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ASSIGNMENT 1

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GRAPH COLORING BASED

1) We can use induction on the no of vertices in the graph which we denote by n. Let $P(n)$ be the proposition that an n-vertex graph with maximum degree of at most k is $(k+1)$ colourable.

BASE CASE

$n = 1$.

A 1-vertex graph has max degree 0 & is 1-colourable.

$\therefore P(1)$ is true.

INDUCTIVE STEP

Assume $P(n)$ is true

Let G be an $(n+1)$ vertex graph with max. degree atmost k.
Remove a vertex 'v' (thus removing all edges incident to it), leaving an n-vertex subgraph. The max. degree of this graph is 'k' & hence is $(k+1)$ colourable.

Now add back vertex 'v'. We can assign 'v' a colour from the set of $(k+1)$ colours that is different from all its adjacent vertices as there are atmost k vertices adjacent to v & thus atleast one of the $(k+1)$ colours is still available.

$\therefore G$ is $(k+1)$ colourable.

$\therefore$ Chromatic no. of a graph will not exceed by more than 1 the max. degree of vertices in a graph.

2) A graph is bicolourable if it is bipartite.
Let $G = (V, E)$ be bipartite, i.e.,
$V = A \cup B$ such that $A \cap B = \phi$ &
all edges $e \in E$ are such that e is of the form $\{a, b\}$
where $a \in A$ & $b \in B$.

Definition of bipartite graph -
A graph G is said to be bipartite if the set of vertices of graph are partitioned into two disjoints set of no two vertices of same set are adjacent.

Suppose G has (at least) one odd cycle C
let length of C be 'n'.

$$C = \{v_1, v_2, \dots, v_n\}$$

Without loss of generality:
Let $v_1 \in A$. It follows that $v_2 \in B$ & hence
$v_3 \in A$.

$$\therefore \forall K \in \{1, 2, \dots, n\}$$

$$v_K \in \begin{cases} A & K \text{ is odd} \\ B & K \text{ is even} \end{cases}$$

But as n is odd, $v_n \in A$
But $v_1 \in A$ & $v_n v_1 \in C_n$
So $v_n v_1 \in E$ which contradicts the assumption that G is bipartite.
$\therefore G$ is bipartite & thus has no odd cycles
G is bicolourable if & only if it has no odd cycle.

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3) Let P be a Hamiltonian path of G, with origin u. Because the path $P-u$ extends to a Hamiltonian path of G, the path P extends to a Hamiltonian cycle C of G.

When C has no chord, $G=C$ is a cycle. So let uv be a chord of C. Then u^-v^- is one too, because $u^-CvuC^{-1}v^-$ is a Hamiltonian path of G, likewise, u^-v^- is a chord of C (where u^- denotes the successor of u on C and u^{--} is the successor of u^-). And if the length of uCv is at least four, uv and u^-v^- are also chords of C, in view of the Hamiltonian path $u^{--}Cv^-u^-v^-C^{-1}v^-u^-uv$ and the fact that $u^-v^- = (u^-)^-v^-$.

When C has a chord uw of length two, let $v=u^-(=w^-)$. Then $vw^- \in E$. Moreover, if $vw^- \in E$, then $vw^{-(-1)} \in E$ in view of the Hamiltonian path $w^{-(-1)}CuwCw^-v$. It follows that v is adjacent to every vertex of G. But then G is complete, because u^-w^- is a chord of length two for all i. If C has no chord of length two, every chord of C is odd; moreover, every odd chord must be present. Thus, $G= K_n$, where $|V(G)| = 2n$.

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4) Let the regions & edges of G be respectively denoted by $r_1, \ldots, r_t$ and $e_1, \ldots, e_m$. Let the vertices of G^* be $r_1^*, \ldots, r_t^*$ and edges be $e_1^*, \ldots, e_m^*$. Then the vertices and edges of G^* are in one to one correspondence with the regions & edges of G and two vertices r^* and s^* in G^* are joined by an edge e^* if and only if the corresponding regions r and s in G have the corresponding edge e as a common edge on their boundary.

