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## UNIT - IV

### \* Impedance:

- The opposition provided by AC circuit to the flow of current is called as Impedance.

$$I = \frac{V}{R}$$

$Z = \frac{V}{I}$  - opposition by resistor - Resistance  $R$

$I = \frac{V}{X_L}$  - opposition by inductor - inductive reactance  $X_L$

$Z = \frac{V}{I}$  - opposition by capacitor - Capacitive reactance  $X_C$

- combined notation for  $X_L$  &  $X_C$  is  $X$

Impedance is combination of  $R \& X$ . Unit -  $\Omega$

$$Z = R + jX \quad \begin{matrix} \text{real part} \\ \text{imaginary part} \end{matrix} \quad \text{- Rectangular}$$

$$Z = |Z| L \phi \quad \begin{matrix} \uparrow \text{magnitude} \\ \uparrow \text{phase} \end{matrix} \quad \text{- Polar}$$

$$\text{magnitude} = \sqrt{R^2 + X^2}$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\text{phase} = \tan^{-1} \left[ \frac{\text{imaginary part}}{\text{Real part}} \right]$$

$$\phi = \tan^{-1} \left[ \frac{X}{R} \right]$$

## \* Impedance Triangle:

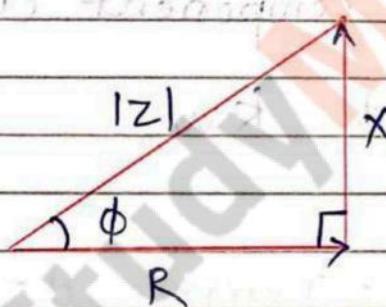
- It is used to draw graphical representation of impedance.

$$Z = R + jX \quad \& \quad |Z| = \sqrt{R^2 + X^2}$$

Here,  $j$  terms indicates that  $R$  &  $X$  are perpendicular to each other. Thus  $R$  &  $X$  form two sides of triangle.

According to pythagoras, third side (hypotenuse)

$$= \sqrt{R^2 + X^2}$$



Angle betw  $|Z|$  &  $R$  is  $\phi$

$$R = |Z| \cos \phi \quad - \text{Horizontal line}$$

$$X = |Z| \sin \phi \quad - \text{Vertical line}$$

$$\tan \phi = \frac{X}{R}$$

$$\phi = \tan^{-1} \left[ \frac{X}{R} \right]$$

## \* Types of power in AC circuit

### 1. Apparent Power (S)

- It is defined as product of RMS value of voltage and RMS value of current.

$$S = V_{\text{rms}} \times I_{\text{rms}} \cdot \text{VA / kVA}$$

### 2. Real or Active Power (P)

- It is defined as the average power taken by AC circuit or product of supply voltage and active component current ( $I \cos \phi$ )

$$P = VI \cos \phi \text{ kW}$$

P - real power

V =  $V_{\text{rms}}$ , I =  $I_{\text{rms}}$ ,  $\phi$  = phase betw. V & I

### 3. Imaginary or Reactive power (Q)

- It is defined as product of supply voltage and reactive component current ( $I \sin \phi$ )

$$Q = VI \sin \phi$$

## \* Power factor ( $\cos\phi$ )

i)  $P \cdot F = \frac{\text{True Power}}{\text{Apparent power}} = \frac{P}{S}$

$$P = VI \cos\phi, S = VI$$

$$P.F. = \frac{VI \cos\phi}{VI} = \cos\phi$$

ii)  $P \cdot F = \cos\phi = \frac{R}{Z}$

iii)  $P \cdot F = \cos\phi = \frac{\text{fundamental comp. of current}}{\text{RMS value of total current}}$

## \* Types of power factor.

### 1) Lagging P.F. :

- when current is lagging behind voltage.  
purely inductive ckt. current lags by  $90^\circ$ .  
 $\phi = 90^\circ$

$$\cos\phi = \cos 90^\circ = 0 \text{ zero lagging p.f.}$$

### 2) Leading p.f

- current leads voltage.

Purely capacitive ckt.

$$P \cdot F = \cos\phi = \cos 90^\circ = 0, \text{ zero leading p.f}$$

### III) Unity power factor

- No phase diff. b/w  $V$  &  $I$ .

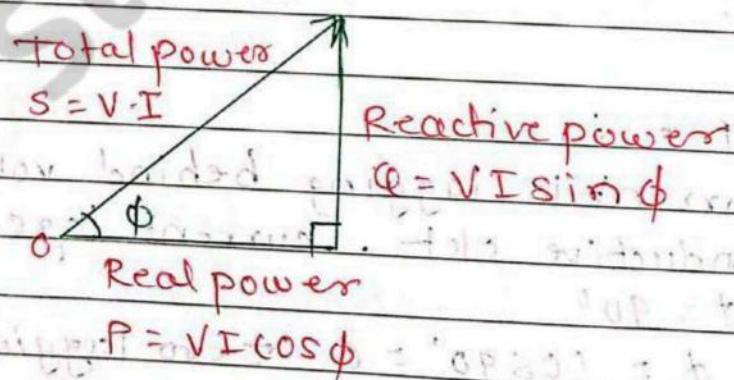
- purely resistive

$$P.F = \cos \phi = \cos(0) = 1$$

### \* Importance of P.F.

- Ideally p.f must be equal to 1.
- High P.F  $\uparrow$  current carrying capacity
- power losses reduced.
- high p.f indicates good power utilization of generator capacity
- P.F play imp. role in voltage regulation

### \* Power Triangle:-

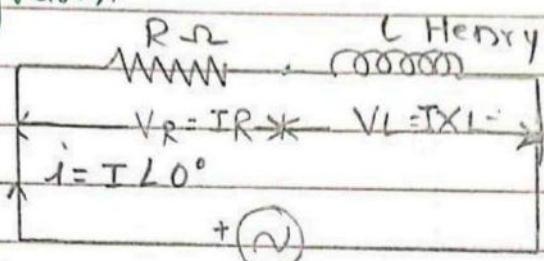


$$\tan \phi = \frac{Q}{P}$$

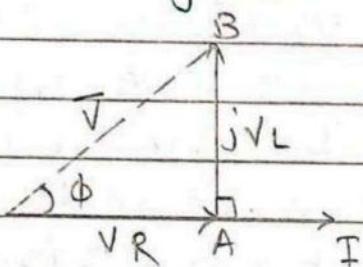
$$\phi = \tan^{-1} \left( \frac{Q}{P} \right)$$

## \* R-L Series Circuit

i) circuit diagram.



ii) phasor diagram.



i) Taking  $\bar{I} = IL \angle \theta$  as ref. phasor, OA & AB represent  $V_R$  &  $V_L$  resp.

ii)  $V_R$  is in phase with  $I_L$  while  $V_L$  leads  $I_L$  by  $90^\circ$

applied voltage  $\bar{V} = \bar{V}_R + \bar{V}_L$  as per KVL law.

iii) Voltage triangle ( $V-\Delta$ )

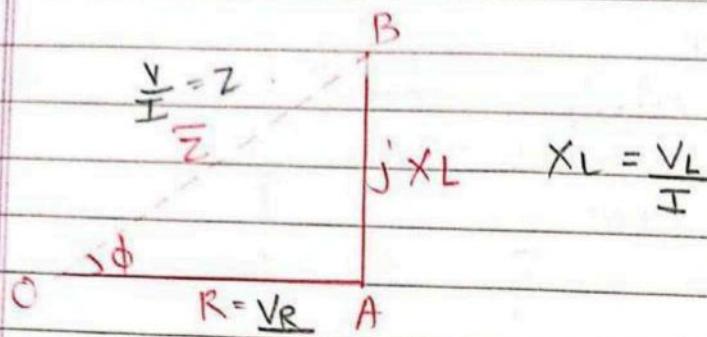
The  $\Delta OAB$  in phasor diagram is called as  $V-\Delta$ .

$$\bar{V} = (V_R + jV_L) = V \angle \phi$$

$$V = \sqrt{V_R^2 + V_L^2} = I \cdot Z$$

$$P.F = \cos \phi = \left( \frac{V_R}{V} \right) \text{ lagging}$$

## v) Impedance Triangle.

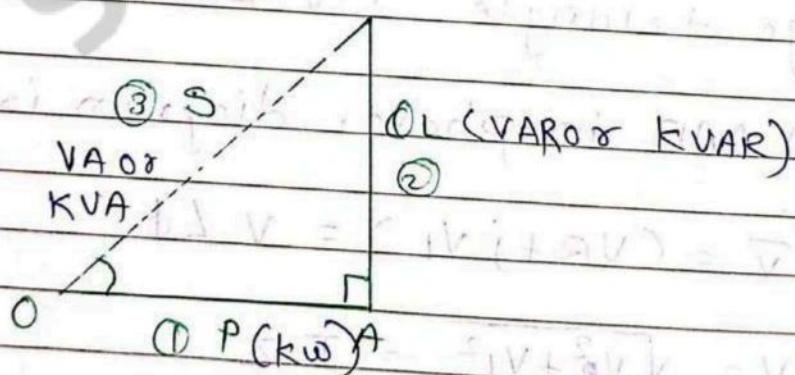


We know,  $\frac{V}{I} = V_R = I \cdot R$ ,  $V_L = I \cdot X_L$ ,  $V = IZ$ .  
 If we devide each side of  $V - \Delta$  by common factor  $I$  & assuming  $I > 1$ , we will get a similar  $\Delta$  known as  $z - \Delta$ , as each sides represents opposition as shown.  $z$  is called total opposition to the current.

$$z = (R + jX_L) = z \angle \phi$$

$$z = \sqrt{R^2 + X_L^2} \cdot \text{pf} = \cos \phi = \left(\frac{R}{z}\right) \text{ lagging}$$

## v) Power Triangle (P-Δ)



If we multiply each side by  $V\Delta$  by a common factor  $I$  & assuming  $I > 1$ , we will get similar but larger  $\Delta$  shown,

$$\text{KVA} = \text{KVA} L + \phi,$$

$$\text{KVA} = \sqrt{(\text{KW})^2 + (\text{KVAR})^2}$$

p.f  $\Rightarrow \left( \frac{\text{KW}}{\text{KVA}} \right)$  lagging

$$S = V \times I = I^2 Z$$

$$\text{Q}_L = V_L \cdot I = I^2 \cdot X_L = VI \sin \phi$$

$$\text{P} = V_R \cdot I = I^2 R = VI \cos \phi$$

vi) Power factor.

