Assignment 02

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1 Overview of the experiment

1.1 Aim of the Assignment

In this Assignment, we study the cascade control of The Jacketed Continuously stirred tank reactor (CSTR). We are calculating the stability margin for a system with a delay in the next question by hand and plotting tools.

1.2 Methods

MATLAB has been used for plotting purposes. Nyquist plot , bode plot and root locus has been plotted using matlab in this assignment.
Using the general formulas of control theory, we calculated some values by hand. For question 1, I have used the internet to go through the process by which CSTR works.

2 Question 1

In this question, I read an article on the internet to have a clear understanding of the workings of CSTR.
link of the website: https://controlguru.com/the-normal-or-standard-pid-algorithm/

2.1 PArt 1

For Primary system:
G1(s) = 1/(10s+1)

time constant = 5 second
kp1 = 3
ki1= 0.1
close loop pole: -0.027, -0.373
damping = 2
natural frequency = 0.1

For secondary system:
G2(s) = 1/(2s+1)
time constant = 2 second
kp2 = 1
ki2= 0.5
close loop pole: -0.5, -0.5
damping = 1
natural frequency = 0.5

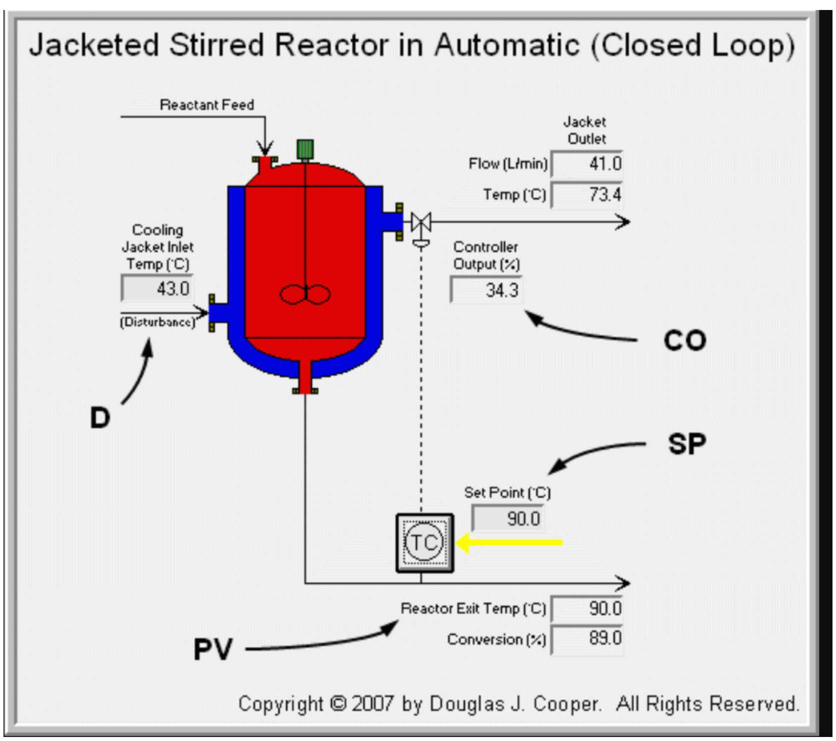


Figure 1: question 1 part 1 diagram

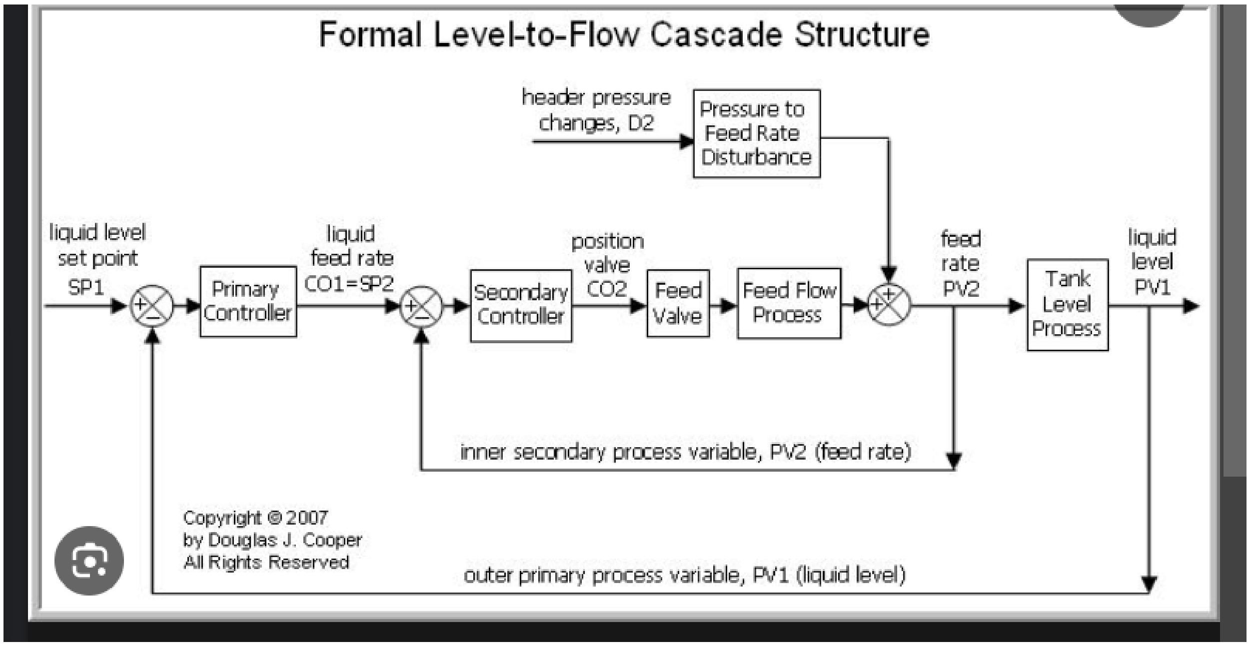


Figure 2: question 1 part 1 cascade structure

```
    s = tf('s');

kp1= 3;
ki1= 0.1;

num = [kp1, ki1];
den = [10,(1+kp1),  ki1];
G1 = tf(num, den);

kp2= 1;
ki2= 0.5;

num = [kp2, ki2];
den = [2,(1+kp2),  ki2];
G2 = tf(num, den);

%rlocus(G2);
%rlocus(G1);
```

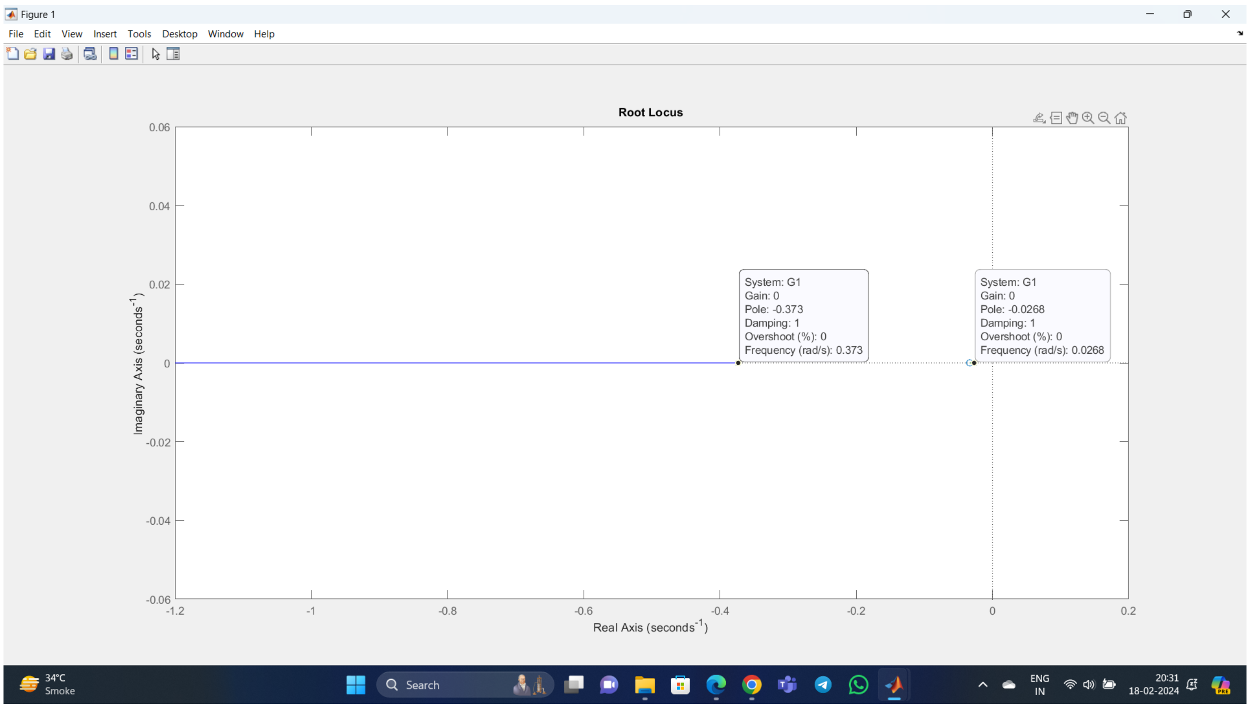


Figure 3: question1 part 1 primary rootlocus

2.2 part 2

```
    s = tf('s');

G1 = 1 / (10*s + 1);
kp1 = 3;
ki1 = 0.1;
C1 = (kp1) + (ki1 / s);

G2 = 1 / (2*s + 1);
kp2 = 1;
ki2 = 0.5;
C2 = (kp2) + (ki2/s);
```

