IIT Bombay
Systems and Control Engineering
Intelligent Feedback and Control

Date: 17.03.24

SC 645 — *Assignment 4* — Maximum Marks: 10

Instructions:

- Submit the answers to this assignment on or before the **deadline** at 11:59 p.m. on 26.03.2024.
- All the results and the associated observations/analysis must be compiled in a single pdf file. This pdf and the associated code must also be submitted in a single zip folder on moodle on the relevant submission link.
  Label this folder in the form: FirstName_RollNumber_AS04.
- Please preserve the code and the report till the end of this semester.
- Assumptions made, if any, must be clearly stated and must be justified.
- After the end of each question, the numbers to the right, in square brackets, indicate marks allotted to it.

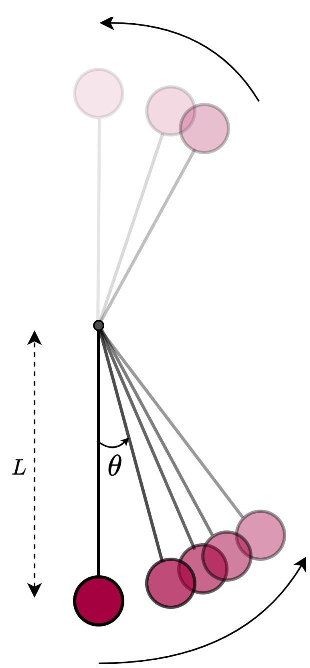

Figure 1: Swinging the pendulum

1. In this assignment, you are supposed to design **gain-scheduled controllers** to **swing and stabilize a pendulum** to its upright position and demonstrate its performance

Question 1 continues on the next page…

via **MATLAB simulations**. Consider the **non-linear pendulum dynamics** as follows,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{L}\sin(x_1) - \frac{b}{mL^2}x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{mL^2}\end{bmatrix}\tau$$

Here, $x_1 = \theta, x_2 = \dot{\theta}$ and the control input is τ. The pendulum dynamics is influenced by both gravity ($'g'$) and air friction. The coefficient of air friction damping is given by $'b'$, and the mass and length of the pendulum are $'m'$ and $'L'$ respectively. The parameter values are as follows,

$$g = 9.81 \quad ; \quad L = 1 \quad ; \quad b = 0.5 \quad ; \quad m = 1$$

The initial state of the pendulum is $\begin{bmatrix}\theta_i & \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}0 & 0\end{bmatrix}$, and the final upright configuration at which the pendulum should be stabilized is $\begin{bmatrix}\theta_f & \dot{\theta}_f\end{bmatrix} = \begin{bmatrix}\pi & 0\end{bmatrix}$.

The following steps are to be followed for Gain scheduling.

- **Linearization:** Suppose the system dynamics is,

  $$\dot{X} = f(X, U) \qquad X \in \mathbb{R}^n, U \in \mathbb{R}^m$$

  The linearized system dynamics is as follows,

  $$\Delta\dot{X} = f(X_i, U_i) + \left(\frac{\partial f}{\partial X}\right)_{X=X_i}\Delta X + \left(\frac{\partial f}{\partial U}\right)_{U=U_i}\Delta U \tag{1}$$

  Here, $\Delta X = X - X_i$ and $\Delta U = U - U_i$.

- **Trimming**: The linearized system dynamics in (1) is converted to the structure of a linear dynamical system around an operating point X_i by computing U_i via solving the following equation.

  $$f(X_i, U_i) = 0$$

- Uniformly sample a set of operating points $\{X_i\}$ for gain scheduling. For each sampled point, the linearized system matrices are as follows,

  $$A_i = \left(\frac{\partial f}{\partial X}\right)_{X=X_i} \quad ; \quad B_i = \left(\frac{\partial f}{\partial U}\right)_{U=U_i}$$

  At each operating point, tune control gains such that they ensure stabilization **without significant overshoots** using the system matrices (A_i, B_i).

- Choose appropriate gains based on the pre-determined set of operating points and the current system configuration for computing the required control inputs to swing the pendulum and achieve stabilization around the upright configuration.

Question 1 continues on the next page...

Your report must consist of and elaborate on the following. $(1+3+3+3)$

- Computations of the linearized system dynamics.
- Control gain tuning, the structure of the control equation, and the reasoning behind how they were tuned based on the prescribed performance requirements.
- Choice of how the sequence of operating points was selected and elaborate on how the implementation of the control algorithm was carried out in MATLAB. What do you think would be the pros and cons of increasing/decreasing the number of operating points around which the gains are determined for control computations?
- Plot all the system states and control inputs as a function of time during stabilization and provide reasoning on the nature of the output plots obtained. *(Carry out the system simulations by adding an additive noise disturbance to the system dynamics. The additive noise should be randomly sampled from a gaussian distribution of mean* 0 *and variance* 0.001*)*

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