Experiment 3: Aero 2 DOF - Pitch and Yaw Control by PID Controller

Group No.: 14 and 15

Siddhant Dongare – 21D070070

Saurav Kumar – 21D070063

Parth Arora – 21D070047

Rajashekhar – 24D0556

EE615: Control and Computation Lab

Course Instructor: Prof. Debasattam Pal

Indian Institute of Technology, Bombay

April 1, 2025

Contents

1 Aim of the Experimrnt **2**
1.1 Model specifications 2

2 Aero 2 - 2 DOF Helicopter Modeling **2**
2.1 Block diagram and review of the concept 2
2.2 Transfer function model 4
2.3 Linear state-space representation 4

3 Aero 2 - 2 DOF PD Control **4**
3.1 Block diagram and review of the concept 4
3.2 Pitch and yaw models 5
3.3 Design of control 5
3.4 Peak time and overshoot equations 6

4 Simulink Models and Results **7**
4.1 Hardware and software simulink models 7
4.2 Results 8

5 Challenges faced and Solutions **10**

6 Inference **10**
6.1 Experimental computed values 11

1 Aim of the Experimrnt

The Objectives of this experiment are:

- To design two proportional integral derivative (PID) controllers in SIMULINK to separately control pitch and yaw.
- To design PID gains according to the following set of specifications:
  - Peak time: $t_p \leq 2.5$s for pitch angle and $t_p \leq 3.5$s for yaw angle.
  - Percent Overshoot: PO $\leq 5\%$ for both axes.
- And even to design a low-pass filter if there is noise in the system.

1.1 Model specifications

Below are the model specifications considered in our experiment:

➢ D_p = Damping about the pitch axis = 0.0020

➢ D_t = Distance between the Aero 2 pivot and center of the rotor = 0.1674

➢ D_y = Damping about the yaw axis = 0.0019

➢ J_p = Total moment of inertia about the pitch axis = 0.0232

➢ J_y = Total moment of inertia about the yaw axis = 0.0238

➢ K_{pp} = Thrust force gain acting on the pitch axis from the pitch/front rotor = 0.0032

➢ K_{py} = Thrust force gain acting on the pitch from the yaw/rear rotor = 0.0014

➢ K_{sp} = Stiffness about the pitch axis = 0.0074

➢ K_{yp} = Thrust force gain acting on the yaw axis from the pitch/front rotor = -0.0032

➢ K_{yy} = Thrust force gain acting on the yaw axis from the yaw/rear rotor = 0.0061

2 Aero 2 - 2 DOF Helicopter Modeling

2.1 Block diagram and review of the concept

The free-body diagram illustration of the Aero 2 system is shown in Fig. 1. The conventions are used for modeling are as follows:

✯ The helicopter is horizontal and parallel with the ground when the pitch angle is zero, i.e., $\theta = 0$.

✯ The yaw angle increases positively ($\dot{\Psi}(t) > 0$) when the body rotates counter-clockwise (CCW) about the z-axis.

✯ The pitch angle increases positively ($\dot{\theta}(t) > 0$) when the body rotates clockwise (CW) about the y-axis, i.e., when the front rotor goes up.

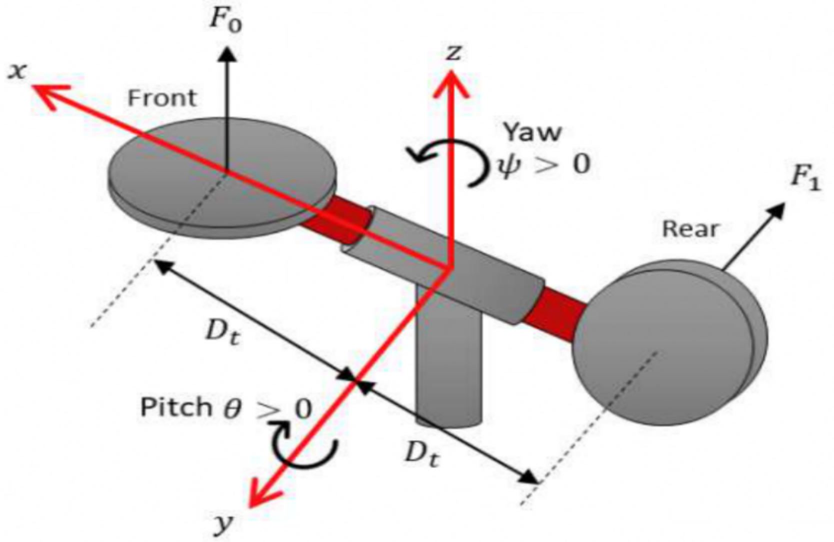


Figure 1: Free-body diagram of Aero 2 Experiment

- ✭ Yaw increases ($\dot{\Psi} > 0$) when the rear/yaw rotor voltage is positive ($V_y > 0$).
- ✭ Pitch increases ($\dot{\theta} > 0$) when the front/pitch rotor voltage is positive ($V_p > 0$).

