NATIONAL UNIVERSITY OF SINGAPORE

MA1301 Introductory Mathematics **Tutorial 10**

1. Two lines L_1 and L_2 have vector equations given respectively by

$$\boldsymbol{r} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} + \lambda(2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) \quad \text{and} \quad \boldsymbol{r} = 4\boldsymbol{i} + \boldsymbol{j} + 10\boldsymbol{k} + \mu(\boldsymbol{i} + 3\boldsymbol{k}).$$

(a) Show that L_1 and L_2 intersects, and find the point of the intersection.

(b) Find the acute angle between L_1 and L_2.

(c) Show that the point $A(3, 3, 7)$ does not lie on the line L_1, and determine the foot of perpendicular from A to L_1.

2. Find an equation of the plane which is parallel to the vectors $\boldsymbol{i} + 2\boldsymbol{k}$ and $3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, and contains the point $(0, -1, -2)$.

3. Consider the planes $\boldsymbol{r} \bullet (\boldsymbol{i} - \boldsymbol{j}) = 3$ and $\boldsymbol{r} \bullet (\boldsymbol{j} + \boldsymbol{k}) = 1$.

(a) Find the acute angle between the two planes.

(b) Find a vector equation of the line of intersection.

4. Consider the plane $\boldsymbol{r} \bullet (\boldsymbol{i} - \boldsymbol{j}) = 0$ and the line $\boldsymbol{r} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k} + \lambda(\boldsymbol{j} + 3\boldsymbol{k})$.

(a) Find the acute angle between the plane and the line.

(b) Find the intersection point of the plane and the line.

5. Find the foot of perpendicular from the given point A to the plane Π, and calculate the distance from A to Π.

(i) $A(5, -3, 4)$, Π: $3x - 4y + z = 5$.

(ii) $A(7, 2, -5)$, Π: $3x - 4z + 9 = 0$.

6. Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be vectors in $\mathbb{R}^3$.

(a) Verify that $|\boldsymbol{u} \times \boldsymbol{v}|^2 + (\boldsymbol{u} \bullet \boldsymbol{v})^2 = |\boldsymbol{u}|^2|\boldsymbol{v}|^2$.

(b) Verify that $(\boldsymbol{u} \times \boldsymbol{v}) \bullet \boldsymbol{w} = (\boldsymbol{v} \times \boldsymbol{w}) \bullet \boldsymbol{u} = (\boldsymbol{w} \times \boldsymbol{u}) \bullet \boldsymbol{v}$.

Answer and Hint

1. (a) $(1, 1, 1)$. (b) $\cos^{-1} \dfrac{\sqrt{15}}{6}$.

(c) $(5, 3, 3)$. *Hint*: Let P be the foot of perpendicular. Express the position vector of P in λ. Find λ by noting that $\overrightarrow{AP}$ is perpendicular to the direction vector of L_1.

2. $-2x + 5y + z = -7$.

3. (a) $\pi/3$. *Hint*: Find the angle between the normal vectors of the planes.

(b) $4\boldsymbol{i} + \boldsymbol{j} + \lambda(-\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$; *Hint*: The directional vector of the line of intersection is perpendicular to the normal vectors of the planes.

4. (a) $\sin^{-1}(1/2\sqrt{5}) \approx 0.225 \approx 12.9°$; *Hint*: First find the angle between the normal vector of the plane and the directional vector of the line.

(b) $(1, 1, 8)$.

5. $(2, 1, 3)$. *Hint*: Write down the equation of the line which passes through the given point and is perpendicular to the plane. Then find its intersection with the plane.

6. Write $\boldsymbol{u} = u_1\boldsymbol{i} + u_2\boldsymbol{j} + u_3\boldsymbol{k}$, $\boldsymbol{v} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + \boldsymbol{v}_3\boldsymbol{k}$, $\boldsymbol{w} = w_1\boldsymbol{i} + w_2\boldsymbol{j} + w_3\boldsymbol{k}$ and expand.