

DELFT UNIVERSITY OF TECHNOLOGY  
FACULTY OF AEROSPACE ENGINEERING  
ROTOR / WAKE AERODYNAMICS - AE4135

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## **Rotor / wake Aerodynamics**

*Assignment 1: Blade Element Theory*

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# Nomenclature

# 1

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$\alpha$	angle of attack at blade element (—)
$\beta$	blade twist angle at blade element (—)
$\lambda$	tip speed ratio (—)
$\rho$	fluid density ( $\text{kg} \cdot \text{m}^{-3}$ )
$\Gamma$	circulation at blade element ( $\text{m}^2 \cdot \text{s}^{-1}$ )
$\Phi$	perceived-wind inflow-angle at blade element (—)
$\Omega$	rotor rotational velocity ( $\text{rad} \cdot \text{s}^{-1}$ )
$a$	axial induction factor (—)
$a'$	azimuthal induction factor (—)
$c$	blade element chord (m)
$C_d$	drag coefficient (—)
$C_l$	lift coefficient (—)
$C_T$	thrust coefficient (—)
Drag	drag force per unit span ( $\text{N} \cdot \text{m}^{-1}$ )
Lift	lift force per unit span ( $\text{N} \cdot \text{m}^{-1}$ )
$F_{\text{azim}}$	azimuthal/tangential force per unit span ( $\text{N} \cdot \text{m}^{-1}$ )
$F_{\text{axial}}$	axial force per unit span ( $\text{N} \cdot \text{m}^{-1}$ )
$N_{\text{blades}}$	number of blades (—)
$V_{\text{axial}} = U_{\text{rotor}}$	axial velocity perceived by blade element, axial velocity at rotor ( $\text{m} \cdot \text{s}^{-1}$ )
$V_P$	velocity perceived by blade element ( $\text{m} \cdot \text{s}^{-1}$ )
$V_{\text{tan}}$	azimuthal/tangential velocity perceived by blade element ( $\text{m} \cdot \text{s}^{-1}$ )

# Introduction 2

Table 2.1: Wind turbine geometrical specifications

Variable	Value
Radius (R)	50 [m]
Number of Blades	3
Blade starts at	0.2 r/R
Twist	$14 \cdot (1 - r/R)$ [degrees]
Blade Pitch	-2 [degrees]
Chord Distribution	$3 \cdot (1 - r/R) + 1$ [m]
Airfoil	DU 95-W-180
Rotor yaw angle	0, 15 and 30 [degrees]

Table 2.2: Wind turbine operational specifications

Variable	Value
Wind speed ( $U_0$ )	10 [m/s]
Tip speed ratio ( $\lambda$ )	6, 8, 10
Rotor yaw angle	0, 15, 30 [degrees]