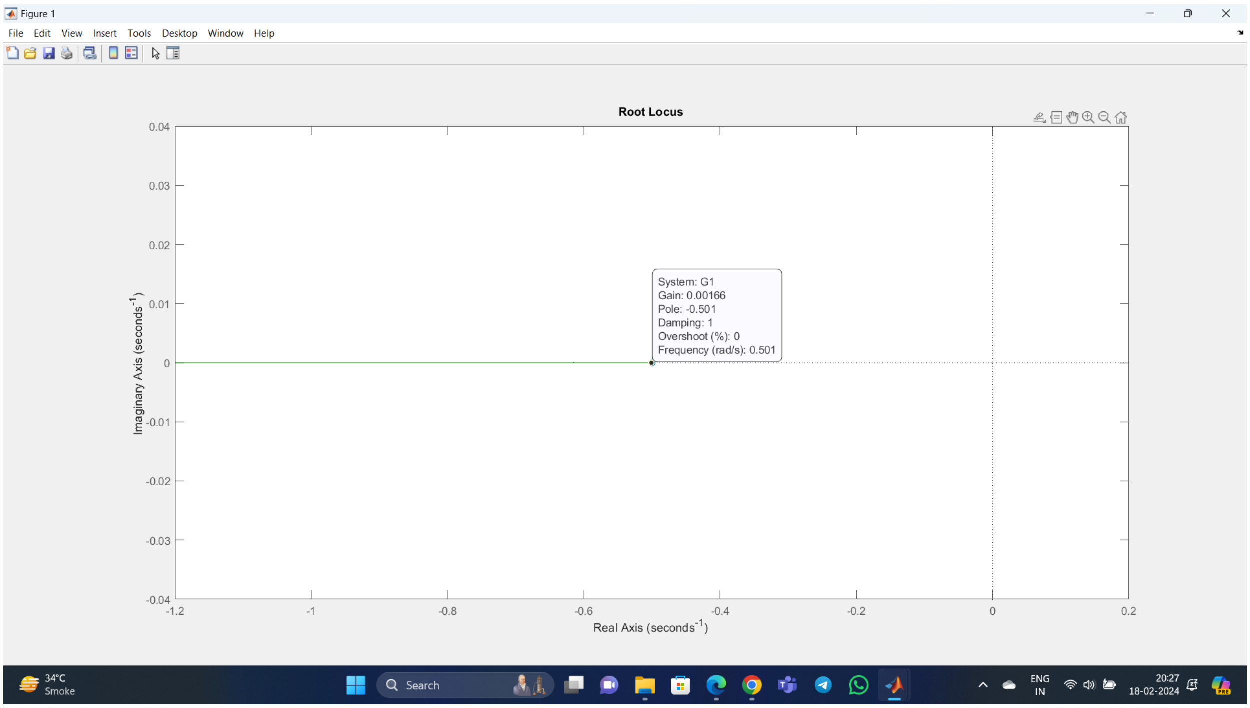


Figure 4: question1 part 1 secondary rootlocus

```
out.yout{1}.Values.plot
hold on
out.yout{2}.Values.plot
```

2.3 part3

With Integral Windup:

The controller output saturates due to constraints on the manipulated variable (MV). Integral Windup occurs, causing overshoot and oscillations in the response. It takes longer for the system to settle to the setpoint.

Without Integral Windup:

The controller output saturates, but the integral term is limited, preventing Integral Windup. The system settles faster to the setpoint without overshoot or oscillations.

Conclusion:
Integral Windup can significantly degrade the performance of a control system, especially in the presence of input constraints or large setpoint changes. It leads to overshoot, oscillations, and longer settling times. Therefore, addressing Integral Windup is crucial for improving system performance and stability.

3 Quesion 2

3.1 part 1

Given transfer function is unstable in a close loop and has a delay term that contributes only to the phase of the system and has no effect on the gain of the system.

gain margin = 13.71 DB (theoretically)
gain margin in MATLAB = 28.1 DB
phase margin = 111.2 degree(theoretically and in Matlab)
delay margin = -1.2 second

Positive Stability Margins:

1. A positive gain margin indicates that the system can tolerate a certain gain increase before becoming unstable.
2. A positive phase margin indicates a certain amount of phase shift can be added to the system before it becomes unstable.
3. These margins imply robustness against parameter variations and disturbances.

