Line Search Methods Analysis

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Step Length

- In computing the step length we face a trade-off.
- We want to choose α_k to give a substantial reduction of f, but we don't want to spend too mush time making the choice.
- Offcourse the ideal choice would be the global minimiser of the univariate function $\phi(.)$ defined by

$$\phi(\alpha) = f(x_k + \alpha p_k), \ \alpha > 0.$$

- But in general, it is too expensive to identify this value.
- It requires too many evaluations of the objective function and/or the gradient to even find a local minimiser to moderate precision.

Step Length

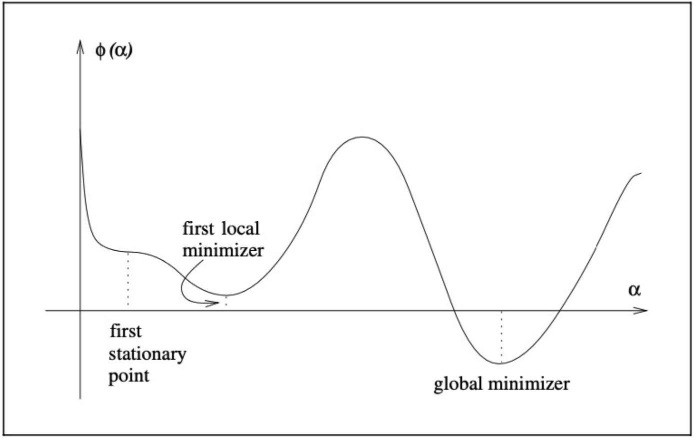

Figure: The ideal step length is the global minimiser

Step Length

- Practically, strategies perform an inexact line search to identify a step length that achieves adequate reductions in f at minimal cost.
- We will discuss these search strategies a little later.
- we will now discuss various termination conditions for line search algorithms and show that effective step lengths need not lie near minimisers of the univariate function $\phi(\alpha)$.
- Is $f(x_k + \alpha_k p_k) < f(x_k)$ good enough to get convergence??
- for example consider the function

$$f(x) = x^2 - 1$$

it has the global minima at $x = 0$, $f = -1$.

Step Length

- Consider a sequence $\{x_k\}$ s.t.

$$f(x_k) = \frac{5}{k}, \quad k = 1, 2, 3, \ldots$$
$$\implies f(x_k) > f(x_{k+1})$$

- The reduction in f at each step is not enough to get it to converge to the minimiser.

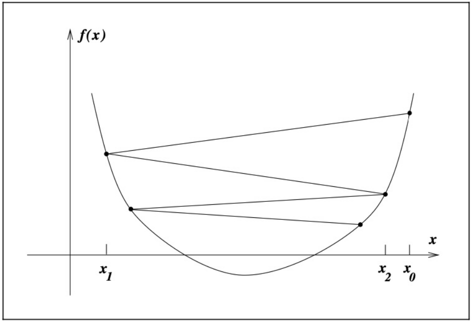

Figure: Insufficient reduction