Objectives

The primary objectives of the lab are:

- To achieve an attenuation of 20 dB, when a noise of 100 Hz frequency is applied
- To design an analog compensator to stabilize the system, i.e. loop shaping of the loop transfer function.

Control Algorithm

The control algorithm is implemented by studying the bode plot of the open loop transfer function and accordingly designing an analog compensator in order to get a desirable gain and phase margin.

Obtaining the bode plot

The input sinusoid is given to the headphones and the output waveform's amplitude and phase(with respect to input) is observed. The gain plot is made by doing $\frac{V_{out}p-p}{V_{in}p-p}$. The phase plot is made from the phase readings on the DSO.

Checking the gain and phase margin

We want an attenuation of 20dB in the open loop, which means we require a gain of 20dB in the open loop bode plot at 100 Hz. We therefore shift the entire magnitude plot upwards until we get 20dB gain at 100Hz. We then see the frequency at which the phase crosses of -180^o and the value of gain at that frequency. If the gain is positive, it means the closed loop system is unstable(negative gain margin) and if it's negative, then the closed loop system is stable(positive gain margin). In our case, we noticed that the gain was positive, hence the closed loop system was unstable and a compensator would need to be designed.

Compensator design

We would want a compensator transfer function that ideally causes no attenuation at 100Hz, and high attenuation near the phase crossover frequency. This is because we want the gain to remain 20dB at 100Hz and become negative close to the phase crossover frequency. We also need to take care of the phase plot affecting the phase plot of our open loop transfer function. A transfer function that has a similar properties to the above mentioned is of the form:

$$H(s) = \frac{as^2 + bs + c}{ds^2 + es + f} \tag{1}$$

Our phase crossover frequency was approximately 3200 Hz and we want attenuation of at least 34dB. Therefore the coefficients a,b,c,d,e,f were found to be of the form:

$$H_1(s) = \frac{s^2 + 4000\pi s + 4\pi * 10^6}{100s^2 + 40000\pi s + 4\pi * 10^6} \tag{2}$$

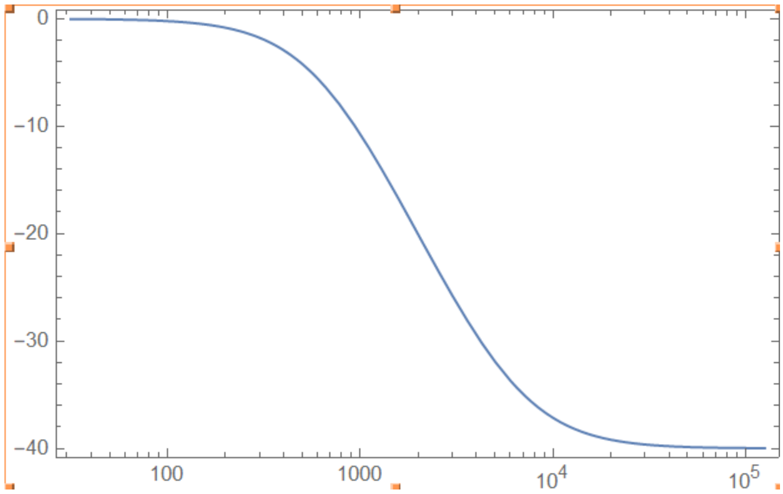

Figure 1: Magnitude of $H_1(s)$

The magnitude plot of the above transfer function is displayed below:
When we cascade this with the bode plot of the headphones we will get a stable system. This is because at 3200Hz, the gain of the compensator is -40dB. Thus the system gets stable as it results in no encirclements of $\frac{-1}{K}$. Here K is the gain we have put initally to get a 20dB gain at 100Hz. The final plot after cascading the compensator is shown in the results section. The circuit is made in the following way for in order to get a stable closed loop:

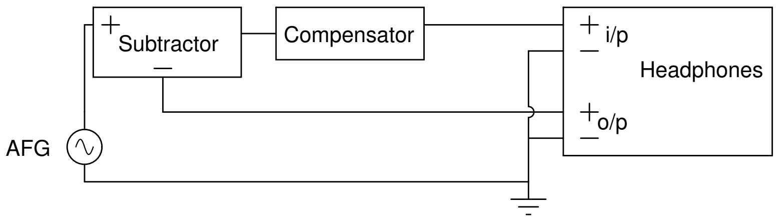

Figure 2: Circuit diagram

Challenges and Solutions

Challenge 1: Obtaining Initial Gain and Phase Plots

Problem: At low frequencies, the output was extremely noisy, making it difficult for the oscilloscope to provide an accurate peak-to-peak voltage and phase difference. This noise led to inconsistent readings and unreliable data for low-frequency measurements.

