

# NATIONAL UNIVERSITY OF SINGAPORE

## MA1301 Introductory Mathematics

## Tutorial 9

1. Relative to the origin  $O$ , the point  $P$  has position vector  $4\mathbf{i} + \mathbf{k}$  and the point  $Q$  has position vector  $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ .

i) Find a vector equation of the line  $L$  passing through  $P$  and  $Q$ .

ii) Determine the two points on  $L$  which are  $\sqrt{35}$  units from  $O$ .

2. Relative to the origin  $O$ , the point  $A$  has position vector  $2\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$  and the point  $B$  has position vector  $6\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ .

The point  $C$  is such that  $\overrightarrow{OC} = 2\overrightarrow{OA}$  and  $D$  is the midpoint of segment  $AB$ .

i) Find the position vectors of  $C$  and  $D$ .

ii) Find a vector equation of the line  $L$  through  $C$  and  $D$ .

iii) Find the point at which  $L$  intersects the line through  $O$  and  $B$ .

iv) Find the angle between  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$ .

3. Compute the following vector products.

(a)  $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ .

(b)  $[(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} + \mathbf{k})] \times (\mathbf{i} + \mathbf{j} + \mathbf{k})$ .

(c)  $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times [(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \times (\mathbf{i} + \mathbf{j} + \mathbf{k})]$ .

4. Let  $A(1, -3, 2)$ ,  $B(0, -4, 5)$  and  $C(5, 0, -3)$  be points in  $\mathbb{R}^3$ .

i) Find a vector equation of the line  $L$  through  $A$  and  $B$ .

ii) Show that  $C$  does not lie on the line  $L$ .

iii) Find the foot of perpendicular from  $C$  to  $L$ , and hence determine the image of  $C$  under a reflection with respect to  $L$ .

iv) Find a vector equation of the image of the line through  $A$  and  $C$  under a reflection with respect to  $L$ .

v) Find the distance from  $C$  to  $L$ .

5. Let  $A(1, 2, 3)$ ,  $B(1, -2, -3)$  and  $C(1, 2, -3)$  be points in  $\mathbb{R}^3$ .

(a) Show that  $A, B$  and  $C$  are not collinear.

(b) Find the area of  $\triangle ABC$ .

### SOLUTIONS AND HINTS

1. i)  $4\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ ; ii)  $(1, 3, -5), (5, -1, 3)$ ; *Hint*: Set  $|\mathbf{r}| = \sqrt{35}$  to find  $\lambda = \pm 2$ .

2. i)  $4\mathbf{i} + 18\mathbf{j} - 12\mathbf{k}, 4\mathbf{i} + 6\mathbf{j}$ ; ii)  $4\mathbf{i} + 6\mathbf{j} + \lambda(\mathbf{j} - \mathbf{k})$ ; iii)  $(4, 2, 4)$ ;

iv)  $\cos^{-1}(1/33) \approx 1.54 \approx 88.3^\circ$ .

3. (a)  $-4\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$ ; (b)  $12\mathbf{i} - 12\mathbf{k}$ ; (c)  $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ .

4. i)  $\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ ;

iii)  $(1, -2, -5)$ ; *Hint*: Let  $P$  be the foot of perpendicular from  $C$  to  $L$ . Then use  $\overrightarrow{CP} \perp \overrightarrow{AB} = 0$  to find  $P(3, -1, -4)$ . Finally, note that  $P$  is the midpoint of  $C$  and  $C'$ , the image of  $C$  under a reflection in  $L$ .

iv)  $\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{j} - 7\mathbf{k})$ ; *Hint*: It is the line through  $P$  and  $C'$ ; v)  $\sqrt{6}$ .

5. (a) *Hint*: Find  $\angle CAB$ ; (b) 12.