1. (a) Twelve points are located on the circumference of a circle. Lines are drawn to connect all possible pairs of points. How many lines are drawn?

   (b) Find the length of the arc of the parabola $x^2 = 4y$ from the vertex to the point $x = 2$, where $x \in R$ and $y \in R$.

   $$[7 + 8 = 15]$$

2. (a) A company manufacturing industrial enclosures is interested in optimizing the powder coating process to minimize the variation in coating thickness around the target value of 56 microns. The models developed for thickness variance and the expected value of thickness are given below. Identify the optimum values of conductivity and powder output which would result in an expected thickness of 56 microns with a minimum variance? To execute the powder coating process, the conductivity and powder output needs to be greater than zero.

   $$\text{Variance} = 9.5 + 2 \times \text{Conductivity}^2 + 3 \times (\text{Powder output})^2$$

   $$\text{Expected Thickness} = 4 \times \text{Conductivity} + 12 \times (\text{Powder output})$$

   (b) What is the interval on which the function $f(x) = \frac{\log (x)}{x}$, $x \in (0, \infty)$ is increasing?

   $$[8 + 7 = 15]$$

3. (a) A student is allowed to select at most n books from a collection of $(2n + 1)$ books. If the number of ways in which he can select at least one book is 63, find the value of n.

   (b) Show that the sequence $\{S_n\}$, where $S_n = \left(1 + \frac{1}{n}\right)^2$, is convergent and that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ lies between 2 and 3.

   $$[7 + 8 = 15]$$

4. (a) If the third, fourth and fifth terms in the expansion of $(a + x)^n$ be 84, 280 and 560 respectively, find the value of a, x and n.

(b) If $x_1, x_2, ... x_{20}$ are in harmonic progression and $x_1, 2, x_{20}$ are in geometric progression, find the value of $\sum_{r=1}^{19} x_r x_{r+1}$.

[7 + 8= 15]

5. (a) John wishes to get from town A to town C via town B. There are three roads connecting town A to town B and 4 roads connecting town B to town C. In how many ways can John get from town A to town C ?

(b) Find the values of λ and μ for which the system of equation

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have no solution.

[6 + 9 = 15]

6. (a) A 1-inch-diameter coin is thrown on a table covered with a grid of lines two inches apart. What is the probability that the coin lands in a square without touching any of the lines of the grid?

(b) One of the subjects of a postgraduate course offered by a reputed institute is handled online by a faculty from another city. Suppose the video conferencing software used by the faculty is 99% reliable, his internet connection is 98% reliable and the internet connectivity at the institute is also 98% reliable, what is the chance of having failure-free online classes? The online classes will work only if the video conferencing software and internet connectivity at the faculty premise and the institute works. Suppose another video conferencing software is available in the market which is also 99% reliable and an alternative internet service provider is

available at faculty's city with 98% reliability. Can purchasing and keeping one more video conferencing software or internet facility or both and keeping them as standby improve the chance of failure-free operation of the online class to 98%?

[7 + 8 =15]

7. (a) *A* and *B* throw a die alternatively till one of them throws a '6' and wins the game. Find their respective probabilities of winning, if *A* starts the game.

(b) Calculate the mean deviation for the arithmetic progression
$a, a + d, a + 2d, a + 3d, ..., a + 2nd$.

[7 + 8 = 15]

8. (a) Customers are used to evaluating preliminary product designs. In the past 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

    i. What is the probability that a product attains a good review?

    ii. If a new design attains a good review, what is the probability that it will be a highly successful product?

    iii. If a product does not attain a good review, what is the probability that it will be a highly successful product?

(b) If

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

find m so that $A^2 = mA - 2B$.

[8 + 7 = 15]