Solution: To address this, we conducted the experiment in a silent environment to reduce external interference. Additionally, we averaged output values at low frequencies to establish a more stable phase difference. For higher frequencies, we adjusted the phase readings by appropriately adding -180° or -360° to ensure a decreasing phase plot, resulting in a more accurate gain and phase profile.

Challenge 2: Designing the Compensator

Problem: Initially, we considered designing a lag-lead compensator to stabilize the system. However, we later realized that a simpler lag compensator would be more effective, as our objective was to achieve a significant gain reduction at 3.2 kHz for a positive gain margin while maintaining no gain reduction at 100 Hz.

Solution: With these constraints in mind, we designed a second-order lag compensator. After several iterations and adjustments, we achieved a response that met our design criteria. This finalized compensator plot was then used for our circuit design.

Challenge 3: Designing the Closed-Loop Circuit

Problem: Initially, we directly connected the positive terminal of the input to the negative terminal of the output in the closed-loop design. This configuration resulted in no output across all frequencies. Upon further investigation, we realized that an internal biasing circuit prevented a direct connection between these terminals.

Solution: To resolve this, we designed a simple subtractor circuit to implement the closed-loop connection. This approach allowed us to properly close the loop without shorting the terminals, leading to the desired output response across frequencies.

Challenge 4: Output Saturation in Initial Compensator Design

Problem: The output of our initial compensator design was saturated across all frequencies, indicating that the output of the operational amplifier was excessively amplified. This saturation likely stemmed from the chosen resistor values in the compensator circuit, which amplified the output beyond the desired levels.

Solution: To address this, we redesigned the subtractor circuit using higher resistor values to control the amplification. Additionally, we incorporated potentiometers to fine-tune the resistance values, allowing us to adjust the circuit until achieving the desired attenuation in our closed-loop system.

Results

System Frequency Response

The following table summarizes the magnitude and phase response of the system obtained

Table 1: Input Data for Frequency Response Analysis

Frequency (Hz)	**20 log(Gain) (dB)**	**Phase (degrees)**
10	-37.08	-
30	-18.42	-
50	-16.14	-
70	-6.74	-
90	-5.68	-
100	-5.04	89
200	-2.62	54.2
300	-3.48	32.2
400	-0.72	8
500	-2.27	-1.44
600	-2.16	-7
700	-2.50	-13.6
800	-2.85	-18.5
900	-3.10	-10.3
1000	-2.97	-10.9
1200	-0.92	-7.8
1400	2.28	-12
1600	3.52	-33
1800	3.29	-40
2000	5.48	-53
2200	6.53	-68
2400	7.60	-85
2600	9.19	-111
2800	9.19	-138
3000	8.03	-161
3200	6.81	-180
3400	4.71	-200
3600	2.92	-215
3800	2.01	-223
4000	0.83	-239
4200	-0.54	-247
4400	-2.85	-274.7
4600	-7.96	-

Bode Plot Analysis

The initial Bode plots of the system before compensator design were generated and are shown in Figure 3. These plots illustrate the system's frequency response, including the gain and phase characteristics.

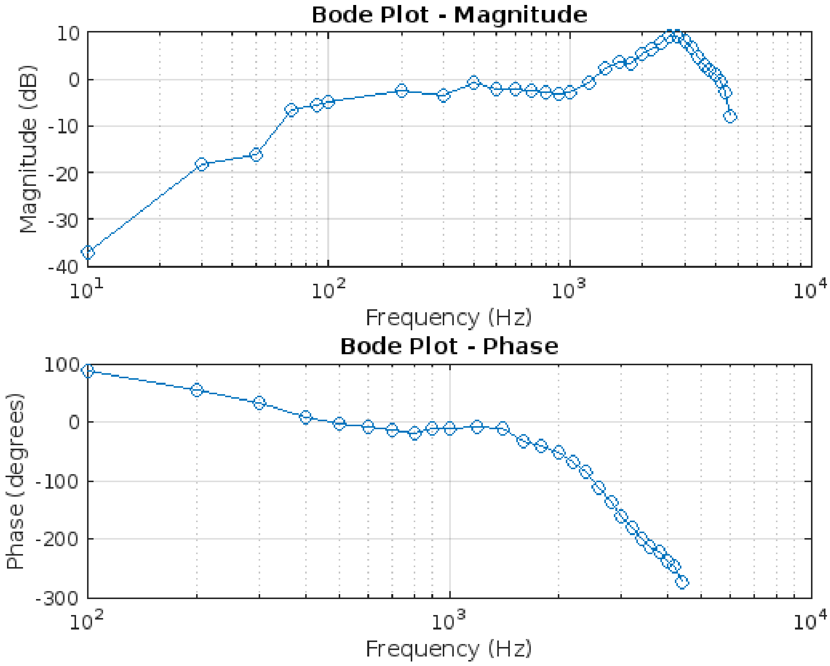

Figure 3: Initial Bode Plots

After implementing the compensator, the new Bode plots were recorded, demonstrating the improved performance of the system. The updated magnitude and phase plots are presented in Figures 4 and 5, respectively while that of the compensator is added in the section for control algorithm

