Homework 4 Part 1 Language Modeling using RNNs

11-785: Introduction to Deep Learning (Spring 2023)

OUT: March 31, 2023 DUE: April 28, 2023, 11:59 PM Early Bonus: April 15, 2023, 11:59 PM

Start Here

• Collaboration policy:

- You are expected to comply with the University Policy on Academic Integrity and Plagiarism.
- You are allowed to help your friends debug
- You are allowed to look at your friends code
- You are not allowed to type code for your friend
- You are not allowed to look at your friends code while typing your solution
- You are not allowed to copy and paste solutions off the internet
- You are not allowed to import pre-built or pre-trained models
- You can share ideas but not code, you must submit your own code. All submitted code will be compared against all code submitted this semester and in previous semesters using MOSS.

We encourage you to meet regularly with your study group to discuss and work on the homework. You will not only learn more, you will also be more efficient that way. However, as noted above, the actual code used to obtain the final submission must be entirely your own.

• Overview/TL;DR:

- Part 1: All of the problems in Part 1 will be graded on Autolab. Download the starter code from Autolab.
- Single Word prediction: Complete the function predict in class TestLanguageModel in the Jupyter Notebook.
- Generation of Sequence: Complete the function generate in the class TestLanguageModel in the Jupyter Notebook.
- Beam Search [Bonus]: Use beam search on prediction and generation tasks.

Homework Objectives

- If you complete this homework successfully, you would ideally have learned
 - Learn how to train Recurrent Neural Network models for generating text.
 - Learn about various techniques to regularize Recurrent Neural Networks like Locked Dropout,
 Embedding Dropout, Weight Decay, Weight Tying, Activity Regularization
 - You will understand the workings of Language Modeling and you will also train one to generate next-word as well as entire sequences!

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IMPORTANT NOTE

The writeup is about modelling written language. Since we may consider either characters or words to be the units of written language, we generally talk in terms of "tokens", where a token may be a word or character, depending on what we model.

The examples provided are in terms of characters (which we will use in HW4P2), but this homework itself (HW4P1) actually models language in terms of words. Please do not let this confuse you; we expect you to generalize the concepts presented using character sequences as examples to apply to word sequences.

The writeup also includes several listings of pseudocode. These are only intended to present the logic behind the described approaches. The pseudocode is *not* in python form, and may not even be directly translatable to python. It is merely a didactic tool; you will have to determine how to implement the logic in python.

And last, but definitely not the least: The writeup is long and detailed and explains the underlying concepts. It includes explanations and pseudocode that are not directly related to the homework, but are intended to be instructive and assist in understanding the homwork. The actual homework problems to solve are listed in two of the sections (or subsections, or subsubsections, we aren't actually sure which). Please read the document carefully and identify these sections.

1 Introduction

Key new concepts: Language modeling, text generation, regularization techniques for RNNs

Mandatory implementation: RNN-based Language model, text prediction and text generation using your trained model, Greedy Decoding.

Restrictions: You may not use any data besides that provided as part of this homework; You may not use the validation data for training; You have to use at least greedy decoding (and optionally beam search) for both text prediction and text generation.

1.1 Overview of Language Modeling

In Homework 4 Part 1, you will learn how to use a Recurrent Network (ideally an LSTM-based network) to model language, and generate text. You will also learn about the various techniques we use to regularize recurrent networks and improve their performance.

Language modelling is, quite literally, the task of modelling language. From a statistical point of view, which is the one we will take, it is the problem of accurately estimating the probability distribution of sentences (or, more precisely, token sequences) in the language¹.

At its core it is no different from, say, finding the probability distribution of a loaded dice, except that unlike the usual dice with only six sides, this dice has infinite sides, one corresponding to every token sequence from the language. This little difference is what complicates matters – unlike the usual dice, where you can store the entire probability distribution in a table with six entries, the distribution for language cannot be stored in a table – we would need an infinite-sized table for it. Instead, we will learn a model for the distribution – specifically, a neural network model. We refer to such models for the distribution of word sequences in a language as a language model.

Here are some things you would be able to do if you knew the entire probability distribution of a discrete random variable:

- 1. Determine the probability of any given outcome;
- 2. Draw samples from the distribution;
- 3. Rank order outcomes by probability to identify the most likely or the n^{th} most likely outcome;
- 4. Compute various expectations and other statistics.

A model-based probability distribution enables 1 and 2 above (we can use it to compute the probability of any given token sequence; we can also use it to draw samples from the distribution), however 3 and 4 are infeasible².

In this homework we will learn how to *train* a neural-network language model from data. We will also learn how to use it to *compute the probabilities* of specified token sequence, and to *draw* (i.e. generate) token sequences from the language.

1.1.1 A note on beginnings and endings

Language models model the distribution of token sequences. A token sequence is a *complete* string of tokens that has a definitive beginning and an end. For instance, consider the following word sequence (where the tokens are words): "four score and". Is this a *complete* word sequence, or is it merely a part of a longer sentence? From its appearance, it looks incomplete – as it is. This sequence is the first three words of the first sentence of Abraham Lincoln's famous Gettysburg address: "Four score and seven years ago our fathers

¹Language models may be estimated over *any* type of tokens, e.g. words, characters, byte-pairs etc. In this homework we will focus on word-based LMs. However, to remain agnostic to the type of unit (character, word, byte-pair etc) whose sequences are being modelled, we will generically refer to the symbols as *tokens*.

²In order to rank order outcomes by probability, or to compute expectations, we must consider *all* infinite possible outcomes, which is generally infeasible.

brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal." When modelling language, we would, in fact, be attempting to model the statistical properties of the *entire* underlying sentence, not merely some arbitrarily chosen section (such as the first three words) of it. Thus, here, "Four score and" is *not* a complete sequence; it is merely a portion of a much longer, complete sequence of words.

On the other hand, it is also entirely likely that somewhere, somewhen, a novice actor playing the part of Lincoln went on stage for the first time and held forth "Four score and", got nervous, and quit and went home. Now the *complete* sentence was merely this short sequence of words.

How do we distinguish between the two? In one case, the three words did *not* constitute the entire word sequence, while in the other case they did.