Negative Stability Margins:

1. A negative gain margin indicates the system is already on the brink of instability and any increase in gain can lead to instability.
2. A negative phase margin implies the system is prone to oscillations and may exhibit poor transient response.
3. These margins indicate instability and poor performance under parameter variations and disturbances.

```
s = tf('s');
num = 1;
den = [1, 0, 2];
G1 = tf(num, den);
G2 = tf(exp(-1.2 * s));

G3 = G1 * G2;

%bode(G3); grid on ;
%[gm, pm , wcg, wpc] = margin(G3);


nyquist(G3)
```

3.2 part 2

System dynamics can change the stability margin. New poles and zeroes can alter the system making it stable or unstable.
Controller Gain:

1. Proportional Gain (Kp):

Increasing proportional gain can enhance system response and reduce steady-state error but may decrease stability margins.
Conversely, decreasing Kp can improve stability margins but may slow down system response.

2. Integral Gain (Ki):

Increasing integral gain can improve steady-state accuracy and reduce offset but may reduce stability margins and introduce oscillations if too high. Decreasing Ki can improve stability margins but may lead to steady-state error in the presence of disturbances.

3. Derivative Gain (Kd):

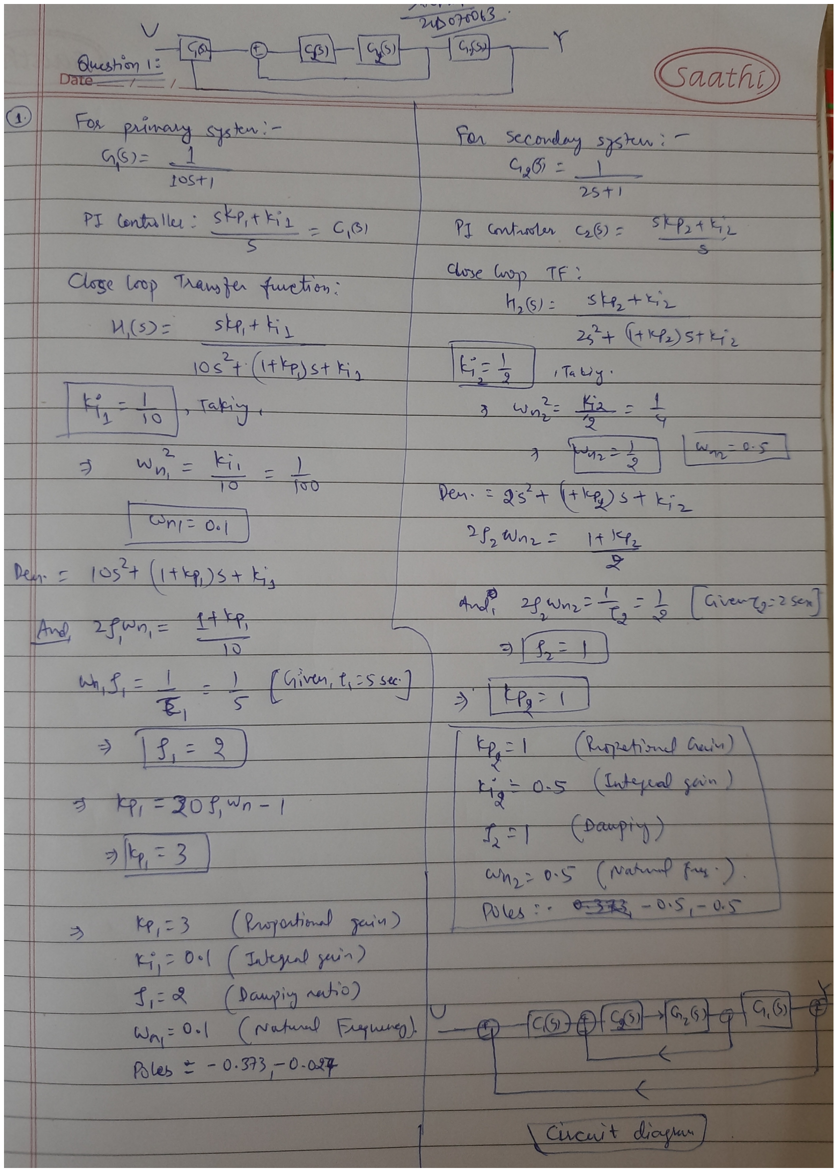
Increasing derivative gain can improve transient response and damping but may destabilize the system if too high, reducing stability margins. Decreasing Kd can enhance stability margins but may result in a slower transient response.

Tuning:
Tuning can be done using the iterative tuning method like the Ziglar- Nicholas method to systematically adjust controller gains while observing system response and stability margins.

4 references

***For question 1, the Control Guru website has articles for jacketed CSTR : link 1: https://controlguru.com/modeling-the-dynamics-of-the-jacketed-stirred-reactor-with-software/

***For question 2, I used YouTube to learn plotting techniques and for basic calculation, I used Professor Debraj's notes from the course EE302 (Basic Control Theory).