$$\text{o.f} = \cos \phi$$

$$\left( \frac{V_R}{V} \right) \Rightarrow V - \Delta$$

$$\left( \frac{R}{Z} \right) \Rightarrow Z - \Delta$$

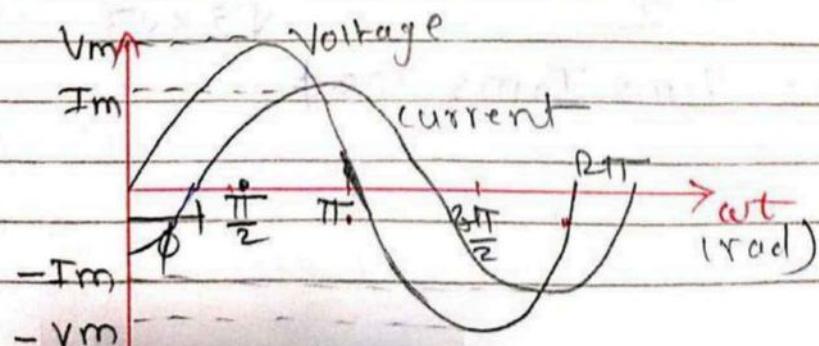
$$\left( \frac{\text{KW}}{\text{KVA}} \right) \Rightarrow \left( \frac{\text{active power}}{\text{apparent power}} \right) \text{ from } P - \Delta.$$

vii) equation of i & v

$$i = I_m \sin \omega t$$

$$v = V_m \sin(\omega t + \phi)$$

viii) Waveform



## Power

$$P = V \times I$$

$$= V_m \sin(\omega t) \times I_m \sin(\omega t - \phi)$$

$$= V_m \cdot I_m \cdot \sin(\omega t) \sin(\omega t - \phi) \quad \text{--- (1)}$$

Use formula

$$\sin A \cdot \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\sin \omega t \cdot \sin(\omega t - \phi)$$

$$= \frac{1}{2} [\cos(\omega t - (\omega t - \phi)) - \cos(\omega t + \omega t - \phi)]$$

$$= \frac{1}{2} [\cos \phi - \cos(2\omega t - \phi)]$$

put this value in eqn (1)

$$P = \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t - \phi)]$$

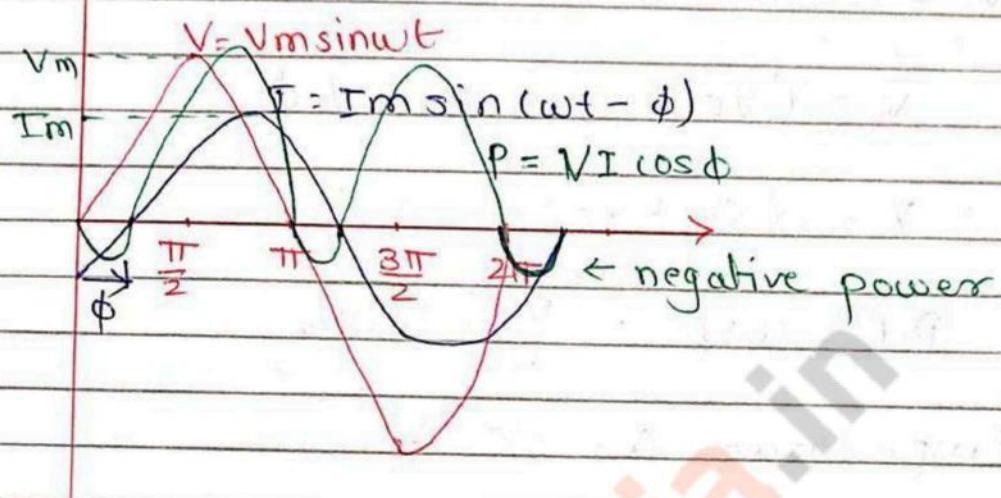
$$P = \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

The second term is cosine term whose average power value over a complete cycle is zero

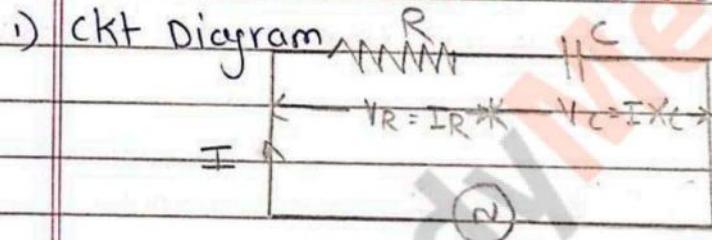
$$\therefore P_{av} = \frac{V_m I_m}{2} \cos \phi = \frac{V_m I_m}{\sqrt{2} \times \sqrt{2}} \cos \phi$$

$$P_{av} = V_{rms} I_{rms} \cos \phi$$

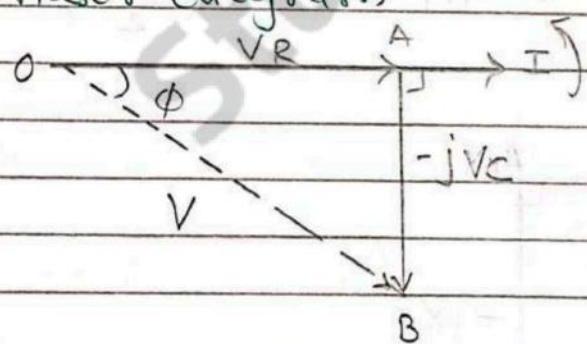
## Power Waveform.



## \* R-C Series Circuit.



2) phasor diagram



$\bar{I} = I \angle 0^\circ$ , OA & OB represent  $V_R$  &  $V_C$  resp.

$V_R$  leads in phase with  $I$  &  $V_C$  lags  $I$  by  $90^\circ$

$$\bar{V} = \bar{V}_R + \bar{V}_C \text{ as per kvl.}$$

### iii) Voltage triangle ( $V-\Delta$ )

$V-\Delta$ , since 3 sides of  $\Delta$  represent voltage.

$$\bar{V} = (V_R - jV_C) = V L - \phi$$

$$V = \sqrt{V_R^2 + V_C^2} = I \cdot Z$$

$$P.f = \cos\phi = \left(\frac{V_R}{V}\right) \text{ leading}$$

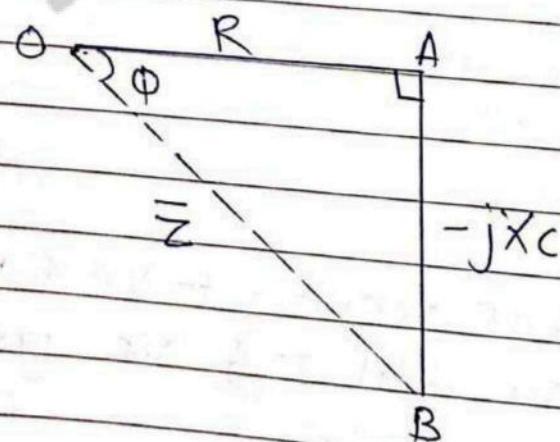
### iv) Impedance $\Delta$ . ( $Z-\Delta$ )

$$V_R = I \cdot R, V_C = I \cdot X_C$$

by divide each side  $V-\Delta$  by common factor  $I$ ,

$$\bar{Z} = (R - jX_C) = Z L - \phi$$

$$Z = \sqrt{R^2 + X_C^2} \quad \therefore P.f = \cos\phi \left(\frac{R}{Z}\right) \text{ leading}$$



$Z-\Delta$

## v) Power triangle

If we multiply each side of  $V - \Delta$  by a common factor  $I$ , we will get similar but larger  $\Delta$  known as  $P\Delta$

$$KVA = (Kw + j KVAR) = KVA L + \phi$$

$$= \sqrt{(Kw)^2 + (KVAR)^2}$$

$$P.f = \left( \frac{Kw}{KVA} \right) \text{ leading}$$

Active power

$$P = V_R \cdot I = I^2 R = V I_A \cos \phi \text{ (kw)}$$

$$\text{input power} S = V \cdot I = I^2 Z$$

Reactive power

$$Q = I \times X_C = V I \sin \phi$$

→ Power factor

$$V - \Delta = \left( \frac{V_R}{V} \right)$$

$$Z - \Delta = \left( \frac{R}{Z} \right)$$

$$P - \Delta = \left( \frac{Kw}{KVA} \right)$$

Equation of current & V

$$i = I_m \sin(\omega t + \phi)$$

$$V = V_m \sin(\omega t + \phi)$$

### \* Power

$$P = V \cdot i$$

$$= V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m \sin \omega t \cdot \sin(\omega t + \phi)$$

Use formula:

$$[\sin A \cdot \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]]$$

$$P = V_m I_m \cdot \frac{1}{2} [\cos(\omega t - (\omega t - \phi)) - \cos(\omega t + \omega t + \phi)]$$

$$P = \frac{V_m I_m}{2} [\cos(-\phi) - \cos(2\omega t + \phi)]$$

$$P = \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t + \phi)]$$

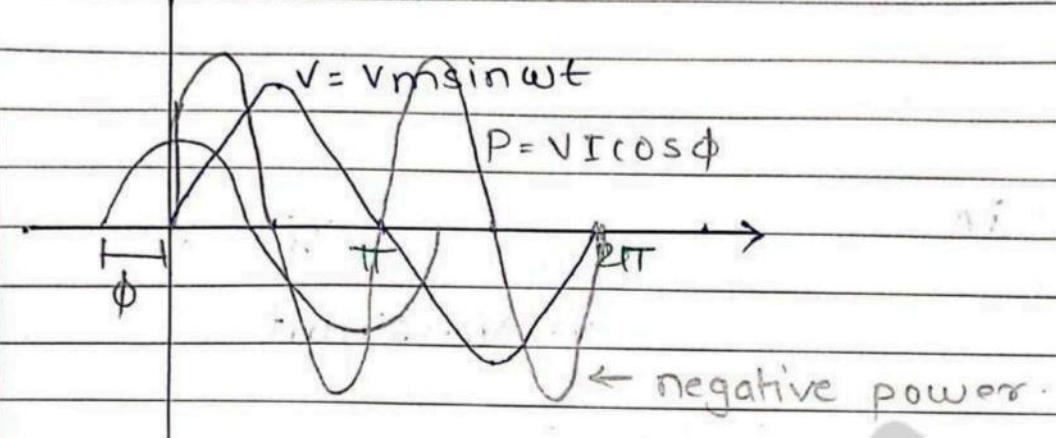
$$P = \frac{V_m I_m}{2} (\cos \phi - \frac{V_m I_m}{2} \cos(2\omega t + \phi))$$

The second term is cosine term whose average value over a complete cycle is zero.

$$P_{av} = \frac{V_m I_m}{2} \cos \phi = \frac{V_m I_m}{R_2 \sqrt{2}} \cos \phi$$

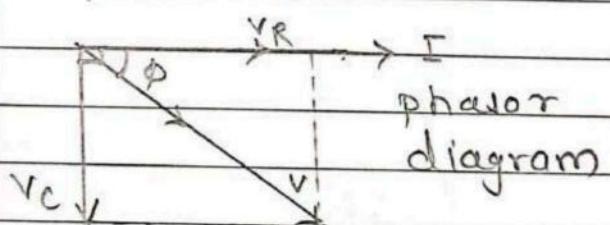
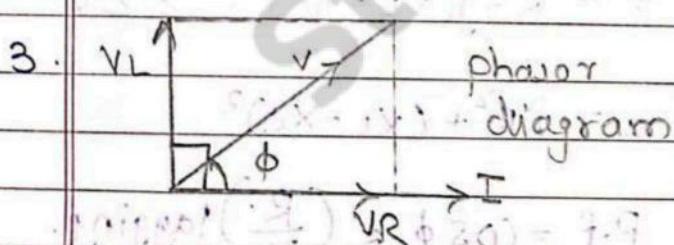
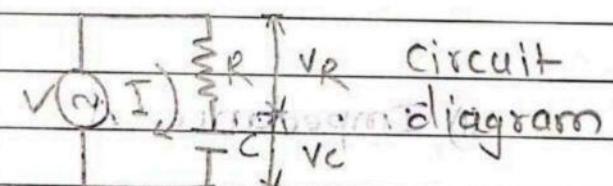
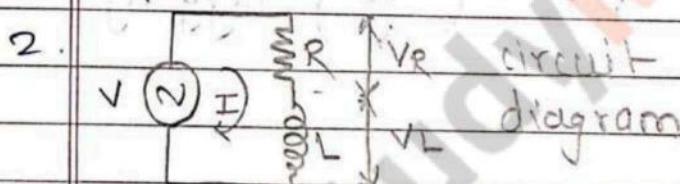
$$P_{av} = V_{rms} \cdot I_{rms} \cos \phi$$

\* Power waveform.



