Engineering Physics

Unit No. 2

QUANTUM PHYSICS

Unit II	Quantum Physics	(08 Hours)
de Broglie hypothesis of matter waves, de Broglie wavelength for a particle accelerated by KE "E" and a charged particle accelerated by PD "V", properties of matter waves; Wave function and probability density, mathematical conditions for wave function, problems on de Broglie wavelength; Need and significance of Schrödinger's equations, Schrödinger's time independent and time dependent equations; Energy of a particle enclosed in a rigid box and related numerical problems; Quantum mechanical tunneling, alpha particle decay, principle and applications of STM; Principles of quantum computing: concept of qbit, superposition and entanglement, comparison of classical & quantum computing, potential applications of quantum computing.		

Prof. S. J. Gadakh

Sample Questions

6 Marks

1. Starting from de Broglie hypothesis, derive Schrodinger's Time Independent wave equation.
2. State the importance of Schrodinger's equations. Derive Schrodinger's Time Dependent wave equation.
3. Derive the equation for the energy of particle is enclosed in a one dimensional rigid box of infinite potential well.
4. Derive the equation of wave function of the particle is enclosed in a one dimensional rigid box of infinite potential well.
5. What is quantum mechanical tunneling effect? Explain the principle and working of Scanning Tunneling Microscope (STM). State the applications of STM.

4 Marks

1. State de Broglie hypothesis. Derive the equation of de Broglie wavelength by analogy with radiation.
2. State de Broglie hypothesis. Derive the equation of de Broglie wavelength in terms of Kinetic Energy of a particle.
3. State de Broglie hypothesis. Derive the equation of de Broglie wavelength for an electron and proton when it is accelerated by a potential difference 'V'.
4. [illegible]
5. State Heisenberg's uncertainty principle. [illegible] it using the concept of narrow [illegible]
6. What is wave function Ψ? Explain the physical significance of $|\Psi|^2$.
7. State and explain the mathematical conditions that a wave function Ψ must satisfy and obey. What is a well behaved wave function?
8. What is the physical significance of $|\Psi|^2$. Explain normalization condition of wave function and state why it should be normalized?
9. What is quantum mechanical tunneling effect? Explain in brief the role of quantum mechanical tunneling in alpha particle decay.
10. What is quantum mechanical tunneling effect? Explain in brief the principle of tunnel diode.
11. What is quantum mechanical tunneling effect? Explain in brief the principle of Scanning Tunneling Microscope.
12. Explain in brief, principles of quantum computing. Explain in brief what would be the potential applications of quantum computing?

Topic:
de Broglie's hypothesis of matter wave

IMP Sample Question

Q1] State de Broglie's hypothesis of matter wave.
-1 or 2 Marks

Ans→ According to de Broglie's hypothesis,
"Every moving particle is associated with a wave, which governs the motion of the particle."
That wave is called as matter wave or de Broglie's wave.

The de Broglie's wavelength is given by

$$\boxed{\lambda = \frac{h}{p}}$$

or

$$\boxed{\lambda = \frac{h}{mv}}$$

Where, h = Plank's constant
m = mass of particle
v = velocity of particle
p = momentum of particle

Topic:

de Broglie's wavelength in terms of kinetic energy.

IMP Sample Question:

Q3] Derive the de Broglie's wavelength in terms of kinetic energy. —— 3 Marks

Ans ⟹ The particle of mass 'm' moving with velocity 'v' then the momentum 'p' of a particle is given by

$$P = mV \quad —— ①$$

and the de Broglie's wavelength

$$\lambda = \frac{h}{P} \quad —— ②$$

Now the kinetic energy of a particle is given by $E = \frac{1}{2} mv^2$

Multiplying and dividing by m then we get, $E = \frac{m^2v^2}{2m}$

But $mv = p$ and $m^2v^2 = p^2$

Then above eq^n becomes

$$E = \frac{p^2}{2m}$$

$$\therefore p^2 = 2mE$$

$$\therefore p = \sqrt{2mE} \quad \text{— (3)}$$

From eq^n (2) & (3) we get

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{— (4)}$$

In eq^n (4) λ is wavelength of moving particle expressed in terms of kinetic energy of particle.

Topic:
de Broglie's wavelength of electron or charged particle.

IMP) Sample Questions:

Q4] Show that the de Broglie's wavelength of charged particle (Proton) is inversely proportional to the square root of the accelerating potential. — 4 Marks

Q Show that the de Broglie's wavelength of an electron is inversely proportional to the square root of the accelerating potential.

Prof. S.J. Gadakh

Ans:

If an electron acquires velocity v on accelerati[ng] it through potential difference of V volts, then the work on the electron is given by

$$W = eV \qquad \text{——} (1)$$

Where, e = charge on electron
V = accelerating potential

This work is converted in kinetic energy of electron

i.e. $E = \frac{1}{2}mv^2 = eV \qquad \text{——} (2)$

If e is in coulombs, m in kg, V in volts then the velocity v will be in m/s.

But $\lambda = \frac{h}{\sqrt{2mE}} \qquad \text{——} (3)$

So, by considering eq^n no (2), above eq^n becomes.

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad \text{——} (4)$$

hence $\lambda \propto \frac{1}{\sqrt{V}}$ is proved

Ignoring relativistic correction, we can take
$m = m_0$
where, m_0 is the rest mass of electron.

$$\therefore \lambda = \frac{h}{\sqrt{2m_0 eV}} \qquad \text{——} (5)$$

Now, h, m_0 and e are universal constants with values

$$h = 6.625 \times 10^{-34} \, J.s$$
$$e = 1.6 \times 10^{-19} C$$
$$m_0 = 9.1 \times 10^{-31} kg$$

Substituting these in above eqn no. 6 we get,

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}} \, m$$

$$\lambda = \frac{12.27}{\sqrt{V}} \times 10^{-10} m$$

$$\lambda = \frac{12.27}{\sqrt{V}} \, A^{o}$$

$$\lambda \propto \frac{1}{\sqrt{V}}$$ is proved

Note - Out of above four quations, combine question is asked in university exam.
Q1 is asked with either Q2, Q3 or Q4.

Prof. S.J. Gadakh

IMP | Topic : Properties of matter wave

Sample Questions :

Q State de Broglie's wave's properties.

Q State the properties of matter wave.

— 4 Marks

Ans: Properties:

1] Lighter the particle, greater would be the would be the wavelength of the matter waves associate with it.

i.e. $\lambda \propto \frac{1}{m}$, for constant v

2] Smaller the velocity of particle, greater would be the wavelength of the matter waves.

i.e. $\lambda = \frac{1}{v}$, for constant m

3] For $v = \infty$, λ becomes zero and for $v = 0$, λ becomes infinity. i.e. the wave becomes indeterminate when $v = 0$. This simply means that matter waves are produced by particles moving with finite velocities.

4] Matter waves are different from electromagnetic waves because these waves are produced by a moving particles which may charged or uncharg where electromagnetic waves are only produced by charged particle.

5] The velocity of matter waves depends on the velocity of particle generating them

$u = \frac{c^2}{v}$ and not constant.

6] Matter waves travel faster than light, because the particle velocity can not exceed the velocity of light C. So the velocity of matter waves

$$u = \frac{c^2}{v}$$ is greater than C.

Topic: Wave Function ψ & probability density $|\psi|^2$

Sample Question:

IMP
Q What is wave function ψ. Explain the physical Significance of $|\psi|^2$. — 4 Marks

Ans:- 1] Wave function ψ, in quantum mechanics, is a wave variable that mathematically describes the wave charecteristics of a particle.

2] The value of the wavefunction of a particle at a given point of space time is related to the likelihood of the particle is being there at that point.

3] ψ is not an observable quantity, therefore ψ it self has no direct physical significance.

4] ψ is function of space and time. i.e. $\psi(x, y, z, t)$.

5] ψ represents the amplitude of matter wave, it has both real and imaginary parts. $\psi = A + iB$.

Physical Significance of $|\psi|^2$.

1] $|\psi|^2$ represents the probability density.

$$|\psi|^2 = \psi\psi^*$$

Where, ψ is wave function and ψ^* is its complex conjugate.

2] $|\psi|^2$ gives probability per unit volume of finding a particle described by wave function ψ at particular time t, at a particular point (x, y, z) contained in the volume.

3] The large value of $|\psi|^2$ means strong probability of finding the particle there.

4] $|\psi|^2 = 0$ means that the particle is absent at that point at time t.

5] It is likelihood of existance of particle somewhere in space is specified by $|\psi|^2$ or ~~$|\psi|$~~ $\psi\psi^*$

6] Probability of finding the particle somewhere in space at all times is unity, hence should satisfy the condition

$$\int_{-\infty}^{+\infty} |\psi|^2 dv = 1$$

or.
$$\iiint_{-\infty}^{+\infty} |\psi|^2 \, dx\, dy\, dz = 1$$

Above equation gives the normalization

Q State the mathematical condition that should be satisfy by wavefunction ψ.

Ans: Mathematical Conditions:-

1] ψ should be normalized wave function.

2] ψ should be single valued function of space and time.

3] ψ should be finite.

4] ψ and its derivatives $\frac{d\psi}{dx}$, $\frac{d\psi}{dy}$, $\frac{d\psi}{dz}$ and $\frac{d\psi}{dt}$ must be continuous everywhere in the region where ψ is defined.

Q State and explain the mathematical conditions that a wave function ψ must satisfy and obey. What is a well behaved wave function?

IMP

Ans: Mathematical conditions:

1] It should be finite.

2] It should be normalized.

3] It should be single valued.

4] It should be continuous.

5] It's derivatives should be continuous.

