DS-GA 1018.001 Probabilistic Time Series Analysis
Homework 5

Due date: Dec 12

Problem 1. (10pt) Show that white noise has a flat spectrum.

Problem 2. (15+10pt) Given an AR(1) process, $x_t = \phi x_{t-1} + w_t$, with $|\phi| < 1$ and white noise variance σ_w^2,
1) compute the corresponding power spectrum, $f_x(\omega)$.
2) show that the autocovariance function can be obtained by inverting $f_x(\omega)$.
Note: see also problem 4.6 from Shumway (pg 232 on tsa4.pdf).

Problem 3. (15pt) Given an MA(1) process, $x_t = w_t - \theta w_{t-1}$, with parameter θ and white noise variance σ_w^2. Compute the corresponding power spectrum, $f_x(\omega)$.

Problem 4. (Extra credit: 30pt) Consider data generated according to the following process:

$$
\begin{aligned}
x_1(t) &= 2\cos(2\pi\omega_1 t) + 3\sin(2\pi\omega_1 t) && (1)\\
x_2(t) &= 4\cos(2\pi\omega_2 t) + 5\sin(2\pi\omega_2 t) && (2)\\
x_3(t) &= 6\cos(2\pi\omega_3 t) + 7\sin(2\pi\omega_3 t) && (3)\\
x(t) &= x_1(t) + x_2(t) + x_3(t) && (4)
\end{aligned}
$$

with $\omega_1 = 6/100$, $\omega_2 = 1/10$ and $\omega_3 = 4/10$.
1) Numerically compute and plot the scaled periodogram of the data for a sequence of length 100.
2) Do the same for a noisy version of the same signal $y(t) = x(t) + 0.1w_t$, with w_t iid white noise with unit variance. Comment on the results.