Series R-L circuit

- R & L(inductor) are connected in series with supply voltage.



- Current lags behind voltage.

5. P.f is lagging

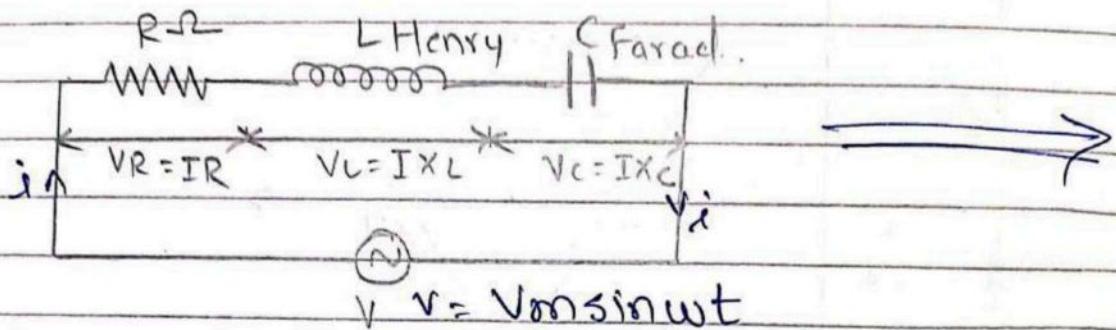
P.f is leading

$$Z = R + jX_L = |Z|L\phi$$

$$Z = R - jX_C = |Z|L - \phi$$

- It passes high freq. current & it passes low freq. so acts as high pass filter, as low pass filter.

## \* R-L-C Series Circuit.



Case I.  $X_L > X_C$

i) Voltage  $\Delta$

$$\bar{V} = V_R + j(V_L - V_C) = V_L \phi$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\text{p.f.} = \left( \frac{V_R}{V} \right) \text{ lagging.}$$

ii) Impedance  $\Delta$

$$\bar{Z} = R + j(X_L - X_C) = z \angle \phi$$

$$z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{p.f.} = \cos \phi = \left( \frac{R}{z} \right) \text{ lagging.}$$

iii) Power  $\rightarrow \Delta$ .

|  |  |
|--|--|
| Apparent power<br>$= VRI$<br>$= I^2 Z$<br>$V_A \text{ or } \text{KVA}$<br>$\phi$ | Reactive or quadrature power<br>$= V_L I = I^2 X (X_L - X_C)$<br>$= VI \sin \phi$<br>(measured in VAR or KVAR) |
| Active or useful power<br>$= V_R I = VI \cos \phi (\text{W or kW})$              |  |

The ac supply given by,

$$V = V_m \sin \omega t$$

The ckt draws a current  $I$ . Due to current  $I$ , there are diff. voltage drops across  $R, L, C$ .

1) Drop across  $R$  is  $V_R = I \cdot R$

2) Drop across  $L$  is  $V_L = I \cdot X_L$

3) Drop across  $C$  is  $V_C = I \cdot X_C$

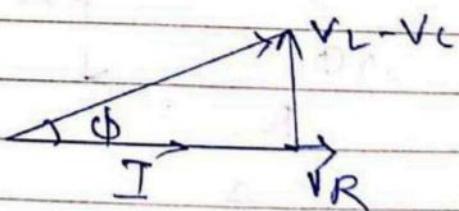
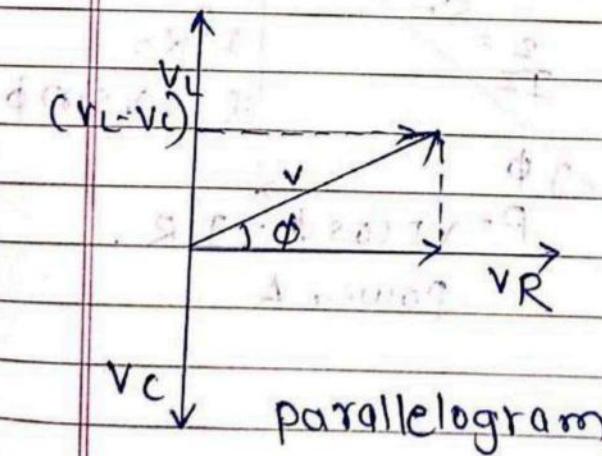
$$\vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C$$

i)  $X_L > X_C$

When  $X_L > X_C$ ,  $V_L$  is greater than  $V_C$ . So resultant of  $V_L$  &  $V_C$  will be directed toward  $V_L$ : i.e leading current  $I$ . Current  $I$  will lag the resultant of  $V_L$  &  $V_C$  i.e  $(V_L - V_C)$

The ckt is said to be inductive.

The phasor sum of  $V_R$  &  $(V_L - V_C)$  gives resultant supply voltage.



from voltage,  $\Delta$  will be  $177.8 \angle -30^\circ$

$$V = \sqrt{(VR)^2 + (VL - VC)^2} \quad \text{using } \Delta \text{ method} = \sqrt{V^2}$$

$$= \sqrt{(IR)^2 + (IXL - IXC)^2}$$

$$V = I \sqrt{R^2 + (XL - XC)^2}$$

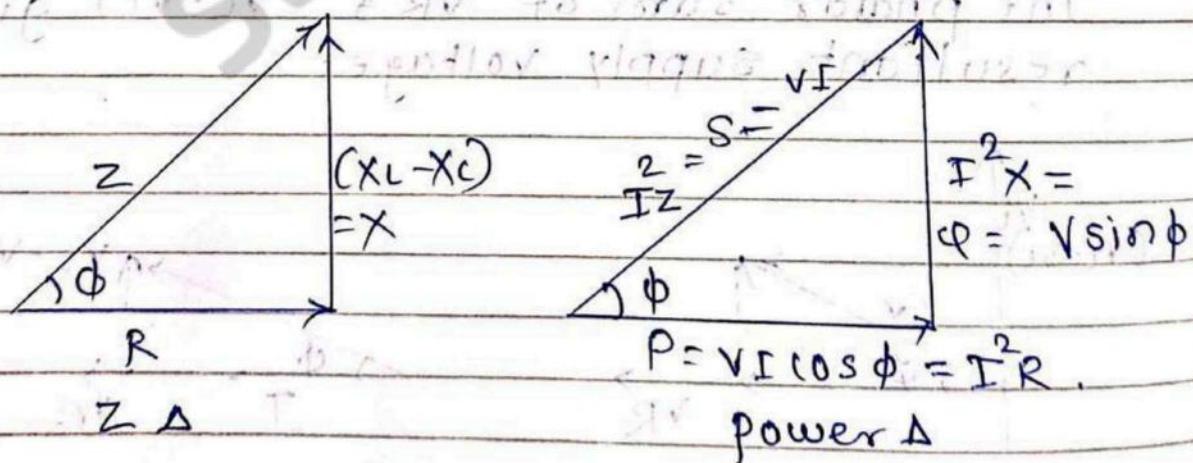
$$V = I \cdot Z$$

$$Z = \sqrt{R^2 + (XL - XC)^2}$$

$$|Z| = R + j(XL - XC)$$

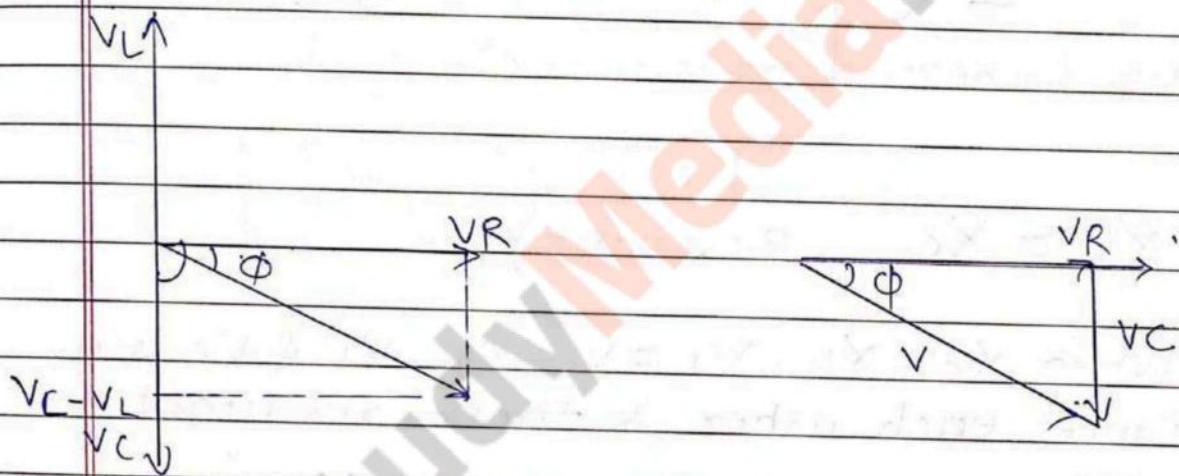
$$i = I_m \sin(\omega t - \phi)$$

as  $i$  lags voltage by  $\phi$ , for  $XL > XC$ .



