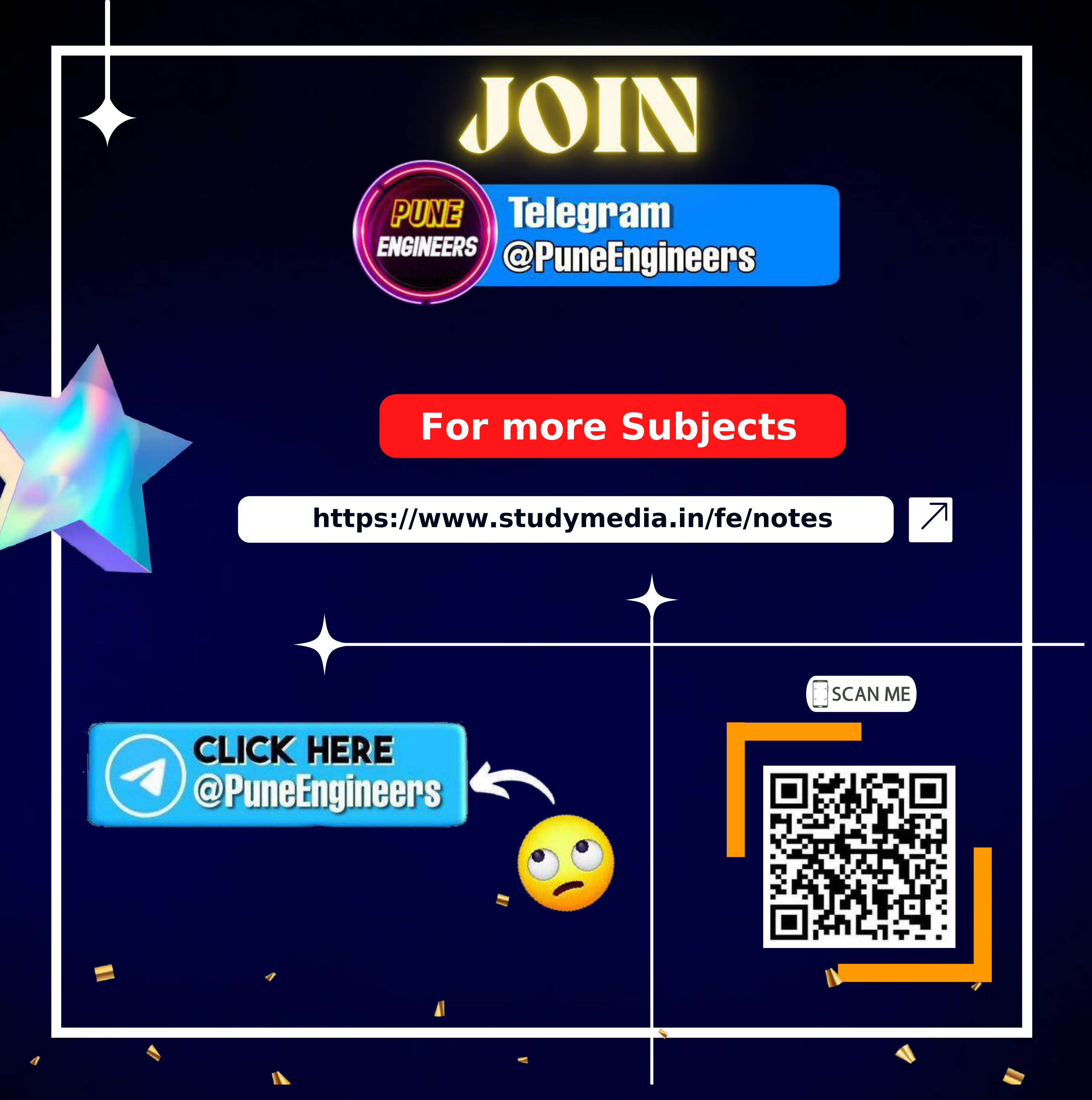
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UNIT V KINETICS OF PARTICLE

Kinetics: Force and Acceleration

• Kinetics is the study of motion of bodies by considering the forces acting on them. In rectilinear motion, acceleration is caused by unbalanced force.

• Newton's second law is used to find acceleration of bodies which are treated as particles. Once acceleration of the body is known, the methods discussed below are used for further analysis.

Newton's Second Law of Motion

• Newton's second law of motion can be stated as follows:

• *The acceleration of a particle is directly proportional to magnitude of the resultant force acting on it and is in the direction of resultant force.*

• Mathematically, it can be written as,

$$\boxed{\sum \vec{F} = m\vec{a}} \qquad \text{... (.1)}$$

Where, m is the mass of the particle. The above equation in two dimensions can be written in scalar form as

$$\boxed{\sum F_x = ma_x} \qquad \text{... (.2)}$$

$$\boxed{\sum F_y = ma_y} \qquad \text{... (.3)}$$

We can rewrite equation (.1) as,

$$\boxed{\sum \vec{F} + (-m\vec{a}) = 0} \qquad \text{... (.4)}$$

• This equation indicates that if a vector $-m\vec{a}$ is added to the forces acting on the particle, we get an equation that resembles equilibrium.

• The vector $-m\vec{a}$, which has same magnitude as $m\vec{a}$ but opposite direction, is called the **inertia vector.**

• We can say that the particle is in a state of dynamic equilibrium. This is called **D'Alembert's principle of dynamic equilibrium.**

• The inertia vector is a measure of the opposition that is offered by an object when we try to change its state of rest or of uniform motion along a straight line.

Problems on kinetics of particles in rectilinear motion can be classified broadly into the following types:

1) Variable force: When atleast one of the forces acting on the particle is variable, the acceleration will be variable.

2) Constant forces: If all forces acting on the object are constant, the acceleration will be constant. For further analysis we have to use kinematical equations

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

3) Dependent motion: If objects are connected by cords/cables then the accelerations of such objects are related. The tensile tes are related using the equilibrium of pulleys and the concept that tensile force throughout a given string is constant in absence of friction. Pulleys can be considered to be equilibrium eventhough they may be accelerated as their mass will be considered negligible.

$$\sum \vec{F} = m\vec{a} = 0$$

4) Relative motion: When an object is kept inside/on another object, the acceleration of the first object can be obtained with respect to the second. Then, knowing the absolute acceleration of the second object, the absolute acceleration of the first object can be obtained.

- **General Procedure for Solving Problems :**

1) Draw free body diagram of the objects in statics. The additional term now that has to be shown is the ma term, which is the effect. To differentiate the effect from the causes, we will show the effect with a double arrow (→)
2) Choose co-ordinate axes parallel and perpendicular to the direction in which motion can take place. For example, if an object is moving on an inclined plane, choose x-axis parallel to inclined plane and y-axis perpendicular to it.
3) Use following equation to find acceleration of an object :

$$\sum F_x = ma_x \text{ and}$$
$$\sum F_y = ma_y$$

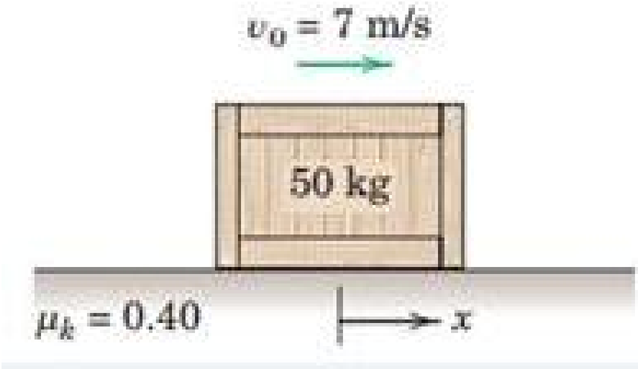
4) If objects are connected through cables, relate their acceleration and tensile forces in connecting cables.
5) Take ma term in the direction of motion so that if '*a*' comes out to be negative it will indicate deceleration.

