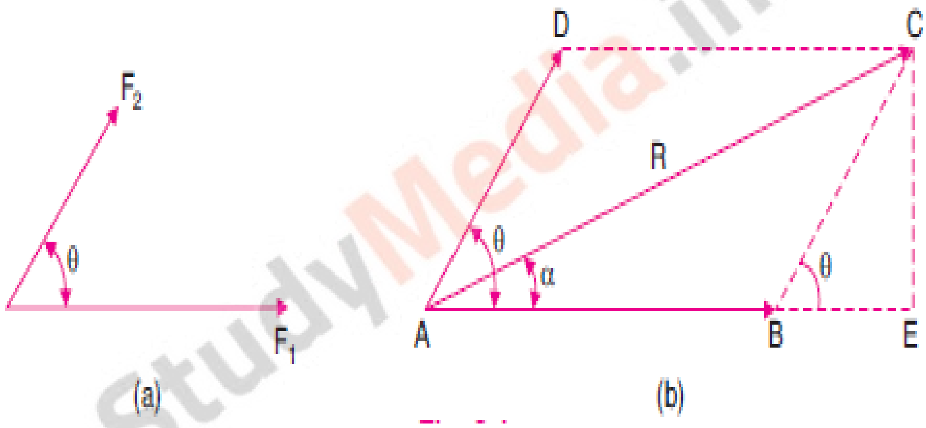
$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta}$$

and $$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

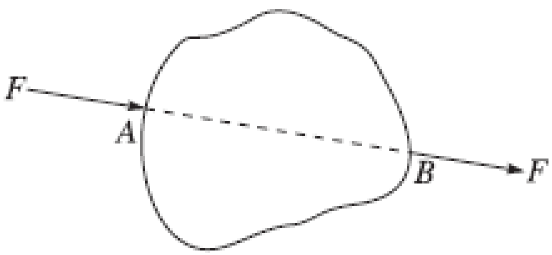
where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

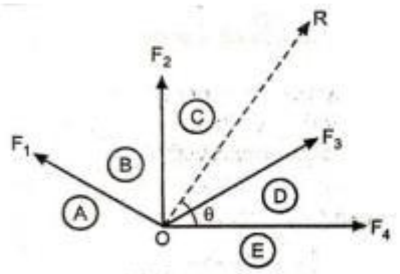
α = Angle which the resultant force makes with one of the forces (say F_1).



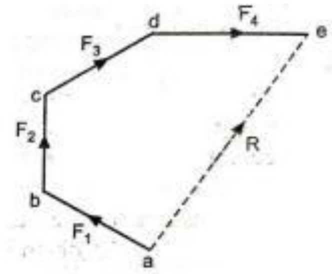
- **Principle of transmissibility** of force states that, "if a force acts at a point on a rigid body, it is assumed to act at any other point on the line of action of force within the same body". As per this principle force of push nature can be made pull by extending the line of a force in opposite quadrant.



- **Law of polygon of forces**. This law states that, "If number of coplanar concurrent forces acting simultaneously on a body, be represented in magnitude and direction by the sides of polygon taken in same order, then their resultant may be represented in magnitude and direction by the closing side of the polygon, taken in opposite order."



(a) Space diagram



(b) Vector diagram

❖ **RESULTANT FORCE :**

If a number of forces, *P*, *Q*, *R* ... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them *i.e.*, which would produce the same effect as produced by all the given forces. This single force is called *resultant force* and the given forces *R* .

ANALYTICAL METHOD FOR RESULTANT FORCE

The resultant force, of a given system of forces, may be found out analytically by the following methods :

1. Parallelogram law of forces. 2. Method of resolution

Example 1 : Calculate the resultant of two concurrent forces of magnitudes of 25 kN and 50 kN with included angle of 55°.

Solution :

1) Magnitude of Resultant

$$R = \sqrt{P^2 + Q^2 + 2PQ \times \cos\theta}$$

$$= \sqrt{25^2 + 50^2 + 2 \times 25 \times 50 \times \cos 55°}$$

$$= \sqrt{625 + 2500 + 1433.94}$$

R = 67.52 kN

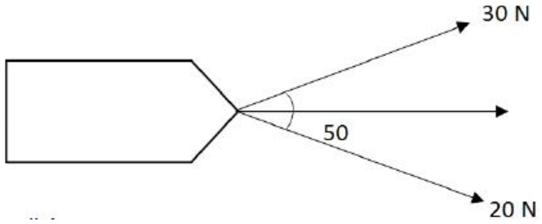
2) Direction of Resultant

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right) = \tan^{-1}\left(\frac{50\sin 55°}{25 + 50\cos 55°}\right)$$

α = 37.34°

(Note: Considering the forces P and Q of same nature).

Example:2 A boat kept in position by two ropes as shown in figure. Find the drag force on the boat.



Answer:

According to law of parallelogram

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{20^2 + 30^2 + 2\times 20\times 30\cos 50} = 45.51N$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{30\sin 50}{20 + 30\cos 50} \qquad \therefore \alpha = 30.32^0$$

Example 3 :**Find the angle between two equal forces of magnitude 300 N each, if their resultant is 150 N.**

Answer :

Using Law of Parellelogram

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$(150)^2 = (300)^2 + (300)^2 + (2\times 300\times 300)\cos\theta$$

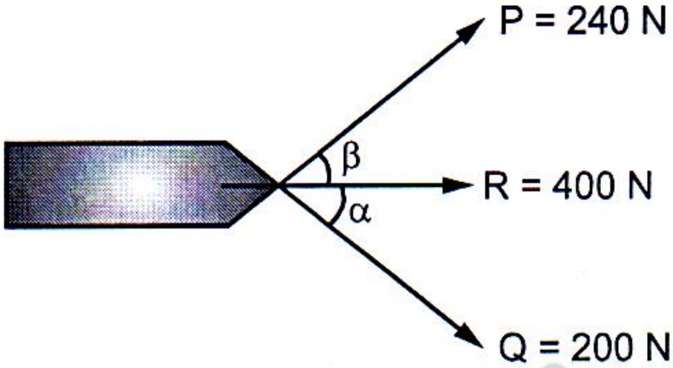
$$22500 = 90000 + 90000 + 180000\cos\theta$$

$$\cos\theta = -\frac{157500}{180000} = -0.875$$

$$\theta = \cos^{-1}(-0.875)$$

$$\boxed{\theta = 151.04^\circ}$$

Example : Resultant force R = 400 N has two component forces P = 240 N and Q = 200 N as shown. Determine direction of component forces P and Q w.r.t. resultant force R.



By Using Law of Parellelogram

$R^2 = P^2 + Q^2 + (2 PQ COS\Theta)$

$400^2 = 240^2 + 200^2 + 2 \times 240 \times 200 COS\Theta$

$160000 = 57600 + 40000 + 96000 COS\Theta$

$62400 = 96000 COS\Theta$

$COS\Theta = 62400 / 96000 = 0.65$

$\boxed{\Theta = 49.45^0}$

$\bar{\alpha} = \tan^{-1} (Q\sin\Theta / P + Q \cos\Theta)$

$= \tan^{-1} (200 \sin 49.45 / 240 + 200 \cos 49.25)$

$\boxed{\bar{\alpha} = 22.29^0}$

$\Theta = \bar{\alpha} + ß$

$49.45 = 22.29 + ß$

$ß = 49.45 - 22.29 = 27.16^0$

$ß = 27.16^0$

❖ METHOD OF RESOLUTION FOR THE RESULTANT FORCE :

Procedure :

Step 1: Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*, ΣF_x) : **{ Sign Rule $\rightarrow$ (+ Ve) $\leftarrow$(-Ve) }**

Step 2:. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.* ΣF_y) : **{ Sign Rule $\uparrow$ (+Ve) $\downarrow$ (-Ve) }**

Step 3 : The resultant *R* of the given forces will be given by the equation :

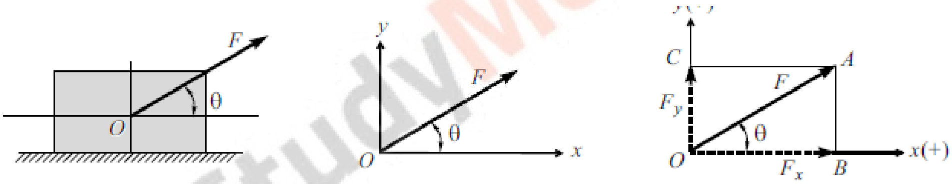
$$R=\sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

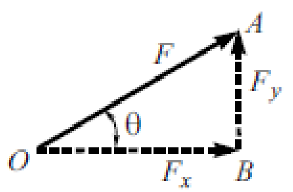
Step 4 :.

$$\text{Direction of Resultant } (\Theta) = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

Note : Use only positive value of ΣF_x and ΣF_x for finding Θ with horizontal x-axis so the value of Θ will be always acute angle.

Resolution of Force into Rectangular Components of Force :





By trigonometric convention, we have the relation of components F_x and F_y with F and θ.

$$\sin\theta = \frac{F_y}{F} \quad \text{and} \quad \cos\theta = \frac{F_x}{F}$$

$$\therefore\ F_y = F\sin\theta \quad \text{and} \quad F_x = F\cos\theta$$

$$F = \sqrt{F_x^{\,2} + F_y^{\,2}} \text{ and } \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right).$$

Sign Conventions

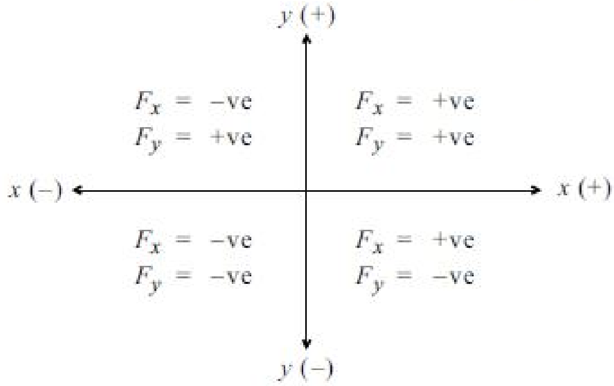
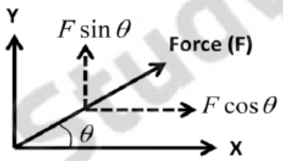


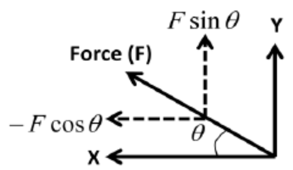
Fig. 2.2.2-v

1. Forces acting horizontally towards right are +ve and left are −ve.
2. Forces acting vertically upward are +ve and downward are −ve.

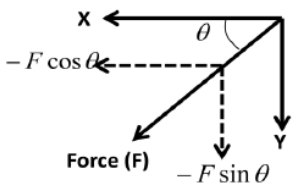
❖ **Resolution of force in rectangular coordinates in different quadrants:**



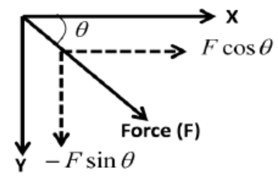
Force (F) in first quadrant



Force (F) in second quadrant



Force (F) in third quadrant



Force (F) in fourth quadrant

Note : · 1) **If resultant is horizontal then** $\Sigma F_y = 0$ **and** $\Sigma F_x = R$

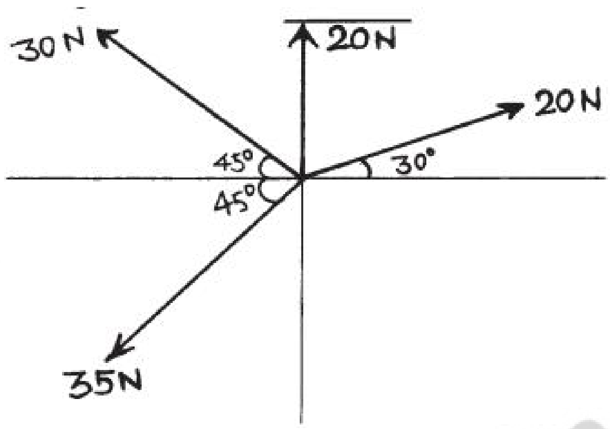
2) **If resultant is vertical then** $\Sigma F_y = R$ **and** $\Sigma F_x = 0$

3) **If resultan is inclined then** $\Sigma Fx = R_x$ **and** $\Sigma Fy = R_y$

❖ **RESULTANT OF -CONCURRENT FORCE SYSTEM** :

Example 1 : Find the magnitude and direction of the resultant force as shown in Fig. No

Answer :



Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$$\Sigma Fx = 20\cos 30 - 30\cos 45 - 35\cos 45$$

$$\Sigma Fx = 17.32 - 21.21 - 24.74$$

$$\Sigma Fx = -28.63\ N$$

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

$$\Sigma Fy = 20 + 20\sin 30 + 30\sin 45 - 35\sin 45$$

$$\Sigma Fy = 20 + 10 + 21.21 - 24.74$$

$$\Sigma Fy = 26.46\ N$$

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

$$R = \sqrt{(-28.63)^2 + (26.46)^2}$$

R= 38.98 N…… (Ans.)

