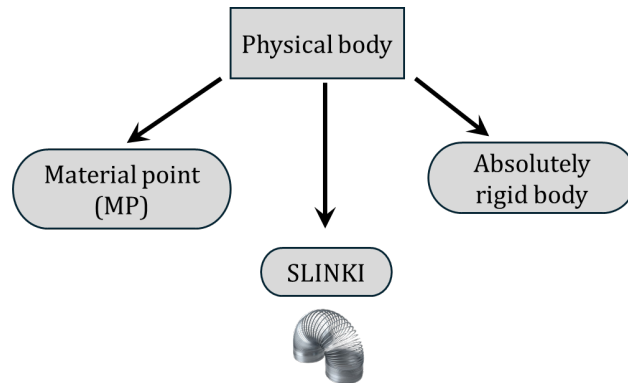


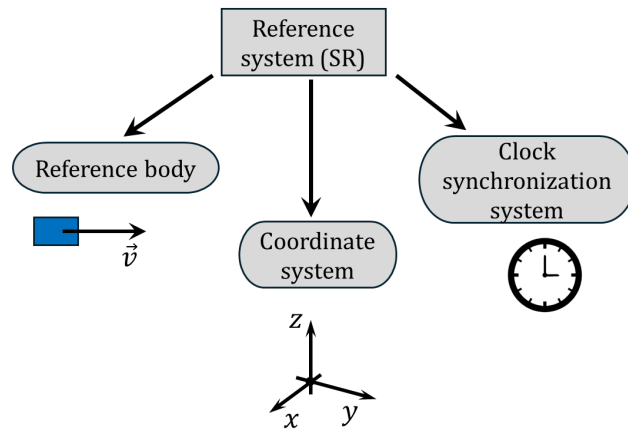
Kinematics

Uniform and uniformly accelerated motion

Df: *Mechanics is a branch of physics that studies the laws of motion and the causes that cause it.*

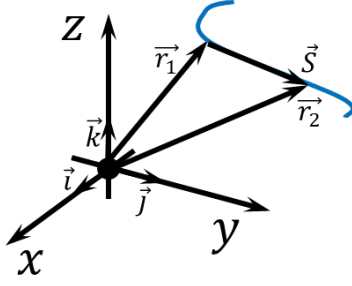


Re: *Reference System (SR)*



NO: *In Newtonian terms: time is absolutely ($t = t'$); Einstein: ($t \neq t'$)*

Df: *Radius of curvature \vec{r} —Directed segment connecting the origin of coordinates with the current position of the material point (MP).*



From the drawing:

$$\vec{r}_2 = \vec{r}_1 + \vec{S}$$

$$\vec{S} = \vec{r}_2 - \vec{r}_1 = \Delta \vec{r}$$

Vector sum:

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$$

$$\boxed{\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}} \quad (1)$$

From the Pythagorean theorem:

$$\boxed{r = \sqrt{x^2 + y^2 + z^2}} \quad (2)$$

Re: Path is a scalar physical quantity equal to the length of the projection of a material point

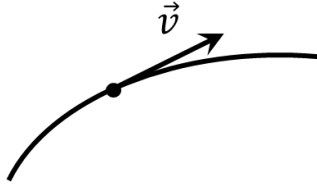
Dimension of the path $[S] = \text{m}$

Re: Displacement is a directed segment connecting the start point and the end point of the point's movement

Rectilinear unidirectional motion

$$\boxed{|\vec{S}| = l} \quad (3)$$

Df: Instantaneous velocity of a material point:



In a very short period of time $\Delta t \rightarrow 0$

$$\boxed{\vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}} \quad (4)$$

$$\Delta \vec{r} = \vec{v} \Delta t$$

The dimension of speed is $[v] = \text{m/s}$

Newton's method split into N parts

$$\Delta \vec{r}_1 = \vec{v}_1 \Delta t_1$$

$$\Delta \vec{r}_2 = \vec{v}_2 \Delta t_2$$

$$\vdots$$

$$\Delta \vec{r}_i = \vec{v}_i \Delta t_i$$

The displacement \vec{S} is found as the vector sum of displacements over small intervals Δt

$$\vec{S} = \vec{r}_1 + \vec{r}_2 + \dots + \vec{r}_n = \sum_{i=1}^{\infty} \vec{v}_i \Delta t_i$$

$$\vec{S} = \vec{v}_1 \Delta t_1 + \vec{v}_2 \Delta t_2 + \dots + \vec{v}_n \Delta t_n$$

$$\boxed{S = \sum_{i=1}^{\infty} \vec{v}_i \Delta t_i} \quad (5)$$

Considering the rectilinear motion

$$\vec{v}_i = \vec{v} = \text{const}$$

$$\vec{v} \sum_{i=1}^{\infty} \Delta t_i = \vec{v}t$$

$$\boxed{\vec{S} = \vec{v}t} \quad (6)$$

Df: Acceleration of a material point is a vector physical quantity equal to the change in velocity over a small period of time Δt