Example :A rigid body is acted upon by a force of 100 N, the velocity of body changes from 15 m/s to 25 m/s during a period of 50 s. Find the mass of body and the distance moved by the body during the time interval
Solution :

$$a = \frac{v-u}{t} = \frac{25-15}{50}$$
$$\therefore \quad a = 0.2 \text{ m/s}^2$$
$$F = ma$$
$$\therefore \quad 100 = m \times 0.2$$
$$\therefore \quad \boxed{m = 500 \text{ kg}}$$
$$v^2 = u^2 + 2as :$$
$$25^2 = 15^2 + 2(0.2)s$$
$$\therefore \quad \boxed{s = 1000 \text{ m}}$$

Example :The 50 kg crate is projected along the floor with an initial speed of 7 m/s at x = 0. The coefficient of kinetic friction μ_k = 0.4. Calculate the time required for the crate to come to rest and corresponding distance x travelled. Refer Fig

Solution :



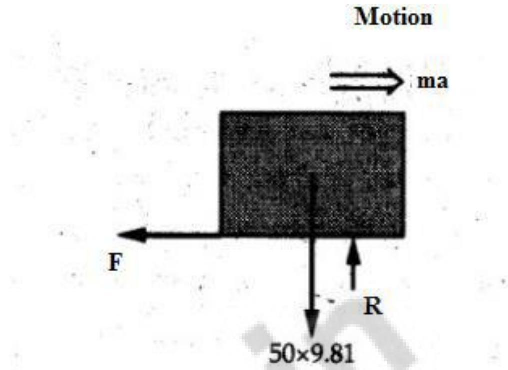
Step 1 :

Σ Fy = 0

R – (50 X 9.81) = 0

R = 490..5 N

FBD



Step 2 :

Σ Fx = ma

-F = 50 X a

- (µ X R) = 50 X a

- (0.4 X 490.5) = 50 X a

a = -3.924 m / s^2

$$\therefore \quad a = -3.924 \text{ m/s}^2$$

$$u = 7 \text{ m/s}, v = 0$$

$$v = u + at :$$

$$0 = 7 + (-3.924)t$$

$$\therefore \quad \boxed{t = 1.784 \text{ s}}$$

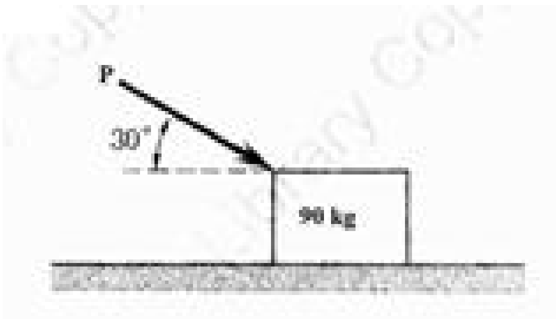
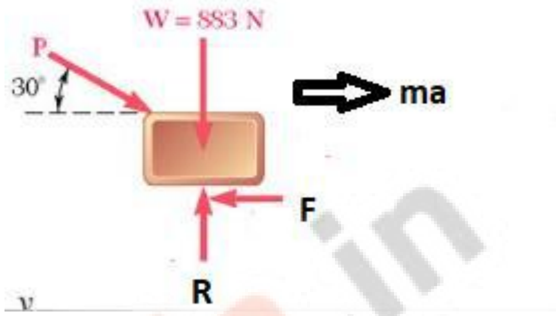
$$v^2 = u^2 + 2as$$

$$0 = 7^2 + 2(-3.924)s$$

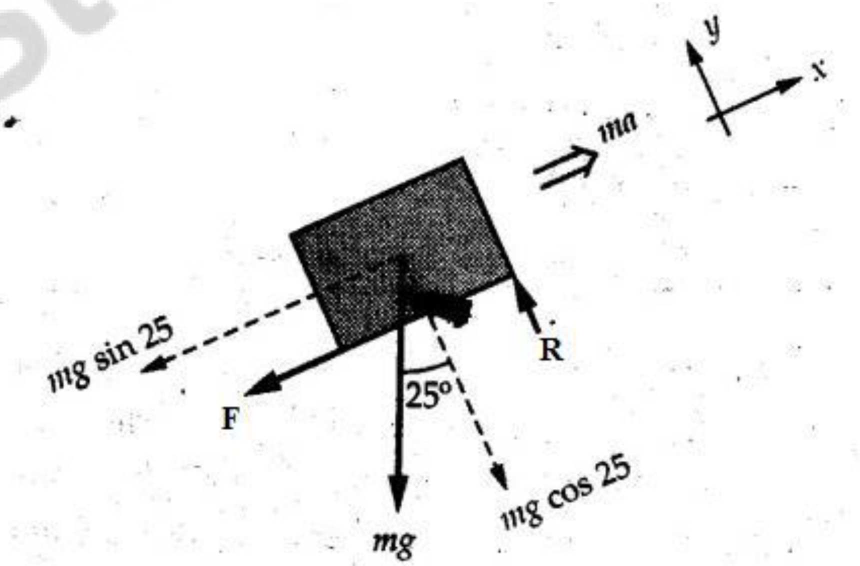
$$\therefore \quad \boxed{s = 6.244 \text{ m}}$$

Example :A 90 kg block rests on a horizontal plane. Find the magnitude of the force P required to give the block an acceleration of 3m/s^2 to the right .the coefficient of friction between the block and the plane is 0.25

Solution : m = 90 kg , a = 3 m/ s ^{2} , μ = 0.25

	FBD
 ***Step 1 :*** **Σ Fy = 0** **R – (90 X 9.81) – (P sin 30) = 0** **R = (P sin 30) + (90 X 9.81)**	 **Step 2 :** **Σ Fx = ma** **Pcos 30 –F = 90 X 3** **Pcos 30 - (μ X R) = 90 X 3** **Pcos 30 - 0.25((P sin 30) + (90 X 9.81) = 90 X 3** **P = 662.22 N**

Example : A body of mass 'm' is projected up a 25° inclined plane with an initial velocity of 15 m/s. If the coefficient of friction μ_k = 0.25, determine how far the body will move up the plane and the time required to reach the highest point.



Step 1 : **Σ Fy = 0** **R – (mgcosθ) = 0** **R = mgcosθ -------- ------------------ (1)** **Step 2 :** **Σ Fx = ma** **-mgsinθ - R = ma** **-mgsinθ - (0.25 X mgcosθ) = ma** **-mg (Sin25 – 0.25 X cos 25) = ma** **- 9.81 (Sin25 – 0.25 X cos 25) = a** **a = -6.369 m/ S ^{2}**	**Step 3 :** $u = 15$ m/s, $v = 0$ $v = u + at$: $0 = 15 + (-6.369)\, t$ $\therefore \boxed{t = 2.355 \text{ s}}$ $v^2 = u^2 + 2as$: $0 = 15^2 + 2(-6.369)\, s$ $\therefore \boxed{s = 17.66 \text{ m}}$

Example : A hockey player hits a puck so that it comes to rest in 9 s after sliding 30 m horizontally on the ice. Determine
a) The initial velocity of the puck,
b) The coefficient of friction between the puck and ice.

