Unit 2.Equilibrium

Introduction, free body diagram, equilibrium of coplanar forces, equilibrium of two forces, three force principle, equilibrium of concurrent, parallel and general force system, type of load, type of support, type of beam and support reaction.

Introduction :

A little consideration will show, that if the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces. A body can be said to be in equilibrium when all the force acting on a body balance each other or in other word there is no net force acting on the body.

The force, which brings the set of forces in equilibrium, is called an equilibrant.

Thus, the necessary and sufficient conditions of equilibrium for a system:
(i) The algebraic sum of the resolved parts of the forces along any direction is equal to zero (i.e., $\Sigma X= 0$),
(ii) The algebraic sum of the resolved parts of the forces along a directional right angles to the previous direction is equal to zero (i.e. $\Sigma Y = 0$), and
(iii) The algebraic sum of the moments of the forces about any point in their plane is equal to zero (i.e. $\Sigma M = 0$).

PRINCIPLES OF EQUILIBRIUM

1. **Two force principle**. As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
2. **Three force principle**. As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. **Four force principle**. As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Types of Forces Acting on a Body :

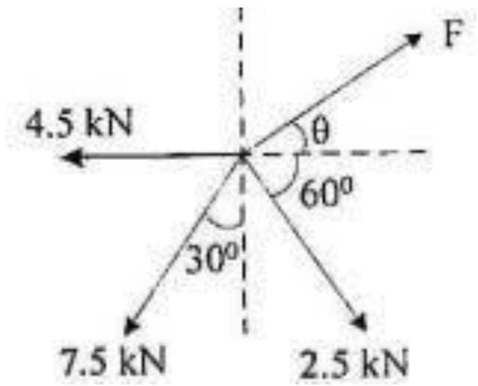
While applying equilibrium equations to a body, it is necessary that all forces acting on a body should be considered. The various forces acting on a body may be grouped as:

- Applied forces : Applied forces are the forces applied externally to a body. Each force has got point of contact with the body. If a person stands on a ladder, his weight is an applied force to the ladder. If a temple car is pulled, the force in the rope is an applied force for the car.
- Non-applied forces : There are two types of non-applied forces

(a) self weight and (b) reactions

TYPE: Equilibrium of Concurrent Force System

Example1 : Determine the magnitude and position of force F so that the force system shown in fig maintain equilibrium



Solution

By Using Equilibrium conditions

$\rightarrow$ (+Ve) $\leftarrow$(-Ve) Σ Fx = 0

Σ Fx = FcosΘ - 4.5 -7.5 X Cos 30+2.5 Cos60 = 0

F CosΘ = 7 ……….(1)

$\uparrow$ (+Ve) $\downarrow$ (-Ve) Σ Fy = 0

Σ Fy = F SinΘ - 7.5 Cos30 – 2.5sin60 = 0

F SinΘ = 8.66 ……..(2)

Divide equation 2 by 1 , we get

tan Θ = 8.66/ 7

Θ = 51.05^0

Put value of Θ in equation 1 we get ,

F = 7 / cos 51.05

F =11.13 N

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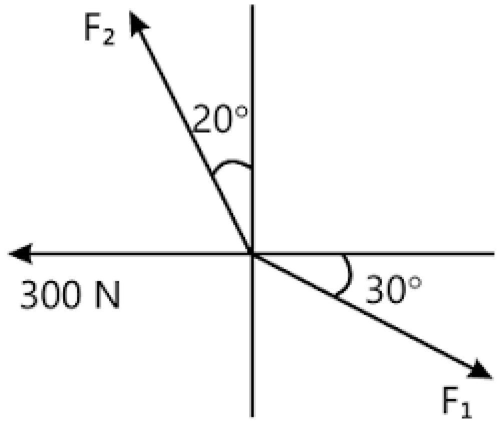
Example2 : Determine the magnitude of F_1 and F_2 so that the particle is in equilibrium



By Using Equilibrium conditions

$\rightarrow$ (+Ve) $\leftarrow$(-Ve) $\Sigma Fx = 0$

$$\Sigma Fx = F_1 \cos 30 - F_2 \sin 20 - 300 = 0$$

$$F_1 \cos 30 - F_2 \sin 20 = 300$$

$$0.866F_1 - 0.342 F_2 = 300$$

Divide above equation by 0.866 , we get

$$F_1 - F_2\ 0.3949 = 346.420 \quad ....(1)$$

$\uparrow$ (+Ve) $\downarrow$ (-Ve) $\Sigma Fy = 0$

$$\Sigma Fy = -F_1 \sin 30 + F_2 \cos 20 = 0$$

$$-0.5 F_1 + 0.939 F_2 = 0$$

Divide above equation by 0.5 , we get

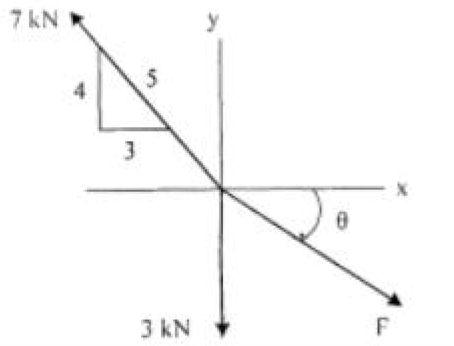
$$-F_1 + 1.878F_2 = 0 \quad ....(2)$$

Solving Equation (1) and (2) , We get ,

F_1 = 438.57 N

F_2 = 233.36 N

Example3: Determine the magnitude and direction ϴ of force F so that the particle is in equilibrium as shown in fig :



Solution : **tan $\acute{\alpha}$ = 4/3** **$\acute{\alpha}$ = 53.13°**

By Using Equilibrium conditions

$\rightarrow$ (+Ve) $\leftarrow$(-Ve) Σ Fx = 0

Σ Fx = Fcosϴ - 7 X Cos 53.13 = 0

F Cosϴ = 4.2 ……….(1)

$\uparrow$ (+Ve) $\downarrow$ (-Ve) Σ Fy = 0

Σ Fy = - F Sinϴ + 7 X sin 53.13 – 3 = 0

F Sinϴ = 2.6 ……..(2)

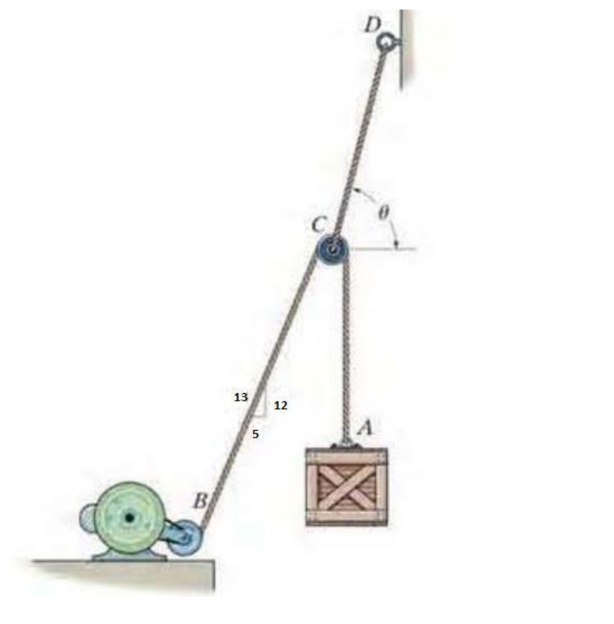
Divide equation 2 by 1 , we get

tanϴ = 2.6/ 4.2

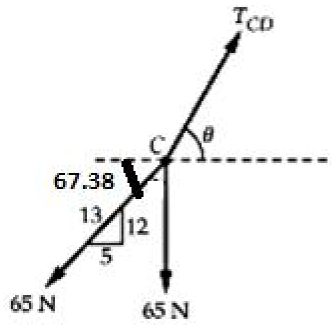
ϴ = 31.76°

Put value of **ϴ in equation 1 we get , F = 4.2 / cos 31.76 = 4.93 N**

Example 4: The motor at B winds up the cord attached to the 65 N crate with a constant speed as shown in fig . Determine the force in cord CD supporting the pulley and the angle Θ for equilibrium . Neglect the size of the pulley.



The F.B.D. of pulley C is shown in Fig.



Inclination of cord with horizontal,

$$\tan^{-1}\left(\frac{12}{5}\right) = 67.38°$$

Solution :

Since forces are in equilibrium,

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

Resolving the forces along x-axis,

$$\Sigma F_x = T_{CD} \cos\theta - 65 \cos 67.38° = 0$$

$$T_{CD} \cos\theta = 25 \qquad \text{... (i)}$$

Resolving the forces along y-axis,

$$\Sigma F_y = T_{CD} \sin\theta - 65 \sin 67.38° - 65 = 0$$

$$T_{CD} \sin\theta = 125 \qquad \text{... (ii)}$$

From equations (i) and (ii),

$$\frac{T_{CD} \sin\theta}{T_{CD} \cos\theta} = \frac{125}{25}$$

$$\tan\theta = 5$$

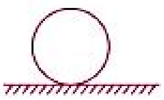
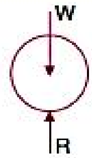
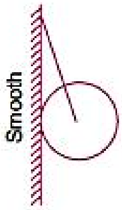
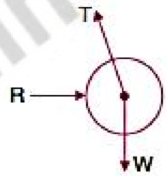
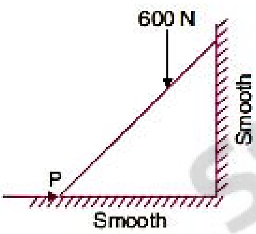
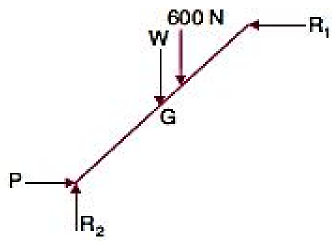
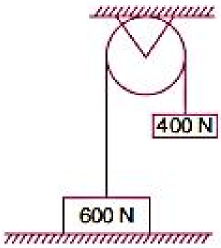
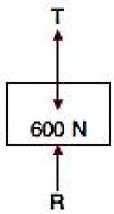
$$\theta = \mathbf{78.7°} \qquad \textbf{... Ans.}$$

Substituting θ = 78.7° in equation (i),

$$T_{CD} = \mathbf{127.5\ N} \qquad \textbf{... Ans.}$$

Free Body Diagram

For the analysis of equilibrium condition it is necessary to isolate the body under consideration from the other bodies in contact and draw all forces acting on the body. For this, first the body is drawn and then all applied forces, self weight and reactions from the other bodies in contact are drawn. Such diagram of the body in which the body under consideration is freed from all contact surfaces and is shown with all the forces on it (including self weight and reactions from other contact surfaces) is called the Free Body Diagram (FBD). Free body diagrams (FBD) are shown for few typical cases in

Reacting bodies	*FBD required for*	*FBD*
	Ball	
	Ball	
	Ladder	
	Block weighing 600 N	

METHODS FOR THE EQUILIBRIUM OF COPLANAR FORCES :

Though there are many methods of studying the equilibrium of forces, yet the following are important from the subject point of view :

1. Analytical method. 2. Graphical method.

ANALYTICAL METHOD FOR THE EQUILIBRIUM OF COPLANAR FORCES :

The equilibrium of coplanar forces may be studied, analytically, by Lami's theorem as discussed below :

LAMI'S THEOREM

Lami's theorem states that *if a body is in equilibrium under the action of only three forces, each force is proportional to the sine of the angle between the other two forces.* Thus for the system of forces shown in Fig.