We do this through the use of *start of sequence* and *end of sequence* markers. A *start of sequence* marker, often represented as <sos> indicates that a token sequence has just begun. The symbol immediately following the <sos> tag is, in fact, the first token in the sequence; the <sos> marker is merely the indicator of the start of a new sequence. Similarly, the *end of sequence* marker, often represented as <eos>, indicates the *end* of a complete sequence. Again, the <eos> tag is not the actual final token in the sequence; it is only a marker that indicates the end of a complete sequence.

Using these tags Abraham Lincoln's complete sentence would be:

<sos> Four score and seven years ago our fathers brought forth on this continent, a new nation,
conceived in Liberty, and dedicated to the proposition that all men are created equal.

Here, the section "Four score and" is clearly not the full sequence, but is a section of a longer sequence. This is also identifiable from visual inspection – "Four score and" by itself is missing the $\langle \cos \rangle$ and $\langle \cos \rangle$ tags, and so cannot be complete.

On the other hand, our nervous actor's complete monologue consisted of

<sos> Four score and <eos>

That was the entire sequence as indicated by the $\langle sos \rangle$ and $\langle eos \rangle$ tags.

As a matter of convenience, it is sometimes simpler to just have the same tag to indicate both the start and end of sequences. With this standard, using the symbol <eos> to tag both, our complete sequences would be.

<eos> Four score and seven years ago our fathers brought forth on this continent, a new nation,
conceived in Liberty, and dedicated to the proposition that all men are created equal.<eos>
<eos> Four score and <eos>

From the perspective of language modelling, this means that we must explicitly include the (<sos> and) <eos> tags to our sequences:

- We must explicitly include <code><sos></code> and <code><eos></code> to the neural network's vocabulary. So, for instance, if we are building a language model over characters, in addition to the alphabet (a-z and A-Z) and punctuation marks and spaces ('.', ',', ' ', etc. which are all part of your text), our vocabulary must be expanded to include <code><sos></code> and <code><eos></code> as well, so now our vocabulary would be: <code>{a-z , A-Z , ',' , ';' , ',' , · ', · <sos>, · <eos>}.</code>
- When training the LM we must include the tags in our training sequences. So, for instance, given a "Four score and" as a complete sequence (the product of our nervous actor), we must actually represent it as "<sos> Four score and <eos>".
- When we use the LM to *compute the probability* of a (complete) token sequence, we must ensure it includes the tags. So, if we were to want to compute the probability of 'Four score and' as a

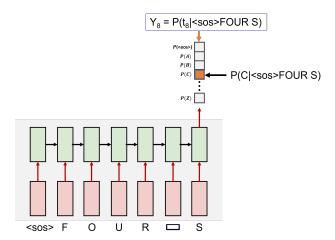


Figure 1: Given the length 7 input "<sos>FOUR S" (including the space), the RNN outputs P(T|<sos>FOUR S), the probability distribution for the eighth character in the sequence given the input. P(C|<sos>FOUR S), the actual probability of having a "C" in the 8th position must be read from this distribution, as shown by the highlighted red box. The RNN figure uses the same pictorial representation as HW3P2.

complete sequence, this must be converted to "<sos> Four score and <eos>", and the probability of the longer five-token sequence must be computed.

• When we *generate* sequences using the LM, the generation can only be considered complete when the terminal <eos> marker is generated. So, "<sos> Four score and" will not be considered a complete generated sequence, but "<sos> Four score and <eos>" will be.

1.1.2 Computing the probability of a complete token sequence

Let us now consider how we might *compute the probability* of a token sequence $\langle sos \rangle$, $t_1, t_2, \dots, t_N, \langle eos \rangle$ (recall that if we are just given a sequence t_1, \dots, t_N we must add $\langle sos \rangle$ at the beginning and $\langle eos \rangle$ at the end of the sequence to make it complete).

We assume that the probability is being computed by a neural-network language model. To do so, we first decompose the probability using Bayes rule as:

$$P(< sos>, t_1, t_2, \cdots, t_N < eos>) = P(< sos>)P(t_1| < sos>)P(t_2| < sos> t_1) \cdots P(t_N| < sos> t_1, \cdots, t_{N-1})P(< eos> | < sos> t_1 \cdots t_N)$$

Here $P(t_n | < sos > t_1, \dots, t_{n-1})$ is the probability that t_n is the n^{th} token in a sequence, given that $< sos > t_1, \dots, t_{n-1}$ are the first n-1 tokens from the beginning (i.e. from < sos >). By this notation, P(< sos >) is the probability that < sos > is the inital token in a complete sequence. Since < sos > is always the beginning of a complete sequence, P(< sos >) = 1.

The rest of the probability terms must be computed by a neural network and have the form $P(t_n | < sos > t_1, \dots, t_{n-1}; \theta)$. Here θ represents the parameters of the network. So using this mechanism, and considering that P(< sos >) is always 1.0, we can write:

$$P(\langle \operatorname{sos}\rangle, \operatorname{t}_1, \operatorname{t}_2, \cdots, \operatorname{t}_N \langle \operatorname{eos}\rangle; \theta) = P(\operatorname{t}_1 | \langle \operatorname{sos}\rangle; \theta) P(\operatorname{t}_2 | \langle \operatorname{sos}\rangle \operatorname{t}_1; \theta) P(\operatorname{t}_3 | \langle \operatorname{sos}\rangle \operatorname{t}_1 \operatorname{t}_2; \theta)$$

$$\cdots P(\langle \operatorname{eos}\rangle | \langle \operatorname{sos}\rangle \operatorname{t}_1 \cdots \operatorname{t}_N; \theta)$$

$$(1)$$

 $P(t_n | < sos >, t_1, \dots, t_{n-1}; \theta)$ looks familiar – it is the probability assigned to symbol t_n by a recurrent network that has obtained the sequence $< sos >, t_1, \dots, t_{n-1}$ as inputs. This is illustrated by Figure 1.

By extrapolation, since the network is recurrent, all of the probabilities $P(t_1 | < sos > ; \theta)$, $P(t_2 | < sos > t_1; \theta)$, $P(t_3 | < sos > t_1t_2; \theta)$, \cdots , $P(< eos > | < sos > t_1 \cdots t_N; \theta)$ can be computed using the same neural network, as shown by Figure 2.