When a voltage is applied to the pitch motor V_p, the front rotor generates a force F_0 that acts normal to the body at a distance D_t from the pitch axis (along the x-axis). As discussed in the Aero2 - 2 DOF Helicopter, due to aerodynamic drag the torque generated from the rotation of the front propeller blade also generates a torque about the yaw-axis (z-axis). That's why conventional helicopters include a tail or anti-torque rotor to compensate for the torque generated about the yaw axis by the large main rotor. Similarly, the rear motor generates a force F_1 that acts on the body at a distance D_t from the yaw axis. This creates torque about the yaw axis and pitch axis.

Based on this, we can represent the motions of the Aero 2 about the horizontal (i.e., when the body is parallel with the ground) using the following equations of motions (EOMs):

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp}\theta = \tau_p,$$
$$J_y\ddot{\Psi} + D_y\dot{\Psi} = \tau_y,$$

where the torques acting on the yaw and pitch axes are

$$\tau_y = K_{yp}D_tV_p + K_{yy}D_tV_y, \text{ and}$$
$$\tau_p = K_{pp}D_tV_p + K_{py}D_tV_y.$$

The parameters used in the EOMs are mentioned above in the model specifications section, such that V_p is the voltage applied to the front pitch rotor, and V_y is the voltage applied to the rear yaw rotor motor. Note that this model is not exhaustive in that it does not take into account all of the system dynamics, e.g., nonlinearities. It is meant to be used for designing linear control systems. However, it does capture the gyroscopic reaction torques that act on each axis from both rotors.

2.2 Transfer function model

Taking the Laplace transform of the equations of motion, we get

$$s^2 J_p \theta(s) + s D_p \theta(s) + K_{sp}\theta = \tau_p, \tag{1}$$

and

$$s^2 J_y \Psi(s) + s D_y \Psi(s) = \tau_y. \tag{2}$$

This is a multiple input multiple output (MIMO) system with two outputs and two inputs. Since the system starts when it is at rest, all the initial conditions are zero, i.e., $\theta(0^-) = 0, \dot{\theta}(0^-) = 0, \Psi(0^-) = 0,$ and $\dot{\Psi}(0^-) = 0$. Given this, we can obtain the following transfer functions describing the system motions relative to the different inputs, i.e.,

$$\Theta(s) = \left(\frac{K_{pp} D_t}{J_p s^2 + D_p s + K_{sp}}\right) V_p(s) + \left(\frac{K_{py} D_t}{J_p s^2 + D_p s + K_{sp}}\right) V_y(s), \tag{3}$$

and

$$\Psi(s) = \left(\frac{K_{yp} D_t}{s^2 J_y + D_y s}\right) V_p(s) + \left(\frac{K_{yy} D_t}{s^2 J_y + D_y s}\right) V_y(s). \tag{4}$$

2.3 Linear state-space representation

The linear state-space equations are:

$$\dot{x} = Ax + Bu, \text{ and } y = Cx + Du.$$

Using the equations of motion, we can define the state-space matrices for the Aero 2 system as,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_{sp}/J_p & 0 & -D_p/J_p & 0 \\ 0 & 0 & 0 & -D_y/J_y \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ D_t K_{pp}/J_p & D_t K_{py}/J_p \\ D_t K_{yp}/J_y & D_t K_{yy}/J_y \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

where $y = [\theta(t), \Psi(t)]^T$ is the output state, $u = [V_p(t), V_y(t)]^T$ is the control input, and $x = [\theta(t), \Psi(t), \dot{\theta}(t), \dot{\Psi}(t)]^T$ is the system state,

3 Aero 2 - 2 DOF PD Control

3.1 Block diagram and review of the concept

Here, a proportional-derivative (PD) control is used to control the pitch and yaw axes to a desired angle. This is a variation of the standard PD control where only the negative velocity is fed back, i.e., not the velocity of the error, and a low-pass filter will be used in-line with the derivative term to suppress measurement noise. The PD control shown in Fig. 2 has the following structure,

