Abstract

This paper presents the concepts and analysis of mass-spring damper system. [1]

1 Introduction

In this paper we will model a simple behaviour of mass-spring damper system and anaylsze system reponses. Figure 1 Shows an mass-spring damper system.

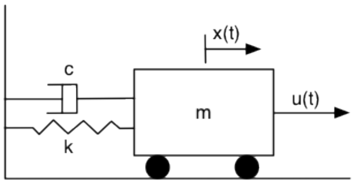

Figure 1: Mass Spring Damper System

Mass 'm' is attached to a spring, property of which is governed by Hooke's Law. $F_s = -k * x$, where k is called Spring Constant, F_s is restoring force applied by spring and x is distance by which string is compressed or stretched. Another component of system is a damper. Suppose we apply a force, say F in a direction, damper will apply a counter force in opposite direction. Therefore, damper acts as kind of shock absorber.

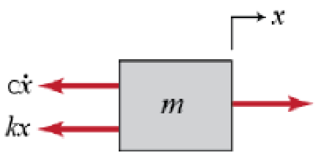

Figure 2: Free Body Diagram

We can use free body diagram to balance the force applied on body. As shown in Figure 1 and Figure 2

$$m\ddot{x} = u(t) - c\dot{x} - kx \tag{1}$$

where:
x : position of mass [m] at time t seconds
m : mass of body [Kg]
c : viscous damping coefficient [Ns/m]
k : spring constant [N/m]
u : force input [N]

Rearranging equation 1 we can write

$$u(t) = m\ddot{x} + c\dot{x} + kx \tag{2}$$

Depending on the values of m, c, and k, the system can be underdamped, overdamped or critically damped. For each case the behaviour of the system will be different.

Equation 2 is a 2^{nd} order differential equation. We will use state-space representation technique to analyze the system. State-space representation is choosen because it leds to first order differential equation and hence make analysis easier.

From 2^{nd} order differential of Equation 2,we rename the dependent variable x and its first derivative as follows:

$$x_1(t) = x(t)$$
$$x_2(t) = \dot{x}(t) = \dot{x_1}(t)$$

where:
x_1 : position of the mass m
x_2 : derivative of x_1 = velocity of the mass (in m/s)

Just from this change of variables we get the first 1st order ode.

$$\dot{x_1}(t) = x_2(t) \tag{3}$$

Substituting into the Equation 1 and noting that:

$$\dot{x_2}(t) = \ddot{x}(t)$$

we get:

$$m\dot{x_2} + cx_2 + kx_1 = u \tag{4}$$

$$\dot{x_2} = -(c/m)x_2 - (k/m)x_1 + (1/m)u \tag{5}$$

Representing Equation ?? and Equation 5 in matrix form, we get:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

Also the output equation which represent the postion of the mass is :

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2 Result

Damping coefficient ζ is a factor which determines the kind of response system is giving.

ζ is calculated as

$$\zeta = \frac{c}{2 * \sqrt{k * m}}$$

System is said to be:

- Underdamped if $\zeta < 1$
- Undamped if $\zeta = 0$
- Critically Damped if $\zeta = 1$
- Overdamped if $\zeta > 1$

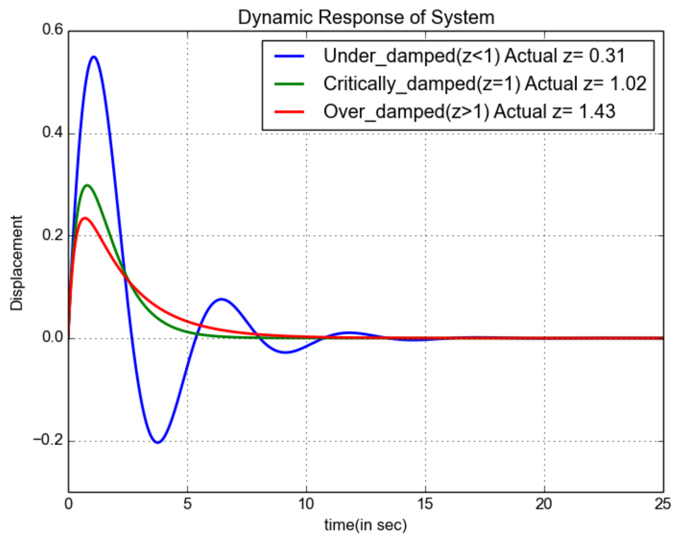

Figure 3: Dynamic Response of System for various values of damping coefficients

Figure 2 shows response of system for different value of ζ

In Figure 2, response to system is shown when $\zeta = 0$. This is done by making the damping coefficient = 0. This can be typically understood as response of a car without shock absorber going on a road. As shown in Figure 2 system never settle down or in other words keep oscillating.

References

[1] Scipy-cookbook matplotlib: lotka volterra tutorial.

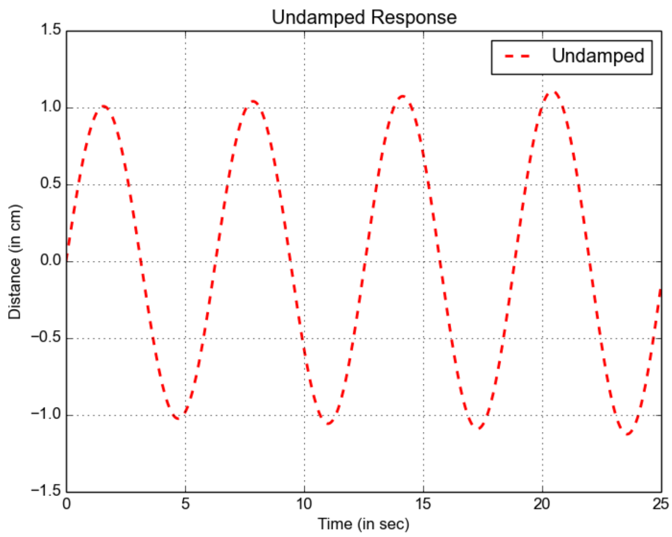

Figure 4: Undamped response of system