DS-GA 1018.001 Probabilistic time series analysis

Midterm prep

Instructor: Cristina Savin

NYU, CNS & CDS

Materials covered

L1: Basics of graphical models

L2-3: ARIMA models

L4-5: LDS, Kalman, EM

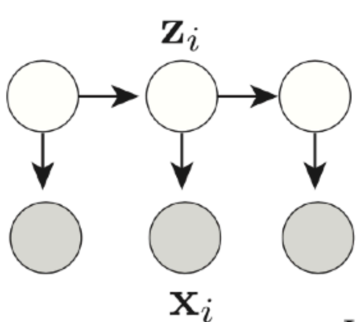
L5: Particle filtering

L6: HMMs

L7: Extensions, related models

Given graphical model write factorization, identify conditional independences

Problem 8. Write down 4 conditional independence relationships implied by the graphical model:



I was looking for a subset of these (as discussed in the HMM lecture):

$$
\begin{aligned}
\mathrm{P}(\mathbf{x}_{1:t}|\mathbf{z}_i) &= \mathrm{P}(\mathbf{x}_{1:i}|\mathbf{z}_i)\,\mathrm{P}(\mathbf{x}_{i+1:t}|\mathbf{z}_i) && (8)\\
\mathrm{P}(\mathbf{x}_{1:i}|\mathbf{z}_{i+1}, \mathbf{x}_{i+1}) &= \mathrm{P}(\mathbf{x}_{1:i}|\mathbf{z}_{i+1}) && (9)\\
\mathrm{P}(\mathbf{x}_{i+2:t}|\mathbf{z}_{i+1}, \mathbf{x}_{i+1}) &= \mathrm{P}(\mathbf{x}_{i+2:t}|\mathbf{z}_{i+1}) && (10)\\
\mathrm{P}(\mathbf{x}_{1:i}|\mathbf{z}_i, \mathbf{z}_{i+1}) &= \mathrm{P}(\mathbf{x}_{1:i}|\mathbf{z}_i) && (11)\\
\mathrm{P}(\mathbf{x}_{i+1:t}|\mathbf{z}_i, \mathbf{z}_{i+1}) &= \mathrm{P}(\mathbf{x}_{i+1:t}|\mathbf{z}_{i+1}) && (12)\\
\mathrm{P}(\mathbf{x}_{1:t}|\mathbf{z}_i, \mathbf{z}_{i+1}) &= \mathrm{P}(\mathbf{x}_{1:i}|\mathbf{z}_i)\,\mathrm{P}(\mathbf{x}_{i+1}|\mathbf{z}_{i+1})\,\mathrm{P}(\mathbf{x}_{i+2:t}|\mathbf{z}_{i+1}) && (13)
\end{aligned}
$$

The factorization of the model would also be ok:

$$
\mathrm{P}(\mathbf{x}_{1:t}, \mathbf{z}_{1:t}) = \mathrm{P}(\mathbf{z}_0) \prod_{i=0:t-1} \mathrm{P}(\mathbf{z}_{i+1}|\mathbf{z}_i) \prod_{i=1:t} \mathrm{P}(\mathbf{x}_i|\mathbf{z}_i) \tag{14}
$$

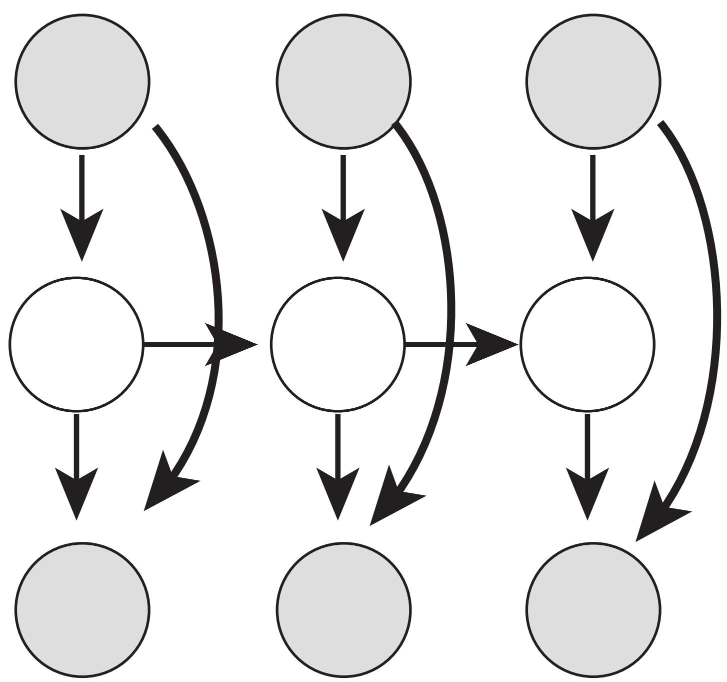
Simpler versions of the above are also acceptable.

