

Let  $A_n = (0, \frac{1}{n})$ , then  $A_{n+1} = (0, \frac{1}{n+1})$ . As  $\frac{1}{n+1} < \frac{1}{n}$  then  $A_{n+1} \subset A_n$ .

Suppose there is some  $x$  that is part of every  $A_n$  and it's the smallest such element.

$$0 < x < \frac{1}{n}$$

$x$  would be in form  $\frac{1}{k}$ , where  $k \in (0, n)$ . But we could find a number  $\frac{1}{2k}$  that is smaller.

$$\frac{1}{2k} < \frac{1}{k}$$

In other words we could find an interval  $(0, \frac{1}{2k})$ , where  $x$  is not a member. But this is contradiction. Hence  $\cap_{n=1}^{\infty} A_n = \emptyset$ .  
QED.