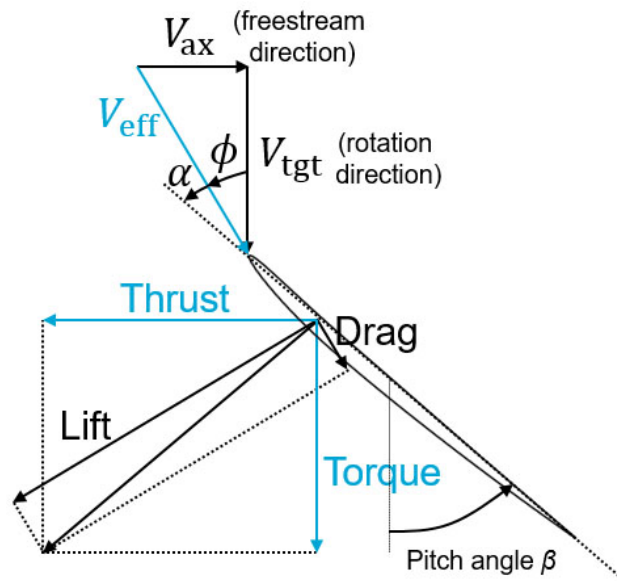


Figure 2.1: Caption

# Flow diagram of the code 3

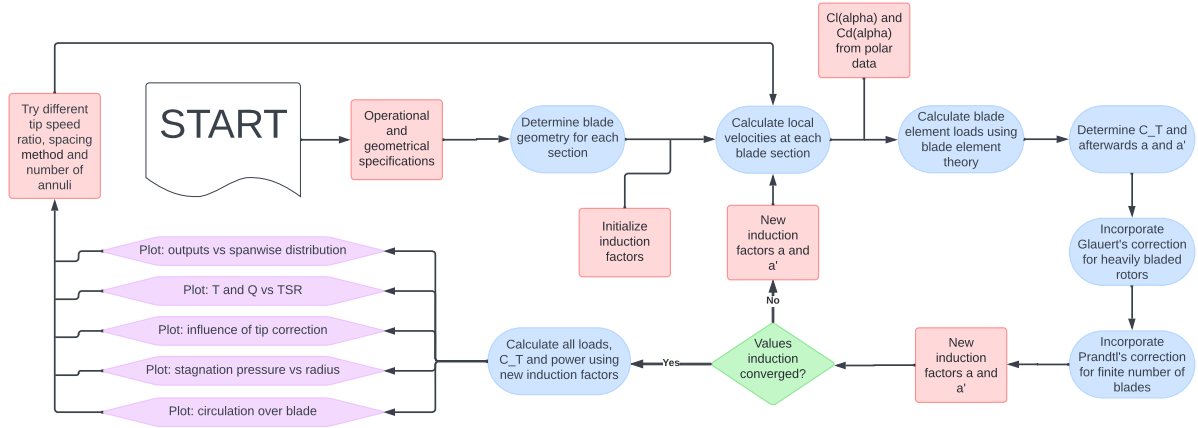


Figure 3.1: Flow diagram of the code

## 3.1 Assumptions

- **Steady Flow:** the flow characteristics are assumed to be independent of time.
- **Inviscid Flow:** the flow is assumed to be inviscid. This means no viscous effects are taken into account.
- **Incompressible Flow:** the flow is assumed to be incompressible. This means the density throughout the streamtube is constant. This enables the use of Bernoulli's equation in locations of a continuous pressure distribution. This also results in the product of area and flow velocity being constant over the flow.
- **2D Flow:** it is assumed that the flow characteristics can accurately be modeled using 2 dimensional flow characteristics.
- **Constant Internal Energy:** it is assumed that the internal energy within the streamtube is constant so there no radiation, convection or conduction occurring.
- **Independent annulus:** the annuli are considered independently of one another. In reality, flow characteristics on one annulus will influence the characteristics on another (cross flow), this effect is ignored.
- **Circular Discs:** the actuator disc in the model is assumed to be of circular shape. In reality slight changes in the shape could occur, these are neglected.
- **Root-section:** it is assumed that the root section till  $r/R = 0.2$  has no influence on the performance of the turbine and can be neglected.

