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"Techno-Social Excellence"
Marthwada Mitramandal's Institute of Technology (MMIT)
Lohgaon, Pune-47

LECTURE NOTES

Faculty :- Mukesh M Sharma

Subject :- Engineering Mathematics - I

Reference for Topic:-

Name of the Book/paper/Website/Other	Author Name	Page No/Link
Heigher Engineering	B.S. Grewal	
Mathematics		
Heigher Engineering	B.V. Ramana	
Mathematics		

Topic Name :- 5] Linear Algebra - Matrices, System of Linear Equations :-

* Rank of Matrix :-

A number r is said to be rank of a matrix A if

a) There exists at least one non-zero minor of order r and

b) Every Minor of order greater that r is zero.

It is denoted by $\rho(A)$.

Sharma M.M
Subject Teacher

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e.g. Rank of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is 2 as $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$

Rank of $\begin{bmatrix} 3 & 2 \\ 4 & 5 \\ 0 & 1 \end{bmatrix}$ is 2 as $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 7 \neq 0$

Rank of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is 2 as $\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = -3 \neq 0$

and $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = 0$

Notes :-

1] The Rank of null Matrix is zero.

2] The Rank of Identity Matrix of order n is n.

3] The Rank of Non-singular matrix A of order n is n.

4] If $A = [a_{ij}]_{m \times n}$ then

$$\text{Rank}(A) = \rho(A) \leq \min\{m, n\}.$$

* Nullity of a Matrix [$n(A)$] :-

let A be any matrix of order $m \times n$

then nullity of $A = m - \rho(A)$

i.e. Nullity of A = No. of Rows of A − Rank of a Matrix A.

It is denoted by $n(A)$.

Thus, $n(A) + \rho(A) =$ Number of Rows of A.

* Elementary transformations :-

A] Elementary row transformations :-

These are totol 3 elementary row transformations :-

i] Interchange of elements of ith row with elements of jth row. It is denoted by R_{ij}.

ii] The multiplication of ith row by a non zero scalar K. It is denoted by KR_i.

iii] Addition of K times elements of jth row in the corresponding element of ith row. It is denoted by $R_i + KR_j$ where K is non-zero scalar.

B] Elementary column transformations :-

i] C_{ij}, Interchange of ith and jth column of a matrix

ii] KC_i, Multiplication of ith column of a matrix with a non zero scalar K.

iii] $C_i + KC_j$, Addition of K times the elements of column j in the corresponding element of ith column.

C] Equivalent matrices :-

If A is any matrix, then a matrix B is called as a matrix equivalent of A if matrix B is obtained from matrix A by applying a finite number of elementary operations. It is denoted by $A \sim B$.

D] Elementary Matrix :-

A Matrix is said to be elementary matrix if it is obtained from Identity matrix by applying only one elementary operation.

* Note :- Elementary transformation does not change rank of a Matrix.

i.e If $A \sim B$ then $\rho(A) = \rho(B)$

Also Rank of elementary matrix of order n is n.

* Submatrix :- A Matrix obtained from given matrix by deleting either a row, or a column or both or none is called as submatrix of given matrix.
* Minor of a Matrix :- The determinant of a square submatrix of given matrix is called a minor of a matrix.

* Row echelon form of a matrix or Echelon form or Cannonical form :-

A Matrix having following properties is said to be in row echelon form if,

i] If there are any zero rows, then they are grouped together at the bottom of the matrix.

ii] In any two successive non zero rows, the leading element (i.e. the first non-zero element of that row) in the lower row occur to the right than the leading element in the heigher row.

e.g. $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 5 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

Then both A and B are in Row echelon form.

$$C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 10 \\ 1 & 5 & 6 \end{bmatrix}, \quad D = \begin{bmatrix} 6 & 2 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

then matrix C is not in row echelon form as a_{31} and a_{32} are non zero also matrix D is not in row echelon form as zero row is not at bottom.

* Working Rule to find Row echelon form of a Matrix :-

1] If entry a_{11} is non-zero then go to next step. If $a_{11}=0$ then look for the non-zero entry in the first column. If there is a non-zero entry in the i^{th} row (i.e. if $a_{i1} \neq 0$) then use R_{1i} operation to get non zero entry at a_{11}.

[Note :- If all the entries of first column are 0, then go to a_{12} and repeat step 1]

2] After getting $a_{11} \neq 0$, make all the other entries in first column equal 0 using row operations.