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \vec{v}'(t)$$

The dimension of acceleration is $[a] = \text{m/s}^2$

Df: With uniformly accelerated motion, the velocity of a material point changes by the same amount over any equal time intervals

$$\Delta \vec{v} = \vec{a} \Delta t$$

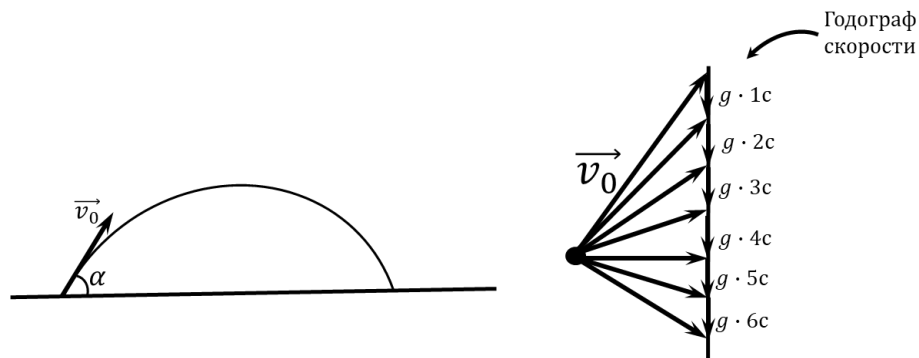
$$\vec{v} - \vec{v}_0 = \vec{a} \Delta t$$

$$\boxed{\vec{v} = \vec{v}_0 + \vec{a} \Delta t} \quad (7)$$

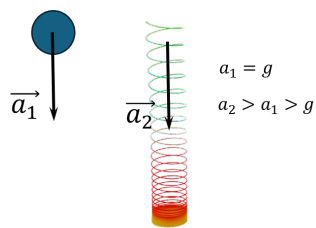
$$\boxed{\vec{S} = \vec{v}_0 t + \frac{\vec{a} t^2}{2}} \quad (8)$$

$$S = \frac{v^2 - v_0^2}{2a}; \quad S = \frac{v + v_0}{2} t$$

Ex:



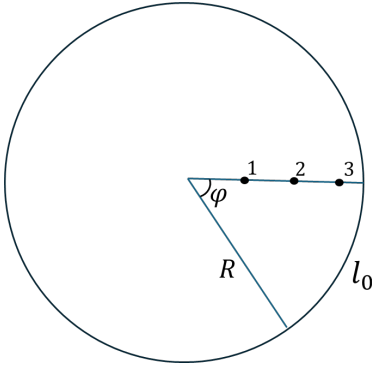
Dem:



"Super free fall"

Rotational motion

Df: The trajectory of a material point is a circle of radius R



$$l_1 < l_2 < l_3$$

Rotation angle φ radians

$$\varphi_1 = \varphi_2 = \varphi_3$$

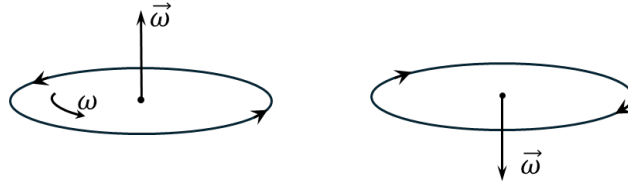
$$\varphi = \frac{l_0}{R}; \quad l_0 = \varphi R \quad (1)$$

Df: Angular velocity of a material point:

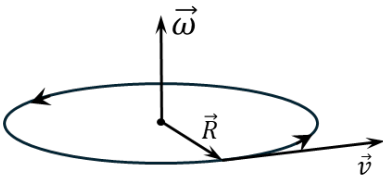
$$\omega = \frac{\Delta\varphi}{\Delta t} = \frac{d\varphi}{dt} \quad (2)$$