$$u = k_p r(t) - k_d \dot{y}(t) - k_p y(t),$$

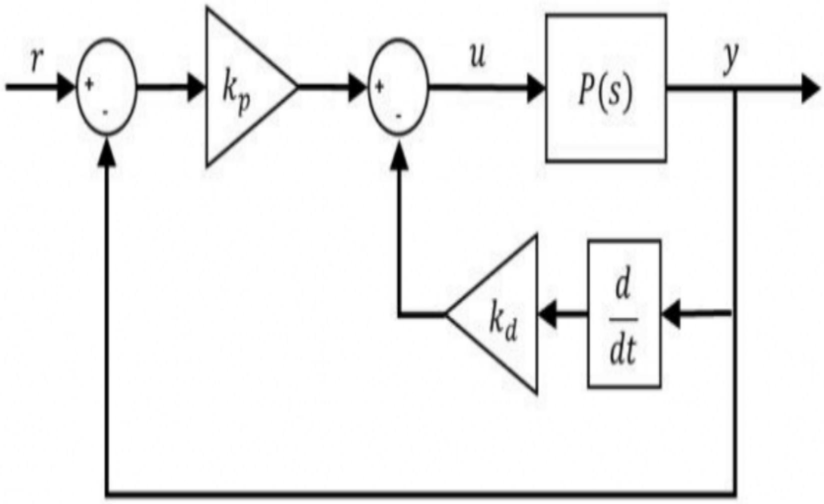


Figure 2: Proportional-Derivative (PD) control

where k_p is the proportional gain, k_d is the derivative/velocity gain, $r = \theta_d(t)$ is the reference pitch angle, $y = \theta(t)$ is the measured pitch angle, and $u = V_p(t)$ is the control input - the applied motor voltage to the front pitch rotor. For the PD control of the yaw axis, the reference is the desired yaw angle $r = \Psi_d(t)$, the measured variable is the yaw angle $y = \Psi(t)$, and the control input is the rear yaw rotor voltage $u = V_y(t)$.

3.2 Pitch and yaw models

The pitch and yaw can be modeled as separate transfer functions:

$$P_p(s) = \frac{\Theta(s)}{V_p(s)} = \frac{K_{pp}D_t}{J_p s^2 + D_p s + K_{sp}}, \tag{5}$$

and

$$P_y(s) = \frac{\Psi(s)}{V_y(s)} = \frac{K_{yy}D_t}{s^2 J_y + D_y s}, \tag{6}$$

3.3 Design of control

The PD controller transfer function can be found by taking the Laplace transform of the equation $u = k_p r(t) - k_p y(t) - k_d \dot{y}(t)$. Since the Aero 2 starts at rest, the initial conditions are zero, i.e. $\theta(0^-) = 0$ and $\dot{\theta}(0^-) = 0$, and we can obtain the following

$$U(s) = k_p R(s) - k_p Y(s) - k_d s Y(s).$$

Given the control input is the front/pitch motor voltage, i.e., $U(s) = V_p(s)$. The closed-loop transfer function can be found by substituting the PD control into the pitch transfer function, and solving for $Y(s)/R(s)$:

$$\frac{Y(s)}{R(s)} = \frac{\Theta(s)}{R(s)} = \frac{K_{pp}K_p D_t}{s^2 J_p + s(D_p + K_d K_{pp} D_t) + K_{sp} + K_{pp} K_p D_t}. \tag{7}$$

Given the standard second-order prototype transfer function

$$\frac{Y(s)}{R(s)} = \frac{\Psi(s)}{R(s)} = \frac{K_p K_{yy} D_t}{s^2 J_y + s(D_y + K_{yy} K_d D_t) + K_p K_{yy} D_t}, \tag{8}$$

we can express the PD control gains based on the required natural frequency w_n, and damping ratio ζ, with the equations as,

$$K_p = \frac{J_p w_n^2 - K_{sp}}{K_{pp} D_t}, \tag{9}$$

and

$$K_d = \frac{2\zeta w_n J_p - D_p}{K_{pp} D_t}. \tag{10}$$

We can apply the same control design for the yaw-axis and get the PD equations as,

$$K_p = \frac{J_y w_n^2}{K_{yy} D_t}, \tag{11}$$

and

$$K_d = \frac{2\zeta w_n J_y - D_y}{K_{yy} D_t}. \tag{12}$$

3.4 Peak time and overshoot equations

We can use the following expressions to obtain the required w_n and ζ from the peak time and overshoot specifications. In a second-order system, the amount of overshoot depends solely on the damping ratio parameter, i.e.,

$$PO = 100 \exp\left\{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right\}.$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as,

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}.$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response. Note that decoupled control design does not take into account the reaction torque effects as documented in the 2 DOF Helicopter modeling. The PD control is designed separately for the pitch and yaw axes, i.e., each axes is treated as a single input single output (SISO) system. This is known as a decoupled controller. As a result, simultaneously controlling the pitch and yaw axes to a desired reference command may yield unexpected motions. For example, large overshoot can be seen in the pitch axis as the yaw is tracking a reference command signal.

The control specifications for PD control design, i.e., PD gains are designed according to the following set of specifications:

❶ Peak time: $t_p \leq 2.5$s for pitch angle and $t_p \leq 3.5$s for yaw angle.

❷ Percent overshoot: $PO \leq 5\%$ for both axes.

