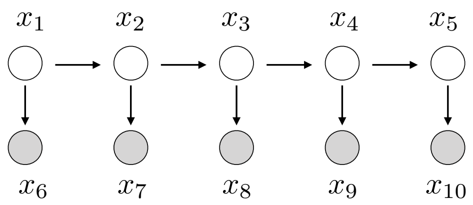
DS-GA 1018.001 Probabilistic Time Series Analysis
Homework 1

Due date: Sept. 29, by 6pm

Problem 1. (10pt) Given the graphical model below, write down the factorization for $P(x_{1:10})$. What happens with the distribution when conditioning on x_3?



Problem 2. (10pt) Consider the *sample mean* of a stationary time series x_t, defined as:

$$\hat{\mu} = \frac{1}{T} \sum_t x_t. \tag{1}$$

Compute the variance of this estimate $Var[\hat{\mu}]$, as a function of T, and the autocovariance function $\gamma(h)$.
Hint: The empirical mean is also a linear combination of random variables, so you can use the formula for the covariance of linear combinations of random variables from the lecture.

Problem 3. (10pt) Confidence bounds for the autocorrelation function: show that the variance of the empirical ACF for white noise with variance σ^2 estimated given T data points is $\frac{1}{T}$.
Hint: Use theorem A.7 from Shumway, and Stoffer (yellow book, pdf on brightspace); alternatively, you can just show it numerically by plotting empirical estimates of the ACF as a function of T.

Problem 4. (5pt+5pt) Identify the following models as ARMA(p, q):

- $x_t = 0.8x_{t-1} - 0.15x_{t-2} + w_t - 0.3w_{t-1}$
- $x_t = x_{t-1} - 0.5x_{t-2} + w_t - w_{t-1}$

Note: watch out for parameter redundancy!

Problem 5. (10pt) Given the AR(2) process with $P(B) = (1 - 0.2B)(1 - 0.5B)$, what is $\rho(h)$? Check your analytical solution against an empirical estimate obtained using the code from the lab.
Hint: Difference equations + initial conditions.