The dimension of angular velocity is $[\omega] = \text{s}^{-1}$

NO: $\vec{\omega}$ is a vector quantity



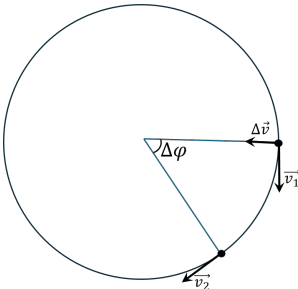
Right Screw Rule



$$v = \frac{\Delta l}{\Delta t} = \frac{R\Delta\varphi}{\Delta t} = \omega R$$

$$\vec{v} = \vec{\omega} \times \vec{R} \quad (3)$$

Ex: Centripetal acceleration: $a = \frac{v\Delta\varphi}{\Delta t} = vR$



$$a = \frac{\Delta v}{\Delta t} = \{\Delta v = v\Delta\varphi\}$$

Ex:

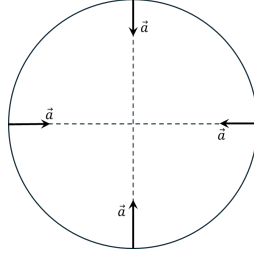
$$2\pi R = vt \quad T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$$

$$2\pi = \omega t \quad \boxed{\omega = \frac{v}{R} \quad v = \omega R} \quad (4)$$

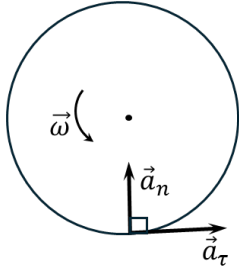
$$\omega = 2\pi\nu$$

NO: Formula for calculating centropetral acceleration a

$$\boxed{a = v \cdot \omega = \omega^2 R = \frac{v^2}{R} = 4\pi^2 \mu^2 R = \frac{4\pi^2 R}{T^2}} \quad (5)$$

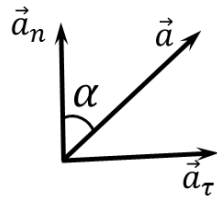


Ex: Uneven circular motion: (Tangential acceleration)



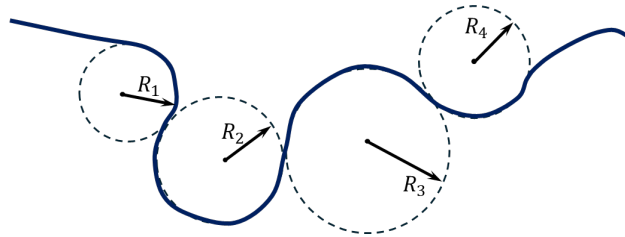
$$\begin{cases} a_\tau = \frac{\Delta v}{\Delta t} \\ v = v_0 + a_\tau t \\ l = v_0 t + \frac{a_\tau t^2}{2} \end{cases} \quad (6)$$

Df: Total acceleration of a material point



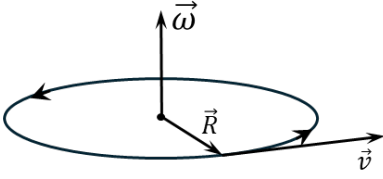
$$\begin{cases} \vec{a} = \vec{a}_n + \vec{a}_\tau \\ a = \sqrt{a_n^2 + a_\tau^2} \\ \tan \alpha = \frac{a_\tau}{a_n} \end{cases} \quad (7)$$

NO: Newton's Method:



An arbitrary trajectory is divided into different arcs of a circle (R_i)

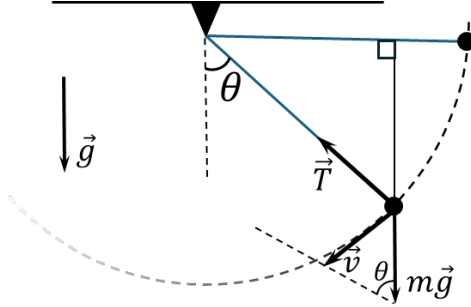
Ex: $a_n = a_{u.c.} = v\omega$



From the drawing:

$$\vec{a}_n = \vec{a}_{u.c.} = \vec{\omega} \times (\vec{\omega} \times \vec{R}) \quad (8)$$

Ex: "Horizontal Swing"