Step 4 : Direction of Resultant :

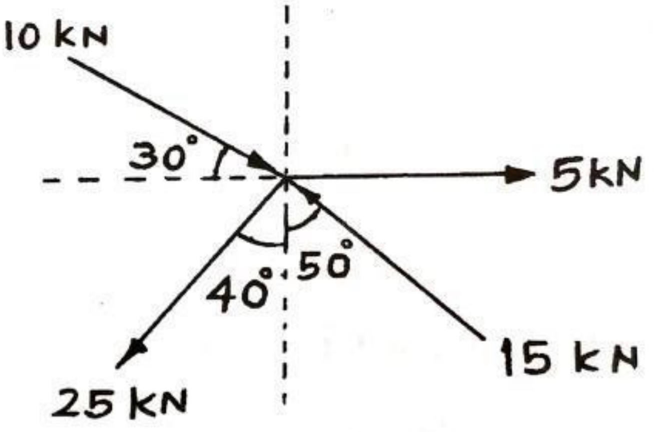
$$\text{Direction of Resultant } (\theta) = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$$\theta = \tan^{-1}\left|\frac{26.46}{-28.63}\right|$$

$$\theta = \tan^{-1}|-0.9242|$$

$$\theta = 42.74^0 \text{ ....(Ans.)}$$

Example 2 : Calculate the magnitude and direction of resultant for the concurrent force system as shown in figure No. Show it on the sketch. Use analytical method only.



Answer :

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

ΣFx = + (10cos30°) – (25cos50°) – (15cos40°) + 5 = - 13.90 kN.

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

ΣFy = - (10sin30°) – (25sin50°) + (15sin40°) = - 14.51 kN.

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2} = \sqrt{(-13.90)^2 + (-14.51)^2}$$

R = 20.09 kN.

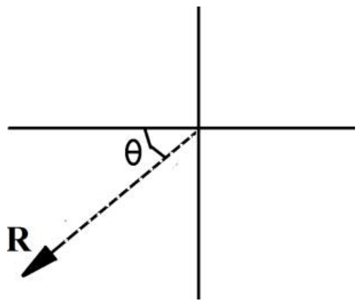
Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right| = \tan^{-1}\left|\frac{14.51}{13.90}\right|$$

θ = 46.23°

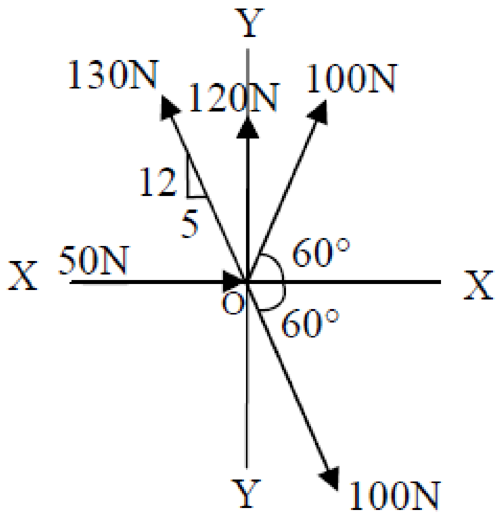
Position of Resultant

Since Σ Fx is –ve and Σ Fy is –ve



Resultant lies in **Third quadrant**

Example 3 Determine magnitude and direction of resultant force of the force system shown in fig.



Answer :

$$\tan\theta = 12/5 = 2.4$$

$$\theta = \tan^{-1}(2.4) = 67.38$$

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$$\Sigma Fx = +100 \text{ Cos } 60^{\circ} - 130\text{Cos } 67.38^{\circ} + 50 + 100\text{Cos}60^{\circ} = +100 \text{ N } (\rightarrow)$$

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

$$\Sigma Fy = +100\text{Sin}60^{\circ} + 120 + 130\text{Sin } 67.38^{\circ} - 100\text{Sin}60^{\circ} = +240 \text{ N } (\uparrow)$$

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

R = 260 N

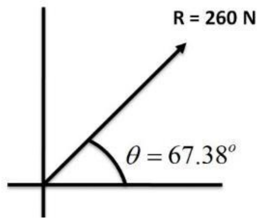
Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$$\theta = 67.38^{o}$$

Position of Resultant :

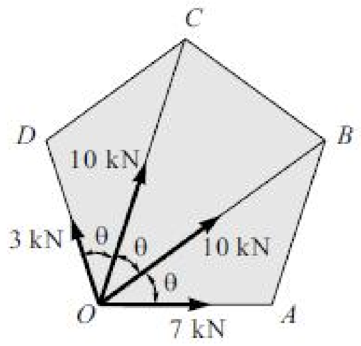
Since Σ Fx is +ve and Σ Fy is +ve , Resultant Lies in First Quadrant

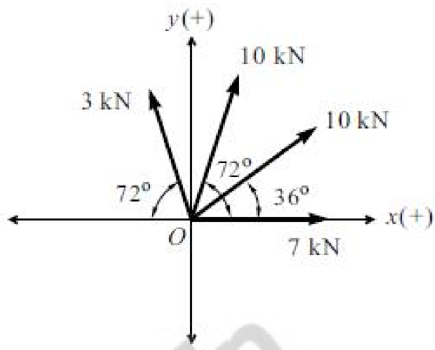


Example 4 : Forces 7 kN, 10 kN, 10 kN and 3 kN, respectively act at one of the angular point of regular pentagon toward the other four points taken in order. Find their resultant completely.

Answer : Regular pentagon is a polygon having five sides of equal length. The point of intersection of two sides is called an angular point.

Regular pentagon is a polygon having five sides of equal length. The point of intersection of two sides is called an angular point. Therefore, pentagon has five angular





Included angles of any regular polygon

$$= 180 - \frac{360}{\text{Number of sides}}$$

For pentagon included angle

$$= 180 - \frac{360}{5} = 108^{\circ}$$

$$\therefore \ \theta = \frac{108}{3} = 36^{\circ}$$

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

ΣFx = 7 + 10 cos 36 + 10 cos 72 - 3 cos 72 = 17.25 KN

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

Fy = 10 sin 36 + 10 sin 72 + 3 sin 72 = 18.24 KN

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

R = 25.10 KN

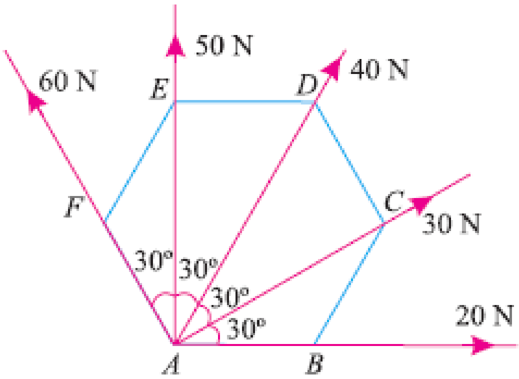
Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$\theta = 46.59^{\circ}$

Example 5 :**The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force**.

Answer :



Answer :

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$$\Sigma Fx = 20 + 30 \cos 30 + 40 \cos 60 - 60 \cos 60 = 36 \text{ N}$$

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

$$\Sigma Fy = 30 \sin 30° + 40 \sin 60° + 50 + 60 \sin 60 = 151.6 \text{ N}$$

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

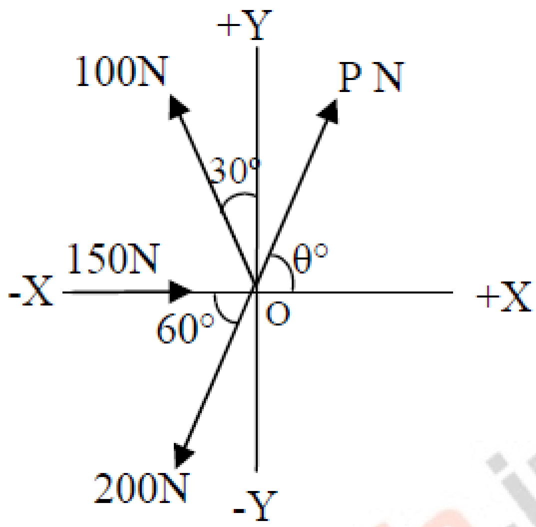
R = 155.8 N

Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$$\theta = 76.6°$$

Example 6: A system of four forces shown in Fig. has resultant 50 kN along + X - axis. Determine magnitude and inclination of unknown force P.



Answer : As the R= 50N & directed along + X – axis

$\Sigma Fx = +50\ N$ and $\Sigma Fy = 0$

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$\Sigma Fx = +150 + P \cos\theta - 100 \cos 60 - 200 \cos 60°$

50 = P Cos θ .... (1)

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

$\Sigma Fy = +P \sin\theta + 100 \sin 60° - 200 \sin 60° = 0$

$\therefore$ **P Sin θ = 86.60 _ _ _ _ _ _ (2)**

From Equation (1) & (2

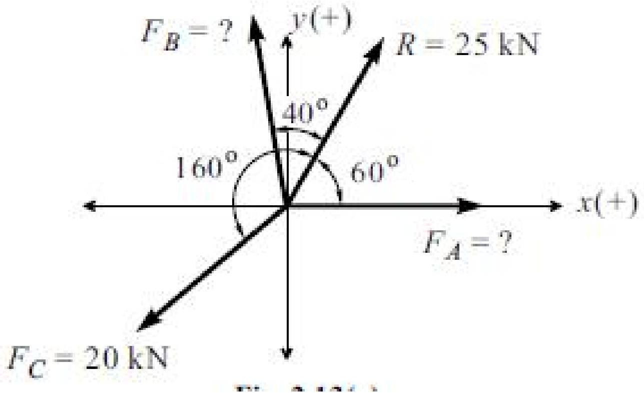
P Sin θ / P Cos θ = 86.60 / 50

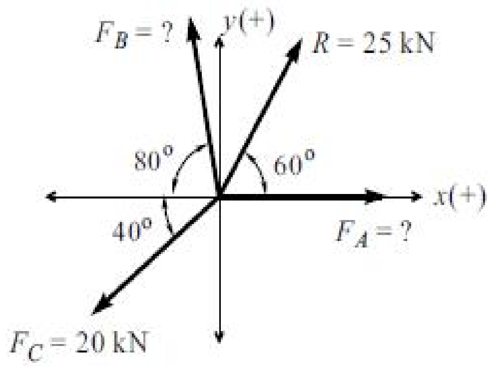
$\therefore$ **tan θ = 1.732**

$\therefore$ **θ = tan⁻¹ (1.732) = 59.99⁰**

$\therefore$ **P = 100 N**

Example 7 : A force R = 25 kN, acting at O, has three components F_A , F_B and F_C as shown in Fig. If FC = 20 kN, find F_A and F_B .





Answer :

I) To Find F_B :

$R_y = \Sigma F_y$

$25 \sin 60 = F_B \sin 80 - 20 \sin 40$

$F_B = 35.04$ KN

II) To Find F_A :

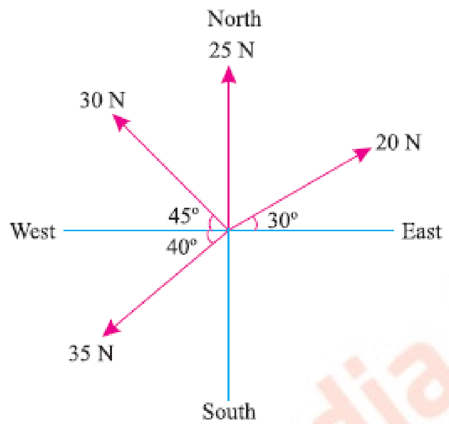
$R_x = \Sigma F_x$

$25 \cos 60 = F_A\ 35.04 \cos 80 - 20 \cos 40$

$F_A = 33.91$ KN

Example 8: The following forces act at a point :
(i) 20 N inclined at 30° towards North of East,
(ii) 25 N towards North,
(iii) 30 N towards North West, and
(iv) 35 N inclined at 40° towards South of West.
Find the magnitude and direction of the resultant force.