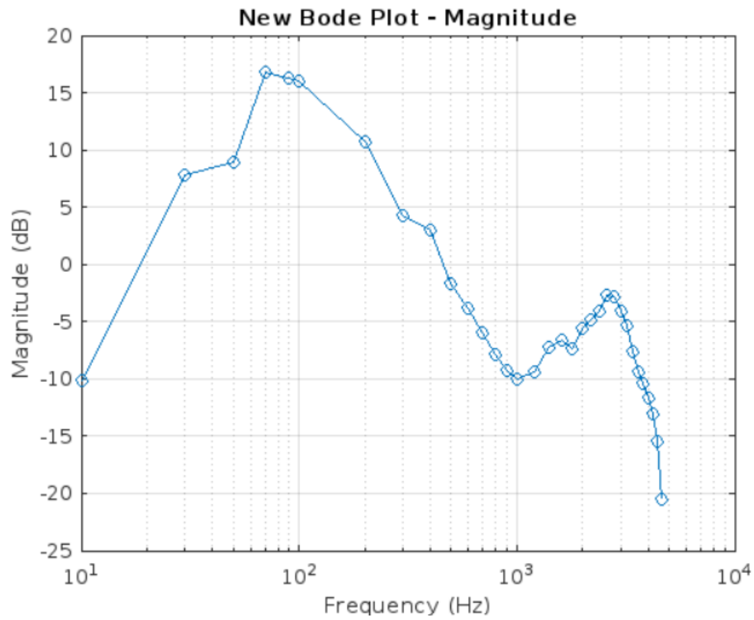

Figure 4: New Bode Magnitude Plot

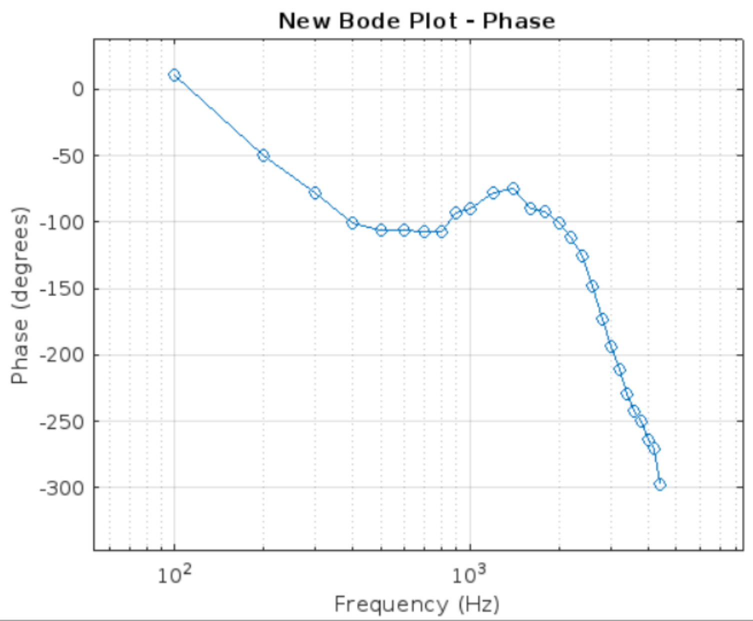

Figure 5: New Bode Phase Plot

Gain and Phase Margins

The gain and phase margins were calculated to assess the stability of the closed-loop system after the compensator implementation. The results are as follows:

- **Gain Margin (dB)**: 3.20 at 2863.61 Hz
- **Phase Margin (degrees)**: 76.22 at 461.85 Hz

These margins indicate a stable system, providing assurance that the designed compensator successfully enhances the stability without compromising performance.

Attenuation Measurement

The system achieved the desired attenuation level of approximately **1/10** at a frequency of **100 Hz**, validating the effectiveness of the noise cancellation circuit. The image for the same is attached at the end of the report.