3] Now go to a_{22}, check weather $a_{22} \neq 0$, if $a_{22} \neq 0$ go to step 3 but if $a_{22}=0$ then look for the non zero entry in the second column below a_{22}. If there is a non zero entry in second column below a_{22} say it is in j^{th} row then use R_{2j} to get non zero entry at a_{22}. If all the entries in the second column below a_{22} are zero then go to a_{32} and repeat the same process of step 2.

3] After doing step 2] go to a_{33} and repeat the same process of step 2.

4] Keep repeating above process untill the matrix gets reduced into row echelon form.

Q.1] Find the row echelon form of the matrix,

$$A = \begin{bmatrix} 2 & 6 & 7 & 8 \\ 2 & 7 & 8 & 9 \\ 4 & 1 & 1 & 1 \end{bmatrix}$$

⇒ Here,

$$A = \begin{bmatrix} 2 & 6 & 7 & 8 \\ 2 & 7 & 8 & 9 \\ 4 & 1 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3 - 2R_1]{R_2 - R_1} \begin{bmatrix} 2 & 6 & 7 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & -11 & -13 & -15 \end{bmatrix}$$

$$\xrightarrow{R_3+11R_2} \begin{bmatrix} 2 & 6 & 7 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

which is the row echelon form of Matrix A.

* Note :-
The number of non zero rows in a row echelon form of a matrix A gives the rank of a matrix.

$\therefore$ $\rho(A)$ = No. of non-zero rows in row echelon form of matrix A.

Also, $\rho(A)$ = No. of Rows of A − No. of zero rows in Raw echelon form of A.

So, rank of A in above example is equal to 3.

* Normal form of a Matrix :-

By performing elementary transformations, we can reduce any nonzero matrix to one of the following forms of that matrix.

$$[I_r \; 0] , \begin{bmatrix} I_r \\ 0 \end{bmatrix} , [I_r], \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

where I_r denotes identity Matrix of order r and 0 denotes null matrix of suitable order. Here, the rank of matrix A is the order of the identity matrix in normal form of A.

* Procedure to find Normal form :-

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

Step 1] Get $a_{11} = 1$ by using elementary transformations.

$\therefore$ we get, $\begin{bmatrix} 1 & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$

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Step 2 :- Get zeros below $a_{11} = 1$ using row operations and R_1 only.

$\therefore$ we get, $\begin{bmatrix} 1 & * & * \\ 0 & * & * \\ 0 & * & * \end{bmatrix}$

Step 3 :- Get zeros to the right of 1 by using column operations and C_1 only.

$\therefore$ we get $\begin{bmatrix} 1 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{bmatrix}$

Step 4 :- Get $a_{22} = 1$ (If possible as for $\begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix}$ it will not be possible) by using elementary transformations and R_2, R_3 or C_2, C_3 without disturbing R_1 and C_1, we get

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & * \\ 0 & * & * \end{bmatrix}$

Step 5] Get zeros below $a_{22} = 1$ and and to right by using elementary transformation and so on.

Q. 1] Reduce the matrix to its normal form and hence find its rank.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$$

$\Rightarrow$ Here, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

$$\xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & -5 & 0 \end{bmatrix}$$

$$\xrightarrow[C_3 - C_1]{C_2 - C_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 2 \\ 0 & -5 & 0 \end{bmatrix}$$

$$\xrightarrow{R_{23}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & -5 & 2 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{5} R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 2 \end{bmatrix}$$

$$\xrightarrow{R_3 + 5R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{\frac{1}{2} R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore A \sim [I_3]$

So, Rank of matrix A is 3.

* PAQ form of a Matrix :-

If A is a given matrix then, ∃ two non singular Matrices P and Q such that PAQ is in normal form

i.e. Normal form of matrix A = PAQ

* Procedure to find two non singular Matrices P and Q such that PAQ is normal form :-

If $A_{m\times n}$ is given matrix then write $A_{m\times n} = I_m A_{m\times n} I_n$ —(i)

Apply elementary row and column on both sides of eq.^{n} (i) to get matrix $A_{m\times n}$ on L.H.S. in its normal form.

while applying row operations apply it on I_m matrix only on R.H.S. and while applying column operations apply it on I_n matrix only on R.H.S.

At the end we get,

Normal form of A = PAQ

i.e. consider,

$$A_{m\times n} = I_m \quad A \quad I_n$$

$A_{m\times n}$ ↓ Apply row and column operations to reduce it to normal form of A

I_m ↓ Apply same row operations only

I_n ↓ Apply same column operations only,

i.e. Normal form of A $= P_m \quad A \quad Q_n$

* Note :-

1] If A is non singular square Matrix then $A^{-1} = QP$.

2] To find row echelon form, we use only row operations, but to Normal form and PAQ form of a matrix, we can use both row as well as column operations.