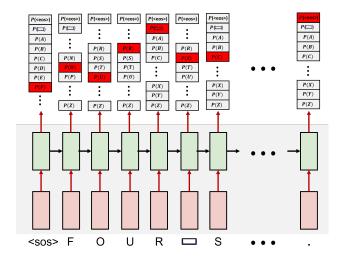


Figure 2: To compute the probability of the entire first sentence of the Gettysburg address, the entire sequences of characters (from the first <sos> until, but not including the final <eos>) is fed into the network. At each step, the network outputs the probability distribution for the next character in the sequence. At each time we select the probability assigned to the next character in the sequence (highlighted in red). The final probability selected is the probability assigned to <eos> at the final period in the sentence. The product of all of the selected (highlighted) probabilities is the total probability of the sentence.

1.1.3 The Token Embedding

The math and the figure above do not clarify how we actually represent the tokens ($\langle sos \rangle$, $\langle eos \rangle$ and the individual tokens, be they words or characters or whatever else) to the network.

The obvious way is to simply represent every token in the vocabulary (including <sos> and <eos>) as a one-hot vector. As discussed in class, this representation assigns the same length to the vector for any word, and the same distance between any two words (i.e. between their vectors). This way, we make no assumptions about the relative distance between words or the relative importance of words.

However, the one-hot vector approach is incredibly wasteful. The issue is the high dimensionality: For example, a vocabulary of one million words would require a million-dimensional vector! In order to deal with this problem, we typically *project* the one-hot vectors down to a lower-dimensional space, using a linear transformation. Thus, each D-dimensional one-hot vector T (D = 1000,000 in our example) is converted to a reduced N-dimensional vector E, by multiplying it by a $N \times D$ projection matrix P, such that E = PT. To implement this, The linear projection P can itself be implemented as a $N \times D$ linear layer, which applies to every input. Note that the actual input to the recurrent layers is now E = PT.

Thus, the *actual* neural network that models language would look like Figure 3, with an explicit projection layer to derive token embeddings, prior to passing them to the recurrent layers.

This gives us the following algorithm for computing the probability of any complete token sequence t_1, \dots, t_N using a recurrent neural network.

- Prepend $\langle sos \rangle$ and append $\langle eos \rangle$ to the token sequence to get $\langle sos \rangle$, $t_1, t_2, \cdots, t_N, \langle eos \rangle$.
- Pass the entire sequence $\langle sos \rangle$, t_1, \dots, t_N through the network. It will compute a probability distribution over the vocabulary (symbol set) of the model at each time step. This vocabulary must include $\langle eos \rangle$ (and $\langle sos \rangle$, if it is separate from $\langle eos \rangle$).
- Within the network, at each timestep $n = 0, \dots, N$ (where $t_0 = \langle sos \rangle$ and $t_{N+1} = \langle eos \rangle$):
 - Compute the embedding for the $n^{\rm th}$ token
 - Pass it into the network to compute the probability distribution for the $(n+1)^{\text{th}}$ token,

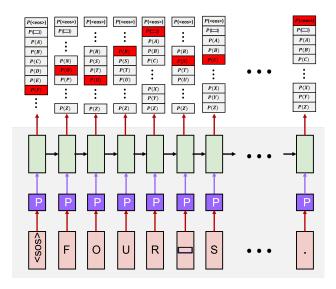


Figure 3: The orange boxes at the bottom represent one-hot embeddings of the input tokens (shown inside the boxes). These are passed through a projection layer (the purple boxes with P) to generate *embedding* vectors that are passed to the recurrent layers.

- Read out the probability for t_{n+1} from the output probability distribution. Note that the final token $t_{N+1} = \langle \cos \rangle$.
- Multiply the sequence of probabilities.

The entire operation is given by the following pseudocode. Note that we are actually computing the *log* probability to avoid underflows. Also the pseudocode is referring to words directly, rather than their dictionary indices (which is what you would have in a real implementation).

Pseudocode 1: Computing the probability of a word sequence

```
# Takes in token sequence T and returns its log probability
function LogProb = ComputeTokenSequenceLogProbability(T)
   LogProb = 0
   h = initial_h # Assuming that h[-1] = hinit is part of the model
   i = 0 # Assuming T[0] = <sos>
   do
        E = P*onehot(T[i]) # P is the projection matrix from one-hot vectors to embeddings
        [Y, h] = RNNstep(h, E)
        LogProb = LogProb + log(Y[T[i+1]])
        i = i + 1
   until (T[i] = <eos>) # Stop when we hit the end of the sequence
   return LogProb
end
```

What is the projection matrix P? There are several ways to approach P, including *finite* approaches like the soft bag approach, which predicts words based on their immediate context, and the skipgram approach, which predicts adjacent words based upon the current word at hand, as explained in the lecture slides. Here, in this homework, we are using RNNs for language modeling, and P will be considered part of the model and learned along with all the other parameters of the network.

1.1.4 Drawing a sample from a Language Model (a.k.a. generating text)

The other key ability required by a model for any distribution is to enable drawing samples from it. Since an LM is actually probability distributions over token sequences, it must enable us to draw samples from

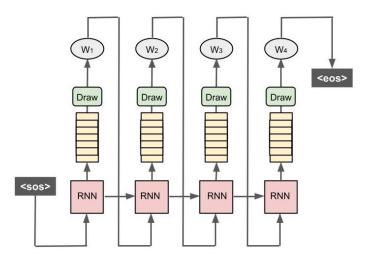


Figure 4: Generating text with the LM. At each time step a word is drawn from the output distribution and input back into the recurrent block at the next time. Although not explicitly shown, we actually input the *embedding* of the word to the RNN block

this distribution as follows:

$$\langle \cos \rangle, t_1, t_2, \cdots, t_N, \langle \cos \rangle \sim P(\langle \cos \rangle, t_1, t_2, \cdots, t_N \langle \cos \rangle; \theta)$$

This is actually the task of *generating text*. So how exactly do we use our network to produce text?

