



$$|V| = 3n$$

to prove triangle packing is NP-Complete

If any problem that is NP converges towards another problem, then the latter problem is NP-Complete

To prove it is NP:

Certificate of polynomial length, Solution can be verified in polynomial time.

Certificate $((C_1a, C_1b, C_1c), (C_2a, C_2b, C_2c), (C_3a, C_3b, C_3c) \dots (C_ka, C_kb, C_kc))$

length of Certificate would be $(3 \times k)$

which is of polynomial length

It can be verified in polynomial time if there exists 'k' monochromatic triangles

So, the problem is NP

→ Now, we need to show that any problem in NP



→ Now, we need to show that any problem in NP reduces to our problem of finding unique triangles

A very famous NP problem is 3DM (3-dimensional

[Its proof was shown by 'karp' matching])

in his paper reducibility among combinatorial problems in 1972

3DM Problem Description

Say, we have a set of elements X, Y, W

let $X = \{x_1, \dots, x_n\}$, $Y = \{y_1, \dots, y_m\}$

$W = \{w_1, \dots, w_k\}$

Say $M \subseteq X \times Y \times W$

↳ generic instance of 3DM



Say $M \subseteq X \times V \times W$

↳ generic instance of 3DM

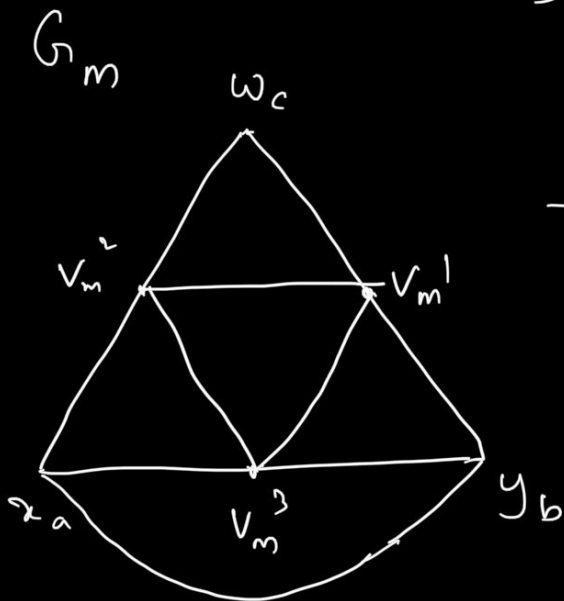
lets say the whole graph is G_M → Single big gadget

for every (x_a, y_b, w_c) combinations from 'M', we

construct a graph $G_m = (V_m, E_m)$

↳ unit gadget

V_m → Contains 3-public vertices,
3-private vertices



→ This graph has about
10 edges

$\{v_m^1, v_m^2, v_m^3\}$

↳ "Private vertices" as they

are not connected with nodes

G_m gadget
Can exist in



V_m, V_m^-, V_m^+

"

each gadget
can exist in
one of two
states

private vertices as they
are not connected with nodes
from other gadgets

(a) Complete State → called Complete state because

(b) Shared State all the nodes in G_m are covered
among themselves forming two
disjoint triangles



is called shared
state because, the

private nodes would be connected among themselves
forming a triangle,

the public nodes must be covered by other
triangles in G_m

→ Given any partition of G_m into triangles,



the smaller gadgets must be either in
complete state or shared state i.e. all the



→ Given any partition of G_m into triangles, all the smaller gadgets must be either in complete state or shared state i.e all the triangles would be disjoint spanning the graph

Proving 3DM converges towards our problem of finding 'k' triangles.

lets assume 'M' has a perfect matching

$$M_1 \subseteq M$$

every time a triple of M is in M_1 , we set the corresponding gadget to complete state. otherwise shared state

The public and private nodes would be connected by the definition of 3DM.



once by the definition of 3DM.

Assume, G_m be partitionable into multiple disjoint triangles and such a partition is 'p'

let us assume M_1 be the set of all gadgets in complete state,

So M_1 covers X, Y, W . Since all triangles in 'p' are disjoint, No gadget shares an edge with other gadgets while in complete state.

Hence the triples of M_1 are also disjoint and are perfect match for 'M'

→ The verification of this would be in the order of size of input dataset which is polynomial in time over the dataset.