$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma} \qquad ...(2.10)$$

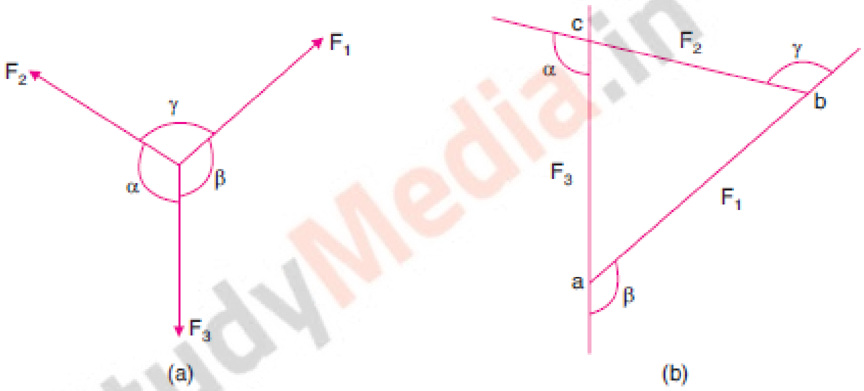


Fig. 2.12

Proof: Draw the three forces F_1, F_2 and F_3 one after the other in direction and magnitude starting from point a. Since the body is in equilibrium the resultant should be zero, which means the last point of force diagram should coincide with a. Thus, it results in a triangle of forces abc as shown in Fig. 2.12 (b). Now the external angles at a, b and c are equal to β, γ and α, since ab, bc and ca are parallel to F_1, F_2 and F_3 respectively. In the triangle of forces abc,

$$ab = F_1$$
$$bc = F_2 \quad \text{and}$$
$$ca = F_3$$

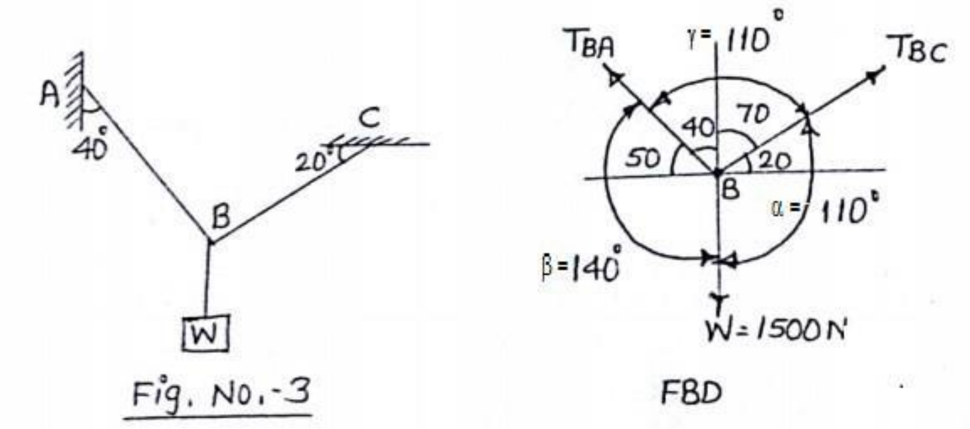
Applying sine rule for the triangle abc,

$$\frac{ab}{\sin(180° - \alpha)} = \frac{bc}{\sin(180° - \beta)} = \frac{ca}{\sin(180° - \gamma)}$$

i.e.,
$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

Example 1 : Calculate the tension induced in the cable used for the assembly shown in Figure No. 3. W = 1500 N.

Solution :



$\alpha = 110°$, $\beta = 140°$ and $\gamma = 110°$

According to Lami's Theorem

$$\frac{T_{BA}}{Sin\ \alpha} = \frac{T_{BC}}{Sin\ \beta} = \frac{W}{Sin\ \gamma}$$

$$\frac{T_{BA}}{Sin\ 110°} = \frac{T_{BC}}{Sin\ 140°} = \frac{1500}{Sin\ 110°}$$

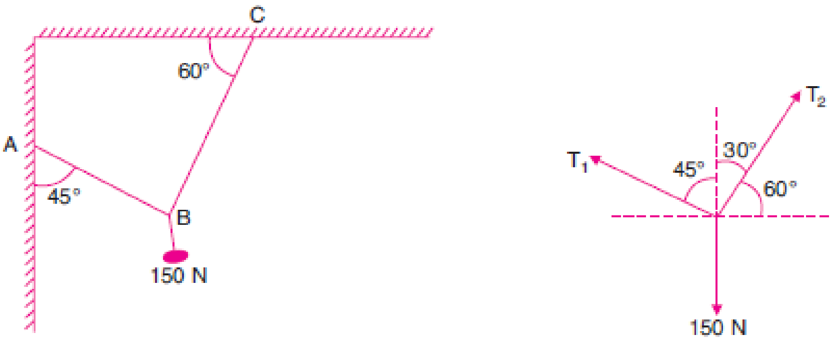
$$\frac{T_{BA}}{Sin\ 110°} = \frac{1500}{Sin\ 110°}$$

$T_{BA} = 1500$ N

$$\frac{T_{BC}}{Sin\ 140°} = \frac{1500}{Sin\ 110°}$$

$T_{BC} = 1026.06$ N

Example 2 : Find the forces developed in the wires, supporting an electric fixture as shown in Fig.



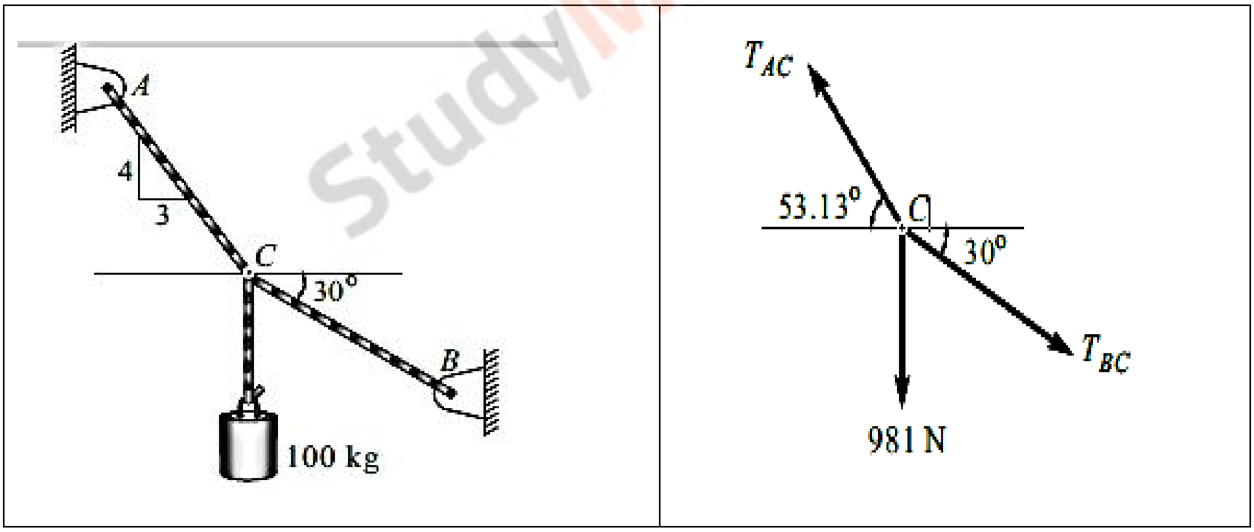
(a) (b)

Solution: Let the forces developed in the wires *BA* and *BC* be *T*1 and *T*2 as shown in Fig. (*b*). Applying Lami's theorem to the system of forces, we get

$$\frac{T_1}{\sin(90° + 60°)} = \frac{T_2}{\sin(180° - 45°)} = \frac{150}{\sin(45° + 30°)}$$

$$T_1 = 77.6 \text{ N} \quad \text{and} \quad T_2 = 109.8 \text{ N} \quad \textbf{Ans.}$$

Example 3:Find the tension in each rope in Fig.



Solution:

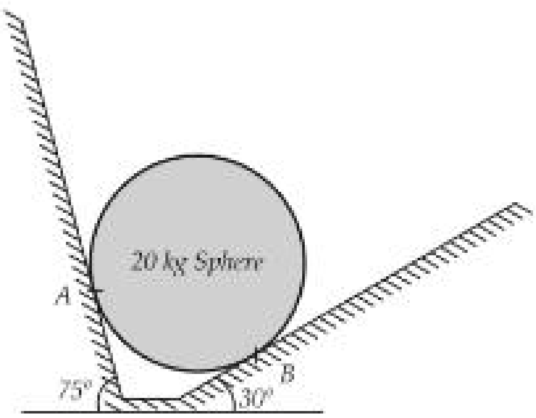
By Lami's theorem,

$$\frac{981}{\sin 156.87^\circ} = \frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 143.13^\circ}$$

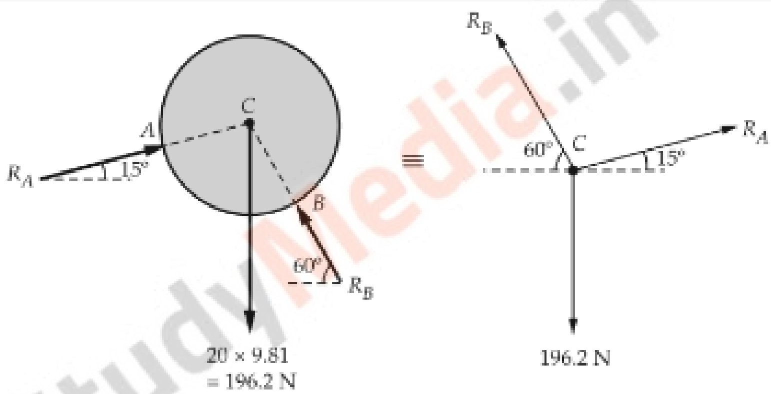
$$T_{AC} = 2162.76 \text{ N}$$

$$T_{BC} = 1498.41 \text{ N} \quad \textit{Ans.}$$

Example 4: The 20 Kg Smooth sphere rest on the two inclines as shown in fig . Determine the contact forces at A and B



Solution : F B D of Sphere are shown below



By Using Lami's Theorem ,

$$\frac{RA}{Sin(150\ )} = \frac{RB}{Sin\ (105)} = \frac{196.2}{Sin\ (105)}$$

By Comparing ,

$$\frac{RA}{Sin(150\ )} = \frac{196.2}{Sin\ (105)}$$

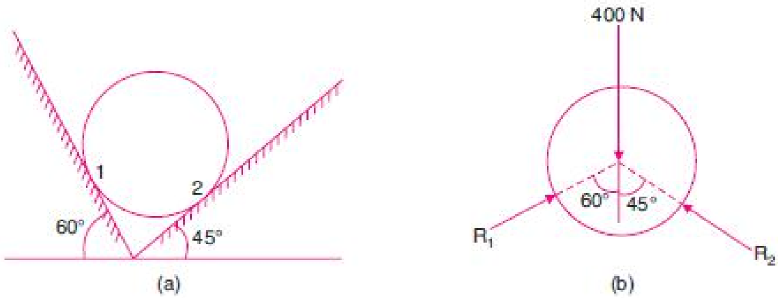
R_A = 101.56 N

By Comparing ,

$$\frac{RB}{Sin\ (105)} = \frac{196.2}{Sin\ (105)}$$

R_B = 196.2 N

Example 5: A 400 N sphere is resting in a trough as shown in Fig (a). Determine the reactions developed at contact surfaces. Assume all contact surfaces are smooth.



$$\frac{R1}{Sin(135\,)} = \frac{R2}{Sin\,120} = \frac{400}{Sin\,105}$$

By Comparing ,

$$\frac{R1}{Sin(135\,)} = \frac{400}{Sin\,105}$$

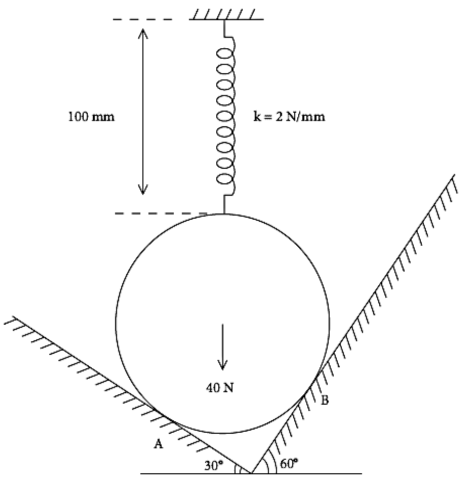
$$\mathbf{R_1 = 292.8\ N}$$

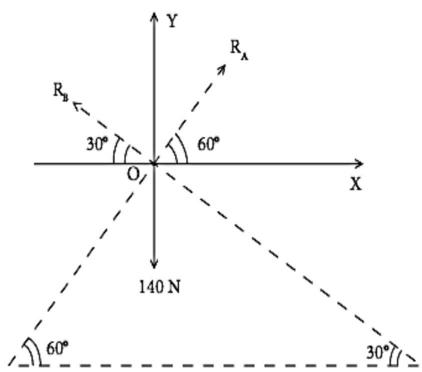
Similarly ,

$$\frac{R2}{Sin\,120} = \frac{400}{Sin\,105}$$

$$\mathbf{R_2 = 358.6\ N}$$

Example 6: Figure shows a sphere resting in a smooth V-shaped groove and subjected to a spring force. The spring is compressed to a spring force. The spring is compressed to a length of 100 mm from its free length of 150 mm. It the stiffness of spring is 2 N/mm, determine the contact reactions at A and B.





FBD of sphere

Solution:

The spring is compressed by = 150 − 100 = 50 mm.

Thus compression force in the spring = 50 × stiffness of spring = 50 × 2 = 100 N

By Lami's Theorem

$$\frac{RA}{Sin(120\,)} = \frac{RB}{Sin\,150} = \frac{140}{Sin\,90}$$

By Comparing

$$\frac{RA}{Sin(120\,)} = \frac{140}{Sin\,90}$$

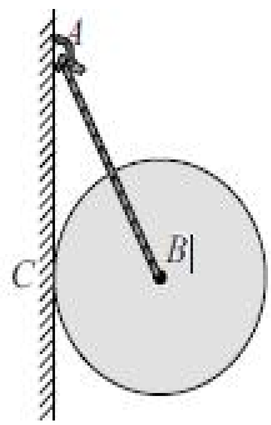
$$RA = 121.24\ N$$

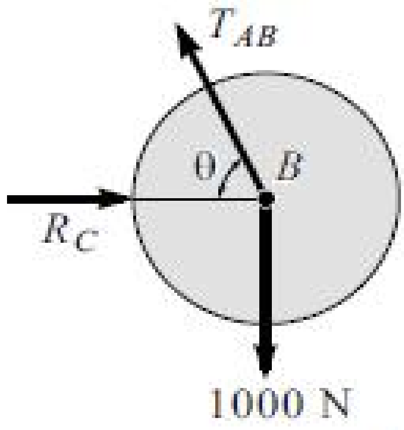
By Comparing

$$\frac{RB}{Sin\,150} = \frac{140}{Sin\,90}$$

$$RB = 70\ N$$

Example 7 : A circular roller of weight 1000 N and radius 20 cm hangs by a tie rod AB = 40 cm and rests against a smooth vertical wall at C as shown in Fig. (a). Determine the tension in the rod and reaction at point C.





