Aero 2: 2 DOF PD Control

Concept Review

In this lab, a proportional-derivative (PD) control is used to control the pitch and yaw axes to a desired angle.

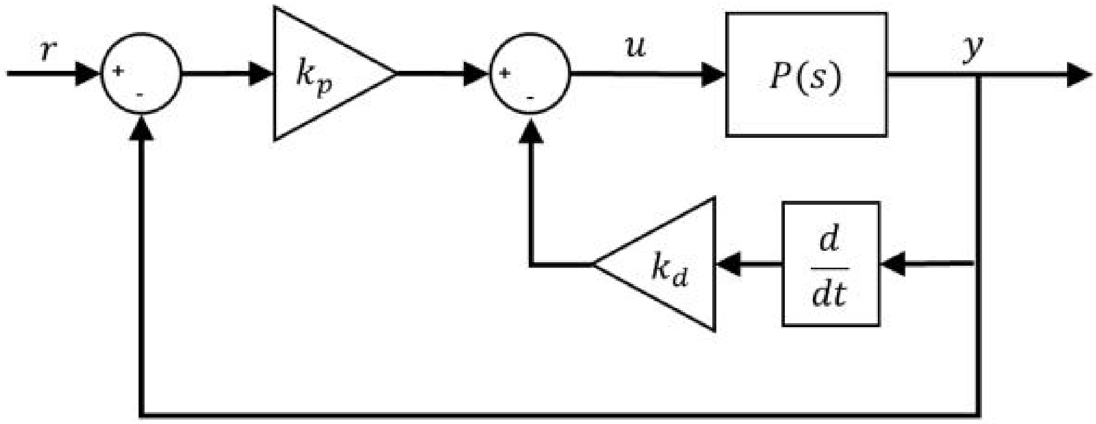

Figure 1: Proportional-Derivative (PD) control

This is a variation of the standard PD control where only the negative velocity is fed back, i.e., not the velocity of the *error*, and a low-pass filter will be used in-line with the derivative term to suppress measurement noise. The PD control shown in Figure 1 has the following structure

$$u = k_p(r(t) - y(t)) - k_d\dot{y}(t), \qquad (1)$$

where k_p is the proportional gain, k_d is the derivative (velocity) gain, $r = \theta_d(t)$ is the reference pitch angle, $y = \theta(t)$ is the measured pitch angle, and $u = V_p(t)$ is the control input - the applied motor voltage to the front pitch rotor.

For the PD control of the yaw axis, the reference is the desired yaw angle $r = \psi_d(t)$, the measured variable is the yaw angle $y = \psi(t)$, and the control input is the rear yaw rotor voltage $u = V_y(t)$.

Pitch and Yaw Models

The pitch and yaw can be modeled as separate transfer functions:

$$P_p(s) = \frac{\Theta(s)}{V_p(s)} = \text{[redacted]}$$

and

$$P_y(s) = \frac{\Psi(s)}{V_y(s)} = \text{[redacted]}$$

Control Design

The PD controller transfer function can be found by taking the Laplace transform of Equation 1. Since the Aero 2 starts at rest, the initial conditions are zero, i.e. $\theta(0^-) = 0$ and $\dot{\theta}(0^-) = 0$, and we can obtain the following

$$U(s) = k_p(R(s) - Y(s)) - k_d s Y(s).$$

Given the control input is the front/pitch motor voltage, $U(s) = V_p(s)$. The closed-loop transfer function can be found by substituting the PD control into the pitch transfer function, and solving for $Y(s)/R(s)$:

$$\frac{Y(s)}{R(s)} = \text{[redacted]} \quad (2)$$

Given the standard second-order prototype transfer function

$$\frac{Y(s)}{R(s)} = \text{[redacted]} \quad (3)$$

we can express the PD control gains based on the required natural frequency, ω_n, and damping ratio, ζ, with the equations

$$k_p = \text{[redacted]} \quad (4)$$

and

$$k_d = \text{[redacted]} \quad (5)$$

We can apply the same control design for the yaw-axis and get the PD equations:

$$k_p = \text{[redacted]} \quad (6)$$

and

$$k_d = \text{[redacted]} \quad (7)$$

Peak Time and Overshoot Equations

We can use the following expressions to obtain the required ω_n and ζ from the peak time and overshoot specifications. In a second-order system, the amount of overshoot depends solely on the damping ratio parameter

$$PO = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad (8)$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad (9)$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

Decoupled control design: Note that this does not take into account the reaction torque effects as documented in the *2 DOF Helicopter Modeling* lab. The PD control is designed separately for the pitch and yaw axes, i.e. each axes is treated as a single-input, single-output (SISO) system. This is known as a *decoupled controller*. As a result, simultaneously controlling the pitch and yaw axes to a desired reference command may yield unexpected motions. For example, large overshoot can be seen in the pitch axis as the yaw is tracking a reference command signal.

PD Control Design

Control Specifications

Design PD gains according to the following set of specifications:

1. Peak time: $t_p \leq 2.5$ s for pitch angle and $t_p \leq 3.5$ s for yaw angle.
2. Percent Overshoot: $PO \leq 5\%$ for both axes.