NATIONAL UNIVERSITY OF SINGAPORE

MA1301 Introductory Mathematics **Tutorial 6**

1. Evaluate the following indefinite integrals.

(a) $\displaystyle\int \left(x+\frac{2}{x^2}\right)\,dx$; (b) $\displaystyle\int \left(\tan^2 x-\sec^2 2x\right)\,dx$;

(c) $\displaystyle\int \frac{2\sin^2 x+1}{\cos^2 x}\,dx$; (d) $\displaystyle\int \frac{3}{2x^2-8x+58}\,dx$;

(e) $\displaystyle\int \frac{1}{\sqrt{6x-x^2-5}}\,dx$; (f) $\displaystyle\int (3\cos x-\sin x)^2\,dx$.

2. Find the equations of the following curves.

(a) The gradient of the function is $3x^2-4x$, and it passes through the point $(1,0)$.

(b) $\dfrac{d^2y}{dx^2}=6x-\dfrac{4}{x^2}$, and the tangent line at $x=1$ is $y=10x-4$.

3. i) Evaluate $\dfrac{d}{dx}(\tan^2 x+2\ln|\cos x|)$.

ii) Use the result in (i) to evaluate $\displaystyle\int_0^{\pi/4}\tan^3 x\,dx$.

4. Using appropriate substitutions, evaluate the following integrals.

(a) $\displaystyle\int \frac{\cos x}{(2\sin x+5)^3}\,dx$; (b) $\displaystyle\int_0^{\ln 2}\frac{\sqrt{4-e^{-x}}}{e^x}\,dx$;

(c) $\displaystyle\int_1^{e/2}\frac{\sqrt{\ln 2x}}{2x}\,dx$; (d) $\displaystyle\int_0^9 \frac{1}{\sqrt{x+x\sqrt{x}}}\,dx$;

(e) $\displaystyle\int_0^{\pi/4}\frac{2+\tan x}{\cos^2 x}\,dx$; (f) $\displaystyle\int \frac{1}{t^2e^{2/t}}\,dt$;

(g) $\displaystyle\int \frac{\tan^{-1}x}{1+x^2}\,dx$; (h) $\displaystyle\int \frac{2}{x\sqrt{4\ln x-\ln^2 x}}\,dx$.

5. Evaluate the following indefinite integrals.

(a) $\displaystyle\int \cos x\cos 9x\,dx$; (b) $\displaystyle\int \frac{8\sin^2 4x}{1+\cos 8x}\,dx$;

(c) $\displaystyle\int \frac{7+4x-2x^2}{(x+2)(x^2+1)}\,dx$; (d) $\displaystyle\int \frac{1}{(x+1)(x+2)(x+3)}\,dx$.

Solutions and Hints

1. (a) $\dfrac{x^2}{2}-\dfrac{2}{x}+C$; (b) $\tan x - x - \dfrac{1}{2}\tan 2x + C$; (c) $2\tan x - x + C$;

(d) $\dfrac{3}{10}\tan^{-1}\dfrac{x-2}{5}+C$; (e) $\sin^{-1}\dfrac{x-3}{2}+C$;

(f) $2\sin 2x + \dfrac{3}{2}\cos 2x + 5x + C$. *Hint*: Use trigonometric identities

$$\sin 2x = 2\sin x\cos x, \quad \cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1.$$

2. (a) $y = x^3 - 2x^2 + 1$;

(b) $y = x^3 + 4\ln|x| + 3x + 2$. *Hint*: Find the value of y and $\dfrac{dy}{dx}$ at $x = 1$.

3. i) $2\tan^3 x$; ii) $\dfrac{1}{2}-\dfrac{1}{2}\ln 2$.

4. (a) $-\dfrac{1}{4(2\sin x+5)^2}+C$. *Hint*: $u = 2\sin x + 5$.

(b) $\dfrac{7}{6}\sqrt{14}-2\sqrt{3}$. *Hint*: $\dfrac{7}{6}\sqrt{14}-2\sqrt{3}$, $\displaystyle\int \frac{\sqrt{4-e^{-x}}}{e^x}\,dx = \frac{2}{3}(4-e^{-x})^{3/2}+C$.

(c) $\dfrac{1}{3}[1-(\ln 2)^{3/2}]$. *Hint*: $u = \ln 2x$, $\displaystyle\int \frac{\sqrt{\ln 2x}}{2x} = \frac{1}{3}(\ln 2x)^{3/2}+C$.

(d) 4. *Hint*: $u = 1+\sqrt{x}$, $\displaystyle\int \frac{1}{\sqrt{x+x\sqrt{x}}}\,dx = 4\sqrt{1+\sqrt{x}}+C$.

(e) $\dfrac{5}{2}$. *Hint*: $u = 2+\tan x$, $\displaystyle\int \frac{2+\tan x}{\cos^2 x}\,dx = \frac{1}{2}(2+\tan x)^2 + C$.

(f) $-\dfrac{1}{2}e^{-2/t}+C$. *Hint*: $u = -\dfrac{2}{t}$. (g) $\dfrac{1}{2}(\tan^{-1}x)^2+C$. *Hint*: $u = \tan^{-1}x$.

(h) $2\sin^{-1}\dfrac{\ln x - 2}{2}+C$. *Hint*: $u = \ln x$.

5. (a) $\dfrac{1}{20}\sin 10x + \dfrac{1}{16}\sin 8x + C$. *Hint*: Use $\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$.

(b) $\tan 4x - 4x + C$. *Hint*: Use $\cos 2\alpha = 2\cos^2\alpha - 1$.

(c) $-\dfrac{9}{5}\ln|x+2| - \dfrac{1}{10}\ln(x^2+1) + \dfrac{22}{5}\tan^{-1}x + C$.

Hint: $\dfrac{7+4x-2x^2}{(x+2)(x^2+1)} = \dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+1}$, $A = -\dfrac{9}{5}$, $B = -\dfrac{1}{5}$, $C = \dfrac{22}{5}$.

(d) $\dfrac{1}{2}\ln|x+1| - \ln|x+2| + \dfrac{1}{2}\ln|x+3| + C$.

Hint: $\dfrac{1}{(x+1)(x+2)(x+3)} = \dfrac{A}{x+1}+\dfrac{B}{x+2}+\dfrac{C}{x+3}$, $A = \dfrac{1}{2}$, $B = -1$, $C = \dfrac{1}{2}$.