Solution

(i) Draw the FBD of the roller

$$\cos\theta = \frac{20}{40}$$

$$\therefore \theta = 60^{\circ}$$

By Lami's Theorem

$$\frac{1000}{Sin(120\,)} = \frac{TAB}{Sin\,90} = \frac{Rc}{Sin\,150}$$

By Comparing ,

$$\frac{1000}{Sin(120\,)} = \frac{TAB}{Sin\,90}$$

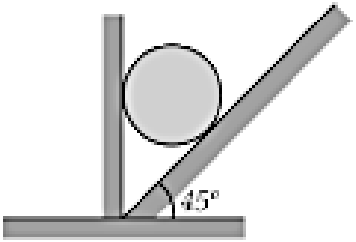
T_{AB} = 1154.70 N

By Comparing ,

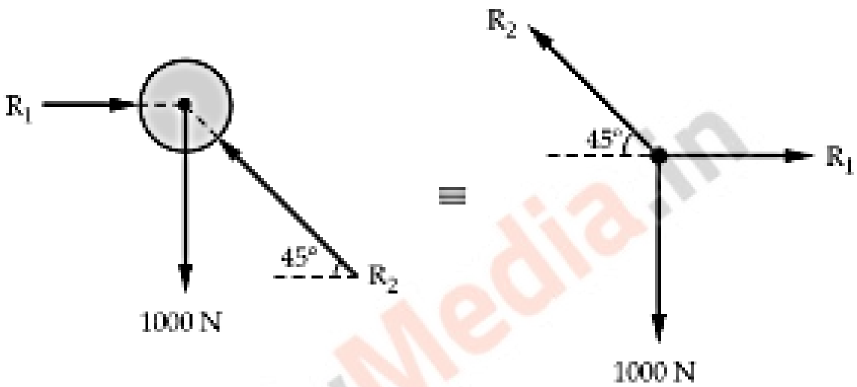
$$\frac{1000}{Sin(120\,)} = \frac{Rc}{Sin\,150}$$

R_c = 577.35 N

Example 8 : A sphere weighing 1000N is Placed in a wrench as shown in fig , find the reactions at the point of contacts



Solution: FBD of sphere as shown in fig



By Lami's Theorem

$$\frac{R1}{Sin(135\,)} = \frac{R2}{Sin\,90} = \frac{400}{Sin\,135}$$

By Comparing ,

$$\frac{R1}{Sin(135\,)} = \frac{400}{Sin\,135}$$

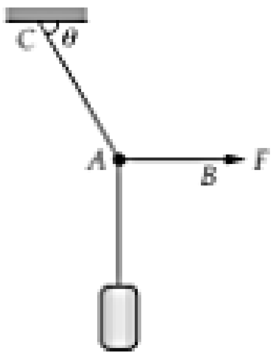
R_1 = 1000 N

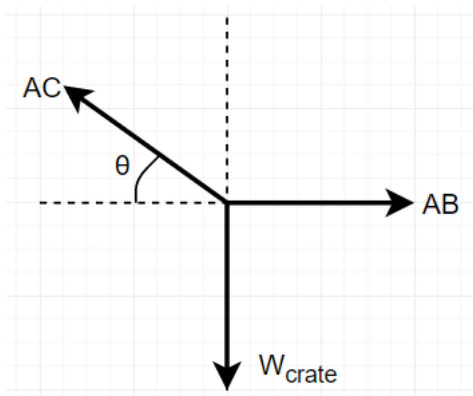
By Comparing ,

$$\frac{R2}{Sin\,90} = \frac{400}{Sin\,135}$$

R_2 = 1414.2 N

Example 9: The 500 N crate is hoisted using the ropes AB and AC . Each rope can withstand a maximum tension of 2500 N before it breaks . If AB always remains horizontal , determine the smallest angle Θ to which the crate can be hoisted. Refer following fig.





FBD

By Lami's Theorem

$$\frac{TAB}{Sin(90+\theta\,)} = \frac{TAC}{Sin\,90} = \frac{500}{Sin\,(180-\theta)}$$

$$\frac{TAB}{Cos\,\theta} = \frac{TAC}{Sin\,90} = \frac{500}{Sin\,\theta}$$

For T_{AB} = 2500N

$$\frac{2500}{Cos\,\theta} = \frac{500}{Sin\,\theta}$$

$$\frac{Sin\,\theta}{Cos\,\theta} = \frac{500}{2500}$$

tan Θ = 0.2

Θ = 11.30^0

Check ,

$$\frac{TAC}{Sin\,90} = \frac{500}{Sin\,\theta}$$

$TAC = 2551N > 2500$

Therefore Θ = 11.30^0 will not satiesfied the given condition

For T_{AC} = 2500N

$$\frac{TAC}{Sin\,90} = \frac{500}{Sin\,\theta}$$

$$\frac{2500}{Sin\,90} = \frac{500}{Sin\,\theta}$$

Sin Θ = 500 / 2500

Θ = 11.53^0

Check ,

$$\frac{TAB}{Cos\,\theta} = \frac{500}{Sin\,\theta}$$

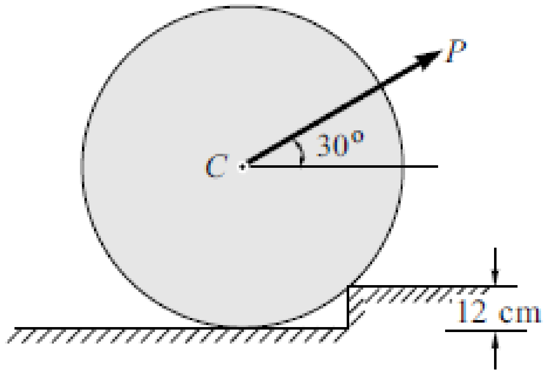
tan Θ = 500 / TAB

T_{AB} = 500 / tan11.53 = 2451N < 2500N

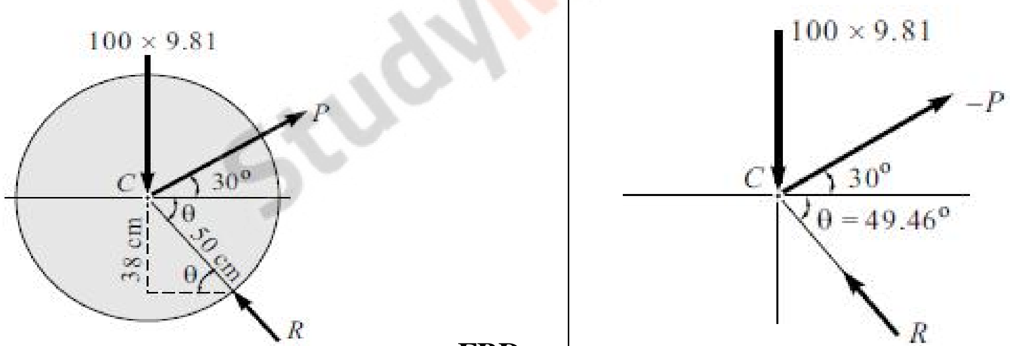
Satisfied the given condition

Therefore Θ = 11.53^0 (Ans)

Example10: Determine the force P applied at 30 to the horizontal just necessary to start a roller having radius 50 cm over an obstruction 12 cm high, if the roller is of mass 100 kg, shown in Fig. (a). Also find the magnitude and direction of P when it is minimum.



Solution : As per the given condition, P is just sufficient to start the roller. At this instant, the roller will not have any pressure on the horizontal surface. Therefore, the surface will not offer any reaction. We can identify that this body is subjected to three forces, viz., 100 x 9.81, P and R



FBD

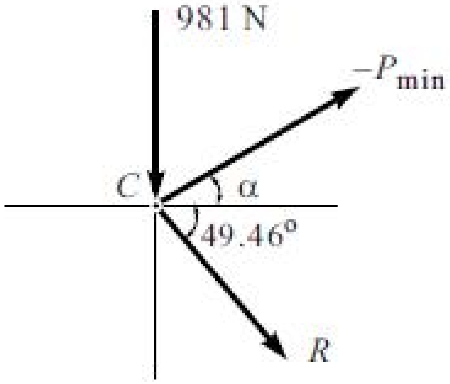
$$\sin\theta = \frac{38}{50} \quad \therefore \theta = 49.46^\circ$$

By Lami Theorem

$$\frac{981}{Sin\,79.46} = \frac{P}{Sin\,139.46}$$

P = 648.57 N

To find Pmin : By Lami's Theorem



$$\frac{981}{Sin(\acute{\alpha} + 49.40\,)} = \frac{Pmin}{Sin\,139.46}$$

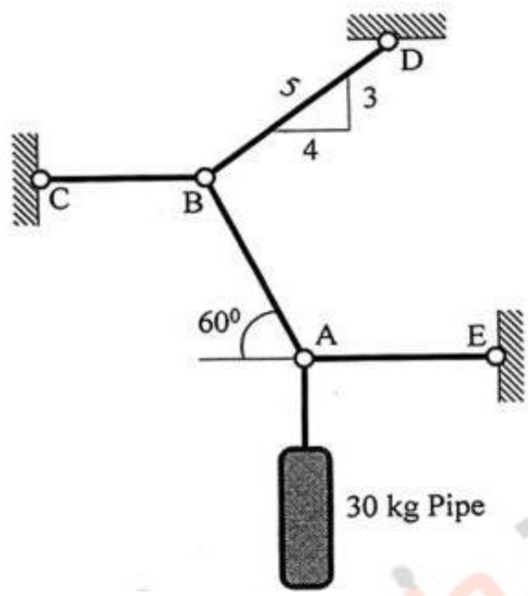
$$\frac{981\ X\ Sin\,139.46}{Sin(\acute{\alpha} + 49.40\,)} = Pmin$$

For P_{min} the denominator should be maximum

$$\frac{981\ X\ Sin\,139.46}{1} = Pmin$$

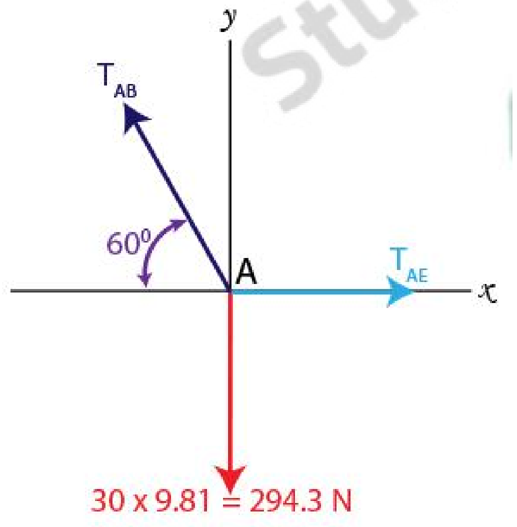
P_{min} = 637.62 N

Example 11: The 30 kg pipe is supported at A by a system of five cords as shown in fig . Determine the force in each cord for equilibrium.



Solution :

Let us draw a free body diagram focusing on ring A.