Solution : **t = 9 s, s = 30 m,**

$v = 0$

$v = u + at$:

$$0 = u + a\,(9) \qquad \ldots (1)$$

$$s = ut + \frac{1}{2} at^2$$

$$30 = u\,(9) + \frac{1}{2} a\,(9^2) \qquad \ldots (2)$$

From equation. (1), $u = -9\,a$

Substituting in equation (2),

$$30 = (-9a)\,(9) + \frac{1}{2} a\,(9^2)$$

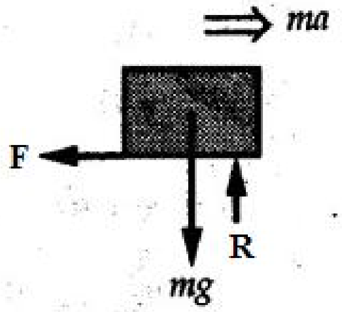
$$30 = -40.5\,a$$

$$\therefore \quad a = 0.741 \text{ m/s}^2$$

$$u = (-9)\,(-0.741)$$

$$\therefore \quad \boxed{u = 6.67 \text{ m/s}}$$

Consider F.B.D. of the puck shown in Fig :



$$\sum F_y = 0 : \quad R - mg = 0$$

$$\therefore \quad R = mg$$

$$\sum F_x = m a_x :$$

$$-\mu_k . R = ma$$

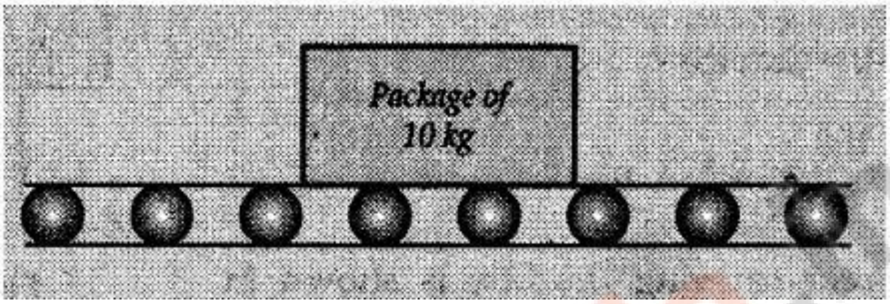
$$-\mu_k (mg) = m(-0.741)$$

$$\mu_k = \frac{0.741}{9.81}$$

$$\boxed{\mu_k = 0.0755}$$

Example : The conveyor belt as shown in Fig. is designed to transport packages of various weights. Each 10 kg package has a coefficient of friction μx = 0.15. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the required time and distance the package will slide on the conveyor before coming to rest.

Solution : Consider that the package is moving towards right. When conveyor stops, package starts moving with velocity 5 m/s towards right due to inertia. Consider F.B.D. of package shown in Fig.



Solution :

$$\sum F_y = 0 :$$

$$R - 10 \times 9.81 = 0$$

$$\therefore \quad R = 98.1 \text{ N}$$

$$\sum F_x = ma_x :$$

$$-0.15\,R = 10a$$

$$\therefore \quad a = -\frac{0.15 \times 98.1}{10} = -1.4715 \text{ m/s}^2$$

$$u = 5 \text{ m/s}, \quad v = 0$$

$$v = u + at :$$

$$0 = 5 + (-1.4715)\,t$$

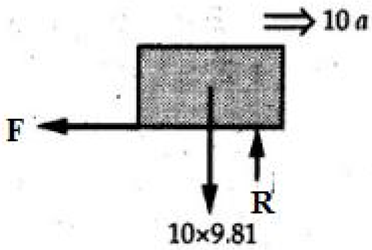
$$\therefore \quad \boxed{t = 3.4 \text{ s}}$$

$$v^2 = u^2 + 2as :$$

$$0 = 5^2 + 2(-1.4715)\,s$$

$$\therefore \quad \boxed{s = 8.495 \text{ m}.}$$

FBD



Example : A elevator being lowered into a mine-shaft starts from rest and attains a speed of 10 m/s within a distance of 15 meters. The elevator alone has a mass of 500 kg and it carries a box of mass 600 kg in it. Find the total tension in the cables supporting the elevator, during this accelerated motion. Also find the total pressure between the box and the floor of the elevator.

Solution :

For the elevator, u = 0, v = 10 m/s, S = 15 m

$v^2 = u^2 + 2as$:

$10^2 = 0 + 2a\,(15)$

$a = 100 / 30 = 33.333$ m/s^2

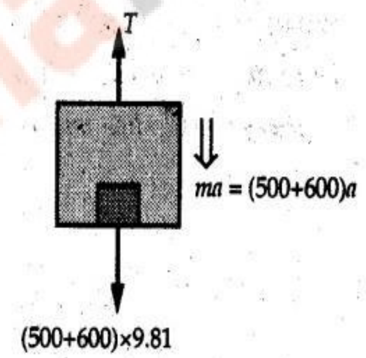
The F.B.D. of elevator carrying the box is shown in Fig

$$\sum F_y = m a_y \ :$$

$$T - 1100 \times 9.81 = -1100\, a$$

$$\therefore \quad T = 1100 \times 9.81 - 1100\,(3.333)$$

$$\therefore \quad \boxed{T = 7124.7 \text{ N}}$$

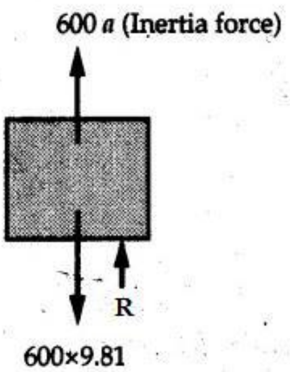


The total pressure between the box and the floor can be obtained from the F.B.D of box shown in Fig. 600 *a* is inertia force directed opposite to acceleration of lift. Acceleration of box with respect to lift is zero.

$$\sum F_y = 0 \ : \quad R + 600a - 600 \times 9.81 = 0$$

$$R = 600 \times 9.81 - 600\,(3.333)$$

$$\therefore \quad \boxed{R = 3886.2 \text{ N}}$$



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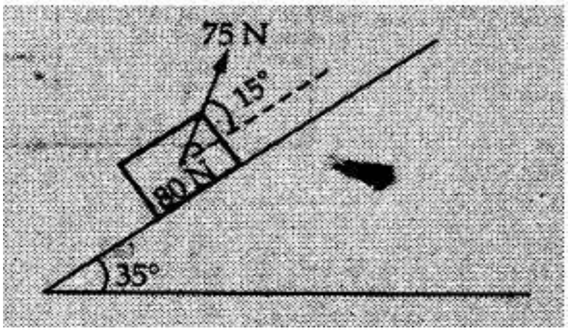
Example : An object weighing 80 N is pulled up on the smooth plane by a 75 N force as shown in Fig.. Determine the velocity of the object after it has moved 4 m.

Solution :



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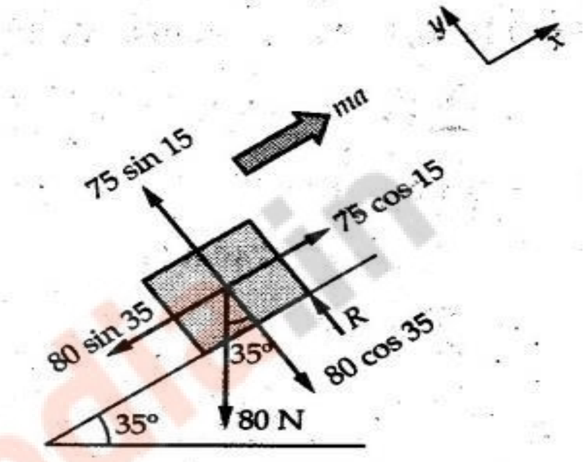


$$\sum F_x = m a_x$$

$$75 \cos 15 - 80 \sin 35 = \frac{80}{9.81} a_x$$

$$\therefore \quad a_x = 3.256 \text{ m/s}^2$$

$$\text{Now,} \quad v^2 = u^2 + 2as = 0 + 2 \times 3.256 \times 4$$

$$\therefore \quad \underline{v = 5.104 \text{ m/s}}$$

Kinetics of Curvilinear Motion :