Answer : The system of given forces is shown in Fig



Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$$\Sigma Fx = 20 \cos 30 - 30 \cos 45 - 35 \cos 40 = -30.70 \text{ N}$$

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

$$\Sigma Fy = 20 \sin 30° + 25 + 30 \sin 45 - 35 \sin 40 = 33.71 \text{ N}$$

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

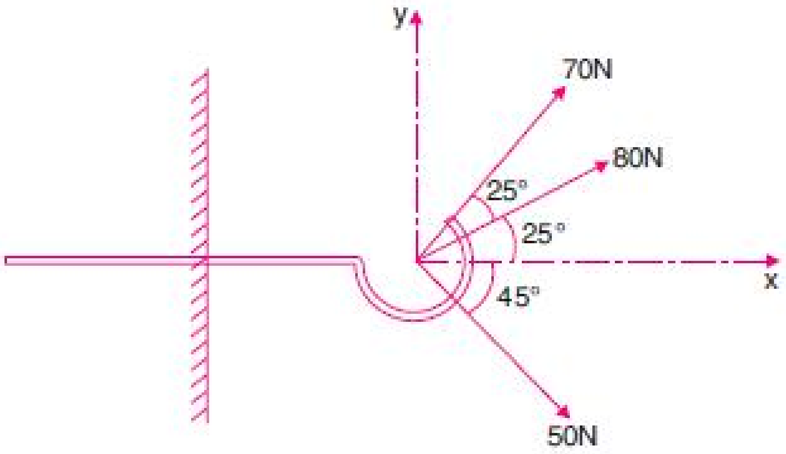
R = 45.59 N

Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$$\theta = 47.67°$$

Example 9: Determine the resultant of the three forces acting on a hook as shown in Fig.



Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$$\Sigma Fx = 50 \cos 45 + 80 \cos 25 + 70 \cos 50 = 152.85 \text{ N}$$

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

$$\Sigma Fy = -50 \sin 45 + 80 \sin 25 + 70 \sin 50 = 52.07 \text{ N}$$

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

R = 161.47 N

Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$$\theta = 71.18°$$

❖ **Moment of Force:** When a force acts upon a body to turn it with respect to a point, the turning effect of force is called as moment of the force.

Mathematically ,

$$M = P \times L$$

P = Force acting on the body

L = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

- The product of magnitude of force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.
- The unit of moment in S.I system is N-m.
- **Clockwise moment** – It is the moment of a force which produces the turning effect of the body in clockwise direction. The clockwise moment is taken as **positive**
- **Anticlockwise moment** – It is the moment of a force which produces the turning effect of the body in anticlockwise direction. The anticlockwise moment is taken as **negative**

❖ **Varignon's theorem of moments (Or Law of Moments) :**

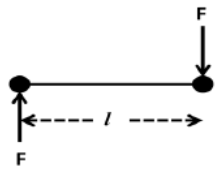
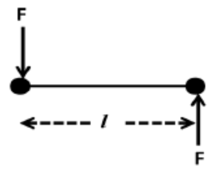
Varignon‟s theorem states, "The algebraic sum of moments of all forces about any point is equal to moment of resultant about the same point"

Let, $\sum MF_A$ = Algebraic sum of moments of all forces about point A

MR_A = Moment of Resultant about point A

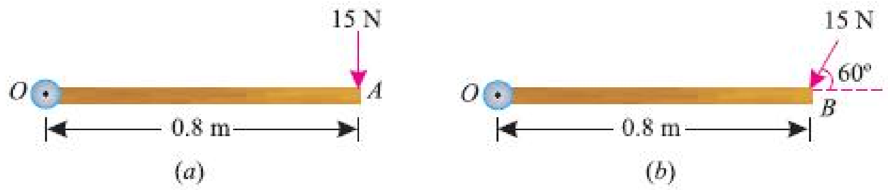
Then, $\sum MF_A = MR_A$

❖ **Couple:** *Two equal and opposite parallel forces acting at different points in a body forms a couple.*

	
Clockwise couple: It is the couple produced by a pair of forces which tends to rotate the body in clockwise direction. It is taken as positive couple.	*Anticlockwise couple*: It is the couple produced by a pair of forces which tends to rotate the body in anticlockwise direction. It is taken as negative couple.

The main characteristics of a couple 1. The algebraic sum of the forces, consisting the couple, is zero. 2. The algebraic sum of the moment of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself. 3. A couple cannot be balanced by a single force, but can be balanced only by a couple, but of opposite sense. 4. Any number of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Example 10 : A force of 15 N is applied perpendicular to the edge of a door 0.8 m wide as shown in Fig (a) . Find the moment of the force about the hinge. If this force is applied at an angle of 60° to the edge of the same door, as shown in Fig. (b), find the moment of this force.



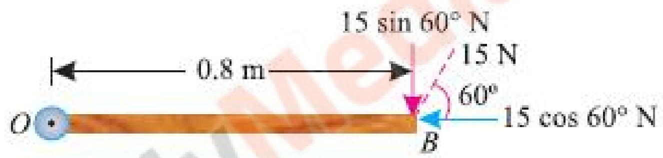
Answer :

P = 15 N and L = 0.8 m

a) Moment when the force acts perpendicular to the door , We know that the moment of the force about the hinge

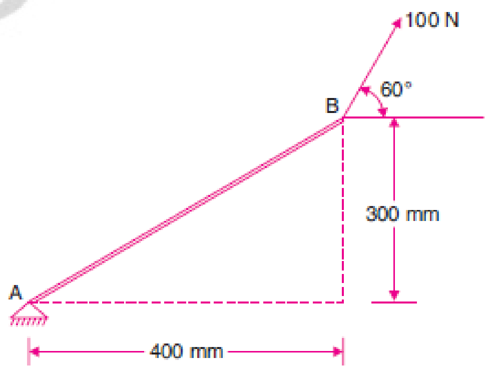
$$M_o = P \times L = 15 \times 0.8 = 12.0 \text{ N-m}$$

b) Moment when the force acts at an angle of 60° to the door



$$M_o = 15 \sin 60 \times 0.8 = 10.4 \text{ N-m}$$

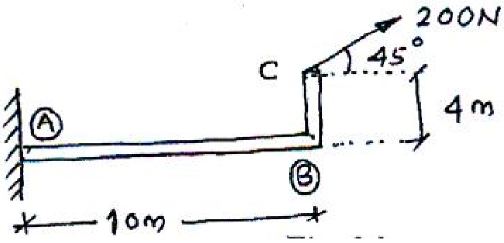
Example 11 . Determine the moment of 100 N force acting at B about moment centre A as shown in Fig.



Answer : $M_A = 100 \cos 60° \times 300 - 100 \sin 60° \times 500$

$= -28301$ **N-mm** (anticlockwise.)

Example 12. Determine the moment of 200 N force about point A and for the bracket as shown in Fig

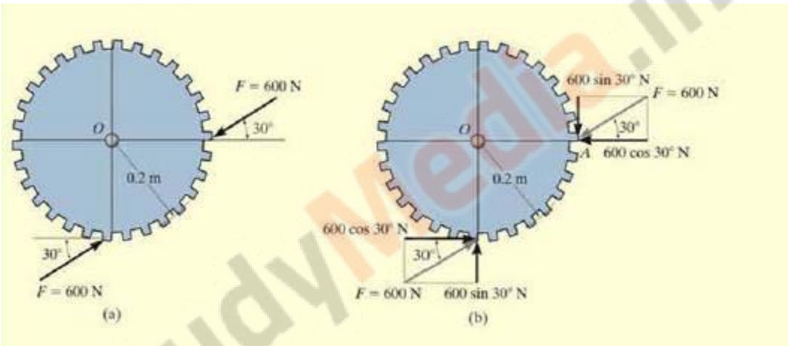


Answer :

Taking Moment About A :

$$M_A = -200 \cos 45 \times 10 + 200 \sin 45 \times 4 = -848.52 \text{ N-m}$$

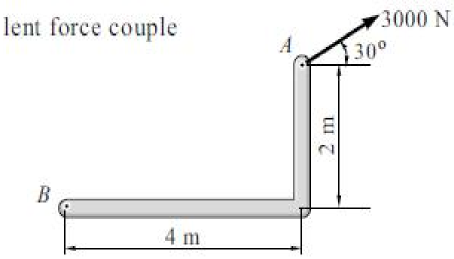
Example11 : Determine the malgnitude and direction of the couple moment acting on the gear in Fig.

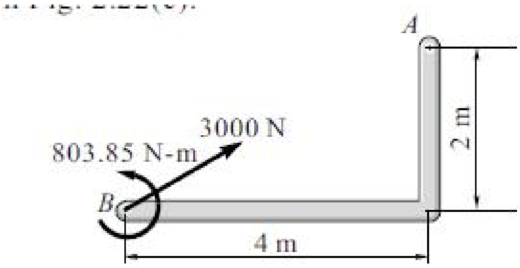


Taking Moment About 0 :

$$M_o = -600 \cos 30 \times 0.2 + 200 \sin 30 \times 0.2 = -43.92 \text{ N-m (Anticlockwise)}$$

Example 13 : Replace the force (3000 N) from point A by equivalent force couple at B. Refer fig :



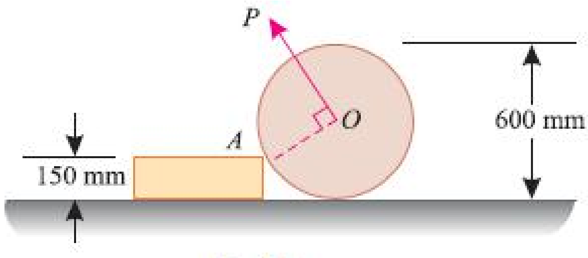


ANSWER :

Couple = Moment of forces about B

$\Sigma M_B = 3000 \sin 30 \times 4 - 3000 \cos 30 \times 2 = 803.85$ N-m

Example 14 : A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Fig. Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth .



Answer : Given : Diameter of wheel = 600 mm; Weight of wheel = 5 kN and height of the block = 150 mm.

Least pull required just to turn the wheel over the corner.

Let P = Least pull required just to turn the wheel in kN.

A little consideration will show that for the least pull, it must be applied normal to AO. The system of forces is shown in Fig. 3.9. From the geometry of the figure, we find that

$$\sin\theta = \frac{150}{300} = 0.5 \quad \text{or} \quad \theta = 30^\circ$$

and

$$AB = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm}$$

Now taking moments about A and equating the same,

$$P \times 300 = 5 \times 260 = 1300$$

$$\therefore \quad P = \frac{1300}{300} = 4.33 \text{ kN} \quad \textbf{Ans.}$$

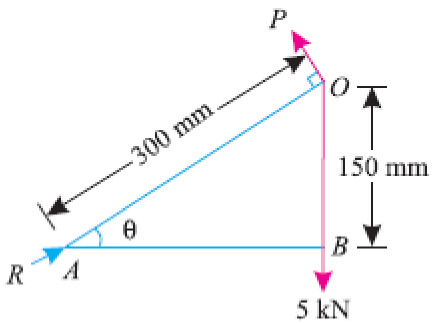


Fig. 3.9.

Reaction on the block

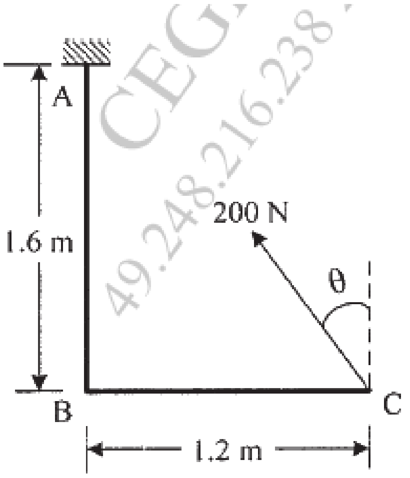
Let R = Reaction on the block in kN.