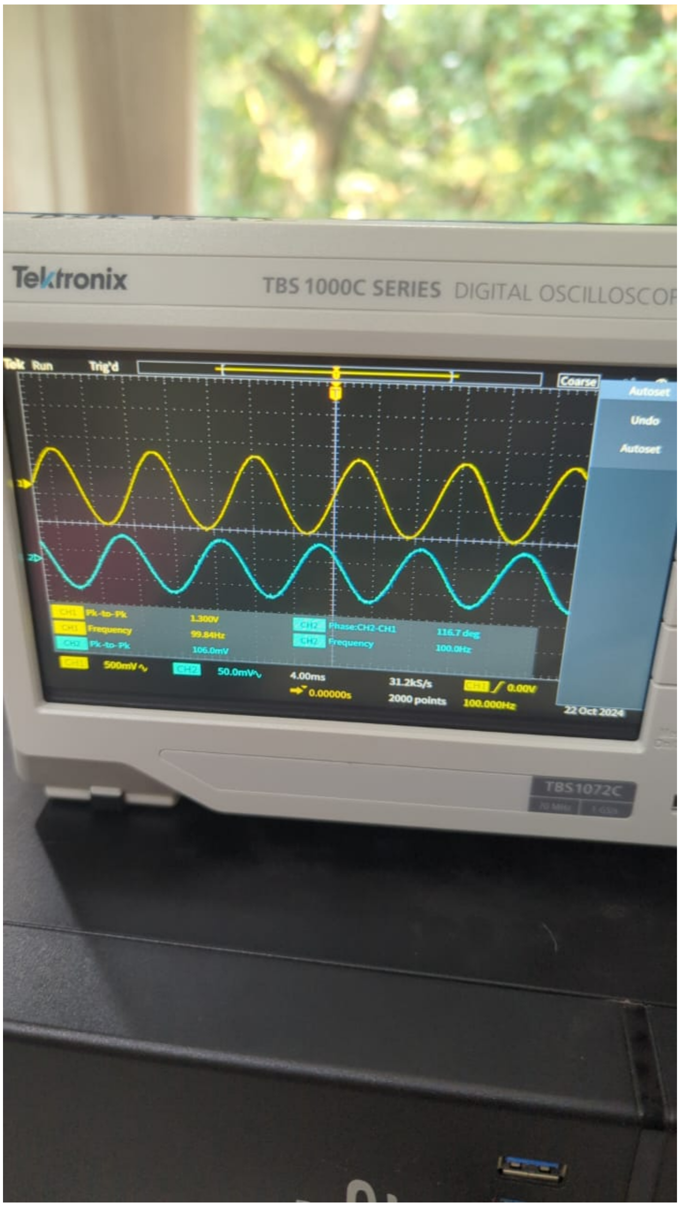

Figure 6: Attenutation results

Observations

During the course of the experiment, several key observations were made regarding the performance and characteristics of the noise cancellation system:

1. **Effectiveness of Noise Cancellation:** The circuit successfully achieved the target attenuation level of **20 dB** at a frequency of **100 Hz**, demonstrating the effectiveness of the designed noise cancellation system.

2. **Bode Plot Analysis:** The initial Bode plots indicated a significant phase lag at higher frequencies, which can lead to instability in the system. However, after the compensator design, the updated Bode plots showed improved gain and phase margins, indicating a more stable system.

3. **Stability Margins:** The calculated gain margin of **3.2** and phase margin of **76.22 degrees** suggest that the system is stable and has a good buffer against variations in system parameters. This indicates that the compensator effectively mitigated potential instabilities.

4. **Phase Shift at Target Frequency:** The phase shift observed at **100 Hz** (89 degrees) indicates that the system was able to operate close to the desired phase alignment necessary for effective noise cancellation, while at higher frequencies, the phase lag increased, necessitating further adjustments.

5. **Performance at Different Frequencies:** The phase and gain measurements varied significantly across different frequencies. For example, a noticeable drop in gain was observed at higher frequencies (e.g., **5000 Hz**), where the gain fell below zero, indicating potential challenges in noise cancellation at those frequencies.

6. **Initial Closed Loop Response:** The system's performance was evaluated under varying noise conditions. Initial testing indicated that, without any compensator, the system struggled to achieve the desired attenuation of 20 dB at 100 Hz, failing to effectively cancel the noise. Upon closing the loop without a compensator, the system exhibited saturation behavior, rapidly switching between high and low output levels. This oscillation indicated instability, as the feedback loop was not effectively controlling the output, leading to inconsistent noise cancellation performance. The calculated Gain Margin was negative, which indicates potential instability. The Phase Margin, when evaluated, further confirmed that the system was at risk of instability. The instability

manifested in audible artifacts, with the headphones producing fluctuations in sound levels rather than a steady noise-canceling effect. This poor performance highlighted the necessity for a robust compensator design. Thus we observe oscillaotry behaviour without any compensator circuit.

Conclusion

The proposed control algorithm effectively stabilises the closed loop system. This is apparent based on the output sinusoid obtained in the closed loop system. If the system was unstable, then no output would be observed in the closed loop system. Attenuation of 20dB was observed at 100Hz and at other frequencies very less attenuation was observed as expected.

THE END