By definition of odd number:

$$(\forall n \in Q)(\exists a \in Q)[n^2 + n + 1 = 2a + 1]$$

Suppose $n^2 + n + 1 = 2a$ (is even).

$$\begin{aligned} n^2 + n + 1 &= 2a \\ n^2 + n &= 2a - 1 \\ n(n+1) &= 2a - 1 \end{aligned}$$

So $n(n+1)$ should be odd.
If $n = 2k$ where $k \in Q$:

$$2k(2k+1) \neq 2a - 1$$

As it's divisible by 2 it can't be odd.
If $n = 2l + 1$ where $l \in Q$:

$$\begin{aligned} (2l+1)((2l+1)+1) &= \\ (2l+1)(2l+2) &= \\ (2l+1)2(l+1) &\neq 2a - 1 \end{aligned}$$

It's also divisible by 2 and can't be odd. But this is a contradiction.
Hence $n^2 + n + 1$ is odd.
QED.