Resolving the forces horizontally and equating the same,

$$R\cos 30^\circ = P\sin 30^\circ$$

$$\therefore \quad R = \frac{P\sin 30^\circ}{\cos 30^\circ} = \frac{4.33 \times 0.5}{0.866} = 2.5 \text{ kN} \quad \textbf{Ans.}$$

EXAMPLE : The lever ABC fixed at A shown in Figure is subjected to a 200 N force at C at $\theta = 30^0$. Find the moment of this force about A. Also find the value of θ for which the moment about A is Zero



SOLUTION :

1) **Moment about A :**

M_A = (200 cos 30 X 1.2) - (200 sin 30 X 1.6) = 47.846 N-m

2) **Value Of ϴ :**

When $M_A = 0$

200 Cosϴ X 1.2 – 200 Sin ϴ X 1.6 = 0

240 Cosϴ = 320 Sinϴ

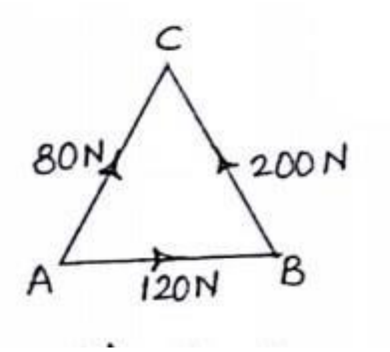
Sin ϴ / Cosϴ = 240 / 320

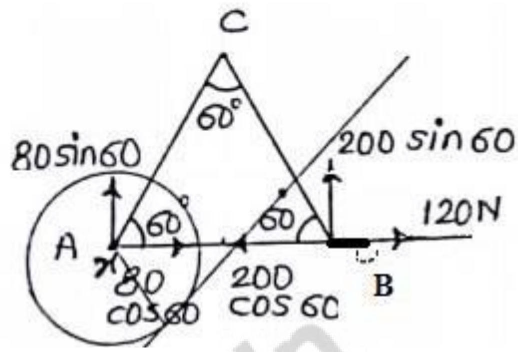
tanϴ = 0.75

ϴ = 36.57°

❖ RESULTANT OF NON-CONCURRENT FORCE SYSTEM :

Example15 : Calculate the resultant and its position wrt. point A for the force system shown in Figure No. AB = BC = CA = 2m





Answer :

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$$\Sigma Fx = 80Cos60 + 120 - 200Cos60 = 60 \text{ N}$$

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

$$\Sigma Fy = 80Sin60 + 200Sin60 = 242.49 \text{ N}$$

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

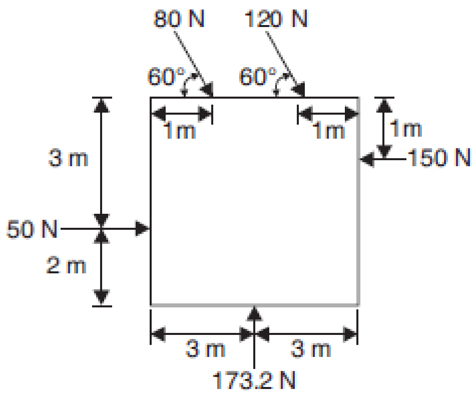
R = 249.80 N

Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$$\theta = 76.10°$$

Example16 : Determine the resultant forces acting on the board as shown in fig :



Answer :

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

$$\Sigma Fx = 80 \cos 60 + 120 \cos 60 - 150 + 50 = 0$$

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

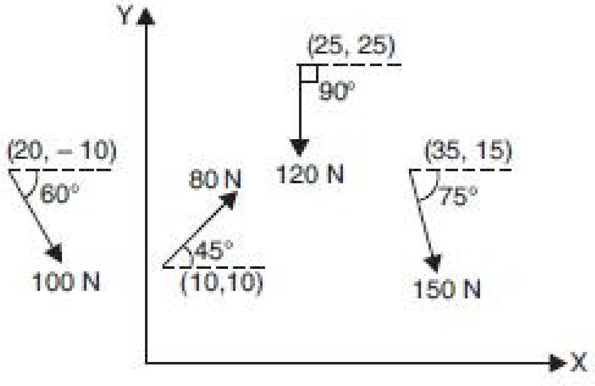
$$\Sigma Fy = -80 \sin 60 - 120 \sin 60 + 173.2 = 0$$

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

R = 0

Example 17 : Determine the resultant forces system as shown in fig



Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

ΣFx = 100 Cos 60 + 80 cos 45 +150 cos 75 = 145.4 N

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

ΣFy = -100 sin60 + 80 sin 45 – 120 - 150 sin 75 = -294.94 N

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

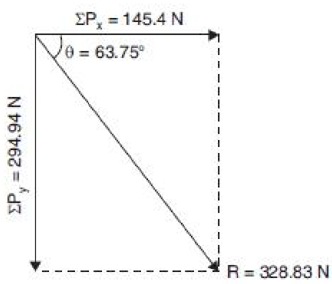
R = 328.83 N

Step 4 : Direction of Resultant :

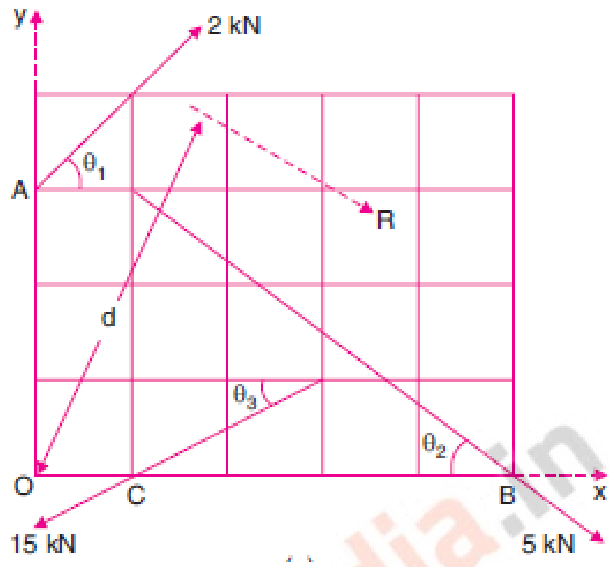
$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$\theta = 63.75°$

Since **ΣFy is negative ,** R lies in Fourth Quadrant :



Example 18 : Find the resultant of the system of coplanar forces acting on a lamina as shown in Fig. Each square has a side of 10 mm.



Answer :

Solution. If θ_1, θ_2 and θ_3 are the inclinations of forces 2 kN, 5 kN and 1.5 kN with respect to x-axis, then

$$\tan\theta_1 = \frac{10}{10} = 1 \qquad \therefore \quad \theta_1 = 45°$$

$$\tan\theta_2 = \frac{30}{40} \qquad \therefore \quad \theta_2 = 36.87°$$

and $$\tan\theta_3 = \frac{10}{20} \qquad \therefore \quad \theta_3 = 26.565°$$

Step 1: Summation of horizontal force: → (+Ve) ←(-Ve)

ΣFx = 2 Cos 45 + 5 cos 36.87 – 1.50 cos 26.565 = 4.072 KN

Step 2: Summation of vertical force : ↑ (+Ve) ↓ (-Ve)

ΣFy = 2 Sin 45 - 5 sin 36.87 - 1.5 sin 26.565 = -2.257 KN

Step 3 : Magnitude Of Resultant :

$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$$

R = 4.655 KN

Step 4 : Direction of Resultant :

$$\theta = \tan^{-1}\left|\frac{\Sigma Fy}{\Sigma Fx}\right|$$

$$\theta = 29\ ^{o}$$

By varignon theorem

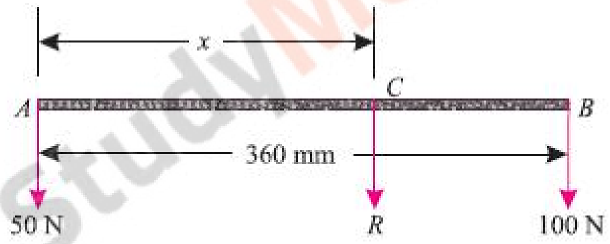
$$\Sigma M_0 = 2 \text{ Cos } 45 \text{ X } 30 + 5 \sin 36.87 \text{ X } 50 + 1.5 \sin 26.565 \text{ X } 10 = 199.13 \text{ KN-mm}$$

$$d = 199.13 / R$$

$$d = 42.8 \text{ mm}$$

METHODS FOR MAGNITUDE AND POSITION OF THE RESULTANT OF PARALLEL FORCES :

Example 19 : Two like parallel forces of 50 N and 100 N act at the ends of a rod 360 mm long. Find the magnitude of the resultant force and the point where it acts.



Answer :

Magnitude of the resultant force :

$$R = 50 + 100 = 150 \text{ N}$$

Point where the resultant force acts

Let x = Distance between the line of action of the resultant force (R) and A (*i.e.* AC) in mm. Now taking clockwise and anticlockwise moments of the forces about C and equating the same,

$50 \times x = 100\ (360 – x) = 36\ 000 – 100\ x$

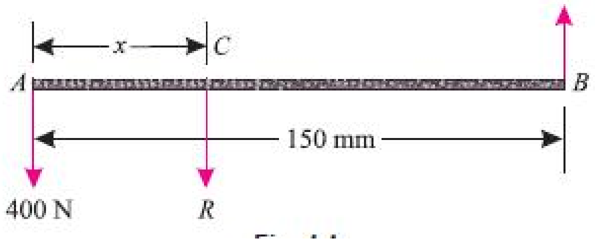
or $150\ x = 36\ 000$

x = 36000 \ 150

x = 240 mm

Example 20 : Two unlike parallel forces of magnitude 400 N and 100 N are acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.

Answer :



Magnitude of the resultant force

Since the given forces are unlike and parallel, therefore magnitude of the resultant force,

$$R = 400 - 100 = 300 \text{ N}$$

Point where the resultant force acts :

Let x = Distance between the lines of action of the resultant force and A in mm.

Now taking clockwise and anticlockwise moments about A and equating the same,

$$300 \times x = 100 \times 150 = 15\,000$$

$$x = 15000 / 300 = 50 \text{ mm}$$

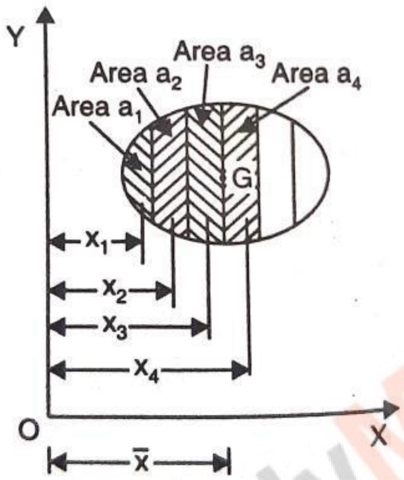
Centroid: It is defined as the point through which the entire area of a plane figure is assumed to act, for all positions of the lamina. e. g. Triangle, Square.

Centre of Gravity: It is defined as the point through which the whole weight of the body is assumed to act, irrespective of the position of a body.
e.g. Cone, Cylinder.

CENTER OF GRAVITY OF PLANE FIGURES BY THE METHOD OF MOMENTS :

Fig shows a plane figure of total area A whose centre of gravity is to be determined. Let this area A is composed of a number of small areas $a1$, $a2$, $a3$, $a4$, ……. etc.

A = $a1$, $a2$, $a3$, $a4$, + …….



Let 1 = The distance of the C.G of the area $a1$ from axis OY

$x2$ = The distance of the C.G of the area $a2$ from axis OY

$x3$ = The distance he C.G of the area $a3$ from axis OY

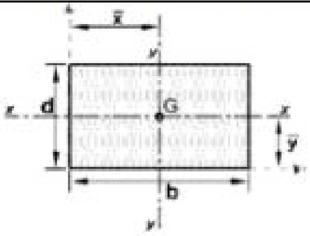
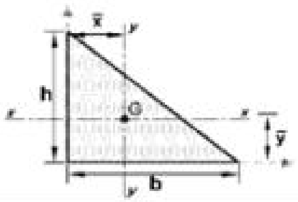
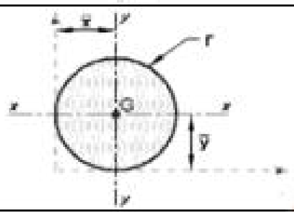
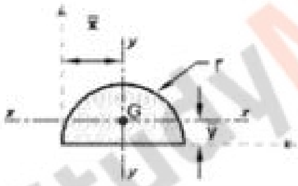
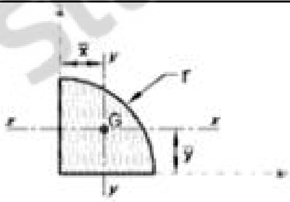
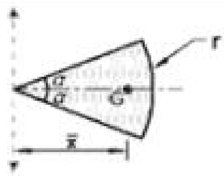
$x4$ = The distance of the C.G of the area $a4$ from axis OY and so

Let x and y be the co-ordinates of the centre of gravity with respect to some axis of reference, then

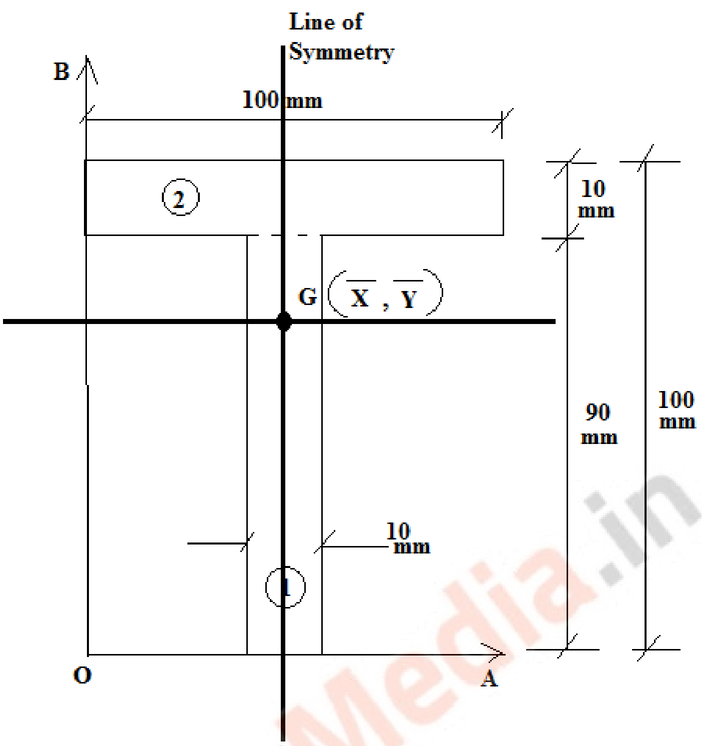
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots\ldots}{a_1 + a_2 + a_3 + \ldots}$$

where a1, a2, a3........ etc., are the areas into which the whole figure is divided x1, x2, x3 ..... etc.,are the respective co-ordinates of the areas a1, a2, a3....... on X-X axis with respect to same axis of reference.
y1, y2, y3....... etc., are the respective co-ordinates of the areas a1, a2, a3....... on Y-Y axis with respect to same axis of the reference

Area(Lamina) Element Centroid– Basic Shape				
Element name	**Geometrical Shape**	**Area**	$\bar{x}$	$\bar{y}$
Rectangle		bd	$\frac{b}{2}$	$\frac{d}{2}$
Triangle		$\frac{1}{2}bh$	$\frac{b}{3}$	$\frac{h}{3}$
Circle		πr^2	r	r
Semicircle		$\frac{\pi r^2}{2}$	r	$\frac{4r}{3\pi}$
Quarter circle		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Circular segment		αr^2 $(\alpha\ in\ radian)$	$\frac{2\ r\sin\alpha}{3\ \ \alpha}$	On Axis of Symmetry

Example21 : Locate the centroid of T section 100 X 100 X 10 mm having total depth of 100 mm.