By Lami's Theorem

$$\frac{TAB}{Sin(90\ )} = \frac{TAE}{Sin\ (150)} = \frac{294.3}{Sin\ (120)}$$

$$\frac{TAB}{Sin(90\ )} = \frac{30\ X9.81}{Sin\ (120)}$$

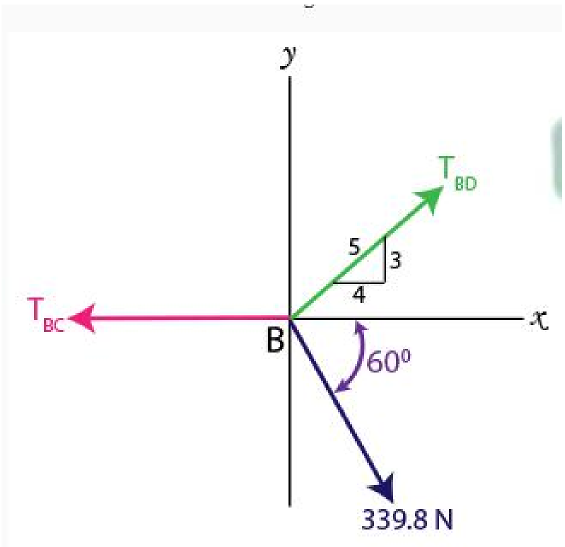
$$TAB\ = 339.82N$$

Similarly ,

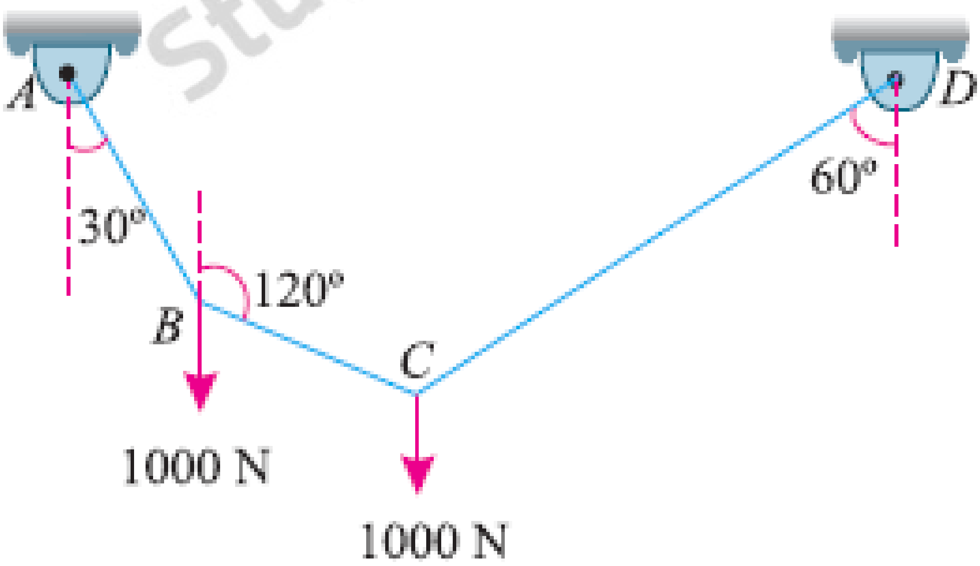
$$\frac{TAE}{Sin\ (150)} = \frac{30\ X9.81}{Sin\ (120)}$$

$$TAE = 169.91\ N$$

Let us draw a free body diagram focusing on ring B.

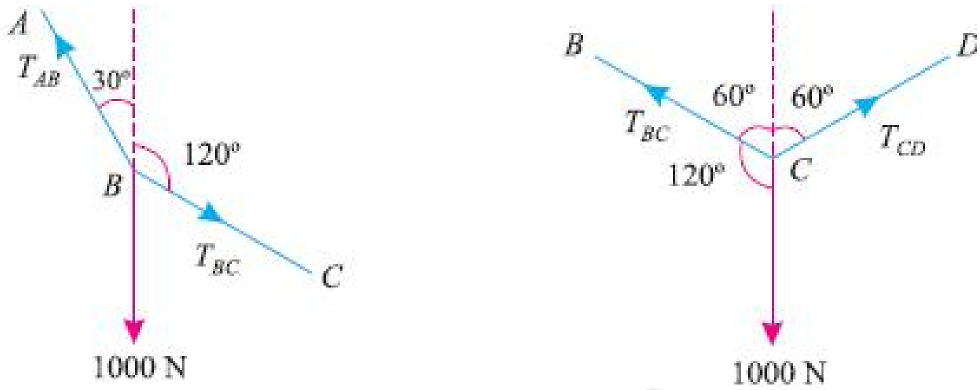
	$\frac{TBC}{Sin(96.87)} = \frac{TBD}{Sin\,(120)} = \frac{TAB}{Sin\,(143.13)}$ $\frac{TBC}{Sin(96.87)} = \frac{339.82}{Sin\,(143.13)}$ $\boldsymbol{TBC = 562.29\ N}$ $\frac{TBD}{Sin(120)} = \frac{339.82}{Sin\,(143.13)}$ $\boldsymbol{TBD = 490.48\ N}$

Example 12 : A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Fig . Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.



Solution:

Let T_{AB} = Tension in the portion AB of the string,
T_{BC} = Tension in the portion BC of the string, and
T_{CD} = Tension in the portion CD of the string.



FBD

Applying Lami's equation at joint B,

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 150°} = \frac{1000}{\sin 150°}$$

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 30°} = \frac{1000}{\sin 30°} \qquad ...[\because \sin(180° - \theta) = \sin\theta]$$

$$\therefore \qquad T_{AB} = \frac{1000 \sin 60°}{\sin 30°} = \frac{1000 \times 0.866}{0.5} = 1732\text{ N} \quad \textbf{Ans.}$$

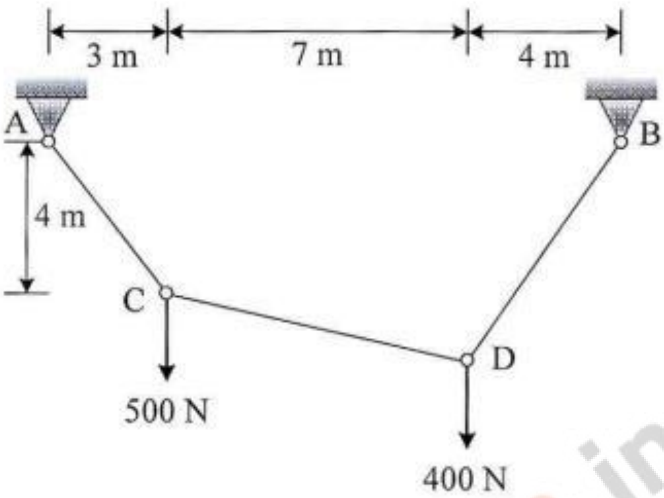
$$T_{BC} = \frac{1000 \sin 30°}{\sin 30°} = 1000\text{ N} \quad \textbf{Ans.}$$

Again applying Lami's equation at joint C,

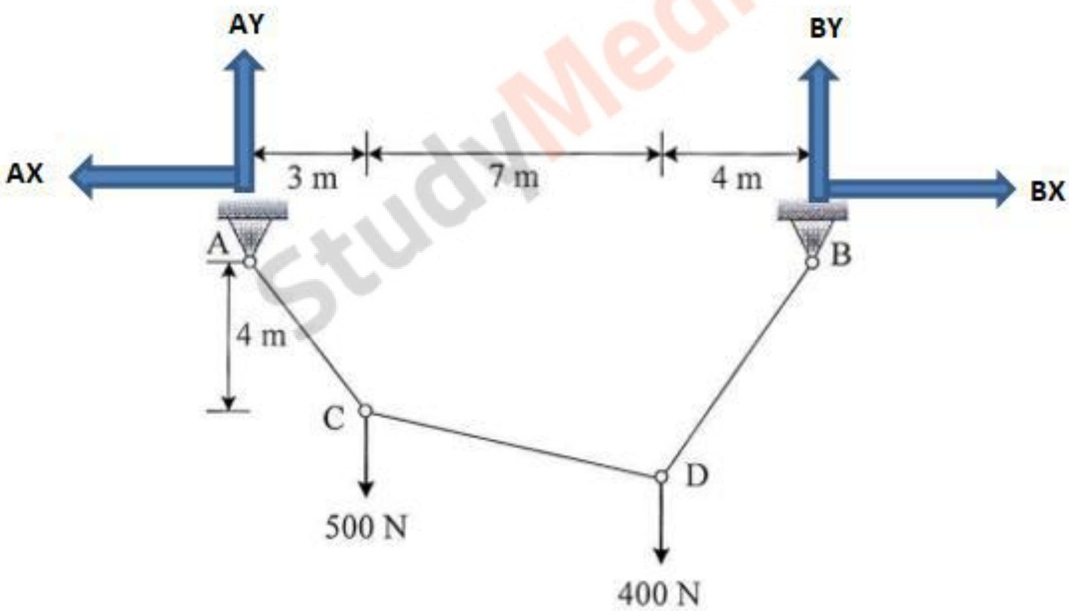
$$\frac{T_{BC}}{\sin 120°} = \frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°}$$

$$\therefore \qquad T_{CD} = \frac{1000 \sin 120°}{\sin 120°} = 1000\text{ N} \quad \textbf{Ans.}$$

Example13: The cable segment support the loading as shown in fig . Determine the support reaction and maximum tension in segment of cable



Solution : FBD of Given system :



By Using Equilibrium conditions

$\rightarrow$ (+Ve) $\leftarrow$(-Ve) Σ Fx = 0

$$\Sigma F_x = -A_X + B_X = 0$$

$$A_X = B_X \quad \ldots\ldots\ldots(1)$$

↑ (+Ve) ↓ (-Ve) $\Sigma F_y = 0$

$$\Sigma F_y = A_Y + B_Y - 500 - 400 = 0$$

$$\mathbf{A_Y + B_Y = 900 \quad ....(2)}$$

$$\Sigma M_A = 0$$

$$(500 \times 3) + (400 \times 10) - (B_Y \times 14) = 0$$

$$\mathbf{B_Y = 392.85\ N}$$

PUT IN EQUATION 2 WE GET

$$A_Y + 392.85 = 900 \quad ....(2)$$

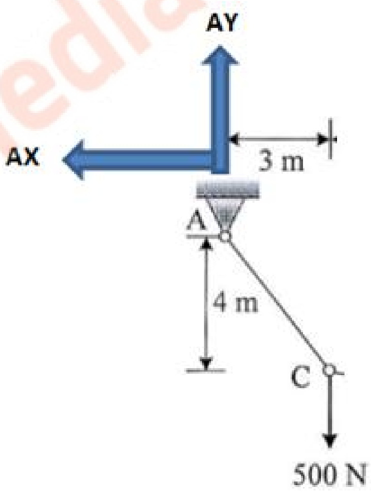
$$\mathbf{A_Y = 507.15\ N}$$

Consider part AC

$\mathbf{M_c} = 0$

- $A_x \times 4 + A_y \times 3 = 0$
- $A_x \times 4 + 507.15 \times 3 = 0$

Ax = 380.36 N



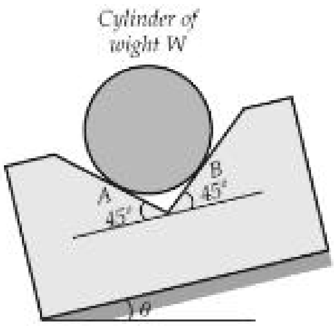
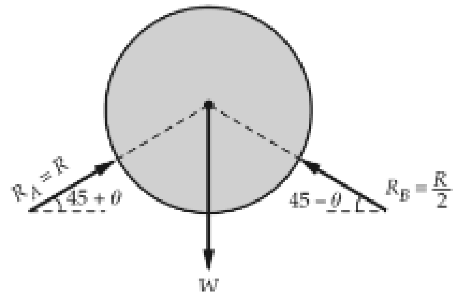
Calculation of Support Reaction :

$$RA = \sqrt{Ax^2 + Ay2} \quad = 633.93\ \text{N}$$

$$RB = \sqrt{Bx^2 + By2} \quad = 546.81\ \text{N}$$

Maximum Tension in segment Cable:
Since
A_y = 507.15N > B_y = 392.85 therefore Maximum tension will be in cable AC

Example 14 : Find the angle of tilt Ɵ with the horizontal so that the contact force at B will be one half that at A for the smooth cylinder as shown in fig :

	FBD
	

Solution

$$\sum F_x = 0$$

$$R \cos(45 + \theta) - \frac{R}{2}\cos(45 - \theta) = 0$$

$$\cos(45 + \theta) = \frac{1}{2}\cos(45 - \theta)$$

$$\cos 45 \cos\theta - \sin 45 \sin\theta = \frac{1}{2}[\cos 45 \cos\theta + \sin 45 \sin\theta]$$

$$\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}\times\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{2}\times\frac{1}{\sqrt{2}}\sin\theta$$

$$\frac{1}{\sqrt{2}}\left(1-\frac{1}{2}\right)\cos\theta = \frac{1}{\sqrt{2}}\left(1+\frac{1}{2}\right)\sin\theta$$

$$\frac{1}{2}\cos\theta = \frac{3}{2}\sin\theta$$

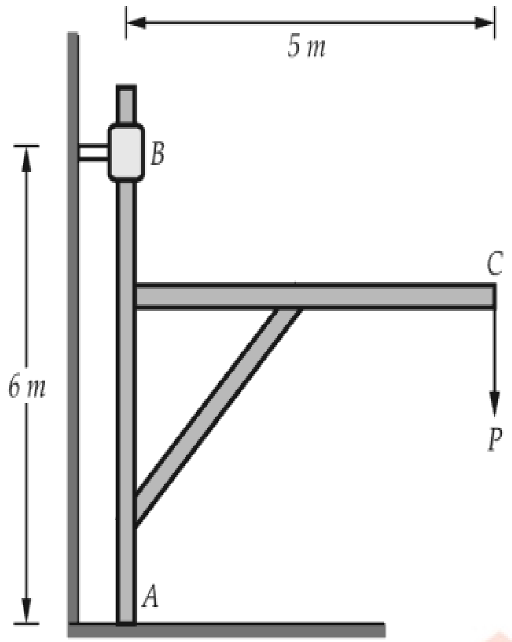
$$\cos\theta = 3\sin\theta$$

$$\tan\theta = \frac{1}{3}$$

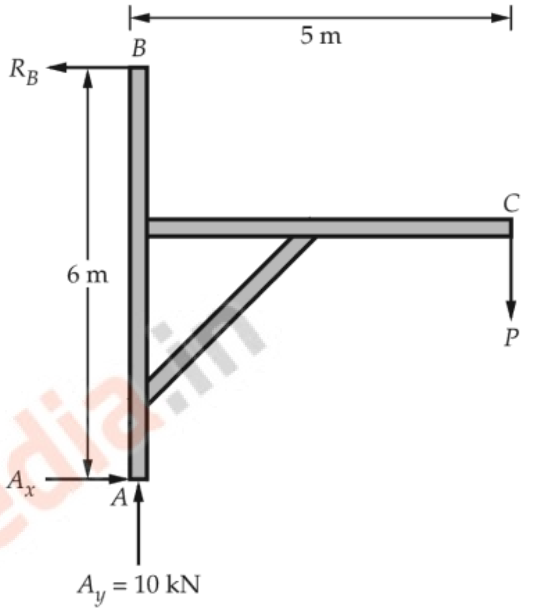
$$\boxed{\theta = 18.435°}$$

Type : Equilibrium of General Force System :

Example1: The wall Crane is Supported by smooth collar at B and Pin at A as Shown in fig . If the vertical component of reaction at A is 10 KN , determine the force P , normal reaction at B and Tangential component of reactions at A.