# Results (axial flow) 4

## 4.1 Reference Data (for $c_l(\alpha)$ and $c_d(\alpha)$ )

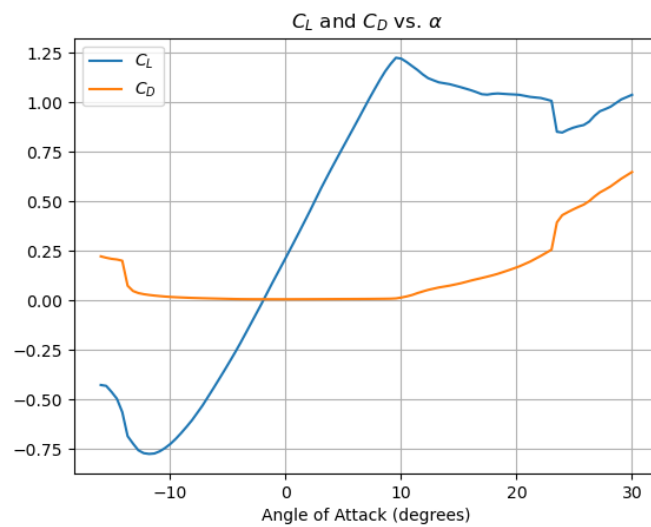


Figure 4.1: Lift Curve and Drag Curve

## 4.2 Corrections

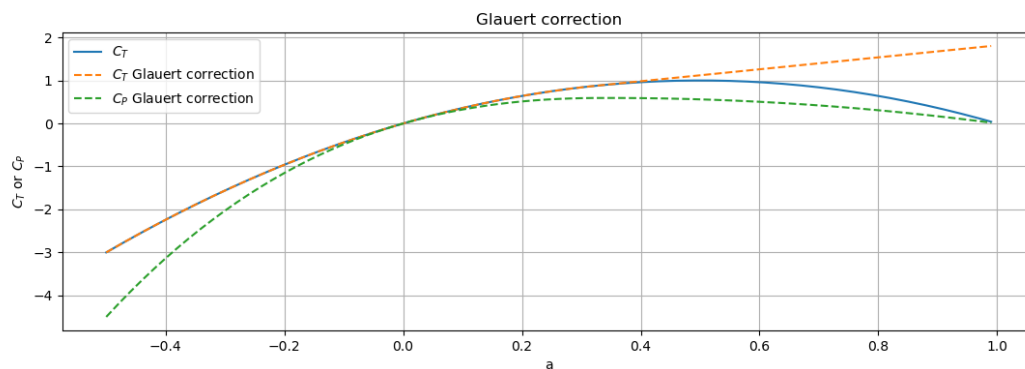


Figure 4.2: Glauert correction

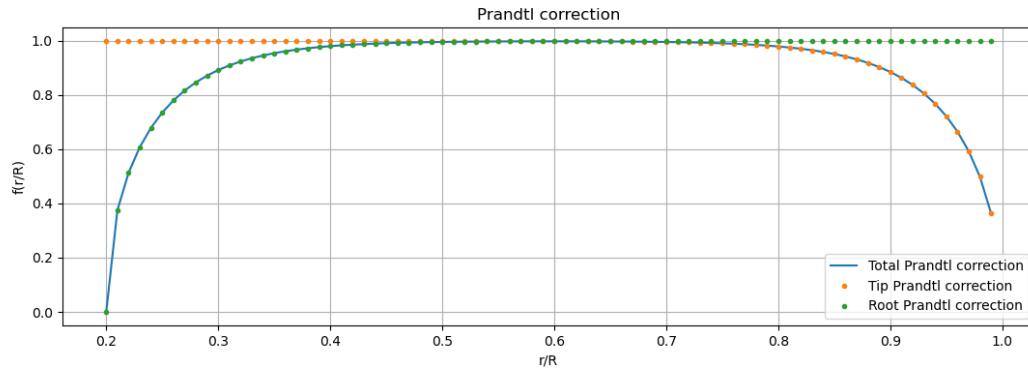


Figure 4.3: Prandtl correction

### 4.3 Angles

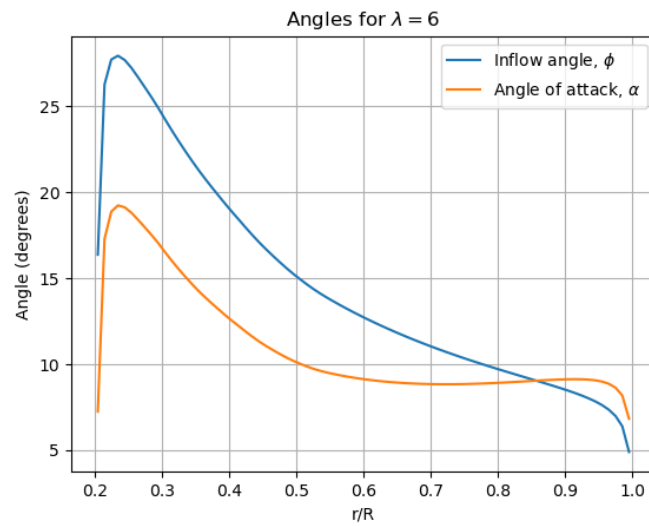


Figure 4.4: Span-wise distribution of Angle of Attack and Inflow Angle for Tip Speed Ratio of 6

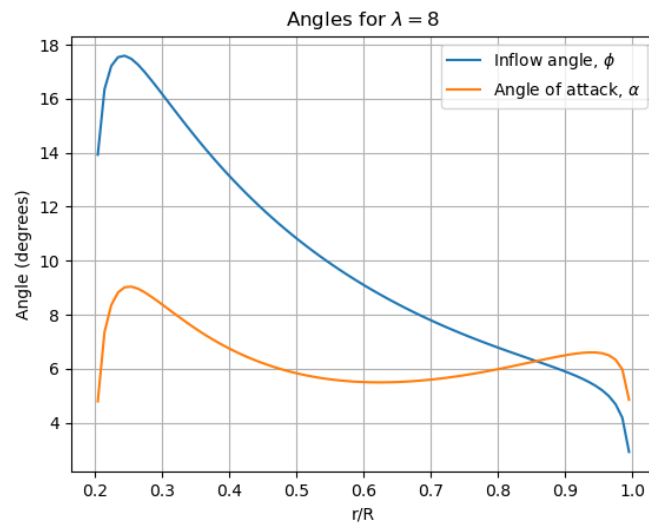


Figure 4.5: Span-wise distribution of Angle of Attack and Inflow Angle for Tip Speed Ratio of 8

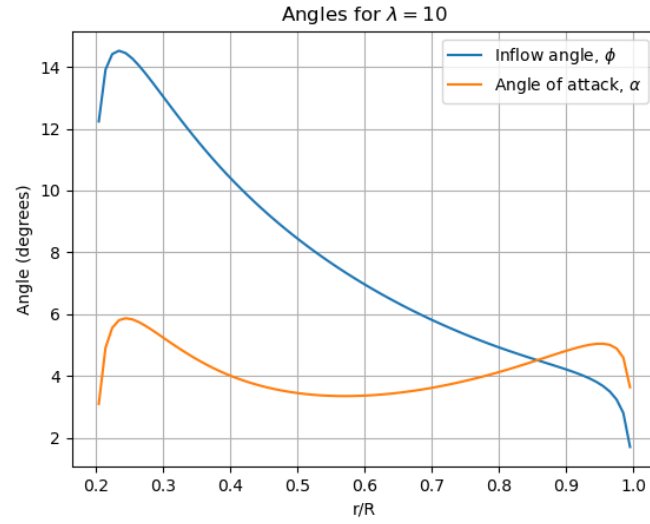


Figure 4.6: Span-wise distribution of Angle of Attack and Inflow Angle for Tip Speed Ratio of 10

#### 4.4 Induction factors

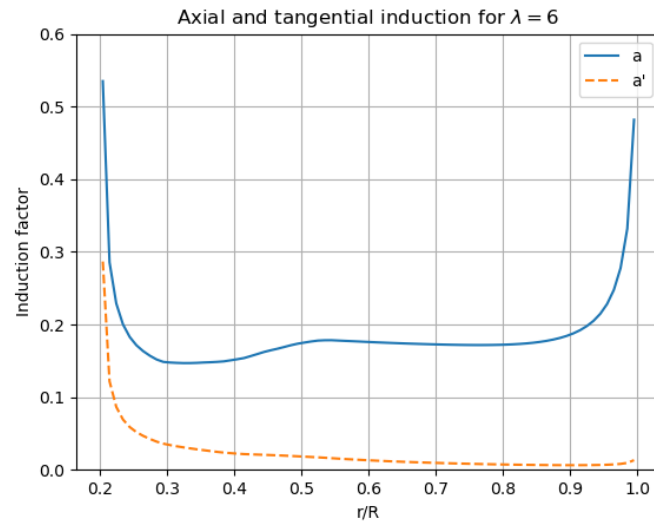


Figure 4.7: Span-wise distribution for Axial and Tangential Induction for Tip Speed Ratio of 6