1) Figure is symmetric @ y-y axis and hence,
   $\bar{x}$ = Maximum horizontal dimension /2
   = 100 / 2
   = 50 mm

2) Area calculation
$A_1 = 90 \text{ X } 10 = 900 \text{ mm}^2$
$A_2 = 100 \text{ X } 10 = 1000 \text{ mm}^2$
$A = A_1 + A_2 = 1900 \text{ mm}^2$

3) Location of $\bar{y}$
$y_1 = 90 / 2 = 45$ mm
$y_2 = 90 + (10/2) = 95$ mm

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A}$$

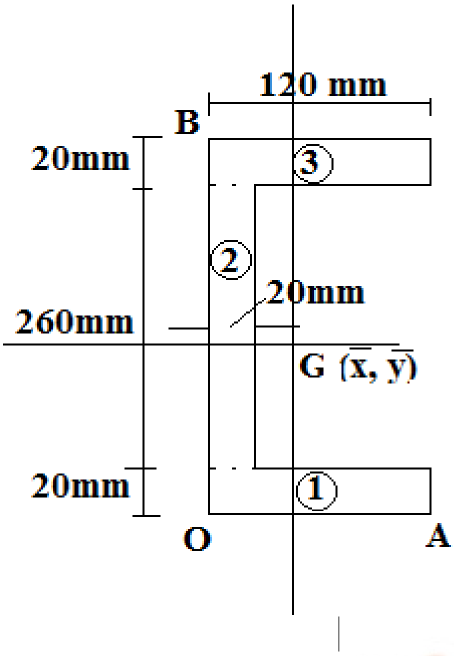
$$\bar{y} = \frac{(900X45) + (1000X95)}{1900}$$

$$\bar{y} = 71.315mm$$

Hence, centroid (G) for given section lies at G($\overline{x}, \overline{y}$)

= (50 mm from OB and 71.315 mm from OA)

Example22 : Find the centroid for a channel section as shown in figure.



1) Figure is symmetric @ x-x axis and hence,

$\bar{y}$ = Maximum vertical dimension /2

= 300 / 2

= 150 mm

2) Area calculation

$A_1 = A_3 = 120 \text{ X } 20 = 2400 \text{ mm}^2$

$A_2 = 260 \text{ X } 20 = 5200 \text{ mm}^2$

$A = A_1 + A_2 + A_3 = 10000 \text{ mm}^2$

3) Location of $\bar{x}$

$x_1 = x_3 = 120 / 2 = 60$ mm

$x_2 = 20 / 2 = 10$ mm

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$

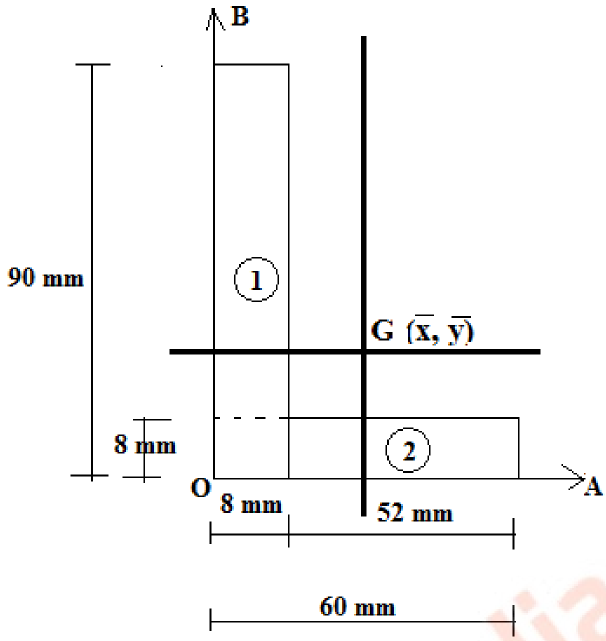
$$\bar{x} = \frac{(2400X60) + (5200X10) + (2400X60)}{10000}$$

$$\bar{x} = 34mm$$

Hence, centroid (G) for given section lies at G($\bar{x}, \bar{y}$)

= (34 mm from OB and 150 mm from OA)

Example23 : Find the centroid of L section 90 X 60 X 8 mm.



1) Area calculation

$A_1 = 90 \text{ X } 8 = 720 \text{ mm}^2$

$A_2 = 52 \text{ X } 8 = 416 \text{ mm}^2$

$A = A_1 + A_2 = 1136 \text{ mm}^2$

2) Location of $\bar{x}$

$x_1 = 8 / 2 = 4$ mm

$x_2 = 8 + (52/2) = 34$ mm

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A}$$

$$\bar{x} = 14.98mm$$

3) Location of $\bar{y}$

$y_1 = 90 / 2 = 45$ mm

$y_2 = 8 / 2 \quad = 4$ mm

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A}$$

$$\bar{y} = 29.98mm$$

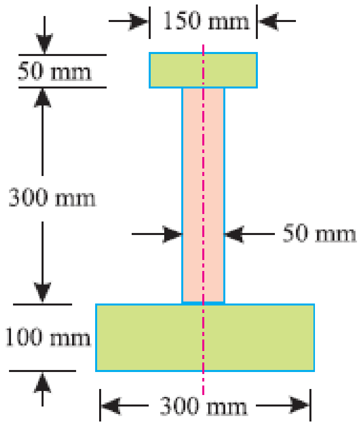
Hence, centroid (G) for given section lies at G($\overline{x}, \overline{y}$)

= (14.98 mm from OB and 29.98 mm from OA)

Example 24: I-section has the following dimensions in mm units :

Bottom flange = 300 × 100   Top flange = 150 × 50   Web = 300 × 50

Determine mathematically the position of centre of gravity of the section.



Let bottom of the bottom flange be the axis of referei

(*i*) *Bottom flange*

$$a_1 = 300 \times 100 = 30\,000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Web*

$$a_2 = 300 \times 50 = 15\,000 \text{ mm}^2$$

and $$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

(*iii*) *Top flange*

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

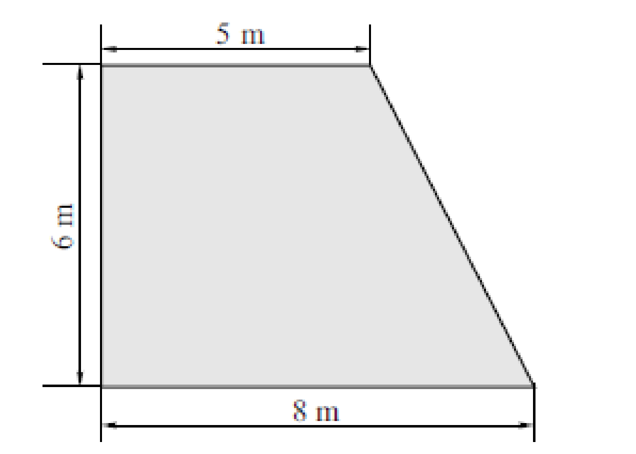
and $$y_3 = 100 + 300 + \frac{50}{2} = 425 \text{ mm}$$

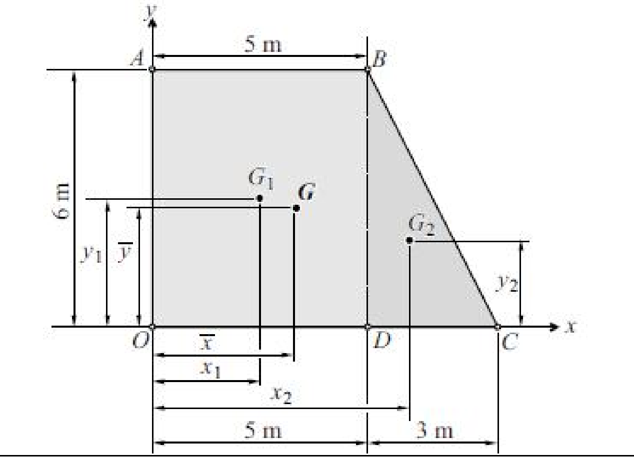
We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30\,000 \times 50) + (15\,000 \times 250) + (7500 \times 425)}{30\,000 + 15\,000 + 7500} = 160.7 \text{ mm} \quad \textbf{Ans.}$$

Example 25: Find the centroid of the shaded area in Fig. (a).





Answer :

1) Consider rectangle OABD :

$$A_1 = 5 \times 6 = 30 \text{ m}^2$$
$$x_1 = \frac{5}{2} = 2.5 \text{ m}$$
$$y_1 = \frac{6}{2} = 3 \text{ m}$$

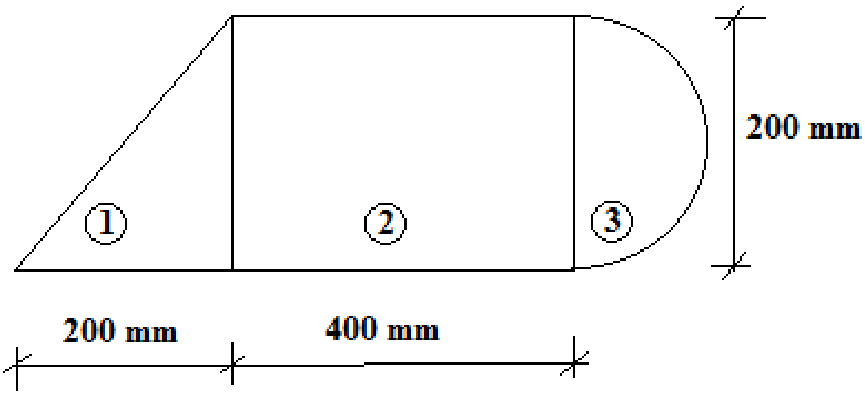
2) Consider triangle DBC :

$$A_2 = \frac{1}{2} \times 6 \times 3 = 9 \text{ m}^2$$
$$x_2 = 5 + \frac{3}{3} = 6 \text{ m}$$
$$y_2 = \frac{6}{3} = 2 \text{ m}$$

3) Centroid :

$$\bar{x} = \frac{30 \times 2.5 + 9 \times 6}{30 + 9} = 3.308 \text{ m}$$
$$\bar{y} = \frac{30 \times 3 + 9 \times 2}{30 + 9} = 2.769 \text{ m}$$

Example 26: Locate the centroid of lamina shown



1) Area calculation

$$A_1 = \frac{1}{2}\times 200\times 200 \quad = 20000 \text{ mm}^2$$

$$A_2 = 400 \text{ x } 200 \quad = 80000 \text{ mm}^2$$

$$A_3 = \frac{\pi\times(100)^2}{2} \quad = 15707.96 \text{ mm}^2$$

$$A = A_1 + A_2 + A_3 \quad = 115707.96 \text{ mm}^2$$

2) Location of $\bar{x}$ (from left side)

$$x_1 = \frac{2}{3}\times 200 = 133.33 \text{ mm}$$

$$x_2 = 200 + \frac{400}{2} = 400 \text{ mm}$$

$$x_3 = 200 + 400 + \left(\frac{4\times 100}{3\times\pi}\right) = 642.44 \text{ mm}$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$

$$= \frac{(20000\times 133.33)+(80000\times 400)+(15707.963\times 642.441)}{115707.963}$$

$$\bar{x} = 386.819 \text{ mm from left side.}$$

3) Location of $\bar{y}$ (from bottom)

$$y_1 = \frac{1}{3} \times 200 = 66.667 \text{ mm}$$

$$y_2 = \frac{200}{2} = 100 \text{ mm}$$

$$y_3 = \frac{200}{2} = 100 \text{ mm}$$

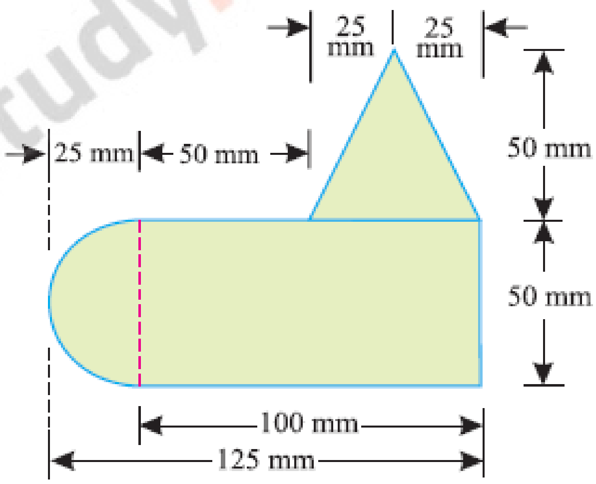
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A}$$

$$= \frac{(20000 \times 66.667) + (80000 \times 100) + (15707.963 \times 100)}{115707.963}$$

$\bar{y} = 94.238$ mm from bottom.