Q. 1] Find non singular Matrices P and Q such that $PAQ = I$, where I is unit matrix and hence find A^{-1} for the matrix

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Higher Engineering Mathematics	B.V. Ramana	
Higher Engineering Mathematics	S.B. Grewal	

Topic Name :- System of Linear Equations

* Linear equation :-

An equation of the form

$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$

where $a_1, a_2, \dots, a_n$ are constants and $x_1, x_2, \dots, x_n$ are variables, is called a linear equation.

e.g. $x_1 - 2x_2 + 3x_3 = 16$, $\sqrt{2}x_1 + 5x_2 = 7$ are linear equations. and $x_1 + \sqrt{x_2} = 1$, $\sin x_1 + x_2 = 5$, $e^{-x_1} + 5x_2 = 7$ are non linear equations.

Sharma M.M.

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* Solution of linear equation :-

A solution of a linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ is a sequence of numbers $u_1, u_2, \ldots, u_n$ such that the equations is satisfied when we put $x_1 = u_1, x_2 = u_2, \ldots, x_n = u_n$.

i.e. $a_1u_1 + a_2u_2 + \cdots + a_nu_n = b$

* System of linear equations :-

consider the below system of equations.

$$\begin{matrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{matrix}$$

is called system of m linear equations in n unknowns.

In Matrix form it is given by,

$AX = B$ where,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

$\uparrow$ Matrix of coefficients $\uparrow$ Matrix of constents. $\uparrow$ Matrix of unknowns

The Augumented matrix $[A : B]$ of above system is given by,

$$[A : B] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

* Non - Homogeneous linear system :-

A linear system $AX = B$ is said to be non homogeneous system if matrix B is non zero or non null Matrix.

eg. $\begin{aligned} 2x + 5y &= 3 \\ 4x + 6y &= 7 \end{aligned}$, $\begin{aligned} 3x + 6y + 9z &= 13 \\ 5x + 7y + 9z &= 0 \\ 7x + 9y + 11z &= 0 \end{aligned}$

are non homogeneous systems.

* Homogeneous system :-

A linear system $AX = B$ is said to be homogeneous system if B is a null matrix or zero matrix i.e, $AX = 0$ is a homogeneous system

eg. $2x + 3y = 0$, $5x + 6y = 0$, $3x + 4y + 5z = 0$, $4x + 6y + 7z = 0$, $8x + 9y + 10z = 0$

$$\begin{matrix} 2x+3y=0 \\ 5x+6y=0 \end{matrix} \quad , \quad \begin{matrix} 3x+4y+5z=0 \\ 4x+6y+7z=0 \\ 8x+9y+10z=0 \end{matrix}$$

are homogeneous system.

* Consistant and Non consistant systems:-

A linear system is said to be consistant if it has solution and it is said to be non consistant if it does not have any solution.

* Consistency conditions :-

1) Consistency of Non homogeneous system:-

A Non homogeneous linear system is consistant if and only if $\rho(A) = \rho(A:B)$

It has unique solution if

$\rho(A) = \rho(A:B) =$ No. of unknowns and

It has infinitly many solutions if

$\rho(A) = \rho(A:B) <$ No. of unknowns.

* Inconsistency of Non-homogeneous system:-

A Non-homogeneous system is inconsistent if $\rho(A) \neq \rho(A:B)$.

* Consistancy of homogeneous system:-

A homogeneous system is always consistant as $\rho(A) = \rho(A:B)$ for any homogeneous system.

A homogeous system has unique solution (which is trivial solution i.e. $x_1 = x_2 = \dots = x_n = 0$) if

$\rho(A) = \rho(A:B) =$ No. of unknowns.

Also a homogeneous system has infinitly many solutions if,

$\rho(A) = \rho(A:B) <$ No. of unknowns.

* Type I :- Non-Homogeneous system of Linear equation having unique solution:-

Q.1] Test for consistency and solve if found consistant.

$x - y - z = 2$, $x + 2y + z = 2$

$4x - 7y - 5z = 2$

$\Rightarrow$ In Matrix form above system is given by,

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \\ 4 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

so, Augumented Matrix is, $[A:B]$

$$\therefore [A:B] = \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 1 & 2 & 1 & 2 \\ 4 & -7 & -5 & 2 \end{array}\right]$$

$$\xrightarrow[R_3 - 4R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 3 & 2 & 0 \\ 0 & -3 & -1 & -6 \end{array}\right]$$

$$\xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & -6 \end{array}\right] \quad \text{--- (i)}$$

which is a row echelon form

Hence, $\rho(A) = \rho(A:B) = 3$ so given system is consistent.

