TOPIC 9
VECTOR ALGEBRA
SCHEMATIC DIAGRAM

Topic	**Concept**	**Degree of importance**	**Refrence** NCERT Text Book Edition 2007
Vector algebra	(i)Vector and scalars	*	Q2 pg428
	(ii)Direction ratio and direction cosines	*	Q 12,13 pg 440
	(iii)Unit vector	* *	Ex 6,8 Pg 436
	(iv)Position vector of a point and collinear vectors	* *	Q 15 Pg 440 , Q 11Pg440 , Q 16 Pg448
	(v)Dot product of two vectors	**	Q6 ,13 Pg445
	(vi)Projection of a vector	* * *	Ex 16 Pg 445
	(vii)Cross product of two vectors	* *	Q 12 Pg458
	(viii)Area of a triangle	*	Q 9 Pg 454
	(ix)Area of a parallelogram	*	Q 10 Pg 455

SOME IMPORTANT RESULTS/CONCEPTS

* Position vector of point $A(x, y, z) = \vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$

* If $A(x_1, y_1, z_1)$ and point $B(x_2, y_2, z_2)$ then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

* If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$; $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

* Unit vector parallel to $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

* Scalar Product (dot product) between two vectors: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$; θ is angle between the vectors

* $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

* If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$

* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a}.\vec{b} = 0$
* $\vec{a}.\vec{a} = \left|\vec{a}\right|^2$
* Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}$
* Vector product between two vectors:

  $\vec{a}\times\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta\,\hat{n}$; $\hat{n}$ is the normal unit vector which is perpendicular to both $\vec{a}$ & $\vec{b}$
* $\hat{n} = \dfrac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$
* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a}\times\vec{b} = 0$
* Area of triangle (whose sides are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|$
* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$) $= \left|\vec{a}\times\vec{b}\right|$
* Area of parallelogram (whose diagonals are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|$

ASSIGNMENTS

(i) Vector and scalars, Direction ratio and direction cosines&Unit vector

LEVEL I

1. If $\vec{a} = \hat{i} + \hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} - 4\hat{j} + 3\hat{k}$ find a unit vector parallel to $\vec{a} + \vec{b}$
2. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$
3. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + \hat{k}$; $\vec{c} = -\hat{i} + \hat{j} + \hat{k}$ find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$

4. Find a unit vector in the direction of the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ **[CBSE 2011]**

5. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$, whose magnitude is 7

LEVEL II

1. Find a vector of magnitude 5 units, perpendicular to each of the vectors ($\vec{a} + \vec{b}$) ,($\vec{a} - \vec{b}$) where

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

2. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

LEVEL – III

1. If a line make α,β, γ with the X - axis , Y– axis and Z – axis respectively, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$
2. For what value of p, is $(\hat{i} + \hat{j} + \hat{k})\,p$ a unit vector?
3. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis
4. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

(ii)Position vector of a point and collinear vectors

LEVEL – I

1. Find the position vector of the midpoint of the line segment joining the points A($5\hat{i} + 3\hat{j}$) and B($3\hat{i} - \hat{j}$).

2. In a triangle ABC, the sides AB and BC are represents by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA.

3. Show that the points (1,0), (6,0) ,(0,0) are collinear.

LEVEL – II

1.Write the position vector of a point R which divides the line joining the points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 2 : 1 externally.

2.Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively, externally in the ratio 1:2. Also, show that P is the mid-point of the line segment RQ

(iii) Dot product of two vectors

LEVEL – I

1.Find $\vec{a} \cdot \vec{b}$ if $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 3\hat{k}$.

2.If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$. Then find the angle between $\vec{a}$ and $\vec{b}$.

3.Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$ **[CBSE 2011]**

LEVEL – II

1. The dot products of a vector with the vectors $\hat{i} - 3\hat{j}$, $\hat{i} - 2\hat{j}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0 , 5 and 8 respectively. Find the vectors.

2. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$.

3. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, find the value of λ.

LEVEL – III

1. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$.

2. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

3. For what values of λ, vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{a} = \lambda\hat{i} - 4\hat{j} + 8\hat{k}$ are
   (i) Orthogonal (ii) Parallel `

4..Find$|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$.

5. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \mu\hat{k}$, find μ, such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

6. Show that the vector $2\hat{i} - \hat{j} + \hat{k}$, $-3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form sides of a right angled triangle.

7.Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 18$.

8. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

9. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each of them being perpendicular

to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$.

(iv) Projection of a vector

LEVEL – I

1. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}\cdot\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

2. Write the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ **[CBSE 2011]**

3.Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

LEVEL – II

1.Three vertices of a triangle are A(0, -1, -2), B(3,1,4) and C(5,7,1). Show that it is a right angled triangle. Also find the other two angles.

2.Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

3.If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non - zero and non – coplanar vectors, prove that $\vec{a} - 2\vec{b} + 3\vec{c}$, $-3\vec{b} + 5\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are also coplanar

LEVEL – III

1.If a unit vector $\vec{a}$ makes angles $\pi/4$, with $\hat{i}$, $\pi/3$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the component of $\vec{a}$ and angle θ.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

3.If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

4.Show that the points A, B, C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angled triangle.

5. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that

$$\text{(i) } \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}| \quad \text{(ii) } \tan\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$$

(vii)Cross product of two vectors

LEVEL – I

1. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 9$. Find $|\vec{a} \times \vec{b}|$

2. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

3. Find $|\vec{x}|$, if $\vec{p}$ is a unit vector and , $(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$.

4. Find p, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

LEVEL – II

1. Find λ, if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

2. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

3. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$.

4. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{a} \times \vec{b})$.

LEVEL – III

1. Find the value of the following: $\hat{i}.(\hat{j} \times \hat{k}) + \hat{i}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$

2. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$, and $\vec{a} \times \vec{b}$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

4. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $(\vec{a} - \vec{d})$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

5. Express $2\hat{i}-\hat{j}+3\hat{k}$ as the sum of a vector parellal and perpendicular to $2\hat{i}+4\hat{j}-2\hat{k}$.

(viii)Area of a triangle & Area of a parallelogram

LEVEL – I

1.Find the area of Parallelogram whose adjacent sides are represented by the vectors

$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$.

2.If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a Parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.

3. Find the area of triangle having the points A(1,1,1) , B(1,2,3) and C(2,3,1) as its vertices.

LEVEL – II

1.Show that the area of the Parallelogram having diagonals ($3\hat{i} + \hat{j} - 2\hat{k}$) and ($\hat{i} - 3\hat{j} + 4\hat{k}$) is $5\sqrt{3}$ Sq units.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices of a ΔABC, show that the area of the ΔABC is

$$\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|.$$

3.Using Vectors, find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5)

[CBSE 2011]

Questions for self evaluation

1.The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| =4$, $|\vec{c}| =5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

3. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

4. Dot product of a vector with $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$, and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5, 8 respectively. Find the vector.

5. Find the components of a vector which is perpendicular to the vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.