

## SBML Model Report

**Model name:**  
**“Komarova2003\_BoneRemodeling”**



May 6, 2016

### 1 General Overview

This is a document in SBML Level 2 Version 3 format. This model was created by Harish Dharuri<sup>1</sup> at July 30<sup>th</sup> 2007 at 8:56 p.m. and last time modified at October ninth 2014 at 4:24 p.m. Table 1 gives an overview of the quantities of all components of this model.

Table 1: Number of components in this model, which are described in the following sections.

Element	Quantity	Element	Quantity
compartment types	0	compartments	1
species types	0	species	7
events	4	constraints	0
reactions	6	function definitions	0
global parameters	13	unit definitions	2
rules	3	initial assignments	0

### Model Notes

This a model from the article:

**Mathematical model predicts a critical role for osteoclast autocrine regulation in the control of bone remodeling.**

Komarova SV, Smith RJ, Dixon SJ, Sims SM, Wahl LM Bone2003 Aug;33(2):206-15 [14499354](#),

**Abstract:**

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Bone remodeling occurs asynchronously at multiple sites in the adult skeleton and involves resorption by osteoclasts, followed by formation of new bone by osteoblasts. Disruptions in bone remodeling contribute to the pathogenesis of disorders such as osteoporosis, osteoarthritis, and Paget's disease. Interactions among cells of osteoblast and osteoclast lineages are critical in the regulation of bone remodeling. We constructed a mathematical model of autocrine and paracrine interactions among osteoblasts and osteoclasts that allowed us to calculate cell population dynamics and changes in bone mass at a discrete site of bone remodeling. The model predicted different modes of dynamic behavior: a single remodeling cycle in response to an external stimulus, a series of internally regulated cycles of bone remodeling, or unstable behavior similar to pathological bone remodeling in Paget's disease. Parametric analysis demonstrated that the mode of dynamic behavior in the system depends strongly on the regulation of osteoclasts by autocrine factors, such as transforming growth factor beta. Moreover, simulations demonstrated that nonlinear dynamics of the system may explain the differing effects of immunosuppressants on bone remodeling in vitro and in vivo. In conclusion, the mathematical model revealed that interactions among osteoblasts and osteoclasts result in complex, nonlinear system behavior, which cannot be deduced from studies of each cell type alone. The model will be useful in future studies assessing the impact of cytokines, growth factors, and potential therapies on the overall process of remodeling in normal bone and in pathological conditions such as osteoporosis and Paget's disease.

The model reproduces Fig 2A and Fig 2B of the paper. Note that the Y-axis scale is not right, the osteoblast steady state is approximately 212 and not 0 as depicted in the figure. Also, there is a typo in the equation for  $x2\_bar$  which has been corrected here. Model successfully tested on MathSBML.

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## 2 Unit Definitions

This is an overview of five unit definitions of which three are predefined by SBML and not mentioned in the model.

### 2.1 Unit substance

**Name** number

**Definition** item

## 2.2 Unit time

**Name** day

**Definition** 86400 s

## 2.3 Unit volume

**Notes** Litre is the predefined SBML unit for volume.

**Definition** l

## 2.4 Unit area

**Notes** Square metre is the predefined SBML unit for area since SBML Level 2 Version 1.

**Definition** m<sup>2</sup>

## 2.5 Unit length

**Notes** Metre is the predefined SBML unit for length since SBML Level 2 Version 1.

**Definition** m

# 3 Compartment

This model contains one compartment.

Table 2: Properties of all compartments.

Id	Name	SBO	Spatial Dimensions	Size	Unit	Constant	Outside
compartment			3	1	litre	<input checked="" type="checkbox"/>	

### 3.1 Compartment `compartment`

This is a three dimensional compartment with a constant size of one litre.

## 4 Species

This model contains seven species. Section 9 provides further details and the derived rates of change of each species.

Table 3: Properties of each species.

Id	Name	Compartment	Derived Unit	Constant	Boundary Condition
x1	Osteoclast	compartment	item	$\square$	$\square$
x2	Osteoblast	compartment	item	$\square$	$\square$
x1_bar	Steady state osteoclast	compartment	item	$\square$	$\square$
x2_bar	Steady state osteoblast	compartment	item	$\square$	$\square$
z	Bone mass	compartment	item	$\square$	$\square$
y1	Cells actively resorbing bone	compartment	item	$\square$	$\square$
y2	Cells actively forming bone	compartment	item	$\square$	$\square$

## 5 Parameters

This model contains 13 global parameters.

Table 4: Properties of each parameter.

