IES MASTER

Institute for Engineers (IES/GATE/PSUs)

GATE 2023

COMPUTER SCIENCE

Detailed Solution

EXAM DATE: 04-02-2023

FORENOON SESSION (09:30 AM-12:30 PM)

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1. We reached the station late, and _____ missed the train.

(a) near (b) nearly

(c) utterly (d) mostly

Sol: (b)

Most appropriate word is nearly, Hence option (b) is correct.

2. Kind : _____ : : Often : Frequently

(By word meaning)

(a) Mean (b) Type

(c) Cruel (d) Kindly

Sol: (b)

Frequently is synonym of often, similarly type is synonym of kind.

Hence, correct answer is (b).

3. A series of natural numbers F_1, F_2, F_3, F_4, F_5, F_6, F_7 .... obeys $F_{n+1} = F_n + F_{n-1}$ for all integers for all integers $n \geq 2$

If $F_6 = 37$, and $F_7 = 60$, then what is F_1?

(a) 4 (b) 5

(c) 8 (d) 9

Sol: (a)

$f_6 = 37$, $f_7 = 60$

$f_{n+1} = f_n + f_{n-1}$, $n \geq 2$

put, $n = 6$

$f_7 = f_6 + f_5$

$60 = 37 + f_5$

$f_5 = 23$

Now, put $n = 5$

$f_6 = f_5 + f_4$

$37 = 23 + f_4$

$f_4 = 14$

Put $n = 4$

$f_5 = f_4 + f_3$

$23 = 14 + f_3$

$f_3 = 9$

put $n = 3$

$f_4 = f_3 + f_2$

$14 = 9 + f_2$

$f_2 = 5$

put, $n = 2$

$f_3 = f_2 + f_1$

$9 = 5 + f_1$

$f_1 = 4$

4. A survey for a certain year found that 90% of pregnant women received medical care at least once before giving birth. Of these women, 60% received medical care from doctors, while 40% received medical care from other healthcare providers.

Given this information, which one of the following statements can be inferred with *certainty*?

(a) More than half of the pregnant women received medical care at least once from a doctor.

(b) Less than half of the pregnant women received medical care at least once from a doctor.

(c) More than half of the pregnant women received medical care at most once from a doctor.

(d) Less than half of the pregnant women received medical care at most once from a doctor.

Sol: (a)

Total women = 100

- Pregnant women recieved medical care at least. Once before giving birth = 90% of 100 = 90
- Out of 90 women 60% received medical care from doctors = 60% of 90 = 54
- While 40% recieved from other health core providers = 40% of 90 = 46
- Here, more than half of pregnant women (54) recieved medical care at least once from a doctor.

5. Looking at the surface of a smooth 3-dimensional object from the outside, which one of the following options is TRUE?

(a) The surface of the object must be concave everywhere.

(b) The surface of the object must be convex everywhere.

(c) The surface of the object may be concave in some places and convex in other places.

(d) The object can have edges, but no corners.

Sol: (c)

6. The country of Zombieland is in distress since more than 75% of its working population is suffering from serious health issues. Studies conducted by competent health experts concluded that a complete lack of physical exercise among its working population was one of the leading causes of their health issues. As one of the measures to address the problem, the Government of Zombieland has decided to provide monetary incentives to those who ride bicycles to work.

Based only on the information provided above, which one of the following statements can be logically inferred with *certainty*?

(a) All the working population of Zombieland will henceforth ride bicycles to work.

(b) Riding bicycles will ensure that all of the working population of Zombieland is free of health issues.

(c) The health experts suggested to the Government of Zombieland to declare riding bicycles as mandatory.

(d) The Government of Zombieland believes that riding bicycles is a form of physical exercise.

Sol: (d)

Goverment of zombieland believes that riding bicycles is a form of physical exercise.

7. Consider two functions of time (t),

$f(t) = 0.01\ t^2$

$g(t) = 4(t)$

where $0 < t < \infty$

Now consider the following two statements :

(i) For some $t > 0$, $g(t) > f(t)$.

(ii) There exists a T, such that $f(t) > g(t)$ for all $t > T$.

Which one of the following options is TRUE?

(a) only (i) is correct

(b) only (ii) is correct

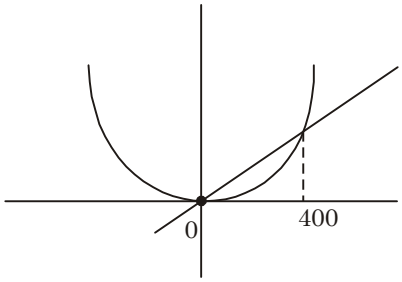
(c) both (i) and (ii) are correct

(d) neither (i) nor (ii) is correct

Sol: (c)

$f(t) = 0.01 \times t^2$ [equation for parabola]

$g(t) = 4t$ [equation for line]



$\Rightarrow$ find points where they both meet,

$\Rightarrow 0.01t^2 = 4t$

$\Rightarrow t[0.01\ t = 4]$

$\Rightarrow t = 0,\ 400$

Now, 2 statement are given $\Rightarrow f(t) < g(t)$

or $f(t) > g(t)$

It is seen from graph that for some value g(t) is greater and for some value f(t) is greater.

8. Which one of the following sentence sequences creates a coherent narrative?

(i) Once on the terrace, on her way to her small room in the corner, she notices the man right away.

(ii) She begins to pant by the time she has climbed all the stairs.

(iii) Mina has bought vegetables and rice at the market, so her bags are heavy.

(iv) He was leaning against the parapet, watching the traffic below.

(a) (i), (ii), (iv), (iii)

(b) (ii), (iii), (i), (iv)

(c) (iv), (ii), (i), (iii)

(d) (iii), (ii), (i), (iv)

Sol: (d)

9. f(x) and g(y) are functions of x and y, respectively, and f(x) = g (y) for all real values of x and y. Which one of the following ptions is necessarily TRUE for all x and y?

(a) f (x) = 0 and g(y) = 0

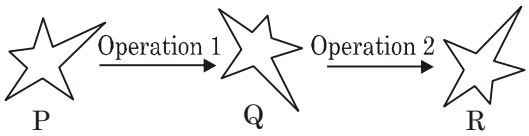
(b) f (x) = g (y) = constant

(c) f (x) ≠ constant and g(y) ≠ constant

(d) f (x) + g(y) = f (x) – g(y)

Sol: (b)

10. Which one of the options best describes the transformation of the 2-dimensional figure P to Q, and then to R, as shown?



(a) Operation 1: A clockwise rotation by 90° about an axis perpendicular to the plane of the figure

Operation 2: A reflection along a horizontal line

(b) Operation 1: A counter clockwise rotation by 90° about an axis perpendicular to the plane of the figure

Operation 2: A reflection along a horizontal line

(c) Operation 1: A clockwise rotation by 90° about an axis perpendicular to the plane of the figure

Operation 2: A reflection along a vertical line

(d) Operation 1: A counter clockwise rotation by 180° about an axis perpendicular to the plane of the figure

Operation 2: A reflection along a vertical line

Sol: (a)

1. Consider the following statements regarding the front-end and back-end of a compiler.

S1: The front-end includes phases that are independent of the target hardware.

S2: The back-end includes phases that are specific to the target hardware.

S3: The back-end includes phases that are specific to the programming language used in the source code.

Identify the CORRECT option.

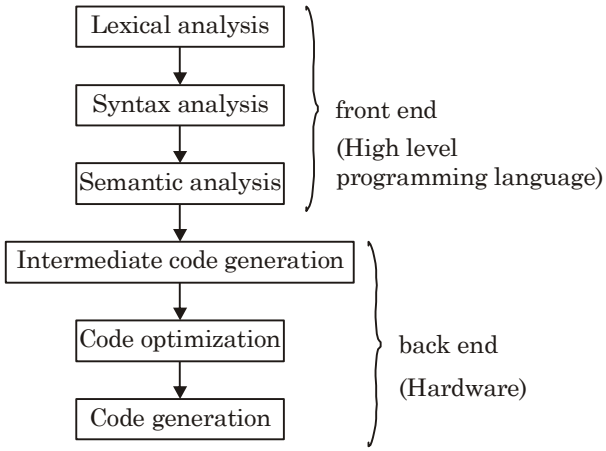
(a) Only **S1** is TRUE.

(b) Only **S1** and **S2** are TRUE.

(c) **S1**, **S2**, and **S3** are all TRUE.

(d) Only **S1** and **S3** are TRUE.

Sol: (b)



- Front end are independent of hardware because front end are specific to the programming language.
- Back end include phases that are specific to hardware.

So, only S_1 and S_2 are true.

2. Which one of the following sequences when stored in an array at locations

$A[1], \ldots, A[10]$ forms a max-heap?

(a) 23, 17, 10, 6, 13, 14, 1, 5, 7, 12

(b) 23, 17, 14, 7, 13, 10, 1, 5, 6, 12

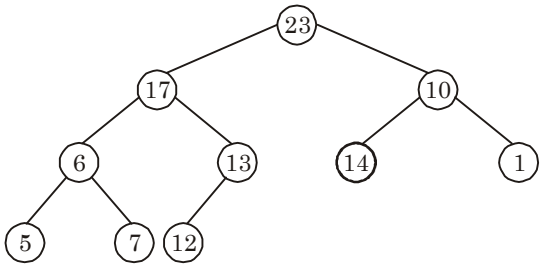
(c) 23, 17, 14, 6, 13, 10, 1, 5, 7, 15

(d) 23, 14, 17, 1, 10, 13, 16, 12, 7, 5

Sol: (b)

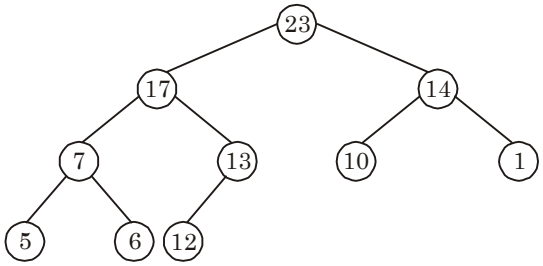
Max Heap: The value of each node is less than or equal to the value of its parent. The maximum value element is placed at the root.

(a)



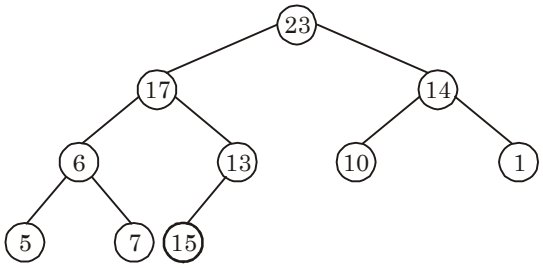
Max heap property not satisfied.

(b)



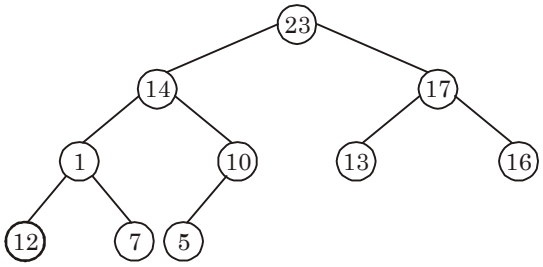
Every node satisfy max heap property.