4 Simulink Models and Results

4.1 Hardware and software simulink models

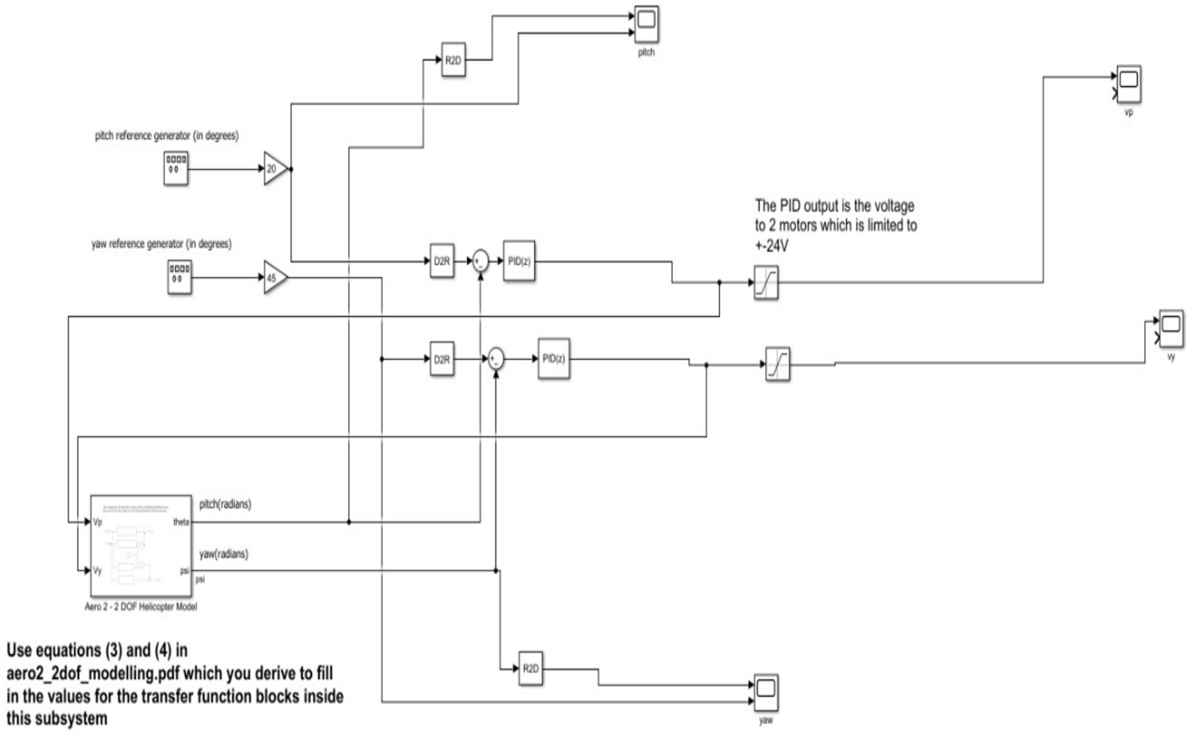


Figure 3: Software block diagram

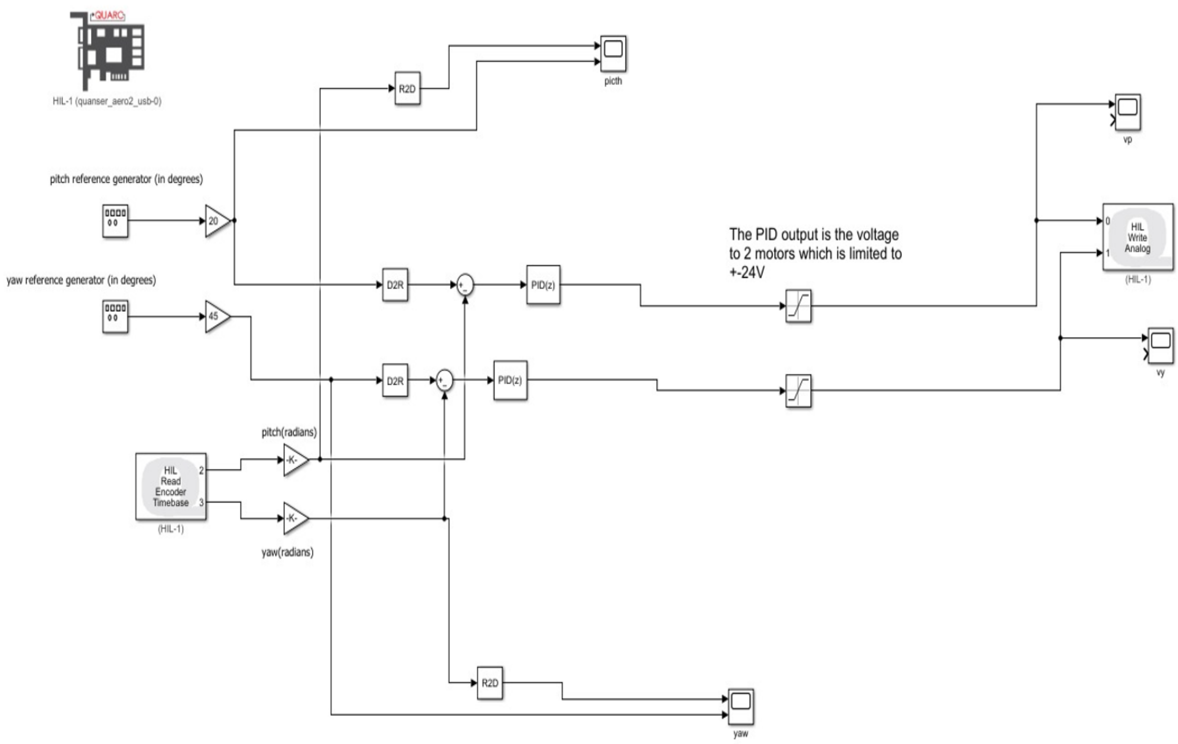


Figure 4: Hardware block diagram

Fig. 3 and Fig. 4 shows the block diagrams that are used in the simulink. Fig. 5 provides computed transfer function blocks, for which we used Eq. (3) and Eq. (4) to derive and filled in the values for those transfer function blocks.