Now, From Eq. (1) R_3 gives, $z = -6$

Now, $z = -6$ together with R_2 gives,

$3y + 2(-6) = 0 \Rightarrow y = 4$

Now, $z = -6$, $y = 4$ together with R_1

gives, $x - (4) - (-6) = 2 \Rightarrow x = 0$

$\therefore$ $x = 0$, $y = 4$ and $z = -6$ is unique solution of given system of linear equations.

* Type 2:- Non homogeneous system having infinitly many solutions:-

Q.1] Examine for consistency and if found consistant solve it.

$2x - 3y + 5z = 1$, $3x + y - z = 2$, $x + 4y - 6z = 1$.

$\Rightarrow$ Above system in Matrix form is given by,

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 1 & -1 \\ 1 & 4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

So, Augumented Matrix is given by,

$$[A:B] = \left[\begin{array}{ccc:c} 2 & -3 & 5 & 1 \\ 3 & 1 & -1 & 2 \\ 1 & 4 & -6 & 1 \end{array}\right]$$

$$\xrightarrow{R_{13}} \left[\begin{array}{ccc:c} 1 & 4 & -6 & 1 \\ 3 & 1 & -1 & 2 \\ 2 & -3 & 5 & 1 \end{array}\right]$$

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$$\xrightarrow[R_3-2R_1]{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & -11 & 17 & -1 \end{array}\right]$$

$$\xrightarrow{R_3-R_2} \left[\begin{array}{ccc|c} 1 & 4 & -6 & 1 \\ 0 & -11 & 17 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{— (ii)}$$

which is in Row echelon form

$\therefore \rho(A) = \rho(A:B) = 2 < \text{No. of unknowns} = 3$.

$\therefore$ Given system is consistent and has infinitely many solutions.

Now, R_2 from eq. (i) gives,

$$-11y + 17z = -1$$

Since this is single equation in two variables, we have to substitute to get the solution so let $z = t$, then,

$$-11y + 17t = -1$$

$$\Rightarrow \boxed{y = \frac{17t+1}{11}}$$

Now, R_1 from eq. (i) together with $z = t$ and $y = \frac{17t+1}{11}$ gives,

$$x + 4\left(\frac{17t+1}{11}\right) - 6t = 1 \Rightarrow \boxed{x = \frac{7-2t}{11}}$$

So, $x = \frac{7-2t}{11}$, $y = \frac{17t+1}{11}$ and $z = t$ are the infinitely many solutions of given system of equations as values of x, y and z depends on t and $t \in \mathbb{R}$ (set of real numbers) so t can have any value and accordingly we get different values of x, y and z.

*. Type 3 :- Non homogeneous Linear systems involving parameter k or λ or μ or a or b :-

Q.1] For which value of λ and k the following system have

i] No solution ii] Unique solution

iii] Infinitely many solutions.

$x+y+z=6$, $x+2y+3z=10$,

$x+2y+kz=\lambda$

$\Rightarrow$ The given system in Matrix form is given by,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \lambda \end{bmatrix}$$

So, Augumented matrix $[A:B]$ is given by,

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & K & \lambda \end{array}\right]$$

$$\xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & K-1 & \lambda-6 \end{array}\right]$$

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & K-3 & \lambda-10 \end{array}\right]$$

i] No solution :- Given system has no solution if and only if $\rho(A) \neq \rho(A:B)$

If $K=3$ and $\lambda \neq 10$ then $\rho(A) = 2$ and $\rho(A:B) = 3$, so $\rho(A) \neq \rho(A:B)$

So, Linear system has no solution if $K=3$ and $\lambda \neq 10$.

ii] Unique solution :- Given system has unique solution if $\rho(A) = \rho(A:B) = n$ where n is number of unknowns which is 3 in this example

$\therefore$ Given system has unique solution if $\rho(A) = \rho(A:B) = 3$

Also, $\rho(A) = \rho(A:B) = 3$ if $K \neq 3$ and any value of λ.

So, Given system has unique solution if $K \neq 3$ and any value of λ.

iii) Infinitely many solutions :-

Given system has infinitely many solutions if $\rho(A) = \rho(A:B) <$ No. of unknowns

i.e $\rho(A) = \rho(A:B) < 3$

Now if $K = 3$ and $\lambda = 10$ then

$\rho(A) = \rho(A:B) = 2 < 3$

So the system will have infinitely many solutions $K = 3$ and $\lambda = 10$.