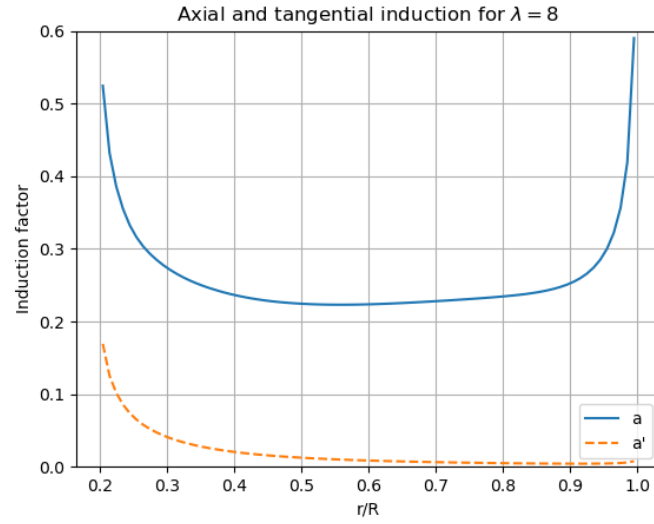


Figure 4.8: Span-wise distribution for Axial and Tangential Induction for Tip Speed Ratio of 8

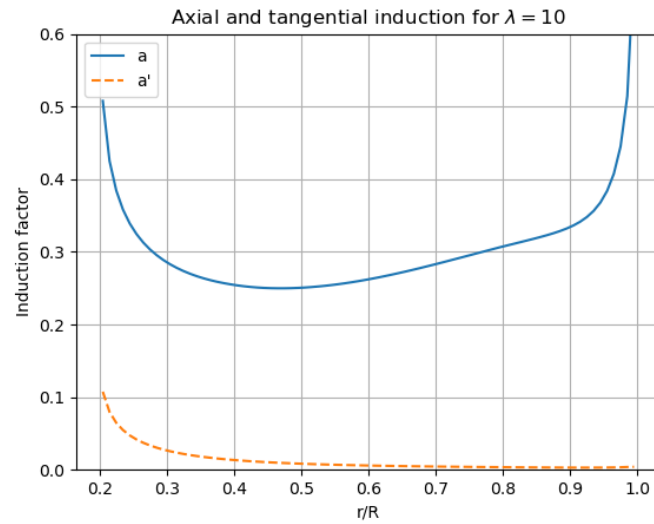


Figure 4.9: Span-wise distribution for Axial and Tangential Induction for Tip Speed Ratio of 10