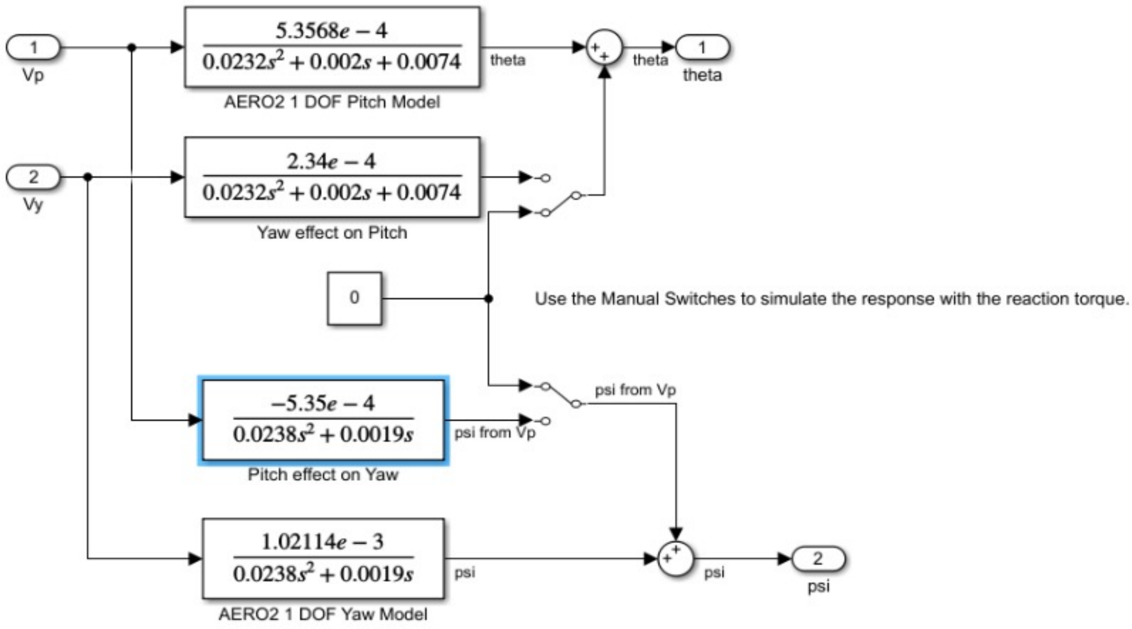
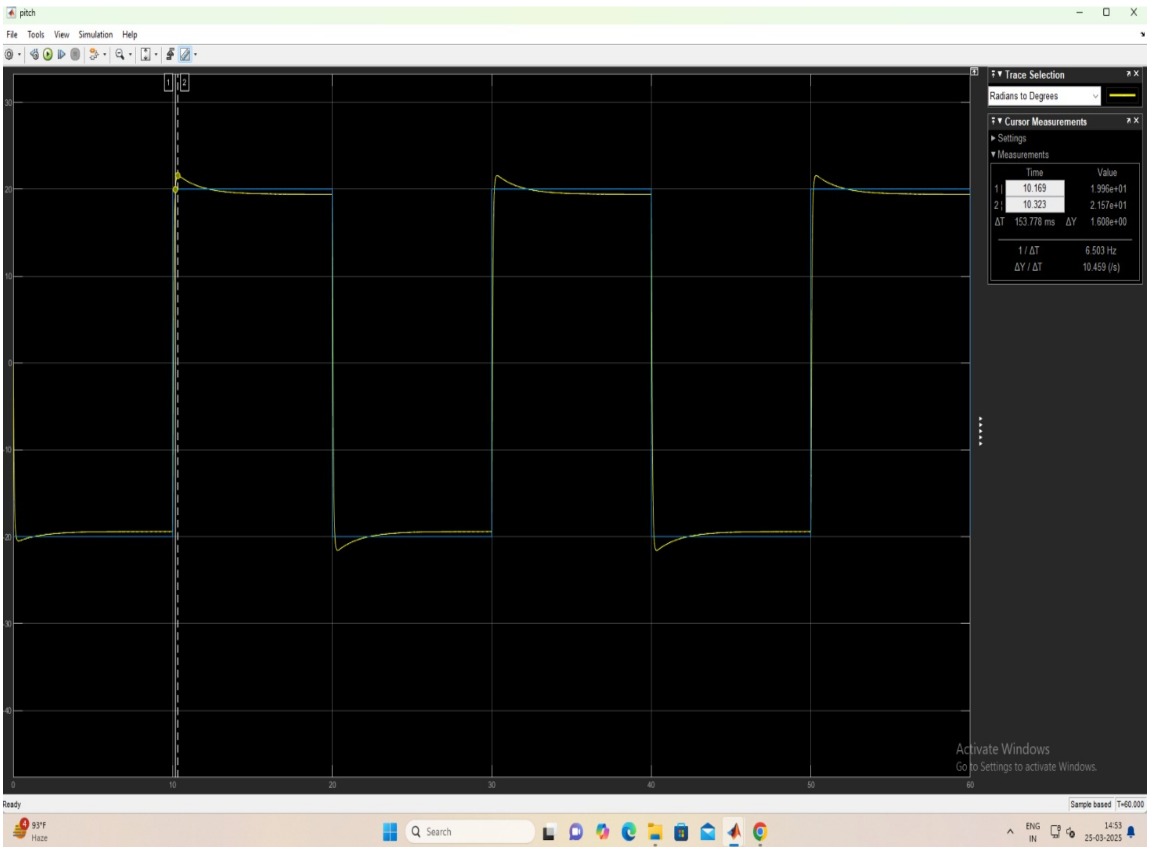