Id	Name	SBO	Value	Unit	Constant
alpha1			3.000		<input checked="" type="checkbox"/>
alpha2			4.000		<input checked="" type="checkbox"/>
beta1			0.200		<input checked="" type="checkbox"/>
beta2			0.020		<input checked="" type="checkbox"/>
g11			0.500		<input checked="" type="checkbox"/>
g21			−0.500		<input checked="" type="checkbox"/>
g12			1.000		<input checked="" type="checkbox"/>
g22			0.000		<input checked="" type="checkbox"/>
k1			0.240		<input checked="" type="checkbox"/>
k2			0.002		<input checked="" type="checkbox"/>
gamma			0.000		<input type="checkbox"/>
flag- _resorption			0.000		<input type="checkbox"/>
flag- _formation			0.000		<input type="checkbox"/>

## 6 Rules

This is an overview of three rules.

### 6.1 Rule `gamma`

Rule `gamma` is an assignment rule for parameter `gamma`:

$$\text{gamma} = g12 \cdot g21 - (1 - g11) \cdot (1 - g22) \quad (1)$$

### 6.2 Rule `x1_bar`

Rule `x1_bar` is an assignment rule for species `x1_bar`:

$$[x1\_bar] = \left( \frac{\text{beta1}}{\text{alpha1}} \right)^{\frac{1-g22}{\text{gamma}}} \cdot \left( \frac{\text{beta2}}{\text{alpha2}} \right)^{\frac{g21}{\text{gamma}}} \quad (2)$$

### 6.3 Rule $x2\_bar$

Rule  $x2\_bar$  is an assignment rule for species  $x2\_bar$ :

$$[x2\_bar] = \left( \frac{\text{beta1}}{\text{alpha1}} \right)^{\frac{g12}{\text{gamma}}} \cdot \left( \frac{\text{beta2}}{\text{alpha2}} \right)^{\frac{1-g11}{\text{gamma}}} \quad (3)$$

## 7 Events

This is an overview of four events. Each event is initiated whenever its trigger condition switches from false to true. A delay function postpones the effects of an event to a later time point. At the time of execution, an event can assign values to species, parameters or compartments if these are not set to constant.

### 7.1 Event `event_0000001`

**Notes** When the osteoclast number is greater than the steady state osteoclast number the flag is set to 1, this insures that in reaction R5,  $y1$  is equal to the difference of  $x1$  and  $x1\_bar$ . When  $x1$  is greater than  $x1\_bar$  the difference is set to zero by setting the flag to zero. The same procedure is used for bone formation.

**Trigger condition**

$$x1 > x1\_bar \quad (4)$$

**Assignment**

$$\text{flag\_resorption} = 1 \quad (5)$$

### 7.2 Event `event_0000003`

**Trigger condition**

$$x1 \leq x1\_bar \quad (6)$$

**Assignment**

$$\text{flag\_resorption} = 0 \quad (7)$$

### 7.3 Event `event_0000002`

**Trigger condition**

$$x2 > x2\_bar \quad (8)$$

**Assignment**

$$\text{flag\_formation} = 1 \quad (9)$$

### 7.4 Event `event_0000004`

**Trigger condition**

$$x2 \leq x2\_bar \quad (10)$$

**Assignment**

$$\text{flag\_formation} = 0 \quad (11)$$

## 8 Reactions

This model contains six reactions. All reactions are listed in the following table and are subsequently described in detail. If a reaction is affected by a modifier, the identifier of this species is written above the reaction arrow.

Table 5: Overview of all reactions

Nº	Id	Name	Reaction Equation	SBO
1	R1	Osteoclast production	$\emptyset \xrightarrow{x2} x1$	
2	R2	Osteoclast removal	$x1 \xrightarrow{\phantom{x1}} \emptyset$	
3	R3	Osteoblast production	$\emptyset \xrightarrow{x1} x2$	
4	R4	Osteoblast removal	$x2 \xrightarrow{\phantom{x2}} \emptyset$	
5	R5	Bone resorption	$z \xrightarrow{x1, x1\_bar} \emptyset$	
6	R6	Bone formation	$\emptyset \xrightarrow{x2, x2\_bar} z$	

### 8.1 Reaction R1

This is a reversible reaction of no reactant forming one product influenced by one modifier.

**Name** Osteoclast production

#### Reaction equation



#### Modifier

Table 6: Properties of each modifier.

Id	Name	SBO
x2	Osteoblast	

#### Product

Table 7: Properties of each product.

Id	Name	SBO
x1	Osteoclast	

#### Kinetic Law

**Derived unit** contains undeclared units

$$v_1 = \text{alpha1} \cdot x1^{g11} \cdot x2^{g21} \quad (13)$$

### 8.2 Reaction R2

This is a reversible reaction of one reactant forming no product.

**Name** Osteoclast removal

#### Reaction equation



#### Reactant



Table 8: Properties of each reactant.

Id	Name	SBO
x1	Osteoclast	

### Kinetic Law

**Derived unit** contains undeclared units

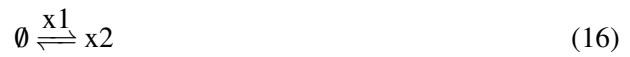
$$v_2 = \text{beta1} \cdot x1 \quad (15)$$

### 8.3 Reaction R3

This is a reversible reaction of no reactant forming one product influenced by one modifier.