Explanation:

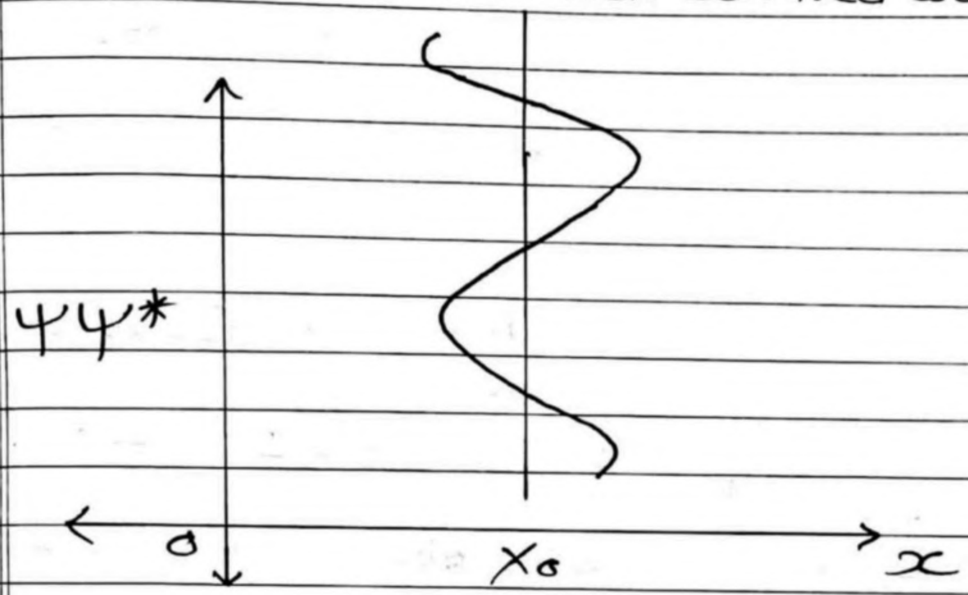
1] Function must be finite:

- The function should be finite everywhere. Even if $x \to \infty$ or $-\infty$, $y \to \infty$ or $-\infty$, $z \to \infty$ or $-\infty$, the wave function should not tends to the infinity.
- It must be remained finite for all values of x, y, z.
- If ψ is infinite, it would imply an infinitely large probability of finding the particle at that point. This would violet the uncertainty principle

ii] The function must be continuous

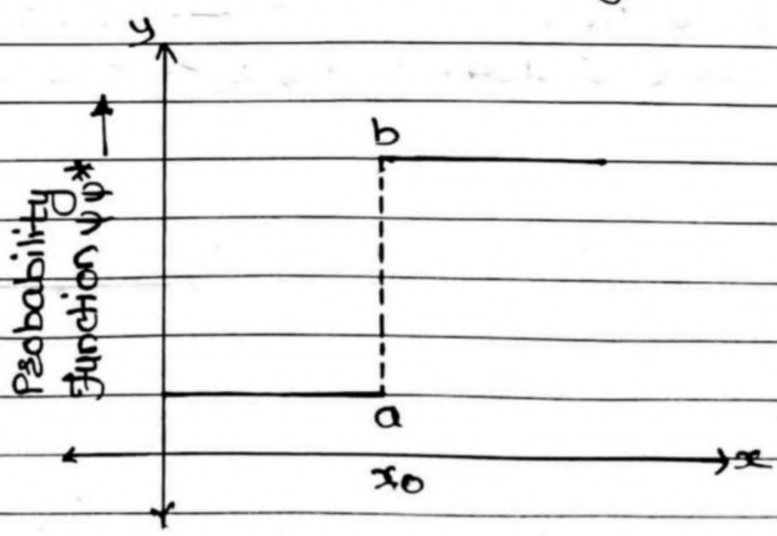
since ψ is related to a real particle, it can not have a discontinuity at any boundry to where potential changes.

- Wave functions satisfying the above mathematical conditions are called well behaved wave functions.



- ψ function should be continue across any boundry. since ψ is related to a physical quantity, it can not have a discontinuity at any point. Therefore we have function ψ & its space derivative $\frac{\partial\psi}{\partial x}$, $\frac{\partial\psi}{\partial y}$ & $\frac{\partial\psi}{\partial z}$.
should be continuous across any boundry.

iii) The function must be single-valued :-



- Any physical quantity can have only one value at a point for this reason, the function related to a physical quantity can not have more than one value at that point.
- If it has more than one value at a point, it means that there is more than one value of probability of finding the particle at that point.

iv) The function should be normalized
- The particle is certainly to be found somewhere in space we must have

$$\iiint |\psi|^2 \, dx\, dy\, dz = 1 \quad -①$$

The triple integral extending over all possible values of x, y and z.

A function ψ satisfying this condition is called a normalized wave function and eqn ① is called as normalization condition.
- Thus, ψ has to be a normalisable function.

Well Behaved Wave Function :
- Well behaved wave function is the wave function which is single valued, continuous and finite.

IMP | Topic : Normalization Condition

Q Explain Normalization condition of wave function and state why it should be normalized.

Ans:→ $|\psi|^2$ represents the probability density.

The large value of $|\psi|^2$ means a strong probability of finding the particle at that point.

- $|\psi|^2 = 0$ means that particle is absent at that point at time 't'

- The probibility of finding the particle within the volume element $dV = dx\,dy\,dz$ will be given by, $P = |\psi|^2 dV$

- The total probability of finding the particle somewhere in space at all times is unity, hence should satisfy the condition.

$$\int |\psi|^2 dV = 1$$

$$\iiint_{\infty}^{+\infty} |\psi|^2 \, dx\,dy\,dz = 1 \quad \text{——— (1)}$$

- This condition means that ψ represents a particle which is certainly found somewhere in space.

- The wave function that satisfies the condition given by eqn. (1) is called as normalized wave function.

- if $$\iiint_{\infty}^{+\infty} |\psi|^2 \, dx\,dy\,dz = 0$$

then particle doesn't exist in a volume $dx\,dy\,dz$, means there is no wave and wave function.

Topic: Schrodinger's Time Independent Wave Equation:

Sample Question

IMP

Q. Starting from de Broglie hypothesis, derive Schrodinger's Time independent wave equation.
— 6 Marks

→ Ans:

- de Broglie's hypothesis states that, "Every moving particle is associated with a wave, which governs the motion of that particle".
- That wave is called de Broglie's wave or matter wave.
- The de Broglie wavelength is given by

$$\lambda = \frac{h}{p} \quad \text{or} \quad \lambda = \frac{h}{mv}$$

- The velocity of matter wave is given by

$$u = \frac{c^2}{v} \quad \text{and} \quad c = \frac{\omega}{k}$$

- The general wave equation for a wave with a wave function ψ, travelling with velocity u in three dimensions is

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right]$$

$$\text{i.e.} \quad \frac{\partial^2 \psi}{\partial t^2} = u^2 \nabla^2 \psi \quad \text{—} ①$$

Where ∇^2 is Laplace Operator which is given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The general solution of equation ① is of the form.

$$\psi(x,y,z,t) = \psi_0(x,y,z)\, e^{-i\omega t} \quad \text{———} ②$$

Where $\psi_0(x,y,z)$ is the amplitude of the wave at point (x,y,z).

The above equation can also be written as

$$\psi(\bar{r},t) = \psi_0(\bar{r})\, e^{-i\omega t} \quad \text{———} ③$$

$$\psi = \psi_0 e^{-i\omega t}$$

Where $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of point (x,y,z).

Differentiating eqn ③ partially with respect to 't' twice,

$$\frac{\partial \psi}{\partial t} = (-i\omega)\,\psi_0\, e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)(-i\omega)\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0\, e^{-i\omega t}$$

But from eqn ③ we have $\psi = \psi_0 e^{-i\omega t}$

$$\therefore \frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$$

Substituting in eqn ①

$$-\omega^2\psi = u^2\nabla^2\psi$$

$$\nabla^2\psi + \frac{\omega^2}{u^2}\psi = 0 \quad \text{——} \quad (4)$$

But $\omega = 2\pi\nu = 2\pi\frac{u}{\lambda}$

$$\frac{\omega}{u} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2} \quad \text{——} \quad (5)$$

From de Broglie's hypothesis of matter waves,

$$\lambda = \frac{h}{p}$$

$$\therefore \quad \lambda^2 = \frac{h^2}{p^2} \quad \text{——} \quad (6)$$

The total energy (E) of particle is a sum of Kinetic Energy $\frac{1}{2}mv^2$ and potential energy V.

$$E = \frac{1}{2}mv^2 + V$$

$$= \frac{m^2v^2}{2m} + V \qquad \because p^2 = m^2v^2$$

$$E = \frac{P^2}{2m} + V$$

$$p^2 = 2m(E-V)$$

Prof. S. J. Gadakh

Substituting in eqn 6

$$\lambda^2 = \frac{h^2}{2m(E-V)}$$

Substituting in eqn 5

$$\frac{\omega^2}{u^2} = 4\pi^2 \frac{2m(E-V)}{h^2}$$

$$\frac{\omega^2}{u^2} = \frac{8\pi^2 m}{h^2}(E-V)$$

Substituting in eqn 4

$$\boxed{\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0} \quad \text{—} 7$$

The quantity $\frac{h}{2\pi}$ appears very Frequently in quantum mechanics and hence is substituted as $\hbar$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad \text{———} 8$$

eqn 7 or 8 is the Schrodinger's time independent wave eqn.

Topic: Schrodinger's Time Dependent Wave Equation

IMP

Q State the importance of schrodinger's equations. Derive Schrodinger's time dependent wave equation. — 6 Marks

→

Importance of Schrodinger's Equations:

- The Schrodinger's equations are used to find the allowed energy levels of quantum mechanical systems.
- The associated wave function gives the probability of finding the particle at certain position.
- The solution to this equation is a wave that describes the quantum aspects of a system.
- Schrodinger's equations are mathematical tools for calculating the energy of electrons and their nature of wave function, probabilit in any active systems by applying proper boundary conditions related to system.

Schrodinger's Time Dependent Wave Equation:-

- Derivation;

The general differential equation for a wave with wave function 'ψ', travelling with velocity 'u' in three dimensions is

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right]$$

i.e. $$\frac{\partial^2 \psi}{\partial t^2} = u^2 \nabla^2 \psi \quad \text{--- (1)}$$

Where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

∇^2 is called Laplace equation

The general solution of eqn no. (1) is of the form

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t} \quad \text{--- (2)}$$

Where $\psi_0(x, y, z)$ is the amplitude of the wave at point (x, y, z). The above eqn can also be written as

$$\psi(\bar{r}, t) = \psi_0(\bar{r})\, e^{-i\omega t} \quad \text{--- (3)}$$

Where, $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of the point (x, y, z).