2)  $X_L < X_C$

When  $X_L < X_C$ , i.e.  $V_L$  is less than  $V_C$  so the resultant of  $V_L$  &  $V_C$  will be directed toward  $V_C$ . Current  $I$  will lead  $(V_C - V_L)$ . This current is said to be capacitive current. And circuit said to be capacitive C.R.T. The phasor sum of  $V_R$  &  $(V_C - V_L)$  gives the resultant supply voltage  $V$ .



from voltage -  $\Delta$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$= \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$V = I \sqrt{R^2 + (X_C - X_L)^2}$$

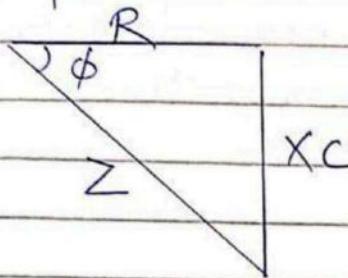
$$\frac{V}{I} = Z$$

$$Z = R - j(X_C - X_L)$$

$$i = I_m \sin \omega t + \phi$$

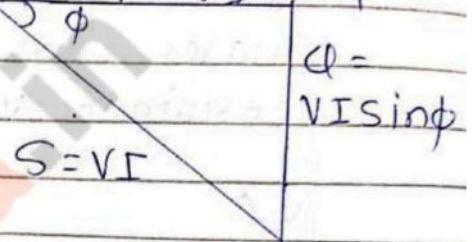
As  $I$  lead voltage by angle  $\phi$  for  $X_L < X_C$

Impedance  $Z$



Power  $A$ .

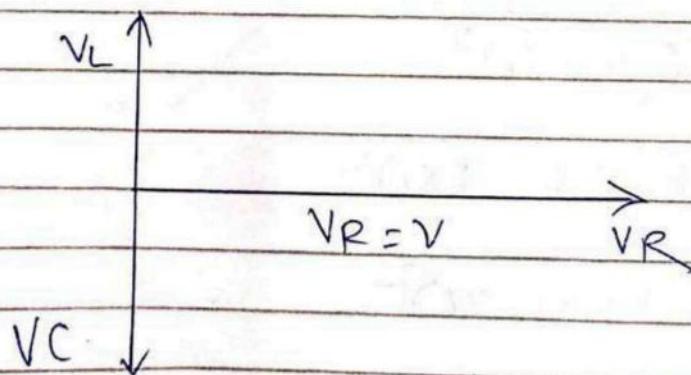
$$P = V I \cos \phi$$



$$3) X_L = X_C \text{ (Resonant)}$$

When  $X_C = X_L$ ,  $V_L = V_C$  so  $V_L$  &  $V_C$  will cancel each other & their resultant is zero.

so  $V_R = V$  in such case & overall circuit is purely resistive in nature.



$$V = V_R = I R$$

$$V = I Z \quad \text{where } R = Z$$

## Power :

Average power consumed by the circuit is,

$P_{avg} = \text{Avg. power consumed by } R + \text{Avg. power consumed by } L + \text{Avg. power consumed by } C.$

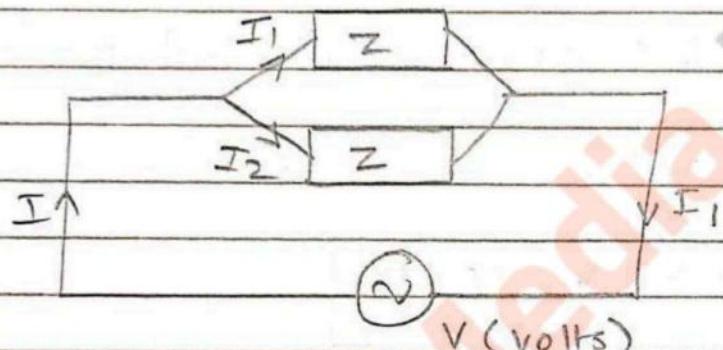
(Pure L & C never consumed power)

$\therefore P_{avg} = \text{Avg. power consumed by } R.$

$$P_{avg} = VI \cos \phi$$

## AC parallel circuit.

A parallel ckt is one in which two or more impedances are connected in parallel across the supply voltage. Each impedance is called branch of a parallel ckt.



AC parallel ckt.

voltage across all the impedances is same as supply voltage of  $V$  volts.

$$I = I_1 + I_2 \quad (\text{phases})$$

$$\frac{V}{Z} = \frac{V}{Z_1} + \frac{V}{Z_2}$$

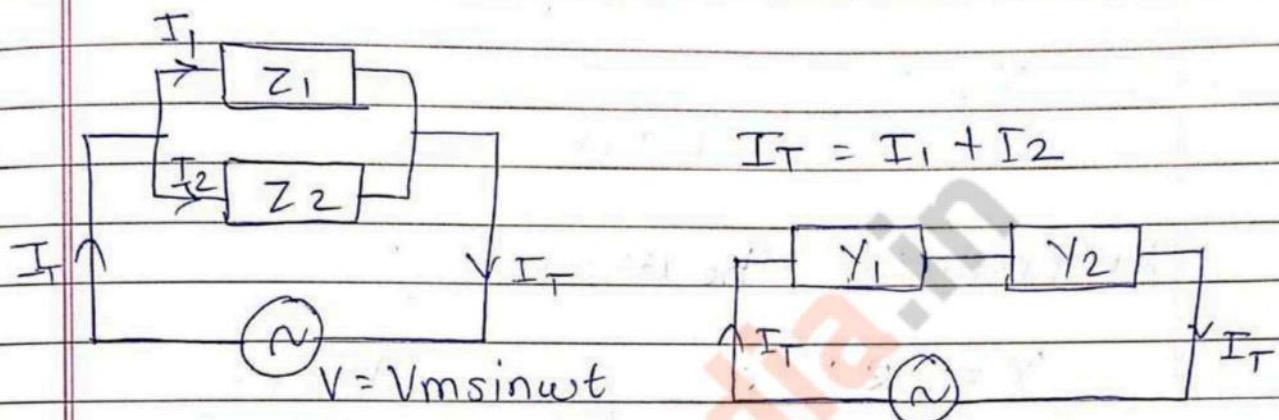
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$Z$  is called equivalent impedance

|                                   |
|-----------------------------------|
| $Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$ |
|-----------------------------------|

## Concept of Admittance

Admittance is defined as reciprocal of the impedance. It is denoted by  $y$  & is measured in siemens or mho.



$$\text{As, } I_T = I_1 + I_2$$

$$I_T = V \times \frac{1}{Z_1} + V \times \frac{1}{Z_2}$$

$$V \times \frac{1}{Z_T} = V \cdot \frac{1}{Z_1} + V \cdot \frac{1}{Z_2}$$

$$VY = VY_1 + VY_2$$

$$Y = Y_1 + Y_2$$

where,  $Y$  is admittance of ckt. The two impedances connected in parallel can be replaced by an eq. ckt where the admittances are connected in series.

## Component of admittance

Consider impedance given as,

$$Z = R + jX$$

$$Y = \frac{1}{Z} = \frac{1}{R + jX}$$

Rationalising the above eq?

$$Y = \frac{R \pm jX}{(R + jX)(R \pm jX)} = \frac{R \pm jX}{R^2 + jX^2}$$

$$= \frac{R}{R^2 + X^2} \pm j \frac{X}{R^2 + X^2}$$

$$Y = \frac{R}{Z^2} + j \frac{X}{Z^2}$$

$$\boxed{Y = G \pm jB}$$

G = conductance ratio of resistance to square of impedance (simens)

B = Susceptance ratio of reactance to square of impedance (simens)

Y = Admittance, Reciprocal of impedance assistance to flow of AC current (simens)

$B_L$  - inductive Susceptance

$$Y = G - j B_L \quad (\phi \text{ is negative})$$

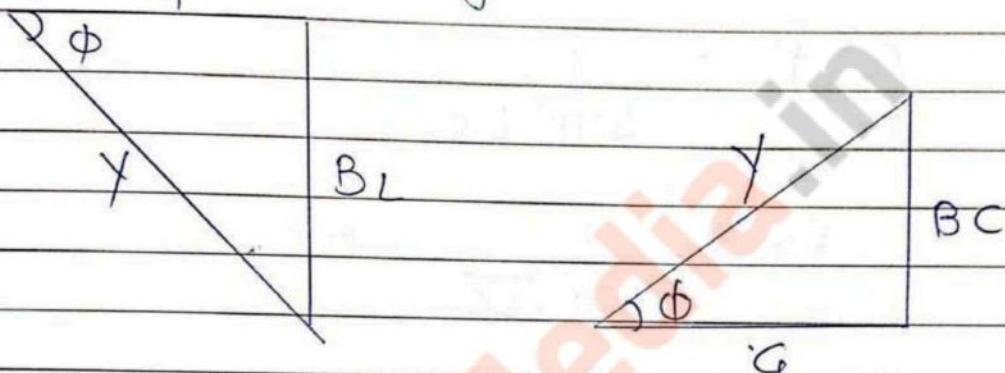
$B_C$  - Capacitive Susceptance

$$Y = G + j B_C \quad (\phi \text{ is positive})$$

\*

Admittance triangle

$G$



RL Admittance  $A$

RC admittance  $A$

-

\* Resonance in RLC series circuit.

As in case of RLC series ckt  $X_L$  &  $X_C$  are functions of  $F$ , when  $F$  will vary  $X_L$  &  $X_C$  vary.

At a certain frequency  $X_L$  becomes equal to  $X_C$ , such a condition when  $X_L = X_C$  for certain freq. is called series resonance.

At resonance the reactive part in the impedance of RLC series ckt is zero.

The freq. at which resonance occurs is called resonant freq. denoted by  $\omega_r$  rad/sec or Hz.

## Expression for Resonant frequency.

Let  $f_r$  be the resonant freq. in Hz,  
at which

$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$(f_r)^2 = \frac{1}{4\pi^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz.}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

As impedance is minimum the current

$I = \frac{V}{Z}$  is max. at series

Resonance  $\Rightarrow (X_L = X_C)$

## \* Summary of RLC circuit

Complex power

AC ckt has three power. These are real reactive & apparent power,  $P, Q, S$ .

$$|S| = \sqrt{P^2 + Q^2}$$

$$\begin{aligned}\phi &= \tan^{-1} \left[ \frac{\sin \phi}{\cos \phi} \right] \\ &= \tan^{-1} \frac{V \sin \phi}{V \cos \phi}\end{aligned}$$

$$\boxed{\phi = \tan^{-1} \left[ \frac{Q}{P} \right]}$$

Apparent power can be expressed in rectangular form.

$$S = P \pm jQ$$

This is called complex power.

The +ve sign indicates lagging nature, of reactive power, while negative sign indicates leading nature of power.