Let G be k-region colorable. We color the vertices in G^* such that each vertex in G^* gets the same colour as assigned to the region r in G. Since, the vertices r^* and s^* are only adjacent in G^* if the corresponding regions r & s are adjacent in G. G^* is k-vertex colorable

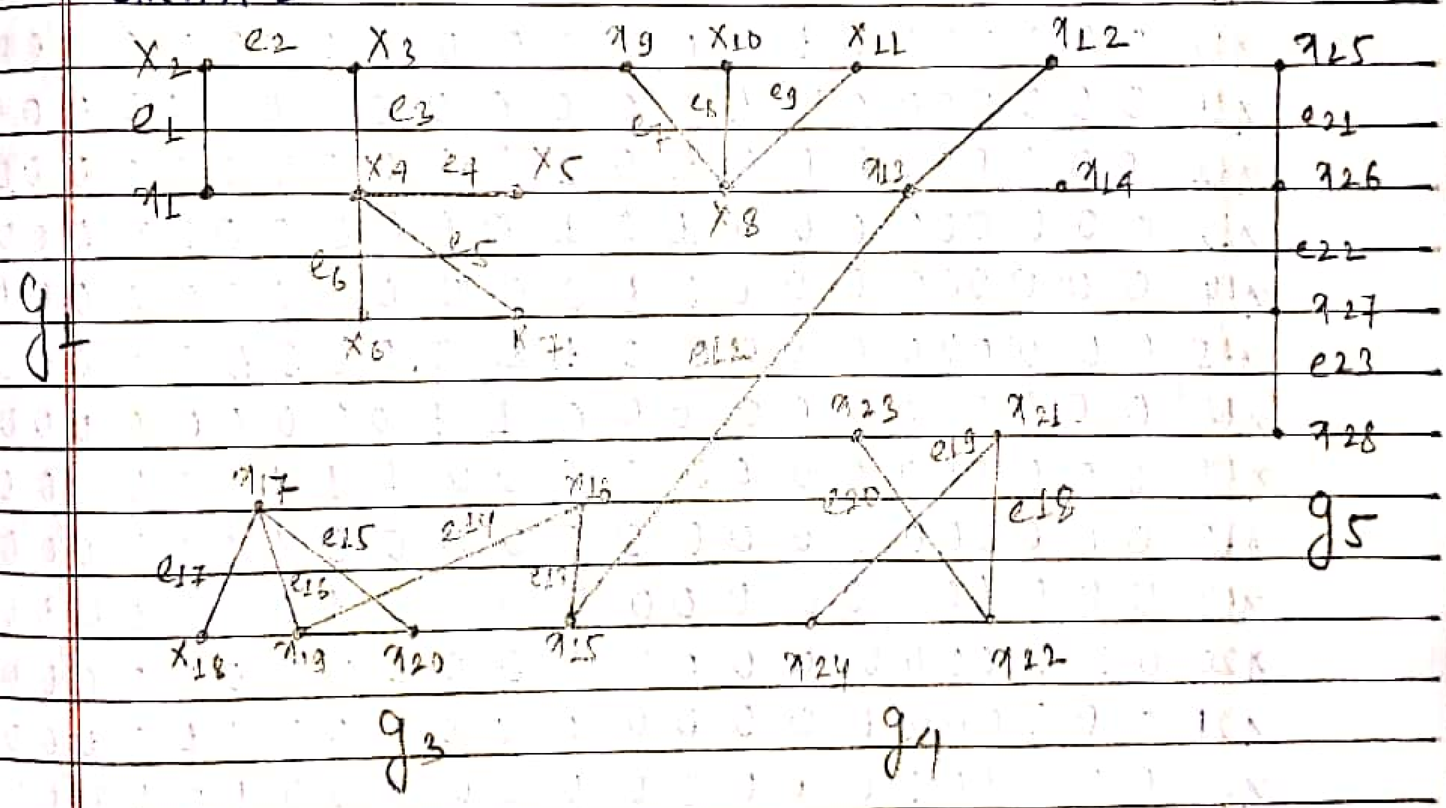
Conversely, let G^* be k-vertex colourable. Now, color the regions of G such that the region r in G gets the same colour as the vertex r^* in G^*. This gives a k-region coloring of G, since the regions r and s are adjacent in G only if the corresponding vertices r^* and s^* are adjacent in G^*.