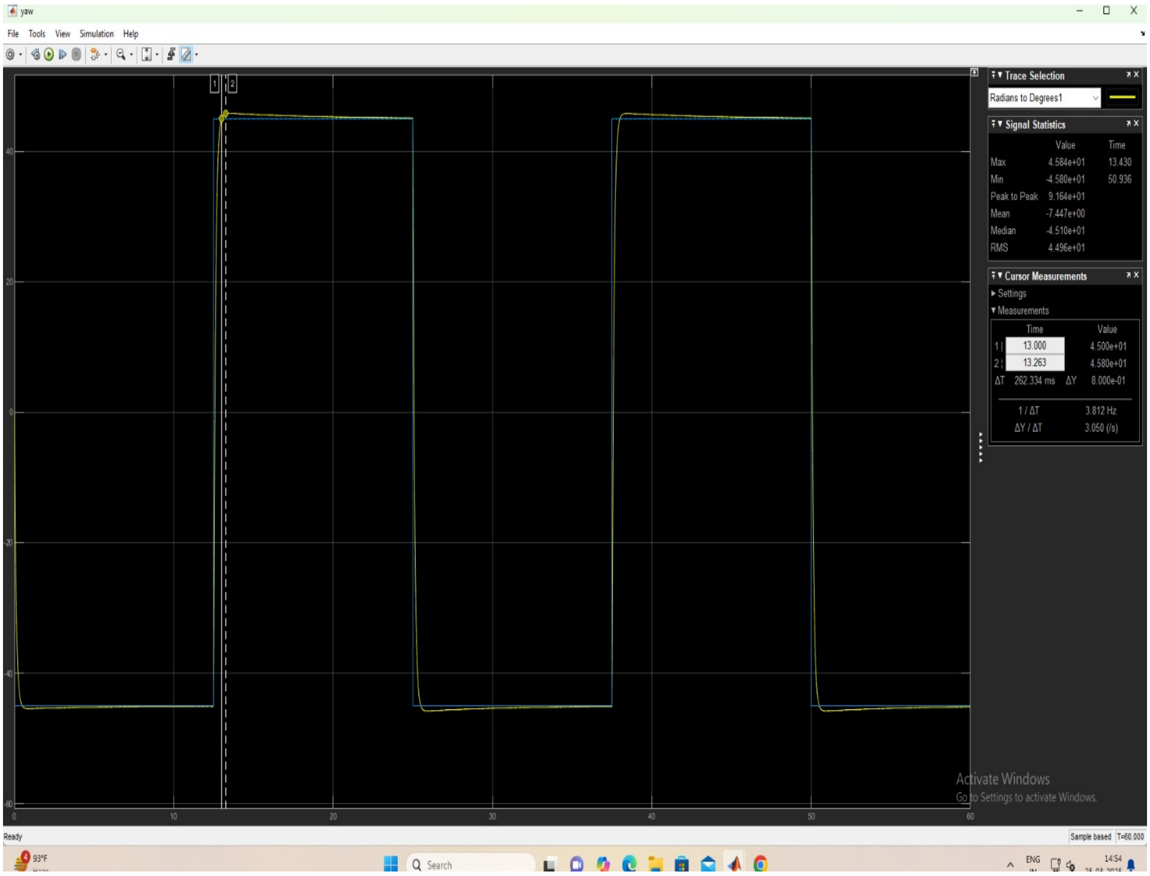


