The Relationship Between Birth and Poverty Rates: A Mathematical Model

Savannah Simmons

December 2021

1 Introduction

When presented with the task of creating my first mathematical model, I wanted to work on something that I have professional experience with and to which I can bring some context. I've worked at nonprofits for several years, initially providing direct services before transitioning to a more administrative role managing data. Particularly in my work at YWCA Utah's domestic violence crisis shelter, I saw cycles that would feel demoralizing and seem inescapable, whether that was the same person returning to shelter, or a young adult seeking services who had stayed in the shelter as a child with their parent years before. There is a lot to be said there about other cyclic circumstances, including those around abuse, but poverty was just as significant in people's need for crisis shelter, if not more so; someone of means who suffers abuse may have more resources before turning to emergency shelter than someone near the poverty line. This was my motivation to attempt to model the complex topic of intergenerational poverty.

After sketching out a model, which I will outline below, I realized that I had achieved my goal of creating a simplified model, but I didn't really have a question to ask it. I then had a conversation about states with trigger laws that would instantly ban abortion should Roe v. Wade be overturned in the impending Supreme Court verdict. We discussed the oft-referenced point that, seeing as many states will keep abortion legal should it be left to them, those of certain financial means will always be able to travel for an abortion should they need one, while others will now be unable to access them. I realized that while my model was simplified, it does have birth rate terms. This is how I arrived at projecting how different birth rates would affect the poverty rates over time.

2 Methods

I opted to model this with a compartmental model with four state variables. The variables are described alongside the diagram of the model (Figure 1), but the general path through the system is that adults in each economic class have a birth rate at which they produce children in the same economic class, obtained

Intergenerational Transfer of Economic Status

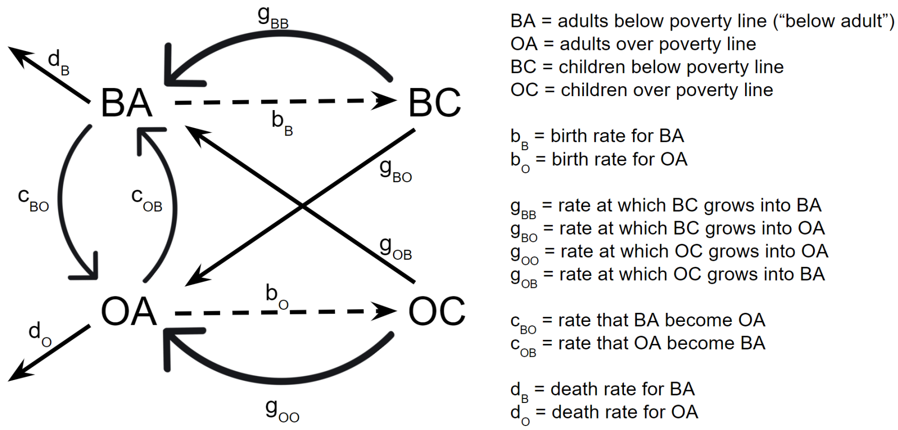

Figure 1: A compartmental model describing adults and children moving through state variables of economic wealth

primarily from census data [1], though this represented birth rates per 1,000 women and was therefore halved. The dashed arrow represents production, not changing states. The children then grow to adults in their same or other economic class weighted by different estimates of correlation of economic status from various studies [2][3]. Typically the two growth rates leaving a child state were set to add to 1/18 in my simulations, but other values can be used if looking to set the cutoff at young adulthood. Adults move across the poverty line at different rates; in my simulations I used .06 for c_{BO} and .01 for c_{OB} to account for the "upward mobility" frequently touted in the United States. This was difficult to find reliable data on because it would require true longitudinal study rather than census totals. Finally, we have death rates for each class of adults [4]. For simplicity and to put an upper bound on the negativity, it was assumed that there would be no child deaths in this model, but those rates could be added. The corresponding differential equations are as follows:

$$\frac{dBA}{dt} = (-d_B - c_{BO})BA + c_{OB}OA + g_{BB}BC + g_{OB}OC$$
$$\frac{dOA}{dt} = c_{BO}BA + (-d_O - c_{OB})OA + g_{BO}BC + g_{OO}OC$$
$$\frac{dBC}{dt} = b_B BA + (-g_{BB} - g_{BO})BC$$
$$\frac{dOC}{dt} = b_O OA + (-g_{OO} - g_{OB})OC$$

These were translated into R code (Appendix A) which, after much adjustment and exploration, was run over a period of 100 years, first with a birth rate of .045 for the BA group and .04 for the OA group, then instead with a .55 rate for BA and OA held the same at .04. This represents a 22% increase in the birth rate, which is likely significantly higher than the increase we would see overall if Roe v. Wade were overturned, and very possibly higher than we would see even with those in poverty. Unfortunately, the projected difference in birth rate is extremely difficult to estimate because even looking at the years before 1973, access to abortion was very different and likely would not reflect the change in legality were it to happen today. Because of this uncertainty, I focused mostly on the relationship between increasing the birth rate and the poverty rate, which should hold similarly regardless of what the actual increase is.