If you recall from Section 1.1.2, the LM models the probability of a token sequence $\langle sos \rangle$, $t_1, t_2, \dots, t_N, \langle eos \rangle$, by incrementally decomposing it using Bayes rule as:

$$P(\langle \operatorname{sos}\rangle, \operatorname{t}_1, \operatorname{t}_2, \cdots, \operatorname{t}_N \langle \operatorname{eos}\rangle; \theta) = P(t_1 | \langle \operatorname{sos}\rangle; \theta) P(\operatorname{t}_2 | \langle \operatorname{sos}\rangle \operatorname{t}_1; \theta) \cdots$$

$$P(t_N | \langle \operatorname{sos}\rangle \operatorname{t}_1, \cdots, \operatorname{t}_{N-1}; \theta) P(\langle \operatorname{eos}\rangle | \langle \operatorname{sos}\rangle \operatorname{t}_1 \cdots \operatorname{t}_N; \theta)$$

Here $P(t_n|<\cos>t_1,\cdots,t_{n-1};\theta)$ is the probability that t_n is the $n^{\rm th}$ token in a sequence, given that $<\cos>t_1,\cdots,t_{n-1}$ are the first n-1 tokens from the beginning (i.e. from $<\cos>$).

What this tells us is that the sequence $\langle sos \rangle, t_1, t_2, \cdots, t_N, \langle eos \rangle$ is drawn from the following probability distribution:

$$<$$
sos $>$, t_1, t_2, \cdots, t_N , $<$ eos $> \sim P(t_1 | <$ sos $>$; $\theta)P(t_2 | <$ sos $> t_1; \theta) \cdots$
$$P(t_N | <$$
sos $> t_1, \cdots, t_{N-1}; \theta)P(<$ eos $> | <$ sos $> t_1 \cdots t_N; \theta)$

It follows naturally from this equation that each token in the sequence must be drawn from its corresponding distribution:

$$t_i \sim P(t_i | \langle sos \rangle t_1 t_2 ... t_{i-1}; \theta)$$

I.e. the generation is itself incremental: each token in the sequence is drawn from its own conditional distribution, and then used to condition the distributions of subsequent tokens (from which they are drawn). In terms of the neural-net LM, this means that the n^{th} token in the sequence must drawn from the distribution output by the network at the n^{th} timestep, and then fed back into the network as input to influence the network outputs for subsequent tokens. The generation procedure is illustrated in Figure 4. Note that the figure does not explicitly show the projection matrix P, which is always implicitly assumed.

The following pseudocode explains the procedure. Note that the first symbol is <sos>. Also note how even the generation uses the embedding matrix P. DrawFromDistribution(Y) is any routine for drawing

a sample from a probability distribution Y. The pseudocode assumes you are directly drawing a token. In reality you would be drawing its index and recovering the actual token from a dictionary.

Pseudocode 2: Generating a token sequence

```
# Generates a sequence TokenSeq
function TokenSeq = Generate()
    TokenSeq[0] = <sos>
    h = hinit # Initial value of hidden state
    i = 0 # Assuming TokenSeq[0] = <sos>
    do
        E = P*onehot(TokenSeq[i]) # P is the projection matrix from one-hot vectors to embeddings
        [Y, h] = RNNstep(h, E)
        NewToken = DrawFromDistribution(Y)
        i = i + 1
        TokenSeq[i] = NewToken
    until (TokenSeq[i] = <eos>) # Stop when we hit the end of the sequence
    return TokenSeq
end
```

1.2 Training the Language Model

Let us now consider how we would train the neural language model that computes $P(t_n | < eos > t_1, \cdots, t_{n-1}; \theta)$.

Recall that the LM is actually just a parametric model for a probability distribution. So we will use the standard approach for training parametric probability-distribution models – maximum likelihood estimation. In the following sections we first outline the concept of maximum-likelihood estimation (MLE) and how it applies to training neural LMs.

1.2.1 Maximum Likelihood Estimation

Maximum-Likelihood Estimation, or MLE is a general method for estimating a parametric probability distribution for a random variable from training data.

Let X be any random variable (e.g. the outcome of a roll of dice, or the height of people in a population, or sentences from a language). Our objective is to estimate the probability distribution of X.

To estimate the probability distribution of the random variable, we we assign it a parametric model $P(X;\theta)$ (where θ are the parameters of the model), such as a category distribution (e.g. for the roll of dice; the parameters are the probabilities of the six sides), a Gaussian (e.g. for the height of people; the parameters are the mean and variance of the Gaussian), or, as in our case, a neural networks (e.g. for Language; here the parameters of the distribution are the parameters of the network).

Then, given a collection of training samples X_1, X_2, \dots, X_N of the variable (e.g. a number of rolls from the dice, or the heights of a number of people from the population, or a large number of sentences from the language), we attempt to estimate the parameter θ such that $P(X;\theta)$ best explains our training data X_1, X_2, \dots, X_N .

The governing principle behind maximum likelihood estimation is that whatever training data we have observed are *highly typical* of the process. In fact, the data are *so* typical that of all the models in the world, the one that assigns them the highest probability is the one we will assume to be their underlying distribution.

So, in keeping with this principle, we choose the parameter θ such that $P(X;\theta)$ assigns the most probability to our training data:

$$\hat{\theta} = \arg\max_{\alpha} P(X_1, X_2, \cdots, X_N; \theta)$$

where $\hat{\theta}$ is our estimate.

Often, we can assume that the individual training instances are independent (and identically distributed). In this case $P(X_1, X_2, \dots, X_N; \theta) = \prod_i P(X_i; \theta)$.

$$\hat{\theta} = \arg\max_{\theta} \prod_{i} P(X_i; \theta)$$

Maximizing a product of many terms can be difficult. To simplify matters, we note that the logarithm converts products to sums. Morever, it is monotonic – the maximum of a function occurs at the same value of the argument as the maximum of the log of the function. This leads us to the following rule for maximum likelihood estimation of the model parameter θ :

$$\hat{\theta} = \arg\max_{\theta} \sum_{i} \log P(X_i; \theta)$$

1.2.2 Applying the MLE to training an LM

We now move to the task of applying the Maximum Likelihood Estimation principle to training on a language model.

Suppose we have a set $S = \{S_1, S_2, \dots, S_K\}$, where each S_i is a training token sequence: $S_i = (\langle sos \rangle, T_{i,1}, T_{i,2}, \dots, T_{i,l_i}, \langle eos \rangle)$, where l_i is the length of the sequence. Recall that you must attach $\langle sos \rangle$ and $\langle eos \rangle$ to the start and end of every sequence you are provided, so we assume here that those are already included in the sequences.