Differentiating eqn no. (3) partially with respect to 't',

$$\frac{\partial \psi}{\partial t} = (-i\omega)\, \psi_0\, e^{-i\omega t}$$

But $\psi = \psi_0\, e^{-i\omega t}$

$$\therefore \frac{\partial \psi}{\partial t} = (-i\omega)\, \psi \quad \text{--- (4)}$$

As $\omega = 2\pi\nu$ and $E = h\nu$

$$\omega = 2\pi \frac{E}{h}$$

From Eqn no. 4

$$\frac{\partial \psi}{\partial t} = \frac{-i2\pi E}{h} \psi$$

$$E\psi = \frac{-h}{i2\pi} \frac{\partial \psi}{\partial t}$$

$$E\psi = \frac{ih}{2\pi} \frac{\partial \psi}{\partial t} \quad \text{--- (5)}$$

But $\hbar = \frac{h}{2\pi}$

$$E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \text{--- (6)}$$

Schrodinger's time independent eqn is

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V)\psi = 0$$

multiplying by $\frac{h^2}{8\pi^2 m}$

$$\frac{h^2}{8\pi^2 m} \nabla^2 \psi + (E - V)\psi = 0$$

$$\frac{h^2}{8\pi^2 m} \nabla^2 \psi + E\psi - V\psi = 0$$

$$-\frac{h^2}{8\pi^2 m} \nabla^2 \psi + V\psi = E\psi$$

But $\hbar = \frac{h}{2\pi}$

$$\therefore \quad -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi$$

From eqⁿ no. (6) we get

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}} \quad \text{— (7)}$$

The above eqⁿ is known as Schrodinger's time dependent wave equation.

Topic: <u>Particle in rigid Box or Infinite Potential Well</u>

|IMP|

Q Derive the equation for the energy of particle is enclosed in a one dimensional rigid box or infinite potential well.

— 6 Marks

⟹ Particle in Rigid Box. or infinite potential well

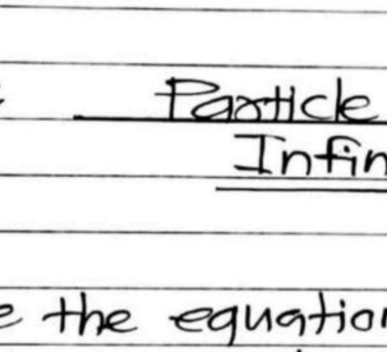


fig (1) Particle in rigid Box

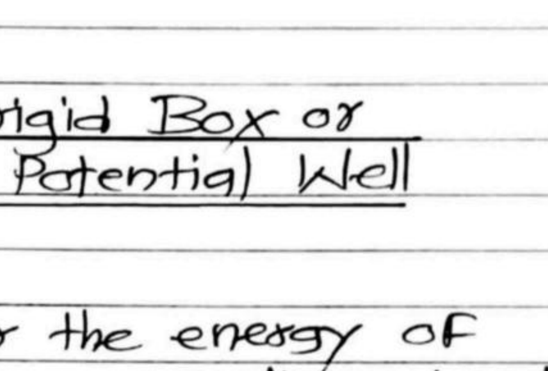


fig (2) Particle in infinite potential well.

- Consider a particle of mass 'm' moving with velocity along the 'x'- direction.

- The motion of particle is restricted between $X=0$ and $x=L$ inside a box bounded by infinitely rigid walls. As shown in fig.

- The particle bounces back and forth between the walls of the box. When it collides with walls it does not lose energy. Hence total energy remains constant.

- STEP-I

- The potential energy of the particle is infinite outside the box and is constant inside the box.

- For convenience, suppose that the potential energy $V=0$ inside the box.

- Thus we have,

$$Vx = \infty \qquad \text{for} \quad x \leq 0 \text{ and } x \geq L$$

$$Vx = 0 \qquad \text{for} \quad 0 < x < L$$

- The variation of potential with x is as shown in fig-②. The particle is inside an infinitely deep potential well.

- ψ is the wave function associated with the particle inside the box.

- We can use schrodinger's time independent wave equation to the free particle inside the box, to find the wave function and energy inside the box.

- The particle can not have infinite amount of energy thats why it can not exist out side the box.

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{—①} \longrightarrow$$ Schrodinger's time independent wave equation.

- Then for $V=0$, the above eq^n will become,

$$\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0 \quad \text{—— ②}$$

- For one dimensional potential well

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

But $p = \hbar k$, $k = \frac{p}{\hbar}$ $\qquad p = \sqrt{2mE}$

$$k^2 = \frac{2mE}{\hbar^2} \quad \text{— ③}$$

Then we get $\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad$ — ④

STEP-II

- The solution of Equation - ④ can be either a sine function or a cosine function.

Then the general solution is,

$$\psi_x = A \sin(kx) + B \cos(kx) \quad — ⑤$$

Where A & B are constant whose values can be determined by applying the boundary conditions. on ψ are,

$$\left.\begin{aligned} \psi &= 0 \quad \text{at} \quad x = 0 \\ \text{and} \quad \psi &= 0 \quad \text{at} \quad x = L \end{aligned}\right\} \quad — ⑦$$

Putting $\psi = 0$ at $x = 0$ in equation ⑤ we have

$$0 = A \sin 0 + B \cos 0$$

$$\therefore \; B = 0$$

$\therefore$ Equation ⑤ can be written as

$$\psi = A \sin kx \quad — ⑦$$

$\boxed{\text{STEP-III}}$

Using the second boundary conditions i.e. $\psi = 0$ at $x = L$ in equation-⑦

we get,

$$0 = A \sin kL$$

$A \neq 0$ since then $\psi = 0$ always

$$\therefore \sin kL = 0$$

$KL = n\pi$ where $n = 1, 2, 3, ---$

$\therefore K = \frac{n\pi}{L}$

STEP - IV

Putting for K from Equation no. 3 we have

$$K = \sqrt{\frac{2mE}{\hbar^2}} = \frac{n\pi}{L} \quad \text{—— (8)}$$

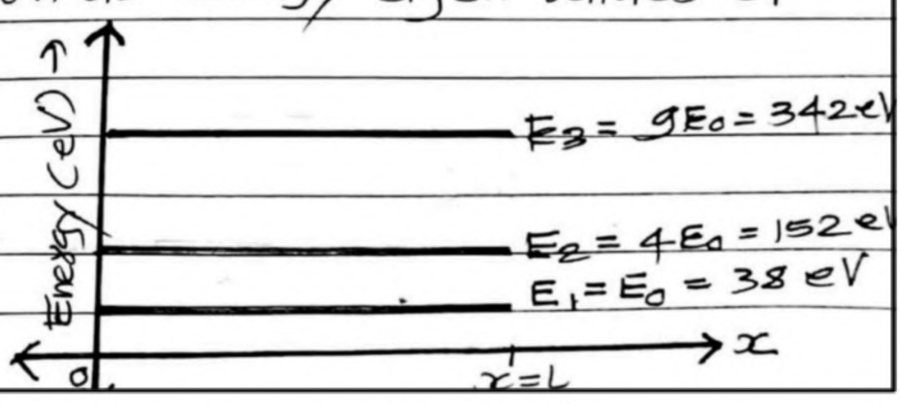
$$\therefore \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

or $$\boxed{E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}} \quad \text{—— (9)}$$

or $$\boxed{E_n = \frac{n^2h^2}{8mL^2}} \quad \text{—— (10)}$$

$$\because \hbar^2 = \frac{h^2}{4\pi^2}$$

- From eqn no. 9 and 10 we see that the particle inside an infinitely deep potential well can have only descrete values of energy Excluding zero.
- These are known as energy eigen values of nth particle.



IMP

Tunneling Effect

Q. State & explain quantum mechanical tunneling effect?

Q. State and prove the tunneling effect. — 4 Marks

⟹ **Quantum Mechanical Tunneling Effect:**

- "The phenomenon of the particles penetrating the potential barrier is called the tunneling effect."
- Classically, particle will be always reflected back

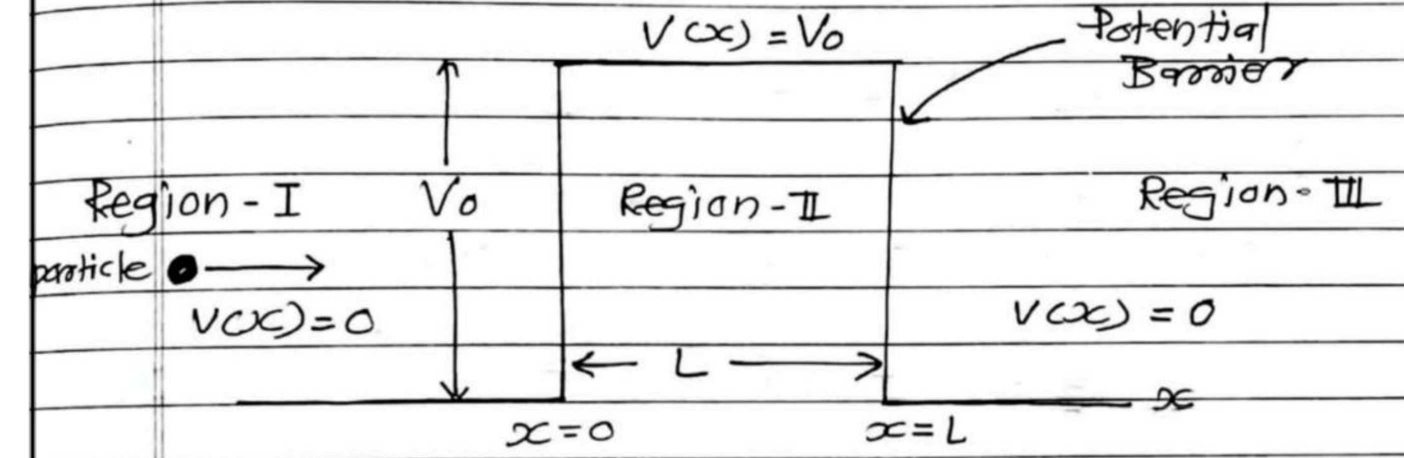
Examples of tunneling Effect:

Tunnel Diode, Alfa Decay, Scanning Tunneling microscope.