# POLYPHASE AC CIRCUITS.

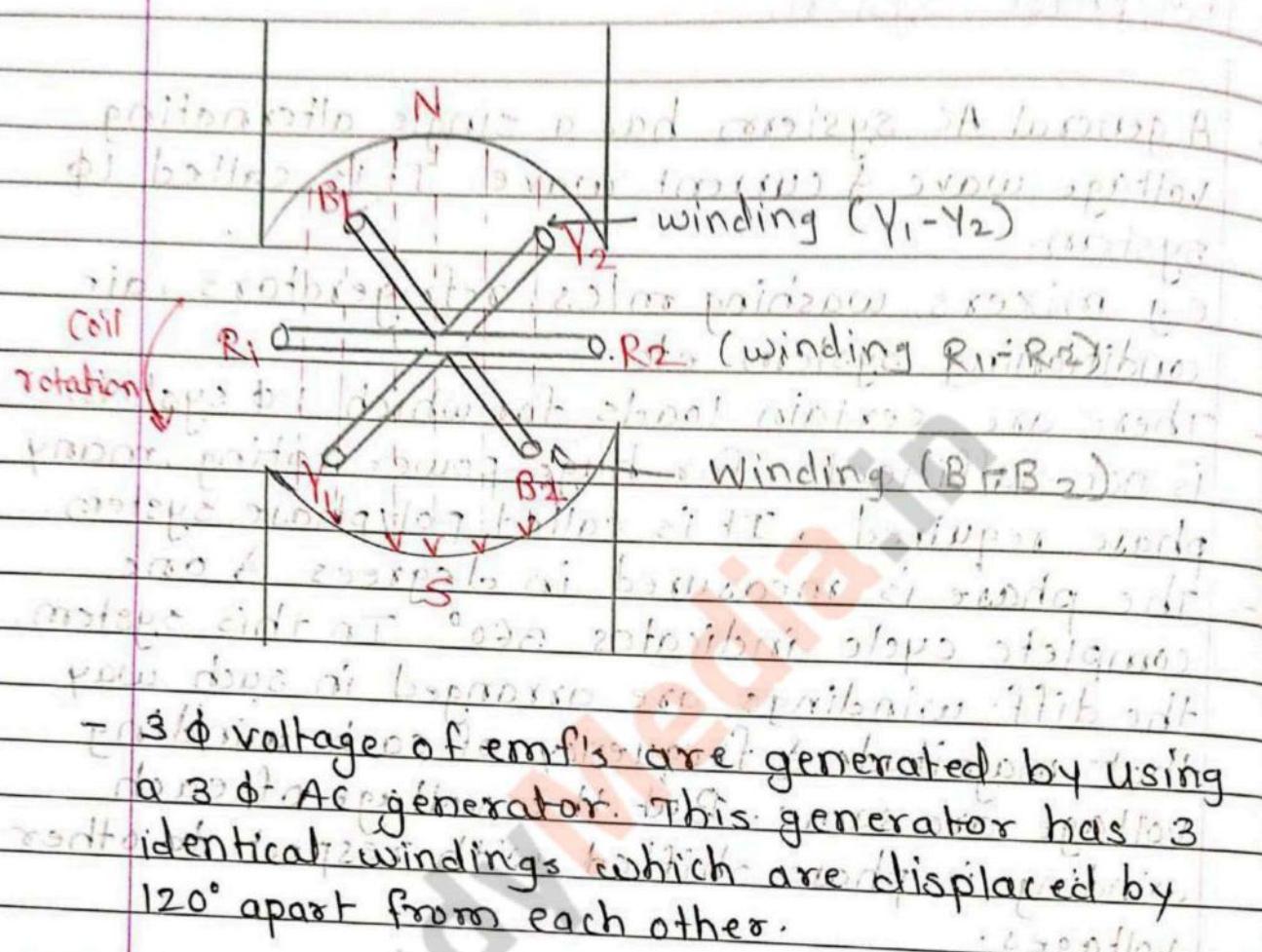
## \* Polyphase system:

- A general AC system has a single alternating voltage wave & current wave. It is called 1φ system.  
e.g mixers, washing m/c's, refrigerators, air conditioning system etc.
- There are certain loads for which 1φ system is not sufficient. For huge power rating, many phase required. It is called polyphase system.
- The phase is measured in degrees & one complete cycle indicates  $360^\circ$ . In this system, the diff. windings are arranged in such way that magnitude & frequency of each winding voltage are same. But the voltage of each winding is phase shifted with respect to other voltages.

## \* Advantages of polyphase system over 1φ system

- i) Constant power
- ii) More output power
- iii) Economical transmission
- iv) Self starting
- v) Uniform torque
- vi) Constant Instantaneous power
- vii) Higher kVA rating
- viii) Small cross-sectional area of conductor
- ix) High power factor & efficiency
- x) Smooth AC output
- xi) Simple parallel operations.

## \* Generation of 3 φ voltages



- 3 φ voltages or emf's are generated by using a 3 φ A.C. generator. This generator has 3 identical windings which are displaced by  $120^\circ$  apart from each other.

## \* Working

- An emf is induced in each coil either by keeping the mag. field stationary and rotating the winding or by keeping the winding stationary and rotating the mag. field.
- The windings  $R_1-R_2$ ,  $Y_1-Y_2$ ,  $(B_1-B_2)$  are mounted on same axis.
- The coils of are rotated in anticlockwise direction. It cuts the mag. field lines & an emf is induced in coil.

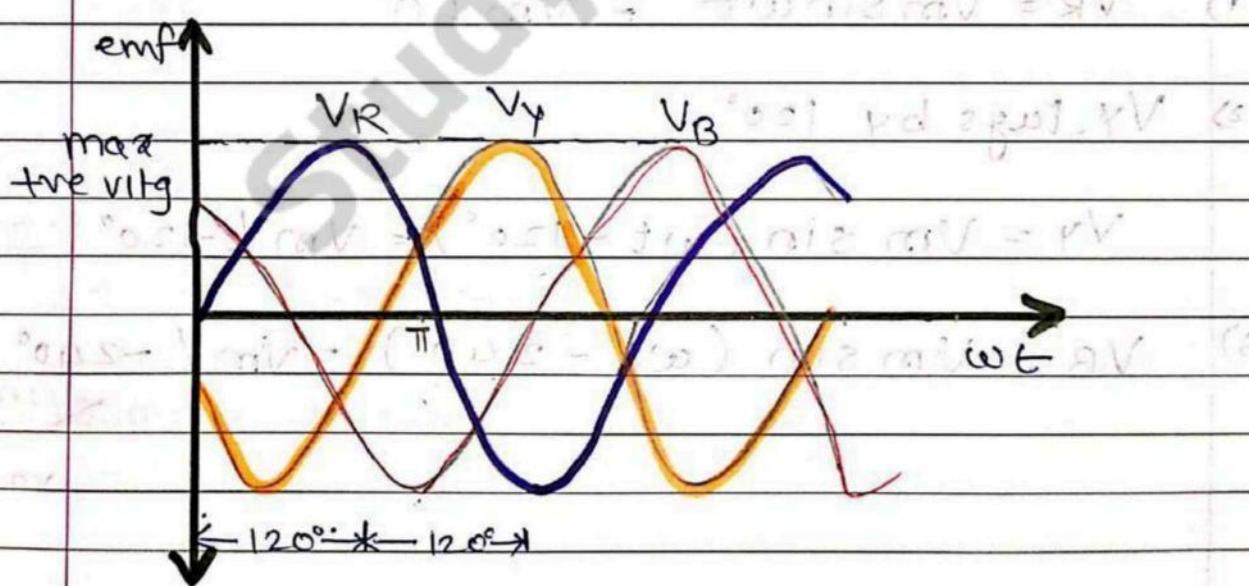
### - Voltage waveform.

$V_R$  = Voltage (emf) induced in winding  $R_1 - R_2$

$$V_Y = -\frac{1}{2} \omega (Y_1 - Y_2)$$

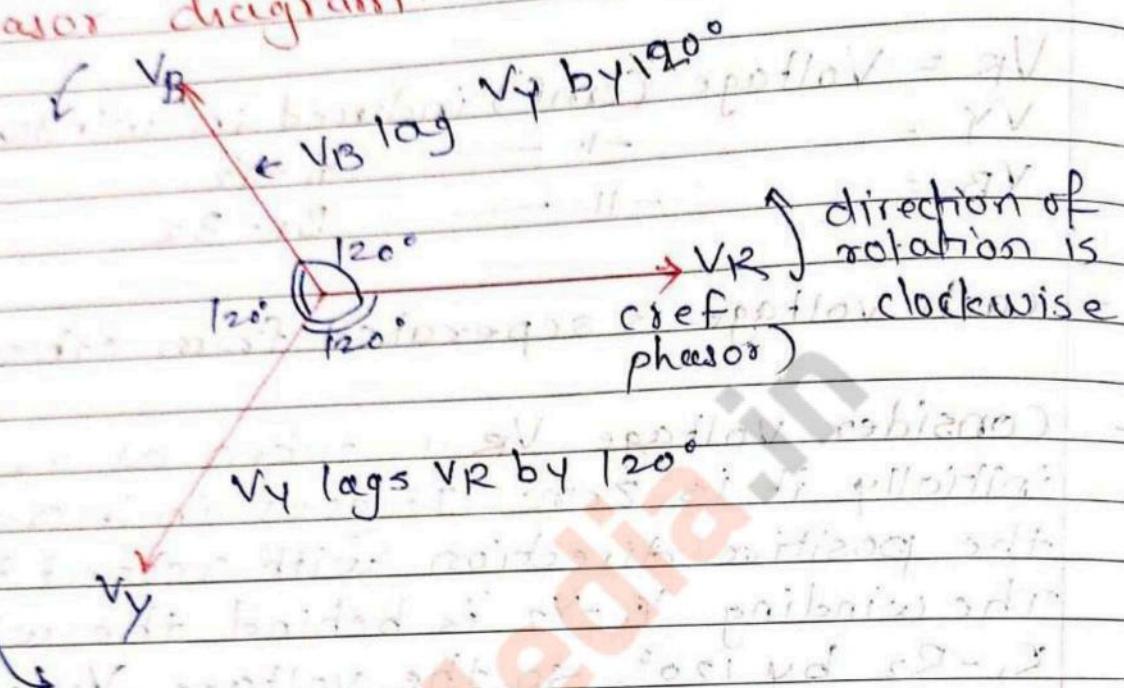
$$V_B = -\frac{1}{2} \omega (B_1 - B_2)$$

- each voltage is separated from other by  $120^\circ$
- Consider voltage  $V_R$  is taken as a ref & initially it is zero. It goes in increasing in the positive direction with respect to  $\omega t$ . The winding  $Y_1 - Y_2$  is behind the winding  $R_1 - R_2$  by  $120^\circ$ . so the voltage  $V_Y$  is initially negative &  $B_1 - B_2$  are behind by  $240^\circ$  so, initially positive.