GRAPH REPRESENTATION IN MATRIX

1.(a) GRAPH 1

$$A(G_1) = \begin{array}{c|ccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \hline x_1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ x_2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ x_4 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}$$

GRAPH 2



$A(G_2) =$

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}
x_1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_3	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_4	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_5	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_6	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_7	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_8	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_9	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{10}	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{11}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{12}	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{13}	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
x_{14}	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
x_{15}	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
x_{16}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
x_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
x_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
x_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
x_{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
x_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
x_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
x_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
x_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
x_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
x_{26}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
x_{27}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
x_{28}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

INCIDENT MATRIX OF EACH COMPONENT OF GRAPH 2

$A(g_1) =$

	e_1	e_2	e_3	e_4	e_5	e_6
x_1	1	0	0	0	0	0
x_2	1	1	0	0	0	0
x_3	0	1	1	0	0	0
x_4	0	0	1	1	1	1
x_5	0	0	0	1	0	0
x_6	0	0	0	0	0	1
x_7	0	0	0	0	1	0

$A(g_2) =$

	e_7	e_8	e_9
x_8	1	1	1
x_9	1	0	0
x_{10}	0	1	0
x_{11}	0	0	1

$A(g_3) =$

	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}
x_{12}	1	0	0	0	0	0	0	0
x_{13}	1	1	1	0	0	0	0	0
x_{14}	0	1	0	0	0	0	0	0
x_{15}	0	0	1	1	0	0	0	0
x_{16}	0	0	0	1	1	0	0	0
x_{17}	0	0	0	0	0	1	1	1
x_{18}	0	0	0	0	0	0	0	1
x_{19}	0	0	0	0	1	0	1	0
x_{20}	0	0	0	0	0	1	0	0

$A(g_4) =$

	e_{18}	e_{19}	e_{20}
x_{21}	1	1	0
x_{22}	1	0	1
x_{23}	0	0	1
x_{24}	0	1	0

$A(g_5) =$

	e_{21}	e_{22}	e_{23}
x_{25}	1	0	0
x_{26}	1	1	0
x_{27}	0	1	1
x_{28}	0	0	1

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(b) Total circuits

1. $\{e_1, e_2\}$
2. $\{e_1, e, e_3\}$
3. $\{e_4, e_5, e_6\}$
4. $\{e_7\}$
5. $\{e_2, e_3, e_4, e_5\}$
6. $\{e_1, e_3, e_4, e_5\}$
7. $\{e_1, e_3, e_6\}$

$B(G_1) =$

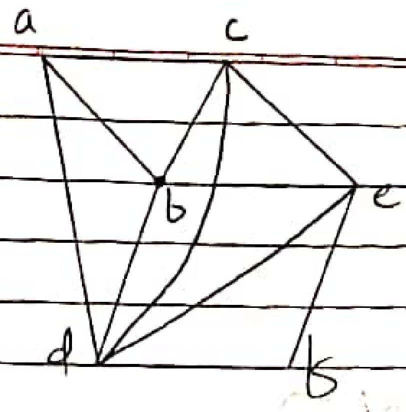
	e_1	e_2	e_3	e_4	e_5	e_6	e_7
1	1	1	0	0	0	0	0
2	0	1	1	0	0	1	0
3	0	0	0	1	1	1	0
4	0	0	0	0	0	0	1
5	0	1	1	1	1	0	0
6	1	0	1	1	1	0	0
7	1	0	1	0	0	1	0

(c)

$P(v_2, v_4) =$

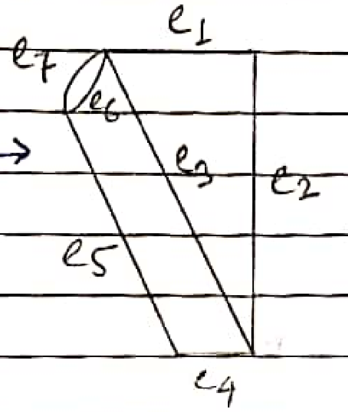
	e_1	e_2	e_3	e_4	e_5	e_6	e_7
1	0	0	1	1	0	0	0
2	0	1	0	0	1	0	0
3	0	0	1	0	1	1	0
4	0	1	0	1	0	1	0