Let us consider the one-dimensional barrier, where the height of barrier is V_0 and width is L.

The potential function is defined as

$$V(x) = 0 \quad \text{for} \quad x \leq 0 \ \& \ x \geq L$$
$$V(x) = V_0 \quad \text{for} \quad 0 < x < L$$



The Schrodinger's time independent equation is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \qquad \text{--- (1)}$$

- consider a particle approaching the potential barrier from the left that is from I^{st} region.

- As the motion of particle is in X-Direction, so $\nabla^2\psi$ can be replaced by $\frac{d^2\psi}{dx^2}$

For region I and III, $V(x) = 0$, therefore eq^n one becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

Let $K^2 = \frac{8\pi^2 m}{h^2} E$ — ②

$\frac{d^2\psi}{dx^2} + K^2\psi = 0$ — ③

The solution of above differential eqn is

For region I i.e. $x \leq 0$

$\psi_I(x) = A e^{ikx} + B e^{-ikx}$ — ④

For region III ie. $x \geq L$

$\psi_{III}(x) = C e^{ikx} + D e^{-ikx}$ — ⑤

In eqn five the first term represents the wave travelling along positive X direction in region III ie. the wave transmitted at $x = L$.

The second term of eqn ⑤ represents that the wave is travelling along negative x-direction in region III, But now wave travel back from infinity in region III,

∴ Taking $D = 0$ in eqn no. ⑤

Then eqn ⑤ Becomes,

$\psi_{III}(x) = C e^{ikx}$ — ⑥

For region II i.e. $0 < x < L$ the $V(x) = V_0$
. So, the equation ① Becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\psi = 0$$

Let $\frac{8\pi^2 m}{h^2}(E - V_0) = k'^2$ — ⑦

$$\therefore \frac{d^2\psi}{dx^2} + k'^2\psi = 0 \quad \text{— ⑧}$$

The solution of above eqn is

$$\psi_{II}(x) = P.e^{ik'x} + Qe^{-ik'x} \quad \text{— ⑨}$$

The first term of the equation represents the wave travelling along positive X-axis in region II, that is the wave transmitted at $X = 0$ and the second term represents the wave travelling along negative X-axis in region-II i.e. the wave reflected ~~along~~ at $x = L$.

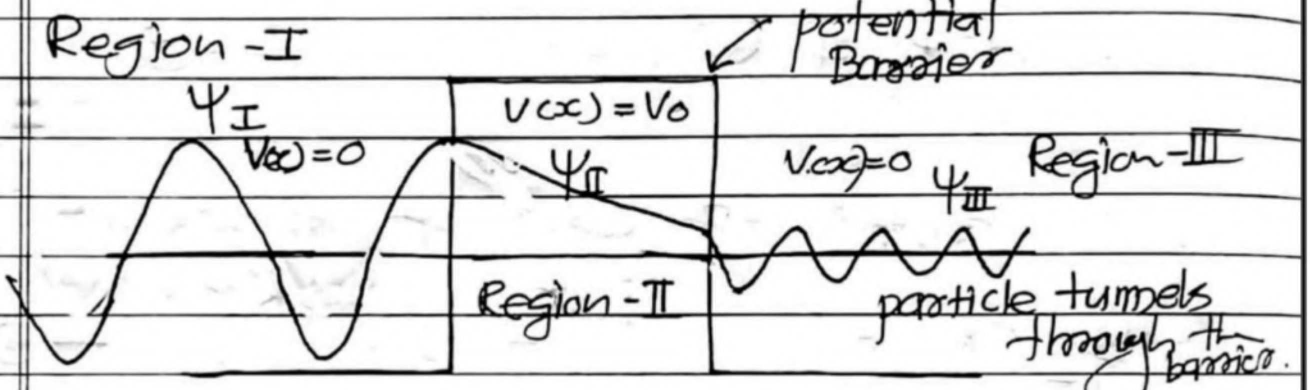
For the determination of constants A, B, C & P & Q the boundary conditions are applied at $x = 0$ and $x = L$ i.e. $\psi_I = \psi_{II}$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_{II}}{\partial x} \quad \text{at } x = 0$$

and $\psi_{II} = \psi_{III}$ $\frac{\partial \psi_{II}}{\partial x} = \frac{\partial \psi_{III}}{\partial x}$ at $X = L$

The property of the barrier penetration is due to the wave nature of matter and is similar to the total internal reflection of light.

It means there is always some probability of transmission through barrier even though the particle has less energy than the potential barrier, this is the tunneling effect.



[IMP]

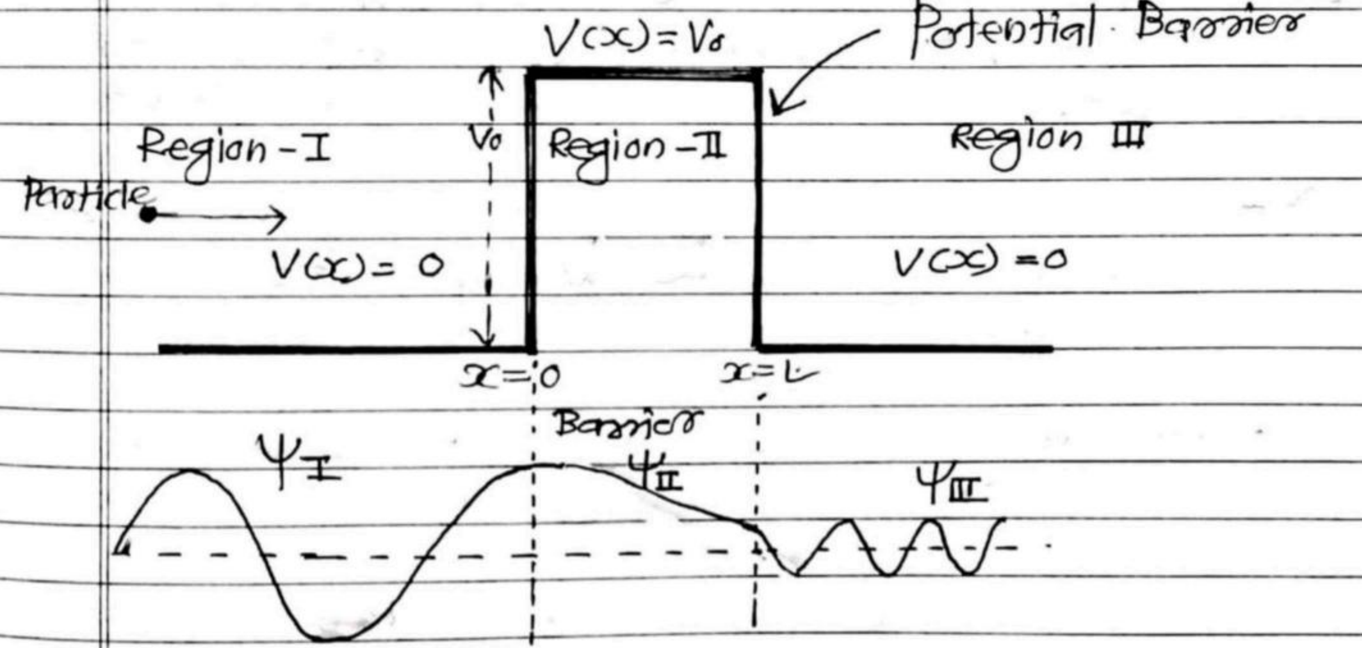
Q What is Quantum Mechanical tunneling Effect? —— 2 Marks

⟹

Quantum Tunneling Effect:

Statement:

If a particle incidenting on the potential barrier with energy less than the height of the potential barrier, there is always some probability of transmission through the barrier. This phenomenon of crossing the barrier is called as tunneling effect.



ψ_I is the wave function of incident particle

ψ_{III} is the wave function of particle after tunneling

- The property of barrier penetration is due to the wave nature of matter and is similar to ~~each~~ total internal reflection.

Region I $\Psi_I = A e + B e$ $\Psi_{III} = D e$

transmitted   Reflected   Tunneled

- consider a particle approaching the potential barrier from the left i.e. from I^{st} region.
- If the particle has energy less than height of potential barrier V_0 that is $E < V_0$, classically the particle will be always reflected back and hence will not penetrate the barrier.

- The probability of penetration increases if $(V_0 - E)$ and L i.e. height and width of potential barrier are smaller.