(c)



Max heap property not satisfied.

(d)



Max heap property not satisfied.

3. Let SLLdel be a function that deletes a node in a singly-linked list given a pointer to the node and a pointer to the head of the list. Similarly, let DLLdel be another function that deletes a node in a doubly-linked list given a pointer to the node and a pointer to the head of the list.

Let n denote the number of nodes in each of the linked lists. Which one of the following choices is TRUE about the worst-case time complexity of SLLdel and DLLdel?

(a) SLLdel is O(1) and DLLdel is O(n)

(b) Both SLLdel and DLLdel are O(log(n))

(c) Both SLLdel and DLLdel are O(1)

(d) SLLdel is O(n) and DLLdel is O(1)

Sol: (d)

Given two functions SLLdel for singly linked list and DLLdel for doubly linked list.

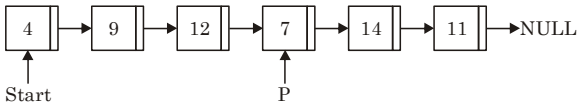
Two pointers are given

(i) start pointer (head of the list)

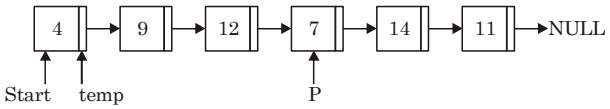
(ii) P pointer point to the node which has to delete.

Since, list is sorted or not is not given. so we assume list is unsorted.

And P pointer points a node that has to delete. Suppose node 7 has to delete.



A node temp is required to traverse the node.

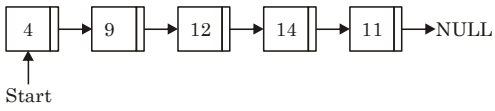


If (temp $\rightarrow$ next $\rightarrow$ data = = P $\rightarrow$ data)

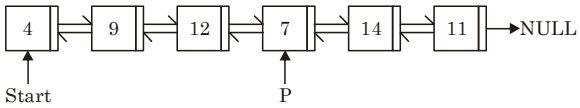
temp $\rightarrow$ next = P $\rightarrow$ next;

free (P);

return start;



But, for doulbly linked list,

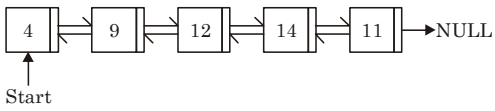


P $\rightarrow$ prev $\rightarrow$ next = P $\rightarrow$ next;

P $\rightarrow$ next $\rightarrow$ prev = P $\rightarrow$ prev;

free (P);

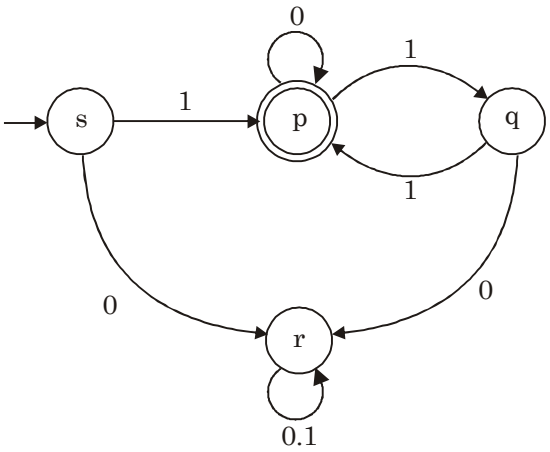
return (start);



Hence, in the singly link list, we need to traverse. So complexity O(n). But in doubly linked list no need to traverse so complexity O(1).

4. Consider the Deterministic Finite-state Automaton (DFA) A shown below. The DFA runs

on the alphabet {0, 1}, and has the set of states {s, p, q, r}, with s beingthe start state and p being the only final state.



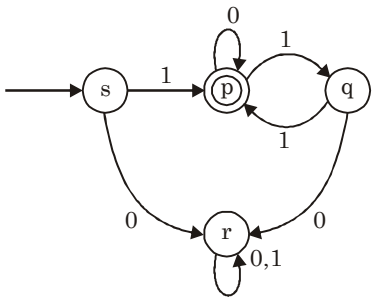
Which one of the following regular expressions correctly describes the language accepted by *A*?

(a) 1(0*11)* (b) 0(0 + 1)*

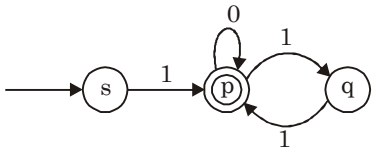
(c) 1(0 + 11)* (d) 1(110*)*

Sol: (c)

Given DFA shown below,



Since state r is the dead state. So it can be removed.



Required regular expression = 1.(0 + 11)*

5. The Lucas sequence L_n is defined by the recurrence relation:

$L_n = L_{n-1} + L_{n-2}$, for $n \geq 3$,

with $L_1 = 1$ and $L_2 = 3$

Which one of the options given is TRUE?

(a) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$

(b) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{3}\right)^n$

(c) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{3}\right)^n$

(d) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

Sol: (a)

$L_n = L_{n-1} + L_{n-2}$, for $n \geq 3$

$L_1 = 1$ and $L_2 = 3$

Put L = 1 and 2 in first option,

$$L_1 = \left(\frac{1+\sqrt{5}}{2}\right)^1 + \left(\frac{1-\sqrt{5}}{2}\right)^1$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$L_2 = \left(\frac{1+\sqrt{5}}{2}\right)^2 + \left(\frac{1-\sqrt{5}}{2}\right)^2$$

$$= (a+b)^2 + (a-b)^2$$

$$= 2\left(a^2 + b^2\right)$$

$$= 2\left(\frac{1}{4} + \frac{5}{4}\right)$$

$$= 2 \times \frac{6}{4}$$

$$= 3$$

So, option (a) is correct

6. Which one of the options given below refers to the degree (or arity) of a relation in relational database systems?

(a) Number of attributes of its relation schema.

(b) Number of tuples stored in the relation.

(c) Number of entries in the relation.

(d) Number of distinct domains of its relation schema.

Sol: (a)

Number of attributes of its relation schema is called degree or arity of a relation.

7. Suppose two hosts are connected by a point-to-point link and they are configured to use Stop-and-Wait protocol for reliable data transfer. Identify in which one of the following scenarios, the utilization of the link is the lowest.

(a) Longer link length and lower transmission rate

(b) Longer link length and higher transmission rate

(c) Shorter link length and lower transmission rate

(d) Shorter link length and higher transmission rate.

Sol: (b)

Since, link utilization can be defined as $\frac{1}{(1+2a)}$

$$a = \frac{T_P}{(T_t)_{data}} = \frac{\text{Propagation delay}}{\text{Transmission time of data}}$$

$$T_P = \frac{l}{V},\ (T_t)_{data} = \frac{F}{B}$$

where B = bandwidth, F = frame length

$$= \frac{1}{1+2\times\frac{T_P}{T_t}}$$

$$= \frac{1}{1+\frac{2\times l}{V}\times\frac{B}{F}}$$

$$= \frac{VF}{VF+2\times l\times B}$$

If link length (l) is longer i.e. a < 1, it means link is inefficiently utilized.

Higher bandwidth leads to higher transmission rate and longer length leads to larger propagation delay. Hence, longer length and higher transmission rate gives lower link utilization.

8. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

Let det(A) and det(B) denote the determinants of the matrices A and B, respectively.

Which one of the options given below is TRUE?

(a) det(A) = det(B)

(b) det(B) = – det(A)

(c) det(A) = 0

(d) det(AB) = det(A) + det(B)

Sol: (b)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

$\Rightarrow A \Rightarrow R_1 \Leftrightarrow R_3$

$$\text{then A becomes} = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

If we interchange an row then $\det(A)\text{becomes}(-\det(A))$

So, $\det(B) = -\det(A)$

9. Consider the following definition of a lexical token **id** for an identifier in a programming language, using extended regular expressions:

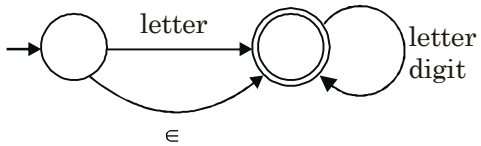
letter → [A-Za-z]

digit → [0-9]

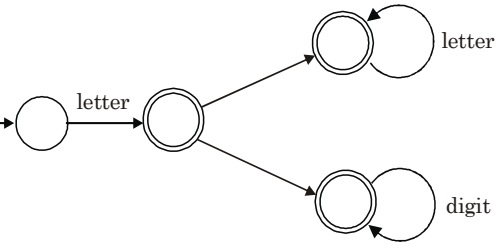
id → **letter** (**letter** | **digit**)*

Which one of the following Non-deterministic Finite-state Automata with $\in$-transitions accepts the set of valid identifiers? (A double-circle denotes a final state).

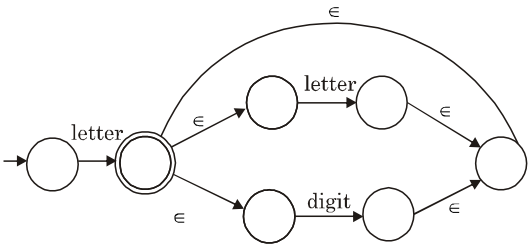
(a)



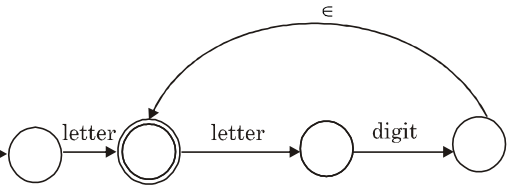
(b)



(c)



(d)



Sol: (c)

Given grammar,

Letter $\rightarrow$ [A-Z a-z]

digti $\rightarrow$ [0–9]

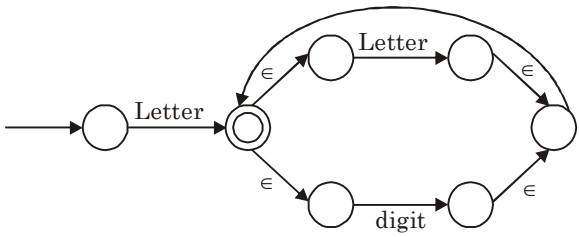
id $\rightarrow$ letter (letter/digit)*

Valid identifier should start with letter and followed by either letter or digit.

(A) First option is wrong because as per NFA given, identifier can be start with digit.

(B) Second option is wrong, because this NFA gives identifier as letter (letter)* or letter (digit)*

(C) Third option is correct it gives correct regular expression as per given grammar.



letter (letter/digit)*

(D) Fourth option is wrong, because this NFA doesn't accept regular expression, letter (letter)*

10. An algorithm has to store several keys generated by an adversary in a hashtable. The adversary is malicious who tries to maximize the number of collisions. Let k be the number of keys, m be the number of slots in the hash table, and k >m.

