

If the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \rightarrow \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML .

$$(\forall \epsilon > 0)(\exists n \in \mathbb{N})(\forall m \geq n)[|a_m - L| < \epsilon] \quad (1)$$

$$(\forall \epsilon > 0)(\exists n \in \mathbb{N})(\forall m \geq n)[|Ma_m - ML| < \epsilon] \quad (2)$$

Proof: By contradiction.

Suppose:

$$\begin{aligned} (\forall m \geq n)[|Ma_m - ML| \geq \epsilon] \\ M|a_m - L| \geq \epsilon \end{aligned} \quad (\text{By algebra})$$

By (1) we could find such $a_m - L$ that equal to $\frac{\epsilon}{2M}$.

$$\begin{aligned} M * \frac{\epsilon}{2M} &\geq \epsilon \\ \frac{\epsilon}{2} &\geq \epsilon \end{aligned}$$

But this is contradiction. Hence we could find n for (2).
QED.