

$$(\exists n \in \mathbb{N})[2 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 2]$$

Proof: By induction.

$n = 1$ :

$$2 = 2^{1+1} - 2$$

$$2 = 2^2 - 2$$

$$2 = 2$$

Assume  $n = k$  holds:

$$2 + 2^2 + 2^3 + \dots + 2^k = 2^{k+1} - 2$$

Show that  $n = k + 1$  holds

$$2 + 2^2 + 2^3 + \dots + 2^k + 2^{k+1} = 2^{(k+1)+1} - 2 \quad (\text{By induction})$$

$$\begin{aligned} 2 + 2^2 + 2^3 + \dots + 2^k + 2^{k+1} &= 2^{k+1} - 2 + 2^{k+1} \\ &= 2 * 2^k - 2 + 2 * 2^k \\ &= 4 * 2^k - 2 \\ &= 2^2 * 2^k - 2 \\ &= 2^{k+2} - 2 \end{aligned}$$

Therefore by induction the equation holds for all  $n$ .

QED.