Topic: Alpha Decay (α - Decay)

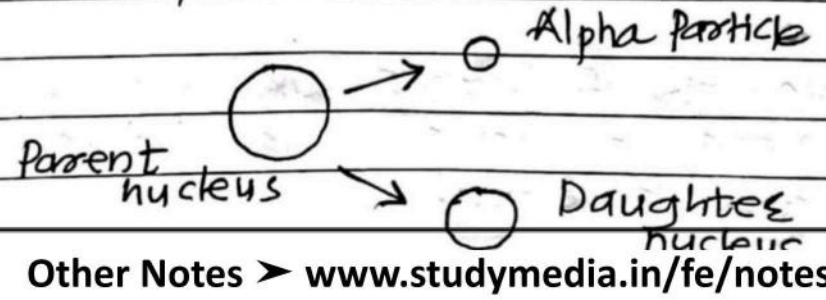
[IMP]

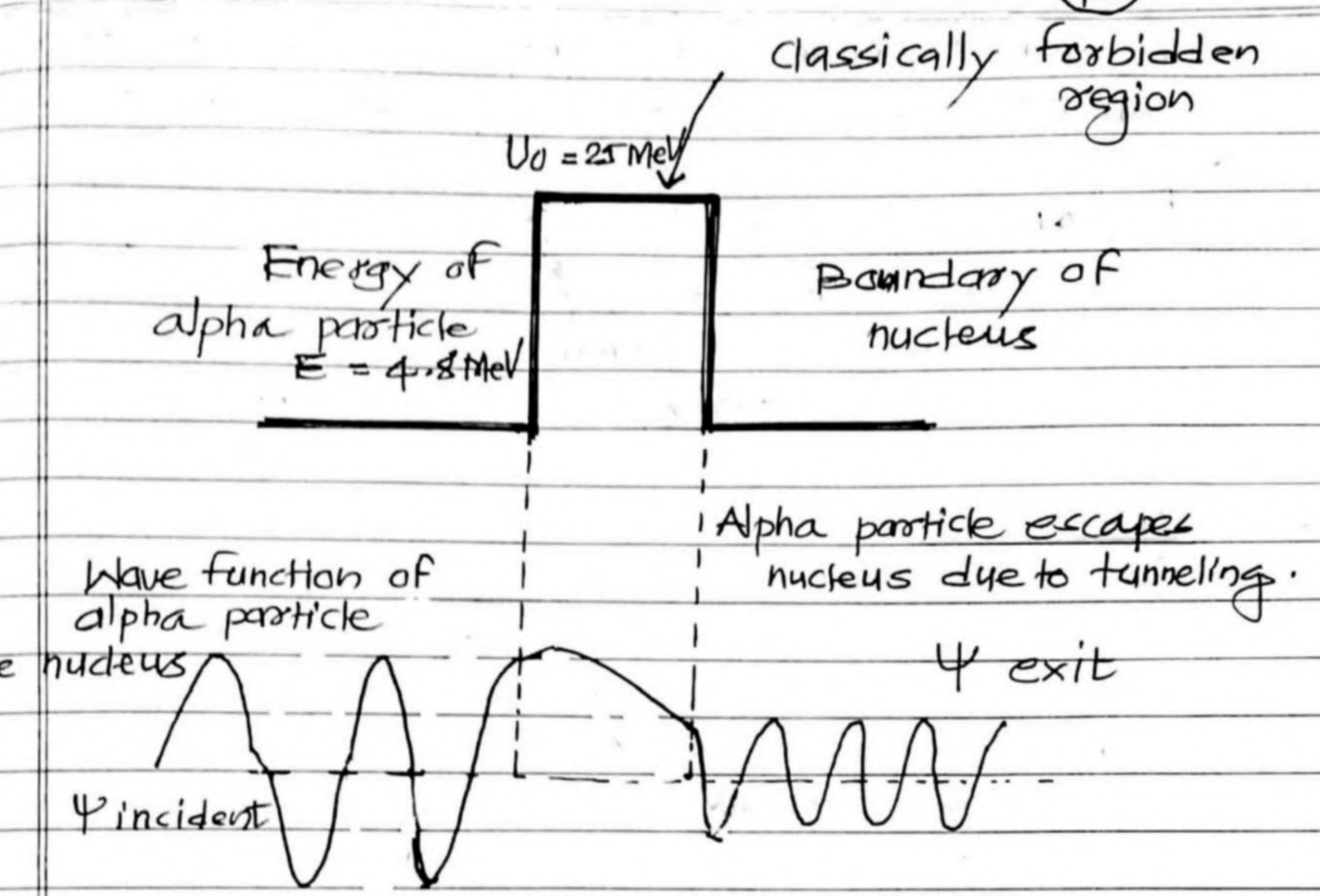
Q Explain in Brief the role of quantum mechanical tunneling effect in alpha particle decay. — 3 Marks

$\Rightarrow$ - Alpha decay is a type of radioactive decay in which an atomic nucleus emits an alpha particle.

- An alpha particle consist of two protons and two neutrons bound together $(_2He^4)$

- An α particle has energy of only around 4.8 MeV and it is bounded in the nucleus with energy around 25 MeV (potential Barrier). Thus classically it is impossible for α-particle to escape from nucleus.





- Quantum mechanically de-Broglie wave associated with alpha particle. The de-Broglie waves associated with alpha particle can penetrate the potential barrier (nucleus) and there is non-zero probability that it will escape from the nucleus.

- The probability of alpha particle is so small i.e. 1 in 10^{38} i.e. alpha particle has to strike potential well of nucleus 10^{38} or more times before it emerges, but it would definitely escape from nucleus.

classmate

Date
Page

IMP Topic: Scanning Tunneling Microscope (STM) (5)

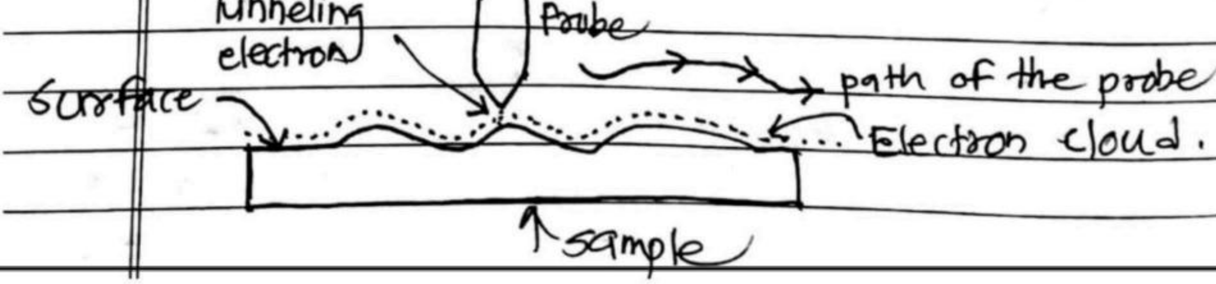
Q - Explain principle & working of scanning tunneling microscope. State the application of STM

Principle of STM (Scanning Tunneling Microscope):

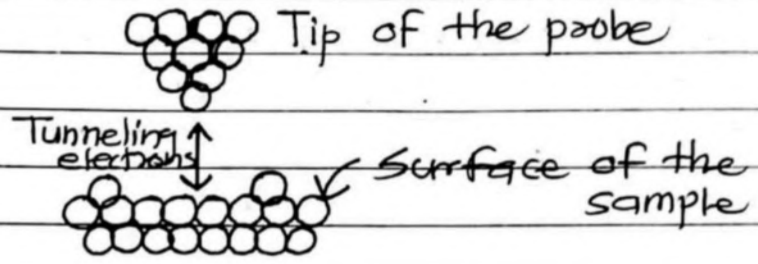
Scanning Tunneling Microscope is an instrument for imaging at atomic level by using the principle of quantum tunneling effect.

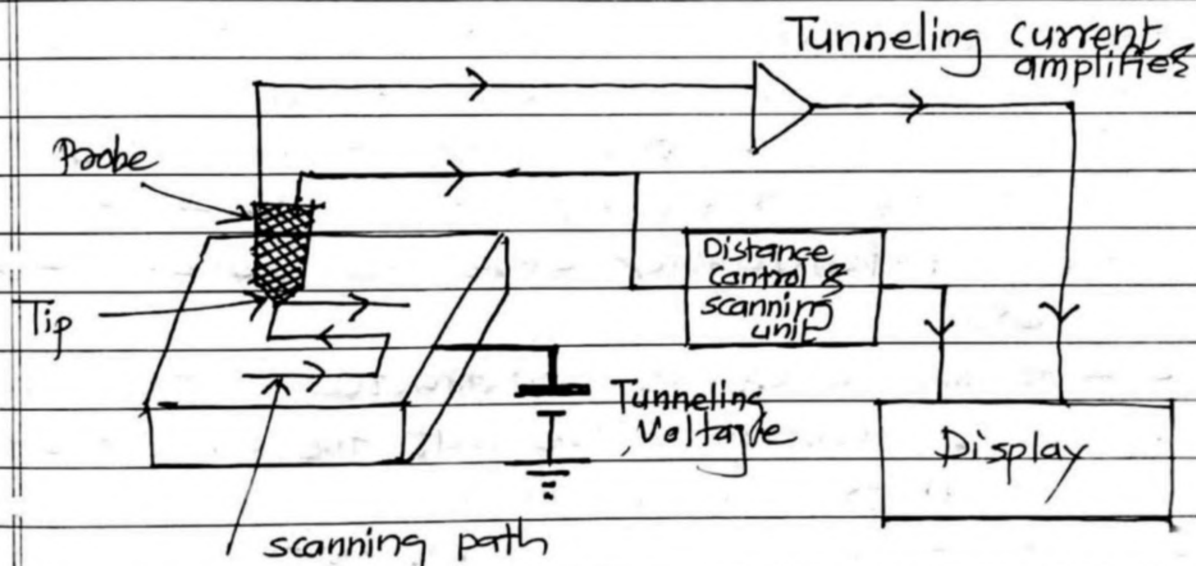
Working:

- The resolution of an STM is 0.1 nm lateral and 0.0 nm depth measurement. With such high resolution, individual atoms of material can be imagined.
- When conducting tip is brought close to the surface to be examined, the bias voltage applied between them will allow electrons to tunnel through the vacuum between them.
- The 'tunneling current' Between them will depend on the width of the barrier i.e. the distance between the probe and the surface if bias voltage between them is constant.
- The voltage can be recorded and displayed simulating the surface of the sample. The probe is moved vertically so that the distance between the probe and the surface remains constant as the probe scans laterally.
- The below fig shows the basic mechanism of STM.



Prof. S.J. Gadakh





Scanning Tunneling microscope.

The applications of STM :-

- STM is used to study friction, surface roughness, defects and surface reaction in material like catalysts.
- STMs are also very important tools in research surrounding semiconductors and microelectronics.
- STM is also used to study surface geometry, molecula

structure, local spin structure.

- STM is also used in single molecular vibration, atom manipulation, nano-chemical reactions.

IMP

Topic: Quantum Computing

Principle of Quantum computing.