## 4.5 Forces

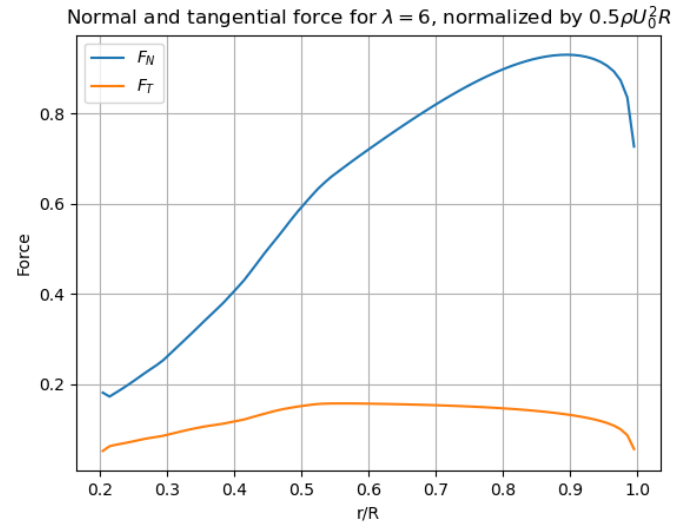


Figure 4.10: Normalized Normal and Tangential Force for Tip Speed Ratio of 6

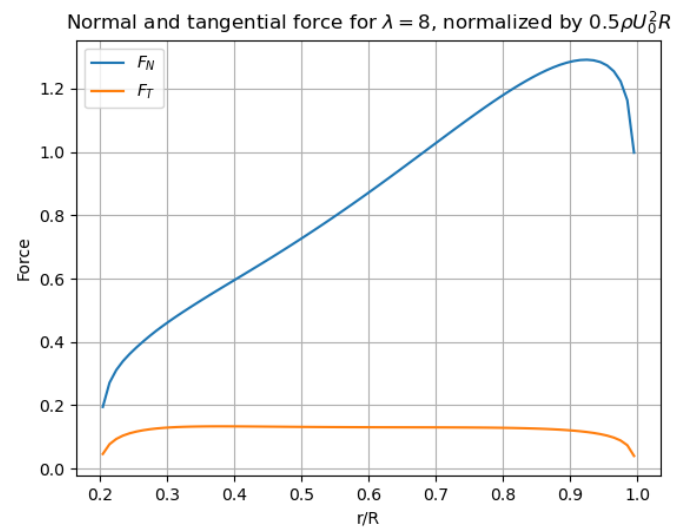


Figure 4.11: Normalized Normal and Tangential Force for Tip Speed Ratio of 8

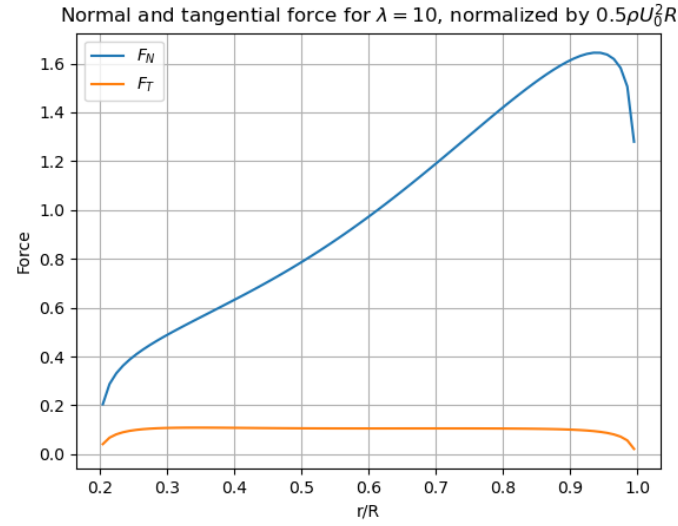


Figure 4.12: Normalized Normal and Tangential Force for Tip Speed Ratio of 10

## 4.6 Circulation

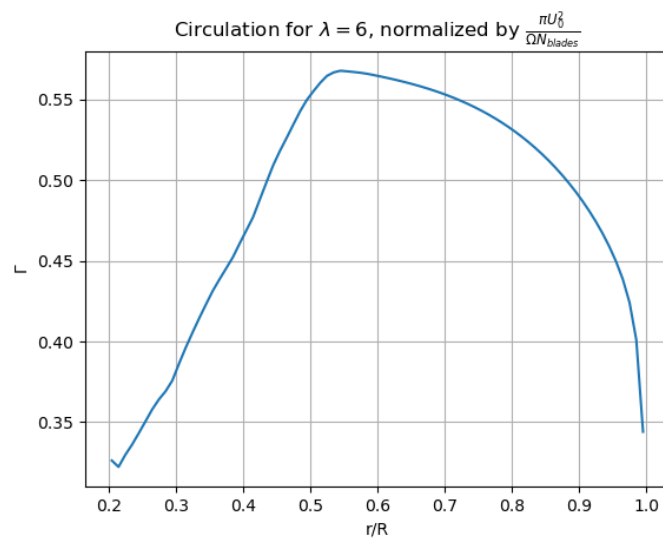


Figure 4.13: Normalized Circulation for Tip Speed Ratio of 6

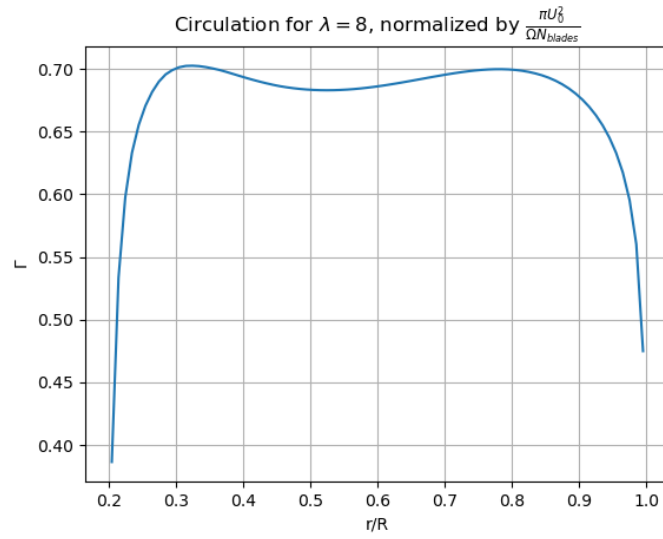


Figure 4.14: Normilzed Circulation for Tip Speed Ratio of 8