If normal and tangential components are used, the equations of motion would be

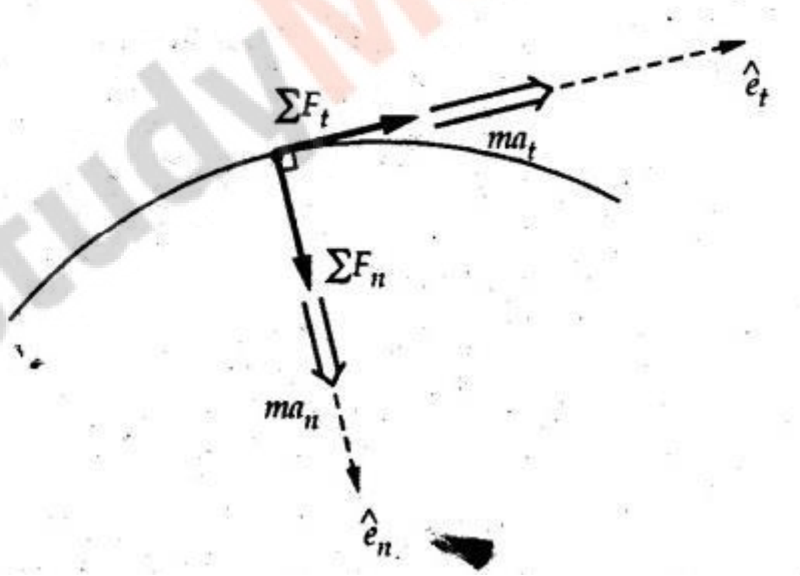
$$\sum F_t = ma_t \quad \text{and}$$

$$\sum F_n = ma_n$$

where $$a_n = \frac{v^2}{\rho}$$

ma_n is called the **centripetal force.**

• a_t, is directed along the tangent in the direction of velocity and a_n is directed towards the centre of curvature as shown in Fig.



$$ma_n = \frac{mv^2}{\rho}$$

Another approach is to use dynamic equilibrium condition in which is shown directed away from the centre (the centrifugal force).

• In this book, we will be using the dynamic equilibrium equation for the normal direction, so that equations of motion will be

$$\sum F_t = ma_t \qquad \text{... (9.10.1)}$$

and

$$\sum F_n = 0 \qquad \text{... (9.10.2)}$$

where mv^2/ρ the centrifugal force will be shown directed away from the centre of curvature. In many problems only equation (9.10.2) will be required.

1. General Procedure for Solving Problems

Draw F.B.D. of object. Resolve all forces in tangential and normal directions. Show centrifugal force directed away from the centre.

$$\sum F_t = ma_t$$

and

$$\sum F_n = 0$$

If radial and transverse components are used, the equations of motion will be

$$\sum F_r = ma_r \qquad \text{... (9.10.3)}$$

and

$$\sum F_\theta = ma_\theta \qquad \text{... (9.10.4)}$$

Example : A motorcycle stuntman drives his motorcycle in a spherical cage in a vertical circle of radius 8 m. Determine the minimum velocity with which he should drive his motorcycle so that he does not lose contact with the cage at top of the circular path.

Solution :

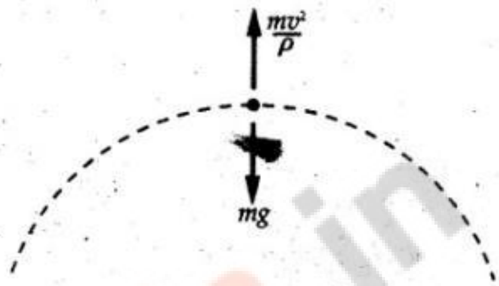
At the top, the centrifugal force mv^2/ρ must balance mg if the motorcycle is not to lose contact with the cage.

$$\therefore \quad mg = \frac{mv^2}{\rho}$$

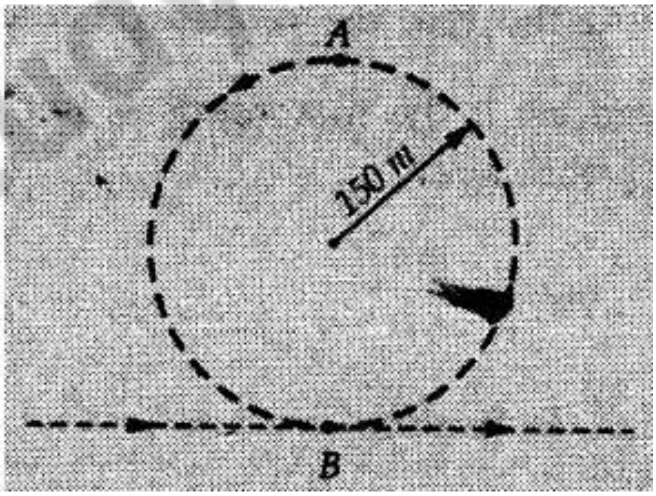
$$v = \sqrt{\rho g}\ ;$$

$$\rho = 8\,\text{m},\ g = 9.81\,\text{m/s}^2$$

$$\therefore \quad v = \sqrt{8 \times 9.81} = 8.86\ \text{m/s}$$



Example : A 70 kg pilot flies in a small plane which moves in a vertical loop of 150 m radius as shown. Knowing that pilot experiences weightlessness at 'A' and has apparent weight 2800 N at 'B', find speed of plane at 'A' and 'B'.



At A, $\sum F_n = 0$:

$$mg - \frac{mv_A^2}{\rho} = 0$$

$$\therefore \quad v_A = \sqrt{\rho g} = \sqrt{150 \times 9.81}$$

$$\therefore \quad \boxed{v_A = 38.36 \text{ m/s} = 138.1 \text{ km/h}}$$

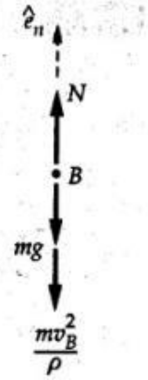
At B, $\sum F_n = 0$:

$$N - mg - \frac{mv_B^2}{\rho} = 0$$

The normal reaction is the apparent weight.

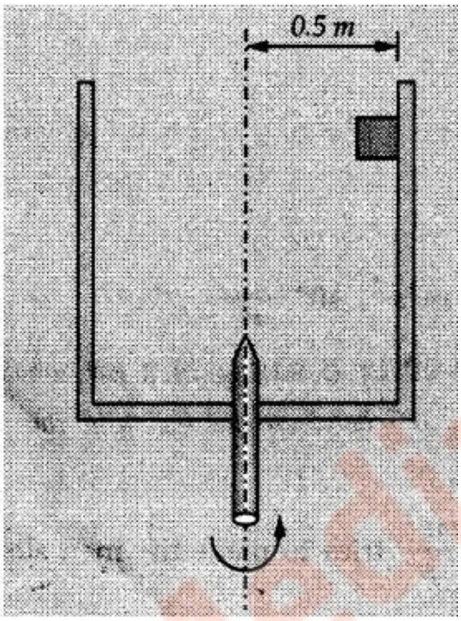
$$\therefore \quad 2800 - 70 \times 9.81 - \frac{70\, v_B^2}{150} = 0$$

$$\therefore \quad \boxed{v_B = 67.29 \text{ m/s} = 242.26 \text{ km/h}}$$



Example : **A cylindrical bowl of 500 mm radius rotates about its vertical axis. Find minimum angular speed in r.p.m. so that a small block placed on the wall of cylinder does not fall down. Assume μ = 0.30 between block and bowl.**

Solution :



Solution :