L2-3: ARIMA models

Given Plot ACF, PACF for a given model

Identify model properties: stationarity, causality, etc

Variations of kalman: missing data, input dependent dynamics



L5: Particle filtering

Problem 11. Consider a generalization of LDS with the usual linear-gaussian latent dynamics:

$$\mathbf{z}_i = \mathbf{A}\mathbf{z}_{i-1} + \mathbf{w}_i \tag{1}$$

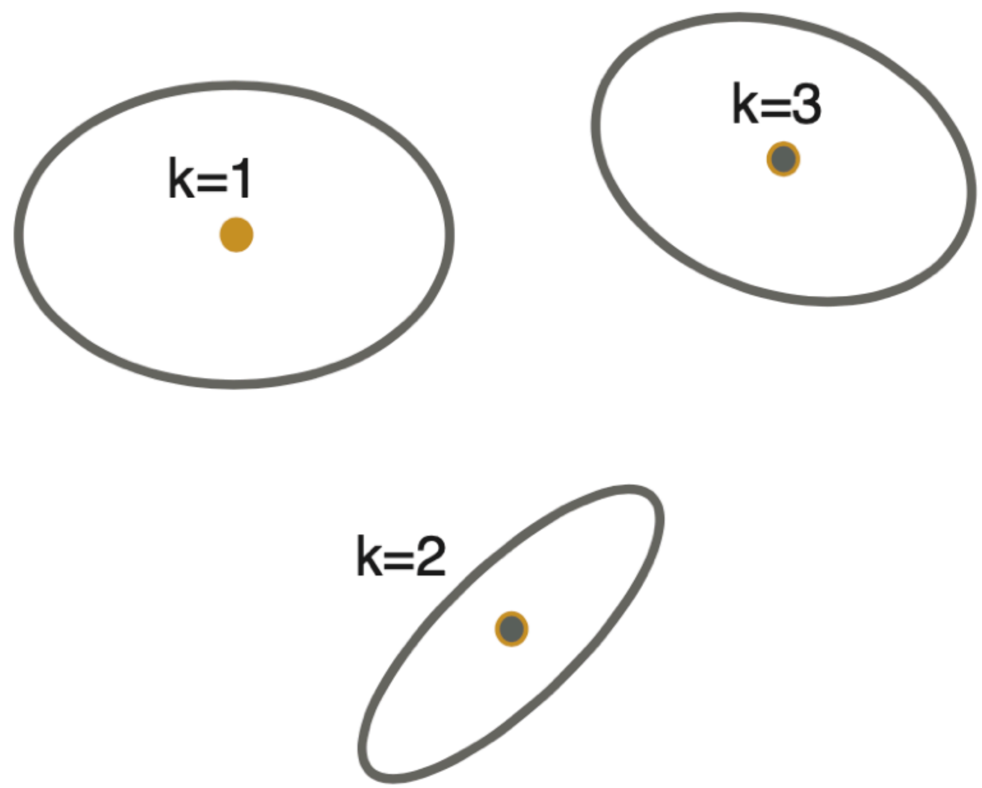
where $\mathbf{z}_0 \sim \mathcal{N}(\mu_0, \Sigma_0)$ and $\mathbf{w}_i \sim \mathcal{N}(0, \mathbf{Q})$, and observations obtained via a a *nonlinear* map $f(\cdot)$with i.i.d. gaussian noise:

$$\mathbf{x}_i = f(\mathbf{z}_i) + \mathbf{v}_i \tag{2}$$

with $\mathbf{v}_i \sim \mathcal{N}(0, \mathbf{R})$. Describe the main steps of the algorithm implementing particle filtering in this model.

L6: HMMs

Problem 6. Given an HMM model with 3 latent states $k = \{1, 2, 3\}$, initial state distribution $\pi = [0.7, 0.2, 0.1]$, the transition probabilities: $A = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.15 & 0.5 & 0.35 \\ 0.25 & 0.05 & 0.7 \end{bmatrix}$, and gaussian observation noise as shown in the figure below draw an example sequence of length 10 that is likely according to the model parameters.