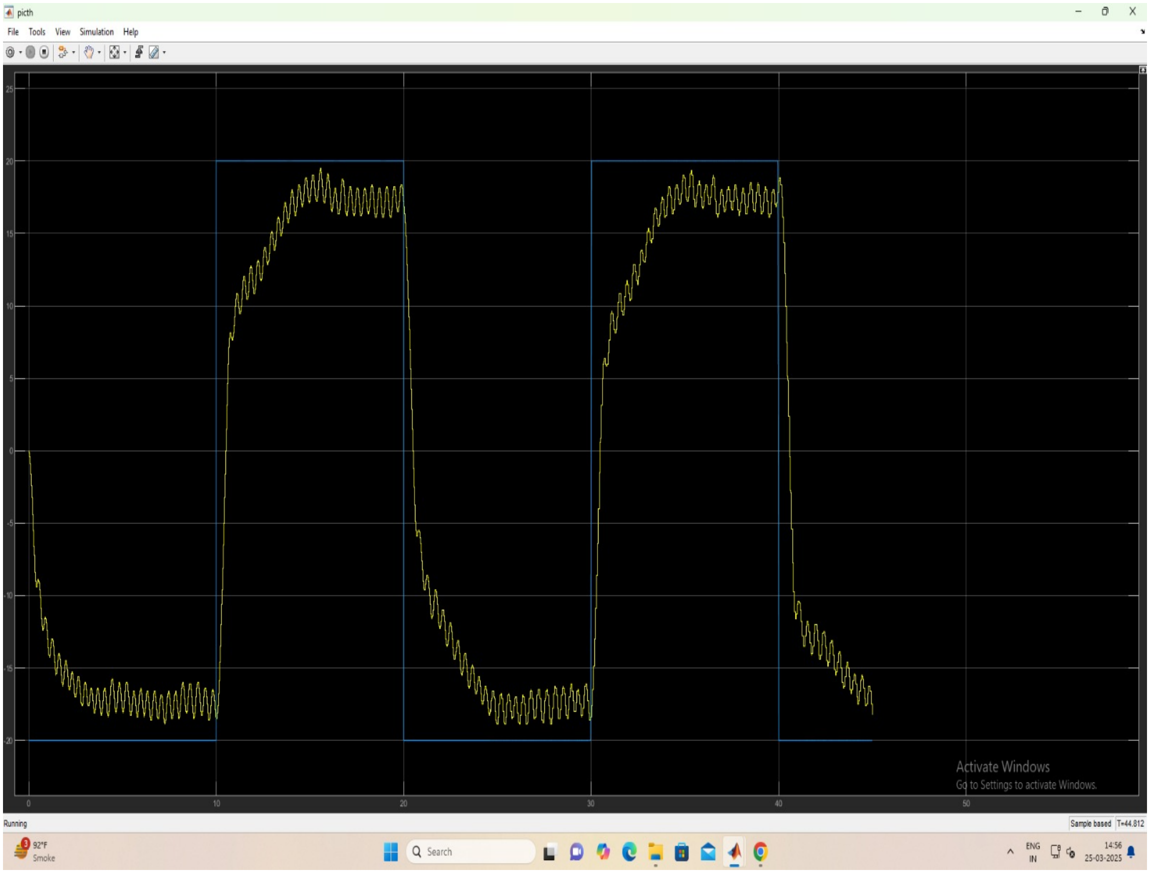
Figure 5: Transfer function blocks

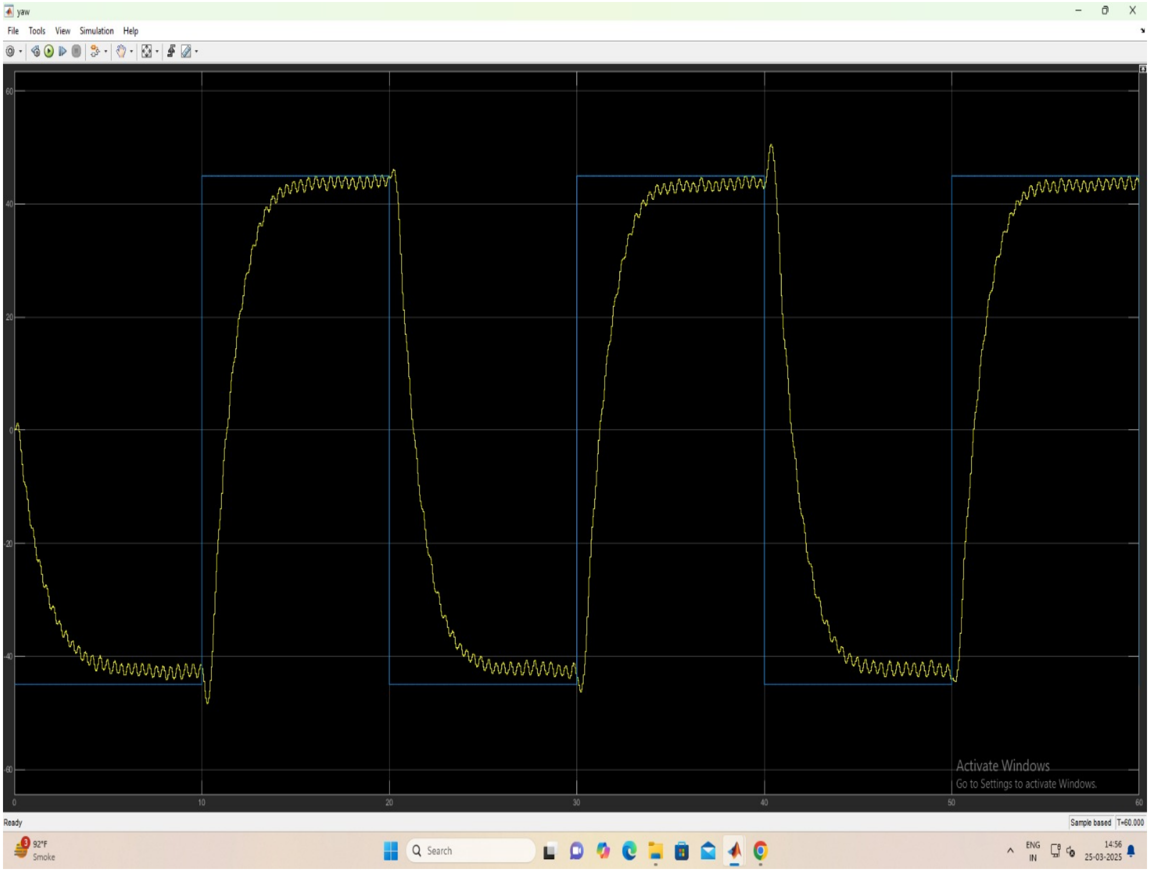
4.2 Results

Below figures shows the results obtained for the pitch and yaw.




yaw
File Tools View Simulation Help
Trace Selection
Radians to Degrees1
Signal Statistics
Value Time
Max 4.584e+01 13.430
Min -4.580e+01 50.936
Peak to Peak 9.164e+01
Mean -7.447e+00
Median -4.510e+01
RMS 4.496e+01
Cursor Measurements
Settings
Measurements
Time Value
1 13.000 4.500e+01
2 13.263 4.580e+01
ΔT 262.334 ms ΔY 8.000e-01
1 / ΔT 3.812 Hz
ΔY / ΔT 3.050 (/s)
Ready
Sample based T=60.000


pitch
File Tools View Simulation Help
Running
Sample based T=44.812



5 Challenges faced and Solutions

- Tuning the PD controller in simulation: Challenge is achieving the required specifications in simulation before implementing on hardware. The solution is fine-tuning the proportional (P) and derivative (D) gains iteratively while ensuring system stability and minimal overshoot.
- Noise in hardware implementation: The actual hardware introduced additional noise affecting the system response. It can be solved by designing an appropriate filter to mitigate noise without significantly affecting the control system performance.
- Nonlinear system dynamics: The model does not account for all system non-linearities, such as gyroscopic effects and aerodynamic forces. Solution is to use a linearized state-space representation and carefully tuning the controller to compensate for unmodeled dynamics.
