1. Find the value of T(2) for the recurrence relation T(n) = 3T(n-1) + 12n, given that T(0)=5.

We are given the recurrence relation:
T(n)=3T(n−1)+12nT(n) = 3T(n-1) + 12nT(n)=3T(n−1)+12n
with the initial condition:
T(0)=5T(0) = 5T(0)=5
We need to find T(2)T(2)T(2).

Step 1: Compute T(1)T(1)T(1)
Using the recurrence relation:
T(1)=3T(0)+12(1)T(1) = 3T(0) + 12(1)T(1)=3T(0)+12(1)
Substituting T(0)=5T(0) = 5T(0)=5:
T(1)=3(5)+12(1)=15+12=27T(1) = 3(5) + 12(1) = 15 + 12 = 27T(1)=3(5)+12(1)=15+12=27

2: Compute T(2)T(2)T(2)
Using the recurrence relation:
T(2)=3T(1)+12(2)T(2) = 3T(1) + 12(2)T(2)=3T(1)+12(2)
Substituting T(1)=27T(1) = 27T(1)=27:
T(2)=3(27)+12(2)=81+24=105T(2) = 3(27) + 12(2) = 81 + 24 = 105T(2)=3(27)+12(2)=81+24=105
Thus, the value of T(2)T(2)T(2) is:
105\boxed{105}

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2. Given a recurrence relation, solve it using the substitution method:

a. T(n) = T(n-1) + c

We are given the recurrence relation:
T(n)=T(n−1)+cT(n) = T(n-1) + cT(n)=T(n−1)+c
where ccc is a constant. We will solve it using the **substitution (iterative) method**.
Step 1: Expand the recurrence
Let's expand T(n)T(n)T(n) by repeatedly substituting for smaller values:
T(n)=T(n−1)+cT(n) = T(n-1) + cT(n)=T(n−1)+c T(n−1)=T(n−2)+cT(n-1) = T(n-2) + cT(n−1)=T(n−2)+c
T(n−2)=T(n−3)+cT(n-2) = T(n-3) + cT(n−2)=T(n−3)+c
Continuing this pattern, after kkk steps, we reach:
T(n−k)=T(n−k−1)+cT(n-k) = T(n-k-1) + cT(n−k)=T(n−k−1)+c
Expanding until we reach the base case at T(0)T(0)T(0), we get:
T(n)=T(n−1)+cT(n) = T(n-1) + cT(n)=T(n−1)+c =(T(n−2)+c)+c=T(n−2)+2c= (T(n-2) + c) + c = T(n-2) + 2c=(T(n−2)+c)+c=T(n−2)+2c =(T(n−3)+c)+2c=T(n−3)+3c= (T(n-3) + c) + 2c = T(n-3) + 3c=(T(n−3)+c)+2c=T(n−3)+3c ⋮\vdots⋮ =T(0)+nc= T(0) + nc=T(0)+nc
Step 2: General Solution
From the pattern, the closed-form solution is:
T(n)=T(0)+ncT(n) = T(0) + ncT(n)=T(0)+nc
where T(0)T(0)T(0) is the initial condition.
Conclusion

Thus, the recurrence relation
T(n)=T(n−1)+cT(n) = T(n-1) + cT(n)=T(n−1)+c
solves to:
T(n)=T(0)+ncT(n) = T(0) + ncT(n)=T(0)+nc
This shows that the function grows linearly with nnn.

b. T(n) = 2T(n/2) + n

We will solve the recurrence relation using the **substitution (iteration) method**:
T(n)=2T(n/2)+nT(n) = 2T(n/2) + nT(n)=2T(n/2)+n
Step 1: Expand the Recurrence
Expanding the recurrence by substituting for T(n/2)T(n/2)T(n/2):
T(n)=2T(n/2)+nT(n) = 2T(n/2) + nT(n)=2T(n/2)+n T(n/2)=2T(n/4)+n/2T(n/2) = 2T(n/4) + n/2T(n/2)=2T(n/4)+n/2
Substituting T(n/2)T(n/2)T(n/2) into the first equation:
T(n)=2(2T(n/4)+n/2)+nT(n) = 2(2T(n/4) + n/2) + nT(n)=2(2T(n/4)+n/2)+n =4T(n/4)+2n= 4T(n/4) + 2n=4T(n/4)+2n
Expanding further:
T(n/4)=2T(n/8)+n/4T(n/4) = 2T(n/8) + n/4T(n/4)=2T(n/8)+n/4 T(n)=4(2T(n/8)+n/4)+2nT(n) = 4(2T(n/8) + n/4) + 2nT(n)=4(2T(n/8)+n/4)+2n =8T(n/8)+3n= 8T(n/8) + 3n=8T(n/8)+3n
Following this pattern, at the kkk-th step:
T(n)=2kT(n/2k)+knT(n) = 2^k T(n/2^k) + k nT(n)=2kT(n/2k)+kn
Step 2: Determine the Base Case
We assume the recurrence reaches the base case at n=1n = 1n=1, where T(1)T(1)T(1) is some constant CCC. This happens when:
n/2k=1n/2^k = 1n/2k=1 2k=n2^k = n2k=n k=log⁡2nk = \log_2 nk=log2n
Substituting k=log⁡2nk = \log_2 nk=log2n into our equation:
T(n)=2log⁡2nT(1)+(log⁡2n)nT(n) = 2^{\log_2 n} T(1) + (\log_2 n) nT(n)=2log2nT(1)+(log2n)n =nT(1)+nlog⁡2n= nT(1) + n \log_2 n=nT(1)+nlog2n
Since T(1)=CT(1) = CT(1)=C, we get:
T(n)=Cn+nlog⁡2nT(n) = Cn + n \log_2 nT(n)=Cn+nlog2n
Step 3: Asymptotic Complexity
Ignoring the constant term, we get:
T(n)=O(nlog⁡n)T(n) = O(n \log n)T(n)=O(nlogn)
Final Answer
T(n)=O(nlog⁡n)T(n) = O(n \log n)T(n)=O(nlogn)