From Equation 1) (Section 1.1.2) the log probability of any sequence S_i is given by:

$$\log P(S_i \theta) = \sum_{t=1}^{l_i+1} \log P(T_{i,t} | < sos >, T_{i,1}, \dots, T_{i,t-1}; \theta)$$
 (2)

Here we have assumed that $T_{i,l_i+1} = \langle eos \rangle$.

Assuming all of the training sequences are independent, the total log probability of the training data is:

$$\log P(S; \theta) = \sum_{i} \log P(S_i; \theta)$$

Note that since the training data S are given, this is actually just a function of θ .

By the MLE principle, our estimate of θ is given by

$$\hat{\theta} = \arg\max_{\theta} \sum_{i} \log P(S_i; \theta)$$

This is given in the form of a maximization. To convert it to the standard loss minimization format, we can define our loss as the negative of $\log P(S;\theta)$ to get:

$$Loss(\theta) = -\sum_{i} \log P(S_i; \theta)$$

where $\log P(S_i; \theta)$ is given by Equation 2. Note that this is just the NLL (negative log likelihood) loss. The actual optimization is performed as

$$\hat{\theta} = \arg\min_{\theta} Loss(\theta)$$

This can be minimized using gradient descent, as always.

The following pseudocode explains how the loss is computed. The *current* estimate of the parameter, θ , is explicitly shown in the code, for clarity. The code is split into two routines, one which computes the loss for a single sequence, and the second which aggregates loss over a set of sequences. Note that the first routine SequenceLoss() is nearly identical to ComputeSequenceProbability() from Section 1.1.3.

Pseudocode 4: LM training loss for a set of training sequences

```
# Give a set of sequences TrainingSequences = {S1, S2, ..., SN} and model $\theta$, computes the
    loss
function Loss = ComputeLoss(TrainingSequences, theta)
    Loss = 0
    for S in TrainingSequences:
        Loss = Loss - SequenceLoss(S, theta)
    end
    return Loss
end
```

1.3 Evaluating your Language Model

How exactly do you evaluate whether the language model you have trained is good?

There are several methods. Here are two simple techniques:

1.3.1 Predicting the next token

The LM learns to compute $P(T_{k+1}| < sos >, T_1, T_2, \cdots, T_k; \theta)$, i.e. the probability distribution of the *next* token in a sequence T_{k+1} , given the entire sequence until the current token T_k . All of the other probabilities are composed from these incremental probabilities. So the obvious way to test the quality of your LM is to simply verify how accurately it computes this term, $P(T_{k+1}| < sos >, T_1, T_2, \cdots, T_k; \theta)$.

Say that the LM has been well trained and has learned the structure of language well. Then, given the first K words of any natural sequence of tokens, it would ideally assign the highest probability (or, at least a high probability) to the true next token. For instance, if it were given the word sequence "<sos> Four Score and Seven Years", and asked to compute the probability distribution of the next word (ref. Figure 1), it would ideally assign the highest probability to the word "ago". (Four score and seven is part of a quote from the Gettysburg address, which begins with "<sos> Four score and seven years ago".) In other words, P(ago|<sos> four score and seven; θ) would ideally be assigned a high probability, if not a probability greater than P(W|<sos> four score and seven; θ) for any other word W.

So, a good test would be as follows:

For a large test set of partial token sequences (or word sequences) of the kind $\langle sos \rangle$ T₁ T₂ ··· T_k, where we also know the next token T_{k+1} in the sequence, compute the probability distribution for the $(k+1)^{th}$ token. If, over the entire test set, the average predicted log probability for the last token (in the sequence)

is large enough, the LM may have been well estimated. (Note that this example is a *supervised* test. We know the next token T_{k+1} , and we are evaluating the LM by how well it predicts it.)

The pseudocode below can be used to compute the (log) probability for any given partial token sequence. The predicted log probability of the actual next token is simply $\log Y[T_{k+1}]$.

Pseudocode 5: Probability distribution of the next token

```
# Given a partial token sequence Tpartial = <sos> t1 t2 ... tK, compute probability distribution
    of next token
function Y = Predict(Tpartial)
    h = initial_h # Initial value of hidden state
    # The partial sequence has symbols Tpartial[0] ... Tpartial[K], where Tpartia[0] = <sos>
    for i = 0:K
        E = P*onehot(Tpartial[i])
        [Y, h] = RNNstep(h, E)
    end
    return Y
```

1.3.2 Sequence Completion

A second method to test the language model is to evaluate how well it *extends* (or completes) partial token sequences. For the purpose of explanation, we will assume tokens are words.

The test scenario: We are given a partial word sequence, e.g. "<sos> four score and". We use the LM to extend N words in the sequence (or to complete the sequence). The initial sequence of words is input to the network. We ignore the first few outputs of the network, and only begin drawing words from the output at the last word in the sequence. Generation continues until the sequence has been extended by N words, or until an <eos> has been encountered. (Figure 5 illustrates this process.)

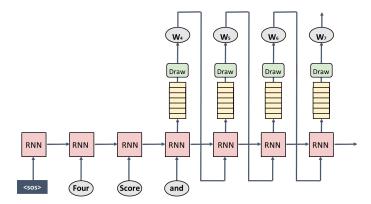


Figure 5: Given input sequence "<sos> four score and" the network must extend or complete the sequence. It does so by feeding in "<sos> four score and" in the first four time steps to the net. When the final word "and" is fed in, we begin drawing words from the output distribution for subsequent words w4, w5 etc.

The pseudocode below explains this procedure. Note that we have not yet defined what DrawToken() does (but you can continue reading to find out how it's defined).