(2) $G =$

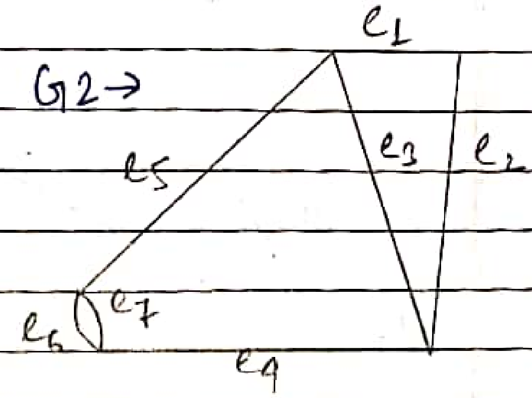


(3)

$G1 \rightarrow$



$G2 \rightarrow$



Total circuits:

1. $\{e_1, e_2, e_3\}$
2. $\{e_1, e_2, e_4, e_5, e_7\}$
3. $\{e_6, e_7\}$
4. $\{e_3, e_4, e_5, e_6\}$
5. $\{e_3, e_4, e_5, e_7\}$
6. $\{e_1, e_2, e_4, e_5, e_6\}$

1. $\{e_1, e_2, e_3\}$
2. $\{e_1, e_2, e_4, e_7, e_5\}$
3. $\{e_6, e_7\}$
4. $\{e_3, e_4, e_6, e_5\}$
5. $\{e_3, e_4, e_7, e_5\}$
6. $\{e_1, e_2, e_4, e_6, e_5\}$

$G1$ & $G2$ are non-isomorphic, connected, simple & non-separable graphs.

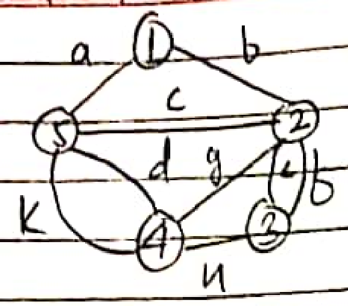
$B(G_1) =$

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
1	1	1	1	0	0	0	0
2	1	1	0	1	1	0	1
3	0	0	0	0	0	1	1
4	0	0	1	1	1	1	0
5	0	0	1	1	1	0	1
6	1	1	0	1	1	1	0

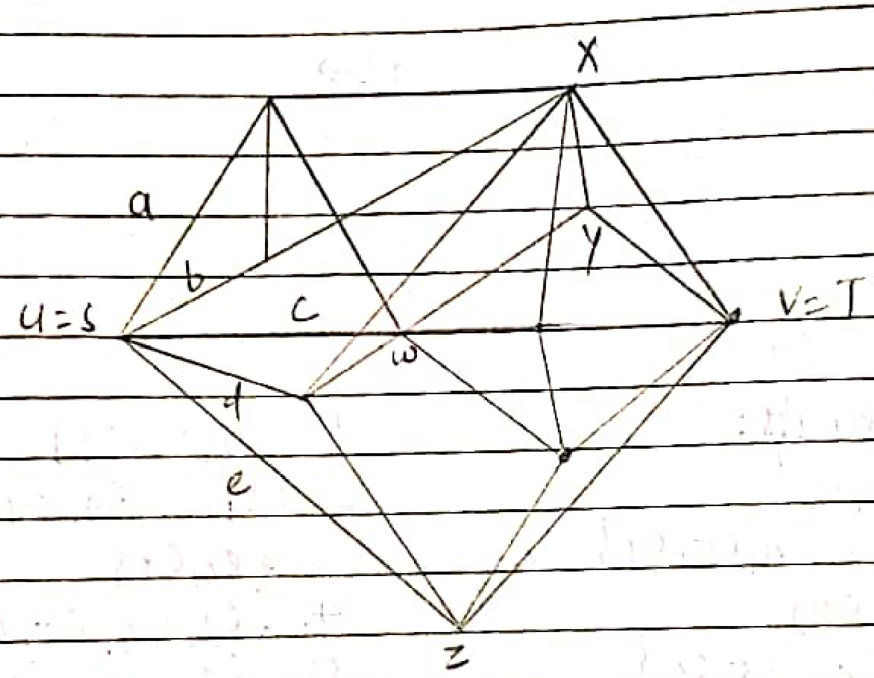
$B(G_2) =$

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
1	1	1	1	0	0	0	0
2	1	1	0	1	1	0	1
3	0	0	0	0	0	1	1
4	0	0	1	1	1	1	0
5	0	0	1	1	1	0	1
6	1	1	0	1	1	1	0