3 Results

After running the simulation, the plots of the results can be seen in Figure 2, and the totals and percentages were:

Current Birth Rate = .045	Increased Birth Rate = .055
BA = 718	BA = 840
OA = 3944	OA = 4187
BC = 446	BC = 627
OC = 2192	OC = 2300
Total adults = 4662	Total adults = 5027
Total children = 2637	Total children = 2927
Total pop = 7299	Total pop = 7954
BA to total adults = 15.3%	BA to total adults = 16.7%
BC to total children = 16.8%	BC to total children = 21.4%
BA + BC to total pop = 15.9%	BA + BC to total pop = 18.4%

Understandably, the total population and the totals for each state variable were larger after 100 years of the increased birth rate, but we are interested in the resulting poverty rates. The adult poverty rate with the increased birth rate was 9% higher than with the unadjusted birth rate, the child poverty rate was 27% higher, and the total poverty rate was 16% higher.

Simulations of Economic Status Over Time

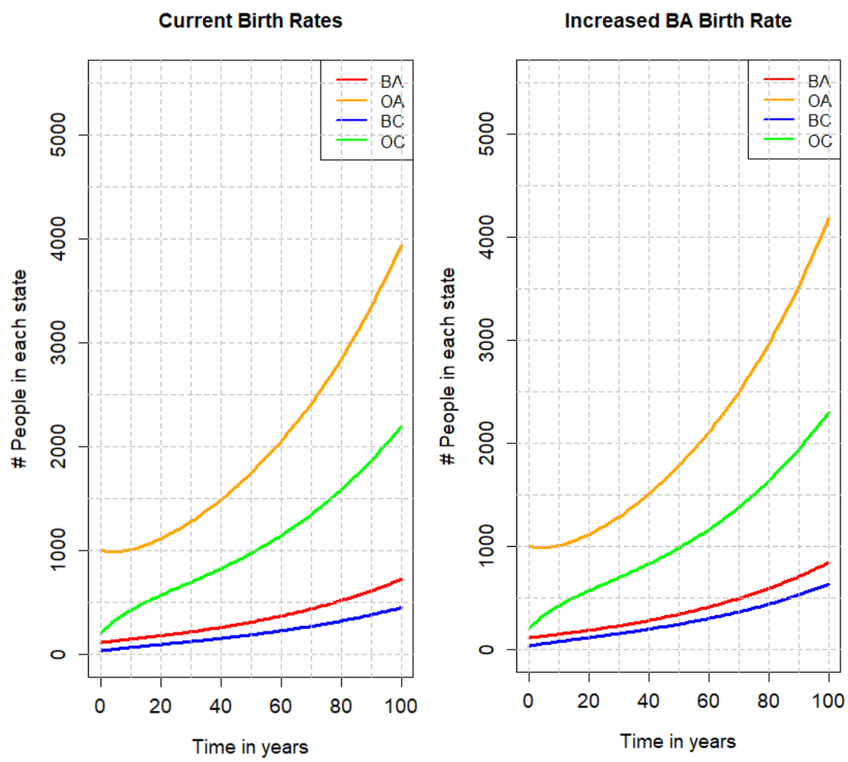

Figure 2: A comparison of 100 years of current birth rates vs. 100 years with 22% increase of the birth rate of those in poverty

4 Discussion

Considering that I included a term for upward mobility, I wasn't necessarily expecting the increase in any of the poverty rates to match the increase in the birth rate. I was somewhat surprised to see the child poverty rate not only match the increase, but surpass it. I will not suggest that these results should influence policy, primarily because I am a modeling child and this model's parameters deserve considerable scrutiny and adjustment, but also because legislative decisions made on abortion are obviously affected by myriad other moral and societal considerations.

I was generally pleased that even with many simplifications, this model can answer certain questions about intergenerational poverty. Of course, there are many more interesting queries to be posed about the topic for which this model likely couldn't be used, like what feeds into each of the change and growth rate parameters, but we can see what the big picture rates and totals look like under circumstances created by those parameters.

4.1 Challenges

There were many different challenges in building this simulation, including simply how to begin. Intergenerational poverty is a weighty, complex topic, and boiling it down to a model that I could work with was tough. I learned that simpler was, counterintuitively, more productive, as every time I tried to add a layer of complexity, I lost the ability to really draw any conclusions at all.

I also ran into some problems simply keeping track of terms. Originally I wanted to hold the population constant over time, before deciding that that was too unrealistic to be useful (particularly adjusting the death rate to keep up with the birth rate). That said, instead of staying constant, my population kept quickly going to 0. I couldn't figure out how I was killing everyone until I realized that I was only including the negatives of c_{OB} and c_{BO} and not their positive counterparts in my differential equations.