Pseudocode 6: Completing a word sequence

```
# Given a partial word sequence Tpartial = <sos> w1 w2 ... wK, generates the next N words
function TokenSeq = Complete(Tpartial)
   h = initial_h # Initial value of hidden state
   # First run the provided partial sequence through the net
   # The partial sequence has symbols Tpartial[0] ... Tpartial[K], where Tpartia[0] = <sos>
   for i = 0:K
```

```
E = P*onehot(Tpartial[i])
       [Y, h] = RNNstep(h, E)
   end
   TokenSeq[0:K] = Tpartial[0:K] # The first K tokens are just copied over
   # Now draw the rest
   i = K
   for j = 1:N
       E = P*onehot(TokenSeq[i]) # P is the projection matrix from one-hot vectors to embeddings
       [Y, h] = RNNstep(h, E)
       NewToken = DrawToken(Y) # PLEASE SEE BELOW FOR AN EXPLANATION OF THIS FUNCTION
       i = i + 1
       TokenSeq[i] = NewToken
       if Tokenseq[i] == <eos>: break
   end
   return TokenSeq
end
```

If the LM is well trained, this extension (or completion) will be *natural*. "Natural", here, is unfortunately hard to quantify. If, however, we had access to an oracle (such as another LM trained on an extremely large amount of data that we could use to score our outputs), we could have that oracle score our completion; and if that oracle were to score our completion as high, the completions could be deemed reasonably natural and our LM could be good.

For such an oracle to work, we must ideally complete the sequence as best as we can. In principle, even a random draw is likely to be plausible if our LM is accurate, and DrawToken(Y) could just be a random draw from the distribution Y (which we referred to as DrawFromDistribution() in Section 1.1.4).

A more reasonable approach, though, is to try to find an extension that is highly probable according to our LM. If the LM is well trained, high-probability extensions are likely to be closer to the distribution of our training data, and are hence more likely to be plausible. The DrawToken(Y) function here would draw the most probable token at each time:

```
function t = DrawToken(Y)
   return Token[argmax(Y)]
end
```

Alternatively, we could try to draw the *most likely* extension of the word sequence. However, as we recall from 1.1, neural-network based LMs do not facilitate identifying the rank-ordering of sentences by probability, nor do they facilitate finding the most probable sentence. There is no clean solution to guessing the sequence, and so we must use some heuristics such as a beam search. We defer the discussion of beam search to HW4P2, as you will not be using it here.

2 Dataset

We will be training an RNN Language Model on the WikiText-2 Language Modeling Dataset, which has been pre-processed for HW4p1 and included in the template tarball:

- vocab.npy: a NumPy file containing the words in the vocabulary
- vocab.csv: a human-readable CSV file listing the vocabulary
- wiki.train.npy: a NumPy file containing training text

We also provide test files in the fixture directory which will make up the test cases for the prediction and generation functions you will implement. You don't need to worry about these files, test scripts that process

them are already set up for you.

The train and validation files each contain an array of articles. Each article is an array of integers, corresponding to words in the vocabulary. There are 579 articles in the training set.

As an example, the first article in the training set contains 3803 integers. The first 6 integers of the first article are [1420 13859 3714 7036 1420 1417]. Looking up these integers in the vocabulary reveals the first line to be: = Valkyria Chronicles III = <eol>

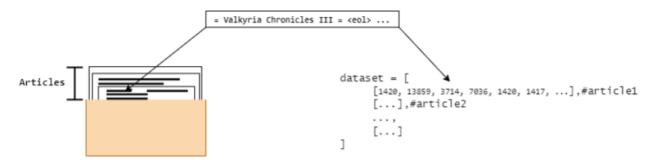


Figure 6: A little visualization of the dataset

<eol> (end of line) is a special token added to the vocabulary to specify the end of a given line.

A preview of the raw dataset (Wikitext 2) is also available on **huggingface.co** via the following link https://huggingface.co/datasets/wikitext/viewer/wikitext-2-raw-v1/train.

The dataset for this homework is provided as a collection of articles. So you may concatenate those articles to perform batching as described in the paper. It is advised to shuffle articles between epochs if you take this approach.

You will also receive a vocabulary file containing an array of strings. Each string is a word in the vocabulary. There are 33,278 vocabulary items.

Refer to Section 5 for more details about the testing procedure.

3 Problems

3.1 Prediction of a Single Word (50 points)

Complete the function prediction in class LanguageModel in the Jupyter Notebook.

This function takes as input a batch of sequences, shaped [batch size, sequence length]. This function should use your trained model and perform a forward pass.

Return your model's guess of what the next token might be. You should return the score for the last timestep from the output of the model. The returned array will assign a probability score to every token in the vocabulary, so the result should be in shape of [batch size, vocabulary size], of dtype float.

These input sequences will be drawn from the unseen test data. Your model will be evaluated based on the score it assigns to the actual next word in the test data. Note that scores should be raw linear output values. Do not apply softmax activation to the scores you return. Required performance for this task is a negative log likelihood of 5.2. The value reported by autolab for the test dataset should be similar to the validation NLL you calculate on the validation dataset. If these values differ greatly, you likely have a bug in your code.

3.2 Generation of a Sequence (50 points)

Complete the function generate in the class LanguageModel in the Jupyter Notebook.

As before, this function takes as input a batch of sequences, shaped [batch_size, sequence_length].

Instead of only scoring the next word, this function should generate an entire sequence of words. The length of the sequence you should generate is provided in the **timesteps** parameter. The returned shape should be [batch_size, timesteps]. This function requires sampling the output at one time-step and incorporate that in the input at the next time-step. You can call the function predict in class TestLanguageModel to get the score of prediction. Please refer to Recitation 8 for additional details on how to perform this operation. Your predicted sequences will be passed through tests that we have curated, and the NLL of your outputs will be calculated.

If your outputs make sense, they will have a reasonable NLL. If your outputs do not reasonably follow the given outputs, the NLL will be poor. Required performance for this task is a negative log likelihood of 2.75.

Note: Unlike Section 1.3.2, the generation function which you are required to implement should not draw $\langle sos \rangle$ or $\langle eos \rangle$ from the vocabulary, although your vocabulary and the embedding layer will include it. We instead require you to generate for a fixed number of timesteps T. Hence, when you are drawing your tokens for completion, please draw from the probability distribution without including $\langle sos \rangle$ and $\langle eos \rangle$. Hint: You can do this by slicing your probability distribution by the actual number of words in the vocabulary before you draw from the distribution.

4 Implementation

We break down the problem to these steps. First, implement the data loader. Then, set up the model with predict and generate functions. After we have both components, we provide guidance on training techniques.

4.1 Data Processing

As mentioned in section 2, the train and validation files for this assignment contain an array of articles. Each article is in turn an array of integers, corresponding to words in the vocabulary. The arrays of "words" are token sequences that our language model aims to model the distribution of.