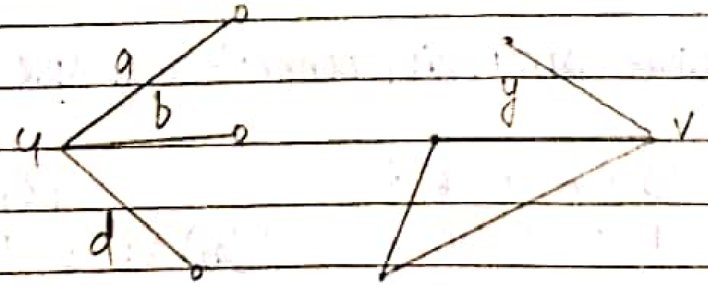
(4). $A(A) =$



MENGERS THEOREM BASED

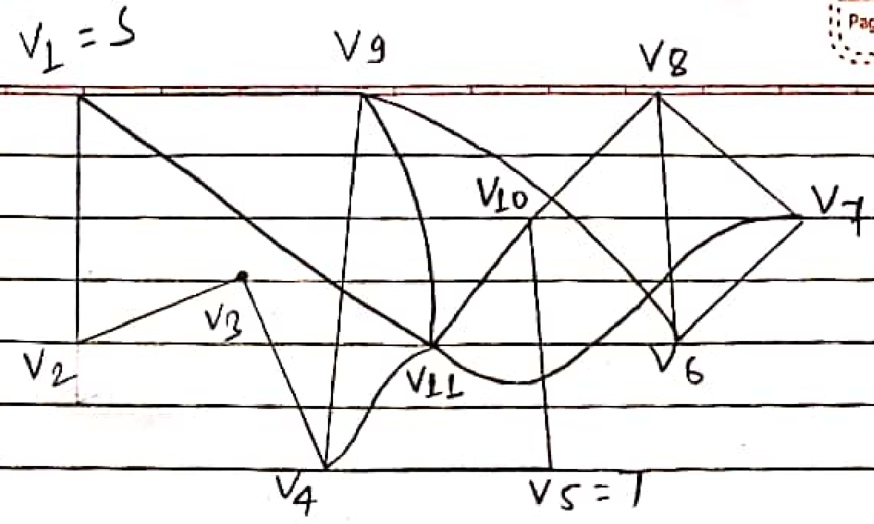


Vertex disjoint paths = 3
(Orange, pink, blue paths)



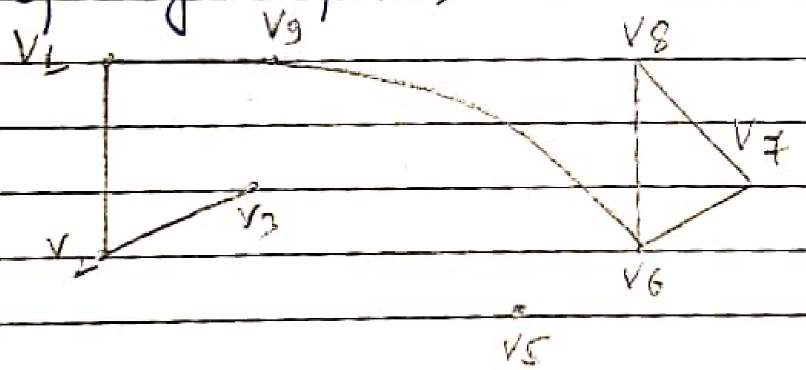
Vertex cut = 3 (by removing w, x, z)

∴ Max. disjoint paths = min vertex cut
This graph satisfies Menger's theorem.



$V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4 \rightarrow V_5$
$V_1 \rightarrow V_{11} \rightarrow V_{10} \rightarrow V_5$

Vertex disjoint paths = 2
(pink, green paths)



Cut-vertices = 3
(by removing V_4, V_{11}, V_{10})
Max. vertex disjoint paths $\neq$ min vertex cut
∴ This graph does not satisfy mengers theorem.