Q.2] Show that the system

$$3x + 4y + 5z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$5x + 6y + 7z = \gamma$$

is consistent only when α, β, γ are in arithmatic progression

* Type IV] Homogeneous system having unique solution (i.e. trivial solution):-

Q.1] Solve the following system of linear equations $x+2y+3z=0$, $2x+3y+z=0$, $4x+5y+4z=0$, $x+y-2z=0$

⇒ Given system in Matrix form is given by,

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 5 & 4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So, Augumented matrix is [A : B]

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 2 & 3 & 1 & 0 \\ 4 & 5 & 4 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow[\substack{R_3-4R_1 \\ R_4-R_1}]{R_2-2R_1} \left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -3 & -8 & 0 \\ 0 & -1 & -5 & 0 \end{array}\right]$$

$$\xrightarrow[R_4 - R_2]{R_3 - 3R_2} \left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

which is in Row-echelon form

Hence $\rho(A) = \rho(A:B) = 3 =$ No. of unknowns

$\therefore$ Given system has unique solution which is trivial solution given by,

$x = 0$, $y = 0$ and $z = 0$.

* Type V] Homogeneous equation having infinitely many solutions:-

Q.1] Examine for non trivial solutions and solve,

$$4x - y + 2z + w = 0$$
$$2x + 3y - z - 2w = 0$$
$$7y - 4z - 5w = 0$$
$$2x - 11y + 7z + 8w = 0$$

$\Rightarrow$ Given system in Matrix form is given by,

$$\begin{bmatrix} 4 & -1 & 2 & 1 \\ 2 & 3 & -1 & -2 \\ 0 & 7 & -4 & -5 \\ 2 & -11 & 7 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So Augumented Matrix $[A:B]$ is given by,

$$[A:B] = \left[\begin{array}{cccc|c} 4 & -1 & 2 & 1 & 0 \\ 2 & 3 & -1 & -2 & 0 \\ 0 & 7 & -4 & -5 & 0 \\ 2 & -11 & 7 & 8 & 0 \end{array}\right]$$

$$\xrightarrow{R_{12}} \left[\begin{array}{cccc|c} 2 & 3 & -1 & -2 & 0 \\ 4 & -1 & 2 & 1 & 0 \\ 0 & 7 & -4 & -5 & 0 \\ 2 & -11 & 7 & 8 & 0 \end{array}\right]$$

$$\xrightarrow[R_4 - R_1]{R_2 - 2R_1} \left[\begin{array}{cccc|c} 2 & 3 & -1 & -2 & 0 \\ 0 & -7 & 4 & 5 & 0 \\ 0 & 7 & -4 & -5 & 0 \\ 0 & -14 & 8 & 10 & 0 \end{array}\right]$$

$$\xrightarrow[R_4 - 2R_2]{R_3 + R_2} \left[\begin{array}{cccc|c} 2 & 3 & -1 & -2 & 0 \\ 0 & -7 & 4 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \text{— (i)}$$

which is in row echelon form

Also, $\rho(A) = \rho(A:B) = 2 < n = \underset{\text{unknowns}}{\text{No. of}} = 4$

and hence it has $n - r = 4 - 2 = 2$ linearly independent solution

It has $n - r = 4 - 2 = 2$ free variables

Now R_2 of eq. (i) gives, $-7y + 4z + 5w = 0$

Since the above system has two free variables, let w and z be these free variables.

Then let $\boxed{z=a}$ and $\boxed{w=b}$, $a,b \in \mathbb{R}$

Then we get,

$$-7y+4z+5w=0$$

$$\Rightarrow -7y+4a+5b=0$$

$$\Rightarrow \boxed{y = \frac{4a+5b}{7}}$$

Also, R_1 of eq.(1) gives,

$$2x+3y-z-2w=0$$

$$\Rightarrow 2x = -3y+z+2w$$

$$\Rightarrow x = \frac{z+2w-3y}{2}$$

$$\Rightarrow x = \frac{a+2b-3\left(\frac{4a+5b}{7}\right)}{2}$$

$$\Rightarrow \boxed{x = \frac{-b-5a}{14}}$$

So, $x = \frac{-b-5a}{14}$, $y = \frac{4a+5b}{7}$, $z=a$ and $w=b$ are infinitely many solutions of given homogeneous system.

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Lohgaon, Pune-411047

LECTURE NOTES

Faculty :- Mukesh M Sharma

Subject :- Engineering Mathematics I

Reference for Topic:-

Name of the Book/paper/Website/Other	Author Name	Page No/Link
Higher Engineering Mathematics	B.S. Grewal	
Higher Engineering Mathematics	B.V. Ramana	

Topic Name :- Linear Dependence and Independence of vectors

* vector :- A column Matrix or Row matrix is called as a vector.