Recall from section 1.1.1 that we need to distinguish partial and complete word sequences through the use of start of sequence and end of sequence markers. <sos>, the start of sequence marker, indicates that a token sequence has just begun and the end of sequence marker, <eos>, indicates the end of a complete sequence.

We could use different tokens for <sos> and <eos>, or we can also use the same tag, say <eos>, to indicate both the start and end of sequences.

Before we proceed with further data loading tasks, add these markers to each text segment in the training corpus.

4.2 DataLoader

As explained in section 1, your main task is to build a language model that you will use to perform text generation, which is the task of predicting the next word given a sequence of preceding words. Thus, your samples should be formatted in a certain way so as to successfully train for that task. Lets see an example.

Article: I eat a banana and an apple everyday including today.

The following may be considered as hyperparameters that you can tune at your convenience.

- sequence length = 3
- batch size = 2

Then one batch consists of 2 of pairs of sequences which are pairs of (input sequence, target sequence).

A batch would have 2 training samples with sequence length 3. The first batch looks like this:

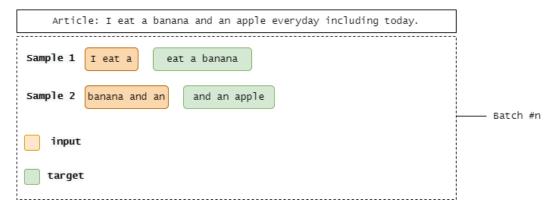


Figure 7: An example of batching and data loading

Pytorch's DataLoader class allows you to format your samples the way you desire. To do so, you will have to overwrite the __iter__ method (See starter notebook).

The __iter__ method should:

- 1. Randomly shuffle all the articles from the WikiText-2 dataset.
- 2. Concatenate all articles in the dataset.
- 3. Divide the complete corpus into inputs and targets, where targets are shifted by a window of 1 word.
- 4. Resize your inputs and targets into batches of articles based on batchsize.
- 5. Run a loop that yields a tuple of (input, target) in every iteration based on sequence length, with both these components being in shape (batchsize, seqlen). Look at iterators in python if this sounds unfamiliar.

4.3 Model Description

Your next task is to fill out the LanguageModel class. The LanguageModel class needs to have the following attributes, which are listed for you in the starter code:

- 1. self.token_embedding: As we mentioned in section 1.1.3, we need to represent the tokens somehow; we need to compute token embeddings. Use the Embedding class from PyTorch to define self.token_embedding. This class is a simple lookup table that stores embeddings of a fixed dictionary and size, and takes in num_embeddings, the size of the dictionary of embeddings and embedding_dim, the size of each embedding vector. Think about the parameters that the LanguageModel takes in (you should add more parameters!) and define num_embeddings and embedding_dim accordingly. The embeddings are randomly initialized and update as your model trains. You can use the instance of the nn.Embedding class (self.token_embedding) to get the corresponding embedding for an input sequence x.
- 2. self.lstm_cells: LSTM cells are recurrent units that are used as the building blocks for the layers of our RNN. PyTorch offers an LSTMCell class (different from the LSTM in HW3P2) which you can use to define these LSTM cells. An LSTM layer is equivalent to an LSTMCell that is looped over T timesteps on a batch of sequences. We use LSTMCells instead of LSTMs to break down the operations involved in generating probability distributions over the vocabulary more atomically (this will feed into HW4P2, where you will use the attention mechanism to add context to generate your probability distributions over vocaubulary). We have pre-written a single LSTMCell layer in self.lstm_cells in the starter code, but you are encouraged to add more layers of LSTM cells to the sequential container. Please refer

to the documentation in PyTorch to understand it's parameters. Once again, consider the parameters that the LanguageModel takes in (you should add more parameters!) and define input_size and hidden_size accordingly.

3. self.token_probability: As mentioned in section 1.1.3, at the nth timestep, our model needs to compute the probability distribution for the (n+1)th token. The output from the rnn_step method is the next hidden state in our RNN, which will have some hidden_size. This tensor will need to be projected to dimension vocab_size, to obtain a "score" for each token in our vocabulary, which self.token_probability will do, after which we will typically perform a softmax operation to obtain a probability distribution over the vocabulary. self.token_probability is defined as a linear layer in our starter code, which takes in two parameters, in_features, the size of each input sample and out_features, the size of each output sample. You are not required to use a softmax function on the raw logits returned by self.token_probability, as after 3 HWs you would be knowing that PyTorch implicitly handles softmax within its CrossEntropyLoss function. Think about what dimensions we are projecting from and into and define in_features and out_features accordingly.

There are also a number of functions within our LanguageModel that you are expected to fill out. These are as follows: rnn_step, predict, generate, and forward.

We will first discuss the rnn_step method. The rnn_step method iterates over our LSTMCell layers, defined in self.lstm_cells. At each iteration, our LSTMCell layers take in a token embedding at time step t and the hidden state at each layer and computes the next hidden state. The method returns the final hidden state output, and our final list of hidden states.

We now come to the predict and generate methods. As mentioned earlier in sections 3.1 and 3.2, predict returns our model's best guess of what the next token might possibly be and generate generates an entire sequence of words to extend the current input sequence. Sections 1.3.1 and 1.3.2 go into great detail on the steps needed to perform these high-level tasks, so please refer to them as well as section 3 when you are implementing this section.

We finally come to the forward method in our LanguageModel class. In this function, the model takes in one batch of inputs, x, from the dataloader, which has shape [batchsize, seglen].

We have defined two lists for you, token_prob_distribution, which will contain probability distributions for all timesteps, and hidden_states_list, to store the hidden states of our LSTM cells at timestep t.

We need to perform the following steps in forward:

- 1. The first step is to obtain the embedding of input x. Think about what attribute of the LanguageModel class stores our embeddings and how we would obtain these embeddings for the input x.
- 2. Following this, we iterate over our the tokens in our sequences, and perform the following steps. For each timestep $t = 0, \dots, N$, where N is the sequence length of our batch of token sequences:
 - (a) Obtain the token embeddings for the input x (all samples in our batch) at the time t.
 - (b) We then need to return the final hidden state output, rnn_out, and our final list of hidden states, hidden_states_list. Which method allows us to do this?
 - (c) As mentioned when previously, rnn_out is the final hidden state in our RNN. This needs to be projected to dimension vocab_size, to obtain a "score" for each token in our vocabulary. Therefore, this step will map the RNN output to the vocabulary's dimension and store it in token_prob_dist_t, the token probability distribution at time t. Think about what attribute of the LanguageModel allows us to perform this projection.
 - (d) Finally, append token_prob_dist_t to a list of token probability distributions.
- 3. Return the list of token probability distributions and the list of hidden states.