Question 1:

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(1.) For primary system:-

$$G_1(s) = \frac{1}{10s+1}$$

PI Controller: $\frac{sk_{p_1} + k_{i_1}}{s} = C_1(s)$

Close loop Transfer function:

$$H_1(s) = \frac{sk_{p_1} + k_{i_1}}{10s^2 + (1+k_{p_1})s + k_{i_1}}$$

$\boxed{k_{i_1} = \frac{1}{10}}$, Taking,

$$\Rightarrow \omega_{n_1}^2 = \frac{k_{i_1}}{10} = \frac{1}{100}$$

$$\boxed{\omega_{n_1} = 0.1}$$

Den. $= 10s^2 + (1+k_{p_1})s + k_{i_1}$

And, $2\zeta_1\omega_{n_1} = \frac{1+k_{p_1}}{10}$

$\omega_{n_1}\zeta_1 = \frac{1}{\tau_1} = \frac{1}{5}$ [Given, $t_1 = 5$ sec.]

$$\Rightarrow \boxed{\zeta_1 = 2}$$

$$\Rightarrow k_{p_1} = 20\zeta_1\omega_n - 1$$

$$\Rightarrow \boxed{k_{p_1} = 3}$$

$\Rightarrow$ $k_{p_1} = 3$ (Proportional gain)

$k_{i_1} = 0.1$ (Integral gain)

$\zeta_1 = 2$ (Damping ratio)

$\omega_{n_1} = 0.1$ (Natural Frequency).

Poles $= -0.373, -0.027$

For secondary system:-

$$G_2(s) = \frac{1}{2s+1}$$

PI controller $C_2(s) = \frac{sk_{p_2} + k_{i_2}}{s}$

Close loop TF:

$$H_2(s) = \frac{sk_{p_2} + k_{i_2}}{2s^2 + (1+k_{p_2})s + k_{i_2}}$$

$\boxed{k_{i_2} = \frac{1}{2}}$, Taking.

$$\Rightarrow \omega_{n_2}^2 = \frac{k_{i_2}}{2} = \frac{1}{4}$$

$$\Rightarrow \boxed{\omega_{n_2} = \frac{1}{2}} \quad \boxed{\omega_{n_2} = 0.5}$$

Den. $= 2s^2 + (1+k_{p_2})s + k_{i_2}$

$$2\zeta_2\omega_{n_2} = \frac{1+k_{p_2}}{2}$$

And, $2\zeta_2\omega_{n_2} = \frac{1}{\tau_2} = \frac{1}{2}$ [Given $\tau_2 = 2$ sec]

$$\Rightarrow \boxed{\zeta_2 = 1}$$

$$\Rightarrow \boxed{k_{p_2} = 1}$$

$k_{p_2} = 1$ (Proportional Gain)

$k_{i_2} = 0.5$ (Integral gain)

$\zeta_2 = 1$ (Damping)

$\omega_{n_2} = 0.5$ (Natural freq.).

