Matrix Chain Multiplication

The problem is not here to multiply but how to multiply so that we get the minimum cost for multiplying the elements.

Suppose there are two matrix . R1 & R2 represents the row of first & second matrix. C1 & C2 represents the row of first & second matrix.

Cost of matrix multiplication = R1*C1*C2

Note:- For Multiplication of matrices . C1==R2

So order in which we multiply the matrices effect the change in cost of matrix multiplication.

No of ways the matrix can be multiplied is given by Catalan number.

We can solve this problem using dynamic programming approach.

For Matrix Multiplication we will use Bottom-Up Approach i.e. Tabulation
Step 1 : -

	1	2	3	4
1	0	120		
2		0	48	
3			0	84
4				0

A1	A2	A3	A4
5 * 4	4 * 6	6 * 2	2 * 7

- Diagonal is zero because the same matrix isn't multiplied with anything .

Two matrix are multiplied :-

- Cost[1][2] = 5*4*6=120
- Cost[2][3] = 4*6*2=48
- Cost[3][4] = 6*2*7=84

For Matrix Multiplication we will use Bottom-Up Approach i.e. Tabulation
Step 1 : -

A1	A2	A3	A4
5 * 4	4 * 6	6 * 2	2 * 7

	1	2	3	4
1	**0**	**120**	**88**	
2		**0**	**48**	
3			**0**	**84**
4				**0**

Three matrix are multiplied :-

- Cost[1][3]

The matrix can be multiplied in two ways :-

a) A1*(A2*A3)

b) (A1*A2)*A3

So,

Cost[1][3] = Cost[1][1]+Cost[2][3]+5*4*2

= 0 + 48 + 40 = 88

Cost[1][3] = Cost[1][2]+Cost[3][3]+5*6*2

= 120 + 0 + 60 =180

For Matrix Multiplication we will use Bottom-Up Approach i.e. Tabulation
Step 1 : -

	A1	A2	A3	A4
	5 * 4	4 * 6	6 * 2	2 * 7

	1	2	3	4
1	0	120	88	
2		0	48	104
3			0	84
4				0

Three matrix are multiplied :-

- Cost[2][4]

The matrix can be multiplied in two ways :-

a) A2*(A3*A4)

b) (A2*A3)*A4

So,

Cost[2][4] = Cost[2][2]+Cost[3][4]+4*6*7

= 0 + 84 + 40 = 252

Cost[2][4] = Cost[2][3]+Cost[4][4]+4*2*7

= 48 + 0 + 56 =104

For Matrix Multiplication we will use Bottom-Up Approach i.e. Tabulation

Step 1 : -

	A1	A2	A3	A4
	5 * 4	4 * 6	6 * 2	2 * 7

	1	2	3	4
1	0	120	88	158
2		0	48	104
3			0	84
4				0

Now Let us see the Formula and calculate

Cost[1][4]=min{Cost[1][1]+Cost[2][4]+5*4*7,
Cost[1][2]+Cost[3][4]+5*6*7,
Cost[1][3]+Cost[4][4]+5*2*7,}
= 158

In General

Cost[i][j]={ Cost[i][k]+Cost[k+1][j]+d1*d2*d3

Where d1= D(i-1) , d2 = dk , d3=dj

Time Complexity O(n^3)