# Advanced Computer Architectures
# Performance formulas

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# 1 Speedup

## 1.1 Performance

$$Performance(x) = \frac{1}{Ex\,Time(x)}$$

## 1.2 Performance ratios

"X is n times faster than Y" means

$$\frac{Ex\,Time(y)}{Ex\,Time(x)} = \frac{Performance(x)}{Performance(y)}$$

## 1.3 Performance percentage

"X is n % times faster than Y" means

$$\frac{Ex\,Time(y)}{Ex\,Time(x)} = 1 + \frac{n}{100}$$

$$Speedup(x, y) = \frac{Performance(x)}{Performance(y)}$$

# 2 Amdhal's law

$$Speedup(E) = \frac{Performance_{with}(E)}{Performance_{without}(E)} = \frac{Ex\,Time_{without}(E)}{Ex\,Time_{with}(E)}$$

$$Ex\,Time_{with} = Ex\,Time_{without} \cdot [(1 - Fraction_{enhanced}) + \frac{Fraction_{enhanced}}{Speedup_{enhanced}}]$$

$$Speedup(E) = \frac{Ex\,Time_{without}(E)}{Ex\,Time_{with}(E)} = \frac{1}{(1 - Fraction_{enhanced}) + \frac{Fraction_{enhanced}}{Speedup_{enhanced}}}$$

# 3 CPU Time - execution time

## 3.1 CPU Time

$$CPU\ time = \frac{\#\ CC}{CC\ Frequency}$$
$$CPU\ time = \#\ CC \cdot CC\ Time$$

$$CPU\ time = \frac{Time}{Program} = \frac{\#Instructions}{Program} \cdot \frac{\#CC}{Instruction} \cdot \frac{Time}{CC}$$
$$CPU\ time = IC \cdot CPI \cdot CC\ Time$$

**IC**: Instructions count, the number of instructions the program is composed, it depends on program - obviously -, compiler and Instruction Set.

**CPI**: Average Cycles per Instruction, this term is reduced with pipelining, it depends on Instruction Set and organization of the processor.

**CC Time**: the time needed to perform a the Clock Cycle (CC), it is the inverse of Clock Frequency. It depends on organization of the processor and the technology used.

## 3.2 Average CPI

$$CPI = \frac{CPU\ time}{IC \cdot CC\ Time} = \frac{CPU\ Time \cdot CC\ Frequency}{IC}$$
$$CPU\ time = CC\ Time \cdot \sum_{i=1}^{IC} CPI_i \cdot \#I_i$$
$$CPI = \sum_{i=1}^{IC} CPI_i \cdot F_i \text{ with F, instruction frequency } F_i = \frac{\#I_i}{IC}$$

# 4 MIPS and MFLOPS

## 4.1 MIPS

Millions of instructions per second

$$MIPS = \frac{IC}{CPU\ time \cdot 10^6} = \frac{CC\ Frequency}{CPI \cdot 10^6}$$

$$CPU\ time = \frac{IC}{MIPS \cdot 10^6}$$

## 4.2 MFLOPS

Millions of floating point operations per second

$$MFLOPS = \frac{IC_{floating\ point}}{CPU\ time \cdot 10^6}$$

# 5 Power consumption

$$P = \frac{E}{t}$$

$$P = \frac{1}{CPU\ time} \cdot \sum_{i=1}^{n} E_i \cdot I_i$$

# 6 Dependability

Mean time to failure (MTTF)
Mean time to repair (MTTR)
Mean time between failures (MTBF)

$$MTBF = MTTF + MTTR$$

Availability

$$A = \frac{MTTF}{MTBF}$$