* Linearly Dependent vectors :-

Given vectors $X_1, X_2, \ldots, X_n$ are said to be linearly independent vectors if there exists scalars $K_1, K_2, \ldots, K_n$ not all zero, such that,

$$K_1 X_1 + K_2 X_2 + \ldots + K_n X_n = 0$$

Sharma M.M

Subject Teacher

* Linearly independent vectors :-

Given vector $x_1, x_2, \ldots, x_n$ are said to linearly independent if

$k_1x_1 + k_2x_2 + \ldots + k_nx_n = 0$ implies

$k_1 = k_2 = \ldots = k_n = 0$.

i.e. $k_1x_1 + k_2x_2 + \ldots + k_nx_n = 0$ only when $k_1 = k_2 = \ldots = k_n = 0$.

* Note :- If any x_i is zero vectors in given vectors then the given vectors are linearly dependent.

* Procedure to check linear dependence and independence of vectors, $x_1, x_2, \ldots, x_n$:-

1] consider the equation

$k_1x_1 + k_2x_2 + \ldots + k_nx_n = 0$

2] Above equation gives us system of homogeneous linear equations.

3] solve the above homogeneous system of linear equations, to get the values of $k_1, k_2, \ldots, k_n$. If all k_i's are 0 then the vectors are linearly independent. otherwise the vectors are linearly dependent.

Q.1] Examine the linear dependence or independence of the following vectors. If dependent find relation among vectors $(1,1,1)$, $(1,2,3)$, $(2,3,8)$.

$\Rightarrow$ Let $x_1 = (1,1,1)$, $x_2 = (1,2,3)$

$x_3 = (2,3,8)$

Consider the equation,

$K_1 x_1 + K_2 x_2 + K_3 x_3 = 0$, where K_1, K_2, K_3 are scalars.

$\Rightarrow K_1(1,1,1) + K_2(1,2,3) + K_3(2,3,8) = (0,0,0)$

$\Rightarrow (K_1 + K_2 + 2K_3, K_1 + 2K_2 + 3K_3, K_1 + 3K_2 + 8K_3) = (0,0,0)$

$$\Rightarrow \begin{aligned} K_1 + K_2 + 2K_3 &= 0 \\ K_1 + 2K_2 + 3K_3 &= 0 \\ K_1 + 3K_2 + 8K_3 &= 0 \end{aligned}$$

In Matrix form above system can be written as,

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 8 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

which is homogeneous system. Also the Augumented Matrix is given by,

$$[A \vdots O] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 8 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 6 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right]$$

which is in Row echelon form

$\therefore \rho(A) = \rho(A:B) = 3 =$ No. of unknowns.

$\therefore$ Given system has trivial solution.

i.e. $K_1 = 0$, $K_2 = 0$ and $K_3 = 0$ is the unique solution.

$\therefore$ Given vectors are linearly independent.

* Shortcut Method :-

$\Rightarrow$ consider, $A = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 8 \end{bmatrix}$

Then $|A| = 1(2\times8 - 3\times3) - 1(1\times8 - 3\times2) + 1(1\times3 - 2\times2)$

$\Rightarrow |A| = 4 \neq 0$

$\therefore$ Given vectors are linearly independent.

Q.2] Examine for linear independence or dependence of vectors. If dependent find the relation among them.

(3, 2, 7), (2, 4, 1), (1, -2, 6)

$\Rightarrow$ let, $x_1 = (3,2,7)$, $x_2 = (2,4,1)$, $x_3 = (1,-2,6)$

consider the equation,

$K_1x_1 + K_2x_2 + K_3x_3 = 0$, where K_1, K_2, K_3 are scalars.

$\Rightarrow K_1(3,2,7) + K_2(2,4,1) + K_3(1,-2,6) = (0,0,0)$

$\Rightarrow (3K_1 + 2K_2 + K_3,\ 2K_1 + 4K_2 - 2K_3,\ 7K_1 + K_2 + 6K_3) = (0,0,0)$

$$\Rightarrow \begin{aligned} 3K_1 + 2K_2 + K_3 &= 0 \\ 2K_1 + 4K_2 - 2K_3 &= 0 \\ 7K_1 + K_2 + 6K_3 &= 0 \end{aligned}$$

In Matrix form above system can be written as,

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & -2 \\ 7 & 1 & 6 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

so, Augumented matrix is given by,

$$[A:0] = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 2 & 4 & -2 & 0 \\ 7 & 1 & 6 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & 4 & -2 & 0 \\ 7 & 1 & 6 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 - 7R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 8 & -8 & 0 \\ 0 & 15 & -15 & 0 \end{array}\right]$$

$$\xrightarrow[\frac{1}{15}R_3]{\frac{1}{8}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{— (i)}$$

which is Row echelon form and hence, $\rho(A) = \rho(A:B) = 2 < 3 =$ No. of unknowns

so, Above system has infinitely many solution.