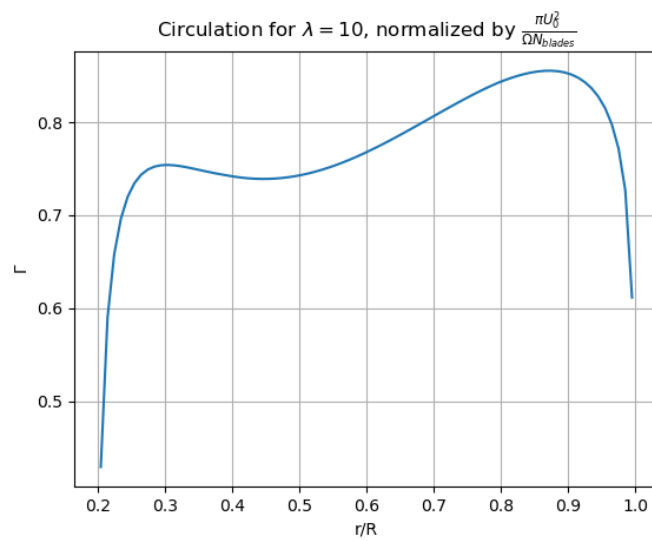


Figure 4.15: Normilzed Circulation for Tip Speed Ratio of 10

## 4.7 Static Pressure

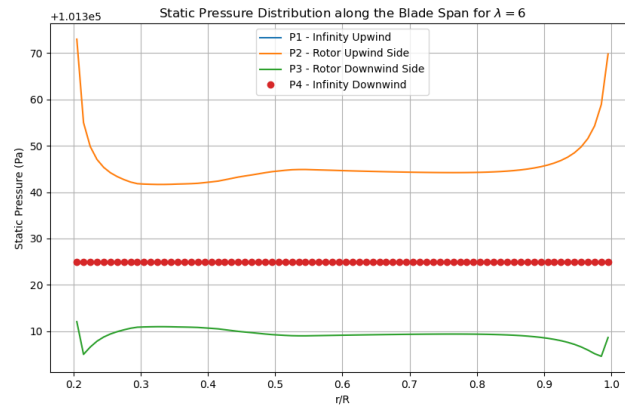


Figure 4.16: Span-wise Static Pressure Distribution for Tip Speed Ratio of 6

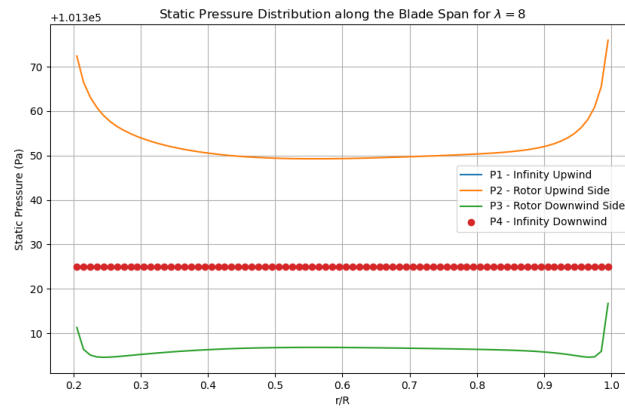


Figure 4.17: Span-wise Static Pressure Distribution for Tip Speed Ratio of 8

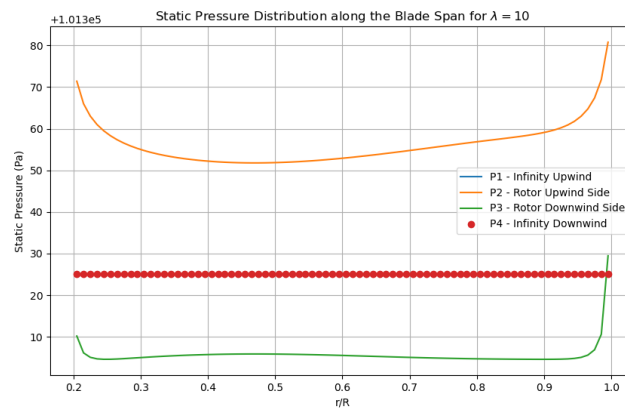


Figure 4.18: Span-wise Static Pressure Distribution for Tip Speed Ratio of 10

## 4.8 Total pressure

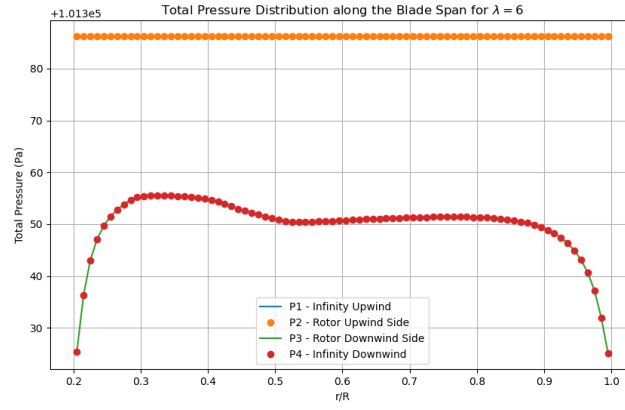


Figure 4.19: Span-wise Total Pressure Distribution for Tip Speed Ratio of 6