FBD



Solution :

$$\sum F_y = 0 : \quad 10 - P = 0$$

$\therefore$ $\boxed{P = 10 \text{ kN}}$ **...Ans.**

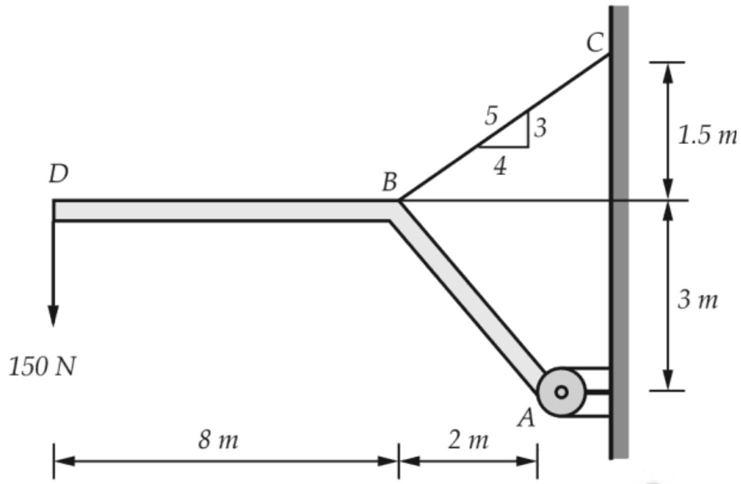
$$\sum M_A = 0 : -R_B \times 6 + P \times 5 = 0$$

$\therefore$ $\boxed{R_B = 8.33 \text{ kN} \leftarrow}$ **...Ans.**

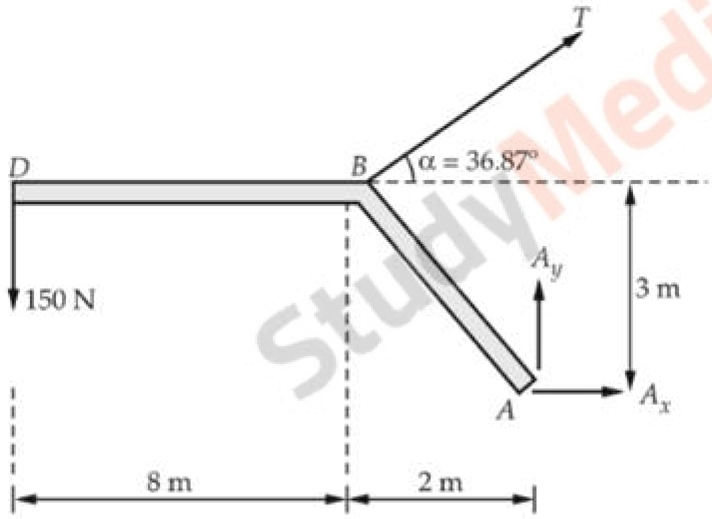
$$\sum F_x = 0 : \quad A_x - R_B = 0$$

$\therefore$ $\boxed{A_x = 8.33 \text{ kN} \rightarrow}$ **...Ans.**

Example 2 : A force of 150 N acts on the end of beam ABD as shown in fig . Determine the magnitude of tension in cable BC to maintain Equilibrium.



Solution:



$$\tan \alpha = \frac{3}{4} \qquad \therefore \alpha = 36.87°$$

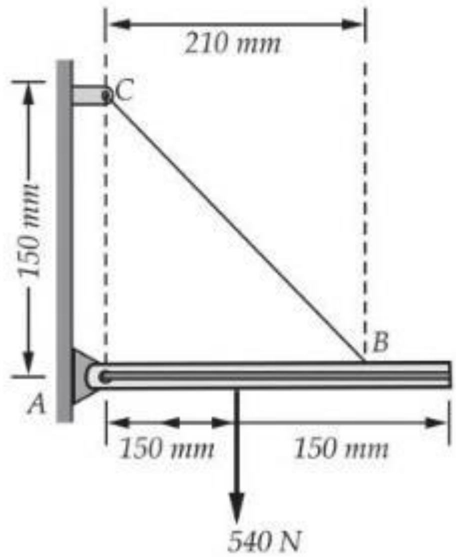
$\sum M_A = 0$:

$$-(150)(10) + (T \times \cos 36.87)(3) + (T \times \sin 36.87)(2) = 0$$

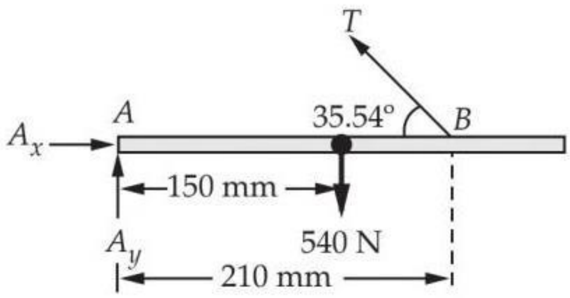
1500 = 3.599 T

$$\therefore \qquad \boxed{T = 416.67 \text{ N}} \qquad \textbf{...Ans.}$$

Example 3: A 300 mm wooden beam weighing 540 N is supported by a pin and bracket at A and by cable BC. Find the reaction at A and tension in cable BC .



FBD



Solution :

The angle made by cable with horizontal is

$$\alpha = \tan^{-1}\left(\frac{150}{210}\right) = 35.54°$$

$\sum M_A = 0$:

$$(540)\ (150) - (T \sin 35.54)\ (210) = 0$$

$$\therefore \qquad \boxed{T = 663.57 \text{ N}} \qquad \text{...Ans.}$$

$\sum F_x = 0 : \quad A_x - T \cos 35.54 = 0$

$$\therefore \qquad A_x = 540 \text{ N}$$

$\sum F_y = 0 : \quad A_y - 540 + T \sin 35.54 = 0$

$$\therefore \qquad A_y = 154.3 \text{ N}$$

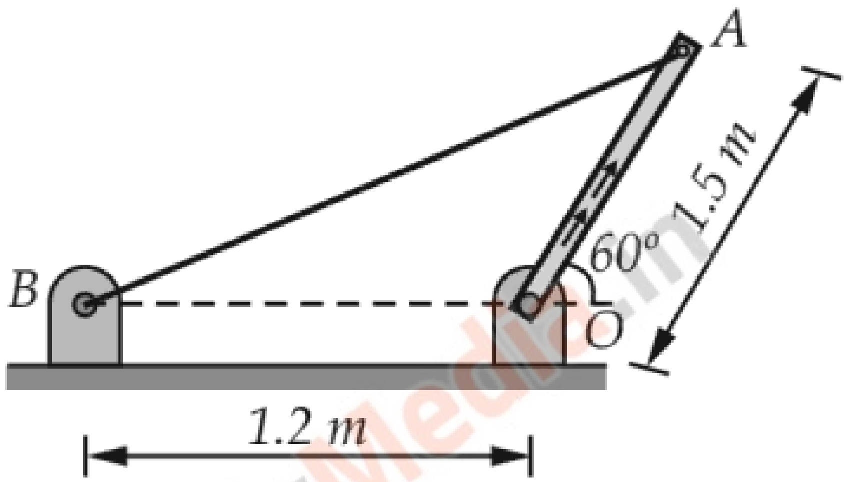
$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{540^2 + 154.3^2}$$

$$\therefore \qquad \boxed{R_A = 561.6 \text{ N}} \qquad \text{...Ans.}$$

$$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{154.3}{540}\right)$$

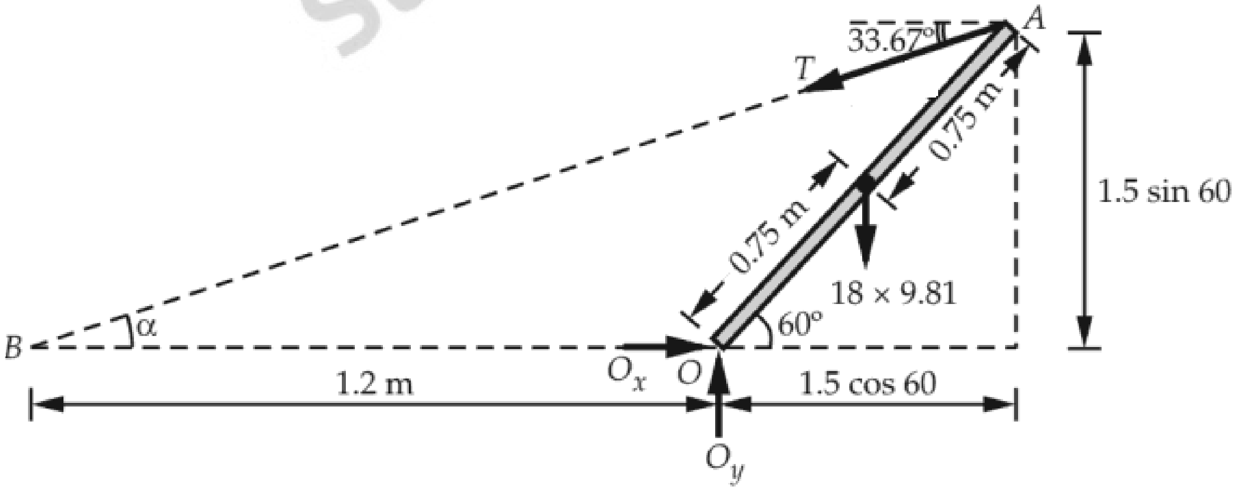
$\therefore$ $$\boxed{\theta_A = 15.95^\circ}$$ **...Ans.**

Example 4: The uniform 18 Kg bar OA is held in position as shown in fig. by the smooth pin at O and the cable AB. Determine the tension in the cable and reaction at O.



Solution :

FBD of the fig as shown in fig :



The angle made by AB with horizontal is

$$\alpha = \tan^{-1}\left[\frac{1.5\sin 60}{1.2 + 1.5\cos 60}\right] = 33.67^\circ$$

The FBD of bar OA is shown in Fig.

$$\sum M_O = 0 :$$

= (18 x 9.81 x 0.375) + (T sin 33.67 x 1.5 cos 60) - (T cos 33.67 X1.5 sin 60) = 0

$$\therefore \quad \boxed{T = 99.53 \text{ N}} \quad \text{... Ans.}$$

$$\sum F_x = 0 :$$

$$O_x - T \cos 33.67 = 0$$

$$\therefore \quad O_x = 82.83 \text{ N}$$

$$\sum F_y = 0 :$$

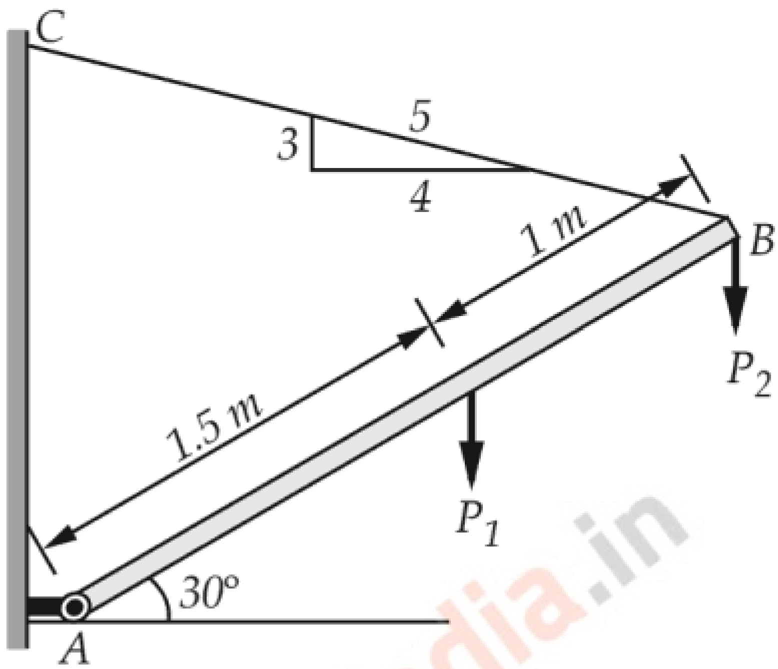
$$O_y - 18 \times 9.81 - T \sin 33.67 = 0$$