Which one of the following is the best hashing strategy to counteract the adversary?

(a) Division method, i.e., use the hash function h(k) = k mod m.

(b) Multiplication method, i.e., use the hash function $h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$, where A is a carefully chosen constant.

(c) Universal hashing method.

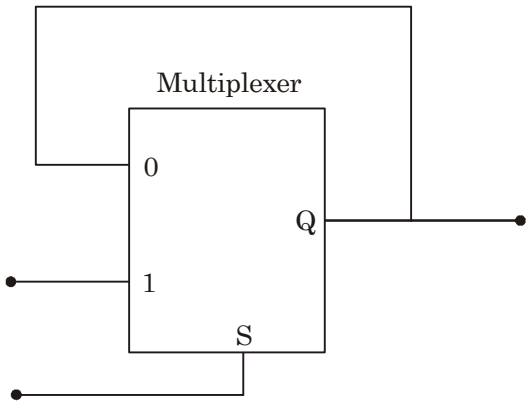
(d) If k is a prime number, use Division method. Otherwise, use Multiplication method.

Sol: (c)

Any fixed hash function is vulnerable to hash the key in the table, because there is a chance that someone choose the key in such a way that all the keys are hashed in same slot, yielding an average retriveval time $\theta(n)$. This is the worst case behaviour.

The only effective way to improve the situation is to choose the hash function randomly in such a way that is independent of the key that are actually going to be stored. This approach is called universal hashing.

11. The output of a 2-input multiplexer is connected back to one of its inputs as shown in the figure.



Match the functional equivalence of this circuit to one of the following options.

(a) D Flip-flop (b) D Latch

(c) Half-adder (d) Demultiplexer

Sol: (b)

$$Q_{n+1} = \bar{S}Q_n + SQ_n$$

$$Q_{n+1} = Q_n(\bar{S} + S)$$

$$Q_{n+1} = Q_n$$

Q_{n+1} is generated after delay.

In a flip-flop, we always consider edge, but the above circuit is level sensitive, therefore, it is equivalent to a D-latch.

12. Which one or more of the following need to be saved on a context switch from one thread (T1) of a process to another thread (T2) of the same process?

(a) Page table base register

(b) Stack pointer

(c) Program counter

(d) General purpose registers

Sol: (b,c,d)

Program counter, general purpose registers and stack painter is updated when a thread T_1, of a process switches context to a thread T_2 of the same process.

13. Which one or more of the following options guarantee that a computer system will transition from user mode to kernel mode?

(a) Function Call (b) malloc Call

(c) Page Fault (d) System Call

Sol: (c,d)

System call and page fault guranatee that a computer system will transition from user mode to kernal mode.

14. Which of the following statements is/are CORRECT?

(a) The intersection of two regular languages is regular.

(b) The intersection of two context-free languages is context-free.

(c) The intersection of two recursive languages is recursive.

(d) The intersection of two recursively enumerable languages is recursively enumerable.

Sol: (a,c,d)

(a) Regular languages are closed under intersection.

$$L_{1(\text{regular})} \cap L_{2(\text{regular})} = L_{(\text{Regular})}$$

So, a is correct.

(b) Context free languages are not closed under intersection and complement.

So, b is incorrect.

(c) Recursive languages are closed under intersection.

$$L_{1(\text{Recursive})} \cap L_{2(\text{Recursive})} = L_{(\text{Recursive})}$$

So, c is correct.

(d) Recursive enumerable languages are closed under intersection.

So d is also correct.

15. Which of the following statements is/are INCORRECT about the OSPF (Open Shortest Path First) routing protocol used in the Internet?

(a) OSPF implements Bellman-Ford algorithm to find shortest paths.

(b) OSPF uses Dijkstra's shortest path algorithm to implement least-cost path routing.

(c) OSPF is used as an inter-domain routing protocol.

(d) OSPF implements hierarchical routing.

Sol: (a,c)

OSPF uses Dijkastra's shortest path algorithm to implements least cost path routing and OSPF is an intra-domain routing protocal and it also implements Hierarchical routing.

So, option a and c are incorrect.

16. Geetha has a conjecture about integers, which is of the form

$$\forall x\left(P(x) \Rightarrow \exists y Q(x,y)\right),$$

where P is a statement about integers, and Q is a statement about pairs of integers. Which of the following (one or more) option(s) would imply Geetha's conjecture?

(a) $\exists x\left(P(x) \wedge \forall y Q(x,y)\right)$

(b) $\forall x \forall y Q(x,y)$

(c) $\exists y \forall x\left(P(x) \Rightarrow Q(x,y)\right)$

(d) $\exists x\left(P(x) \wedge \exists y Q(x,y)\right)$

Sol: (b,c)

$$L : \forall x\left[P(x) \rightarrow \exists y\, Q(x,y)\right]$$

"For every x if P(x) is true then there exists some y such that Q(x,y) will be true.

Option (i) $\exists x\left(P(x) \wedge \forall y\, Q(x,y)\right)$

"For some x, P(x) is true and for all y Q(x,y) is true which is not implies L"

(ii) $\forall x \forall y\, Q(x,y)$

"For every x and every y Q(x,y) is true which implies L"

(iii) $\exists y\, \forall x \left[P(x) \rightarrow Q(x,y)\right]$

"These exists some y such that for every x if P(x) is true then Q(x,y) is also true which implies L"

(iv) $\exists x P(x) \wedge \exists y Q(x,y)$

"There exists some x for which P(x) is true and also for some y Q(x,y) is true which cannot implies L"

17. Which one or more of the following CPU scheduling algorithms can potentially cause starvation?

(a) First-in First-Out

(b) Round Robin

(c) Priority Scheduling

(d) Shortest Job First

Sol: (c,d)

Starvation is a problem where high priority process keep executing and low priority processes get blocked for indefinite time.

first-in-first-out : In this scheduling $\Rightarrow$ Every process will get a chance on the basis of arrival. So, this scheduling doesn't suffer from starvation.

Round robin : Round robin will give equal opportunity for each process. So process will never face the problem of starvation.

Shortest job first and priority scheduling suffer from starvation

18. Let

$f(x) = x^3 + 15x^2 - 33x - 36$

be a real-valued function.

Which of the following statements is/are TRUE?

(a) f(x) does not have a local maximum.

(b) f(x) has a local maximum.

(c) f(x) does not have a local minimum.

(d) f(x) has a local minimum.

Sol: (b,d)

$f(x) = x^3 + 15x^2 - 33x - 36$. real valued function

$f'(x) = 3x^2 + 30x - 33$

$f''(x) = 6x + 30$

$\Rightarrow f'(x) = 0$

$3x^2 + 30x - 33 = 0$

$x^2 + 10x - 11 = 0$

$x^2 + 11x - x - 11 = 0$

$x(x-1) + 11(x-1) = 0$

$(x-1)(x+11) = 0$

$x = -11, 1$

$\Rightarrow f''(x) = 6x + 30$, at $x = -11$

$f''(-11) = 6(-11) + 30$

$f''(-11) = -66 + 30$

$f''(-11) = -36$

$f''(-11) < 0$

$-36 < 0$

at $x = -11$

local maxima

$f''(x) = 6x+30$, at $x = 1$

$f''(1) = 6.1 + 30$

$f''(1) = 36$

$f''(1) > 0$

at $x = 1$, local minima

19. Let f and g be functions of natural numbers given by $f(n) = n$ and $g(n) = n^2$. Which of the following statements is/are TRUE?

(a) $f \in O(g)$ (b) $f \in \Omega(g)$

(c) $f \in o(g)$ (d) $f \in \Theta(g)$

Sol: (a,c)

$f(n) = n$

$g(n) = n^2$

(a) $f \in O(g)$ $(\text{Big} - \text{oh})$

$n \le C \cdot n^2$ True

(b) $f \in \Omega(g)$ (omega)

$n \ge C.n^2$ False

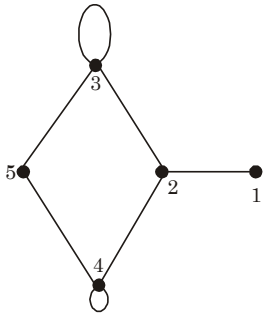
(c) $f \in o(g)$ $(\text{small} - \text{oh})$

$n < C \cdot n^2$ True

(d) $f \in \theta(g)$ (Theta notation)

$n \neq n^2$ False

20. Let A be the adjacency matrix of the graph with vertices {1, 2, 3, 4, 5}.



Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5 be the five eigenvalues of A. Note that these eigenvalues need not be distinct.

The value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 =$ _____

Sol: (2)

Adjacency Matrix A =

	1	2	3	4	5
1	0	1	0	0	0
2	1	0	1	1	0
3	0	1	1	0	1
4	0	1	0	1	1
5	0	0	1	1	0

$\because$ sum of eigen values of A = trace of A

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 0+0+1+1+0$

So, the value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 2$

21. The value of the definite integral

$\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(4x^2y - z^3)\, dz\, dy\, dx$

is _____(Rounded off to the nearest integer).

Sol: (0)

$$\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1} 4x^2y - z^3 \;\; dx \;\; dy \;\; dz$$

$$\left[\text{use} \Rightarrow \int 1dx = x\right] \qquad ...(2)$$

$$\left[\text{use} \Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1}\right] \qquad ...(1)$$

$$\Rightarrow \int_{-3}^{3}\int_{-2}^{2}\left[\frac{4x^3y}{3} - z^3x\right]_{-1}^{1} dy\, dz$$

$$\Rightarrow \int_{-3}^{3}\left(\int_{-2}^{2}\left(\frac{4y}{3} - z^3\right) - \left(\frac{-4y}{3} + z^3\right)\right) dy\, dz$$

$$\Rightarrow \int_{-3}^{3}\int_{-2}^{2} \frac{8y}{3} - 2z^3 \;\; dy\, dz$$

$$\Rightarrow \int_{-3}^{3}\left[\frac{8y^2}{2\times 3} - 2z^2y\right]_{-2}^{2} dz$$

$$\Rightarrow \int_{-3}^{3} \frac{16}{3} - 4z^3 - \frac{16}{3} - 4z^3 \; dz$$

$$\Rightarrow \int_{-3}^{3} -8z^3 \; dz$$

$$\Rightarrow \left[\frac{-8\times 2^4}{4}\right]_{-3}^{3}$$

$$\Rightarrow -2\times(3)^4 - \left((-2)\times(-3)^4\right)$$

$$\Rightarrow 0$$

22. A particular number is written as 132 in radix-4 representation. The same number in radix-5 representation is _____.

Sol: (110)

Given (132) in radix-4 representation. First convert (132)4 to decimal (radix 10)

$1\times4^2+3\times4^1+2\times4^0 = 16+12+2$

$= 30$

Now convert $(30)_{10}$ to radix 5 representation

5	30	
5	6	0
	1	1

$(30)_{10} = (110)_5$

$\therefore (132)_4 = (30)_{10} = (110)_5$

23. Consider a 3-stage pipelined processor having a delay of 10 ns (nanoseconds),20 ns, and 14 ns, for the first, second, and the third stages, respectively. Assume that there is no other delay and the processor does not suffer from any pipeline hazards. Also assume that one instruction is fetched every cycle.