**Name** Osteoblast production

### Reaction equation



### Modifier

Table 9: Properties of each modifier.

Id	Name	SBO
x1	Osteoclast	

### Product

Table 10: Properties of each product.

Id	Name	SBO
x2	Osteoblast	

### Kinetic Law

**Derived unit** contains undeclared units

$$v_3 = \text{alpha2} \cdot x1^{g12} \cdot x2^{g22} \quad (17)$$

#### 8.4 Reaction R4

This is a reversible reaction of one reactant forming no product.

**Name** Osteoblast removal

#### Reaction equation



#### Reactant

Table 11: Properties of each reactant.

Id	Name	SBO
x2	Osteoblast	

#### Kinetic Law

**Derived unit** contains undeclared units

$$v_4 = \text{beta2} \cdot x2 \quad (19)$$

#### 8.5 Reaction R5

This is a reversible reaction of one reactant forming no product influenced by two modifiers.

**Name** Bone resorption

#### Reaction equation



#### Reactant

Table 12: Properties of each reactant.

Id	Name	SBO
z	Bone mass	

#### Modifiers

Table 13: Properties of each modifier.

Id	Name	SBO
x1	Osteoclast	
x1_bar	Steady state osteoclast	

### Kinetic Law

**Derived unit** contains undeclared units

$$v_5 = \text{flag\_resorption} \cdot k_1 \cdot (x_1 - x_1\_bar) \quad (21)$$

## 8.6 Reaction R6

This is a reversible reaction of no reactant forming one product influenced by two modifiers.

**Name** Bone formation

### Reaction equation



### Modifiers

Table 14: Properties of each modifier.

Id	Name	SBO
x2	Osteoblast	
x2_bar	Steady state osteoblast	

### Product

Table 15: Properties of each product.

Id	Name	SBO
z	Bone mass	

### Kinetic Law

**Derived unit** contains undeclared units

$$v_6 = \text{flag\_formation} \cdot k_2 \cdot (x_2 - x_2\_bar) \quad (23)$$

## 9 Derived Rate Equations

When interpreted as an ordinary differential equation framework, this model implies the following set of equations for the rates of change of each species.

Identifiers for kinetic laws highlighted in gray cannot be verified to evaluate to units of SBML substance per time. As a result, some SBML interpreters may not be able to verify the consistency of the units on quantities in the model. Please check if

- parameters without an unit definition are involved or
- volume correction is necessary because the `hasOnlySubstanceUnits` flag may be set to `false` and `spacialDimensions` > 0 for certain species.

### 9.1 Species `x1`

**Name** Osteoclast

**Initial amount** 11 item

This species takes part in four reactions (as a reactant in [R2](#) and as a product in [R1](#) and as a modifier in [R3](#), [R5](#)).

$$\frac{d}{dt}x1 = v_1 - v_2 \quad (24)$$

### 9.2 Species `x2`

**Name** Osteoblast

**Initial amount** 212 item

This species takes part in four reactions (as a reactant in [R4](#) and as a product in [R3](#) and as a modifier in [R1](#), [R6](#)).

$$\frac{d}{dt}x2 = v_3 - v_4 \quad (25)$$

### 9.3 Species `x1_bar`

**Name** Steady state osteoclast

**Initial amount** 0 item

**Involved in rule** [x1\\_bar](#)

This species takes part in one reaction (as a modifier in [R5](#)) and is also involved in one rule which determines this species' quantity.

#### 9.4 Species $x2\_bar$

**Name** Steady state osteoblast

**Initial amount** 0 item

**Involved in rule**  $x2\_bar$

This species takes part in one reaction (as a modifier in [R6](#)) and is also involved in one rule which determines this species' quantity.

#### 9.5 Species $z$

**Name** Bone mass

**Initial amount** 100 item

This species takes part in two reactions (as a reactant in [R5](#) and as a product in [R6](#)).

$$\frac{d}{dt}z = v_6 - v_5 \quad (26)$$

#### 9.6 Species $y1$

**Name** Cells actively resorbing bone

**Initial amount** 0 item

This species does not take part in any reactions. Its quantity does hence not change over time:

$$\frac{d}{dt}y1 = 0 \quad (27)$$

#### 9.7 Species $y2$

**Name** Cells actively forming bone

**Initial amount** 0 item

This species does not take part in any reactions. Its quantity does hence not change over time:

$$\frac{d}{dt}y2 = 0 \quad (28)$$

SBML<sup>2</sup>TeX was developed by Andreas Dräger<sup>a</sup>, Hannes Planatscher<sup>a</sup>, Dieudonné M Wouamba<sup>a</sup>, Adrian Schröder<sup>a</sup>, Michael Hucka<sup>b</sup>, Lukas Endler<sup>c</sup>, Martin Golebiewski<sup>d</sup> and Andreas Zell<sup>a</sup>. Please see <http://www.ra.cs.uni-tuebingen.de/software/SBML2LaTeX> for more information.

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