Poles: ~~0.373~~, $-0.5, -0.5$

Circuit diagram

Figure 5: question1 part 1 calculations

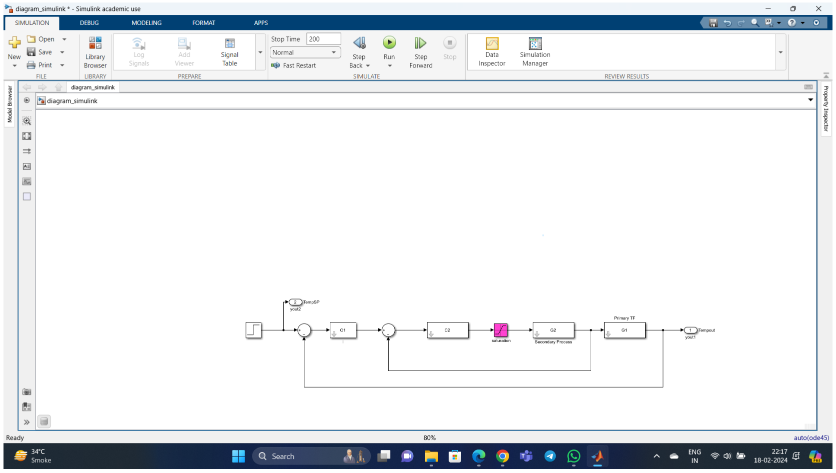


Figure 6: q1 part2 block diagram

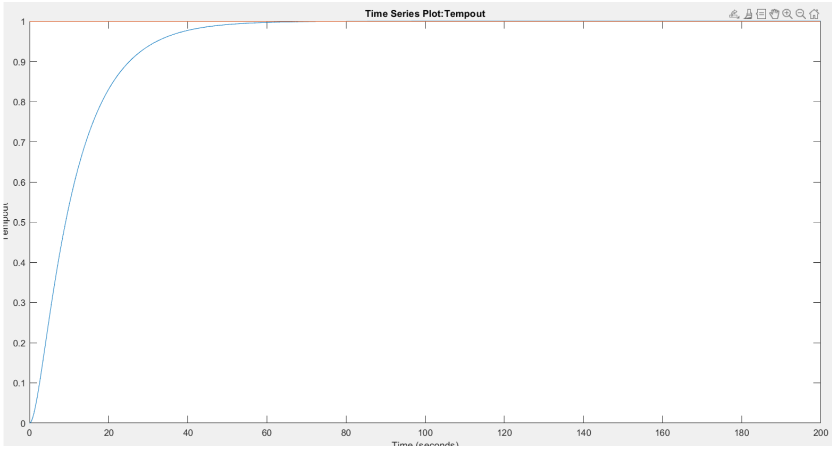


Figure 7: q1 part2 step output

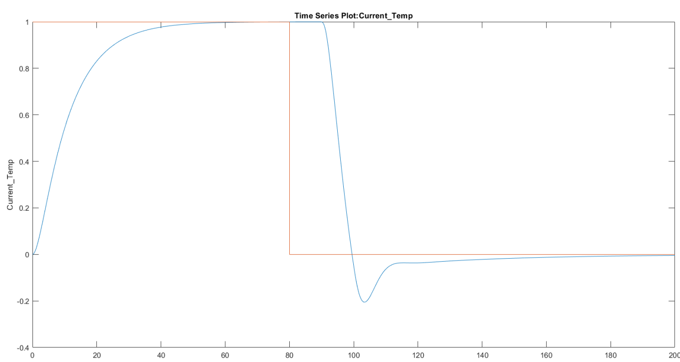
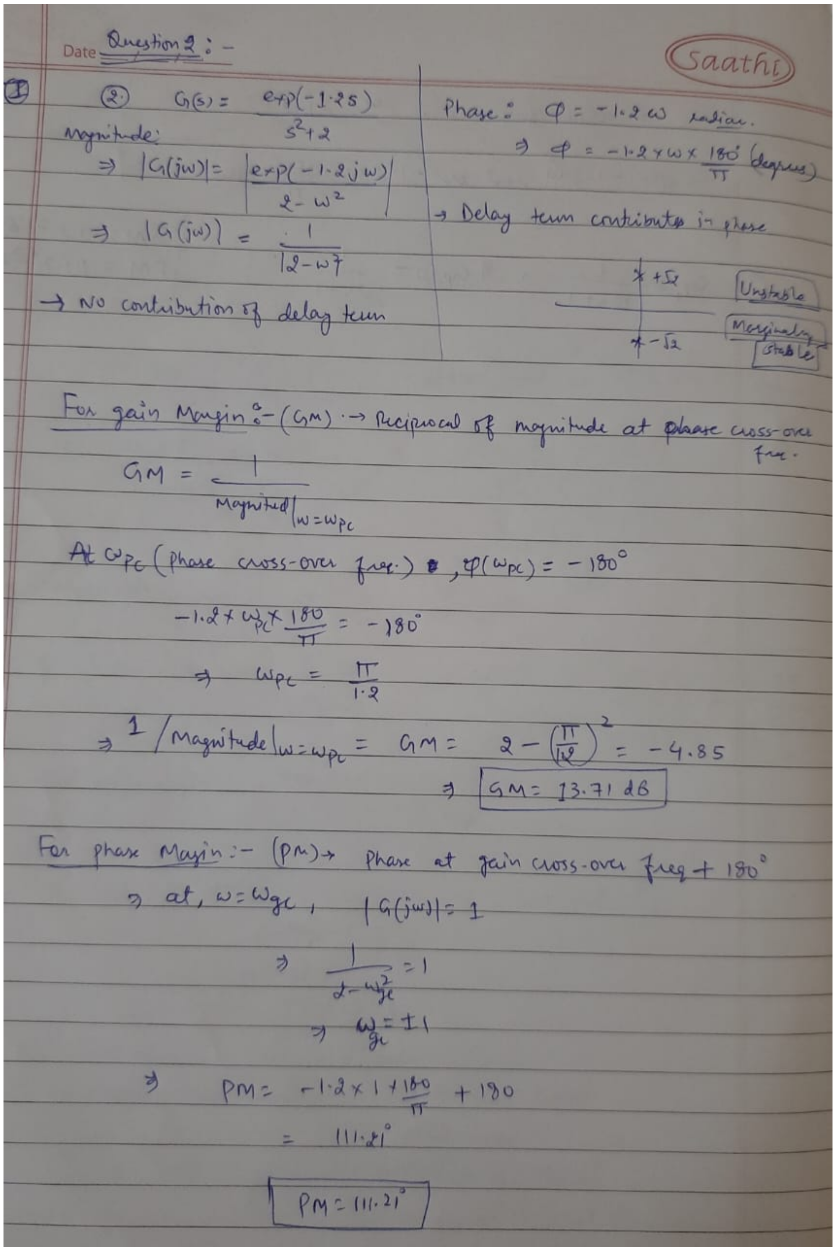