The total execution time for executing 100 instructions on this processor is _____ns.

Sol: (2040)

t_p = max (stage delay) + buffer delay

t_p = max (10,20,14) + 0 [buffer delay = 0]

t_p = 20

k = 3

n = 100 instruction

Execution time = $(k + n - 1)t_p$

= $(3 + 100 - 1) \times 20$

= 102×20

= 2040 ns

24. A keyboard connected to a computer is used at a rate of 1 keystroke per second. The computer system polls the keyboard every 10 ms (milli seconds) to check for a keystroke and consumes 100 μs (micro seconds) for each poll. If it is determinedafter polling that a key has been pressed, the system consumes an additional 200 μs to process the keystroke. Let T_1 denote the fraction of a second spent in polling and processing a keystroke.

In an alternative implementation, the system uses interrupts instead of polling. An interrupt is raised for every keystroke. It takes a total of 1 ms for servicing an interrupt and processing a keystroke. Let T_2 denote the fraction of a second spent in servicing the interrupt and processing a keystroke.

The ratio $\frac{T_1}{T_2}$ is . (Rounded off to one decimal place).

Sol: (10.2)

Keyboard connected with computer system uses at a rate of 1 keystroke per second.

i.e., is 1 sec only 1 key stroke is possible.

In polling method, computer system polls the device, i.e keyboard every 10 μsec.

It means, In 1000 msec 1 key stroke possible and $\frac{1000}{10} = 100$ times polling possible in 1 sec.

And for each poll, 100 μsec is spent.

So, time spent in polling is 1 sec is (100×100μsec)

= $10^4 \times 10^{-6}$ sec.

= 10 msec.

If it is determined that key has been pressed after polling then time required to process the keystroke is 200 μsec.

T_1 = 10 msec + 200 μsec

= 10 msec + 0.2 msec

= 10.2 msec

Now, for interrupt driven I/O, the interrupt is raised for every keystroke.

And servicing an interupt and processing keystrokes takes 1 msec.

T_2 = 1 msec.

$\frac{T_1}{T_2} = 10.2$

25. The integer value printed by the ANSI-C program given below is_____ .

```
#include<stdio.h>
int funcp(){
        static int x = 1;
        x++;
        return x;
}
int main(){
        int x,y;
        x = funcp();
        y = funcp()+x;
        printf("%d\n", (x+y));
        return 0;
}
```

Sol: (7)

x in fun( ) is static, persists it's value across the function calls.

main ( )	x	y
	0	0

(i) x = fun( )

fun( )
x x = fun( ) = 2
~~1~~ 2 x = 2

(ii) y = fun( ) + x = 3 + 2 = 5

fun( )
x
~~2~~ 3 y = 5

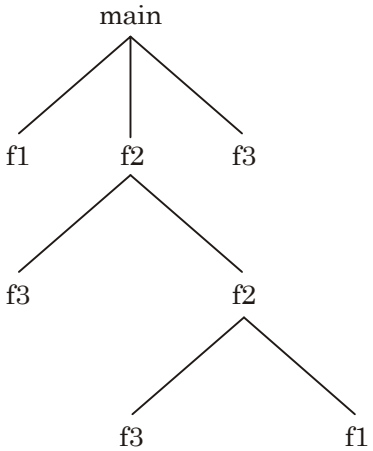
(iii)printf("%d\n",(x+y)) = 2+5 = 7

26. Consider the following program :

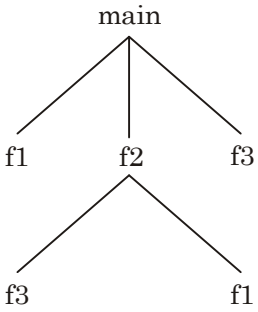
```
int main()                    int f1( )
{                             {
    f1();                         return(1);
    f2(2);                    }
    f3();
    return(0);
}
int f2 (int X)                int f3 ( )
{                             {
    f3 ( ) :                      return (5) ;
    if (X == 1)               }
        return f1 ( ) ;
    else
        return (X*f2(X-1)) ;
}
```

Which one of the following options represents the activation tree corresponding to the main function?

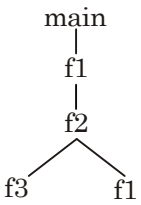
(a)



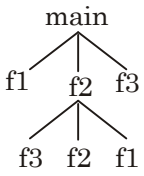
(b)



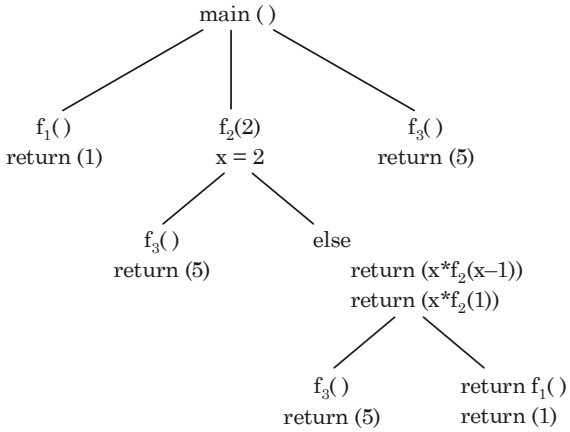
(c)



(d)

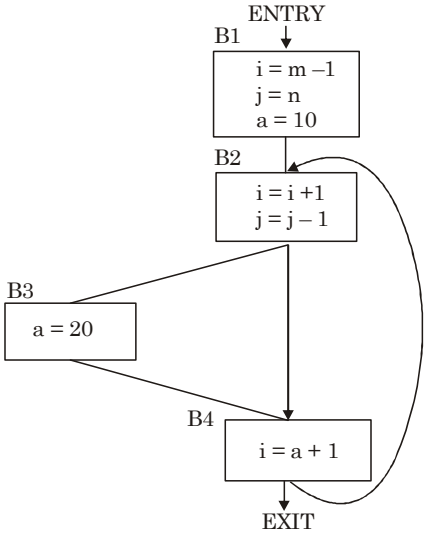


Sol: (a)



Hence, a is correct.

27. Consider the control flow graph shown.



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Which one of the following choices correctly lists the set of *live* variables at the exit point of each basic block?

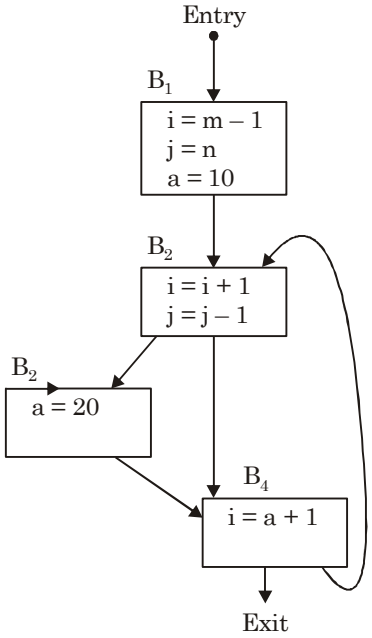
(a) B1: { }, B2: {a}, B3: {a}, B4: {a}

(b) B1: {i, j}, B2: {a}, B3: {a}, B4: {i}

(c) B1: {a, i, j}, B2: {a, i, j}, B3: {a, i}, B4: {a}

(d) B1: {a, i, j}, B2: {a, j}, B3: {a, j}, B4: {a, i, j}

Sol: (d)



- It is given that to find live variables at the exit point of each basic block.
- To find a live variables of B_1, we start the exit point of B_1 means from B_2 to so on.
- x is a live variable at statement Si, if :

(i) There is a statement Si using x.

(ii) There is a path from Si to Sj.

(iii)There is no new defination to x before Sj.

	i	j	m	n	a
B_1	live	live	dead	dead	live
B_2	dead	live	dead	dead	live
B_3	dead	live	dead	dead	live
B_4	live	live	dead	dead	live

B_1 : {i, j, a}, B_2 : {a, j}

B_3 : {a, j}, B_4 : {a, i, j}

28. Consider the two functions incr and decr shown below.

```
incr( ){              decr ( ){
    wait(s);              wait(s);
    X = X+1;              X = X-1;
    signal(s);            signal(s);
}                     }
```

There are 5 threads each invoking incr once, and 3 threads each invoking decr once, on the same shared variable X. The initial value of X is 10.

Suppose there are two implementations of the semaphore s, as follows:

I-1: s is a binary semaphore initialized to 1.

I-2: s is a counting semaphore initialized to 2.

Let V1, V2 be the values of X at the end of execution of all the threads with implementations **I-1**, **I-2**, respectively.

Which one of the following choices corresponds to the minimum possible values of V1, V2, respectively?

(a) 15, 7 (b) 7, 7

(c) 12, 7 (d) 12, 8

Sol: (c)

Given two implementation

I-1 : S = 1

I-2 : S = 2

In implementation I-1, binary semaphore is initialized by 1.

If five process and three process execute in any order to their respective code, the value of x will be 12.

In implementation I-2, counting semaphore is initialized to 2.

If thread t_1 execute decr( ).

```
decr( ){
wait(s); /*s=1*/
x=x–1; /*x=9*/
signal(s); /*s=2*/
}
```

Suppose thread t_2 enter into critical section, after t_1

```
decr( ){
wait(s); /*s=1*/
x=x–1; /*x=8*/
signal(s); /*s=2*/
}
```

And finally thread t_3 enter into critical section.

And it also compute variable, x = x –1, before assigning the resultant value x = 7 to memory, incr( ) operation start. After executing all five execution, x = 13. Now, return goes to decr( ) operation where assignment of value x is pending. So final value x = 7 overwrite x = 13.

Hence, minimum value of V_1 and V_2 is, (12,7)

29. Consider the context-free grammar G below

$S \rightarrow aSb \mid X$

$X \rightarrow aX \mid Xb \mid a \mid b$,

where S and X are non-terminals, and a and b are terminal symbols. The starting non-terminal is S.

Which one of the following statements is CORRECT?

