Time Complexity Analysis

1 Key Variables

- S: Total number of **unique symptoms** across all diseases.
- \bullet D: Total number of **diseases**.
- M: Number of user-entered symptom primes in one run.

Our goal is to understand the computational costs in two phases:

- 1. Precomputation (Once at Startup)
- 2. Per-Analysis (When User Inputs Symptoms)

2 Phase 1: Precomputation

2.1 1. Generating S Prime Numbers

We first generate the first S primes. If we use the naive primality check (dividing by all integers up to \sqrt{n}), the approximate time can be viewed as:

$$\mathcal{O}(S \times \sqrt{p_S}),$$

where p_S is roughly the size of the S-th prime. For small S (e.g., 10–20), this is very fast. For larger S, it can become a bottleneck (though generating primes is well-understood and easily computable using more complex hardware).

2.2 2. Building Disease SQFs & an Inverted Index

- Disease SQF: Multiply the prime for each symptom in a disease. For each of the D diseases, this is roughly $\mathcal{O}(\text{number of symptoms per disease}) \leq \mathcal{O}(S)$.
- Inverted Index: A dictionary from prime \rightarrow (set of diseases). Inserting entries involves the same pass over each disease's symptoms, again $\mathcal{O}(D \cdot S)$ in the worst case.

Overall, precomputation is dominated by:

$$\mathcal{O}(S\sqrt{p_S}+D\cdot S)$$
.

With small S and D, this is typically very quick.

3 Phase 2: Per-Analysis (User Input)

When a user provides M symptom primes, we do:

- 1. Validation and Parsing (checking each prime against known primes): $\mathcal{O}(M)$.
- 2. Matching with the Inverted Index:
 - For each of the M user primes, look up the corresponding diseases in a hash map (average $\mathcal{O}(1)$).
 - If every prime happened to map to most of the D diseases, this step is at worst $\mathcal{O}(M \cdot D)$.
 - In many cases, it is much faster because not all primes appear in all diseases.
- 3. Optional Graph Visualization: checks each (prime, disease) pair to draw an edge if relevant. In a worst-case scenario, this can be another $\mathcal{O}(M \times D)$ operation. Typically fast for small M and D.

Hence, each user query takes up to:

$$\mathcal{O}(M + (M \times D) + (M \times D)) = \mathcal{O}(M \times D).$$

For small M and D, this is generally quite efficient.

4 Summary

- Precomputation (One-Time): $\mathcal{O}(S\sqrt{p_S} + D \cdot S)$
- Per-Analysis (Each Time a User Inputs M Symptoms): $\mathcal{O}(M \times D)$

When S and D are small (e.g. $S \approx 10$ –20, $D \approx 5$ –10), both phases run quickly. If S or D grow large, the following become potential bottlenecks:

- 1. **Prime Generation** via naive trial division. (Use a faster sieve if S is large.)
- 2. Matching/Visualization if $M \times D$ becomes large, causing many lookups and edge plots.