DS-GA 1018.001 Probabilistic Time Series Analysis
Homework 2

Due date: Oct 25, by 6pm

Problem 1. LDS model, 10p
Consider a special case of LDS with $\mathbf{C} = \mathbf{I}$ and $\mathbf{R} = \sigma^2\mathbf{I}$, where $\mathbf{I}$ denotes the identity matrix. Show that in the limit where there is no observation noise the best estimate for latent $\mathbf{z}_i$ is to simply use the observation $\mathbf{x}_i$: formally, in the limit when $\sigma^2 \to 0$ the posterior for $\mathbf{z}_i$ has mean $\mathbf{x}_i$ and vanishing variance.

Problem 2. LDS with inputs, 10p
Consider the standard parametrization of the LDS model, with a change of the horizontal arrow, namely

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{y}_t + \mathbf{w}_t,$$

where matrix $\mathbf{B}$ is an additional model parameter and $\mathbf{y}_t$ is an *observed* input vector. How do the Kalman filtering updates change for this variation?

Problem 3. LDS inference with missing observations, 10p
Consider a variation of the original LDS graphical model with one single missing value $\mathbf{x}_j$. Everything else is as in the original; the only difference is that the graphical model loses the downward observation arrow and the corresponding $\mathbf{x}_j$.) How do the Kalman filtering/smoothing updates change?

Problem 4. Particle filtering, 20p
Consider the usual LDS model, but where inference is done using particle filtering instead of the traditional Kalman filter. A) Write down pseudocode for the particle updates. B) Given the generated samples, $\{\mathbf{z}_i^{(k)}\}_{k=1:K,i=1:t}$, how would you go about computing the quantities $\mu_{i|i}$, $\Sigma_{i|i}$ and $\mathbb{E}[\mathbf{z}_i\mathbf{z}_{i+1}^\top]$?
Hint: Use the general form from the lecture, and plug in the expressions for the different probabilities of the LDS model. The mean and variance can be written as expectations and approximated accordingly.