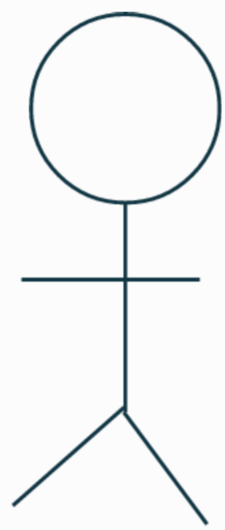
Vectors

Physics Student

Vectors are arrows pointing in space. what defines it is it's length & direction.



CS Student

Vectors are ordered list of numbers.

In Linear Algebra,

Whenever you hear vector, Think about arrow in coordinate system. In physics vector is anywhere on space. But in linear algebra it's always from origin. There are co-ordinate of vectors, they are pair of number which tells how to get from tail to tip.

$\begin{bmatrix} x \\ y \end{bmatrix}$

x → how far in x-axis, +ve :- R, -ve :- L

y → how far in y-axis, +ve :- ↑, -ve :- ↓

Vector Addition

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$$

Vector Multiplication

On multiplying, it causes scaling of vector.

Span & Basis

The span of $\vec{v}$ & $\vec{w}$ is the set of all their linear combinations.

$$a\vec{v} + b\vec{w}$$

The basis of a vector space is a set of linearly independent vectors that span the full space.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

Linear Transformation

$\underbrace{\text{Transformation}}_{\text{function}}$

$$\begin{bmatrix} 5 \\ 7 \end{bmatrix} \quad L(\vec{v}) \quad \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Vector input   Vector Output

Determinant

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \Rightarrow ad - bc$$

Inverse

$$\frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Rank

No of dimension in the output or column space.

Dot Product

$$\begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 2 \\ 8 \end{bmatrix}$$

Dot Product

Cross Product

Determinant Method

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - B_y A_z) - \hat{j}(A_x B_z - B_x A_z) + \hat{k}(A_x B_y - B_x A_y)$$

Eigen Values & Eigen Vector

Let A be the square matrix of order $n \times n$, then a number (Real or Complex) λ is said to be Eigen value of matrix A if there exist a column matrix X of order $n \times 1$ such that

$$AX = \lambda X$$

$X \rightarrow$ Eigen vector of A

$\lambda \rightarrow$ Eigen value of A

$$AX - \lambda X = 0$$

$$(A - \lambda I)X = 0$$

Working to find Eigen Value & Eigen Vector

Characteristics eqn of matrix A:

$$|A - \lambda I| = 0$$

Square matrix

Solution of characteristic eqn is called eigen values

Eigen Vector

$$\lambda = \lambda_1, \lambda_2, \lambda_3$$

$$(A - \lambda_1 I)X = 0$$

Solution of I for any Eigen value λ_1 i.e X is called eigen vector of matrix A corresponding to $\lambda = \lambda_1$