By far the greatest challenge was finding what my parameters should actually be. Finding reliable data felt impossible, even for apparently simple statistics like birth rates. When searching for birth rates broken down by income or economic status, they were sometimes different by orders of magnitude from the birth rate for the general population. Then, when I did finally get what seemed like accurate parameters, the numbers that I got out of the base simulation (with the current birth rates) were wildly unrealistic. For example, the poverty rate might shoot up to 50%, or down to 0. I freely admit that I tweaked the parameters to get reasonable outcomes in my base simulation, which is why I know that this model would need likely need serious modification before being truly useful.

4.2 Simplifying Assumptions and Next Steps

One simplifying assumption that was mentioned earlier was excluding child deaths. More significant than discrete simplifications, however, was the overall disregard for what goes into the parameters. The most interesting questions are about how and why the change and growth factors are what they are. Simply slapping a .06 on c_{BO} glosses over the most significant aspects of the dynamics of poverty. On a similar note, this was a purely deterministic model, which, while probably unrealistic for about any model, is particularly nonviable for this topic. Next steps for improving this model should definitely include examining the accuracy and dynamics of the parameters and injecting them with some degree of stochasticity.

Related to the simplifying of the parameters, this model doesn't account for the snowballing that can happen when poverty grows in an area and social services are overrun. As affordable housing, food banks, case management, certain jobs, etc., dry up, those on the verge of poverty can lose the resources they need to keep afloat. It would be interesting to add a component analogous to our queuing model for social services.

On the topic of being on the verge of poverty, one huge simplifying assumption is treating poverty as a binary. One's economic well-being is obviously a spectrum, and families don't go from easily meeting their needs to suddenly needing emergency shelter. A more accurate version would include accounting for that spectrum. While making it truly continuous would be tricky, adding a third set of states to break up economic status and including the queuing model mentioned above for resources would be a good place to start.

Finally, possibly the greatest flaw in this setup is in its very structure; as a compartmental model, there is no case study component. The rate at which an adult who was once a poor child crosses the poverty line should probably differ from that of an adult who was the child of a well-off family, but there is no way to account for it. An agent-based model may be more appropriate for this topic, or it should at least be examined in conjunction with longitudinal data on specific individuals for context.

Appendix A: R Code

Actual code available upon request since LaTeX's R formatting will make the strings look like they're coming out of a demon if copied and pasted.

```
require(deSolve)

rhs <- function(t, x, parms){
        BA <- x["BA"]
        OA <- x["OA"]
        BC <- x["BC"]
        OC <- x["OC"]

        dBA <- gOB*OC + gBB*BC - dB*BA - cBO*BA + cOB*OA
        dOA <- gBO*BC + gOO*OC - dO*OA - cOB*OA + cBO*BA
        dBC <- bB*BA - gBO*BC - gBB*BC
        dOC <- bO*OA - gOB*OC - gOO*OC

        return(list(c(dBA,dOA,dBC,dOC)))
}
# N is just general scope here to adjust graph
N <- 1000

bB <- .055
bO <- .04

g <- 1/18
gBB <- g * .95
gBO <- g * .05
gOO <- g * .97
gOB <- g * .03


dB <- 1/62
dO <- 1/68

cOB <- .01
cBO <- .06

init <- c(110,1000,30,200)
names(init) <- c("BA","OA","BC","OC")

tmax <- 100
times <- seq(from=0,to=tmax,by=1)

povout <- as.data.frame(ode(y = init,times = times, func = rhs))
```

```
xmax <- tmax
ymax = 5*N + 500

par(mfrow=c(1,2))
plot(BA ~ time, povout, type="l", lwd=3, xlim=c(0,xmax), ylim=c(0,ymax),
     ylab="# People in each state", main="Increased BA Birth Rate",
     xlab = "Time in years", cex.axis=1.2, cex.lab=1.2, col="red")

lines(OA ~ time, povout, type="l", lwd=3, xlim=c(0,xmax), col="orange")
lines(BC ~ time, povout, type="l", lwd=3, xlim=c(0,xmax), col="blue")
lines(OC ~ time, povout, type="l", lwd=3, xlim=c(0,xmax), col="green")
legend(x="topright", c("BA","OA","BC","OC"), col= c("red","orange",
            "blue","green"), lwd= 2)

# Manually adding grid because the built-in is stupid
abline(h = c(0,500,1000,1500,2000,2500,3000,3500,4000,4500,5000,5500),
     lty = 2, col = "grey")
abline(v = c(0,10,20,30,40,50,60,70,80,90,100),  lty = 2, col = "grey")

BAf <- povout[[2]][tmax+1]
OAf <- povout[[3]][tmax+1]
BCf <- povout[[4]][tmax+1]
OCf <- povout[[5]][tmax+1]
Af <- BAf + OAf
Cf <- BCf + OCf
Bf <- BAf + BCf
Pop <- Af + Cf

print(tail(povout))

cat("\nTotal adults = ", Af, "\n")
cat("Total children = ", Cf, "\n")
cat("Total pop = ", Pop, "\n")
cat("BA to total adults = ", BAf / Af, "\n")
cat("BC to total children = ", BCf / Cf, "\n")
cat("BA + BC to total pop = ", Bf / Pop, "\n")
```