Problem Sheet 4

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Q. 1 Consider the following optimisation problem:

$$\min_x f(x) \qquad \text{subject to } \; Ax = b, \tag{1}$$

where A is a $m \times n$ matrix and $m \leq n$ and x is a vector in $\mathbb{R}^n$ and b is a vector in $\mathbb{R}^m$. If A has full row rank show that (1) is equivalent to an unconstrained optimisation problem given by:

$$\min_{x_N} h(x_N) \stackrel{\text{def}}{=} f\left(P \begin{bmatrix} B^{-1}b - B^{-1}Nx_N \\ x_N \end{bmatrix}\right), \tag{2}$$

where $x_B \in \mathbb{R}^m$ and $x_N \in \mathbb{R}^{N-m}$ are some appropraite vectors, P, B and N are respectively $n \times n$, $m \times m$ and $(n-m) \times m$ matrices.

Q. 2 Consider the quadratic program:

$$\begin{aligned} \min_x q(x) &= \frac{1}{2}x^T G x + x^T c \\ \text{subject to } \; Ax &\geq b, \quad \bar{A}x = \bar{b}, \end{aligned} \tag{3}$$

where G is symmetric and positive semidefinite. Write down the KKT conditions for the above problem.

Q. 3 Consider the following quadratic optimisation problem

$$\min_x q(x) = (x_1 - 1)^2 + (x_2 - 2.5)^2 \tag{4}$$

$$\text{subject to } \quad x_1 - 2x_2 + 2 \geq 0, \tag{5}$$

$$-x_1 - 2x_2 + 6 \geq 0, \tag{6}$$

$$-x_1 + 2x_2 + 2 \geq 0, \tag{7}$$

$$x_1 \geq 0, \tag{8}$$

$$x_2 \geq 0. \tag{9}$$

Refer the constraints, in order, by indices 1 through 5. Starting with the feasible point $x^0 = (2, 0)^T$ and the intial working set $\mathcal{W}_0 = \{3\}$. Write iterations of primal active-set method and inturn solve the problem under consideration.

Q. 4 Write down the similarities and difference between the effect of the quadratic-penalty function and non-smooth l_1 penalty function used for optimisation of constrained optimisation problems. Specifically, do mention about their convergence.

Q. 5 Consider the equality constrained optimisation problem

$$\min_x f(x) \quad \text{subject to } c_i(x) = 0,\ i \in \mathcal{E}. \tag{10}$$

Define the augmented Lagrangian is as:

$$\mathcal{L}_A(x, \lambda; \mu) \stackrel{\text{def}}{=} f(x) - \sum_{i \in \mathcal{E}} \lambda_i c_i(x) + \frac{\mu}{2} \sum_{i \in \mathcal{E}} c_i^2(x). \tag{11}$$

Note the similarities and disimilarities between the augumented Lagragian function, the Lagrangian function, and the quadratic penalty fucntion