NATIONAL UNIVERSITY OF SINGAPORE

MA1301 Introductory Mathematics **Tutorial 2**

1. (a) Find the smallest positive integer n for which the sum of the first n terms of the following arithmetic sequence

$$5, 8, 11, 14, \ldots$$

exceeds 1500.

(b) Find the smallest positive integer n for which the sum of the first n terms of the geometric sequence

$$64, 32, 16, \ldots$$

exceeds 127.95.

2. Evaluate the following telescoping sums.

(a) $\displaystyle\sum_{r=1}^{99} \lg\left(\frac{r+1}{r}\right)$.

(b) $\displaystyle\sum_{r=2}^{N} \frac{1}{4r^2-1}$.

3. i) Show that for any positive integer r,

$$(r+1)! - r! = r(r!).$$

ii) Use (i) to show that

$$\sum_{r=1}^{m} r(r!) = (m+1)! - 1.$$

4. i) Obtain the binomial expansion of $(2-3x)^6$ in ascending powers of x.

ii) Determine the coefficient of x^3 in the expansion of $(2-3x)^8$.

iii) Find the value of the constant a for which the coefficient of x in the expansion of $(1+ax)(2-3x)^6$ is zero.

5. Find the constant term in the binomial expansion of the following.

(a) $\left(3x - \dfrac{1}{x}\right)^{12}$.

(b) $\left(\frac{x}{2}+\frac{4}{\sqrt{x}}\right)^9$.

6. Use the binomial theorem to show that for all natural number n,

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}=2^n.$$

7. i) Expand $\sqrt{1-x}$ up to and including the term in x^2.

ii) By taking $x=\frac{1}{64}$ in the expansion of (i), deduce that $\sqrt{7}\approx\frac{10837}{4096}$.

8. i) Express $\frac{x}{x^2-3x+2}$ in the partial fraction form.

ii) Show that, if x is so small that x^3 and higher powers of x can be neglected, then

$$\frac{x}{x^2-3x+2}\approx\frac{1}{2}x+\frac{3}{4}x^2.$$

9. Show that for sufficiently small x,

$$\sqrt{\frac{4-x}{1+x}}\approx 2-\frac{5}{4}x+\frac{55}{64}x^2.$$

Solutions and Hints

1. (a) $n=31$; (b) $n=12$.

2. (a) 2. *Hint*: $\lg\frac{a}{b}=\lg a-\lg b$.

(b) $\frac{1}{6}-\frac{1}{2(2N+1)}$. *Hint*: Convert $\frac{1}{4r^2-1}$ into the partial fraction form.

4. i) $64-576x+2160x^2-4320x^3+4860x^4-2916x^5+729x^6$.

ii) -48384; iii) $a=9$.

5. i) 673 596; ii) 43008. *Hint*: The constant term is the coefficient of x^0.

6. *Hint*: Choose proper values for a and b in the binomial theorem.

7. i) $1-\frac{1}{2}x-\frac{1}{8}x^2+\cdots$. *Hint*: $\sqrt{1-x}=[1+(-x)]^{1/2}$.

8. i) $-\frac{1}{x-1}+\frac{2}{x-2}$. *Hint*: Write $\frac{x}{x^2-3x+2}=\frac{A}{x-1}+\frac{B}{x-2}$ and determine A, B.

ii) *Hint*: $-\frac{1}{x-1}=[1+(-x)]^{-1}$ and $\frac{2}{x-2}=-\left[1+\left(-\frac{x}{2}\right)\right]^{-1}$.

9. *Hint*: $\sqrt{4-x}=2\left[1+\left(-\frac{x}{4}\right)\right]^{1/2}$ and $\frac{1}{\sqrt{1+x}}=(1+x)^{-1/2}$. Find their approximations up to the term in x^2. Note that $\sqrt{\frac{4-x}{1+x}}=\sqrt{4-x}\,\frac{1}{\sqrt{1+x}}$.