c. T(n) = 2T(n/2) + c

We will solve the recurrence relation:
T(n)=2T(n/2)+cT(n) = 2T(n/2) + cT(n)=2T(n/2)+c
using the **substitution (iteration) method**.
Step 1: Expand the Recurrence
Expanding the recurrence iteratively:

T(n)=2T(n/2)+cT(n) = 2T(n/2) + cT(n)=2T(n/2)+c T(n/2)=2T(n/4)+cT(n/2) = 2T(n/4) + cT(n/2)=2T(n/4)+c
Substituting T(n/2)T(n/2)T(n/2) into the first equation:
T(n)=2(2T(n/4)+c)+cT(n) = 2(2T(n/4) + c) + cT(n)=2(2T(n/4)+c)+c =4T(n/4)+2c+c=4T(n/4)+3c= 4T(n/4) + 2c + c = 4T(n/4) + 3c=4T(n/4)+2c+c=4T(n/4)+3c
Expanding further:
T(n/4)=2T(n/8)+cT(n/4) = 2T(n/8) + cT(n/4)=2T(n/8)+c T(n)=4(2T(n/8)+c)+3cT(n) = 4(2T(n/8) + c) + 3cT(n)=4(2T(n/8)+c)+3c =8T(n/8)+4c+3c=8T(n/8)+7c= 8T(n/8) + 4c + 3c = 8T(n/8) + 7c=8T(n/8)+4c+3c=8T(n/8)+7c
Following this pattern, at the kkk-th step:
T(n)=2kT(n/2k)+(2k−1)cT(n) = 2^k T(n/2^k) + (2^k - 1)cT(n)=2kT(n/2k)+(2k−1)c
Step 2: Determine the Base Case
We assume the recurrence reaches the base case at n=1n = 1n=1, where T(1)T(1)T(1) is some constant CCC. This happens when:
n/2k=1n/2^k = 1n/2k=1 2k=n2^k = n2k=n k=log⁡2nk = \log_2 nk=log2n
Substituting k=log⁡2nk = \log_2 nk=log2n into our equation:
T(n)=2log⁡2nT(1)+(2log⁡2n−1)cT(n) = 2^{\log_2 n} T(1) + (2^{\log_2 n} - 1)cT(n)=2log2 nT(1)+(2log2n−1)c
Since 2log⁡2n=n2^{\log_2 n} = n2log2n=n, we get:
T(n)=nT(1)+(n−1)cT(n) = nT(1) + (n - 1)cT(n)=nT(1)+(n−1)c
If T(1)=CT(1) = CT(1)=C, then:
T(n)=nC+(n−1)cT(n) = nC + (n - 1)cT(n)=nC+(n−1)c
Step 3: Asymptotic Complexity
Ignoring the constant terms, we get:
T(n)=O(n)T(n) = O(n)T(n)=O(n)
Final Answer
T(n)=O(n)T(n) = O(n)T(n)=O(n)

d. T(n) = T(n/2) + c

We will solve the recurrence relation:
T(n)=T(n/2)+cT(n) = T(n/2) + cT(n)=T(n/2)+c
using the **substitution (iteration) method**.

Step 1: Expand the Recurrence
Expanding iteratively:
T(n)=T(n/2)+cT(n) = T(n/2) + cT(n)=T(n/2)+c T(n/2)=T(n/4)+cT(n/2) = T(n/4) + cT(n/2)=T(n/4)+c T(n/4)=T(n/8)+cT(n/4) = T(n/8) + cT(n/4)=T(n/8)+c
Following this pattern, at the kkk-th step:
T(n)=T(n/2k)+kcT(n) = T(n/2^k) + kcT(n)=T(n/2k)+kc

Step 2: Determine the Base Case

We assume the recurrence reaches the base case at n=1n = 1n=1, where T(1)T(1)T(1) is some constant CCC. This happens when:

n/2k=1n/2^k = 1n/2k=1 2k=n2^k = n2k=n k=log2nk = \log_2 nk=log2n

Substituting k=log2nk = \log_2 nk=log2n into our equation:

T(n)=T(1)+(log2n)cT(n) = T(1) + (\log_2 n)cT(n)=T(1)+(log2n)c

Since T(1)=CT(1) = CT(1)=C, we get:

T(n)=C+clog2nT(n) = C + c \log_2 nT(n)=C+clog2n

Step 3: Asymptotic Complexity

Ignoring the constant term CCC, we get:

T(n)=O(logn)T(n) = O(\log n)T(n)=O(logn)

Final Answer

T(n)=O(logn)T(n) = O(\log n)T(n)=O(logn)