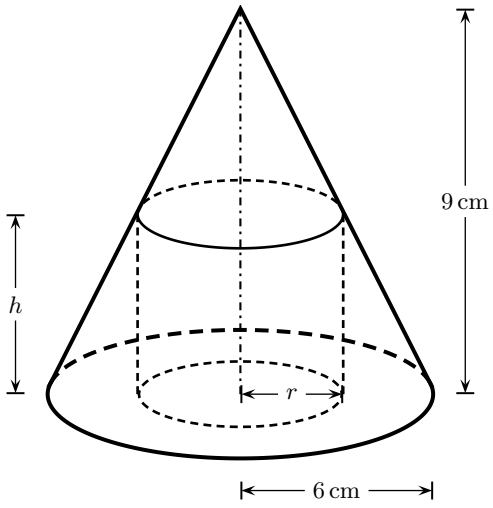
NATIONAL UNIVERSITY OF SINGAPORE

MA1301 Introductory Mathematics **Tutorial 5**

1. (a) A closed rectangular box that has a square base and a capacity of $8\,\text{ft}^3$ is to be constructed using the least amount of materials. Find the dimensions of the box.

(b) An open rectangular box that has a square base and a capacity of $8\,\text{ft}^3$ is to be constructed using the least amount of materials. Find the dimensions of the box.

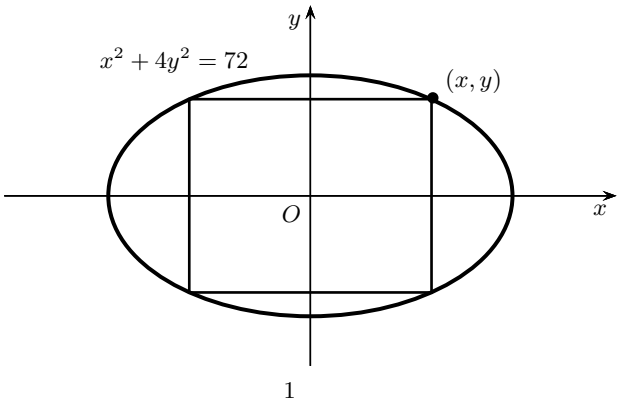
2. A right circular cone has a base radius of 6 cm and a height of 9 cm. Find the volume of the largest cylinder that can be fitted in the cone.



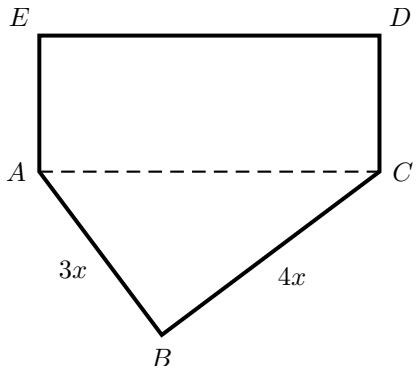
3. A rectangle of maximum area is to be inscribed in the ellipse with equation

$$x^2 + 4y^2 = 72.$$

Find the area of such a rectangle.



4. A piece of wire of length 32 cm is bent to from the pentagon $ABCDE$, where $ACDE$ is a rectangle and $\triangle ABC$ is a right-angled triangle such that $\angle ABC = 90°$, $AB = 3x$ cm and $BC = 4x$ cm.



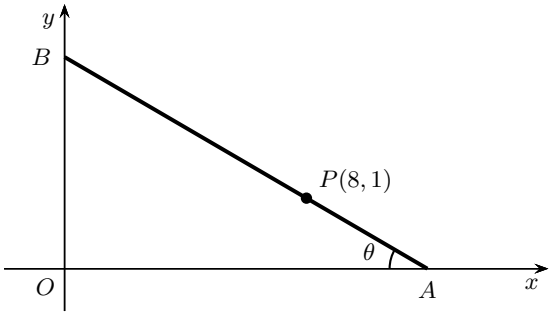
i) Express the length of AC and CD in terms of x.

ii) Show that the area enclosed by the wire is given by $A = 80x - 24x^2$.

iii) Find the stationary value of A.

iv) Show that the stationary value of A in (iii) is a maximum.

5. A line passes through the point $P(8, 1)$ and cuts the positive part of x- and y-axis at points A and B respectively. Let $\theta = \angle BAO$, where O is the origin.



i) Show that the length of the line segment AB is given by $L = \dfrac{1}{\sin\theta} + \dfrac{8}{\cos\theta}$.

ii) Find the stationary value of L.

iii) Suppose θ is increasing at a constant rate of 1 radian per second. Calculate the rate at which L is changing at the instant when $\theta = \dfrac{\pi}{4}$.

Solutions and Hints

1. *Hint*: Let x be the side of the base square and h the height. Express h in terms of x.

(a) $x = 2\,\text{ft}$, $h = 2\,\text{ft}$. *Hint*: Express the area $2x^2 + 4xh$ in terms of x.

(b) $x = 2\sqrt[3]{2}\,\text{ft}$, $h = \sqrt[3]{2}\,\text{ft}$. *Hint*: Express the area $x^2 + 4xh$ in terms of x.

2. $48\pi\,\text{cm}^3$. *Hint*: Let r be the radius and h the height of the cylinder. Using triangles, derive $\dfrac{6-r}{6} = \dfrac{h}{9}$ or $\dfrac{r}{6} = \dfrac{9-h}{9}$. Then express $\pi r^2 h$ in terms of r only, or of h only.

3. $72\,\text{cm}^2$. *Hint*: Let (x, y) be the vertex of the rectangle in the first quadrant. Express y in terms of x, and hence the area $4xy$ in terms of x. Alternatively, express x in terms of y, and hence $4xy$ in terms of y.

4. i) $5x$, $16 - 6x$; ii) $80x - 24x^2$; iii) $\dfrac{200}{3}$; iv) *Hint*: Show that $\left.\dfrac{d^2A}{dx^2}\right|_{x=5/3} < 0$.

5. i) *Hint*: $L = AP + PB$, $AP = \dfrac{1}{\sin\theta}$ and $PB = \dfrac{8}{\cos\theta}$;

ii) $5\sqrt{5}$. *Hint*: Set $\dfrac{dL}{d\theta} = 0$ to obtain $\tan\theta = \dfrac{1}{2}$; iii) $7\sqrt{2}$.