Figure 8: q1 part2 integral wind



Question 2 :-

(2) $G(s) = \frac{\exp(-1.2s)}{s^2+2}$

Magnitude:

$\Rightarrow |G(j\omega)| = \left|\frac{\exp(-1.2j\omega)}{2-\omega^2}\right|$

$\Rightarrow |G(j\omega)| = \frac{1}{|2-\omega^2|}$

→ No contribution of delay term

Phase: $\phi = -1.2\omega$ radian.

$\Rightarrow \phi = -1.2 \times \omega \times \frac{180^\circ}{\pi}$ (degrees)

→ Delay term contributes in phase

$+\sqrt{2}$ Unstable

$-\sqrt{2}$ Marginally stable

For gain Margin:- (GM) → Reciprocal of magnitude at phase cross-over freq.

$GM = \frac{1}{\text{Magnitude}|_{\omega=\omega_{pc}}}$

At ω_{pc} (phase cross-over freq.), $\phi(\omega_{pc}) = -180^\circ$

$-1.2 \times \omega_{pc} \times \frac{180}{\pi} = -180^\circ$

$\Rightarrow \omega_{pc} = \frac{\pi}{1.2}$

$\Rightarrow 1/\text{Magnitude}|_{\omega=\omega_{pc}} = GM = 2 - \left(\frac{\pi}{1.2}\right)^2 = -4.85$