(a) The language generated by G is (a + b)*

(b) The language generated by G is a*(a + b)b*

(c) The language generated by G is a*b*(a + b)

(d) The language generated by G is not a regular language

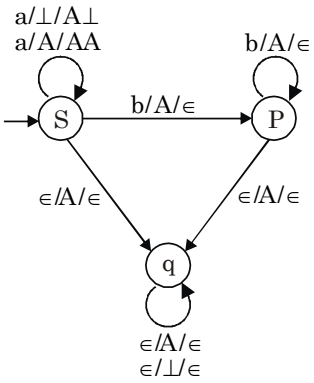
Sol: (b)

$S \rightarrow aSb \mid x$

$X \rightarrow aX \mid Xb \mid a \mid b = a^*(a+b)b^*$

The language generated by above grammar is $a^*(a+b)b^*$

30. Consider the pushdown automaton (PDA) P below, which runs on the input alphabet {a, b}, has stack alphabet {⊥, A}, and has three states {s, p, q}, with s being the start state. A transition from state u to state v, labelled c/X/γ, where c is an input symbol or ∈, X is a stack symbol, and γ is a string of stack symbols, represents the fact that in state u, the PDA can read c from the input, with *X* on the top of its stack, pop *X* from the stack, push in the string γ on the stack, and go to state v. In the initial configuration, the stack has only the symbol ⊥ in it. The PDA accepts by empty stack.



Which one of the following options correctly describes the language accepted by P?

(a) $\{a^m b^n \mid 1 \le m \text{ and } n < m\}$

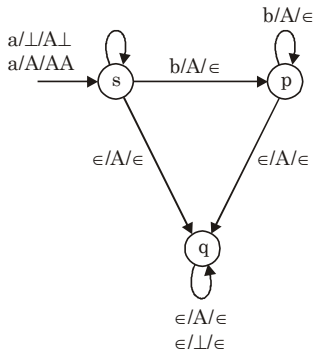
(b) $\{a^m b^n \mid 0 \le n \le m\}$

(c) $\{a^m b^n \mid 0 \le m \text{ and } 0 \le n\}$

(d) $\{a^m \mid 0 \le m\} \cup \{b^n \mid 0 \le n\}$

Sol: (a)

Given PDA,



In this PDA, number of input 'a' must be greater than 'b' i.e. n < m.

So, option (a) is correct.

31. Consider the given C-code and its corresponding assembly code, with a few operands U1–U4 being unknown. Some useful information as well as the semantics of each unique assembly instruction is annotated as inline comments in the code. The memory is byte-addressable.

```
//C-code
int a[10], b[10], i;
// int is 32-bit
for (i=0; i<10; i++)
    a[i] = b[i] * 8;
```

; assembly code (; indicates comments)

; r1–r5 are 32-bit integer registers

; initialize r1=0, r2=10

; initialize r3, r4 with base address of a, b

L01: jeq r1, r2, end ; if(r1==r2) goto end

L02: lw r5, 0(r4) ; r5 <- Memory[r4+0]

L03: shl r5, r5, U1 ; r5 <- r5 << U1

L04: sw r5, 0(r3) ; Memory[r3+0] <- r5

L05: add r3, r3, U2 ; r3 <- r3+U2

L06: add r4, r4, U3

L07: add r1, r1, 1

L08: jmp U4 ; goto U4

L09: end

Which one of the following options is a CORRECT replacement for operands in the position (U1, U2, U3, U4) in the above assembly code?

(a) (8, 4, 1, L02)

(b) (3, 4, 4, L01)

(c) (8, 1, 1, L02)

(d) (3, 1, 1, L01)

Sol: (b)

Since memory is byte addressable. And integer is 32-bit i.e. 4-bytes. So to increment variable 4 byte need to add.

L05: add r_3, r_3, U2; $r_3 \leftarrow r_3 + U2$

Here U2 = 4

Similarly U3 = 4

L08 = jmp U4; goto U4

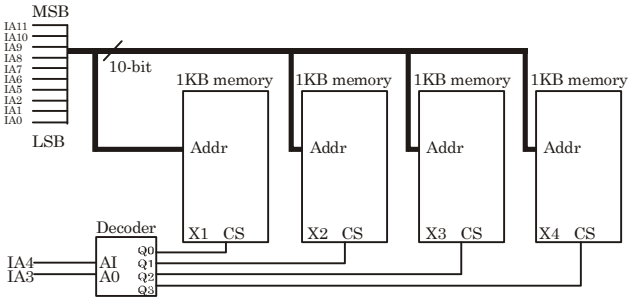
This instruction move the control to comparision condition.

So, U4 = L01

Hence option (B) is correct.

U1 = 3, U2 = 4, U3 = 4, U4 = L01

32. A 4 kilobyte (KB) byte-addressable memory is realized using four 1 KB memory blocks. Two input address lines (IA4 and IA3) are connected to the chip select (CS) port of these memory blocks through a decoder as shown in the figure. The remaining ten input address lines from IA11–IA0 are connected to the address port of these blocks. The chip select (CS) is active high.



The input memory addresses (IA11–IA0), in decimal, for the starting locations (Addr=0) of each block (indicated as X1, X2, X3, X4 in the figure) are among the options given below. Which one of the following options is CORRECT?

(a) (0, 1, 2, 3)

(b) (0, 1024, 2048, 3072)

(c) (0, 8, 16, 24)

(d) (0, 0, 0, 0)

Sol: (c)

Given input memory address (IA11-1A0)

Chip select selects the blcok. Where chip select input is given as IA4 and IA3.

Start address of each block can be decided by memory address with chip select input.

	IA11	IA10	...	IA5	IA4	IA3	IA2	IA1	IA0
X_1	0	0	...	0	0	0	0	0	0
X_2	0	0	...	0	0	1	0	0	0
X_3	0	0	...	0	1	0	0	0	0
X_4	0	0	...	0	1	1	0	0	0

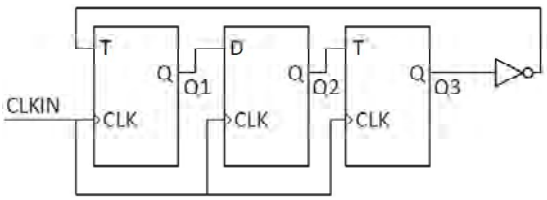
X1 = 0

X2 = 8

X3 = 16

X4 = 24

33. Consider a sequential digital circuit consisting of T flip-flops and D flip-flops as shown in the figure. CLKIN is the clock input to the circuit. At the beginning, Q1, Q2 and Q3 have values 0, 1 and 1, respectively.



Which one of the given values of (Q1, Q2, Q3) can NEVER be obtained with this digital circuit?

(a) (0, 0, 1) (b) (1, 0, 0)

(c) (1, 0, 1) (d) (1, 1, 1)

Sol: (a)

Truth table for D flip flop

D	Q_n	Q_{n+1}
0	0	0
0	1	0
1	0	1
1	1	1

Truth table for T flip flop

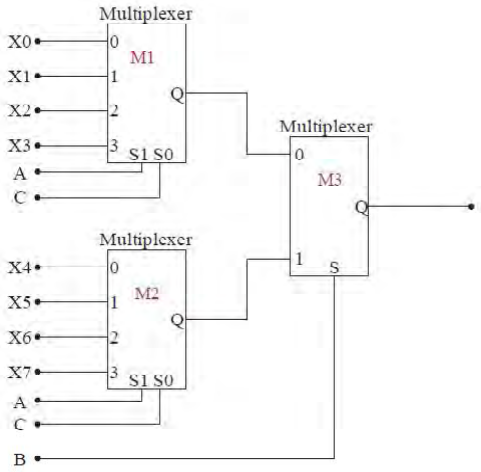
T	Q_n	Q_{n+1}
0	0	0
0	1	1
1	0	1
1	1	0

Q_1	Q_2	Q_3	$T_1 = \bar{Q}_3$	$D_2 = Q_1$	$T_3 = Q_2$
0	1	1	0	0	1
0	0	0	1	0	0
1	0	0	1	1	0
0	1	0	1	0	1
1	0	1	0	1	0
1	1	1	0	1	1
1	1	0	1	1	1
0	1	1	0	0	1

States involved : 011, 000, 100, 010, 101, 111, 110, 011

The state (0,0,1) can never be obtained with this digital circuit.

34. A Boolean digital circuit is composed using two 4-input multiplexers (M1 and M2) and one 2-input multiplexer (M3) as shown in the figure. X0–X7 are the inputs of the multiplexers M1 and M2 and could be connected to either 0 or 1. The select lines of the multiplexers are connected to Boolean variables A, B and C as shown.



Which one of the following set of values of (X0, X1, X2, X3, X4, X5, X6, X7) will realise the Boolean function $\bar{A} + \bar{A}.\bar{C} + A.\bar{B}.C$?

(a) (1, 1, 0, 0, 1, 1, 1, 0)

(b) (1, 1, 0, 0, 1, 1, 0, 1)

(c) (1, 1, 0, 1, 1, 1, 0, 0)

(d) (0, 0, 1, 1, 0, 1, 1, 1)

Sol: (c)

Given Boolean functino

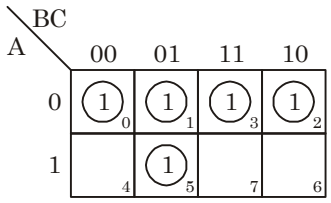
$$f(ABC) = \bar{A} + \bar{A}\cdot\bar{C} + A\cdot\bar{B}\cdot C$$

Simplify the given Boolean function

$$= \bar{A}(1+\bar{C}) + A\cdot\bar{B}\cdot C$$

$$= \bar{A} + A\cdot\bar{B}\cdot C$$

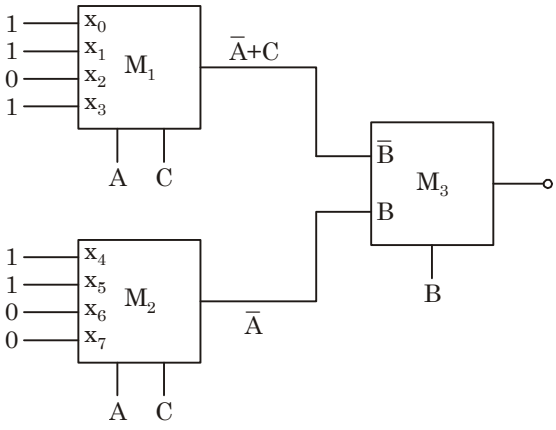
$$f(ABC) = \bar{A} + \bar{B}\cdot C \quad \text{(using absorption property)}$$



$$f(ABC) = \Sigma m(0,1,2,3,5)$$

Now, Implement usign 2×1 MUX

	$\bar{A}\bar{C}$	$\bar{A}C$	$A\bar{C}$	AC	
$\bar{B}$	⓪	①	4	⑤	$\bar{A}+C$
B	②	③	6	7	$\bar{A}$



So, $(x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7) = (1,1,0,1,1,1,0,0)$

35. Consider the IEEE-754 single precision floating point numbers P = 0xC1800000 and Q = 0x3F5C2EF4.

Which one of the following corresponds to the product of these numbers (i.e., P × Q), represented in the IEEE-754 single precision format?

(a) 0x404C2EF4 (b) 0x405C2EF4

(c) 0xC15C2EF4 (d) 0xC14C2EF4

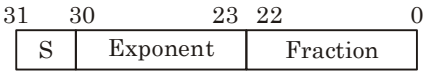
Sol: (c)

Given floating point number in hexa decimal format.