- Quantum computing focuses on the principles of quantum theory that, which deals with the modern physics that explain the behavior of matter and energy of an atomic & subatomic level.

- Quantum computing makes use of quantum phenomenon, such as quantum bits, superposition and entanglement to perform data operations.

- Computing in this manner essentially tackles extremely difficult tasks that ordinary computer can not perform on their own.

- A number of elemental particles such as electrons or protons can be used, with either their charge or polarization acting as representation of 0 or/and 1. Each of these particles is known as a quantum bit, or qubit.
- A qubit is a unit of quantum information.

- The processing power of quantum computes is measured in teraflops or trillions of of logical operations per second.

Prof. S.J. Gadakh

Potential Applications of Quantum Computing

1] Artificial Intelligence:

- Artificial intelligence requires analysis of data from large datasets of image, video and text. This data is available in vast quantity.
- For analyzing and processing this huge data, traditional computers would require thousands years.
- Quantum computers would be able to process this data in few seconds.

2] Software Testing:

- For very large software programs or large ASIC chips that have billions of transistors, it becomes difficult and expensive to verify them for correctness.
- There can be billions of different possibilities of errors in codes and it is impossible for Classical computes to check every single one simulation.
- Quantum computes can provide much better dealing with these tasks with great improvement

3] Cyber Security:

- In the era of deep penetration of internet, cyber security is one of the biggest challenges

Malware and viruses spread through internet within fraction of seconds, which is difficult for classical computers to deal with.
- Various techniques to deal with cyber security threats can be developed using some of the quantum machine learning approches to recognize the threats earlier and reduce the damage.

4] Drug Design:
- It requires a kind of trial and error method for many of the drugs to understand how they will react.
- These methods are very expensive, complex and require much processing time.
- Using quantum computers the process can be simulated more effectively.

(i) De-Broglie wavelength $\lambda = \frac{h}{mv}$

(ii) Energy of photon $E = h\upsilon = \frac{hc}{\lambda}$

(iii) De-Broglie wavelength in terms of K.E.

$$\lambda = \frac{h}{\sqrt{2\,mE}}$$

(iv) De-Broglie wavelength of an electron

$$\lambda = \frac{12.26}{\sqrt{V}}\ A^\circ$$

Solved Examples

Ex. 8.10.1

The energy of photon is 5.28×10^{-19} J . Calculate frequency and wavelength.

Soln. :

$$E = h\,\upsilon$$

$$\upsilon = \frac{E}{h} = \frac{5.28 \times 10^{-19}\ J}{6.625 \times 10^{-34}\ J-S}$$

$$= 7.96 \times 10^{14}\ Hz$$

$$\text{Again } \upsilon = \frac{c}{\lambda}$$

$$\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^{8}\ m/s}{7.96 \times 10^{14}}$$

$$\lambda = 0.3768 \times 10^{-6}\ m = 3768 \times 10^{-10}\ m$$

$$= 3768\ A^\circ$$ **...Ans.**

Ex. 8.10.2

Electrons moving with a speed of 7.3×10^{7} m/s have wavelength of 0.1A°. Calculate Planck's constant.

Soln. :

Given : $\lambda = 0.1A^\circ = 0.1 \times 10^{-10}\ m,$

$v = 7.3 \times 10^{7}\ m/s$

Formula required : $\lambda = \frac{h}{mv}$

$$h = \lambda \cdot mv$$
$$= 0.1 \times 10^{-10} \times 9.1 \times 10^{-31} \times 7.3 \times 10^{7}$$
$$= 6.643 \times 10^{-34}\ \text{J.s} \quad \text{...Ans.}$$

Ex. 8.10.3

At what velocity the De Broglie wavelength of an alpha particle is equal to the wavelength of 1 KeV X-ray photons? Given mass of alpha particle is four times the mass of a proton. Mass of proton = 1.67×10^{-27} kg.

Soln. :

Given : $E = 1\ \text{keV},\ m = 1.67 \times 10^{-27}\ \text{kg}$

Formula required : $\lambda = \frac{h}{mv} = \frac{hc}{E};\ \lambda = \frac{h}{mv}$

For a photon, the energy

$$E = h\upsilon = \frac{h \cdot c}{\lambda}$$

∴ Wavelength of the photon is

$$\lambda = \frac{hc}{E} \quad \text{...(1)}$$

The De Broglie wavelength of the alpha particle of mass m and velocity v is

$$\lambda = \frac{h}{mv} \quad \text{...(2)}$$

∴ As per the data, Equation (1) = Equation (2)

$$\frac{h}{mv} = \frac{hc}{E}$$

∴ velocity of alpha particle having same energy as 1 KeV X - ray photon is

$$v = \frac{E}{mc} = \frac{10^{3} \times 1.6 \times 10^{-19}}{4 \times 1.67 \times 10^{-27} \times 3 \times 10^{8}}$$

$$\therefore\ v = 0.0798 \times 10^{3}\ \text{m/s} = 79.8\ \text{m/s} \quad \text{...Ans.}$$

Ex. 8.10.4

Find the De Broglie wavelength of 10 KeV electrons.

Soln. :

Given : $E = 10\ \text{KeV} = 10 \times 10^{3}\ \text{e V}$
$= 10 \times 10^{3} \times 1.6 \times 10^{-19}\ \text{J}$

Formula required : $\lambda = \frac{h}{\sqrt{2mE}}$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10^{4} \times 1.6 \times 10^{-19}}}$$

$$= 1.227 \times 10^{-11}\ \text{m} = 0.1227\ \text{A}^\circ \quad \text{...Ans.}$$

Ex. 8.10.5

De Broglie wavelength of electron in monoenergetic beam is 7.2×10^{-11} metres. Calculate the momentum and energy of electrons in the beam in electron volts.

Soln. :

Given : $\lambda = 7.2 \times 10^{-11}$ m.

Formulae required : Momentum $p = \frac{h}{\lambda}$,

Energy $E = \frac{p^2}{2m}$

(i) $$p = \frac{6.63 \times 10^{-34}}{7.2 \times 10^{-11}} = 0.9208 \times 10^{-23}\ \text{kg m/s}$$

(ii) $$E = \frac{(0.9208 \times 10^{-23})^2}{2 \times 9.1 \times 10^{-31}} = 0.04658 \times 10^{-15}\ \text{J}$$

$$= \frac{0.04658 \times 10^{-15}}{1.6 \times 10^{-19}}\ \text{eV}$$

$$E = 0.0291 \times 10^{4}\ \text{eV} \quad \text{...Ans.}$$

Ex. 8.10.6

Find the K.E. of a neutron which has a wavelength of 3A°. At what angle will such a neutron undergo first order Bragg reflection from a calcite crystal for which the grating space is 3.036 A°? Mass of neutron = 1.67×10^{-27} kg.

Soln. :

Given : $\lambda = 3\text{A}^\circ = 3 \times 10^{-10}$ m
$d = 3.036\ \text{A}^\circ = 3.036 \times 10^{-10}$ m
$m = 1.67 \times 10^{-27}$ kg.

Formulae required : Wavelength of neutron is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$2d \sin\theta = n\lambda$$

∴ Neutron energy

$$E = \frac{h^2}{2m\lambda^2}$$

$$\therefore\ E = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (3 \times 10^{-10})^2}$$

$$= 1.449 \times 10^{-21}\ \text{J}$$

For Bragg reflection, $2\,d \sin\theta = n\lambda$

$$n = 1$$

$$\therefore\ \sin\theta = \frac{\lambda}{2d} = \frac{3 \times 10^{-10}}{2 \times 3.036 \times 10^{-10}} = 0.4940$$

$$\therefore\ \theta = \sin^{-1}(0.4940) = 29^\circ\ 36' \quad \text{...Ans.}$$

Ex. 8.10.7

Determine the velocity and kinetic energy of a neutron having De Broglie wavelength 1.0A° (mass of neutron is 1.67 $\times 10^{-27}$ kg).

Soln. : We have,

$$\lambda = \frac{h}{mv}, \quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 10^{-10}}$$

$$v = 3.97 \times 10^{3} \text{ m/s}$$

Again we can write,

$$\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow E = \frac{h^2}{2m\lambda^2}$$

$$E = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (10^{-10})^2}$$

$$= 13.16 \times 10^{-21} \text{J}$$

$$= 8.225 \times 10^{-2} \text{ eV}$$

$$(\because 1\text{eV} = 1.6 \times 10^{-19} \text{ J})$$

...Ans.

Ex. 8.10.8 SPPU - May 14

What accelerating potential would be required for a proton with zero initial velocity to acquire a velocity corresponding to its de-Broglie wavelength of 10^{-10} m.

[Given : $m_p = 1.67 \times 10^{-27}$ kg].

Soln. :

Given : $\lambda = 10^{-10}$ m, $m_p = 1.67 \times 10^{-27}$ kg

$e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ Js

Formula required :

$$\lambda = \frac{h}{\sqrt{2\, m_p eV}}$$

$$\therefore \lambda = \frac{h^2}{2\, m_p eV}$$

$\therefore$ Accelerating potential of proton should be

$$V = \frac{h^2}{2\, m_p e\lambda^2}$$

$$= \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times (10^{-10})^2}$$

$$V = \frac{4.396 \times 10^{-67}}{5.344 \times 10^{-66}} = 82.25 \times 10^{-3}$$

$$= 8.225 \times 10^{-2} \text{ V}$$

Ex. 8.10.9

Calculate the wavelength of electron of energy 291 eV.

Soln. :

Given : $E = 291 \text{ eV} = 291 \times 1.6 \times 10^{-19}$ J

Formula required : $\lambda = \frac{h}{\sqrt{2mE}}$

The electron wavelength is,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 291 \times 1.6 \times 10^{-19}}}$$

$$= 0.7202 \times 10^{-10} \text{ m}$$

$$\therefore \lambda = 0.7202 \text{ A}°$$

...Ans.

Ex. 8.10.10

Calculate the De Broglie wavelength of a neutron having kinetic energy of 1eV.

