$|V| = 3n$

to prove triangle packing is NP-Complete

If any problem that is NP converges towards another problem, then the latter problem is NP-Complete

To prove it is NP:

Certificate of polynomial length, Solution can be verified in polynomial time.

Certificate $((C_1a, C_1b, C_1c), (C_2a, C_2b, C_2c), (C_3a, C_3b, C_3c) \_\_\_\_ \to (C_na, C_nb, C_nc))$

length of Certificate would be $(3 \times k)$ which is of polynomial length

It can be verified in polynomial time if there exists 'k' monochromatic triangles

So, the problem is NP

→ Now, we need to show that any problem in NP

→ Now, we need to show that any problem in NP reduces to our problem of finding unique triangles

A very famous NP complete problem is 3DM (3-dimension matching)

[Its proof was shown by 'karp' in his paper reducibility among combinatorial problems in 1972

3DM Problem Description

Say, we have a set of elements X, Y, W

let $X = \{x_1, \ldots, x_q\}$, $Y = \{y_1, \ldots, y_q\}$

$W = \{w_1, \ldots, w_q\}$

Say $\underbrace{M \subseteq X \times Y \times W}$

↳ generic instance of 3DM

Say $\underbrace{M \subseteq X \times Y \times W}$,

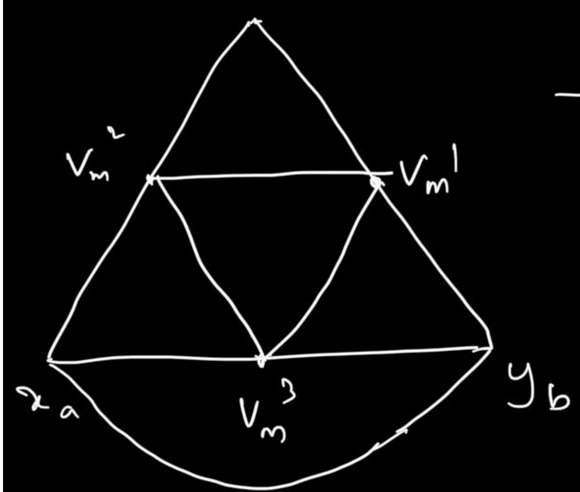
↳ generic instance of 3DM

lets say the whole graph is $\boxed{G_M}$ → Single big gadget

for every (x_a, y_b, w_c) combinations from 'M', we construct a graph $G_m = (V_m, E_m)$ → unit gadget

V_m → Contains 3-public vertices,
3-private vertices

G_m



→ This graph has about 10 edges

$\underbrace{v_m^1, v_m^2, v_m^3}$

↳ "Private vertices" as they are not connected with nodes

each gadget Can exist in

v_m , v_m' , v_m''

"Private vertices" as they are not connected with nodes from other gadgets

each gadget can exist in one of two states

ⓐ Complete State → Called Complete state because all the nodes in G_m are covered among them selves forming two disjoint triangles

ⓑ Shared State

is called shared state because, the private nodes would be connected among themselves forming a triangle,

the public nodes must be covered by other triangles in G_M

→ Given any partition of G_m into triangles,

the smaller gadgets must be either in complete state ~~or shared state~~ i.e all the

→ Given any partition of G_m into triangles, all the smaller gadgets must be either in complete state or shared state i.e all the triangles would be disjoint spanning the graph

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Proving 3DM converges towards our problem of finding 'k' triangles.

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lets assume 'M' has a perfect matching $M_1 \subseteq M$

every time a triple of M is in M_1, we set the corresponding gadget to complete state. otherwise shared state

The public and private nodes would be connected by the definition of 3DM.

once by the definition of 3DM.

Assume, G_M be partitionable into multiple disjoint triangles and such a partition is 'P'

let us assume M_1 be the set of all gadgets in complete state,

So M_1 covers X, Y, W. Since all triangles in 'P' are disjoint, No gadget shares an edge with other gadgets while in complete state.

Hence the triples of M_1 are also disjoint and are perfect match for 'M'

→ The verification of this would be in the order of size of input dataset which is polynomial in time over the dataset.