$$\therefore \quad O_y = 231.76 \text{ N}$$

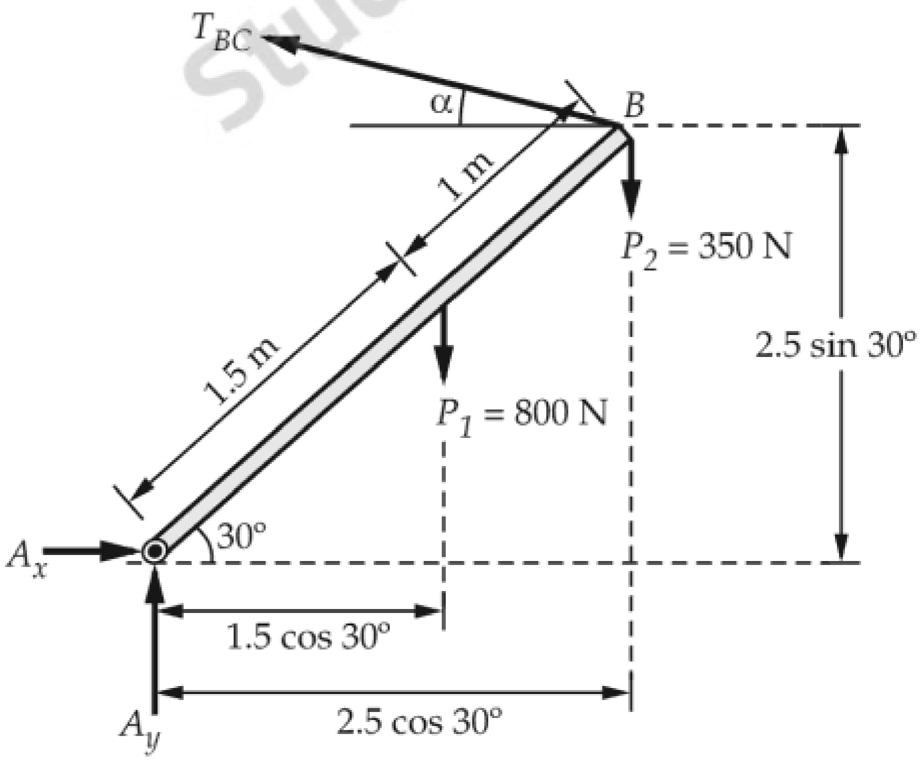
$$\therefore \quad R_O = \sqrt{O_x^2 + O_y^2} = \sqrt{82.83^2 + 231.76^2}$$

$$\therefore \quad \boxed{R_O = 246.116 \text{ N}} \quad \text{...Ans.}$$

Example 5 : The boom supports the two vertical loads P_1 = 800 N and P_2 = 350 N as shown in fig. Determine the tension in cable BC and component of reaction at A.



Sol. : The F.B.D. of beam is shown in Fig.



$$\tan\alpha = \frac{3}{4} \qquad \therefore \alpha = 36.87^\circ$$

$$\sum M_A = 0 :$$

$$(800)\ (1.5 \cos 30) + (350)\ (2.5 \cos 30)$$

$$-\left(T_{BC} \times \cos 36.87\right)\ (2.5 \sin 30)$$

$$-\left(T_{BC} \times \sin 36.87\right)\ (2.5 \cos 30) = 0$$

$$\therefore \qquad \boxed{T_{BC} = 781.63\ \text{N}} \qquad \text{...Ans.}$$

$$\sum F_x = 0 :$$

$$A_x - T_{BC} \times \cos 36.87 = 0$$

$$\therefore \qquad A_x = 781.63 \times \cos 36.87$$

$$\therefore \qquad \boxed{A_x = 625.3\ \text{N} \rightarrow} \qquad \text{...Ans.}$$

$$\sum F_y = 0 :$$

$$A_y - 800 - 350 + T_{BC} \times \sin 36.87 = 0$$

$$\therefore \qquad \boxed{A_y = 774.82\ \text{N} \uparrow} \qquad \text{...Ans.}$$

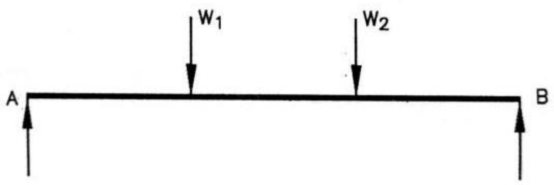
❖ **Types of load**

1) Point load
2) Uniformly distributed load
3) Uniformly varying load

1) Point load

• Load concentrated on a very small length compare to the length of the beam, is known as point load or concentrated load. Point load may have any direction.

• For example truck transferring entire load of truck at 4 point of contact to the bridge is point load.

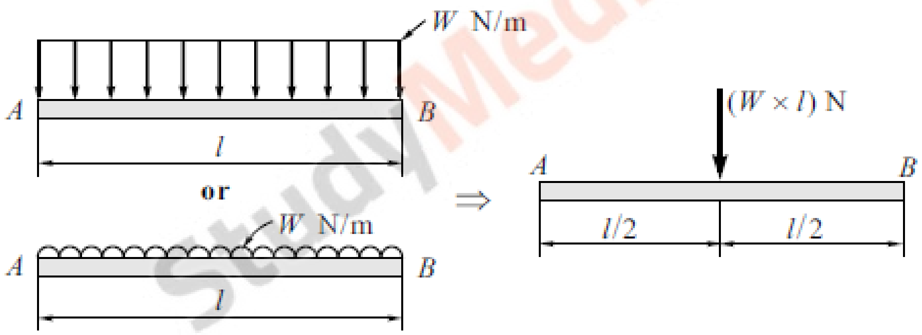


2) Uniformly distributed load

• Load spread uniformly over the length of the beam is known as uniformly distributed load.

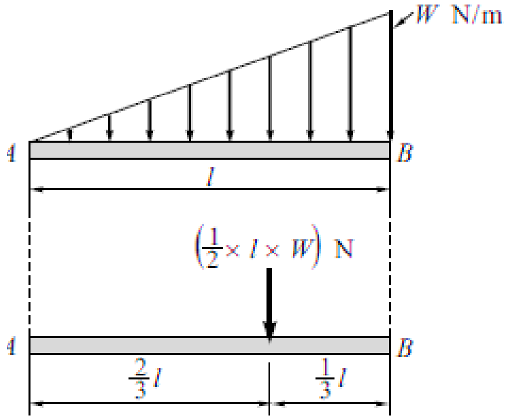
• Water tank resting on the beam length

• Pipe full of water in which weight of the load per unit length is constant.



3) Uniformly varying load

Load in which value of the load spread over the length if uniformly increasing or decreasing from one end to the other is known as uniformly varying load. It is also called triangular load.

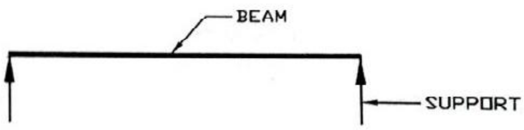


❖ **Beam :**

In a structure, horizontal member which takes transverse load in addition to other loading is called beam. In engineering structures like bridges, beam is one of the important structural member. In trusses and frames, pin-joined members take only tensile or compressive load. Beam is capable to take all types of load, i.e., transverse load, tensile load, compressive load, twisting load, etc. Further beams may carry different types of transverse load such as point load, uniformly distributed load, uniformly varying load, etc.

Types Of Beam :

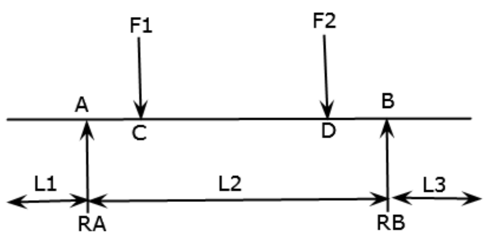
1) Simple supported Beam: If the both the ends of the beam are simply supported then it is called as Simple Supported Beam.



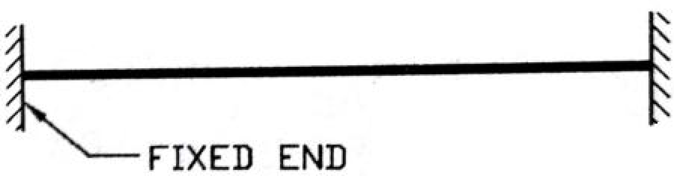
2. Cantilever Beam: If the beam is fixed at one end and free at other end, then it is called as cantilever Beam.

3. Continuous Beam: If the beam is supported at more than two points then it is called as continuous Beam.

4. Overhanging Beam: If the end portion of the beam is extended beyond the support then it is called as overhanging Beam.

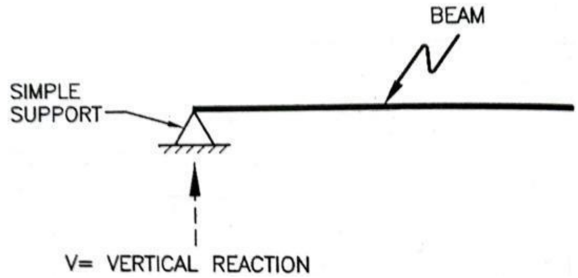


5. Fixed Beam: If the both ends beam are fixed in the wall then it is called as fixed or constrained or built in Beam.

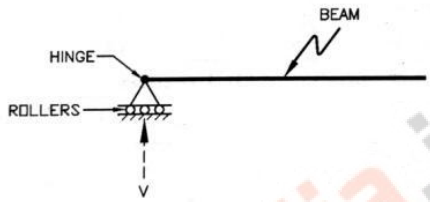


❖ **Type of support**

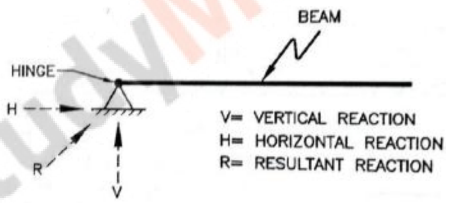
1) **Simple support :** In this type of support beam is simply supported on the support. There is no connection between beam and support. Only vertical reaction will be produced.



2) **Roller support :** Here rollers are placed below beam and beam can slide over the rollers. Reaction will be perpendicular to the surface on which rollers are supported.
This type of support is normally provided at the end of a bridge.

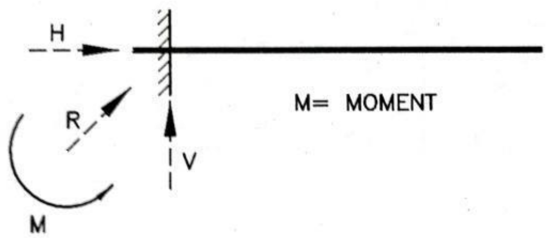


3) **Hinged support :** Beam and support are connected by a hinge.Beam can rotate about the hinge Reaction may be vertical, horizontal or inclined.

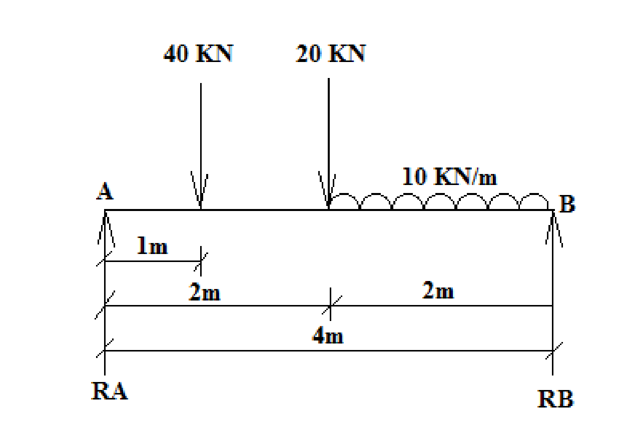


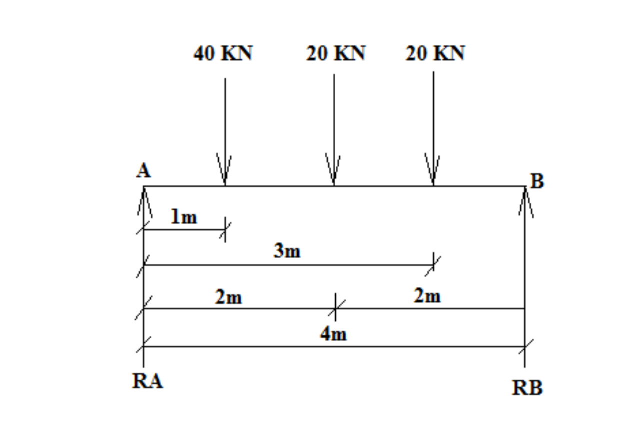
Fixed support

• Beam is completely fixed at end in the wall or support. Beam cannot rotate at end. Reactions may be vertical, horizontal, inclined and moment.



1: A beam of span 4 m is simply supported at it's end. It carries a concentrated loads of 40 KN & 20 KN at 1 m & 2 m from left hand support respectively. It carries udl of 10 kN/m for 2 m from the right end. Determine the reactions at support.