Also, R_2 of eq.(i) gives,

$$K_2 - K_3 = 0 \Rightarrow K_2 = K_3$$

let, $K_2 = t$ then $K_2 = K_3 \Rightarrow K_3 = t$

Also, R_1 of eq.(i) gives,

$$K_1 - 2K_2 + 3K_3 = 0$$

$$\Rightarrow K_1 = -t$$

$\therefore$ $K_1 = -t$, $K_2 = t$ and $K_3 = t$ are the infinitely many solutions of above system of equations.

let $t = 1$, then $K_1 = -1$, $K_2 = K_3 = 1$ and hence the given vectors are <u>linearly dependent</u>.

To find the relation:-

Consider the equation $K_1 X_1 + K_2 X_2 + K_3 X_3 = 0$

$\Rightarrow -tX_1 + tX_2 + tX_3 = 0$

$\Rightarrow -X_1 + X_2 + X_3 = 0 \quad (\because t \neq 0)$

$\Rightarrow \boxed{X_2 + X_3 = X_1}$ is the required relation among vectors.

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LECTURE NOTES

Faculty :- Mukesh M Sharma

Subject :- Engineering Mathematics I

Reference for Topic:-

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Higher Engineering Mathematics	B.S. Grewal	
Higher Engineering Mathematics	B.V. Ramana	

Topic Name :- Linear and orthogonal Transformation :-

* Linear Transformation :-

A General linear transformation is,

$$y_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n$$
$$y_2 = a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n$$
$$\vdots$$
$$y_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n$$

or

$$Y = AX$$

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where,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Notes :-

1] Here A is called as Matrix of linear transformation and $|A|$ is Modulus of linear transformation

2) $|A| = 0$ then the transformation is singular. and $|A| \neq 0$ then the transformation is non singular.

3] If $|A| \neq 0$ then the inverse transformation of $Y = AX$ is given by $X = A^{-1}Y$.

4) If $Y = AX$ is linear transformation from X to Y and $Z = BY$ is linear transformation from Y to Z then $Z = (BA)X$ is linear transformation from X to Z.

5] If matrix A is orthogonal then $Y = AX$ is orthogonal transformation

6) Every square matrix defines a linear transformation

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Q.1] Given the transformation $Y = PX$ where

$X = [x_1, x_2, x_3]$

$$P = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix} \Rightarrow P^{-1} = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix}$$

Find the co-ordinates in X corresponding to the co-ordinates (1, 1, −1) in Y.

$\Rightarrow$ Here the given transformation is, $Y = PX$

So, $X = P^{-1}Y$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore x_1 = 1,\ x_2 = -2,\ x_3 = 1$ are co-ordinates of X corresponding to the co-ordinates (1, 1, −1) in Y.

Q.2] A transformation from the variables x_1, x_2, x_3 to y_1, y_2, y_3 is given by $Y = AX$ and another transformation from y_1, y_2, y_3 to z_1, z_2, z_3 is given by, $Z = BY$ where,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$$

obtain the transformation from x_1, x_2, x_3 to z_1, z_2, z_3.

$\Rightarrow$ Given transformations are $Y = AX$, $Z = BY$

So, $Z = BY = B(AX) = (BA)X$

$$\therefore \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & -1 \\ -1 & 9 & -1 \\ -3 & 14 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow \quad z_1 = x_1 + 4x_2 - x_3$

$z_2 = -x_1 + 9x_2 - x_3$

$z_3 = -3x_1 + 14x_2 - x_3$ is the required transformation from x_1, x_2, x_3 to z_1, z_2, z_3.

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LECTURE NOTES

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Reference for Topic:-

Name of the Book/paper/Website/Other	Author Name	Page No/Link
Higher Engineering Mathematics	B.S. Grewal	
Higher Engineering Mathematics	B.V. Ramana	

Topic Name :- Orthogonal Transformation :-

* Orthogonal Matrix :-

A Matrix A is said to be orthogonal if $AA^T = A^TA = I$

i.e. $A^{-1} = A^T$

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$

Note :-

1] A is orthogonal $\Rightarrow |A| = \pm 1$

2] Product of orthogonal matrices is orthogonal

3] Transpose and inverse of orthogonal Matrix is orthogonal

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* orthogonal transformation :-

The linear transformation $Y = AX$

where,

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

is called orthogonal transformation if it transforms $x_1^2 + x_2^2 + \dots + x_n^2$ into $y_1^2 + y_2^2 + \dots + y_n^2$

If $Y = AX$ is orthogonal transformation then A is orthogonal Matrix

* Properties of orthogonal Matrix :-

1] Product of two orthogonal matrices is orthogonal Matrix.