- Coupling between pitch and yaw motions: Challenge faced is, voltage applied to one rotor affects both pitch and yaw due to cross-coupling effects. It can be resolved by using a multiple input multiple output (MIMO) control strategy to manage interactions between the axes.

6 Inference

- The simulation model successfully approximates the system dynamics, but real-world implementation introduces additional complexities such as noise and nonlinearities.

- Proper PD tuning in the simulation phase significantly improves hardware performance but requires further adjustments when deployed.
- Cross-coupling between pitch and yaw needs to be managed using a control strategy that considers both degrees of freedom.
- The designed filter effectively reduces noise, improving the overall system stability.

6.1 Experimental computed values

Below are the calculated values that we obtained for this experiment.

- Hardware:
  Pitch $\rightarrow K_p$=700, K_d=800,
  Yaw $\rightarrow K_p$=700, K_d=280.
- Simulation:
  Pitch $\rightarrow K_p$=450, K_d=700,
  Yaw $\rightarrow K_p$=35.346, K_d=200.
- Calculated:
  Pitch $\rightarrow K_p$=123.2, K_d=101.4,
  Yaw $\rightarrow K_p$=36.28, K_d=39.30.

Such that, for pitch $t_p = 1.1$ sec, PO = 5%, and for yaw t_p=1 sec, PO=3% are considered.

$\Rrightarrow$ $\frac{\Theta}{V_p} = \frac{5.3568e-5}{0.0232s^2+0.002s+0.0074}$, and $\frac{\Theta}{V_y} = \frac{2.34e-4}{0.0232s^2+0.002s+0.0074}$.

$\Rrightarrow$ $\frac{\Psi}{V_y} = \frac{1.02114e-3}{0.0238s^2+0.0019s}$, and $\frac{\Psi}{V_p} = \frac{-5.35e-4}{0.0238s^2+0.0019s}$.