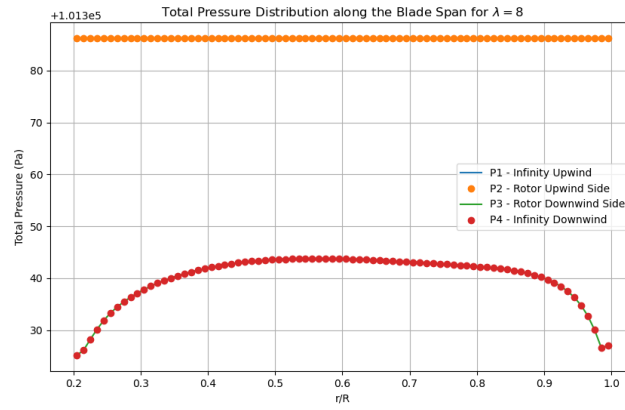


Figure 4.20: Span-wise Total Pressure Distribution for Tip Speed Ratio of 8

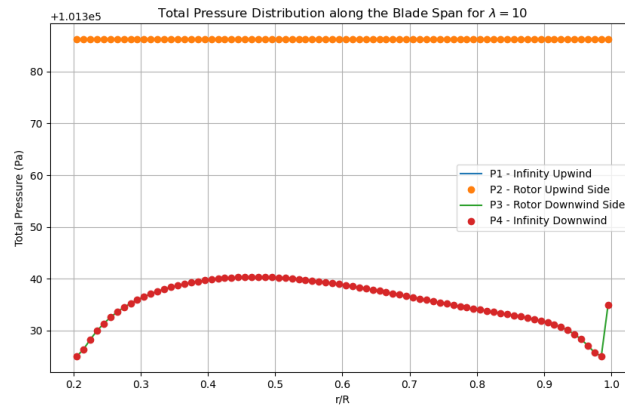


Figure 4.21: Span-wise Total Pressure Distribution for Tip Speed Ratio of 10

## 4.9 Thrust and torque vs TSR

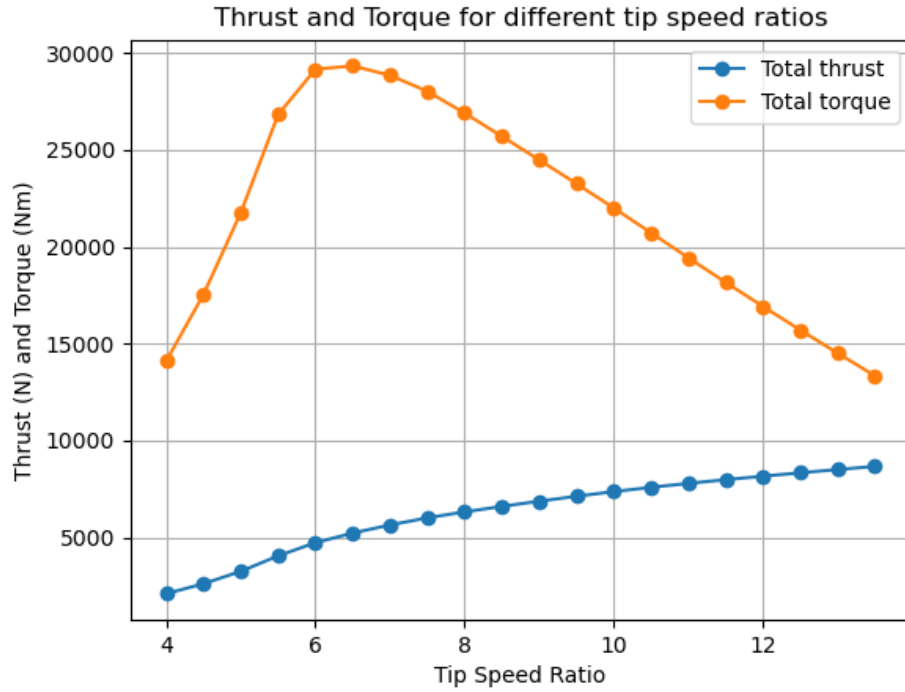


Figure 4.22: Thrust and Torque function of Tip Speed Ratio

## 4.10 Thrust vs number of annuli

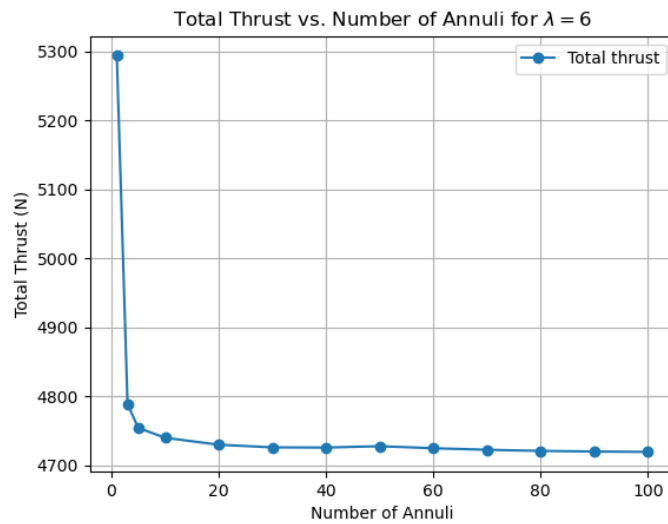


Figure 4.23: Thrust function of Annuli for Tip Speed Ratio of 6

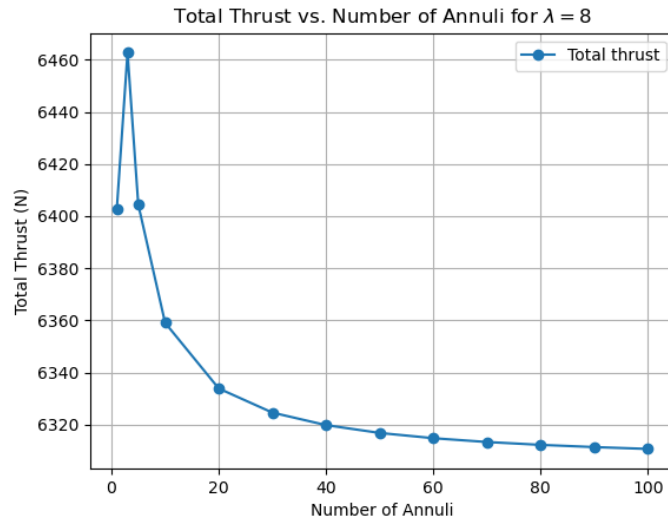


Figure 4.24: Thrust function of Annuli for Tip Speed Ratio of 8

