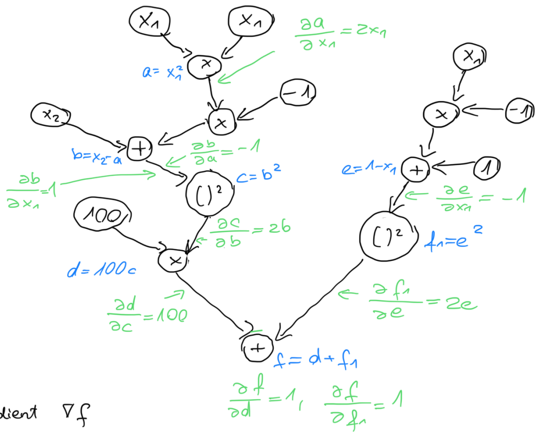
Blatt 3 A b)

$f(x) = 100\,(x_2 - x_1^2)^2 + (1-x_1)^2$

<u>Ges:</u> gerichteter azyklischer Graph, Gradient ∇f der Rosenbrock-Funktion

<u>Lsg:</u>

$$\underbrace{\underbrace{100\,\underbrace{(\underbrace{x_2-\underbrace{x_1^2}_{=a}}_{x_2-a=b})^2}_{b^2=c}}_{100c=d}+\underbrace{(\underbrace{1-x_1}_{e})^2}_{e^2=f_1}}_{d+f_1=f}$$



Gradient ∇f

$\frac{\partial a}{\partial x_1} = 2x_1$, $\frac{\partial b}{\partial x_2} = 1$, $\frac{\partial b}{\partial a} = -1$, $\frac{\partial c}{\partial b} = 2b$,

$\frac{\partial d}{\partial c} = 100$, $\frac{\partial e}{\partial x_1} = -1$, $\frac{\partial f_1}{\partial e} = 2e$, $\frac{\partial f}{\partial d} = 1$,

$\frac{\partial f}{\partial f_1} = 1$

$$\frac{\partial f}{\partial x_2} = \frac{\partial f}{\partial d}\,\frac{\partial d}{\partial c}\,\frac{\partial c}{\partial b}\,\frac{\partial b}{\partial x_1} = 1 \cdot 100 \cdot 2b \cdot 1$$

$$= 200\,(x_2 - x_1^2)$$

$$\left.\frac{\partial f}{\partial x_1}\right|_d = \frac{\partial f}{\partial d}\,\frac{\partial d}{\partial c}\,\frac{\partial c}{\partial b}\,\frac{\partial b}{\partial a}\,\frac{\partial a}{\partial x_1} = 1 \cdot 100 \cdot 2b \cdot (-1) \cdot 2x_1$$

$$= -400\,x_1\,(x_2 - x_1^2)$$

$$\left.\frac{\partial f}{\partial x_1}\right|_{f_1} = \frac{\partial f}{\partial f_1}\,\frac{\partial f_1}{\partial e}\,\frac{\partial e}{\partial x_1} = 1 \cdot 2e \cdot (-1) = -2 \cdot (1 - x_1)$$

$$\Rightarrow \quad \frac{\partial f}{\partial x_1} = -400x_1\,(x_2 - x_1^2) - 2\,(1 - x_1)$$

$$\operatorname{grad}(f) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_1} \end{pmatrix} = \begin{pmatrix} -400x_1\,(x_2 - x_1^2) - 2(1 - x_1) \\ 200\,(x_2 - x_1^2) \end{pmatrix}$$