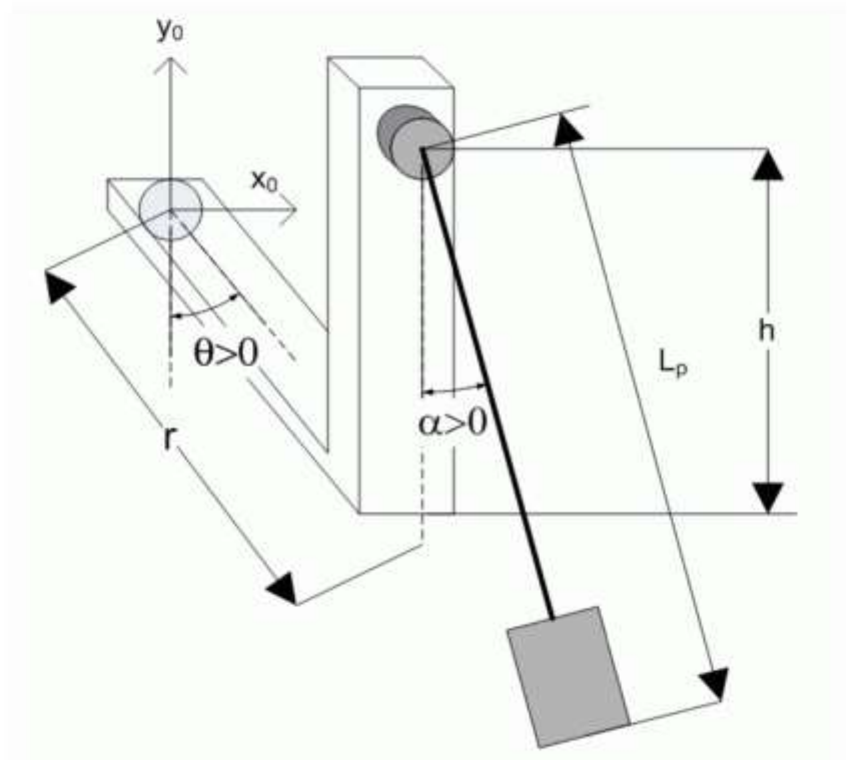
EE615: Control and Computation Lab

Exp 1: Rotary Inverted Pendulum

Design a swing up controller and stabilization controller for the inverted pendulum.

Suggested Steps (your choice)

- Derive Non-linear Model



$$\frac{d^2}{dt^2}\theta(t) = -\frac{M_p^{\,2}\, g\, l_p^{\,2}\, r\cos(\theta(t))\,\alpha(t)}{(M_p r^2 \sin(\theta(t))^2 - J_{eq} - M_p r^2)\,J_p - M_p\, l_p^{\,2}\, J_{eq}} - \frac{J_p M_p r^2 \cos(\theta(t))\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2}{(M_p r^2 \sin(\theta(t))^2 - J_{eq} - M_p r^2)\,J_p - M_p\, l_p^{\,2}\, J_{eq}} - \frac{J_p\,\tau_{output} + M_p\, l_p^{\,2}\,\tau_{output}}{(M_p r^2 \sin(\theta(t))^2 - J_{eq} - M_p r^2)\,J_p - M_p\, l_p^{\,2}\, J_{eq}}$$

$$\frac{d^2}{dt^2}\alpha(t) = -\frac{l_p M_p(-J_{eq}\, g + M_p r^2 \sin(\theta(t))^2 g - M_p r^2 g)\,\alpha(t)}{(M_p r^2 \sin(\theta(t))^2 - J_{eq} - M_p r^2)\,J_p - M_p\, l_p^{\,2}\, J_{eq}} - \frac{l_p M_p r \sin(\theta(t))\, J_{eq}\left(\frac{d}{dt}\theta(t)\right)^2}{(M_p r^2 \sin(\theta(t))^2 - J_{eq} - M_p r^2)\,J_p - M_p\, l_p^{\,2}\, J_{eq}} + \frac{l_p M_p r\,\tau_{output}\cos(\theta(t))}{(M_p r^2 \sin(\theta(t))^2 - J_{eq} - M_p r^2)\,J_p - M_p\, l_p^{\,2}\, J_{eq}}$$

where the torque generated at the arm pivot by the motor voltage V_m is

$$\tau_{output} = \frac{K_t\left(V_m - K_m\left(\frac{d}{dt}\theta(t)\right)\right)}{R_m}.$$

Notation

Symbol	Description	Value	Unit
M_p	Mass of the pendulum assembly (weight and link combined).	0.027	kg
l_p	Length of pendulum center of mass from pivot.	0.153	m
L_p	Total length of pendulum.	0.191	m
r	Length of arm pivot to pendulum pivot.	0.08260	m
J_m	Motor shaft moment of inertia.	3.00E-005	$kg \cdot m^2$
M_{arm}	Mass of arm.	0.028	kg
g	Gravitational acceleration constant.	9.810	m/s^2
J_{eq}	Equivalent moment of inertia about motor shaft pivot axis.	1.23E-004	$kg \cdot m^2$
J_p	Pendulum moment of inertia about its pivot axis.	1.10E-004	$kg \cdot m^2$
B_{eq}	Arm viscous damping.	0.000	N·m/(rad/s)
B_p	Pendulum viscous damping.	0.000	N·m/(rad/s)
R_m	Motor armature resistance.	3.30	Ω
K_t	Motor torque constant.	0.02797	N·m
K_m	Motor back-electromotive force constant.	0.02797	V/(rad/s)

Design Swing-up control

- Method is your choice
- Study literature for suitable swing-up control
- Energy based Swing-up control (https://web.ece.ucsb.edu/~hespanha/ece229/references/AstromFurutaAUTOM00.pdf)

Linearize the model (for stay-up control)

State-Space Matrix	Expression
A	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{r M_p^2 l_p^2 g}{J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2} & -\dfrac{K_t K_m (J_p + M_p l_p^2)}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} & 0 \\ 0 & \dfrac{M_p l_p g (J_{eq} + M_p r^2)}{J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2} & \dfrac{-M_p l_p K_t r K_m}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} & 0 \end{bmatrix}$
B	$\begin{bmatrix} 0 \\ 0 \\ \dfrac{K_t (J_p + M_p l_p^2)}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} \\ \dfrac{M_p l_p K_t r}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} \end{bmatrix}$
C	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
D	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Design state-feedback controller

- use LQR based controller design