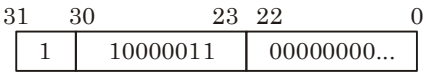
P = 0×C1800000

Q = 0×3F5C2EF4

IEEE standard floating-point formats.



P = 0×C1800000



Sign bit = 1 (-ve)

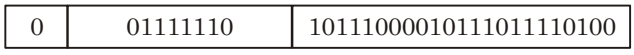
Single precision exponent is represented in excess-127 format.

Actual exponent = 132 – 127 = 4

Fractional or mantina part is 0.

P = $-(1.0)\times 2^{+4}$

Q = 0×3F5C2EF4



S B.E Fraction/Mantissa

Given format is in biased exponent.

We need to convert it is actual exponent.

Actual exponent = 126 – 127

= –1

Sign bit = 0 (+ve)

Actual exponent = –1

Fraction = .10111000010111011110100

Q = $(1.10111000010111011110100) \times 2^{-1}$

Data = P×Q

$-(1.0)\times 2^{+4} \times (1.1011.100001011011110100)\times 2^{-1}$

Data = $-(1.10111000010111011110100)\times 2^{+3}$

Sign bit = 1

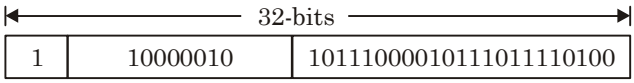
Biased exponent = Actual exponent + 127

= 3 + 127

= 130

Fraction/Mantina = 10111000010111011110100

IEEE standard floating-point format,



Sign Biased exponent Mantissa

1100 0001 0101 1100 0010 1110 1111 0100

C 1 5 C 2 E F 4

P×Q = 0XC15C2EF4

36. Let A be a priority queue for maintaining a set of elements. Suppose A is implemented using a max-heap data structure. The operation Extract-Max(A) extracts and deletes the maximum element from A. The operation Insert(A,key) inserts a new element key in A. The properties of a max-heap are preserved at the end of each of these operations.

When *A* contains *n* elements, which one of the following statements about the worst case running time of these two operations is TRUE?

(a) Both Extract-Max(A) and Insert(A, key) run in O(1).

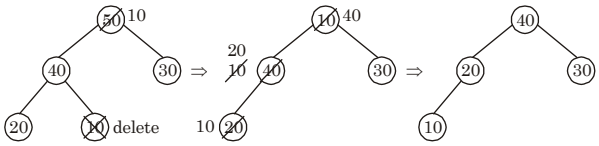
(b) Both Extract-Max(A) and Insert(A, key) run in O(log(n)).

(c) Extract-Max(A) runs in O(1) whereas Insert(A,key) runs in O(n).

(d) Extract-Max(A) runs in O(1) whereas Insert(A,key) runs in O(log(n)).

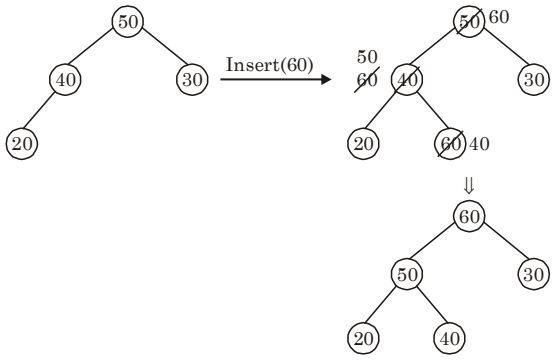
Sol: (b)

Extract_Max(A): Extracts and deletes the maximum element from A



Delete the root node data and replaced it by last level leaf data to maintain the max-heap property, perform top to bottom max-heapify operation, it will take O(logn) time.

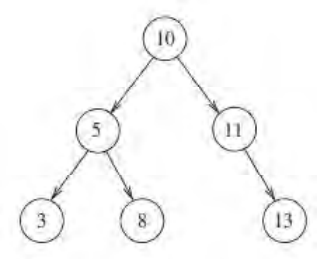
Insert(A, Key): Inserts a new element key in A.



Insert an element to the last level of max heap. If the inserted value is larger than parent node, then we have to perform bottom-to-top max heapify and it will take O(logn) time.

So, both extract max(A) and insert(A, key) run in O(logn) time.

37. Consider the C function foo and the binary tree shown.



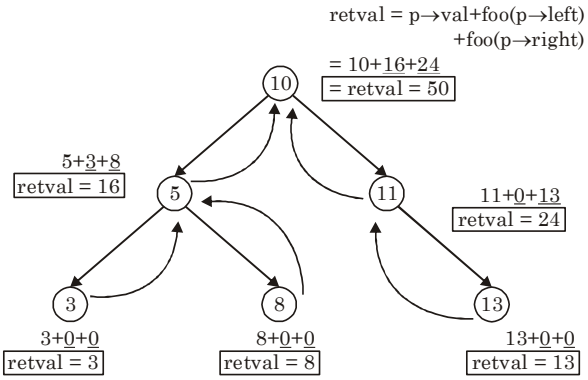

```
typedef struct node {
    int val;
    struct node *left, *right;
} node;
int foo(node *p) {
    int retval;
    if (p == NULL)
        return 0;
    else {
        retval = p->val + foo(p->left) + foo (p->right);
        printf("%d ", retval);
        return retval;
    }
}
```

When foo is called with a pointer to the root node of the given binary tree, what will it print?

(a) 3 8 5 13 11 10 (b) 3 5 8 10 11 13

(c) 3 8 16 13 24 50 (d) 3 16 8 50 24 13

Sol: (c)



Output : 3 8 16 13 24 50

38. Let U = {1, 2, . . . , n}, where n is a large positive integer greater than 1000. Let k be a positive integer less than n. Let A,B be subsets of U with $|A| = |B| = k$ and $A \cap B = \varnothing$ We say that a permutation of U separates A from B if one of the following is true.

– All members of A appear in the permutation before any of the members of B.

– All members of B appear in the permutation before any of the members of A.

How many permutations of U separate A from B?

(a) n!

(b) $\binom{n}{2k}(n-2k)!$

(c) $\binom{n}{2k}(n-2k)!(k!)^2$

(d) $2\binom{n}{2k}(n-2k)!(k!)^2$

Sol: (d)

$u = \{1, 2, 3, ... n\}$ $\forall n > 2000$

- $|A|=|B| = k$ & $A \cap B = \varnothing$
- K is a positive integer less than n.
- $|A|= k$ such that $A \subseteq u$
- $|B|= k$ such that $B \subseteq u$
- Number of permutation for all element of A appear before element of B.

  ${}^{n}C_{2k} * (n-2k)! \times k! \times k!$
- Number of permutation for all element of B appear before element of A.

  ${}^{n}C_{2k} * (n-2k)! \times k! \times k!$

Total permutation = $2 \times {}^{n}C_{2k} (n \times 2k)! \times k! \times k!$

39. Let $f : A \to B$ be an onto (or surjective) function, where A and B are nonempty sets. Define an equivalence relation $\sim$ on the set A as

$$a1 \sim a_2 \text{ if } f(a_1) = f(a_2) ,$$

where $a_1, a_2 \in A$. Let $\varepsilon = \{[x] : x \in A\}$ be the set of all the equivalence classes under $\sim$. Define a new mapping $F : \varepsilon \to B$ as

F([x]) = f(x), for all the equivalence classes [x] in ε.

Which of the following statements is/are TRUE?

(a) F is NOT well-defined.

(b) F is an onto (or surjective) function.

(c) F is a one-to-one (or injective) function.

(d) F is a bijective function.

Sol: (b,c,d)

- Option A is false because each classes of x follow equivalence relation is uniquely mapped with some element of x in a function f. Each function is well-defined.
- A function is said to be one-one if no two element in x are mapped to some element of y.[x], [y] are follow Distinct equivalence, So they follow one-one property.
- A function is onto because each element of co-domain B is associated with some element of domain E in a function f.
- If f is one-one and onto, then f is bijective function.

40. Suppose you are asked to design a new reliable byte-stream transport protocol like TCP. This protocol, named myTCP, runs over a 100 Mbps network with Round Trip Time of 150 milliseconds and the maximum segment lifetime of 2 minutes.

Which of the following is/are valid lengths of the **Sequence Number** field in the myTCP header?

(a) 30 bits (b) 32 bits

(c) 34 bits (d) 36 bits

Sol: (b,c,d)

Given round trip time = 150 millisecond.

Maximum segment lifetime = 2 minutes.

Network bandwidth = 100 mbps

We need to calculate number of bytes sent within maximum segment life time.

In 1 second 100×10^6 bit sent.

i.e. $\frac{100\times10^6}{8} = 125\times10^5$ bytes

So, in 120 second = $120\times125\times10^5$ bytes

= 15000×10^5 bytes

Length of sequence number field

$\geq \lceil \log_2(15\times10^8) \rceil$

$\approx \lceil (30.48) \rceil$

= 31 bits

41. Let X be a set and 2^X denote the powerset of X. Define a binary operation Δ on 2^X as follows:

$$A \Delta B = (A - B) \cup (B - A)$$

Let $H = (2^X, \Delta)$. Which of the following statements about H is/are correct?

(a) H is a group.

(b) Every element in H has an inverse, but H is NOT a group.

(c) For every $A \in 2^X$, the inverse of A is the complement of A.

(d) For every $A \in 2^X$, the inverse of A is A.

Sol: (a,d)

Let x be a set and 2^x is a power set of x

$A \Delta B = (A - B) \cup (B - A)$ for $A, B \in 2^x$

$H = (2^x, \Delta)$

Check H for each property.

(i) H satisfies the closure property under Δ

(ii) H satisfies the associative property under Δ such that

$A \Delta (B \Delta C) = (A \Delta B) \Delta C$

(iii) H satisfies the identity property such that '$\varnothing$', is the identity element.

$A \Delta e = \varnothing$

(iv) H satisfies inverse property such that inverse of A = A

$A \Delta \varnothing = \varnothing = (A - \varnothing) \cup (\varnothing - A)$

$= A \cup \varnothing$

$= A$

So, H is a group and inverse of A is A.

42. Suppose in a web browser, you click on the www.gate-2023.in URL. The browser cache is empty. The IP address for this URL is not cached in your local host, so a DNS lookup is triggered (by the local DNS server deployed on your local host) over the 3-tier DNS hierarchy in an

iterative mode. No resource records are cached anywhere across all DNS servers.

Let RTT denote the round trip time between your local host and DNS servers in the DNS hierarchy. The round trip time between the local host and the web server hosting www.gate-2023.in is also equal to RTT. The HTML file associated with the URL is small enough to have negligible transmission time and negligible rendering time by your web browser, which references 10 equally small objects on the same web server.

Which of the following statements is/are CORRECT about the minimum elapsed time between clicking on the URL and your browser fully rendering it?

(a) 7 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.

(b) 5 RTTs, in case of persistent HTTP with pipelining.