Let m = Mass of the block

$$\sum F_n = 0$$

$$-R + m\rho\omega^2 = 0$$

$$R = m\rho\omega^2$$

$$\sum F_y = 0$$

$$\mu R - mg = 0$$

$$\mu(m\rho\omega^2) - mg = 0$$

$$\omega = \sqrt{\frac{g}{\mu\rho}} = \sqrt{\frac{9.81}{(0.3)(0.5)}}$$

$$\therefore \quad \omega = 8.087 \text{ rad/s} = 8.087 \times \frac{60}{2\pi} \text{ r.p.m}$$

$$\therefore \quad \boxed{\omega = 77.225 \text{ r.p.m.}}$$

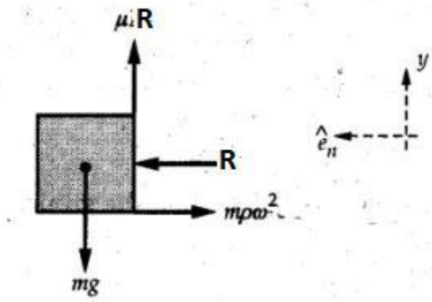
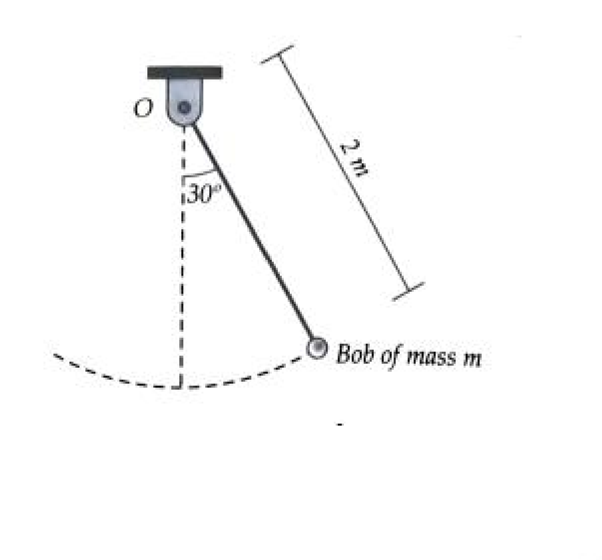


Fig. 9.10.7 (a)

Example : The bob of a 2 m pendulum describes an arc of circle a in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown times in Fig. find the velocity and acceleration of bob in that position.



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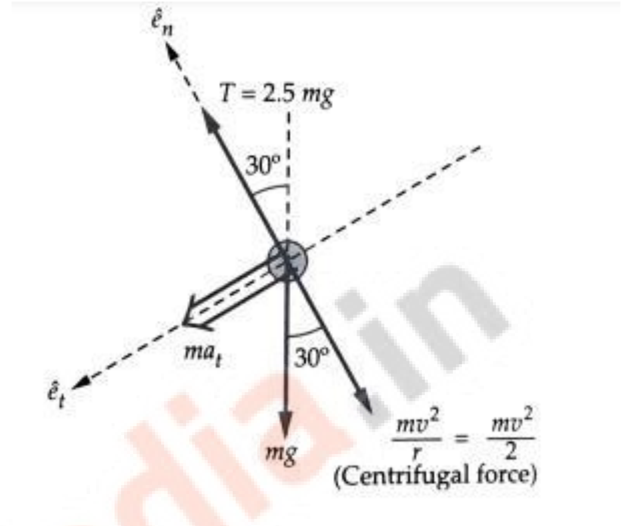


Solution :

$$\sum F_t = ma_t :$$

$$mg \sin 30 = ma_t$$

$$\therefore \quad a_t = g \sin 30 = 9.81 \sin 30$$

$$\therefore \quad a_t = 4.905 \text{ m/s}^2$$

$$\sum F_n = 0 :$$

$$T - mg \cos 30 - \frac{mv^2}{r} = 0$$

$$2.5 \, mg - mg \cos 30 = \frac{mv^2}{2}$$

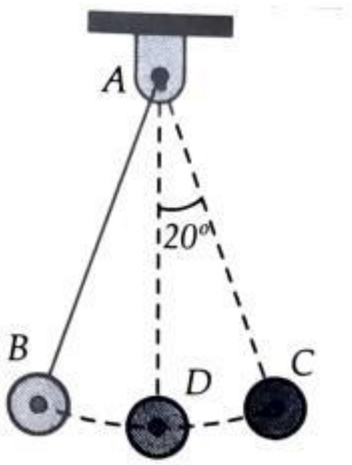
$$\therefore \quad v = 5.662 \text{ m/s}$$

$$a_n = \frac{v^2}{r} = \frac{5.662^2}{2} = 16.03 \text{ m/s}^2$$

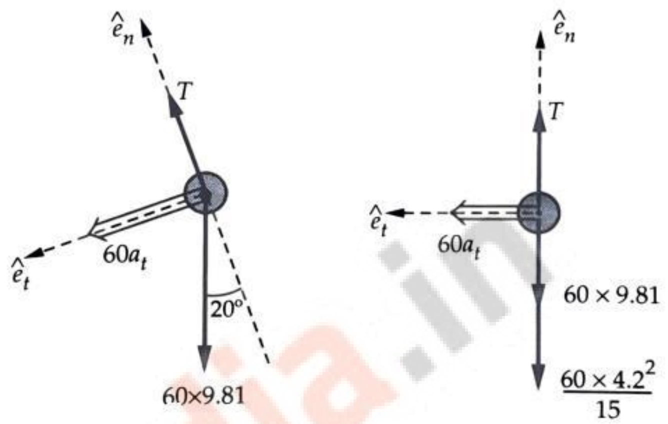
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.905^2 + 16.03^2}$$

$$\therefore \quad a = 16.764 \text{ m/s}$$

Example : A 60 kg ball B is attached to a 15 m long steel cable and strings in a vertical arc as shown in Fig. Determine the tension in the cable a) At the top C of the swing, and b) At the bottom D of the swing where the speed of the ball is 4.2 mls.



F.B.D. of the ball at C and D is shown in Fig: At the top of the swing,



Solution :

$$v = 0 \Rightarrow \frac{mv^2}{\rho} = 0$$

$$\sum F_n = 0$$

$$T - 60 \times 9.81 \cos 20 = 0$$

$$\therefore \quad \boxed{T = 553.1 \text{ N}}$$

...Ans.

F.B.D. of ball at point D is shown in Fig.

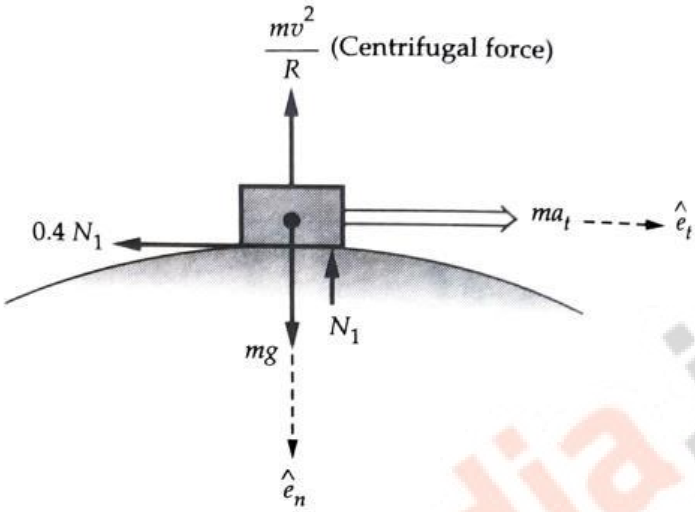
$$\sum F_n = 0$$

$$T - 60 \times 9.81 - \frac{60 \times 4.2^2}{15} = 0$$

$$\therefore \quad \boxed{T = 659.16 \text{ N}}$$

Example : An automobile is moving on a hump with radius = 50m , along a road , at a speed 40 KMPH . Knowing that the coefficient of friction between tyres and road surface is 0.4 , determine the tangential acceleration produced if brakes are suddenly applied.

