NATIONAL UNIVERSITY OF SINGAPORE

MA1301 Introductory Mathematics   **Tutorial 1**

1. It is given that the third and seventh terms of an arithmetic sequence are 175 and 105 respectively.

    i) Find the first term and common difference of this sequence.

    ii) Find the number n so that the sum of the first n terms of the sequence is zero.

2. The sum of three consecutive terms of a geometric sequence is 28 and their product is 512. Find these numbers.

3. The sum of the first four terms of a geometric sequence is 3.75, and the sum to infinity is 4. Find the first term and common ratio of the sequence.

4. The common difference of an arithmetic sequence is 1, and the common ratio of a geometric sequence is 3. A new sequence is formed by adding the corresponding terms of these two sequences. Suppose that the second and fourth terms of the new sequence are 12 and 86 respectively.

    i) Find the nth term of the new sequence.

    ii) Find the sum of the first n terms of the new sequence.

5. Find the value of x for which

$$3x + 6, \quad x + 2 \quad \text{and} \quad 34 - x$$

are successive terms of a geometric sequence.

6. An arithmetic sequence with first term a and common difference d $(\neq 0)$ is given such that its third, sixth and tenth terms are successive terms of a geometric sequence.

    i) Show that $a = 7d$.

    ii) Find the ratio of the sum of the first 30 terms to the sum of the first 10 terms of the arithmetic sequence.

7. Express the recurring decimal $0.345\,345\,345\cdots$ as a rational number.

8. The first two terms of a geometric sequence are $x + 2$ and $x^2 - 2x - 8$.

i) Find the range of x for which the sum of the sequence to infinity exists.

ii) Find the sum of the sequence to infinity, in terms of x, when it exists.

9. Express the following in $\sum$ notations and evaluate them.

(a) $\ln 2 + \ln 4 + \ln 8 + \ln 16 + \cdots + \ln 2048$,

(b) $3 + 5 + \dfrac{25}{3} + \dfrac{125}{9} + \dfrac{625}{27} + \cdots$ up to $2n$ terms.

Solutions and Hints

1. i) First term: 210; common difference: $-\dfrac{35}{2}$;   ii) 25.

2. $4, 8, 16$ (or $16, 8, 4$). *Hint*: Let a be the first term and r the common ratio. Then the three numbers are a, ar and ar^2.

3. i) First term: 2; common ratio: 0.5;   or ii) First term: 6; common ratio: -0.5.

4. i) $n + 1 + 3^n$.

   ii) $\dfrac{1}{2}n(n+3) + \dfrac{3}{2}(3^n - 1)$. *Hint*: The sum of the first n terms of the new sequence is the sum of that of the arithmetic sequence and that of the geometric sequence.

5. 25. *Hint*: If r is the common ratio, then $r = \dfrac{x+2}{3x+6}$ and $r = \dfrac{34-x}{x+2}$.

6. i) *Hint*: If a, b, c are successive terms of a geometric sequence, then $\dfrac{a}{b} = \dfrac{b}{c}$;   ii) $\dfrac{129}{23}$.

7. $\dfrac{115}{333}$.

8. i) $3 < x < 5$. *Hint*: First find the common ratio r and then use the fact that the sum to infinity exists if and only if $|r| < 1$;   ii) $\dfrac{x+2}{5-x}$.

9. (a) $\displaystyle\sum_{i=1}^{11} \ln(2^i) = 66 \cdot \ln 2$. *Hint*: $\ln(a^b) = b \cdot \ln a$ for $a > 0$.

   (b) $\displaystyle\sum_{i=1}^{2n} 3\left(\dfrac{5}{3}\right)^{i-1} = \dfrac{9}{2}\left[\left(\dfrac{5}{3}\right)^{2n} - 1\right]$.