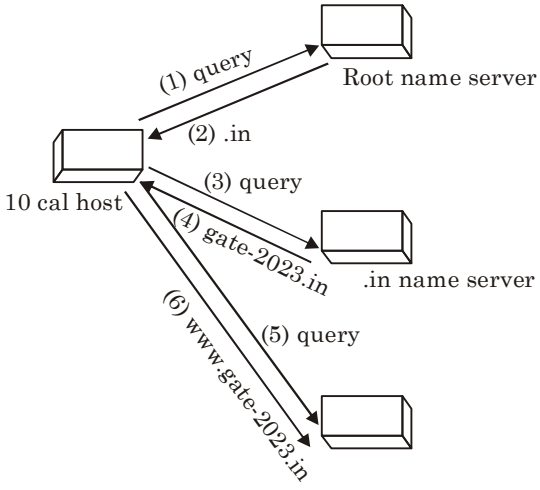
(c) 9 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.

(d) 6 RTTs, in case of persistent HTTP with pipelining.

Sol: (c,d)

Given URL,

www.gate-2023.in



In persistent connection, first of all connection (TCP connection) is established between local host and name server. After connection establishment local host send DNS query and server send query. In this connection, connection is persist.

So, there are 3RTT possible in connection establishment b/w local host and servers. And 3RTT possible in DNS query and their response. So total 6RTT possible is case of persistent HTTP with pipelining.

But in non-persistent connection, connection is not persist, i.e. after getting response connection terminated.

So, number of RTT's possible is,

3RTT possible for iterative request/response.

3RTT possible for connection establishment.

3RTT possible for connection termination.

Total number of RTT's = 9

43. Consider a random experiment where two fair coins are tossed. Let A be the event that denotes HEAD on both the throws, B be the event that denotes HEAD on the first throw, and C be the event that denotes HEAD on the second throw. Which of the following statements is/are TRUE?

(a) A and B are independent.

(b) A and C are independent.

(c) B and C are independent.

(d) Prob(B | C) = Prob(B)

Sol: (c,d)

Two fair coins toss = outcome is HH, HT, TH, TT

$$A : \text{Head on both toss} \Rightarrow P(A) = \frac{1}{4}$$

$$B : \text{Head on 1st throw} \Rightarrow P(B) = \frac{1}{2}$$

$$C : \text{Head on second throw} \Rightarrow P(C) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4},\ P(B \cap C) = \frac{1}{4},\ P(A \cap C) = \frac{1}{4}$$

To show A & B are independent

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

$$\Rightarrow P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$\frac{1}{8} \neq \frac{1}{4}$ then A&B are not independent.

$\Rightarrow$ To show B&C are independent

$$\Rightarrow P(B \cap C) = P(B) \cdot P(C)$$

$$P(B)\cdot P(C)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

$\frac{1}{4}=\frac{1}{4}$ then C&B are independent

$\Rightarrow$ To show A&C are independent

$\Rightarrow P(A\cap C)=P(A)\cdot P(C)$

$$P(A)\cdot P(C)=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

$\frac{1}{8}\neq\frac{1}{4}$

A,C are not independent.

If B&C are independent then

$$\Rightarrow P\left(\frac{B}{C}\right)=\frac{P(B\cap C)}{P(C)}$$

$$\Rightarrow \frac{P(B)\cdot P(C)}{P(C)}$$

$\Rightarrow P(B)$

$$P\left(\frac{B}{C}\right)=P(B)$$

44. Consider functions **Function 1** and **Function 2** expressed in pseudocode as follows:

Function 1

```
while n > 1 do
  for i = 1 to n do
      x = x + 1;
  end for
  n = n/2;
end while
```

Function 2

```
for i = 1 to 100 * n do
  x = x + 1;
end for
```

Let $f_1(n)$ and $f_2(n)$ denote the number of times the statement "x = x + 1" is executed in **Function 1** and **Function 2**, respectively.

Which of the following statements is/are TRUE?

(a) $f_1(n)\in\Theta(f_2(n))$ (b) $f_1(n)\in o(f_2(n))$

(c) $f_1(n)\in co(f_2(n))$ (d) $f_1(n)\in O(n)$

Sol: (a,d)

Given two function,

Function_1

```
while n>1 do
    For 1 to n do
        x = x + 1
    end for
    n = ⌊n/2⌋
    end while
```

variable n becomes half for every termination of for loop and for loop iterate n times.

$$f_1(n)=n+\frac{n}{2}+\frac{n}{2^2}+\ldots$$

$$=n\left(1+\frac{1}{2}+\frac{1}{2^2}+\ldots\right)$$

$=\theta(n)$

Function_2

```
for i = 1 to 100*n do
    x = x + 1;
end for
```

$f_2(n)=n\times 100=\theta(n)$

Hence, $f_1(n)\in\theta(f_2(n))$

$f_1(n)\in O(n)$

45. Let G be a simple, finite, undirected graph with vertex set $\{v_1, \ldots, v_n\}$. Let $\Delta(G)$ denote the maximum degree of G and let N = {1, 2, . . .} denote the set of all possible colors. Color the vertices of G using the following greedy strategy:

for i = 1, . . . , n

color(v_i) $\leftarrow$ min{j $\in$ N : no neighbour of v_i is colored j}

Which of the following statements is/are TRUE?

(a) This procedure results in a proper vertex coloring of G.

(b) The number of colors used is at most $\Delta(G)+1$.

(c) The number of colors used is at most $\Delta(G)$.

(d) The number of colors used is equal to the chromatic number of *G*.

Sol: (a,b)

Given G be a simple finete directed graph with vertex set $\{V_1, \ldots, V_n\}$

ΔG : Maximum degree of graph.

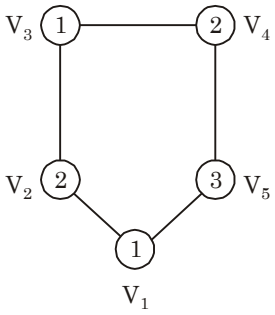
N be the set of all colors.

N = {1, 2, ...}

Color $(V_i) \leftarrow \min\{j \in N$: no neighbour of v_i is colored j}

As per given greedy strategy, no neighbour or directly connected vertex has coloured. It means this procedure results in a proper vertex colouring of G.

Take one example of graph.



Suppose vertex V_1 is coloured by 1. Now V_2 and V_5 can't be coloured by 1. So, minimum number remaining in the set is 2. Hence vertex V_2 is coloured by 2.

Now, vertex V_3 can't be coloured with 2, so minimum number except 2 in the set is 1. Hence number 1 is assigned to vertex V_3.

Vertex V_4 can't be coloured with 1, so it is coloured by minimum number except 1, minimum number 2 is assigned to V_4.

And finally, vertex V_5 can't be coloured with 1 and 2. So minimum number except 1 and 2 is 3. So number 3 is assigned to vertex V_5.

$\Delta G = 2$

N = {1,2,3}

Hence, the number of colors used is atmost $\Delta G + 1$.

46. Let U = {1, 2, 3}. Let 2^U denote the powerset of U. Consider an undirected graph G whose vertex set is 2^U. For any A,B $\in 2^U$, (A,B) is an edge in G if and only if (i) $A \neq B$, and (ii) either $A \subseteq B$ or $B \subseteq A$. For any vertex A in G, the set of all possible orderings in which the vertices of G can be visited in a Breadth First Search (BFS) starting from A is denoted by B(A).

If $\varnothing$ denotes the empty set, then the cardinality of $B(\varnothing)$ is _____ .

Sol: (5040)

Given, U = {1,2,3}

2^U denote the powerset of U.

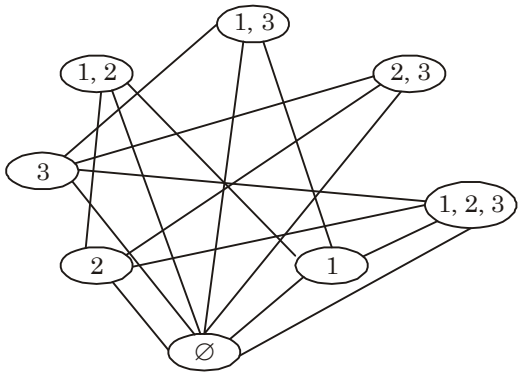
$V = \{\varnothing, 1, 2, 3, (1,2), (1,3), (2,3), (1,2,3)\}$

$A, B \in 2^U$

A, B is an edge in G if and only if

(i) $A \neq B$

(ii) either $A \subseteq B$ or $B \subseteq A$



Node $\varnothing$ connect all the node in the graph.

Cardinality of $B(\varnothing) = 7! = 5040$

47. Consider the following two-dimensional array D in the C programming language, which is stored in row-major order :

```
int D[128][128];
```

Demand paging is used for allocating memory and each physical page frame holds 512 elements of the array D. The Least Recently Used (LRU) page-replacement policy is used by the operating system. A total of 30 physical page frames are allocated to a process which executes the following code snippet:

```
for (int i = 0; i < 128; i++)
    for (int j = 0; j < 128; j++)
        D[j][i] *= 10;
```

The number of page faults generated during the execution of this code snippet is _____.

Sol: (4096)

Given array,

int D[128] [128]

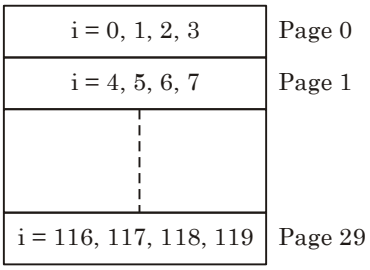
Page frame holds 512 elements of the array D. A total 30 physical page frames are allocated to a process which executes the following code snippet.

```
for (int i = 0, i < 128; i++)
```

for (int j = 0; j < 128; j++)

D[j] [i]* = 10;

for each i, loop execuetes 128 times.



Since 30 page frames allocated to the process.

Each physical frame holds 512 elements.

And array access is in column major order.

D[j] [i] * = 10

The first 4 rows belongs to one page.

Since memory contain 30 page frames, and array contains 32 pages.

Page required for array = $\frac{128}{4} = 32$

No. of page fault = 128×32 = 4096

48. Consider a computer system with 57-bit virtual addressing using multi-level tree-structured page tables with L levels for virtual to physical address translation. The page size is 4 KB (1 KB = 1024 B) and a page table entry at any of the levels occupies 8 bytes.

The value of L is_____ .

Sol: (5)

Virtual address = 57 bit

Page size = 4 KB (1 KB = 1024 B)

Page table entry size = 8 B

Page tables will L levels (L) = ?