4.4 Training and Regularization Techniques

Since the WikiText-2 data set for HW4p1 has been pre-processed, we can focus on breaking the text down into tokens for training the RNN. As you may have noticed in HW3, a good initialization significantly improves training time and the final validation error.

To achieve better performance on your language model, you will have to use regularization methods such as Locked Dropout, Weight tying, Embedding Dropout, and also Data Augmentation. For structuring and training your model, we would like you to follow the protocols in Regularizing and Optimizing LSTM Language Models as closely as you are able to, in order to guarantee maximal performance. You are not expected to implement every method in this paper, and the regularization techniques (below) will be sufficient to achieve performance to pass on Autolab.

Ensure your implementation is consistent with the batching strategy you choose (batch first or batch last), since you may be required to transpose the outputs of your prediction and/or generation models.

4.4.1 Locked Dropout

In standard dropout, a new binary dropout mask is sampled every time when the dropout function is called. For example, the input x0 to an LSTM at timestep t=0 receiving a different dropout mask than the input x1 fed to the same LSTM at t=1.

A variant of this, locked dropout, available in torchnlp, also called variational dropout (Gal & Ghahramani, 2016), samples a binary dropout mask only once upon the first call. It then repeatedly uses that locked dropout mask for all inputs and outputs of the LSTM within the forward and backward pass.

A thing to notice is that each sample within the mini-batch should use a unique dropout mask. It will ensure the diversity in the elements dropped out. Please also think about on which dimension you should apply the mask.

4.4.2 Embedding Dropout

Embedding dropout (Gal & Ghahramani (2016)) is to perform dropout on the embedding matrix at a word level. The dropout is broadcasted across all the word vector's embedding. The remaining non-dropped-out word embeddings are scaled by $\frac{1}{1-p_e}$ where p_e is the probability of embedding dropout. Embedding dropout is performed for a full forward and backward pass, which means all occurrences of a specific word will disappear within that pass.

Note that if you use torch.nn.Embedding for the embedding layer, the input to that layer is an integer and not a vector. Therefore, to implement embedding dropout, we need to first convert the input to a one-hot vector (look into torch packages for how), apply regular Dropout on it, and then manually multiply it with the weights of the embedding layer to complete the conversion to embeddings.

4.4.3 Weight Tying

Weight tying (Inan et al., 2016; Press & Wolf, 2016) shares the weights between the embedding and softmax layer, substantially reducing the total parameter count in the model. The technique has theoretical motivation (Inan et al., 2016) and prevents the model from having to learn a one-to-one correspondence between the input and output, resulting in substantial improvements to the standard LSTM language model.

To implement weight tying, you can manually link the weight component of two different layers to be the same object so that they will have the same weights.

4.4.4 Activation Regularization (AR)

Activation Regularization is regularization performed on activation functions. Different from L2-Regularization, which is performed on weights, AR penalizes activation functions that are significantly larger than 0. It is defined as:

$$\alpha L_2(m\odot h_t)$$

where m is the dropout mask, L_2 is L2 norm, h_t is the output of the RNN at timestep t, and α is a scaling coefficient.

4.4.5 Temporal Activation Regularization (TAR)

Temporal Activation Regularization is a kind of slowness regularization for RNNs that penalizes the model from producing large changes in the hidden state. It is defined as:

$$\beta L_2(h_t - h_{t+1})$$

where m is the dropout mask, L_2 is L2 norm, h_t is the output of the RNN at timestep t, and β is a scaling coefficient.

To implement AR and TAR, you need to calculate these regularization terms from layers you'd like to be regularized, and add any such term as an additional loss to the raw loss accumulated over the training samples. The AR and TAR term should only be applied to the output of the final RNN layer as opposed to being applied to all layers.

5 Testing and Submission

In the handout you will find a template Jupyter notebook that also contains tests that you run locally on the dev set to see how your network is performing as you train. In other words, the template contains a test that will run your model and print the generated text. If that generated text seems like English, then the test on Autolab will likely pass.

Our tests are not overly strict, so you can work your way to a performance that is sufficient to pass Autolab using only a subset of the methods specified in the aforementioned paper.

These tests require that you train the model on your own and submit a tarball (handin.tar) containing the code capable of running the model, generating the predictions and plotting the loss curves.

The root of the tar file should have a directory named hw4 containing your module code. You can use the following command from the handout directory to generate the required submission tar.

make runid=<your run id> epoch=<epoch number>

You can find the run ID in your Jupyter notebook (its just the CPU time when you ran your experiment). You can choose the best epoch using <code>epoch_number</code>. You will then have to submit the file <code>handin.tar</code> to autolab to get your score for the assessment.

Within the Jupyter Notebook, hw4/training.ipynb, there are TODO sections that you need to complete.

Every time you run training, the notebook creates a new experiment folder under experiments/ with a run_id (which is CPU clock time for uniqueness). All of your model weights and predictions will be saved there.

The notebook trains the model, prints the Negative Log Likelihood (NLL) on the dev set and creates the generation and prediction files on the test dataset, per epoch.

Your solutions will be autograded by Autolab. In order to submit your solution, create a tar file containing your code.

Important: The average model will likely take around 10 or more epochs, to achieve a validation NLL below 5.0 (On the prediction task). The Autolab tests require a performance of 5.4 (or less).

Performance reported in the paper is 4.18, so you have room for error. However, with a good set of hyper-parameters and some well chosen regularization techniques (mentioned above), you can get an NLL below 5.0 in no more than 2 epochs.

Warning: The classes provided for training your model are given to help you organize your training code. You shouldn't need to change the rest of the notebook, as these classes should run the training, save models/predictions and also generate plots. If you do choose to diverge from our given code (maybe implement early stopping for example), be careful.

Good luck!