$\Rightarrow \boxed{GM = 13.71\ dB}$

For phase Margin:- (PM) → Phase at gain cross-over freq + 180°

$\Rightarrow$ at, $\omega = \omega_{gc}$, $|G(j\omega)| = 1$

$\Rightarrow \frac{1}{2-\omega_{gc}^2} = 1$

$\Rightarrow \omega_{gc} = \pm 1$

$\Rightarrow PM = -1.2 \times 1 \times \frac{180}{\pi} + 180$

$= 111.21^\circ$

$\boxed{PM = 111.21^\circ}$

Figure 9: question 2 part 1 calulations

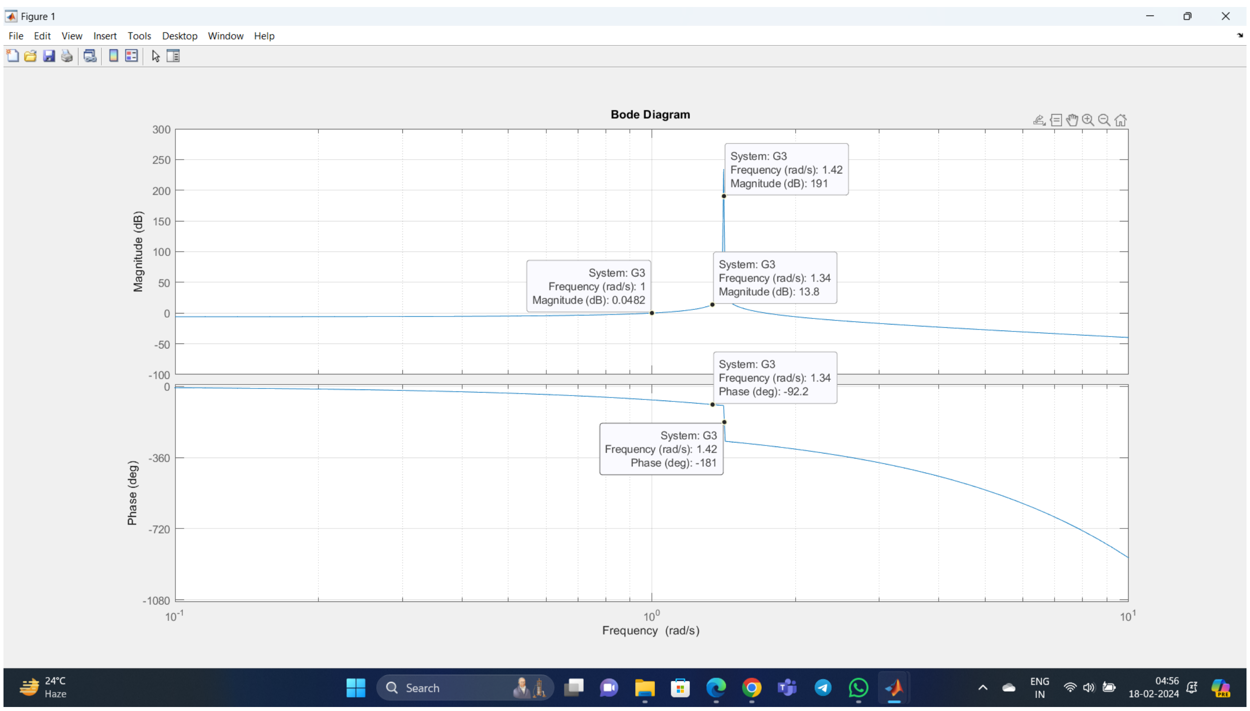


Figure 10: question 2 part 1 bodeplot

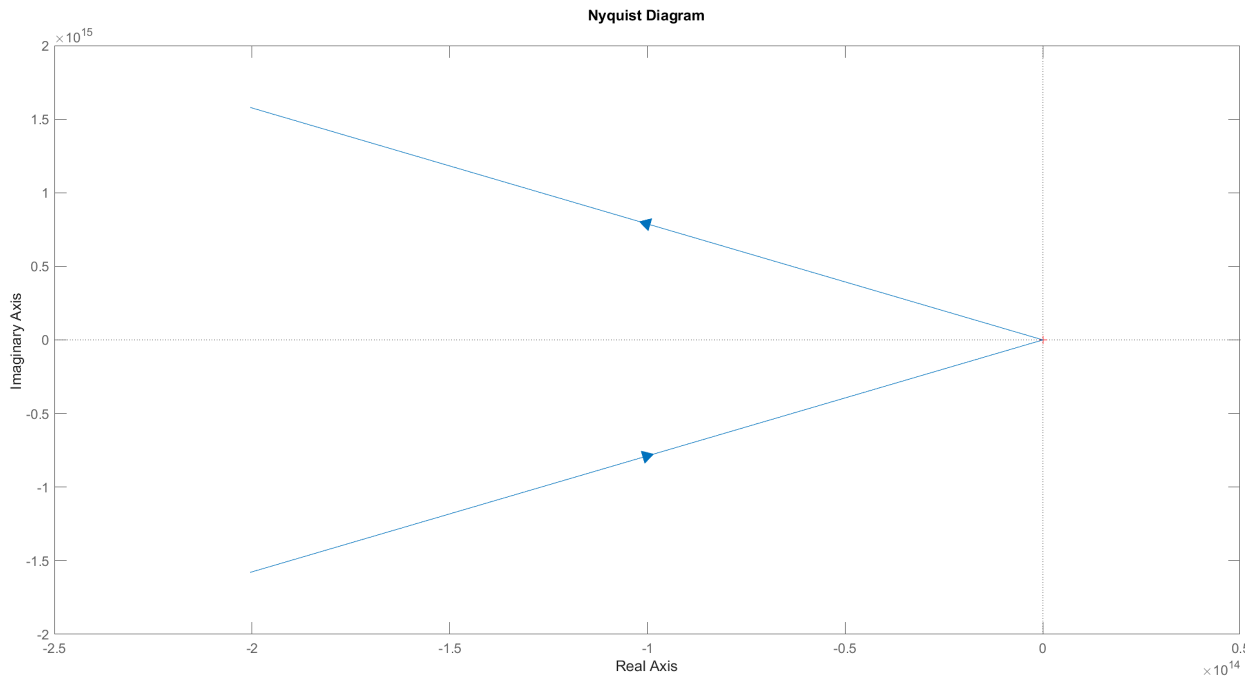


Figure 11: question 2 part 1 Nyquist plot

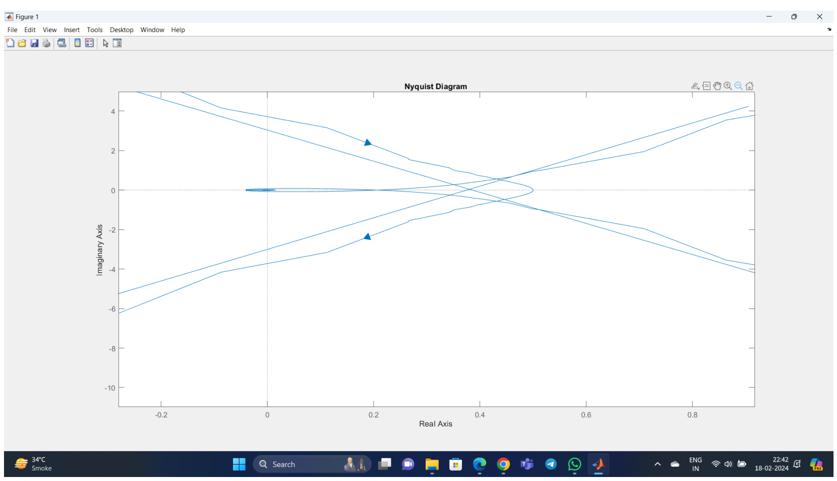


Figure 12: q1 part1 nyquist plot zoomed