Voltage waveform for 3φ

### Phasor diagram



### Mathematical equations

$$1) VR = V_m \sin \omega t = V_m L 0^\circ$$

$$2) Vy \text{ lags by } 120^\circ$$

$$Vy = V_m \sin (\omega t - 120^\circ) = V_m L -120^\circ \left(\frac{2\pi}{3}\right) \text{ rad.}$$

$$3) VB = V_m \sin (\omega t - 240^\circ) = V_m L -240^\circ \left(\frac{4\pi}{3}\right) \text{ rad.}$$

## Symmetrical system:

If the magnitude & freq. of all 3 voltages are same and voltage displaced by  $120^\circ$  from each other, then it is called symmetrical.

## Unsymmetrical system:

A system said to be unsymmetrical when any one of voltages are unequal in magnitude and time angle is not equally displaced.

## Balanced load

A load is said to be balanced when the load connected in each phase are same in magnitude as well as in phase & some parameters.

## Unbalanced load

A load is said to be unbalanced when the load connected in each phase are not same in magnitude or phase.

## \* Phase Sequence :

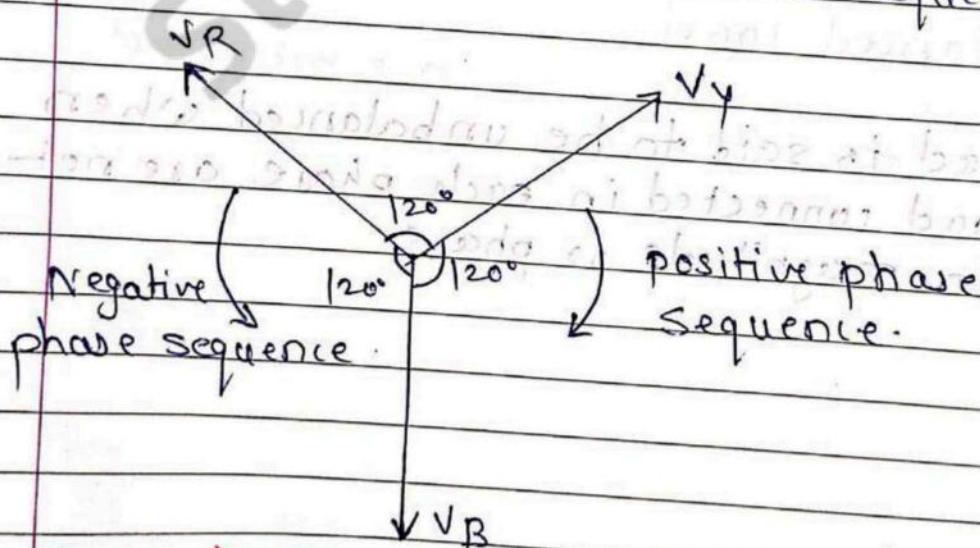
- The sequence by which voltage in three phase attain maximum positive value, is called as phase sequence.

### 1. Positive phase sequence

If max value attended by phase voltage is in the sequence  $V_R, V_Y, V_B$  then it is called positive phase sequence.

### 2. Negative phase sequence

If the max. value attended by phase voltage is in the sequence  $V_R, V_B, V_Y$  then it is called negative phase sequence. also called as anticlockwise sequence.



### Application :

- In parallel operation of alternator.
- change the direction of AC motor.

\* **Line current (IL)**

The current passing thr' any line R,Y,B is called line current. As the current flowing thr' each line is equal to current flowing thr' corresponding branch. Line current denoted as IL.

\* **phase current (Iph)**

The current flowing thr' each of the 3d winding.

\* **phase voltage (Vph)**

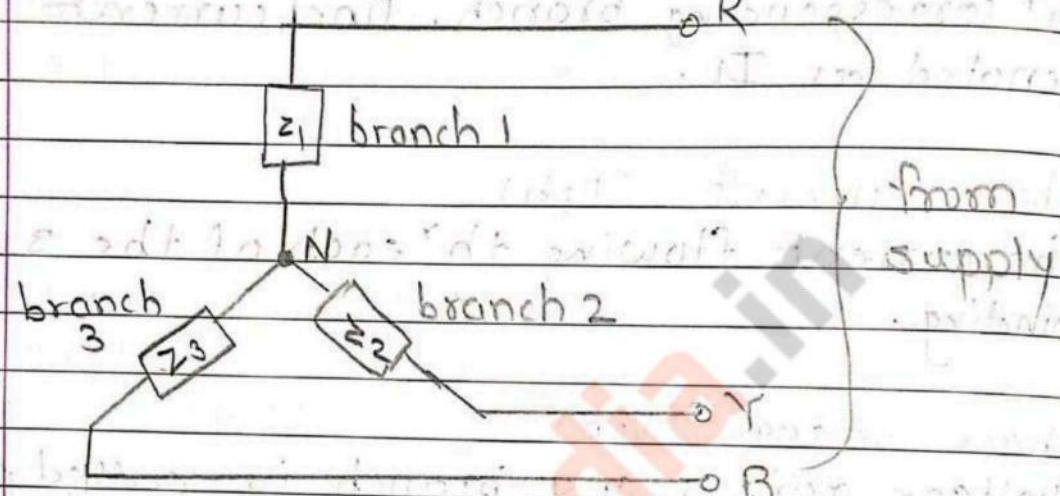
Voltage across any branch is called phase voltage

\* **line voltage (VL)**

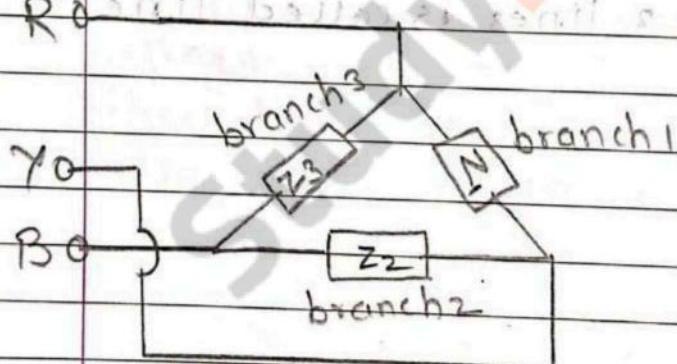
voltage across 2 lines is called line voltage

## \* Types of load connections.

### \* Star connection



### \* Delta connected load



### \* Balanced load -

- The load is said to balanced load if the following conditions are satisfied:

- i) The magnitudes of all impedance ( $z_1$ ,  $z_2$ ,  $z_3$ ) are equal & p.f is same in all phase.
- ii) phase angles bet'n impedances are equal.
- iii) The nature of all impedance is same. that means all impedances are inductive, all are

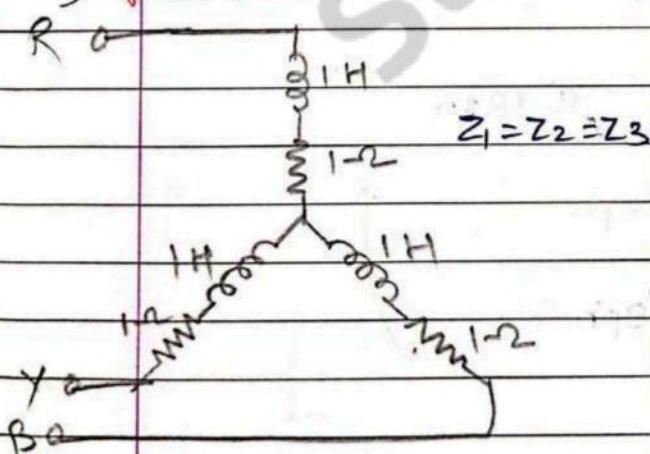
capacitive or all are resistive.

2. In case of balanced load, all parameters like phase voltages, phase current, line voltage & line current having same magnitude, Similarly the phase shift between each other is  $120^\circ$ .

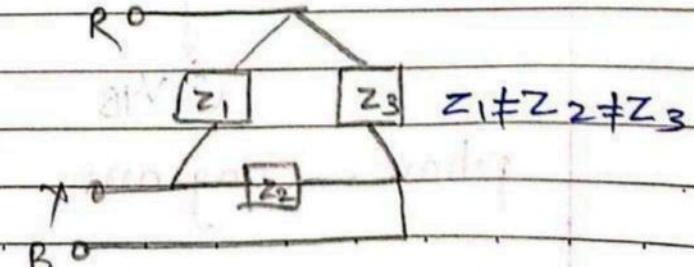
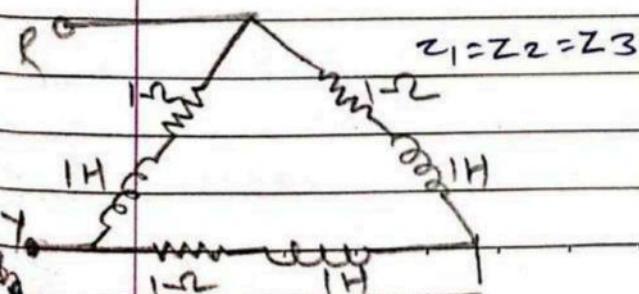
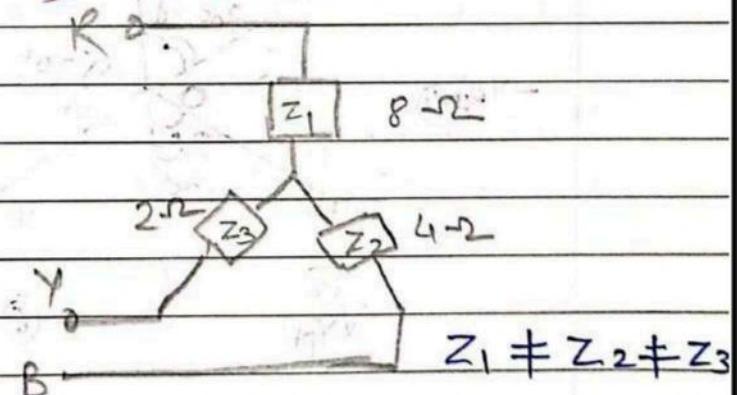
### \* Unbalanced load :

- When the magnitude of all impedances are unequal, as well as phase angles between them are unequal then such a load is called as unbalanced load.
- The phase voltage & phase current are not equal, as well as they are not phase shifted from each other by  $120^\circ$ .