G = (386.819 mm, 94.238 mm)

Example 27: A uniform lamina shown in Fig. consists of a rectangle, a circle and a triangle.



Determine the centre of gravity of the lamina. All dimensions are in mm.

Let left edge of circular portion and bottom face rectangular portion be the axes of reference.

(*i*) *Rectangular portion*

$$a_1 = 100 \times 50 = 5000 \text{ mm}^2$$

$$x_1 = 25 + \frac{100}{2} = 75 \text{ mm}$$

and $$y_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Semicircular portion*

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(25)^2 = 982 \text{ mm}^2$$

$$x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4 \text{ mm}$$

and $$y_2 = \frac{50}{2} = 25 \text{ mm}$$

(*iii*) *Triangular portion*

$$a_3 = \frac{50 \times 50}{2} = 1250 \text{ mm}^2$$

$$x_3 = 25 + 50 + 25 = 100 \text{ mm}$$

and $$y_3 = 50 + \frac{50}{3} = 66.7 \text{ mm}$$

We know that distance between centre of gravity of the section and left edge of the circular portion,

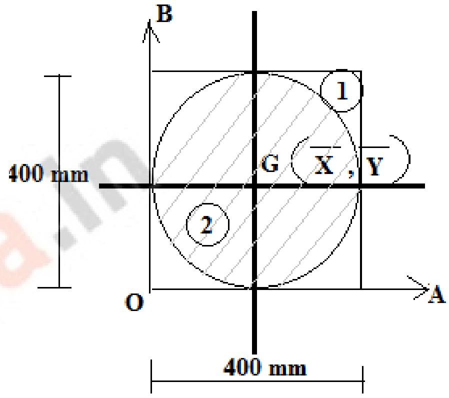
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250}$$

$$= 71.1 \text{ mm} \qquad \textbf{Ans.}$$

❖ CENTRE OF GRAVITY OF SECTIONS WITH CUT OUT HOLES :

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} \quad \text{and} \quad \bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

Example 28 : A square of 400 mm side from which a circle of 400 mm diameter is cutoff from the center. Find centroid of the remaining area.



1) Area calculation

$A_1 = 400 \text{ X } 400 = 160000 \text{ mm}^2$

$A_2 = (\pi / 4) \text{ X } (400)^2 = 125663.706 \text{ mm}^2$

$A = A_1 - A_2 = 34336.293 \text{ mm}^2$

2) Location of $\bar{x}$

$x_1 = 400 / 2 = 200 \text{ mm}$

$x_2 = 400 / 2 = 200 \text{ mm}$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A}$$

$$\bar{x} = 200 mm$$

3) Location of $\bar{y}$

$y_1 = 400 / 2 = 200 \text{ mm}$

$y_2 = 400 / 2 = 200 \text{ mm}$

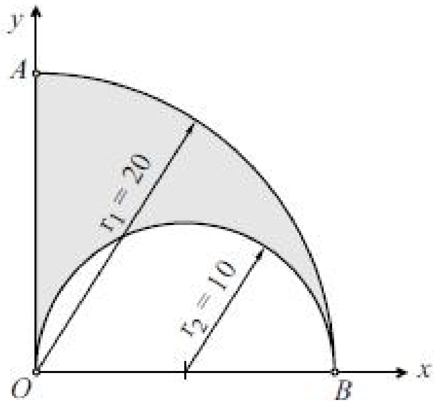
$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A}$$

$$\bar{y} = 200 mm$$

Hence, centroid (G) for given section lies at G($\bar{x}$,

= (200 mm from OB and 200 mm from OA)

Example29 : Find the coordinates of the centroid of the area shown in Fig..



(i) Divide the given area into two subareas as shown.

(ii) Quarter circle OAB : part ①

$$A_1 = \frac{\pi r_1^2}{4} = \frac{\pi \times 20^2}{4} = 314.16 \text{ cm}^2$$

$$x_1 = y_1 = \frac{4r_1}{3\pi} = \frac{4 \times 20}{3\pi}$$

$$\therefore x_1 = y_1 = 8.49 \text{ cm}$$

(iii) Semicircle OB : part ②

$$A_2 = \frac{\pi r_2^2}{2} = \frac{\pi \times 10^2}{2} = 157.08 \text{ cm}^2$$

$$x_2 = 10 \text{ cm}$$

$$y_2 = \frac{4r_2}{3\pi} = 4.244 \text{ cm}$$

(iv) Centroid of the given shaded area is given as

$$\overline{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{314.16 \times 8.49 - 157.08 \times 10}{314.16 - 157.08}$$

$$\overline{x} = 6.98 \text{ cm}$$

$$\overline{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{314.16 \times 8.49 - 157.08 \times 4.244}{314.16 - 157.08}$$

$$\overline{y} = 12.74 \text{ mm}$$

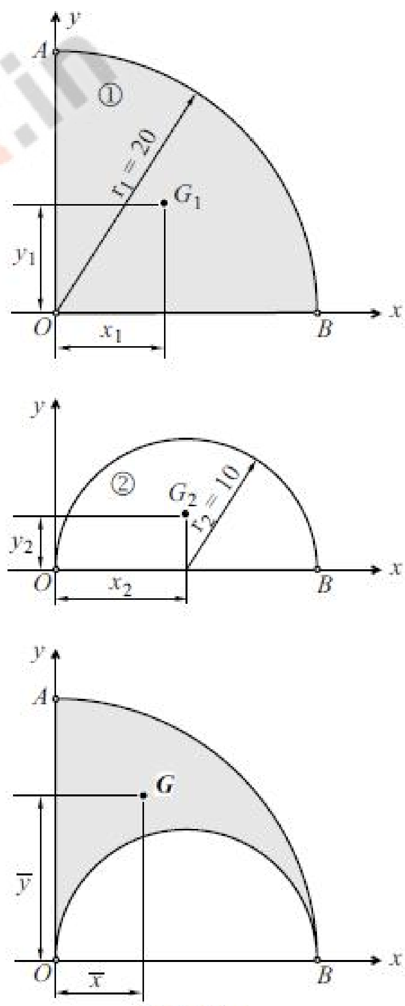
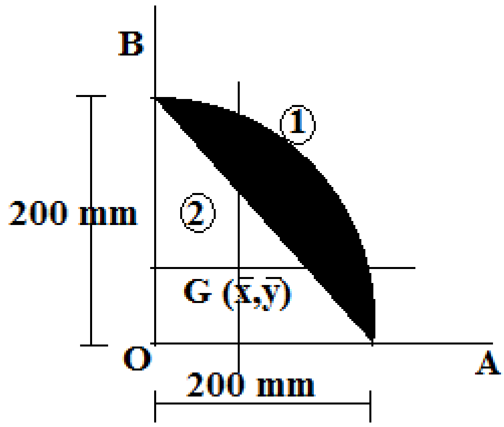


Fig. 5.3(b)

$\therefore$ Coordinates of centroid w.r.t. origin O are $\boldsymbol{G}$ (6.98, 12.74) cm. ***Ans.***

Example30 : Find the centroid of the shaded area of a lamina as shown in figure.



1) Let, Fig. 1 – Quarter circle and Fig. 2 – Triangle

$$A_1 = \frac{\pi r^2}{4} = \frac{\pi (200)^4}{4} = 31415.93 mm^2$$

$$A_2 = \frac{1}{2} bh = \frac{1}{2} X 200 X 200 = 20000 mm^2$$

$$A = A_1 - A_2 = 11415.93 mm^2$$

2) $\bar{x}$ calculation

$$x_1 = \frac{4r}{3\pi} = \frac{4X200}{3\pi} = 84.88 mm$$

$$x_2 = \frac{b}{3} = \frac{200}{3} = 66.67 mm$$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A} = \frac{(31415.93 X 84.88) - (20000 X 66.67)}{11415.93}$$

$$\bar{x} = 116.78 mm$$

3) $\bar{y}$ calculation

$$y_1 = \frac{4r}{3\pi} = \frac{4X200}{3\pi} = 84.88 mm$$

$$y_2 = \frac{b}{3} = \frac{200}{3} = 66.67 mm$$

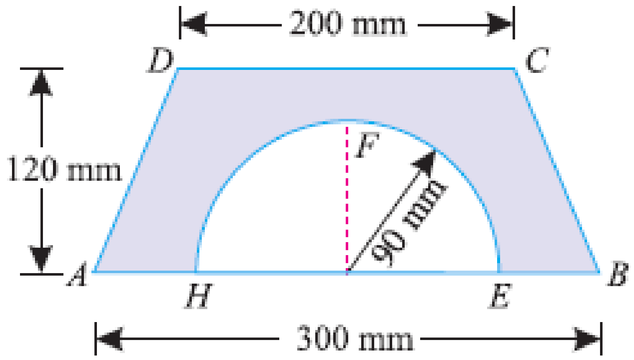
$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A} = \frac{(31415.93 X 84.88) - (20000 X 66.67)}{11415.93}$$

$$y = 116.78 mm$$

Hence, centroid (G) for given section lies at G($\overline{x, y}$)

= (116.78 mm from OB and 116.78 mm from OA)

Example31 : A semicircle of 90 mm radius is cut out from a trapezium as shown in Fig Find the position of the centre of gravity of the figure.



Solution. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Now consider two portions of the figure *viz.*, trapezium *ABCD* and semicircle *EFH*.

Let base of the trapezium *AB* be the axis of reference.

(*i*) *Trapezium ABCD*

$$a_1 = 120 \times \frac{200 + 300}{2} = 30\ 000\ \text{mm}^2$$

and
$$y_1 = \frac{120}{3} \times \left(\frac{300 + 2 \times 200}{300 + 200}\right) = 56\ \text{mm}$$

(*ii*) *Semicircle*

$$a_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times (90)^2 = 4050\pi\ \text{mm}^2$$

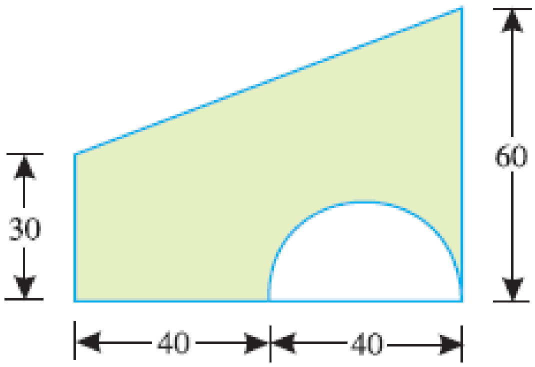
and
$$y_2 = \frac{4r}{3\pi} = \frac{4 \times 90}{3\pi} = \frac{120}{\pi}\ \text{mm}$$

We know that distance between centre of gravity of the section and *AB*,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(30\ 000 \times 56) - \left(4050\pi \times \frac{120}{\pi}\right)}{30\ 000 - 4050\pi}\ \text{mm}$$

$= 69.1$ mm **Ans.**

Example32 : A semicircular area is removed from a trapezium as shown in Fig. (dimensions in mm) Determine the centroid of the remaining area (shown hatched).



Solution. As the section in not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the area. Split up the area into three parts as shown in Fig. 6.25. Let left face and base of the trapezium be the axes of reference.

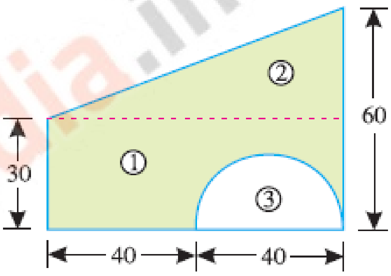


Fig. 6.25.

