1. A chicken lays n eggs. Each egg independently does or doesn't hatch, with probability p of hatching. For each egg that hatches, the chick does or doesn't survive (independently of the other eggs), with probability s of survival. Let $N \leftarrow \text{Bin}(n, p)$ be the number of eggs which hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch but don't survive (so $X + Y = N$). Find the marginal PMF of X, and the joint PMF of X and Y . Are they independent?

Solution

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Solution 1 :

Given information :

Total eggs = n
hatching Probability = P
survival Prob = S

X = no of survives
Y = no of not survives

a) Marginal PMF

using binomial Theorem

Y / X	0	1	2 .....
0	0	0	1 − P(x)
1	0	1−P(x)	0
2	1−P(x)	0	0
⋮			

Cont .....

hatch - survive Prob. is : PS

survive Prob of X chicks over n eggs = ?

$$P(X_i) = \binom{n}{X_i} PS^{(X_i)} (1-PS)^{n-X_i}$$

So, Marginal PMF of X_i

$$P_X(X_i) = 1 - P(X_i)$$

b) Joint PMF

$$P_{XY}(X,Y) = \begin{cases} 1 - P(X) \\ X \in R_X \end{cases}$$