NATIONAL UNIVERSITY OF SINGAPORE

MA1301 Introductory Mathematics **Tutorial 3**

(1) Differentiate the following functions with respect to x. Simplify your answers.

(a) $y = \tan^5(\sqrt{x})$;

(b) $y = \dfrac{2x^2 + 3x + 15}{x^2 + x + 5}$;

(c) $y = \cos^{-1}(\ln x)$;

(d) $y = \sqrt{x}\,\sin(e^x)$;

(e) $y = \csc(\ln x) - e^{\sec 2x}$;

(f) $y = \cot^5\left(\dfrac{1}{\sqrt{x}}\right)$;

(g) $y = \dfrac{1 + \sin x}{1 - \sin x}$;

(h) $y = \dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$.

(2) Using implicit differentiation, find $\dfrac{dy}{dx}$ in terms of x and y.

(a) $x^2 + 4xy + y^2 = 20$;

(b) $\sin(x^2 y) + e^{x-2y} = 7$;

(c) $\ln x + \ln y + xy = 3$.

(3) Using logarithmic differentiation, differentiate the following with respect to x.

(a) $y = x^{\ln x}$;

(b) $y = (\sin x)^{\tan x}$;

(c) $y = \dfrac{\sqrt{e^{-2x}\sin 4x}}{(x^2 + 2)^3}$.

(4) Find the first and second derivatives of y with respect to x. Simplify your answers.

(a) $x = e^t + 1$, $y = te^t$, $\quad t$ is the parameter.

(b) $x = \sin\theta$, $y = 1 - 2\cos\theta$, $\quad \theta$ is the parameter.

(c) $x = t - \sin t$, $y = 1 - \cos t$, $\quad t$ is the parameter.

Solutions and Hints

1. (a) $\dfrac{5}{2}\,\dfrac{\tan^4(\sqrt{x})\sec^2(\sqrt{x})}{\sqrt{x}}$; (b) $-\dfrac{x^2+10x}{(x^2+x+5)^2}$; (c) $-\dfrac{1}{x\sqrt{1-\ln^2 x}}$;

(d) $\dfrac{\sin(e^x)}{2\sqrt{x}}+\sqrt{x}\,\cos(e^x)\,e^x$; (e) $-\dfrac{\csc(\ln x)\,\cot(\ln x)}{x}-2e^{\sec 2x}\sec 2x\,\tan 2x$;

(f) $\dfrac{5}{2}\cot^4\left(\dfrac{1}{\sqrt{x}}\right)\csc^2\left(\dfrac{1}{\sqrt{x}}\right)\dfrac{1}{x\sqrt{x}}$; (g) $\dfrac{2\cos x}{(1-\sin x)^2}$; (h) $\dfrac{8}{(e^{2x}+e^{-2x})^2}$.

2. (a) $-\dfrac{x+2y}{2x+y}$; (b) $-\dfrac{2xy\cos(x^2y)+e^{x-2y}}{x^2\cos(x^2y)-2e^{x-2y}}$; (c) $-\dfrac{y}{x}$.

3. (a) $2x^{\ln x-1}\ln x$; (b) $(\sin x)^{\tan x}\,[\sec^2 x\,\ln(\sin x)+1]$;

(c) $\dfrac{\sqrt{e^{-2x}\sin 4x}}{(x^2+2)^3}\cdot\left[-1+2\cot 4x-\dfrac{6x}{x^2+2}\right]$.

4. (a) $1+t$, e^{-t}; (b) $2\tan\theta$, $2\sec^3\theta$; (c) $\dfrac{\sin t}{1-\cos t}$, $-\dfrac{1}{(1-\cos t)^2}$.