Solution :



$$\sum F_n = 0 :$$

$$mg - \frac{mv^2}{R} - N_1 = 0$$

$$N_1 = mg - \frac{mv^2}{R}$$

$$= m\left[9.81 - \frac{\left(40 \times \frac{5}{18}\right)^2}{50}\right]$$

$$N_1 = 9.463\ m$$

$$\sum F_t = ma_t$$

$$-0.4\ N_1 = ma_t$$

$$-0.4 \times 9.463\ \text{m} = ma_t$$

$$\therefore \quad \boxed{a_t = -3.785\ \text{m/s}^2}$$

Kinetics of Particle :Energy Methods

Work done (U)= $F \times s$

S. I. unit : Joule or N·m

- Work done is a scalar quantity.
- **Power :** It is the rate of doing work

$$\therefore \quad \text{Power} = \frac{F \times s}{t} = F \times v$$

S. I. unit : $\frac{N \cdot m}{sec}$ OR watt

- **Energy :** It is the capacity to do work.

S. I. unit : Joule OR N·m.

- Energy is a scalar quantity.
- Types of energy :

a) Potential Energy (P.E.) : This is the energy of a particle due to its position.

$$P.E. = mgh$$

b) Kinetic Energy (K.E.) : This is the energy of a particle due to its motion.

- **Work-energy principle :** $U_{1\to2} = KE_2 - KE_1$

General Procedure for Solving Problems

1) Draw F.B.D. of the object.

ii) Resolve all forces into two mutually perpendicular directions - parallel and perpendicular to displacement.

iii) Identify velocities at initial position 1 and final position 2 to write KE_1, and KE_2.

iv) Calculate U_{1-2}, ie, the work done by all forces parallel to the direction of displacement. If force is in the direction of displacement then the work done will be positive. If force is in a direction opposite to displacement, work done is negative. Work done by forces perpendicular to displacement is zero.

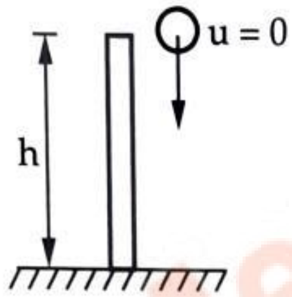
v) Use the work-energy principle to find required quantities.

Note: The work energy principle cannot be used to find acceleration or forces for which the work done is zero.

Example : *A 2 kg stone is dropped from a height h and strikes the ground with a velocity of 24 m/s. Using work energy principle find the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped.*

Sol. : At the top

$$K.E_1 = 0$$



At the ground,

$$K.E_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 24^2$$

$\therefore$ $\boxed{K.E_2 = 576 \text{ J}}$ **...Ans.**

By work-energy principle,

$K.E_1$ + Work done by all forces = $K.E_2$

Only gravitational force acts on the stone.

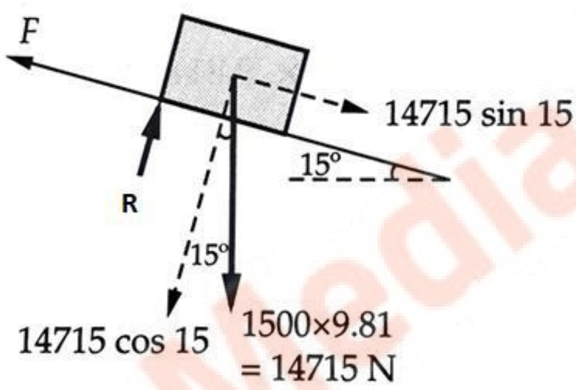
Work done by gravity = $mgh = 2 \times 9.81 \times h$

$\therefore$ $0 + 2 \times 9.81h = 576$

$\therefore$ $\boxed{h = 29.36 \text{ m}}$ **...Ans.**

EXAMPLE : *A car of mass 1500 kg is moving down a hill having a slope of 15° to the horizontal. At the time, when the car is moving at a speed of 10 m/s, the driver applies the brakes. Calculate the average force applied parallel to the hill slope that will stop the car in a distance of 30 m. Use work energy principle.*

Solution :



$$\text{Initial K.E.} = \frac{1}{2} \times 1500 \times 10^2$$

$$\text{Final K.E.} = 0$$

Work done by all forces = 14715 sin 15 × 30 − F × 30

By work-energy principle,

Initial K.E. + Work done by all forces = Final K.E.

$$\therefore \quad \frac{1}{2} \times 1500 \times 10^2 + [14715 \sin 15 \times 30 - F \times 30] = 0$$

$$\therefore \quad \boxed{F = 6308.5 \text{ N}}$$

EXAMPLE : *The car having mass of 2 Mg is originally travelling at 2 m/s. Determine the distance it must be travelled by a force F = 4 kN in order to attain a speed of 5 m/s. Neglect the friction.*

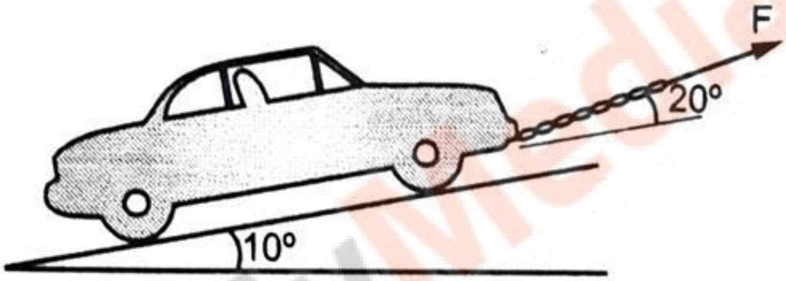
Solution :

Given data :

Mass of car, m = 2 Mg = 2000 kg

Initial velocity of car, u = 2 m/s

Final velocity of car, v = 5 m/s



Let d be the distance travelled by the car along the inclined plane. Considering F.B.D. of car and using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2} \times 2000 \times 2^2 + 4000 \cos 20 \times d - 2000 \times 9.81 \sin 10 \times d = \frac{1}{2} \times 2000 \times 5^2$$

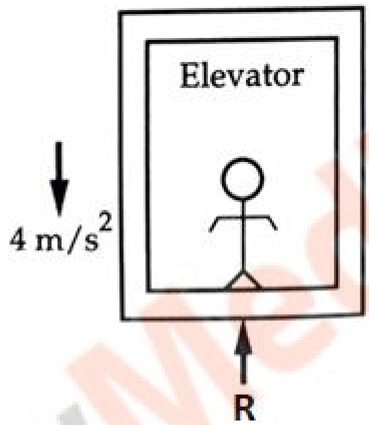
$$4000 + 3758.77\,d - 3406.97\,d = 25000$$

$$351.8\,d = 21000$$

$$\therefore \quad d = \mathbf{59.69\ m}$$

EXAMPLE : *A woman having a mass of 70 kg stands in an elevator which has a downward acceleration of 4 m/s^2 starting from rest. Determine the work done by her weight and the normal force which the floor exerts on her when the elevator descends 6 m.*

SOLUTION : FBD OF ELEVATOR AS SHOWN IN FIG :



As the weight acts in downward direction and the displacement is also downwards, the works done is positive.

$$W = m\,g \times h = 70 \times 9.81 \times 6$$

$\therefore$ $\boxed{W = 4120.2 \text{ J}}$ ...Ans.

Normal force $R = mg - ma = 70 \times 9.81 - 70 \times 4$

$\therefore$ $= 406.7 \text{ N} \uparrow$

$\therefore$ Work done $W_1 = -406.7 \times 6$

$\therefore$ $\boxed{W_1 = -2440.2 \text{ J}}$ ...Ans.

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EXAMPLE : *Two blocks are connected by an inextensible string as shown in Fig. . If the system is released from rest, determine the velocity of the block A after it has moved 2 m by work energy principle. The coefficient of friction between block A and the plane is* $\mu_s = 0.25$.