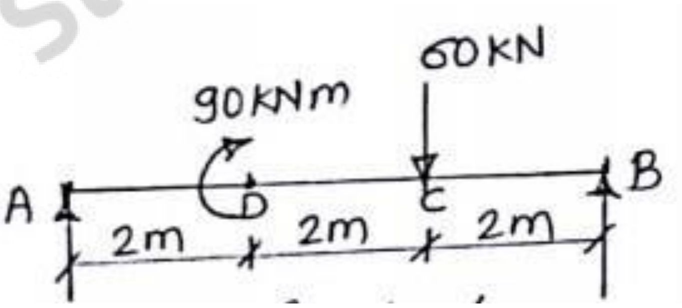


Solution:

Applying equilibrium conditions

Step 1 : Taking Moment at LHS A (↻+ve , ↺−ve) Taking moment of all forces @ point A (RA x 0) + (40 X 1) + (20 X 2) + (20 X 3) – (RB X 4) = 0 **RB = 35 KN**	**Step 2 : Summation of vertical force :** Σ Fy = 0 RA – 40 – 20 - 20 + RB = 0 RA + RB = 80 KN -------(1) Putting value of RB in eqn. (1) RA + 35 = 80 **RA = 45 KN**

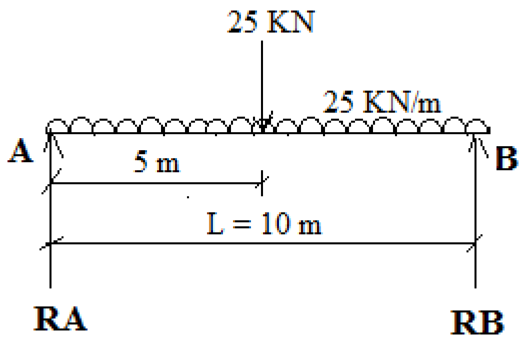
2: Calculate the reaction of beam loaded as shown in Figure No.

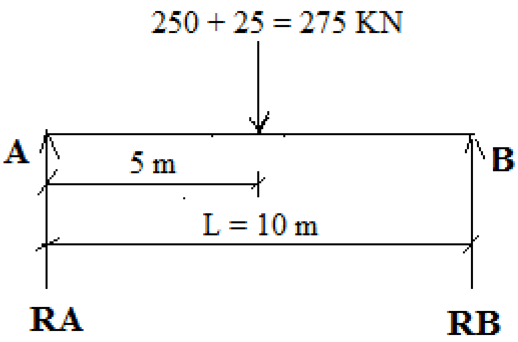


Applying equilibrium conditions

Step 1 : Taking Moment at LHS A. (↻+ve , ↺−ve) $\Sigma M_A = 0$ 90 +(60 x 4) – (R_B x 6) = 0 0 + 90 + 240 – 6R_B = 0 **R_B = 55 kN**	**Step 2 : Summation of vertical force :** ↑ (+Ve) ↓ (-Ve) ΣFy = 0 R_A – 60 + R_B = 0 R_A – 60 + 55 = 0 **R_A = 5 kN**

3: A simply supported beam of span 10 m carries a centre load of 25 KN & a udl of 25 KN/m throughout. Find support reaction.

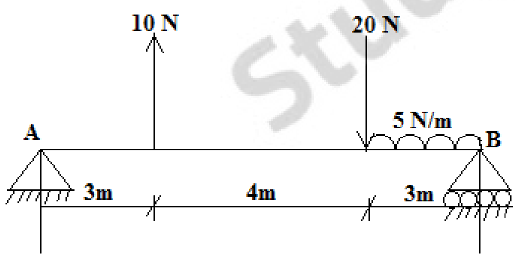


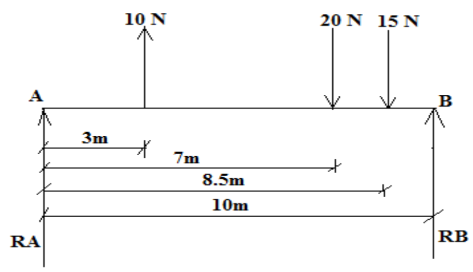


Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS A	**Step 2 : Summation of vertical force :**
(↻+ve , ↺−ve) $\Sigma M_A = 0$ Taking moment of all forces @ point A (275 X 5) – (RB X 10) = 0 1375 = 10 RB RB = 137.5 KN	↑ **(+Ve)** ↓ **(-Ve)** $\Sigma Fy = 0$ RA – 275 + RB = 0 RA + RB = 275 KN -------(1)

4: Find the support reactions of simply supported beam shown in Figure.

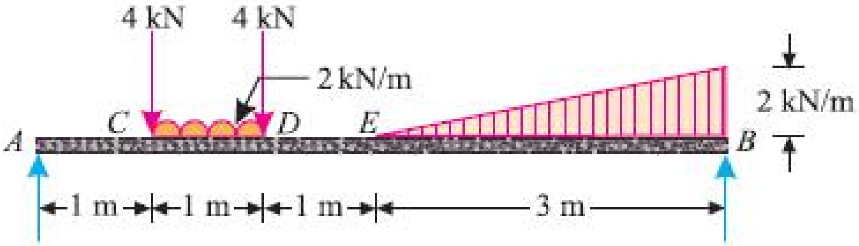




Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS A	**Step 2 : Summation of vertical force :**
(↻+ve , ↺−ve) $\Sigma M_A = 0$ Taking moment of all forces @ point A - (10 X 3) + (20 X 7) + (15 X 8.5) – (RB X 10) = 0 **RB = 23.75 N**	↑ **(+Ve)** ↓ **(-Ve)** $\Sigma Fy = 0$ RA + 10 – 20 - 15 + RB = 0 RA + RB = 25 N -------(1) Putting value of RB in eqn. (1) RA + 23.75 = 25 **RA = 1.25 N**

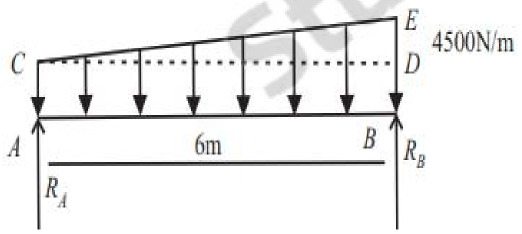
5: A simply supported beam AB of 6 m span is subjected to loading as shown in Fig .

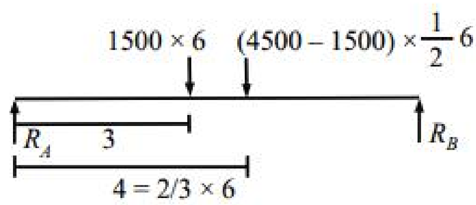


Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS A (↻+ve , ↺−ve) Σ MA = 0 Taking moment of all forces @ point A (4X1) + (2 X 1.5) + (4 X 2) + (3 X 5) - (RB X 6) = 0 **RB = 5 KN**	**Step 2 : Summation of vertical force :** **↑ (+Ve) ↓ (-Ve)** Σ Fy = 0 RA + RB - 4- 2- 4- 3 = 0 RA + RB = 13 -------(1) Putting value of RB in eqn. (1) RA + 5 = 13 **RA = 8 KN**

6: Determine the reactions at supports of simply supported beam of 6m span carrying increasing load of 1500N/m to 4500N/m from one end to other end.

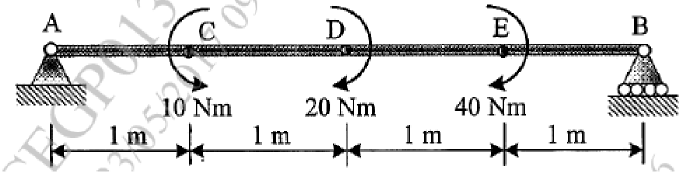




Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS A (↻+ve , ↺−ve) Σ MA = 0 Taking moment of all forces @ point A (9000X3) + (9000X4) - (RB X 6) = 0 **RB = 10500N**	**Step 2 : Summation of vertical force :** **↑ (+Ve) ↓ (-Ve)** Σ Fy = 0 RA + RB -9000-9000 = 0 RA + RB = 18000 -------(1) Putting value of RB in eqn. (1) RA + 10500 = 18000 **RA = 7500 N**

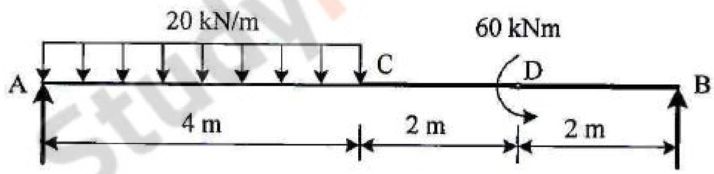
7. Determine reaction at A and B for the beam loaded and supported as Shown in fig .Moments are act at point C, D and E



Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS A (↺+ve , ↻−ve) $\Sigma M_A = 0$ Taking moment of all forces @ point A -10 +20 + 40 - (RB X 4) = 0 **R_B = 12.5 N**	**Step 2 : Summation of vertical force :** ↑ (+Ve) ↓ (-Ve) $\Sigma F_y = 0$ $R_A + R_B = 0$ $R_A + 12.5 = 0$ **R_A = -12.5 N**

8. Determine the support reaction for the beam loaded and supported as shown in fig:

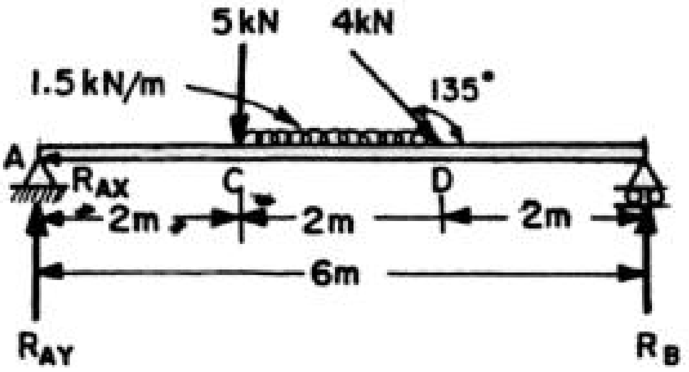


Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS A (↺+ve , ↻−ve) $\Sigma M_A = 0$ Taking moment of all forces @ point A (80X2) - 60 - (RB X 8) = 0 **R_B = 12.5 KN**	**Step 2 : Summation of vertical force :** ↑ (+Ve) ↓ (-Ve) $\Sigma F_y = 0$ RA + RB – 80 = 0 R_A + 12.5 - 80 = 0 **R_A = 67.5 KN**

9. A beam AB 6m long is loaded as shown in fig. Determine the reaction at A and B by analytical method.

Given :



The reaction R_A will be inclined as the beam is hinged at A and carries inclined load.

Let R_{AX} = Horizontal component of reaction R_A

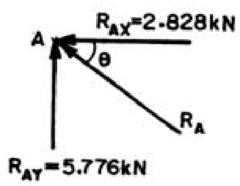
R_{AY} = Vertical Component Of reaction R_A

First resolve the inclined load 4 KN into horizontal and vertical components

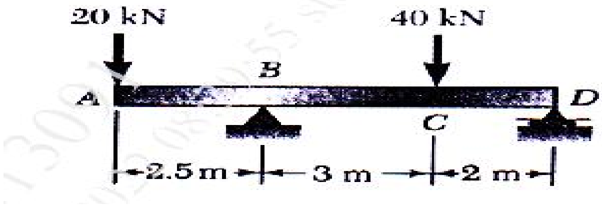
Horizontal Component of 4 KN at D = 4 cos45 = 2.828 KN →

Vertical Component of 4 KN at D = 4 sin 45 = 2.828 KN ↓

Applying equilibrium conditions

Step 1 : Taking Moment at LHS A (↺+ve , ↻−ve) $\Sigma M_A = 0$ Taking moment of all forces @ point A (5X2) + (4sin45 X4) +(1.5 X 2 X 3) – (R_B X 6) = 0 **R_B = 5.052 KN**	**Step 2 : Summation of vertical force :** **↑ (+Ve) ↓ (-Ve)** $\Sigma Fy = 0$ R_{AY} + 5.052 - 2.828 - 3 - 5 = 0 **R_{AY} = 5.776 KN** **Step 3: Summation of horizontal force: → (+Ve) ←(-Ve)** $\Sigma Fx = 0$ - R_{AX} - 2.828 = 0 **R_{AX} = 2.828 KN**
Step4: Reaction at A is Given by : **$R_A = \sqrt{RAX^2 + RAY^2}$ = 6.43 KN** **Let Θ is angle made by R_A with x direction** **tanΘ = (R_{AY} / R_{AX}) = 2.024** **Θ = 63.9°**	

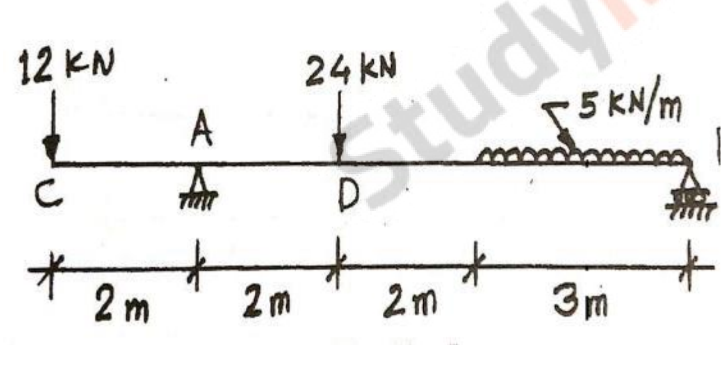
10.The I joist supports 20 kN and 40 kN on bean AB of span 7.5 m, as shown in Fig. 2c. Determine the support reactions at hinge B and roller D.