Soln. :

De Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2mE}}$

Given : $h = 6.63 \times 10^{-34}$ J-s, $m = 1.67 \times 10^{-27}$kg,

$E = 1\text{eV} = 1 \times 1.6 \times 10^{-19}$ J

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19}}}$$

$$= 2.87 \times 10^{-11}\text{m} = 0.287\text{A}°$$

...Ans.

Ex. 8.10.11

A proton and an α particle are accelerated by the same potential difference. Show that the ratio of the De Broglie wavelengths associated with them is $2\sqrt{2}$. Assume the mass of alpha particle to be 4 times the mass of proton.

Soln. :

Given : $m_\alpha = 4m_p$,

m_α = mass of α - particle,

m_p = mass of proton,

Let q_α = charge of particle,

q_p = charge of proton

$\therefore \quad q_\alpha = 2\, q_p$

Let $V \rightarrow$ Accelerating potential for the particles.

Formula required : $\lambda = \frac{h}{\sqrt{2\, m\, e\, V}}$

$$\therefore \lambda_p = \frac{h}{\sqrt{2\, m_p \cdot q_p V}}$$

and $$\lambda_\alpha = \frac{h}{\sqrt{2\, m_\alpha \cdot q_\alpha V}}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2\, m_p \cdot q_p V}} \frac{\sqrt{2\, m_\alpha \cdot q_\alpha V}}{h}$$

$$= \sqrt{\frac{m_\alpha}{m_p}} \cdot \sqrt{\frac{q_\alpha}{q_p}}$$

$$= \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$$

...Proved.

Ex. 8.10.12 SPPU - May 13

Calculate the wavelength associated with 1 MeV proton.

Soln. :

Formulae :

$$h = \frac{h}{\sqrt{2\, m_p\, E}} = \frac{h}{\sqrt{2\, m_p\, eV}}$$

Given : $V = 10^6\ V$

$$h = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 10^6 \times 1.6 \times 10^{-19}}}$$

$$= 2.868 \times 10^{-14}\ m. \qquad \text{...Ans.}$$

Ex. 8.10.13

Find the De-Broglie's wavelength of a neutron of energy 12.8 MeV.

Given : mass of neutron $= 1.675 \times 10^{-27}$ kg

Soln. : The De-Broglie wavelength λ for a particle of mass m is given by,

$$\lambda = \frac{h}{\sqrt{2m\, E}}$$

Given :

$$E = 12.8 \times 10^6 \times 1.6 \times 10^{-19}\ J$$

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 12.8 \times 10^6 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 8.0 \times 10^{-15}\ m = 8.0 \times 10^{-5}\ A^\circ \qquad \text{...Ans.}$$

Ex. 8.10.14

What potential difference must be applied to an electron microscope to obtain electrons of wavelength 0.3 Aº ?

Soln. :

$$\lambda = \frac{12.27}{\sqrt{V}}\ A^\circ \quad \text{but} \quad \lambda = 0.3\ A^\circ$$

$$\sqrt{V} = \frac{12.27}{0.3}$$

$$V = \left(\frac{12.27}{0.3}\right)^2 = 1672.81 \text{ volts} \qquad \text{...Ans.}$$

Ex. 8.10.15

Calculate the velocity and De-Broglie Wavelength of an α-particle of energy 1 Kev.

Soln. :

$$\lambda_\alpha = \frac{h}{\sqrt{2\, m_\alpha\, E}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 1 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$= 45.34 \times 10^{-14}\ m = 0.0045\ A^\circ$$

$$\lambda_\alpha = \frac{h}{m_\alpha v_\alpha}$$

$$v_\alpha = \frac{h}{m_\alpha \lambda_\alpha} = \frac{6.63 \times 10^{-34}}{4 \times 1.67 \times 10^{-27} \times 4.5 \times 10^{-13}}$$

$$v_\alpha = 0.22 \times 10^6 = 2.2 \times 10^5\ m/s \qquad \text{...Ans.}$$

Ex. 8.10.16

An electron has kinetic energy equal to its rest mass energy. Calculate De-Broglie's wavelength associated with it.

Soln. :

Data : $h = 6.63 \times 10^{-34}$ JS

$m = 9.1 \times 10^{-31}$ kg.

$c = 3 \times 10^8$ m/sec.

$\lambda = ?,\qquad E = m_o c^2$

Formula :

$$\lambda = \frac{h}{\sqrt{2\, mE}} \qquad \therefore\ \lambda = \frac{h}{\sqrt{2\, m^2 c^2}}$$

$\therefore\ E = mc^2.$

$$\lambda = \frac{h}{\sqrt{2} \times mc} = \frac{6.63 \times 10^{-34}}{\sqrt{2} \times 9.1 \times 10^{-31} \times 3 \times 10^8}$$

$$= 0.1715 \times 10^{-11}\ m. \qquad \text{...Ans.}$$

Ex. 8.10.17 :

In a T.V Set, electrons are accelerated by a p.d. of 10 kV. What is the wavelength associated with these electrons ?

Soln. :

Given formulae :

$$\lambda = \frac{h}{\sqrt{2\, m\, eV}} = \frac{12.3}{\sqrt{V}}\ A^\circ = \frac{12.3}{\sqrt{10^4}}\ A^\circ$$

$$\lambda = 0.12\ A^\circ \qquad \text{...Ans.}$$

Ex. 8.10.18

Find the de Broglie's Wavelength Associated with Monoenergetic Electron Beam having momentum 10^{-23} kg m/s.

Soln. :

Given : $p = 10^{-23}$ kg-m/s

Formula :

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{10^{-23}} = 6.63 \times 10^{-11} m$$

$$= 0.663 \times 10^{-10} m = 0.663\ A^\circ \qquad \text{...Ans.}$$

Ex. 8.10.19

Which has a shorter wavelength 1 eV photon or 1 eV electron ? Calculate the value and explain.

Soln. : For photon, $E = h\nu = \frac{hc}{\lambda}$

$$\lambda_{ph} = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19}}$$

$$= 1.2431 \times 10^{-6} \text{ m} = 12431 \text{ A}^\circ$$

For electron, $\lambda_e = \frac{h}{\sqrt{2 m_e E}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}}$$

$$\lambda_e = 1.228 \times 10^{-9} \text{ m} = 12.28 \text{ A}^\circ \quad \text{...Ans.}$$

$\lambda_{ph} > \lambda_e$, So, frequency of electron is more, hence more effective.

Ex. 8.10.20

At what Kinetic energy an electron will have a wavelength of 5000 A° ?

Soln. : **Formula Required :** $\lambda = \frac{h}{mv}$

Data Given :

$$\lambda = 5000\text{A}^\circ = 5000 \times 10^{-10} \text{ m.}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$h = 6.63 \times 10^{-34} \text{ J-S.}$$

So, $v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 5 \times 10^{-7}}$

$$= 1.457 \times 10^{3} \text{ m/s}$$

$$\text{K.E.} = \frac{1}{2} mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.457 \times 10^{3})^2$$

$$= 9.66 \times 10^{-25} \text{ J} \quad \text{...Ans.}$$

Ex. 8.10.21 SPPU - May 15

Calculate de-Broglie wavelength of 10 ke V protons in A. U.

Soln. :

Given : $E = 10 \text{ KeV} = 10 \times 10^{3} \times 1.6 \times 10^{-19} \text{J}$

$m_p = 1.67 \times 10^{-27}$ kg.

Formulae :

$$\lambda = \frac{h}{\sqrt{2 m_p E}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 10^{4} \times 1.6 \times 10^{-19}}}$$

$$= 2.86 \times 10^{-13} \text{ m.} = 2.86 \times 10^{-3} \text{ A}^\circ \quad \text{...Ans.}$$

Ex. 8.10.22 SPPU - Dec. 14

Calculate the De Broglie wavelength of electron having kinetic energy 1 KeV.

Soln. :

Formula : $\lambda_e = \frac{12.26}{\sqrt{V}}$ A°

Data given : $V = 10^{3}$ V

so $\lambda_e = \frac{12.26}{\sqrt{10^3}}$ A° = 0.38A°

Ex. 8.10.23 SPPU - Dec. 15

Calculate the de Broglie wavelength of electron of energy 1 keV.

Soln. : **Formulae :**

$$\lambda_e = \frac{12.26}{\sqrt{V}} \text{ A}^\circ = \frac{12.26}{\sqrt{10^3}} = 0.38 \text{ A}^\circ$$

Ex. 8.10.24 SPPU - May 16

Calculate de Broglie wavelength for a proton moving with velocity 1 percent of velocity of light.

Soln. :

Given : $m_p = 1.67 \times 10^{-27}$ kg

$h = 6.63 \times 10^{-34}$ J- s

Velocity proton = 1 % velocity of light

$$= \frac{1}{100} \times 3 \times 10^{8} = 3 \times 10^{6} \text{ m/s}$$

De – Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 3 \times 10^{6}} = 1.32 \times 10^{-13} \text{ m}$$

Ex. 8.10.25 SPPU- May 17

Calculate the energy (in eV) with which a proton has a acquire De-Broglie wavelength of 0.1 Å.

Soln. :

Given : $m_p = 1.673 \times 10^{-27}$ kg, $\lambda_p = 0.1 \times 10^{-10}$ m

De-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

For proton,

$$\lambda_p = \frac{h}{\sqrt{2m_p E}}$$

$$E = \frac{h^2}{2m_p \lambda_p^2}$$

$$= \frac{(6.626 \times 10^{-34})^2}{2 \times 1.673 \times 10^{-27} \times (0.1 \times 10^{-10})^2}$$

$$E = \frac{4.39 \times 10^{-67}}{3.346 \times 10^{-49}} = 1.312 \times 10^{-18} \text{ J}$$

$$E = 8.2 \text{ eV}$$

Solved Examples

Ex. 9.9.1

Find the lowest energy level and momentum of an electron in one dimensional potential well of width 1A°.

Soln. :

Given : $L = 1A^\circ = 10^{-10}$m.