$a(\theta) - ?$

Newton's Second Law

$$m\vec{a} = m\vec{g} + \vec{T} + \vec{F}_c \quad (F_c \ll mg)$$

$$m(\vec{a}_n + \vec{a}_\tau) = mg \sin \theta$$

$$a_\tau = g \sin \alpha \quad (9)$$

Law of conservation of energy

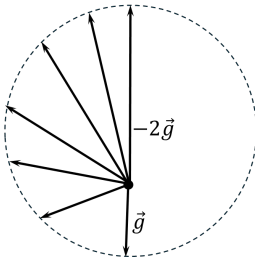
$$mgh = \frac{mv^2}{2}$$

$$gl \sin \theta = \frac{v^2}{2}$$

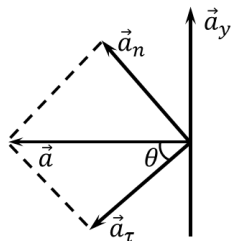
$$a_n = \frac{v^2}{R} = 2g \cos \theta \quad (10)$$

$$a = \sqrt{a_n^2 + a_\tau^2} = \sqrt{g^2 \sin^2 \alpha + 4g^2 \cos^2 \alpha} = g\sqrt{1 + 3 \cos^2 \alpha} \quad (11)$$

Df: Hodograph of acceleration of a material point (set of end points of a vector)



NO: At the moment of time when the vector of total acceleration is horizontal, its projection on the vertical axis is equal to O



$$a_n \cos \alpha = a_\tau \sin \alpha$$

$$2g \cos^2 \alpha = g \sin^2 \alpha$$

$$\theta = \arctan \sqrt{2} \approx 55^\circ \quad (12)$$

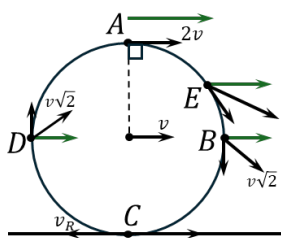
Df: Angular acceleration of a material point (β, ε)

$$\boxed{\beta = \frac{\Delta\omega}{\Delta t}} \quad (13)$$

The dimension of angular acceleration is $[\omega] = \text{s}^{-2}$ With $\beta = \text{const}$

$$\boxed{\begin{aligned} \omega &= \omega_0 + \beta t \\ \varphi &= \varphi_0 + \omega_0 t + \frac{\beta t^2}{2} \end{aligned}} \quad \begin{aligned} v &= \omega R \\ \frac{\Delta v}{\Delta t} &= \frac{\Delta\omega R}{\Delta t} \\ \beta &= \frac{a_\tau}{R} \end{aligned} \quad (14)$$

Ex: Rolling without slipping



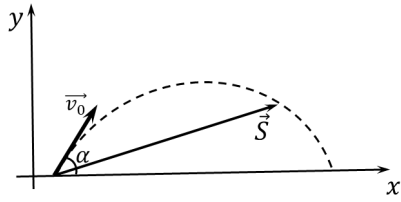
\vec{v} — translational motion velocity
 \vec{v}_R — rotational motion velocity

$$\boxed{\vec{V} = \vec{v} + \vec{v}_R} \quad (15)$$

C — instantaneous center of rotation
 $v_c = 0 \Rightarrow$ without slipping

$$\boxed{v = \omega R; \quad \omega = \frac{v}{R}}$$

Movement at an angle to the horizon

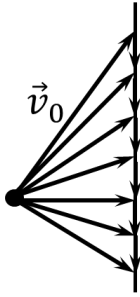


Newton's Second Law:

$$m\vec{a} = m\vec{g} + \vec{T} + \vec{F}_c \quad (F_c \ll mg)$$

$$\vec{a} = \vec{g} = \text{const}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{g}t$$



Speed hodograph:

$$t_n = 2t_1 = \frac{2v_0 \sin \alpha}{g} \quad (1)$$

$$\begin{cases} v_x(t) = v_0 \cos \alpha \\ v_y(t) = v_0 \sin \alpha - gt \end{cases}$$

$$0 = v_0 \sin \alpha - gt$$

$$t_{\text{flight}} = \frac{v_0 \sin \alpha}{g} \quad (2)$$

$$\vec{S} = \vec{v}_0 t - \frac{gt^2}{2}$$

$$\begin{cases} x(t) = v_0 \cos \alpha \cdot t \\ y(t) = v_0 \sin \alpha \cdot t - \frac{gt^2}{2} \end{cases} \quad (3)$$

Equation of trajectory:

$$t = \frac{x}{v_0 \cos \alpha}$$

$$y(x) = x \tan \alpha - \frac{g \cdot x^2}{2 \cdot v_0^2 \cos^2 \alpha} \quad (4)$$

Flight range:

$$S = v_0 \cos \alpha \cdot \frac{2v_0 \sin \alpha}{g} = \frac{v_0^2}{g} (2 \sin \alpha \cos \alpha)$$

$$S = \frac{v_0^2}{g} \sin 2\alpha \quad (5)$$