The HDG local system can be written as:

$$\begin{pmatrix} M & C & E \\ B & D & F \\ K & G & H \end{pmatrix} \begin{pmatrix} \delta q \\ \delta u \\ \delta \hat{u} \end{pmatrix} = \begin{pmatrix} Rq \\ Ru \\ R\hat{u} \end{pmatrix} \tag{1}$$

Assuming $Rq = 0$, we have:

$$\begin{pmatrix} M & C & E \\ B & D & F \\ K & G & H \end{pmatrix} \begin{pmatrix} \delta q \\ \delta u \\ \delta \hat{u} \end{pmatrix} = \begin{pmatrix} 0 \\ Ru \\ R\hat{u} \end{pmatrix} \tag{2}$$

To eliminate δq, we take the first equation:

$$M\delta q - C\delta u + E\delta\hat{u} = 0 \tag{3}$$

$$\delta q = M^{-1}(C\delta u - E\delta\hat{u}) \tag{4}$$

Substituting into the second equation:

$$B\delta q + D\delta u + F\delta\hat{u} = Ru \tag{5}$$

$$BM^{-1}(C\delta u - E\delta\hat{u}) + D\delta u + F\delta\hat{u} = Ru \tag{6}$$

$$(BM^{-1}C + D)\delta u + (F - BM^{-1}E)\delta\hat{u} = Ru \tag{7}$$

Substituting into the third equation:

$$K\delta q + G\delta u + H\delta\hat{u} = R\hat{u} \tag{8}$$

$$KM^{-1}(C\delta u - E\delta\hat{u}) + G\delta u + H\delta\hat{u} = R\hat{u} \tag{9}$$

$$(KM^{-1}C + G)\delta u + (H - KM^{-1}E)\delta\hat{u} = R\hat{u} \tag{10}$$

Forming the reduced system

$$\begin{pmatrix} \tilde{D^*} & \tilde{F^*} \\ \tilde{G^*} & \tilde{H^*} \end{pmatrix} \begin{pmatrix} \delta u \\ \delta\hat{u} \end{pmatrix} = \begin{pmatrix} Ru \\ R\hat{u} \end{pmatrix} \tag{11}$$

Where

$$\tilde{D^*} = BM^{-1}C + D \tag{12}$$

$$\tilde{F^*} = F - BM^{-1}E \tag{13}$$

$$\tilde{G^*} = KM^{-1}C + G \tag{14}$$

$$\tilde{H^*} = H - KM^{-1}E \tag{15}$$

$$\tag{16}$$