2] Transpose of orthogonal Matrix is orthogonal

3] Inverse of orthogonal Matrix is orthogonal

4] If A is orthogonal Matrix then $|A| = \pm 1$

Q.1] Show that $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

is orthogonal Matrix.

$\Rightarrow$ Here, $A^T = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$

Now,

$$AA^T = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\therefore AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore$ Given Matrix is orthogonal

Q.2] Find values of a, b, c if

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$$ is orthogonal.

$\Rightarrow$ since matrix A is orthogonal,

so, $AA^T = I$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{9}\begin{bmatrix} 5+a^2 & 4+ab & -2+ac \\ 4+bc & 5+b^2 & 2+bc \\ -2+ac & 2+bc & 8+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \frac{1}{9}(5+a^2) = 1 \Rightarrow 5+a^2 = 9 \Rightarrow a^2 = 4 \Rightarrow a = \pm 2$

$\frac{1}{9}(5+b^2) = 1 \Rightarrow 5+b^2 = 9 \Rightarrow b^2 = 4 \Rightarrow b = \pm 2$

$\frac{1}{9}(8+c^2) = 1 \Rightarrow 8+c^2 = 9 \Rightarrow c^2 = 1 \Rightarrow c = \pm 1$

so, Given matrix will be orthogonal if

$a = \pm 2$, $b = \pm 2$ and $c = \pm 1$.

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Topic Name :- Applications in Engineering.

* Matrix transformation :-

Matrix transformations are used in computer graphics. Using CAD we can transform the actual design of a model to its efficient display on screen. The procedure of transformation from the model co-ordinate system to the screen co-ordinate system requires three transformations.

i] Scaling ii] Rotation iii] Translation.

Sharma M.M.
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1] Scaling :-

The transformation is $X = tx$, $Y = ty$ and $Z = tz$.

This changes the scale from (x, y, z) co-ordinate system to (X, Y, Z) co-ordinate system. In Matrix form it is given by,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & t \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Here, t is scaling factor.

2] Rotation :-

The rotation about x axis in Matrix form is given by,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The rotation about y axis in Matrix form is given by,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The rotation about z axis is given by,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

3] Translation :-

If (x, y, z) are the co-ordinates of ox, oy, oz axis and (X, Y, Z) are the co-ordinates of P w.r.t. the new axis by shifting the origin to the point (u, v, w) w.r.t. ox, oy, oz then

$X = x - u,\ Y = y - v,\ Z = z - w.$

In Matrix form it is given by,

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -u \\ 0 & 1 & 0 & -v \\ 0 & 0 & 1 & -w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

So,

1] If the rotation is about z axis by angle θ and origin is shifted to (u, v, w) then the combined transformation in matrix form is given by,

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$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & -u \\ -\sin\theta & \cos\theta & 0 & -v \\ 0 & 0 & 1 & -\omega \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Similarly if rotation is about x axis by angle θ and origin is shifted to (u, v, ω) then, in Matrix form it is given by,

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -u \\ 0 & \cos\theta & \sin\theta & -v \\ 0 & -\sin\theta & \cos\theta & -\omega \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Also if rotation is about y axis by angle θ and origin is shifted to (u, v, ω) then in Matrix form it is given by

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & -u \\ 0 & 1 & 0 & -v \\ -\sin\theta & 0 & \cos\theta & -\omega \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Q.1] The co-ordinates of a point P are (50, 50, 50). If the origin is shifted to the point (5, -2, 3) and Rotation is about z axis through 45°. then find the new co-ordinates of point P.

⇒ we know if the rotation is about z-axis and translation is (u, v, w) then the rotation and translation together is given by,

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & -u \\ -\sin\theta & \cos\theta & 0 & -v \\ 0 & 0 & 1 & -w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Here, $x = 50$, $y = 50$, $z = 50$, and $\theta = 45$

$u = 5$, $v = -2$, $w = 3$

so, we get,

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45 & \sin 45 & 0 & -5 \\ -\sin 45 & \cos 45 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 50 \\ 50 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & -5 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 50 \\ 50 \\ 1 \end{bmatrix}$$

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