Note: Write the sequence of latent states first, then chose the observations. Mark first observation with $\otimes$*, use* $\times$ *to denote the following data points, and draw lines between subsequent observations.*

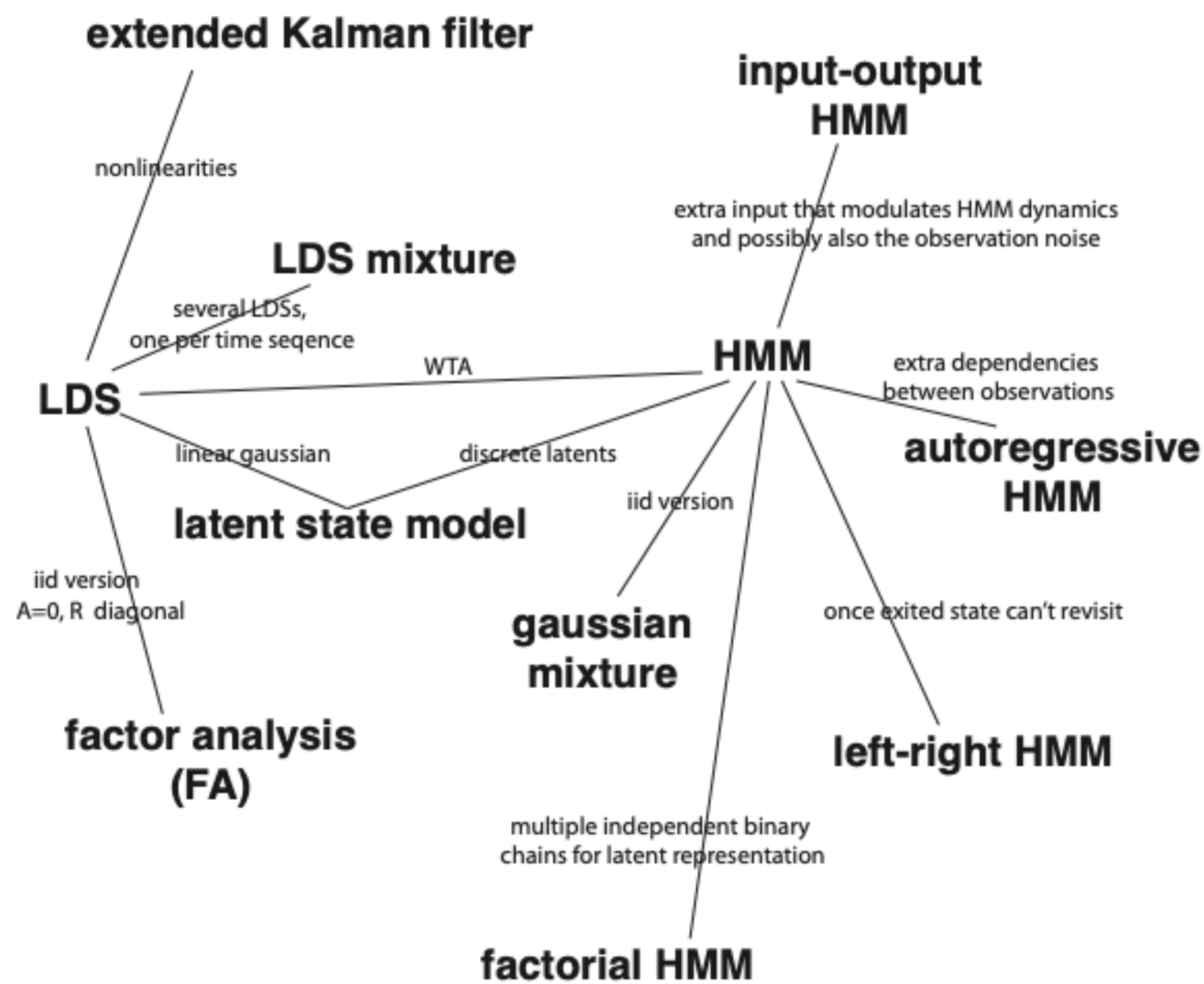
Problem 14. What are the similarities and differences between the alpha-beta and Viterbi algorithms?

Solution:
Both are message passing algorithms for inference in HMMs, but alpha-beta computes posterior marginals

while Viterbi computes most likely sequence, i.e. a MAP estimate. Practically, this means that in alpha-beta we average across all possible paths (sum), while in Viterbi we consider only the most likely path (max).

L7: Extensions, related models

Problem 11. Identify the relationships between the models below. Draw arrows between related models. For each write down the link between the two; these could take the form 'model B is generalisation of model A, in parameter limit ...', or 'model C generalizes model D by adding extra feature ...'.)



Problem 10. Identify the relationships between the concepts below. Draw arrows between related concepts and annotate them with text describing relationship.