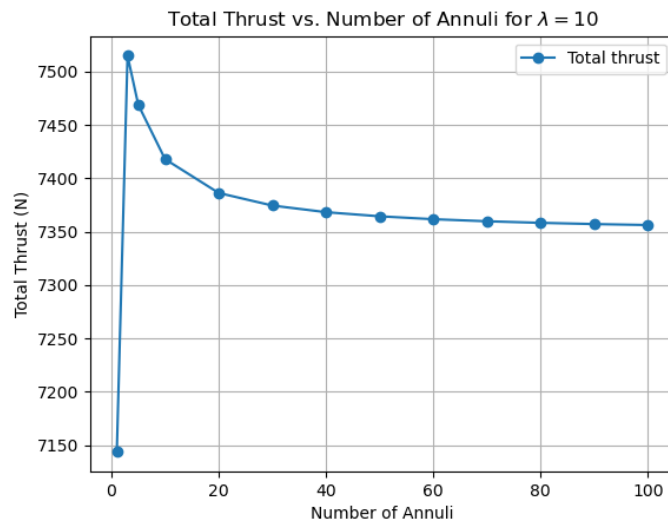


Figure 4.25: Thrust function of Annuli for Tip Speed Ratio of 10



#### 4.11 Effect of spacing method on convergence

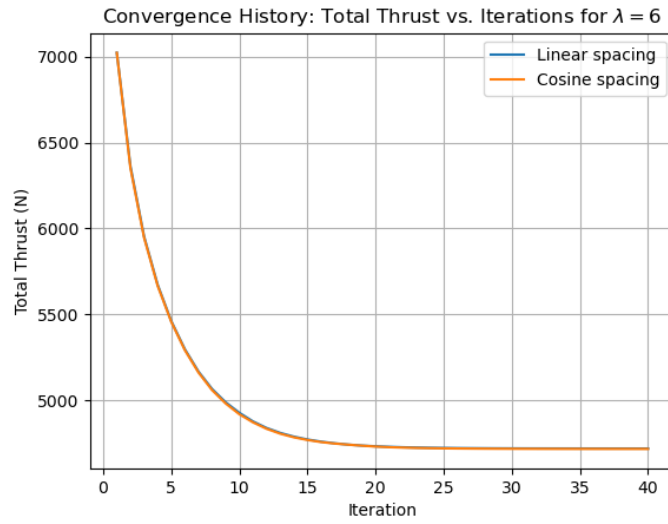


Figure 4.26: Thrust Convergence for Tip Speed Ratio of 6

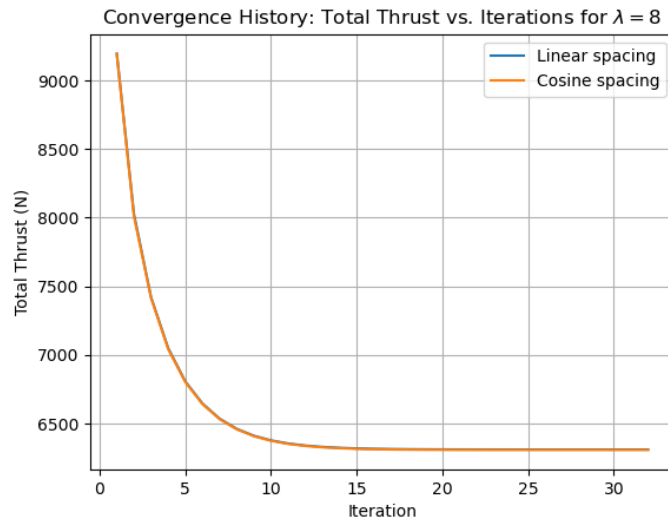


Figure 4.27: Thrust Convergence for Tip Speed Ratio of 8

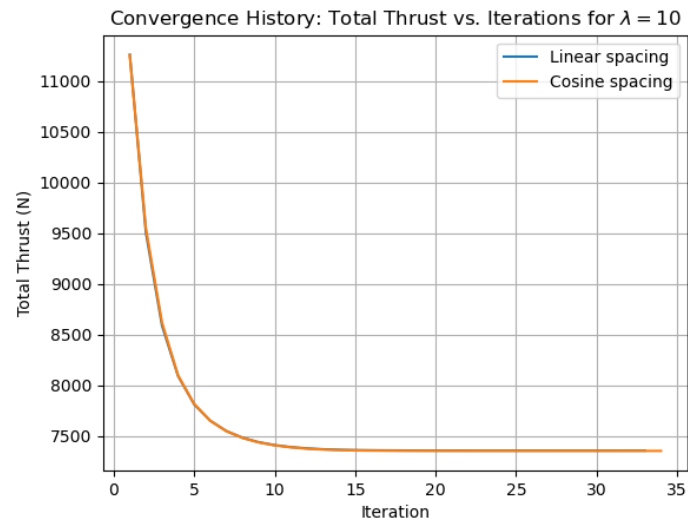


Figure 4.28: Thrust Convergence for Tip Speed Ratio of 10

## Conclusion 5

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# Python Script 6

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