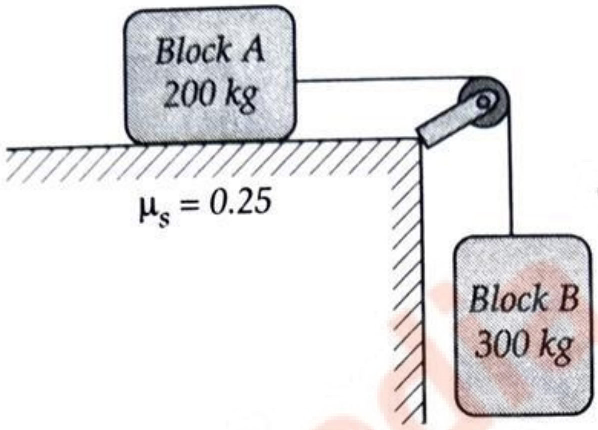


Fig.

Sol. : By work-energy principle,

$$\text{Work done by all forces} = K.E_f - K.E_i$$

$$K.E_i = 0 \quad \text{and} \quad v_A v_B = v$$

$$\therefore \quad m_B g \times s - \mu_K m_A g \times s = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2$$

$$300 \times 9.81 \times 2 - 0.25 \times 200 \times 9.81 \times 2 = \frac{1}{2} \times 200 v^2 + \frac{1}{2} \times 300 v^2$$

$$\therefore \quad \boxed{v = 4.429 \text{ m/s}}$$

EXAMPLE : : *Block A has a weight of 300 N and block B has a weight of 50 N. Determine the speed of block A after it moves 1.5 m down the plane, starting from rest by work energy principle. Neglect the friction and mass of the pulleys.*
Refer Fig.

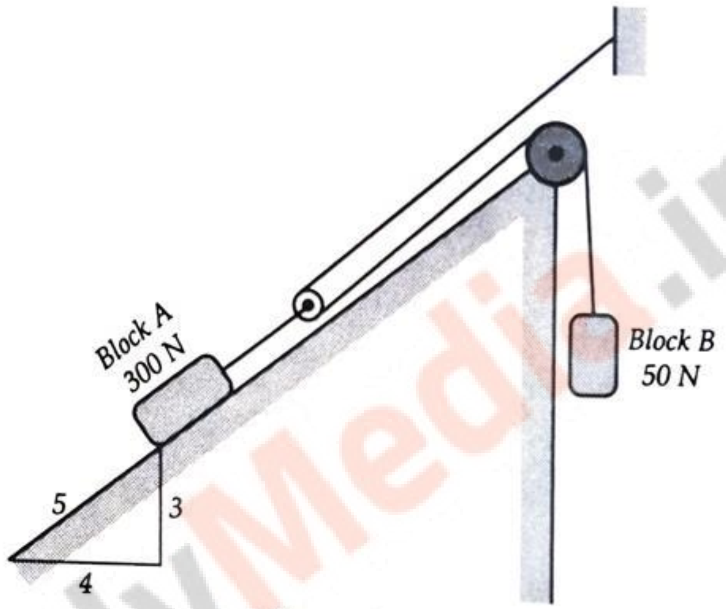


Fig.

Sol. : The external forces acting on the system are shown in Fig.

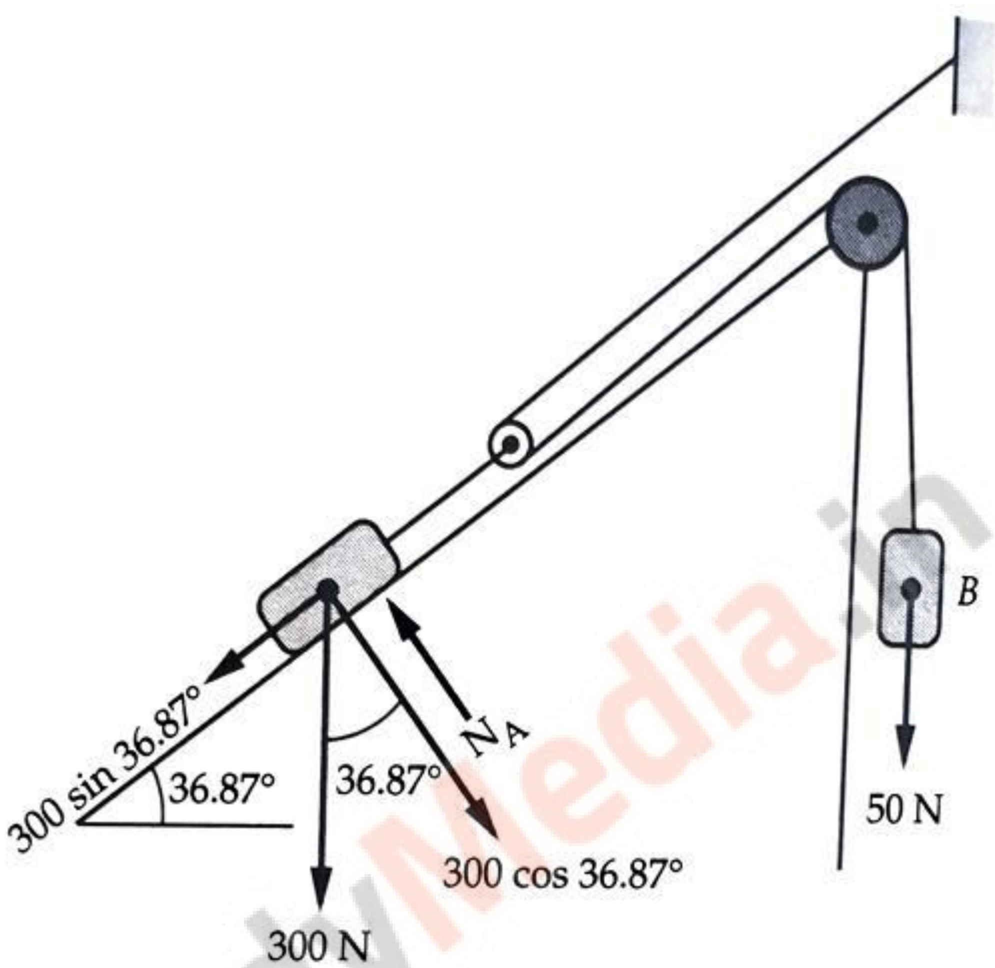
If A travels 1.5 m down the plane, B travels 3 m upward. Also, if v is speed of A, speed of B will be 2 v.

R_y work - energy principle,

Initial K.E. + Work done by all forces = Final K.E

$$\text{Initial K.E} = 0$$

$$\text{Work done} = 300 \sin 36.87 \times 1.5 - 50 \times 3 = 120 \text{ J}$$



$$\text{Final K.E} = \frac{1}{2}\left(\frac{300}{9.81}\right)v^2 + \frac{1}{2}\left(\frac{50}{9.81}\right)(2v)^2$$

$$= 25.484\,v^2$$

$$\therefore \quad 0 + 120 = 25.484\,v^2$$

$$\therefore \quad \boxed{v = 2.17 \text{ m/s}}$$

Conservative Forces and Potential Energy :

If the work done by a force does not depend on the path along which the particle is taken from one point to another, the force is called conservative force. In such a case the work done depends on the initial and final positions of the particle.

• For example, the work done by weight ($U_{1-2} = -W\ \Delta y$) depends only on the difference of height Δy.

• Even if the object is taken from A to B along different paths ACB, AEB or ADB, the work done remains same. Similarly, the work done by spring force $\left(U_{1\text{-}2} = \frac{1}{2}\,kx_1^2 - \frac{1}{2}kx_2^2\right)$ depends only on positions x_1 and x_2 and hence spring force is a conservative force.