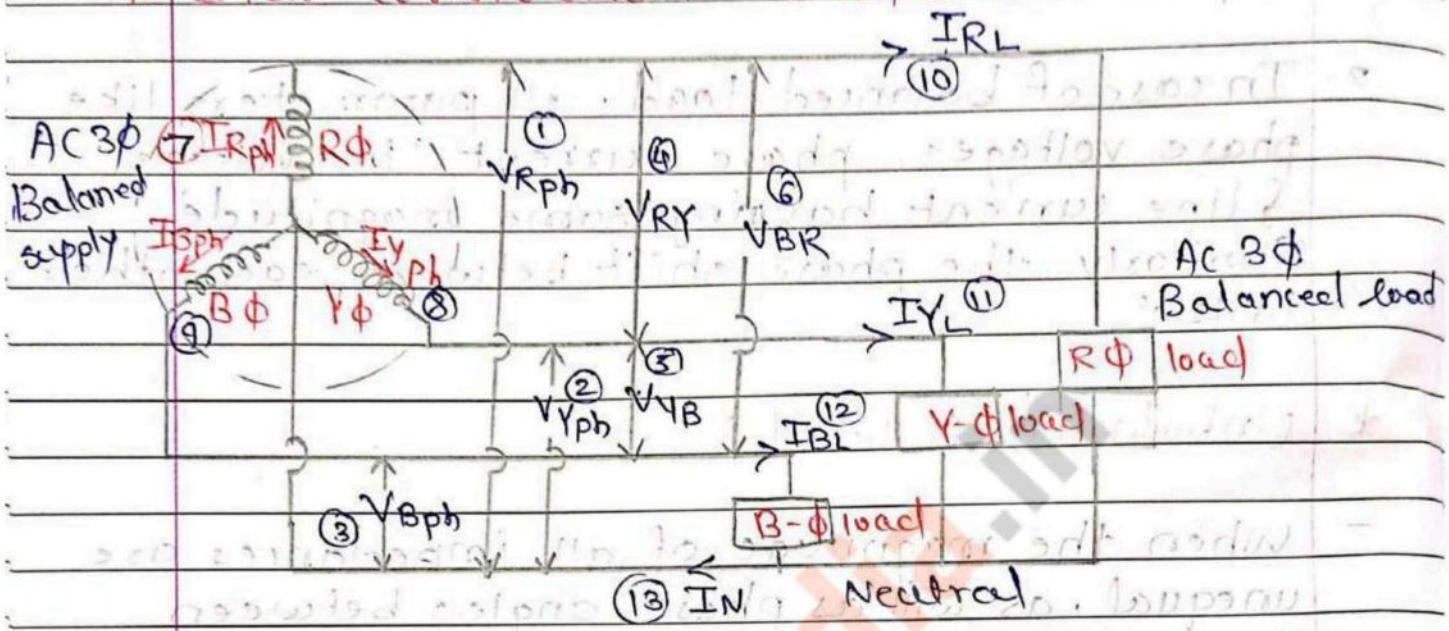
a) balanced load



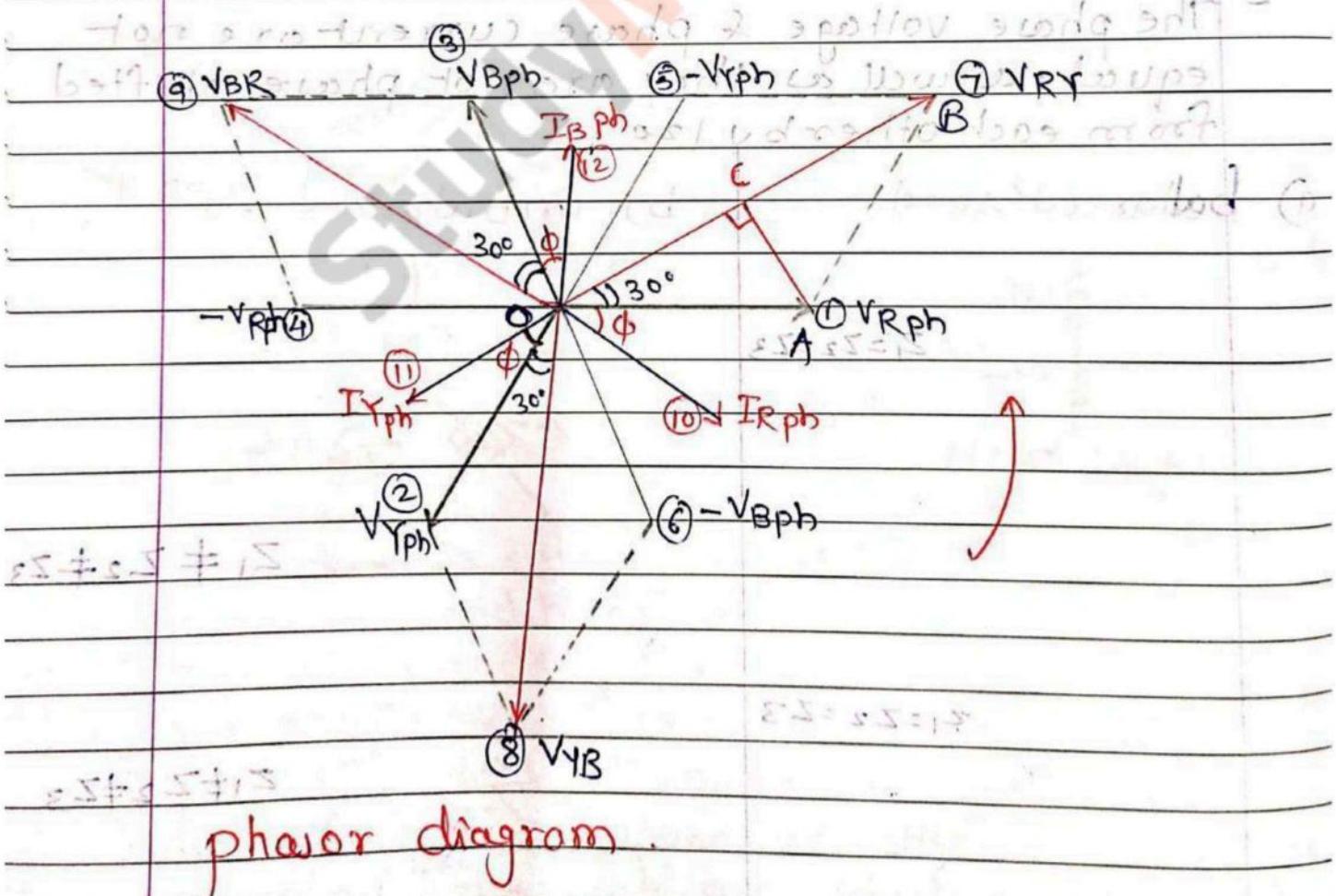
b) unbalanced load



## \* Star connection (Wye connection)



## \* Relationship between line & phase voltages



Applying KVL bet<sup>n</sup> R & Y lines

$$\bar{V}_{RY} = \bar{V}_{Rph} - \bar{V}_{Yph}$$

similarly,

$$\bar{V}_{YB} = \bar{V}_{Yph} - \bar{V}_{Bph}$$

$$\bar{V}_{BR} = \bar{V}_{Bph} - \bar{V}_{Rph}$$

- a. In phasor diagram OA & OB represent  $V_{Rph}$  &  $V_{RY}$  resp.

$$\therefore OB = V_{RY} = 2(OC) = 2(OA \cdot \cos 30^\circ)$$

$$= 2 \left( V_{Rph} \cdot \frac{\sqrt{3}}{2} \right) = \sqrt{3} V_{Rph}$$

$V_L = \sqrt{3} V_{ph}$

$V_L$  leads respectivey  $V_{ph}$  by  $30^\circ$ .

- 2) Relationship bet<sup>n</sup> line & phase current

From connection dia. it is seen that each phase of supply is connected in series with respective load

$I_L = I_{ph}$

- 3) Power consumed

The total power in  $3\phi$  system is sum of power in each of  $3\phi$

a) Total active power

$$P_T = 3 V_{ph} I_{ph} \cos \phi_{ph}$$

$$= 3 \left( \frac{V_L}{\sqrt{3}} \right) I_L \cos \phi_{ph}$$

$$\therefore P_T = \sqrt{3} V_L I_L \cos \phi_{ph} \quad \text{where } I_L = I_{ph}$$

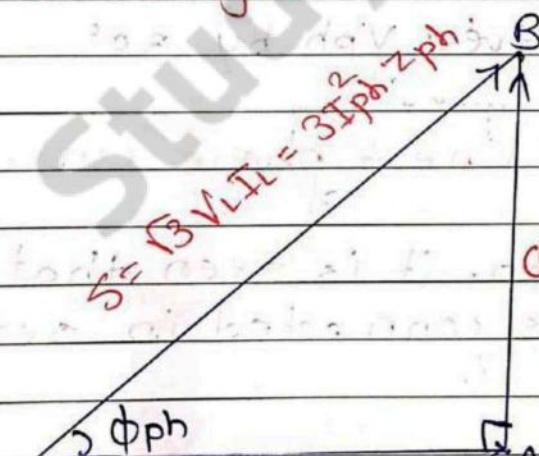
b) Total reactive power

$$Q_{TL} = \sqrt{3} V_L I_L \sin \phi_{ph}$$

c) Apparent power

$$S_T = \sqrt{3} V_L I_L$$

\* Power triangle

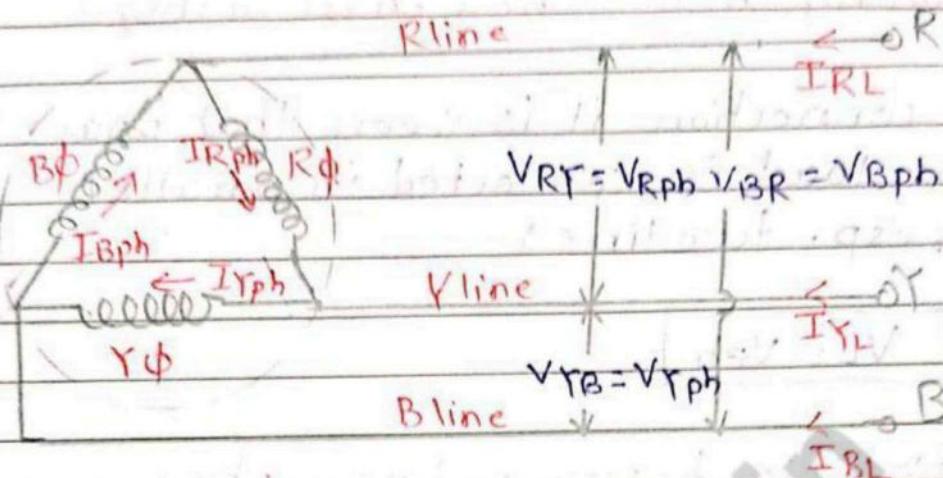


$$\begin{aligned}
 P &= \sqrt{3} V_L I_L \cos \phi \\
 &= 3 I_{ph}^2 R_{ph}
 \end{aligned}$$