Formulae required : $E_n = \dfrac{n^2h^2}{8mL^2}$, $E = \dfrac{p^2}{2m}$

The energy of an electron in a potential well of width L is,

$$E_n = \frac{n^2h^2}{8mL^2}$$

The lowest energy level corresponds to n = 1. Hence it is E_1

$$\text{Hence,} \quad E_1 = \frac{h^2}{8mL^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$= 6.038 \times 10^{-18}\ \text{J}$$

$$\therefore\ E_1 = \frac{6.038 \times 10^{-18}\ \text{J}}{1.6 \times 10^{-19}} = 38\ \text{ev},$$

$E_2 = 4\,E_1 = 152$ ev and $E_2 - E_1 = 114$ ev

The momentum p is given as per the relation.

$$E = \frac{p^2}{2m} \quad \therefore P = \sqrt{2mE} \quad \therefore P_1 = \sqrt{2mE_1}$$

$\therefore \quad P_1 = \sqrt{2mE_1} =$

$\sqrt{2 \times 9.1 \times 10^{-31} \times 6.038 \times 10^{-18}}$

$\therefore \quad P_1 = 33.045 \times 10^{-25}$ kg.m/s ...**Ans.**

Ex. 9.9.2

An electron is bound in a one dimensional potential box which has a width 2.5×10^{-10} m. Assuming the height of the box to be infinite, calculate the lowest two permitted energy values of the electron.

Soln. : We have, $E_n = \frac{n^2h^2}{8mL^2}$

Given : $h = 6.63 \times 10^{-34}$ J-s, $m = 9.1 \times 10^{-31}$ kg, $L = 2.5 \times 10^{-10}$ m

$$E_n = \frac{n^2(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$= 9.66 \times 10^{-19}\ n^2 \text{Joule.}$$

$$= \frac{9.66 \times 10^{-19}}{1.6 \times 10^{-19}} = 6.04n^2 \text{eV.}$$

First lowest permitted energy level (n = 1) = 6.04 eV

Second lowest permitted energy level (n = 2)

$= 6.04 \times 2^2 = 24.16$eV ...**Ans.**

Ex. 9.9.3

Lowest energy of an electron trapped in a potential well is 38 eV. Calculate the width of the well.

Soln. :

Given : $E_1 = 38$ eV

Formula required : $E_n = \frac{n^2 h^2}{8mL^2}$

We have lowest energy of a electron.

$$E_n = \frac{n^2 h^2}{8\ mL^2}$$

For lowest energy n = 1

$$E_1 = \frac{h^2}{8mL^2}$$

$$L^2 = \frac{h^2}{8mE_1}$$

$$= \frac{(6.63 \times 10^{-38})^2}{8 \times 9.1 \times 10^{-31} \times 38 \times 1.6 \times 10^{-19}}$$

$$= 9.93 \times 10^{-21}$$

$L = 0.996$A° ≈ 1A° ...**Ans.**

Ex. 9.9.4

Calculate first two energy eigen values of an electron in eV which is confined to a box of length 2 A. U.

Soln. :

Given : $L = 2$A° $= 2 \times 10^{-10}$m.

Formula : $E_n = \frac{n^2 h^2}{8mL^2}$

$$E_n = \frac{n^2(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= 1.50 \times 10^{-18} n^2 J = 9.43\ n^2 \text{ eV}$$

First two given values are,

$E_1 = 9.43$eV

$E_2 = 9.43 \times 4$ eV $= 37.72$ eV ...**Ans.**

Ex. 9.9.5

An electron is bound by a potential which closely approaches an infinite square well of width 1 A°. Calculate the lowest three permissible energies (in electron volts) the electron can have.

Soln. : Formula

$$E = \frac{n^2 h^2}{8m\ L^2}, \text{ Given } L = 1 \text{ A°} = 10^{-10} \text{ m}$$

$$= \frac{(6.63 \times 10^{-34})^2 \times n^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2\ J}$$

$$= \frac{0.6037 \times 10^{-17}\ n^2}{1.6 \times 10^{-19}} \text{ eV}$$

The lowest three permissible energies of electron correspond to n = 1, 2, 3 respectively.

$E_1 = 38 \times (1)^2$ eV $= 38$ eV

$E_2 = 38 \times (2)^2$ eV $= 152$ eV

$E_3 = 38 \times (3)^2$ eV $= 342$ eV ...**Ans.**

Ex. 9.9.6 SPPU - May 12, May 13, Dec. 14, May 15

An electron is trapped in a rigid box of width 2A°. Find its lowest energy level and momentum. Hence find energy of the 3rd energy level.

Soln. : Formulae : $E_n = \frac{n^2 h^2}{8m\ L^2}$

For lowest energy level n = 1

$L = 2$ A° $= 2 \times 10^{-10}$ m

$$E_1 = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= \frac{43.95 \times 10^{-68}}{291 \times 10^{-51}} = 1.5105 \times 10^{-18} \text{ J}$$

$$= \frac{15.105 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 9.44 \text{ eV}$$

$F_1 = 9.5$ eV

$$P_1 = \sqrt{2\ m\ E_1}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 9.5 \times 1.6 \times 10^{-19}}$$

$$= 1.663 \times 10^{-24} \text{ kg-m/s}$$

and $E_3 = 9 \times 9.5$ eV $= 85.5$ eV (as n = 3) ...**Ans.**

Ex. 9.9.7 SPPU - Dec. 12

An electron is bounded by an infinite potential well of width 2×10^{-8} cm. Calculate the lowest two permissible energies of an electron.

(Given : $h = 6.64 \times 10^{-34}$ J – sec., $m = 9.1 \times 10^{-31}$ kg)

Soln. : Formula : $E_n = \frac{n^2 h^2}{8m L^2}$

Data given : $n = 1$, $h = 6.64 \times 10^{-34}$ J-S ,

$m = 9.1 \times 10^{-31}$ kg, $L_1 = 2 \times 10^{-10}$ m

$$\text{So, } E_1 = \frac{1^2 \times (6.64 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= 1.5 \times 10^{-18} \text{ J} = 9.44 \text{ ev} \approx 9.5 \text{ ev}$$

$$\text{and } E_2 = 9.44 \times 4 = 37.79 \text{ ev} \approx 38 \text{ ev}$$

Ex. 9.9.8

Assuming Atomic Nucleus to be a rigid box (infinite potential well), calculate the ground state energy of an electron if it existed inside the nucleus.

(Given Planck's Constant = 6.63×10^{-34} j – s, Mass of the Electron = 9.1×10^{-31} kg. and size of the nucleus ~ 10^{-15} m. Using this result, argue that electron cannot exist inside the nucleus. Given, maximum binding energy per nucleon = 8.8 Me V)

Soln. : Formula $E_n = \frac{n^2 h^2}{8 \text{ m } L^2}$

Data given $h = 6.63 \times 10^{-34}$ J–s

$m_e = 9.1 \times 10^{-31}$ kg.

$L = 10^{-15}$ m.

$$\text{So, } E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-15})^2}$$

$$= 6.038 \times 10^{-8} \text{ J}/1.6 \times 10^{-13} \text{ Mev}$$

$$= 3.77 \times 10^{5} \text{ Mev}$$ **...Ans.**

If an electron exists inside the nucleus, then its ground state energy = 3.7×10^5 mev, but maximum binding energy per nucleon = 8.8 mev, so, electron cannot exist inside the nucleus.

Ex. 9.9.9 SPPU - Dec. 13

Calculate the energy difference between the ground state and first excited state of an electron in the rigid box of length 1 A°.

Soln. : Formula : $E_n = \frac{n^2 h^2}{8m L^2}$

For first excited state n = 2

For ground state n = 1

$$\text{So, } E_1 = \frac{1^2 \times (6.64 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$= 6.038 \times 10^{-18} \text{ Joules}$$

$$= \frac{6.038 \times 10^{-18}}{1.6 \times 10^{-19}} = 38 \text{ eV}$$

$$E_2 = \frac{2^2 \times (6.64 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2} = 152 \text{ eV}$$

Hence, $E_2 - E_1 = 114$ eV

Ex. 9.9.10 SPPU - Dec. 15

The lowest energy of an electron trapped in a rigid box is 4.19 eV. Find the width of the box in A.U.

Soln. : Formulae : $E_1 = \frac{n^2 h^2}{8m\, l^2}$

Data given : $E_1 = 4.19 \text{ eV} = 4.19 \times 1.6 \times 10^{-19}$ J,

$n = 1$, $m = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J-s

$$l^2 = \frac{n^2 h^2}{8m E_1} =$$

$$\frac{1^2 \times (6.63 \times 10^{-34})2}{8 \times 9.1 \times 10^{-31} \times 4.19 \times 1.6 \times 10^{-19}}$$

$$= 0.090 \times 10^{-18} = 9.0 \times 10^{-20}$$

$$l = 3 \times 10^{-10} \text{ m} = 3\text{A}°$$ **...Ans.**

Ex. 9.9.11 SPPU - May 16

A neutron is trapped in an infinite potential well of width 10^{-14} m. Calculate its first energy eigen value in eV.

Soln. :

Data Given : $m_n = 1.67 \ 10^{-27}$ kg ,

$h = 6.63 \times 10^{-34}$ J- s

Width of potential well $L = 10^{-14}$ m, n = 1

$$\text{Formula } E_n = \frac{n^2 h^2}{8 \text{ mL}^2}$$

$$= \frac{1 \times (6.63 \times 10^{-34})^2}{8 \times 1.67 \times 10^{-27} \times (10^{-14})^2} \text{ J}$$

$$E_1 = 3.276 \times 10^{-14} \frac{1}{1.6 \times 10^{-19}}$$

$$= 2.045 \times 10^{5} \text{ eV} = 0.2045 \text{ MeV}$$

Ex. 9.9.12 SPPU - May 17

A neutron is trapped in an infinite potential well of width 1 Å. Calculate the values of energy and momentum in its ground state.

Soln. :

Given : $m_n = 1.675 \times 10^{-27}$ kg

Width of potential well, $L = 1 \times 10^{-10}$ m

$E_n = ?$, $P_n = ?$ for n = 1

Energy of an infinite potential well,

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

For ground state, n – 1