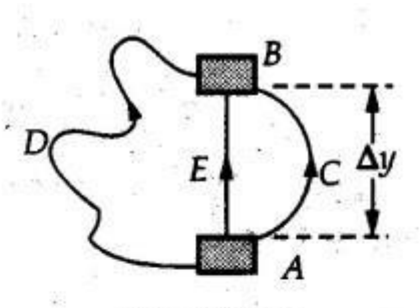


Fig. 9.3.11

• Friction is a non-conservative force. The work done by friction depends on the path.

• Potential energy of a particle is due to the position of the DC particle (Kinetic energy is due to velocity).

• It is defined for the conservative forces for which the work done depends only on the initial and final position like the gravitational force and the spring force.

• In mechanics we come across two types of potential energies the gravitational potential energy and the potential energy in a spring (the spring potential energy).

Gravitational Potential Energy (G.P.E) :

• G.P.E. = $mgh = Wh$

where h is measured from a reference level. Although, any reference level can be taken, it is more convenient to take lower of the two positions as reference so that gravitational potential energy at the lower position becomes zero.

Spring Potential Energy (S.P.E)

- $$S.P.E. = \frac{1}{2}kx^2$$

where k = Spring constant and

x = Deformation of spring.

5. Conservation of Energy

• Principle of conservation of energy states that under the action of only conservative forces, the sum of kinetic energy and potential energy of a particle remains constant.

• If a particle under the action of only conservative forces is taken from position 1 to position 2,

$$\boxed{K.E_1 + G.P.E_1 + S.P.E_1 = K.E_2 + G.P.E_2 + S.P.E_2} \quad \text{... (9.3.8}$$

• Conservation of energy implies that during the motion of particle, the kinetic energy will be converted to potential energy or potential energy to kinetic energy so as to keep the total energy constant.

• An important result which can be remembered while solving problems is the velocity attained by an object falling freely under gravity through height '*h*', starting from a state of rest. Its potential energy gets converted to kinetic energy.

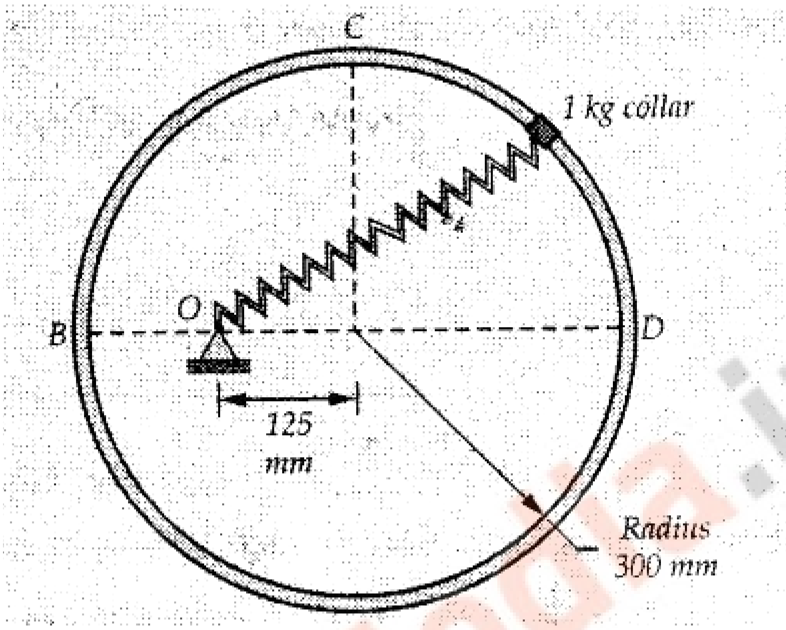
$$mgh = \frac{1}{2}mv^2$$

$$\therefore \quad \boxed{h = \frac{v^2}{2g}} \quad \text{... (9.3.9)}$$

or

$$\therefore \quad \boxed{v = \sqrt{2gh}} \quad \text{... (9.3.10)}$$

EXAMPLE : A 1 kg collar is attached to a spring of stiffness 250 N/m and can slide freely without friction on a circular rod in horizontal plane. Spring is undeformed when collar is at B. Knowing that collar passes through point D with speed 1.8 m/s, determine velocity of collar as it passes through point C and point B.



Solution:

As the rod is in horizontal plane, there is no change in height

Undeformed length of collar = OB = 300 – 125 = 175 mm.

Deformation of spring at B = $x_B = 0$

At D, $x_D = (300 + 125) - 175$

= 250 mm.

$$\text{At C,}\quad x_C = \sqrt{125^2 + 300^2} - 175$$

$\therefore x_C = 150$ mm

Using conservation of energy for D and C,

$$K.E_D + S.P.E_D = K.E_C + S.P.E_C$$

$$\frac{1}{2}(1)(1.8^2) + \frac{1}{2}(250)(0.25^2) = \frac{1}{2}(1)v_C^2 + \frac{1}{2}(250)(0.15^2)$$

$$\therefore \quad \boxed{v_C = 3.64 \text{ m/s}}$$

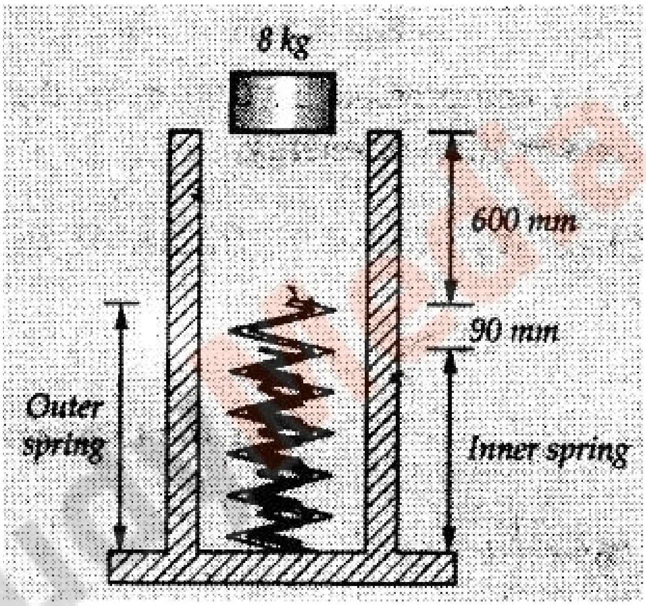
Using conservation of energy for D and B,

$$K.E_D + S.P.E_D = K.E_B + S.P.E_B$$

$$\frac{1}{2}(1)(1.8^2) + \frac{1}{2}(250)(0.25)^2 = \frac{1}{2}(1)v_B^2 + 0$$

$$\therefore \quad \boxed{v_B = 4.34 \text{ m/s}}$$

EXAMPLE : A 8 kg plunger is released from 600 mm height onto nested springs. Outer and inner springs have constants 4 KN/m and 12 KN/m respectively. Calculate maximum deformation of outer spring.



If outer spring is compressed by 'x', inner spring will be compressed by $(x - 0.09)$.

By conservation of energy,

$K.E_1 + G.P.E_1 + S.P.E_1 = K.E_2 + G.P.E_2 + S.P.E_2$.

Taking the final position as reference, the height of initial point is $h = (x + 0.6)$,

$$\therefore 0 + (8)(9.81)(x + 0.6) + 0 = 0 + 0 + \frac{1}{2}(4000)(x^2) + \frac{1}{2}(12000)(x - 0.09)^2$$

$$78.48x + 47.088 = 2000x^2 + 6000(x^2 - 0.18x + 0.0081)$$

$$8000x^2 - 1158.48x + 1.512 = 0$$

$$\therefore \quad \boxed{x = 0.1435 \text{ m} = 143.5 \text{ mm}}$$