NATIONAL UNIVERSITY OF SINGAPORE

MA1301 Introductory Mathematics **Tutorial 4**

1. Find the tangent and normal lines of the following curves at the indicated points.

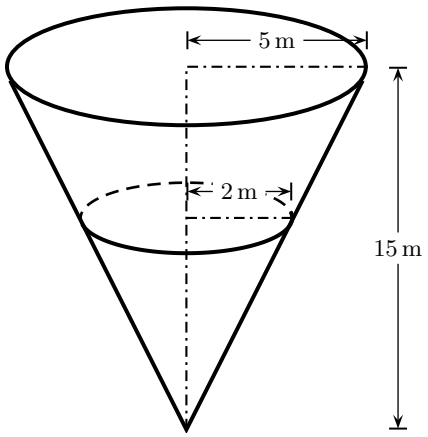
(a) $y = 4x - x^3 + 2$ at $x = -1$.

(b) $x^2 + y^2 - 6x + 2y = 0$ at the origin $(0, 0)$.

2. Use the method of linear approximation to estimate the following numbers.

(a) $\sqrt[3]{8.01}$; (b) $\sin^{-1}(0.49)$.

3. Water is poured into an inverted right circular conical container of base radius 5 m and height 15 m at a rate of 12 m^3/min. Find the rate at which the water level is rising when the radius of the water surface is 2 m.



4. The volume of a cube is increasing at a constant rate of 5 cm^3 per second. Find the rate at which the total surface of the cube is increasing at the instant when the volume is 216 cm^3.

5. The volume of a spherical balloon is increasing at a rate of 10 m^3 per second. Find the rate at which its surface area is increasing at the instant when the radius is 5 m.

6. Find two nonnegative numbers whose sum is 20 and the sum of whose cubes is a minimum.

7. A solid cylinder with a volume of 128π cm^3 is to be manufactured with minimum total surface area. Find the base radius and the height of the cylinder.

8. A solid cylinder of radius r cm is surmounted by a solid hemisphere of the same radius. If the volume of this solid is to be fixed at $576\pi\,\text{cm}^3$, determine the value of r for which the total surface area of the solid has the least value.

Solutions and Hints

1. (a) $y = x$, $y = -x - 2$; (b) $y = 3x$, $y = -\dfrac{1}{3}x$.

2. (a) $\dfrac{2401}{1200}$. *Hint*: Use $f(x) = x^{1/3}$ at $x = 8$.

(b) $\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{150}$. *Hint*: Use $f(x) = \sin^{-1} x$ at $x = 0.5$.

3. $\dfrac{3}{\pi} \approx 0.955\,\text{m/min}$. *Hint*: At time t, let r and h be the base radius and the height of the cone respectively. Then use similar triangles to express r in h, and hence express the volume V in terms of h.

4. $\dfrac{10}{3} \approx 3.33\,\text{cm}^2/\text{s}$. *Hint*: At time t, let x be the side of the cube. Then express the volume and the surface area of the cube in terms of x.

5. $4\,\text{m}^2/\text{s}$. *Hint*: At time r, let r be the radius of the sphere. Then express the volume and the surface area of the sphere in terms of r.

For Questions 6, 7 and 8, it is necessary to verify the maximality/minimality by the second derivative test.

6. 10, 10. *Hint*: Let one number be x. Then the other is $20 - x$. Minimize $x^3 + (20 - x)^3$.

7. Radius 4 cm, height 8 cm. *Hint*: Let r and h be the base radius and height respectively. Use the given volume to express h in terms of r. Then express $2 \cdot \pi r^2 + 2\pi rh$ in terms of r and find its minimum.

8. $\dfrac{12}{\sqrt[3]{5}} \approx 7.02\,\text{cm}$. *Hint*: Let r and h be the base radius and height respectively. Use the given volume to express h in terms of r. Then express $\pi r^2 + 2\pi rh + \dfrac{1}{2} \cdot 4\pi r^2$ in terms of r and find its minimum.