(i) *Rectangle*

$$a_1 = 80 \times 30 = 2400 \text{ mm}^2$$

$$x_1 = \frac{80}{2} = 40 \text{ mm}$$

and $$y_1 = \frac{30}{2} = 15 \text{ mm}$$

(ii) *Triangle*

$$a_2 = \frac{80 \times 30}{2} = 1200 \text{ mm}^2$$

$$x_2 = \frac{80 \times 2}{3} = 53.3 \text{ mm}$$

and $$y_2 = 30 + \frac{30}{3} = 40 \text{ mm}$$

(iii) *Semicircle*

$$a_3 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(20)^2 = 628.3 \text{ mm}^2$$

$$x_3 = 40 + \frac{40}{2} = 60 \text{ mm}$$

and $$y_3 = \frac{4r}{3\pi} = \frac{4 \times 20}{3\pi} = 8.5 \text{ mm}$$

We know that distance between centre of gravity of the area and left face of trapezium,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 40) + (1200 \times 53.3) - (628.3 \times 60)}{2400 + 1200 - 628.3}$$

$= 41.1$ mm **Ans.**

Similarly, distance between centre of gravity of the area and base of the trapezium,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 15) + (1200 \times 40) - (628.3 \times 8.5)}{2400 + 1200 - 628.3}$$

$= 26.5$ mm **Ans.**

❖ **Moment of Inertia (M. I)**

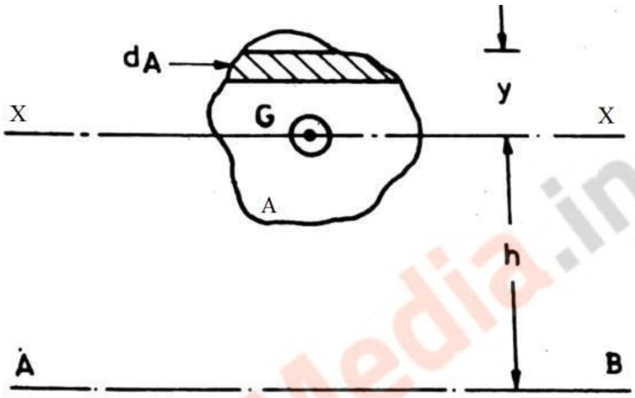
Moment of inertia of a body about any axis is equal to the product of the area of the body and square of the distance of its centroid from that axis.

OR

Moment of inertia of a body about any axis is defined as the sum of second moment of all elementary areas about that axis.

Unit- mm^4, cm^4, m^4

❖ **Parallel axis theorem:** It states, the moment of inertia of a plane section about any axis parallel to the centroidal axis is equal to the moment of inertia of the section about the centroidal axis plus the product of the area of the section and the square of the distance between the two axes.



$$I_{AB} = I_{XX} + Ah^2$$

Where,

I_{AB} =MI about axis AB which is parallel to XX axis.

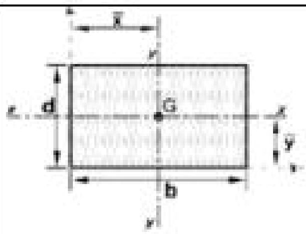
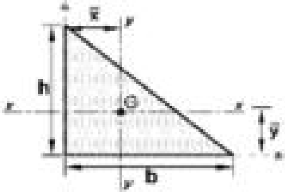
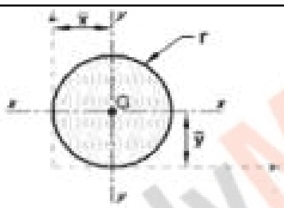
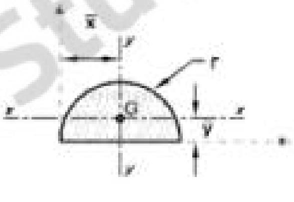
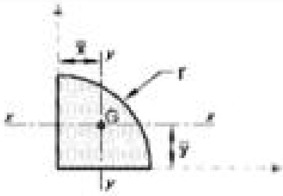
I_{xx} = MI about horizontal centroidal axis.

A= Area of the section.

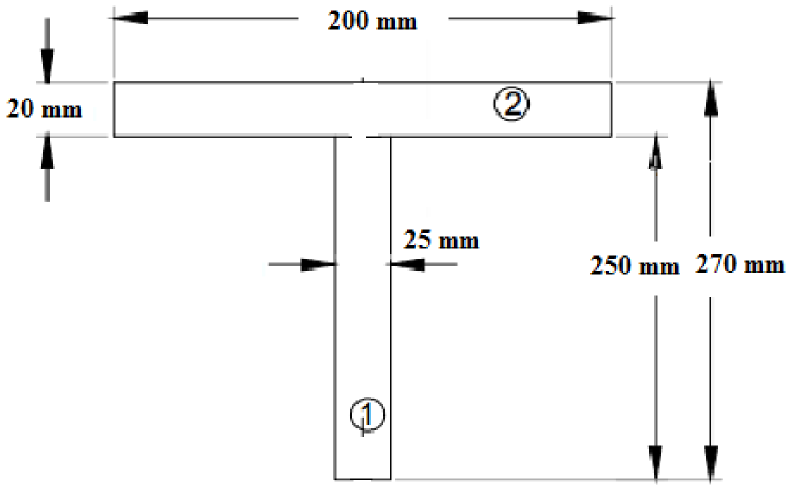
h = Distance between the two axes AB and XX .

❖ **Perpendicular Axis Theorem:** It states that, if I_{XX} and I_{YY} are the moments of inertia of a plane section about the two mutually perpendicular axes meeting at O, then the moment of inertia I_{ZZ} about the third axis ZZ perpendicular to the plane and passing through the intersection of XX and YY is equal to addition of moment of inertia about X-X and Y-Y axes.

$$I_{zz} = I_{xx} + I_{yy}$$

Area (Lamina) Element – Moment of Inertia (Basic Shape)				
Element name	**Geometrical Shape**	**Area**	I_{xx}	I_{yy}
Rectangle		bd	$\frac{bd^3}{12}$	$\frac{db^3}{12}$
Triangle		$\frac{1}{2}bh$	$\frac{bh^3}{36}$	$\frac{hb^3}{36}$
Circle		πr^2	$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{64}$
Semicircle		$\frac{\pi r^2}{2}$	$0.11\ r^4$	$\frac{\pi d^4}{128}$
Quarter circle		$\frac{\pi r^2}{4}$	$0.055\ r^4$	$0.055\ r^4$
d= diameter				

Example33 : Calculate the moment of inertia about its both centroidal axes of a T – section having flange 200 mm x 20 mm and web 250 mm x 25 mm. Overall depth is 270 mm.



Solution:

As given T-section is symmetrical about Y-Y axis,

$$\bar{x} = \frac{\text{Flange width}}{2} = \frac{200}{2} = 100\,\text{mm}$$

$$A_1 = 250 \times 25 = 6250\,\text{mm}^2$$

$$A_2 = 200 \times 20 = 4000\,\text{mm}^2$$

$$y_1 = \frac{250}{2} = 125\,\text{mm}$$

$$y_2 = 250 + \frac{20}{2} = 260\,\text{mm}$$

$$\bar{y} = \frac{(A_1 \times y_1) + (A_2 \times y_2)}{(A_1 + A_2)}$$

$$= \frac{(6250 \times 125) + (4000 \times 260)}{(6250 + 4000)}$$

$$= 177.683\,\text{mm from bottom}$$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= (I_{G_1} + A_1 \times h_1^2) + (I_{G_2} + A_2 \times h_2^2)$$

Here, $h_1 = \bar{y} - y_1 = 177.68 - 125 = 52.68\,\text{mm}$

$h_2 = y_2 - \bar{y} = 260 - 177.68 = 82.32\,\text{mm}$

$$I_{xx} = \left(\frac{bd^3}{12} + A_1 \times h_1^2\right) + \left(\frac{bd^3}{12} + A_2 \times h_2^2\right)$$

$$= \left(\frac{25 \times 250^3}{12} + (6250 \times 52.68^2)\right) + \left(\frac{200 \times 20^3}{12} + (4000 \times 82.32^2)\right)$$

$$\boxed{I_{xx} = 77.136 \times 10^6\,\text{mm}^4}$$

$$I_{yy} = I_{yy_1} + I_{yy_2}$$
$$= \left[\frac{db^3}{12}\right]_1 + \left[\frac{db^3}{12}\right]_2$$
$$= \frac{250\times25^3}{12} + \frac{20\times200^3}{12}$$
$$\boxed{I_{yy} = 13.658\times10^6\ \text{mm}^4}$$

Example 34: Calculate the moment of inertia of a L- section about the XX axis passing through the center of gravity for a section as shown in fig. No.

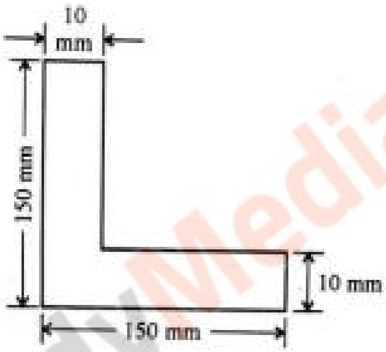


Fig. 1

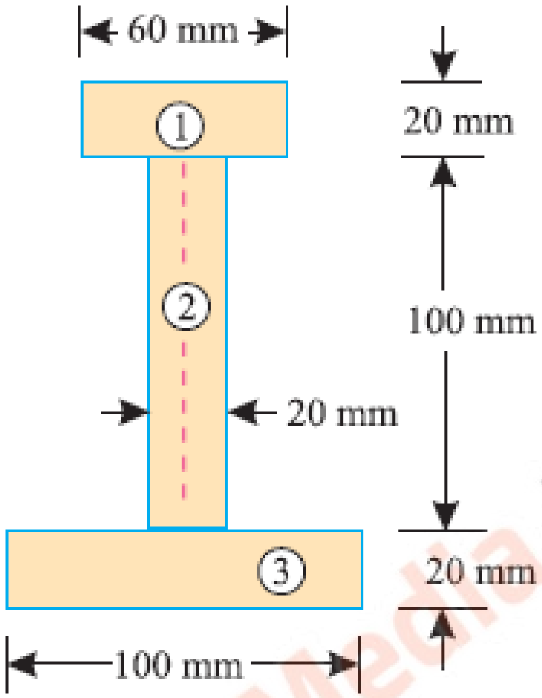
$$a_1 = 150\times10 = 1500\ mm^2 \quad y_1 = \frac{150}{2} = 5mm$$

$$a_2 = 10\times140 = 1400mm^2 \quad y_2 = 10 + \frac{140}{2} = 80mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(1500\times5)+(1400\times80)}{1500+1400} = 41.21mm\ from\ the\ base.$$

$$I_{xx} = I_{xx_1} + I_{xx_2} = (I_G + ah^2)_1 + (I_G + ah^2)_2$$
$$I_{xx} = \left(\frac{bd^3}{12} + ah^2\right)_1 + \left(\frac{bd^3}{12} + ah^2\right)_2$$
$$I_{xx} = \left(\frac{150\times10^3}{12} + \left(1500\times(41.21-5)^2\right)\right)_1 + \left(\frac{10\times140^3}{12} + \left(1400\times(80-41.21)^2\right)\right)_2$$
$$I_{xx} = (1979246.15)_1 + (4393196.407)_2$$
$$I_{xx} = 6372442.557mm^4$$

Example35 : An I-section is made up of three rectangles as shown in Fig. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.



