

Given a running stream of numbers, compute mean & variance at any given point.

Soln

Suppose the number are,

$x_1, x_2, x_3, x_4, \dots, x_n$

$$E(X) = M = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2}, \text{ Std dev} = \sigma^2$$

We can't use this formula, because if at any time when numbers increases or decreases the mean changes.

So,

$\mu \rightarrow \mu_{\text{new}}$

Time :- $O(N)$

So this a slower process. Computing mean time & again.

So, a better solution is :-

$$\begin{aligned}
\sigma^2 &= \frac{1}{n} \left[\sum_{i=1}^n (x_i - \mu)^2 \right] \\
&= \frac{1}{n} \left[\sum_{i=1}^n x_i^2 + \sum \mu^2 - 2\mu \sum_{i=1}^n x_i \right] \\
&= \frac{1}{n} \left[\sum x_i^2 + n\mu^2 - 2\mu \sum_{i=1}^n x_i \right] \\
&= \frac{\sum x_i^2}{n} + \mu^2 - 2\mu^2 \left[\because \frac{\sum x_i}{n} = \mu \right]
\end{aligned}$$

$$\sigma = \sqrt{E(X^2) - E(X)^2}$$

Mean when all
number are squared

So, now it is easier.

$$E(X), \text{ Mean} = \frac{\sum x}{N}$$

$$E(x^2) = \frac{\sum x^2}{N}$$

For each new number
add the number and
get the result