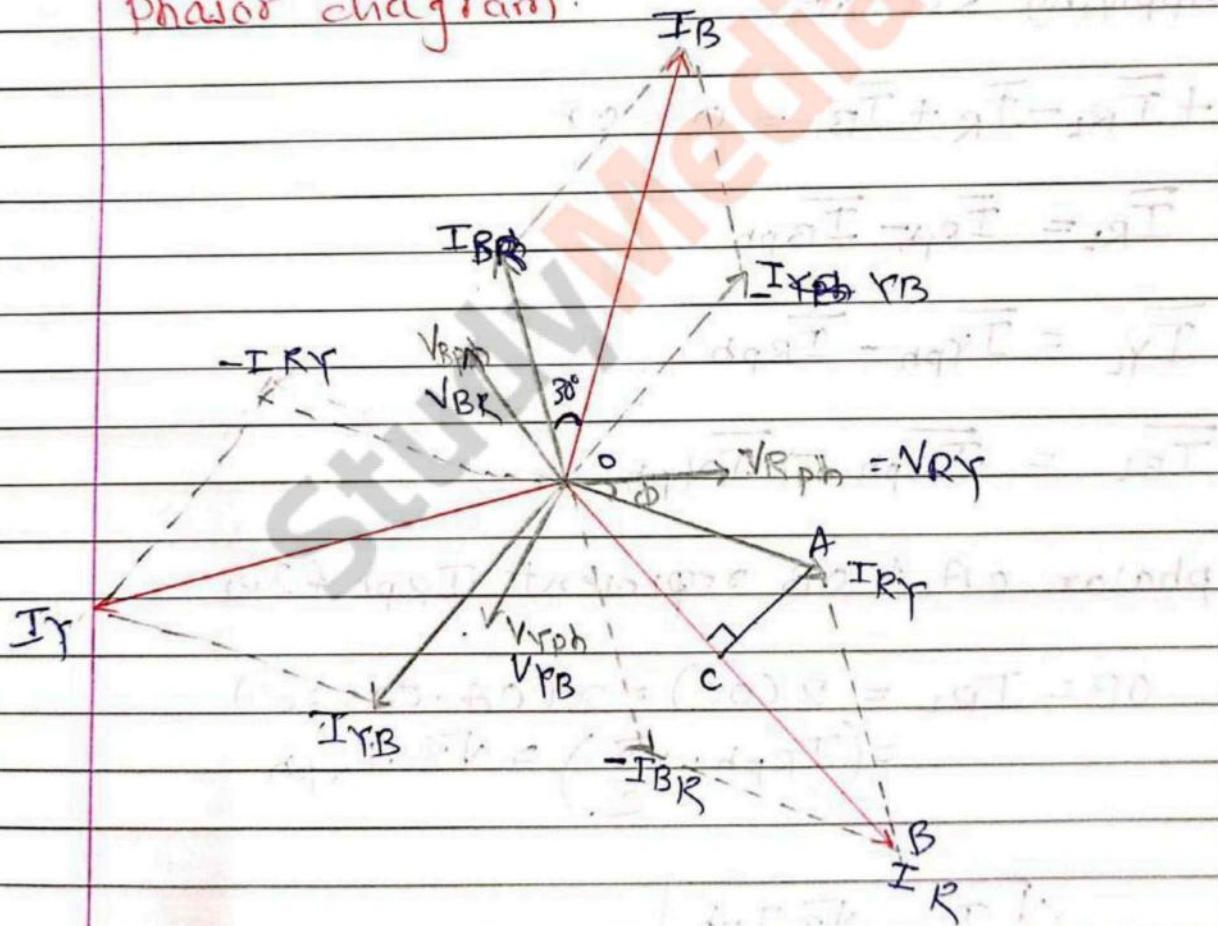
$$Q = \sqrt{3} V_L I_L \sin \phi$$

$$= 3 I_{ph}^2 X_{Lph}$$

## \* Delta (mesh) connection.



Phasor diagram.



### \* Relationship b/w line & phase voltage

From connection it is seen that phase of the load is connected in parallel with resp. two lines

$$V_L = V_{ph}$$

### \* relationship between line & phase current

Applying KCL at

$$+ \bar{I}_{RL} - \bar{I}_R + \bar{I}_B = 0 \text{ or}$$

$$\bar{I}_{RL} = \bar{I}_{Rph} - \bar{I}_{Bph}$$

$$\bar{I}_{YL} = \bar{I}_{Rph} - \bar{I}_{Bph}$$

$$\bar{I}_{BL} = \bar{I}_{Bph} - \bar{I}_{Yph}$$

phasor OA & OB represent  $\bar{I}_{Rph}$  &  $\bar{I}_{BL}$

$$\begin{aligned} OB &= I_{R1} = 2(\text{oc.}) = 2(OA \cdot \cos 30^\circ) \\ &= \left( I_{Rph} \cdot \frac{\sqrt{3}}{2} \right) = \sqrt{3} \cdot I_{Rph} \end{aligned}$$

$$\therefore I_L = \sqrt{3} I_{ph}$$

| M        | T | W | T | F | S | S     |
|----------|---|---|---|---|---|-------|
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Power consumed

$$P = 3 V_{ph} I_{ph} \cos \phi$$

$$= \sqrt{3} V_L I_L \cos \phi$$

$$Q = \sqrt{3} V_L I_L \sin \phi$$

$$S = \sqrt{3} V_L I_L$$

### Star connection

1. Starting or finishing end connected together to form neutral point.

2. There is neutral or star point

3) 3 phase 4 wire system

4. Line current = phase current  
 $I_L = I_{ph}$

$$5. V_L = \sqrt{3} V_{ph}$$

$$6. P = \sqrt{3} V_L I_L \cos \phi$$

7. speed of motor are slow  
 $1/\sqrt{3}$  voltage received

8. less insulation required  
 as low voltage

9. need low no. of turns,  
 save copper

10. Power transmission, is  
 general & typical to  
 be used.

### Delta connection

end of each coil is connected to start of another coil.

No neutral point

3 phase 3 wire system.

Line voltage = phase voltage  
 $V_{ph} = V_L$

$$I_L = \sqrt{3} I_{ph}$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

High speed, each phase get total line voltage.

High insulation required.

need more no. of turns.

In power distribution & industries connection used.

problem based on star connected load.

1. 3 identical impedances each of  $6+j8$  are connected in star across a  $\phi$  400V, 50Hz ac supply calculate

i) line current, ii) Active power iii)  $\phi$ .

$$\rightarrow V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$Z_{ph} = 6+j8 = 10/53.13^\circ$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94 \angle 0^\circ}{10 \angle 53.13^\circ} = 23.09 \angle -53.13^\circ$$

In star connection

$$I_{ph} = I_L = 23.09 \angle -53.13^\circ$$

Active power

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$= \sqrt{3} \times 400 \times 23.09 \times \cos(-53.13)$$

$$P = 959.8 \text{ W}$$

$$Q = \sqrt{3} V_L I_L \sin \phi$$

$$= \sqrt{3} \times 400 \times 23.09 \times \sin(-53.13)$$

$$Q = 12800 \text{ VAR}$$

2) Three coils each having a series resistance of  $20\ \Omega$  & reactance  $15\ \Omega$  are connected in star to a  $400\text{V}$ ,  $3\phi$ ,  $50\text{Hz}$  supply. If line current, p.f & power supplied.

$$\rightarrow R_{ph} = 20\ \Omega, \quad X_L = 15\ \Omega$$

$$V_L = 400\text{V}, \quad f = 50\text{Hz}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{V}$$

$$I_{Lph} = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{(20 + j15)} = 25.236.87$$

$$I_L = 9.2 L - 36.87^\circ$$

$$\begin{aligned} \text{Power factor} &= \cos \phi \\ &= \cos(-36.87) \\ &= 0.8 \text{ lag.} \end{aligned}$$

$$\begin{aligned} P &= \sqrt{3} V_L I_L \cos \phi \\ &= \sqrt{3} \times 400 \times 9.2 \times 0.8 \\ P &= 5099.15 \text{ W.} \end{aligned}$$

$$\begin{aligned} Q &= \sqrt{3} V_L I_L \sin \phi \\ &= \sqrt{3} \times 400 \times 9.2 \times 0.6 \end{aligned}$$

$$Q = 3824.48 \text{ VAR.}$$

problem based on  $\Delta$  connection

- 1) Three identical coils each having a resistance 15-2 & an inductance of 0.03 H are in delta across 400V, 50Hz supply. Det.  
 1)  $I_{ph}$ , 2)  $I_L$  3) power consumed (P.D)

$$\rightarrow R_{ph} = 15.2, L = 0.03 \text{ H}$$

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.03 \\ = 9.42 \Omega$$

$$Z = R + jX \\ = 15 + j9.42$$

$$Z_{ph} = 17.71 \angle 32.12^\circ \text{ ohm} = \text{d.v}$$

$$V_{ph} = V_{ph} = 400 \text{ V. } 0.45 = \text{d.v. } = \text{d.v}$$

$$V = I_{ph} \times Z_{ph}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{17.71 \angle 32.12^\circ} \text{ A. } = \text{d.v.}$$

$$I = 22.58 \angle -32.12^\circ$$

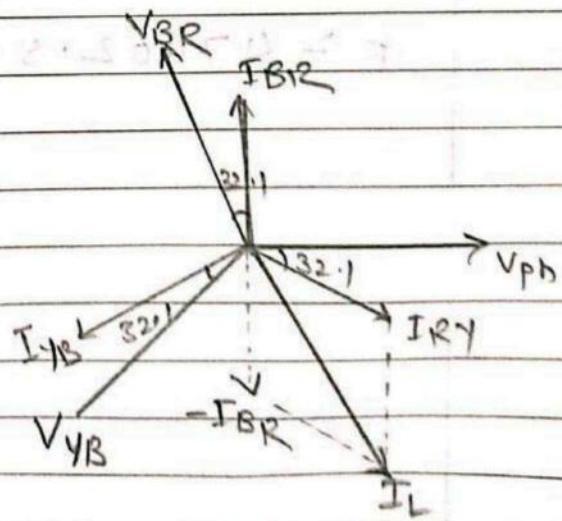
$$I_L = \sqrt{3} I_{ph} \\ = \sqrt{3} \times 22.58$$

$$I_L = 39.10 \text{ A.}$$

$$P = \sqrt{3} V_L I_L \cos \phi \\ = \sqrt{3} \times 400 \times 39.10 \times 0.84$$

$$P = 22942.9 \text{ W.}$$

$$P = 22766.79 \text{ W.}$$



2) 3 coils each of  $5\Omega$  R &  $6\Omega$   $X_L$  are connected in delta across  $\phi$  L  $440V$ ,  $50Hz$ . cal. current drawn, p.f & power consumed.

$$\rightarrow R_{ph} = 5\Omega, X_L = 6\Omega, V_L = 440,$$

$$Z = R + jX = 5 + j6$$

$$\sum Z = 7.81 \angle 50.19^\circ$$

$$V_{ph} = I_{ph} \cdot Z_{ph}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{7.81 \angle 50.19} = 56.33 \angle -50.19^\circ$$

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 56.33$$

$$I_L = 97.56 A$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$= \sqrt{3} \times 440 \times 97.56 \times 0.64$$

$$P = 47602.5 W$$