Problem Sheet 1

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Q. 1 Define a line search algorithm. What are the key components for a line search method. What are the necessaary conditions on these components, violating those the algorithm fails to converge.

Q. 2 Suppose that $\tilde{f}(z) = f(x)$, where $x = Sz + s$ for some $S \in \mathbb{R}^{n\times n}$ and $s \in \mathbb{R}^n$. Show that

$$\nabla \tilde{f}(z) = S^T \nabla f(x), \qquad \nabla^2 \tilde{f}(z) = S^T \nabla^2 f(x) S. \tag{1}$$

Q. 3 Let $f : \mathbb{R}^n \to \mathbb{R}$. Assume that f is twice continuously differentiable. Explicitly derive the steepest descent direction i.e. the direction in which maximum reduction takes place from any point x_0.

Q. 4 Write down the conditions to be enforced on the matrix B which is an approiximation to the Hessian in a quasin Newton method, for the search direction to be a descent direction. Show that under these conditions the quasin Newton direction is a descent direction.

Q. 5 Suppose f is the following quadratic function

$$f(x) = \frac{1}{2} x^T \mathcal{Q} x - b^T x, \tag{2}$$

where $\mathcal{Q}$ is symmetric and positive definite. Find the minimiser of the function. Prove that it is unique. Compute α such that it uniquely minimises the univariate $\phi(\alpha) = f(x - \alpha \nabla f)$ for any fixed x.