Page table size = No. of pages * Page table entry size

1st level :

No. of pages = $\frac{2^{57}}{2^{12}} = 2^{45}$

Page table size = 2^{45} * 8B

= $2^{45} \times 2^3$ B

$\boxed{\text{Page table size} = 2^{48}\text{B}}$

2nd level :

No. of pages = $\frac{2^{48}}{2^{12}} = 2^{36}$

Page table size = $2^{36} \times 8$B

$\boxed{\text{Page table size} = 2^{39}\text{B}}$

3rd level :

No. of pages = $\frac{2^{39}}{2^{12}} = 2^{27}$

Page table size = $2^{27} \times 8$ B

= $2^{27} \times 2^3$ B

$\boxed{\text{Page table size} = 2^{30}\text{B}}$

4th level :

No. of pages = $\frac{2^{30}}{2^{12}} = 2^{18}$

Page table size = $2^{18} \times 8$ B

$\boxed{\text{Page table size} = 2^{21}\text{B}}$

5th level :

No. of pages = $\frac{2^{21}}{2^{12}} = 2^9$

Page table size = $2^9 \times 8$ B = 2^{12} B

At level 5th, page table fit's into 1 page (4 KB)

$\therefore$ Value of L is 5.

49. Consider a sequence a of elements $a_0 = 1$, $a_1 = 5$, $a_2 = 7$, $a_3 = 8$, $a_4 = 9$, and $a_5 = 2$. The following operations are performed on a stack S and a queue Q, both of which are initially empty.

I: push the elements of a from a0 to a5 in that order into S.

II: enqueue the elements of a from a0 to a5 in that order into Q.

III: pop an element from S.

IV: dequeue an element from Q.

V: pop an element from S.

VI: dequeue an element from Q.

VII: dequeue an element from Q and push the same element into S.

VIII: Repeat operation VII three times.

IX: pop an element from S.

X: pop an element from S.

The top element of S after executing the above operations is_____ .

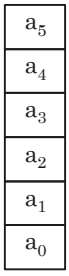
Sol: **(8)**

Given sequence a of element,

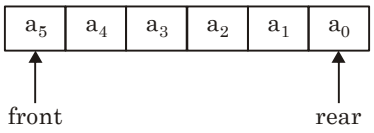
$a_0 = 1,\ a_1 = 5,\ a_2 = 7,\ a_3 = 8,\ a_4 = 9,\ a_5 = 2$

Initial condition:

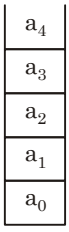
(I)



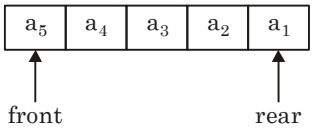
(II)



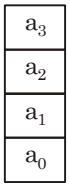
(III)



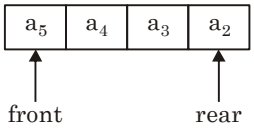
(IV)



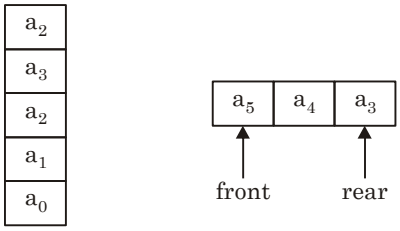
(V)



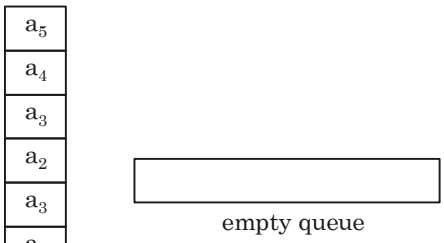
(VI)



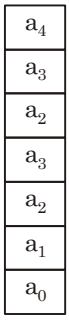
(VII)



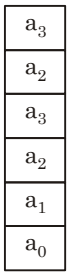
(VIII)



(IX)



(X)



The top element of S after executing the above operation is $\underline{a_3 = 8}$

50. Consider the syntax directed translation given by the following grammar and semantic rules. Here N, I, F and B are non-terminals. N is the starting non-terminal, and #, 0 and 1 are lexical tokens corresponding to input letters "#", "0" and "1", respectively. X.val denotes the synthesized attribute (a numeric value) associated with a non-terminal X. I_1 and F_1 denote occurrences of I and F on the right hand side of a production, respectively. For the tokens 0 and 1, 0.val = 0 and 1.val = 1.

$N \rightarrow I\ \#F$ N.val = I.val + F.val

$I \rightarrow I_1 B$ $\quad I.val = (2\ I_1.val) + B.val$

$I \rightarrow B$ $\quad I.val = B.val$

$F \rightarrow BF_1$ $\quad F.val = \frac{1}{2}(B.val + F_1.val)$

$F \rightarrow B$ $\quad F.val = \frac{1}{2} B.val$

$B \rightarrow 0$ $\quad B.val = 0.val$

$B \rightarrow 1$ $\quad B.val = 1.val$

The value computed by the translation scheme for the input string

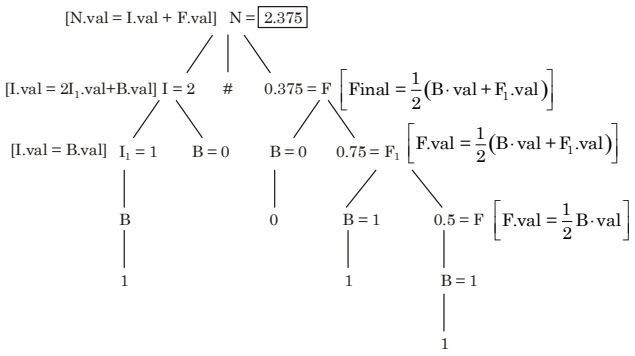
10#011

is_____. (Rounded off to three decimal places)

Sol: (2.375)

We have given an SDT and input is 10# 011.

Now, create a syntax tree and scan top to bottom and left to right.



51. Consider the following table named Student in a relational database. The primary key of this table is rollNum.

Student

Roll Num	Name	Gender	Marks
1	Naman	M	62
2	Aliya	F	70
3	Aliya	F	80
4	James	M	82
5	Swati	F	65

The SQL query below is executed on this database

SELECT *

FROM Student

WHERE gender = 'F' AND

marks > 65;

The number of rows returned by the query is _____ .

Sol: (2)

Roll Num	Name	Gender	Marks	
1	Naman	M	62	× (male)
2	Aliya	F	70	√
3	Aliya	F	80	√
4	James	M	82	× (male)
5	Swati	F	65	× (not grater than 65)

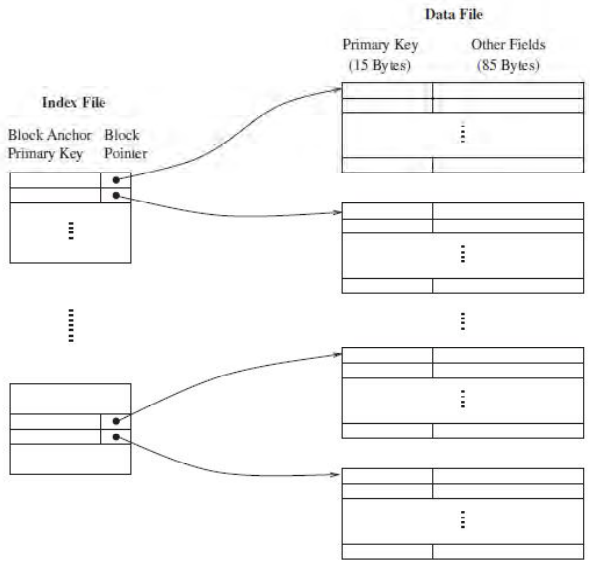
Query:

Select * from student where gender = 'F' AND marks > 65

Returned relation

Roll Num	Name	Gender	Marks
2	Aliya	F	70
3	Aliya	F	80

52. Consider a database of fixed-length records, stored as an ordered file. The database has 25,000 records, with each record being 100 bytes, of which the primary key occupies 15 bytes. The data file is block-aligned in that each data record is fully contained within a block. The database is indexed by a primary index file, which is also stored as a block-aligned ordered file. The figure below depicts this indexing scheme.



Suppose the block size of the file system is 1024 bytes, and a pointer to a block occupies 5 bytes. The system uses binary search on the index file

to search for a record with a given key. You may assume that a binary search on an index file of b blocks takes [$\log_2$ b] block accesses in the worst case.

Given a key, the number of block accesses required to identify the block in the data file that may contain a record with the key, in the worst case, is_____ .

Sol: (6)

Given, Number of records in the database = 25,000

Record size = Primary key size + pointer to a block size

= (15 + 5) bytes

Block size = 1024 bytes

No. of index record in 1-block = $\left\lfloor \frac{1024}{20} \right\rfloor = 51$

No. of blocks in the index file = $\frac{25000}{51} \approx 50$

The number of block access required to identify the block in the data = $\lceil \log_2 b \rceil$

= $\lceil \log_2 50 \rceil$

= 6 block access

53. Consider the language L over the alphabet {0, 1}, given below:

L = {w ∈ {0, 1}* | w does not contain three or more consecutive 1's}.

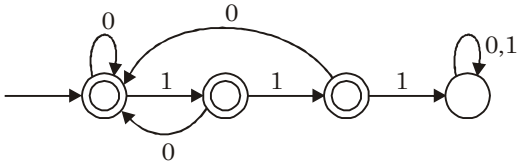
The minimum number of states in a Deterministic Finite-State Automaton (DFA) for L is_____ .

Sol: (4)

Given language,

L = {w ∈ {0,1} * w does not contain three or more consecutive 1's}

It means, w contains two or less number of 1's.



The minimum number of states = 4

54. An 8-way set associative cache of size 64 KB (1 KB = 1024 bytes) is used in a system with 32-bit address. The address is sub-divided into TAG, INDEX, and BLOCK OFFSET.

The number of bits in the TAG is_____ .

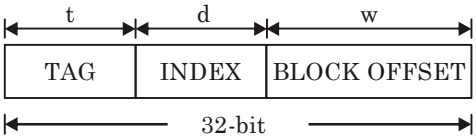
Sol: (19)

Given 8-way set associative cache of size 64 KB

Suppose size of block = 2^w

Number of sets = $\frac{64 \times 2^{10}}{2^w \times 8} = \frac{2^{16}}{2^3 \times 2^w} = 2^{13-w}$

A system has 32-bit address,



TAG = 32 – (d + w)

d = 13 – w

TAG = 32 – (13 – w + w)

= 19 bits

55. The forwarding table of a router is shown below.

Subnet Number	Subnet Mask	Interface ID
200.150.0.0	255.255.0.0	1
200.150.64.0	255.255.224.0	2
200.150.68.0	255.255.255.0	3
200.150.68.64	255.255.255.224	4
Default		0

A packet addressed to a destination address 200.150.68.118 arrives at the router. It will be forwarded to the interface with ID _____.

Sol: (3)

- Router check the destination address and perform bitwise AND with subnet mask of all interfaces and if the resulting network address match with corresponding interface then router send packet to that interface.

  Destination address = 200.150.68.118

- (200.150.68.118) ∧ (255.255.0.0) = (200.150.0.0)
- (200.150.68.118) ∧ (255.255.254.0) = (200.150.64.0)
- (200.150.68.118) ∧ (255.255.255.0) = (200.150.68.0)

All three mask producing the same network ID. Then select the one with greater number of ones will be selected and the packet will be forward there.

So 255.255.255.0 have greater number of ones then interface 3 is used for sending the packet.