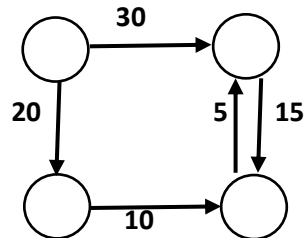


Network Flow

Let us imagine a connected, directed and weighted graph.



Vertices → Junction
Edges → Pipe

We can imagine this graph as a pipeline. You can imagine these vertices as junction and the edges as the pipe through which any material can flow. These vertices act as splitting point of the flow.

In a pipe-network graph we have two special vertices known as source and sink.

Imagine a material flowing through this system from source and following some path to reach sink. The source material is produced from source at steady rate. The flowing material is consumed at the sink. And the rate at this material flows is also steady.

The flow of the material at any point in the system is defined the rate at which material flows into the system.

Terminologies Regarding Network Flow

Capacity of an Edge

Flow is the exact amount of material flowing through the edge whereas capacity is the maximum amount of material that can flow through that edge.

Suppose there are two vertices A and B. Let, the capacity between A and B is X.

Then $\text{flow}(f) \leq X$.

Residual Capacity

Residual capacity is defined as the difference of total capacity and the flow of material through a given edge.

Residual Graph

If the given graph is represented in the form of residual capacity of edge then such graph is called Residual Graph.

Augmented Path

A path from source to sink such that the residual capacity on every edge is greater than zero.

Suppose there are vertices A, B, C and D. And the weight distributed on the edges are 2,4,0 then this path can't be augmented path.

The flow from A-B is 3, B-C is 10, C-D is 2, D-E is 4.

For this augmented path, the maximum flow will be the weight of the edge with minimum capacity or minimum flow. So here the edge with minimum capacity is 2 so the maximum flow will be 2.

Ford Fulkerson Algorithm

Important Points

1. We can't send the flow greater than the max capacity of an edge
2. For conservation of flow at any vertex other than source or sink the amount of flow going in must be equal to amount of flow going out.
3. For any augmented path the maximum flow is equal to the edge with minimum weight.
4. Any path we are considering as augmented path should have non-zero backward edge and a non-full forward edge.

Objective: - Maximize the flow of given network graph and we should keep in our mind no edges should go into source and no edges should come out of source.

Algorithm:

- i. Setup our directed residual graph with edge capacity as original graph weight.
- ii. Initialise a variable `max_flow=0`
- iii. while(there exist any augmented path from source to sink)
{
 1. Find 'f', the minimum edge weight along the current path.
 2. Decrease the capacity of all the forward edges by f.
 3. Increase the capacity of all the backward edges by f.
 4. `max_flow+=f`}
- iv. `print(max_flow)`

How we will find augmented path?

Depth First search is used for finding all path from source to sink. And we will apply Ford's Fulkerson algorithm on every path.

If we use BFS in Ford's Fulkerson algorithm then it is known as **Edmond Karp Algorithm**.

Application of Min Cut Problem

In a flow network or any given network graph, an S-T cut is a cut that requires the source and the sink to be in different subset. A cut in a graph is a division in a graph that makes it divided into more than one component.

In a network flow graph an S-T cut is a cut that requires source and sink to be in different cut where S component will describe the component which have source and respectively T component is for sink. It consists of all edges going from the source side to the sink side. Source side is the S-component and sink side is the T-component.

The capacity of an S-T cut is defined as the sum of the capacity of each edge in the cut.

So to find the minimum cut in the flow network we will use

Ford Fulkerson's Algorithm

1. Run the ford algorithm and calculate the final residual graph
2. Find the set of vertices that are reachable from the source. So all the vertices that will be reachable directly from the source will belong to the S-Component in which the source is present.
3. All the edge which are from a reachable vertex to a non-reachable vertex are inside minimum cut edge. We will print all such edges. Non reachable vertices are those vertices which belong to sink component.

Capacity of minimum cut is amount of maximum flow.