(*i*) *Rectangle* 1

$$a_1 = 60 \times 20 = 1200 \text{ mm}$$

and $$y_1 = 20 + 100 + \frac{20}{2} = 130 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

(*iii*) *Rectangle* 3

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_3 = \frac{20}{2} = 10 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis,

$$= I_{G1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2] = 5786 \times 10^3 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{20 \times (100)^3}{12} = 1666.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis,

$$= I_{G2} + a_2 h_2^2 = (1666.7 \times 10^3) + [2000 \times (9.2)^2] = 1836 \times 10^3 \text{ mm}^4$$

Now moment of inertia of rectangle (3) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G3} = \frac{100 \times (20)^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (3) and X-X axis,

$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

∴ Moment of inertia of rectangle (3) about X-X axis,

$$= I_{G3} + a_3 h_3^2 = (66.7 \times 10^3) + [2000 \times (50.8)^2] = 5228 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3) = 12\,850 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

We know that the distance between centre of gravity of the section and bottom face,

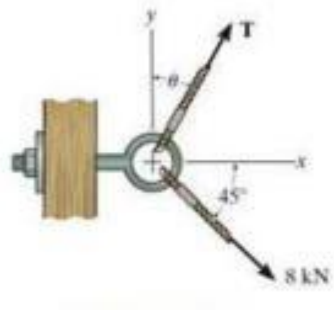
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} \text{ mm}$$

$$= 60.8 \text{ mm}$$

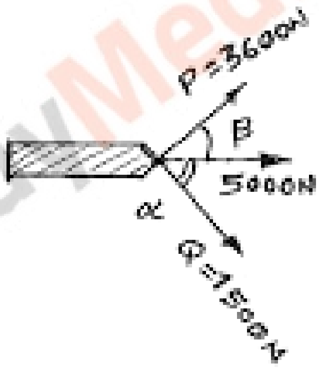
Question Bank

1.State and derive "Law of Parallelogram" of forces with sketch

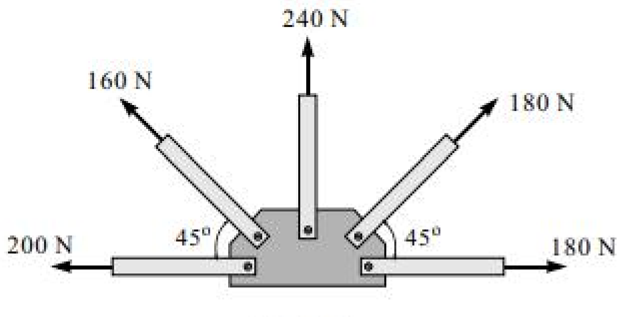
2. If $\Phi = 30^0$ and T = 6 KN , determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



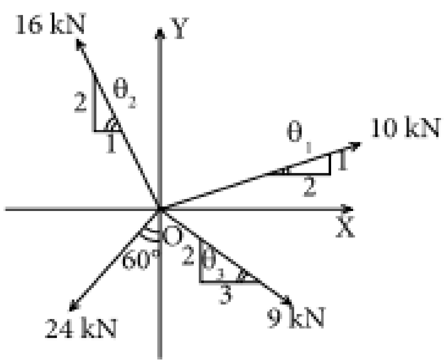
3.Resultant force R = 5000 N has two component forces 'P' = 3600 N and 'Q' = 1500 N as shown in Fig. . Determine direction of component forces 'P' and 'Q' w. r. to resultant force 'R'



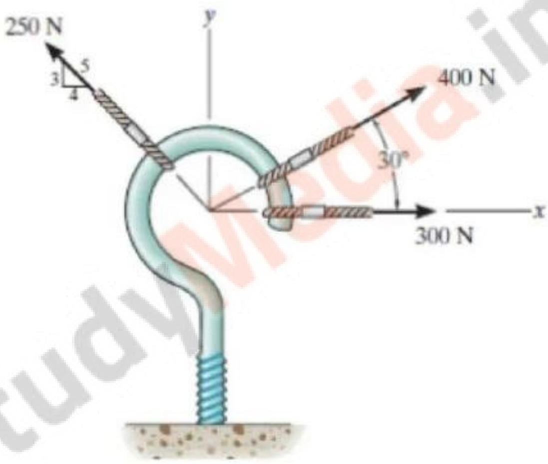
4 A gusset plate of roof truss is subjected to five forces as shown in fig. Determine the magnitude and direction of the resultant force.



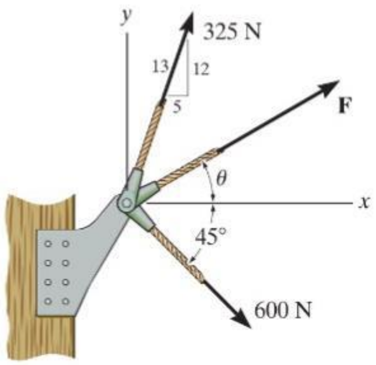
5 Determine the resultant of the concurrent coplanar force system acting at point ‘O’ as shown in fig



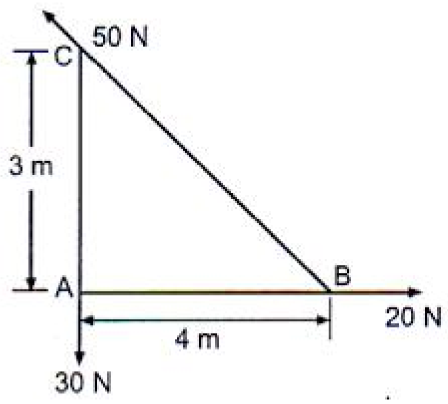
6.Determine the magnitude and direction of the resultant force of the system as shown in fig.



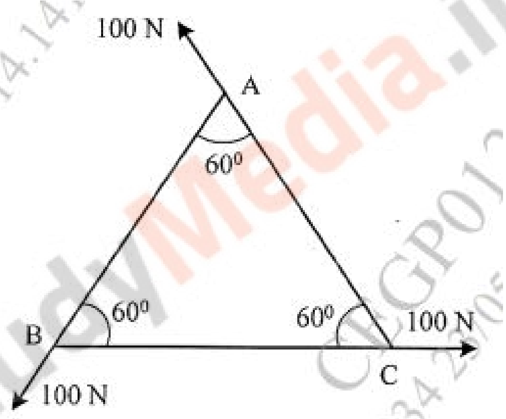
7. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis. Determine the magnitude of F and its direction



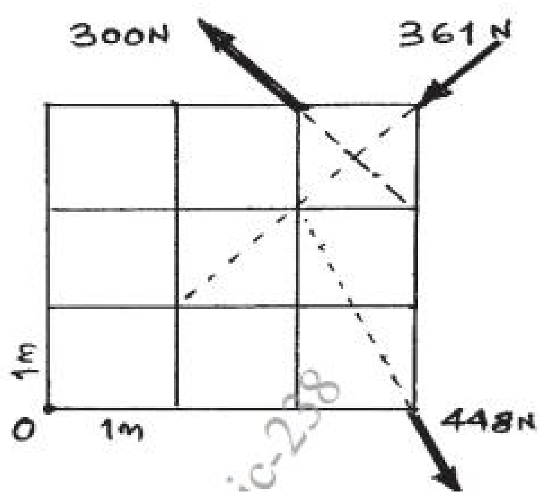
7. Determine resultant of the force system as shown below w.r.to A. [Forces acting are 50, 20, 30 N]



8: Determine the magnitude and direction of resultant with reverence to point A for the force system as shown in Fig. if side of equilateral triangle is 1m.



9 .Determine the magnitude, direction and position of the resultant force for the given three forces acting as shown in Fig., w.r. to 'O'

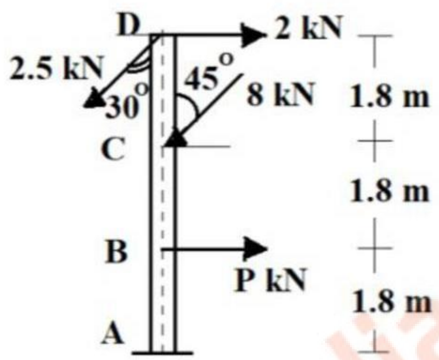


10.State and derive Varignon's theorem

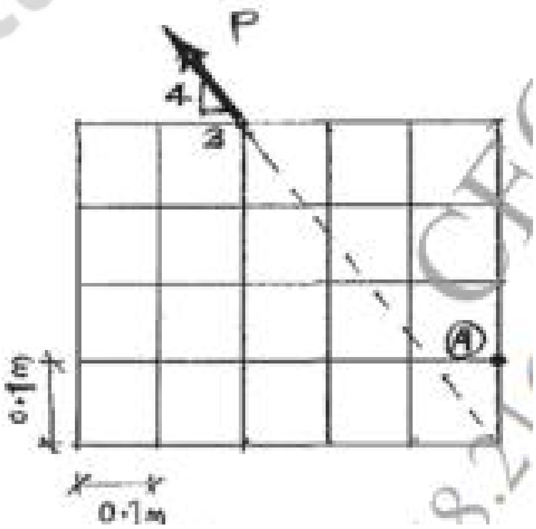
11.What is Couple? Give any three characteristics of couple with sketch.

12.Diffrentiate Moment and Couple with a sketch

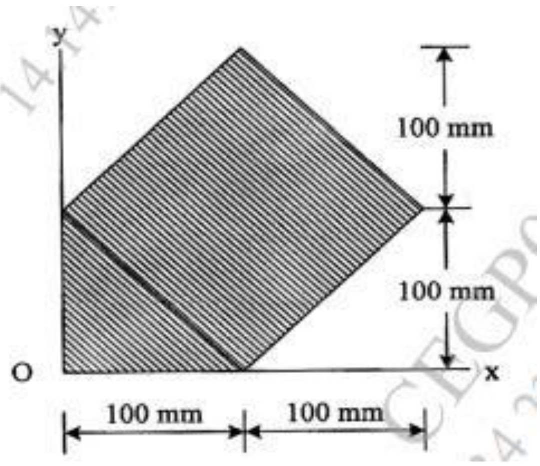
13.Determine the magnitude and sense of a horizontal force 'P' to be applied at 'B' which will keep the vertical rod ABCD in equilibrium as shown in Fig. . Take length AB = BC = CD = 1.8 m.



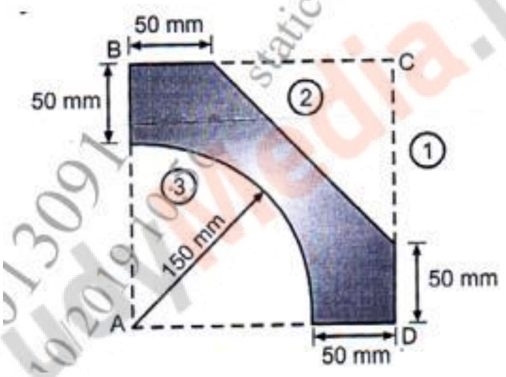
14. Force 'P' is acting on the plate which is divided in to squares of 0.1 m as shown in Fig. . The moment of force 'P' about point 'A' is 30 Nm clockwise. Determine the magnitude of force 'P



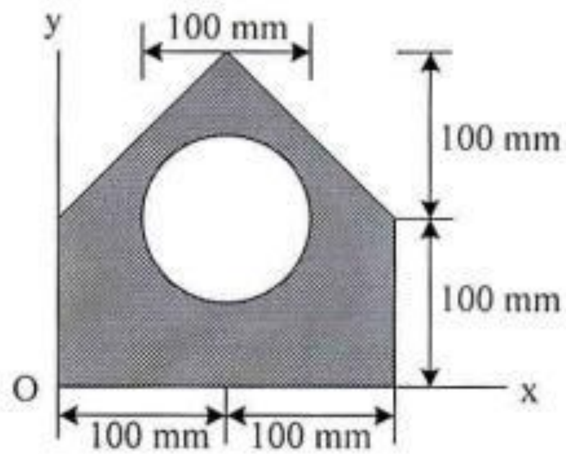
15. Define centroid and center of gravity .Determine the position of centroid of the shaded area with respect to origin O as shown in fig



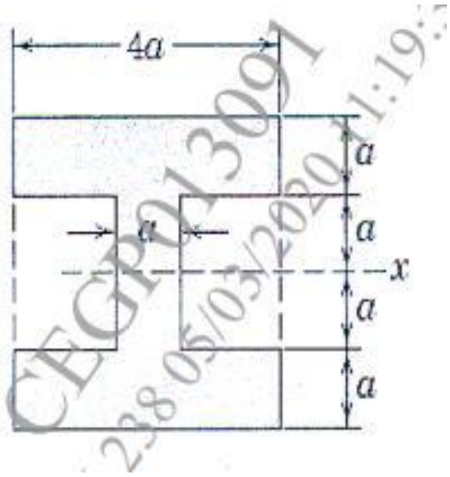
16 . Analyze and locate the position of centroid for the plane lamina as shown in Fig. w.r.to 'A'.



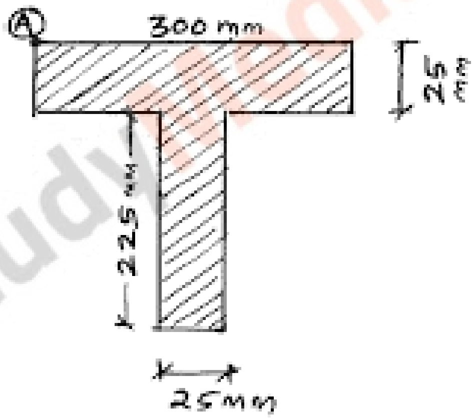
17. Determine the y coordinate of centroid of the shaded area as shoen in fig



18. Define moment of inertia and determine the M. I. of the composite Figure, if a = 40 mm with respect to x - axis as shown in Fig. a



19 Analyze and locate the position of centroid for the plane lamina as shown in Fig. w.r.to 'A'. Also determine the moment of inertia of the shaded portion with respect to x-x axis (horizontal) passing through the centroid



20. Analyze and locate the position of centroid for the plane lamina as shown in Fig. w.r.to 'O'. Also determine the moment of inertia of the shaded portion with respect to y-y axis (vertical) passing through the centroid