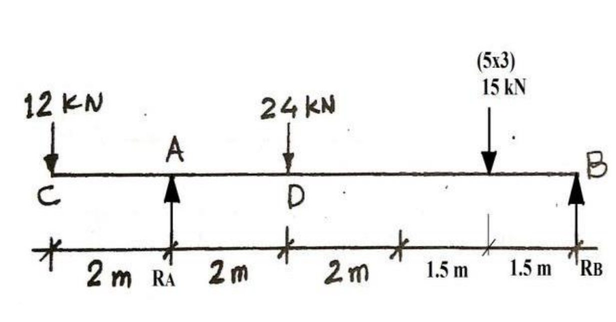


Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS B (+ve , −ve) $\Sigma M_A = 0$ Taking moment of all forces @ point A - (20X2.5) + (40X3) - (RD X 5) = 0 **R_B = 14 KN**	**Step 2 : Summation of vertical force :** **↑ (+Ve) ↓ (-Ve)** $\Sigma Fy = 0$ RA + RB – 20 - 40 = 0 R_A + 14 - 60 = 0 **R_A = 46 KN**

11.Calculate the support reactions of beam loaded as shown in figure No.

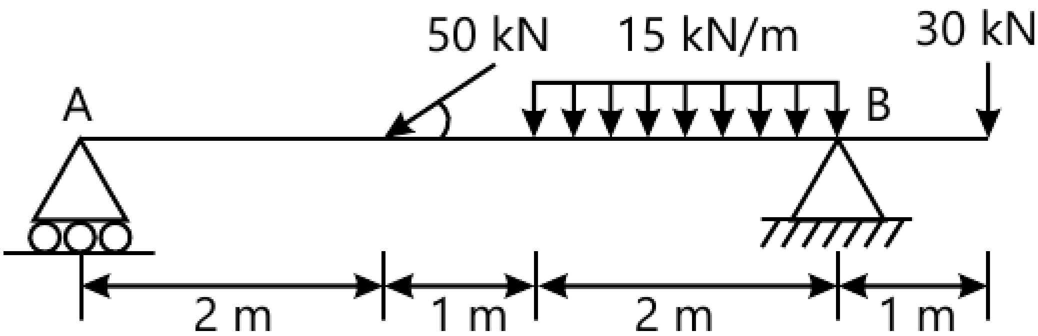




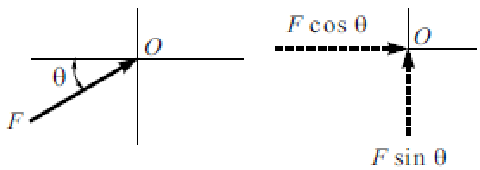
Solution:
Applying equilibrium conditions

Step 1 : Taking Moment at LHS A (+ve , −ve) $\Sigma M_A = 0$ Taking moment of all forces @ point A - (12 x 2) + (R_A x 0) + (24 x 2) + (15 x 5.5) – (R_B x 7) = 0 **R_B = 15.214 kN**	**Step 2 : Summation of vertical force :** **↑ (+Ve) ↓ (-Ve)** $\Sigma Fy = 0$ R_A – 12 – 24 – 15 + R_B = 0 R_A – 51 + 15.214 = = 0 **R_A = 35.786 kN**

12. **Determine the support reaction for the beam loaded and supported as shown in Fig. 50 kN force is inclined at 30° to horizontal..**

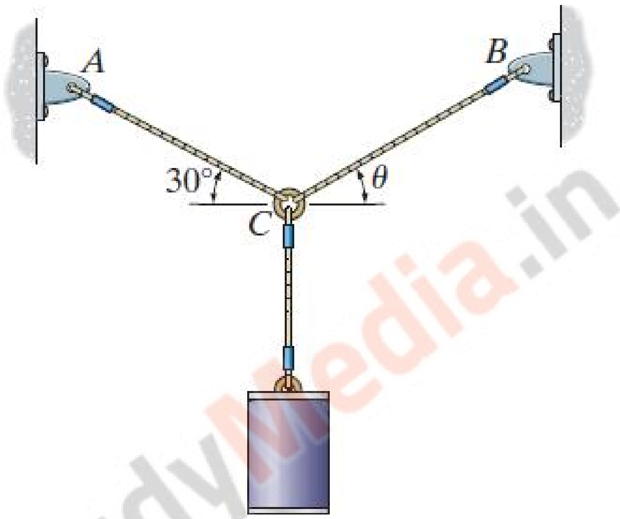


Solution:
Applying equilibrium conditions

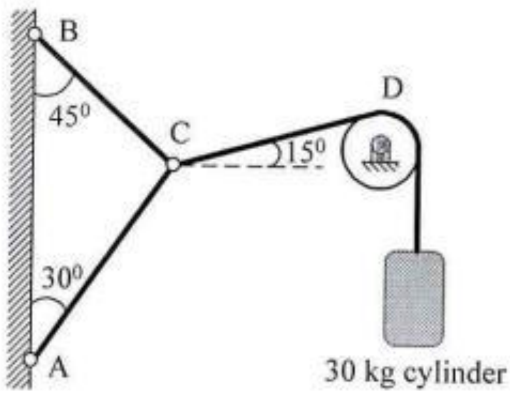
Step 1 : Taking Moment at RHS B (↻+ve , ↺−ve) $\Sigma M_B = 0$ Taking moment of all forces @ point B $(50 \text{ Sin}30 \times 3) - (30 \times 1) + (30 \times 1) - (R_A \times 5) = 0$ $\mathbf{R_A = 15\ KN}$	**Step 2 : Summation of vertical force :** ↑ (+Ve) ↓ (-Ve) $\Sigma Fy = 0$ $\mathbf{R_{B\,y} + 15 - 50 \sin 30 - 30 - 30} = 0$ $\mathbf{R_{B\,y} = 70\ KN}$ **Step 3: Summation of horizontal force:** → (+Ve) ←(-Ve) $\Sigma Fx = 0$ $\mathbf{R_{B\,x} - 50 \cos 30 = 0}$ $\mathbf{A_x = -\ 43.3\ KN}$
Step4: Reaction at A is Given by : $\mathbf{R_A = \sqrt{RB\,x^{2} + RB\,y^{2}} = 82.31\ KN}$ **Let ϴ is angle made by R_A with x direction** $\mathbf{\tan\Theta = (R_{BY} / R_{BX}) = 70 / 43.3}$ $\mathbf{\Theta = 58.26^0}$	

Question Bank

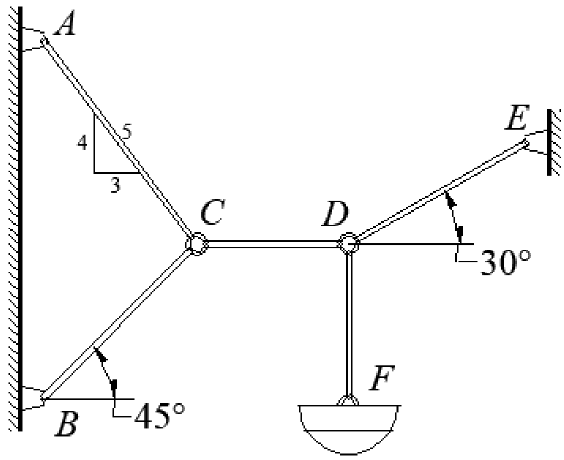
1. State the condition of equilibrium for coplanar concurrent forces
2. Explain the concept of Free body diagram (FBD) giving example.
3. State and explain Lami's Theorem.
4. Draw and explain with neat sketches of various types of Beam
5. Draw neat sketches of the various types of supports used in engineering mechanics and mention the number of unknown reaction components for each case.
6. Explain how Uniformly distributed load (UDL) and Uniformly Varying load is converted into a point load with sketch
7. Determine the tension developed in wires CA and CB required for equilibrium of the 10kg cylinder as shown in Figure. Take ϴ = 40°.



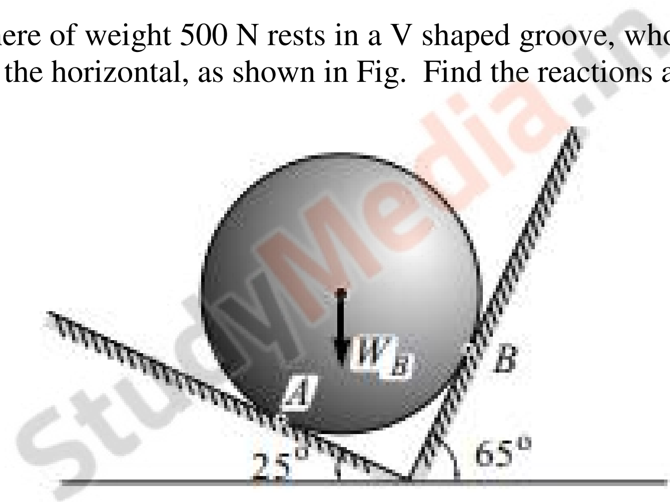
8. Three cables are joined at the junction C as Shown In Fig . Determine the tension in cable AC and BC caused by the weight of the 30Kg cylinder.



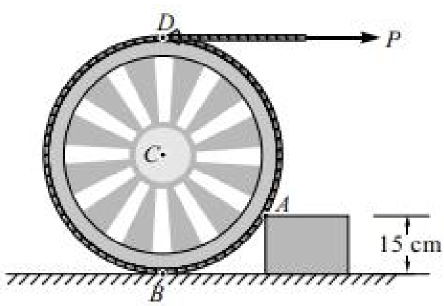
9. Knowing that lamp attached at D is, $m_f = 20$ kg, determine the tension in each segment of the cable loaded and supported as shown in Fig



10. A smooth sphere of weight 500 N rests in a V shaped groove, whose sides are inclined at 25° and 65° to the horizontal, as shown in Fig. Find the reactions at A and B



11. A uniform wheel of 60 cm diameter and weighing 1000 N rest against a rectangular block 15 cm high lying on a horizontal plane as shown in Fig. . It is to be pulled over the block by a horizontal force P applied to the end of a string wound round the circumference of the wheel. Find force P when the wheel is just about to roll over the block.



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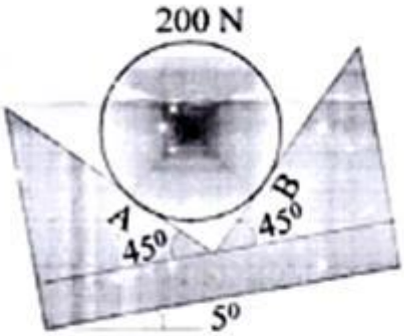
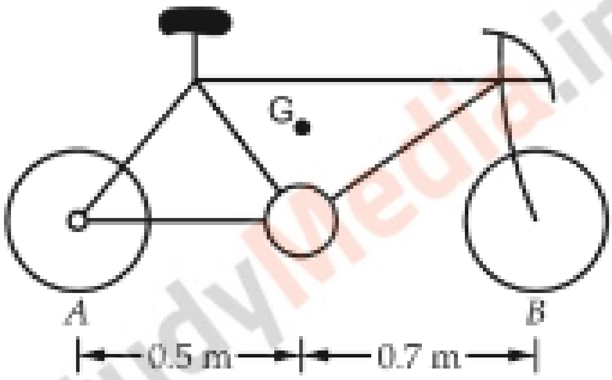
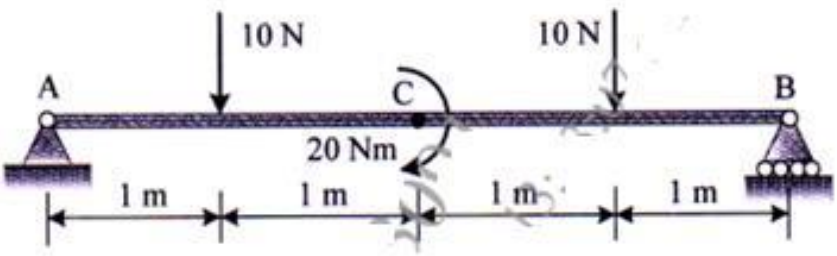
12.Find the reaction exerted at A and B on the sphere of 200 N kept in a trough as shown in Fig.



13. The weight of the cycle is 500 N which act at center of gravity G as shown in Fig. 1 a. Determine the normal reaction at A and B when the cycle is in equilibrium.



14 . Find support reaction at A and B for the beam AB as shown in Fig.



15 : The lever BCD is hinged at C and is attached to a Control rod at B .If P= 200N . Determine i) The tension in rod AB ii) The reaction at C.