

**DS-GA 1018.001 Probabilistic Time Series Analysis**  
**Homework 1**

**Due date: October 11, by 2pm**

**Problem 1.** (5pt) Consider the *sample mean* of a stationary time series  $x_t$ , defined as:

$$\hat{\mu} = \frac{1}{T} \sum_t x_t. \quad (1)$$

Compute the variance of this estimate  $\text{Var}[\hat{\mu}]$ , as a function of  $T$ , and the autocovariance function  $\gamma(h)$ .  
*Hint: The empirical mean is also a linear combination of random variables, so you can use the formula for the covariance of linear combinations of random variables from the lecture.*

**Problem 2.** (10pt) Confidence bounds for the autocorrelation function: show that the variance of the empirical ACF for white noise with variance  $\sigma^2$  estimated given  $T$  data points is  $\frac{1}{T}$ .

*Hint: Use theorem A.7 from Shumway, and Stoffer (yellow book, pdf on brightspace); alternatively, you can just show it numerically by plotting empirical estimates of the ACF as a function of  $T$ .*

**Problem 3.** (10pt) For an MA(1),  $x_t = w_t + \theta w_{t-1}$  show that the autocorrelation function  $|\rho_x(1)| \leq 0.5$ , for all  $\theta$ . For which values  $\theta$  is it maximum/minimum?

**Problem 4.** (5pt+5pt) Identify the following models as ARMA( $p, q$ ):

- $x_t = 0.8x_{t-1} - 0.15x_{t-2} + w_t - 0.3w_{t-1}$
- $x_t = x_{t-1} - 0.5x_{t-2} + w_t - w_{t-1}$

*Note: watch out for parameter redundancy!*

**Problem 5.** (5pt) Having observed a sequence  $\{x_1, x_2, \dots, x_t\}$  we are trying to predict a future observation  $x_{t+h}$ , with  $h \geq 1$ . How well / far can one predict into the future if the data comes from a a) MA(3) and b) AR(1) model.

*Hint: think of the functional form of the optimal estimator and/or the corresponding graphical model.*

**Problem 6.** (10pt) Given the AR(2) process with  $P(B) = (1 - 0.2B)(1 - 0.5B)$ , what is  $\rho(h)$ ? Check your analytical solution against an empirical estimate obtained using the code from the lab.

*Hint: Difference equations + initial conditions.*