

DS-GA 1018.001 Probabilistic Time Series Analysis
Homework 5

Due date: Dec 17

Problem 1. (10pt) Show that white noise has a flat spectrum.

Problem 2. (25pt) Given an AR(1) process, $x_t = \phi x_{t-1} + w_t$, with $|\phi| < 1$ and white noise variance σ_w^2 ,

1) compute the corresponding power spectrum, $f_x(\omega)$.

2) show that the CCF can be obtained by inverting $f_x(\omega)$.

Note: see also problem 4.6 from Shumway (pg 232 on tsa4.pdf).

Problem 3. (15pt) Given an MA(1) process, $x_t = w_t - \theta w_{t-1}$, with parameter θ and white noise variance σ_w^2 . Compute the corresponding power spectrum, $f_x(\omega)$.

Problem 4. (Bonus 30pt) Consider the following simulated data:

$$x_1(t) = 2 \cos(2\pi\omega_1 t) + 3 \sin(2\pi\omega_1 t) \quad (1)$$

$$x_2(t) = 4 \cos(2\pi\omega_2 t) + 5 \sin(2\pi\omega_2 t) \quad (2)$$

$$x_3(t) = 6 \cos(2\pi\omega_3 t) + 7 \sin(2\pi\omega_3 t) \quad (3)$$

$$x(t) = x_1(t) + x_2(t) + x_3(t) \quad (4)$$

with $\omega_1 = 6/100$, $\omega_2 = 1/10$ and $\omega_3 = 4/10$.

1) Compute and plot the scaled periodogram of the data for a sequence of length 100.

2) compute and plot the scaled periodogram for a noisy version of the same signal $y(t) = x(t) + 0.1w$, with w white noise with unit variance.